Commodity taxation in a differentiated oligopoly

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May 1992, revised January 1993

*We thank Firouz Gahvari, Maurice Marchand and the participants at seminars at GREMAQ (Toulouse), the University of Kansas, Universität Wien and Vanderbilt University, for useful comments.

Abstract

We introduce commodity taxation into a duopoly model of vertical product differentiation. Our setting differs from the existing literature in two respects. First, we consider price competition and, second, we allow for endogenous product selection. As usual, firms choose the *quality* of their products prior to making their pricing decisions. We first show that a *uniform* ad valorem tax, where the same *rate* applies to all variants of the product, lowers both equilibrium qualities and distorts the allocation of consumers between firms. More interestingly, we then establish that a number of familiar properties are no longer valid in our setting. In particular, it appears that the tax *lowers* the consumer prices of both variants and that the decrease in consumer prices is larger than the decrease in production costs brought about by the reduction in qualities. Even more surprisingly, it turns out that a small uniform ad valorem tax whose proceed are redistributed in a lump-sum way is always welfare-improving over the no-tax equilibrium.

Finally, we allow for the possibility of a non-uniform tax with different rates applying to different variants of the commodity. It turns out that depending on the parameter values such a differentiation of tax rates may or may not be desirable on welfare grounds. If a welfare improvement is possible through a non-uniform tax, it is always the high-quality variant which must be taxed at a higher rate.

1 Introduction

Commodity taxation and its impact on prices and welfare has been extensively studied in the public finance literature. Most of these studies deal with the incidence of a tax or the design of optimal tax policies within the framework of a competitive economy [see Auerbach (1987) and Kotlikoff and Summers (1987) for surveys]. Monopolies and monopolistic competition have also received some attention, especially in the tax incidence literature [see e.g. Atkinson and Stiglitz (1980), p. 206–213, and Lockwood (1990)]. However, very little attention has been devoted to the empirically relevant case of oligopolistic markets. And, the few exception that there are deal primarily with Cournot oligopolies; that is, they consider a setting with a homogeneous product where the firms' strategic variables are quantities [see Katz and Rosen (1983), Seade (1987), Stern (1987) and Dierickx *et al.* (1988) for issues of tax incidence and Myles (1987, 1989) for optimal taxation problems].

To the best of our knowledge, the impact of commodity taxation on the firms' product selection *in an oligopoly* has not been considered so far. Yet, this is an important issue since changes in product specification brought about by a commodity tax may affect welfare not only directly, but also indirectly through the determination of the equilibrium prices. This paper is an attempt to (partially) fulfill this gap. More precisely, we study commodity taxation in a duopoly model of vertical product differentiation [see Eaton and Lipsey (1989) for a recent survey about product differentiation]. Our setting differs in two respects from the existing studies. First, we assume that the firms' strategic variables are prices.[1] Second, we allow for endogenous product selection.[2] As usual, firms choose the *quality* of their products prior to making their pricing decisions. The explicit modeling of the firm's quality choice allows us to deal with two important issues. First, there is the question of how, if at all, commodity taxation will affect the equilibrium product selection. Specifically, one would want to know if the presence of the tax results in lower or in higher product qualities. Second, and more generally, it is interesting to investigate how endogenous product selection affects the impact of commodity taxes on prices and welfare and how this affects the design of an optimal tax policy.

Our model is based on a specification introduced by Mussa and Rosen (1978). We concentrate on an industry in which each of the two firms offers a single variant of a differentiated commodity. Firms choose qualities first and then prices. Production costs increase in quality. All consumers agree that a higher quality is better so that if all variants are sold at the same price everyone buys (at most) one unit of the one with the higher quality. Nevertheless, consumers have different preferences over qualities:

[1] Gual (1986) also considers price competition but in a different context.

[2] Kay and Keen (1983) do allow for endogenous product selection but consider different market structures. They determine the optimal structure of commodity taxes in a *competitive environment* where qualities are endogenous and in a horizontal differentiation model of *monopolistic competition.*

they differ in their *marginal willingness to pay for quality.* Following Mussa and Rosen, we assume that preferences are quasi-linear so that the marginal utility of income is constant and the same for all individuals.[3] This assumption allows us to neglect issues related to the distribution of income and to concentrate on efficiency aspects. It also implies that while our analysis seems to have a strong partial equilibrium flavor, it could easily be embedded into a simple general equilibrium model.[4]

Our results are as follows. We first show that a *uniform* ad valorem tax, where the same *rate* applies to all variants of the product, lowers both equilibrium qualities and distorts the allocation of consumers between firms. More interestingly, we then establish that a number of familiar properties that emerge in a competitive industry, or even in a duopoly with exogenously given products, are no longer valid in our setting. In particular, it appears that the tax *lowers* the consumer prices of both variants and that the decrease in consumer prices is larger than the decrease in production costs brought about by the reduction in qualities. Even more surprisingly, it turns out that a small uniform ad valorem tax whose proceed are redistributed in a lump-sum way is always welfare-improving. Hence, the standard result that a small *subsidy* to a Cournot oligopoly improves welfare is reversed in our setting.[5]

Finally, we allow for the possibility of a non-uniform tax with different rates applying to different variants of the commodity. It turns out that depending on the parameter values such a differentiation of tax rates may or may not be desirable on welfare grounds. If a welfare improvement is possible through a non-uniform tax, it is always the high-quality variant which must be taxed at a higher rate. Hence, with endogenous qualities, the case for higher taxes on the top variant of a product can be made on efficiency grounds alone.

While our results are to a large extend at odds with conventional wisdom in the theory of taxation, they do not appear to be mere model-specific curiosa. Our analysis indicates that taxation may affect the economy through channels which are not usually considered in the literature. In particular, it suggests that the impact of taxation on firm's quality choices may have unexpected effects. While the relative magnitudes (or even the signs) of the various effects may to some extend be model-specific, there is no reason to assume that they would vanish altogether in a different setting—as long,

[3]This approach to vertical differentiation differs from the specifications used by Gabszewicz and Thisse (1979) and Shaked and Sutton (1982) who assumes that all consumers have the same preferences but different income, with richer people being willing to pay more for quality than poor individuals.

[4]In general equilibrium models with imperfect competition, there is usually a circular relationship between a firm's price and quality decisions and its profits and demand, and this complicates the modeling substantially (see e.g. Hart (1985)). This problem does not arise here. Because preferences are quasi-linear there are no income effects and the demand for the differentiated commodity will not depend on firm's profits. However, not very much would be gained by such an exercise as it would result in much heavier notation. We therefore focus on a single market.

[5]See Besley (1989), Konishi, Okuna-Fujiwara and Suzumura (1990), and Besley and Suzumura (1992).

of course, as the model allows for endogenous product selection. Hence, we believe that the importance of our results goes beyond our specific model. At the very least, they show that the issue of commodity taxation under imperfect competition deserved further investigation.

