Optimal timing of CCS policies with heterogeneous energy consumption sectors

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Abstract

Using the Chakravorty et al. (2006) ceiling model, we characterize the optimal consumption paths of three energy resources: dirty oil, which is non-renewable and carbon emitting; clean oil, which is also non-renewable but carbon-free thanks to an abatement technology, and solar energy, which is renewable and carbon-free. The resulting energy-mix can supply the energy needs of two sectors. These sectors differ in the additional abatement cost they have to pay for consuming clean rather than dirty oil, as Sector 1 (industry) can abate its emissions at a lower cost than Sector 2 (transport). We show that it is optimal to begin by fully capturing Sector 1's emissions before the ceiling is reached. Also, there may be optimal paths along which the capture devices of both sectors must be activated. In this case, Sector's 1 emissions are fully abated first, before Sector 2 abates partially. Finally, we discuss the way heterogeneity of abatement costs causes sectoral energy price paths to differ.

Keywords: Energy resources; Carbon stabilization cap; Heterogeneity; Carbon capture and storage; Air capture.

JEL classifications: Q32, Q42, Q54, Q58.

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1 Introduction

Among all the alternative abatement technologies aiming at reducing anthropogenic carbon dioxide emissions, Carbon Capture and Sequestration (CCS) is of particular interest (IPCC, 2005 and 2007). Even if the efficiency of this technology is still under assessment[1], current engineering estimates suggest that CCS could be a credible cost-effective approach for eliminating most of the emissions from coal and natural gas power plants (MIT, 2007). Following this line of argument, Islegen and Reichelstein (2009) point out that CCS has considerable potential to reduce CO_2 emissions at a "reasonable" social cost, given the social cost of carbon emissions predicted for a business-as-usual scenario. CCS is also intended to play a major role in limiting the effective carbon tax, or the market price for CO_2 emission permits under a cap-and-trade system. The crucial point is then to estimate how far the CO_2 price would have to rise before the managers of power plants would find it advantageous to install the CCS technology rather than buy emission permits or pay a carbon tax. The International Energy Agency (2006) estimates such a break-even price in the range of \$30-90/tCO_2 (depending on the technology). However, assuming reasonable advances in the technology, projected CCS cost should drop to around \$25/tCO_2 by 2030.

The deployment capacity of CCS strongly depends upon the type of the energy users, or carbon emitters involved. Obviously, capturing emissions from a natural gas-fired power plant will be cheaper than capturing emissions from vehicles powered by this fossil fuel. More generally, CCS technology has been proved to be better adapted to large point sources of pollution such as power plants or large-scale manufacturing than to small and scattered emitters such as transportation, individual residence heating or agricultural activities. Although in this last case filtering CO_2 flows would be indirectly technically possible by using e.g. air capture, this technology is still prohibitively costly. Keith (2009) underlined that while this technology costs more than CCS, it enables the treatment of small and

[1] CCS technology consists in filtering CO_2 fluxes at the source of the emissions. For this purpose, in fossil energy-fueled power plants for instance, scrubbers are installed next to the top of chimney stacks. Carbon is next sequestered in reservoirs, such as depleted oil and gas fields or deep saline aquifers. However, as mentioned by Herzog (2011), the idea of separating and capturing CO_2 from the flue gas of power plants did not originate with climate change concerns. The first commercial CCS plants were built in the late 1970s in the United States to achieve enhanced oil recovery operations, where CO_2 is injected into oil reservoirs to increase the pressure and thus the output of the reservoir.

mobile emission sources, an advantage that may compensate for the intrinsic difficulty of capturing carbon from the air. Estimates of marginal cost of chemical air capture[2] range from \$100-200/tCO_2, which is higher than the cost of alternative solutions for emissions reduction such as CCS. They are also higher than current estimates of the social cost of carbon, which range from about \$7-85/tCO_2. But, as concluded by Barrett (2009), bearing the cost of chemical air capture could become profitable in the future under constraining cap-and-trade scenarios. For the time being, air capture is a somewhat extreme example. However, even when considering the CCS technology, abatement costs can differ among energy users, depending upon the location of the storage site and the type of reservoirs (Hamilton et al., 2009).

This paper addresses the question of the heterogeneity of energy users regarding their abatement costs. It examines how this heterogeneity affects the optimal energy consumption and price paths as well as the timing of abatement policies. To tackle this issue, we use the "ceiling model" developed by Chakravorty et al. (2006) and extended to the specific CCS abatement device by Lafforgue et al. (2008-a and 2008-b).

The model can be briefly described as follows. We consider two sectors of energy consumption which differ in the cost of the abatement technology they can use. Their energy needs can be supplied by three types of energy resources that are perfect substitutes. The first type is non-renewable and carbon-emitting (dirty oil), the second is also non-renewable but does not contribute to climate change thanks to a specific abatement device (clean oil). The third energy source is renewable and carbon-free (solar). The problem is to characterize the optimal path of the energy-mix of each sector, given that the atmospheric carbon stock should not exceed some critical ceiling. This energy-mix choice results from the comparison of the respective full marginal cost of each energy option. Both the marginal extraction cost of oil and the marginal cost of solar energy are constant and the same in each sector. However, oil is assumed to be cheaper than solar. Producing clean oil requires

[2]Currently, chemical air capture is probably the most credible process for capturing carbon directly from the atmosphere (Barrett, 2009). This technology consists in bringing air into contact with a chemical "sorbent" (an alkaline liquid). The sorbent absorbs CO_2 in the air, and the chemical process then separates the CO_2 and recycles the sorbent. The captured CO_2 is stored in geological deposits just as is done in the case of CCS technology used in power plants. Otherwise, the most obvious way to reduce the atmospheric carbon concentration would be to exploit the process of photosynthesis by increasing forested areas or changing agricultural processes. However, this is not the type of device we want to consider in the present paper.

an additional cost of carbon capture which varies between the two sectors. This cost is assumed to be higher in Sector 2 than in Sector 1, and constant in both cases. Furthermore, since the patterns of the optimal paths strongly depend upon the cost of solar energy as compared with the full cost of clean oil, we examine various possibilities depending on whether the solar cost is high, intermediate or low. Lastly, we assume that when a sector abates its emissions, carbon is stockpiled in very large reservoirs. As in Chakravorty et al. (2006), this suggests a generic abatement scheme of unlimited capacity.

It is important to note that the ceiling constraint can be relaxed owing to two mitigation options. The first consists in substituting clean oil for dirty oil and the second in substituting solar energy for dirty oil. Each option both delays the (endogenous) point in time at which the ceiling constraint begins to be effective and relaxes this constraint once it is binding.

The key results of the paper are: i) Irrespective of Sector 2's ability to capture its emissions, it may be optimal to begin Sector 1's abatement before the atmospheric carbon concentration cap is attained.[3] ii) Due to the abatement cost differential between the sectors, it is also optimal to capture Sector 1's entire emissions before the ceiling is reached. These first two results, obtained irrespective of the level of solar energy cost, contrast with Chakravorty et al. (2006) and Lafforgue et al. (2008-a and 2008-b) who consider a single type of energy user and a single abatement technology. iii) In the optimal scenarios where both sectors consume clean oil, Sector 2 must start to abate its emissions when the ceiling constraint begins to apply and it still needs to abate only partially. iv) The sectoral prices of the energy-mix may be different.

The paper is organized as follows. Section 2 presents the model. In Section 3 we lay down the social planner program and derive the optimality conditions. Section 4 examines the case in which only Sector 1 consumes clean oil along the optimal path and discusses the optimal scenarios depending on the level of solar energy cost. In Section 5 we characterize the optimal path in the case where each sector consumes clean oil. In Section 6 we focus on the specific problem of air capture. In this case, although Sector 2 does not have access

[3] This result is in accordance with Coulomb and Henriet (2010) who show that in a model with a single abatement technology, when technical constraints make it impossible to capture emissions from all energy consumers, and if such emissions are large enough, CCS should be used before the ceiling is reached.

to CCS technology, it can capture the carbon directly from the atmosphere. Finally, we conclude in the last section.

2 The model

Let us consider a stationary economy with two sectors, indexed by $i = 1, 2$, in which the instantaneous gross surpluses derived from energy consumption are the same.[4] For an equal energy consumption q in both sectors, $q_1 = q_2 = q$, the sectoral surplus $u_1(q)$ and $u_2(q)$ are thus equal: $u_1(q) = u_2(q) = u(q)$. We assume that this common function u is twice continuously differentiable, strictly increasing, strictly concave, with $\lim_{q\downarrow 0} u'(q) = +\infty$ and $\lim_{q\uparrow +\infty} u'(q) = 0$. We denote by $p(q)$ the sectoral marginal gross surplus function $u'(q)$ and by $q(p) = p^{-1}(q)$, the direct demand function of the sector.

Energy can be supplied by two primary resources, a potentially polluting non-renewable resource (oil) and a carbon-free renewable resource (solar).

Clean and dirty oil

Let $X(t)$ be the available stock of oil at time t and X^0 be the initial endowment. Each sector can consume either "dirty oil" or "clean oil".

Consuming dirty oil implies some carbon emissions that are proportional to its use. Let ζ be the unitary pollutant content of dirty oil so that the emission flow of sector i amounts to ζx_{id}, where x_{id} is the dirty oil consumption of this sector. We denote by c_x the average delivery cost of oil, assumed to be constant and the same in both sectors. This cost includes the extraction cost of the resource, the cost of industrial processing (crude oil refining) and the transportation cost.

The consumption of clean oil is carbon-free but is also costlier than the consumption of dirty oil. We denote by s_i the average cost that has to be borne by sector i in addition to c_x for using clean rather than dirty oil. This cost is assumed to be constant and smaller in Sector 1 than in Sector 2: $0 < s_1 < s_2$.[5] In other words, Sector 1 has access to a cheaper

[4] Since the focus of the paper is on the effect of heterogeneity on the abatement costs, all the other sectoral characteristics are assumed to be the same in order to highlight the effects of this sole difference.

[5] s_i is an average cost per unit of oil and may be seen as a cost of capture and storage. The CCS cost per unit of carbon captured in sector i amounts to s_i/ζ. It is assumed to be constant. For non-linear cost functions, see Amigues et al. (2012).

abatement technology than Sector 2. In both sectors we assume that carbon emissions are stockpiled into reservoirs whose capacities are unlimited so that no additional rent has to be charged.[6]

Denoting by x_{ic} the consumption of clean oil in sector i, the dynamics of X must satisfy:

$$\dot{X}(t) = -\sum_{i=1,2} [x_{ic}(t) + x_{id}(t)] \tag{1}$$

$$X(0) = X^0 \quad \text{and} \; X(t) \geq 0 \tag{2}$$

$$x_{ik}(t) \geq 0, \; i = 1, 2 \; \text{ and } \; k = c, d. \tag{3}$$

Pollution stock

Let $Z(t)$ be the stock of carbon in the atmosphere at time t, and Z^0 its initial level. The atmospheric pollution stock is fed by the emissions from dirty oil consumption. Moreover, we assume that this stock is self-regenerating at a constant proportional rate α, $\alpha > 0$.

The pollution damage may be neglected if the pollution stock does not exceed some critical level $\bar{Z}$. Above this threshold, the damage is supposed to be infinitely high.[7] Put differently, we assume that a carbon cap policy is prescribed to prevent catastrophic damage which would be infinitely costly and that this policy consists in forcing the atmospheric stock to stay below $\bar{Z}$. Thus the dynamics of Z must satisfy:

$$\dot{Z}(t) = \zeta[x_{1d}(t) + x_{2d}(t)] - \alpha Z(t) \tag{4}$$

$$Z(0) = Z^0 < \bar{Z} \quad \text{and} \; \bar{Z} - Z(t) \geq 0 \tag{5}$$

When the atmospheric carbon stock reaches its critical level, i.e. when $Z(t) = \bar{Z}$, the total dirty oil consumption is constrained to be at most equal to $\bar{x}_d = \alpha\bar{Z}/\zeta$, where $\bar{x}_d = x_{1d} + x_{2d}$. Since the two sectors have the same gross surplus function, each of them must consume the same quantity of dirty oil $x_{id} = \bar{x}_d/2$, for $i = 1, 2$, when the ceiling

[6] In order to focus on the abatement options for each sector and their respective costs, we ignore the consideration that reservoirs might have limited capacity. The question of the size of carbon sinks and of the time profile for filling them is addressed by Lafforgue et al. (2008-a) and (2008-b) in models with carbon cap, and by Ayong Le Kama et al. (2013) in a model with a standard damage function.

