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Building Reputation for Contract Renewal: Implications for Performance Dynamics and Contract Duration

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Abstract

We study how career concerns affect the dynamics of incentives in a multi-period contract, when the agent's productivity can evolve exogenously (random shocks) or improve endogenously through investment.

We show that incentives are stronger and performance is higher when the contract approaches its expiry date. Contrary to common wisdom, long-term contracts may strengthen reputational effects whereas short-term contracting may be optimal when investment has persistent, long-term effects.

Keywords: Career concerns, contract duration, contract renewal, reputation and dynamic incentives.

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1 Introduction

In January 2006, the university canteen for staff members of an Italian Economics Department was serving overcooked pasta and oily tomato sauce. The canteen was run by a private contractor on a 5-year contract, expiring in December 2006. From November 2006, higher standards were observed; the pasta being cooked *al dente* and some delicious sauces being served. There were rumors that a new chef had been hired. In the following December the university decided to renew the contract with the catering company, but the higher standards did not last: by the end of January 2007, the quality of the food had gone back to its initial level.

Similar features have been observed in other sectors such as sports or public procurement. In professional basketball, using data on individual performance (points scored, rebounds, assists, etc.) covering the 1980s and 1990s, Stiroh (2007) found that performance improved significantly in the year before signing a multi-year contract, and declined after the contract was signed.[1] The same pattern was noted in professional baseball.[2] In the UK railway industry, using a panel of the 25 franchisees providing passenger services in 1997-2000, Affuso and Newbery (2000) found that contractors' voluntary investment in rolling stocks increased – presumably leading to better service – towards the end of the contracts. And in the French water industry, Chong, Huet and Saussier (2006) found that contracts near expiry date had lower prices compared to other contracts, all things being equal.[3]

In this paper, we first provide a rationale for such performance patterns;[4] it is based on information decay, due e.g. to random productivity shocks. For example, a catering company using natural ingredients may benefit from a change in market attitude towards organic food; an athlete may have an accident that affects his ability to play; a private contractor may experience

[1]He also found that players were strongly rewarded for improvements in performance in their contract year: a one-point increase in a player's scoring average, for example, was associated with an annual salary increase of over $300,000. Sen and Rice (2008) document a similar pattern, using performance and contract data from a panel of NBA players from 1991 to 2006.

[2]Using a sample of free agent contracts signed in the US between 2000 and 2004, baseball analysts observed that, with the exception of superstar players who had already established themselves as perennial caliber players, hitting players' performance was exceeding expectations in the last year of their contract and declining drastically in the following three years. See http://baseballanalysts.com/archives/2006/03/longterm_free_a.php and also http://expertvoices.nsdl.org/cornell-info204/2010/03/10/winners-curse-in-mlb-free-agency/

[3]While the study relies on a cross-section analysis of 1102 French local public authorities in 2001, their finding suggests that operators raise the performance of their public-service provision, and thus the payoff of the public authority, by reducing their prices as the expiry date of the contract approaches.

[4]The labour literature has provided ample empirical evidence and rationale as to why performance may be higher (or absenteeism may be lower) before tenure or promotion, but lower afterwards (see e.g. Lazear, (2004), Ichino and Riphahn (2005)). Less attention, however, has been devoted to performance patterns within the pre-promotion or pre-tenure period.

improved productivity thanks to technological change; and so on. Because of such information decay, recent performance provides more information than past performance about the agent's (current) productivity. This in turn induces the agent to improve his performance when a new contract is to be signed, e.g. by investing (hiring a new chef for a catering company, adopting a better training technique for an athlete, investing in a new technology for a company, and so on), or by exerting effort (spending more time in the kitchen for a chef, undertaking more intense training for an athlete, hiring temporary staff for a company, and so on).

To capture this insight in as simple way, we consider in Section 2 a baseline model in which an agent provides a good or service for two periods, followed by competition for the agent's services in the next period. In each period, the agent's performance is observable but nonverifiable and depends on his unobservable innate productivity (the agent's "type"). Over time, this productivity may change exogenously (random shock), or endogenously through nonverifiable investment.

In this setting, the agent cannot be explicitly incentivized to invest, as performance and investment are nonverifiable, but career concerns create an implicit incentive, as by providing a good performance the agent enhances his future market value. Yet, due to information decay, the agent never invests until the last period of the contract; in that period, however, the agent invests if needed when information is sufficiently persistent (so that current performance provides a good enough signal of future productivity and performance), the future matters (that is, the agent's discount factor is high enough) and the investment is not too expensive. As a result, expected performance improves over time during the contract.

We show that these insights apply as well when the agent interacts with a single buyer, as it is often the case in public procurement.[5] We then consider the case where the agent can also exert effort to improve current performance. In the second period, the agent favours effort over investment whenever the former is cheaper; this, in turn, creates incentives to invest in the first period, although expected performance still weakly improves over time. Thus, for products or services with large potential for performance improvement, such as catering services, public services, consultancy, or sports, both long-term investment and short-term effort can take place – the former at the beginning of a contract and the latter towards the end.

In the second part of the paper, we derive the implications of these insights for optimal contract duration. We consider an infinite repetition of our baseline model. Increasing the

[5] This links our paper to the repeat-purchase mechanism for experience goods pioneered by Klein and Leffler (1981). The incentives generated by contract renewal were initially investigated by Taylor and Wiggings (1997).

duration of the contracts has then two opposite effects on performance: on the one hand, it reduces the frequency of investment, because as before the agent only invests in the last period of a contract; on the other hand, it increases the intensity of the investment in these last periods, because it limits the crowding out effect of future investments. The optimal contract duration depends on the relative importance of these two effects: for example, longer contracts are desirable when it is important to foster the intensity of investment in the last contracting periods, which is the case when investment is costly, information decay is high, or the weight given to the future (as measured by the discount factor) is low. Thus, contrary to common wisdom, long-term contracting is optimal when there is a need for greater implicit incentives, whereas short-term contracting is optimal when noncontractible investment has persistent, long term effects.

Our paper incorporates information decay in a multi-period contract with career concerns. In Holmström (1982), the agent exerts effort to increase performance, in an attempt to influence market beliefs about his type. However, whilst this literature also explores information decay due to switching types,[6] the focus has been on stationary environments.[7] We focus instead on performance patterns over the life of the contracts.[8] In addition, by allowing for endogenous type-switching, we are able to study the choice between such (long-term) productivity investment and (short-term) performance improvement effort, as well as their interaction over time. Board and Meyer-ter-Vehn (2010) also introduce endogenous switching types in a career concerns settings, but their focus is on the value of reputation under different market learning hypotheses, rather than on performance patterns.

Our paper also contributes to the literature on the effects of contract duration, or the frequency of evaluations, on incentives.[9] On the one hand, longer contracts alleviate moral hazard problems by facilitating consumption smoothing (Lambert (1983)) and ease hold-up and ratchet effects in the presence of specific investment (Laffont and Tirole (1993)). On the other hand, shorter contracts increase the flexibility to use new information as it comes along

[6] See for example Mailath and Samuelson (2001) and Phelan (2006). As discussed in depth by Bar-Isaac and Tadelis (2008), for reputational concerns to be sustained over time, the market must never fully learn the type of the agent. This may hold for example if types exogenously change over time or if there is finite memory.

[7] Hidden action models of relational contracting have also focused on stationary environments rather than on performance dynamics; see MacLeod (2007) for a survey.

[8] Information decay and career concerns have been used to explain performance dynamics in other contexts – e.g., earnings smoothing in Fudenberg and Tirole (1995); pre-election policies in Cukiermann and Meltzer (1986) and Martinez (2009); and political budget cycle in Rogoff (1990).

[9] A link also exists with the literature in repeated games with imperfect monitoring that looks at the effect of frequency of interaction (or the effect of shortening the period length) on incentives; see e.g. Abreu, Milgrom, and Pearce (1991).

(Ellman (2006)) and reduce the gain from defection from implicit agreements (Shapiro (1983), Strausz (2009) and Calzolari and Spagnolo (2009)), making them optimal when the agent's hidden actions do not generate persistent effects.[10] This literature has mostly focused on moral hazard issues. By allowing also for heterogeneous productivity types and information decay, we obtain quite different implications.

The structure of the paper is as follows. Section 2 presents the baseline model where there is competition for the service of an agent. The agent can invest to enhance his productivity and investment costs are linear. It also analyzes convex investment costs. Section 3 extends our results to a setting where the principal has all the bargaining power; it also discusses the case where the agent can choose between effort and investment. Section 4 studies optimal contract duration in an infinitely repeated version of the baseline model with investment only. Section 5 concludes and provides some testable predictions. Some of the more technical proofs are in the Appendix.

2 Baseline model

We first focus here on the dynamics of incentives provided by career concerns, when the agent's productivity can evolve both exogenously (random shocks) and endogenously (investment). We consider a baseline model in which an agent provides a good or service for two periods ($t = 1, 2$), followed by competition for the agent's services in the next period ($t = 3$). As in Holmström (1982):

- the agent's "performance" (e.g., output, quality, and so forth) is observable but nonverifiable; it depends on the agent's innate productivity (his "type"), which for technological reasons may exogenously change over time;
- at the end of the two periods, the agent's payoff depends on market beliefs about his productivity.

We depart from Holmström's model, however, by allowing the agent to make a nonverifiable (and possibly unobservable) investment to enhance his productivity.

[10] Evidence on the determinants of contract duration shows that contracts are longer when relationship-specific investment is important (Joskow (1987)) and shorter in periods of higher uncertainty (Masten and Crocker (1985) and Saussier (1999)), which is consistent with the benefit of shorter contracts in the presence of information decay. The findings of Bandiera (2007) are also consistent with the idea that contract length is chosen to provide incentives.

2.1 Framework

Productivity and performance. In each period t, the agent's performance q_t can take two values: high (H) or low (L). The realized performance depends on: (i) the agent's productivity type, θ_t, which can also take two values, high (H) or low (L), and (ii) investment, which costs c to the agent; we denote by $i_t \in [0,1]$, the probability that the agent actually invests in period t. We assume that, absent investment, the productivity θ_t **evolves** over time according to a Markov process, with transition probability $\rho \geq 1/2$. We further assume that current type and investment affect current performance and future types in a similar way:

- if $\theta_t = H$ or $i_t = 1$, then

$$q_t = H \text{ and } \theta_{t+1} = \begin{cases} H & \text{with probability } \rho \\ L & \text{with probability } 1-\rho \end{cases};$$

- if instead $\theta_t = L$ and $i_t = 0$, then

$$q_t = L \text{ and } \theta_{t+1} = \begin{cases} L & \text{with probability } \rho \\ H & \text{with probability } 1-\rho \end{cases}.$$

An interpretation of this process is that the agent can invest to improve his productivity, which enhances both current performance and the distribution of his productivity in the next period. It follows that current performance constitutes a "sufficient statistic" for the agent's future productivity and performance. The parameter ρ reflects information decay: a lower value of ρ denotes faster changing technologies and therefore a lower probability that, absent any investment, the agent's productivity in period $t+1$ remains the same as in period t.

Payoff. As productivity and performance are nonverifiable, they do not affect the agent's payoff; this payoff thus depends only on the agent's market value in period 3, which increases with his expected performance in that period, $q_3^e \equiv E[q_3 \mid q_1, q_2]$. For the sake of exposition, we will simply assume that the agent appropriates q_3^e – later on, the agent's market value will also depend on exogenous variables such as the parties' discount factor and endogenous variables such as the expected investment in future periods. Denoting by δ the discount factor of the agent, his objective is thus:

$$\Pi = \sum_{t=1}^{t=3} \delta^{t-1} \pi_t = -ci_1 - \delta c i_2 + \delta^2 q_3^e.$$

Timing. The timing of the game is as follows. Periods $t=1$ and $t=2$: (i) θ_t is realized and observed by the agent; (ii) the agent chooses whether to invest; (iii) q_t is realized and publicly observed.[11] Then in period $t = 3$, the agent obtains q_3^e.

2.2 Performance dynamics

We first note that the agent's expected payoff only depends on q_2:

Proposition 1 *The payoff of the agent in period* 3 *increases with his performance in the previous period,* q_2*:* $\pi_3 = q_3^e(q_2)$*, given by*

$$q_3^e(q_2) = E[q_3 \mid q_2] = \begin{cases} L + \rho\Delta & \text{if } q_2 = H, \\ L + (1-\rho)\Delta & \text{if } q_2 = L. \end{cases} \tag{1}$$

Proof. This follows directly from the fact that: (i) q_2 provides a sufficient statistic for θ_3, the agent's productivity at $t = 3$; and (ii) period 3 being the last period, the agent never invests in that period. ■

In period 2 a low-type agent ($\theta_2 = L$) thus chooses i_2 so as to maximize:

$$\Pi_2^L = -ci_2 + \delta\{L + [i_2\rho + (1-i_2)(1-\rho)]\Delta\} = \delta[L + (1-\rho)\Delta] + (b_2 - c)i_2,$$

where

$$b_2 \equiv (2\rho - 1)\delta\Delta. \tag{2}$$

The optimal investment is thus $i_2^* = 1$ if $c < b_2$, and $i_2^* = 0$ if $c > b_2$.

The expected payoff of a high-type agent ($\theta_2 = H$) is instead equal to $\Pi_2^H = \delta(L + \rho\Delta)$. Therefore, in period 1 a low-type agent ($\theta_1 = L$) will choose i_1 so as to maximize:

$$\begin{aligned} \Pi_1^L &= -ci_1 + \delta\left\{i_1\left[\rho\Pi_2^H + (1-\rho)\Pi_2^L\right] + (1-i_1)\left[(1-\rho)\Pi_2^H + \rho\Pi_2^L\right]\right\} \\ &= (1-\rho)\Pi_2^H + \rho\Pi_2^L + (b_1 - c)i_1, \end{aligned}$$

where the marginal benefit is now

$$b_1 \equiv (2\rho - 1)\delta\left(\Pi_2^H - \Pi_2^L\right) = (2\rho - 1)\delta[b_2 - (b_2 - c)i_2^*]. \tag{3}$$

Thus, in each period $t = 1, 2$, the agent chooses his investment i_t so as to maximize $(b_t - c)i_t$; the marginal benefit b_t is however lower in period 1 (that is, $b_1 < b_2$), for two reasons:

- information decay ($\rho < 1$) and discounting ($\delta < 1$) reduce the direct return from investment, as reflected by the additional factor $(2\rho - 1)\delta$ in (3);

[11] The analysis does not depend here on whether the principal observes the productivity θ_t and/or the investment decision i_t: observing the performance q_t eliminates any relevant information asymmetry. In later sections, in which the agent may temporarily increase performance, observing only that performance maintains some ambiguity.

