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« A Cheap-talk Model with Multiple Free-riding Audiences: Reference to Global Environmental Protections »

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A Cheap-talk Model with Multiple Free-riding Audiences: Reference to Global Environmental Protections

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Abstract

This paper presents a cheap-talk one-sender-multiple-receiver model in which audiences free-ride on each other in the context of global environmental protections. The sender observes the magnitude of damage of emission, and sends the same message simultaneously to all audiences, who then play a game to determine individual emission level. The sender may find it impossible to credibly send the truth when externality is large enough because of the incentive to correct free-riding behavior. If a private club is established for sharing information, the sender's information with more countries may not be optimal because the sender is less truthful when the club is larger.

KEYWORDS: Cheap Talk, Externality, Environmental Protections

JEL Classification: D82, H41

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1 Introduction

Environmental protection has recently become a hot issue in the agenda of many governments. Cross-border environmental problems are especially annoying because cross-jurisdiction cooperation is needed. South Korea and Japan are the victims of acid rain caused by the fast-growing and heavily polluting industries in the East of China. The development of the Arctic Ocean may benefit countries nearby but may have huge adverse impact onto the global climate. The atmosphere, the sea level, the global temperature and the biodiversity we enjoy are all subject to different extents of damage or pollution. Authorities are not specialists and have to gather useful information for policy-making. Environmental experts possess expertise and try to explore any opportunities in the political arena to pursue their interests. The asymmetry of information casts doubts on any information transmission between policymakers and information providers.

Bjorn Lomborg had written a controversial book in 1998 (translated and published in English in 2001) that presented a stunning challenge to the environmentalists' claims over many issues such as overpopulation, deforestation, species loss and global warming. He states that environmental groups frequently distorted scientific evidence and uniformly biased errors toward portraying a worse trend. *Greenpeace* had once claimed that "it is expected that half of the Earth's species are likely to disappear within the next seventy-five years."[1] The correct figure is about 0.7% in 50 years. *World Wild Fund for Nature* (WWF) once in 1997 proclaimed that two-thirds of the world's forests have been lost[2]. This estimate contrasted with other sources, which range from 20 to 25%[3]. Many other examples can be found in Lomborg's book.

Lomborg's book generated a lot of debates. Zywicki (2003) suggested that we could explain environmental lobbyists' behavior by private interest rather than public interest motivation. His main explanation is "bad news" help selling environmental protections. Reports of bad news induce more public monetary contribution, which has become the main source of fund for many groups and institutes to survive. This paper is not going to tell whether their claims are correctbut to present a positive analysis of the incentive to bias.

Information transmission from an expert, the sender, to a policymaker, the receiver, for political decision makings is considered as a type of lobbying behaviour. Lobbying is defined as an action taken to influence government's or authority's policy-making. Lobbying could be done through two channels. Lobbyists can exert influence on decision makers through spending money, either for pleasing the political parties and their members, or on campaigns educating the public. A second channel is to provide expertise to decision makers and the public. Experts, who possess private information, could take advantage of this and bias decisions towards their preferred results.

[1]This quote is from Lomborg who took if from Greenpeace's website. The link has been removed because of Lomborg's criticism.

[2]World Wild Fund for Nature. (1997). *Global Annual Forest Report 1997.*

[3]Twenty percent is taken from Lomborg's book who quoted Goudie's estimate from "*The Human Impact on the Nature Environment*" (1993).

With relatively more abundant resources, trade associations usually exert influence through monetary contributions. Other groups, such as environmental groups, lack resources to lobby through monetary contributions but usually exert their influence through sending essential information to policymakers. However, it is not an easy task for a policymaker to verify the truthfulness of any information sent by the experts. It is particularly true for environmental issues due to the complexity involved. Groups may deliberately mislead authorities toward more aggressive decisions in favor of the groups.

I analyse the information transmission between an expert and policymakers through a cheap-talk model. Cheap talk refers to costless information transmission. For usual signaling games, senders could be separated by their cost type. Proper contractual arrangements are then able to differentiate different types of senders. For cheap talk games, this is basically impossible. The efficiency of information transmission is dictated by the sender's willingness to be truthful. Any conflicts of interests between the sender and the audiences will disturb the transmission of information. There is a continuum type of senders. To one extreme, the expert is a joint-welfare maximizer. This type of sender refers to those international institutions, which balance benefits of production and costs of pollution and strive for the wellness of the globe. To another extreme, the expert is a pollution-minimizer. This type of sender refers to some environmental interest groups, which strive for better and better environmental protections without much consideration of the adverse effect of production reduction. In a cheap-talk game with a binary signal concerning the damage of pollution, the ex-ante truthfulness of the message is influenced by the magnitude of the externality. Then we will introduce a social planner to limit the size of a club within which the message is shared. It may not be optimal to share the expert's message with all countries because a larger club reduces the ex-ante probability of having a truthful sender.

In this paper, the conflict of interest between the sender and the audiences comes from the free-riding incentive of the audiences on others' reduction on emission. This incentive induces countries to emit more than the socially optimal level. An information provider has an incentive to bias the information to induce less emission. This motive may even make a joint-welfare maximizer unable to credibly send information to governments, who anticipate a biased information when the free-riding incentive is too large. Meanwhile, all countries have to decide how much pollution to emit by weighting the benefit and the cost of emission, given information available. An expert is ex-ante less trustworthy if pollution is more "public", measured by how much pollution one country suffers per unit of emission by another country. The model could explain why some environmental groups tend to bias scientific evidences toward a dimmer future. The model provides a fresh viewpoint of how we should digest the information released by experts and environmental groups. It suggests to any central authority, who expects the experts to be not totally trustworthy, to limit the share of information so as to improve efficiency of information transmission.

The paper is organized as follows. Next section presents a brief literature review and is followed by the basic model. Section 4 analyzes the optimal limit of information sharing and is followed by a conclusion.

2 Literature Review

The literature of informational lobbying is built upon the seminal paper by Crawford and Sobel (1982) who considered a costless communication environment (cheap talk) in which a sender observes the true state of the world and then sends a signal to a receiver who then determines a variable that determines payoffs of both. The main result is, the signal is more informative if the sender and the receiver are closer in their preferences. If they coincide in preferences, signal is totally informative. Since then a strand of literature has discussed the relationship between a sender and a receiver. Gilligan and Krehbiel (1987) extended the model toward collective decision making in which a committee acts as a sender and a floor acts as a receiver. They concluded that the use of restrictive amendment procedures raises the committee's incentive to specialize to collect relevant information in policies and also enhances the informational role of the committee. Austen-Smith (1993) built on Gilligan and Krehbiel's model and included a lobby group into the picture. Crombez (2002) modified Austen-Smith's model and applied it to the legislative process in the European Union.

Farrell and Gibbons (1989) were the first authors to look at a cheap-talk model with two audiences and presented an analysis on how the presence of another audience regulates the truthfulness of signals. The sender observes the true state of the world and then communicates with two audiences. A main result is, whenever the sender is honest with two audiences in private, she has incentives for honesty in public. In their analysis, an audience's payoff does not depend on the action taken by another audience. In this current paper, we are going to relax this assumption and to allow audiences to interact in a common-pool game.

