A General Equilibrium Entrepreneurial Theory of Firm Formation Based on Risk Aversion

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We construct a theory of competitive equilibrium under uncertainty using an entrepreneurial model with historical roots in the work of Knight in the 1920s. Individuals possess labor which they can supply as workers to a competitive labor market or use as entrepreneurs in running a firm. All entrepreneurs have access to the same risky technology and receive all profits from their firms. In the equilibrium, more risk averse individuals become workers while the less risk averse become entrepreneurs. Less risk averse entrepreneurs run larger firms and economy-wide increases in risk aversion reduce the equilibrium wage. A dynamic process of firm entry and exit is stable. The equilibrium is efficient only if all entrepreneurs are risk neutral. Inefficiencies in the number of firms and in the allocation of labor to firms are traced to inefficiencies in the risk allocation caused by institutional constraints on risk trading. In a second best sense which accounts for these constraints, the equilibrium is efficient.

I. Introduction

The recent work on the economics of uncertainty has failed to achieve general agreement as to the goals which motivate firm behavior under

uncertainty. The criteria guiding firm decision making which have been proposed and studied in the existing literature include expected profit maximization and expected utility of profit maximization as well as maximization of the firm's stock market value. Difficulties with each of these criteria have led to a discussion ("Symposium on the Optimality of Competitive Capital Markets" [1974]) of the conditions under which there exists a criteria for firm decision making which achieves unanimous approval of stockholders. They have also led to the study of other more sophisticated criteria for firm decision making. The paper of Drèze (1974) is one in which this latter approach is taken. Each of these subsequent approaches has achieved only limited success. For example, unanimity can be achieved only in limited technological circumstances. Similarly, the equilibria of Drèze are not always efficient in the "second best" sense of Diamond (1967).

In this paper we construct a competitive general equilibrium theory of the firm under uncertainty which is based on an entrepreneurial model having its historical roots in the work of Knight (1921). The entrepreneurial model permits us simultaneously to use the expected utility maximization criterion and to provide a justification for its use. This is accomplished by assuming that for each firm there is an expected utility maximizing entrepreneur who makes decisions for the firm. Furthermore, the model uses a free-entry assumption to endogenously determine the number of firms and the identity of the entrepreneurs who run them. It also permits us to identify the individual characteristics of individuals who choose to become entrepreneurs.

In the model, individuals are assumed to have a choice between operating a risky firm or working for a riskless wage. There are, of course, many factors which should influence this choice. The most important ones would include entrepreneurial ability, labor skills, attitudes toward risk, and initial access to the capital required to create a firm. The present paper focuses on risk aversion as the determinant which explains who becomes an entrepreneur and who works as a laborer. The equilibrium which is shown to exist has the property that less risk averse individuals become entrepreneurs, while the more risk averse work as laborers.

In addition to providing an explanation for the identity of entrepreneurs, the entrepreneurial model can also be used to study several issues of traditional interest to economists. One of these is the process of firm entry and exit.

Specifically, using the model described below, it is possible to analyze the dynamics of firm entry and exit in a general equilibrium context. This can be done using a formalization of a tâtonnement process which is analogous to that commonly used to study the stabil-

ity of competitive equilibrium. While our stability analysis is less complete and more special than the analysis in the stability literature for competitive equilibrium, it nevertheless introduces an element, specifically firm entry and exit, which this literature was unable to incorporate. Furthermore, this element is introduced while retaining the general equilibrium framework and the basic price-adjustment process. In our general equilibrium process, as in the tâtonnement process used in the competitive equilibrium literature, prices (in our case, wages) adjust to (labor) market disequilibrium by rising when there is excess supply. Earlier formalizations of the entry-exit process, specifically, those in Quandt and Howrey (1968) and Brock (1972), were partial equilibrium models. They were also based on formalizations of the adjustment process which, while similar in spirit, differed in detail from the tâtonnement price-adjustment process used in the competitive equilibrium framework. For example, in the papers by Quandt and Howrey and by Brock, the dynamic variable is the number of firms in an industry. The industry grows when profits (or excess profits) are positive; it contracts when profits are negative.

Another traditional question which can be investigated using the entrepreneurial model concerns the determinants of the distribution of firm size. Specifically, an entrepreneur's attitude toward risk can be related to the size of the firm which he operates. While it might be conjectured that more risk averse entrepreneurs run smaller firms, this is not always true. However, it does follow when the production function satisfies certain conditions which are spelled out in theorem 3 below.

It is also possible to use this model to study one determinant of the distribution of income between workers and entrepreneurs. Specifically, it can be shown that, under certain conditions, the equilibrium wage level would be depressed if the economy's population became more risk averse.

Finally, it is possible to investigate the efficiency of the equilibrium of the entrepreneurial model. In general the equilibrium is inefficient and the inefficiency takes three forms: risks are maldistributed, firms are operated at the wrong levels, and there is an inappropriate number of firms. It is shown, however, that all of these forms of inefficiency occur because there are institutional constraints embodied in the model which prohibit an efficient allocation of risks when entrepreneurs are risk averse. This is seen in two ways. First, the equilibrium is efficient if, in equilibrium, all entrepreneurs are risk neutral. Second, we follow Diamond (1967) and Radner (1968) and investigate the efficiency of equilibrium in a second best or "limited" sense which permits a less than completely efficient allocation of risks. Although the "limited efficiency" approach taken in this paper is in

the same spirit as those adopted by Diamond and by Radner, it employs a different concept of limited efficiency than the ones they employed. Thus we accept the specific institutional constraints imposed by our equilibrium model on the distribution of risk and ask only that, given these constraints, all other decisions be made efficiently (Pareto optimally). The constraints on risk trading imposed in taking this approach are, in fact, stronger than those introduced by Diamond and by Radner. It is possible, however, to show that if, in defining limited efficiency, these constraints are imposed, then the equilibrium is efficient in the limited sense.

Because all of the inefficiencies which may arise in an equilibrium can be traced to the institutional constraints on risk trading, it is reasonable to conjecture that the efficiency properties of the equilibrium will be substantially improved by the introduction of at least some market opportunities for risk sharing among entrepreneurs and between entrepreneurs and workers. The introduction of a stock market in which the entrepreneur can raise capital for the purpose of financing his input purchases would be one way of providing additional opportunities for risk sharing. Sharecropping arrangements provide another device by which risks are, in fact, often shared. This is especially true in agricultural economies. The present paper does not investigate the issues which arise when either of these risk sharing possibilities becomes available. In a subsequent paper (Kihlstrom and Laffont 1978), we have, however, succeeded in studying these extensions of the entrepreneurial model discussed here. The emphasis there is on the stock market as a device for risk sharing. It is specifically shown that the introduction of a stock market does, indeed, result in equilibrium allocations which are efficient in a stronger sense than that considered here. Specifically they are efficient in the sense of Diamond.

This paper concludes with a brief summary of our results and a discussion of their relationship to Knight's above-mentioned entrepreneurial theory.

II. The Model

The set of agents is identified with the interval [0,1]. If $\alpha \epsilon [0,1]$, individual α has the von Neumann Morgenstern utility function $u(I,\alpha)$ where I represents income, and $I \epsilon [0,\infty)$. For all $I \geq 0$, the first and second derivatives u_I and u_{II} exist and are continuous. The marginal utility u_I is positive and nonincreasing, that is, $\mu_{II} \leq 0$. Thus all agents are risk averse or indifferent to risk.

We also assume that the Arrow (1971)-Pratt (1964) absolute risk

aversion measure is nondecreasing in α. More precisely, if α exceeds β, then agent α is at least as risk averse as agent β in the sense that

$$r(I,\alpha) = -\frac{u_{II}(I,\alpha)}{u_I(I,\alpha)} \geqq -\frac{u_{II}(I,\beta)}{u_I(I,\beta)} = r(I,\beta) \qquad (1)$$

for all $I \in (0, \infty)$.

Each agent can become an entrepreneur and use without cost a technology defined by a continuous production function $y = g(L,x)$ where $y \geqq 0$ is output, $L \geqq 0$ is the labor input, and x is the value taken by a nondegenerate random parameter $\tilde{x}$ with support $[0, \bar{x}]$, $0 < \bar{x} < +\infty$.

The marginal product g_L is assumed to be continuous and positive on $[0,+\infty) \times (0, \bar{x}]$. The second derivative is continuous and nonpositive on $[0,+\infty) \times [0, \bar{x}]$. Thus g exhibits nonincreasing returns to scale for each x. In addition, $g(0,x) = g(L,0) = 0$ for all $x \in [0, \bar{x}]$ and $L \in [0,+\infty)$, while $g(L,x) > 0$ on $(0,+\infty) \times (0, \bar{x}]$.

A variety of interpretations of the random variables $\tilde{x}$ is possible. In all of these interpretations, the stochastic distribution of $\tilde{x}$ is assumed to be the same for all firms. On the one extreme we can assume that the random variables which determine the output of each firm are stochastically independent. At the other extreme they can be perfectly correlated. In this case, not only is the distribution of $\tilde{x}$ the same for all firms, but the same random variable $\tilde{x}$ influences the output of all firms. Intermediate cases occur when the $\tilde{x}$'s are correlated but not perfectly correlated. In each of these alternative interpretations, all individuals are assumed to have the same beliefs about the distribution of $\tilde{x}$, that is, the distribution of $\tilde{x}$ is objective.

