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Mass Retailers' Advertising Strategies Faced with Different Competitor Store Formats: Commodity Stores or Hard Discounts.

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Abstract

A retailer has different opportunities to advertise in the media: emphasizing the store image or promoting specifically its private label (PL). In the first option, advertising benefits all products sold, whereas in the alternative, only store brands are concerned by image improvement.

We analyze the retailer's advertising campaign strategy distinguishing its competitor's format: commodity store or hard-discounter. PL quality is endogenous and chosen according to the competitor's product range.

We show the retailer prefers to advertise the store rather than its PL. However, this strategy may push commodity stores out of the market and reduce social welfare.

JEL Codes : L15, M37, L41.

Keywords: Advertising, Retailing, Competition.

Introduction

Advertising campaigns constitute an important non-price strategy for retailers to develop their sales. Indeed, the retailing sector devotes 1.5 % of its total turnover to advertising expenditures. This represents 2'962 millions € for France in 2011, which places retailers as the first rank among sectors in terms of budget spent (followed by the Automobile industry and food manufacturers).[1] In table 1, the advertising expenditures of , the four biggest French retailing chains (Carrefour, Leclerc, Intermarché and Auchan) are reported in terms of the different advertising media used (press, radio, TV, external displays, internet[2]

Marketer	Press	Radio	TV	External displays	Internet
Carrefour	20.21	40.71	16.96	17.64	4.48
E. Leclerc	19.85	45.7	9.29	10.7	4.46
Intermarché	14.6	58.61	14.46	6.96	5.38
Auchan	9.53	58.31	13.23	12.3	6.63

Table 1: Advertising distribution of retailers in 2010
(source Kantar Media)

Globally, retailers use the different media the same way, indicating that they do not differentiate their strategies for advertising. Radio is the most important media for retail advertising (about 50%). TV is the prevailing media for advertising by agrofood manufacturers (more than 75%) whereas this media represents less than 17% of retailers' advertisements. This can be explained by the fact that TV has been a new media for distributors since January 2007, when France repealed an old law from 1968 forbidding retailers from broadcasting their advertisements on TV. [3]

[1] Advertising expenditures are computed over the 10 first months of 2011 in order to avoid Christmas effect.
[2] These ratailers are among the top 20 marketers.
[3] The French decree of the 7th October 2003 opened retailing advertising to local broadcasting for January 2004 and to national TV in January 2007.

Over the last 30 years, retailers have become "double agents" by not only reselling brand manufacturers' goods but also by introducing their own private labels (also named store brands), the market share of which reached 35% in France in 2010 according to the Private Label Manufacturers' Association statistical yearbook. The consequence is that competition between retailers has become twofold: on the one hand, retailers compete with each other (intrabrand store competition) and on the other hand, national brands compete with private labels within each retail store (interbrand competition). Because of this dual role (as retailers and as store brand manufacturers), distributors have to choose between two possible messages in their advertising campaign. They can emphasize the general store image, irrespectively of the products sold in the store (like Carrefour and the 'blue line' campaign) or they can communicate specifically on their private labels (PL) in order to increase interbrand competition.[4] Retailers' advertising strategy thus exhibits a specificity generating a particular trade-off that brand manufacturers do not face.

The economic literature on advertising is quite vast (Bagwell, 2008). Textbooks generally distinguish three kinds of advertising campaign related to their impact on consumers' utility. The first two are the most common: informative advertising and persuasive advertising. Informative advertising (Milgrom & Roberts, 1986) provides general information about the product advertised to consumers (existence, characteristics, etc.). By enhancing the consumers' potential choice set, the resulting demand becomes more elastic or consumers make firms compete more. This kind of advertising is thus seen as welfare enhancing as it reduces the firms' market power. Persuasive advertising (Braithwaite, 1928), however, alters consumers' tastes by increasing product differentiation and their willingness-to-pay for the good advertised, leading to a less elastic demand. It can result in higher prices

[4] Carrefour, a major French retailing group with more than 25% of market share, initiated in 2009 a TV advertising campaign promoting their new 'blue line' concept. At cash tills, when the queue exceeds the blue line on the ground, Carrefour commits to opening more tills in order to reduce the waiting time to the cash desk. Intermarché and Leclerc, two other French big retailers, clearly based their advertising contents on their store brands by promoting their value for money compared to equivalent branded products.

because of reduced competition or by deterring new firms, resulting in lower social welfare. Becker & Murphy (1993) characterized a third kind of advertising called "complementary advertising". In their view, the intensity of advertising becomes a part of consumers' utility by defining a 'social image' linked to the consumption of the advertised good, but nonetheless generating a positive externality on competing goods for consumers. The social welfare effects of such advertising are ambiguous.

Most articles considering manufacturers' advertising find a positive link with average retail prices as reported in Steiner (1998). The presence of a private label in the retailer's supply does not jeopardize this result as shown by Soberman and Parker (2006). However, there are not many articles about retailers' advertising strategies *per se* in the presence of private labels. Karray and Martín-Herrán (2008) study a particular framework where local monopolist retailers only provide institutional advertising increasing willingness-to-pay for all products sold in-store (national brand and private label). In the absence of analytical solutions, they show through numerical simulations that the effect of persuasive advertising on final prices is ambiguous: it brings down total demand while increasing product differentiation. Karray and Martín-Herrán (2009) develop a model of vertical relationships where a retailer sells a national brand and a private label perceived as horizontally differentiated in their characteristics. The advertising investments are made by the manufacturer for the national brand and by the retailer for the store image. Each kind of advertisement alters consumers' tastes for both products simultaneously. Their conclusion is that the retailer may limit his store advertising investment since it increases competition with the national brand product, generating lower prices and thus lower revenues. From an empirical point of view, the main question relates to the efficiency of retailers' advertising. Reiley and Lewis (2011) find that Yahoo! ads promoting a Video On Demand (VOD) website significantly increase VOD

demand and are very profitable. They also note that the sales effects remain persistent for weeks even in the absence of renewed advertising.

The dual nature of retailers enhances their possible advertising strategy in the sense that they can decide to promote either their store image or their own brand. One of the determinants of the choice between these alternatives may be the nature of the rival retailers they are competing with. Traditional mass retailers face two different kinds of competitors when selling agrofood goods: commodity stores (CS) or hard-discounters (HD).[5]

The objective of our article is to investigate how retailers' image positioning influences the choice of advertising. It is also to find out whether there are any anticompetitive effects of mass retailers' advertising on their competitors. In the framework we develop, advertising is mainly persuasive (changing preferences across retailers).

In section 1, we present the framework by characterizing retailers' supply as well as consumer preferences. Section 2 and section 3 analyze the competitive equilibrium in advertising strategy between mass retailers with respectively commodity stores/hard-discounts. Section 4 concludes.

1. The framework

1.1 Retailers' supply

We consider two vertically differentiated retailers, R_1 and R_2. Retailer 1 sells two goods: the national brand (NB) with quality q_{NB} and the private label product (PL) with quality q_{PL}. It is assumed that the quality of the PL is chosen by R_1 but it is still lower than that of the NB: $q_{\mathrm{PL}} < q_{\mathrm{NB}}$. This is a classic assumption in the economic literature on PL

[5] Commodity stores refer to traditional stores or small convenience stores, mainly selling food products. Hard-discounters are characterized by everyday-low-pricing, narrow choice and no service.

quality (Mills, 1995, 1998 and Bontems *et al.*, 1999).[6] This generates a higher willingness-to-pay for national brands than for private labels (see Bell, 2000 or Bergès *et al.*, 2009). Retailer 2 sells only one good (G2) with quality q_{G2}.

We consider the following alternative cases for this retailer: R2 is either a hard discounter (HD) or a commodity store (CS). A commodity store is generally a small store located downtown contrary to supermarkets and hypermarkets.

When R1 is facing a CS, one can consider that they both sell the same NB good, but since the quality perceived by consumers depends on the store's characteristics (CS location close to consumers' home, in-store services provided), it could finally result in $q_{PL} < q_{NB} < q_{G2} = 1$. Conversely, when R_2 is a HD, we pragmatically assume that quality levels are such that $q_{G2} < q_{PL} < q_{NB} = 1$.[7]

We assume that the two retailers face the same cost function, linear in the total quantity produced (X) but quadratic in the quality (q_i): $C(X, q_i) = \frac{{q_i}^2}{2} X$. Note that the marginal cost of quality is constant in the quantity produced but increasing in the level of quality chosen.