- in addition, there is *crowding out*: the investment expected in period 2, i_2^*, further reduces the incentives to invest in period 1 (as $b_2 \geq c$ when $i_2^* > 0$).

This leads to:

Proposition 2 *The agent never invests in period* 1*, and he invests in period* 2 *if and only if he has a low type and the cost is sufficiently low:*

$$i_t^* = 1 \text{ if and only if } t = 2,\ \theta_2 = L, \text{ and } c \leq c^*,$$

where $c^* \equiv (2\rho - 1)\,\delta\Delta > 0$*. The incentive to invest in period* 2 *therefore increases with the degree of information persistence* ρ*, the discount factor* δ *and the performance differential* Δ*, relative to the cost* c*.*

Proof. From the above analysis, in period 2 the agent invests if $c < c^* = b_2$, and does not invest if $c > c^*$ (in the limit case where $c = c^*$, the agent is indifferent between investing or not); the comparative statics follow directly. We now show that the agent never invests in period 1. When $c < c^*$, the agent invests in period 2 whenever $\theta_2 = L$; thus, investing in period 1 would have no impact on q_2 (and thus on q_3^e), and although this would reduce the probability of having to invest in period 2, the expected benefit from cost saving is only $b_1 = (2\rho - 1)\,\delta c < c$. Conversely, when $c > c^*$, the agent prefers not to invest in period 2; but then, it does not pay to invest in period 1 either, as the benefit is even lower (due to information decay) and it comes later – that is, we then have $b_1 = (2\rho - 1)^2\,\delta^2\Delta = (2\rho - 1)\,\delta c^* < c$. ■

An implication of Proposition 2 is that, on average, the agent's performance improves as the market evaluation approaches: the expected performance in period 1 is equally likely to be H or L, as the agent never invests in that period; in the second period, the performance is instead equal to H with probability 1 when $c < c^*$. Therefore:

Corollary 1 *The expected performance (weakly) increases as the market evaluation approaches:* $q_1^e \leq q_2^e$*, with a strict inequality when* $c < c^*$*.*

Proof. The agent's productivity is initially high or low with equal probability. As he never invests in period 1, his expected performance is:

$$q_1^e = \frac{L + H}{2} = L + \frac{\Delta}{2}.$$

By contrast, in period 2, the performance is low only if the agent's type is low *and* he does not invest; as a result, the expected performance is

$$q_2^e = \frac{\rho\left(\left(1-i_2^*\right)L + i_2^*H\right) + (1-\rho)\,H}{2} + \frac{\rho H + (1-\rho)\left(\left(1-i_2^*\right)L + i_2^*H\right)}{2} = L + \left(1+i_2^*\right)\frac{\Delta}{2}.$$

The conclusion ($q_2^e \geq q_1^e$) follows from $i_2^* \geq 0$ (with a strict inequality when $c > c^*$). ■

2.3 Discussion

As performance and investment are nonverifiable, the agent cannot be explicitly incentivized to provide high performance. However, career concerns create an implicit incentive, as past performance affects the market value of the agent in period 3. Due to information decay, this implicit incentive is stronger in period 2 than in period 1. This effect is here extreme and actually nullifies investment incentives in period 1: performance in that period has no impact on market beliefs, as period 2 performance provides a sufficient statistic for the agent's productivity in period 3.[12]

Information decay affects not only the temporal pattern of the implicit incentive, but also its strength. In case of full decay ($\rho = 1/2$), past performance tells nothing about future productivity; it thus has no impact on market beliefs, and the agent then never invests. As ρ increases, past performance becomes more and more informative about the agent's future productivity, which in turn gives the agent greater incentives. Investment incentives remain however suboptimal: although the agent fully internalizes here the impact of his investment on his performance in period 3, he ignores the short-term impact on the performance in periods 1 and 2 – this is particularly clear in period 1, where the agent never invests, but applies as well to period 2.

Before concluding this section, we briefly comment on the assumptions of this baseline model. That performance and productivity levels are all binary simplifies the exposition but does not play a critical role. The assumption that the agent knows his type is also not necessary to generate the investment and performance patterns that we have described. If θ_t were not observed by the agent either, we would still observe investment only in period 2, as performance in that period would still provide a sufficient statistic for predicting performance in period 3.[13]

[12] In this baseline model, $\rho < 1$ is not necessary to generate this investment pattern: due to discounting, even with $\rho = 1$, the agent never invests in period 1, whilst he might invest in period 2. As we will see, discounting no longer suffices to generate this pattern in case of repeated interaction over time.

[13] We provide a formal analysis of such a variant of our baseline model in Online Appendix A. Interestingly, in this alternative baseline model, the incentive to invest increases over time even in the absence of information

Likewise, while the "sufficient statistic" feature greatly simplifies the analysis, we would expect similar investment and performance patterns with alternative processes determining current productivity as a function of performance history, such as moving averages or other higher-order Markov process, as long as recent performance provides more reliable information about future productivity.

For simplicity we also assume that investment is only valuable in case of low productivity; more generally, investment could be socially desirable for both types of agents, and conversely both types could have an incentive to invest. For example, consider a variant where the probability of a high performance is additively separable in past productivity and current investment (e.g., it is of the form $\psi\theta_t + \varphi i_t$, with $\psi, \varphi > 0$). Enhancing their market value then gives *both* types of agents the same investment incentive;[14] however, current performance still provides a sufficient statistic for future productivity, and thus these investment incentives remain concentrated in period 2.

In the same vein, the binary investment decision introduces "constant returns to scale", which exacerbates the investment dynamics. Decreasing returns to scale would create a motive for investment smoothing, but the pattern remains "backloaded". To see this, the following proposition considers the case where, when $\theta_t = L$, upgrading performance with probability $i \leq 1$ costs $k(i)$ (in the baseline model, $k(i) = ci$):

Proposition 3 *When investment costs $k(.)$ are convex and satisfy $k(0) = k'(0) = 0$, a low-type agent invests in both periods, and investment levels increase with the degree of information persistence ρ, the discount factor δ and the performance differential Δ. The agent however invests less in period 1 than in period 2, and the expected performance thus still increases as the market evaluation approaches: $q_1^e < q_2^e$. In particular, the agent never fully invests in period 1: $i_1^* < 1$.*

Proof. See Appendix A. ■

As in the baseline model, in each period t a low-type agent chooses i_t so as to maximize $b_t i_t - k(i_t)$, where b_1 and b_2 are respectively given by (3) and (2). When $k(.)$ is convex, these optimization problems have unique solutions, which are both positive when $k(0) = k'(0) = 0$: due to decreasing returns, it is worth investing also in period 1, in order to smooth investment

decay or of discounting, as there is an additional "option value" in waiting and learning about the agent's type before deciding whether to invest.

[14] Introducing cost or benefit complementarities between productivity and investment could even generate greater incentives for a high productivity agent.

cost over the lifetime of the contract. However, $b_1 < b_2$, due to information decay and discounting as well as crowding-out, and thus the agent still invests less in period 1 than in period 2; in particular, because of crowding-out, $i_1^* < 1$: $i_1^* = 1$ would imply $i_2^* = 1$, and it never pays to invest merely to save on the cost of investing in the future.

3 Variant and extensions

This section considers two variants of our baseline model: in Section 3.1, the agent has a continuing relationship with a single principal; Section 3.2 extends instead the baseline model by allowing also for temporary performance improvements.

3.1 Principal-agent relationship

Our baseline model keeps in line with standard career concern models, in that competition for the agent's services in period 3 enables the agent to appropriate the expected value of his investment for that period. This setup is relevant for situations (e.g., the market for managers) in which several "buyers" (the firms) compete for the same agent (the manager). But in many contexts (e.g., public procurement for refuse collection, water distribution, prison services, and so forth) the agent (a private contractor) provides specialized services that have mainly one buyer (the public authority). However, our analysis carries over to such contexts, even if the principal has the bargaining power, as long as the agent benefits from enhancing the principal's beliefs about his expected performance. To see this, we develop here a simple model where information asymmetries generate a rent for the agent, making it worthwhile to convince the principal to renew the contract.

Assume that a principal can either delegate the provision of a public service to the agent or keep it in house:

- If the agent provides the service in period t at price p_t, the principal obtains a net payoff $q_t - p_t$ and the agent obtains $p_t - \phi_t - ci_t$, where ϕ_t denotes the agent's operating cost. This cost is random and can take two values, $\underline{\phi}$ and $\overline{\phi}$, with respective probabilities α and $1 - \alpha$; its realization is privately observed by the agent.

- Keeping the provision in-house yields instead V for the principal, which is also random and uniformly distributed over a range $\left[\underline{V}, \overline{V}\right]$, and 0 for the agent.

As in the baseline model, the agent is already working for the principal in periods 1 and 2, but at the beginning of period 3 a negotiation now takes place as follows:

- The principal has the bargaining power: having observed the value V of in-house provision, she makes a take-it-or-leave-it price offer to the agent.
- The agent's participation depends on the realization of his operating cost ϕ, which he observes before accepting or not the principal's offer.

The principal can thus secure V by keeping the provision in house, or offer the agent a more or less attractive price, covering either both cost levels or only the lower one: in particular, if she is very optimistic about the agent's future performance, she will be willing to offer a good price covering $\overline{\phi}$, and leave a rent $\overline{\phi}-\underline{\phi}$ to the agent in case of a low cost. This, in turn, induces the agent to invest when needed in order to enhance the principal's beliefs, which yields the same investment and performance patterns as in the baseline model:[15]

Proposition 4 *The expected rent of the agent in period* 3 *increases with his previous performance:*

$$E\left[\pi_3 \mid q_1, q_2\right] = \lambda \left(q_3^e\left(q_2\right) - \hat{q}\right),$$

where $q_3^e\left(q_2\right)$ *is given as before by* (1)*, and* $\lambda \equiv \alpha\left(\overline{\phi}-\underline{\phi}\right)/\left(\overline{V}-\underline{V}\right)$ *measures the sensitivity of the agent's rent to his expected productivity, compared with a performance threshold* $\hat{q} \equiv \underline{V} + 2\overline{\phi} - \underline{\phi}$.

It follows that the investment incentives, and thus the equilibrium investment and performance, have the same pattern as in the baseline model: the agent never invests in period 1*, and in period* 2*, he invests in case of low productivity if* $c < c_\lambda^* \equiv \lambda\left(2\rho - 1\right)\delta\Delta$.

Proof. In period 3, the principal's relevant options are:

[15] In a previous version of this paper (Iossa and Rey (2010)), we also considered investment aiming at reducing cost rather than improving performance. Investment incentives then decrease, rather than increase, as the renewal date approaches, and this for two reasons: first, the agent benefits more directly from cost reductions than from performance improvements; second, whereas the principal is willing to offer a higher price when she expects a good performance, she is on the contrary likely to insist on a low price when she expects the agent to face a low cost. In a similar vein, in Lewis (1986) the threat of cancellation induces the contractor to make nonverifiable cost-reducing investment in the early periods of a contract, to assure that the project will be extended.

This is also related to the ratchet effect identified in regulatory settings – see e.g. Pint (1992) and Laffont and Tirole (1993). There, the firm reduces productivity efforts before a price review, fearing that exhibiting low costs would induce the regulator to tighten regulation. Note however that investment and performance patterns would be as in our baseline model if the contractor's prices were not regulated; this may explain the findings of Chong, Huet and Saussier (2006) mentioned in the introduction.

- Offering a low price $p_3 = \underline{\phi}$, which the agent accepts only when $\phi_3 = \underline{\phi}$; this gives the principal an expected payoff equal to $\alpha\left(q_3^e - \underline{\phi}\right) + (1-\alpha) V$;
- Or offering a high price $p_3 = \overline{\phi}$ to secure the agent's services, which yields an expected payoff equal to $q_3^e - \overline{\phi}$.

The agent obtains a positive payoff only in the latter case, where he obtains a rent $\overline{\phi} - \underline{\phi}$ with probability α; and this option is favored by the principal only when she is sufficiently optimistic about the agent's productivity, namely, when:

$$q_3^e - \overline{\phi} > \max\left\{\alpha\left(q_3^e - \underline{\phi}\right) + (1-\alpha) V, V\right\} \Longleftrightarrow q_3^e > V + \underline{\phi} + \frac{\overline{\phi} - \underline{\phi}}{1-\alpha}.$$

The agent's expected rent is thus:

$$E\left[\pi_3 \mid q_1, q_2\right] = \Pr\left(V \leq q_3^e - \underline{\phi} - \frac{\overline{\phi} - \underline{\phi}}{1-\alpha}\right)\alpha\left(\overline{\phi} - \underline{\phi}\right) = \frac{\alpha\left(\overline{\phi} - \underline{\phi}\right)}{\overline{V} - \underline{V}}\left(q_3^e - \underline{\phi} - \frac{\overline{\phi} - \underline{\phi}}{1-\alpha} - \underline{V}\right).$$

The agent's rent is thus of the form $\lambda\left(q_3^e - \hat{q}\right)$, where the ratio $\lambda \equiv \alpha\left(\overline{\phi} - \underline{\phi}\right) / \left(\overline{V} - \underline{V}\right)$ and the performance threshold $\hat{q} \equiv \underline{V} + \underline{\phi} + \frac{\overline{\phi} - \underline{\phi}}{1-\alpha}$ are independent of the agent's behavior.