The price of output is 1 and labor is hired at a competitive wage w. It is assumed that the demands of entrepreneurship preclude additional work by agents who choose to operate a firm. Thus agents have a choice. They can become entrepreneurs and receive an uncertain income or they can work and receive the market wage w. If an individual becomes an entrepreneur and employs L workers he will receive profits equal to

$$g(L,\tilde{x}) - wL. \qquad (2)$$

To avoid the difficulties associated with the problem of bankruptcy we assume that all individuals begin with A units of income and that they are unable to hire workers who cannot be paid if $\tilde{x} = 0$. Thus L must be less than or equal to A/w.

An individual who becomes an entrepreneur will choose to employ

$L(w,\alpha)$ workers where $L(w,\alpha)$ is the L value in $[0, A/w]$ which maximizes

$$Eu\left(A + g(L,\tilde{x}) - wL, \alpha\right). \tag{3}$$

Our assumptions on u and g guarantee that $L(w,\alpha)$ exists. If either $u_{II} < 0$ or $g_{LL} < 0$, then $L(w,\alpha)$ will be unique. When entrepreneur α faces the wage w and employs $L(w,\alpha)$ workers, his profits are random and equal to

$$\tilde{\pi}(w,\alpha) = g\left(L(w,\alpha), \tilde{x}\right) - wL(w,\alpha). \tag{4}$$

If the wage is w, agent α will choose to be an entrepreneur when

$$Eu\left(A + \tilde{\pi}(w,\alpha), \alpha\right) \geqq u(A + w,\alpha). \tag{5}$$

He will be a worker at wage w if

$$Eu\left(A + \tilde{\pi}(w,\alpha), \alpha\right) \leqq u(A + w, \alpha), \tag{6}$$

and he will be indifferent if the equality holds in (5) and (6).

Equilibrium is reached when the labor market clears. At the equilibrium wage, the labor demanded by all agents who choose to become entrepreneurs equals that supplied by agents who choose to enter the labor market.

Formally, an equilibrium is a partition $\{\Delta,\Gamma\}$ of $[0,1]$ and a wage w, that is, a pair $(\{\Delta,\Gamma\},w)$; for which

(E.1)   labor supply equals labor demand in the sense that

$$\int_\Delta L(w,\alpha)\mu(d\alpha) = \mu(\Gamma)$$

where μ is Lebesgue measure and

(E.2)   for all $\alpha \in \Delta$ (5) holds and for all $\alpha \in \Gamma$ (6) holds.

III. The Existence and Uniqueness of Equilibrium

We can now prove that an equilibrium exists. The first step is to define $w(\alpha)$, the certainty equivalent wage which makes agent α indifferent between the two activities—work and entrepreneurship. Formally, $w(\alpha)$ is defined by

$$Eu\left(A + \tilde{\pi}(w(\alpha), \alpha), \alpha\right) = u\left(A + w(\alpha), \alpha\right). \tag{7}$$

The properties of $w(\alpha)$ are established in the lemma which follows. These properties will permit us to describe the structure of the equilibrium in a way which simplifies the existence proof. Further interpretive remarks follow the formal statement of the lemma.

Lemma

Assume that for each I, $r(I,\alpha)$ is an increasing function of α.[1] Also assume that either $g_{LL} < 0$ or $u_{II} < 0$. Then:

i) For each $\alpha \in [0,1]$, $Eu(A + \tilde{\pi}(w,\alpha), \alpha) - u(A + w, \alpha)$ is a continuous monotonically decreasing function of w.

ii) $w(\alpha)$ is a well-defined function of α, that is, for each $\alpha \in [0,1]$, $w(\alpha)$ exists and is unique. In addition $w(\alpha) > 0$.

iii) If $w > (<) w(\alpha)$, then $Eu(A + \tilde{\pi}(w,\alpha), \alpha) < (>) \mu(A + w,\alpha)$.

iv) If $\alpha > \beta$, then $w(\alpha) < w(\beta)$.

v) If $\beta > (<) \alpha$, then $Eu(A + \tilde{\pi}(w(\alpha), \beta), \beta) < (>) u(A + w(\alpha), \beta)$.

vi) If $0 < w < w(\beta)$, then $L(w,\beta) > 0$.

This is true, in particular, if $w = w(\alpha)$ where $\alpha > \beta$.

Remark 1

Result iv asserts that more risk averse individuals are induced to become workers at lower wages than less risk averse agents. In order to interpret result v, note that agent α will be the marginal entrepreneur if the equilibrium wage is $w(\alpha)$. Result v asserts that all individuals who are more (less) risk averse than the marginal entrepreneur will be workers (entrepreneurs). This result implies that in any equilibrium, there will be a marginal entrepreneur $\hat{\alpha}$ for whom $w(\hat{\alpha})$ is the equilibrium wage. The set of entrepreneurs Δ will be the interval $[0,\hat{\alpha}]$ and the set of workers Γ will be $(\hat{\alpha},1]$. The problem of finding an equilibrium then reduces to the problem of finding a marginal entrepreneur $\hat{\alpha}$ for whom E.1 holds when $w = w(\hat{\alpha})$, $\Delta = (0,\hat{\alpha}]$, and $\Gamma = (\hat{\alpha},1]$, that is, for whom $\int_0^{\hat{\alpha}} L(w(\hat{\alpha}),\alpha)\mu(d\alpha) = 1 - \hat{\alpha}$.

PROOF.—(i) The assumptions made about u and g guarantee that $Eu(A + g(L,\tilde{x}) - wL, \alpha)$ is a strictly concave continuous function of L and a continuous function of w.

To prove monotonicity, note first that for each nonnegative L, the monotonicity of u implies that

$$Eu(A + g(L,\tilde{x}) - wL, \alpha) < Eu(A + g(L,\tilde{x}) - w'L, \alpha) \qquad (8)$$

when $w > w'$. Maximizing over L on each side of inequality (8) implies the inequality

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[1] If $r(I,\alpha)$ is nondecreasing but not strictly increasing the strict inequalities in iii, iv, and v are replaced by weak inequalities.

$$Eu\big(A + \tilde{\pi}(w,\alpha), \alpha\big) = \max Eu\big(A + g(L,\tilde{x}) - wL, \alpha\big)$$

$$\frac{A}{w} \geqq L \geqq 0$$

$$\leqq \max Eu\big(A + g(L,\tilde{x}) - w'L, \alpha\big) \qquad (9)$$

$$\frac{A}{w'} \geqq L \geqq 0$$

$$= Eu\big(A + \tilde{\pi}(w',\alpha), \alpha\big)$$

when $w > w'$. Thus $Eu\big(A + \tilde{\pi}(w,\alpha), \alpha\big)$ is nonincreasing and $Eu\big(A + \tilde{\pi}(w,\alpha), \alpha\big) - u(A + w,\alpha)$ is monotonically decreasing.

ii) It is easily seen that $Eu\big(A + \tilde{\pi}(w,\alpha), \alpha\big) - u(A + w,\alpha) > 0$ when $w \simeq 0$. If, on the other hand, w is large, then
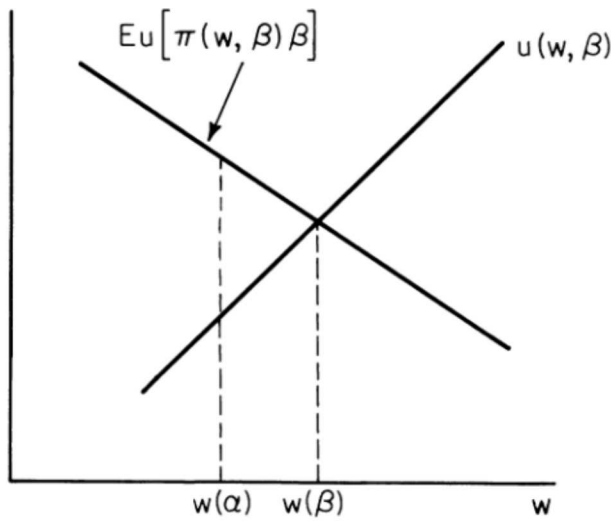
$$g(L,x) - wL \leqq \max g(L,x) \simeq 0$$

$$0 \leqq L \leqq \frac{A}{w} \qquad (10)$$

$$0 \leqq x \leqq \bar{x}$$

and equation (6) will hold. Because of the continuity established in i, the intermediate value theorem implies the existence of a positive wage $w(\alpha)$ which satisfies (7). The monotonicity established in i implies the uniqueness of $w(\alpha)$. Monotonicity also implies inequality iii. Figure 1 illustrates the situation.
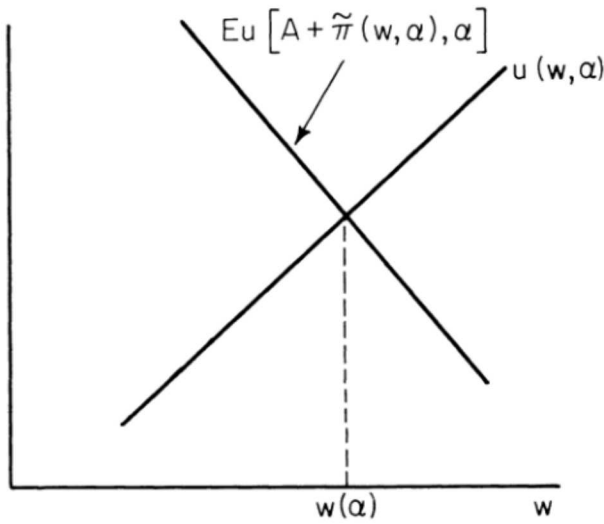
FIG. 1