In addition to the choice of PL quality, retailer R_1 can implement an advertising campaign. He must decide between two kinds of advertising message: store *versus* product (denoted SA *vs* PA). The store advertisement results in an increase in store image that modifies the consumer's utility, as soon as he/she consumes the NB or the PL sold by this specific retailer R_1. The product advertisement impacts only on the private label by increasing

[6] Empirical analysis (Dodds et al., 1991) shows that brand names have a positive effect on perception of quality and willingness to pay. This article focuses on private labels that mimic NB products but often sell at a lower price. It does not apply to high-quality private labels.

[7] In both cases the high quality level is normalized to 1.

the PL product's quality as perceived by the consumer.[8] We will consider the mass-media campaign as a fixed cost for the retailer, depending only on the advertisement's intensity, like the duration of broadcasting (TV, radio), irrespective of the quantity sold.

1.2 Consumer preferences

Faced with the choice set $\{NB, PL, G2\}$., each consumer buys at most one unit of either good: preferences are of the Mussa-Rosen (1978) type. Each consumer is indexed by a parameter θ measuring his taste for quality, and θ is uniformly distributed in the interval $[0,1]$. The consumer characterized by θ derives a utility $U_i = \theta.q_i - p_i$ from consuming a unit of good of quality q_i sold at price p_i, where $i = \{NB, PL, G2\}$. Utility is zero if neither good is bought. Faced with retailers' prices, the consumer classically chooses the product that provides the highest level of utility.

Regarding advertising, if the intensity of the advertisement is defined by μ, the two possible advertising strategies of R_1 (SA or PA) impact in the following way on consumers' utility:

- For store advertising, R_1's image is increased by μ^{SA} according to quality taste but independently of the good bought in R_1. The utility function becomes: $U_i = \theta.\mu^{SA} + \theta.q_i - p_i$ for $i = \{NB1, PL\}$.
- In the case of product advertising, only the perceived PL quality increases from q_{PL} to $(q_{PL} + \mu^{PA})$. The utility derived from PL consumption thus becomes: $U_{PL} = \theta(q_{PL} + \mu^{PA}) - p_{PL}$.

[8] Irrespective of the strategy chosen, SA or PA, both advertisements are in fact "persuasive advertising" in the sense that they increase consumers' willingness-to-pay either for R1 in the case of SA, or for the PL in the case of PA. In this framework, because of the implicit assumption of perfect information by consumers (retailers' products range and prices are known before the purchase), there is no scope for "informative advertising".

Moreover, regardless of the strategy chosen by R_1, we assume that the fixed cost of advertising necessary to finance a mass-media campaign of intensity μ^j is $c(\mu^j) = \frac{(\mu^j)^2}{2}$ for $j = \{SA, PA\}$.

The timing of the game is as follows:

- In the first stage, retailer R_1 chooses the quality of the private label good according to the quality of the national brand (q_{NB}) he sells, and the product quality of his competitor (q_{G2}).
- In the second stage, R_1 chooses its advertising strategy (SA or PA), as well as its intensity (μ^j).
- In the third stage, retailers R_1 and R_2 compete on prices.

Note that this timing is consistent with retailers' commitments in the sense that quality definition is more irreversible than a retailer's advertising campaign or final prices.

We now turn to the analysis of the preferred advertising campaign depending on whether R_1 faces a CS competitor or a HD rival.

2. Competition against a Commodity Store

We consider the situation where retailer R_1 competes with a commodity store. We characterize the commodity store such that, at the same price, consumers would buy the product G2 even if the intrinsic characteristics of products NB and G2 were the same. This is due to the fact that, for example, retailer 2 benefits from a better geographical position (city center with higher population density). We first define the benchmark equilibrium characterized by the fact that retailer R_1 does not advertise. The choice of PL quality (q_{PL}) is

made by R_1 according to the quality of the NB (q_{NB}) and that of G2 (q_{G2}), respecting the following quality ranking: $q_{PL} < q_{NB} < q_{G2} = 1$. We then look for the optimal advertising strategies and derive the equilibrium chosen by R_1.

2.1 Benchmark

We first compute demands by characterizing the indifferent marginal consumers. $\theta_{0/PL} = \frac{p_{PL}}{q_{PL}}$ denotes the marginal consumer who is indifferent between buying the PL or nothing, $\theta_{PL/NB} = \frac{p_{NB} - p_{PL}}{q_{NB} - q_{PL}}$ is the marginal consumer who is indifferent between buying the PL or the NB in R_1 and $\theta_{NB/G2} = \frac{p_{G2} - p_{NB}}{1 - q_{NB}}$ is the marginal consumer who is indifferent between buying the NB product or the product G2. Demands for products are thus defined by:

$$D_{G2} = 1 - \frac{p_{G2} - p_{NB}}{1 - q_{NB}} \; ; \; D_{NB} = \frac{p_{G2} - p_{NB}}{1 - q_{NB}} - \frac{p_{NB} - p_{PL}}{q_{NB} - q_{PL}} \; ; \; D_{PL} = \frac{p_{NB} - p_{PL}}{q_{NB} - q_{PL}} - \frac{p_{PL}}{q_{PL}} ; \; D_0 = \frac{p_{PL}}{q_{PL}}$$

Retailers' profits are given by:

$$\Pi_1^B = D_{NB}\left(p_{NB} - \frac{q_{NB}^2}{2}\right) + D_{PL}\left(p_{PL} - \frac{q_{PL}^2}{2}\right) \text{ and } \Pi_2^B = D_{G2}(p_{G2} - 1)$$

These definitions are valid only if: $\theta_{0/PL} < \theta_{PL/NB} < \theta_{NB/G2}$ (such conditions are checked at equilibrium). Competition in prices between R_1 and R_2, for a given PL quality, leads to:

$$p_{PL} = \frac{1}{4} q_{PL}\left(\frac{6}{4 - q_{NB}} - (q_{NB} - q_{PL})\right); p_{NB} = \frac{3\, q_{NB}}{8 - 2\, q_{NB}} \text{ and } p_{G2} = \frac{3}{4 - q_{NB}} - \frac{q_{NB}}{2}$$

Incorporating these equilibrium prices and maximizing Π_1^B with respect to q_{PL} results in: $q_{PL}^*(q_{NB}) = \frac{q_{NB}}{2}$. Classically, the higher the NB quality R_1 sells, the higher the PL product quality should be in order to increase final prices (market power). Note that the degree of

differentiation of products sold by R_1 increases with the level of quality. Indeed, in order to attract consumers with low taste for quality, the intensity of the increase in PL quality is lower than the NB one since there is no other competitor in the low-quality range.

2.2 Store Advertising

Given the market configuration, store advertising impacts directly on two marginal consumers: the one who is indifferent between buying the private label product or nothing ($\theta_{0/PL} = \frac{p_{PL}}{q_{PL}+\mu^{SA}}$ and the one who is indifferent between buying the NB at R_1 or the higher quality good at R_2 ($\theta_{NB/G2} = \frac{p_{G2}-p_{NB}}{1-(q_{NB}+\mu^{SA})}$). We assume that the choice of advertising intensity still maintains the original range of qualities: $q_{PL} + \mu^{SA} < q_{NB} + \mu^{SA} < q_{G2} = 1$.

Demands for products and profits for firms are:

$D_{G2} = 1 - \frac{p_{G2}-p_{NB}}{1-(q_{NB}+\mu^{SA})}$; $D_{NB} = \frac{p_{G2}-p_{NB}}{1-(q_{NB}+\mu^{SA})} - \frac{p_{NB}-p_{PL}}{q_{NB}-q_{PL}}$; $D_{PL} = \frac{p_{NB}-p_{PL}}{q_{NB}-q_{PL}} - \frac{p_{PL}}{q_{PL}+\mu^{SA}}$; $D_0 \frac{p_{PL}}{q_{PL}+\mu^{SA}}$

$\Pi_1^{SA} = D_{NB}\left(p_{NB} - \frac{q_{NB}^2}{2}\right) + D_{PL}\left(p_{PL} - \frac{q_{PL}^2}{2}\right) - c(\mu^{SA})$, with $c(\mu^{SA}) = \frac{(\mu^{SA})^2}{2}$ and $\Pi_2^{SA} = D_{G2}(p_{G2} - 1)$

The equilibrium, solved numerically by computing the optimal prices (Appendix A.1) and then the optimal SA intensity (given the PL quality decided in the benchmark case), is depicted in Figure 1.