[7] Taking into account non negligible damage for $Z < \bar{Z}$ would not change the main qualitative properties of the optimal paths as shown in Amigues et al. (2011).

constraint (5) is binding and when neither of them uses clean oil.

We assume that it may be optimal to use clean oil in each sector – and therefore to abate carbon emissions – in order to delay the point in time at which the ceiling begins to constrain oil consumption and/or to relax this constraint once it begins to apply, i.e. $c_x + s_1 < c_x + s_2 < u'(\bar{x}_d)$.

Solar energy

Solar energy is a perfect substitute for oil. It is available at a constant average cost c_y which is assumed to be the same for each sector and to be larger than c_x. Hence, denoting by y_i its consumption in sector i, the sectoral aggregate energy consumption amounts to $q_i = x_{ic} + x_{id} + y_i$.

As we shall see, the structures of the optimal paths strongly depend upon solar energy cost. Thus three intervals of average cost have to be distinguished: high, when $c_y > u'(\bar{x}_d/2)$; intermediate, when $u'(\bar{x}_d/2) > c_y > u'(\bar{x}_d)$; and low, when $u'(\bar{x}_d) > c_y$.

We denote by $\tilde{y}$ the solar consumption rate solving $u'(y) = c_y$. This rate $\tilde{y}$ reads as the optimal sectoral consumption of solar energy when oil is exhausted and absent any constraint on its use. That is, assuming that its natural supply is large enough, at least as large as $2\tilde{y}$, in which case no rent has ever to be charged for its use. The only physical constraint on the y_i's are then the non-negativity constraints:

$$y_i(t) \geq 0, \quad i = 1, 2 \tag{6}$$

Social welfare and discounting

If (5) is satisfied, the social welfare function W writes as the sum of the sectoral net surpluses discounted at some constant social rate ρ, $\rho > 0$. Otherwise, it is equal to $-\infty$ (that is if the critical threshold $\bar{Z}$ is overshot).

3 Social planner problem and optimality conditions

The problem of the social planner consists in maximizing W subject to the various constraints introduced above. Denoting by S_i the instantaneous net surplus of sector i, $S_i(x_{ic}, x_{id}, y_i) = u(x_{ic} + x_{id} + y_i) - [c_x + s_i]x_{ic} - c_x x_{id} - c_y y_i$, the social planner has to solve the following program (P):

$$(P): \quad \max_{\{x_{ic}, x_{id}, y_i\}} \int_0^\infty \left\{ \sum_{i=1,2} S_i(x_{ic}(t), x_{id}(t), y_i(t)) \right\} e^{-\rho t} dt$$

subject to (1)-(6).

We denote by λ_X the costate variable of X, by λ_Z the costate variable of Z in absolute value, by γ's the Lagrange multipliers associated with the non-negativity constraints on the control variables, by ν_X the multiplier associated with the non-negativity constraint on X and by ν_Z the multiplier associated with the ceiling constraint on Z. Omitting the time index for notational convenience, the current value Lagrangian $\mathcal{L}$ of program (P) is:

$$\begin{aligned} \mathcal{L} \ = \ & \sum_{i=1,2} S_i(x_{ic}, x_{id}, y_i) - \lambda_X \sum_{i=1,2} \sum_{k=c,d} x_{ik} - \lambda_Z \left[\zeta \sum_{i=1,2} x_{id} - \alpha Z \right] \\ & + \nu_X X + \nu_Z [\bar{Z} - Z] + \sum_{i=1,2} \sum_{k=c,d} \gamma_{ik} x_{ik} + \sum_{i=1,2} \gamma_{iy} y_i \end{aligned} \tag{7}$$

The first-order conditions for optimality are:

$$u'(x_{ic} + x_{id} + y_i) = c_x + s_i + \lambda_X - \gamma_{ic}, \quad i = 1,2 \tag{8}$$

$$u'(x_{ic} + x_{id} + y_i) = c_x + \zeta\lambda_Z + \lambda_X - \gamma_{id}, \quad i = 1,2 \tag{9}$$

$$u'(x_{ic} + x_{id} + y_i) = c_y - \gamma_{iy}, \quad i = 1,2 \tag{10}$$

$$\dot{\lambda}_X = \rho\lambda_X - \nu_X \tag{11}$$

$$\dot{\lambda}_Z = (\rho + \alpha)\lambda_Z - \nu_Z \tag{12}$$

together with the associated complementary slackness conditions and the following transversality conditions:

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_X(t)X(t) = 0 \quad \text{and} \quad \lim_{t\uparrow\infty} e^{-\rho t}\lambda_Z(t)Z(t) = 0 \tag{13}$$

Remarks

Since c_x is constant, the shadow marginal cost of the stock of oil must grow at the social rate of discount. Defining $\lambda_{X0} \equiv \lambda_X(0)$, from (11), we get the following well known result:

$$X(t) > 0 \Rightarrow \lambda_X(\tau) = \lambda_{X0}e^{\rho\tau}, \quad \tau \in [0, t] \tag{14}$$

The transversality conditions (13) imply that if oil has some positive initial value $\lambda_{X0} > 0$, then it must be exhausted along the optimal path, i.e. $\lim_{t\uparrow\infty} X(t) = 0$.

Next, since $Z^0 < \bar{Z}$, there is some initial maximum time interval $[0, \underline{t}_Z)$ during which the ceiling constraint is not active, hence $\nu_Z = 0$, so that from (12):

$$\lambda_Z(t) = \lambda_{Z0}e^{(\rho+\alpha)t}, \quad t \in [0, \underline{t}_Z) \tag{15}$$

where $\lambda_{Z0} \equiv \lambda_Z(0)$ and $\underline{t}_Z$ is the first date at which $Z(t) = \bar{Z}$. Clearly, once the ceiling constraint is no longer active, λ_Z must be nil:[8]

$$\lambda_Z(t) = 0, \quad t \in [\bar{t}_Z, \infty) \tag{16}$$

where $\bar{t}_Z$ is the last date at which $Z(t) = \bar{Z}$.

Solving strategy of the social planner

In order to characterize the optimal paths, the first problem is to determine which sector, if any, has to consume clean oil. Note that from (8) and (9), and under the assumption that oil has to be consumed, each sector i must use either only dirty oil or only clean oil at any time t, depending on whether $\zeta\lambda_Z(t)$ is lower or higher than s_i. This suggests the following test.

[8] This characteristic is standard under the assumption of a linear natural regeneration process of the atmospheric carbon stock. For non-linear decay functions, see e.g. Toman and Withagen (2000).

First, solve a modified social planner program in which the use of clean oil is not possible in any sector. Let $\lambda_Z^1(t)$, for any $t \geq 0$, be the trajectory of the shadow marginal cost of the pollution stock of this program, and $\bar{\lambda}_Z^1$ be the maximum value of λ_Z^1 along its trajectory. Consequently, either $\zeta\bar{\lambda}_Z^1 > s_1$ and it would be preferable to use clean oil in Sector 1 during a certain time interval, or $\zeta\bar{\lambda}_Z^1(t) \leq s_1 < s_2$ and clean oil would never be used in any sector.

Assuming that $\zeta\bar{\lambda}_Z^1 > s_1$, the next step consists in solving a second modified program in which consuming clean oil is possible in Sector 1 but not in Sector 2. Let $\lambda_Z^2(t)$, $t \geq 0$, be the new trajectory of λ_Z and $\bar{\lambda}_Z^2$ its maximum value. If we now apply the same test for Sector 2, we conclude that either $\zeta\bar{\lambda}_Z^2 \leq s_2$ and only Sector 1 uses clean oil, or $\zeta\bar{\lambda}_Z^2 > s_2$ and clean oil is used simultaneously in both sectors.

In the following sections, we will successively characterize the case where only Sector 1 consumes clean oil (Section 4) and next, the case where both sectors have to abate their emissions (Section 5).

Notations

For clearer readability, we introduce the following additional notations. We first denote by $p^F(t, \lambda_{X0})$ the common component of the clean and dirty oil full marginal cost: $p^F(t, \lambda_{X0}) \equiv c_x + \lambda_{X0}e^{\rho t}$, where F stands for free of tax and/or CCS cost.

In the figures to come, $p_i(t)$ denotes the energy full marginal cost for sector i, that is: $p_i(t) \equiv \min\left\{p^F(t, \lambda_{X0}) + \min\left\{\zeta\lambda_Z(t), s_i\right\}, c_y\right\}$, $i = 1, 2$.

Last, we use the following generic notations for the critical dates in the different scenarios:

- $\underline{t}_Z$ and $\bar{t}_Z$ are the dates at which the ceiling constraint begins and ceases to be active respectively.

- $\underline{t}_{ic}$ and $\bar{t}_{ic}$, $i = 1, 2$, are the dates at which sector i begins and ceases to use clean oil respectively, or equivalently, begins and ceases to capture either some part or the totality of its potential carbon emissions.

- $\tilde{t}$ is the time at which $p^F(t, \lambda_{X0}) + s_1 = u'(\bar{x}_d)$, if it exists.

- $\bar{t}_x$ is the time at which the initial oil endowment X^0 is exhausted.

- t_y is the date from which only solar energy is exploited.

Note that in some scenarios several critical dates might be confused, such as when the solar energy cost is high, formally when $c_y > u'(\bar{x}_d/2)$, so that $\bar{t}_x = t_y$ as we shall see in the next section.

4 Optimal policies with abatement only in Sector 1

This case arises when the solving strategy tests introduced above result in $\zeta\bar{\lambda}_Z^1 > s_1$ and $\zeta\bar{\lambda}_Z^2 \leq s_2$. Several types of optimal paths may occur depending on whether $\underline{t}_{1c}$ is smaller or equal to $\underline{t}_Z$ and depending on the cost level of the solar substitute. We first examine the family of scenarios where Sector 1 deploys carbon capture before the time at which the ceiling constraint begins to be active, that is $\underline{t}_{1c} < \underline{t}_Z$. These scenarios imply that the sectoral energy consumer prices p_1 and p_2 can differ during certain periods. Next, we consider the scenarios where Sector 1 begins to use clean oil at the precise time $\underline{t}_Z$ at which the pollution stock reaches its critical level. In such a case the sectoral energy prices are always identical. Note that the case where carbon capture is deployed after $\underline{t}_Z$ cannot be optimal with constant marginal costs (see Lafforgue et al., 2008-a, p.593).

4.1 Sector 1's abatement starts before $\underline{t}_Z$

Let us consider successively the cases of high, intermediate and low average solar energy costs. We show that the results of the former case strongly contrast with those of the two latter cases, insofar as the aggregate consumption of dirty oil has to be shared out between the sectors during some phases at the ceiling when solar energy is competitive. In the high solar cost case, this allocation is always strictly determined, whereas in the two other cases the global constraint on dirty oil consumption may give rise to an infinite number of feasible allocations when solar energy is competitive and when the constraint on the pollution stock is active at the same time.

4.1.1 The high solar cost case: $u'(\bar{x}_d/2) < c_y$

To proceed as simply as possible, we reason graphically by considering Figure 1 below, which plots the paths $p^F(t, \lambda_{X0})$, $p^F(t, \lambda_{X0}) + s_1$ and $p^F(t, \lambda_{X0}) + s_2$. In this figure, each path can be obtained from the other by a vertical translation. Moreover λ_{X0} is set small enough such that the trajectories $p^F(t,\lambda_{X0})$ and $p^F(t,\lambda_{X0})+s_1$ cross the horizontal lines $u'(\bar{x}_d)$, $u'(\bar{x}_d/2)$ and c_y at some finite dates. Furthermore, $\zeta\lambda_{Z0} < s_1$ in such a way that the path $p^F(t,\lambda_{X0})+\zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ starts below the path $p^F(t,\lambda_{X0})+s_1$, but crosses this last path at a time $\underline{t}_{1c}$ which is earlier than $\underline{t}_Z$ at which it crosses the horizontal line $u'(\bar{x}_d)$. A last feature of Figure 1 is that, at time $\underline{t}_Z$, $p^F(t,\lambda_{X0})+s_2 > u'(\bar{x}_d)$.