The present paper is also related to environmental economics. Environmental problems are often analyses as common-pool problems in economics (Ostrom, 1990; Ostrom et al., 1994; and many others). The literature mainly focuses on how economic agents could solve the problem. This paper sticks to the negative presumption that common property is going to be over-exploited by selfish economic agents and instead analyses how an information sender talks to those free-riding audiences.

This model can also be compared to Bramoullé and Treich (2009)'s work on commons problem, who show that uncertainty can lower pollution emissions. In their model, countries are risk-averse and the damage of pollution is uncertain. Countries, facing a rise in risk, will cut back pollution as a form of insurance. My focus is how a possibly biased expert sends information to policymakers. A social planner may prefer some countries to be ignorant because the uncertainty will induce those ignorant governments to reduce emission.

My paper is also related to the literature of international environmental agreements (IEAs), which began with Barrett (1994). His main focus is to find the size of self-enforcing IEAs as no country can be forced to sign an IEA. Diamantoudi and Sartzetakis (2002) reach a more pessimistic conclusion that the number of signatories of a self-enforcing IEA is no greater than four. Barrett (2013) shows that uncertainty easily breaks any coordination between countries. This paper also highlights the importance of uncertainty in forming IEAs. My paper suggest that if uncertainty can be solved beforehand or information transmission becomes more efficient, we would expect more and more stable and effective IEAs to emerge.

3 The Model

3.1 Outline of the Model

The model is divided into two parts. We first establish the analysis of the production decision of audiences and the message decision of the information provider. The state of the world concerning the damage of pollution is binary and is observed perfectly by the sender. Countries do not observe the magnitude of the damage and the type of the sender. Ex-ante the probability of encountering a untruthful sender is higher when the magnitude of the externality rises.

Then, we introduce a central authority that can limit the size of a club, within which the sender's information is shared. The central authority may not want to establish a grand club to include everyone because the sender is ex-ante less trustworthy when the club is larger. Limiting the size of the club helps improve information transmission and hence total welfare.

3.2 The Economic Environment

3.2.1 Audiences

There are $N > 1$ audiences. Receiver $n \in N$, which could be considered as a country or a government, is going to maximize the following utility function, or domestic welfare, by choosing e_n:

$$\begin{aligned} U_n &= e_n - (1+d)\left(c\left(e_n\right) + \alpha v\left(\sum_{n'=1}^{N} e_{n'}\right)\right) \\ & \quad \text{for } n = 1, 2, ..., N \end{aligned}$$

where e_n can be interpreted as emission, pollution level or production. Production incurs environmental costs, which can be separated into two components. $c\left(e_n\right)$ is the local environmental cost solely borne by country n. A country's pollution also adds to the total pollution stock that adversely impacts all countries. Pollution is thus a public "bad". $v\left(\sum_{n'=1}^{N} e_{n'}\right)$ is the global environmental cost function. $\alpha \in [0, \widehat{\alpha}]$ is an exogenous parameter, which measures the magnitude of the adverse impact of global emission on domestic welfare. α can also interpreted as the measure of externality, or spillover effect of domestic pollution. A country will only bear part of the environmental cost induced by her emission. When α increases, the divergence of marginal social cost and marginal private cost enlarges. A more public environmental problem is associated with a higher α. d is a random variable that governs the damage of pollution onto the social welfare.

For simplicity, we take

$$c\left(e_n\right) = \frac{e_n^2}{2} \tag{1}$$

$$v\left(\sum_{n'=1}^{N} e_{n'}\right) = \frac{\left(\sum_{n'=1}^{N} e_{n'}\right)^2}{2} \tag{2}$$

3.2.2 Sender

There is only one sender, or expert, whose type θ is drawn by nature from the cumulative distribution function $F\left(\theta\right)$ over the support $[0,1]$. Throughout the game, the type of the sender is unknown to all audiences The sender perfectly observes, and is known to observe, the state of the world d. A type-θ sender's utility function is as follows:

$$W_\theta = \sum_{n=1}^{N}\left\{\theta\left[e_n - \left(1+d\right)\frac{e_n^2}{2}\right] - \left(1+d\right)\frac{\alpha}{2}\left(\sum_{n'=1}^{N} e_{n'}\right)^2\right\}$$

If $\theta = 1$, the sender is a joint-welfare maximizer, whose utility is perfectly aligned with the utilitarian social welfare. Moving towards another extreme, the sender is biased towards minimizing global emission. θ could also be interpreted as the relative weight given to the local production benefit.

The sender cannot choose the emission level directly but can indirectly influence the countries' choices by sending a message $m \in \{L, H\}$ concerning the damage parameter d to all audiences.

3.2.3 State of the World d

The value of d is drawn by nature at the beginning of the game according to the probability distribution function

$$g\left(d\right) = \begin{cases} \pi & \text{if } d = d_H \\ 1-\pi & \text{if } d = d_L \end{cases}$$

with $1 > \pi > 0$. Although receivers do not observe the state of the world, they hold the same prior about d. Denote the prior expectation of d by $d_0 \equiv \pi d_H + \left(1-\pi\right) d_L$ and β_m the belief of the expectation of θ after receiving a message sent by the sender, $m \in \{L, H\}$. From now on we normalize that $d_L = 0$.

The sender sends a binary message $m \in \{L, H\}$ to all receivers after observing d. All receivers believe that

$$\begin{aligned} E[d \mid m = L] &= \beta_L \quad (3) \\ E[d \mid m = H] &= \beta_H \quad (4) \end{aligned}$$

After receivers receive the message, they play a simultaneous pollution game to determine individual emission level, determining payoffs of N receivers and the sender.

The three-stage game is summarized as follows. At stage 0, nature independently draws the state of the world $d \in \{0, d_H\}$ and the type of the sender $\theta \in [0, 1]$. At stage 1, S observes θ and then sends the same public message m to all N receivers. At stage 2, each receiver chooses e_n through a simultaneous pollution game, which determines payoffs of all.

3.3 Production Stage

We solve the game by backward induction. In the second stage, countries choose e_n to maximize their own utility given the message sent by the sender. Countries, however, do not take into account the adverse impact of production onto others. Thus they maximize their own domestic welfare. Denote the total emission level except n by e_{-n}, the F.O.C is as follows.

$$\begin{aligned} 1 &= (1 + \beta_m) [e_n + \alpha (e_n + e_{-n})] \quad (5) \\ & \forall n \in N \text{and } m = H, L \end{aligned}$$

As countries are identical and they update their belief in the same way, the F.O.C becomes

$$1 = (1 + \beta_m) (1 + \alpha N) \widetilde{e} (m, \beta_m) \text{ for } m = L, H \quad (6)$$

and hence

$$\widetilde{e} (m, \beta_m) = \frac{1}{(1 + \beta_m) (1 + \alpha N)}$$

$\widetilde{e} (m, \beta_m)$ is the Nash-equilibrium emission level of each country for $m = L, H$. One can also easily see that the S.O.C is satisfied.

Some comparative statics are worth mentioning at this stage.