2 The model

We consider a Mussa-Rosen type model of vertical product differentiation. The basic setting is as follows:

1. There is a continuum of consumers whose types are identified by $\theta \in [a, b] \subset \mathcal{R}_+$; θ is uniformly distributed over $[a, b]$. The (conditional) indirect utility of a consumer of type θ is given by:

$$V_\theta(p, q) = \begin{cases} y - p + k + \theta q & \text{if he buys one unit of a product of quality } q \\ & \text{at price } p \\ y & \text{otherwise,} \end{cases}$$

   where y is the consumer's income and k a constant large enough for all consumers to buy in equilibrium.[6] Note that θ is the marginal willingness to pay for quality.

2. There are two firms in the industry. Firm i offers a variant of quality $q_i \in \mathcal{R}_+$ at price p_i $(i = 1, 2)$. Without loss of generality, it is assumed that $q_1 \leq q_2$.

3. The marginal cost of firm i is independent of quantity (total cost is linear in quantity), but quadratic in quality: $MC(q_i) = cq_i^2$. Since the unit of quality is arbitrary it is chosen for $b - a$ to be equal to one.[7]

4. The government's instrument is an *ad valorem* tax (i.e., a commodity tax where the per-unit tax is proportional to the price level). First, a uniform tax, with the same rate applying to all the variants of the product, is considered. In section 5, this assumption is relaxed and the government is allowed to impose a variant-specific rate, t_i, which can be negative, as well as positive, so that a subsidy is also allowed. Formally, the tax rate faced by firm i is t_1 if $q_i < q_j$ and t_2 otherwise. Note that to impose the variant-specific tax, the government does not have to observe the *precise* qualities. It is sufficient to observe the *ranking* of the qualities.

[6] For simplicity in exposition, income is assumed to be the same for all consumers. The results of this paper hold true if income differs across individuals. The assumption that all consumers buy in equilibrium will be discussed in section 6.

[7] The constraint $q \geq 0$ can now be justified as follows. Since cost is quadratic it would be decreasing in quality for negative values of q_i, which does not make sense.

5. Let $\overline{\theta}$ be the index of the *marginal consumer*, who is indifferent between buying variant 1 at price p_1 and variant 2 at price p_2:

$$\overline{\theta} = \frac{p_2 - p_1}{q_2 - q_1}. \tag{1}$$

Consumers to the left of $\overline{\theta}$ buy from firm 1 while those to the right buy from firm 2. Therefore, the market demand for variant 1 is given by $D_1 = \overline{\theta} - a$ while the market demand for variant 2 is $D_2 = (1 + a) - \overline{\theta}$.[8] The profit of firm i is then given by

$$\pi_i = [(1 - t_i)p_i - MC(q_i)]D_i.$$

Define $\tau_i \equiv 1/(1 - t_i)$ and rewrite the profit as:

$$\pi_i = \frac{1}{\tau_i}[p_i - \tau_i MC(q_i)]D_i. \tag{2}$$

We use τ_i instead of t_i throughout the paper because this change of variable simplifies the presentation. Note also that equation (2) has an interesting implication. It shows that if $\tau_1 = \tau_2 = \tau$ the equilibrium with taxes and cost cq^2 is the same as the equilibrium that would prevail without taxes but with a cost function τcq^2. We use this property below, in particular when addressing the issue of existence of an equilibrium.

Finally, it is often convenient to consider the space of "preferred qualities" rather than the space of consumer types. The preferred quality of a consumer θ is defined as the value of q that maximizes his utility *if all potential variants are offered at marginal cost.* Formally,

$$q^{\theta} = \arg\max(\theta q - cq^2) = \frac{\theta}{2c}.$$

Preferred qualities are uniformly distributed over $[a/2c, (a + 1)/2c]$.

3 First best allocation

We consider a utilitarian social welfare function. In our setting this is equivalent to using total surplus (sum of consumer and producer surplus).[9] First, it is readily verified

[8] We assume $a < \overline{\theta} < a + 1$. This inequality is necessarily satisfied at the equilibrium since each firm can always choose a quality that guarantees to itself a positive demand. A complete specification of the payoff functions must, of course take into account the cases where this inequality is violated but nothing is gained here from such an exercise.

[9] This equivalence relies on the (traditional) assumption that the owners of the firms are among the consumers. The distribution of profits does not matter since utility is linear in income (with the same marginal utility for everyone).

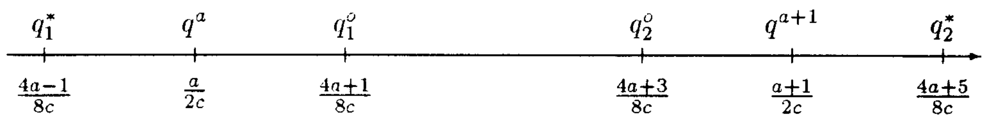

Figure 1: The interval of preferred qualities, optimal qualities (q_1^o and q_2^o), and the no-tax equilibrium (q_1^* and q_2^*)

that an optimal allocation of consumers between firms *given* q_1 *and* q_2 requires that all individuals with $\theta < \tilde{\theta}$ consume variant 1, while all individuals with $\theta > \tilde{\theta}$ consume variant 2, where

$$\tilde{\theta} = \frac{c(q_2^2 - q_1^2)}{q_2 - q_1} = c(q_1 + q_2) \tag{3}$$

is the index of the marginal consumer *when both variants are sold at marginal cost.* The optimal[10] qualities, q_1^o and q_2^o can then be obtained by solving the following problem:

$$\max_{q_1, q_2} \int_a^{\tilde{\theta}} (\theta q_1 - c q_1^2) d\theta + \int_{\tilde{\theta}}^{a+1} (\theta q_2 - c q_2^2) d\theta,$$

The solution is given by:

$$q_1^o = \frac{4a+1}{8c}, \qquad q_2^o = \frac{4a+3}{8c}, \qquad \tilde{\theta}^o = a + \frac{1}{2}, \tag{4}$$

where $\tilde{\theta}^o$ denotes the value of $\tilde{\theta}$ (as defined by (3)) corresponding to the optimal qualities q_1^o and q_2^o.

Figure 1 is helpful for understanding the intuition behind this solution. It represents the optimal qualities in the interval of preferred qualities. The length of this interval is $1/(2c)$ and, not surprisingly, the optimal qualities are "located" respectively at the first and third quartiles of this interval.

[10] Note that the optimum is derived *given the number of variants that are supplied.*

As a final remark it should be said that the optimum is not only characterized by the qualities, but also by the appropriate allocation of consumers between firms.[11] It follows that having q_1^o and q_2^o as an equilibrium outcome is *not* sufficient to guarantee optimality. An additional requirement is that the equilibrium prices are such that the "right" allocation of consumers between firms is achieved (i.e., $\overline{\theta} = \tilde{\theta}$). This also shows that, even though total demand for the product is perfectly inelastic, prices continue to affect welfare since they determine the allocation of consumers between firms.

