## Forthcoming

# Journal of Economic Theory 

# The value of switching costs * 

Gary Biglaiser<br>University of North Carolina, Chapel Hill<br>Jacques Crémer<br>Toulouse School of Economics<br>(GREMAQ, CNRS and IDEI)<br>Gergely Dobos<br>Gazdasági Versenyhivatal (GVH)

October 30, 2012

[^0]
#### Abstract

We study a dynamic model with an incumbent monopolist and entry in every subsequent period. We first show that if all consumers have the same switching cost, then the intertemporal profits of the incumbent are the same as if there was only one period. We then study the consequences of heterogeneity of switching costs. We prove that even low switching cost customers have value for the incumbent: when there are more of them its profits increase as their presence hinders entrants who find it more costly to attract high switching cost customers.


Key words: switching cost, dynamic competition, incumbency, entry.

In many industries, the market power of incumbents is protected by the switching costs that consumers have to incur when they purchase from an entrant. This paper focuses on the consequences of the presence of low switching cost customers; we find that their presence hinders entrants, who find it more costly to attract high switching cost customers.

As we will discuss in Section 2, in the simplest static economic model of switching costs, with one incumbent and at least two entrants, heterogeneity of switching costs does not matter. If a proportion $\alpha>0$ of the agents have switching cost $\sigma>0$, while the others have no switching cost, the incumbent will charge $\sigma$ and its profits will be equal to $N \times(\alpha \sigma)$, the average switching cost of all the consumers, multiplied by the number of agents, $N$. We show that this result changes drastically in a dynamic model in which there are new potential competitors in every period; the more skewed the distribution of switching costs, the greater the profits of the incumbent. To the best of our knowledge, this fact and the importance of the distribution of switching costs has not been recognized in the literature, despite the existence of a significant body of theory which explores the consequences of consumer switching costs. (We discuss the literature below in Section 1.)

Our results have implications for managerial practice and for public policy. Entrants should beware of not pricing aggressively while attracting footloose consumers who will not stick with them when they increase prices. Antitrust authorities should take into account the whole distribution of switching costs (including the switching costs of consumers who decide to purchase from entrants) when determining whether incumbent firms are behaving anticompetitively.

We conduct our analysis by studying a series of models with the following features: a) the switching costs of consumers are invariant over time; b) at the start of the 'game' there is a single incumbent firm; and c) there is entry (at least potentially) in every period. Following much of the literature, we assume that only short term contracts are used and that switching costs do not depend on the firm from which consumers purchase.

In Section 2, we introduce our analysis by considering the case where all consumers have the same switching cost $\sigma$. In a one period model, the incumbent would charge $\sigma$, and, assuming that the mass of consumers is equal to 1 , its profit would also be equal to $\sigma$. We embed this static model in a dynamic framework and show that in equilibrium aggregate discounted profit over all periods is also equal to $\sigma$, whether the number of periods is finite or, subject to stationarity assumptions, infinite. In the latter case, this implies that the profit of the incumbent is equal to the value of a flow of per period payments equal to $(1-\delta) \sigma$, not to $\sigma$ ! Although this result is very easy to prove, and is implicit in some of the literature, we feel that it is
worth stressing as it shows that switching costs are a leaner cash cow than sometimes assumed.

The bulk of our analysis of the heterogeneity of switching costs can be found in Section 3. A proportion $\alpha \in(0,1)$ of consumers have a switching cost equal to $\sigma>0$, while the others have no switching cost. We identify the (stationary) equilibrium of the infinite horizon model. As opposed to the case where all consumers have the same switching cost, the intertemporal profit of the incumbent is greater than the one period profit, although it is smaller than the value of an infinite stream of one period profits. We prove that even zero switching cost customers have value for the incumbent, despite the fact that they never purchase its product after entry has occurred: the profits of the incumbent increase when, keeping fixed the number of high switching cost consumers, there are more zero switching cost customers. Indeed, their presence hinders entrants who find it more costly to attract high switching cost customers. ${ }^{1}$

In section 4, we examine alternative entry assumptions: we first study the case where there is only one entrant in every period, and then the case where the number of entrants is random. Finally, while in the rest of the paper we assume that price discrimination based on purchasing histories is not possible, in 4.3 we show that this is not essential for our results and that our results still hold if we allow firms to price discriminate.

The conclusion discusses further research as well as policy implications.

## 1 Literature

The literature has distinguished switching cost models proper and subscription models: in switching cost models, firms must charge the same price to both current and new consumers, while in subscription models they can offer different prices to consumers with different purchase histories. Switching cost models were introduced in the economics literature by [18] (see the surveys of the theoretical literature in [19], Annex A of [20], and [13], and the discussion of policy implications in National Economic Research Associates [20, especially Annex C]). [10] initiated the investigation of subscription models. We present our model as a switching cost model, but, as discussed in the introduction and in 4.3, our results also hold for subscription models.

Much of the switching cost literature focuses on two-period duopsony

[^1]models in which firms choose between charging a high price in order to extract rents from their customers and charging a low price in order to attract customers from their rivals. [14], [3], [21], and [1] study infinite horizon switching cost models, in each of these cases with two firms and homogenous switching costs; ${ }^{2}$ they focus on the evolution of market shares and on the effect of switching costs on prices (see also [16]).

In this framework, [17] shows that higher switching costs may make entry more likely, by inducing incumbents to abandon the hope of attracting the customers of other incumbents and therefore choosing higher prices. Farrell and Klemperer [13, p. 1997] explain that
> "the firm must balance the incentive to charge high prices to 'harvest' greater current profits ... against the incentive for low prices that 'invest' in market share and hence increase future profits."

Recently, [12] have studied the interaction between these two effects in an infinite horizon model where a single consumer has random utility and firms have differentiated products; their focus is on empirics, and, through the use of simulation methods, they provide numerical examples where prices fall when switching costs increase. In a theoretical investigation of a simplified version of this model, $[7,8]$ has shown that for low switching costs, the incentives of firms to invest in the acquisition of new customers outweigh their 'harvesting' incentives; as a result an increase in switching costs leads to lower prices and to lower profits. ${ }^{3,4}$

We also find conditions under which higher switching costs lead to lower profits for the incumbent, but the reasons are very different from those stressed in the literature. Whereas previous authors have assumed a fixed number of firms, there is entry in our models, and therefore competition is more intense. Thus, the incumbent has no incentive to invest in the acquisition of new customers, on which it can only make zero profits - indeed, in equilibrium, the incumbent does not try to "recover" the consumers that it has lost to other firms. In other words, our comparative statics are entirely the consequence of the heterogeneity of switching costs, and of the fact that low switching cost customers protect the incumbent from entry.

[^2]The paper that is closest to ours is [22]. It analyzes a finite horizon subscription model where consumers have different switching costs and where there is free entry. In his primary model, consumers draw new switching costs from identical, independent distributions in each period. He shows that free entry limits the advantages of incumbency and that a firm makes zero expected profits from the consumers that it attracts from its rivals. In an extension, Taylor examines a two period model with two types of consumers who draw their switching cost (as before, independently in each period) from different distributions. His focus is on the incentives of consumers to build a reputation of having low switching cost in order to get better offers in the future, while consumers in our model do not have an incentive to build a reputation.