[Figure 1. Optimal path supporting scenarios where clean oil is used only by Sector 1, with an abatement beginning before $\underline{t}_Z$. The high solar cost case: $u'(\bar{x}_d/2) < c_y$]

The optimal scenario suggested by Figure 1 is a seven-phase scenario.

- Phase 1, before the ceiling and without any clean oil use: $[0,\underline{t}_{1c})$

During this phase, $\zeta\lambda_Z(t) = \zeta\lambda_{Z0}e^{(\rho+\alpha)t} < s_1 < s_2$, hence dirty oil and only dirty oil is used in both sectors. The phase ends at time $\underline{t}_{1c}$ when $\zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}_{1c}} = s_1$, i.e. when the marginal shadow cost of the pollution induced by dirty oil use equals the additional marginal cost of abatement in Sector 1.

Note that during this phase the energy price is the same for each sector: $p_i(t) = p^F(t,\lambda_{X0})+\zeta\lambda_{Z0}e^{(\rho+\alpha)t}$, $i=1,2$. Moreover, $x_{id}(t) = q\left(p^F(t,\lambda_{X0})+\zeta\lambda_{Z0}e^{(\rho+\alpha)t}\right) > \bar{x}_d$, $i=1,2$, and since $Z^0 < \bar{Z}$, the pollution stock must increase during this phase. However, the existence of such a phase requires that, at time $\underline{t}_{1c}$, the critical level is not yet attained: $Z(\underline{t}_{1c}) < \bar{Z}$.

- Phase 2, before the ceiling with full abatement of Sector 1's emissions: $[\underline{t}_{1c},\underline{t}_Z)$

During this phase, $s_1 < \zeta\lambda_{Z0}e^{(\rho+\alpha)t} < s_2$ hence Sector 1 uses clean oil exclusively while Sector 2 still uses only dirty oil. Consequently, the two sectoral prices now differ, $p_1(t) = p^F(t,\lambda_{X0})+s_1 < p_2(t) = p^F(t,\lambda_{X0})+\zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ resulting in greater energy consumption in Sector 1 than in Sector 2. Since at $\underline{t}_{1c}$ the pollution stock is lower than $\bar{Z}$ and since $x_{2d} = q\left(p^F(\underline{t}_{1c},\lambda_{X0})+\zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}_{1c}}\right) > \bar{x}_d$, this stock is still increasing at least at the beginning of the phase. The phase ends at time $\underline{t}_Z$ when $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = u'(\bar{x}_d)$ and, simultaneously, the pollution stock reaches the stabilization cap $\bar{Z}$.

- Phase 3, at the ceiling with full abatement of Sector 1's emissions: $[\underline{t}_Z, \tilde{t})$

During this first phase at the ceiling, $\zeta\lambda_Z(t) = u'(\bar{x}_d) - p^F(t, \lambda_{X0}) > s_1$ and also $\zeta\lambda_Z(t) < s_2$, hence Sector 1 uses only clean oil while Sector 2 consumes only dirty oil, as during the previous phase. However, since the ceiling constraint is active, the dirty oil consumption by Sector 2 is bounded from above by $\bar{x}_d$. Consequently this sector is the only one that has to bear the burden of the constraint: $x_{2d}(t) = \bar{x}_d$. The shadow marginal cost of pollution $\lambda_Z(t)$ decreases as $p^F(t, \lambda_{X0})$ increases and the phase ends at time $\tilde{t}$ when $\zeta\lambda_Z(t) = s_1$.

- Phase 4, at the ceiling with partial abatement of Sector 1's emissions: $[\tilde{t}, \bar{t}_{1c})$

During this second phase at the ceiling, $\zeta\lambda_Z(t) = s_1$. Since $\zeta\lambda_Z(t) < s_2$, Sector 2 consumes only dirty oil while Sector 1, being indifferent, uses a mix of clean and dirty oil. Now the burden of the ceiling constraint is borne simultaneously by both sectors: $x_{1d}(t) + x_{2d}(t) = \bar{x}_d$. Both sectors also consume the same amount of energy: $x_1(t) = x_{1c}(t) + x_{1d}(t) = q\left(p^F(t, \lambda_{X0}) + s_1\right) = x_{2d}(t) = x_2(t)$, implying that $x_{1c}(t) = 2q\left(p^F(t, \lambda_{X0}) + s_1\right) - \bar{x}_d$ decreases, $x_{1d}(t) = \bar{x}_d - q\left(p^F(t, \lambda_{X0}) + s_1\right)$ increases and $x_{2d}(t)$ decreases. Sector 1 thus gradually substitutes dirty for clean oil.

The phase ends at time $\bar{t}_{1c}$ when $p^F(t, \lambda_{X0}) + s_1 = u'(\bar{x}_d/2)$. At this time, $x_{1d}(t) = x_{2d}(t) = \bar{x}_d/2$ and $x_{1c}(t) = 0$. From this time onwards, Sector 1 must in turn use only dirty oil, as clean oil becomes too costly in relative terms.

- Phase 5, at the ceiling and without abatement of Sector 1's emissions: $[\bar{t}_{1c}, \bar{t}_Z)$

This is the last phase at the ceiling. Since now $\zeta\lambda_Z(t) = u'(\bar{x}_d/2) - p^F(t, \lambda_{X0}) < s_1 < s_2$, both sectors use only dirty oil and share equally the burden of the ceiling constraint: $x_{1d}(t) = x_{2d}(t) = \bar{x}_d/2$. The phase ends at time $\bar{t}_Z$ when $p^F(t, \lambda_{X0}) = u'(\bar{x}_d/2)$, i.e. when $\lambda_Z(t) = 0$, which indicates the end of the period at the ceiling.

- Phase 6, post-ceiling phase of oil exhaustion: $[\bar{t}_Z, t_y)$

This phase is a pure Hotelling regime during which only oil is consumed by both sectors as in the initial phase, but now with $\lambda_Z(t) = 0$. Since $p^F(t, \lambda_{X0}) > u'(\bar{x}_d/2)$, we get $x_{1d}(t) + x_{2d}(t) = 2q\left(p^F(t, \lambda_{X0})\right) < \bar{x}_d$ and the ceiling constraint is no longer active. The phase ends at time t_y when $p^F(t, \lambda_{X0}) = c_y$ and the oil stock is exhausted at the same time.

- Phase 7, permanent solar energy consumption: $[t_y, +\infty)$

From t_y onwards, solar energy is competitive and $y_1(t) = y_2(t) = \tilde{y}$.

The following proposition states the existence of such a path.

Proposition 1 *Assume that $u'(\bar{x}_d/2) < c_y$, that λ_{X0} and λ_{Z0} generate the full marginal cost paths $p^F(t, \lambda_{X0})$, $p^F(t, \lambda_{X0}) + s_i$, $i = 1, 2$, and that $p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ has the properties plotted in Figure 1. Furthermore the carbon stabilization cap $\bar{Z}$ is attained when $p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t} = u'(\bar{x}_d)$ and the initial oil endowment is exhausted when $p^F(t, \lambda_{X0}) = c_y$. Under these conditions the above seven-phase scenario is the solution of the social planner problem.*

Proof: Clearly, there are non-negative multipliers $\gamma_{ik}(t)$, $i = 1, 2$, $k = c, d$, $\nu_X(t)$ and $\nu_Z(t)$, $t \geq 0$, such that the first-order conditions (8)-(12) and the transversality conditions (13) are all satisfied. This is obvious for the Lagrange multipliers γ_{ik} associated with the control variables. Next, we can show that $\nu_X(t) = 0$, $t \geq 0$ is the right candidate and that the optimal trajectory of $\nu_Z(t)$ is given by:

$$\nu_Z(t) = \begin{cases} 0 & , t \in [0, \underline{t}_Z) \\ \left\{(\rho+\alpha)[u'(\bar{x}_d) - c_x] - \alpha\lambda_{X0}e^{\rho t}\right\}/\zeta & , t \in [\underline{t}_Z, \tilde{t}) \\ (\rho+\alpha)s_1/\zeta & , t \in [\tilde{t}, \bar{t}_{1c}) \\ \left\{(\rho+\alpha)[u'(\bar{x}_d/2) - c_x] - \alpha\lambda_{X0}e^{\rho t}\right\}/\zeta & , t \in [\bar{t}_{1c}, \bar{t}_Z) \\ 0 & , t \in [\bar{t}_Z, \infty) \end{cases} \tag{17}$$

Lastly, since the program (P) is convex, the first-order conditions (8)-(12) are sufficient and have a unique solution. ■

Since the proofs of all the other forthcoming propositions are basically the same, they will not be repeated in the next sections.

Discussion

As far as abatement is concerned, it would also be possible to have Sector 1's full abatement starting from the initial date. The first phase of the scenario illustrated in Figure 1 would then be similar to the second phase, with clean oil consumed exclusively by Sector 1 and dirty oil by Sector 2.

Assume for instance that the social planner is facing the initial conditions Z^*, $Z^0 < Z^* < \bar{Z}$, and X^*, $X^* < X^0$, corresponding respectively to the pollution stock level and the available oil stock at time t^*, $\underline{t}_{1c} < t^* < \underline{t}_Z$, and starting from $Z(0) = Z^0$ and $X(0) = X^0$ as initially considered. The optimal scenario associated with these new initial conditions is then proved to be the continuation of the initial scenario from t^* onwards: at time t, any variable in the scenario corresponding to the new initial conditions takes its value at time $t + t^*$ in the original scenario.

The same remark applies to all the cases we will examine hereafter. We have chosen to systematically present the longest possible scenario corresponding to the case under study, beginning with a phase of dirty oil consumption in both sectors.

The pattern of the optimal scenario is the result of two main rules. The first is the Herfindahl (1967) least cost principle which predicts the introduction of solar energy only when oil has been exhausted. More generally this least cost principle gives priority to the cheapest energy source, i.e. dirty oil, once the ceiling constraint is no longer binding. The second driving force results from the dynamics of energy prices under the Hotelling rule. The energy price never decreases through time, implying that if carbon capture has to be deployed, it has to be at the maximum rate initially. The result is the full capture of emissions by Sector 1 once the profitability threshold condition concerning price and cost is verified. The progressive depletion of the resource stock causes an increase in the scarcity cost of oil, $\lambda_X(t)$, an incentive for Sector 1 to cut its abatement cost and revert gradually to dirty oil. Such an outcome could not arise if there were an infinite supply of oil, i.e. if $\lambda_X(t) = 0$. With an infinite oil endowment, Sector 1 should never stop fully capturing its emissions, the ceiling constraint binding forever. The pattern of carbon capture in this scenario is thus the consequence of the Hotelling scarcity effect when combined with the optimal pollution accumulation pattern resulting from a global constraint on atmospheric carbon concentration. The same logic is at work in the scenarios that are examined below, although with different consequences.

4.1.2 The intermediate solar cost case: $u'(\bar{x}_d) < c_y < u'(\bar{x}_d/2)$

This case is illustrated in Figure 2.

[Figure 2. Optimal path supporting scenarios where clean oil is used only by Sector 1, with an abatement beginning before $\underline{t}_Z$. The intermediate solar cost case: $u'(\bar{x}_d) < c_y < u'(\bar{x}_d/2)$]

The optimal path is now a six-phase scenario. The first four phases are similar to the first four phases depicted in Figure 1, meaning that Sector 1's emissions again begin to be captured before the ceiling is reached.