FIG. 2

iv) We use the fact that $w(\alpha)$ is the certainty equivalent of $\tilde{\pi}(w(\alpha), \alpha)$. We also define $w(\alpha,\beta)$ to be the certainty equivalent of $\tilde{\pi}(w(\alpha), \alpha)$ for agent β. Pratt's (1964) theorem 1 is now applied to prove that $\beta > (<) \alpha$ implies $w(\alpha,\beta) < (>) w(\alpha)$. The monotonicity of $u(w,\beta)$ in w then guarantees that

$$Eu\big(A + \tilde{\pi}(w(\alpha), \alpha), \beta\big) = u\big(A + w(\alpha,\beta), \beta\big) < (>) u\big(A + w(\alpha), \beta\big) \tag{11}$$

when $\beta > (<) \alpha$.

Now note that, by definition of $\tilde{\pi}(w,\beta)$,

$$Eu\big(A + \tilde{\pi}(w(\alpha), \beta), \beta\big) \geqq Eu\big(A + \tilde{\pi}(w(\alpha), \alpha), \beta\big). \tag{12}$$

When $\beta < \alpha$ inequalities (11) and (12) combine to yield

$$Eu\big(A + \tilde{\pi}(w(\alpha), \beta), \beta\big) > u\big(A + w(\alpha), \beta\big). \tag{13}$$

Figure 2 illustrates what is easily proven; that inequality iv is a consequence of inequality (13), the equality defining $w(\beta)$, and the fact that $Eu\big(\tilde{\pi}(w,\beta), \beta\big) - u(w,\beta)$ decreases monotonically in w.

v) Inequality v follows immediately from iii and iv.

vi) Since iii implies that $Eu\big(A + \tilde{\pi}(w, \beta)\, \beta\big) > u\big(A + w, \beta\big)$, if $w(\beta) > w$, $\tilde{\pi}(w, \beta) = g\big(L(w, \beta), \tilde{x}\big) - wL(w, \beta)$ must exceed $w > 0$ with positive probability. This is impossible if $L(w, \beta) = 0$. ||

In the discussion of existence, the analysis is restricted to cases which satisfy assumption A: $u(I,\alpha)$ is everywhere a continuous function of α.

THEOREM 1.[2]—Assume that for each I, $r(I,\alpha)$ is a nondecreasing function of α. Also assume that either $g_{LL} < 0$ or $u_{II} < 0$. Under assumption A an equilibrium exists.

PROOF.—Under assumption A it can be shown that our assumptions guarantee that $L(w,\alpha)$ and $w(\alpha)$ are continuous functions of α on $[0,1]$. Thus for each $w \in [w(0),w(1)]$ and $\hat{\alpha} \in [0,1]$, $\int_0^{\hat{\alpha}} L(w,\alpha)d\alpha$ exists.

We can now find an $\hat{\alpha}*$ such that

$$\int_0^{\hat{\alpha}*} L(w(\hat{\alpha}*), \alpha)d\alpha = 1 - \hat{\alpha}*. \tag{14}$$

Note that $\int_0^{\hat{\alpha}} L(w(\hat{\alpha}), \alpha)d\alpha - (1 - \hat{\alpha})$ is a continuous function of $\hat{\alpha}$ which is negative when $\hat{\alpha} = 0$ and positive (by vi of the lemma) when $\hat{\alpha} = 1$. The intermediate value theorem implies the existence of an $\hat{\alpha}*$ satisfying 14.

Now we can define

$$(\{\Delta,\Gamma\}, w) = (\{[0,\hat{\alpha}*], (\hat{\alpha}*,1], w(\hat{\alpha}*)) \tag{15}$$

or

$$(\{\Delta,\Gamma\}, w) = (\{[0,\hat{\alpha}*), [\hat{\alpha}*,1], w(\hat{\alpha}*)).$$

For these entrepreneur, worker, wage combinations v of the lemma implies that condition E.2 holds while E.1 reduces to (14). ‖

The next theorem gives conditions under which the equilibrium is unique.

THEOREM 2.—Assume that for each I, $r(I,\alpha)$ is a nondecreasing function of α. Also assume that either $g_{LL} < 0$ or $u_{II} < 0$. If, in addition, $L(w,\alpha)$ is a decreasing function of w, then the equilibrium is unique.

PROOF.—Because of the lemma, and for reasons discussed in remark 1, the equilibrium occurs at an $\hat{\alpha}$ for which (14) holds. In addition, the lemma implies that $L(w,\alpha) > 0$ for all α and all $w \leq w(\alpha)$, and that $w(\hat{\alpha}) \geq w(\hat{\alpha}')$ if $\hat{\alpha} < \hat{\alpha}'$. Then since $L(w,\alpha)$ is a decreasing function of w, $\hat{\alpha} < \hat{\alpha}'$ implies

$$\int_0^{\hat{\alpha}'} L(w(\hat{\alpha}'),\alpha)d\alpha = \int_0^{\hat{\alpha}} L(w(\hat{\alpha}'),\alpha)d\alpha$$

$$+ \int_{\hat{\alpha}}^{\hat{\alpha}'} L(w(\hat{\alpha}'),\alpha)d\alpha > \int_0^{\hat{\alpha}} L(w(\hat{\alpha}),\alpha)d\alpha.$$

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[2] This theorem can be proved without assumption A. The assumption is made solely to permit a simple existence proof.

Thus labor demand at $w(\hat{\alpha})$, $\int_0^{\hat{\alpha}} L(w(\hat{\alpha}),\alpha)d\alpha$, is a strictly increasing function of $\hat{\alpha}$. Furthermore, labor supply at $w(\hat{\alpha})$, $(1 - \hat{\alpha})$, is a strictly decreasing function of $\hat{\alpha}$. Therefore excess demand at $w(\hat{\alpha})$, $\int_0^{\hat{\alpha}} L(w(\hat{\alpha}),\alpha)d\alpha - (1 - \hat{\alpha})$, is a strictly increasing function of $\hat{\alpha}$ and there can be only one $\hat{\alpha}$ at which excess demand can equal zero, that is, at which (14) can hold. $\Vert$

Conditions under which $L(w,\alpha)$ is a decreasing function of the wage w are discussed in remark 4 at the end of the following section on comparative statics.

IV. Comparative Statics

Having established the existence of an equilibrium, it is now possible to study its properties. Specifically, we can first ask how a firm's size, as measured by its labor demand, is related to the risk averseness of the entrepreneur running the firm. It might be expected that more risk averse entrepreneurs operate smaller firms, that is, use less labor than less risk averse entrepreneurs. Theorem 3 gives conditions under which this expected result obtains. The conditions require that a change in x must affect output and the marginal product of labor in the same way; if an increase in x raises output it must also raise the marginal product of labor. One important special case in which this condition holds occurs when the uncertainty enters multiplicatively

$$g((L, x) = xh(L).^3 \tag{16}$$

THEOREM 3.—Assume that $L(w,\alpha) < A/w$. If $g(L,x)$ and $g_L(L,x)$ are both monotonically increasing or both monotonically decreasing functions of x, then $L(w,\alpha)$ is a monotonically decreasing function of α.

The proof is essentially the same as that given in Baron (1970) and is not reproduced.

We can now ask to what extent it is possible to describe the influence of individual attitudes toward risk and of technological parameters on the equilibrium. In general, not much can be said about the effect of these parameters on the number of firms, a variable of particular interest. But for the purpose of studying the distribution of wealth between workers and entrepreneurs it is important to know how these parameters influence the wage. What can be shown is that, under certain reasonable conditions, an increase in individual risk aversion reduces the wage.

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[3] This is the case considered by Baron (1970). In Baron's paper, x is interpreted as price. Equation (16) is also included in the class of cases studied by Diamond. In Diamond's terminology, (16) represents a case of stochastic constant returns to scale.

THEOREM 4.—If (i) in equilibrium, all entrepreneurs are identical, (ii) either $g_{LL} < 0$ or $u_{II} < 0$, (iii) $g(L,x)$ and $g_L(L,x)$ are both monotonically increasing (or decreasing) functions of x, and (iv) $L(w,\hat{\alpha})$ is an interior solution and a decreasing function of w, then an increase in the Arrow-Pratt absolute risk aversion measure $r(I,\hat{\alpha})$ for all I, lowers the equilibrium wage.