In our models, each consumer has switching costs that are constant over time, which seems a reasonable approximation of reality in many cases, and this implies that it is harder for an entrant to attract the more valuable consumers, those with higher switching costs, than to attract the less valuable customers. As in Taylor, the presence of low switching cost consumers hurts the high switching cost consumers, but in our model, we show that it can also increase the incumbent's profit.

Finally, in our models, incumbent firms find it just as difficult as entrants to attract customers of other firms. Therefore, incumbent firms, just like entrants, make zero profits on customers of other firms, and in equilibrium they ignore them when choosing the price they charge. This is the reason why our model generates exactly the same results if we transform it into a subscription model.

## 2 Homogenous switching costs: "you cannot get rich on switching costs alone"

In this section, we consider a repeated version of the most standard textbook model of switching cost, with one incumbent and entry in every period. We show that, in equilibrium, the profit of the incumbent is equal to its profit in the one period version of the game. This is true for all equilibria when there are a finite number of periods, and for stationary equilibria when there are an infinite number of periods. We begin by presenting the one period version of the model and then turn to the repeated game with a finite number of periods.

There is a finite set of $N$ consumers indexed by $n=1, \ldots N$, and a good which can be supplied by a number of firms, as we will describe below. In
previous periods, all the consumers have bought from the Incumbent ${ }^{5}$ firm $I$ (we will capitalize Incumbent to denote the firm which is the incumbent at the beginning of period 1). We do not study the process by which firm $I$ became the Incumbent, but only the continuation game after entry becomes possible. In general, at least some of the incumbency rents which we identify would have been dissipated in the competition to become the Incumbent.

Each consumer has a perfectly inelastic demand for one unit of the good. In this section only, all consumers have the same switching cost $\sigma$. This switching cost is incurred every time a consumer changes from one supplier to another. It reflects industry wide similarities or compatibilities between products, rather than idiosyncrasies of specific sellers. This implies, for instance, that our comparative statics results which describe the consequences in changes of the switching costs bear on circumstances where the cost of changing between any pair or products increase or decrease.

### 2.1 One period

Let us consider first a one period model and at least two entrants who can enter the market at zero cost. The main focus of our study is the following "Bertrand" game:
Stage 1: The Incumbent and the entrants set prices;
Stage 2: The consumers choose from which firm to buy.
All of our results also hold true, and are sometimes easier to establish, in the "Stackelberg" version of this game:
Stage 1: The Incumbent sets a price;
Stage 2: The entrants set their prices;
Stage 3: The consumers choose from which firm to buy.
In the one period case, these models are exactly equivalent to standard models of competition where the incumbent has a quality or cost advantage equal to $\sigma$.

In all the paper, we will identify subgame perfect equilibria in undominated strategies.

Assuming, as we will throughout this paper that all firms have zero marginal cost, it is easy to prove that, in both the Bertrand and Stackelberg versions of the game, there is (essentially) only one equilibrium, where the Incumbent ${ }^{6}$ charges $\sigma$, the entrants 0 , and all consumers buy from the Incumbent whose profit is $N \sigma$.

[^3]
### 2.2 Dynamic model with a finite number of periods

We will now show that in the repeated version of the game, the discounted intertemporal profit of the Incumbent is the same as in the one period version of the game: it is still equal to $N \sigma$. One can only pocket the switching cost once. ${ }^{7}$

This is very easy to prove when there are two periods. Formally, we expand the game of 2.1 in the following way. The set of second period incumbents is composed of all the firms that have sold at least one unit in the first period. For definitiveness, we assume that the firms that have sold no unit in the first period "drop out" of the game. ${ }^{8}$ In period 2, there is a finite set containing at least two entrants. In the Stackelberg version of our model, if there are several incumbents at the start of period 2, they all announce their prices simultaneously in stage 1 of the second period.

We assume that firms cannot commit to prices beyond the current period and, until 4.3, that firms cannot discriminate between consumers.

In equilibrium, whether in the Bertrand or Stackelberg model, all second period incumbents charge $\sigma$, and make profits equal to $\sigma$ multiplied by the number of their first period customers. Therefore, the lower bound of the prices that entrants can charge in the first period and not make negative profits is $-\delta \sigma$, where $\delta \in(0,1]$ is the discount rate. ${ }^{9}$ Consumers know that all incumbents will charge $\sigma$ in the second period. Hence, firm $I$ will be able to "keep" its customers only by charging a price less than or equal to $-\delta \sigma+\sigma$. It is straightforward to show that it indeed charges this price and "keeps" all its customers, under Bertrand or Stackelberg competition. Hence its discounted

[^4]profit is
$$
N[(-\delta \sigma+\sigma)+\delta \sigma]=N \sigma .
$$

An easy proof by induction shows that the same result holds with any finite number of periods.

### 2.3 Stationary dynamic model with an infinite number of periods

We now show that the results of 2.2 also hold true in the pure strategy stationary solutions of the infinite horizon version of this model ${ }^{10}$ (we now assume $\delta<1$ ).

Strategies are defined as usual: in the first stage of the first period the Incumbent chooses a price in the Stackelberg version of the game or the Incumbent and the entrants choose prices in the Bertrand version. Thereafter players choose their moves as a function of the history of the game. We identify stationary strategies which we define in the following way: there exists a price $p^{*}$ and a price $p_{E}$ such that along the equilibrium paths of the game and of every subgame in every period all incumbents charge $p^{*}$ and every entrant charge $p_{E}$, where "incumbents" are all the firms which sold at least one unit in the previous period. (When in section 3, we consider different types of consumers and mixed strategies, we will need to complete this definition.)

Thus, the strategies of the firms only depend on whether they sold to consumers in the past period. They do not depend on the prices charged in the past, on the firms' identities, or how many incumbents were in the market this period or in the past period.

We now state and prove the main result ${ }^{11}$ of this section.
Proposition 1. In both the Stackelberg and the Bertrand models, when all consumers have the same switching costs $\sigma$, the intertemporal discounted profit of the Incumbent is equal to $N \sigma$, whatever the number of periods.

[^5]Proof. In 2.2, we have shown that the result holds when there is a finite number of periods. We prove it now for an infinite horizon in the Bertrand case. The Stackelberg case is very similar and somewhat easier to prove.

We first establish that $\sigma(1-\delta)$ is an upper bound on $p^{*}$. By stationarity, whether consumers purchase from an incumbent or an entrant, in future periods they will have the same opportunities to purchase the good at a total cost equal to $\min \left\{p^{*}, p_{E}+\sigma\right\}$. Hence, in the current period, consumers will necessarily choose to purchase from an entrant who charges $p^{\prime}<p^{*}-\sigma$. Therefore, for any $\varepsilon>0$, an entrant who would charge $p^{*}-\sigma-\varepsilon$ would attract customers, and obtain profits equal to the number of consumers it attracted multiplied by

$$
p^{*}-\sigma-\varepsilon+\frac{\delta p^{*}}{1-\delta}
$$

Writing that this expression is negative for all $\varepsilon>0$ yields $p^{*} \leq \sigma(1-\delta)$.
We now show that $\sigma(1-\delta)$ is also a lower bound on $p^{*}$, which will complete the proof. In any period, the lowest priced entrant must charge $p^{*}-\sigma$ : otherwise the incumbent(s) could increase its (their) price(s) without loosing customers. If the entrant attracted customers at this price it would make profits equal to the mass of these customers multiplied by $p^{*}-\sigma+\delta p^{*} /(1-\delta)$, which must be non negative for $p^{*}-\sigma$ to be undominated.