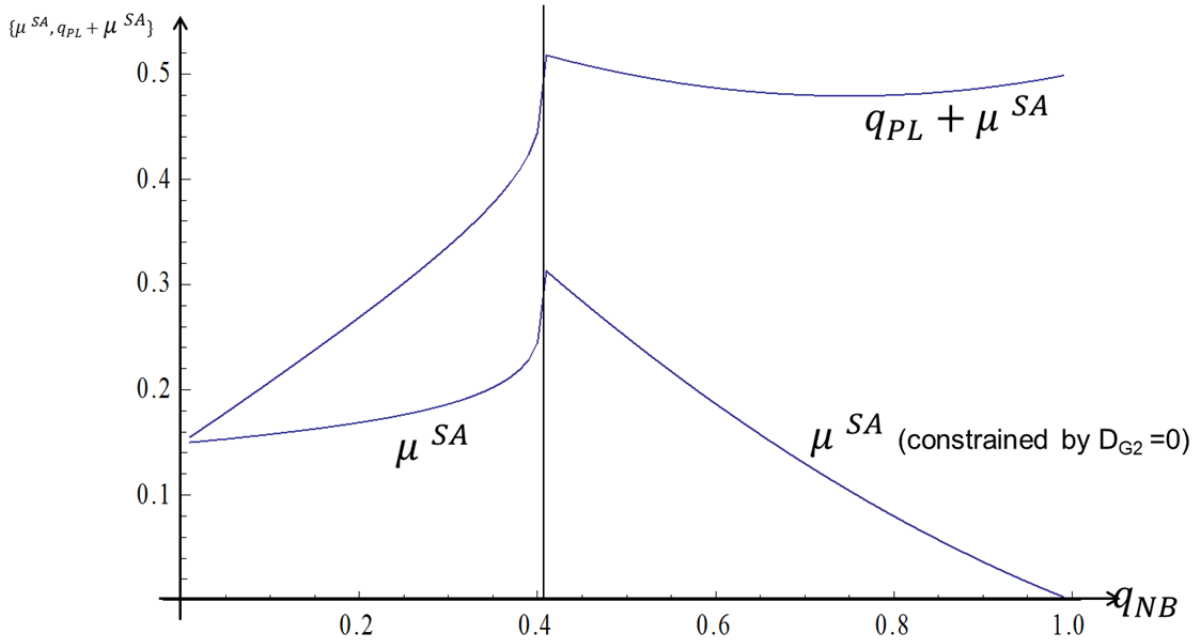

Figure 1: Optimal SA intensity with respect to the NB product.

First, it is interesting to note that an increase in the NB quality sold by R_1 results in an increase in store advertising intensity. This comes from two effects. On the one hand, reducing product differentiation between the NB and G2 increases competition with R_2, leading to a lower G2 demand but also to a lower NB price; on the other hand, a SA increase allows R_1 to restore good PL value for money (and improve market coverage) in order to temper the PL price increase generated by the rise in q_{PL}.

Second, there exists a range of NB quality ($q_{NB} > 0.42$) such that SA intensity annihilates the commodity store's demand for good G2. Imposing the constraint that the rival retailer's demand for G2 should be at least positive generates a decrease in μ^{SA}.

Therefore, for a sufficiently high q_{NB}, R_1 is able to push R_2 out of the market by selecting an appropriate $(\tilde{p}_{NB}, \mu^{SA})$ such that $D_{G2}(\tilde{p}_{NB}, \mu^{SA}, p_{G2}) = 0$. R_1, in this regime, will behave as a monopoly. The limit price for the NB such that R_2's demand is nil is defined by:

$$D_{G2}(\tilde{p}_{NB}, \mu^{SA}, p_{G2}) = 0 \Leftrightarrow \tilde{p}_{NB}(\mu^{SA}) = 1 + q_{NB} + \mu^{SA} - \frac{-10 + {q_{NB}}^2}{2(q_{NB} + \mu^{SA} - 4)}$$

In order to keep R_2 out of the market, R_1 has to set a relatively low price for its NB good generating an opportunity cost for this strategy. However, the possibility of store advertising is a relief for R_1 in the sense that the limit NB price is increasing in μ^{SA}.[9] The higher the advertising intensity, the lower the opportunity cost of keeping R_2 out of the market, since the limit price required for such action increases.

Numerically computing the new monopoly equilibrium (denoted SAm, Appendix A.2) leads to optimal store advertising depicted Figure 2.

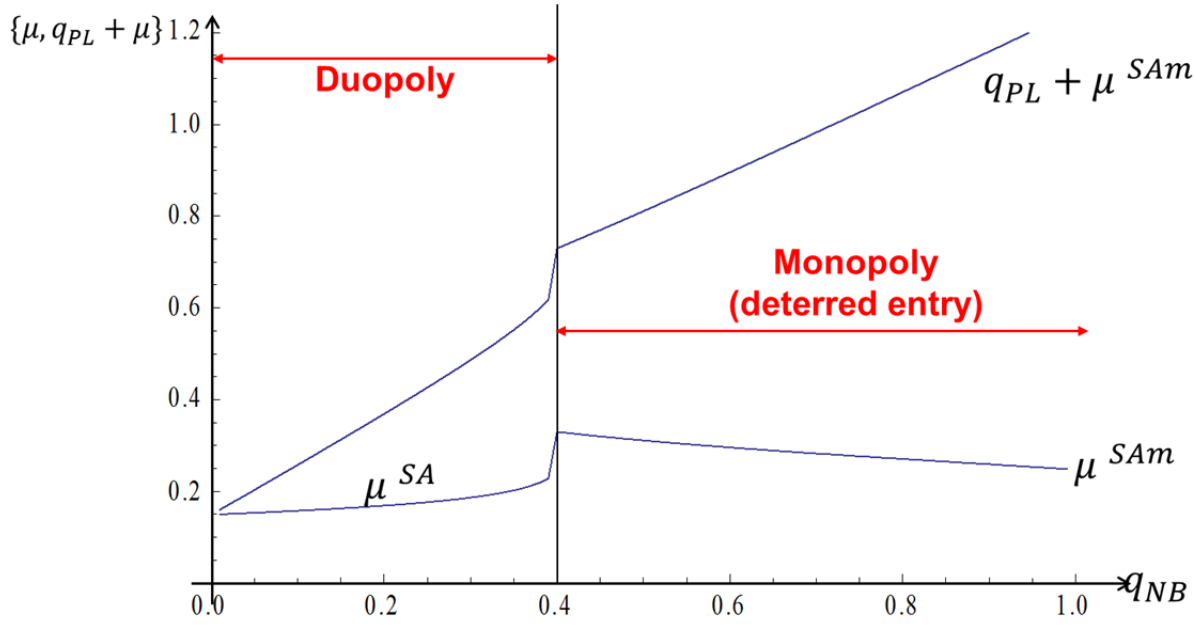

Figure 2: Optimal SAm intensity in the absence of R_2.

In the monopoly section, we first find that $\mu^{SAm} > \mu^{SA}$. Since there is no longer a competitor selling a higher-quality good, R_1 uses store advertising to increase perceived quality (and to lower the opportunity cost of excluding R_2) and benefits from high-valuation consumer rent extraction. However, when q_{NB} rises, μ^{SAm} decreases because the absence of competition gives R_1 no incentive to maintain a differentiation in quality with his rival through the NB product. A second consequence of R_1 being a monopoly is the decrease in market coverage because of market power exertion.

[9] One can check that $\frac{\partial(\tilde{p}_{NB}(\mu^{SA}))}{\partial\mu^{SA}} = 1 + \frac{-10+q_{NB}{}^2}{2(-4+q_{NB}+\mu^{SA})^2} > 0$ since $q_{NB} + \mu^{SA} < 1$.

2.3 Product Advertising

The effect of product advertising is comparable to an increase in perceived PL quality (respecting the constraint: $q_{PL} + \mu^{PA} < q_{NB}$). So only marginal consumers with this good in their set choice are directly concerned by the advertisement: $\theta_{0/PL} = \frac{p_{PL}}{q_{PL}+\mu^{PA}}$ and $\theta_{PL/NB} = \frac{p_{NB}-p_{PL}}{q_{NB}-(q_{PL}+\mu^{PA})}$. Demands are as follows:

$$D_{G2} = 1 - \frac{p_{G2} - p_{NB}}{1 - q_{NB}};\ D_{NB} = \frac{p_{G2} - p_{NB}}{1 - q_{NB}} - \frac{p_{NB} - p_{PL}}{q_{NB} - (q_{PL} + \mu^{PA})};\ D_{PL} = \frac{p_{NB} - p_{PL}}{q_{NB} - (q_{PL} + \mu^{PA})} - \frac{p_{PL}}{q_{PL} + \mu^{PA}};\ D_0 = \frac{p_{PL}}{q_{PL} + \mu^{PA}}$$

The computed equilibrium (Appendix A.3) is depicted inFigure 3.