Remark 2

The intuitive basis for this result is as follows. Since, in equilibrium, workers are the most risk averse individuals, an economy-wide increase in risk aversion increases the supply of workers and this tends to lower the wage. This tendency is reinforced by demand changes implied by theorem 3 which applies because of assumption iii. Specifically, theorem 3 implies that an increase in the entrepreneurs' aversion to risk reduces the demand for labor.

PROOF.--If, in equilibrium, $L(w,\hat{\alpha})$ is an interior solution, the first-order maximization condition for the marginal entrepreneur is

$$Eu_I(A + g(L,\tilde{x}) - wL, \hat{\alpha})g_L(L,\tilde{x}) = Eu_I(A + g(L,\tilde{x}) - wL, \hat{\alpha})w \quad (17)$$

where $L = L(w,\hat{\alpha})$.

We also have

$$u(A + w, \hat{\alpha}) = Eu(A + g(L,\tilde{x}) - wL, \hat{\alpha}) \quad (18)$$

at $L = L(w,\hat{\alpha})$.

The equilibrium conditions (17) and (18) imply relationships between L and w which are described in figure 3. The relationship implied by (17) is downward sloping because of assumption iii. As indicated, (18) implies that w is a function of L which reaches its minimum when it intersects the line defined by (17). This is proved by differentiating (18) implicitly to obtain

$$\frac{dw}{dL} = - \frac{Eu_I(A + g(L,\tilde{x}) - wL, \hat{\alpha}) [g_L(L,\tilde{x}) - w]}{u_I(A + w,\hat{\alpha}) + Eu_I(A + g(L,\tilde{x}) - wL, \hat{\alpha})L} . \quad (19)$$

The differentiation is justified by the implicit function theorem because the denominator in (19) is positive. The numerator is zero when (17) holds. The second order sufficient condition for the entrepreneurial maximization problem is satisfied because of condition ii. Thus, as reflected in figure 3, the numerator in (19) is positive (negative) and dw/dL is negative (positive) when $L < (>) L(w, \hat{\alpha})$.

Now suppose that $r(I,\hat{\alpha})$ increases for every I, then theorem 3 above guarantees that $L(w,\hat{\alpha})$ is lower for each w. Also, reasoning similar to that employed in the proof of the lemma guarantees that for each L,
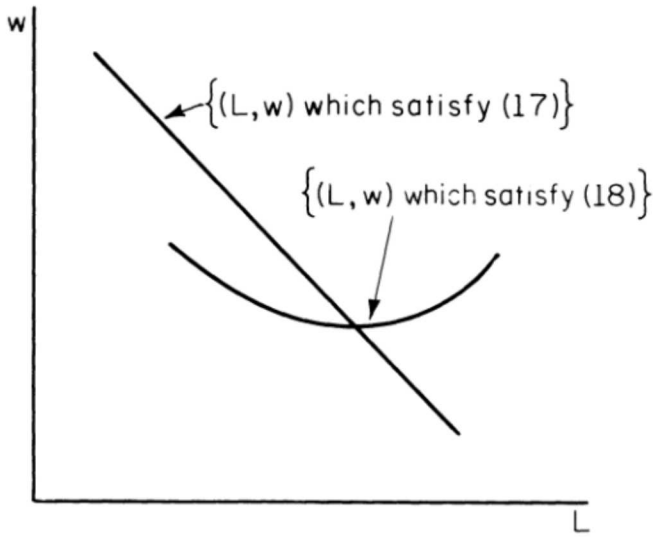
FIG. 3

the wage level w at which (18) holds is also reduced. Thus the r increase affects the relationships between L and w implied by (17) and (18) as shown in figure 4. As a result the equilibrium wage must decline. ||

We can now make several observations which we formalize as remarks.

Remark 3

A similar proof applies if $L(w,\hat{\alpha})$ is always an increasing function of w.

Remark 4

There are several important cases in which $L(w,\hat{\alpha})$ is indeed a decreasing function of w. These occur when either (a) $r(I,\hat{\alpha})$ is a constant function of I, (b) $g(L,x)$ and $g_L(L,x)$ are both increasing (or both decreasing) functions of x and $r(I,\hat{\alpha})$ is a nonincreasing function of I, or (c) $g(L,x)$ satisfies (16) and

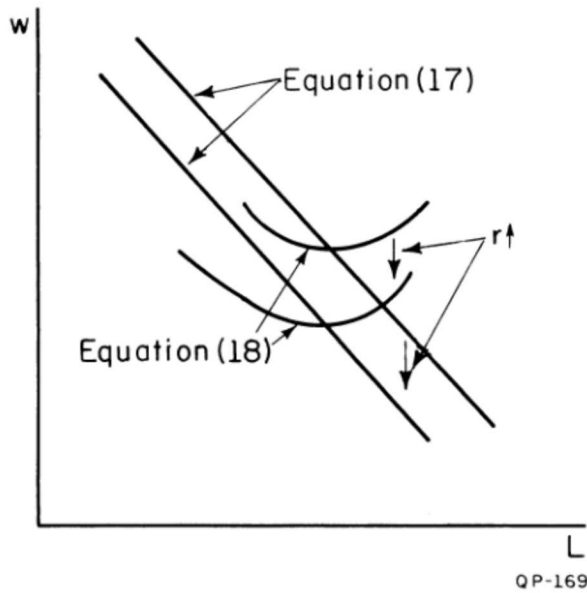$$-\frac{Iu_{II}(A + I, \hat{\alpha})}{u_I(A + I, \hat{\alpha})} \leqq 1$$

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Fig. 4

for all I. The condition imposed on $u(\cdot,\hat{\alpha})$ in c asserts that when $u(A + I, \hat{\alpha})$ is considered as a function of I it has Arrow-Pratt relative risk aversion less than or equal to one.

PROOF OF REMARK 4.—Implicitly differentiating (17) we obtain

$$\frac{dL}{dw} = -\frac{-LEu_{II}(A + g(L,\tilde{x}) - wL, \hat{\alpha})[g_L(L,\tilde{x}) - w]}{D} - \frac{-Eu_I(A + g(L,\tilde{x}) - wL, \hat{\alpha})}{D} \quad (20)$$

where $D = Eu_{II}(A + g(L,\tilde{x}) - wL, \hat{\alpha})[g_L(L,\tilde{x}) - w]^2 + Eu_I(A + g(L,\tilde{x}) - wL, \hat{\alpha})g_{LL}(L,\tilde{x})$. The second-order condition for the entrepreneur's maximization problem (which is implied by condition ii of theorem 4) guarantees that the implicit function theorem applies to justify the implicit differentiation. This condition also asserts that the denominator in (20) is negative. In general, the sign of the numerator in (20) is ambiguous since the first term is ambiguous. (The second term is negative.) In case a, however, the first term is

$$LEu_{II}(A + g(L,\tilde{x}) - wL,\hat{\alpha}) [g_L(L,\tilde{x}) - w]$$
$$= - rLEu_I (A + g(L,\tilde{x}) - wL,\hat{\alpha}) [g_L(L,\tilde{x}) - w] \quad (21)$$

which equals zero because of the first-order condition (17). Thus in case a, the numerator is negative as is dL/dw.

In case b, the first term can be shown to be nonpositive by an argument similar to that used by Baron (1970).

In case c, the fact that $h''(L) \leqq 0$ can be used to obtain

$$[h'(L)L\tilde{x} - wL] \leqq h(L)\tilde{x} - wL \tag{22}$$

and

$$-u_{II}(A + h(L)\tilde{x} - wL, \hat{\alpha})[h'(L)L\tilde{x} - wL]$$
$$\leqq -u_{II}(A + h(L)\tilde{x} - wL, \hat{\alpha})[h(L)\tilde{x} - wL]. \tag{23}$$

When this inequality is combined with c, the numerator in (20) is negative. ||

V. Dynamics

In this brief section, we consider the stability of a tâtonnement adjustment process similar to that used in studying the stability of competitive equilibrium. In this process, the wage is assumed to adjust to labor market disequilibrium by rising when there is excess demand and falling when there is excess supply. Specifically,

$$\frac{dw}{dt} = \phi\left[\int_0^{\hat{\alpha}(w(t))} L(w(t), \alpha)d\alpha - (1 - \hat{\alpha}(w(t)))\right] \tag{24}$$

where ϕ is a differentiable increasing function such that $\phi(0) = 0$ and where $\hat{\alpha}(w)$ satisfies the equation

$$Eu(\bar{\pi}(w, \hat{\alpha}(w)), \hat{\alpha}(w)) = u(w, \hat{\alpha}(w)). \tag{25}$$

We define

$$V(w) = \left(\phi\left[\int_0^{\hat{\alpha}(w)} L(w, \alpha)d\alpha - [1 - \hat{\alpha}(w)]\right]\right)^2 \tag{26}$$

to be the Lyapunov function. Then[4]

$$\frac{d}{dt}V(w(t)) = 2\phi'\left[\int_0^{\hat{\alpha}(w)} L(w, \alpha)d\alpha - (1 - \hat{\alpha}(w))\right]$$
$$\times \left\{\left[L(w(t), \hat{\alpha}(w(t))) + 1\right]\hat{\alpha}'(w(t))\right. \tag{27}$$
$$\left. + \int_0^{\hat{\alpha}(w(t))} \frac{\partial L}{\partial w}(w(t), \alpha)d\alpha\right\}\left(\frac{dw}{dt}\right)^2.$$

Now $\phi' > 0$ by assumption, and $\alpha'(w(t))$ is negative because of the lemma. Thus if $\partial L/\partial w < 0$, $d/dt V(w(t))$ is negative and $w(t)$ converges to the equilibrium wage.