Thus, the only possible equilibrium price is if $p^{*}=\sigma(1-\delta)$.
We now show that there does exist an equilibrium. If the incumbents charge $\sigma(1-\delta)$, it is a best response for the entrants to charge $-\sigma \delta$ and for consumers to all purchase from their respective incumbents. By the construction above, clearly there can be no profitable deviation.

## 3 Heterogenous switching costs

We now turn to the main theme of the article: the consequences of heterogenous switching costs. In this section, we study a model with two types of consumers: $N_{h}$ high switching cost (HSC) consumers, and $N_{\ell}$ low switching cost (LSC) consumers who have a switching cost ${ }^{12}$ equal to 0 . There are therefore $N=N_{h}+N_{\ell}>0$ consumers. We call $\alpha=N_{h} / N$ the proportion of HSC consumers. We keep the same assumptions on entry as in 2.3 (see also footnote 10).

With such a population of consumers, in the one period model, entrants would charge 0 while the incumbent would charge $\sigma$ and obtain a profit equal to $N_{h} \sigma=N \times(\alpha \sigma)$ : under the assumptions of this section, its profits are the

[^6]average switching cost of consumers multiplied by their number. We analyze the infinite horizon version of this game.

### 3.1 Results

As in the model of section 2 where consumers all have the same switching cost, we restrict attention to equilibria that satisfy stationarity conditions, and where players do not use weakly dominated strategies.

In section 2 , stationarity required that in any period, all incumbents, i.e., all firms that had sold to at least one consumer in the previous period, charged $p^{*}$. With different types of consumers, we need to redefine the notion of incumbent: we will say that for any period $t \geq 2$ an incumbent is a firm which has sold a unit of the good to at least one HSC consumer. We also need to redefine the notion of entrants. A "generalized entrant" in period $t$ is either a period $t$ entrant or a firm who has sold only to LSC consumers in period $t-1$. To lighten the terminology, we will use the term "entrant" to refer to "generalized entrants" and will specify "period $t$ entrant" when we want to refer to a firm which was not present in period $t-1$.

With this change of definition, we adapt the definition of stationarity as follows. First, in the Stackelberg case where there are pure strategy equilibria, all incumbents, that is all firms who have sold at least one unit of the good to one HSC consumer in the previous period, charge the same price, whether or not they also sold to a LSC customer (which they will never do in equilibrium!). All entrants also charge the same price. In the Bertrand competition case of 3.2.2, where no pure strategy equilibrium exists, we look for equilibria such that all incumbents use the same mixed strategy. We also assume that the distribution of the lowest price charged by any entrant is the same in every period.

The following proposition summarizes our results.
Proposition 2. In the infinite horizon model, where $N_{h}=\alpha N$ consumers have switching costs equal to $\sigma>0$, while the remaining consumers have 0 switching costs, under either Stackelberg or Bertrand competition the expected profit of the incumbent is

$$
\begin{equation*}
\Pi=N \times \frac{\alpha \sigma}{1-\delta+\alpha \delta} \tag{1}
\end{equation*}
$$

Proposition 2 yields interesting comparative statics, which we discuss in 3.3.

Although they lead to the same profits for the incumbent, the equilibria under Bertrand and Stackelberg competition are very different. Under

Stackelberg competition, the Incumbent offers the same price in every period, and HSC consumers never change suppliers. In Bertrand competition, the incumbent and the entrants play mixed strategies, and in each period there is a strictly positive probability that all the HSC consumers change suppliers. The next subsection presents the proof of Proposition 2. It can be skipped by readers mostly interested in the consequences of our analysis, which are discussed in 3.3.

### 3.2 Proof of Proposition 2

### 3.2.1 Analysis of Stackelberg competition

We show that

$$
p^{S}=\alpha \sigma \frac{1-\delta}{1-\delta+\alpha \delta}
$$

is the equilibrium price that the Incumbent charges in the first period and which incumbents charge in any subgame. Along the equilibrium path, the Incumbent sells to all the HSC consumers and to no LSC consumer.

We first show that $p^{S}$ is an upper bound on the equilibrium price $p^{*}$. Let $p_{E}$ the minimum of the prices charged by any entrant (this minimum exists as there are a finite number of entrants in each period). By stationarity, if $p_{E}<p^{*}-\sigma$ all consumers purchase from one of the lowest price entrants. The profits of one of these entrants is equal to the number of consumers which it has attracted multiplied by $p_{E}+\alpha \delta p^{*} /(1-\delta)$. If this profit per customer were strictly positive, at least one of the lowest price entrants would find it profitable to slightly undercut the others. Therefore,

$$
\left(p^{*}-\sigma\right)+\frac{\delta \alpha p^{*}}{1-\delta} \leq 0 \Longleftrightarrow p^{*} \leq \sigma \frac{1-\delta}{1-\delta+\alpha \delta}=p^{S}
$$

To show that $p^{S}$ is a lower bound on $p^{*}$, we show that if $p^{*}<p^{S}$ a deviation by the incumbent to any $p^{\prime} \in\left(p^{*}, p^{S}\right)$ would be profitable. Indeed, entrant(s) who would respond by charging $p^{\prime}-\sigma$ or less would generate aggregate discounted profits of at most $\left[p^{\prime}-\sigma+\delta \alpha p^{*} /(1-\delta)\right] N$. (This is their profit if they attract all the HSC consumers.) If both $p^{\prime}$ and $p^{*}$ are strictly smaller than $p^{S}$, this expression is strictly negative. Therefore, at least one of the entrants would be making strictly negative profits; the deviation by the incumbent is profitable, as entrants would not be able to respond profitably.

We have therefore proven that in any equilibrium $p^{*}=p^{S}$
Now, we show by construction that there does exist such an equilibrium. In period 1 , the incumbent charges $p^{S}$ and entrants charge 0 . All the HSC consumers purchase from the incumbent while all the LSC consumers purchase
from one of the lowest priced entrants. In period $t>1$, the incumbents charges $p^{S}$ while the entrants charge 0 . All consumers purchase from the firm from which they purchased the previous period. By construction, there are no profitable deviations. This proves the theorem in the Stackelberg case.

### 3.2.2 Analysis of Bertrand competition

In the Bertrand game, there is no equilibrium in which the incumbent charges $p^{S}=-\delta \Pi+\sigma$ and at least one entrant charges $p^{S}-\sigma=-\delta \Pi$. Indeed, if the incumbent did not retain all the HSC customers, he would have incentives to decrease its price; if it retained them, the entrant would attract only the LSC consumers, who generate no profit in future periods, at a negative price. More generally, it is easy to show that there is no pure strategy equilibrium of the game, but we will still be able to show that the profits of the incumbent are equal to the profits in Stackelberg competition.