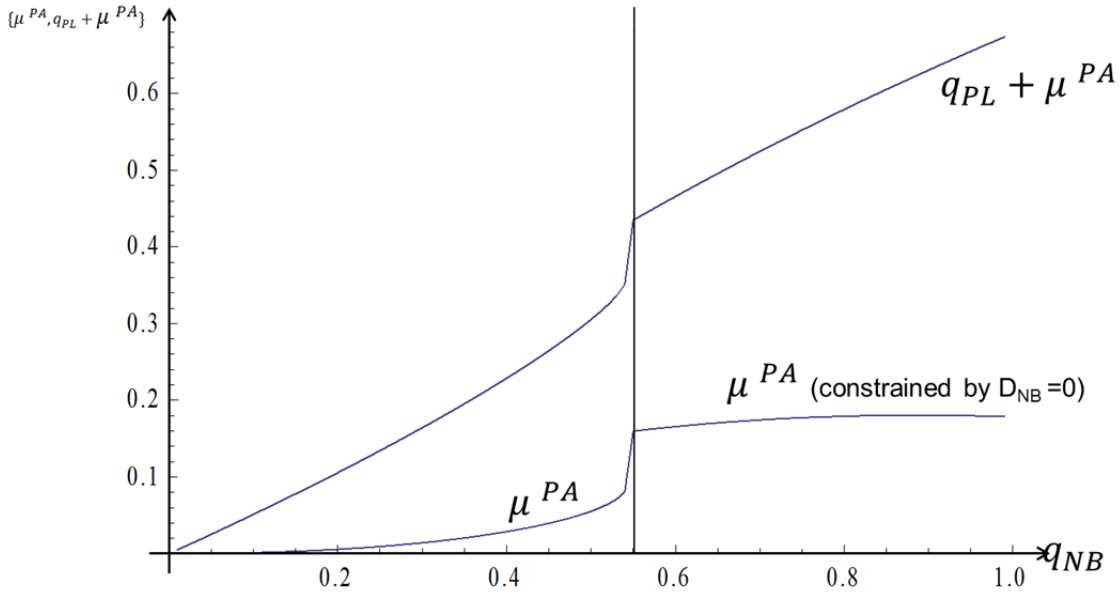

Figure 3: Optimal PA intensity with respect to the NB product.

As q_{NB} increases, retailer R_1 also increases its store brand advertising μ^{PA}, resulting into too little differentiation between R_1's products, possibly leading to no NB demand at the end (when $q_{NB} > 0.55$).

2.4 Equilibrium Advertising Strategy

In this section, R_1 was challenged by a retailer selling a higher quality product. R_1 is thus concerned not only about market coverage (and the null demand) but also about competition with R_2. Total R_1 demand will thus depend not only on $\theta_{G2/PL}$ but also on $\theta_{0/PL}$.

Irrespective of the advertising strategy, R_1's demand rises while R_2's demand decreases. Additionally, market coverage increases due to the pro-competitive effect of publicity which increases the value for money.

Advertising is always a profitable strategy for R_1 as shown in Table 2.[10]

Proposition 1: *R_1 always chooses the Store Advertising strategy. For a sufficiently high quality level of the NB product, R_1 can force R_2 to leave the market by increasing its store advertising intensity.*

Proof: *Table 3 shows that $\Pi_1^{SA} > \Pi_1^{PA} > \Pi_1^{B}$, and for $q_{NB} > 0.42$, $\Pi_1^{SAm} > \Pi_1^{SA}$ while $\Pi_2^{SA} = 0$.*

Qualities		Benchmark		Store Advertising				Product Ad.	
				Duopoly (SA)		Monopoly (SAm)			
q_{NB}	q_{PL}	Π_1^B	Π_2^B	μ^{SA}	Π_1^{SA}	μ^{SAm}	Π_1^{SAm}	μ^{PA}	Π_1^{PA}
0.05	0.025	0.007	0.058	0.153	0.018	0.447	-0.041	0.000	0.007
0.1	0.05	0.012	0.053	0.158	0.024	0.424	-0.021	0.001	0.012
0.15	0.075	0.018	0.049	0.163	0.029	0.403	-0.005	0.003	0.018
0.2	0.1	0.022	0.045	0.169	0.034	0.384	0.010	0.006	0.022
0.25	0.125	0.025	0.041	0.176	0.038	0.368	0.022	0.009	0.025
0.3	0.15	0.028	0.037	0.187	0.042	0.354	0.033	0.014	0.028
0.35	0.175	0.031	0.033	0.202	0.045	0.341	0.041	0.021	0.031
0.4	0.2	0.032	0.030	0.245	0.049	0.330	0.048	0.029	0.033
0.45	0.225	0.033	0.026	0.284	0.053	0.320	0.054	0.040	0.034
0.5	0.25	0.034	0.023	0.250	0.056	0.311	0.059	0.055	0.035
0.55	0.275	0.034	0.020	0.218	0.057	0.303	0.062	0.160	0.037
0.6	0.3	0.033	0.017	0.187	0.056	0.296	0.064	0.166	0.038
0.65	0.325	0.032	0.014	0.158	0.054	0.289	0.065	0.171	0.038
0.7	0.35	0.031	0.012	0.130	0.051	0.283	0.065	0.174	0.039
0.75	0.375	0.029	0.009	0.104	0.046	0.277	0.065	0.177	0.039
0.8	0.4	0.027	0.007	0.080	0.041	0.271	0.063	0.179	0.039
0.85	0.425	0.024	0.005	0.058	0.035	0.265	0.061	0.180	0.039
0.9	0.45	0.022	0.003	0.037	0.029	0.259	0.059	0.181	0.038
0.95	0.475	0.019	0.001	0.018	0.022	0.253	0.056	0.180	0.037
0.98	0.49	0.017	0.001	0.007	0.018	0.250	0.054	0.179	0.037

Table 2: Advertising Equilibrium in the CS case (grey zone is not relevant at equilibrium).

[10] The figures in Table 3 result from numerical resolution.

The argument of SA *vs* PA choice is that the PA strategy reduces NB demand whereas it is the product on which R_1 makes the higher unit margin. Note that as in Mills (1995) or Bontems et al. (1999), the unit margin on the NB is higher than on the PL, but this is reversed when considering relative margins. This result is validated by empirical studies on the competition between NB and PL, such as those by Dhar and Hoch (1997), Ward et al. (2002), Chintagunta (2002) or Ailawadi and Harlam (2004).

Additionally, by choosing the SA strategy, R_1 can increase its profits by becoming a monopoly on the market as long as the NB quality is high enough. In this case, the excess of store advertising generates an anticompetitive outcome by decreasing the number of firms on the market, namely by making the commodity store disappear.

Proposition 2: *An increase in store advertising (SA) induces an increase in both of R_1's products prices (p_{NB} and p_{PL}), while R_2 must lower its price (p_{G2}) due to stronger competition.*

Proof:

$$\frac{dp_{PL}}{d\mu^{SA}} = 1 + \frac{(4+q_{PL}-q_{NB})(-10+{q_{NB}}^2)}{4(-4+q_{NB}+\mu^{SA})^2} > 0 \text{ (found with numerical resolution)}$$

$$\frac{dp_{NB}}{d\mu^{SA}} = 1 + \frac{-10+{q_{NB}}^2}{(-4+q_{NB}+\mu^{SA})^2} > 0 \text{ (found with numerical resolution)}$$

$$\frac{dp_{G2}}{d\,\mu^{SA}} = \frac{-10+{q_{NB}}^2}{2(-4+q_{NB}+\mu^{SA})^2} < 0$$

The increase in store advertising intensity results in an increased willingness-to-pay for the PL and the NB, translating into higher prices to finance the advertising campaign since the PL-NB quality differential does not change. However, R_2's price decreases in order to limit the decrease in demand due to a higher competitive supply from R_1.

The second structural possibility is that R_1 may be facing a hard-discounter with lower perceived quality. We thus analyze the same advertising strategies in this new set-up.

3. Competition against a Hard-Discounter

In this situation, retailer R_1 sells a high quality product (NB) and a private label. The competitor he faces is a HD who sells a low quality product G2: $q_{G2} < q_{PL} < q_{NB} = 1$. Solgaard and Hansen (2003) found that consumers' perception of HD was lower than that of mass-retailers, translating into a lower willingness-to-pay.[11] We first define the benchmark equilibrium (choice of PL quality), characterized by the fact that retailer R_1 does not advertise. We then look for the optimal advertising strategies and derive the equilibrium chosen by R_1.

