Université de Toulouse

# THESE

En vue de l'obtention du

## DOCTORAT DE L'UNIVERSITÉ DE TOULOUSE

Délivré par l'Université Toulouse 1 Capitole
Discipline : Sciences Economiques

---

Présentée et soutenue par

**Mathias LAFFONT**

Le 28 janvier 2013

Titre :

# Costs and Prices in Electricity Transport

---

JURY

Anna CRETI, Professeur, Université Paris 10 Nanterre La Défense
Thomas-Olivier LEAUTIER, Professeur, Université Toulouse 1
Wilfried SAND-ZANTMAN, Professeur, Université Toulouse 1
Bert WILLEMS, Associate Professor, Tilburg University

---

Ecole doctorale : Toulouse School of Economics
Unité de recherche : GREMAQ - TSE
Directeur de Thèse : Claude CRAMPES

**L'Université n'entend ni approuver, ni désapprouver les opinions particulières du candidat.**

*A ma famille et, plus particulièrement, à ma femme.*

# Remerciements

Cette thèse représente le point final de mon parcours universitaire qui débuta à Toulouse 1 Capitole en 2002. Après quelques années parisiennes au cours desquelles j'ai rencontré de véritables amis, j'ai eu l'opportunité de regagner ma terre natale afin de réaliser mon master 2 et ma thèse. Au cours de ces années j'ai eu l'occasion d'apprendre beaucoup, de découvrir le monde de la recherche et de rencontrer de nombreuses personnes avec lesquelles j'ai toujours pris plaisir à échanger. Ce fut une expérience humaine et professionnelle très enrichissante. Ces quelques lignes ne pourront pas exprimer toute la reconnaissance que j'éprouve aujourd'hui.

Tout d'abord, je souhaite exprimer ma gratitude à Claude Crampes pour avoir accepté de diriger cette thèse et pour m'avoir donné la chance de travailler avec lui sur le chapitre 2. Je tiens à le remercier pour le soutien moral qu'il m'a apporté durant les années écoulées ainsi que pour ses conseils et suggestions qui ont toujours été précieux. Je lui suis reconnaissant pour le temps consacré à superviser mon travail et pour m'avoir éclairé sur le secteur de l'électricité. Pour tout cela, je lui suis redevable et j'espère que notre collaboration a été aussi agréable pour lui qu'elle l'a été pour moi.

Je souhaite aussi remercier ici les membres du jury: Anna Creti, Bert Willems, Thomas-Olivier Léautier et Wilfried Sand-Zantman. Anna Creti et Bert Willems m'ont fait l'honneur d'être les rapporteurs de cette thèse. Je les remercie vivement pour le temps qu'ils ont consacré à lire et commenter mes travaux.

Le chapitre 4 de la thèse a été co-écrit avec Wilfried Sand-Zantman. J'aimerais le remercier pour m'avoir donné l'opportunité de travailler avec lui. La contribution de Wilfried va bien au-delà de ce chapitre: il a su me guider en master 2 quant au choix de mon directeur de thèse et m'a associé à l'un de ses cours. Je tiens également à présenter mes remerciements à Alban Thomas pour ses explications concernant la distribution d'eau. Cet échange m'a permis de comprendre les enjeux de la distribution dans des secteurs différents.

Cette thèse n'aurait été possible sans l'Université Toulouse 1 Capitole qui m'a offert une allocation de recherche et un poste d'assistant de recherche et d'enseignement. L'Ecole d'Economie de Toulouse m'a aussi permis d'obtenir un poste de lecteur pour finir ma thèse. Je remercie aussi l'ensemble des membres du GREMAQ pour leur disponibilité et leur écoute. Je souhaite associer à cette thèse Michel-Benoit Bouissou, présent lors de mes premières années à l'université, qui m'a fait confiance lorsque j'étais à la recherche de travaux dirigés. Merci à Aude pour ses conseils avisés et sa gentillesse qui rendent la vie d'un doctorant de Toulouse bien plus simple et agréable. Merci aussi à Fabian pour les discussions agréables que nous avons eues au détour des couloirs.

Je souhaite remercier mes "collègues" qui ont partagé ces dernières années et quelques pauses café: Daniel, Mehtap, Julien (tu auras ta revanche au squash!), Anna, Samuele, Jiangli, Marion, Oscar, Ahmat, Kyriacos, Paulo et tous les autres. Je pense aussi à mes amis: Q., Toto, Raoul, Alec, Guigui, Bosket et Thib's qui ont partagé les années passées à Paris et qui restent, malgré l'éloignement, présents dans ma vie; Fabien(s), Brocchi, Rémy et Yoyo pour les moments partagés sur un terrain de handball et au S$^{t}$ Jé et Max avec qui on ne compte plus les années d'amitié (et les tirs sur le poteau). Merci à vous tous pour les bons moments passés et pour ceux à venir.

Ma gratitude s'adresse bien évidemment à mes parents qui m'ont toujours guidé et encouragé dans les moments importants de ma vie. Je ne les remercierai jamais assez pour ce qu'ils ont fait pour moi. Merci à ma soeur, Floriane, pour avoir su m'écouter et pour la complicité que nous avons. Merci à Arnaud pour ses petites blagues et à Agathe pour nos appels FaceTime. Je remercie aussi mon frère, Axel, pour ses connaissances en physique et pour ce qu'il représente pour moi. Courage Axel, ce n'est que le début pour toi! Je souhaite associer à ces remerciements ma famille et mes beaux-parents, Gina et Florenso pour leur gentillesse et leur soutien. Merci aussi à Karine et David (alias le roi du barbecue) pour les bons moments passés à Biarritz, Arcachon et Bordeaux... Léa et Manon, je vous remercie pour vos petites attentions.

Enfin, si je n'avais qu'une personne à remercier, ce serait ma femme, Julie. Merci pour ta confiance, ton soutien et tes encouragements pendant ces années de thèse. Tu as su être patiente avec moi, ce qui n'est pas toujours chose aisée... Je te remercie pour le bonheur que tu m'apportes jour après jour, les fous rires que nous avons et l'amour que tu me portes. Tu mérites amplement que je te dédie cette thèse.

*Mathias Laffont*
*Janvier 2013*

# Abstract

This dissertation is focused on the economics of electricity transport and consists in four independent chapters. Chapter 1 proposes a general introduction to the electricity sector. In particular, we present the distribution activity and the main missions of the distributors in France and in Europe.
Chapter 2 provides an economic view on how the connection to a distribution network should be priced when the operator considers the spatial distribution of consumers. It highlights the impact of public service constraints on the investment in service quality, the size of the network and the connection fee. The main ingredient is the geographical dispersion of potential consumers in the distribution area and the costs linked to this dispersion as opposed to those common to all connected customers.
Chapter 3 addresses the problem of the management of electric thermal losses. We analyze how a specific design incentivizes the network operators to increase network efficiency. We consider a two-node/one-line model to show the importance of thermal losses in the merit order. We then compare two types of management implemented in Europe and compare their impact on the optimal level of consumption and investment.
Chapter 4 analyzes the impact of feed-in tariffs in an open economy model and studies the consequences of transmission constraints. We consider different types of energy sources, renewable and non-renewable, used in each of two countries. We assume that producing electricity thanks to renewable energies creates a positive local externality and question the relevance of a coordinated policy for the promotion of renewable energy in a world of limited connections.



**Keywords**: distribution network, capacity constraint, electricity, externality, infrastructure, policy coordination, quality, renewables, thermal losses.

# Contents

# List of Figures

# Chapter 1

# Introduction

Durant les deux dernières décennies, l'industrie électrique, comme auparavant le secteur des transports et celui des télécommunications, a connu dans de nombreux pays une série de changements conduisant à l'ouverture des marchés de gros et de détail et à la séparation de ces activités[1]. Jusqu'alors, les quatre activités du secteur (production, transport, distribution et commercialisation) étaient intégrées au sein d'une même entreprise, souvent nationalisée. L'entreprise verticalement intégrée était considérée comme un monopole naturel car elle nécessite d'importants investissements initiaux. Cependant, seules les deux activités de réseau, le transport et la distribution, sont des monopoles naturels.

Ces activités de réseaux sont considérées comme des facilités essentielles. Elles constituent un goulet d'étranglement entre la production et la commercialisation, activités concurrentielles, et les consommateurs finals. Ainsi, afin de développer de la concurrence dans les activités de production et de commercialisation, les activités de réseaux ont été séparées et régulées de manière à garantir leur transparence et un accès non-discriminant à l'ensemble des producteurs et des fournisseurs[2]. La figure 1.1 schématise l'incidence de la libéralisation sur le secteur de l'électricité.

Ce chapitre introductif propose une vue générale du secteur de l'électricité, une présentation plus détaillée de la distribution d'électricité et une synthèse des articles composant la thèse. Les deux premiers chapitres étudient les problématiques de tarification d'accès à un réseau de distribution et de gestion de couverture des pertes en réseau

[1]Le Chili fut le premier pays à privatiser le secteur électrique à la fin des années 1970. Une présentation complète de la dérégulation et libéralisation du secteur de l'électricité peut être trouvée dans les livres écrits par Gomez et Rothwell [2003] ou pour ce qui est de la réforme mise en place en Europe par Glachant et Lévêque [2009].

[2]Pour cela, le Parlement et le Conseil européen ont fixé la ligne que les 15 pays membres devaient adopter dans la directive 96/92/CE.

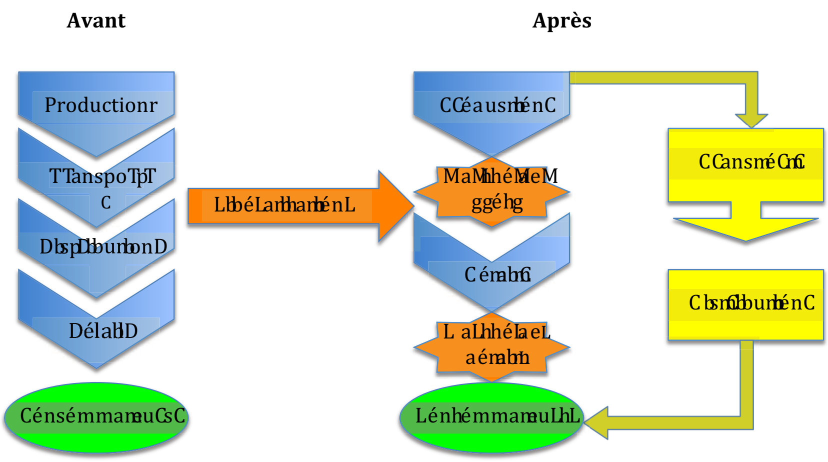



Figure 1.1: Libéralisation de l'industrie électrique

présentes dans la distribution d'électricité. Le dernier chapitre, quant à lui, aborde le principe de subsidiarité lors de la promotion des énergies renouvelables.

## 1.1 Présentation du secteur

### La demande d'électricité

L'électricité est un bien particulier car les consommateurs ne consomment pas le "produit électricité" mais les services que ce dernier procure comme la lumière, le chauffage ou le fonctionnement d'appareils domestiques et industriels. De plus, l'électricité est un bien homogène car on ne peut différencier un électron produit dans une centrale d'un autre produit dans une autre centrale ou même importé. Seule la qualité d'approvisionnement (temps moyen de coupures par an par exemple) importe pour les consommateurs. Compte tenu des services que l'électricité fournit et le manque de substitut à ce bien à équipement donné pour certains usages, la variation des prix a peu d'influence sur la demande d'électricité[3]. L'élasticité prix faible s'explique aussi par le peu de consommateurs observant réellement les variations du prix de l'électricité sur le marché. La figure 1.2 montre la part des clients encore aux tarifs réglementés en France (par type de clients).

Enfin, la quantité demandée d'électricité est caractérisée par sa forte volatilité durant le jour et l'année. Lors d'une journée, la consommation d'électricité dépend des horaires

[3]Bernstein et Griffin [2005] présentent une étude sur les différences régionales d'élasticité aux Etats-Unis. Ils montrent, qu'au niveau national, l'élasticité prix de la demande résidentielle d'électricité varie entre $-0.24$ à court terme et $-0.31$ à long terme.

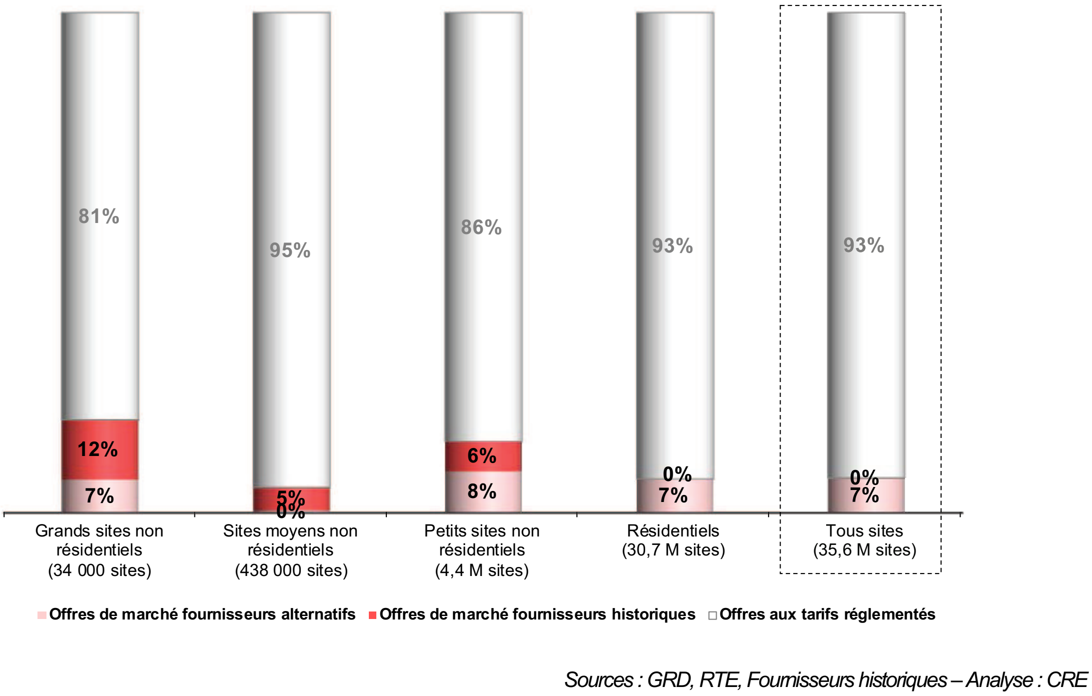



Figure 1.2: Répartition des sites par type d'offres en France au 30 septembre 2012

de travail. En hiver, en Europe, le pic de consommation se produit à 19h alors que l'été ce pic a lieu vers 13h. La sensibilité aux variations climatiques dépend du taux de pénétration des appareils électriques de chauffage ou des climatiseurs[4]. En France où le chauffage électrique est très présent, lorsque les températures diminuent de 1 °C en hiver, la consommation d'électricité augmente de 2300 $MW$[5].

### La production et la commercialisation

Cette grande variabilité de la demande nécessite un outil de production capable de s'adapter car l'électricité ne peut se stocker en grande quantité à faible coût. Il faut donc des centrales électriques de natures différentes. On distingue:

i) les centrales dites de base qui requièrent un investissement initial très important et bénéficient d'un coût marginal faible. Pour être rentables, ces centrales doivent produire le plus souvent possible. C'est le cas des centrales nucléaires;

ii) les centrales dites de pointe dont l'investissement initial est faible mais dont le coût marginal est important. Ces centrales ne produisent qu'épisodiquement mais elle sont rentables car elles sont appelées lorsque le prix de l'électricité est élevé.

[4]Bessec et Fouquau [2007] analysent l'impact de la température sur la consommation d'électricité.

[5]Le détail de ces données peut être trouvé sur le site du transporteur français d'électricité, RTE: http://clients.rte-france.com/lang/fr/visiteurs/vie/courbes_methodologie.jsp.

Afin de compenser l'impossibilité de stocker l'électricité, les producteurs stockent les énergies sources comme le charbon, le gaz ou encore les éléments radioactifs. Les Stations de Transfert d'Energie par Pompage (STEP) utilisent l'électricité lorsqu'elle est peu chère (en période de base) pour pomper de l'eau servant par la suite à produire de l'électricité lorsque la demande et le prix de l'électricité, sont élevés[6].

Enfin, le paysage de la production d'électricité change avec le développement et la promotion des productions délocalisées grâce aux énergies renouvelables. La particularité de ce type d'énergies est son intermittence et la difficulté de prédire leur occurrence. L'instabilité créée par cette intermittence renforce le besoin d'avoir des technologies de secours (Ambec et Crampes [2012]) et accroît la volatilité du prix sur le marché de gros (Green et Vasilakos [2010]).

Tout comme la production d'électricité, la commercialisation a été ouverte à la concurrence lors des deux dernières décennies. En France, depuis 2007, la totalité des consommateurs, résidentiels ou non, peuvent choisir leur fournisseur d'électricité, même si cette option reste marginale (cf. figure 1.2). Joskow et Tirole [2006] ou Green [2003] étudient différents aspects de la concurrence sur le marché de détail (importance des contrats de long terme, manque de réactivité de la demande). Alors qu'en France les compteurs font partie des attributions des distributeurs, dans certains pays comme au Royaume-Uni, les fournisseurs d'électricité sont en charge de l'installation et de la gestion des compteurs de leurs clients. De manière générale, les fournisseurs d'électricité sont très impliqués dans la gestion de la demande. Pour éviter les périodes de pointe ils peuvent proposer des contrats d'effacement de la demande lorsque celle-ci est trop importante. L'aplatissement de la courbe de charge qui en résulte permet de réduire à la fois la volatilité des prix et les problèmes de congestion sur le réseau de transport d'électricité.

**Le transport d'électricité**

Le réseau de transport d'électricité relie les producteurs aux "postes sources", points d'entrée du réseau de distribution. Certains gros consommateurs industriels sont cependant connectés au réseau de transport[7].

Cette activité est un monopole naturel (sa duplication serait plus coûteuse que les gains espérés) et est contrainte par des lois physiques telles que les lois des noeuds

[6] Crampes et Moreaux [2010] analysent l'efficience de ce type de technologie.

[7] En 2009, ils étaient 525 clients industriels connectés au réseau de transport français, représentant 15, 4% de la consommation totale. Source: `www.rte-france.com`.

et des mailles de Kirchhoff et la loi d'Ohm. Les transporteurs d'électricité utilisent principalement du courant alternatif à haute, voire très haute, tension. Ce courant facilite les changements successifs de tension et l'utilisation de très haute tension qui réduit les pertes par effet Joule. Le courant continu, quant à lui, engendre moins de pertes en ligne mais nécessite des installations plus coûteuses. Il n'est donc rentable que pour de très grandes distances ou des projets souterrains ou sous-marins comme l'Interconnexion France-Angleterre ou le projet Inelfe[8].

Le transport d'électricité est un des outils majeurs de la mise en concurrence de la production d'électricité et du développement du marché de gros. Cependant, les phénomènes de congestion[9] peuvent limiter la concurrence entre producteurs car ils isolent des zones, augmentant ainsi le pouvoir de marché des producteurs qui y sont localisés. Stoft [1997], Hogan [1997], Joskow et Tirole [2000, 2005] ou encore Borenstein *et al.* [2000] étudient l'impact stratégique que les contraintes de transport ont sur la concurrence dans la production de l'électricité.

Enfin, les réseaux de transport d'électricité sont des outils pour le développement du marché unique de l'électricité en Europe grâce aux interconnexions entre les pays de l'Union européenne.

## 1.2 La distribution d'électricité

**Présentation générale de la distribution d'électricité**

Le réseau de distribution d'électricité permet d'acheminer l'électricité prélevée au réseau de transport vers les consommateurs finals, grâce à une structure arborescente. Le courant y circulant est un courant alternatif à basse ou moyenne tension. Le réseau est constitué: i) de postes sources, situés à la frontière avec le réseau de transport; ii) de lignes à moyenne tension (HTA) et à basse tension (BT) allant jusqu'au compteur des clients raccordés et iii) de transformateurs HTA/BT. La figure 1.3 présente les données du réseau du distributeur français Electricité Réseau Distribution France (ERDF) à la fin de l'exercice 2011. La distribution d'électricité dépend fortement des caractéristiques géographiques de la région considérée. En effet, les coûts d'installation et de maintenance d'une ligne électrique sont très différents en plaine, en région montagneuse, en ville et en campagne. Pourtant pour des raisons réglementaires le service doit être identique dans tous les types de régions.

[8]Ce projet, dont une partie est enterrée, vise à augmenter la capacité d'interconnexion entre la France et l'Espagne.

[9]Une ligne électrique a une capacité de transport maximale définie par sa résistance thermique.