The general plan of the proof is a follows. We prove that, if $\Pi$ is the (expected) profit of the incumbent, then $-\delta \Pi+\sigma>0$ belongs to the support of the distribution of prices that it announces; furthermore when it chooses this price, its HSC customers purchase its product with probability 1 and LSC customers always purchase from one of the entrants. This implies that Eq. (1) holds. (More precisely, we will show that $-\delta \Pi+\sigma$ is the lower bound on the support of prices charged by the incumbent, and that when it chooses a price arbitrarily close to this lower bound, it 'keeps' the HSC customers with probability arbitrarily close to 1.) We conclude the proof by computing the distribution of prices and by showing that an equilibrium exists.

The incumbents choose prices on an interval whose lower bound is $\underline{b}_{I}$ and whose upper bound is $\bar{b}_{I}$. In any state of nature, the effective constraint on the incumbents comes from the smallest price charged by an entrant. We call $\underline{b}_{E}$ and $\bar{b}_{E}$ the lower and upper bounds of the distribution of this smallest price. We begin by computing these bounds.

We proceed through a series of claims.
Claim 1. Incumbents have strictly positive profits, which implies $\underline{b}_{I}>0$.
Proof. We assume that incumbents have profits equal to 0 , and show that this lead to a contradiction. Because we are looking for stationary equilibria, any entrant who would attract consumers would become an incumbent in the next period, and therefore make 0 profits. Therefore charging a negative price is a dominated strategy for entrants and competition among them leads them to announce a price of 0 .

Now assume that an incumbent deviates in period $t$ and charges $p^{\prime} \in$ $(0,(1-\delta) \sigma)$. An upper bound for the total cost that an HSC consumer
can incur by purchasing from the incumbent in period $t$ and switching in period $t+1$ is $p^{\prime}+\delta(\sigma+C)$, where $C$ is his expected cost of purchase along the equilibrium path. This is strictly smaller than $\sigma+\delta C$, which is a lower bound of the cost that he would incur by purchasing from an entrant in period $t$. Therefore there exists a profitable deviation for the incumbent.

Claim 2. In any period, the expected discounted profit of entrants is equal to 0 and $\bar{b}_{E} \leq 0$.

Proof. The expected discounted profits of entrants are the same for all prices in $\left(\underline{b}_{E}, \bar{b}_{E}\right)$. If the distribution of $p_{E}$ does not have a mass point at $\bar{b}_{E}$, then the expected profit of an entrant who chooses this price is 0 , as he has zero sales with probability 1 . If the distribution of $p_{E}$ has a mass point at $\bar{b}_{E}$, an entrant which charges $\bar{b}_{E}$ cannot make a strictly positive profit: charging a price slightly below $\bar{b}_{E}$ would yield even greater profits, which cannot be true in equilibrium because of competition between entrants. This proves that the profits of entrants are not positive and therefore are equal to 0 . By Claim 1, with $\bar{b}_{E}>0$, an entrant could make a positive profit by charging a price in $\left(0, \min \left\{\underline{b}_{I}, \bar{b}_{E}\right\}\right)$, attract the LSC consumers (and, maybe, some HSC consumers) with probability 1 and make strictly positive profits, which establishes the contradiction.

By Claims 1 and 2, LSC consumers never buy from an incumbent. Therefore, the expected profit of an incumbent is independent of the number of its LSC customers in the previous period. This enables us to define, without ambiguity, $\Pi$ as the profit of an incumbent from whom all the HSC consumers purchased in the previous period. Any incumbent would have discounted profits equal to $\Pi / N$ multiplied by the number of its customers in the previous period. We will now proceed to demonstrate that the incumbent equilibrium profit is the same in the Bertrand model as in the Stackelberg model.

We now establish the lower bounds on prices charged by incumbents and entrants.

Claim 3. $\underline{b}_{E}=-\delta \Pi$ and $\underline{b}_{I}=-\delta \Pi+\sigma$.
Proof. Clearly, $\underline{b}_{E} \geq-\delta \Pi$; otherwise whenever $p_{E} \in\left[\underline{b}_{E},-\delta \Pi\right)$, the aggregate expected discounted profits of the lowest price entrants would be strictly negative. Furthermore, by announcing any price strictly smaller than $\underline{b}_{E}+\sigma$, the incumbent "leaves money on the table" and therefore $\underline{b}_{I} \geq \underline{b}_{E}+\sigma$.

If we had $\underline{b}_{E}>\delta \Pi$, by choosing a price in $\left(-\delta \Pi, \underline{b}_{E}\right)$ an entrant would with probability 1 be the lowest price entrant and underprice all the incumbents by more than $\sigma$. It would make strictly positive profits, which contradicts Claim 2. Thus, $\underline{b}_{E}=-\delta \Pi$ (and $\bar{b}_{E}>-\delta \Pi$, as entrants use a mixed strategy).

With $\underline{b}_{I}>-\delta \Pi+\sigma$, an entrant could make positive profits by choosing a price in $\left(-\delta \Pi, \min \left\{\underline{b}_{I}-\sigma, \bar{b}_{E}\right\}\right)$, which contradicts Claim 2.

We now compute the profits of the Incumbent. We will need the following lemma.

Claim 4. When $p_{I}$, the price charged by an incumbent, converges to $\underline{b}_{I}$ from above, the probability that it sells to all its HSC customers of the previous period converges to 1 .

Proof. The probability that all its HSC customers from the previous period purchase from an incumbent is decreasing in the price $p_{I}$ that it charges; let $\eta$ be limit as $p_{I}$ converges from above to $\underline{b}_{I}$ as the probability that past HSC customers will choose to purchase from the incumbent. The profit of the incumbent, if it sold the good to a proportion $\zeta$ of consumers in the previous period, converges to ${ }^{13}$

$$
\zeta\left(\eta N_{h} \underline{b}_{I}+\eta \delta \Pi\right)<\zeta\left(N_{h} \underline{b}_{I}+\delta \Pi\right)
$$

By Claim 3, it can guarantee itself a profit arbitrarily close to $\zeta\left(N_{h} \underline{b}_{I}+\delta \Pi\right)$ by charging a price below, but very close to $b_{I}$, which establishes that $\eta$ must be 1 .

Claim 4 implies that there is no mass point at $\underline{b}_{I}$ in the distribution of the prices charged by incumbents. Furthermore, it implies that we have $\Pi=N_{h} \underline{b}_{I}+\delta \Pi$, which implies that (1) holds.

We now must prove that there exists such an equilibrium. To do this we begin by deriving the distribution of prices that must prevail in any equilibrium.

Distribution of prices in equilibrium Assuming that an equilibrium does exist, we compute the distribution of $p_{E}$ and $p_{I}$. Below, we show that there indeed exists an equilibrium corresponding to these distributions.