4 The no-tax and the single tax rate equilibria

In this section it is assumed that $\tau_1 = \tau_2 = \tau$ because the government cannot, for informational or legal reason, use a non-uniform tax. Qualities and prices are determined at the subgame perfect equilibrium of the two stage game in which firms choose, first, qualities and, then, prices. As usual, the game is solved by backward induction. The actual derivation of the equilibrium is standard and is omitted here.[12] We derive the equilibrium for any $\tau > 0$ that is, for any tax rate less than 1. The no-tax equilibrium is obtained from these expression by setting $\tau = 1$. We concentrate on the equilibrium values of the variables which are relevant for our problem: q_1, q_2 and $\overline{\theta}$. Prices are also given. but their impact on welfare is already captured by $\overline{\theta}$ which, as known from (1), depends on prices.

Two cases must be distinguished: $a > 1/4$ and $a \leq 1/4$. The equilibrium values of the different variables are denoted by *.

Case 1: $a > 1/4$

The equilibrium of the quality game is unique and given by:[13]

$$q_1^* = \frac{4a-1}{8c\tau}, \qquad q_2^* = \frac{4a+5}{8c\tau}, \tag{5}$$

[11]Strictly speaking the specification of an allocation should also include the distribution of income among consumers. This can be ignored here since individual utility is linear in income and the social welfare function is additive.

[12]See Tirole (1988, ch. 6) for the zero cost case and Moorthy (1988) for the quadratic cost case when the whole market is not served. There is no taxation in these models, but it can easily be introduced by using the property that the equilibrium with a tax τ and a cost parameter c is the same as the one with a cost parameter τc and no tax. The existence issue is discussed below (see footnote 13).

[13]These qualities are derived by maximizing firm 1's profits on $[0, q_2]$ and firm 2's profits on $[q_1, +\infty[$ (there is only one solution to this problem). To prove that this is indeed an equilibrium one has to check that firm 1 cannot improve its profits by choosing a higher quality than q_2^* and similarly that firm 2 does not want to choose $q_2 < q_1^*$. For firm 1 this can be done by determining it's optimal quality given that $q_2 = q_2^* = (4a+5)/8c\tau$ and subject to $q_1 \geq q_2$. It is a straightforward exercise to show that the solution to this problem is $q_1 = (4a+7)/8c\tau$. The corresponding profit for firm 1 is $1/144c\tau^2$ which is less than $3/16c\tau^2$, its profit at q_1^*. Proceeding along the same lines for firm 2 completes the proof and shows that (q_1^*, q_2^*) is indeed a Nash equilibrium. (This also follows directly from Cremer and Thisse (1991)).

while the corresponding equilibrium prices are

$$p_1^* = \frac{1}{4c\tau}[a^2 - \frac{a}{2} + \frac{25}{16}], \qquad p_2^* = \frac{1}{4c\tau}[a^2 + \frac{5a}{2} + \frac{49}{16}], \tag{6}$$

so that

$$\overline{\theta}^* = a + \frac{1}{2}. \tag{7}$$

One striking feature of the equilibrium is that both (consumer) prices *decrease* with τ and, hence, with the tax rate. It is tempting to explain the price decrease by the fact that both qualities decrease, hence reducing unit production costs. This explanation is, however, incomplete since price decreases are larger than cost reductions. The additional reason is to be found in the narrowing of the quality gap, which intensifies price competition. In turn, firms choose to narrow their quality gap because the space of preferred qualities narrows down monotonically with c in the no-tax case, and with τ in the uniform tax case since the corresponding outcome is equivalent to the no-tax case when the marginal cost is τc.

Figure 1 can be used to compare the no-tax equilibrium (obtained by setting $\tau = 1$) to the optimal allocation. In the no-tax equilibrium both qualities are outside of the interval of "preferred qualities": the lower quality is too low while the higher quality is too high. As mentioned in section 3, for given qualities q_1 and q_2, an optimal allocation of consumers requires the marginal consumer to be at $\tilde{\theta} = c(q_1 + q_1)$ (see (3)). Using (5) this condition is satisfied *if and only if* $\tau = 1$, that is, if there is no tax. Hence, *a commodity tax distorts the allocation of consumers between firms.* Specifically, if $\tau > 1$ we have $\overline{\theta} > \tilde{\theta} = (a + 1/2)/\tau$ which implies that the market share of firm 1 is too large. A subsidy ($\tau < 1$), on the other hand, results in $\overline{\theta} < \tilde{\theta}$.

This result, together with the fact that both q_1^* and q_2^* are decreasing in τ (see (5)), imply that a tax affects welfare in three different ways:

1. it lowers the high quality q_2^* (a positive effect),
2. it lowers the low quality q_1^* (a negative effect),
3. it results in a misallocation of consumers between firms (a negative effect).

If qualities were exogenously given, the tax would be necessarily welfare-reducing. However, with endogenous qualities, the tax has one positive effect and there is some potential for welfare improvements. To further investigate this possibility, one can express social welfare as a function of τ:

$$S(\tau) \equiv \int_a^{a+\frac{1}{2}} (\theta q_1^* - c q_1^{*2})\, d\theta + \int_{a+\frac{1}{2}}^{a+1} (\theta q_2^* - c q_2^{*2})\, d\theta, \tag{8}$$

where q_1^* and q_2^* are given by (5).[14] The function $S(\tau)$ is strictly concave on $\mathcal{R}_+$. Differentiating (8) with respect to τ,

$$\frac{dS(\tau)}{d\tau} = -\frac{16a^2(\tau-1) + 16a(\tau-1) + 7\tau - 13}{3\tau^3 c}. \tag{9}$$

Setting this expression equal to zero and solving yields the *optimal uniform tax rate*

$$\tau^u = \frac{16a^2 + 16a + 13}{16a^2 + 16a + 7} \tag{10}$$

The following properties hold:

1. Since $dS(1)/d\tau = 6/(32c) > 0$, a "small" tax is *always* welfare improving over the no-tax equilibrium. To understand this result it is useful to discuss the relative strength of the three effects described above. First, the distortion in the allocation of consumers between firms (effect 3) is negligible when t rises from zero (i.e., τ rises from one). This is because the no-tax equilibrium achieves an optimal allocation of consumers (for given qualities) which implies that the change in $\overline{\theta}$ resulting from the tax has no first-order effect on welfare.[15] Thus, the overall welfare impact is only determined by the first effect (positive) associated with the decrease in q_2^* and by the second effect (negative) associated with the decrease in q_1^*. The first effect dominates since, as τ increases from one, q_2^* decreases faster than q_1^* (see (5)).

   This result is quite surprising and it further strengthens our claim that product differentiation does have a significant impact on the welfare properties of taxation. In Cournot competition with a homogenous product one typically obtains exactly the opposite result: a small *tax* reduces welfare while a small *subsidy* is welfare-improving. And, in Cournot models this property appears to be quite robust; in particular, it continues to hold for two stage games.[16]

[14] It is worth emphasizing that the specification of $S(\tau)$ relies on the assumption that the proceeds of the tax are redistributed among the consumers.