As before, (i) the agent has no incentive to invest in period 3, as this is the last contracting period; and (ii) the performance observed in period 2, q_2, provides a sufficient statistic for the agent performance in period 3. As a result, $q_3^e = q_3^e(q_2)$ as given by (1). As the agent's rent is equal to $\lambda\left(q_3^e(q_2) - \hat{q}\right)$, the analysis is then formally the same as in the baseline model, scaling the expected benefits from the investment, b_1 and b_2, by a factor λ: that is, in each period t a low-productivity agent ($\theta_t = L$) now seeks to maximize $(\lambda b_t - c)\, i_t$. The conclusion follows. ■

3.2 Signal jamming

Let us return to the baseline model, in which the agent appropriates the expected performance in period 3 (that is, $\pi_3 = E\left[q_3 \mid q_1, q_2\right]$), but suppose now that, in each period t, a low-type agent ($\theta_t = L$) can choose between investing in lasting improvements, as before, or exerting effort to generate short-term effects:[16] exerting effort costs $\gamma < c$ but only improves current performance ($q_t = H$), and does not affect the distribution of the agent's type in the next period (that is, θ_{t+1} coincides with θ_t with probability ρ, regardless of the agent's effort).[17]

[16] For example, the catering company running the university canteen may increase the quality of the food served in a given period by asking its chef to spend more time in the kitchen in that period (effort). Alternatively, the company could hire a new chef (investment) bringing long-term quality improvements.

Effort may also only affect *perceived* performance – for example, athletes may use more Performance Enhancing Drugs (PEDs); we thank Andy Skrzypacz for pointing this out to us.

[17] If $\gamma \geq c$, the agent would always favor effort over investment, and the analysis would be the same as in the baseline model.

Obviously, the agent will never invest or exert effort in period 3 (as this is the last period), and in the previous periods he will never do so either when having a high productivity ($\theta_t = H$). Consider now the behavior of a low productivity agent in period 2 ($\theta_2 = L$). The agent will never invest in that period, as exerting effort would have the same impact on the market belief and would be less costly to produce. He may however exert effort in order to enhance market beliefs:

- If $q_2 = L$, the market infers that the agent has a low productivity in period 2, and thus expects in period 3 a performance

$$E\left[q_3 \mid q_1, q_2 = L\right] = L + (1-\rho)\,\Delta.$$

- By contrast, $q_2 = H$ entertains ambiguity, since a high performance can be the result of productivity or effort. Let ν_{q_1} denote the market belief, as a function of the performance $q_1 \in \{H, L\}$ observed in period 1, that the agent has a high productivity in period 2 when observing $q_2 = H$. The expected performance in period 3, is then given by:

$$\begin{aligned} E\left[q_3 \mid q_1, q_2 = H\right] &= \nu_{q_1}\left(\rho H + (1-\rho)\,L\right) + \left(1-\nu_{q_1}\right)\left((1-\rho)\,H + \rho L\right) \\ &= L + (1-\rho)\,\Delta + (2\rho - 1)\,\nu_{q_1}\Delta. \end{aligned}$$

The expected payoff of a low-type agent, as a function of his past performance q_1 and of his current effort e_2, is therefore:

$$\Pi_2^L\left(e_2; q_1\right) = \delta\left[L + (1-\rho)\,\Delta\right] + \left((2\rho-1)\,\delta\nu_{q_1}\Delta - \gamma\right)e_2.$$

As a result, in period 2, effort will be undertaken if:

$$\gamma < (2\rho - 1)\,\delta\nu_{q_1}\Delta. \tag{4}$$

Intuitively, the market will be more optimistic if it observes a high performance in period 1 ($\nu_H \geq v_L$); it follows that the agent has more incentive to exert effort in that case: denoting by e_{q_1} the equilibrium probability of effort in period 2, as a function of the performance $q_1 \in \{H, L\}$ observed in period 1, we expect $e_H \geq e_L$.

When instead the agent has a high productivity in period 2, his expected payoff is $\Pi_2^H\left(q_1\right) = \delta E\left[q_3 \mid q_1, q_2 = H\right]$. Consider now the behavior of a low-type agent in period 1 ($\theta_1 = L$). If the agent does neither exert effort nor invest, then $q_1 = L$ and, in period 2:

- with probability $1-\rho+\rho e_L$, the performance is $q_2 = H$, leading the market to expect a high productivity with probability ν_L;

- with probability $\rho(1-e_L)$, the performance is $q_2 = L$, thus revealing the low productivity of the agent.

Thus, the agent's expected payoff from neither investing nor exerting effort in period 1 is:

$$\begin{aligned}\Pi_1^L\Big|_{i_1=e_1=0} &= \delta\left[(1-\rho)\,\Pi_2^H(L) + \rho\Pi_2^L(e_L;L)\right] \\ &= \delta^2\left[L+(1-\rho)\Delta\right] + (1-\rho+\rho e_L)\,\nu_L\,(2\rho-1)\,\delta^2\Delta - \rho e_L\delta\gamma. \qquad (5)\end{aligned}$$

If instead the agent invests in period 1, he enhances both his current performance q_1 from L to H and his future productivity; there is then no point exerting effort, and his expected payoff becomes:

$$\begin{aligned}\Pi_1^L\Big|_{\substack{i_1=1\\e_1=0}} &= \delta\left[\rho\Pi_2^H(H) + (1-\rho)\,\Pi_2^L(e_H;H)\right] - c \\ &= \delta^2\left[L+(1-\rho)\Delta\right] + \left[\rho+(1-\rho)\,e_H\right]\nu_H\,(2\rho-1)\,\delta^2\Delta - (1-\rho)\,e_H\delta\gamma - c. \qquad (6)\end{aligned}$$

Finally, the agent may choose to exert effort rather than to invest, in which case he only enhances his current performance q_1 and his expected payoff becomes:

$$\begin{aligned}\Pi_1^L\Big|_{\substack{i_1=0\\e_1=1}} &= \delta\left[(1-\rho)\,\Pi_2^H(H) + \rho\Pi_2^L(e_H;H)\right] - \gamma \\ &= \delta^2\left[L+(1-\rho)\Delta\right] + \left[1-\rho+\rho e_H\right](2\rho-1)\,\nu_H\delta^2\Delta - \rho e_H\delta\gamma - \gamma. \qquad (7)\end{aligned}$$

Comparing these options yields:

Proposition 5 *Suppose that the agent can invest at cost c and/or exert effort at cost $\gamma < c$; then:*

- *In period 2, the agent never invests but exerts effort with positive probability in case of low productivity ($\theta_2 = L$) when this is not too costly (namely, if $\gamma \leq c^* = (2\rho-1)\,\delta\Delta$; in period 1, the agent favours investment whenever its relative cost is not excessive (namely, if $c/\gamma < 1+(2\rho-1)\,\delta$), and effort otherwise.*

- *The incentive to enhance performance increases in each period with the discount factor δ and the performance differential Δ; in addition, it increases:*
  - *over time: $e_1^* > 0$ or $i_1^* > 0$ imply $e_H^* = 1$ and $e_L^* > 0$, whereas we can have $e_H^* > 0$, $e_L^* > 0$ and $e_1^* = i_1^* = 0$;*

– *and with the agent's reputation:* $e_H^* \geq e_L^*$, *with a strict inequality whenever* $0 < e_L^* < 1$.

- *In the absence of information decay* ($\rho = 1$)*, performance remains constant over time:* $q_1 = q_2$*; with information decay* ($\rho < 1$)*, performance increases over time:* $q_1^e \leq q_2^e$*, with a strict inequality whenever costs are in a moderate range.*[18]

Proof. See Online Appendix B. ■

Long-term investment and short-term effort are substitutes in each period, and in the second period the agent favors effort over investment, as the former is a cheaper way to deliver high performance. However, in contrast with our baseline model, here the effort exerted in period 2 does not "crowd out" the incentives in the previous period: as q_2 becomes less informative about the agent's type, delivering good performance becomes valuable also in period 1.[19] In the first period, the agent favors investment over effort when the cost difference is small enough, as investment reduces the likelihood of having to exert effort in the next period. In the second period, the agent has more incentives to hide bad news when he is supposed to be good: $e_H \geq e_L$.[20]

Contrary to the baseline model where the agent can only invest, the expected performance increases as the market evaluation approaches only when there is (limited) information decay, $(1/2 <)\, \rho < 1$: discounting alone does not suffice.[21] Indeed, with $\rho = 1$, either performance is high in every period or it is low; thus, once $q_1 = L$ is observed, there is no incentive to exert effort in period 2:[22] $e_L = 0$ (and investing is not credible).

[18] Namely, when (i) $\gamma \leq (2\rho - 1)\,\delta\Delta$, so that the agent exerts effort in period 2, but (ii) c and γ are not too low, so that $e_1^*, i_1^* < 1$.

[19] Our formulation allows for "asymmetric ambiguity": a bad performance L « reveals » a low productivity, whereas a good performance H entertains some ambiguity, as it may be the result of the agent's effort. More generally, incentives to exert effort remain stronger in the second period whenever, due to information decay, productivity estimates are ranked as "$LL \leq HL < LH \leq HH$".

[20] The literature on life-cycle effects in reputational models shows that incentives may either decrease or increase with existing reputation; see Bar-Isaac and Tadelis (2008) for a survey. Board and Meyer-ter-Vehn (2010) highlight the role of the learning process: when bad news reveal the agent's type, as in our framework, agents with better reputation have higher incentives to invest. The opposite result obtains when instead there is perfect learning upon good news.

[21] In Grossman and Shapiro (1985), discounting alone explains why firms may devote more resources to a project as the project nears completion. In their model, investment yields no return until the project is completed, thus the marginal discounted benefit of investing increases as the project nears completion. In Rice and Sen (2008), where both the agent and the principal progressively learn the agent's type, discounting suffices to generate increasing effort over time; however, the implications on the performance pattern are ambiguous.

[22] Information persistence fosters incentives in period 1 and has also a positive impact on incentives in period 2, for *given* market beliefs at the beginning of that period. However, information persistence also exacerbates those beliefs: when $q_1 = L$, they become more pessimistic as ρ increases, which reduces effort incentives; as a

4 Optimal contract duration

In this section we extend the analysis to study the impact of contract duration on incentives and performance dynamics. For this purpose we plug the baseline model of Section 2 into an infinitely repeated framework; that is, in each period $t = 1, 2, ...$: if the agent is already productive ($\theta_t = H$) or invests ($i_t = 1$), then current performance is high ($q_t = H$) and, in the next period, the productivity is high ($\theta_{t+1} = H$) with probability ρ; otherwise, the current performance is low ($q_t = L$) and, in the next period, the productivity is high only with probability $1 - \rho$.

Suppose that the agent renegotiates the terms of his contract every n periods. At the beginning of each contracting period $\tau = 1, n+1, 2n+1, ...$, competition for the agent's services then allows him to appropriate the full expected value generated by the contract, of the form:

$$v(\mathbf{q}_{\tau-1}) = E\left[\sum_{t=1}^{t=n} \delta^{t-1} q_{\tau+t-1} \mid \mathbf{q}_{\tau-1}\right],$$

where $\mathbf{q}_{\tau-1} = \{q_1, ..., q_{\tau-1}\}$ denotes the agent's performance history at the beginning of period τ. As in the baseline model, the most recent performance $q_{\tau-1}$ provides a sufficient statistic for the history $\mathbf{q}_{\tau-1}$. Therefore, it is only in the last period of a contract that the agent can have an incentive to invest. We will further focus on Markovian equilibria in which the agent's investment strategy, and thus the market expectations, are stationary;[23] we will denote by i_n the probability that the agent invests in case of low productivity in the last period of a contract.

Consider a contract running from $t = 1$ to $t = n$, following a performance $q \in \{H, L\}$ in the last period of the previous contract. Due to information persistence ($\rho > 1/2$), this past performance affects the agent's expected productivity in this contract. Indeed, we have:

Lemma 1 *The probability p_t^q of a high productivity in period t ($\theta_t = H$), as a function of the performance $q \in \{H, L\}$ observed in the last period of the previous contract, is given by:*

$$p_t^H \equiv \frac{1 + (2\rho - 1)^t}{2}, p_t^L \equiv 1 - p_t^H.$$

Proof. See Appendix B. ■

result, $e_L = 0$ when ρ is close to 1 (by contrast, e_H increases with ρ, as the effect on market beliefs also tends to foster incentives when $q_1 = H$).

[23] The analysis would be much less tractable when allowing for short-term performance-enhancing efforts, as in Section 3.2; market beliefs then depend on the entire performance history, and this in turn implies that effort incentives vary over time.

As the agent never invests before the last period, the probability of a high performance in period $t \in \{1, ..., n-1\}$ is then simply $\mu_t^q = p_t^q$; in the last period ($t = n$), this probability becomes

$$\mu_n^q = p_n^q + (1 - p_n^q)\, i_n.$$

Thus, the value v^q generated by the contract is given by:

$$v^q = \sum_{t=1}^{n} \delta^{t-1} \left(L + \mu_t^q \Delta\right) \tag{8}$$

Finally, the agent's overall expected payoff, evaluated at the beginning of the contract, can thus be expressed as, for $q \in \{H, L\}$:

$$\Pi^q = v^q - \delta^{n-1} (1 - p_n^q)\, c i_n + \delta^n \left[\mu_n^q \Pi^H + (1 - \mu_n^q)\, \Pi^L\right]. \tag{9}$$

This equation determines recursive conditions that determine Π^H and Π^L, which we can use to analyze the agent's incentives to invest in the last period of the contract. When facing a low productivity in period n, the agent will then make his investment decision so as to maximize $(b_n - c)\, i_n$, where

$$b_n \equiv \delta \left(\Pi^H - \Pi^L\right).$$

Using (9), in equilibrium the expected benefit b_n can be written as:

$$b_n = \delta \frac{v^H - v^L + \delta^{n-1} \left(p_n^H - p_n^L\right) c i_n}{1 - \delta^n \left(\mu_n^H - \mu_n^L\right)}. \tag{10}$$

where i_n corresponds here to the equilibrium investment level in the last periods of future contracts. Good performance in period $t = n$ brings three types of benefit to the agent:

- It increases the expected performance of the next contracts by

$$\delta \left(v^H - v^L\right) = \sum_{t=1}^{n} \delta^t \left(\mu_t^H - \mu_t^L\right) \Delta = \sum_{t=1}^{n} (2\rho - 1)^t \delta^t \Delta - i_n (2\rho - 1)^n \delta^n \Delta,$$

  As before, this benefit increases with the performance differential Δ and the discount factor δ (which has also a direct positive effect on b_n, reflected in the first factor δ in (10)), but now it also suffers from crowding-out: current investment has less impact on future performance if it is anticipated that the agent will invest again when needed in the future.