The differences between these two scenarios occur at the end of phase 4. In the present case at $\bar{t}_{1c}$, $p^F(t, \lambda_{X0}) + s_1 = c_y$, contrary to what has been observed at the end of the fourth phase in the previous scenario where we found $p^F(t, \lambda_{X0}) + s_1 = u'(\bar{x}_d/2)$ at time $\bar{t}_{1c}$. Remember that during this fourth phase, since $Z_t = \bar{Z}$, the aggregate consumption of dirty oil is constant and equal to $\bar{x}_d$, while the aggregate total consumption of oil is larger than $\bar{x}_d$: $x_{1d}(t) + x_{2d}(t) = \bar{x}_d$. Sector 2 uses only dirty oil: $x_2(t) = x_{2d}(t) = q\left(p^F(t, \lambda_{X0}) + s_1\right) > \bar{x}_d/2$ and Sector 1 uses a mix of clean and dirty oil: $x_{1c}(t) = 2q\left(p^F(t, \lambda_{X0}) + s_1\right) - \bar{x}_d > 0$ and $x_{1d}(t) = \bar{x}_d - q\left(p^F(t, \lambda_{X0}) + s_1\right) > 0$. At the end of the phase, since now $p^F(t, \lambda_{X0}) + s_1 = c_y < u'(\bar{x}_d/2)$, we have $x_{1c}(\bar{t}_{1c}) > 0$, contrary to the case illustrated in Figure 1 where $x_{1c}(\bar{t}_{1c})$ is nil.

The fifth phase $[\bar{t}_{1c}, \bar{t}_x)$ is still a phase at the ceiling during which the aggregate consumption of dirty oil is locked at $\bar{x}_d$: $x_{1d}(t) + x_{2d}(t) = \bar{x}_d$. Since $c_y < u'(\bar{x}_d/2)$, the aggregate consumption of energy amounts to $2\tilde{y}$, which is larger than $\bar{x}_d$. The difference $2\tilde{y} - \bar{x}_d$ is supplied by solar energy since the marginal cost of clean oil in Sector 1 is higher than the marginal cost of solar energy: $p^F(t, \lambda_{X0}) + s_1 > c_y$.[9] The distribution of the dirty

[9]Since both c_x and c_y are constant, dirty oil and solar energy may only be simultaneously used during a phase at the ceiling. A generalization of this result to the case of a damage function that increases with the atmospheric carbon stock can be found in Hoel and Kverndokk (1996) and Tahvonen (1997). In particular, using a stock-dependent marginal extraction cost, but a constant marginal cost of the backstop, Tahvonen (1997) shows that there can be a multiplicity of simultaneous energy-use scenarios.

oil aggregate consumption between the sectors, hence the correlative distribution of the solar energy aggregate consumption, is of no importance.

The shadow marginal cost of the pollution stock remains positive, $\lambda_Z(t) = \left[c_y - p^F(t, \lambda_{X0})\right]/\zeta > 0$, meaning that the ceiling constraint still applies. This fifth phase ends when $\lambda_Z(t) = 0$. At this point in time, the initial oil endowment must be completely exhausted, and the ceiling must be definitively left: $\bar{t}_x = \bar{t}_Z$.

The sixth and last phase $[\bar{t}_x, \infty)$ is the phase of exclusive and definitive use of solar energy: $q_i(t) = y_i(t) = \tilde{y}$, $i = 1, 2$.

The following proposition concludes the examination of this intermediate solar cost case.

Proposition 2 *Assume that $u'(\bar{x}_d) < c_y < u'(\bar{x}_d/2)$, that λ_{X0} and λ_{Z0} generate the full marginal cost paths $p^F(t, \lambda_{X0})$, $p^F(t, \lambda_{X0}) + s_i$, $i = 1, 2$, and that $p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ has the properties plotted in Figure 2. Moreover the carbon stabilization cap $\bar{Z}$ is attained when $p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t} = u'(\bar{x}_d)$ and the initial oil endowment is exhausted when $p^F(t, \lambda_{X0}) = c_y$. Under these conditions the above six-phase scenario is optimal.*

4.1.3 The low solar cost case: $c_y < u'(\bar{x}_d)$

The case is illustrated in Figure 3. The important new feature of the figure is that we have $\zeta\lambda_{Z0}e^{(\rho+\alpha)t_y} < s_2$ at time t_y at which $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = c_y - p^F(t, \lambda_{X0})$.

[Figure 3. Optimal path supporting scenarios where clean oil is used only by Sector 1, with an abatement beginning before $\underline{t}_Z$. The low solar cost case: $c_y < u'(\bar{x}_d)$]

The optimal scenario is a sequence of five phases. The first two phases are similar to the phases obtained in the previous scenarios. Once again, Sector 1 starts to capture its emissions at time $\underline{t}_{1c}$ before the ceiling constraint begins to apply. However, the present scenario diverges from the previous ones at the end of the second phase during which Sector 1 fully abates and Sector 2 uses only dirty oil. In the present case at t_y, the end of phase 2, the full marginal cost of dirty oil matches the level c_y and the ceiling is attained at the same time: $t_y = \underline{t}_Z$. Since $c_y < u'(\bar{x}_d)$, the dirty oil consumption rate of Sector 2 amounts to $\tilde{y}$ being larger than $\bar{x}_d$: $x_d(t_y) = \tilde{y} > \bar{x}_d$.

The third phase $[t_y, \bar{t}_{1c})$ is a phase at the ceiling where Sector 1 consumes only clean oil, while Sector 2 combines dirty oil with $x_{2d}(t) = \bar{x}_d$ and solar energy with $y_2(t) = \tilde{y} - \bar{x}_d$, since it bears the burden of the ceiling constraint alone. During this phase, the shadow marginal cost of the pollution stock is still positive as the constraint is still active: $\lambda_Z(t) = \left[c_y - p^F(t, \lambda_{X0})\right]/\zeta > s_1$. Because $\lambda_Z(t)$ is decreasing, the phase ends at time $\bar{t}_{1c}$ when $\zeta\lambda_Z(t) = s_1$. From this date onwards, capturing Sector 1's carbon emissions becomes too costly.

The fourth phase $[\bar{t}_{1c}, \bar{t}_Z)$ is similar to the fifth phase in the previous scenario, as illustrated in Figure 2. The ceiling constraint still applies and no sector may use clean oil because it is too costly, hence $x_{1d}(t) + x_{2d}(t) = \bar{x}_d$. The remaining energy needs of the two sectors are supplied by solar energy: $y_1(t) + y_2(t) = 2\tilde{y} - \bar{x}_d$. Again, the way dirty oil and solar energy are shared out between the two sectors is a matter of indifference.

The shadow marginal cost of the pollution stock is positive, $\lambda_Z(t) = \left[c_y - p^F(t, \lambda_{X0})\right]/\zeta$, and it declines to 0 at the end of the phase. At this time, the stock of oil must be exhausted: $\bar{t}_Z = \bar{t}_x$.

The fifth and last phase $[\bar{t}_Z, \infty)$ is the usual infinite phase of exclusive solar energy consumption.

Proposition 3 *Assume that $c_y < u'(\bar{x}_d)$, that λ_{X0} and λ_{Z0} generate the full marginal cost paths $p^F(t, \lambda_{X0})$, $p^F(t, \lambda_{X0}) + s_i$, $i = 1, 2$, and that $p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ has the properties plotted in Figure 3. Furthermore the critical pollution stock $\bar{Z}$ is reached when $p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t} = c_y$ and the stock of oil is exhausted when $p^F(t, \lambda_{X0}) = c_y$. Under these conditions, the above five-phase scenario is optimal.*

4.2 Sector 1's abatement starts at $\bar{t}_Z$

Such policies are possible, provided that when $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = s_1$, we have $\min\{u'(\bar{x}_d/2), c_y\} > p^F(t, \lambda_{X0}) + s_1 > u'(\bar{x}_d)$ as the same time as the ceiling is attained. Figure 4 illustrates the high solar cost case $c_y > u'(\bar{x}_d/2)$, and Figure 5 the intermediate solar cost case

$u'(\bar{x}_d/2) > c_y > u'(\bar{x}_d)$. This scenario may not occur under the low solar cost assumption $c_y < u'(\bar{x}_d)$, which will be explained later.

[Figure 4. Optimal path supporting scenarios where clean oil is used only by Sector 1, with an abatement beginning at $\underline{t}_Z$. The high solar cost case: $u'(\bar{x}_d/2) < c_y$]

[Figure 5. Optimal path supporting scenarios where clean oil is used only by Sector 1, with an abatement beginning at $\underline{t}_Z$. The intermediate solar cost case: $u'(\bar{x}_d) < c_y < u'(\bar{x}_d/2)$]

In Figures 4 and 5 the first two phases of the optimal scenarios are the same. First, each sector consumes exclusively dirty oil up to the time $\underline{t}_Z = \underline{t}_{1c}$ where the atmospheric carbon stock hits the cap $\bar{Z}$. At the same time $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = s_1$, which implies that abatement may now be a competitive option for Sector 1. Now, $x_{1d}(\underline{t}_{1c}) = x_{2d}(\underline{t}_{1c}) = q\left(p^F(\underline{t}_{1c}, \lambda_{X0}) + s_1\right) < \bar{x}_d$ since $p^F(\underline{t}_{1c}, \lambda_{X0}) + s_1 > u'(\bar{x}_d)$.

It should by now be clear that the assumption $p^F(\underline{t}_{1c}, \lambda_{X0}) + s_1 = p^F(\underline{t}_{1c}, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}_{1c}} > u'(\bar{x}_d)$ on which Figures 4 and 5 are based is crucial. If $p^F(\underline{t}_{1c}, \lambda_{X0}) + s_1$ is lower than $u'(\bar{x}_d)$, the second phase of the above two scenarios cannot occur.

The second phase occurs at the ceiling. Because $p^F(t, \lambda_{X0}) + s_1 < \min\{u'(\bar{x}_d/2), c_y\}$ the burden of the ceiling constraint has to be borne by both sectors. This result stems from the fact that $q\left(p^F(t, \lambda_{X0}) + s_1\right) > \bar{x}_d/2$ and also that $q\left(p^F(t, \lambda_{X0}) + s_1\right) < \bar{x}_d$, resulting in $x_{1c}(t) = 2q\left(p^F(t, \lambda_{X0}) + s_1\right) - \bar{x}_d$, $x_{1d}(t) = \bar{x}_d - q\left(p^F(t, \lambda_{X0}) + s_1\right)$, $x_{2c}(t) = 0$ and $x_{2d}(t) = q\left(p^F(t, \lambda_{X0}) + s_1\right)$.

The contrasting features between Figures 4 and 5 are the same as those distinguishing Figures 1 and 2. The phases occurring after the date $\bar{t}_{1c}$ at which Sector 1 stops abating its emissions in Figure 4 (respectively Figure 5) are the same as in Figure 1 (resp. Figure 2).

Finally, note that both sectors permanently face the same full marginal cost of energy.

Proposition 4 *For the optimal scenarios in which Sector 1 is the only sector using clean oil, two cases can occur:*

i) Sector 1 begins to abate its emissions before the ceiling is reached. In this case its full marginal cost of energy is lower than Sector 2's during the first two phases of Sector 1's clean oil consumption;

ii) Sector 1 begins to abate when the ceiling is attained and then the full marginal cost of energy is the same in both sectors during any phase of the scenario.

5 Optimal policies with abatement in both sectors

The case of abatement in both sectors arises when the solving strategy test in Section 3 results in $\zeta\bar{\lambda}_Z^2 > s_2$. In this case the sectoral full marginal costs of energy are necessarily distinct during the phases of simultaneous abatement. This results from the fact that the additional marginal abatement cost is smaller in Sector 1 than in Sector 2, which means that Sector 1 will necessarily abate if Sector 2 does so.

The existence of such scenarios requires the assumption that when $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = s_2$, $p^F(t, \lambda_{X0}) + s_2 < \min\{u'(\bar{x}_d), c_y\}$, as illustrated in Figures 6, 7 and 8 below for the high, intermediate and low solar energy cost cases. This characteristic contrasts with the case of the scenarios developed in Section 4, where abatement in Sector 2 was never optimal, since the above inequality was reversed (see Figures 1 to 5).