3.1 Benchmark

We first compute demands by characterizing the indifferent marginal consumers. We denote $\theta_{0/G2} = \frac{p_{G2}}{q_{G2}}$ the marginal consumer who is indifferent between buying the hard-discount good G2 in R_2 or nothing, $\theta_{G2/PL} = \frac{p_{PL} - p_2}{q_{PL} - q_2}$ the marginal consumer who is indifferent between buying the hard-discount good G2 or the PL product in R_1 and $\theta_{PL/NB} = \frac{p_{NB} - p_{PL}}{1 - q_{PL}}$ the marginal consumer who is indifferent between buying the NB product or the PL product in R_1.[12] Then, demands for products are defined by:

$$D_{NB} = 1 - \frac{p_{NB} - p_{PL}}{1 - q_{PL}};\ D_{PL} = \frac{p_{NB} - p_{PL}}{1 - q_{PL}} - \frac{p_{PL} - p_{G2}}{q_{PL} - q_{G2}};\ D_2 = \frac{p_{PL} - p_{G2}}{q_{PL} - q_{G2}} - \frac{p_{G2}}{q_{G2}} \text{ and } D_0 = \frac{p_{G2}}{q_{G2}}$$

[11] They estimated a multinomial discrete choice Bayesian logit using data from a survey of grocery shopping in the greater Copenhagen metropolitan area (Denmark).
[12] Recall that $q_{NB} = 1$.

These definitions are valid only if: $\theta_{0/G2} < \theta_{G2/PL} < \theta_{PL/NB}$. It translates into conditions on prices that have to be checked at the equilibrium.

Retailers' profits are given by:

$$\Pi_1^B = D_{NB}.\left(p_{NB} - \frac{1}{2}\right) + D_{PL}.(p_{PL} - \frac{q_{PL}^2}{2}) \text{ and } \Pi_2^B = D_{G2}.(p_{G2} - \frac{q_{G2}^2}{2})$$

Using backward induction, that is, maximizing π_1 with respect to (p_{NB}, p_{PL}) while maximizing π_2 with respect to p_{G2} leads to:

$$\mathrm{p_{G2}} = -\frac{3}{-4+\mathrm{q1}} - \frac{\mathrm{q1}}{2} \;;\; \mathrm{p_{NB}} = \frac{3\mathrm{q1}}{8-2\mathrm{q1}} \text{ and } \mathrm{p_{PL}} = \frac{1}{4}\mathrm{q}(\mathrm{q} - \frac{6}{-4+\mathrm{q1}} - \mathrm{q1})$$

Resulting in:

$$\pi_1^{B*}(q_{PL}, \mathrm{q_{G2}}) = \frac{8{q_{PL}}^4(-2+\mathrm{q_{G2}}) + {\mathrm{q_{G2}}}^2 - q_{PL}\mathrm{q_{G2}}(8+3\mathrm{q_{G2}}) + {q_{PL}}^3(16+(8-13\mathrm{q_{G2}})\mathrm{q_{G2}}) + {q_{PL}}^2(16+\mathrm{q_{G2}}(-40+(35-4\mathrm{q_{G2}})\mathrm{q_{G2}}))}{16(-4q_{PL}+\mathrm{q_{G2}})^2}$$

and $\pi_2^{B*}(q_{PL}, \mathrm{q_{G2}}) = \frac{q_{PL}(q_{PL}-\mathrm{q_{G2}})(2+q_{PL}-\mathrm{q_{G2}})^2\mathrm{q_{G2}}}{4(-4q_{PL}+\mathrm{q_{G2}})^2}$

Retailer R_1 maximizes his profit with respect to q_{PL}. Because analytical expressions cannot easily derive the optimal q_{PL}^*, we use numerical resolution imposing $\mathrm{q_{PL}} < 1$. Figure 4 represents the optimal PL quality with respect to the quality of the HD since the quality of the NB is set to 1.

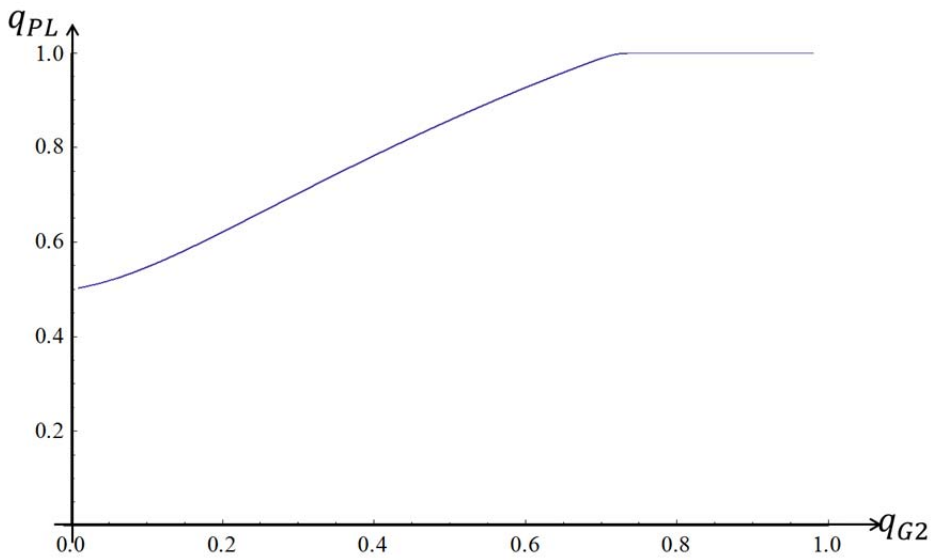

Figure 4: Optimal PL quality with respect to the HD product.

Due to the classical competition effect, it is quite logical that PL quality increases with competitor's product characteristics, note however that product differentiation decreases as long as q_{G2} rises. For a high quality of the HD product ($q_{G2} > \frac{3}{4}$), the choice of optimal PL characteristics leads to too little differentiation between NB and PL, and given that the NB price is higher than the PL price, the national brand demand becomes nil. Such a region is not relevant for our analysis.

3.2 Store Advertising

Store advertising impacts directly on the marginal consumer who is indifferent between buying the hard-discount good G2 or the PL product. The characterization of this consumer is now: $\theta_{G2/PL} = \frac{p_{PL} - p_2}{q_{PL} + \mu - q_2}$. The other marginal consumers are only affected by the price effect, regardless of the choice between NB and PL (since both products are sold in R1), or between G2 or nothing (quality is not concerned by the advertising). Demands for products and profits for firms are now defined by:

$$D_{NB} = 1 - \frac{p_{NB} - p_{PL}}{1 - q_{PL}} \; ; \; D_{PL} = \frac{p_{NB} - p_{PL}}{1 - q_{PL}} - \frac{p_{PL} - p_{G2}}{q_{PL} + \mu^{SA} - q_{G2}} \; ; \; D_{G2} = \frac{p_{PL} - p_{G2}}{q_{PL} + \mu^{SA} - q_{G2}} - \frac{p_{G2}}{q_{G2}} ; \; D_0 = \frac{p_{G2}}{q_{G2}}$$

$$\Pi_1^{SA} = D_{NB} \cdot (p_{NB} - \tfrac{1}{2}) + D_{PL} \cdot (p_{PL} - \tfrac{q_{PL}^2}{2}) - c(\mu^{SA}), \text{ with } c(\mu^{SA}) = \tfrac{(\mu^{SA})^2}{2} \text{ and } \Pi_2^{SA} = D_{G2} \cdot (p_{G2} - \tfrac{q_{G2}^2}{2})$$

Using backward induction, that is, maximizing π_1 with respect to (p_{NB}, p_{PL}) while maximizing π_2 with respect to p_{G2} leads to $\pi_1^{SA*}(q_{PL}, q_{G2}, \mu^{SA})$ and $\pi_2^{SA*}(q_{PL}, q_{G2}, \mu^{SA})$, analytical expressions are given in Appendix B.1. Incorporating the optimal PL quality from the benchmark and maximizing Π_1^{SA} with respect to μ^{SA} leads to the optimal SA advertising intensity depicted in Figure 5.