[4] If we assume that u_α and $u_{l\alpha}$ exist and are continuous, repeated application of the implicit function theorem implies that $\hat{\alpha}(w)$ is a differentiable function of w.

These results are summarized in the following theorem.

THEOREM 5.—Assume that for each I, $r(I,\alpha)$ is a decreasing function of α and that either $g_{LL} < 0$ or $u_{II} < 0$. Also assume that u_α and $u_{I\alpha}$ exist and are continuous. If $L_w(w,\alpha)$ exists and is negative, then $w(t)$ converges to the unique equilibrium wage.

Remark 5

In the standard explanation for firm entry and exit, which does not admit the possibility of uncertain profits, reductions (increases) in profits caused by falling (rising) demand or increases (decreases) in cost result in exit (entry). In our entrepreneurial model demand changes are not explicitly considered and cost changes are introduced by wage changes. In addition, changes in the return to nonentrepreneurial activities, specifically labor, also cause entry or exit. Again these changes are embodied in wage changes.

The fact that returns to nonentrepreneurial activities influence firm entry and exit is a reflection of the general equilibrium nature of our formalization. In this framework, an individual's decision to enter as an entrepreneur or exit to become a worker is made after the expected utility of the random profits available to entrepreneurs has been compared to the utility of the nonrisky wages earned by workers. In the formalizations of Quandt and Howrey (1968) and of Brock (1972), firms decide to enter if (excess) profits can be made. This is appropriate when there is no uncertainty and no opportunity cost to entry other than capital costs. In our model both of these complications are present. Profits are random and the opportunity cost of becoming an entrepreneur is lost wages.

Remark 6

Since the model of the adjustment process studied here is analogous to that employed in the literature on competitive equilibrium, it is subject to the same criticisms. Specifically, the dynamic wage change equations are not explained by an underlying model of maximizing behavior. In the paper by Smith (1974), the dynamic equations which describe the process of firm entry and exit are explicitly obtained from a maximization model.

VI. Efficiency of Equilibrium

In this section, two concepts of efficiency are studied. The first is unconstrained Pareto optimality in the sense of Arrow (1964) and

Debreu (1959). The second is a constrained version of Pareto optimality in which the institutional constraints on risk trading implicit in our concept of equilibrium are imposed on all allocations. The reasons for studying constrained optima will be suggested by the analysis of unconstrained optima. We will show that because of the institutional restrictions embodied in our equilibrium concept, asking for unconstrained optimality is, in general, clearly asking for too much. There are, nevertheless, interesting cases in which an equilibrium is efficient in an unconstrained sense. In addition, it is possible to specify the nature of the unconstrained inefficiencies.

Before proceeding to the formal discussion it is convenient to introduce special assumptions which are employed to simplify the analysis of unconstrained efficiency. Specifically, we now assume that $\tilde{x}$ is the same random variable for all firms, that is, that the random variations in the firms' outputs are perfectly correlated. This assumption will be sufficient to permit an intuitive explanation of the inefficiencies occurring in our model. Furthermore, a general treatment would take us beyond the scope of the paper. The reader should note however that this assumption is used only in the discussion of unrestricted efficiency. In the subsequent discussion of restricted efficiency, no assumptions are made about the dependence or independence of the returns to different firms.

As a preliminary to the formal discussion, we define an unrestricted feasible allocation as a specification of Γ and Δ and of functions $\nu: \Delta \to [0,\infty)$ and $y(\cdot,x):[0,1] \to [0,\infty)$, for each x, which satisfy the conditions

$$\int_\Delta \nu(\alpha)\mu(d\alpha) = \mu(\Gamma) \tag{28}$$

and

$$\int_0^1 y(\alpha,x)\mu(d\alpha) = \int_\Delta g(\nu(\alpha),x)\mu(d\alpha) + A \tag{29}$$

for each x.

The ν specifies the allocation of labor to firms. Equation (28) asserts that labor supply equals demand. The function $y(\cdot,x)$ describes the allocation of income to individuals in each state. The constraints (29) require that, in each state x, the supply of the commodity equals demand.[5]

The Pareto-optimal allocations can be studied by introducing arbitrary linear social welfare functions. Specifically, let λ be an arbitrary

---

[5] Notice that the notation embodies the assumption that the output of all firms is affected by the same random variable $\tilde{x}$. Specifically, a "state of nature" is completely defined by x, the value taken by $\tilde{x}$. If different firms were affected by different random variables, the description of a "state" would have to specify the value taken by each of these variables.

Lebesgue measurable function $\lambda:[0,1] \to [0,1]$. The corresponding social welfare function is

$$\int_0^1 \lambda(\alpha)Eu\big(y(\alpha,\tilde{x}),\alpha\big)\mu(d\alpha). \tag{30}$$

If Γ, Δ, ν, and $y(\cdot,\cdot)$ are chosen to maximize (30) subject to the constraints (28) and (29), the result is a Pareto-optimal allocation. In order to describe the Pareto optimal allocations, we study the solutions to these maximization problems for arbitrary λ functions.

First notice that it is possible for a planner who wishes to maximize (30) subject to the constraints (28) and (29) to ignore the identity of individuals who become workers and entrepreneurs and concern himself only with the number of entrepreneurs, that is, the number of firms. A similar simplification is possible in choosing ν; only the distribution of labor to firms matters; it is unimportant which entrepreneur runs which firm. This makes it possible to establish a convention that facilitates the comparison of efficient allocations with equilibrium allocations. Specifically, we can assume that in making his choice of Γ and Δ, the planner simply chooses an individual $\hat{\alpha}$ ($\hat{\alpha}$ can also be interpreted as the number of firms) and then assigns $\Delta = [0,\hat{\alpha}]$ and $\Gamma = (\hat{\alpha},1]$.

The second simplification which is possible in the discussion of unconstrained optimality is introduced because g exhibits decreasing returns to scale. Under this assumption, efficiency requires that every firm produce the same amount. If this were not true, output in each state x could be increased if labor were transferred from a high output firm to a low output firm. This transfer would increase output because of the differences in labor's marginal productivity (in every state x) which would result from the initial inequality of the outputs of the two firms considered.[6]

Since $\gamma(\alpha)$ must be the same for all entrepreneurs (28) reduces to

$$\nu(\alpha) = \frac{1 - \hat{\alpha}}{\hat{\alpha}} \tag{31}$$

for all $\alpha \epsilon [0,\hat{\alpha}]$. Using this result, (29) becomes

$$\int_0^1 y(\alpha,x)\mu(d\alpha) = \hat{\alpha}g\left(\frac{1 - \hat{\alpha}}{\hat{\alpha}},x\right) + A. \tag{32}$$

---

[6] This result can be derived immediately by writing the Euler equation corresponding to the maximization with respect to $\nu(\cdot)$. We get $E\delta(\tilde{x})g_L(\nu(\alpha),\tilde{x}) = \delta_0$ where δ_0 is the multiplier associated with (28) and $\delta(x)$ are the multipliers associated with (29). Thus $E\delta(\tilde{x})g_L(\nu(\alpha),\tilde{x}) = E\delta(\tilde{x})g_L(\nu(\alpha'),\tilde{x})$ for all $\alpha \le \hat{\alpha}$ and $\alpha' \le \hat{\alpha}$. Since g_L is a decreasing function of L for each x, $\nu(\alpha') \ne \nu(\alpha)$ would make this equality impossible.

The program for obtaining Pareto-optimal allocations reduces to

$$\max_{(\hat{\alpha},\mu(\cdot,\cdot))} \int_0^1 \lambda(\alpha)\, Eu\,\left(y(\alpha,\bar{x}),\alpha\right)\mu(d\alpha)$$

subject to (32) for all x. The first-order conditions are

$$\lambda(\alpha)\pi(x)u_I\big(y(\alpha,x),\alpha\big) = \delta(x), \text{ for all } \alpha \text{ and all } x \qquad (33)$$

and

$$\int_0^{\bar{x}} \delta(x)\, g\!\left(\frac{1-\hat{\alpha}}{\hat{\alpha}},x\right)dx = \int_0^{\bar{x}} \frac{\delta(x)}{\hat{\alpha}}\, g_L\!\left(\frac{1-\hat{\alpha}}{\hat{\alpha}},x\right)dx \qquad (34)$$

where $\delta(x)$ is the multiplier associated with the resource constraint in state x, and $\pi(x)$ is the value of the objective probability density function at x.

Using the value of $\delta(x)$ defined by (33) and inserting in (34) we obtain, after taking the expectation over x,

$$\frac{1}{\hat{\alpha}} E\, u_I\big(y(\alpha,\bar{x}),\alpha\big) g_L\!\left(\frac{1-\hat{\alpha}}{\hat{\alpha}},\bar{x}\right) = E\, u_I\big(y(\alpha,\bar{x}),\alpha\big) g\!\left(\frac{1-\hat{\alpha}}{\hat{\alpha}},x\right) \quad (35)$$

for every α.