[Figure 6. Optimal path supporting scenarios where clean oil is used by both sectors, with abatement beginning before $\underline{t}_Z$ in Sector 1 and at $\underline{t}_Z$ in Sector 2. The high solar cost case: $u'(\bar{x}_d/2) < c_y$]

[Figure 7. Optimal path supporting scenarios where clean oil is used by both sectors, with abatement beginning before $\underline{t}_Z$ in Sector 1 and at $\underline{t}_Z$ in Sector 2. The intermediate solar cost case: $u'(\bar{x}_d) < c_y < u'(\bar{x}_d/2)$]

[Figure 8. Optimal path supporting scenarios where clean oil is used in both sectors, with abatement beginning before $\underline{t}_Z$ in Sector 1 and at $\underline{t}_Z$ in Sector 2. The low solar cost case: $c_y < u'(\bar{x}_d)$]

Whatever the cost of the solar substitute, the first three phases of the scenarios are the same. The distinguishing features of the following phases are similar to the differences observed in the scenarios depicted by Figures 1, 2 and 3 when Sector 1 is the only sector using clean oil. For this reason we focus the analysis on these first three phases.

Phase 1, for $t \in [0, \underline{t}_{1c})$, is the usual initial phase during which both sectors use only dirty oil and the pollution stock increases since $x_{id}(t) = q\left(p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}\right) > \bar{x}_d$, $i = 1, 2$. The phase ends when $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = s_1$, and abatement becomes a competitive option for Sector 1. The pollution stock stays below the cap $\bar{Z}$.

During the second phase, for $t \in [\underline{t}_{1c}, \underline{t}_{2c})$, Sector 1 uses only clean oil and Sector 2 only dirty oil. Since $x_{2d} = q\left(p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}\right) > \bar{x}_d$ and initially $Z(\underline{t}_{1c}) < \bar{Z}$, the atmospheric carbon stock is still increasing. The phase ends when $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = s_2$ and, simultaneously, $Z(t) = \bar{Z}$, implying $\underline{t}_{2c} = \underline{t}_Z$. Given the characteristic that has been emphasized above, i.e. $p^F(t, \lambda_{X0}) + s_2 < \min\{u'(\bar{x}_d), c_y\}$, we get $x_{2d}(\underline{t}_{2c}) = q\left(p^F(t, \lambda_{X0}) + s_2\right) > \bar{x}_d$ at the beginning of the next phase.

During phase 3, for $t \in [\underline{t}_{2c}, \bar{t}_{2c})$, the economy is constrained by the carbon stabilization cap. The abatement option being comparatively cheap for Sector 1, this sector uses only clean oil: $x_{1d}(t) = 0$ and $x_{1c}(t) = q\left(p^F(t, \lambda_{X0}) + s_1\right)$. Sector 2 bears the burden of the ceiling constraint alone and consumes a mix of clean and dirty oil: $x_{2d}(t) = \bar{x}_d$ and $x_{2c}(t) = q\left(p^F(t, \lambda_{X0}) + s_2\right) - \bar{x}_d$. Moreover, since $p_2(t) = p^F(t, \lambda_{X0}) + s_2 = u'(x_{2c}(t) + \bar{x}_d)$ is increasing, the clean oil consumption of Sector 2 decreases during this phase. Time-differentiating this last equality, we get $\dot{x}_{2c}(t) = \rho\lambda_{X0}e^{\rho t}/u''(x_{2c}(t) + \bar{x}_d) < 0$. The phase ends when $p^F(t, \lambda_{X0}) + s_2 = u'(\bar{x}_d)$ in the high and intermediate solar energy cost cases (see Figures 6 and 7, respectively), or when $p^F(t, \lambda_{X0}) + s_2 = c_y$ in the low solar energy cost case (see Figure 8).

The subsequent phases are:

- the same phases 4 to 7 as the phases described in paragraph 4.1.1 when the solar energy cost is high;

- the same phases 4 to 6 as the phases described in paragraph 4.1.2 when the solar energy cost is intermediate;

- the same phases 4 to 5 as the phases described in paragraph 4.1.3 when the solar energy cost is low.

We conclude as follows:

Proposition 5 *In the optimal scenarios where both sectors have to consume clean oil, for any level of solar energy cost, Sector 1 must begin to capture its carbon emissions before the ceiling is attained. On the other hand, Sector 2 begins to partially abate when the ceiling constraint begins to be active.*

6 Direct capture from the pollution stock: the air capture option

Let us now assume that Sector 2 is not able to capture its potential emissions at their source – hence it cannot directly use clean oil – but that society as a whole can capture the carbon directly from the atmospheric pollution stock. We denote by $a(t)$ the instantaneous carbon capture rate from the atmosphere and by c_a the associated average capture cost assumed to be constant.

The dynamics of the oil and pollution stocks are now:

$$\dot{X}(t) = -x_{1c}(t) - \sum_i x_{id}(t) \tag{18}$$

$$\dot{Z}(t) = \zeta \sum_i x_{id}(t) - a(t) - \alpha Z(t) \tag{19}$$

$$a(t) \geq 0 \tag{20}$$

Define the instantaneous net surplus S_1 of Sector 1 as in Section 3 and the surplus S_2 of Sector 2 by:

$$S_2(x_{2d}(t), y_2(t)) = u(x_{2d}(t) + y_2(t)) - c_x x_{2d}(t) - c_y y_2(t)$$

The new social planner program becomes:

$$\max_{\{x_{id}, y_i, x_{1c}, a\}} \int_0^\infty \{S_1(x_{1c}(t), x_{1d}(t), y_1(t)) + S_2(x_{2d}(t), y_2(t)) - c_a a(t)\} e^{-\rho t} dt$$

subject to the constraints (18)-(20), (2), (3), (5) and (6).

The optimality conditions (8), (9) and (10) corresponding respectively to Sector 1's energy choices x_{1c}, x_{1d} and y_1, remain the same, as do the conditions (11), (12) and (13). Concerning the optimality conditions applying to Sector 2's choices, (8) no longer exists and (9) and (10) must be rewritten as:

$$u'(x_{2d}+y_2) = c_x + \lambda_X + \zeta\lambda_Z - \gamma_{2d} \tag{21}$$

$$u'(x_{2d}+y_2) = c_y - \gamma_{2y} \tag{22}$$

Finally, denoting by $\gamma_a(t)$ the Lagrange multiplier associated with the non-negativity constraint on a, the optimality condition related to this last command variable is:

$$c_a = \lambda_Z(t) + \gamma_a(t) \tag{23}$$

together with the corresponding complementary slackness condition.

Assume that $a(t) > 0$ during some time interval. Then $\gamma_a(t) = 0$, $c_a = \lambda_Z(t)$ implying that $\dot{\lambda}_Z(t) = 0$ and, from (12), we get: $\nu_Z(t) = (\rho+\alpha)c_a > 0$. This situation is possible if and only if $Z(t) = \bar{Z}$. Thus, direct capture from the atmospheric pollution stock is seen to occur only during some phases at the ceiling, implying that $\dot{Z}(t) = 0$ and, equivalently, that $a(t) = \zeta\sum_i x_{id}(t) - \alpha\bar{Z}$.

Assume furthermore that $s_1 < \zeta c_a$, i.e. CCS is cheaper for Sector 1 than air capture technology. Sector 1 must therefore consume only clean oil and $a(t) = \zeta x_{2d}(t) - \alpha\bar{Z}$. Since Sector 2 consumes only dirty oil and $\lambda_Z = c_a$, it follows that $x_{2d}(t) = q\left(p^F(t,\lambda_{X0}) + \zeta c_a\right)$. To make the analogy with the initial model, Sector 2's oil consumption clearly reads as the amount of oil that Sector 2 should consume if it had access to clean oil at an additional marginal cost $s_2 = \zeta c_a$. In this case it would use $\bar{x}_d$ units of dirty oil and $q\left(p^F(t,\lambda_{X0}) + s_2\right) - \bar{x}_d$ units of clean oil as during all the phases $[\underline{t}_{2c}, \bar{t}_{2c})$ in Figures 6, 7 and 8 in Section 5. The flow of potential emissions that must be captured to meet this clean oil consumption rate amounts to $\zeta\left[q\left(p^F(t,\lambda_{X0}) + s_2\right) - \bar{x}_d\right] = \zeta q\left(p^F(t,\lambda_{X0}) + s_2\right) - \alpha\bar{Z}$. This is precisely the flow of direct atmospheric carbon capture when Sector 2 only has access to air capture at the cost $c_a = s_2/\zeta$. Thus, the effect on the economy is exactly the same as if Sector 2 had access to clean oil at an additional marginal cost $s_2 = \zeta c_a$.

Proposition 6 *When Sector 2 only has access to air capture at a constant average cost c_a, $\zeta c_a > s_1$, the optimal paths of the full marginal costs of clean and dirty oil in Sector 1 and the optimal path of the full marginal cost of energy in Sector 2 are the same as in the case where Sector 2 has access to clean oil at an average additional cost $s_2 = \zeta c_a$. The sectoral energy consumption paths and the atmospheric pollution stock are the same in both cases.*

7 Conclusion

Using the Chakravorty et al. (2006) model, we have determined the optimal timing of CCS policies for an economy composed of two kinds of energy users differing in the cost of the abatement technology they have access to. In all cases the marginal cost of CCS is constant, but capturing carbon emissions is more costly in Sector 2 than in Sector 1. Both sectors face a global maximal atmospheric carbon concentration constraint.

In this framework we have shown that carbon sequestration carried out by Sector 1 must begin strictly before the atmospheric carbon stock reaches its critical threshold. Furthermore Sector 1's emissions have to be fully abated during a first time phase with constant marginal cost of abatement and a stationary demand schedule. This result stands in contrast to the findings of Chakravorty et al. (2006) who concluded that abatement should begin only when the atmospheric ceiling has been attained in a model with a single sector and a single abatement technology.

The difference appears to be a consequence of the heterogeneity of the abatement costs of the energy users. This heterogeneity constrains the potential of CCS to be at most capable of absorbing the emissions of Sector 1 and thus to be always smaller than the total carbon emissions of fossil energy consumers. In a constant CCS cost setting there is no limitation on the amount of abated emissions below the gross emission level. In a case where Sector 2's emissions alone would drive atmospheric concentration up to its maximum threshold, full emission abatement by Sector 1 appears to be the only optimal choice for the economy. Furthermore, with or without the possibility of abatement in Sector 2, delaying the development of CCS by Sector 1 beyond the time when the atmospheric carbon stock reaches its maximum level is always suboptimal. However, even with abatement in Sector 2, the total carbon emission flow from the two sectors remains only partially abated, resulting in a time phase during which the atmospheric carbon constraint limits the fossil fuel consumption possibilities of the two sectors.

Note also that when both sectors have to capture their emissions, abatement in Sector 2 is undertaken only after the beginning of the atmospheric carbon ceiling phase and that this abatement effort is always smaller than its gross contribution to carbon emissions. This result is now in accordance with Chakravorty et al. (2006).

For the sake of computational convenience, we have assumed constant marginal cost. In a similar ceiling model with a single sector of energy consumption, Amigues et al. (2012) explore more sophisticated CCS cost functions that depend either on the flow of sequestration or on the cumulated sequestration. Considering first a flow-dependent and increasing marginal cost, they show that optimal abatement must begin during the pre-ceiling phase. In this case, carbon sequestration both delays the time at which the ceiling constraint begins to be active and relaxes this constraint once active. Moreover the optimal sequestration flow first increases during the pre-ceiling phase and then decreases during the phase at the ceiling. Next, they investigate the case of stock-dependent cost functions, which implies two contrasting effects: a scarcity and a learning effect. The scarcity effect generated by an increasing marginal cost function conveys the idea that it becomes more and more costly to store carbon emissions as the stock already sequestered increases. Conversely, the learning effect, obtained if the marginal cost function is decreasing, implies that the deployment of CCS technology improves as the installed capacity increases. In both cases, they show that it is never optimal to deploy CCS before the ceiling is reached. However, these two effects have contrasting impacts on the pattern of the energy price. Under the learning effect, the time path of the energy price can decrease locally while it always increases under the scarcity effect.

It is interesting to observe that the economy may experience a complex dynamic pattern of energy prices while being constrained by the atmospheric carbon ceiling. With a constant abatement unit cost, the energy price at the consumer stage is composed of a sequence of constant price phases separated by increasing price phases. This complex shape translates into the time profile of the carbon tax implemented to meet the atmospheric concentration objective.