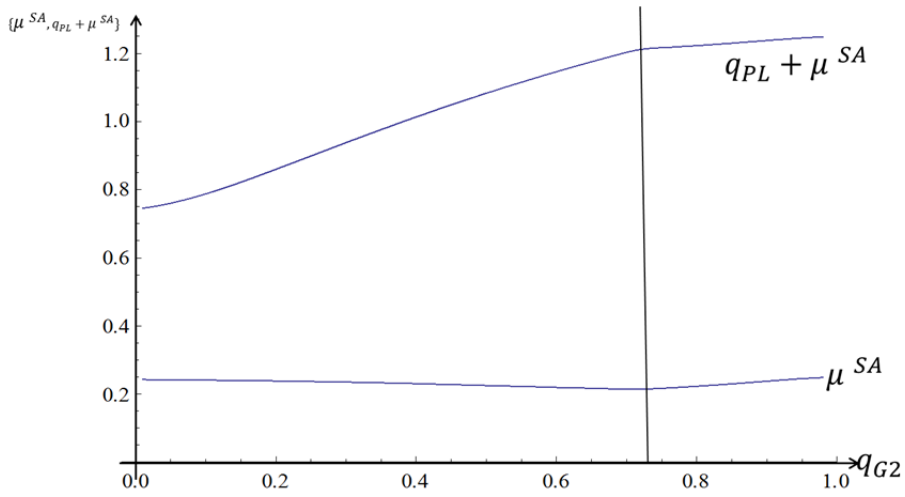

Figure 5: Optimal SA intensity with respect to the HD product.

We observe a small decrease in μ^{SA} as q_{G2} increases due to a lower product differentiation between G2 and PL, leading to lower prices and thus lower revenues. Indeed, the trade-off on advertising consisting in restricting competition on prices through an increase in μ^{SA}, and financing the cost of a higher advertising investment results in a lower μ^{SA}. In addition, an increase in advertising intensity would translate into more PL quantity sold to the detriment of the NB good, whereas the retailer's margin is higher on the NB product. Both effects concordantly results in fewer rents to finance store advertising.

3.3 Product Advertising

Product advertising reinforces consumers' confidence in the quality of the PL. We assume that the intensity of advertising is such that the perceived PL quality remains lower than the NB, translating into $q_{PL} + \mu^{PA} < q_{NB} = 1$. Product advertising directly impacts on two marginal consumers: the consumer who is indifferent between buying the hard-discount good G2 or the PL product, and the consumer who is indifferent between buying the PL product or the NB. The new characterizations of these consumers are: $\theta_{G2/PL} = \frac{p_{PL} - p_{G2}}{(q_{PL}+\mu^{PA}) - q_{G2}}$ and

$\theta_{PL/NB} = \frac{p_{NB} - p_{PL}}{1-(q_{PL}+\mu^{PA})}$. This kind of advertising has no direct effect on the choice between the PL product and the NB sold at R_1. Demands for each good become:

$$D_{NB} = 1 - \frac{p_{NB} - p_{PL}}{1-(q_{PL}+\mu^{PA})} \; ; \; D_{PL} = \frac{p_{NB} - p_{PL}}{1-(q_{PL}+\mu^{PA})} - \frac{p_{PL} - p_{G2}}{(q_{PL}+\mu^{PA}) - q_{G2}} \; ; \; D_2 = \frac{p_{PL} - p_{G2}}{(q_{PL}+\mu^{PA}) - q_{G2}} - \frac{p_{G2}}{q_{G2}} ; \; D_0 = \frac{p_{G2}}{q_{G2}}$$

Retailer R_1 chooses the amount of advertisement μ^{PA} maximizing his profits: $\Pi_1^{PA} = D_{NB}\,(p_{NB} - \frac{q_{NB}^2}{2}) + D_{PL}\,(p_{PL} - \frac{q_{PL}^2}{2})D_{PL} - c(\mu^{PA})$ while R_2's profit is given by: $\Pi_2^{PA} = D_{G2}(p_{G2} - \frac{q_2^2}{2})$. Figure 6 depicts the equilibrium value of the PA equilibrium (see Appendix B.2).

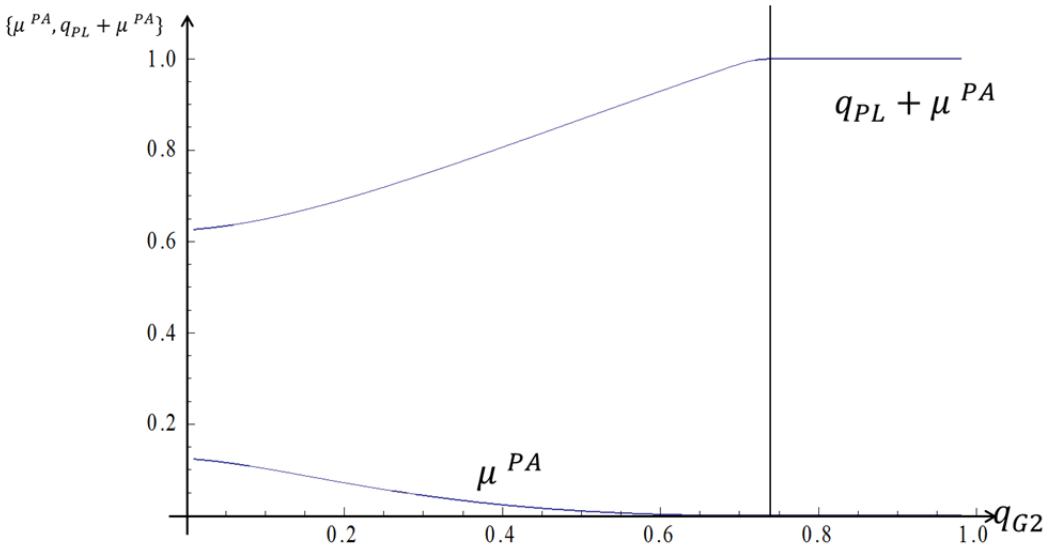

Figure 6: Optimal PA intensity with respect to the HD product.

Like in the previous equilibrium, as q_{G2} increases, we observe a small decrease in μ^{PA} due to the same product differentiation effect. There exists however a threshold such that differentiation between goods becomes so small that revenues cannot finance a positive PA investment.

3.4 Equilibrium Advertising Strategy

R_1 is a retailer faced with a HD competitor, implying that by selling a NB and a PL, he is the retailer selling the "good quality" range (devoted to high-willingness-to-pay consumers). The main competition issue between R_1 and the HD will thus concern the position of the marginal consumer $\theta_{G2/PL}$ defining R_1's total demand. Note that for a given q_{G2}, $\mu^{PA} < \mu^{SA}$ because PA lowers differentiation between the NB and PL, thereby increasing interbrand competition and thus resulting in lower rents from the NB. Also, regardless of the advertising strategy, R_1's demand rises while R_2's demand decreases. However, market coverage increases due to the pro-competitive effect of publicity, lowering R_2 price.

Advertising is always a profitable strategy for R_1 as shown in Table 3.[13]

Proposition 3: *R_1 chooses the Store Advertising strategy irrespective of PL quality.*

Proof: *Table 3 shows that $\Pi_1^{SA} > \Pi_1^{PA} > \Pi_1^{B}$.*

*S*uch a choice allows R_1, on the one hand, to keep constant the differentiation between products within his store (as in the benchmark) and on the other hand to differentiate his PL from the HD product more. Unlike store advertising, the product advertising strategy would cannibalize retailer R_1's NB sales, which are those on which he makes the highest margin. Karray and Martín-Herrán (2009) found a similar result in a different context: when the NB manufacturer can also advertise his product and when PL advertising generates a positive externality on the competing product, the retailer should limit his PL advertising investment in order to restrict an increase in store competition between NB and PL.

[13] The figures in Table 2 result from numerical resolution.

Additionally, demand for the NB is not affected by the store advertising strategy with respect to the benchmark. This is the result of R_1's internalization of the publicity impact on the NB/PL marginal consumer, $\theta_{PL/NB} = \frac{p_{NB} - p_{PL}}{1 - q_{PL}}$, translated by a constant price differential between both goods since the quality gap does not change by definition of the SA.