- It saves on the cost of future investment, ci_n, as it reduces the probability of having to invest in the last period of the next contracts by:

$$\left(p_n^H - p_n^L\right)\delta^n = (2\rho - 1)^n \delta^n.$$

  This benefit increases with the degree ρ of information persistence as well as with the discount factor δ.

- It raises the probability of enjoying Π^H rather than Π^L in the following negotiations by:

$$\delta^n \left(\mu_n^H - \mu_n^L\right) = (2\rho - 1)^n \delta^n (1 - i_n).$$

- This benefit increases again with the degree ρ of information persistence as well as with the discount factor δ, but also suffers from crowding-out, as future investments reduces the impact of current investment on future performance.

Building on this analysis, there exists a unique equilibrium, characterized as follows :

Proposition 6 *Under $n-$period contracting, the agent never invests during the first $n-1$ periods of a contract; in the n^{th} period:*

- *If $c \geq \bar{c} \equiv \frac{(2\rho-1)\delta\Delta}{1-(2\rho-1)\delta}$, the agent never invests ($i_n^* = 0$);*

- *If $c \leq \underline{c}_n \equiv \frac{1-(2\rho-1)^{n-1}\delta^{n-1}}{1-(2\rho-1)^n\delta^n}\bar{c}$, then the agent invests with probability 1 in case of low productivity ($i_n^* = 1$);*

- *If $\underline{c}_n < c < \bar{c}$, the agent invests with probability $i_n^* = \hat{\imath}_n \in (0,1)$ in case of low productivity, where*

$$\hat{\imath}_n = \frac{1-(2\rho-1)^n \delta^n}{(2\rho-1)^n \delta^n}\left(\frac{(2\rho-1)\delta}{1-(2\rho-1)\delta} - \frac{c}{\Delta}\right). \tag{11}$$

  This equilibrium level of investment i_n^* increases with the contract duration n, the degree of information persistence ρ, the discount factor δ and the relative performance benefit Δ/c.

Proof. Due to crowding out, the expected benefit $b_n = b_n(i_n)$ decreases as i_n increases. Therefore, in equilibrium $i_n^* = 0$ if $c > \bar{c} = b_n(0)$, $i_n^* = 1$ if $c < \underline{c}_n = b_n(1)$, and if $c \in (b_n(1), b_n(0))$ then $i_n^* = \hat{\imath}_n$, such that $b_n(\hat{\imath}_n) = c$. Straightforward computations lead to the above expressions. ■

In equilibrium, the agent invests when the performance differential Δ is large enough, compared to the investment cost c, or when the agent puts enough weight on the future (δ high) and information is persistent (ρ high) – what matters is the "effective" discount rate measured by $(2\rho - 1)\,\delta$. To interpret the bound $\bar{c}$, note that for low investment levels: (i) there is not much of crowding out; and (ii) the benefit of saving on future investment costs is also low. Therefore, the agent invests with positive probability whenever the cost of doing so is lower than the "full" impact it has on future performance:

$$c < \delta \sum_{k=1}^{\infty} \delta^{(k-1)n} \left(v^H - v^L \right) \Big|_{i_n^*=0} = \sum_{t=1}^{\infty} \delta^t \left(2\rho - 1\right)^t \Delta = \bar{c}.$$

Obviously, this value increases with δ, ρ and Δ, and thus it is natural to expect investment to increase as well with these parameters. Increasing the investment levels however generates crowding-out, which tends to reduce the impact of current investment on future performance. Yet this crowding-out never completely annihilates the benefit of investment: even if the agent invests with probability 1 at the end of the next contract, investing today still has a positive effect on expected performance during the first $n-1$ periods of the next contract (but only that one); in addition, investing today now also allows the agent to save on future investment costs. As a result, even if the agent always invests in case of low productivity at the end of the next contracts, it is still worth investing if δ, ρ and Δ are large enough, namely, if:

$$c < \delta\; v^H - v^L \Big|_{i_n^*=1} + \left(2\rho - 1\right)^n \delta^n c \Leftrightarrow c < \frac{\delta\; v^H - v^L \Big|_{i_n^*=1}}{1 - \left(2\rho - 1\right)^n \delta^n} = \frac{\sum_{t=1}^{n-1} \left(2\rho - 1\right)^t \delta^t \Delta}{1 - \left(2\rho - 1\right)^n \delta^n} = \underline{c}_n.$$

Finally, extending the length of the contracts limits crowding-out – as future investment comes later and less often; as a result, i_n^* increases with the contract duration n.

We now study the implications for optimal contract duration. As competition for the agent's services allows him to appropriate all the surplus, in equilibrium the expected welfare coincides with the agent's *ex ante* expected payoff:

$$W_n = \frac{\Pi_n^H + \Pi_n^L}{2},$$

where Π_n^q denotes the equilibrium expected payoff at the beginning of a contract under $n-$period contracting, given the performance $q \in \{H, L\}$ in the last period of the previous contract. Using

(9), we have:

$$W_n = v_n - \delta^{n-1} c \frac{i_n^*}{2} + \delta^n \left[W_n + \frac{i_n^*}{2} \left(\Pi_n^H - \Pi_n^L \right) \right]$$
$$= \frac{v_n}{1-\delta^n} + \frac{\delta^{n-1}}{1-\delta^n} \frac{i_n^*}{2} \left[\delta \left(\Pi_n^H - \Pi_n^L \right) - c \right], \tag{12}$$

where $v_n = \frac{v_n^H + v_n^L}{2}$ denotes the equilibrium expected performance generated by the first contract under $n-$period contracting.

When investment is too costly ($c \geq \bar{c}$), the agent never invests: $i_n^* = 0$ for any $n \geq 1$. The expected welfare is then equal to

$$W_n = \frac{1}{1-\delta} \frac{L+H}{2},$$

whatever the duration of the contracts. In what follows, we thus focus on the case where $c < \bar{c}$.

When instead contracts are sufficiently long that $i_n^* = 1$, the expected welfare decreases as the contract length n further increases, which then only delays the next periods of investment. As the agent never invests in the early periods of a contract, and moreover fails to internalize the impact of investment on current performance even in the last period, there is always underinvestment. It is thus never optimal to choose a duration n larger than what is needed to induce $i_n^* = 1$, as this would only make the investment less frequent, without any off-setting effect on the investment intensity in the last periods of the contracts.

For shorter durations where $i_n^* = \hat{\imath}_n < 1$, the second term in (12) is nil;[24] in that case, the equilibrium expected welfare coincides with the *ex ante* expected performance generated by the first contract, which from (8) is given by:

$$W_n = \frac{v_n}{1-\delta^n} = \frac{\sum_{t=1}^{n} \delta^{t-1} \frac{L+H}{2} + \delta^{n-1} \frac{\hat{\imath}_n}{2} \Delta}{1-\delta^n} = \frac{1}{1-\delta} \frac{L+H}{2} + \frac{\delta^{n-1}}{1-\delta^n} \hat{\imath}_n \frac{\Delta}{2}. \tag{13}$$

Increasing the contract duration n has then two opposite effects on expected welfare:

- it *increases the intensity* of investment in the last period of a contract: $\hat{\imath}_n$ increases with n;
- but it *reduces its frequency*: $\frac{\delta^{n-1}}{1-\delta^n}$ decreases as n increases.

[24] This is obvious if $i_n^* = 0$; if $i_n^* = \hat{\imath}_n \in (0,1)$, the agent is indifferent between investing or not in the last period of a contract, and thus $\delta \left(\Pi_n^H - \Pi_n^L \right) = c$.

It turns out that the intensity effect dominates: it is always optimal to increase n so as to increase $\hat{\imath}_n$ despite the reduced frequency of its occurrence. It is therefore optimal to make contracts just long enough to induce $i_n^* = 1$. As the investment benefit is more important when the parties are more patient and/or there is more persistence, this reduces the minimal duration needed to induce $i_n^* = 1$. Instead, an increase in the relative cost c/Δ makes it more difficult to encourage investment, which increases the minimal duration needed to induce $i_n^* = 1$.

As the expressions for the expected performance are similar to those for the expected welfare (in particular, the impact of n is the same), the analysis carries over for the impact of contract duration on performance. Building on these insights, we have:

Proposition 7 *When $c \geq \bar{c}$, the agent never invests for any $n \geq 1$; the duration of the contracts thus has no impact on performance or welfare. When instead $c < \bar{c}$, expected welfare and performance are both maximized by making the contracts just long enough to yield $i_n^* = 1$. As an increase in the discount factor (δ) and/or persistence (ρ) encourages investment, whereas an increase in the relative cost $\frac{c}{\Delta}$ discourages it, the optimal duration $\hat{n}\left((2\rho - 1)\,\delta, \Delta/c\right)$ decreases as the discount factor δ, the relative benefit Δ/c, and the degree ρ of information persistence increase.*

Proof. See Appendix C. ■

That longer contracts may generate greater implicit incentives is opposite to what current wisdom suggests. The key here is that, as the agent never invests in the first periods of a contract, increasing the length of the contracts attenuates the crowding-out effects associated with future investment, which in turn fosters the incentives to invest in the last period of a contract.

That short-term contracting is optimal when information persistence is high is also counterintuitive. When ρ is high, investments have long-term effects, and one may have expected long-term contracts to be optimal. This would for example be the case if the agent benefitted from the investment during the execution of the contract (e.g., investment reduces operating costs); he would then invest at the beginning of the contracts, and even more so if the contracts get longer. Here, however, the agent invests only in the last periods of the contracts, to enhance his market value for the next contracts; as information persistence makes it easier to induce the agent to invest in these last periods, it becomes more important to increase their frequency, through shorter contracts.

5 Conclusion

Underperformance in agency relationships may be a serious problem when performance is non-verifiable and therefore cannot be contracted upon. However, when performance is (at least partially) the result of the agent's productivity, the agent may wish to build a reputation by working harder or by investing to enhance his type; career concerns may then ease the moral hazard problem.

We build on these insights to study the performance dynamics that career concerns generate in multi-period contracts with volatile environments, and derive the implications for contract duration. Environment volatility generates information decay, which in turn fosters the agent's incentives as contract renewal approaches. As a result, performance improves towards the end of the contracts.

Further, and opposite to what the current wisdom suggests, longer contracts generate greater implicit incentives, as they reduce crowding out effects. As a result, longer contracts can be optimal when the agent would otherwise be reluctant to invest, e.g., when the environment is highly volatile, making the benefits from investment rather short-lived. Conversely, shorter contracts can be optimal when investment has long-term effects on performance.

Keeping in line with standard career concern models, we mainly focused here on the case where several principals compete for the same agent; however, we also showed in Section 3.1 that the analysis applies as well to situations in which the agent interacts with a single principal, as in the case of public procurement.[25] These insights highlight the importance of granting some discretion to public authorities in the selection of their contractors. By making past performance relevant for future contract opportunities, this discretion fosters the contractor's incentives to deliver good performance. This can be particularly important for public services, such as educational services, clinical services and nursing homes, which involve many noncontractible dimensions.[26]

Our model provides a number of testable predictions. A first prediction is that performance (and/or investment or effort) improves significantly as contract renewal date approaches. A

[25] In a previous version of the paper, we also considered the infinite repetition of the principal-agent relationship presented in that Section; see Iossa and Rey (2010).

[26] The importance of granting discretion to public procurer to allow reputational forces to operate was initially stressed by Kelman (1990). When he was the head of public procurement during the Clinton administration, Kelman adopted more flexible purchasing practices common in the private sector, among which giving more weight to suppliers' past performance. Since the Federal Acquisitions Streamlining Act in 1994, US Federal Departments and Agencies are expected to record past contractors' performance evaluations and share them through common platforms for use in future contractor selection. See Spagnolo (2012) for further details.

second prediction is that performance falls immediately after the contract is renewed, as the agent then faces less reward for improving performance. The decline in performance is greater for longer term contracts, for which the incentive to enhance performance before the contract is signed is further increased. Third, the performance pattern is exacerbated in environments with larger stakes from contract renewal or greater information persistence, as in sectors with slow technological progress, stable environments, or in sports. Fourth, in procurement contexts we should expect these patterns to be more pronounced when the procurement authority can exert her discretion and rely on nonverifiable information about the contractor's past performance, as is more often the case for private than for public procurement. Fifth, for industries in which both short-term performance-enhancing efforts and long-term productivity investments are relevant options, effort should take place towards the end of the contractual relationship whereas investment should rather take place earlier in the relationship. Finally, for contracts where performance is difficult to verify and explicit incentives are hard to enforce, longer contracts should be observed when the contract stakes are high, investment is highly valuable, and the environment is not too volatile.

Throughout the paper we have restricted our attention to a single agent's investment incentives. It would be interesting to extend the analysis to competing agents with heterogeneous types and study how the duration of the contract may be affected by the competition among agents.

Appendix

A Proof of Proposition 3

In period 2, maximizing $b_2 i_2 - k(i_2)$ leads to:

- if $k'(1) \leq b_2$, then $i_2^* = 1$;
- if $k'(1) > b_2$, then $i_2^* = \hat{\imath}_2^*(b_2)$, such that $k'(\hat{\imath}_2^*) = b_2$.