[15] To see it, note that the welfare loss resulting from the misallocation of consumers between firms is given by:

$$WL(\tau) \equiv \int_a^{(a+\frac{1}{2})\frac{1}{\tau}} (\theta q_1^* - c q_1^{*2})\, d\theta + \int_{(a+\frac{1}{2})\frac{1}{\tau}}^{a+1} (\theta q_2^* - c q_2^{*2})\, d\theta - S(\tau)$$

where $(a + 1/2)/\tau$ is the value of $\tilde{\theta}$ corresponding to q_1^* and q_2^*. It is readily verified that the first two terms on the right-hand side give the level of welfare that would be achieved if consumers were allocated optimally between firms (given qualities). Clearly $WL(1) = 0$ and, because $\tilde{\theta} q_1^* - c q_1^{*2} = \tilde{\theta} q_2^* - c q_2^{*2}$, we also have $dWL(0)/d\tau = 0$ so that an increase in τ from 1 has no first-order effect on WL

[16] See Besley and Suzumura (1992) who consider a two stage, homogenous product setting where the firms first choose their level of (cost-reducing) R&D investment, and then engage in Cournot competition.

2. The misallocation effect ceases to be negligible as τ rises, so that the welfare gain is reduced (this shows why $S(\tau)$ is concave). When τ is large enough this effect becomes dominant and further tax increases are welfare-reducing.

3. The optimal tax rate, $t^u = 1-1/\tau^u$, is monotone decreasing in a over $[1/4, \infty[$ with the limit values $t^u(1/4) = 1/3$ and $\lim_{a\to\infty} t^u(a) = 0$. Intuitively, this relationship can be understood as follows. First, the quality misallocation which occurs at the no-tax equilibrium, $(q_2^* - q_2^o) = (q_1^o - q_1^*) = 1/(4c)$, is independent of a. The reduction in q_2^* achieved by a given tax rate, however, increases with a (see (5)). The combination of these two facts tends to decrease the optimal tax rate. Second, the distortion in the allocation of consumers created by a given tax rate, is an increasing function of a.[17] Hence, the combined effect of an increase in a is to reduce t^u

4. Let $\overline{\tau} \equiv (4a+5)/(4a+3)$ be the value of τ which would result in $q_2^* = q_2^o$ (i.e., the higher quality is "brought down" to its optimal level). Clearly, $\tau^u < \overline{\tau}$ and $q_2^*(\tau_u) > q_2^o$ for all $a > 1/4$, so that the optimal uniform tax rate does not yield the optimal quality for firm 2. This property has an obvious interpretation which is related to a well-known result in the second best literature (see Lipsey and Lancaster (1956–7) for a pioneering contribution): The optimal tax is set by trading-off different distortions and it is not optimal to remove a single one completely, as this would be too costly in terms of the other distortions.

Case 2: $a \leq 1/4$

In this case, (5) is no longer the equilibrium of the two stage game since the value of the low quality would be negative. The equilibrium is now given by:

$$q_1^* = 0, \qquad q_2^* = \frac{a+2}{3c\tau}, \qquad \overline{\theta}^* = \frac{7a+5}{9}, \tag{11}$$

with the corresponding prices

$$p_1^* = \frac{(5-2a)(a+2)}{27c\tau}, \qquad p_2^* = \frac{5(a+2)^2}{27c\tau}. \tag{12}$$

Note that when $a = 1/4$ the expression given in (11) coincide with those in (5). For $a < 1/4$ firm 1 is at a corner solution and the constraint $q \geq 0$ is binding.

As in the previous case, we start by comparing the no-tax equilibrium to the optimum. It turns out that $q_1^* < q_1^o$ and $q_2^* > q_2^o$ continues to hold. However, it is no longer

[17] Formally we have:

$$\overline{\theta} - \hat{\theta} = \left(a + \frac{1}{2}\right)\left(1 - \frac{1}{\tau}\right).$$

true that the no-tax equilibrium results in an optimal allocation of consumers between firms. Using (11) and the index of the marginal consumer $\tilde{\theta}$ at the optimum, we see that $\tilde{\theta} = (a+2)/3 > \overline{\theta} = (7a+5)/9$ for any $a < 1/4$. Hence, a tax can now improve the allocation of consumers between firms. This is because the tax reduces $\tilde{\theta}$ while it leaves $\overline{\theta}$ unchanged. Accordingly, as long as $\tilde{\theta} > \overline{\theta}$ the tax reduces this distortion. If the tax becomes "too large" we return to the case $\tilde{\theta} < \overline{\theta}$ and a further increase in the tax rate increases the distortion. It thus appears that the welfare implications of a tax are quite different from those discussed in the previous case: the second effect (the reduction in q_1^*) does not occur, while the sign of the third effect (the allocation of consumers between firms) is reversed, at least initially.

To determine the optimal tax rate we maximize the social welfare function which for $a < 1/4$ is given by:

$$S(\tau) \equiv \int_a^{\frac{7a+5}{9}} (\theta q_1^* - c q_1^{*2})\, d\theta + \int_{\frac{7a+5}{9}}^{a+1} (\theta q_2^* - c q_2^{*2})\, d\theta, \tag{13}$$

with q_1^* and q_2^* given by (11). Differentiating (13) with respect to τ yields:

$$\frac{dS(\tau)}{d\tau} = -\frac{2(a+2)(\tau(8a^2+23a+14) - 6(a+2)^2)}{243 c \tau^3}, \tag{14}$$

so that the optimal tax rate is:

$$\tau^u = \frac{6(a+2)}{8a+7}. \tag{15}$$

The following properties are easily obtained:

1. By evaluating (14) at $\tau = 1$ it is seen that $\partial S(1)/\partial\tau > 0$. A small tax continues to be welfare-improving. This is not surprising since such a tax has only positive effects on welfare.

2. Let us define

$$\overline{\tau}^q = \frac{8(a+2)}{3(4a+3)},$$

that is, the value of τ for which $q_2^* = q_2^o$. Similarly, we define $\overline{\tau}^\theta$ as the value of τ resulting in $\tilde{\theta} = \overline{\theta}$:[18]

$$\overline{\tau}^\theta = \frac{3(a+2)}{7a+5}.$$

One can easily show the following inequality:

$$\overline{\tau}^\theta < \tau^u < \overline{\tau}^q$$

[18] Note in passing that $\overline{\tau}^\theta < \overline{\tau}^q$ which implies that as the tax increases $\tilde{\theta} = \overline{\theta}$ is realized before $q_2^* = q_2^o$.