The carbon tax must increase over time before the ceiling is reached. Note that Sector 1 escapes the tax when fully abating its emissions and bears a comparatively lower sequestration cost. The environmental constraint burden is transferred to Sector 2. Such a discrepancy between sectors is justified by the fact that Sector 2 benefits from the carbon sequestration efforts of Sector 1, a sort of positive "external" effect of Sector 1 upon Sector 2. Of course this is not a real external effect, since it operates through the carbon price. But this observation opens interesting policy questions with regard to the use of carbon regulation in order to develop non-polluting transportation devices, like the electric car when electricity comes from power generation plants that use CCS technology.[10] During the ceiling phase, the carbon tax has an overall decreasing shape which goes down to zero at the end of the phase. However this general shape is actually composed of a complex sequence of phases with decreasing rates, separated by phases with constant rates. These latter phases correspond respectively to Sector 2's abatement phase and to the partial abatement phase of Sector 1 which follows its full carbon abatement phase.

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[10]See e.g. Chakravorty et al. (2011).

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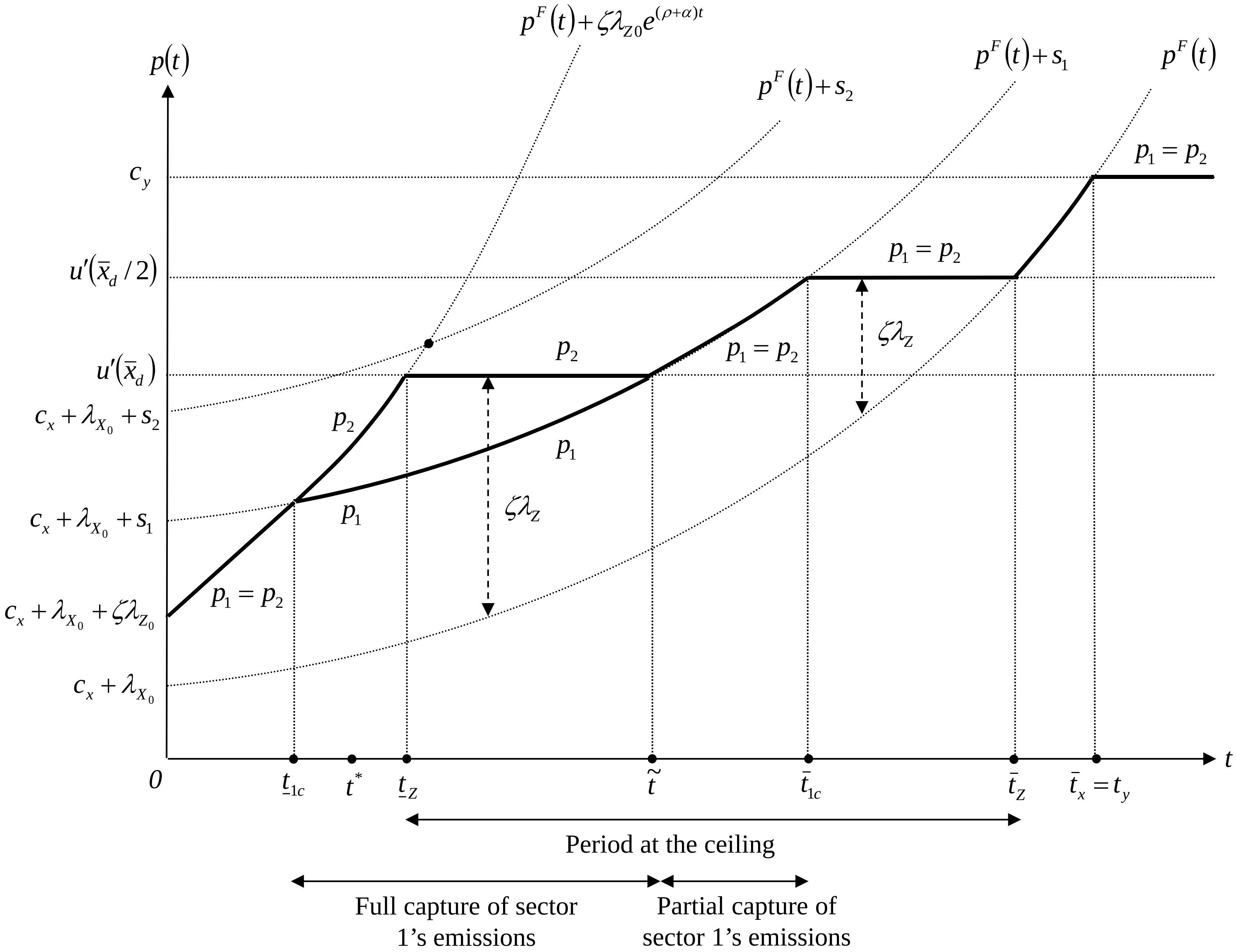
Figure 1



$p^F(t)+\zeta\lambda_{Z0}e^{(\rho+\alpha)t}$
$p(t)$
$p^F(t)+s_2$
$p^F(t)+s_1$
$p^F(t)$
c_y
$p_1 = p_2$
$u'(\bar{x}_d/2)$
$p_1 = p_2$
$\zeta\lambda_Z$
p_2
$p_1 = p_2$
$u'(\bar{x}_d)$
$c_x+\lambda_{X_0}+s_2$
p_2
p_1
$\zeta\lambda_Z$
$c_x+\lambda_{X_0}+s_1$
p_1
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$c_x+\lambda_{X_0}$
0
t
$\underline{t}_{1c}$
t^*
$\underline{t}_Z$
$\tilde{t}$
$\bar{t}_{1c}$
$\bar{t}_Z$
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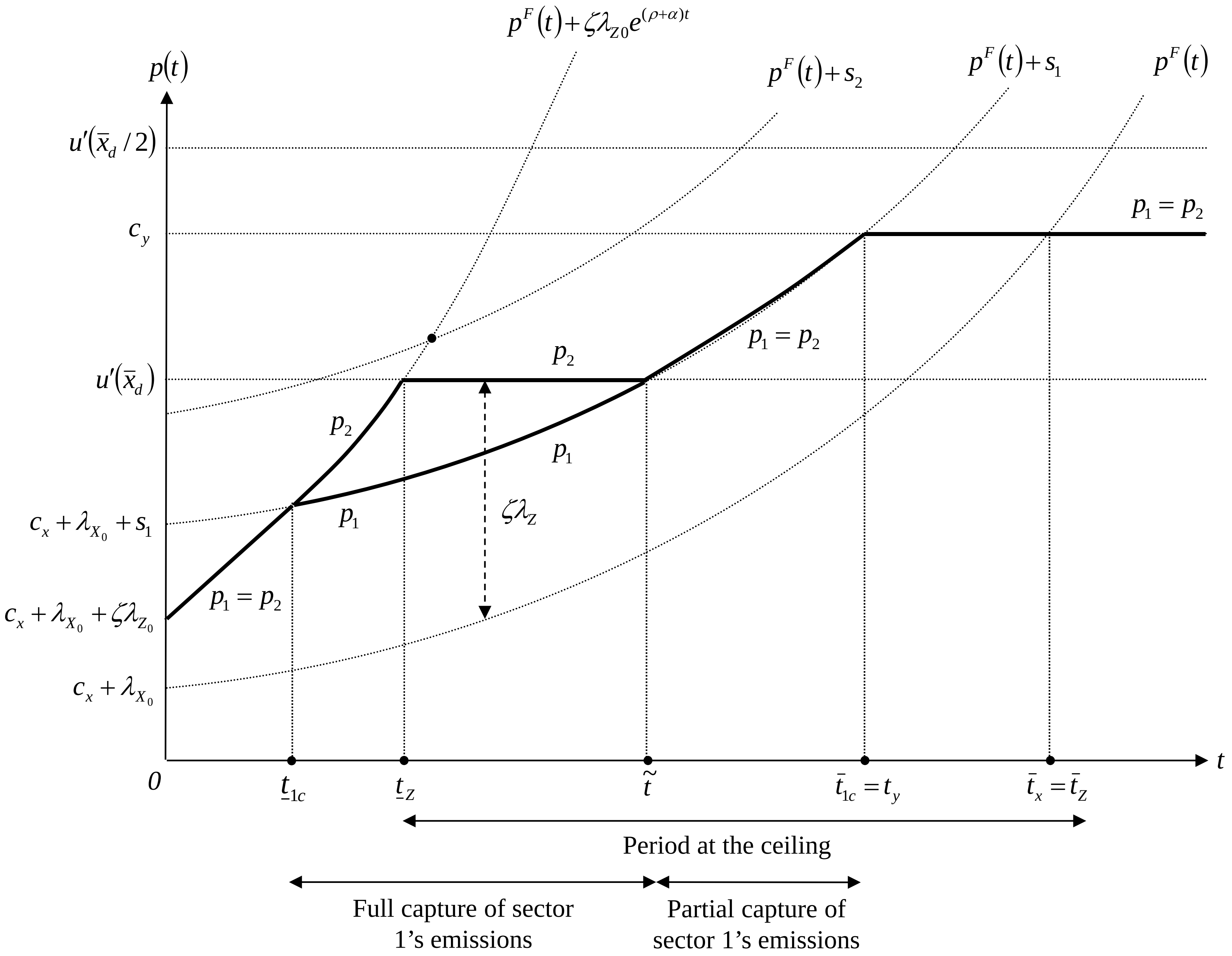
Period at the ceiling
Full capture of sector 1’s emissions
Partial capture of sector 1’s emissions

Figure 2



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$p^F(t)+s_1$
$p^F(t)$
$u'(\bar{x}_d/2)$
c_y
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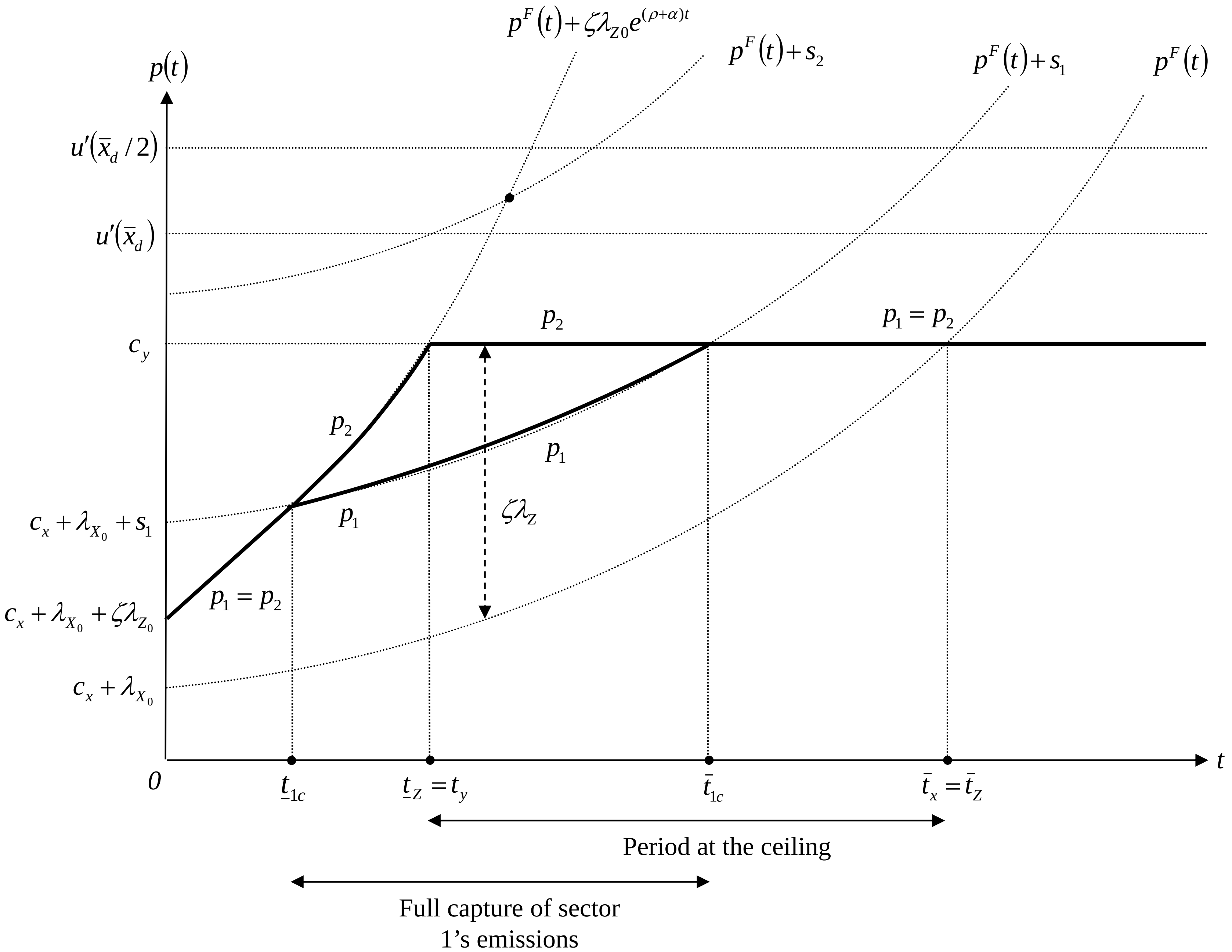
p_2
$p_1 = p_2$
$u'(\bar{x}_d)$
p_2
p_1
$\zeta\lambda_Z$
$c_x+\lambda_{X_0}+s_1$
p_1
$c_x+\lambda_{X_0}+\zeta\lambda_{Z_0}$
$p_1 = p_2$
$c_x+\lambda_{X_0}$
t
0
$\underline{t}_{1c}$
$\underline{t}_Z$
$\tilde{t}$
$\bar{t}_{1c}=t_y$
$\bar{t}_x=\bar{t}_Z$
Period at the ceiling
Full capture of sector 1's emissions
Partial capture of sector 1's emissions