Lemma 1: a) $\frac{\partial \widetilde{e}(m,\beta_m)}{\partial \alpha} = \frac{-N}{(1+\beta_m)(1+\alpha N)^2} < 0$, *b)* $\frac{\partial \widetilde{e}(m,\mu_m)}{\partial N} = \frac{-\alpha}{(1+\beta_m)(1+\alpha N)^2} < 0$, *and c)* $\frac{\partial \widetilde{e}(m,\beta_m)}{\partial \beta_m} = \frac{-1}{(1+\alpha N)(1+\beta_m)^2} < 0$.

A rise in α, which measures how large the impact of global pollution is, reduces pollution emission. As the number of countries increases, pollution emission decreases. It is because each country takes into account the externality generated by others and reduces pollution as a result of a rise in αN. A rise in β_m, the updated expectation of d given m, lowers emission because a higher β_m refers to an expectation of larger damage at each given level of pollution emission.

Imagine that countries could now join force and solve the coordination problem. The co-ordinated individual emission level $e^{JF}(m, \beta_m)$, which is different from the first-best social optimal level without informational constraint, will be characterised by the following F.O.C.:

$$1 = (1 + \beta_m)\left(1 + \alpha N^2\right) e^{JF}(m, \beta_m) \text{ for } m = L, H$$

and hence

$$e^{JF}(m, \beta_m) = \frac{1}{(1 + \beta_m)(1 + \alpha N^2)}$$

One could easily see that the individual Nash equilibrium emission levels is at least as high as the socially optimal level, i.e. $\widetilde{e}(m, \beta_m) \geq e^{JF}(m, \beta_m)$. They are equal if $\alpha = 0$ or $N = 1$. We will come back to the comparison of the Nash equilibrium and the socially optimal level.

3.4 Message Stage

Taking countries' Nash equilibrium choices as given, a type-θ sender chooses between $m = L$ and $m = H$ to maximize W_θ:

$$\max_{m \in \{L,H\}} W_\theta(m \mid d) = N\left[\theta \widetilde{e}(m, \beta_m) - \theta(1 + d) c(\widetilde{e}(m, \beta_m)) - (1 + d) \alpha v(N \widetilde{e}(m, \beta_m))\right]$$

If $d = d_H$, sending $m = H$ and inducing belief of the state of the world β_H is beneficial to the sender if the following incentive constraint if fulfilled:

$$W(m = H \mid d = d_H) \quad \geq \quad W(m = L \mid d = d_H) \tag{7}$$

Similarly for $\theta = \underline{\theta}$, sending $m = L$ is beneficial to S if

$$W(m = L \mid d = d_L) \quad \geq \quad W(m = H \mid d = d_L) \tag{8}$$

One can see that whether these two conditions hold depends on the belief held by countries after receiving m. Here we have to clarify the equilibrium notion used in sorting out the solution.

3.5 Equilibrium

We are going to focus on the Perfect Bayesian Equilibrium (PBE) in which

1. Each country's strategy is optimal given the choices of other countries, sender's strategy and her own belief β_m.

   - $e_n \in \underset{\widehat{e}_n}{\text{argmax}} U_n\left(\widehat{e}_n \mid e_1, e_2, ..., e_{n-1}, e_{n+1}, ..., e_N, m, \beta_m\right)$

2. The sender's strategy is optimal given countries' strategy.

   - $m \in \underset{\widehat{m}}{\text{argmax}} W\left(\widehat{m} \mid e_1, e_2, ..., e_N\right)$

3. The belief β_m is derived from the sender's strategy using Bayes' rule if possible.

Before discussing types of equilibrium and the possibility of multiple equilibria, we first list all possible beliefs the audiences could hold. In a binary-message system, three cases and seven sub-cases are possible.

1. Both messages are informative: a) $\beta_H > d_0 > \beta_L$ or b) $\beta_L > d_0 > \beta_H$
2. Both messages are uninformative: $\beta_H = \beta_L = d_0$
3. Only one message is informative: a) $\beta_H > d_0 = \beta_L$, b) $\beta_H = d_0 > \beta_L$, c) $\beta_L > d_0 = \beta_H$ or d) $\beta_L = d_0 > \beta_H$

Case (3) is inconsistent with Bayes' rule. Take sub-case (3a) as an an example. As $\beta_L = d_0$ implies that the audiences believe that $m = L$ carries no information at all, it must be because receiving $m = H$ also gives no additional information. But $\beta_H > d_0$ implies that the audiences believe that the message $m = H$ must carry some information to the audiences, which is different from the information derived from $m = L$. It is a contradiction to Bayes' rule.

Case (1a) and (1b) are symmetrical. Which one of two arises depends on the interpretation of the messages. For simplicity, we suppose the following:

Assumption 1 (A1): $\beta_H \geq d_0 \geq \beta_L$

It means whenever the audiences receive $m = H$ or $m = L$ respectively, they know that the sender is trying to signal that $d = d_H\,(d_L)$. Hence a message $m = H\,(L)$ can never induce a belief $\beta_H < d_0\,(\beta_L > d_0)$. With (A1), we can ignore sub-case (1b). Two possibilities are left: (1a) and (2). Simply call them separating belief and pooling belief respectively.

Lemma 2: Only possible beliefs at equilibrium are $\beta_H > d_0 > \beta_L$ and $\beta_H = \beta_L = d_0$.

Two kinds of equilibria are possible. A separating equilibrium is a PBE in which the sender sends different messages in different states. When the sender decides to separate messages in two different states, the message must allow the audiences to extract some information from it. Thus $\beta_H > d_0 > \beta_L$.

Another type of equilibrium is a pooling equilibrium. It is also a PBE in which the sender cannot credibly transmit any information in both states. The message is believed to be uninformative. Audiences take their decision based on the prior expectation d_0. A pooling equilibrium arises only when it is believed that the sender cannot credibly separate the two states. For instance, if the sender can generate a higher payoff by sending $m = H$ when $d = 0$ while the audiences are holding separating beliefs, the sender's message $m = H$ is not credible. Only the pooling equilibrium is possible.

The equilibrium is unique because a single belief cannot induce both types of equilibria. The only question left is to determine the equilibrium belief under different sets of parameters. Notice that the sender may prefer the pooling uninformative outcome in one state but the separating informative outcome in another. By Lemma 2, however, It is impossible and hence a separating equilibrium will arise if and only if the sender does not want to deviate in both states while the audiences are holding the separating belief.

3.6 Choice of Message m

First we are going to show that $\beta_L = d_L = 0$. That is to say, whenever audiences receive $m = L$, they are sure that the state of the world is d_L. It happens only when the sender has no incentive to bias when $d = d_H$ and all audiences anticipate the sender's behavior.

> *Lemma 3: When $d = d_H$, the sender must send $m = H$.*
>
> Proof: First consider $\theta = 1$. We know $e^{JF}(H, d_H) < \widetilde{e}(H, d_H)$ when $\alpha \neq 0$ and $N > 1$. By A1, we know $\widetilde{e}(H, \beta_H) < \widetilde{e}(L, \beta_L)$. Thus $e^{JF}(H, d_H) < \widetilde{e}(H, d_H) \leq \widetilde{e}(H, \beta_H) < \widetilde{e}(L, \beta_L)$. As the total welfare function is strictly concave with the maximum at $e^{JF}(H, d_H)$, the sender has no incentive to send $m = L$ to induce a higher emission level. This argument carries forward to any type $\theta < 1$ because the θ-optimal emission level must be lower than $e^{JF}(H, d_H)$. Q.E.D.