The intuition behind this relation is clear. As long as $\tau < \overline{\tau}^{\theta}$ a tax increase is necessarily welfare improving, while for $\tau > \overline{\tau}^{q}$ any tax increase reduces welfare (since it creates two negative effects). Therefore, the optimal tax rate can only be in the range $[\overline{\tau}^{\theta}, \overline{\tau}^{q}]$ where there is a tradeoff between the positive and the negative effect.

The main results of this section is summarized in the following proposition.

Proposition 1 *Consider the vertically differentiated duopoly described in section 2 and a commodity tax whose proceeds are redistributed among the consumers. Further assume that both variants are taxed at the same rate. Then,*
(i) an increase in the tax rate decreases the qualities of both variants if $a > 1/4$ while it decreases the higher quality and leaves the lower quality unchanged if $a \leq 1/4$;
(ii) an increase in the tax rate decreases the (consumer) prices of both variants for all values of a;
(iii) a small increase in the tax rate from zero is welfare-improving for all values of a.

5 Non-uniform taxes

We have seen in the previous section that welfare can be improved by imposing a uniform tax. This welfare improvement arises because the tax can affect the equilibrium product selection which is inefficient at the no-tax equilibrium. In this section we investigate whether an additional welfare improvement can be achieved by using a non-uniform tax and, if yes, which variant should be taxed at a higher rate.

The government now has two instruments, τ_1 and τ_2. Therefore, one might be tempted to think that the optimal policy is trivial and simply consists in choosing τ_1 and τ_2 to induce the optimal qualities as an equilibrium. We start by showing that this is not true. The argument uses the following lemmas which provide the (candidate) equilibrium qualities as a function of the tax rates for the cases $a \geq 1/4$ and $a < 1/4$, respectively. Both are proved in Appendix 1.

Lemma 1 *Assume $a > 1/4$. If an equilibrium in pure strategies exist, the equilibrium qualities are given by*

$$q_1^*(\tau_1, \tau_2) = \begin{cases} \dfrac{a}{6c\tau_1} + \dfrac{5\tau_1 - 2\tau_2}{12c\tau_1(\tau_2 - \tau_1)} - \dfrac{\sqrt{D}}{12c\tau_1(\tau_2 - \tau_1)} & \text{for } \tau_1 \neq \tau_2 \\ \dfrac{4a-1}{8c\tau_1} & \text{for } \tau_1 = \tau_2 \end{cases} \tag{16}$$

and

$$q_2^*(\tau_1,\tau_2) = \begin{cases} \dfrac{a}{6c\tau_2} + \dfrac{7\tau_2 - 4\tau_1}{12c\tau_2(\tau_2-\tau_1)} - \dfrac{\sqrt{D}}{12c\tau_2(\tau_2-\tau_1)} & \text{for } \tau_1 \neq \tau_2 \\ \dfrac{4a+5}{8c\tau_2} & \text{for } \tau_1 = \tau_2, \end{cases} \tag{17}$$

where

$$D = 4a^2(\tau_1^2 - 2\tau_1\tau_2 + \tau_2^2) + 8a(2\tau_1^2 - 2\tau_1\tau_2 - \tau_2^2) + 16\tau_1^2 - 11\tau_1\tau_2 + 4\tau_2^2. \tag{18}$$

Lemma 2 *Assume $a \leq 1/4$. If an equilibrium in pure strategies exist, the equilibrium qualities are given by:*

$$q_1^* = 0, \quad \text{and} \quad q_2^* = \frac{a+2}{3c\tau_2}. \tag{19}$$

These lemmas are then used in Appendix 2 to prove Proposition 2 which shows that the optimal qualities can *never* be achieved as an equilibrium.

Proposition 2 *Whatever a and c, it is never possible to find τ_1 and τ_2 such that (q_1^o, q_2^o) is achieved as an equilibrium.*

The idea behind the proof is that the system of two equations in (τ_1, τ_2) obtained by setting (16) equal to q_1^o and (17) equal to q_2^o does not have a solution.[19] In any event, even if it were possible to achieve the optimal qualities this may not be desirable if the tax distorts the allocation of consumers between firms.

Even though the first-best is not achievable with two tax rates, some welfare gains from using a non-uniform tax can be expected. To see this, consider first the case where $a > 1/4$. We have shown that a uniform tax lowers the qualities of both variants, but that only the decrease in the quality of variant 2 is socially desirable. It seems therefore reasonable to conjecture that it is optimal to tax variant 2 at a higher rate than variant 1 in order to achieve a more substantial decrease in q_2 relative to the decrease in q_1. One may even think of taxing variant 2 while subsidizing variant 1, since this could bring about both a decrease in q_2 and an increase in q_1.

In spite of its intuitive appeal, this conjecture might not be true for the following reasons. First, it has to be shown how a non-uniform tax affects existence of an equilibrium. Since the argument used in the previous section does not work with non-uniform taxes, there is at least some potential for non-existence and, as will become clear below, this may restrict the ability of the government to differentiate tax rates. Second, a non-uniform tax is likely to introduce further distortions in prices and, hence, in the allocation of consumers between firms. Third, it must be checked how $\tau_1 \neq \tau_2$ does affect the equilibrium and whether the expected effects on qualities are obtained. These arguments call for a more formal analysis, using the expressions provided in Lemma 1. This is done in the next subsection.

[19] To be precise, this argument proves the result for $a > 1/4$. For $a \leq 1/4$ one has to use (19) and the proof is trivial.

Case 1: $a > 1/4$

The expressions in Lemma 1 have been derived as follows. The value q_1^* is calculated in such a way that it maximizes firm 1's profits over $[0, q_2^*]$ while q_2^* maximizes firm 2's profits over $[q_1^*, \infty[$.[20] From the fact that these are only necessary conditions and from a simple examination of (16) and (17), we can see that there are (at least) two reasons for which a (pure strategy) equilibrium in the quality game may not exist.[21]

1. If the difference between the two tax rates becomes large it is possible that q_2^* is only a local maximum and that firm 2 would want to select a lower quantity than firm 1. In this case, it is readily verified that there is no equilibrium.

2. The two functions are only defined if $D \geq 0$. By studying the sign of (18) it is seen that D is nonnegative if τ_2/τ_1 is not "too large"; for instance, if $a = 1$ $D \geq 0$ if and only if $\tau_2 \leq (4/3)\tau_1$.

Clearly, $D < 0$ implies that the firms' best reply curves in qualities do not intersect. It can be seen from Appendix 2 that this problem is responsible for the impossibility result stated in Proposition 2: the requirement $D \geq 0$ prevents a differentiation of tax rates that would be sufficient to achieve the optimal qualities.