Qualities		Benchmark	Store Advertising		Product Advertising	
q_{G2}	q_{PL}	Π_1^B	μ^{SA}	Π_1^{SA}	μ^{PA}	Π_1^{PA}
0.05	0.518	0.072	0.242	0.101	0.116	0.086
0.1	0.547	0.066	0.241	0.094	0.103	0.079
0.15	0.582	0.060	0.240	0.088	0.087	0.072
0.2	0.621	0.054	0.239	0.081	0.072	0.065
0.25	0.662	0.049	0.237	0.075	0.057	0.057
0.3	0.703	0.043	0.235	0.069	0.044	0.050
0.35	0.743	0.038	0.233	0.063	0.033	0.043
0.4	0.783	0.033	0.231	0.057	0.024	0.037
0.45	0.821	0.029	0.228	0.052	0.016	0.031
0.5	0.858	0.025	0.225	0.047	0.010	0.026
0.55	0.893	0.021	0.222	0.042	0.006	0.022
0.6	0.927	0.017	0.220	0.038	0.003	0.018
0.65	0.959	0.014	0.217	0.034	0.001	0.014
0.7	0.989	0.012	0.215	0.030	0	0.012
0.75	1	0.009	0.217	0.028	0	0.009
0.8	1	0.007	0.223	0.026	0	0.007
0.85	1	0.005	0.230	0.026	0	0.005
0.9	1	0.003	0.238	0.026	0	0.003
0.95	1	0.001	0.245	0.028	0	0.001
0.98	1	0.001	0.249	0.030	0	0.001

Table 3: Advertising Equilibrium in the HD case.

As mentioned earlier, when q_{G2} is high enough (greater than 0.7), the optimal q_{PL} systematically leads to no differentiation between qualities of the NB and the private label, which is an irrelevant situation.

Proposition 4: *An increase in store advertising (SA) induces an increase in both of R_1's product prices (p_{NB} and p_{PL}), while R_2 must lower his price (p_{G2}) due to stronger competition.*

Proof:

$$\frac{dp_{PL}}{d\mu^{SA}} = \frac{dp_{NB}}{d\mu^{SA}} = \frac{1}{2} - \frac{q_{G2}(2{q_{PL}}^2 + (-3 + q_{G2})q_{G2})}{2(-4q_{PL} + q_{G2} - 4\,\mu^{SA})^2}$$

$$= \frac{8q_{PL}(q_{PL} - q_{G2}) + 2{q_{PL}}^2(1 - q_{G2}) + 6{q_{PL}}^2 + 3{q_{G2}}^2 + {q_{G2}}^2(1 - q_{G2}) + 8\mu^{SA}(q_{PL} - q_{G2}) + 24q_{PL}\mu^{SA} + 16{\mu^{SA}}^2}{2(-4q_{PL} + q_{G2} - 4\,\mu^{SA})^2} > 0$$

$\frac{\mathrm{dp_{G2}}}{\mathrm{d}\,\mu^{\mathrm{SA}}} = -\frac{\mathrm{q_{G2}}(2{\mathrm{q_{PL}}}^2+(-3+\mathrm{q_{G2}})\mathrm{q_{G2}})}{(-4\mathrm{q_{PL}}+\mathrm{q_{G2}}-4\,\mu^{\mathrm{SA}})^2} < 0$ (found with numerical resolution)

The increase in store advertising intensity results in a quality improvement of the PL and the NB, translating into higher prices to finance the advertising campaign since the PL-NB quality differential does not change. However, R_2's price decreases in order to limit the decrease in demand due to a higher competitive supply from R_1.

4. Concluding Remarks

First, regardless of the nature of the competitor store format (CS or HD), the mass-retailer always prefers to advertise his store image rather than his own-brand products. This is due to the fact that, as usual in this kind of framework, the unit margin on NB is higher than that on the PL.[14] Therefore, the retailer has no incentive to increase his PL demand (partially to the detriment of NB demand) by choosing a product advertising campaign. Whereas private label advertising is observed on mass-media (TV, radio, press, etc.), it is never chosen by R_1 in our model. This stems from the fact that, in our model, we only focus on "pure" product advertising strategy. Actually, product strategy may be wider in the sense that the retailer's image may also be enhanced, therefore benefiting also other products sold to a lesser extent.

[14] In the vertical differentiated quality model, the high-quality good exhibits a higher unit margin than the low-quality one since the structure of the cost is common to both products.

Second, some considerations about social welfare have to be made. Irrespective of the nature of competition that R_1 faces (CS or HD), consumers' surplus as well as social welfare are higher with 'store strategy' advertising than in the benchmark. The reason is that advertising, in our model, is always utility-improving. The market power R_1 gains from more product differentiation, translating into higher final prices, is always overridden by an increase in the value for money. Furthermore, market coverage always increases because either R_2's price falls (HD case) or the value for money of the PL increases (in the CS case). Only retailer R_2 is worse-off when R_1 implements a SA strategy. Moreover, R_1 always benefits from advertising whatever the strategy implemented. Also, when the retailer increases his store advertising investment, it results in an increase in both NB and PL prices. This effect is also present in Karray and Martín-Herrán (2009) but with severe limitations due to their specification of consumer preferences towards advertisement and horizontal differentiated goods. In our model this finding is independent of the magnitude of differentiation between goods and also integrates the endogeneity of PL quality by the retailer.

Third, from a more general perspective, we showed that allowing retailers to mass-advertise may result in the exclusion of commodity stores (representing an anti-competitive outcome) but not the exclusion of the hard-discounter. The intuition is that advertising may be a way to improve perceived quality and thus may reduce the opportunity cost generated by a limit pricing policy to exclude the rival, making the exclusion strategy profitable. It is worth bearing in mind that this strategy arises when the quality of the national brand and the private label is high enough, or in other words, when the commodity store does not possess enough of a specific advantage compared to the mass retailer (transportation costs are low making location not so important in consumers' preferences). In addition, the fact that the retailer is

not able to deter HD stores relies on the range of product offered, where the HD has the low-quality good and thus does not face competition for the low-WTP consumers. In France, we have observed a decline in commodity stores (from 30% in 1980 down to 4% in 2009) while hard-discount retailers have gained more than 11% of market share over this same period.[15] A partial explanation may be that the increase in PL quality (reducing the quality gap with national brands in commodity stores) has made advertising strategies by mass-retailers more harmful towards commodity stores, accelerating their demise.

Some extensions of the model should be considered in order to get a better picture of the economic mechanisms at play in advertising strategies. For instance, another possibility is to take into account that the majority of NB manufacturers use advertising and this may impact not only on R_1 but also on R_2's sales. Another issue concerns vertical relationships since the advertising strategies of the retailer may differ from the one the manufacturer would choose, which mainly aims to increase NB demand. This requires consideration of a more complex framework to take into account different objectives between manufacturers and retailers regarding advertising strategies, but also to include intermediate prices so that the manufacturer may influence retailers' choices.

[15] This trend is general in Europe as described by Colla (2004).

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Appendix A

(Competition with commodity stores)

A.1 Equilibrium in prices and profits with the SA strategy:

$$p_{G2}^{SA}(q_{PL}, q_{NB}) = 2 - \frac{-10 + {q_{NB}}^2}{2(-4 + q_{NB} + \mu)}$$

$$p_{NB}^{SA}(q_{PL}, q_{NB}) = \frac{\mu(-3 + 2\mu) + q_{NB}(-3 + 4\mu)}{2(-4 + q_{NB} + \mu)}$$

$$p_{PL}^{SA}(q_{PL}, q_{NB}) = \frac{(q_{PL}(-6 + q_{PL}(-4 + q_{NB}) - (-4 + q_{NB})q_{NB}) - 6\mu + (4 + q_{PL} - q_{NB})(q_{PL} + q_{NB})\mu + 4\mu^2)}{(4(-4 + q_{NB} + \mu))}$$

$$\pi_1^{SA}(q_{PL}, q_{NB}) = \frac{1}{144}\left(-9(16 + (q_{PL} - q_{NB})(q_{PL} + {q_{NB}}^2) + \frac{9{q_{PL}}^4}{q_{PL} + \mu} - \frac{12(-10 + {q_{NB}}^2)^2}{(-4 + q_{NB} + \mu)^2} - \frac{(-10 + {q_{NB}}^2)(-86 + 5{q_{NB}}^2)}{-4 + q_{NB} + \mu} - \frac{4(-1 + {q_{NB}}^2)^2}{-1 + q_{NB} + \mu}\right)$$

$$\pi_2^{SA}(q_{PL}, q_{NB}) = -\frac{(2 + (-3 + q_{NB})q_{NB} - 3\mu)^2}{4(-4 + q_{NB} + \mu)^2(-1 + q_{NB} + \mu)}$$