Whenever $d = d_H$, the sender must send $m = H$ so as to persuade the audiences to take a more aggressive step in reducing emission. Due to the divergence of the social cost and private cost, the total emission level of the Nash-equilibrium outcome must always be higher than the socially optimal level. The sender would only try to bias the state upward, but not downward.

Lemma 3 implies that audiences expect that the true state of the world must be $d = 0$ upon receiving $m = L$. The emission level and hence the payoff to the sender associated with the message $m = L$

are unchanged even the sender decides to randomize strategies when $d=0$. In other words, the sender can always induce the payoff level associated with $m=L$. Playing a mixed-strategy does not generate a higher payoff.

However, whether (8) holds is ambiguous. We are now looking for a threshold type of sender, denoted by $\overline{\theta}$, who are indifferent between sending $m=L$ and $m=H$, when $d=0$. A type-θ sender is going to tell the truth if

$$\theta \geq \frac{\alpha\left[v\left(N\widetilde{e}\left(L,0\right)\right)-v\left(N\widetilde{e}\left(H,\beta_H\right)\right)\right]}{\widetilde{e}\left(L,0\right)-c\left(\widetilde{e}\left(L,0\right)\right)-\widetilde{e}\left(H,\beta_H\right)+c\left(\widetilde{e}\left(H,\beta_H\right)\right)} \equiv \overline{\theta}$$

β_H is indeed a function of $\overline{\theta}$ and $F\left(\overline{\theta}\right)$ is the expected probability of encountering an untruthful sender when $d=0$. By Bayes' rule, we obtain

$$\beta_H = \frac{\pi d_H}{\pi+(1-\pi)F\left(\overline{\theta}\right)}$$

The equilibrium value of $\overline{\theta}$ is obtained by solving this fixed-point problem. With our quadratic cost function restrictions, we obtain

$$\overline{\theta} = \min\left\{\frac{\alpha N^2\left[\widetilde{e}\left(L,0\right)+\widetilde{e}\left(H,\beta_H\right)\right]}{2-\left[\widetilde{e}\left(L,0\right)+\widetilde{e}\left(H,\beta_H\right)\right]},1\right\}$$

Call the R.H.S of the above expression ϕ and replace $\overline{\theta}$ by θ'. Differentiating $\phi\left(\theta'\right)$ with respect to θ' , we obtain

$$\frac{\partial\phi\left(\theta'\right)}{\partial\theta'} = \begin{cases} \frac{2\alpha N^2}{\left[2-\left(\widetilde{e}(L,0)+\widetilde{e}(H,\beta_H)\right)\right]^2}\frac{\partial\widetilde{e}(H,\beta_H)}{\partial\beta_H}\frac{\partial\beta_H}{\partial\theta'}>0 & \text{if } \phi<1 \\ 0 & \text{if } \phi=1 \end{cases}$$

because of Lemma 1c. We can then draw the mapping $\phi:[0,1]\rightarrow[0,1]$ in a $\phi-\theta'$ space. ϕ is increasing in θ' and its intersection with the 45-degree line is the solution $\overline{\theta}$ of this fixed-point problem. An intersection must exist over $\theta'\in[0,1]$ but an intersection over $\theta'\in[0,1)$ may only exist if α is small enough, i.e. the adverse impact of global pollution is small.

Lemma 4: There exists $\overline{\theta}\in(0,1)$ if $\alpha N\neq 0$ and $\alpha<\frac{\pi d_H}{N[(2+d_0)N-2(1+d_0)]}$.

Proof: Because of $\frac{\partial\phi(\theta')}{\partial\theta'}>0$, for an interior solution to exist, we only require $\phi(0)>0$ and $\phi(1)<1$. We know $\frac{\alpha N^2[\widetilde{e}(L,0)+\widetilde{e}(H,d_H)]}{2-(\widetilde{e}(L,0)+\widetilde{e}(H,d_H))}>0$ if $\alpha\neq 0$. To have $\phi(1)<1$, we require

$$\frac{\alpha N^2\left[\widetilde{e}\left(L,0\right)+\widetilde{e}\left(H,d_0\right)\right]}{2-\left[\widetilde{e}\left(L,0\right)+\widetilde{e}\left(H,d_0\right)\right]} < 1$$

or

$$\alpha < \frac{\pi d_H}{N\left[(2+d_0)\,N - 2\,(1+d_0)\right]}$$

Q.E.D.

Suppose $\overline{\theta} \in (0,1)$. We obtain the following proposition.

Proposition 1: An increase in the magnitude of the adverse impact of global emission, i.e. a rise in α, raises the threshold value of θ, denoted by $\overline{\theta}$. A sender of type $\theta > \overline{\theta}$ will be truthful in both states. Otherwise, the sender lies when $d = 0$.

Proof: Going through some algebra, we obtain

$$\frac{\partial \phi}{\partial \alpha} = \frac{N^2\left(\widetilde{e}\,(L,0)^2 - \widetilde{e}\,(H,\beta_H)^2\right)}{\left[2-\left(\widetilde{e}\,(L,0)+\widetilde{e}\,(H,\beta_H)\right)\right]^2} = \frac{\frac{N^2}{(1+\alpha N)^2}\frac{\beta_H(2+\beta_H)}{(1+\beta_H)^2}}{\left[2-\left(\widetilde{e}\,(L,0)+\widetilde{e}\,(H,\beta_H)\right)\right]^2} > 0$$

Thus, the interception with the 45 degree line must be above the original point. Hence, $\frac{\partial \overline{\theta}}{\partial \alpha} > 0$. Q.E.D.

Given that the damage is low, i.e. $d = 0$, when α is large enough, the sender finds deviation from truthtelling beneficial. Receivers anticipate this incentive and will not believe the sender's message. A separating equilibrium now becomes impossible. That is, when one's emission has a larger impact on the environment, the sender is tempted to bias information upward to correct both the externality effect and the free-riding incentive. The sender cannot credibly send any information because countries expect the $m = H$ in both states. Efficient information transmission completely breaks down.

When $\overline{\theta} = 1$, all types of sender, including the type $\theta = 1$, are tempted to bias information upward when $d = 0$. Even a joint-welfare maximizer is untruthful.

Corollary 1: A joint-welfare maximizer, i.e. $\theta = 1$, sends $m = H$ if

$$\alpha > \frac{2-\left[\widetilde{e}\,(L,0)+\widetilde{e}\,(H,\beta_H)\right]}{N^2\left[\widetilde{e}\,(L,0)+\widetilde{e}\,(H,\beta_H)\right]}$$

because she is not able to commit not to lie when $d = 0$.

Information transmission is completely inefficient when $\overline{\theta} = 1$. Countries will not believe the message received and hence stick to the prior expectation of d to determine the emission level. Back to the comparison with the co-ordinated outcome, it is clear that a joint-welfare maximizing sender will have no incentive to lie to receivers in both states if they solve the coordination problem.

Lemma 5: If countries can join force and coordinate among themselves to commit to the co-ordinated individual emission level, a joint-welfare maximizing sender, i.e. $\theta = 1$, will be truthful in both states. Thus $e^{JF}(m,d) = e^{FB}(d)$.