Since either of the two problems can arise if tax rates are sufficiently different, the ability of the government to differentiate tax rates is restricted. Not surprisingly, this complicates the optimal taxation problem substantially and the derivation or even characterization of the optimal values of τ_1 and τ_2 become (at least analytically) intractable. However, since an equilibrium necessarily exists when $\tau_1 \approx \tau_2$ the possibility of welfare improving tax changes starting from $\tau_1 = \tau_2$ can be investigated without encountering the non-existence problem. The functions q_1^* and q_2^* given by (16) and (17) being continuously differentiable at $\tau_1 = \tau_2$, we can start by studying their derivatives evaluated at $\tau_1 = \tau_2 = \tau$.[22] and check whether we get the expected effects of tax changes on qualities. They are given by

$$\frac{\partial q_1^*}{\partial \tau_1} = -\frac{16a^2 + 64a - 17}{96c\tau^2} < 0 \qquad (20a)$$

$$\frac{\partial q_1^*}{\partial \tau_2} = \frac{16a^2 + 16a - 5}{96c\tau^2} > 0 \qquad (20b)$$

[20] More details, including a formal definition of the relevant profit functions, are provided in Appendix 1

[21] We rule out the possibility of a quality equilibrium in mixed strategies an the ground of realism.

[22] These derivatives are calculated by deriving the expression valid for $\tau_1 \neq \tau_2$ with respect to τ_i and then taking the limit of this derivative for $\tau_1 \to \tau_2$. Since the functions are continuously differentiable this is a legitimate way to proceed.

$$\frac{\partial q_2^*}{\partial \tau_1} = -\frac{16a^2 + 16a - 5}{96c\tau^2} < 0 \tag{20c}$$

$$\frac{\partial q_2^*}{\partial \tau_2} = \frac{16a^2 - 32a - 65}{96c\tau^2} \gtrless 0 \tag{20d}$$

The signs of the expressions are given for $a > 1/4$. Expression (20a) has the expected sign. The intuition behind (20b) is less obvious, but it also suggests that the conjecture could be valid (a decrease in τ_1 would tend to decrease q_1^* and this would be further reinforced by an increase in τ_2). The remaining two expressions are more problematic. Equation (20c) suggests that a decrease in τ_1 tends to increase q_2^* which is not the desired effect. Finally, the sign of (20d) depends on a. It is negative (and has the expected sign) if $a < 13/4$ while it is positive otherwise. To sum up, these results provide some indication that the conjecture might be valid (at least in the neighborhood of a uniform tax) but they also show that a more detailed analysis is required.

We redefine the social welfare function as:

$$S(\tau_1, \tau_2) = \int_a^{\overline{\theta}^*} (\theta q_1^* - c q_1^{*2})\, d\theta + \int_{\overline{\theta}^*}^{a+1} (\theta q_2^* - c q_2^{*2})\, d\theta, \tag{21}$$

where $\overline{\theta}^*$ is the index of the marginal consumer.[23] We can then derive the expressions for the change in social welfare due to changes in the tax rates. These expression are in general very complicated and long but they are much simpler when evaluated at $\tau_1 = \tau_2 = \tau$:[24]

$$\frac{\partial S}{\partial \tau_1} = -\frac{(4a(\tau - 1) + \tau + 1)(4a - 1)}{64c\tau^3}, \tag{22a}$$

$$\frac{\partial S}{\partial \tau_2} = -\frac{(4a(\tau - 1) + 3\tau - 5)(4a + 5)}{64c\tau^3}. \tag{22b}$$

It follows that:

$$\frac{\partial S}{\partial \tau_1} < 0 \qquad \text{if} \quad \tau > \frac{4a - 1}{4a + 1} \tag{23a}$$

and

[23] The complete expression for $\overline{\theta}^*$ as a function of τ_1 and τ_2 is very long. For the sake of our calculations it was sufficient to express it as a function of q_1^* and q_2^* which depend themselves on the two tax rates. One has

$$\overline{\theta}^* = \frac{c[\tau_2 (q_2^*)^2 - \tau_1 (q_1^*)^2] + (2a + 1)(q_2^* - q_1^*)}{3(q_2^* - q_1^*)}.$$

[24] The expressions also reflect the impact of the change in the marginal consumer. This is because if $\tau \neq 1$, a change in the marginal consumer does have a (first order) effect on welfare since we start from a situation where the allocation of consumers between firms is not optimal (see section 4).

(23b)

$$\frac{\partial S}{\partial \tau_2} < 0 \qquad \text{if} \quad \tau > \frac{4a+5}{4a+3} \tag{23c}$$

Some simple manipulations lead to:

$$\frac{4a-1}{4a+1} < 1 < \tau^u \leq \frac{4a+5}{4a+3}, \tag{24}$$

where τ^u is the optimal uniform tax as given in expression (10). If $a > 1/4$, the last inequality in (24) is also strict. The following proposition then follows directly from (23a), (23c) and (24)

Proposition 3 *Assume $a > 1/4$. A welfare improvement is achieved starting from $\tau_1 = \tau_2 = \tau$ with*

$$\frac{4a-1}{4a+1} < \tau \leq \frac{4a+5}{4a+3},$$

by marginally increasing τ_2 or decreasing τ_1 or by a combination of these two policies. In particular such a welfare improvement is possible starting from $\tau_1 = \tau_2 = 1$, the no-tax equilibrium, or from $\tau_1 = \tau_2 = \tau^u$, the equilibrium achieved with the optimal uniform tax rate.

This proposition shows that local welfare improvements can be achieved starting from a large set of uniform tax rates by imposing a higher tax rate on the high quality variant. In particular, this result holds at two interesting points namely, the no-tax equilibrium and the equilibrium achieved under an optimal uniform tax. Since τ^u results in a higher value of social welfare than any other uniform tax the proposition also implies, that for $a > 1/4$ a non-uniform tax scheme is always desirable. Roughly speaking Proposition 3 says that the top variant of a product should be taxed more heavily than the basic variant. Interestingly, this result arises solely for reasons of *efficiency* and does not rely on *equity* consideration. Recall, indeed, that there are no benefits to redistribution in our setting.

Thus, our conjecture appears to go through, even though the differential tax may not have the desired impact on q_2^* (see expressions (20c)–(20d) and the discussion provided there). The increase in q_1^* (see (20a)–(20b)) turns out to be the dominant effect on welfare, and its positive impact outweighs any other distortion that a tax rate differentiation might introduce. It remains to be seen whether similar results can be obtained for $a < 1/4$.

Case 2: $a \leq 1/4$

When $a \leq 1/4$, a uniform tax does not affect the choice of the low quality since $q_1^* = 0$. A non-uniform tax could be desirable if it could bring about an equilibrium where $q_1 > 0$

(for instance by subsidizing variant 1) or if it could be used to correct the distortion in the allocation of consumers between firms that exists both in the no-tax as well as in the uniform tax equilibrium (see section 4).

The candidate equilibrium point for this case is given by (19) in Lemma 2. Note that this expression is identical to (11), the single tax rate equilibrium, with τ_2 playing the same role in (19) as τ in (11). Existence can be established for $\tau_2 \geq \tau_1$, but when $\tau_2 < \tau_1$ non-existence arises when the tax differential becomes large enough for firm 1 to prefer a quality higher than q_2^* (for q_2^* given by (19)).