Using the zero profit condition for the entrants the distribution of prices announced by the incumbents satisfy

$$
\begin{array}{r}
G_{I}\left(p_{E}+\sigma\right) \times\left[(1-\alpha) N p_{E}\right]+\left(1-G_{I}\left(p_{E}+\sigma\right)\right) \times\left[N p_{E}+\delta \Pi\right]=0 \quad \forall p_{E} \in\left(\underline{b}_{E}, \bar{b}_{E}\right) \\
\Longrightarrow G_{I}\left(p_{I}\right)=\frac{N\left(p_{I}-\sigma\right)+\delta \Pi}{\alpha N\left(p_{I}-\sigma\right)+\delta \Pi} \quad \forall p_{I} \in\left(\underline{b}_{I}, \bar{b}_{I}\right) . \tag{2}
\end{array}
$$

[^7]Because $\lim _{p_{I} \rightarrow b_{I}+} G_{I}\left(p_{I}\right)=0 \underset{p_{I} \rightarrow \bar{b}_{I_{-}}}{\text {and }} \lim _{I}\left(p_{I}\right)=1$, the function $G_{I}$ has no mass point.

Similarly, the distribution $G_{E}$ of $p_{E}$ is determined by the fact that the profits of the incumbent are equal to $\Pi$ for all prices in $\left[\underline{b}_{I}, \bar{b}_{I}\right]$, and therefore

$$
\begin{equation*}
G_{E}\left(p_{E}\right)=1-\frac{\Pi}{\alpha N\left(p_{E}+\sigma\right)+\delta \Pi} \quad \forall p_{E} \in\left(\underline{b}_{E}, \bar{b}_{E}\right) . \tag{3}
\end{equation*}
$$

Because

$$
\lim _{p_{E} \rightarrow 0^{-}} G_{E}(0)=\frac{\alpha N \sigma-(1-\delta) \Pi}{\alpha N \sigma+\delta \Pi}=\frac{\alpha \delta}{1+\alpha \delta}<1
$$

the distribution $G_{E}$ has a mass point at $p_{E}=\underline{b}_{E}=0$. We can implement by distribution $G_{E}\left(p_{E}\right)$, by having each of the $k$ generalized entrants independently choosing their price according to distribution $G_{E, k}\left(p_{E}\right)=$ $1-\sqrt[k]{1-G_{E}\left(p_{E}\right)}$.

Existence of an equilibrium We have proved that if there exists an equilibrium that satisfies our assumptions, the distribution of prices must satisfy Eq. (2) and (3). We now prove that there does indeed exist such an equilibrium; this is a proof by construction: we exhibit the strategies followed by the agents.

In this equilibrium a) the consumers who buy from an entrant always buy from the same (lowest price) entrant and b) consumers who are indifferent between purchasing from this entrant and from the incumbent from which they purchased in the previous period purchase from the incumbent. The analysis which we have conducted to derive (2) and (3), shows that these strategies are best responses for all the agents when there is only one incumbent.

We need to examine the continuation equilibrium when there are several incumbents, for two reasons: a) for the consumers to only purchase from the Incumbent in equilibrium, it must be the case that it is not a profitable deviation for HSC consumers to purchase from another firm than other HSC consumers; b) we have imposed the requirement that all incumbents, i.e., all firms that have sold to HSC consumers in the previous period, use the same pricing strategy and we need to show that there exists an equilibrium with this property. As we will see, the fact that the strategies of the firms satisfy b) provides an easy proof of point a).

Let us therefore assume that in one period there are $n \geq 2$ incumbents. It is straightforward to see that if the distribution of $p_{E}$ is $G_{E}$, then all the incumbents are indifferent between all prices in $\left[\underline{b}_{I}, \bar{b}_{I}\right]$. We now show that the profits of the lowest price entrants are equal to 0 if all the incumbents choose the strategy described by (2).

Let $N^{i}$ be the number of HSC consumers of incumbent $i=1 \ldots, K$ in the previous period. The lowest price entrant sells to all the LSC consumers and to the HSC consumers who were in the previous period clients of firms who choose in the current period a price $p_{i}>p_{E}+\sigma$. Because it will follow the same strategy as a unique incumbent, and because the distribution of prices of the entrant is independent of the number of incumbents, its profits per HSC customer discounted to the beginning of next period will be $\delta \Pi / N_{h}$.

Therefore, for given prices by the incumbents, the profit of the entrant is

$$
\begin{aligned}
N_{\ell} p_{E}+\sum_{\left\{i \mid p_{i}>p_{E}+\sigma\right\}}\left(N^{i} p_{E}+\frac{N^{i}}{N_{h}} \delta \Pi\right) & =N_{\ell} p_{E}+\sum_{\left\{i \mid p_{i}>p_{E}+\sigma\right\}} N^{i} \times\left(p_{E}+\frac{N^{i}}{N_{h}} \delta \Pi\right) \\
& =N_{\ell} p_{E}+\sum_{i} s_{i}\left(p_{i}\right)\left(p_{E}+\frac{\delta \Pi}{\alpha N}\right),
\end{aligned}
$$

where $s_{i}$ is the random variable, of expected value $N^{i}\left(1-G_{I}\left(p_{E}+\sigma\right)\right)$, that takes the value $N_{i}$ for $p_{i}>p_{E}+\sigma$ and 0 otherwise. The $p_{i}$ 's are independently distributed, and therefore the expected value of $\sum s_{i}\left(p_{i}\right)$ is $\alpha N\left(1-G_{I}\left(p_{E}+\sigma\right)\right)$, and the expected profit of the lowest price entrant, conditional of the fact that it has chosen a price of $p_{E}$, is

$$
(1-\alpha) N p_{E}+\left(\alpha N p_{E}+\delta \Pi\right)\left(1-G_{I}\left(p_{E}+\sigma\right)\right),
$$

which, by Eq. (2), is equal to 0 .
Because all incumbents use the same pricing strategy HSC consumers have no incentive to deviate from the focal strategy described above: in subsequent periods, they would face the same distribution of prices both from the firm they purchased from in previous periods and from the entrants.

### 3.3 Economic consequences of Proposition 2

Proposition 2 yields interesting profit comparisons and comparative statics, which we summarize in the following corollary.

Corollary 1. Under the conditions of Proposition 2:
$i . \Pi$ is greater than the profit of the incumbent in the one period model, $N \alpha \sigma$, but smaller than the value of an infinite stream of one period profits, $N \alpha \sigma /(1-\delta)$.
ii. $\Pi$ is strictly smaller than $N \sigma$, but $\lim _{\delta \rightarrow 1} \Pi=N \sigma$ for all $\alpha$.
iii. $\Pi$ is increasing in $\alpha, \sigma$ and $\delta$;
$i v$. for a given average level of consumer switching costs, $\alpha \sigma$, the profit of the incumbent, $\Pi$, is decreasing in $\alpha$;
v. adding LSC consumers without changing the number of HSC consumers increases $\Pi$;
vi. under Stackelberg competition, the utility of HSC consumers is an increasing function of $\alpha$.