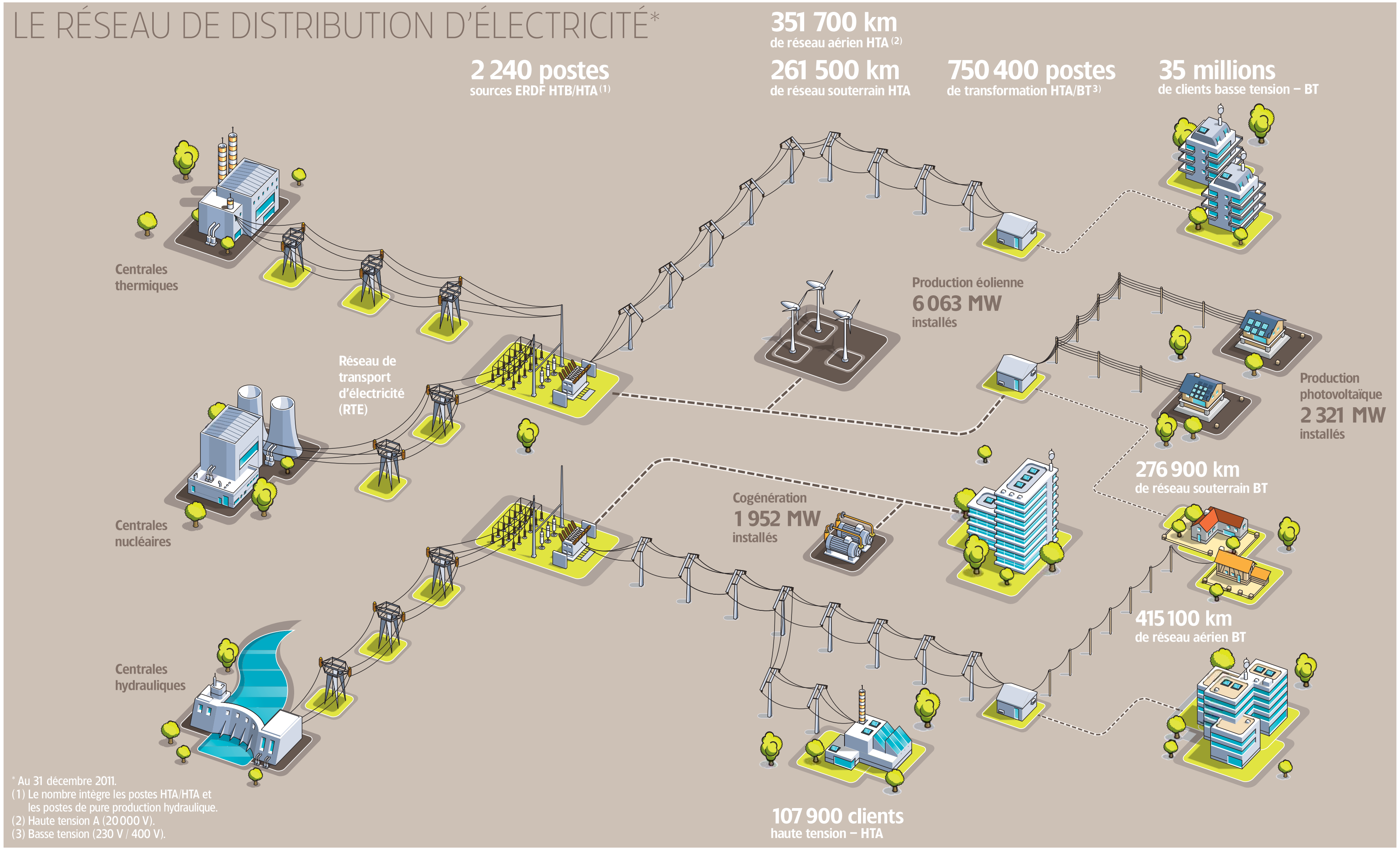



Figure 1.3: Le réseau de distribution d'ERDF (Source: Rapport d'activité 2011 d'ERDF)

Historiquement, les réseaux de distribution principalement organisés en régie communale ont été intégrés dans un monopole public de l'électricité. La nationalisation des distributeurs a eu lieu le 8 avril 1946 avec la création d'un monopole public Electricité de France (EDF). Par la suite, la volonté de créer un marché unique européen de l'énergie a entraîné plusieurs réformes impulsées notamment par les directives de la Commission européenne du 19 décembre 1996 et du 26 juin 2003. Ces directives communautaires ont été transposées dans les différents droits nationaux. En France, la loi du 10 février 2000 a imposé de séparer la distribution d'électricité des autres activités du secteur. Les missions des gestionnaires de réseau de distribution ont été définies par la loi du 9 août 2004 et inscrites dans le contrat de service public signé le 24 décembre 2005 par le gouvernement français et EDF, monopole historique français. Enfin, la loi du 7 décembre 2006 a conduit à la création le 1er janvier 2008 d'ERDF, filiale détenue à 100% par EDF. La séparation est ainsi effective sur le pan juridique, opérationnel et comptable bien qu'elle ne le soit pas sur le plan patrimonial.

La distribution d'électricité est un monopole local naturel nécessitant d'importants coûts d'installation. Il serait donc économiquement inefficace d'avoir une duplication du réseau de distribution dans une région. Bien que l'activité ne soit pas concurrentielle, l'Union européenne a démontré sa volonté de voir se développer la mise en concurrence des distributeurs pour l'acquisition des contrats de concession. Ce type de concurrence dans les services publics a notamment été étudié par Aubert *et al.* [2006] concernant le service d'eau et par Amaral *et al.* [2008] au sujet des réseaux de transports publics. Le droit français ne prévoit pas la publication et la mise en concurrence des concessions d'électricité. Cette différence entre le droit communautaire et le droit national est apparue en 2009, lorsque la ville de Paris a dû renégocier son contrat de concession d'électricité[10].

Le nombre de gestionnaires de réseau par pays est très variable en Europe. En Allemagne, plus de neuf cents entreprises, les Stadtwerke, sont en charge de la distribution d'électricité. Au Royaume-Uni, on dénombre sept distributeurs indépendants (cf. figure 1.4). En France, il existe, en plus d'ERDF, 160 entreprises locales de distribution (ELD) ayant le statut de régie ou même de société anonyme. Ces ELD ne sont en charge que de 5% de l'électricité distribuée, le reste étant de la responsabilité d'ERDF.

[10]Dans le cadre d'une délégation de service public, la loi L. 1411-12 du code général des collectivités territoriales stipule que l'autorité concédante n'est pas soumise à une procédure de publicité, et donc de mise en concurrence, lorsque la loi institue un monopole au profit d'une entreprise comme ce fut le cas pour la distribution d'électricité en France avec la loi n° 2000-108 du 10 février 2000. La ville de Paris qui était une des premières à renouveler son contrat de concession a finalement prolongé le contrat d'ERDF plutôt que de conclure un nouveau contrat.

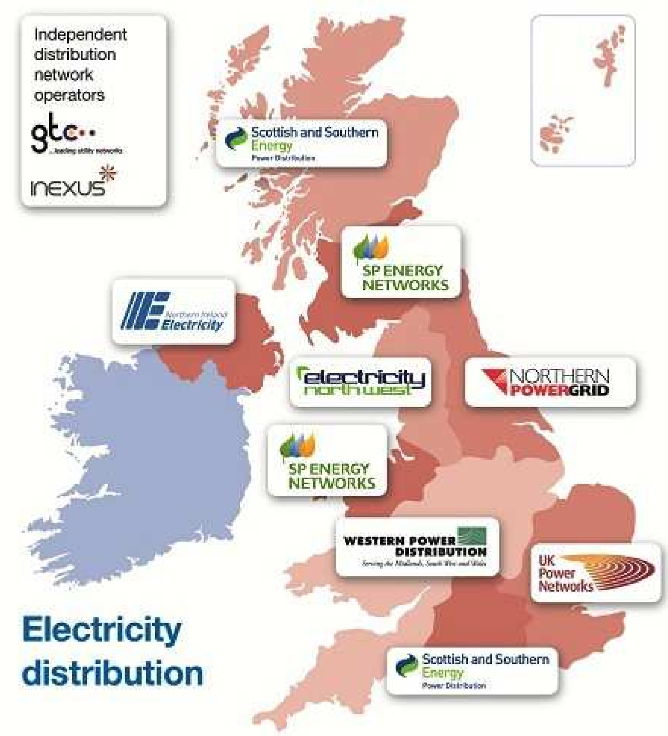



Figure 1.4: Distributeurs locaux d'électricité au Royaume-Uni (Source: National Grid)

Les gestionnaires de réseau font face à deux niveaux de régulation de leur activité. Tout d'abord, les autorités concédantes, c'est-à-dire les municipalités, détenant tout ou partie des actifs du réseau, exercent une régulation locale à travers les contrats de concessions. De plus, elles contrôlent et participent aux investissements des gestionnaires de réseau[11]. En France, les autorités concédantes peuvent se regrouper en syndicat afin d'unifier le service proposé dans les communes. Par exemple, la totalité des communes de la Haute-Garonne à l'exception de la ville de Toulouse, soit 588 sur 589 communes, se sont regroupées au sein du syndicat départemental de l'électricité de Haute-Garonne (SDEHG) afin de gérer leur relation avec ERDF. Les distributeurs sont aussi soumis à une régulation nationale de l'Etat et du régulateur. Ces deux autorités fixent le tarif permettant de couvrir les coûts de l'activité et tentent d'harmoniser les pratiques sur le territoire national. La régulation choisie en France et au Royaume-Uni est une régulation de type price-cap qui ajuste le tarif à chaque période afin de prendre en compte l'inflation et un gain de productivité du distributeur souhaité par le régulateur. La formule d'ajustement de ce tarif est donc décrite par la relation:

[11] Concernant la distribution d'eau, les contrats liant les communes et les distributeurs d'eau relèvent plus du principe de l'affermage (négociation bilatérale des tarifs) que des contrats de concession. Dans ce cadre, certains investissements sont pris totalement en charge par les communes.

$RPI - X\%$, où $RPI$ est le Retail Price Index rendant compte de l'inflation et $X$ le gain de productivité du gestionnaire de réseau. Jamasb et Politt [2003] ont analysé la manière de fixer le facteur $X$, grâce notamment aux données de référence (benchmark) observées dans des pays ou régions similaires.

En France, la troisième version du Tarif d'Utilisation des Réseaux Publics d'Electricité (TURPE) sera appliquée jusqu'au 31 juillet 2013. Le TURPE, fixé par le gouvernement sur proposition de la Commission de Régulation de l'Energie (CRE), dépend de la puissance souscrite, de la quantité d'énergie soutirée, des saisons, des jours (ouvrés ou non) et des heures de consommation. Il respecte deux principes: i) le principe du timbre-poste qui garantit un tarif indépendant de la distance parcourue par l'électricité, et ii) le principe de péréquation tarifaire, le TURPE étant identique quelle que soit la zone de distribution considérée. Cette situation est propre à la France car, bien que le principe du timbre-poste soit appliqué en Europe grâce à l'article 14 du règlement CE n°714/2009 de la Commission européenne, la péréquation tarifaire résulte du droit national (article L121-1 du Code de l'Energie). Ce principe n'est pas appliqué par exemple en Allemagne où le tarif d'acheminement est différent selon les régions. Le TURPE représente 90% des revenus d'ERDF et correspond à environ 42% de la facture hors taxe d'un consommateur résidentiel[12], la moyenne européenne étant de 43%. Enfin, ce tarif permet d'assurer et de piloter les missions des distributeurs d'électricité.

**Missions des gestionnaires de réseau de distribution d'électricité**

On peut distinguer quatre missions qu'un gestionnaire de réseau de distribution doit remplir: i) garantir un libre accès aux différents fournisseurs du marché de détail, ii) assurer l'intégration des productions délocalisées, iii) améliorer la qualité de service et la sûreté du réseau et iv) limiter les pertes en réseau.

Afin de promouvoir la concurrence entre fournisseurs, les gestionnaires de réseau doivent leur garantir un accès non-discriminatoire. Les distributeurs doivent être des acteurs neutres du marché de l'électricité. Pour garantir la transparence d'accès au réseau en France, ERDF, le gouvernement et la CRE ont signé un code de bonne conduite encadrant les activités d'ERDF. La CRE s'assure en outre que les images d'ERDF et d'EDF, monopole historique et acteur sur le marché de détail, soient bien distinctes. Il doit en être de même pour leurs locaux. Avec l'ouverture du marché de détail et l'augmentation du nombre de fournisseurs, les gestionnaires de réseau en charge du comptage collectent une quantité d'information beaucoup plus impor-

[12] A titre de comparaison, le tarif d'acheminement de l'eau en France correspond à 48% du prix final de l'eau.

tante. Ils doivent comptabiliser les injections et soutirages pour chaque fournisseur et établir la facture d'accès au réseau pour ces fournisseurs. Le comptage devient donc de plus en plus lourd et pousse les gestionnaires de réseau de distribution à développer les réseaux dits intelligents. La troisième directive concernant le marché interne européen (directive 2009/72/CE) stipule que, d'ici à 2020, 80% des compteurs devront être intelligents. Afin d'atteindre de tels objectifs, ERDF a lancé une phase de tests sur 250 000 compteurs communicants de type Linky. En septembre 2011, ce projet a été généralisé à l'ensemble des consommateurs français[13]. ERDF participe en outre à un projet européen, Grid4EU, avec cinq autres distributeurs (Čez, Enel, Iberdrola, RWE et Vattenfall), douze industriels et six universités et centres de recherches afin de partager leur expérience grâce à six projets pilotes dans les pays représentés[14]. En plus de faciliter la collecte et la circulation d'information, les réseaux intelligents permettent de mieux connaître les profils des consommateurs et d'accroître leur réactivité face aux variations du prix de l'électricité. Les gestionnaires de réseau sont donc des acteurs de la gestion de demande[15].

Enfin, le développement de compteurs communicants devrait faciliter l'intégration des sources de productions délocalisées. L'intégration de ces sources de production délocalisée a pour but de développer la part des énergies renouvelables dans le mix énergétique[16]. Les distributeurs sont donc des acteurs de la politique énergétique en Europe. Ces productions sont raccordées le plus généralement au réseau basse tension mais certaines fermes éoliennes dont la production est plus importante sont directement raccordées au réseau moyenne tension. Les distributeurs sont obligés de donner la priorité à l'injection des productions délocalisées, sans qu'il y ait un ordre de mérite des producteurs. Cela peut créer de fortes instabilités dans le réseau de distribution. Pour le seul réseau d'ERDF, il existe, au 31 décembre 2011, 232 636 sites de production raccordés dont 86 000 ont été raccordés au cours de la seule année 2011.

Historiquement, le gestionnaire de réseau devait distribuer l'électricité venant du réseau de transport jusqu'aux consommateurs finals, l'électricité circulant alors de l'amont vers l'aval. Avec le raccordement de productions délocalisées, l'électricité circule dans les deux sens dans le réseau de distribution, ce qui modifie les modes opératoires de

[13]Le coût d'un tel projet est estimé à 4,5 milliards d'euros.

[14]Les détails de ce projet sont disponibles sur le site: `http://www.grid4eu.eu`.

[15]Cependant, la rentabilité de tels projets est remise en cause notamment par Léautier [2012].

[16]Cet objectif est stipulé, en France, dans la loi du 13 juillet 2005 de programme fixant les orientations de la politique énergétique (POPE), loi qui transpose notamment dans le droit national les dispositions législatives de la Directive 2002/91/CE du 16 décembre 2002.

leurs gestionnaires. De plus, ces énergies sont intermittentes et leurs occurrences ne peuvent être anticipées de manière certaine. L'intégration de productions délocalisées dans le réseau entraîne donc une grande instabilité des flux d'électricité circulant dans le réseau et nécessite d'importants investissements afin de raccorder les producteurs, installer de nouveaux systèmes de commande et améliorer la qualité de service.

La qualité du réseau se mesure principalement par la durée annuelle moyenne de coupure par client raccordé au réseau basse tension (critère SAIDI, System average interruption duration index). Cette durée se calcule généralement hors évènements exceptionnels et hors coupures liées au réseau de transport afin de se focaliser sur la performance des distributeurs. En France, la durée moyenne de coupure s'est stabilisée après plusieurs années consécutives de hausse. En 2011, on comptait en moyenne 71 minutes de coupure sur les réseaux gérés par ERDF, soit la durée moyenne de coupure constatée en 2007 (72 minutes), alors qu'elle était de 64 minutes en 2005[17]. La principale cause mise en avant concernant la dégradation de la qualité du réseau de distribution est la baisse des investissements réalisés par ERDF dans le rajeunissement du réseau[18]. Le rapport sur la qualité de l'électricité publié par la CRE en octobre 2010 montre que les investissements réalisés par ERDF ont baissé entre 1992 et 2004. Pour arrêter le phénomène d'augmentation de la durée annuelle moyenne de coupure, la CRE a intégré dans la troisième version du TURPE en 2009 une incitation forte aux investissements[19]. Les contrats de concession permettent aussi d'inciter les distributeurs à investir d'avantage[20]. L'amélioration de la qualité du réseau passe donc par le rajeunissement du réseau et la réduction des incidents climatiques grâce à l'enfouissement des lignes. Actuellement 39% du réseau de distribution en France est enterré contre environ 75% en Allemagne. La figure 1.5 présente la situation actuelle concernant l'enfouissement des ouvrages électriques à basse tension et à moyenne tension en Europe.

Même si l'enfouissement de lignes permet de limiter les incidents météorologiques et donc d'améliorer la qualité du réseau, la performance énergétique des lignes enfouies

[17]Selon le cinquième rapport sur la qualité de l'énergie en Europe publié par le Council of European Energy Regulators (CEER), la France possède un des réseaux les plus fiables en Europe.

[18]On considère qu'au delà de 30 ans, âge moyen des lignes et transformateurs du réseau français, la fréquence des incidents s'accroît.

[19]En 2011, ERDF a investi 1352 millions d'euros afin de renforcer le réseau et d'améliorer la qualité de service. Source: Rapport d'activité ERDF 2011.

[20]La délégation de service public à un acteur privé grâce au partenariat public-privé et la manière d'inciter cet acteur privé ont été étudiées par Hart [2003], Martimort et Pouyet [2008] et Maskin et Tirole [2008].

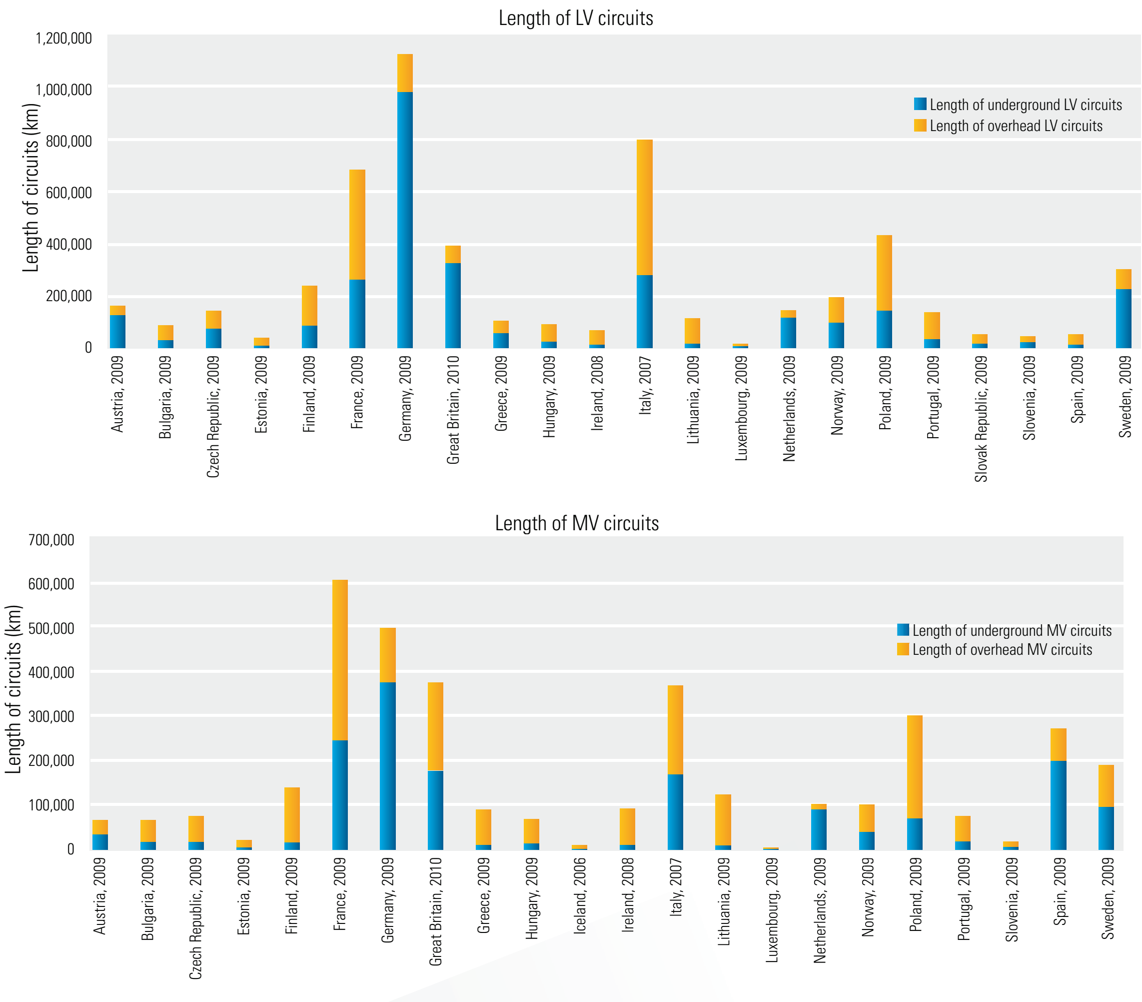



Figure 1.5: Longueur des lignes HTA et BT enfouies et aériennes (Source: CEER)

en termes de pertes par effet Joule est remise en question. La consultation publique menée par la CRE pour la quatrième version du TURPE mentionne l'impact négatif de l'enfouissement des lignes sur le niveau de pertes. Comme dans toute activité de distribution[21], de l'électricité se dissipe lors de son acheminement aux consommateurs finals. Cette dissipation se produit principalement par échauffement des matériaux, c'est-à-dire par effet Joule. Même si les pertes sont aussi présentes dans le réseau transport, la possibilité d'utiliser une tension élevée dans ces réseaux permet de réduire le volume des pertes par effet Joule[22]. En France, le taux de pertes en ligne sur le réseau de transport est d'environ 2% de l'électricité consommée alors que dans le réseau de distribution le taux moyen est de 6%. Les pertes sont sensibles aux matériaux utilisés mais

[21]Dans la distribution d'eau, on estime à 20-25% la perdition dans le réseau en France.

[22]Dans le cas de la distribution d'eau, l'augmentation de la pression accroît les fuites d'eau et donc le coût des pertes dans ces réseaux.

aussi à la quantité d'énergie qui circule sur le réseau. Les pertes thermiques, par effet Joule, évoluent avec le carré de la puissance injectée dans le réseau. Selon les pays, les distributeurs supportent la charge financière et la couverture des pertes d'électricité. Selon Mme Michèle Bellon, présidente du directoire de ERDF, la compensation des pertes représente 1,5 milliard d'euros par an[23]. Afin de réduire les pertes en ligne, les distributeurs peuvent changer et rajeunir les installations de leur réseau intégrant ainsi de nouveaux matériaux et de nouvelles technologies. Une meilleure gestion de la demande permet aussi de limiter les périodes de pointe pendant lesquelles la puissance injectée est importante et donc les pertes par effet Joule. L'impact des productions délocalisées sur les pertes n'est pas clair. En période de pointe, injecter de l'électricité de l'aval vers l'amont permettrait de réduire la puissance injectée dans le réseau et donc les pertes en ligne. En revanche, sachant qu'on ne peut prédire la période de production et que les distributeurs ont l'obligation d'injecter ces productions dans le réseau, elles peuvent augmenter les pertes en période de faible consommation[24].

Les quatre missions d'un gestionnaire de réseau ne sont pas indépendantes. Par exemple, lorsqu'un distributeur doit raccorder de nouveaux producteurs ou de nouveaux consommateurs, il installe de nouveaux ouvrages avec des matériaux plus performants, ce qui renforce le réseau, améliore l'efficacité énergétique des ouvrages et permet d'intégrer des technologies communicantes.

## 1.3 Synthèse des chapitres

• Le deuxième chapitre de cette thèse, co-écrit avec Claude Crampes, est consacré à la tarification de l'accès à un réseau de distribution, lorsque la taille de celui-ci n'est pas déterminée. Même si cette analyse est applicable à des réseaux de distribution variés, par exemple la distribution d'eau, de gaz, nous illustrons nos propos en nous focalisant sur la distribution d'électricité. Même si les pays développés ont choisi de connecter l'ensemble des consommateurs d'électricité, cette solution n'est pas optimale lorsque les consommateurs ont accès à une source de production locale comme une éolienne, des panneaux photovoltaïques. Par ailleurs 20% de la population mondiale n'a toujours pas accès à l'électricité. Outre la détermination de la taille optimale du réseau, nous étudions les effets des contraintes de services publics sur le tarif de

[23]Cet entretien est mentionné dans le rapport n°667 du Sénat.