It follows that i_2 (weakly) increases with $b_2 = (2\rho - 1)\delta\Delta$. In period 1, the agent maximizes $b_1 i_1 - k(i_1)$, where b_1 is of the form:

$$b_1 = \sigma\left[b_2 - \max_{i_2 \leq 1}\{b_2 i_2 - k(i_2)\}\right],$$

where $\sigma \equiv (2\rho - 1)\delta < 1$. It follows that $b_1 < b_2$. Therefore:

- if $k'(1) \geq b_2 (> b_1)$, then $i_1^* = \hat{\imath}_1^*(b_1)$, such that $k'(\hat{\imath}_1^*) = b_1$; $b_1 < b_2$ and $k'' > 0$ then together imply $i_1^* = \hat{\imath}_1^*(b_1) < i_2^* = \hat{\imath}_2^*(b_2) < 1$.
- if $k'(1) < b_2$, then $i_2^* = 1$ and thus $b_1 = \sigma k(1) < k(1) < k'(1)$, where the last inequality stems from $k(0) = 0$ and $k'' > 0$. It follows that i_1 is again equal to $\hat{\imath}_1^*(b_1) < i_2^* = 1$.

We thus have $i_1^* < i_2^* \leq 1$. It remains to establish the comparative statics for i_1^*. They follow from:

$$b_1 = (2\rho - 1)\delta\left[(2\rho - 1)\delta\Delta - \max_{i_2 \leq 1}\{(2\rho - 1)\delta\Delta i_2 - k(i_2)\}\right],$$

which, using the envelope theorem, yields:

$$\frac{\partial b_1}{\partial \Delta} = \sigma(1 - i_2^*)\frac{\partial b_2}{\partial \Delta} > 0,$$
$$\frac{\partial b_1}{\partial \sigma} = \sigma(1 - i_2^*)\frac{\partial b_2}{\partial \sigma} + \left[b_2 - \max_{i_2}\{b_2 i_2 - k(i_2)\}\right] > 0,$$

where the last inequality stems from $\frac{\partial b_2}{\partial \sigma} > 0$ and:

$$\max_{i_2 \leq 1}\{b_2 i_2 - k(i_2)\} < \max_{i_2 \leq 1}\{b_2 i_2\} = b_2.$$

B Proof of Lemma 1

By construction, $p_1^H = \rho$ and, for $t > 1$:

$$p_t^H = \rho p_{t-1}^H + (1-\rho)\left(1 - p_{t-1}^H\right) \implies 2p_t^H - 1 = (2\rho - 1)\left(2p_{t-1}^H - 1\right).$$

Therefore, by iteration:

$$2p_t^H - 1 = (2\rho-1)^{t-1}\left(2p_1^H - 1\right) = (2\rho-1)^t \implies p_t^H = \frac{1}{2} + \frac{(2\rho-1)^t}{2}.$$

Similarly, $p_1^L = 1 - \rho$ and, for $t > 1$:

$$\begin{aligned} p_t^L = \rho p_{t-1}^L + (1-\rho)\left(1 - p_{t-1}^L\right) &\implies 2p_t^L - 1 = (2\rho-1)^{t-1}\left(2p_1^L - 1\right) = -(2\rho-1)^t \\ &\implies p_t^L = \frac{1}{2} - \frac{(2\rho-1)^t}{2} = 1 - p_t^H. \end{aligned}$$

C Proof of Proposition 7

When the contracts are sufficiently lengthy to ensure that $i_n^* = 1$, from (12) the expected welfare is equal to:

$$\begin{aligned} W_n &= \frac{v_n}{1-\delta^n} + \frac{\delta^{n-1}}{1-\delta^n}\frac{1}{2}\left[\delta\left(\Pi_n^H - \Pi_n^L\right) - c\right] \\ &= \frac{\frac{1-\delta^n}{1-\delta}\frac{L+H}{2} + \delta^{n-1}\frac{\Delta}{2}}{1-\delta^n} + \frac{\delta^{n-1}}{1-\delta^n}\frac{1}{2}\left[\frac{1-(2\rho-1)^n\delta^n}{1-(2\rho-1)\delta}(2\rho-1)\delta\Delta - (2\rho-1)^n\delta^n(\Delta - c) - c\right] \\ &= \frac{1}{1-\delta}\frac{L+H}{2} + \chi_n\frac{1}{2\delta}\left(\frac{\Delta}{1-(2\rho-1)\delta} - c\right), \end{aligned}$$

where

$$\chi_n \equiv \frac{\delta^n\left(1-(2\rho-1)^n\delta^n\right)}{1-\delta^n}.$$

In Online Appendix C we show that χ_n decreases when n increases; hence it is never optimal to choose a duration n larger than what is needed to induce $i_n^* = 1$.

When instead $i_n^* = \hat{\imath}_n < 1$ using (11) and (13) the equilibrium welfare is of the form:

$$W_n = \frac{1}{1-\delta}\frac{L+H}{2} + \xi_n\left(\frac{(2\rho-1)\delta}{1-(2\rho-1)\delta} - \frac{c}{\Delta}\right)\frac{\Delta}{2\delta},$$

where

$$\xi_n \equiv \frac{\delta^n}{1-\delta^n}\frac{1-(2\rho-1)^n\delta^n}{(2\rho-1)^n\delta^n} = \frac{1-(2\rho-1)^n\delta^n}{(2\rho-1)^n(1-\delta^n)}$$

increases with n – see Online Appendix D. Therefore, the optimal duration consists of choosing n so that $\hat{\imath}_n = 1$. The comparative statics follows from Proposition 6, and in particular from i_n^* increasing with n, Δ/c, δ and ρ.

A similar analysis can be made for the evaluation of the expected performance generated by the contract. As shown in Online Appendix E, as long as the equilibrium investment is $i_n^* = \hat{\imath}_n < 1$, the expected total discounted value of performance is of the form:

$$\begin{aligned} V_n &= \frac{1}{1-\delta}\frac{L+H}{2} + \frac{\delta^n}{1-\delta^n}\hat{\imath}_n \frac{1}{1-(1-(2\rho-1)\,\delta)\,\frac{c}{\Delta}}\frac{\Delta}{2\delta} \\ &= \frac{1}{1-\delta}\frac{L+H}{2} + \xi_n\left(\frac{(2\rho-1)\,\delta\Delta}{1-(2\rho-1)\,\delta} - c\right)\frac{1}{\Delta-(1-(2\rho-1)\,\delta)\,c}\frac{\Delta}{2\delta} \end{aligned}$$

whereas for $i_n^* = 1$, it is equal to:

$$V_n = \frac{1}{1-\delta}\frac{L+H}{2} + \chi_n \frac{1}{1-(2\rho-1)\,\delta}\frac{\Delta}{2\delta}.$$

As ξ_n increases whereas χ_n decreases when n increases, it follows that, to maximize this expected total performance, it is optimal to choose n so that $\hat{\imath}_n = 1$.

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Online Appendix
Not for publication

A Alternative baseline model without adverse selection

We consider here a variant of our baseline model in which the type θ_t is not observable by the agent either; in each period, the agent thus bases his investment decision only on his performance history.

As before, enhancing performance in period 2 from $q_2 = L$ to $q_2 = H$ increases the agent's expected payoff by $\Delta\Pi_3 = (2\rho - 1)\,\delta\Delta$. Consider now period 2, following a given investment decision $\hat{\imath}_1 \in \{0, 1\}$ taken in period 1 (here $\hat{\imath}_1$ denotes the decision eventually taken by the agent in period ; it is equal to 1 with probability i_1 and to 0 with probability $1 - i_1$), a given performance q_1 observed in that period (which is thus such that $q_1 = H$ if $\theta_1 = H$ or $\hat{\imath}_1 = 1$, and $q_1 = L$ otherwise).

If $q_1 = L$, then the agent and the market infer $\theta_1 = L$, and thus anticipate $\theta_2 = L$ with probability ρ. The agent thus chooses i_2 so as to maximize:

$$\begin{aligned}
\Pi_2^L &= -ci_2 + \delta\left\{(1 - \rho + \rho i_2)(L + \rho\Delta) + \rho(1 - i_2)(L + (1 - \rho)\Delta)\right\} \\
&= \delta(L + \rho\Delta) - ci_2 - \rho(1 - i_2)(2\rho - 1)\delta\Delta \\
&= \delta(L + \rho\Delta) - (2\rho - 1)\rho\delta\Delta + \left(b_2^L - c\right)i_2, \qquad (14)
\end{aligned}$$

where:

$$b_2^L \equiv (2\rho - 1)\,\rho\delta\Delta. \qquad (15)$$

The optimal investment is thus $i_2^L = 1$ if $c < b_2^L$, and $i_2^L = 0$ if $c > b_2^L$.

If instead $q_1 = H$, then the agent and the market anticipate $\theta_2 = H$ with probability ρ. The agent thus chooses i_2 so as to maximize:

$$\begin{aligned}
\Pi_2^H &= -ci_2 + \delta\left\{(\rho + (1 - \rho)i_2)(L + \rho\Delta) + (1 - \rho)(1 - i_2)(L + (1 - \rho)\Delta)\right\} \\
&= \delta(L + \rho\Delta) - ci_2 - (1 - \rho)(1 - i_2)(2\rho - 1)\delta\Delta \\
&= \delta(L + \rho\Delta) - (2\rho - 1)(1 - \rho)\delta\Delta + \left(b_2^H - c\right)i_2, \qquad (16)
\end{aligned}$$

where:

$$b_2^H \equiv (2\rho - 1)(1 - \rho)\,\delta\Delta. \qquad (17)$$

The optimal investment is thus $i_2^H = 1$ if $c < b_2^H$, and $i_2^H = 0$ if $c > b_2$. Note that $b_2^H < b_2^L$ whenever there is information decay ($\rho < 1$):

$$b_2^L - b_2^H = (2\rho - 1)^2 \delta\Delta > 0.$$

Therefore, substituting for the equilibrium values of i_2 in (14) and (16):

- If $c < b_2^H$, the agent always invests in period 2: $i_2^H = i_2^L = 1$; his payoff is then:

$$\Pi_2^L = \Pi_2^H = \delta (L + \rho\Delta) - c.$$

- If instead $b_2^H < c < b_2^L$, the agent invests in period 2 only when $q_1 = L$: $i_2^H = 0 < i_2^L = 1$; his payoff is then:

$$\begin{aligned} \Pi_2^L &= \delta (L + \rho\Delta) - c, \\ \Pi_2^H &= \delta (L + (1 - 2\rho (1 - \rho)) \Delta). \end{aligned}$$

- Finally, if $c > b_2^L$, the agent never invests in period 2: $i_2^H = i_2^L = 0$; his payoff is then:

$$\begin{aligned} \Pi_2^L &= \delta (L + 2\rho (1 - \rho) \Delta), \\ \Pi_2^H &= \delta (L + (1 - 2\rho (1 - \rho)) \Delta). \end{aligned}$$

Consider now period 1. The agent chooses i_1 so as to maximize:

$$\Pi_1 = -ci_1 + \delta \left\{ \frac{1 + i_1}{2} \Pi_2^H + \frac{1 - i_1}{2} \Pi_2^L \right\} = \delta \frac{\Pi_2^H + \Pi_2^L}{2} + (b_1 - c)\, i_1.$$

where

$$b_1 \equiv \delta \frac{\Pi_2^H - \Pi_2^L}{2}. \tag{18}$$

The expected payoff differential $\Pi_2^H - \Pi_2^L$ is by construction non-negative[27] and, using (**14**) and (**16**), can be expressed as:

$$\begin{aligned} \Pi_2^H - \Pi_2^L &= (2\rho - 1)^2 \delta\Delta + \left(b_2^H - c\right) i_2^H - \left(b_2^L - c\right) i_2^L \\ &= \left(1 - i_2^L\right) (2\rho - 1)^2 \delta\Delta + \left(c - b_2^H\right) \left(i_2^L - i_2^H\right). \end{aligned}$$

Therefore:

[27] We have:

$$\begin{aligned} \Pi_2^L &= \max_{i_2} \{\delta (L + \rho\Delta) - ci_2 - \rho (1 - i_2) (2\rho - 1) \delta\Delta\} \\ &\le \max_{i_2} \{\delta (L + \rho\Delta) - ci_2 - (1 - \rho) (1 - i_2) (2\rho - 1) \delta\Delta\} = \Pi_2^H. \end{aligned}$$

- If $c < b_2^H$, the agent always invests in period 2: $i_2^H = i_2^L = 1$, and thus $\Pi_2^L = \Pi_2^H$; hence, the agent does not invest in period 1: $i_1^* = 0$.

- If instead $b_2^H < c < b_2^L$, the agent invests in period 2 only when $q_1 = L$: $i_2^H = 0 < i_2^L = 1$, and thus:
$$b_1 = \delta \frac{c - b_2^H}{2} < c,$$
implying again that the agent does not invest in period 1: $i_1^* = 0$.

- Finally, if $c > b_2^L$, the agent never invests in period 2: $i_2^H = i_2^L = 0$, and thus:
$$b_1 = \frac{(2\rho - 1)^2 \delta^2 \Delta}{2} < b_2^L = (2\rho - 1)\rho\delta\Delta, \tag{19}$$
implying again that the agent does not invest in period 1 (as here $c > b_2^L > b_1$).

This leads to:

Proposition 8 *(i)The agent never invests in period* 1*, but invests in period* 2 *if the cost is sufficiently low:*
$$i_t^* = 1 \textit{ if and only if } t = 2 \textit{ and } c \leq b_2^{q_1},$$
where q_1 *denotes the performance observed in period* 1 *and* $b_2^L \equiv (2\rho - 1)\rho\delta\Delta > b_2^H \equiv (2\rho - 1)(1 - \rho)\delta\Delta$ 0.