Part $i$ of the proposition show that, contrary to what happens when all consumers have the same switching costs, the intertemporal profit is not equal to the one period profit, but is greater; however the per period profit is smaller in the infinite horizon model than in the one period model. The discounted profit is equal to the one period profit discounted at the rate $\delta(1-\alpha) .{ }^{14}$ Part $i i$ shows that when the agents are very patient, the profit of the incumbent is independent of the proportion $\alpha$ of HSC consumers, whereas in the one period model profits are proportional to $\alpha$. As we will explain below, LSC consumers, who always purchase from the lowest price entrant, make it more costly to attract profitable HSC customers away from the incumbent.

Parts $i i i$ and $i v$ of the corollary are obvious from Eq. (1). Part $v$ is easy to prove. Assume that we add $\eta>0$ LSC consumers; the total number of consumers becomes $N^{\prime}=N+\eta$ and the proportion of HSC consumers becomes $\alpha^{\prime}=\alpha N /(N+\eta)$. The new profits of the Incumbent are

$$
\Pi^{\prime}=(N+\eta) \frac{\alpha^{\prime} \sigma}{1-\delta+\alpha^{\prime} \delta}=\frac{\alpha N \sigma}{1-\delta+\frac{\alpha}{1+\eta} \delta},
$$

which is increasing in $\eta$. Lsc consumers are valuable to the incumbent, even though they never purchase its product, as they make it more costly for entrants to make aggressive discounts in order to attract HSC customers.

We now provide an intuition for why adding LSC consumers in the market increases the profit of the Incumbent's despite the fact that it never sells to these consumers. ${ }^{15}$ Entrants are willing to price below marginal cost and make short term losses to attract HSC consumers whom they can "exploit" in the future. However, any offer which is attractive to HSC consumers is even more attractive to LSC consumers, who do not generate any future profits. Hence, their presence reduces the willingness of entrants to price aggressively and benefits the Incumbent.

As we discussed at the end of 3.1, the profits of the Incumbent are the same in the Bertrand and in the Stackelberg models, and in both of them the profits of the entrants are smaller. On the other hand, social welfare is lower

[^8]with Bertrand competition, because resources are wasted in switching. This implies that consumer welfare is also lower with Bertrand competition.

Furthermore, in the mixed strategy Bertrand equilibrium the incumbent trades off the risk of keeping the HSC consumers at a high price with the risk of losing them to an entrant. Similarly, the entrants trade off the risk of undercutting the incumbent's price by more than $\sigma$ and attracting the HSC consumers and making a positive profit with the loss of pricing below marginal costs and attracting only the LSC consumers. Thus, in the Bertrand model we see stochastic turnover of the incumbent which is not present in the Stackelberg model.

## 4 Limited entry and price discrimination

Up to this point, we have assumed that there are at least two potential entrants in every period, but in many industries with important switching costs there is relatively little entry. The main aim of this section is to explore the ways in which our analysis can be adapted to take into account limited entry.

In 4.3, we show that the results of the paper are not changed if firms can discriminate between old and new customers.

The Supplementary material for this article [5] contains more extensive proofs and discussions of the material in this section.

### 4.1 A single entrant in each period

In this subsection, we assume that there is a single entrant in each period; for simplicity, we discuss our results using the Stackelberg model.

If consumers all have the same switching cost $\sigma$ the proof of Proposition 1 still holds. The equilibrium profit of the Incumbent is $N \sigma$ and it sells to all consumers in every period.

With heterogenous consumers, after the first period, nothing is changed as the firm(s) who have acquired LSC consumers in previous periods compete with the entrants. On the other hand, it is not an equilibrium strategy for the Incumbent to charge $p^{S}$ in the first period. If it charges $p_{I}$ slightly greater than $p^{S}$, the first period entrant finds it more profitable to charge "slightly less" than $p_{I}$ and attract all the LSC consumers and none of the HSC consumers than to charge slightly less than $p_{I}-\sigma$.

As the first period is fundamentally different from subsequent periods, we must adapt our definition of stationarity. We define two states. In State 2, at least one firm sold only to LSC consumers in the previous period. Otherwise
we are in State 1, where the game starts. We identify equilibria where State 1 occurs only in the first period.

In State 2, the current period entrant and the firm(s) who sold only to LSC consumers in previous periods compete to attract HSC consumers. The reasoning of section 3.2.1 can be reused essentially unchanged and incumbents charge $p^{S}$. The only caveat is that we must ensure that there is no profitable deviation that sends the system back in State 1; we do this below.

To understand pricing in State 1, consider the first period. The incumbent charges the greatest $p^{I}$ such that the entrant is indifferent between charging a price slightly below $p_{I}$ and attracting all the LSC consumers and charging slightly below $p_{I}-\sigma$ and attracting all the consumers. This $p_{I}$ satisfies $N\left(p_{I}-\sigma\right)+\delta \Pi_{1}=N_{\ell} p_{I}$, where $\Pi_{1}$ is equilibrium profit of an incumbent in State 1 ; it satisfies $\Pi_{1}=N_{h} p^{1}+\delta \Pi_{2}$ where $p^{1}$ is the price charged by the incumbent in period 1 and $\Pi_{2}$ is the equilibrium profit of an incumbent in State 2 if has sold to all the HSC consumers in the previous period; by our discussion in the previous paragraph, $\Pi_{2}$ is equal to the $\Pi$ of Eq. (1). Simple manipulations yield $\Pi_{1}$ and $p^{1}$. Because it does not face any competition in the market for LSC consumers, the first period entrant makes strictly positive profits, $N_{l} p^{1}$.

Having only a single entrant in period 1 allows the Incumbent to obtain greater profits than in our base model; thus the intertemporal profit is here also greater than the flow profit. The effects of changes in $\delta$ are different from the base model: the profit of the Incumbent is the same for $\delta=0$ and $\delta=1$ and convex in $\delta$.

Transitions from State 2 to State 1 We must still show that there is an equilibrium such that the system remains in state 2 once it is there. If the system enters period $t$ in State 2, at least two firms do not have any HSC consumers. Whatever price these firms charge, there exists a continuation equilibrium such that not all the firms have at least one HSC consumer in period $t$. In that equilibrium all HSC consumers who switch firms purchase from the same firm. There is no incentive for these HSC consumers to deviate and purchase from another firm which has no HSC customer: this would not decrease the price paid in the current period, but may increase the price they pay in period $t+1$ if this purchase moves the system from State 2 to State 1.

### 4.2 A random number of entrants in each period

Consider now a model when the number of entrants is random. In period 1, and as long as there has been no previous entry, there there is a single entrant with probability $q_{1}$, two or more entrants with probability $q_{2}$ and no entry
with probability $1-q_{1}-q_{2}=1-q>0$. For simplicity, if there is entry in a period, there is at least one entrant in every subsequent period. Incumbents know whether entry has occurred when they set their price.

To eliminate the possibility of infinite prices, consumers assign a value $V$ to the good, with $V$ large enough that the results above hold. As in 4.1, we assume Stackelberg timing.

As in 4.1, with homogenous consumers. once entry has occurred the present discounted value of the incumbent's profit is $N \sigma$. Let $\bar{\Pi}$ be the expected profit of the incumbent before it knows whether there will be entry in the current period if there was no entry in previous periods: $\bar{\Pi}=N q \sigma+(1-q)(N V+\delta \bar{\Pi})$. The relationship between one period profits and infinite horizon profits are the same as in point $i$. of Proposition 2.