[24]Une note plus détaillée sur les pertes dans le réseau de distribution français est disponible en annexe A.

connexion et sur le niveau d'investissement en qualité que réalisent les opérateurs de réseau. Les paramètres structurant ce chapitre sont la répartition géographique des consommateurs dans la zone de distribution et les coûts engendrés par ces différentes localisations.

Nous montrons que, en présence d'une production alternative locale, l'optimum consiste à ne pas connecter l'ensemble des individus présents dans la zone de distribution. La décentralisation de cet optimum conduit à un tarif qui présente deux inconvénients majeurs: i) en tarifant au coût marginal kilométrique, le gestionnaire du réseau dégage un profit négatif et ii) ce prix croît avec la distance entre la tête du réseau de distribution et le foyer de consommation. Cette relation croissante entre la distance parcourue par l'électricité et le prix payé va à l'encontre du principe du "timbre-poste" défendu par les hommes politiques.

Dans la suite du chapitre, nous prenons en compte ces contraintes de services publics et de financement et analysons l'impact qu'elles ont sur la taille du réseau, la tarification et l'investissement permettant d'améliorer la qualité du réseau. La contrainte d'équilibre budgétaire induit un tarif d'accès plus élevé et un investissement moindre. Il en résulte que les clients potentiels du réseau payent plus pour être connectés et bénéficient d'une qualité moindre. Moins de consommateurs souhaitent donc être connectés au réseau de distribution, réduisant ainsi la taille du réseau à installer. D'autre part, nous montrons qu'une tarification unique de type "timbre-poste" couvrant l'ensemble des coûts du gestionnaire conduit à une situation en coin: soit tous les consommateurs choisissent d'être connectés au réseau (si le tarif est faible), soit personne ne souhaite être connecté (lorsque le tarif de connexion est élevé).

Enfin, nous abordons les avantages d'un tarif binôme qui permet d'atteindre la taille optimale du réseau tout en couvrant les coûts du gestionnaire de réseau. Ce type de tarif a, de plus, un effet redistributif car les clients situés près de la tête de réseau financent une partie de la connexion des consommateurs qui en sont éloignés, ce qui réduit les différences de tarifs entre les clients connectés au réseau.

• Le troisième chapitre étudie les différents modes de gestion des pertes en ligne dans les réseaux de distribution. Les réseaux de transport et de distribution sont très énergivores en raison des pertes en ligne. La volonté des gouvernements et de l'Union européenne est de réduire la consommation des réseaux en luttant contre les pertes en ligne afin de diminuer leur empreinte carbone. La motivation de ce chapitre est notamment de montrer quel est l'impact du mode de gestion des pertes choisi.

Nous analysons d'abord, grâce à un modèle constitué d'une ligne et deux noeuds, l'impact des pertes en ligne sur l'ordre de mérite de centrales situées à des noeuds différents. En présence de pertes en ligne, une centrale présentant un coût marginal faible mais située loin des lieux de consommation n'est pas forcément efficiente pour satisfaire la demande.

Dans la suite du chapitre, nous nous focalisons sur le réseau de distribution et la manière dont les pertes en ligne sont gérées. Le mode de gestion des pertes fait partie des consultations publiques menées pour la quatrième version du TURPE après avoir fait l'objet d'un rapport publié par la CRE en mars 2010. Nous regardons en particulier quel mécanisme de gestion incite au mieux les gestionnaires de réseau à accroître l'efficacité énergétique du réseau. Nous comparons deux types de gestion utilisés en Europe. Le premier, le plus répandu, suppose que les pertes en ligne représentent un coût variable de l'activité de distribution. La couverture des pertes est alors prise en charge par le gestionnaire de réseau. Le second mode de gestion considère que les pertes sont une externalité de la vente d'électricité et, donc, le producteur ou le fournisseur supporte la charge financière induite par la vente d'électricité. Le gestionnaire de réseau, quant à lui, est régulé sur la base d'un volume cible de perte sur le réseau. Ce dernier mode de gestion est notamment appliqué au Royaume-Uni. L'analyse se porte sur l'incidence des modes de gestion sur les niveaux de consommation d'électricité et d'investissement permettant de réduire les pertes en ligne. Connaissant les caractéristiques de la demande d'électricité, il apparaît qu'il est plus efficient de considérer que les pertes en ligne sont un coût variable de la distribution d'électricité et d'attribuer la responsabilité de la couverture des pertes aux gestionnaires de réseau. Cette solution permet de réduire le niveau de pertes en ligne, à niveau de consommation donné.

• Le dernier chapitre de cette thèse, co-écrit avec Wilfried Sand-Zantman, étudie l'interaction existante entre les politiques de promotion des énergies renouvelables et le degré d'interconnexion entre deux pays. La coopération énergétique entre Etats membres a fait l'objet d'une directive publiée par le Parlement européen et le Conseil européen (directive 2009/28/CE). Cependant, l'intérêt d'une telle coopération dépend largement des contraintes de transmission existantes entre les pays de l'Union européenne.

Afin d'illustrer nos propos, nous considérons que la politique de promotion des énergies renouvelables s'appuie sur des prix de rachat attractifs. Nous supposons l'existence de deux pays, interconnectés, dans lesquels la production d'électricité est assurée par des énergies fossiles, non-renouvelables ou par des énergies dites renouvelables. Les

technologies utilisant les énergies renouvelables sont supposées plus coûteuses mais la production d'électricité grâce à ces énergies crée une externalité locale positive. La seule différence entre les deux pays est leur évaluation de cette externalité, ce qui induit un échange d'électricité entre ces pays.
En supposant que la capacité de l'interconnexion est non contraignante, la mise en place de politiques non-coordonnées, et donc de tarifs de rachat différents selon les pays, augmente la production d'électricité basée sur les énergies renouvelables et renverse le flux d'échange entre les deux pays par rapport à celui de l'optimum de premier rang. Dans le cas d'une politique coordonnée, il n'y a pas d'échange net d'électricité entre les deux pays et le tarif coordonné subventionné ne dépend pas de la capacité de l'interconnexion.
Sans contrainte de connexion, une politique uniforme est préférée à la décentralisation des choix politiques. Cependant lorsqu'il existe des contraintes de transmission entre les pays, l'intérêt d'avoir une politique uniforme n'est plus évident. Nous montrons que plus la contrainte est forte plus il est préférable de décentraliser les choix de tarifs de rachat et donc de préserver le principe de subsidiarité.

# Chapter 2

# Pricing the connection to a distribution network*

## 2.1 Introduction

The economic literature on network industries is mostly focussed on the use and development of installed capacity (such as production plants, transformers, lines and pipes) without a great interest on the geographical features of the infrastructure. Take the example of the electricity industry. There exists an important economic literature on electricity wholesale markets (e.g. Borenstein, Bushnell and Wolak [2002]; Fabra, von der Fehr and Harbord [2006]), on capacity markets (e.g. Creti and Fabra [2007]), on the ownership and management of the transmission grid (e.g. Joskow and Tirole [2005]), on transport pricing (e.g. Crampes and Laffont [2001]) and on retail supply (e.g. Joskow and Tirole [2007]). By contrast, there is little economic research on electricity distribution, in particular the constraints imposed by the geographical dispersion of potential consumers. The existing literature can essentially be separated into two branches. The first one concerns the design of concession contracts for a natural monopoly activity[1]. The second group deals with benchmarking techniques to fix tariffs and to control the quality of service (e.g. Jamasb and Pollitt [2007]). Related to this branch of the literature, Heng et al. [2009] study optimal pricing in a distribution network depending on technical characteristics. They look at the impact of network security on the investment cost and thus on the long run incremental cost.

In our research, we focus on the length of lines or pipes as a proxy of the operating cost of a distributor. In the Industrial Organization literature, the design of a

*This chapter is jointly written with Claude Crampes.

[1]See for instance Crampes and Estache [1998].

network, especially its spatial characteristics, is considered as exogenous, and, in the case of contract theory, a secondary concern. However, before contracting with local authorities on tariffs and expected profits, the operators must negotiate on how many people should be connected to the network, their location, as well as on the topological features of the infrastructure. In some industries (electricity, telecom, water), one observes that in most developed countries, almost all consumers are already connected to the grid. However, it is not the case all around the world. For instance, in Africa or in India, the electrification rate is still low[2]. Therefore, it makes sense to consider models where the number of connected client is endogenous. In this paper, we assume that the network is not installed yet and thus its size has to be determined. Our paper also addresses the pricing strategy of a monopolist when consumers are spatially dispersed. Some papers in the industrial organization literature determine the optimal price by taking into account the dispersion of consumers. Beckmann [1976] considers an arbitrary spatial distribution of the consumers and compares mill pricing, uniform delivered pricing and local discriminatory pricing. Spulber [1981] determines the optimal non-linear price of a good which is then transported. In these papers, the transport price is proportional to the distance between "the mill" and the consumer like in our paper the distance is between a consumer and a substation. However, we are not interested in pricing a good that is transported, rather pricing the access to a distribution network. We adopt the transporter point of view rather than the producer's. The papers of Oi [1971] and Coyte and Lindsey [1988] study two-part tariffs from a monopoly point of view. The fixed part of the tariff corresponds to a membership fee, that is consumers pay a right to buy a product (e.g. the number of rides in Oi [1971]). In these approaches, consumers can choose the quantity they buy and consume. In our paper, the consumers can decide wether to be connected or not but they cannot choose the "quantity of line consumed". The distance between them and the entry point of energy or water or IC services is exogenously given.

Even though the model we present can be used in many network industries (water, natural gas, cable TV, etc.), it can be helpful to motivate it by focussing on a specific one, namely electricity distribution. Electricity distribution is a natural monopoly activity[3]. Indeed, distribution requires a huge investment in infrastructure and a rather low operating cost. For example, the marginal cost of distributing electricity to con-

[2] See the electrification rates in appendix B.1.

[3] However, as shown in Saplacan [2008], a more detailed analysis allows to identify some possibility of competition.

sumers is just the cost of thermal losses[4]. Absent any security concern, the duplication of the infrastructure would then be suboptimal. On top of this horizontal view, electricity distribution entails vertical effects. The distribution network is an infrastructure that no retailer can bypass. Therefore it is an essential facility for the provision of electricity. If the network is vertically integrated with energy retail, the owner has some interests in proposing discriminatory access conditions to its competitors. Therefore, it is an industry where a high level of regulation is required. For example, the European Commission encourages the Member States governments to unbundle the activities considered as essential facilities from the ones in which competition is implementable[5]. Nevertheless, in several Member States, distributors are still bundled with the historical incumbents who are competing in the retail market against non integrated suppliers. The essential facility question is not addressed in this paper. Neither we discuss the ownership unbundling of distribution networks.

The distribution of electricity, natural gas, water, etc. is constrained by the geographical characteristics of the region under consideration. The cost to install and maintain lines and pipes is very different in mountain regions and in plain regions. In spite of that, public service regulation can oblige the operator to provide the same service everywhere or to deliver electricity, gas and water to a specified number of consumers. Clearly, the economic modeling of such activity has to include both social and spatial requirements. We do so by building a model aimed at computing the optimal size of a network and determining the connection fee. The critical number of individuals connected to the network results from the consumers' decision upon either to be connected or to consume locally[6]. We analyze how the network should be managed at first best, and we contrast the results to the choices of an independent private operator. We show that the independent operator charges each consumer more than at first best and provides a lower level of quality. We then address different cases where the operator must comply with different public service constraints.

The paper is organized as follows. In section 2, we set up the model. We present the

[4]The quantity injected at the head of the network is larger than the quantity consumed. This difference is due to thermal losses created by the so-called "Joule effect". Thermal losses increase with respect to the distance between the head of the network and electricity takers and with respect to the quantity injected into the network. In order to compensate thermal losses, the operator of the network may be obliged to purchase energy blocks. This essential feature of the distribution activity is addressed in chapter 3 and appendix A.

[5]See Directive 2009/72/EC, article 26.

[6]Local consumption of electricity comes from a local generation plant. As regards water, it comes from a well. Cable TV can be replaced by satellite, etc.

hypotheses on consumers' behavior, and we determine the optimal level of investment and the optimal tariff of access. This provides a benchmark to see how decentralization and public service constraints may distort the operator's decisions away from the first-best solution.

Section 2.3 takes budget balancing into consideration. Indeed, because of the public opinion and/or for legal reasons, the first-best solution that incurs huge financial losses is not viable. We first determine the second-best linear price that balances the operator's budget and then the second-best investment in quality. Section 2.4 deals with two-part pricing. We first show that two-part tariffs allow to implement first best and we analyze the distributive effects of this type of tariff. In section 2.5 we conclude and suggest further developments.

## 2.2 Optimal design of a distribution network

For electricity, water, TV programs and so on, consumers need to have either a local supplier or a connection to remote suppliers. The spatial dispersion of consumers is a key issue in the distribution activity. Each consumer is defined by $\theta$ which stands for the distance between the consumption point and the entry point in the distribution network. The density of agents located at distance $\theta$ is denoted $f(\theta)$ and the cumulative distribution by $F(\theta)$. The general form of consumers spatial dispersion can be illustrated by the density:

$$f(\theta) = a + b\theta \tag{2.2.1}$$

The linear form of the density function allows to capture the main characteristics of the region and to simplify computations. One can distinguish different types of regions depending on where consumers are concentrated with respect to the head of the network. In some regions, the population is concentrated far from the head of the network. The farther from the head, the larger the number of consumers. It results in $b > 0$. The opposite is a region where consumers are concentrated close to the head of the network ($b < 0$). The features of the latter are those of a valley. There is a high concentration of population along the first kilometers of line and then the population is increasingly scattered. The simplest type of region is a suburb. In this kind of area, the population is uniformly distributed, which means $b = 0$. This regional characterization allows to compare connection and tariff solutions and to address the question of resource equalization among heterogeneous distribution zones. Nevertheless, in this paper, we only analyze the simplest spatial configuration, that is the uniform distribution case.

In section 2.2.1, we expose the hypothesis concerning the behavior of consumers. In section 2.2.2, we then determine the optimal size of the network and the optimal investment in service quality.

### 2.2.1 Consumer behavior

We assume that consumers are free to choose between two sources of electricity (or water, or natural gas, etc.). The first source is the electricity coming from the network that conveys energy from the transformer located at the border between the high-voltage transport grid and the distribution grid. The alternative is local self-production. In other words, consumers choose between being connected or not.

We assume that all consumers within a region are identical, except for their location. A consumer who chooses to be connected to the network has a gross utility defined by:

$$U(q, s) \tag{2.2.2}$$

where $q$ is the quantity of electricity consumed and $s$ the quality of the service. It is an increasing and concave function in each argument. The quality of service is an increasing function of the equipment installed in the network (transformers, type of lines). The larger the equipment denoted by $K$, the higher the quality of the service provided by the distributor. For connected consumers, $K$ is a public good. Since the utility function $U(.,.)$ increases with the quality of service, $U(.,.)$ is increasing with the level of common equipment $K$ installed in the network.

Once connected, a consumer buys from his retailer the quantity:

$$q(p_e, K) = arg \max_q U(q, s(K)) - p_e q \tag{2.2.3}$$

where $p_e$ is the unit price of electricity. One can easily check that this function is decreasing with $p_e$ and increasing with $K$. We define the indirect net utility of a consumer when he/she is connected by:

$$v(p_e, K) \stackrel{def}{=} U(q(p_e, K), s(K)) - p_e q(p_e, K) \tag{2.2.4}$$

It is easy to check that $\partial v / \partial p_e < 0$ and $\partial v / \partial K > 0$.

An agent not connected to the network has an indirect net utility $v(p_a)$, where $p_a$ is the cost of the self-produced electricity or the price of electricity from an independent source. This electricity may come from wind turbines, solar-plants, water mills, etc. We assume that this independent electricity provides the same type of service to consumers wherever they are located. If we are in a situation such as $p_a << p_e$, for instance because agents are in a windy area or a very sunny one, then we can have $v(p_a) > v(p_e, K)$ even though $K$ is very large. If so, connection to a distribution network is not economically profitable. In what follows, we assume that $p_a > p_e$ and thus it is *a priori* profitable to install a network, with geographical characteristics that remain to be determined.

The only difference between consumers is their location identified by $\theta$ of density $f(\theta)$. The installation and maintenance cost is equal to $c\theta$ for a group of consumers located at $\theta$, so the per-consumer maintenance cost is equal to $c\theta/f(\theta)$. We assume that the size of the network, i.e. the number of potential customers, is normalized to 1[7].

The type of network we are analyzing follows a hub-and-spoke pattern, that is consumers at the same distance are connected through the same line. We do not consider networks where consumers at different distances are linked by (partially) common lines. In the latter, a consumer who wants to be connected generates both costs which are specific and others that are common to all the consumers connected upstream of his location[8]. Actual electricity, natural gas and water distribution networks are a mix of these two types of network.

### 2.2.2 First-best connection and investment

We determine the number of connected consumers and the optimal level of capital $K$ installed by a well-informed and benevolent social planner. We first identify the consumers who shall be connected to the network. Knowing the connection and maintenance costs, the consumers for whom connection is socially beneficial are such that:

$$v(p_e, K) - \frac{\theta c}{f(\theta)} \geqslant v(p_a) \tag{2.2.5}$$

Defining the marginal consumer (or group of consumers) as the one indifferent between being connected to the public network or being supplied by the alternative source, the

[7] This assumption should be relaxed in an extension of the model because real distribution costs also depend on the number of connected consumption sites.

[8] It would mean that the lines are partially common and partially individual. Here, we assume that all the common costs are captured by variable $K$.

maximal distance of connection is defined by the implicit relation:

$$\widetilde{\theta}(c, K) = \frac{[v(p_e, K) - v(p_a)]f(\widetilde{\theta})}{c} \tag{2.2.6}$$

This distance is an increasing function of the quality of service, therefore of $K$, as well as of the energy price difference $p_a - p_e$. It is obvious that the higher the quality of service, the greater the distance of the marginal consumer since connected consumers have an increased gross surplus. Concerning the difference in prices, if the alternative source of electricity is more and more costly compared to the energy delivered by the network, it is more socially beneficial to be connected to the public network.

The social welfare created by the connection of consumers with $\theta$ no larger than $\widetilde{\theta}$ to the public network is:

$$S(c, K) = \int_0^{\widetilde{\theta}} \left[ v(p_e, K) - v(p_a) - \frac{\theta c}{f(\theta)} \right] dF(\theta) - rK \tag{2.2.7}$$

where $r$ represents the unit cost of the common equipment to maintain the network quality. Hence, we have two types of costs: $rK$ is common to all connected consumers; the other cost $\theta c$ is due to the connection of consumers at distance $\theta$. We assume that the costs are independent of the number of people connected. In order to have explicit results, we mainly focus on the case where consumers are uniformly distributed between 0 and $\bar{\theta}$ in the zone of distribution[9]. With this specification social welfare is:

$$S(c, K) = \frac{\left[ v(p_e, K) - v(p_a) \right]^2}{2c\bar{\theta}^2} - rK \tag{2.2.8}$$

This function is decreasing w.r.t. $\bar{\theta}$. Indeed, keeping all other variables constant, if the dispersion of the population increases (or equivalently if the population density decreases), there are fewer spots of consumption for which connection is socially profitable.

The social planner's problem is to determine the size of the common equipment influencing the service quality for connected consumers:

$$\max_K \; S(c, K) \tag{2.2.9}$$

[9]This type of statistical distribution is rather good for suburbs. It allows to simplify computation while giving good intuitions. By contrast, it is misleading as regards other geographical distributions.

The first order condition is given by:

$$\frac{v(p_e, K^*) - v(p_a)}{c\bar{\theta}^2}\frac{\partial v(p_e, K^*)}{\partial K} \leqslant r \tag{2.2.10}$$

Three cases can occur:
i) where the left-hand-side of the inequality is strictly lower than $r$ for any value of $K$, no distribution network shall be installed. It is optimal to provide everybody with local energy. For example, it is the case when $v(p_e, 0) < v(p_a)$ and $S(.)$ is concave in $K$; ii) the problem has a positive interior solution given by the equality in (2.2.10) only if the net-utility function, $v(p_e, K)$, is strongly concave in $K$[10]; iii) otherwise, all consumers must be connected, $\widetilde{\theta}(c, K) = \bar{\theta}$, and the installed equipment is derived from equation (2.2.6), that is $\widehat{K} = \arg_K\{v(p_e, K) = c\bar{\theta}^2 + v(p_a)\}$.
In the next paragraph, we rather explore the interior solution (case ii) defined by:

$$\frac{v(p_e, K^*) - v(p_a)}{c\bar{\theta}^2}\frac{\partial v(p_e, K^*)}{\partial K} = r \tag{2.2.11}$$

Under the assumption of strong concavity of the net-utility function in $K$, the optimal capital to be installed is decreasing in $r$, $\bar{\theta}$ and $c$ and is increasing in $p_a - p_e$, the price differential between electricity coming from the alternative source and electricity delivered by the network.
To decentralize this optimal choice with a linear connection fee, one has to find a price with two properties: i) all consumers located at a distance $\theta \leqslant \widetilde{\theta}(c, K^*)$ must have the incentive to be connected while the others must choose energy outside the network, and ii) at this price the distributor invests up to $K^*$.
The simplest solution to reach these two conditions is to charge each consumer a price per kilometer of line equal to the marginal cost of installation and maintenance, that is $p^* = c/f(\theta)$. It allows to meet the first condition: for a given $K$, at price $p^*$ the marginal consumer who demands connection is $\widetilde{\theta}(c, K)$. However, there appear two majors drawbacks with this solution:

- the net profit of the operator is:

$$\int_0^{\widetilde{\theta}} p^*\theta dF(\theta) - \int_0^{\widetilde{\theta}} c\theta d\theta - rK^* = -rK^* < 0 \tag{2.2.12}$$

Therefore the operator will ask for subsidies and this may prove hardly possible

[10]In words, the utility from a higher quality of service increases very quickly with $K$ for small value of $K$ and then it stagnates as if quality improvements were quickly exhausted. See the proof of the second order condition in appendix B.2.

(and may be illegal);

- the charge paid by each consumer depends on the density of consumers at distance $\theta$ and is likely to be seen as a discriminatory solution by politicians whereas, in fact, it only reflects the costs induced by distance[11]. Moreover, if consumers may move, it can result in a undesirable geographical location of consumers with a population highly concentrated around the source node where connection is cheaper.

Hence, under the combined pressure of public opinion and administrative authorities, first best solution is not viable. We now consider the obligation to balance the operator's budget.

## 2.3 Optimal network under budget constraint

Assume that the only constraint is budget balancing. We keep the hypothesis that the bill can be proportional to distance. Under linear pricing, the second-best solution is given by:

$$\max_{p,K} \ \widehat{S}(p,K) \ s.t. \int_0^{\widehat{\theta}(p,K)} \left(p - \frac{c}{f(\theta)}\right)\theta dF(\theta) - rK \geqslant 0$$

where the marginal consumer is located at:

$$\widehat{\theta}(p,K) = \frac{v(p_e,K) - v(p_a)}{p} \tag{2.3.1}$$

and the social surplus is:

$$\widehat{S}(p,K) = \int_0^{\widehat{\theta}} \left[v(p_e,K) - v(p_a) - \frac{\theta c}{f(\theta)}\right] dF(\theta) - rK \tag{2.3.2}$$

We successively determine the optimal linear price and the optimal level of capital to be installed under the constraint of budget balancing.