(ii) The incentive to invest in period 2 *therefore increases with the discount factor* δ *and the performance differential* Δ*, relative to the cost* c*; it however decreases with the performance observed in the first period.*

(iii) Following a bad performance, the incentive to invest in period 2 *increases with the degree of information persistence* ρ*; following a good performance, however, it increases with information persistence only when this is initially low (namely, in the range* $\rho < 3/4$*).*

The insights of our baseline model thus carry over. Part (i), for instance, follows directly from the fact that the performance in period 2 is a sufficient statistic for predicting the performance in period 3. Interestingly, however, the incentive to invest increases over time even in the absence of information decay or of discounting: there is an additional "option value" in waiting and observing q_1 before deciding whether to invest.[28] Likewise, in part (ii) the comparative

[28] This is reflected in the term $\rho - \frac{1}{2} < \rho$ in (19), rewritten as:
$$b_1 = \delta \left(\rho - \frac{1}{2}\right)(2\rho - 1)\delta\Delta < b_2^L = (2\rho - 1)\rho\delta\Delta.$$

statics on δ, Δ and c build on the same intuition as before; in addition, incentives decrease with existing reputation as the payoff from investing (net of c) is the same regardless of past performance, whilst the payoff from not investing is lower when past performance is low. Part (iii) follows from information persistence raising the payoff from investing, and lowering (resp., raising) the payoff from not investing when past performance is low (resp., high).

An implication of Proposition 8 is that, on average, the agent's performance improves as the market evaluation approaches: the expected performance in period 1 is equally likely to be H or L, as the agent never invests in that period; in the second period, the performance is more likely to be good when the agent invests with positive probability, which is the case when $c < b_2^L$. Therefore:

Corollary 2 *The expected performance (weakly) increases as the market evaluation approaches:* $q_1^e \leq q_2^e$*, with a strict inequality when* $c < b_2^L$*.*

Proof. The agent's productivity is initially high or low with equal productivity. As he never invests in period 1, his expected performance is:

$$q_1^e = \frac{L+H}{2} = L + \frac{\Delta}{2}.$$

By contrast, in period 2, the performance is low only if the agent's type is low *and* he does not invest; as a result, the expected performance is

$$\begin{aligned} q_2^e &= \frac{\left[\rho + (1-\rho)\, i_2^H\right] H + (1-\rho)\left(1 - i_2^H\right) L}{2} + \frac{\left[1 - \rho + \rho i_2^L\right] H + \rho\left(1 - i_2^L\right) L}{2} \\ &= L + \left(1 + (1-\rho)\, i_2^H + \rho i_2^L\right) \frac{\Delta}{2}. \end{aligned}$$

The conclusion ($q_2^e \geq q_1^e$) follows from $i_2^H, i_2^L \geq 0$ (with a strict inequality for i_2^L when $c > b_2^L$). ■

B Proof of Proposition 5

As already noted: (i) the agent never invests in period 2, as exerting effort is less costly and produces the same benefit on his expected payoff; and (ii) in period 1, the agent may either invest or exert effort, but never does both. What remains to be determined is: (i) the probability e_q of effort in period 2, as a function of the performance $q \in \{L, H\}$ observed in period 1; and (i) the probability of effort e_1, or of investment i_1, in the first period.

B.1 No information decay ($\rho = 1$)

We first consider the case where $\rho = 1$. When $\theta_1 = H$, the agent then remains productive forever and thus never needs to invest or exert effort. If instead $\theta_1 = L$, and the agent neither invests nor exerts efforts in period 1 then, observing $q = L$ in period 1 reveals that the agent has a low productivity; as investing is not credible in period 2, the market will then anticipate that the agent will still have a low productivity in period 3, and thus the agent never exerts effort in period 2. The agent can instead enhance his performance and the market beliefs, either by investing in period 1, or by exerting effort in both periods (exerting effort in period 1 only would be a dominated strategy, as a low performance in period 2 would reveal his low productivity).

In a candidate equilibrium in which the agent invests in period 1, the market then believes that the agent is highly productive and the gain from investment is $\delta^2\Delta - c$. As the agent could deviate either by exerting effort in both periods or do nothing, it follows that investing constitutes an equilibrium strategy if

$$\delta^2\Delta - c \geq 0, \delta^2\Delta - (1+\delta)\,\gamma.$$

In a candidate equilibrium in which the agent exerts effort with probability e_1 in period 1, the market then remains uncertain about the agent's productivity and the gain from investment is $\delta^2\Delta/\left(1+e_1\right) - (1+\delta)\,\gamma$. As the agent could deviate either by investing or by doing nothing, it follows that exerting effort constitutes an equilibrium strategy if

$$\frac{\delta^2\Delta}{1+e_1} - (1+\delta)\,\gamma \geq 0, \frac{\delta^2\Delta}{1+e_1} - c.$$

Finally, in a candidate equilibrium in which the agent neither invests nor exerts effort, by deviating the agent would convince the market that he has a high productivity; therefore, doing nothing constitutes an equilibrium strategy if

$$0 \geq \delta^2\Delta - c, \delta^2\Delta - (1+\delta)\,\gamma.$$

Comparing these conditions, we obtain that:

- if $c < (1+\delta)\,\gamma$, then the agent never exerts effort, and invests in period 1 when he has a low productivity if $c < \delta^2\Delta$ (in the limit case where $c = \delta^2\Delta$, then the agent invests with any probability $i_1 \in [0,1]$);
- if $c > (1+\delta)\,\gamma$, then the agent never invests, and exerts effort in both periods when he has a low productivity if $\gamma < \frac{\delta^2\Delta}{2(1+\delta)}$ (in the limit case where $\gamma = \frac{\delta^2\Delta}{2(1+\delta)}$, then the agent

exerts effort in period 1 with any probability $e_1 \in [0,1]$, in which case he exerts effort also in period 2);

- if $c = (1+\delta)\gamma$, then:
  - if $c < \delta^2\Delta$, there exists an equilibrium in which the agent invests in period 1 (or with any probability $i_1^* \in [0,1]$ in the limit case where $c = \delta^2\Delta$);
  - if $\gamma < \frac{\delta^2\Delta}{1+\delta}$, there exists an equilibrium in which the agent exerts effort in period 1 with some probability $e_1^* > 0$, in which case he exerts effort also in period 2 (if $\gamma < \frac{\delta^2\Delta}{2(1+\delta)}$, then $e_1^* = 1$);
- in all cases, the performance remains constant over time: $q_1 = q_2 = H$ if the agent invests in period 1 or exerts effort in both periods, and $q_1 = q_2 = L$ otherwise.

B.2 Information decay ($\rho < 1$)

We now turn to the case of information decay: $\rho \in (1/2, 1)$. In period 2 a low-productivity agent exerts effort if condition (4) holds, which in equilibrium amounts to:

$$\gamma \leq G_q(e_q) \equiv (2\rho - 1)\,\delta\nu_q(e_q)\,\Delta, \tag{20}$$

where $\nu_q(e_q)$, the equilibrium market belief following $q \in \{L, H\}$ in period 1 and H in period 2, is given by:[29]

$$\nu_H(e_H) \equiv \frac{\rho + (1-\rho)\,e_1}{\rho + (1-\rho)\,e_1 + (1-\rho+\rho e_1)\,e_H}, \tag{21}$$

$$\nu_L(e_L) \equiv \frac{1-\rho}{1-\rho+\rho e_L}. \tag{22}$$

As ν_q decreases when e_q increases, the equilibrium effort of a low-productivity agent in period 2, following a performance $q \in \{L, H\}$ in period 1, is given by:

$$e_q^* = \begin{cases} 0 \text{ if } \gamma \geq G_q(e_q = 0), \\ 1 \text{ if } \gamma \leq G_q(e_q = 1), \\ \hat{e}_q \text{ s.t. } G_q(\hat{e}_q) = \gamma \text{ otherwise.} \end{cases} \tag{23}$$

Furthermore, $\nu_H(0) = \nu_L(0)$ and, for $e > 0$, $\nu_H(e) > \nu_L(e)$:

$$\frac{\nu_H(e)}{\nu_L(e)} = \frac{1 + \frac{\rho}{1-\rho}e}{1 + \frac{1-\rho+\rho e_1}{\rho+(1-\rho)e_1}e} > 1, \tag{24}$$

[29] ν_H and thus G_H depend on both e_H and e_1; when there is no risk of confusion, we drop the argument e_1 to simplify the exposition.

where the inequality stems from $\frac{1-\rho+\rho e_1}{\rho+(1-\rho)e_1} < \frac{\rho}{1-\rho}$ (which amounts to $(1-\rho)^2 < \rho^2$). Therefore, $G_H(0) = G_L(0) = (2\rho-1)\delta\Delta$ and $G_H(e) > G_L(e)$ for $e > 0$. It follows that: (i) e_H^* and e_L^* are both positive when $\gamma < (2\rho-1)\delta\Delta$, and they are both equal to 0 if $\gamma \geq (2\rho-1)\delta\Delta$; (ii) $e_H^* \geq e_L^*$, with a strict inequality whenever $0 < e_L^* < 1$.

Next, we note that the agent neither invests nor exerts effort in period 1 when, in case of a high performance in period 1, it has little incentive to exert effort in period 2:

Lemma 2 *Suppose* $e_H^* < 1$. *Then* $e_1^* = i_1^* = 0$.

Proof. To see this, note first that $e_H^* < 1$ implies $e_L^* (\leq e_H^*) < 1$; therefore, in period 2, and for any given performance $q \in \{L, H\}$ observed in period 1, the equilibrium market belief ν_q^* must be such that agent either does not exert effort (whenever $e_q^* = 0$), or is indifferent between exerting effort or not (if $e_q^* = \hat{e}_q$); it follows that in period 1, the expected payoffs from investing, exerting effort, and doing nothing are respectively given by:

$$\Pi_1^L(i_1 = 1, e_1 = 0) = \delta^2[L + (1-\rho)\Delta] + \rho\nu_H^*(2\rho-1)\delta^2\Delta - c,$$
$$\Pi_1^L(i_1 = 0, e_1 = 1) = \delta^2[L + (1-\rho)\Delta] + (1-\rho)\nu_H^*(2\rho-1)\delta^2\Delta - \gamma,$$
$$\Pi_1^L(i_1 = e_1 = 0) = \delta^2[L + (1-\rho)\Delta] + (1-\rho)\nu_L^*(2\rho-1)\delta^2\Delta.$$

Therefore, the benefit from investing or exerting effort cannot exceed

$$\rho\nu_H^*(2\rho-1)\delta^2\Delta < \nu_H^*(2\rho-1)\delta\Delta,$$

where the right-hand side does not exceed γ (and thus c) when $e_H^* < 1$. ■

As $e_H^* < 1$ implies $e_1^* = 0$, this requires:

$$\gamma > G_H(e_H = 1)|_{e_1^*=0} = \rho(2\rho-1)\delta\Delta. \quad (25)$$

Conversely, under (25), we must have $e_1^* = 0$: to see this, note that $G_H(e_H = 1) = (2\rho-1)\delta\nu_H(e_H = 1)$ where:

$$\nu_H(e_H = 1) = \frac{\rho + (1-\rho)e_1}{1 + e_1}$$

decreases as e_1 increases; but then $e_1^* > 0$ would imply $e_H^* = 1$ and thus require $\gamma < G_H(e_H = 1) \leq G_H(e_H = 1)|_{e_1^*=0} = \rho(2\rho-1)\delta\Delta$, a contradiction. It follows that, under (25), there exists a unique equilibrium, in which the agent (i) never invests, (ii) never exerts effort in period 1, and (iii) in period 2:

- if $\gamma \geq (2\rho - 1)\,\delta\Delta$, the agent never exerts effort;
- if $(2\rho - 1)\,\delta\Delta > \gamma > \rho\,(2\rho - 1)\,\delta\Delta$ then, given the performance $q \in \{L, H\}$ observed in period 1, the agent exerts effort with probability $\hat{e}_q$ in case of low productivity.

From now on, we focus on the case $\gamma \leq \rho\,(2\rho - 1)\,\delta\Delta$. In period 2 we must then have

- $e_H^* = 1$: $e_H^* < 1$ would imply $e_1^* = 0$ and thus require $\gamma > G_H\,(e_H = 1)|_{e_1^*=0} = \rho\,(2\rho - 1)\,\delta\Delta$, a contradiction.
- $e_L^* > 0$: $e_H^* = 1 > 0$ implies $G_L\,(0) = G_H\,(0) > \gamma$; we thus have:
  - if $\gamma \leq G_L\,(1) = (1-\rho)\,(2\rho - 1)\,\delta\Delta$, $e_L^* = 1$;
  - otherwise $e_L^* = \hat{e}_L > 0$ is such that $\gamma = G_L\,(\hat{e}_L) = (2\rho - 1)\,\delta\nu_L\,(\hat{e}_L)\,\Delta$, or:

$$\gamma = \frac{1-\rho}{1-\rho+\rho\hat{e}_L}\,(2\rho - 1)\,\delta\Delta \iff \hat{e}_L = \frac{1-\rho}{\rho}\left(\frac{(2\rho-1)\,\delta\Delta}{\gamma} - 1\right). \tag{26}$$

In period 1, the comparison between the expected payoffs from investing and from exerting effort, given respectively by (6) and (7) shows, using $e_H^* = 1$, that the agent favors effort over investment when:

$$c > [1 + (2\rho - 1)\,\delta]\,\gamma, \tag{27}$$

and favors instead investment over effort when the inequality is reversed.