This model identifies one important reason behind the incentive to bias information. If countries can cooperate and commit to the individual socially optimal emission level, a joint-welfare maximizing information provider will be truthful in both states. Failure to solve coordination problem is actually one of the main causes of the low efficiency of information transmission concerning environmental risks and protections. A joint-welfare maximizing sender cannot credibly send any information to audiences even though observing the true state of the world. It implies that to improve the efficiency of information transmission some institutional or contractual arrangements are necessary. The problem could be reduced to two questions. First, the social planner should explore any possible arrangements to tackle free-riding problem. Second, the social planner should try to screen out biased information providers. Indeed it is not easy to screen away bad senders. It requires an ability to verify the truthfulness of the message afterward and a repeated-game framework or a truthfulness-based compensation scheme.

Lomborg (1998) has discussed some reasons behind exaggeration of environmental problems. To my best knowledge, I have not read any theoretical work relating the incentive to bias information toward an alarming situation to free-riding strategic concern. My model provides a positive analysis behind the incentive to send an alarming signal to policymakers in the context of externality problem.

4 Optimal Limit of Information Sharing

In the previous section, the sender sends a public message to all countries. My next question is, it is possible to establish a “club” to limit information sharing that helps improve information transmission? Consider there exists a joint-welfare maximizing central authority, whose only decision is to determine the size of a club of countries among which the sender's information is shared. We assume that excluded countries have no access to the sender's information. Club members, however, could not commit to any emission level. A co-ordination problem still exists. We are looking for a second-best optimal size of the club, where the social welfare is maximized given the asymmetry of information and the lack of commitment device, not to be confused with the first-best outcome in which information is perfect and the co-ordination problem is resolved.

The game is modified as follows. At stage 0, nature draws $d \in \{0, d_H\}$ and the type of the sender $\theta \in [0,1]$ according to $F(\theta)$. At stage 1, the central authority chooses a proportion $\gamma \in [0,1]$ of countries who are included in the club. Here we ignore integer problem. At stage 2, the sender observes d and sends $m = L$ or $m = H$ to the club exclusively. At stage 3, all countries play a pollution game to determine the individual and global emission levels. The game ends. For the sake of simplicity, we consider γ as a continuous variable.

4.1 Production Stage

Given γ, the two groups of countries, namely the club group C and fringe group $N \setminus C$, play a simultaneous pollution game. Denote the emission level by informed countries after receiving message m by $e_c(m, \beta_m; \gamma)$ and the emission level by the fringe countries $e_f(\emptyset, \beta_\emptyset; \gamma)$. The club group's emission decision is governed by the following F.O.C:

$$1 = (1 + \beta_m)\left[e_c(m, \beta_m; \gamma) + \alpha N\left(\gamma e_c(m, \beta_m; \gamma) + (1-\gamma)\, e_f(\emptyset, \beta_\emptyset; \gamma)\right)\right] \text{ for } m = L, H \tag{9}$$

For the fringe countries, they do not receive any information. Their F.O.C is as follows:

$$\begin{aligned} 1 &= \pi(1 + d_H)\left[e_f(\emptyset, \beta_\emptyset; \gamma) + \alpha N\left(\gamma e_c(H, \beta_H; \gamma) + (1-\gamma)\, e_f(\emptyset, \beta_\emptyset; \gamma)\right)\right] \\ &\quad + (1-\pi)\left[e_f(\emptyset, \beta_\emptyset; \gamma) + \alpha N\left(\gamma e_c(L, \beta_L; \gamma) + (1-\gamma)\, e_f(\emptyset, \beta_\emptyset; \gamma)\right)\right] \end{aligned} \tag{10}$$

The Nash equilibrium is a triple $\{\widetilde{e}_f(\emptyset, \beta_\emptyset; \gamma), \widetilde{e}_c(H, \beta_H; \gamma), \widetilde{e}_c(L, \beta_L; \gamma)\}$, which is obtained by solving this three-equation three-unknown system. We are going to show that the fringe countries will emit exactly the same as the Nash equilibrium emission level without any additional information, i.e. the level associated with the prior expectation of θ.

Lemma 6: The Nash equilibrium emission level of fringe countries $\widetilde{e}_f(\emptyset, d_0; \gamma)$ is invariant to the size of the club γ and is exactly the same as the Nash emission level as if no sender ever exists.

Proof: In the Appendix.

Lemma 7: a) $\widetilde{e}_c(L, \beta_L; \gamma) > \widetilde{e}_f > \widetilde{e}_c(H, \beta_H; \gamma)$ for $\forall\gamma \in (0,1)$; b) $\frac{\partial \widetilde{e}_H}{\partial \gamma} > 0$ and $\frac{\partial \widetilde{e}_L}{\partial \gamma} < 0$ for $\forall\gamma \in (0,1)$.

Proof: Lemma 7a is immediate by A1. And because of $\beta_H \geq d_0$,

$$\begin{aligned} \frac{\partial \widetilde{e}_c(H, \beta_H; \gamma)}{\partial \gamma} &= \frac{\alpha N}{(1 + \alpha\gamma N)^2}\left[\frac{1}{1 + d_0} - \frac{1}{1 + \beta_H}\right] > 0 \\ \frac{\partial \widetilde{e}_c(L, \beta_L; \gamma)}{\partial \gamma} &= \frac{\alpha N}{(1 + \alpha\gamma N)^2}\left[\frac{1}{1 + d_0} - 1\right] < 0 \end{aligned}$$

Q.E.D.

4.2 Message Stage

Similarly we can show that the sender will not bias information downward regardless of γ.

Lemma 8: All types of sender will not bias information downward, i.e. sending $m = L$ when $d = d_H$, for all values of γ.

Proof: In the Appendix.

Thus $\beta_L = 0$. A type-θ sender is going to tell the truth when $d = 0$ if

$$\theta \geq \frac{\alpha \left[v\left(\gamma N \widetilde{e}\left(L,0;\gamma\right) + \left(1-\gamma\right) N \widetilde{e}_f\right) - v\left(\gamma N \widetilde{e}\left(H,\beta_H;\gamma\right) + \left(1-\gamma\right) N \widetilde{e}_f\right)\right]}{\gamma \left[\widetilde{e}\left(L,0;\gamma\right) - \widetilde{e}\left(H,\beta_H;\gamma\right) - c\left(\widetilde{e}\left(L,0;\gamma\right)\right) + c\left(\widetilde{e}\left(H,\beta_H;\gamma\right)\right)\right]}$$

With the explicit functions we assume, we obtain the threshold value of θ as follows:

$$\overline{\theta}_\gamma = \min\left\{\frac{\alpha N^2 \left[\gamma\left(\widetilde{e}\left(L,0;\gamma\right) + \widetilde{e}\left(H,\beta_H;\gamma\right)\right) + 2\left(1-\gamma\right)\widetilde{e}_f\right]}{2 - \left(\widetilde{e}\left(L,0;\gamma\right) + \widetilde{e}\left(H,\beta_H;\gamma\right)\right)}, 1\right\}$$

Again this is a fixed-point problem as β_H is a function of $\overline{\theta}$. Name the R.H.S of the above expression ϕ_γ and replace $\overline{\theta}$ by θ'. We are going to repeat the steps to prove that a fixed-point exists.