[11]The case where consumers are uniformly distributed is an exception since $\forall\theta \in [0,\bar{\theta}], f(\theta) = 1/\bar{\theta}$. By contrast, when density decreases with the distance from the head of the network like in a valley, the price per kilometer should increase with the distance.

### 2.3.1 Second-best linear price

The social welfare function, $\widehat{S}(.)$, is an increasing function of the last consumer to be connected, $\widehat{\theta}$. On the other hand, the maximal distance $\widehat{\theta}(p, K)$ is a decreasing function of the price (per kilometer). Thus, the second-best linear price is the smallest price compatible with the budget constraint of the operator. Consequently, the connection fee per unit of distance to charge is equal to the average cost; that is the price that solves

$$\int_0^{\widehat{\theta}(p,K)} \left(p - \frac{c}{f(\theta)}\right)\theta dF(\theta) - rK = 0 \tag{2.3.3}$$

In the specific case of a uniform distribution between 0 and $\bar{\theta}$, the price per kilometer is (see appendix B.3.1):

$$p^{SB} = \frac{1 - \sqrt{1 - 4\bar{\theta}Ac}}{2A} \tag{2.3.4}$$

$$\text{with} \qquad A = \frac{2rK\bar{\theta}}{[v(p_e, K) - v(p_a)]^2} \tag{2.3.5}$$

The second-best linear price $p^{SB}$ exists if and only if $A \in \left]0, \frac{1}{4\bar{\theta}c}\right]$. Under this condition[12], the second-best price belongs to the interval $\left[c, 2\bar{\theta}c\right]$.

For a given level of capital $K$, the price $p^{SB}$ increases with respect to $c$, $r$ and $\bar{\theta}$. Indeed, when the spatial dispersion $\bar{\theta}$ increases, there are fewer spots of consumption that should be connected and the total population the operator can charge for the common costs $rK$ is reduced. Therefore, it has to charge connected consumers more. Furthermore, $p^{SB}$ decreases when the difference $v(p_e, K) - v(p_a)$ raises, given $K$. This relation is surprising at first sight only. Indeed, at second best, the operator's objective is not to make the highest profit, but rather to cover its costs, $rK$. So, the larger the difference $v(p_e, K) - v(p_a)$ given the common cost to recoup, the larger the number of households connected to the public network. When the number of consumers is higher, the operator has to charge each consumer less in order to balance its budget.
Finally, the effect of $K$ on the second-best price is ambiguous. In fact, if $K$ is larger, the operator has to raise more and more resources ($A$ is an increasing function of $K$) to cover the costs of the installed capital. On the other hand, if $K$ increases, the utility

[12]Notice that if $K \to 0$, the budget constraint is trivially met by solving (2.3.3) with $K = 0$, that is $p^{SB} = \frac{c\hat{\theta}^2/2}{\int_0^{\hat{\theta}} \theta dF(\theta)}$. At the other end of the validity interval, the infrastructure costs are so high that pricing at average cost prevents consumers from connecting to the network.

to be connected to the public network is greater, that is the difference $v(p_e, K) - v(p_a)$ is higher. As we have seen before, this implies that the number of potential consumers is larger and the per-capita contribution is smaller. However, we know from section 3.3.1 that the function $v(p_e, K)$ has to be strongly concave in $K$ in order to have an interior solution. Consequently, we may consider that the case where $p^{SB}$ is an increasing function of $K$ is probably the rule rather than the exception[13].

Knowing the second-best price, we now compute the optimal level of investment under budget constraint.

### 2.3.2 Investment in quality

Taking into account the price $p^{SB}$ that allows the operator to balance its budget, the best choice in terms of common equipment is given by:

$$\max_K \int_0^{\widehat{\theta}(p^{SB},K)} \left[ v(p_e, K) - v(p_a) - \frac{\theta c}{f(\theta)} \right] dF(\theta) - rK \tag{2.3.6}$$

When consumers are uniformly distributed, the program can be written as follows:

$$\max_K \left( \frac{p^{SB}}{\bar{\theta}} - \frac{c}{2} \right) \left( \widehat{\theta}(p^{SB}, K) \right)^2 - rK \tag{2.3.7}$$

The first order condition that gives $K^{SB}$, is (for the details, see appendix B.3.3):

$$-\frac{\widehat{\theta}^2}{\bar{\theta}} \frac{\partial p^{SB}}{\partial K} \left( \frac{p^{SB} - \bar{\theta} c}{p^{SB}} \right) + 2\widehat{\theta} \left( \frac{p^{SB}}{\bar{\theta}} - \frac{c}{2} \right) \frac{\partial \widehat{\theta}}{\partial K} = r \tag{2.3.8}$$

Assume first that $\dfrac{\partial p^{SB}}{\partial K}$ is negligible. To compare $K^{SB}$ and $K^*$, using (2.3.1), we can write from equations (2.2.11) and (2.3.8) that:

$$\frac{v(p_e, K^*) - v(p_a)}{c\bar{\theta}^2} \frac{\partial v(p_e, K^*)}{\partial K} = \frac{v(p_e, K^{SB}) - v(p_a)}{p^{SB}} \frac{\left(2p^{SB} - \bar{\theta} c\right)}{\bar{\theta} p^{SB}} \frac{\partial v(p_e, K^{SB})}{\partial K} \tag{2.3.9}$$

Given that $\dfrac{1}{\bar{\theta} c} \geqslant \dfrac{2p - \bar{\theta} c}{p^2}$, we have[14]:

$$\Big[ v(p_e, K^*) - v(p_a) \Big] \frac{\partial v(p_e, K^*)}{\partial K} \leqslant \Big[ v(p_e, K^{SB}) - v(p_a) \Big] \frac{\partial v(p_e, K^{SB})}{\partial K} \tag{2.3.10}$$

[13] See an example in appendix B.3.2.

[14] This is true for any $\{\bar{\theta}, c\}$, knowing that $\dfrac{1}{\bar{\theta} c} p^2 - 2p + \bar{\theta} c$ is always positive.

where the function $v(.)$ is strongly concave in $K$, so that the function $\left[[v(p_e, K) - v(p_a)]\frac{\partial v}{\partial K}\right]$ is decreasing in $K$. Then inequality (2.3.10) is satisfied only for $K^{SB} \leqslant K^*$. If now, we consider that $\frac{\partial p^{SB}}{\partial K} \neq 0$, equation (2.3.9) becomes:

$$\frac{v(p_e, K^*) - v(p_a)}{c\bar{\theta}^2}\frac{\partial v(p_e, K^*)}{\partial K} = -\frac{\widehat{\theta}^2}{\bar{\theta}}\frac{\partial p^{SB}}{\partial K}\left(\frac{p^{SB} - \bar{\theta}c}{p^{SB}}\right) + \frac{v(p_e, K^{SB}) - v(p_a)}{p^{SB}\bar{\theta}}\frac{\left(2p^{SB} - \bar{\theta}c\right)}{p^{SB}}\frac{\partial v(p_e, K^{SB})}{\partial K} \tag{2.3.11}$$

Keeping the assumption of a price increasing w.r.t. $K$, the equality in (2.3.11) is reached only if $K^{SB}$ is lower than when $\frac{\partial p^{SB}}{\partial K}$ is negligible. Indeed, reducing the level of installed capital would increase the marginal utility $\partial v(p_e, K^{SB})/\partial K$[15], and then this would compensate the fact that $\frac{\partial p^{SB}}{\partial K} \neq 0$. In the other case, when $\frac{\partial p^{SB}}{\partial K} < 0$, we cannot give a clear answer concerning the ranking between $K^{SB}$ and $K^*$.
To sum up, the budget constraint of the network operator implies that the tariff per kilometer is higher than in the first-best solution (where $p^* = c/f(\theta)$). Moreover, the optimal level of equipment that should be installed is lower than in the first-best solution[16]. Hence, the size of the network under the budget constraint is reduced for two reasons as illustrated in figure 2.1:

- a lower level of installed capital reduces the willingness to pay of potential consumers; then the marginal consumer is nearer to the source point, which means that the number of connected people decreases;
- the level of price, higher than the marginal cost per consumer $c/f(\theta)$, weakens the incentive of consumers to be connected. Here again, the marginal consumer is closer to the entry point of the distributed products $\left(\widehat{\theta} < \widetilde{\theta}\right)$.

## 2.4 Two-part tariff

For practical and/or political reasons, uniform pricing independently of distance is an obligation for the managers of distribution networks in many industries. Hereafter, we show that the constraint of uniform linear price joint with the obligation of budget balancing leads to a corner solution: either everyone or nobody is to be connected.

[15] Remember that the net utility function is concave in $K$.
[16] As said repeatedly, this is the most likely outcome but it is not guaranteed, since the effect of the capital installed on the price $p^{SB}$ is ambiguous.

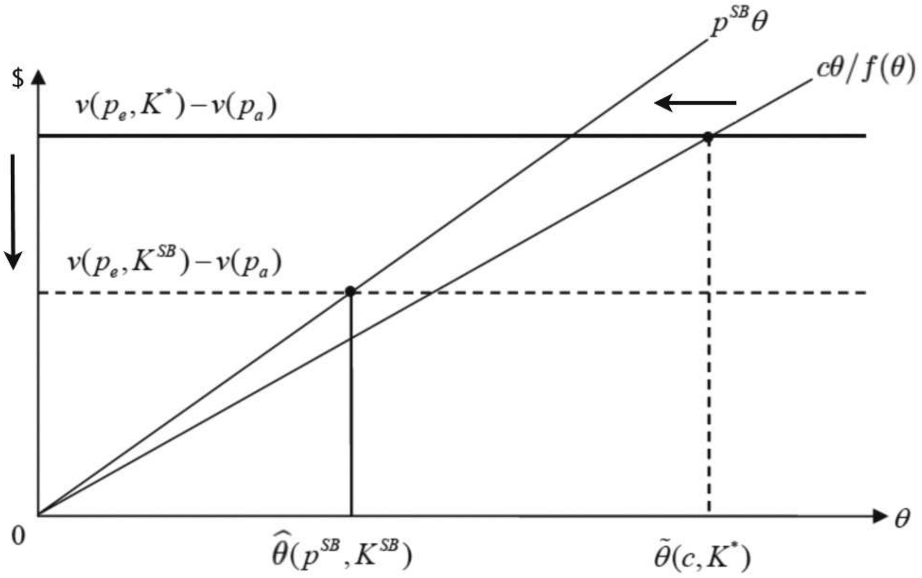



Figure 2.1: Linear tariff and budget constraint

Then, we assume that the price constraint only applies to the common costs and we introduce a two-part tariff. We consider two solutions to approximate first best: either pricing distance at marginal cost or serving the same number of consumers as at first best. In the latter case, the tariff can be viewed as an equalization tool from the consumers who are located close to the head of the network to those who are at remote locations.

### 2.4.1 Uniform linear tariff and two-part tariff

A uniform linear price $p^u$ covering all the costs is such that:

$$p^u F\Big[\theta^u(p^u, K)\Big] - \int_0^{\theta^u(p^u,K)} \frac{c\theta}{f(\theta)}\, dF(\theta) - rK \geqslant 0$$
$$\theta^u(p^u, K):\ v(p_e, K) - p^u = v(p_a)$$

Since the only difference between agents is the distance between their location and the head of the network, if the operator fixes a uniform price per consumer, all potential consumers will react in the same way. If the price $p^u$ is low, everyone would like to be connected to the network since $v(p_e, K) - p^u > v(p_a)$ for all $\theta$. Otherwise, nobody would want to be in the network. That is, depending on the value of $p^u$, either $\theta^u = 0$ or $\theta^u = \bar{\theta}$. These two solutions are far from the optimal criterion established previously. In other words, under a uniform linear tariff, the choice of the network size is a matter of politics[17].

Remember that the distributor has to balance his budget. When everybody is connected, he fixes a price $p^u$ to cover total cost:

$$p^u \geqslant cE(\theta) + rK$$

where $E(\theta) = \int_0^{\bar{\theta}} \theta \, d\theta$ is the average distance between consumers and the head of the network. The optimal level of investment is determined by maximizing social welfare:

$$\max_K \left[ v(p_e, K) - v(p_a) \right] - cE(\theta) - rK \tag{2.4.1}$$

So that $K^u$ is the solution to:

$$\frac{\partial v(p_e, K^u)}{\partial K} = r \tag{2.4.2}$$

Now, depending on the cost parameters and the utility function, we face two cases:

- $v(p_e, K^u) - v(p_a) \geqslant cE(\theta) + rK^u$; then the operator can connect everyone and fix a price $p^u$ between these two values without losing money;

- $v(p_e, K^u) - v(p_a) < cE(\theta) + rK^u$; to persuade consumers to be connected, the operator should fix a uniform price per consumer so low that its accounts would not be balanced.

Note that both cases may appear in a given delivery area, since costs are highly related to the geographical area under consideration and the advantage to be connected depends on the difference $p_e - p_a$, that is on the price $p_a$ of local sourcing.

An intermediate solution between a linear tariff per kilometer and a uniform fee per consumer rests on the combination of a fixed part and a variable one, that is on a two-part tariff. The operator implements a tariff $p_f + \theta p$ for consumers connected at distance $\theta$ from the entry point of the delivered product in the network. Under

[17] In developed countries, the political choice is generally to connect everyone to electricity networks (compare the two last line of the table in appendix B.1). Universal service obligations also exist in water distribution where it can be justified by public health considerations. By contrast, there is no obligation to connect agents in natural gas or cable-tv distribution.

two-part tariff, the marginal consumer is defined by:

$$\breve{\theta}(p_f, p, K) = \frac{\Delta v(K) - p_f}{p}$$
$$\text{where} \qquad \Delta v(K) \stackrel{def}{=} v(p_e, K) - v(p_a)$$

In what follows, we successively consider two types of two-part tariff, each having a characteristic of the first-best solution. We first consider a two-part tariff constrained by a service obligation, namely the operator has to serve the same area (same marginal consumer $\widetilde{\theta}(c, K)$) and invest the same amount of capital $K^*$ as in the first-best solution. Then, we consider a two-part tariff for which the coefficient of the variable part is equal to the marginal cost per kilometer.

### 2.4.2 Second-best two-part tariff

In the case of uniform distribution, we know that the social welfare function is maximum at $\widetilde{\theta}(c, K^*) = \Delta v(K^*)/c\bar{\theta}$ since it is the first-best solution. With a two-part tariff, the marginal consumer $\breve{\theta}(p_f, p, K^*) = \Delta v(K^*)/c\bar{\theta}$ can be reached by any combination of $p$ and $p_f$ such that

$$p_f + \frac{p\Delta v(K^*)}{c\bar{\theta}} - \Delta v(K^*) = 0 \qquad (2.4.3)$$
$$\text{where } p > 0 \quad \text{and} \quad p_f < \Delta v(K^*)$$

For any pair of prices above this frontier, the marginal consumer is smaller than $\widetilde{\theta}(c, K^*)$, that is closer to the head of the network. For instance, if $p$ and $p_f$ satisfy $p_f + \frac{p\Delta v(K^*)}{2c\bar{\theta}} - \Delta v(K^*) = 0$, the marginal consumer is located too close, at $\widetilde{\theta}(c, K^*)/2$.

We now examine under which conditions a pair of prices satisfying first best could also balance the budget of the operator. The budget constraint is:

$$\breve{p}_f \frac{\widetilde{\theta}(c, K^*)}{\bar{\theta}} + \frac{\widetilde{\theta}(c, K^*)^2}{2}\left(\frac{\breve{p}}{\bar{\theta}} - c\right) - rK^* = 0 \qquad (2.4.4)$$

Replacing $p_f$ from (2.4.3) we obtain:

$$\frac{\breve{p}}{2\bar{\theta}}\left(\frac{\Delta v}{c\bar{\theta}}\right)^2 + \left(\Delta v - \breve{p}\frac{\Delta v}{c\bar{\theta}}\right)\frac{\Delta v}{c\bar{\theta}^2} - \frac{c}{2}\left(\frac{\Delta v}{c\bar{\theta}}\right)^2 - rK^* = 0$$
$$\Leftrightarrow \frac{\Delta v^2}{2(c\bar{\theta})^2}\left[c - \frac{\breve{p}}{\bar{\theta}}\right] - rK^* = 0$$

It clearly appears that this equation can be satisfied only for $\breve{p} < c\bar{\theta}$, i.e. if the variable part of the tariff is below the marginal cost per kilometer.

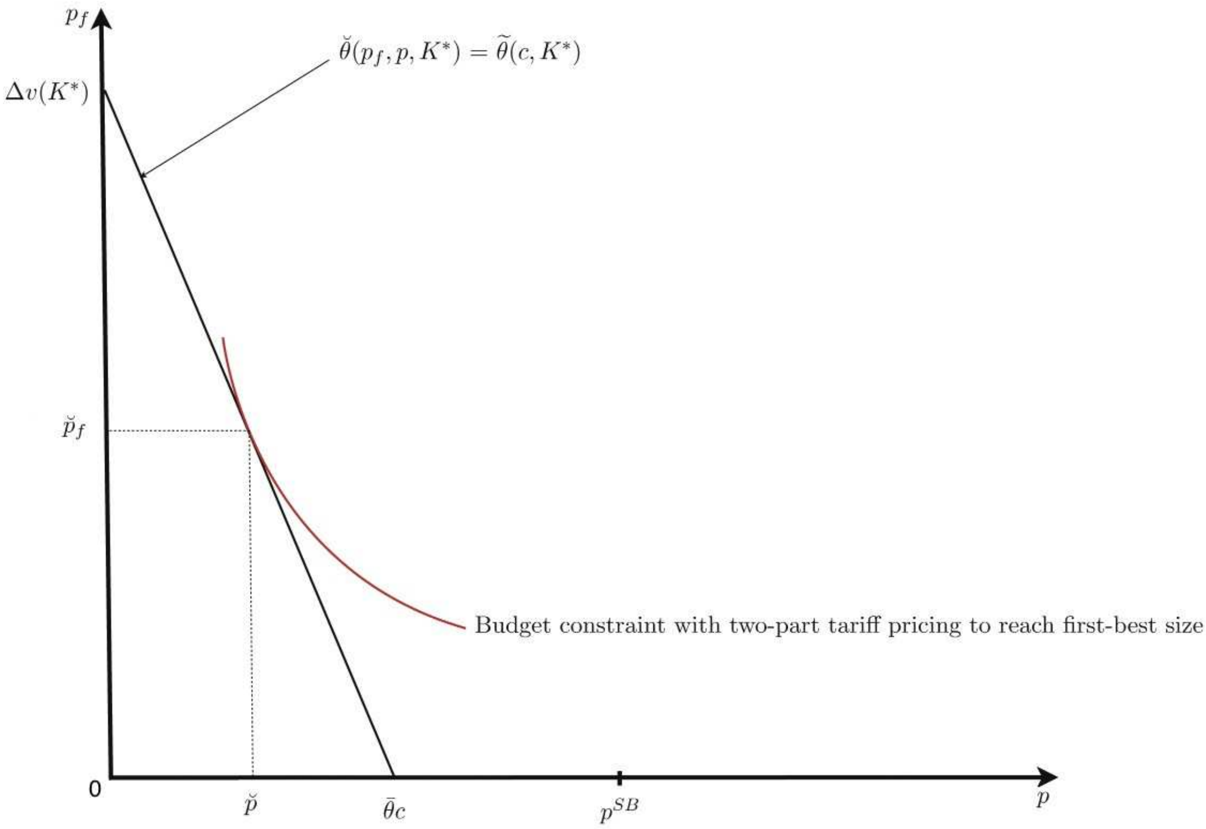



Figure 2.2: Two-part tariff

As illustrated in figure 2.2, whereas a second-best linear price would require $p^{SB} > p^* = c\bar{\theta}$, the two-part tariff $(\breve{p}_f, \breve{p})$ requires $\breve{p} < c\bar{\theta}$ and the financial equilibrium is reached thanks to the fixed part of the tariff. In Oi [1971], the fixed part of the tariff is used to extract the consumer surplus. He studies the case in which the variable part of the tariff is lower than the marginal cost. A monopoly may fix such a tariff to keep all consumers in the market and to maximize profits by increasing the fixed part. In our paper, the operator does not want to maximize his profit. He just wants to balance the budget. Besides, the price strategy aims here at increasing the number of consumers for whom the connection to the network is profitable.

### 2.4.3 Characteristics of the second-best two-part tariff

From the former section, the pair $(\breve{p}_f,\ \breve{p})$ that maximizes welfare under the budget balancing constraint satisfies the simultaneous system[18]:

$$\begin{cases} \breve{p}_f \dfrac{\widetilde{\theta}(c,K^*)}{\bar{\theta}} + \dfrac{\widetilde{\theta}(c,K^*)^2}{2}\left(\dfrac{\breve{p}}{\bar{\theta}} - c\right) - rK^* = 0 \\ \\ \breve{p}_f = \Big(v(p_e,K^*) - v(p_a)\Big)\left(1 - \dfrac{\breve{p}}{\bar{\theta}c}\right) \end{cases} \tag{2.4.5}$$

Solving this system, we obtain:

$$\begin{cases} \breve{p} = \bar{\theta}c\big(1 - A^*\bar{\theta}c\big) & \text{per kilometer} \\ \\ \breve{p}_f = \big(v(p_e,K^*) - v(p_a)\big)A^*\bar{\theta}c & \text{per consumer} \end{cases} \tag{2.4.6}$$

where $A^*$ is the function defined in (2.3.4) valued at $K^*$.
Clearly the price per kilometer $\breve{p}$ is lower than the marginal cost per consumer $\bar{\theta}c$. The fixed part of the tariff plays two roles: i) it allows the operator to balance its accounts and ii) it is an equalization tool among customers. Indeed, since it is required both to connect as many clients as at first best and to balance the budget, the operator decreases the price per kilometer of marginal consumers. Therefore he prices the kilometer below marginal cost. As compared with first best and with second best under linear price, the clients close to the head of the network are worse off since they have little advantage from paying less for the small number of kilometers of line necessary to connect them; indeed they consume only a small amount of this service. On the other hand, they have to pay a fixed part which is higher for two reasons: i) it is used to balance the accounts and ii) the common cost is bigger since the installed equipment at first best is more important than under second-best linear pricing. Consequently, customers located close to the grid head subsidize consumers located far from it. The graph of figure 2.3 illustrates this redistribution.

### 2.4.4 Two-part tariff with marginal cost pricing per kilometer

Pricing below marginal cost can be politically unfeasible as it results in the reallocation analyzed in the former section. Assume then that some regulation imposes that the variable part of the tariff charged to each consumer must reflect the unit kilometer

[18] See appendix B.4.1.