Suppose first that (27) holds, in which case the agent never invests ($i_1^* = 0$). Comparing (5) and (7), in period 1 the agent then exerts effort if:

$$\gamma \leq \frac{(1-\rho+\rho e_H^*)\,\nu_H^* - (1-\rho+\rho e_L^*)\,\nu_L^*}{1 + (e_H^* - e_L^*)\,\rho\delta}\,(2\rho - 1)\,\delta^2\Delta,$$

which, using $e_H^* = 1$ and (22), can be written as:

$$\gamma \leq G_E\,(e_1; e_L^*) \equiv \frac{\nu_H^*\,(e_1) - (1-\rho)}{1 + (1 - e_L^*)\,\rho\delta}\,(2\rho - 1)\,\delta^2\Delta,$$

where:

$$\nu_H^*\,(e_1) \equiv \nu_H\,(e_H^* = 1; e_1) = \frac{\rho + (1-\rho)\,e_1}{1 + e_1}.$$

As ν_H^*, and thus G_E, decreases as e_1 increases, it follows that $e_1^* > 0$ when (using $\nu_H^*\,(0) = \rho$):

$$\gamma < G_E\,(0; e_L^*) = \frac{(2\rho-1)^2\,\delta^2\Delta}{1 + (1 - e_L^*)\,\rho\delta}.$$

Two cases can be distinguished, depending on the value of e_L^*:

- If $\gamma \leq G_L(1) = (1-\rho)(2\rho-1)\delta\Delta$, then $e_L^* = 1$ and :

$$G_E(0; e_L^* = 1) = (2\rho - 1)^2 \delta^2 \Delta,$$

which lies below $(1-\rho)(2\rho-1)\delta\Delta$ if and only if $(2\rho-1)\delta < 1-\rho$, or

$$\rho < \hat{\rho} \equiv \frac{1+\delta}{1+2\delta}.$$

- If $(1-\rho)(2\rho-1)\delta\Delta < \gamma \leq (2\rho-1)\delta\Delta$, then $e_L^* = \hat{e}_L$ and:

$$\gamma < G_E(0; e_L^* = \hat{e}_L) = \frac{(2\rho-1)^2 \delta^2 \Delta}{1 + \delta\rho\left(1 - \frac{1-\rho}{\rho}\left(\frac{(2\rho-1)\delta\Delta}{\gamma} - 1\right)\right)} \iff \gamma < \frac{\rho(2\rho-1)\delta^2\Delta}{1+\delta},$$

where the right-hand side is lower than $\rho(2\rho-1)\delta\Delta$, and lies above $(1-\rho)(2\rho-1)\delta\Delta$ if and only if $\rho > \hat{\rho}$:

$$\frac{\rho(2\rho-1)\delta^2\Delta}{1+\delta} - (1-\rho)\delta(2\rho-1)\Delta = (\rho - \hat{\rho})\delta(2\rho-1)\Delta\frac{1+2\delta}{1+\delta}.$$

Summing-up, we have: $e_1^* > 0$ if

$$\gamma < \hat{\gamma} \equiv \begin{cases} (2\rho-1)\delta^2\Delta & \text{if} \quad \rho < \hat{\rho} \\ \dfrac{\rho(2\rho-1)\delta^2\Delta}{1+\delta} & \text{if} \quad \rho \geq \hat{\rho} \end{cases},$$

and $e_1^* = 0$ if $\gamma \geq \hat{\gamma}$.

Likewise, we have $e_1^* = 1$ when

$$\gamma \leq G_E(1; e_L^*) = \frac{(2\rho-1)^2\delta^2}{1 + (1-e_L^*)\rho\delta}\frac{\Delta}{2}.$$

Therefore:

- If $\gamma \leq G_L(1) = (1-\rho)(2\rho-1)\delta\Delta$, then $e_L^* = 1$ and:

$$G_E(1; e_L^* = 1) = (2\rho-1)^2\delta^2\frac{\Delta}{2},$$

which lies below $(1-\rho)(2\rho-1)\delta\Delta$ if and only if

$$\rho < \tilde{\rho} \equiv \frac{2+\delta}{2(1+\delta)},$$

as can be seen from:

$$\begin{aligned}(2\rho-1)^2\delta^2\frac{\Delta}{2} - (1-\rho)(2\rho-1)\delta\Delta &= [(2\rho-1)\delta - 2(1-\rho)]\frac{(2\rho-1)\delta\Delta}{2} \\ &= (\rho - \tilde{\rho})(2\rho-1)\delta(1+\delta)\Delta,\end{aligned}$$

- If $(1-\rho)(2\rho-1)\delta\Delta < \gamma \leq (2\rho-1)\delta\Delta$, then $e_L^* = \hat{e}_L$ and:

$$\gamma < G_E(1; e_L^* = \hat{e}_L) = \frac{(2\rho-1)^2\delta^2}{1+\delta\rho\left(1-\frac{1-\rho}{\rho}\left(\frac{(2\rho-1)\delta\Delta}{\gamma}-1\right)\right)}\frac{\Delta}{2} \iff \gamma < (2\rho-1)\frac{\delta^2\Delta}{2(1+\delta)},$$

where the right-hand side is lower than $\rho(2\rho-1)\delta\Delta$, and lies above $(1-\rho)(2\rho-1)\delta\Delta$ if and only if $\rho > \tilde{\rho}$:

$$(2\rho-1)\frac{\delta^2\Delta}{2(1+\delta)} - (1-\rho)(2\rho-1)\delta\Delta = (\rho-\tilde{\rho})(2\rho-1)\delta\Delta.$$

Summing-up, we have: $e_1^* = 1$ if:

$$\gamma \leq \tilde{\gamma} \equiv \begin{cases} (2\rho-1)\delta^2\frac{\Delta}{2} & \text{if} \quad \rho < \tilde{\rho} \\ (2\rho-1)\frac{\delta^2}{1+\delta}\frac{\Delta}{2} & \text{if} \quad \rho \geq \tilde{\rho} \end{cases},$$

and $e_1^* < 1$ if $\gamma > \tilde{\gamma}$.

Suppose now that:

$$c < [1+(2\rho-1)\delta]\gamma, \tag{28}$$

in which case the agent never exerts effort in period 1 (i.e., $e_1^* = 0$), as he favors investment over effort in that period. We thus have:

$$\nu_H^* = \frac{\rho}{\rho+(1-\rho)e_H^*}, \nu_L^* = \frac{1-\rho}{1-\rho+\rho e_L^*}.$$

Using this and $e_H^* = 1$, in period 1 the expected payoffs from investing and from not investing are respectively given by:

$$\Pi_1^L(i_1 = e_1 = 0) = \delta^2[L+(1-\rho)\Delta] + (1-\rho)(2\rho-1)\delta^2\Delta - \delta\rho\gamma e_L^*,$$
$$\Pi_1^L(i_1 = 1, e_1 = 0) = -c + \delta^2[L+(1-\rho)\Delta] + \rho(2\rho-1)\delta^2\Delta - \delta(1-\rho)\gamma.$$

Therefore, the agent indeed invests in period 1 if:

$$c \leq G_I(e_L^*) \equiv (2\rho-1)^2\delta^2\Delta + [\rho e_L^* - (1-\rho)]\delta\gamma. \tag{29}$$

Two cases can again be distinguished, depending on the value of e_L^*:

- If $\gamma < G_L(1) = (1-\rho)(2\rho-1)\delta\Delta$, then $e_L^* = 1$ and (29) becomes:

$$c \leq G_I(e_L^* = 1) = (2\rho-1)^2\delta^2\Delta + (2\rho-1)\delta\gamma.$$

  This condition is less demanding than (28) as long as:

$$\gamma < (2\rho-1)^2\delta^2\Delta.$$

  Hence, in the range $\gamma < (1-\rho)(2\rho-1)\delta\Delta$):

- if $\rho \geq \hat{\rho}$, which implies $1-\rho < (2\rho-1)\,\delta$, (29) is always less demanding than (28);
- if $\rho < \hat{\rho}$, (29) is less demanding than (28) as long as $\gamma < (2\rho-1)^2\,\delta^2\Delta$, and more demanding for $(2\rho-1)^2\,\delta^2\Delta < \gamma < (1-\rho)\,(2\rho-1)\,\delta\Delta$.

- If $(1-\rho)\,(2\rho-1)\,\delta\Delta < \gamma \leq (2\rho-1)\,\delta\Delta$, then $e_L^* = \hat{e}_L$ and (29) becomes:

$$c \leq G_I\left(e_L^* = \hat{e}_L\right) = \delta^2\,(2\rho-1)\,\rho\Delta - 2\,(1-\rho)\,\delta\gamma.$$

As the right-hand side decreases when γ increases, this condition is more demanding than (28) for

$$\gamma > \frac{\rho\,(2\rho-1)\,\delta^2\Delta}{1+\delta},$$

where the right-hand side is lower than $(2\rho-1)\,\delta\Delta$ and, as already noted, lies above (resp., below) $(1-\rho)\,(2\rho-1)\,\delta\Delta$ if $\rho > \hat{\rho}$ (resp., $\rho < \hat{\rho}$). Hence, in the range $(1-\rho)\,(2\rho-1)\,\delta\Delta < \gamma \leq (2\rho-1)\,\delta\Delta$:

  - if $\rho \geq \hat{\rho}$, (29) is less demanding than (28) as long as $(1-\rho)\,(2\rho-1)\,\delta\Delta < \gamma < \frac{\rho(2\rho-1)\delta^2\Delta}{1+\delta}$, and more demanding for $\frac{\rho(2\rho-1)\delta^2\Delta}{1+\delta} < \gamma < (2\rho-1)^2\,\delta^2\Delta$;
  - if $\rho < \hat{\rho}$, (29) is always more demanding than (28).

Summing-up:

- If $\gamma \geq (2\rho-1)\,\delta\Delta$, the agent never invests nor exerts effort.

- If $(2\rho-1)\,\delta\Delta > \gamma > \rho\,(2\rho-1)\,\delta\Delta$ then the agent (i) never invests, (ii) never exerts effort in period 1, and (iii) in period 2, given the performance $q \in \{L, H\}$ observed in period 1, the agent exerts effort with probability $\hat{e}_q$ in case of low productivity.

- If $\gamma < \rho\,(2\rho-1)\,\delta\Delta$, then the agent exerts effort with positive probability in case of low productivity in period 2: $e_H^* = 1$ and $e_L^* > 0$; in addition, in case of low productivity in period 1:

  - If $\rho > \hat{\rho}$, then the agent (see Figure 1a):

    * invests ($i_1^* = 1$) if

$$\frac{c}{\gamma} < 1 + (2\rho-1)\,\delta \text{ and } c < \rho\,(2\rho-1)\,\delta^2\Delta - 2\,(1-\rho)\,\delta\gamma,$$

* exerts instead effort with positive probability ($e_1^* > 0$) if

$$\frac{c}{\gamma} > 1 + (2\rho - 1)\,\delta \text{ and } \gamma < \frac{\rho\,(2\rho-1)\,\delta^2\Delta}{1+\delta},$$

with $e_1^* = 1$ if $\gamma \leq \tilde{\gamma}$, and $e_1^* < 1$ if $\gamma > \tilde{\gamma}$,

* neither invest nor exert effort otherwise.

– If $\rho < \hat{\rho}$, then the agent (see Figure 1b):

* invests ($i_1^* = 1$) if $c/\gamma < 1 + (2\rho - 1)\,\delta$ and

$$c < \max\left\{(2\rho-1)^2\,\delta^2\Delta + (2\rho-1)\,\delta\gamma, \rho\,(2\rho-1)\,\delta^2\Delta - 2\,(1-\rho)\,\delta\gamma\right\},$$

* exerts effort with positive probability ($e_1^* > 0$) if $c/\gamma > 1 + (2\rho - 1)\,v$ and

$$\gamma < (2\rho-1)^2\,\delta^2\Delta,$$

with $e_1^* = 1$ if $\gamma \leq \tilde{\gamma}$, and $e_1^* < 1$ if $\gamma > \tilde{\gamma}$,

* and neither invest nor exert effort otherwise.

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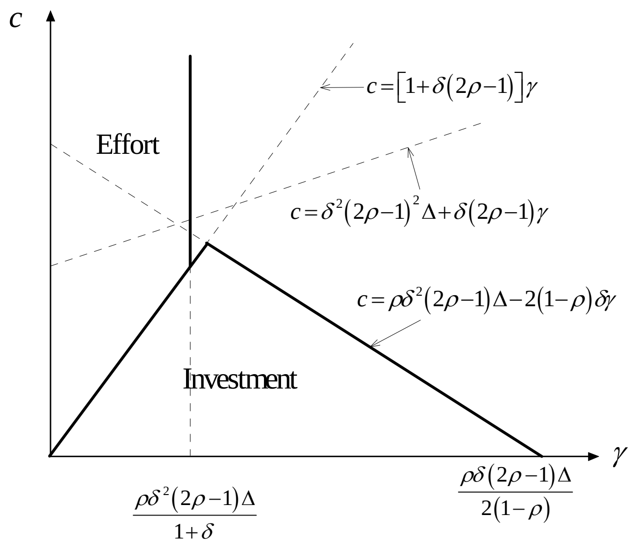

Figure 1a: $\rho > \hat{\rho}$

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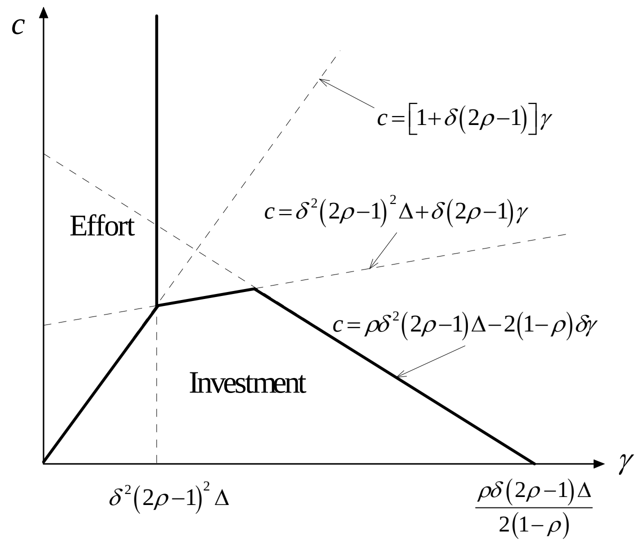

Figure 1b: $\rho < \hat{\rho}$

The comparative statics follow. In particular, it can be checked that the region in which there is either investment or effort in case of low productivity in period 1 expands as either δ or Δ increases.