Using Lemma 2, it is easy to show that the marginal consumer in equilibrium (if any) is given by:

$$\bar{\theta} = \frac{7a + 5}{9}$$

which is independent of both tax rates. It then follows that the only role played by τ_1 is that it can generate non-existence of a quality equilibrium. When such equilibrium exists, it is independent of τ_1 and identical to the outcome obtained with a uniform tax of $\tau = \tau_2$. Hence, for $a < 1/4$ no welfare improvement can be achieved by using a non-uniform tax, and the best the government can do is to adopt the optimal uniform tax rate τ^u.[25] This establishes the following proposition.

Proposition 4 *Assume $a \leq 1/4$. The social welfare function (21) is maximized at* $\tau_1 = \tau_2 = \tau^u = 6(a+2)/(8a+7)$.

Intuitively speaking this result is somewhat surprising, though not completely unexpected. Recall indeed that with $a > 1/4$ the welfare improving impact of a tax rate differentiation was closely related to its (beneficial) effect on the lower quality. Now, for small values of a the lower quality under uniform taxation is given by a corner solution: $q_1^* = 0$. So, there is little hope that a increase in q_1^* (and hence a positive impact on welfare) could be achieved through a *small* tax rate differentiation. The more surprising part of the result is that this corner solution will prevail (as long as an equilibrium exists) no matter how large the difference between tax rates is.

6 Concluding remarks

We have so far considered only a particular type of commodity taxation: an ad valorem tax. Another type of taxation which is empirically relevant and which has been studied extensively in competitive and monopolistic frameworks, is a quantity tax or excise tax. This form of taxation is characterized by the fact that the *per unit* tax is constant and independent of the price. It is well known that both types of taxes are equivalent in competitive markets, while they have different implications in a monopoly. The

[25] To be precise, a non-uniform tax can do as well as the optimal uniform tax (provided that an equilibrium exists), *but it cannot do any better.*

following two questions thus arise quite naturally. First, one would want to know how, if at all, our results would change if a quantity tax were considered instead of the ad valorem tax. Second, it appears interesting to investigate which of these two types of taxation is the "best" instrument in our setting. Even though a detailed study of these issues is too long to be given here, we can gain some insight from the following partial results.

First, it is easy to show that a *uniform* quantity tax has no impact on equilibrium qualities and on welfare.[26] If the government's choice is restricted to uniform tax schemes it is thus clear that an ad valorem tax is better. The welfare-superiority of an ad valorem tax over a per-unit tax which has been established for monopolies and Cournot oligopolies (see Suits and Musgrave (1953) and Delipalla and Keen (1992)) thus continues to hold in our setting. Interestingly, however, the result arises for completely different reasons. In the monopoly and Cournot oligopoly cases, the ad valorem tax is better because it creates *less distortions*. In our setting, the ad valorem tax dominates because it creates *more* distortions. It does affect product selection and the allocation of consumers between firms, while the quantity tax basically acts like a lump-sum tax.

A *non-uniform* quantity tax on the other hand does affect the equilibrium qualities. Let u_1 denote the per unit tax on variety 1 and u_2 the per unit tax on variety 2. One can show that the equilibrium qualities depend only on $u_1 - u_2$ and not on the values of u_1 and u_2 independently. In other words, only the difference between the per unit taxes matters. Assuming $a > 1/4$ it can then be established that starting from $u_1 = u_2 = 0$, a local welfare improvement can be achieved by taxing the high quality variant alone or equivalently by imposing a non-uniform tax with $u_2 > u_1$. There is thus again a case for non-uniform taxation. A more precise comparison of non-uniform quantity and ad valorem taxes is analytically untractable and would in any event go well beyond the scope of this paper.

Our final comment is about the assumption that every consumer buys the good in equilibrium. Under this assumption the allocational role of prices is merely to determine which consumer buys which variant of the commodity. Hence, an increase in both prices which does not change the index of the marginal consumer has no impact on welfare. If, however, reservation prices were lower so that some consumers would not buy in equilibrium, prices would have the additional role to determine the size of the market. In that case, a price increase that leaves the index of the marginal consumer unchanged *does* decrease welfare because it results in less consumers buying the commodity. Incorporating these considerations into our setting would substantially complicate the analysis, especially in the non-uniform tax case. However, based on our result that prices *decrease* when a tax is imposed, one would not expect this to affect our main results. Specifically, one may conjecture that the welfare improving property of a uniform

[26] Prices do increase, but since both prices increase by the same amount, the allocation of consumers between firms is not affected.

tax would be reinforced rather than being reversed because the increase in market size would create an additional positive effect on welfare. While this conjecture provides us with some confidence in the robustness of our results it is clear that incorporating an endogenous market size is the natural next step in the study of commodity taxation in differentiated oligopolies.

References

Atkinson, A.B. and J.E. Stiglitz (1980) *Lectures on Public Economics.* New-York: McGraw-Hill.

Auerbach, A. (1985) "The theory of excess burden and optimal taxation," in *Handbook of Public Economics Vol 1*, ed. Auerbach, A. and M. Feldstein. Amsterdam: North-Holland, 61–127.

Besley, T. (1989) "Commodity taxation and imperfect competition: A note on the effects of entry," *Journal of Public Economics*, **40**, 359–367.

Besley, T. and K. Suzumura (1992) "Taxation and welfare in an oligopoly with strategic commitment," *International Economic Review*, **33**, 413–431.

Cremer, H. and J.-F Thisse (1991) "Location models of horizontal differentiation: A special case of vertical differentiation models," *Journal of Industrial Economics*, **39**, 383–390.

Delipalla, S. and M. Keen (1992) "The comparison between ad valorem and specific taxes under imperfect competition," *Journal of Public Economics*, **49**, 351–368.

Dierickx, I., C. Matutes and D. Neven (1988) "Indirect taxation and Cournot equilibrium," *International Journal Industrial Organization*, **6**, 385–399.

Eaton, B.C. and R.G. Lipsey (1989) "Product differentiation," in *Handbook of Industrial Organization Vol 1*, ed. Schmalensee R. and R.D. Willig. Amsterdam: North-Holland, 723–768.

Gabszewicz, J. and J.-F. Thisse (1979) "Price competition quality and income disparities," *Journal of Economic Theory*, **20**, 340–359.

Gual, J. (1986) "A note on the impact of the value added tax on a differentiated duopoly," mimeo, University of California, Berkeley.

Hart, O. (1985) "Imperfect competition in General equilibrium: An overview of recent work," in *Frontiers of Economics*, ed. Arrow, K and S. Honkapohja. Oxford: Basil Blackwell, 221-256.

Katz, M. and H. Rosen (1985) "Tax analysis in an oligopoly model," *Public Finance Quarterly*, **13**, 3–19.

Kay, J.A. and M.J. Keen (1983) "How should commodities be taxed?," *European Economic Review*, **23**, 339–358.