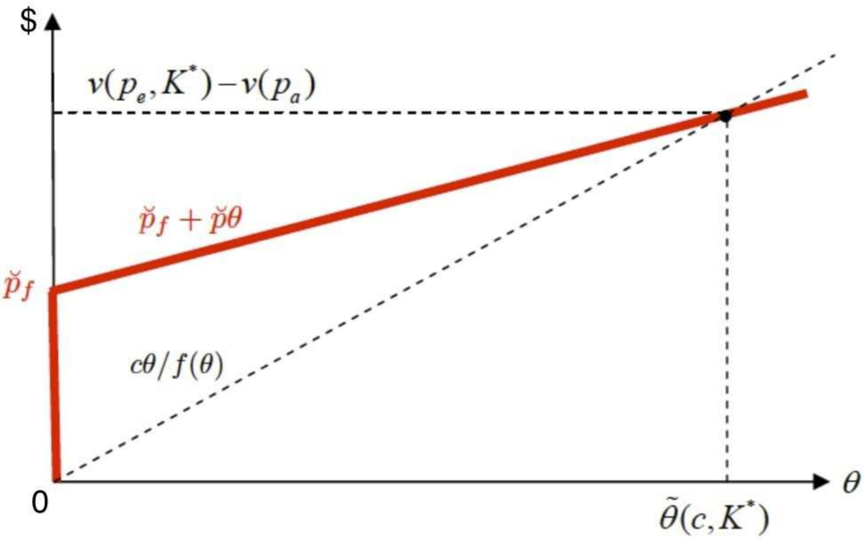



Figure 2.3: Two-part tariff under service constraint

cost of connection, $p = c\bar{\theta}$. Hence, the operator is left with financing the common costs, $rK$, through the fixed part of the tariff. One possible allocation of the common cost is the average value[19]:

$$p_f = \frac{rK}{F\left[\breve{\theta}(p_f, c, K)\right]} \text{ where } \breve{\theta}(p_f, p, K) = \frac{(v(p_e, K) - v(p_a) - p_f) f\left(\breve{\theta}\right)}{c}$$

In the case where consumers are uniformly distributed between 0 and $\bar{\theta}$, the fixed part is equal to:

$$p_f = \frac{v(p_e, K) - v(p_a)}{2}\left(1 - \sqrt{1 - 2\bar{\theta}Ac}\right) \tag{2.4.7}$$

where A is defined in equation (2.3.4). The fixed part of the tariff is an increasing function of both the marginal cost of the capital installed $r$ and the density of consumers. Indeed, if potential consumers are highly scattered, the number of agents for whom connection is socially profitable is low. Then, the operator has to increase the fixed fee per capita in order to cover the common costs. Moreover, when the marginal cost per kilometer $c$ increases, the marginal consumer is nearer the head of the network. Then there will be less consumers connected to the network. Hence, the operator has to charge each connected consumer more in order to cover the common cost $rK$. Here again, the impact of a variation of $K$ on the fixed part $p_f$ is ambiguous. However, as-

[19]See the details in appendix B.4.2.

suming that the fixed part of the tariff is an increasing function of the level of capital installed is a reasonable conjecture.

Given the coefficients of the two-part tariff constrained by the pricing rule for kilometers, the optimal level of capital under the uniform distribution of $\theta$ is determined by the following program:

$$\max_{K} \frac{\check{\theta}(p_f, c, K)}{\bar{\theta}} \left[ \check{\theta}(p_f, c, K) \frac{c\bar{\theta}}{2} + p_f \right] - rK \tag{2.4.8}$$

The optimal level of capital is $K^f$, implicitly defined by:

$$-\frac{p_f}{c\bar{\theta}^2} \frac{\partial p_f}{\partial K} + \frac{v(p_e, K^f) - v(p_a)}{c\bar{\theta}^2} \frac{\partial v(p_e, K^f)}{\partial K} = r \tag{2.4.9}$$

The second term in the left-hand-side of equation (2.4.9) has the same form as the marginal gain in the first-best solution (see equation (2.2.11)). Since we have assumed that the fixed part of the tariff is an increasing function of the level of capital, the first term in the LHS is negative. Thus, to satisfy the equality, we must have:

$$\left[ v(p_e, K^f) - v(p_a) \right] \frac{\partial v(p_e, K^f)}{\partial K} > \left[ v(p_e, K^*) - v(p_a) \right] \frac{\partial v(p_e, K^*)}{\partial K}$$

Hence, $K^f < K^*$. We observe the same bias as in the linear tariff, but the distortion is, *a priori*, weaker since the cost per kilometer is constrained to be equal to the marginal cost per consumer $c/f(\theta)$.

## 2.5 Concluding remarks

In this paper, we have explored the economic foundations of connection pricing to distribution networks. We have determined when it is and when it is not optimal to connect all consumers to the network. Depending on the common costs and on the energy price differential, $p_a - p_e$, the marginal consumer is close or far from the head of the network. The implementation of first best through linear tariffs would entail two drawbacks: i) the net profit of the operator would be negative and ii) the charge paid by each consumer would be proportional to the distance, which could be viewed as discriminatory on legal grounds.

To skip these drawbacks, we have studied the behavior of a regulated distributor as regards the linear price level and the amount of capital installed. As suspected, under the obligation to balance the budget, the number of consumers for whom the connection to the network is optimal is lower than at first best. Moreover, for activities with public service obligations, the distributors face additional regulatory constraints. We have studied the implementation of a uniform tariff across customers as well as a two-part tariff. When the operator implements a two-part tariff with a variable part equal to the marginal cost per kilometer for each consumer, the distortion with first best is slighter than in the non uniform linear tariff case. Finally, when the distributor has to serve as many consumers as at first best, we have found that a two-part tariff requires a variable part lower than the marginal cost per kilometer and the financial equilibrium is reached thanks to the fixed part of the tariff, creating cross-subsidies from consumers located close to the head of the network to those located at the end of lines or pipes.

Starting from this general framework, the model can be developed to include some specific characteristics of the different types of distribution networks, for example leakage for water distribution and thermal losses for electricity distribution. A second avenue of research is the reallocation of resources between heterogeneous regions when they are managed by the same operator or under the same regulatory obligations. Finally, we have considered activities in which the flows are unidirectional (from the head of the network to the final customers). However, in some activities like electricity distribution[20], the flows become bidirectional due to decentralized production units using renewables (photovoltaic panels, windmills). The model can be accommodated to include embedded intermittent sources of electricity and to see how these new energy sources may change the design of the network and the behavior of the distributors as regards pricing and investing.

[20]In water distribution, for physical and sanitary reasons, it is impossible to inject water in the network except at the pipe terminal.

# Chapter 3

# Alternative designs for the management of electric thermal losses

## 3.1 Introduction

Electricity distribution and transmission networks are the largest consumers of energy in most countries because of the necessity to compensate energy losses[1]. Losses are more important in distribution (around 6% of total consumption in France) than in transmission (around 2.5%) especially due to higher voltage and bigger diameter of wires used in transmission networks. According to the Commission de Régulation de l'Energie[2](report March 2010), in France, between 2009 and 2012 thermal losses have added up to 2 billions euros per year to the final consumers' bill. Therefore, reducing thermal losses is a key challenge for the transport activity according to the objectives fixed by the European Union and governments of Member States.

Regulators have explored alternative solutions in order to reduce the financial burden due to thermal losses. In the United Kingdom, even though the regulator encourages network operators to reduce their volume of losses by fixing a target based on historical performances, producers and retailers undergo the financial charges related to losses. This type of regulation is the exception rather than the rule, especially because it

[1]There are two types of electricity losses: i) technical losses are due to the length of the lines, how the network is designed and the material used for the lines and transformers; ii) non-technical losses correspond to non-recorded flows, metering errors or thefts. In developing countries, the proportion of non-technical losses may reach 50% (see for instance, the article "Lights off", in The Economist, February 11th, 2012). We only consider the first type of losses in this analysis.

[2]Called CRE in the following.

is difficult to disentangle non-technical losses and technical losses. In most European countries, Distribution Network Operators (DNO) are in charge of covering energy losses in distribution, under the supervision of the industry regulator and additional political constraints. For example, in some countries DNOs are obliged to buy the missing energy from the wholesale market because it allows to increase the volume traded[3].

In its 2010 report, the CRE compares different regulation schemes for thermal losses. The report considers:

- different roles for the agents: is the producer/retailer or the TNO/DNO in charge of the operation?
- different timing for the purchase of losses: short term (less than 3 years) and long term (between 3 and 6 years).

Following on this report we propose a microeconomic analysis of the management of thermal losses in transport networks and its impact on the level of investment in the infrastructure. The main question is whether the producer/supplier model is more efficient than the TNO/DNO model. From a theoretical point of view, the issue is close to the congestion problem, as both are a matter of externalities. Indeed, thermal losses vary with the square of the total energy flow. Then, when one firm changes its energy injection, the effect on losses depends on the quantity injected by all firms. Depending on the location of producers and consumers, the externality may be either positive or negative like in congestion problems.

In this paper, we compare three different situations concerning the management of thermal losses close to the ones existing in Europe:

i) there is only one operator that produces and distributes electricity, and covers thermal losses (the historical organization);

ii) the system operator is in charge of electricity distribution and thermal losses compensation whereas the producer sells separately to final consumers (close to the French model);

iii) the producer/retailer bears the financial burden due to thermal losses while the DNO is regulated on the volume of losses in its network (the English model).

There is little economic analysis of thermal losses. However, some works on congestion are useful references. For instance, Benitez [2004] studies transmission constraint in

[3]This is the case in France.

a two-node model where there is consumption at one node and consumption and two production plants at the other. He determines quantity competition equilibria in presence of a transmission constraint. The symmetric producers face an infinite local demand and an export-constrained one. Even if the transmission constraint may be close to the analysis of thermal losses, the modeling of the network is not close enough to the distribution network in which the electricity goes from an entry point (typically the end of the transmission network) to final consumers.

Willems [2002] also studies a two-node model where consumption and production are at different nodes. The thermal losses problem can be analyzed in a model similar to the one used by Willems, as his network modeling may describe a distribution network. However, he supposes that a dedicated market is created in order to ensure the allocation of transmission rights. Such a market does not exist for thermal losses, as, when the entities in charge have to buy energy-package, they do it on the electricity market. Covering thermal losses is a way to revitalize the demand-side of the electricity market.

Like congestion, thermal losses may create a distortion of competition and the producers may develop strategic behaviors to increase their market power. Joskow and Tirole [2005] or Borenstein, Bushnell and Stoft [2000] present the impact of congestion on market power. Hence, a producer may have an interest to increase its production to saturate a line and to increase its market power on its local demand. We do not take into account this strategic behavior in this paper.

The analysis of pumped storage by Crampes and Moreaux [2010] is also related. In the transformation of electricity to water and then water to electricity, 25% of the initial energy power is lost. Then, time transport, i.e. storage, has energy losses much larger than node transport. It takes a large difference in costs at several dates to make it profitable.

Schweppe *et al.* [1988] measure thermal losses in an electric network as a function of the quantity injected. Most of the time, thermal losses are defined as a percentage of the total quantity consumed. In line with this definition, we use a formula based on the quantity withdrawn from the grid[4].

The discussion on the type of management used for covering thermal losses comes from three regulator reports, two from national regulators, French and British, and a public consultation of the European regulators' group. In 2003, the British regulator, the Office of Gas and Electricity Markets (Ofgem) has launched a consultation concerning thermal losses on the distribution network. The aim was to find the determinants of

[4] More elements concerning thermal losses are presented in section 3.2.2 and appendix A.

thermal losses, the part that can be controlled and the optimal incentive schemes to reduce these losses. The French regulator has implemented an independent working group on thermal losses that has proposed a similar analysis in 2010. After presenting the importance of thermal losses in electricity networks and especially in distribution, they both discuss the different types of management that may be implemented in order to reduce thermal losses. The French report argues that the most efficient way (or the least bad solution) to compensate thermal losses is when Distribution Network Operators are in charge of the operation. Finally, the report of the European Regulators' Group for Electricity and Gas (ERGEG) in 2008 and the comments made by Eurelectric (an association of actors of the electric industry) in 2008 propose an overview of practices concerning the treatment of thermal losses across Europe.

In section 3.2, we set up a modeling of thermal losses. Section 3.3 is dedicated to the determination of the first-best outcome when losses are taken into account and the impact of thermal losses on the merit order that is a key tool in the liberalized electricity industry. Section 3.4 considers the vertically-integrated monopoly outcome which corresponds to the historical management of thermal losses. Section 3.5 presents and compares the most common management forms in Europe to determine which is the most efficient concerning the reduction of thermal losses. Finally, we conclude and present further research paths in section 3.6.

## 3.2 Modeling losses in electricity networks

### 3.2.1 Hypotheses and notations

For most of our analysis, we do not need to specify the topological characteristics of the network. We will introduce a simple one-line-two-nodes network later in the analysis.

**Consumers behavior** They consume a total quantity of electricity $Q_c$ and obtain a utility equal to $S(Q_c)$. The utility function has the usual feature, that is: $S(.) > 0$, $S'(.) > 0$ and $S''(.) < 0$. Consumers face a price $p_c$ per unit of consumed electricity and a unit price $t$ for the distribution of the quantity consumed. The quantity consumed $Q_c$ is expressed in $kW$. Knowing that we have a single period here, having the consumption expressed in $kWh$ or in $kW$ is equivalent[5].

[5]This assumption is also meaningful as long as the DNO is able to meter consumption every hour.

**Market structure** As we want to compare the impact of thermal losses' management on the level of consumption and investment in the market, and knowing that we assess the case of the public or private monopoly, we assume that production is always made by a single producer. This allows to get rid off the impact of competition on the industry. The producer has a marginal cost $c$ that may correspond, in reality, to the operating cost of the latest power plant called. It sells electricity at a unit price $p_c$.

**Externality** Depending on the nature of nodes (consumption and/or production), the externality created by thermal losses is either positive or negative. For a given demand, if there is production at each node of a line, as in the congestion problem, the externality is positive. Indeed, if producers at both nodes inject electricity, thanks to the netting effect, this tends to reduce thermal losses since the net flow that circulates on the line is decreased. This is true if we consider electricity transmission where electricity goes both ways. By contrast, in a distribution network where producers are localized at only one node, the externality is negative. The quantity injected by one producer impacts negatively the other production (Willems congestion modeling [2002]).

### 3.2.2 Measuring thermal losses

As in all transport activities, the transmission and distribution of electricity bring about losses. The quantity injected at the head of the network is larger than the quantity withdrawn at consumption nodes. In electricity networks, this difference is due to thermal losses created by the "Joule effect[6]".
These thermal losses are defined by the Joule-effect equation:

$$L \stackrel{def}{=} \rho Q_c^2$$

where $Q_c$ is the quantity consumed and $\rho$ is a parameter reflecting the resistance of the line. This parameter $\rho$ depends on the length of the line and its physical characteristics (diameter of wires, material used, etc). By installing less energy-consuming lines, a DNO reduces the value of $\rho$. The resistance parameter $\rho(K)$ is then a decreasing and convex function of the investment expenditures, that is the increase of quality and quantity of installed capital, $k = dK$. Thanks to the data of the main French

[6] Actually, there exist other effects that create electricity technical losses but we only focus on the "Joule effect" which is the main source. In France, around 80% of the technical losses are due to this effect (CRE, 2010).

distributor ERDF, we can set that the parameter $\rho$ is positive and very low[7].
In this paper, thermal losses depend on the quantity drawn and consumed. One could assume that losses are an increasing function of the quantity produced and injected (as in Schweppe *et al.* [1988]). In fact, it is neither the quantity injected nor the quantity consumed that matters. The quantity that induces thermal losses, changes at each meter of the line. Losses are higher close to the injection node than at the withdrawal node. The volume of energy in the line decreases due to thermal losses and then the electricity quantity in the line is progressively smaller, which implies smaller thermal losses, and so on down to the end of the line.
In this specification, we do not integrate non thermal losses or the fact that the energy used to cover losses creates thermal losses. Indeed, covering losses increases the quantity of electricity that runs through the line and thus increases the volume of thermal losses.

## 3.3 First-best outcome and merit order

### 3.3.1 First-best outcome

Social welfare in presence of thermal losses is:

$$W = S(Q_c) - cQ_c - c\rho Q_c^2 - rK$$

where $c$ is the unit production cost and $r$ the unit investment cost.

#### 3.3.1.a Level of consumption at first best

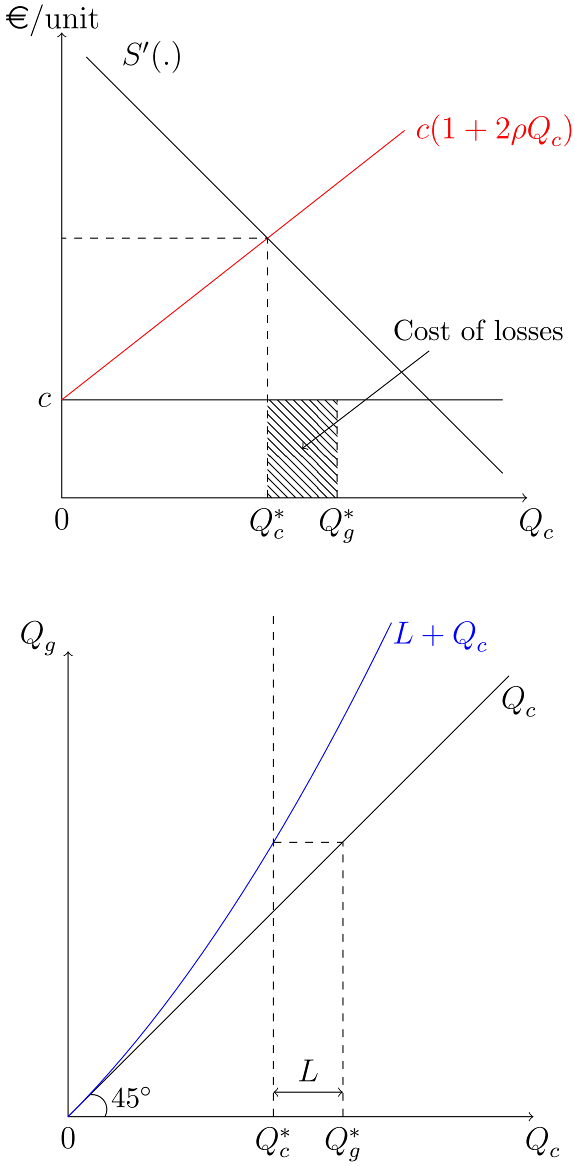
In the short term, the level of installed capital $K$ is fixed so that $\rho$ is given. Social welfare is maximized for a quantity $Q_c^*$ determined by the first-order condition:

$$S'(Q_c^*) - c - 2c\rho Q_c^* = 0 \tag{3.3.1}$$

Graphically, the first-best outcome is determined as follows:
In the upper part of figure 3.1, the quantity to consume is determined by the crossing of the marginal surplus of consumers and the marginal cost of the electricity produced and transported, that is the line $c(1 + 2\rho Q_c)$. The lower part of figure 3.1 represents the link between the quantity consumed and the quantity that has to be produced.

[7]Some data of ERDF concerning $\rho$ are available in table A.1. $\rho$ is around $10^{-9}kW$ per $kW$ transported on average.

Figure 3.1: First-best outcome




The curve $L + Q_c$ depicts the total production, that is the quantity consumed plus the level of thermal losses as a function of consumption. Using the 45° line, one can identify the optimal quantity generated $Q_g^*$ on the X-axis. Putting back this value in the upper part of figure 3.1, one may identify the cost of thermal losses represented by the hatched zone. This cost is equal to the difference between the quantity consumed and the quantity produced multiplied by the unit cost of electricity, i.e. $c.(Q_g^* - Q_c^*)$.

**Remark 1.** *From a normative point of view, if the total quantity consumed $Q_c^*$ has to be produced by $N$ firms, each producing $q_c^i(N)$, the optimal number of firms at first* *best would be determined by:*

$$\max_N \; W(N) = S(Nq_c^i(N)) - cNq_c^i(N) - c\rho[Nq_c^i(N)]^2$$

*It is important to note that the last term is NOT* $-c\rho N(q_c^i(N))^2$*, because the volume of losses is the result of the aggregate production. Then, the production cost incurred by each firm being just proportional to its output, by the envelop theorem, we have that the optimal number of firms is undetermined at first best.* $N$ *corresponds to the number of production plants and has no link to the number of lines that connect consumers to production sources. The total volume of thermal losses is only affected by the number of lines transporting electricity. In this model, one can assume that only one line exists to ensure the transport of electricity. Then, for a given level of production, the number of firms does not affect the total level of thermal losses and the first-best outcome.*

**Remark 2.** *In a competitive framework, managing thermal losses is a profitable activity. Indeed, if the competitive price of electricity is* $p^* = S'(Q_c^*)$*, the net profit of competitive firms is*

$$p^*Q_c^* - cQ_g^* = c(1 + 2\rho Q_c^*)Q_c^* - c(Q_c^* + \rho(Q_c^*)^2) = cL^* > 0$$

*With a cost of production increasing with the square of consumption, inframarginal rents are two times the cost of losses.*

#### 3.3.1.b Level of investment at first best

Let us now determine the optimal level of investment. The resistance parameter $\rho$ is a decreasing function of the installed capital $K$. Then, $K^*$ is given by:

$$\frac{\partial W}{\partial K} = 0 \Leftrightarrow \frac{dQ_c^*}{dK}\big(S'(Q_c) - c - 2c\rho Q_c\big) - c\rho'(K)\Big[Q_c(K)\Big]^2 - r = 0$$

Using the envelop theorem and equation (3.3.1), the optimal level of capital that should be installed, $K^*$ is determined by:

$$-c\rho'(K^*)\Big[Q_c(K^*)\Big]^2 = r \qquad (3.3.2)$$

Equation (3.3.2) reflects the equality between the marginal cost of installing an additional unit of capital, i.e. $r$, and its marginal benefits coming from the decrease in the operating cost generated by thermal losses, i.e.$-c\rho'(K^*)\Big[Q_c(K^*)\Big]^2$.

To complete the first-best analysis, let us look at the second-order condition in the long term. This condition is such that:

$$\begin{aligned} & - c\rho''(Q_c)^2 - 2c\rho' Q_c Q_c' < 0 \\ \Leftrightarrow\ & \rho'' Q_c + 2\rho' Q_c' > 0 \end{aligned} \tag{3.3.3}$$

Remember that $\rho' < 0$ and $\rho'' > 0$. The derivative of $Q_c$ with respect to $K$, coming from the first-order condition (equation (3.3.1)), is given by:

$$\begin{aligned} & S''(.)dQ_c - 2c\rho dQ_c - 2cQ_c\rho' dK = 0 \\ \Leftrightarrow\ & Q_c'(K) \overset{def}{=} \frac{dQ_c}{dK} = \frac{2cQ_c\rho'}{S''(.) - 2c\rho} > 0 \end{aligned} \tag{3.3.4}$$

The larger the investment in infrastructure, the larger the quantity that should be consumed. Therefore, to be satisfied, the second-order condition (3.3.3) imposes that: i) $|\rho'|$ and $Q_c'$ are not too large; and/or ii) $\rho''$ and $Q_c$ are large. Otherwise, we have a corner solution, i.e. the social planner decides not to increase the quantity of capital in the network.