With heterogenous consumers, the incumbent's expected profit $\Pi^{I}$ satisfies $\Pi^{I}=\left(1-q_{1}-q_{2}\right)\left(N V+\delta \Pi^{I}\right)+q_{1} \Pi_{1}+q_{2} \Pi$. It is a probability adjusted convex combination of facing no entrant, a single entrant, and at least two entrants in the current period.

### 4.3 Price discrimination

Up until now, as discussed in the introduction, we have assumed that firms could not price discriminate between consumers on the basis of their purchasing histories: we have studied switching cost as opposed to subscription models. This does not affect the results. Indeed, suppose that we allowed an incumbent firm in period $t$ to have different prices for its old and new customers. In this case, the incumbent firm is exactly in the same position as an entrant as far as attracting new LSC consumers goes. In all our discussion, except for the discussion in 4.1, the profits of entrants are equal to zero, and the Incumbent would not gain from being able to price discriminate. ${ }^{16}$

## 5 Conclusion

A significant body of theory explores the consequences of consumer switching costs: it highlights the role of "bargain then rip-off" pricing patterns, where a firm makes very profitable introductory offers and raises its price in subsequent periods. To the best of our knowledge, the fact that the distribution of switching costs changes considerably the way in which these strategies play

[^9]out has not been pointed out. We hope the present paper will contribute to close this gap. Taking into account the heterogeneity of switching costs has enabled us to identify very rich strategic interactions between the incumbent and the entrants and led to surprising comparative statics.

Our results should affect policy analysis. For instance, the liberalized UK domestic gas and electricity markets analyzed by NERA in [20] appears to broadly fit the context we consider: the product is homogeneous, discrimination between old and new customers was not an option, and entrants had to attract customers away from the historical incumbent (British Gas and the public electricity suppliers) as there were practically no unattached customers. Entrants offered prices below cost, and a fortiori below those of the incumbent(s), which saw their market share decrease. Our analysis shows that information on the distribution of switching costs, for which no data is given, should have been gathered and that its consequences for the strategy of the entrants should have been considered.

Our results should also have consequences for the empirical work which tries to estimate the consequences of changes of switching costs on prices. For instance, [23] examines the effect introducing number portability for toll free calling (if consumers change phone companies they can keep their same number). This reduces the switching costs of buyers of toll free services. If consumers have different switching costs and these switching costs are affected differentially by the change to number portability, then not only do average switching cost fall, but the distribution of switching cost changes. Similarly, [15] examine a switching cost model in banking. Our work demonstrates that when the empirical work does not take into account how the distribution of switching cost changes, then the estimated model maybe misspecified.

On the theoretical side, we have used a very stark model, with entry in every period whereas much of the literature on switching costs has emphasized models where a limited number of incumbents compete over time, trying to vie for each other's consumers. In the rest of this conclusion, we briefly discuss some extensions of the model.

In an earlier version of this article, we allowed for a continuum of consumers. Due to measurability issues, this led to a more complicated definition of stationarity, but led to an equilibrium which takes exactly the same form as with a finite number of consumers. Thus, our results are robust to assuming that consumers are 'small'.

In a companion piece, we allow the low switching cost, $\sigma_{\ell}$, to take a strictly positive value rather than being equal to 0 as in the present paper. Due to technical issues involving the definition of equilibrium and stationarity, we are only able to solved the model for a two period model. For a large range of parameters, the profit of the Incumbent is decreasing in $\sigma_{\ell}$. Indeed,
if in the first period all or nearly all the HSC consumers purchase from the Incumbent, the entrants, even those with a small number of HSC consumers in there clientele will choose to charge $\sigma_{\ell}$ in the second period. This provides an incentive for HSC customers to purchase from an entrant in the second period, and the larger $\sigma_{\ell}$, the more will do so. Thus, the larger $\sigma_{l}$ the more HSC consumers the Incumbent loses in the first period, and one can show that this induces a decrease in its profits.

We finish by discussing a few extension and venues for future research.
The first extension would be to allow for the presence of fixed costs for entrants. Assume that the fixed cost are incurred before any firm sets its price, and that it is small enough that there is entry. Then, as in the standard Bertrand model, the only equilibria are mixed strategy equilibria. When at least two potential entrants actually enter, the pricing decisions in the continuation game will be exactly the same as without entry costs. With zero or one entrant, on the other hand, prices will be higher. We believe that, in general, the presence of switching costs would mitigate the effects studied in this paper. Indeed, an increase in the number of LSC consumers or an increase in $\sigma_{L}$ will lead to a higher probability that there will be at least two entrants, and therefore to lower prices.

Another interesting avenue for future research is to assume that in each period some consumers are replaced with new consumers who are initially not tied to any firm. ${ }^{17}$ To get the flavor of what the equilibrium would look like, we make the following observations, assuming that the proportion of new consumers is small. First, an incumbent with some consumers who have purchased in the past will not try to attract new consumers, and, as in the main body of the paper, only the lowest priced entrants will attract new consumers. The number of incumbents will therefore grow from period to period, and their market shares will shrink. Furthermore, entrants should price as aggressively as when there were no new consumers. This will lead to lower incumbent prices and profits.

We have done some very preliminary work on a two period model where the distribution of switching costs is drawn from a continuous distribution. Additional complications occur when doing comparative statics in this model, since entrants in period 1 will become incumbents in period 2 and will lose some consumers in period 2 if the lowest bound on switching cost is small enough. This may be an interesting avenue to pursue more fully in the future.

We have not been able to identify the equilibria in a infinite horizon model,

[^10]except in the case where the switching cost of the LSC consumers is equal to 0 . Solving this problem raises interesting, but difficult, questions: in particular, we are not sure that a stationary equilibrium exists; we do not even know the appropriate definition of stationarity for that case.

Finally, network externalities often play a role similar to switching costs they have sometimes been called 'collective' switching costs. In future work, we plan to study models where agents have different trade-offs between size of network and prices; we believe that phenomena similar to those analyzed in the current paper can be identified.