Using (33) for two different x values, say x_1 and x_s, and for two different α values, say α and β, we also obtain

$$\frac{u_I\big(y(\alpha,x_s),\alpha\big)}{u_I\big(y(\alpha,x_1),\alpha\big)} = \frac{u_I\big(y(\beta,x_s),\beta\big)}{u_I\big(y(\beta,x_1),\beta\big)} \qquad (36)$$

for all s and all α, β.

Condition (35) can be viewed as that which determines the efficient $\hat{\alpha}$, that is, the optimal division of individuals between workers and entrepreneurs. In the special case where (16) holds, that is, when there are stochastic constant returns to scale, (35) reduces to

$$\hat{\alpha} = \left[h'\!\left(\frac{1-\hat{\alpha}}{\hat{\alpha}}\right)\right]\Big/\left[h\!\left(\frac{1-\hat{\alpha}}{\hat{\alpha}}\right)\right] \qquad (37)$$

which is the $\hat{\alpha}$ level which maximizes the output $\hat{\alpha}h\,((1-\hat{\alpha})/\hat{\alpha})x$ for every x. The input level $1-\hat{\alpha}/\hat{\alpha}$ is also the one which would be chosen in an allocation which is efficient in Diamond's second-best sense.

The conditions (36) are those which characterize efficient allocations of contingent claims to the output produced.

It is clear from the preceding discussion and from the conditions (35) and (36) that there will be several sources of inefficiency in an equilibrium of the type defined above. The most obvious relates to the point made earlier that in an efficient allocation all firms should be producing equal amounts if returns to scale are diminishing for each x. In general, the equilibrium will be characterized by entrepreneurs

who have varying attitudes toward risk. For that reason different entrepreneurs will produce different outputs. This is one source of inefficiency. Note, however, that it will fail to arise if all entrepreneurs have the same utility function and therefore the same attitude toward risk.

A second type of inefficiency arises because of the fact that only entrepreneurs bear risks in equilibrium. This is the institutional constraint on the allocation of risk bearing of which we spoke earlier. Thus, in general, the conditions (36) cannot be satisfied if there are risk averse entrepreneurs. A special case in which this problem does not arise occurs when all entrepreneurs are indifferent to risk. In that case condition (36) holds in equilibrium because of the linearity of entrepreneurs' utility functions and the fact that workers bear no risk. We will return to discuss this case more completely later.

The third source of inefficiency which requires more discussion is the optimal choice of $\hat{\alpha}$. To discuss this problem in an appropriate setting it seems necessary to consider an equilibrium in which all entrepreneurs are the same and produce the same output. This eliminates inefficiencies of the first type mentioned and makes it possible to ask if (35) might be satisfied.

To study this question, recall that (17) is the necessary condition for entrepreneurial expected utility maximization. In general, (17) differs from (35) because, as we shall see below,

$$
\hat{\alpha} E u_I\!\left(A + g\!\left(\frac{1 - \hat{\alpha}}{\hat{\alpha}}, \tilde{x}\right) - w\!\left(\frac{1 - \hat{\alpha}}{\hat{\alpha}}\right), \hat{\alpha}\right) g\!\left(\frac{1 - \hat{\alpha}}{\hat{\alpha}}, \tilde{x}\right)
$$
$$
\neq w E u_I\!\left(A + g\!\left(\frac{1 - \hat{\alpha}}{\hat{\alpha}}, \tilde{x}\right) - w\!\left(\frac{1 - \hat{\alpha}}{\hat{\alpha}}\right), \alpha\right). \tag{38}
$$

(In an equilibrium in which all entrepreneurs have the same utility function, $L(w(\hat{\alpha}), \hat{\alpha}) = \dfrac{1 - \hat{\alpha}}{\hat{\alpha}}$ since supply equals demand.)

Also recall that, in equilibrium, w satisfies (18) where $L = \dfrac{1 - \hat{\alpha}}{\hat{\alpha}}$. Thus w is the certainty equivalent of the random variable $g(\dfrac{1 - \hat{\alpha}}{\hat{\alpha}}, \tilde{x})$ $- w(\dfrac{1 - \hat{\alpha}}{\hat{\alpha}})$. When entrepreneurs are risk averse, condition (18) implies that

$$
w = E g\!\left(\frac{1 - \hat{\alpha}}{\hat{\alpha}}, \tilde{x}\right) - w\!\left(\frac{1 - \hat{\alpha}}{\hat{\alpha}}\right) - \rho \tag{39}
$$

where ρ is a positive risk premium. Rearranging, we obtain that

$$
\frac{1}{\hat{\alpha}} w = E g\!\left(\frac{1 - \hat{\alpha}}{\hat{\alpha}}, \tilde{x}\right) - \rho. \tag{40}
$$

Substituting (40) in (17) yields

$$\frac{1}{\hat\alpha}Eu_I(A + g(L,\tilde x) - wL, \hat\alpha)g_L(L,\tilde x) = \frac{1}{\hat\alpha}wEu_I(A + g(L,\tilde x) - wL, \hat\alpha)$$

$$= [Eg(L,\tilde x)][Eu_I(A + g(L,\tilde x) - wL, \hat\alpha)] - \rho Eu_I(A + g(L,\tilde x) - wL, \hat\alpha) \tag{41}$$

where $L = \dfrac{1-\hat\alpha}{\hat\alpha}$. Risk aversion ($u_{II} < 0$) also implies that

$$0 > c = \mathrm{cov}(g(L,\tilde x), u_I(A + g(L,\tilde x) - wL, \hat\alpha))$$

$$= Eg(L,\tilde x)u_I(A + g(L,\tilde x) - wL, \hat\alpha) \tag{42}$$

$$-[Eg(L,\tilde x)][Eu_I(A + g(L,\tilde x) - wL, \hat\alpha)]$$

where, again $L = (1 - \hat\alpha)/\hat\alpha$. Combining (41) and (42) we obtain

$$\frac{1}{\hat\alpha}Eu_I(A + g(L,\tilde x) - wL, \hat\alpha)g_L(L,\tilde x) = Eg(L,\tilde x)u_I(A + g(L,\tilde x) - wL, \hat\alpha)$$

$$- c - \rho Eu_I(A + g(L,\tilde x) - wL, \hat\alpha), \tag{43}$$

with $L = (1 - \hat\alpha)/\hat\alpha$. Note that (43) and (35) are the same if the covariance c equals the negative of $\rho u_I\left(A + g\left(\dfrac{1-\hat\alpha}{\hat\alpha}, \tilde x\right) - w\left(\dfrac{1-\hat\alpha}{\hat\alpha}\right), \hat\alpha\right)$.

In this case the equilibrium is efficient. Otherwise, risk aversion causes two types of errors. One of these, measured by the term $- \rho u_I\left(A + g\left(\dfrac{1-\hat\alpha}{\hat\alpha}, \tilde x\right) - w\left(\dfrac{1-\hat\alpha}{\hat\alpha}\right), \hat\alpha\right)$, is introduced by the "entry condition" (18) and tends to cause the right-hand side (RHS) of (35) to exceed the left-hand side (LHS). The other type of error is measured by c. It enters through the entrepreneurial maximization condition (17) and it tends to make the RHS of (35) smaller than the LHS in equilibrium. To identify the direction of the effect which each of these errors has on the choice of $\hat\alpha$ consider the case in which (16) holds so that the optimal choice of $\hat\alpha$ is independent of the preferences and the income distribution. Recall that in this case $\hat\alpha$ should be chosen to maximize output in each state and that (35) reduces to the first-order condition for this output maximization problem. It is also easy to verify that when the LHS of (35) exceeds the RHS in equilibrium, then the derivative, $h\left(\dfrac{1}{\hat\alpha} - 1\right)x - \dfrac{1}{\hat\alpha}h'\left(\dfrac{1}{\hat\alpha} - 1\right)x$, of output with respect to $\hat\alpha$ is negative at the equilibrium. It is then clear from figure 5 that in this case the equilibrium $\hat\alpha$ is too large to be efficient. Thus there are too many firms in equilibrium when the error, $-c$, introduced by the
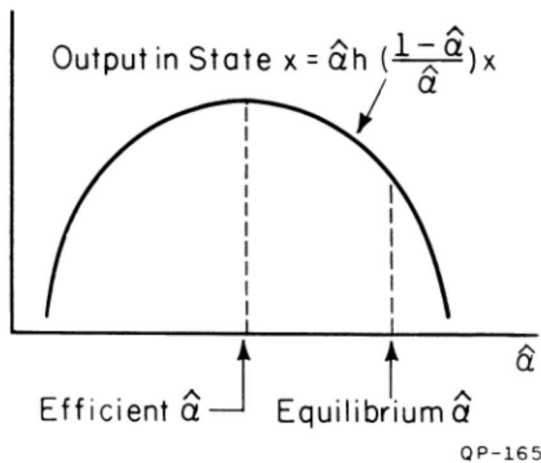
FIG. 5

entrepreneur's equilibrium condition, outweighs $-\rho E u_I\left(A + g\left(\frac{1 - \hat{\alpha}}{\hat{\alpha}},\right.\right.$

$\left.\left.\tilde{x}\right) - w\left(\frac{1 - \hat{\alpha}}{\hat{\alpha}}\right), \hat{\alpha}\right)$, the error introduced by the entry condition. Similar reasoning leads us to conclude that there are too few entrepreneurs when the error introduced by the entry condition outweighs the error introduced by the entrepreneurial first-order condition. These conclusions coincide with intuition. On the one hand risk aversion should cause too few individuals to become entrepreneurs and this should operate through the entry condition. On the other hand risk aversion on the part of those who become entrepreneurs reduces labor demand when (16) holds (recall theorem 3). The error caused by entrepreneurial risk aversion should reduce the equilibrium labor demand and the equilibrium wage. The low wage creates an incentive for too many individuals to become entrepreneurs.