Finally, we look at the impact of the investment in capital on the quantity produced, $Q_g$. This impact is given by:

$$\begin{aligned} \frac{dQ_g}{dK} &= \frac{dQ_c}{dK} + \frac{d(\rho Q_c^2)}{dK} \\ &= \left[\frac{(1 + 2\rho Q_c)2c}{S''(.) - 2c\rho} + Q_c\right]\rho' Q_c \end{aligned}$$

The investment in capital reduces the quantity produced as long as the expression into brackets is positive. This condition holds if and only if:

$$-Q_c S''(.) > 2c(1 + \rho Q_c) \tag{3.3.5}$$

As long as the marginal surplus of consumers is strongly decreasing ($S''(.) << 0$) or when the marginal cost of production is not too high[8], increasing the level of installed capital allows to reduce the quantity of electricity produced, $Q_g$, whereas the quantity of electricity consumed, $Q_c$, increases. In other words, under these conditions, when the level of installed capital increases, the quantity consumed increases and thermal losses significantly decrease. The decrease of $\rho(K)$ overcompensates the increase of $Q_c$ and the network is less energy-consuming.

[8]From section 3.2.2, and table A.1, we know that $\rho$ is much lower than 1.

### 3.3.2 Thermal losses and the merit order

Taking into account thermal losses may change the order in which power plants are called. To illustrate this point, the analysis of this section is limited to the short run[9] and we consider a two-node/one-line network with a single producer in North who is financially in charge of thermal losses generated. Consumers are located in South. The marginal cost of the producer in North is $c_n$.

The first-best outcome $Q_c^*$, defined in section 3.3.1, can be decentralized thanks to a price $p_s$ such that:

$$p_s = S'(Q_c^*) = c_n(1 + 2\rho Q_c^*) \tag{3.3.6}$$

At this price, the profit of the producer in North is given by:

$$\begin{aligned}
\Pi_n(Q_c^*) &= p_s Q_c^* - c_n\left(Q_g^*\right) \\
\Leftrightarrow \qquad &= c_n(1 + 2\rho Q_c^*)Q_c^* - c_n\left(Q_c^* + \rho(Q_c^*)^2\right) \\
\Leftrightarrow \Pi_n(Q_c^*) &= c_n\rho(Q_c^*)^2
\end{aligned}$$

Due to the existence of thermal losses, the producer makes a positive profit equal to:

$$\Pi_n(Q_c^*) = c_n L_n \tag{3.3.7}$$

where $L_n$ represents the level of losses on the line associated to the consumption of $Q_c^*$ units of electricity at the South node. The profit of the producer is then, in the short run, equal to the cost of losses because with a function equal to the square of quantities, losses generate inframarginal rents.

Let us assume now that a new plant is build in South. This plant has a higher marginal cost $c_s > c_n$[10]. A priori, absent production capacity or congestion constraint, this new plant is useless. However, as the plant is located at the consumption spot, the production (and the consumption) of a unit of electricity produced in the South plant does not induce thermal losses. Hence, beyond a given volume of consumption, the merit order we had implicitly in section 3.3.1 with only the Northern plant available is changed and the South plant is called. The new merit order is illustrated by figure 3.2.

The red line represents the new merit order of the plants. The total quantity consumed,

[9] we consider neither the optimal level of capital that should be installed in the network nor how this level is financed.

[10] That can illustrate the case of thermal power plants installed closer to consumption points although they have a higher short-run marginal cost.

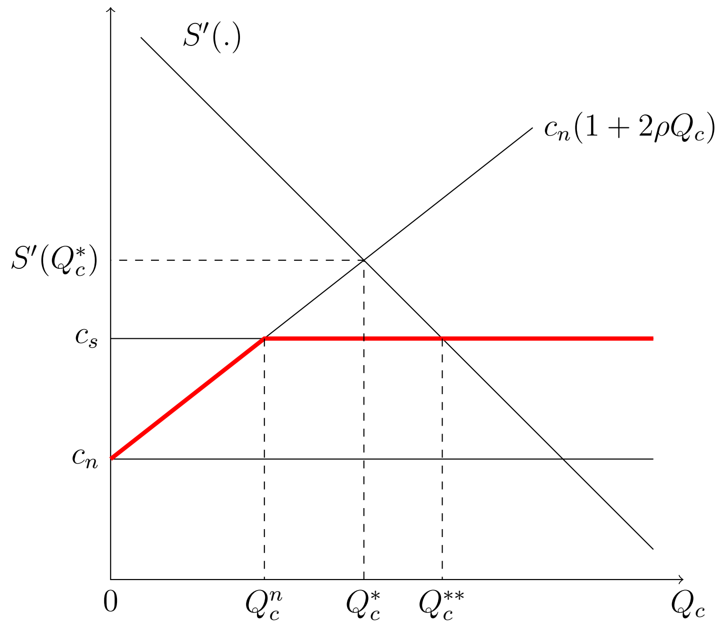



Figure 3.2: Merit order with a production plant in South

$Q_c^{**}$, has increased compared to the first-best outcome defined in section 3.3.1. The part of the total consumption produced by the production plant in North, $Q_c^n$, is determined by the equality of supplying costs, that is:

$$\begin{aligned} c_s &= c_n(1+2\rho Q_c^n) \\ \Leftrightarrow Q_c^n &= \frac{c_s - c_n}{2\rho c_n} \end{aligned} \tag{3.3.8}$$

The level of thermal losses created by the transmission of $Q_c^n$ is $L(Q_c^n) \overset{def}{=} \rho(Q_c^n)^2 < L_n$ where $L_n$ is defined in (3.3.7). This outcome can be decentralized thanks to market mechanism at nodal prices $p_s^{**} = c_s > p_n = c_n$. At these prices, the profits of the two producers are such that:

$$\begin{cases} \Pi_s = 0 \\ \Pi_n = c_n L(Q_c^n) \end{cases}$$

The producer in South makes zero profit, the producer in North faces lower profit whereas the consumers are better off. Installing a power plant close to the consumption node reduces the inframarginal revenue created by thermal losses earned by the firm in charge of the energy transport from the production plant in North to the consumption node. This point illustrates why it is essential to take into account thermal losses when determining the merit order. Similarly, when the governments, producers and regulators decide to build new generation plants, it is essential to internalize the cost of thermal losses. For instance, one may compare a low-cost plant plus the cost of the line and the cost of thermal losses and a power plant with a higher marginal cost but closer to the consumption spot and then without thermal losses.

We have seen that thermal losses generate an important cost in the electricity industry and its management generates profits. In what follows, we will focus on the distribution activity. We discuss who is in charge of this cost and the resulting effect on the level of investment.

## 3.4 Monopoly management

### 3.4.1 Private monopoly outcome

In this part, we study the decentralization of production, distribution and supply to a single private agent like in the traditional integrated firm case. As at first best, we first determine the short term outcome, that is the quantity sold by the monopoly, $Q_c^m$, solving:

$$\max_{Q_c} \Pi^m = p_c\Big(Q_c\Big)Q_c - cQ_c - c\rho\Big[Q_c\Big]^2$$

where $p_c(Q) \overset{def}{=} S'(Q)$ is the demand function. At the solution point $Q_c^m$, the first-order condition is:

$$p_c'(.)Q_c^m + p_c(.) - c - 2c\rho(K)Q_c^m = 0 \tag{3.4.1}$$

From the consumer surplus maximization, we know that: $S'(Q_c^m) = p_c(Q_c^m) = p_c^m$. Then, for a given level of capital, it is easy to see that $Q_c^m < Q_c^*$, where $Q_c^*$ is the first-best consumption level determined by (3.3.1). As usual, the decentralization to a private monopoly creates a loss of efficiency (represented by the term $Q_c p_c'(.) < 0$), absent at first best. Using the elasticity of final demand $\eta = \dfrac{-p}{qp'}$, one can rewrite (3.4.1) as

$$\frac{p_c^m - c}{p_c^m} = \frac{1}{\eta^m} + \frac{2c\rho Q_c^m}{p_c^m} \tag{3.4.2}$$

which shows, in a Ramsey's mode, that the monopolist obtains benefits from both market power (when the demand elasticity $\eta$ is low) and the inframarginal rents due to losses. We now turn to the long-run analysis. The optimal level of capital that the private monopoly has to install is determined by

$$\max_{K} \Pi^m = p_c\Big(Q_c^m(K)\Big)Q_c^m(K) - cQ_c^m(K) - c\rho\Big[Q_c^m(K)\Big]^2$$

where $Q_c^m$ is the function of $K$ determined by equation (3.4.1). Profit maximization leads to the level $K^m$ such that:

$$-c\rho'(K^m)\Big[Q_c(K^m)\Big]^2 = r \tag{3.4.3}$$

The monopoly applies the same optimal rule as at first best given by equation (3.3.2), but the investment is different from $K^*$ as it produces a quantity lower than $Q^*$. Starting from the comparison of $Q_c^m$ and $Q_c^*$, we can write:

$$\begin{aligned} Q_c^m &< Q_c^* \\ \Leftrightarrow -c\rho'(K^*)\Big[Q_c^m\Big]^2 &< -c\rho'(K^*)\Big[Q_c^*\Big]^2 = r = -c\rho'(K^m)\left[Q_c(K^m)\right]^2 \end{aligned}$$

by equations (3.3.2) and (3.4.3). Then $\rho'(K^*) > \rho'(K^m)$ and since $\rho'(K)$ is an increasing function ($\rho'' > 0$), it follows that

$$K^m < K^*$$

This result is standard in public service studies, as the decentralization to a non-regulated private agent cannot be done without a loss of efficiency. Then, the private monopoly invests less in the reduction of losses than the optimal volume. However, we also know that it produces less. Therefore, it potentially generates less thermal losses ceteris paribus. Consequently, the ranking of $L^m$ and $L^*$ cannot be asserted.

### 3.4.2 Public monopoly outcome

We have seen that the decentralization to a private monopoly is not satisfying on efficiency grounds. We now study the decentralization to a public monopoly to determine the second-best outcome. The problem that faces a public monopoly in charge of losses can be defined by

$$\begin{aligned} &\max_{Q_c,K} \; S(Q_c) - cQ_c - c\rho(K)\Big[Q_c\Big]^2 - rK \\ &\quad s.t. \; p_c(Q_c)Q_c - cQ_c - c\rho(K)\Big[Q_c\Big]^2 - rK \geqslant 0 \end{aligned}$$

Because of the inframarginal rents, the constraint may be not binding. Then, the public monopoly can implement the first-best outcome and choose a price $p_c(Q_c)$ such that:

$$p_c(Q_c^*) = S'(Q_c^*) = c(1 + 2\rho Q_c^*)$$

The corresponding level of capital is defined by equation (3.3.2). The point is to know whether the budget constraint is binding or not. The first-best solution is reachable as long as

$$\begin{aligned} &\Pi^* \geqslant 0 \\ \Leftrightarrow\ & p_c(Q_c^*)Q_c^* - cQ_c^* - c\rho(K^*)\left[Q_c^*\right]^2 - rK^* \geqslant 0 \\ \Leftrightarrow\ & c\rho(K^*)\left[Q_c^*\right]^2 \geqslant rK^* \end{aligned}$$

Using equation (3.3.2), the previous condition becomes:

$$\begin{aligned} c\rho(K^*)\left[Q_c^*\right]^2 &\geqslant -c\rho'(K^*)\left[Q_c^*\right]^2 K^* \\ \Leftrightarrow -\rho'(K^*) &\leqslant \frac{\rho(K^*)}{K^*} \\ \Leftrightarrow \xi(K^*) &\leqslant 1 \end{aligned} \tag{3.4.4}$$

where $\xi(K) \overset{def}{=} -\rho'(K)\dfrac{K}{\rho(K)}$. As long as the marginal resistance (i.e. the variation of the resistance of the network for an additional unit of capital corrected for the sign) is lower than the average resistance of the network at $K^*$, the constraint is not binding and first best can be achieved. From the discussion following equations (3.3.3) and (3.3.4), we have seen that the second-order condition requires to have $|\rho'|$ not too large, which makes the condition above easier to reach. Then, the condition (3.4.4) is satisfied for small values of installed capital $K$, for which the break-even constraint is not binding[11].

By contrast, if the first-best outcome is not reachable, the optimal quantity of electricity consumed, the optimal level of capital at second best and the shadow value of the budget constraint $\lambda$ are determined by the budget balancing condition plus the

[11] See appendix C.1.

two following equations:

$$\begin{cases} \dfrac{p_c(Q_c^{sb}) - c}{p_c(Q_c^{sb})} = \dfrac{\lambda}{1+\lambda}\left[\dfrac{1}{\eta^{sb}} + \dfrac{2c\rho Q_c^{sb}}{p_c^{sb}}\right] \\ -c\rho'(K^{sb})\Big[Q_c(K^{sb})\Big]^2 = r \end{cases} \tag{3.4.5}$$

The first equation of the system above, the short-term outcome, is a classical Ramsey pricing result accommodated to take the marginal thermal losses into account. We see that it differs from (3.4.2) by the values of $\rho$ and $\eta^{sb}$ and by the coefficient $\dfrac{\lambda}{1+\lambda} < 1$, where $\lambda$ represents the Lagrange multiplier. The higher $\lambda$, the higher the energy price and the lower the quantity consumed.
We summarize our results in the following proposition where $^*$ identifies first best, $sb$ second best and $m$ private monopoly variables.

**Proposition 3.1.**
*The quantity consumed $Q_c$, the capital invested in losses reduction $K$ and the resulting resistance of lines $\rho$ are such that:*

$$\begin{cases} Q_c^m < Q_c^{sb} < Q_c^* \\ K^m < K^{sb} < K^* \\ \rho^m > \rho^{sb} > \rho^* \end{cases} \tag{3.4.6}$$

Note that as $\rho' < 0$, the total effect on the thermal losses level, or on the average loss in the network, $\rho Q_c$, is undetermined without a specification of the function $\rho(K)$.

## 3.5 Thermal losses in the unbundled industry

Nowadays, due to the liberalization of the electricity industry, the historical model of the vertically integrated incumbent does not exist anymore. Transmission and distribution networks are separated from production and retail[12]. To go further in the economic analysis of thermal losses, we must inspect this organizational novelty. Considering the European case, two types of management for thermal losses are studied depending on who is in charge of compensating for energy losses. We successively consider the case where the demand is stationary (sections 3.5.1 and 3.5.2) then the case where demand is time-varying (section 3.5.3).

[12] At least, the separation is effective from an operational point of view.

### 3.5.1 The DNO is in charge

This is the type of losses management most used in Europe. It is the framework implemented in France. DNOs buy energy packages on the energy market or through call for tenders. The transparency of the process guarantees that DNOs cannot choose a particular power plant to compensate the energy lost so that they face the same price than consumers, i.e. $p_c$[13]. They receive a tariff equal to $t$ per unit distributed, fixed by the regulatory authority ex-ante. This tariff always comes from a price-cap regulation system. In France, $t$ is given by the TURPE (tarif d'utilisation des réseaux publics d'électricité) fixed by the government following a proposal made by the energy regulator, CRE. Actually, it is a two-part tariff. The variable part of the tariff depends on the quantity consumed and the fixed part is based on the power contracted by the consumers. However, without hazard or demand cycles, it is sufficient to only consider the variable part of the tariff to describe the impact of the distribution cost from a consumer point of view. The TURPE aims at covering the operating cost, only represented here by energy losses, and the investment cost, $rK$ in our model.
Let us first determine the optimal level of consumption under this legal framework.

#### 3.5.1.a Production and consumption

To assess the outcome of this management rule for thermal losses, we have to determine the agents' behavior. Let me first determine the quantity that consumers and the DNO want to buy. The consumption level depends on the price of a unit of electricity, $\widehat{p}_c$ and the cost of using the network per unit of electricity, $t$. The consumers' net utility is then maximized if and only if:

$$S'(\widehat{Q}_c) = \widehat{p}_c + t$$

The quantity consumed is then a decreasing function of both the price of electricity, $\widehat{p}_c$ and the distribution tariff, $t$. Total demand also includes demand coming from the DNO, equal to thermal losses, i.e. $L = \rho\widehat{Q}_c^2$. We assume that even if the industry is unbundled, it still remains a monopoly for the production of electricity. This can be motivated by the dominant position that incumbents still have in European countries.

[13]In France, the new law (NOME, n° 2010-1488 of December 7th, 2010) and the order of November 25th, 2011, give distributors the right to buy, from August 1st, 2013, the energy they need to cover thermal losses from installed nuclear sources at a price defined by the ARENH (Accès Régulé à l'Electricité Nucléaire Historique), which is lower than the market price. More details concerning the law NOME and ARENH are at: `http://www.legifrance.gouv.fr` and `http://www.cre.fr/dossiers/la-loi-nome#section4`.

Then, the quantity generated, $Q_g$, is equal to: $Q_g = \widehat{Q}_c + \rho\widehat{Q}_c^2$. The program of the monopoly is, for a given t:

$$\max_{\widehat{p}_c} (\widehat{p}_c - c)Q_g$$

The first-order condition of such a program can be written as:

$$(\widehat{p}_c - c)\frac{dQ_g}{d\widehat{p}_c} + Q_g = 0 \tag{3.5.1}$$

The evolution of $Q_g$ with respect to $\widehat{p}_c$ depends on the evolution of $\widehat{Q}_c$. We have:

$$\frac{dQ_g}{d\widehat{p}_c} = \frac{d\widehat{Q}_c}{d\widehat{p}_c}(1 + 2\rho\widehat{Q}_c) = -\widehat{\eta}\frac{\widehat{Q}_c}{\widehat{p}_c}(1 + 2\rho\widehat{Q}_c)$$

where $\widehat{\eta}$ is the elasticity of the quantity consumed by final consumers with respect to price. If we inject this result into equation (3.5.1), the FOC of the monopoly program becomes:

$$\begin{aligned} \widehat{Q}_c + \rho\widehat{Q}_c^2 &= (\widehat{p}_c - c)(1 + 2\rho\widehat{Q}_c)\widehat{\eta}\frac{\widehat{Q}_c}{\widehat{p}_c} \\ \Leftrightarrow \qquad \frac{\widehat{p}_c - c}{\widehat{p}_c} &= \frac{1}{\widehat{\eta}}\frac{1 + \rho\widehat{Q}_c}{1 + 2\rho\widehat{Q}_c} \end{aligned} \tag{3.5.2}$$

The second ratio of the right-hand side in the previous equation is a loss factor integrating the average and the marginal level of losses. Knowing that the marginal level of losses (at the denominator) is twice the average level of losses (at the numerator), equation (3.5.2) results in $\frac{\widehat{p}_c - c}{\widehat{p}_c} < \frac{1}{\widehat{\eta}}$. From the first line of proposition 3.1, we know that $p_c^m > p_c^{sb} > c$. If the price elasticity is non decreasing in price $p_c$, we have that $p_c^m > \widehat{p}_c$[14].

To compare $p_c^{sb}$ and $\widehat{p}_c$, we need stronger restrictions. For instance, if $\eta$ is non decreasing in price and $\lambda$ is large enough, we have: $p_c^{sb} > \widehat{p}_c$[15]. To have $\lambda$ large means that the inframarginal rents are not sufficient to cover the cost $rK$. Then, the public monopoly has to fix a higher price $p_c^{sb}$. However, when the price of electricity consumed $p_c$ is separated from the transport cost $t$, the fixed cost $rK$ may be covered by a regulated tariff $t$ high[16]. In this case, the level of consumption when the distributor is in charge

[14]The proof is in appendix C.2.

[15]This is true as long as $\lambda > \frac{-(p_c^{sb})'}{2c\rho}$. More details are given in appendix C.3.1.

[16]From the optimization of the consumers' surplus it comes that $t$ and $p_c^{sb}$ have the same impact

of thermal losses, $\widehat{Q}_c$, is greater than the level of consumption in the public monopoly case, $Q_c^{sb}$.

To complete the analysis of this type of thermal losses management, the optimal level of investment in capital has to be determined.

### 3.5.1.b Optimal level of investment

In this context, the distributor has two types of cost: i) the operating cost linked to the purchase of electricity in order to cover energy losses; and ii) the investment cost. By increasing the level of installed capital, the distributor reduces the effective level of thermal losses and, then, its operating cost. Thus again, there exists an arbitrage between the operating cost and the investment cost. The optimal level of investment is a long term decision coming from the following program:

$$\max_{\widehat{K}} -r\widehat{K} + t\widehat{Q}_c - \widehat{p}_c\rho\left(\widehat{K}\right)\widehat{Q}_c^2$$

Under regulated tariff $t$, the first-order condition associated to this program can be written as:

$$-\widehat{p}_c\rho'\left(\widehat{K}\right)\left[\widehat{Q}_c\right]^2 = r \tag{3.5.3}$$

The first-order condition is close to the ones found in section 3.4 except that the marginal cost $c$ appearing in the conditions of section 3.4 is replaced by the price of electricity $\widehat{p}_c$. Starting from the comparison of the quantities consumed, $Q_c^m$ and $\widehat{Q}_c$, we can write:

$$Q_c^m < \widehat{Q}_c$$
$$\Leftrightarrow -c\rho'(K^m)\left[Q_c^m\right]^2 < -\widehat{p}_c\rho'(K^m)\left[\widehat{Q}_c\right]^2$$

Both sides of the above inequality are decreasing functions of $K$ and are evaluated at the same level of capital. Following the first-order condition, $-\widehat{p}_c\rho'\left(\widehat{K}\right)\left[\widehat{Q}_c\right]^2 = r$ requires a level of installed capital $\widehat{K}$ such that:

$$K^m < \widehat{K}$$

However, comparing with the public monopoly outcome is less obvious because we

on the quantity consumed, i.e. $\dfrac{dQ_c}{dp_c} = \dfrac{dQ_c}{dt} = \dfrac{1}{\dfrac{\partial^2 S(Q_c)}{\partial {Q_c}^2}}$.

may have either $p_c^{sb} > \widehat{p}_c$ or $\widehat{p}_c > p_c^{sb}$. The simplest case is when $p_c^{sb} > \widehat{p}_c$, that is when $\widehat{Q}_c > Q_c^{sb}$. Knowing that:

$$-\widehat{p}_c\rho'\left(\widehat{K}\right)\left[\widehat{Q}_c\right]^2 = r = -c\rho'(K^{sb})\left[Q_c^{sb}\right]^2 \tag{3.5.4}$$

we can compare the optimal level of capital installed either by the regulated monopoly or by the distributor when it has to cover thermal losses. Equation (3.5.4) is equivalent to:

$$\frac{\rho'(K^{sb})}{\rho'(\widehat{K})} = \frac{\widehat{p}_c}{c}\frac{\left[\widehat{Q}_c\right]^2}{[Q_c^{sb}]^2} \tag{3.5.5}$$

Using the price and the quantity comparison of section 3.5.1.a, it is easy to see that the right hand side of equation (3.5.5) is larger than 1. Then, knowing that $\rho'(.)$ is a negative and increasing function of $K$, we can write that:

$$\begin{aligned} & \rho'(K^{sb}) < \rho'(\widehat{K}) \\ \Leftrightarrow \quad & K^{sb} < \widehat{K} \end{aligned} \tag{3.5.6}$$

As long as the price set by the vertically integrated public monopoly is larger than the one set in the case where the DNO is in charge of thermal losses, the latter will have a network with higher quality and less thermal losses.