Lemma 9: There exists $\overline{\theta}_\gamma \in (0,1)$ if $\alpha N \neq 0$ and α is small enough.

Proof: In the Appendix.

We are interested in analyzing the impact of a change of γ on $\overline{\theta}_\gamma$.

Proposition 2: $\overline{\theta}_\gamma$ is increasing in γ if $d_0 = \pi d_H > \frac{\beta_H}{2+\beta_H}$. Ex-ante the sender is more likely to lie if the club is larger.

Proof:

$$\frac{\partial \phi_\gamma}{\partial \gamma} = \frac{-\alpha N^2}{\left(2 - \widetilde{e}\left(L,0;\gamma\right) - \widetilde{e}\left(H,d_0;\gamma\right)\right)^2 \left(1+\alpha\gamma N\right)^2} \left(\frac{2}{1+d_0} - \frac{1}{1+\beta_H} - 1\right)\left(1 + \frac{1}{1+\beta_H}\right)$$

For $\frac{\partial \phi_\gamma}{\partial \gamma} > 0$, we require $d_0 > \frac{\beta_H}{2+\beta_H}$ so that $\left(\frac{2}{1+d_0} - \frac{1}{1+\beta_H} - 1\right) < 0$. As a result, the new fixed-point must lie above the original equilibrium. Q.E.D.

Throughout the paper, we assume the following:

Assumption 2 (A2): $d_0 = \pi d_H > \frac{\beta_H}{2+\beta_H}$.

As β_H is bounded below by the prior belief, β_H must be at least as high as d_0. If β_H is close enough to d_0, this assumption must hold because $d_0 > \frac{d_0}{2+d_0}$. If β_H takes its maximum value, i.e. d_H, this assumption holds if $\pi > \frac{1}{3}$. That is, if the probability of having a high state of damage is high enough, A2 must hold and $\overline{\theta}_\gamma$ is increasing in γ. Thus, the expected probability of having an untruthful sender when the state is low, i.e. $F\left(\overline{\theta}_\gamma\right)$, is increasing in the size of the club γ. Proposition 2 is central to this paper. Ex-ante the sender is more likely to lie if the size of the informed group is larger.

4.3 Optimal Size of the Club $\gamma^\star$

After fixing the emission decision of two types of countries and the message decision of the sender, we move one step backward to analyze the decision of club size by the overseeing international authority.

Denote by $U^c(m, \beta_m, \gamma; d)$ and $U^f(m, \beta_m, \gamma; d)$ the utility of the club countries and fringe countries respectively. The international authority maximizes the expected utilitarian welfare function, i.e. $W = \sum_{n=1}^{N} U_n$, by choosing γ.

$$
\begin{aligned}
\max_{\gamma} E[W] &= \pi N \left[\gamma U^c(H, \beta_H, \gamma; d_H) + (1-\gamma) U^f(H, \beta_H, \gamma; d_H)\right] \\
&+ (1-\pi)\left(1 - F\left(\overline{\theta}_\gamma\right)\right) N \left[\gamma U^c(L, 0, \gamma; 0) + (1-\gamma) U^f(L, 0, \gamma; 0)\right] \\
&+ (1-\pi) F\left(\overline{\theta}_\gamma\right) N \left[\gamma U^c(H, \beta_H, \gamma; 0) + (1-\gamma) U^f(H, \beta_H, \gamma; 0)\right]
\end{aligned}
$$

An interior solution $\gamma^\star$ must fulfill the following F.O.C.

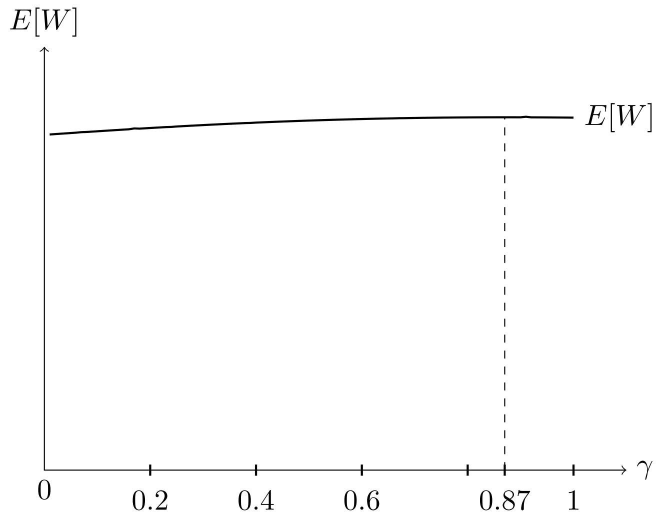
$$
\begin{aligned}
&\pi \left[U^c(H, \beta_H, \gamma^\star; d_H) - U^f(H, \beta_H, \gamma^\star; d_H) + \gamma^\star \frac{\partial U^c(H, \beta_H, \gamma^\star; d_H)}{\partial \gamma} + (1-\gamma^\star)\frac{\partial U^f(H, \beta_H, \gamma^\star; d_H)}{\partial \gamma}\right] \\
&+ (1-\pi)\left[U^c(L, 0, \gamma^\star; 0) - U^f(L, 0, \gamma^\star; 0) + \gamma^\star \frac{\partial U^c(L, 0, \gamma^\star; 0)}{\partial \gamma} + (1-\gamma^\star)\frac{\partial U^f(L, 0, \gamma^\star; 0)}{\partial \gamma}\right] \\
&- (1-\pi) F\left(\overline{\theta}_\gamma\right)\left[U^c(L, 0, \gamma^\star; 0) - U^f(L, 0, \gamma^\star; 0) + \gamma^\star \frac{\partial U^c(L, 0, \gamma^\star; 0)}{\partial \gamma} + (1-\gamma^\star)\frac{\partial U^f(L, 0, \gamma^\star; 0)}{\partial \gamma}\right] \\
&+ (1-\pi) F\left(\overline{\theta}_\gamma\right)\left[U^c(H, \beta_H, \gamma^\star; 0) - U^f(H, \beta_H, \gamma^\star; 0) + \gamma^\star \frac{\partial U^c(H, \beta_H, \gamma^\star; 0)}{\partial \gamma} + (1-\gamma^\star)\frac{\partial U^f(H, \beta_H, \gamma^\star; 0)}{\partial \gamma}\right] \\
&+ (1-\pi) f\left(\overline{\theta}_\gamma\right)\frac{\partial \overline{\theta}_\gamma}{\partial \gamma}\left\{\gamma^\star\left[U^c(H, \beta_H, \gamma^\star; 0) - U^c(L, 0, \gamma^\star; 0)\right] + (1-\gamma^\star)\left[U^f(H, \beta_H, \gamma^\star; 0) - U^f(L, 0, \gamma^\star; 0)\right]\right\} \\
&= 0
\end{aligned}
$$

The last term can be rearranged as

$$
(1-\pi) f\left(\overline{\theta}_\gamma\right)\frac{\partial \overline{\theta}_\gamma}{\partial \gamma}\left\{\gamma^\star\left(1-\overline{\theta}_\gamma\right)\left[\widetilde{e}_c(H, \beta_H; \gamma^\star) - \frac{\widetilde{e}_c(H, \beta_H; \gamma^\star)^2}{2} - \widetilde{e}_c(L, \beta_L; \gamma^\star) + \frac{\widetilde{e}_c(L, \beta_L; \gamma^\star)^2}{2}\right]\right\} < 0
$$

If $\overline{\theta}_\gamma$ is exogenously determined, i.e. $\frac{\partial \overline{\theta}_\gamma}{\partial \gamma} = 0$, equilibrium is pinned down by equalizing the first fourth terms to zero. An additional negative term in the equilibrium condition must pull down the optimal size of the club $\gamma^\star$. As the expected probability of having an untruthful sender is increasing in the club size, the international authority prefers to reduce the club size to improve

Figure 1: Expected Welfare against γ



the efficiency of information transmission. It is not easy to examine comparative statics from the F.O.C analytically. We turn to numerical analysis in the following section.