We can now consider several important special cases. The first such case is that in which all entrepreneurs have constant absolute risk aversion in the sense of Arrow-Pratt, that is, $u(I, \alpha_1) = -e^{-rI}$, for some $r > 0$; and the production function is

$$g(L, x) = L^\gamma x \qquad (44)$$

where $\gamma \in (0, 1)$. In this case, (35) reduces to

$$\hat{\alpha} = 1 - \gamma, \qquad (45)$$

and the efficient labor input per firm is

$$\frac{1 - \hat{\alpha}}{\hat{\alpha}} = \frac{\gamma}{1 - \gamma}. \qquad (46)$$

The equilibrium conditions (17) and (18) combine to yield

$$0 = \frac{E\tilde{x}e^{-rL\gamma\tilde{x}}}{Ee^{-rL\gamma\tilde{x}}} + \frac{1}{r\gamma L^{\gamma-1}(1+L)} \log (Ee^{-r\tilde{x}L\gamma}) \qquad (47)$$

where $L = (1 - \hat{\alpha})/\hat{\alpha}$.

It can be shown that the RHS of (47) is a decreasing function of L and that it is negative if $\gamma/(1 - \gamma)$ is substituted for L. Thus the efficient L level exceeds the equilibrium level. As a result there are too many firms in equilibrium.

Note that in this class of examples the efficient number of firms approaches zero if γ approaches one, that is, if returns to scale become constant. The limiting case in which returns to scale are indeed constant is one in which, in general, there are too many entrepreneurs. The efficiency analysis just carried out does not apply to this case because it assumes that $g_{LL} < 0$ for all L. (The existence of equilibrium proof does apply if $u_{II} < 0$.) It is possible to analyze this case directly however by substituting

$$g(L,x) = h(L)x = kLx \qquad (48)$$

in (34). The output in state x then becomes $k(1 - \hat{\alpha})x$ which is clearly maximized for each x when $\hat{\alpha} = 1$. Thus the measure of the optimal set of entrepreneurs is zero and almost all individuals should work. This result occurs because the technology set of the economy is the same if there is either one entrepreneur, some larger entrepreneur set of measure zero, or a set of positive measure.

There is one important case in which the equilibrium is efficient. That is the case already mentioned in which all entrepreneurs are indifferent to risk. Since the preceding discussion makes it clear that entrepreneurial risk aversion causes the errors which result in a nonoptimal equilibrium level for $\hat{\alpha}$, it should not now be surprising that the equilibrium is efficient when entrepreneurs are not risk averse. For that case, we have already noted that the distribution of risk is efficient, that is, condition (36) is satisfied. The linearity of the utility function implies then that $u_I(\cdot, \alpha)$ is independent of $y(\alpha, x)$, so that (35) reduces to

$$\hat{\alpha}Eg\left(\frac{1-\hat{\alpha}}{\hat{\alpha}}, \tilde{x}\right) = Eg_L\left(\frac{1-\hat{\alpha}}{\hat{\alpha}}, \tilde{x}\right). \qquad (49)$$

For the same reason (17) reduces to

$$Eg_L\left(\frac{1-\hat{\alpha}}{\hat{\alpha}}, \tilde{x}\right) = w. \qquad (50)$$

In addition, risk indifference implies that $\rho = 0$ so that (40) becomes

$$w = \hat{\alpha}Eg\left(\frac{1-\hat{\alpha}}{\hat{\alpha}}, \tilde{x}\right). \qquad (51)$$

Taken together the equilibrium equations (50) and (51) imply (49) and the equilibrium therefore satisfies all of the conditions for efficiency.

The preceding discussion of efficiency suggests that entrepreneurial risk aversion is the source of all of the observed inefficiencies. When entrepreneurs are risk averse the equilibrium is not only characterized by an inefficient distribution of risk; there will, in general, also be too many or too few firms and they will not employ the correct number of workers. In fact, it is well known that an inefficient distribution of risk is inevitable with any equilibrium in which some subgroup of risk averse individuals (in this case, the entrepreneurs) bear all of the risks. Suppose, however, that we concede the inevitability of a maldistribution of risks and ask if, given this kind of inefficiency, the other aspects of the equilibrium might be efficient. Specifically, let us accept the fact that entrepreneurial risks cannot be shared and require only an optimal division of individuals between entrepreneurial activities and labor and of labor between firms. Is it then possible that in this restricted sense the equilibrium is efficient?

In order to pose this question formally we define a restricted feasible allocation to be a specification of Γ, Δ together with two functions $\nu : \Delta \to [0,\infty)$ and $\xi : [0,1] \to [-A,\infty)$ which satisfy the equations (28),

$$\int_0^1 \xi(\alpha) d\alpha = 0, \qquad (52)$$

and $A + g(\nu(\alpha),x) + \xi(\alpha) \geq 0$ for all x and $\alpha \in \Delta$.

The function $\nu(\alpha)$ specifies the labor to be employed by entrepreneur α, that is, for each $\alpha \in \Delta$, $\nu(\alpha)$ is α's labor demand. Equation (28) expresses the equality of labor supply and demand. The function ξ specifies the amount of a certain payment made to each α. Equation (52) is a supply-demand equality for these payments. It guarantees that the resources exist to make all payments. The final condition rules out bankruptcy.

The difference between this concept of restricted feasibility and the notion of unrestricted feasibility should be noted. In the definition of unrestricted feasibility, the payments made to individuals are contingent on the state of nature x. As in an Arrow-Debreu economy, the only constraint on the allocation of contingent claims is that supply must equal demand in each state. As a result, risks can be completely reallocated. In contrast, a restricted feasible allocation specifies, for each α, a payment $\xi(\alpha)$ which is not state contingent; it does not permit a reallocation of risks. As a result, the distribution of risks implied by a restricted feasible allocation has two important features. First, as in equilibrium, workers bear no risks; entrepreneurs bear all risks. In addition, the distribution of risk among entrepreneurs is

determined by the distribution of labor since, for entrepreneurs, $y(\alpha, x)$ is restricted to equal $A + g(v(\alpha), x) + \xi(\alpha)$. This feature is also shared with equilibrium allocations of risk since, in equilibrium, $y(\alpha, x) = g(L(w, \alpha), x) - wL(w, \alpha) + A$ if α is an entrepreneur. These considerations permit us to observe that the conditions defining restricted feasibility do indeed embody the institutional constraints on risk trading implicit in our equilibrium concept.

It should also be noted that this definition of restricted feasible allocations does not employ any explicit or implicit assumptions about the independence or dependence of the $\tilde{x}$'s which enter the production functions of different firms.

An allocation Λ which is restricted feasible is said to be restricted efficient if there is no other restricted feasible allocation Λ^* which Pareto dominates Λ.

We can now prove that an equilibrium is efficient in the restricted sense just defined. It should first be emphasized again that for this theorem we can and will drop the assumption that the $\tilde{x}$ is the same for all firms. In fact, it is not necessary for the theorem to make any assumptions about the dependence or independence of the random variables which enter different firm's production functions.[7]

THEOREM 6.—An equilibrium is restricted efficient.

PROOF.—In an equilibrium allocation

$$
\begin{aligned}
\Delta &= [0, \hat{\alpha}) \\
\Gamma &= [\hat{\alpha}, 1] \\
\nu(\alpha) &= L(w(\hat{\alpha}), \alpha) \text{ and} \\
\xi(\alpha) &= \begin{cases} -wL(w(\hat{\alpha}), \alpha) & \text{if } \alpha \epsilon \Gamma \\ w & \text{if } \alpha \epsilon \Delta. \end{cases}
\end{aligned}
\tag{53}
$$

Now consider some other allocation $\Lambda^* = (\Gamma^*, \Delta^*, \nu^*, \xi^*)$ which Pareto dominates the equilibrium. To express this domination formally it is necessary to first partition the set of individuals into four sets:

i) those in $\Delta^* \cap \Delta$,
ii) those in $\Delta^* \cap \Gamma$,
iii) those in $\Gamma^* \cap \Gamma$, and
iv) those in $\Gamma^* \cap \Delta$.