By contrast, when $\widehat{p}_c > p_c^{sb}$, the investment comparison, i.e. determining if the ratio (3.5.5) is lower or larger than 1, is not so clear. When $\widehat{p}_c > p_c^{sb}$, we know that: $\widehat{p}_c > c$ and $\left(\widehat{Q}_c\right)^2 < \left(Q_c^{sb}\right)^2$. The comparison of the optimal level of capital that should be installed again depends on the price elasticity of the electricity demand. Indeed, if the price elasticity is low, a difference in price would induce a small gap between the quantity consumed. In this case, even though $\widehat{p}_c > p_c^{sb}$, the quantities consumed would be very close and then, $-\widehat{p}_c\rho'\left(\widehat{K}\right)\left[\widehat{Q}_c\right]^2$ would be greater than $-c\rho'(K^{sb})\left[Q_c^{sb}\right]^2$, which means that the optimal level $\widehat{K}$ installed by the independent DNO would be larger than the one installed by the regulated monopoly, $K^{sb}$, i.e. $\widehat{K} > K^{sb}$. However, if the quantities are very sensitive to price variations, then even if the difference in price is small, the difference in quantities consumed would be very important. With a the quantity effect, i.e. the difference between $\widehat{Q}_c$ and $Q_c^{sb}$ more important than the difference in cost, i.e. between $\widehat{p}_c$ and $c$[17], we would have: $K^{sb} > \widehat{K}$.

Different studies published on price elasticity in electricity sector show that for small consumers (connected to the distribution network) price elasticity is quite low, i.e.

[17] $\widehat{p}_c$ is the cost of electricity bought to cover thermal losses when the DNO is in charge of them whereas $c$ is the cost of covering thermal losses in the public monopoly case.

lower than 1 in absolute value[18]. Therefore, it seems that, when $\widehat{p}_c > p_c^{sb}$, the DNO in charge of thermal losses invests more in the network than the regulated monopoly, and we have:

$$\widehat{K} > K^{sb} \tag{3.5.7}$$

Now, let me see the other management design existing in Europe, i.e. when the producer or the retailer is, a priori, in charge of thermal losses.

### 3.5.2 The producer/retailer is in charge

Only five European countries are using this type of loss management: Ireland, Italy, Portugal, Spain and the United-Kingdom[19]. Even if the details may differ across countries, the principle is the same. DNOs do not bear the financial charges of thermal losses, a priori, even though they have to respect a target expressed as a global volume of losses. They may be rewarded or penalized if the effective volume of losses in the network is greater than a target determined by historical data. Producers and/or retailers are assumed to be in charge of losses compensation. In Portugal for instance, producers are in charge of thermal losses and their production is multiplied by a coefficient corresponding to a standardized level of losses. In Spain, the retailers have to pay for a standardized level of losses. The part paid by each retailer is a percentage of its clients' consumption. If the effective level of losses is above the benchmark, then the DNOs pay for the difference. In the United Kingdom, both retailers and producers are financially charged.

Hereafter, we consider a mix of these different applications, i.e. we assume that the monopolist producer is in charge of thermal losses as long as the global volume of thermal losses is lower than a fixed level. Otherwise, the producer covers losses up to the standardized level and the DNO is in charge of the difference and buys it on the energy market. As in the previous section, let me first determine the level of electricity that is consumed.

#### 3.5.2.a Optimal level of consumption

Let $\overline{L}$ be the target fixed by the regulator on historical data. To determine the optimal levels of investment and of consumption, we assume here that we know whether $L < \overline{L}$

[18]See for instance a report published in June 2010 by the regulatory authority of Belgium, CREG, that sums up different studies on electricity price-elasticity. "*Etude relative à la faisabilité de l'instauration d'une tarification progressive de l'électricité en Belgique*".

[19]More details are available in the report of Eurelectric, *"Comments on the ERGEG Position Paper for Public Consultation on Treatment of Losses by Network Operators"* of October 8th, 2008.

or $L \geqslant \overline{L}$ as it is the case when demand is stationary.
When the effective volume of losses is lower than the historical average (or the standardized level), only the producer is in charge of paying for thermal losses. By contrast when the effective volume is greater than the standardized level, both the producer and the distributor pay for losses. Then, to determine the optimal level of consumption one has to define the different components of the total demand depending on the effective level of thermal losses. As in the "DNO-in-charge" case, final consumers equalize their marginal utility of consuming $Q_c$ units of electricity to the marginal price of this consumption. So, for final consumers, we still have: $S'(Q_c) = p_c + t$. The second component of total demand is the quantity of electricity that the distributor buys in order to cover thermal losses. Let $\overline{L}$ stand for the regulated benchmark. The distributor's demand of energy follows:

$$D^d = \max\{0, \rho Q_c^2 - \overline{L}\}$$

Then, the total demand, $Q_d$, that the monopoly has to satisfy is given by:

$$Q_d = \begin{cases} Q_c(p_c + t) + \rho[Q_c(p_c + t)]^2 - \overline{L} & \text{if } L \geqslant \overline{L} \\ Q_c(p_c + t) & \text{otherwise} \end{cases}$$

Let me treat the producer's behavior in the two different cases that depend on the level of thermal losses.

- $\underline{L < \overline{L}}$:

  In this case, the monopoly is fully in charge of thermal losses. Its short-run program is the same as the one of the historical vertically integrated monopoly assessed in section 3.4, i.e.:

$$\max_{p_c} (p_c - c)Q_c - c\rho Q_c^2$$

  The first-order condition associated to this program is then given by:

$$\frac{p_c - c}{p_c} = \frac{1}{\eta} + \frac{2c\rho Q_c}{p_c} > \frac{1}{\eta} \tag{3.5.8}$$

  Comparing with (3.4.2), we see that the outcomes, the price and the quantity consumed, are the same as the ones seen in section 3.4, for a given level of installed capital, that is: $Q_c = Q_c^m$, $p_c = p_c^m$, and $\eta = \eta^m$.

- $\underline{L \geqslant \overline{L}}$:

  In this case, both the monopoly and the distributor pay for thermal losses. The monopolist program can be written as:

$$\max_{\widetilde{p}_c}(\widetilde{p}_c - c)\widetilde{Q}_c - c\rho\widetilde{Q}_c^2 + \widetilde{p}_c(\rho\widetilde{Q}_c^2 - \overline{L})$$

  The optimal price for the monopoly when the effective level of thermal losses is greater than the standardized one is such that:

$$\frac{\widetilde{p}_c - c}{\widetilde{p}_c} = \frac{1}{\widetilde{\eta}}\,\frac{1 + \rho\widetilde{Q}_c - \overline{L}/\widetilde{Q}_c}{1 + 2\rho\widetilde{Q}_c} < \frac{1}{\widetilde{\eta}} \qquad (3.5.9)$$

  Ceteris paribus, the price decided by the monopoly in this case is lower than the one fixed when the effective level of thermal losses is below the threshold $\overline{L}$. As energy losses induce an inframarginal rent, the monopoly may reduce its price in order to increase the level of consumption and consequently thermal losses in the network. The monopolist has a profit by selling a quantity $(L - \overline{L})$ to the DNO at a price $\widetilde{p}_c > c$. Increasing losses will increase the rent linked to these losses. Then, the level of consumption in this case, $\widetilde{Q}_c$, is greater than $Q_c^m$.

Which case is relevant depends on the demand regime. During off-peak period, demand, and then, thermal losses are weak and the price follows equation (3.5.8). From appendix C.2, it is easy to see that $\widetilde{p}_c < p_c^m$ and that the quantities consumed are such that: $\widetilde{Q}_c > Q_c^m$. By contrast, during on-peak periods, i.e. when demand is high, the price is determined by equation (3.5.9). If $\overline{L} = 0$, then we are permanently in the second situation where the price is determined by (3.5.9), which is, for such a value of $\overline{L}$, equal to equation (3.5.2) since the DNO is fully responsible for the compensation of energy losses. We observe that $\frac{\widetilde{p}_c - c}{\widetilde{p}_c}$ is a decreasing function of $\overline{L}$, then, as soon as, $\overline{L} > 0$, the price comparison entails: $\widetilde{p}_c < \widehat{p}_c$ and, for the quantity consumed, we have: $\widetilde{Q}_c > \widehat{Q}_c$.

**Proposition 3.2.**
*When losses overpass the regulated target $\overline{L}$, for a given level of installed capital, the ranking of prices and levels of consumption in the different types of management is such that:*

$$\begin{cases} p_c^m > \widehat{p}_c > \widetilde{p}_c \\ \widetilde{Q}_c > \widehat{Q}_c > Q_c^m \end{cases} \qquad (3.5.10)$$

One has to note that, in reality, $\overline{L}$ is endogenous and depends on the investment made in previous years. Then, in this type of management, the DNOs may anticipate the impact of their behavior on the future effort that they have to exert to reduce thermal losses.

### 3.5.2.b Optimal level of investment

Again, we successively consider the cases where $L < \overline{L}$ and $L \geqslant \overline{L}$.

- $\underline{L < \overline{L}}$:

  In this case, thermal losses have no impact on the DNO's behavior since they are covered by the producer/retailer. The program of the DNO can be written as:

$$\max_K tQ_c(K,t) - rK \tag{3.5.11}$$

  where $Q_c(K,t)$ is defined by $S'(Q_c) = p_c + t$ and $p_c$ follows equation (3.5.8). The first-order condition linked to this program is:

$$t\frac{dQ_c}{dK} = r \tag{3.5.12}$$

  From the study of the short-term outcome, this first-order condition can be rewritten as[20]:

$$t\ \frac{\partial Q_c^m}{\partial p_c^m}\ \frac{dp_c^m}{d\rho(K)}\ \rho'(K) = r$$

$$\Leftrightarrow t\ \frac{\partial Q_c}{\partial p_c^m}\ \frac{2cQ_c^m}{1-(Q_c^m)'\left(-(p_c^m)'+2c\rho\right)}\ \rho'(K) = r$$

$$\Leftrightarrow -c\rho'(K)\left[Q_c^m\right]^2\ A = r \tag{3.5.13}$$

  where $A = \dfrac{2t\eta^m}{p_c^m\left[1-(Q_c^m)'(-(p_c^m)'+2c\rho)\right]}$. As long as $A < 1$[21], it is easy to compare the optimal level of investment in this case and the one chosen by the private monopoly. Indeed, the decision rules are such that:

$$-c\rho'(\breve{K})\left[Q_c^m\right]^2\ A = r = -c\rho'(K^m)\left[Q_c^m\right]^2$$

[20] For more details, see appendix C.4.1.

[21] This is true for $t$ such that $t < p_c^m$. See appendix C.4.2.

This equality is satisfied for $\breve{K}$ such that

$$\breve{K} < K^m$$

Consequently, the level of investment is weaker than the one made by the private monopoly and, by transitivity, lower than all the levels of investment analyzed so far.

- $\underline{L \geqslant \overline{L}}$:

In this part, the distributor, in charge of the infrastructure investment, is partially involved in the compensation of thermal losses. Its program follows:

$$\max_{\widetilde{K}} t\widetilde{Q}_c - \widetilde{p}_c\rho\Big(\widetilde{Q}_c\Big)^2 + \widetilde{p}_c\overline{L} - rK$$

After simplification due to the envelop theorem, the first-order condition associated to this program is:

$$-\widetilde{p}_c\rho'(\widetilde{K})\Big(\widetilde{Q}_c\Big)^2 = r \tag{3.5.14}$$

The first-order condition that determines the optimal level of investment has the same shape as in the others cases, except the one above when $L < \overline{L}$. It is easy to compare the optimal investment in this case, $\widetilde{K}$, and the one decided by a vertically integrated private monopoly. From the discussion on the prices comparison in the previous section, we can write that:

$$\begin{gathered}\widetilde{Q}_c > Q_c^m \\ \Leftrightarrow -\widetilde{p}_c\rho'(K^m)\Big(\widetilde{Q}_c\Big)^2 > -c\rho'(K^m)\Big(Q_c^m\Big)^2 = r\end{gathered}$$

Knowing that the left-hand side of equation (3.5.14) is a decreasing function of $\widetilde{K}$, the first-order condition is satisfied if and only if, $\widetilde{K}$ is such that:

$$\widetilde{K} > K^m$$

The most interesting comparison concerns the optimal level of investment in this case and the one decided when the DNO is in charge of thermal losses, i.e. $\widehat{K}$.

It is easy to see that the decision rules are close in these two cases, as we have:

$$-\widetilde{p}_c\rho'(\widetilde{K})\left(\widetilde{Q}_c\right)^2 = r = -\widehat{p}_c\rho'(\widehat{K})\left(\widehat{Q}_c\right)^2 \tag{3.5.15}$$

We know from section 3.5.2.a that when $\overline{L} = 0$, the price and the quantity consumed in the two cases are the same. Then, the left and the right function in (3.5.15) are identical, and equating it to $r$ gives

$$\widetilde{K} \equiv \widehat{K} \tag{3.5.16}$$

Moreover, we have shown that $\widetilde{p}_c$ is a decreasing function of $\overline{L}$ and, thus, that $\widetilde{Q}_c$ is an increasing function of $\overline{L}$[22]. Then, the evolution of $\widetilde{K}$ depends on the price elasticity of electricity demand. If the price elasticity is low, the increase of $\overline{L}$ has a strong effect on the price $\widetilde{p}_c$ but little impact on the quantity $\widetilde{Q}_c$. In this case, the price $\widetilde{p}_c$ is lower than the price $\widehat{p}_c$ whereas the quantities consumed in both cases are close. Then, we can write that, for a given level of investment $K$:

$$-\widetilde{p}_c\rho'(K)\left(\widetilde{Q}_c\right)^2 < -\widehat{p}_c\rho'(K)\left(\widehat{Q}_c\right)^2$$

To have the equality of equation (3.5.15), the levels of investment of the two cases should be ranked such that:

$$\widetilde{K} < \widehat{K}$$

By contrast, in the less likely case where the price elasticity of demand is very high, an increase of $\overline{L}$ would have a small effect on $\widetilde{p}_c$, but this small decrease of the price would imply an important increase of the consumers' demand, such that:

$$-\widetilde{p}_c\rho'(K)\left(\widetilde{Q}_c\right)^2 > -\widehat{p}_c\rho'(K)\left(\widehat{Q}_c\right)^2$$

Here, ranking the two levels of investment leads to:

$$\widetilde{K} > \widehat{K}$$

The comparison of the optimal level of investment is summarized in figure 3.3.

[22]Remember that the relation between the price $\widetilde{p}_c$ and $\overline{L}$ is given by equation (3.5.9).

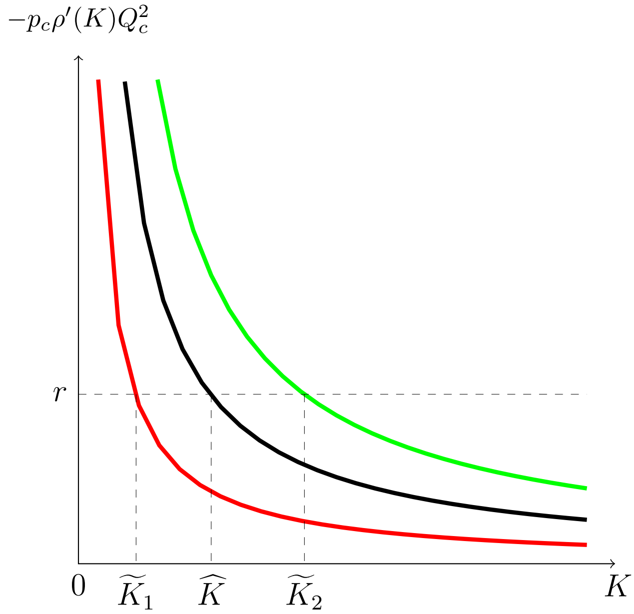



Figure 3.3: Optimal level of investment

The black curve represents the investment function $-\widehat{p}_c\rho'(K)\left(\widehat{Q}_c\right)^2$ whereas the red one (respectively the green one) represents $-\widetilde{p}_c\rho'(K)\left(\widetilde{Q}_c\right)^2$ when the price elasticity is low (respectively high). According to the study quoted in section 3.5.1.b, the price elasticity of electricity is low (i.e. lower than 1). Then, we can conclude that, when the producer is in charge of thermal losses and when thermal losses are above the threshold $\overline{L}$, the optimal level of investment is lower than when the DNO in charge of paying for all thermal losses: $\widetilde{K}_1 < \widehat{K}$.

## 3.5.3 Fluctuating demand

We analyze the case of a producer in charge of thermal losses when the demand is fluctuating. When $\overline{L} = 0$, thermal losses are always covered by the DNO and this case is equivalent to the DNO in charge case. We determine the optimal level of consumption and investment in both cases.

### 3.5.3.a Optimal level of consumption

**The producer/retailer is in charge**

We have seen in the previous section that being above or below $\overline{L}$, fixed exogenously, could be representative of on-peak and off-peak periods. Let $\tau$ be an index of different periods of consumption, between 0 and $\tau_{max}$.

The consumer's surplus is given by $S(Q_c, \tau)$ when they consume $Q_c$ in state $\tau$. We assume that, in this case, the surplus function is such that: $\dfrac{\partial S}{\partial \tau} > 0$ and $\dfrac{\partial^2 S}{\partial \tau \partial Q_c} > 0$. The demand function $Q_c(p_c, t, \tau)$ is then given by:

$$\frac{\partial S}{\partial Q_c} = p_c + t$$

From the study of the surplus function, we have: $\dfrac{\partial Q_c}{\partial \tau} = -\dfrac{\dfrac{\partial^2 S}{\partial \tau \partial Q_c}}{\dfrac{\partial^2 S}{\partial {Q_c}^2}} > 0$.

In this specification, the level of thermal losses is defined by:

$$L(p_c, t, \tau, K) \stackrel{def}{=} \rho(K) Q_c(p_c, t, \tau)^2$$

It comes that the level of thermal losses is a decreasing function of $p_c$, $t$ and $K$ and is increasing in $\tau$. We have seen that the regime is changing if thermal losses $L$ are below or above $\overline{L}$, so let us define:

$$\tau(p_c, t, \overline{L}, K) \stackrel{def}{=} \arg_\tau \left\{ L(p_c, t, \tau, K) = \overline{L} \right\} \tag{3.5.17}$$

This state of nature $\tau(p_c, t, \overline{L}, K)$ is an increasing function of $p_c$, $t$, $K$ and $\overline{L}$. To simplify the notations, we do not mention the variable $t$ in $\tau(.)$, $L(.)$ and $Q_c(.)$. When the DNO is in charge of thermal losses, like in France, $\tau(p_c, t, \overline{L}, K)$ is equal to 0.

The producer cannot wait to know the realization of $\tau$ to fix $p_c$. Then, its program is:

$$\begin{aligned}
&\max_{p_c} \quad \int_0^{\tau(p_c, \overline{L}, K)} \left[ (p_c - c) Q_c(p_c, \tau) - c\rho(K) Q_c(p_c, \tau)^2 \right] f(\tau) d\tau \\
&\qquad + \int_{\tau(p_c, \overline{L}, K)}^{\tau_{max}} \left[ (p_c - c)\big( Q_c(p_c, \tau) + \rho(K) Q_c(p_c, \tau)^2 \big) - p_c \overline{L} \right] f(\tau) d\tau \\
&\Leftrightarrow \max_{p_c} \quad (p_c - c) E[Q_c(p_c, \tau)] - cE[L(p_c, \tau, K)] + p_c \int_{\tau(p_c, \overline{L}, K)}^{\tau_{max}} \left[ \rho(K) Q_c(p_c, \tau)^2 - \overline{L} \right] f(\tau) d\tau
\end{aligned} \tag{3.5.18}$$

The first-order condition associated to this program that determines $\mathring{p}_c \stackrel{def}{=} p_c(\overline{L}, K)$, is:

$$\begin{aligned}
&E[Q_c(\mathring{p}_c,\tau)] + (\mathring{p}_c - c)E\left[\frac{\partial Q_c(\mathring{p}_c,\tau)}{\partial \mathring{p}_c}\right] - cE\left[\frac{\partial L(\mathring{p}_c,\tau,K)}{\partial \mathring{p}_c}\right] \\
&+ \int_{\tau(\mathring{p}_c,\overline{L},K)}^{\tau_{max}} \left[\rho(K)Q_c(\mathring{p}_c,\tau)^2 - \overline{L}\right] f(\tau)d\tau + \mathring{p}_c \int_{\tau(\mathring{p}_c,\overline{L},K)}^{\tau_{max}} \frac{\partial L(\mathring{p}_c,\tau,K)}{\partial \mathring{p}_c} f(\tau)d\tau \\
&- \mathring{p}_c\Big[L\big(\mathring{p}_c,\tau(\mathring{p}_c,\overline{L},K),K\big) - \overline{L}\Big]\, f\big(\tau(\mathring{p}_c,\overline{L},K)\big)\frac{\partial \tau(\mathring{p}_c,\overline{L},K)}{\partial \mathring{p}_c} = 0
\end{aligned} \tag{3.5.19}$$

The first line is equivalent to equation (3.5.8) expressed in expected value whereas the second line represents the incentive that the producer has to reduce $\mathring{p}_c$. By reducing $\mathring{p}_c$, the producer increases the volume of thermal losses and then the inframarginal rents earned by selling electricity to the DNO. The third line is equal to zero since, following (3.5.17), $L\big(\mathring{p}_c,\tau(\mathring{p}_c,\overline{L},K),K\big) = \overline{L}$. The first-order condition of the producer can be rewritten:

$$\begin{aligned}
&E[Q_c(\mathring{p}_c,\tau)] + (\mathring{p}_c - c)E\left[\frac{\partial Q_c(\mathring{p}_c,\tau)}{\partial \mathring{p}_c}\right] - cE\left[\frac{\partial L(\mathring{p}_c,\tau,K)}{\partial \mathring{p}_c}\right] \\
&+ \int_{\tau(\mathring{p}_c,\overline{L},K)}^{\tau_{max}} \left[\rho(K)Q_c(\mathring{p}_c,\tau)^2 - \overline{L}\right] f(\tau)d\tau + \mathring{p}_c \int_{\tau(\mathring{p}_c,\overline{L},K)}^{\tau_{max}} \frac{\partial L(\mathring{p}_c,\tau,K)}{\partial \mathring{p}_c} f(\tau)d\tau = 0
\end{aligned} \tag{3.5.20}$$

When $\overline{L}$ is such that $\tau(\mathring{p}_c,\overline{L},K) = \tau_{max}$, only the first line of equation (3.5.20) remains and the producer fixes a price equivalent, in expected value, to the private monopoly price $p_c^m$ determined by (3.5.8).