A.2 Equilibrium profits with the SAm strategy when R_1 is a monopoly:

$$\pi_1^{SAm}(q_{PL}, q_{NB}) = \frac{1}{64}(-4(16 + (q_{PL} - q_{NB})(q_{PL} + {q_{NB}}^2) + \frac{4{q_{PL}}^4}{q_{PL} + \mu} - \frac{4(-10 + {q_{NB}}^2)^2}{(-4 + q_{NB} + \mu)^2} + \frac{-380 + 68{q_{NB}}^2 - 3{q_{NB}}^4}{-4 + q_{NB} + \mu} - \frac{(2 + {q_{NB}}^2)^2}{q_{NB} + \mu})$$

$$\pi_2^{SAm}(q_{PL}, q_{NB}) = 0$$

A.3 Equilibrium in prices and profits with the PA strategy:

$$p_{G2}^{PA}(q_{PL}, q_{NB}) = -\frac{3}{-4 + q1} - \frac{q1}{2}$$

$$p_{NB}^{PA}(q_{PL}, q_{NB}) = \frac{3q1}{8 - 2q1}$$

$$p_{PL}^{PA}(q_{PL}, q_{NB}) = \frac{q_{PL}(-6 + q_{PL}(-4 + q_{NB}) - (-4 + q_{NB})q_{NB}) - (6 + (-4 + q_{NB})q_{NB})\mu}{4(-4 + q_{NB})}$$

$$\pi_1^{PA}(q_{PL}, q_{NB}) = \frac{1}{16}\left(\frac{q_{NB}(36 - q_{NB}(60 + (-6 + q_{NB})q_{NB}(2 + q_{NB})))}{(-4 + q_{NB})^2} + \frac{{q_{PL}}^4}{q_{PL} + \mu} - \frac{({q_{PL}}^2 - {q_{NB}}^2)^2}{q_{PL} - q_{NB} + \mu}\right)$$

$$\pi_2^{PA}(q_{PL}, q_{NB}) = -\frac{(-2 + q_{NB})^2(-1 + q_{NB})}{4(-4 + q_{NB})^2}$$

Appendix B

(Competition with hard discounters)

B.1 Equilibrium in prices and profits with the SA strategy:

$$p_{NB}^{SA}(q_{PL}^{*}q_{PL}, q_{G2}) = \frac{1}{8}\left(6 - 3q_{G2} + {q_{G2}}^{2} + 4\mu + \frac{q_{G2}(2{q_{PL}}^{2} + (-3 + q_{G2})q_{G2})}{4q_{PL} - q_{G2} + 4\mu}\right)$$

$$p_{PL}^{SA}(q_{PL}, q_{G2}) = \frac{(q_{PL} + \mu)(2q_{PL}(2 + q_{PL}) + (-4 + q_{G2})q_{G2} + 4\mu)}{8q_{PL} - 2q_{G2} + 8\mu}$$

$$p_{G2}^{SA}(q_{PL}, q_{G2}) = \frac{q_{G2}({q_{PL}}^{2} + 2q_{PL}(1 + q_{G2}) + 2(q_{G2}(-1 + \mu) + \mu))}{8q_{PL} - 2q_{G2} + 8\mu}$$

$$\pi_1^{SA}(q_{PL}, q_{G2}) = \frac{1}{144}(-9q_{PL}(-1 + q_{PL} + {q_{PL}}^{2}) - \frac{3q_{G2}(2{q_{PL}}^{2} + (-3 + q_{G2})q_{G2})^{2}}{(-4q_{PL} + q_{G2} - 4\mu)^{2}} + \frac{4({q_{PL}}^{2} - {q_{G2}}^{2})^{2}}{q_{PL} - q_{G2} + \mu}$$
$$+ \frac{(10{q_{PL}}^{2} + (15 - 7q_{G2})q_{G2})(2{q_{PL}}^{2} + (-3 + q_{G2})q_{G2})}{4q_{PL} - q_{G2} + 4\mu} + 9(1 + 2(-1 + q_{G2})q_{G2} + 4\mu))$$

$$\pi_2^{SA}(q_{PL}, q_{G2}) = \frac{q_{G2}(\mu + q_{PL})(2\mu + q_{G2}^{2} + q_{PL}(2 + q_{PL}) - 2q_{G2}(1 + \mu + q_{PL}))^{2}}{4(\mu - q_{G2} + q_{PL})(q_{G2} - 4(\mu + q_{PL}))^{2}}$$

B.2 Equilibrium in prices and profits with the PA strategy:

$$\mathrm{p}_{\mathrm{NB}}^{\mathrm{PA}}(q_{PL}, q_{G2}) = \frac{1}{8}\left(6 + \frac{2\mathrm{q}_{\mathrm{G2}}(\mathrm{q}_{\mathrm{PL}}(-6 + \mathrm{q}_{\mathrm{PL}} + 2\mathrm{q}_{\mathrm{G2}}) + 2(-3 + \mathrm{q}_{\mathrm{G2}})\mu)}{4\mathrm{q}_{\mathrm{PL}} - \mathrm{q}_{\mathrm{G2}} + 4\mu}\right);$$

$$\mathrm{p}_{\mathrm{PL}}^{\mathrm{PA}}(q_{PL}, q_{G2}) = \frac{(\mathrm{q}_{\mathrm{PL}} + \mu)(2\mathrm{q}_{\mathrm{PL}}(2 + \mathrm{q}_{\mathrm{PL}}) + (-4 + \mathrm{q}_{\mathrm{G2}})\mathrm{q}_{\mathrm{G2}} + 4\mu)}{8\mathrm{q}_{\mathrm{PL}} - 2\mathrm{q}_{\mathrm{G2}} + 8\mu};$$

$$\mathrm{p}_{\mathrm{G2}}^{\mathrm{PA}}(q_{PL}, q_{G2}) = \frac{\mathrm{q}_{\mathrm{G2}}({\mathrm{q}_{\mathrm{PL}}}^{2} + 2\mathrm{q}_{\mathrm{PL}}(1 + \mathrm{q}_{\mathrm{G2}}) + 2(\mathrm{q}_{\mathrm{G2}}(-1 + \mu) + \mu))}{8\mathrm{q}_{\mathrm{PL}} - 2\mathrm{q}_{\mathrm{G2}} + 8\mu};$$

$$\pi_1^{\mathrm{PA}}(\mathrm{q}_{\mathrm{PL}}, \mathrm{q}_{\mathrm{G2}}) = \frac{1}{144}(18(-1 + \mathrm{q}_{\mathrm{G2}})\mathrm{q}_{\mathrm{G2}} - \frac{3\mathrm{q}_{\mathrm{G2}}(2{\mathrm{q}_{\mathrm{PL}}}^{2} + (-3 + \mathrm{q}_{\mathrm{G2}})\mathrm{q}_{\mathrm{G2}})^{2}}{(-4\mathrm{q}_{\mathrm{PL}} + \mathrm{q}_{\mathrm{G2}} - 4\mu)^{2}} - \frac{9(-1 + {\mathrm{q}_{\mathrm{PL}}}^{2})^{2}}{-1 + \mathrm{q}_{\mathrm{PL}} + \mu} + \frac{4({\mathrm{q}_{\mathrm{PL}}}^{2} - {\mathrm{q}_{\mathrm{G2}}}^{2})^{2}}{\mathrm{q}_{\mathrm{PL}} - \mathrm{q}_{\mathrm{G2}} + \mu}$$
$$+ \frac{(10{\mathrm{q}_{\mathrm{PL}}}^{2} + (15 - 7\mathrm{q}_{\mathrm{G2}})\mathrm{q}_{\mathrm{G2}})(2{\mathrm{q}_{\mathrm{PL}}}^{2} + (-3 + \mathrm{q}_{\mathrm{G2}})\mathrm{q}_{\mathrm{G2}})}{4\mathrm{q}_{\mathrm{PL}} - \mathrm{q}_{\mathrm{G2}} + 4\mu})$$

$$\pi_2^{\mathrm{PA}}(\mathrm{q}_{\mathrm{PL}}, \mathrm{q}_{\mathrm{G2}}) = \frac{\mathrm{q}_{\mathrm{G2}}(\mathrm{q}_{\mathrm{PL}} + \mu)({\mathrm{q}_{\mathrm{PL}}}^{2} - 2\mathrm{q}_{\mathrm{PL}}(-1 + \mathrm{q}_{\mathrm{G2}}) + {\mathrm{q}_{\mathrm{G2}}}^{2} + 2\mu - 2\mathrm{q}_{\mathrm{G2}}(1 + \mu))^{2}}{4(-4\mathrm{q}_{\mathrm{PL}} + \mathrm{q}_{\mathrm{G2}} - 4\mu)^{2}(\mathrm{q}_{\mathrm{PL}} - \mathrm{q}_{\mathrm{G2}} + \mu)}$$