If α is in $\Delta^* \cap \Delta$, then Pareto dominance of Λ^* implies that

$$
\begin{aligned}
Eu(A &+ g(L(w(\hat{\alpha}), \alpha), \tilde{x}) - w(\hat{\alpha})L(w(\hat{\alpha}), \alpha), \alpha) \\
&\leq Eu(A + g(\nu^*(\alpha), \tilde{x}) + \xi^*(\alpha), \alpha).
\end{aligned}
\tag{54}
$$

---

[7] Note that unlike the definition of unrestricted feasibility, the definition of restricted feasibility embodies no implicit assumption about the dependence of the $\tilde{x}$'s faced by different firms.

By definition of $L(w(\hat{\alpha}), \alpha)$ then

$$\xi^*(\alpha) \geqq - w(\hat{\alpha})\nu^*(\alpha). \tag{55}$$

If α is in $\Delta^* \cap \Gamma$, then again Pareto dominance of Λ^* implies

$$Eu(A + g(L(w(\hat{\alpha}), \alpha), \tilde{x}) - wL(w(\hat{\alpha}), \alpha), \alpha)$$
$$\leqq u(A + w(\hat{\alpha}), \alpha) \tag{56}$$
$$\leqq Eu(A + g(\nu^*(\alpha), \tilde{x}) + \xi^*(\alpha), \alpha)$$

and again (55) holds.

If α is in $\Gamma^* \cap \Gamma$, then

$$u(A + w(\hat{\alpha}), \alpha) \leqq u(A + \xi^*(\alpha), \alpha) \tag{57}$$

and

$$w(\hat{\alpha}) \leqq \xi^*(\alpha). \tag{58}$$

Finally, if α is in $\Gamma^* \cap \Delta$, then

$$u(A + w(\hat{\alpha}), \alpha) \leqq Eu(A + g(L(w(\hat{\alpha}), \alpha), \tilde{x}) - w(\alpha)L(w(\hat{\alpha}), \alpha), \alpha)$$
$$\leqq u(\xi^*(\alpha), \alpha) \tag{59}$$

so that (58) holds in this case also.

We have established that for all $\alpha \epsilon \Delta^*$, (55) holds while for all $\alpha \epsilon \Gamma^*$, (58) holds. In fact, a similar argument guarantees that since Λ^* Pareto dominates the equilibrium, there must either be a Δ^* subset of positive measure on which (55) holds with a strict inequality or a Γ^* subset of positive measure on which (58) holds with a strict inequality. Thus

$$\int_0^1 \xi^*(\alpha)d\alpha > \left[- \int_{\Delta^*} \nu^*(\alpha)d\alpha + \mu(\Gamma^*) \right] w(\hat{\alpha}). \tag{60}$$

Inequality (60) implies that Λ^* cannot be restricted feasible. Specifically, because of (60), equations (28) and (52) cannot hold simultaneously. We have therefore shown that the equilibrium is restricted efficient because it cannot be Pareto dominated by a restricted feasible allocation. ‖

Restricted efficiency is similar in spirit to Diamond's (1967) constrained efficiency, that is, efficiency under the given constraints on the risk allocation. Here we have imposed more constraints than in Diamond's model of the stock market since, in our approach, entrepreneurs are not allowed to share any risk with workers or other entrepreneurs. However, we get restricted efficiency without any technological assumption such as those imposed by Diamond's assumption of stochastic constant returns to scale. A similar result is obtained in Kihlstrom and Laffont (1978), where risk sharing is in-

troduced by the existence of markets for shares to firms. The resulting equilibrium is shown to be efficient in the sense of Diamond. The efficiency theorem involves no restrictive assumptions about technology. Specifically, it does not require stochastic constant returns to scale.

VII. Summary

In this paper we have introduced a simple general equilibrium model of firm formation in which production requires entrepreneurial as well as normal labor inputs. Workers receive fixed wages while entrepreneurs receive risky profits. Individuals decide whether to become entrepreneurs or workers by comparing the risky returns of entrepreneurship with the nonrisky wage determined in the competitive labor market. The wage adjusts to the point where the supply of workers is equal to the entrepreneurial demand for labor.

Although we have not discussed the interpretation of the entrepreneurs' contribution to the productive process we have implicitly or explicitly made assumptions about the nature of this contribution. The primary assumption is that the relationship between output and the entrepreneurial labor input is characterized by an indivisibility. Specifically, each firm requires a unit of entrepreneurial labor regardless of how much normal labor it employs and how much it produces. In this sense, the expenditure of entrepreneurial labor can be viewed as a set-up cost. Normally, the nonconvexities introduced by indivisibilities in general and set-up costs in particular cause problems when the existence of equilibrium is studied. In our model, this problem is avoided, as it can be in general (see, e.g., Aumann 1966), by assuming that the set of individuals is a continuum.

One possible interpretation of our model is that the entrepreneur contributes managerial and organizational skills. In Knight's words he performs the "function of exercising responsible control."[8] In fact, our entire model can be viewed as a formalization, for a special case, of Knight's discussion of the entrepreneur.[9] In our model, an entrepreneur is characterized by two activities. He supplies entrepreneurial inputs and bears the risks associated with production. In

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[8] Knight (1921), p. 278.

[9] Knight's view of the entrepreneur as well as the view formalized here are rather different from that set forth by Schumpeter (1934, 1939). Schumpeter viewed the entrepreneur as an innovator. (See, e.g., the discussion on pp. 132–36 of Schumpeter [1934].) His view of the entrepreneur's contribution and of his compensation is in essence dynamic. He also specifically asserts on p. 137 of Schumpeter (1934) that "the entrepreneur is never a risk bearer." For a more modern discussion of the entrepreneur and his role (or lack of one) in economic theory, see Baumol (1968).

Knight's treatment the entrepreneur makes the same contributions "with the performance of his peculiar twofold function of (*a*) exercising responsible control and (*b*) securing the owners of productive services against uncertainty and fluctuation in their incomes."[10] Knight's view of the labor market and of an individual's decision to become a worker or an entrepreneur also appears to be similar to that formalized here. This is illustrated by the discussion on pages 273–74 of Knight (1921). Specifically, he asserts that, "the laborer asks what he thinks the entrepreneur will be able to pay, and in any case will not accept less than he can get from some other entrepreneur, or *by turning entrepreneur himself*. In the same way the entrepreneur offers to any laborer what he thinks he must in order to secure his services, and in any case not more than he thinks the laborer will be worth to him, *keeping in mind what he can get by turning laborer himself*."[11] He continues: "Since in a free market there can be but one price on any commodity, a general wage rate must result from this competitive bidding."[12]

Our model represents only a special case of Knight's view because we assume that all individuals are equal in their ability to perform entrepreneurial as well as normal labor functions. They differ only in their willingness to bear risks. Knight emphasizes ability as well as "willingness [and] power to give satisfactory guarantees"[13] as factors determining the supply of entrepreneurs. In our model, the size of the initial income A can be interpreted as a measure of an individual's power to guarantee wages by bearing risk. We have assumed here that all individuals are alike in their possession of this "power." An interesting alternative interpretation can be made by explaining the differences in risk aversion as arising from differences in wealth. Suppose, for example, that all entrepreneurs have the same utility function which is decreasingly risk averse (in the absolute sense of Arrow-Pratt). Then the differences in the willingness to bear risk— that is, to "give satisfactory guarantees" in Knight's words—will be determined by initial wealth. Thus if A varies across individuals, the more risk averse individuals will also be those who initially are the poorest. Assuming that the constraints $L \leq A/w$ are never binding in equilibrium our model can then be reinterpreted as predicting that entrepreneurs are those who are initially wealthy. (Of course the opposite prediction can be obtained by making the less generally accepted assumption that the common utility function is increasingly risk averse.) With this interpretation, the possession of wealth which

[10] Knight (1921, p. 278).
[11] Ibid., pp. 273–74 (emphasis added).
[12] Ibid., p. 274.
[13] Ibid., p. 283.

provides additional power to give satisfactory guarantees also makes an individual more willing to bear risk.

To complete the analogy between our results and Knight's discussion note that our theorem 4, which relates the equilibrium wage to the level of entrepreneurial risk aversion, was in a sense anticipated by the discussion of Knight (1921, p. 283) which concludes that "entrepreneur income, being residual, is determined by the demand for these other [productive] services, which demand is a matter of self-confidence of entrepreneurs as a class. . . ."[14]

This paper has extended the classical results concerning the existence and stability of equilibrium to the entrepreneurial model. We have also described the nature of the equilibrium's inefficiencies and identified the institutional constraints on risk trading as the source of these inefficiencies. These results establish that it is possible to construct an internally consistent general equilibrium model of entrepreneurially operated firms. In fact, this analysis should only be viewed as a first step in the construction of a general equilibrium entrepreneurial theory of the firm under uncertainty. As presented here it is perhaps best viewed as a description of equilibrium in a world of small businesses or farms.

The next step is to incorporate a stock market into our analysis. Once a stock market is embedded in the model, we can use it to ask interesting questions about the interaction between a modern firm's financial and productive decisions. Furthermore, the introduction of a stock market represents an institutional change that facilitates risk trading and thereby eliminates some of the inefficiencies that occur at an equilibrium. The extension of the model in this direction has been studied in a subsequent paper (Kihlstrom and Laffont 1978).

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[14] Ibid.

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