We now compare the expected performance in the two periods. Obviously, the expected performance remains unchanged and equal to $q_t^e = \frac{L+H}{2} = L + \frac{\Delta}{2}$ in the absence of any investment or effort, which occurs when $\gamma \geq (2\rho - 1)\,\delta\Delta$. Let us now focus on the case where $\gamma < (2\rho - 1)\,\delta\Delta$, in which case $e_H^*, e_L^* > 0$.

- If $e_1^* = i_1^* = 0$, then

$$\begin{aligned} q_1^e &= L + \frac{\Delta}{2} \\ q_2^e &= L + \frac{1}{2}\left[\rho + (1-\rho)\, e_H^*\right]\Delta + \frac{1}{2}\left(1 - \rho + \rho e_L^*\right)\Delta \\ &= L + \left[1 + (1-\rho)\, e_H^* + \rho e_L^*\right]\frac{\Delta}{2}. \end{aligned}$$

  Therefore:

$$q_2^e - q_1^e = \left[(1-\rho)\, e_H^* + \rho e_L^*\right]\frac{\Delta}{2} > 0,$$

  where the inequality follows from $e_H^*, e_L^* > 0$.

- If $e_1^* > i_1^* = 0$, then $e_H^* = 1$ and $e_L^* > 0$; therefore:

$$\begin{aligned} q_1^e &= L + \frac{1 + e_1^*}{2}\Delta, \\ q_2^e &= L + \frac{1}{2}\Delta + \frac{1}{2}\left[1 - \rho + \rho\left(e_1^* + (1 - e_1^*)\, e_L^*\right)\right]\Delta, \end{aligned}$$

  and thus:

$$q_2^e - q_1^e = (1 - e_1^*)\left[1 - \rho\,(1 - e_L^*)\right]\frac{\Delta}{2}.$$

  Therefore, $q_2^e \geq q_1^e$, with a strict inequality whenever $e_1^* < 1$.

- Finally, if $i_1^* > e_1^* = 0$, then again $e_H^* = 1$ and $e_L^* > 0$, and thus

$$\begin{aligned} q_1^e &= L + \frac{1}{2}\,(1 + i_1^*)\,\Delta, \\ q_2^e &= L + \frac{1}{2}\,(1 + i_1^*)\,\Delta + \frac{1}{2}\,(1 - i_1^*)\,(1 - \rho + \rho e_L^*)\,\Delta, \end{aligned}$$

  which yields:

$$q_2^e - q_1^e = (1 - i_1^*)\left[1 - \rho\,(1 - e_L^*)\right]\frac{\Delta}{2}.$$

  Therefore, $q_2^e \geq q_1^e$, with a strict inequality whenever $i_1^* < 1$.

C The factor χ_n decreases with n

The factor χ_n is of the form

$$\chi_n = \delta^n x_n,$$

where

$$x_n \equiv \frac{1 - (2\rho - 1)^n \delta^n}{1 - \delta^n}$$

decreases as n increases; to show this, it suffices to note that $\log(x_n)$ indeed decreases as n increases:[30]

$$\begin{aligned}
\frac{d \log(x_n)}{dn} &= \frac{d}{dn}\left(\log\left(1 - (2\rho - 1)^n \delta^n\right) - \log\left(1 - \delta^n\right)\right) \\
&= \frac{\delta^n \log(\delta)}{1 - \delta^n} - \frac{(2\rho - 1)^n \delta^n \log((2\rho - 1)\delta)}{1 - (2\rho - 1)^n \delta^n} \\
&= f(\delta) - f(\delta(2\rho - 1)),
\end{aligned}$$

where $f(z) \equiv \frac{z^n}{1 - z^n} \log(z)$ is an increasing function of z for $z < 1$:

$$f'(z) = \frac{z^n\left(nz^{n-1}\log z + z^{n-1}\right) - nz^{n-1}\log z}{(1 - z^n)^2} = \frac{z^n z^{n-1} - (1 - z^n)\, nz^{n-1}\log z}{(1 - z^n)^2} > 0.$$

D The factor ξ_n increases with n

$$\xi_n \equiv \frac{\delta^n}{1 - \delta^n}\frac{1 - (2\rho - 1)^n \delta^n}{(2\rho - 1)^n \delta^n} = \frac{1 - (2\rho - 1)^n \delta^n}{(2\rho - 1)^n (1 - \delta^n)}$$

$$\log(\xi_n) \equiv \log(\delta^n) - \log((2\rho - 1)^n \delta^n) - \log(1 - \delta^n) + \log(1 - (2\rho - 1)^n \delta^n)$$

$$\begin{aligned}
\frac{d \log(\xi_n)}{dn} &= \frac{\delta^n \log(\delta)}{\delta^n} - \frac{(2\rho - 1)^n \delta^n \log((2\rho - 1)\delta)}{(2\rho - 1)^n \delta^n} + \frac{\delta^n \log(\delta)}{1 - \delta^n} - \frac{(2\rho - 1)^n \delta^n \log((2\rho - 1)\delta)}{1 - (2\rho - 1)^n \delta^n} \\
&= \log(\delta) - \log((2\rho - 1)\delta) + f(\delta) - f((2\rho - 1)\delta),
\end{aligned}$$

where the functions $\log(z)$ and $f(z) = \frac{z^n}{1 - z^n}\log(z)$ are both increasing in z.

[30] Note:

$$\frac{da^n}{dn} = \frac{d(\exp(n \log a))}{dn} = (\exp(n \log a))(\log a) = a^n (\log a).$$

E Expected quality

Let $\bar{V}_n$ (resp., $\underline{V}_n$) denote the expected value of the quality generated by the contract when the quality was H or the agent invested in period 0. We have:

$$\bar{V}_n = \bar{v}_n + \delta^n \left[\bar{\mu}_n \bar{V}_n + (1 - \bar{\mu}_n) \underline{V}_n\right], \tag{30}$$

$$\underline{V}_n = \underline{v}_n + \delta^n \left[\underline{\mu}_n \bar{V}_n + \left(1 - \underline{\mu}_n\right) \underline{V}_n\right], \tag{31}$$

and thus:

$$\begin{aligned} V_n &= \frac{\bar{V}_n + \underline{V}_n}{2} \\ &= v_n + \delta^n \left[V_n + \left(\mu_n - \frac{1}{2}\right) \left(\bar{V}_n - \underline{V}_n\right)\right] \\ &= \frac{v_n + \delta^n \left(\mu_n - \frac{1}{2}\right) \left(\bar{V}_n - \underline{V}_n\right)}{1 - \delta^n} \\ &= \frac{v_n + \delta^n \frac{i_n}{2} \left(\bar{V}_n - \underline{V}_n\right)}{1 - \delta^n}. \end{aligned} \tag{32}$$

From (30) and (31), we have:

$$\begin{aligned} \bar{V}_n - \underline{V}_n &= \bar{v}_n - \underline{v}_n + \delta^n \left(\bar{\mu}_n - \underline{\mu}_n\right) \left(\bar{V}_n - \underline{V}_n\right) \\ &= \frac{\bar{v}_n - \underline{v}_n}{1 - \left(\bar{\mu}_n - \underline{\mu}_n\right)} \\ &= \frac{\bar{v}_n - \underline{v}_n}{1 - (2\rho - 1)^n \delta^n (1 - i_n)}, \end{aligned}$$

which, using

$$\begin{aligned} \bar{v}_n - \underline{v}_n &= \sum_{t=1}^{n} \delta^{t-1} \left(\bar{p}_t - \underline{p}_t\right) \Delta - \delta^{n-1} \left(\bar{p}_n - \underline{p}_n\right) i_n \Delta, \\ &= \sum_{t=1}^{n} (2\rho - 1)^t \delta^{t-1} \Delta - (2\rho - 1)^n \delta^{n-1} i_n \Delta \\ &= \frac{1 - (2\rho - 1)^n \delta^n}{1 - (2\rho - 1) \delta} (2\rho - 1) \Delta - (2\rho - 1)^n \delta^{n-1} i_n \Delta \\ &= \left(\frac{1 - (2\rho - 1)^n \delta^n}{1 - (2\rho - 1) \delta} - (2\rho - 1)^{n-1} \delta^{n-1} i_n\right) (2\rho - 1) \Delta, \end{aligned}$$

leads to:

$$\bar{V}_n - \underline{V}_n = \frac{\frac{1 - (2\rho - 1)^n \delta^n}{1 - (2\rho - 1)\delta} - (2\rho - 1)^{n-1} \delta^{n-1} i_n}{1 - (2\rho - 1)^n \delta^n (1 - i_n)} (2\rho - 1) \Delta. \tag{33}$$

$$= \frac{v_n + \delta^n \frac{i_n}{2} \left(\bar{V}_n - \underline{V}_n\right)}{1 - \delta^n}$$

Combining (32) and (33) then yields:

$$V_n = \frac{v_n + \delta^n i_n \frac{\frac{1-(2\rho-1)^n \delta^n}{1-(2\rho-1)\delta} - (2\rho-1)^n \delta^{n-1} i_n}{1-(2\rho-1)^n \delta^n (1-i_n)} (2\rho - 1) \frac{\Delta}{2}}{1 - \delta^n},$$

where

$$v_n = \sum_{t=1}^{n} \delta^{t-1} \frac{L+H}{2} + \delta^{n-1} \frac{\hat{\imath}_n}{2} \Delta = \frac{1-\delta^n}{1-\delta} \frac{L+H}{2} + \delta^{n-1} \hat{\imath}_n \frac{\Delta}{2}.$$

This leads to:

$$V_n = \frac{\frac{1-\delta^n}{1-\delta} \frac{L+H}{2} + \delta^{n-1} i_n \frac{\Delta}{2} + \delta^n i_n \frac{\frac{1-(2\rho-1)^n \delta^n}{1-(2\rho-1)\delta} - (2\rho-1)^{n-1} \delta^{n-1} i_n}{1-(2\rho-1)^n \delta^n (1-i_n)} (2\rho - 1) \frac{\Delta}{2}}{1 - \delta^n}$$

$$= \frac{1}{1-\delta} \frac{L+H}{2} + \frac{\delta^{n-1} i_n \frac{\Delta}{2} + \delta^{n-1} i_n \frac{\frac{1-(2\rho-1)^n \delta^n}{1-(2\rho-1)\delta} - (2\rho-1)^{n-1} \delta^{n-1} i_n}{1-(2\rho-1)^n \delta^n (1-i_n)} (2\rho - 1) \delta \frac{\Delta}{2}}{1 - \delta^n}$$

$$= \frac{1}{1-\delta} \frac{L+H}{2} + \frac{\delta^{n-1} i_n \frac{\Delta}{2}}{1 - \delta^n} \left(1 + \frac{\frac{1-(2\rho-1)^n \delta^n}{1-(2\rho-1)\delta} - (2\rho-1)^{n-1} \delta^{n-1} i_n}{1 - (2\rho-1)^n \delta^n (1-i_n)} (2\rho - 1) \delta\right).$$

For $i_n = i_n^* = 1$, this boils down to:

$$V_n^* = \frac{1}{1-\delta} \frac{L+H}{2} + \frac{\delta^{n-1} \frac{\Delta}{2}}{1-\delta^n} \left(1 + \left(\frac{1-(2\rho-1)^n \delta^n}{1-(2\rho-1)\delta} - (2\rho-1)^{n-1} \delta^{n-1}\right) (2\rho-1) \delta\right)$$

$$= \frac{1}{1-\delta} \frac{L+H}{2} + \frac{\delta^{n-1} \frac{\Delta}{2}}{1-\delta^n} \left(\frac{1-(2\rho-1)^n \delta^n}{1-(2\rho-1)\delta}\right)$$

$$= \frac{1}{1-\delta} \frac{L+H}{2} + \chi_n \frac{1}{1-(2\rho-1)\delta} \frac{\Delta}{2\delta}.$$

As χ_n decreases when n increases, the expected quality V_n^* decreases as contracts last longer than needed to induce $i^* = 1$.

For $i_n = i_n^* = \hat{\imath}_n \leq 1$, we have

$$V_n = \frac{1}{1-\delta} \frac{L+H}{2} + \frac{\delta^{n-1} \hat{\imath}_n \frac{\Delta}{2}}{1-\delta^n} \left(1 + \frac{\frac{1-(2\rho-1)^n \delta^n}{1-(2\rho-1)\delta} - (2\rho-1)^{n-1} \delta^{n-1} \hat{\imath}_n}{1-(2\rho-1)^n \delta^n (1-\hat{\imath}_n)} (2\rho-1) \delta\right),$$

where, using (11) and after simplification

$$1 + \frac{\frac{1-(2\rho-1)^n \delta^n}{1-(2\rho-1)\delta} - (2\rho-1)^{n-1} \delta^{n-1} \hat{\imath}_n}{1-(2\rho-1)^n \delta^n (1-\hat{\imath}_n)} (2\rho-1) \delta = 1 + \frac{\frac{c}{\Delta}}{\frac{1}{1-(2\rho-1)\delta} - \frac{c}{\Delta}}.$$

Therefore:

$$
\begin{aligned}
V_n &= \frac{1}{1-\delta}\frac{L+H}{2} + \frac{\delta^n}{1-\delta^n}\hat{\imath}_n \frac{1}{1-\left(1-\left(2\rho-1\right)\delta\right)\frac{c}{\Delta}}\frac{\Delta}{2\delta} \\
&= \frac{1}{1-\delta}\frac{L+H}{2} + \xi_n\left(\frac{\left(2\rho-1\right)\delta}{1-\left(2\rho-1\right)\delta} - \frac{c}{\Delta}\right)\frac{1}{1-\left(1-\left(2\rho-1\right)\delta\right)\frac{c}{\Delta}}\frac{\Delta}{2\delta},
\end{aligned}
$$

which, as ξ_n, increases with n.