## References

[1] E. T. Anderson, N. Kumar, and S. Rajiv. A comment on: ‘Revisiting dynamic duopoly with consumer switching costs'. Journal of Economic Theory, 116(1):177-186, 2004.
[2] G. Arie and P. L. E. Grieco. Who pays for switching costs? Unpublished manuscript, Northwestern University, 2010.
[3] A. Beggs and P. Klemperer. Multiperiod Competition and Switching Costs. Econometrica, 60:651-666, 1992.
[4] G. Biglaiser and J. Crémer. Equilibria in an infinite horizon game with an incumbent, entry and switching costs. International Journal of Economic Theory, 7(1):65-76, 22011.
[5] G. Biglaiser, J. Crémer, and G. Dobos. Supplementary material for "the value of switching costs", October 2012. URL www. cremeronline.com/ research/SupMatBCD.pdf.
[6] J. Bouckaert, H. Degryse, and T. Provoost. Enhancing market power by reducing switching costs. CESifo Working Paper 2449, Munich, 2008.
[7] L. Cabral. Switching costs and equilibrium prices. Unpublished manusrcript, available at http://luiscabral.org, 2008.
[8] L. Cabral. Small Switching Costs Lead to Lower Prices. Journal of Marketing Research, 46(4):449-451, Aug. 2009.
[9] J. Chen. How do switching costs affect market concentration and prices in network industries? Unpublished manuscript, 72011.
[10] Y. Chen. Paying Customers to Switch. Journal of Economics and Management Strategy, 6:877-897, 1997.
[11] Y. Chen and J. Xie. Cross-market network effect with asymmetric customer loyalty: Implications for competitive advantage. Marketing Science, 26(1):52-66, January-February 2007.
[12] J.-P. Dubé, G. J. Hitsch, and P. Rossi. Do Switching Costs Make Markets Less Competitive? Journal of Marketing Research, 46(4):435-445, Aug. 2009.
[13] J. Farrell and P. Klemperer. Coordination and Lock-In: Competition with Switching Costs and Network Effects. In M. Armstrong and R. Porter, editors, Handbook of Industrial Organization, volume 3, pages 1967-2072. North-Holland, 2007.
[14] J. Farrell and C. Shapiro. Dynamic Competition with Switching Costs. The RAND Journal of Economics, 19(1):123-137, Spring 1988.
[15] M. Kim, D. Kliger, and B. Vale. Estimating switching costs: the case of banking. Journal of Financial Intermediation, 12(1):25-56, Jan. 2003.
[16] P. D. Klemperer. Markets with consumer switching costs. PhD thesis, Graduate School of Business, Stanford University, 1986.
[17] P. D. Klemperer. Entry Deterrence in Markets with Switching Costs. Economic Journal, 97:99-117, 1987.
[18] P. D. Klemperer. Markets with Consumer Switching Costs. Quarterly Journal of Economics, 102:375-394, 1987.
[19] P. D. Klemperer. Competition When Consumers Have Switching Costs. Review of Economic Studies, 62:515-539, 1995.
[20] National Economic Research Associates. Switching Costs. Report prepared for the Office of Fair Trading and the Department of Trade, UK, 2003.
[21] A. J. Padilla. Revisiting Dynamic Duopoly with Consumer Switching Costs. Journal of Economic Theory, 67(2):520-530, Dec. 1995.
[22] C. R. Taylor. Supplier Surfing: Competition and Consumer Behavior in Subscription Markets. The RAND Journal of Economics, 34(2):223-246, Summer 2003.
[23] V. B. Viard. Do switching costs make markets more or less competitive? The case of 800 -number portability. The Rand Journal of Economics, 38 (1):146-163, Mar. 2007.


[^0]:    *We would like to thank Andrew Clausen, Philipp Kircher, George Mailath, Curt Taylor, Paul Klemperer, and the participants at seminars where this paper has been presented, especially those of the Economics department at the University of Cambridge. We are especially grateful to the co-editor and anonymous referees for their very helpful comments that have greatly improved the paper.

    Biglaiser: gbiglais@email.unc.edu,
    Crémer: jacques@cremeronline.com,
    Dobos: gdobos@gmail.com.
    Corresponding author: Jacques Crémer, Toulouse School of Economics (IDEI \& GREMAQ), 21 allée de Brienne, 31015 Toulouse Cedex 6. Tel: 05611286 33; jacques@cremeronline.com

[^1]:    ${ }^{1}$ However, if the proportion of zero switching cost consumers increase keeping fixed the total number of consumers (and therefore decreasing the number of high switching cost consumers) the profits of the incumbent decrease - high switching cost consumers are still more valuable than zero switching cost consumers.

[^2]:    ${ }^{2}$ Beggs and Klemperer assume that consumers are horizontally differentiated, but that, once they have purchased from a firm, they never buy from another firm.
    ${ }^{3}$ [2] also provide a theoretical analysis of [12] but, unlike Cabral (and like Dubé et al.), they assume that the consumers are myopic. (Our consumers are forward looking.) See also [9] for a simulation analysis with myopic consumers.
    ${ }^{4}$ In a subscription model based on [10], [6] also show that higher swithching costs can lead to lower profits.

[^3]:    ${ }^{5}$ In the dynamic version of the model, there could be, in some periods $t>1$, several incumbents, i.e., firms who have sold goods to a positive number of consumers in previous period.
    ${ }^{6}$ The results would be the same with several incumbents.

[^4]:    ${ }^{7}$ Although the model we use is a trivial extension of the most elementary model of switching costs, we have not found in the literature a clear statement of what happens when this game is repeated, with new entrants in every period; almost all of the literature focuses on the case of duopsony, where the same two firms compete again each other period after period.
    ${ }^{8}$ Formally, if a firm has zero sales in period 1, in period 2 it has zero sales whatever the price it announces. All our results would hold if the entrants who do not sell in one period are also present in future periods. They would also hold if new entrants appeared in period 2 only if all period 1 entrants sold at least one unit of the good in period 1.
    ${ }^{9}$ This negative price should be interpreted as a discount below marginal cost. Thus, like much of the literature, we assume that entrants can charge prices below marginal cost, and this is often the case in commercial practice.

    In some cases $-\delta \sigma$ could be greater than the marginal cost, and, in practice, the entrant may not be able to offer the full discount that we assume, for instance because consumers have free disposal. The constraint which this imposes on the strategies of the firms have been studied recently by [11]. Throughout the paper, we assume that this constraint does not bind.

[^5]:    ${ }^{10}$ As above, we assume that in every period there are least two entrants (although this is not strictly necessary, see section 4 ). As in 2.2 (see footnote 8 ), it is easier to think of the firms that did not sell anything in a period dropping out of the game: if their sales are equal to zero in period $t$, in any subsequent period they have zero sales whatever price they announce. This ensures that the number of "active" firms does not become infinite,
    ${ }^{11}$ In a companion paper, [4] prove that there exist many other equilibria of this game which satisfy a weaker version of stationarity: although the outcome of the game is stationary (with prices in each period as low as 0 or as high as $\sigma$ ), after a deviation incumbents may charge prices different from the prices along the equilibrium path.

[^6]:    ${ }^{12}$ See the conclusion for the consequences of assuming that the LSC consumers have strictly positive switching costs.

[^7]:    ${ }^{13}$ If some of the HSC consumers do not purchase from the incumbent, none of the LSC consumers will.

[^8]:    ${ }^{14}$ In a companion paper, we analyze the same model with only two periods. The discounted profit of the Incumbent is $N \alpha \sigma(1+\delta-\alpha \delta)$ : this is again the value of a flow of the one period profit discounted at the rate $\delta(1-\alpha)$.
    ${ }^{15} \mathrm{We}$ thank the editor for suggesting this intuition.

[^9]:    ${ }^{16}$ Even in the case of one entrant, it would actually not gain from being able to price discriminate. It is only in the first period that the entrant makes a positive profit, and in the first period, the Incumbent has no scope for discrimination as all the consumers have the same purchasing history.

[^10]:    ${ }^{17}$ In an interesting working paper, Rhodes (2012) analyzes an overlapping generation, Hotelling model where all consumers have the same switching costs. He assumes for most of the paper that a consumer's location in the second period is independent of their first period location.