Figure 3



$p^F(t)+\zeta\lambda_{Z0}e^{(\rho+\alpha)t}$
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$p^F(t)+s_1$
$p^F(t)$
$p(t)$
$u'(\bar{x}_d/2)$
$u'(\bar{x}_d)$
c_y
p_2
$p_1 = p_2$
p_2
p_1
p_1
$\zeta\lambda_Z$
$c_x+\lambda_{X_0}+s_1$
$c_x+\lambda_{X_0}+\zeta\lambda_{Z_0}$
$p_1 = p_2$
$c_x+\lambda_{X_0}$
0
t
$\underline{t}_{1c}$
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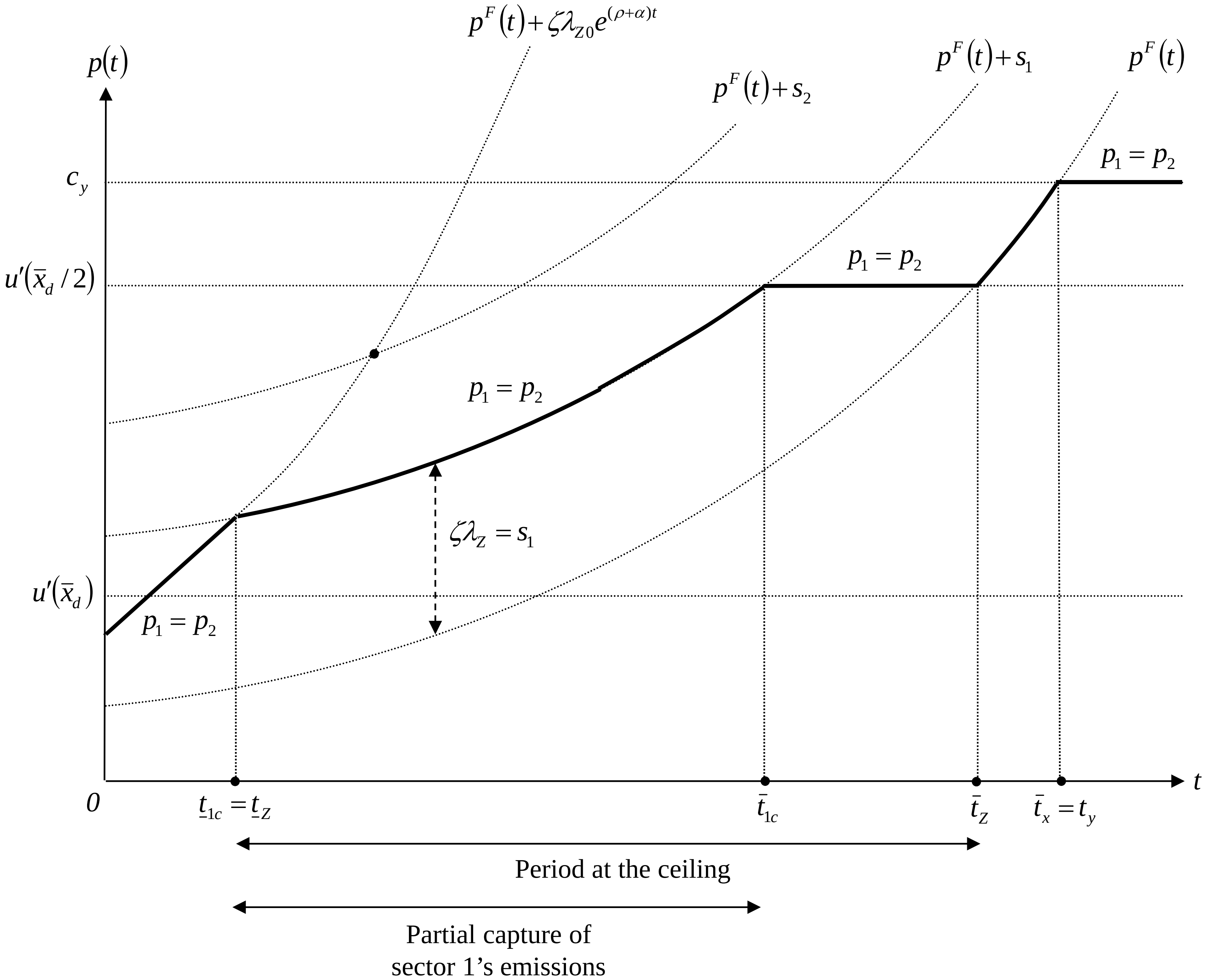
$\bar{t}_{1c}$
$\bar{t}_x = \bar{t}_Z$
Period at the ceiling
Full capture of sector 1's emissions

Figure 4



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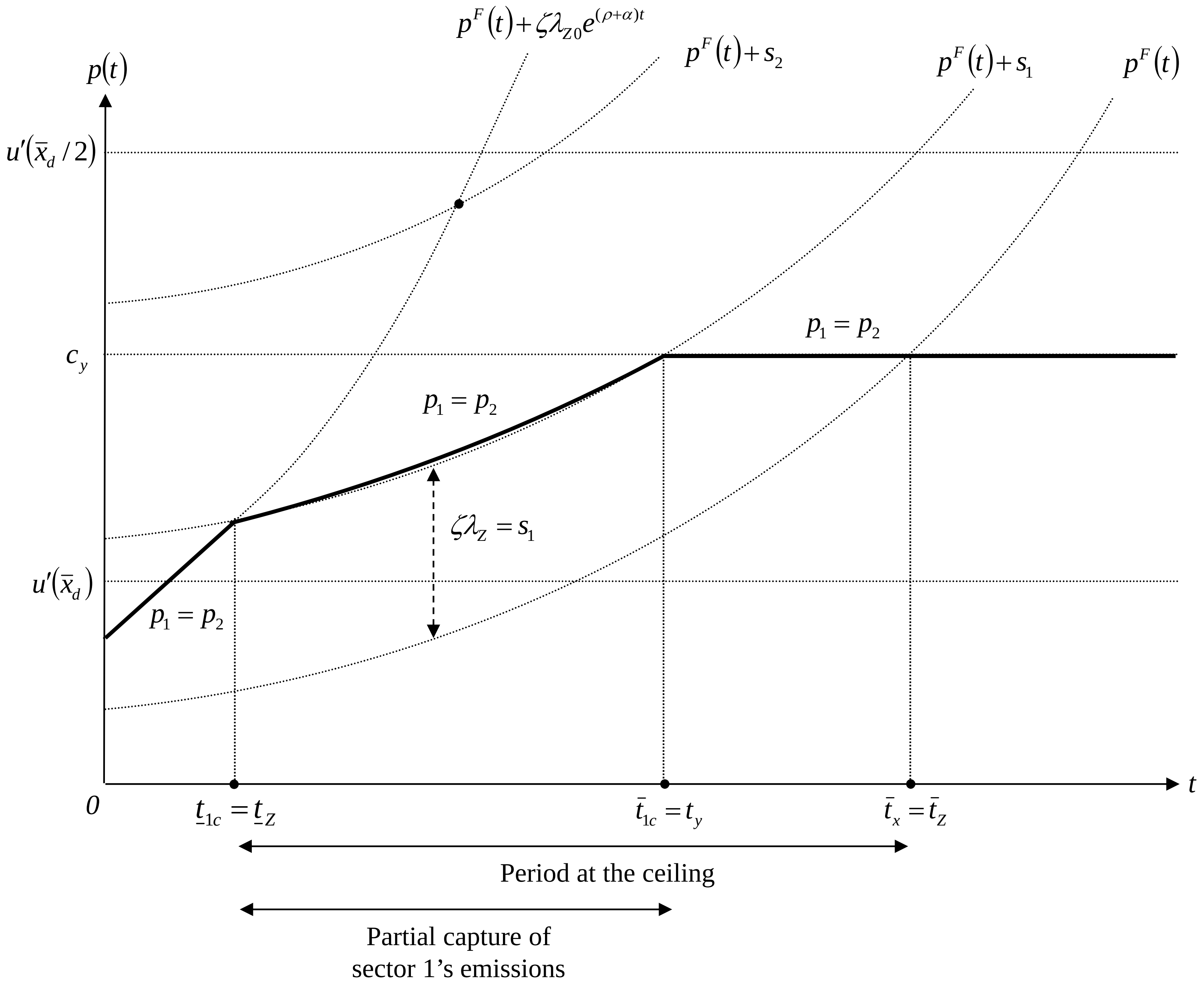
$p^F(t)+s_1$
$p^F(t)$
c_y
$p_1 = p_2$
$u'(\bar{x}_d/2)$
$p_1 = p_2$
$p_1 = p_2$
$\zeta\lambda_Z = s_1$
$u'(\bar{x}_d)$
$p_1 = p_2$
t
0
$\underline{t}_{1c} = \underline{t}_Z$
$\bar{t}_{1c}$
$\bar{t}_Z$
$\bar{t}_x = t_y$
Period at the ceiling
Partial capture of
sector 1's emissions