Konishi, H., M. Okuno-Fujiwara and K. Suzumura (1990) "Oligopolistic competition and economic welfare: A general equilibrium analysis of entry regulation and tax-subsidy schemes," *Journal of Public Economics*, **42**, 67–88.

Kotlikoff, L. and L. Summers (1987) "Tax incidence," in *Handbook of Public Economics Vol 2*, ed. Auerbach, A. and M. Feldstein. Amsterdam: North-Holland, 1043–1092.

Lipsey, R. and K. Lancaster (1956–7) "The general theory of second best," *Review of Economic Studies*, **24**, 11–32.

Lockwood, B. (1990) "Tax incidence, market power, and bargaining structure," *Oxford Economic Papers*, **42**, 187–209.

Moorthy, S. (1988) "Product and price competition in a duopoly," *Marketing Science*, **7**, 141–168.

Mussa, M. and S. Rosen (1978) "Monopoly and product quality," *Journal of Economic Theory*, **18**, 301–317.

Myles, G. (1987) "Tax design in the presence of imperfect competition: An example," *Journal of Public Economics*, **34**, 367–378.

Myles, G. (1989) "Ramsey rules for economies with imperfect competition," *Journal of Public Economics*, **38**, 95–115.

Seade, J. (1987) "Profitable cost increases and the shifting of taxation: Equilibrium responses of markets in oligopoly," mimeo.

Shaked, A. and J. Sutton (1982) "Relaxing price competition through product differentiation," *Review of Economic Studies*, **49**, 3–13.

Stern, N. (1987) "The effects of taxation, price control and Government contracts in oligopoly and monopolistic competition," *Journal of Public Economics*, **32**, 133–158.

Suits, D.B. and R.A. Musgrave (1953) "Ad valorem and unit taxes compared," *Quarterly Journal of Economics*, **67**, 598–604.

Tirole, J. (1988) *The Theory of Industrial Organization*. Cambridge, Massachusetts: The MIT Press.

Appendices

A Proof of Lemmas 1 and 2

As usual, the game is solved by backward induction. We thus start with the second stage of the game where firms chose prices given qualities. We use the index e to refer to the values of the different variables at the equilibrium of this stage. Straightforward algebra shows that the price equilibrium is given by

$$p_1^e(q_1, q_2; \tau_1, \tau_2) = \frac{2\tau_1 c q_1^2 + \tau_2 c q_2^2 + (1-a)(q_2 - q_1)}{3},$$

$$p_2^e(q_1, q_2; \tau_1, \tau_2) = \frac{\tau_1 c q_1^2 + 2\tau_2 c q_2^2 + (a+2)(q_2 - q_1)}{3},$$

and the index of the corresponding marginal consumer is[27]

$$\overline{\theta}^e = \frac{\tau_2 c q_2^2 - \tau_1 c q_1^2 + (2a+1)(q_2 - q_1)}{3(q_2 - q_1)}.$$

Equilibrium profits are given by

$$\pi_1^e(q_1, q_2; \tau_1, \tau_2) = \frac{[\tau_2 c q_2^2 - \tau_1 c q_1^2 + (1-a)(q_2 - q_1)]^2}{9(q_2 - q_1)}, \tag{A1}$$

$$\pi_2^e(q_1, q_2; \tau_1, \tau_2) = \frac{[\tau_1 c q_1^2 - \tau_2 c q_2^2 + (a+2)(q_2 - q_1)]^2}{9(q_2 - q_1)}. \tag{A2}$$

We can now consider the first stage of the game. If both qualities are *strictly* positive at a subgame perfect Nash equilibrium they must satisfy $\partial \pi_i^e / \partial q_i$ for $i = 1, 2$. Using (A1) and (A2) these conditions can be written as

$$2(q_2 - q_1)[2\tau_1 c q_1 + (1-a)] = [\tau_2 c q_2^2 - \tau_1 c q_1^2 + (1-a)(q_2 - q_1)] \tag{A3}$$

$$2(q_2 - q_1)[-2\tau_2 c q_2 + (a+2)] = [\tau_1 c q_1^2 - \tau_2 c q_2^2 + (a+2)(q_2 - q_1)]. \tag{A4}$$

Combining these expressions one can show that if $\tau_1 \neq \tau_2$ the system has two solutions vectors (q_1, q_2), but that only one of them satisfies the second order conditions for both firms. This solution is given by the first expressions in (16) and (17). Note that if $\tau_1 = \tau_2$ both sides of (A3) and (A4) can be divided by $(q_2 - q_1)$ and the solution is considerably simplified.

[27] We assume $q_2 \neq q_1$ and $a < \overline{\theta} < a + 1$. We have, of course, taken the cases into account where either of these condition is violated. Nothing would be gained by entering into those details here

It is now easy to check that q_1^* as given by (16) is strictly positive if and only if $a > 1/4$. Hence, (q_1^*, q_2^*) given by (16) and (17) is the only candidate for an *interior* equilibrium. To complete the proof of Lemma 1 it is now sufficient to rule out the possibility of a corner solution for $a > 1/4$. This is easily done by showing that the best reply of firm 2 to $q_1 = 0$ is $q_2 = (a+2)/(3c\tau_2)$ and that the best reply of firm 1 to this value of q_2 is always strictly positive if $a > 1/4$.

Lemma 2 now follows immediately. For $a \leq 1/4$ the right hand side of (16) is negative (zero for $a = 1/4$). This rules out an interior equilibrium. To complete the proof, it is the sufficient to show that firm 1's best reply to $q_2 = (a+2)/(3c\tau_2)$ is $q_1 = 0$.

B Proof of Proposition 2

If $a \leq 1/4$ the proof is trivial and follows directly from Lemma 2. For the case of $a > 1/4$ we have to show that the system of two equations in (τ_1, τ_2)

$$\begin{cases} q_1^*(\tau_1,\tau_2) = \dfrac{4a+1}{8c} \\ q_2^*(\tau_1,\tau_2) = \dfrac{4a+3}{8c} \end{cases} \tag{A5}$$

obtained by setting (16) equal to q_1^o and (17) equal to q_2^o does *not* have a solution.

We use the fact that (16) and (17) give one of the solution to the system of equations (A3)–(A4). Hence, a *necessary* condition for a vector (τ_1, τ_2) to be a solution to (A5) is that it solves (A3)–(A4) in which q_i's are replaced by q_i^o's. Simple algebra shows that the so-obtained system of equations has a *unique* solution given by

$$\tau_1 = \frac{20a+1}{3(4a+1)}, \qquad \tau_2 = \frac{20a+19}{3(4a+3)}. \tag{A6}$$

Hence, *if* there is a solution to (A5) then it must be (A6). A simple substitution then shows that the τ_i's given in (A6) *do not satisfy (A5)*. It follows that (A5) does not have a solution and this completes the proof of Proposition 2.