4.4 Numerical Analysis

For effective comparison, we set our benchmark case as follows: $\alpha = 0.0005$, $N = 10$, $\pi = 0.5$, and $d_H = 1$. The second-best optimal size $\gamma^\star \approx 0.87$ and the expected social welfare level $E[W] \approx 3.3342$, as shown in Figure 1, where the horizontal axis refers to the club size and the vertical axis the social welfare.

We first analyse the impact of a change of α, the magnitude of the adverse impact of the global pollution, on the optimal size of the club. Illustrated in Figure 2, where the vertical axis represents the club size, an increase in α reduces the optimal size of the club $\gamma^\star$. When domestic pollution exerts larger adverse impact on the global environment, the international authority should shrink the club size to improve information transmission and hence welfare. This result sheds light onto finding a possible solution to improve information transmission and facilitate cross-border co-operation. Allowing the experts or informational providers to have access to all countries is not necessarily beneficial. Limiting the size of informed group can help lessen free-riding behavior and hence raise the incentive of information providers to present a more trustworthy assessment of the problems. Due to lack of information, the fringe countries will act according to the prior expectation of the damage parameter. Two effects are in play. First, no matter what the state of the world and the message are, the emission level of fringe countries is in fact fixed. No strategic interactions exist between the informed countries and the fringe countries. Free-riding behaviors are then less severe. Second, uninformed countries produce less with the prior expectation, hence, the sender would find it less attractive to see production shrink further.

Figure 2: Impact of Change of α on $\gamma^\star$

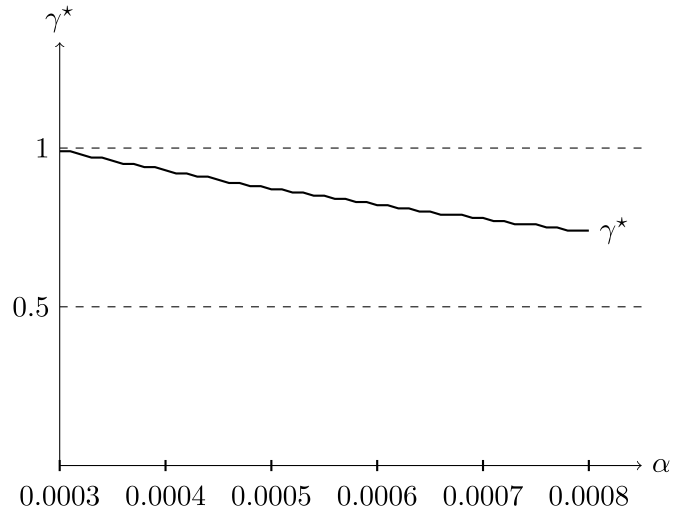
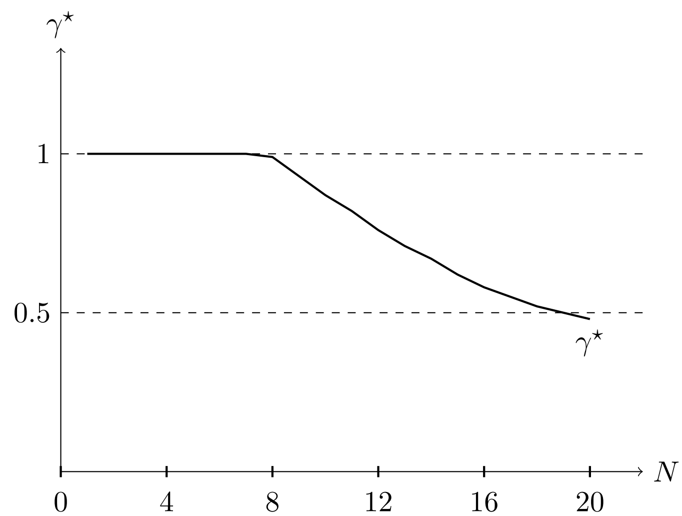


Figure 3: Impact of Change of N on $\gamma^\star$



An increase in the total number of countries N induces an effect siilar to the of a rise in α as shown in Figure 3 because the intensity of free-riding behavior is increasing in N.

An increase in π, the probability of the high state of damage, enlarges the optimal size of the club as shown in Figure 4. Holding π unchanged and increasing d_H generates similar result as shown in Figure 5. An increase in π or d_H reduces $\overline{\theta}_\gamma$, i.e. the sender is ex-ante more trustworthy, and hence the central authority tends to establish a larger club as the benefit of keeping some audiences ignorant diminishes.

Figure 4: Impact of Change of π on $\gamma^\star$

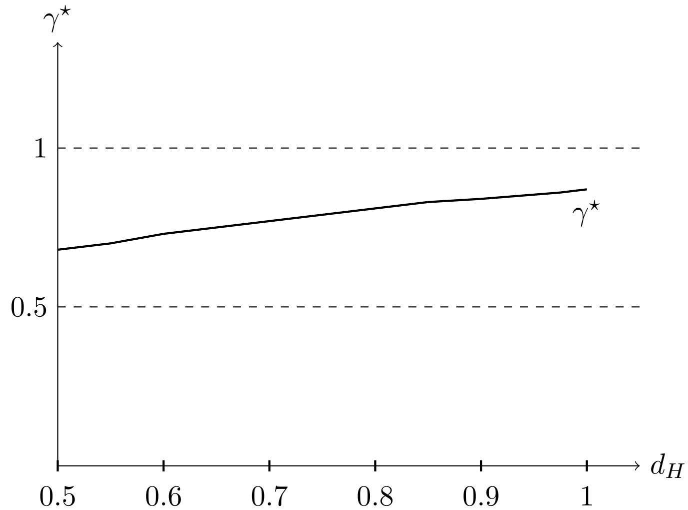
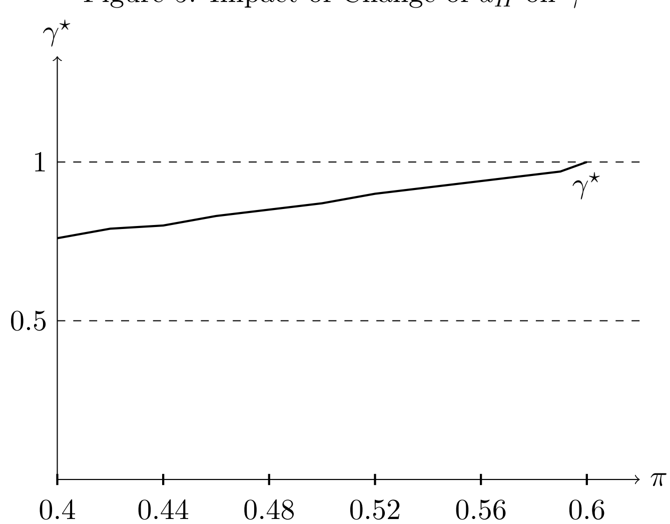