Figure 5



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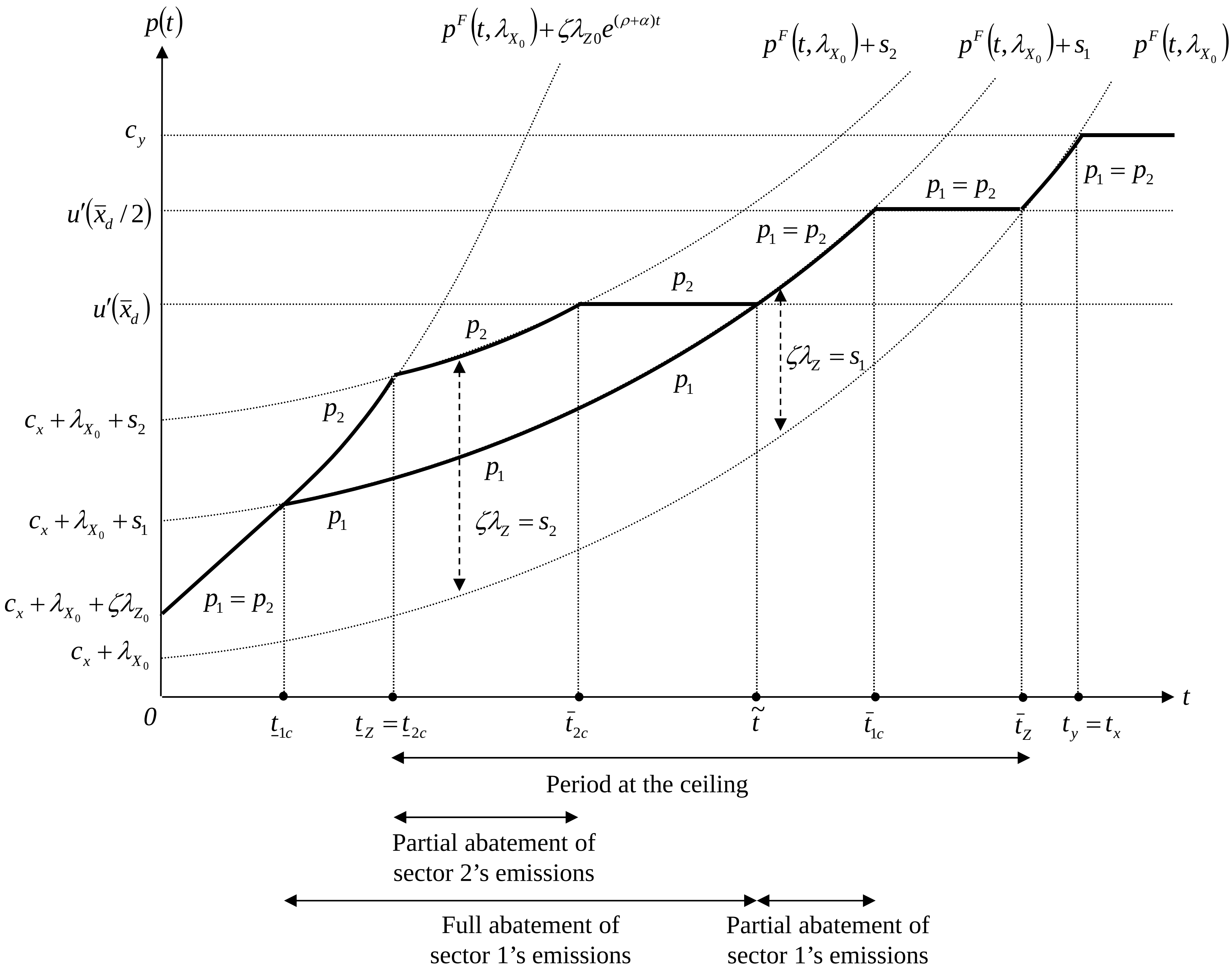
$p^F(t)+s_1$
$p^F(t)$
$p(t)$
$u'(\bar{x}_d/2)$
c_y
$u'(\bar{x}_d)$
$p_1 = p_2$
$p_1 = p_2$
$p_1 = p_2$
$\zeta\lambda_Z = s_1$
0
t
$\underline{t}_{1c} = \underline{t}_Z$
$\bar{t}_{1c} = t_y$
$\bar{t}_x = \bar{t}_Z$
Period at the ceiling
Partial capture of
sector 1's emissions

Figure 6



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$p^F(t, \lambda_{X_0}) + s_1$
$p^F(t, \lambda_{X_0})$
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$u'(\bar{x}_d)$
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p_2
p_1
$\zeta\lambda_Z = s_2$
$\zeta\lambda_Z = s_1$
0
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$\underline{t}_Z = \underline{t}_{2c}$
$\bar{t}_{2c}$
$\tilde{t}$
$\bar{t}_{1c}$
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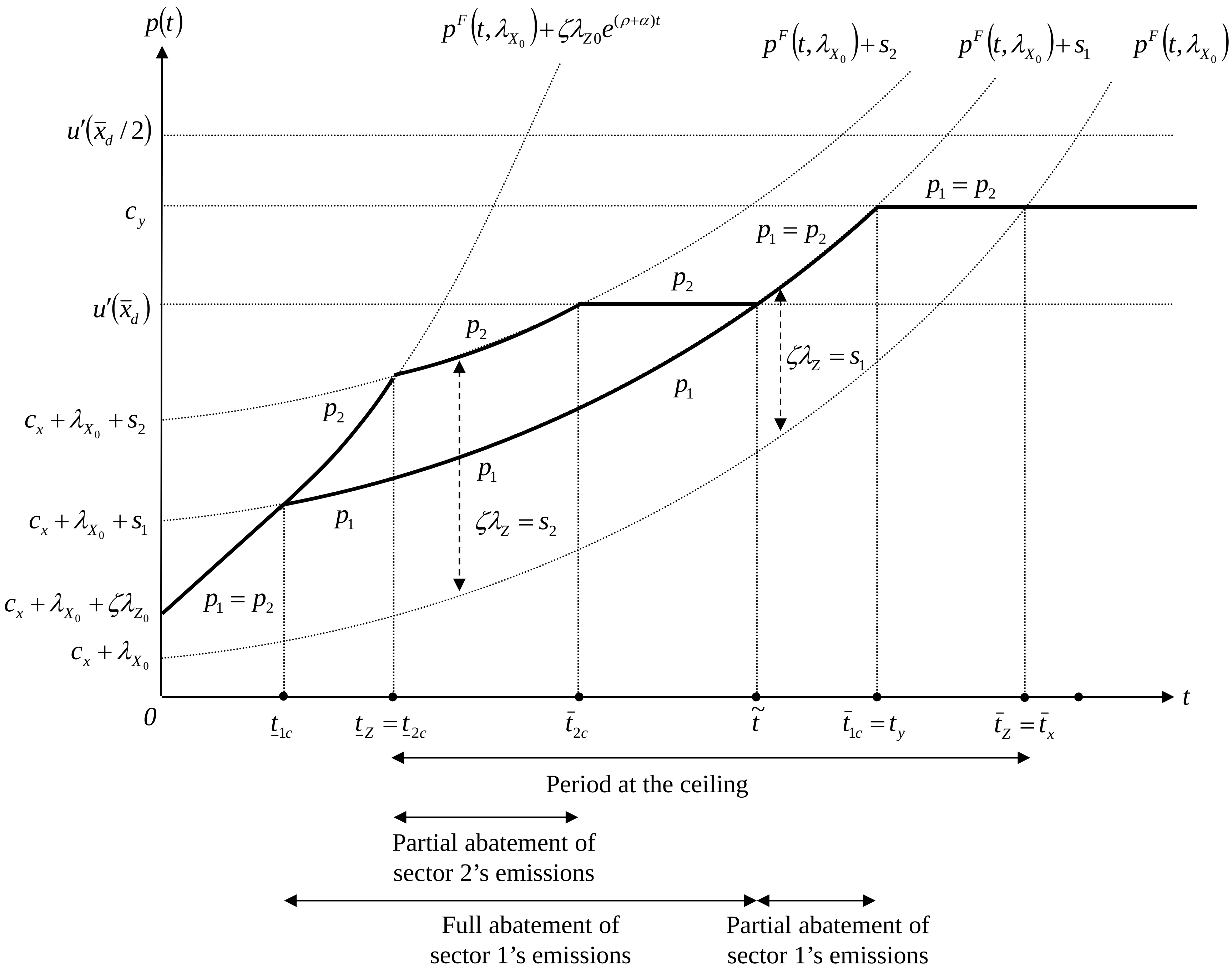
Period at the ceiling
Partial abatement of sector 2's emissions
Full abatement of sector 1's emissions
Partial abatement of sector 1's emissions

Figure 7



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$u'(\bar{x}_d/2)$
c_y
$u'(\bar{x}_d)$
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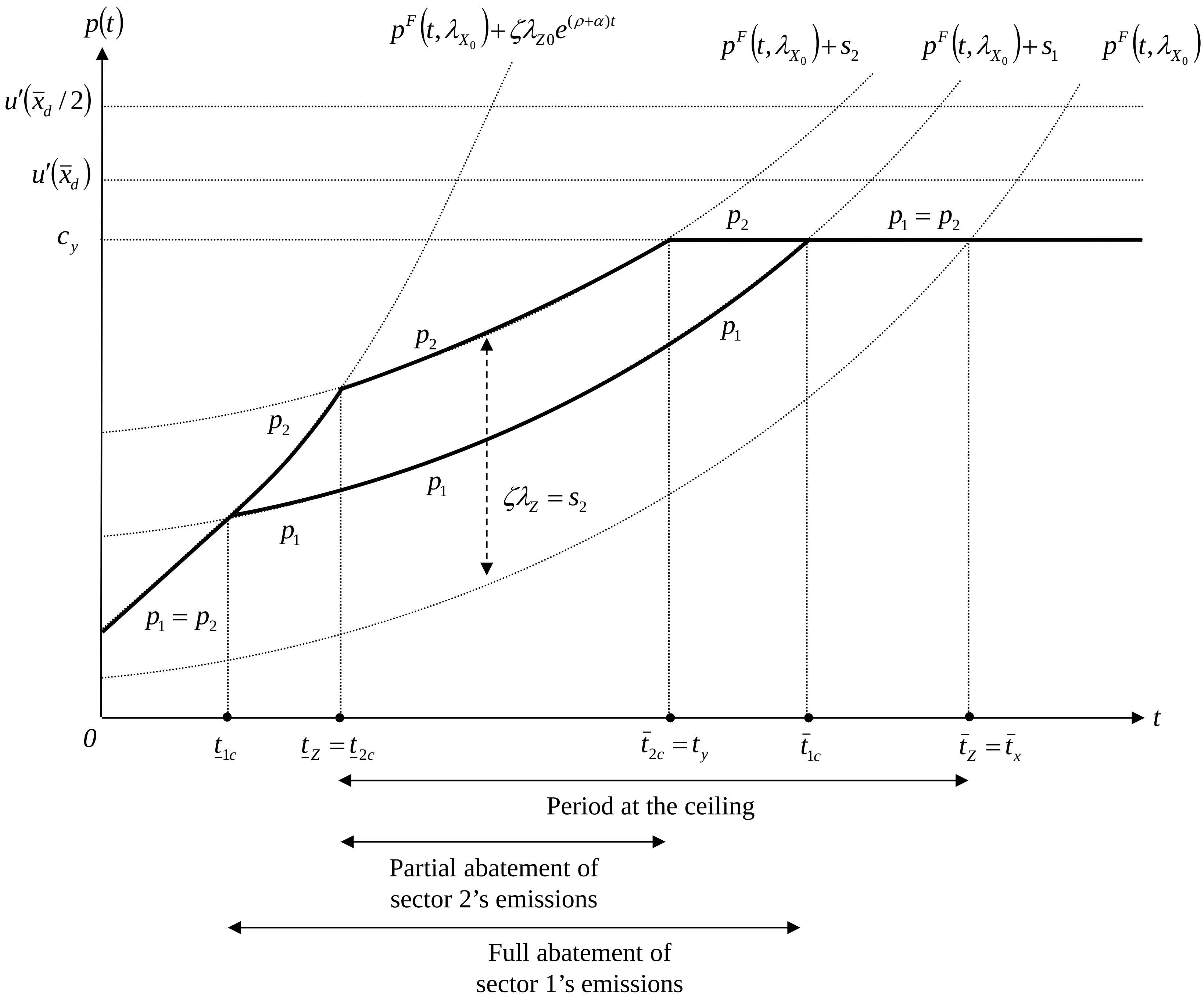
$\tilde{t}$
$\bar{t}_{1c}=t_y$
$\bar{t}_Z=\bar{t}_x$
t
Period at the ceiling
Partial abatement of sector 2's emissions
Full abatement of sector 1's emissions
Partial abatement of sector 1's emissions

Figure 8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$p^F(t,\lambda_{X_0})+s_1$
$p^F(t,\lambda_{X_0})$
$u'(\bar{x}_d/2)$
$u'(\bar{x}_d)$
c_y
p_2
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p_2
p_1
p_2
p_1
$\zeta\lambda_Z = s_2$
p_1
$p_1 = p_2$
t
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$\underline{t}_Z = \underline{t}_{2c}$
$\bar{t}_{2c} = t_y$
$\bar{t}_{1c}$
$\bar{t}_Z = \bar{t}_x$
Period at the ceiling
Partial abatement of
sector 2's emissions
Full abatement of
sector 1's emissions