Figure 5: Impact of Change of d_H on $\gamma^\star$



5 Conclusion

This paper presents a cheap-talk model with a single sender and multiple free-riding audiences framed in the context of environmental protections. The main conclusion is that if the pollution induces a larger externality the sender is less likely to be truthful. The sender is tempted to bias information so as to correct the free-riding incentive. Even if the sender is a welfare-maximizer, the sender may find it impossible to transmit credible information. All countries react as if there were no information provider. Then we extend the basic model to include a welfare-maximizing social planner who can set up a club to limit the extent of information sharing. We find that sharing the sender's information to all countries may not be beneficial especially when the number of countries involved in the pollution and the magnitude of externality are high. This result suggests that by setting up a club within which information is shared could be an effective solution to first enhance the truthfulness of the information and second to induce more devoted environmental protections.

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6 Appendix

6.1 Proof of Lemma 6

Condition (10) is rearranged as follows.

$$1 = (1 + \pi d_H)[1 + \alpha(1-\gamma)] e_f(\varnothing, \beta_\varnothing; \gamma) + \alpha\gamma N [\pi(1+d_H) e_c(H, \beta_H; \gamma) + (1-\pi) e_c(L, \beta_L; \gamma)] \tag{11}$$

By substituting (9) into (10), we obtain

$$\begin{aligned}
1 &= \pi(1+d_H)\left[e_f(\varnothing, \beta_\varnothing; \gamma) + \frac{1}{1+d_H} - e_c(H, \beta_H; \gamma)\right] \\
&\quad + (1-\pi)[e_f(\varnothing, \beta_\varnothing; \gamma) + 1 - e_c(L, \beta_L; \gamma)] \\
1 - (1+d_0) e_f(\varnothing, \beta_\varnothing; \gamma) &= \pi - \pi(1+d_H) e_c(H, \beta_H; \gamma) + (1-\pi) - (1-\pi) e_c(L, \beta_L; \gamma) \\
e_f(\varnothing, \beta_\varnothing; \gamma) &= \frac{\pi(1+d_H)}{1+d_0} e_c(H, \beta_H; \gamma) + \frac{1-\pi}{1+d_0} e_c(L, \beta_L; \gamma)
\end{aligned}$$

Putting back to (11), we obtain

$$\begin{aligned}
1 &= (1+d_0)[1+\alpha(1-\gamma)]\widetilde{e}_f(\varnothing, \beta_\varnothing; \gamma) + (1+d_0)\alpha\gamma\widetilde{e}_f(\varnothing, \beta_\varnothing; \gamma) \\
1 &= (1+d_0)[1+\alpha N\widetilde{e}_f(\varnothing, \beta_\varnothing; \gamma)]
\end{aligned}$$

and

$$\widetilde{e}_f(\varnothing, \beta_\varnothing; \gamma) = \widetilde{e}_f(\varnothing, d_0)$$

That is to say, a fringe country will stick to the prior expectation of d, d_0, and $e_f(\varnothing, d_0; \gamma)$ is invariant to γ. This result indeed depends very much on the restrictions $c''' = 0$ and $v''' = 0$. Q.E.D.

Therefore we can from now on denote $e_f(\varnothing, d_0; \gamma)$ by $\widetilde{e}_f$. Solving the system, we get

$$\begin{aligned}
\widetilde{e}_f &= \frac{1}{(1+d_0)(1+\alpha N)} \\
\widetilde{e}_c(H, \beta_H; \gamma) &= \frac{1}{(1+\beta_H)(1+\alpha\gamma N)} - \frac{\alpha(1-\gamma)N}{(1+d_0)(1+\alpha\gamma N)(1+\alpha N)} \\
\widetilde{e}_c(L, \beta_L; \gamma) &= \frac{1}{(1+\beta_L)(1+\alpha\gamma N)} - \frac{\alpha(1-\gamma)N}{(1+d_0)(1+\alpha\gamma N)(1+\alpha N)}
\end{aligned}$$

6.2 Proof of Lemma 8

We know $\widetilde{e}_c\left(L,\beta_L;\gamma\right) > \widetilde{e}_f > \widetilde{e}_c\left(H,\beta_H;\gamma\right)$ for $\forall\gamma \in (0,1)$. Thus deviating from $m = H$ to $m = L$ must increase emission level. Again first we focus on the type $\theta = 1$. Suppose countries in the club join force and co-operate to determine individual emission level. In other words, we are searching for the individual emission level that maximizes the social welfare function, given that $m = H$ and a proportion of $1-\gamma$ of countries is left uninformed. Denote this level by $e_c^{JF}\left(H,\beta_H;\gamma\right)$, we obtain

$$e_c^{JF}\left(H,\beta_H;\gamma\right) = \frac{1}{\left(1+\beta_H\right)\left(1+\alpha\gamma N^2\right)} - \frac{\alpha\left(1-\gamma\right)N^2}{\left(1+\alpha\gamma N^2\right)}\widetilde{e}_f$$

Then we have $e_c^{JF}\left(H,\beta_H;\gamma\right) > \widetilde{e}_c\left(L,\beta_L;\gamma\right) > \widetilde{e}_c\left(H,\beta_H;\gamma\right)$. Therefore a joint-welfare maximizer will never send $m = L$ when $d = d_H$. For $\theta < 1$, the sender prefers an even lower emission level than $e_c^{JF}\left(H,\beta_H;\gamma\right)$. Q.E.D.

6.3 Proof of Lemma 9

Because of $\frac{\partial\phi_\gamma(\theta')}{\partial\theta'} > 0$, for an interior solution to exist, we only require $\phi_\gamma\left(0\right) > 0$ and $\phi_\gamma\left(1\right) < 1$. We know $\phi_\gamma\left(0\right) > 0$ if $\alpha \neq 0$. For $\phi_\gamma\left(1\right) < 1$, we require

$$\alpha \quad < \quad \frac{2-\left(\widetilde{e}\left(L,0;\gamma\right)+\widetilde{e}\left(H,d_0;\gamma\right)\right)}{\gamma N^2\left(\widetilde{e}\left(L,0;\gamma\right)+\widetilde{e}\left(H,d_0;\gamma\right)\right)+2\left(1-\gamma\right)N^2\widetilde{e}_f}$$

or

$$\begin{aligned}\alpha \quad < \quad & \frac{\sqrt{(d_0^2\gamma^2+4d_0\gamma+4)N^2-(4d_0^2\gamma^2-(2d_0^2-4d_0)\gamma+4d_0+8)N+4d_0^2\gamma^2-4d_0^2\gamma+d_0^2+4d_0+4}}{\left(2d_0+4\right)\gamma N^2-4(1+d_0)\gamma N} \\ & +\frac{-(d_0\gamma+2)N+2d_0\gamma+d_0+2}{\left(2d_0+4\right)\gamma N^2-4(1+d_0)\gamma N}\end{aligned}$$

Q.E.D.