### The DNO is in charge

When the DNO always covers thermal losses, $\overline{L} = 0$ and $\tau(\mathring{p}_c, 0, K)$ is equal to 0. So, equation (3.5.20) becomes:

$$E[Q_c(p_c,\tau)] + (p_c - c)E\left[\frac{\partial Q_c(p_c,\tau)}{\partial p_c}\right] + (p_c - c)E\left[\frac{\partial L(p_c,\tau,K)}{\partial p_c}\right] + E[L(p_c,\tau,K)] = 0$$

This equation corresponds to equation (3.5.2), which determines $\widehat{p}_c$, in expected value. When $\overline{L}$ becomes strictly positive, i.e. when both the producer and the DNO may be in charge of thermal losses, $\mathring{p}_c$ is determined by a combination of the rules, in expected value, determining $p_c^m$ and $\widehat{p}_c$, depending on the weight of each regime. In this case, the quantity consumed when both regimes coexists is lower than the quantity consumed when only the monopoly is in charge of thermal losses but higher than the quantity consumed when the DNO covers thermal losses.

#### 3.5.3.b Optimal level of investment

**The producer/retailer is in charge**

When demand is fluctuating, the DNO may be in charge of a part of thermal losses and it may have to compensate electricity losses. It has to take into account the two regimes described in section 3.5.2.a, so its program is:

$$\max_K \; -rK + \int_0^{\tau(\overline{L},K)} tQ_c\big(\mathring{p}_c,\tau\big)f(\tau)d\tau + \int_{\tau(\overline{L},K)}^{\tau_{max}} \Big[tQ_c\big(\mathring{p}_c,\tau\big) - \mathring{p}_c\big(L\big(\mathring{p}_c,\tau,K\big)-\overline{L}\big)\Big]f(\tau)d\tau$$
$$\Leftrightarrow \max_K \; -rK + tE\Big(Q_c\big(\mathring{p}_c,\tau\big)\Big) - \mathring{p}_c\int_{\tau(\overline{L},K)}^{\tau_{max}} \Big(L\big(\mathring{p}_c,\tau,K\big)-\overline{L}\Big)f(\tau)d\tau \tag{3.5.21}$$

The first-order condition associated to the DNO's program is:

$$r = \quad tE\left[\frac{\partial \mathring{Q}_c}{\partial K}\right] - \frac{d\mathring{p}_c}{dK}\int_{\tau(\overline{L},K)}^{\tau_{max}} \Big(L\big(\mathring{p}_c,\tau,K\big)-\overline{L}\Big)f(\tau)d\tau - \mathring{p}_c\int_{\tau(\overline{L},K)}^{\tau_{max}} \frac{\partial L\big(\mathring{p}_c,\tau,K\big)}{\partial K}f(\tau)d\tau$$
$$+\,\mathring{p}_c\Big[L\big(\mathring{p}_c,\tau(\mathring{p}_c,\overline{L},K),K\big)-\overline{L}\Big]\,f\big(\tau(\mathring{p}_c,\overline{L},K)\big)\frac{\partial\tau(\mathring{p}_c,\overline{L},K)}{\partial K}$$

The second line of this first-order condition is equal to 0 because, by definition (equation (3.5.17)), $L\big(\mathring{p}_c,\tau(\mathring{p}_c,\overline{L},K),K\big)=\overline{L}$. So, the optimal level of investment when the demand is fluctuating, is given by:

$$r = -\,\mathring{p}_c\rho'(K)\int_{\tau(\overline{L},K)}^{\tau_{max}} [Q_c(\mathring{p}_c,\tau)]^2 f(\tau)d\tau$$
$$+\,\frac{d\mathring{p}_c}{dK}\left[tE\left[\frac{\partial \mathring{Q}_c}{\partial \mathring{p}_c}\right] - \int_{\tau(\overline{L},K)}^{\tau_{max}} \Big(L\big(\mathring{p}_c,\tau,K\big)-\overline{L}\Big)f(\tau)d\tau - \mathring{p}_c\int_{\tau(\overline{L},K)}^{\tau_{max}} \frac{\partial L\big(\mathring{p}_c,\tau,K\big)}{\partial \mathring{p}_c}f(\tau)d\tau\right] \tag{3.5.22}$$

The first line is equivalent to equation (3.5.14) that determines $\widetilde{K}$ and it matters only when the DNO is in charge of a part of thermal losses, i.e. when $L > \overline{L}$. The second line is the effect of the investment made by the DNO on the price set by the producer. If the level of investment modifies the price, then the DNO has to take into account the price effects on the quantity consumed and on the volume of thermal losses. All these effects are represented in the expression into brackets, which is negative.

Let us assume that $\overline{L}$ is such that $\tau(\overline{L},K)=\tau_{max}$. When thermal losses are always covered by the producer, the optimal level of investment, $\widehat{K}$ is determined by:

$$r = tE\left[\frac{\partial \mathring{Q}_c}{\partial K}\right]$$

This rule is equivalent, in average, to the one described in equation (3.5.12) that determines $\breve{K}$.

**The DNO is in charge**

When thermal losses are only covered by the DNO, the optimal level of investment is determined by equation (3.5.22) when $\overline{L} = 0$ and is given by:

$$r = -\mathring{p}_c \rho'(K) E\big(\mathring{Q}_c^2\big) + \frac{d\mathring{p}_c}{dK}\left[tE\left[\frac{\partial \mathring{Q}_c}{\partial \mathring{p}_c}\right] - E\Big(L\big(\mathring{p}_c, \tau, K\big)\Big) - \mathring{p}_c E\bigg(\frac{\partial L(\mathring{p}_c, \tau, K)}{\partial \mathring{p}_c}\bigg)\right] \tag{3.5.23}$$

The sign of $\frac{d\mathring{p}_c}{dK}$ matters and influences the optimal level of investment[23]. Let us first assume that $\frac{d\mathring{p}_c}{dK}$ is negligible. Here, the optimal level of investment is determined by:

$$r = -\mathring{p}_c \rho'(K) E\big[Q_c(\mathring{p}_c, \tau)^2\big] \tag{3.5.24}$$

This relation, that defines $\mathring{K}_0$, is equivalent, in expected value, to equation (3.5.3) that determines $\widehat{K}$.

If we consider now that $\frac{d\mathring{p}_c}{dK} \neq 0$, the optimal level of investment is determined by equation (3.5.23). If $\frac{d\mathring{p}_c}{dK} > 0$, equation (3.5.23) would be negatively affected by the term into brackets and then the equality is reached for a level of investment $\mathring{K}_+$ lower than $\mathring{K}_0$. Indeed, when the DNO invests, the price $\mathring{p}_c$ increases. This price increase leads to a lower quantity consumed and a lower level of inframarginal rents linked to thermal losses. Then, the DNO may have a lower level of investment because its investment would increase the price which strengthens the effect of investment in the thermal losses reduction.

On the other hand, if $\frac{d\mathring{p}_c}{dK} < 0$, the brackets would have a positive impact on the left-hand side of equation (3.5.23). The optimal level of investment, $\mathring{K}_-$, determined by equation (3.5.23) in this case, is such that $\mathring{K}_- > \mathring{K}_0$. In fact, in this case, when the DNO increases the level of installed capital, the price chosen by the producer decreases. The monopolist's decision tends to increase the quantity consumed and the volume of thermal losses. Then, the DNO has to be more aggressive to reduce thermal losses and to overcompensate the incentive of the producer to reduce its price in order to increase its inframarginal rents. To summarize, when the DNO is in charge of thermal

[23]See appendix C.4.3.

losses, depending on the sign of $d\mathring{p}_c/dK$, we have:

$$\mathring{K}_+ < \mathring{K}_0 < \mathring{K}_-$$

**Comparison of the two cases**

When $0 < \tau(\overline{L}, K) < \tau_{max}$, the producer and, for $\tau > \tau(\overline{L}, K)$, the DNO are in charge of thermal losses. The optimal level of investment is determined by equation (3.5.22). In this case, under assumption of a low price elasticity, the first line of the equation determining $\mathring{K}$ is lower than the first term of the right-hand side of equation (3.5.23) because:

$$E\big[Q_c(\mathring{p}_c, \tau)^2\big] > \int_{\tau(\overline{L},K)}^{\tau_{max}} [Q_c(\mathring{p}_c, \tau)]^2 f(\tau) d\tau$$

The expression into brackets is also lower as:

$$E\Big(L\big(\mathring{p}_c, \tau, K\big)\Big) + \mathring{p}_c E\bigg(\frac{\partial L(\mathring{p}_c, \tau, K)}{\partial \mathring{p}_c}\bigg) > \int_{\tau(\overline{L},K)}^{\tau_{max}} \bigg[\Big(L\big(\mathring{p}_c, \tau, K\big) - \overline{L}\Big) + \mathring{p}_c \frac{\partial L\big(\mathring{p}_c, \tau, K\big)}{\partial \mathring{p}_c}\bigg] f(\tau) d\tau$$

When $\tau(\overline{L}, K)$ is low and when $d\mathring{p}_c/dK$ is positive (respectively negative), the optimal level of investment, $\mathring{K}$ is lower than $\mathring{K}_+$ (respectively $\mathring{K}_-$). Then, when $\tau(\overline{L}, K)$ increases the level of investment decreases and becomes closer to $\widehat{K}$.

**Proposition 3.3.**

*When demand is fluctuating:*

*i) if the price is an increasing function of the investment level, $\mathring{K}$ is such that:*

$$\widehat{K} < \mathring{K} < \mathring{K}_+$$

*ii) if the price is a decreasing function of the investment level, $\mathring{K}$ is such that:*

$$\widehat{K} < \mathring{K} < \mathring{K}_-$$

*In both cases, for a non-stationary demand, under the assumption of a low price elasticity, the DNO-in-charge case leads to a higher level of investment than the producer-in-charge case.*

## 3.6 Conclusion

Like congestion in transmission, thermal losses are an important and costly part of the distribution activity. Even if we can fight against losses and reduce them, they will never disappear. It is important to implement the best management of this externality created by the use of electrical network. To do so, we have developed a microeconomic analysis modeling the debate on the optimal type of thermal losses management. We have based our analysis on the state of the art in Europe.

We have first determined the first-best outcome and found that this outcome does not determine a finite number of firm. We have presented, thanks to a simple example, the importance to take into account energy losses in the determination of the merit order. Thermal losses may change the merit order by overcompensating the difference between marginal costs.

We have then focused the analysis on the distribution activity which is the one generating most thermal losses. We have first considered the vertically integrated public and private monopolies to present the historical management types of thermal losses. Then, we have modeled the management framework implemented in European countries, i.e. when the DNO is in charge of thermal losses vs. when, a priori, the producer/retailer is in charge of covering these losses. At first sight, even if the management designs are different, the investment rules in all cases are close. The difference comes from the price levels that are influenced by the management type and the level of consumption. Using the characteristics of the electricity demand, that has a low price elasticity, the comparison of the two management designs leads to prefer the model of the DNO-in-charge because the level of investment is higher than the one of the producer/retailer-in-charge's case or the outcome of the vertically integrated regulated monopoly and being higher, it is closer to first best. The countries that have chosen this type of management (the majority in Europe) are then better off.

For further research, one may relax the assumption of a monopoly in the production of electricity. Starting from the results found, if competition is strengthened in production (for instance Cournot competition), the level of investment when the DNO is in charge would be higher. On the other hand, the impact of competition when the producer/retailer is in charge needs more appropriate research. More importantly, the model can be developed by integrating embedded sources of electricity, localized at the consumption node. Being closer to consumers, local sources may reduce the volume of thermal losses in distribution networks thanks to netting effect.

# Chapter 4

# Promoting renewable energy in a common market with transmission constraint*

## 4.1 Introduction

The debate concerning the promotion of renewable sources of energy mainly focuses on the policy instruments (feed-in tariffs, green certificates, quotas, ...) that a country should implement. However, even if the European Commission has published a legal framework[1] advocating energy cooperation between member States, the impact of international trade on the production of renewable energies has been understudied. Our objective is to fill this gap and to look at the merit of cooperation when transmission capacities between countries are limited. Indeed, we assess how the transmission capacity installed in a common market may influence the optimal degree of cooperation on energy policy.

We use a model where two countries are linked by a line with a fixed limited capacity. Each country has two different sources of energy, renewable (RES) and non-renewable (NRE). Moreover, the two countries value differently electricity produced by a renewable source. The home country has a higher valuation of the electricity generation thanks to RES than the foreign one. We analyze the autarky case and the social optima with unlimited and limited connections in order to provide several benchmarks. Then, we look at the decentralized equilibrium under unlimited connections and, as

*This chapter is co-written with Wilfried Sand-Zantman.

[1] See for instance the directive 2009/28/EC of the European Parliament and of the Council on the promotion of the use of energy from renewable sources.

a function of the externality parameter, distinguish different market equilibria. These different situations are used to compare the gains of implementing coordinated and non-coordinated policies, where coordination amounts to choose the same policy in both countries. Finally, we assess the more common situation with limited transmission capacities. We derive the optimal behavior of private agents in order to determine the optimal level of coordination in the promotion of renewable energy sources. We show that the optimal level of coordination depends on the transmission capacity existing between the two countries.

The promotion of renewable energy in an international context has been recently addressed by Wand and Leuthold [2009] and Garcia and Alzate [2010]. These papers analyze the impact of promotional instruments within a country whereas our paper is based on a two-country model and insists on the possibility of implementing a coordinated policy. Voogt and Uyterlinde [2004] present a European-wide analysis of the international trade benefits in the European Union[2] but do not take into account the transmission constraints existing between countries.

The specific role of congestion, and therefore of transmission constraints, has been discussed in Joskow and Tirole [2005] who consider the congestion and the potential creation of market power when the line linking two zones is congested. In the same vein, Borenstein, Bushnell and Stoft [2000] analyze the benefits of increasing the transmission capacity between two regions in a deregulated electricity industry. They conclude that increasing the transmission capacity may strengthen competition in both regions. Contrary to this paper, we do not have market power in the two regions considered and we have two different sources of energy (RES and NRE). Moreover, there is no international trade feature in the above-mentioned papers, since the two regions cannot and do not choose specific and differentiated policies.

This chapter is organized as follows. The model is described in section 4.2. Section 4.3 is devoted to defining the benchmarks of our analysis, deriving the social optimum and the competitive equilibrium in the autarky case as well as the social optimum when the two countries are interconnected. In section 4.4, we assess the impact of the promotion policies in a decentralized way when there is no connection constraint. In section 4.5, we analyze the decentralized equilibrium under limited connections and describe the optimal level of coordination that countries should have. Finally, section 4.6 summarizes the findings of our paper and concludes.

[2]The targets mentioned are defined in the Third Energy Package. See: `ec.europa.eu/energy/gas_electricity/legislation/third_legislative_package_en.htm`

## 4.2 The model

In this section, we introduce the building blocks of our model and present the main assumptions of the article.

**The electricity good** Although electricity is homogenous for consumers, its generation can take several forms (biomass, nuclear, coal, oil, water, ect). For our purpose, we will differentiate between two types of electricity generating processes: either from renewable sources (RES) or from non-renewable sources (NRE). We assume that those two types are produced by two different industries, or at least with different technics.

**Market structure** We assume that the market, eventually the markets since both energies can be differentiated, is competitive. From a national perspective, this may not be obvious but in an international market with large interconnection capacity, this simplifying assumption makes sense.

**The countries** We consider two countries, called domestic and foreign. All variables (production, cost, export) with a "star" are related to the foreign country. Even if we assume that gross consumer surplus (and hence the demand functions) are identical across countries, there exist some differences in country's parameters. Let us detail some points more precisely.

**Demand** For consumers, we consider that only the total amount of energy matters, not the origin[3]. Hence, the gross surplus from consuming $q$ units of energy is $U(q)$, with $U$ strictly increasing and concave. Let $q$ and $q^*$ denote the total quantity of electricity consumed respectively at home and abroad.

**Production** Let $x$ ($x^*$ abroad) be the energy from NRE and $y$ ($y^*$ abroad) the energy from RES. In the domestic country, the cost function for the NRE is $cx^2/2$ and the cost function for RES is $\gamma y^2/2$ with $0 < c < \gamma$. In the foreign country, the cost structure is the same as at home, then the cost function for the NRE is $c^*x^2/2$ and $\gamma^* y^2/2$ with $0 < c^* < \gamma^*$ for the RES. Most of the time, it appears that the cost of producing an additional unit of RES is null. Actually, the marginal cost considered in our paper is the long run marginal cost of both types of production (RES and NRE).

[3]The number of consumers for whom the origin of the energy matters is marginal even though there exist some contracts ensuring that the energy consumed is "green". Moreover, consumers cannot distinguish RES and NRE for electricity.

By this way, we incorporate the evaluation of the investment needed to increase the RES generation. Moreover, as we mainly focus on the externality induced by RES, the production costs differ between NRE and RES but not across countries. Hence, we assume that $c = c^* = 1$ and $\gamma = \gamma^* > 1$.

**The externality** RES and NRE are not only different with respect to the costs of production but also to the benefits (or damage) their production generates[4]. We assume that every produced unit of RES generates a positive local externality of $\beta$ in the domestic country and $\beta^*$ in the foreign country. This positive effect measures the future environmental cost avoided, or the additional job created[5]. One of the countries, let say the home country, has a higher externality than the other, that is: $\beta > \beta^*$. To simplify the analysis, we assume that $\beta^* = 0$.

**International trade** Some exchanges of energy can occur between the two countries. Nevertheless, those exchanges are constrained by the size of the interconnection capacity, denoted $K$. If $K$ increases, the constraint of the line decreases between both countries. If the domestics country's exports for both types of energy are $(x^E, y^E)$ and the foreign country's are $(x^{*E}, y^{*E})$, we must have

$$x^E + y^E \leq K \qquad \text{(CT1)}$$

$$x^{*E} + y^{*E} \leq K \qquad \text{(CT2)}$$

Even though, the standard laws of physics could accommodate less stringent rules based on net flows, dealing with gross flows is more realistic as regulators do not integrate netting effect when they compute the balance of the system. However, in what follows, we assume with no significant loss of generality that one kind of energy is not traded in both directions. For this reason, when we can define the sign of the net-effect of trade, we will put the lower variable equal to zero in order to simplify computation.

[4] We could assume that the externality concerns the consumption of electricity. However, we think that consuming one unit of electricity coming from NRE or consuming one unit of electricity produced thanks to RES has the same impact on the environment.

[5] An alternative way and somewhat equivalent is to consider the negative externalities induced by the production of NRE. Since we focus on the promotion of RES rather than on the refrain of NRE, this modeling seems more appropriate.

## 4.3 Benchmark cases

### 4.3.1 Social optimum and competitive equilibrium in autarky

**Social optimum**

Let us first define the social optimum of this economy. Note that in this case, it is sufficient to analyze one country, say the home country. One has to take into account the positive externality generated by the production of RES. The program of a benevolent social planner is then

$$\max_{x,y} U(x+y) + \beta y - \frac{x^2}{2} - \gamma\frac{y^2}{2}$$

The social optimum is therefore such that $U'(x+y) = \gamma y - \beta = x$.

**Competitive equilibrium**

Let us now consider the situation where both types of energy are not differentiated in the market. Therefore, there is a unique price $p$ clearing the electricity market.

On the demand side, only the total of energy produced, irrespective of the origin, matters. Therefore, the consumer's utility depends on $q = x+y$. The demand function is then defined by the standard condition:

$$U'(q) = p \Leftrightarrow q = U'^{-1}(p)$$

On the supply side, two different industries produce energy but both face the same price $p$. For example, the producers' programs in the home country are given by:

$$\max_{x} px - \frac{x^2}{2} \text{ and } \max_{y} py - \gamma\frac{y^2}{2}$$

The maximization of profits for each of them leads to $x = p$ for the NRE and $\gamma y = p$ for the RES.

The competitive equilibrium $(p, x, y)$ is then given by the following relationship $U'(x+y) = p = x = \gamma y$. Since $U'$ is decreasing, the industry specialized in the NRE produces more than the one producing the RES.

In the home country, the competitive equilibrium differs from social optimum in two dimensions. First, the production of energy is too small compared with social optimum. This simply comes from the fact the positive externality of producing the RES is not considered in a competitive economy. Second, the way the global amount of energy is shared between RES and NRE is not optimal, with a bias toward the latter[6].

[6]In the foreign country, as we have assumed that $\beta^* = 0$, there is no difference between the social

The social optimum in the home country can be decentralized by subsidizing the production of RES, with a unit transfer "$s$" equal to the marginal external benefit, i.e. $s = \beta$. The next section will show how this subsidy affects the production and the trade of energy in a common energy market.

### 4.3.2 Social optimum with interconnection

**The case of unlimited connection**

When energy markets are perfectly interconnected, the production in one country can flow "freely" to the other one. For each country, a common social planner would choose not only the amount of energy to produce, but also which type of energy and the amount of production to be exported or imported. Using the notations defined previously, the social optimum program is given by

$$\begin{aligned} \max \quad & U\left(x - x^E + y - y^E + x^{*E} + y^{*E}\right) + \beta y \\ & + U\left(x^* - x^{*E} + y^* - y^{*E} + x^E + y^E\right) \\ & - \frac{x^2}{2} - \gamma\frac{y^2}{2} - \frac{x^{*2}}{2} - \gamma\frac{y^{*2}}{2} \end{aligned}$$

subject to the positivity of all variables.

Suppose that the transmission capacity is large enough to avoid any congestion. Then, the first-best optimum is given by

$$U'(q) = U'(q^*) = x = \gamma y - \beta = x^* = \gamma y^* \tag{4.3.1}$$

Using our parameter specifications, it is straightforward to derive the main characteristics of the social optimum. From the first-best optimum in equation (4.3.1), we know that the demand function is the same for both countries. Therefore, marginal surplus and consumptions are equalized across countries and the production levels are such that $x = x^*$ and $y > y^*$. The energy production is greater in the domestic country than in the foreign country because the externality at home is greater than abroad. Then, the net energy flow goes from the domestic country to the foreign one. The

optimum and the competitive equilibrium in Autarky.

consumption levels are given by

$$\begin{cases} q = x + \dfrac{x+\beta}{\gamma} - y^E \\ q^* = x + \dfrac{x}{\gamma} + y^E \end{cases} \tag{4.3.2}$$

with $U'(q) = U'(q^*) = x = x^*$. Knowing that $q = q^*$, the net export of RSE from the home country to the foreign one is given by:

$$q - q^* = 0 \Leftrightarrow x + \frac{x+\beta}{\gamma} - y^E - x - \frac{x}{\gamma} - y^E = 0 \Leftrightarrow y^E = \frac{\beta}{2\gamma}$$

Inter-country as well as intra-country marginal production costs are equalized. This relationship ensures productive efficiency. It appears that the transmission constraints are not binding as long as the capacity of the line $K$ is greater than $\beta/2\gamma$.

**The case of limited connection**

As interconnections are in fact limited, it is important to consider the transmission constraints. As the net flows go from the domestic country to the foreign country, the binding constraint is (CT1). The optimal productions are defined by the following expression:

$$q = x + y - K > q^* = x^* + y^* + K$$
$$U'(q) = x = \gamma y - \beta < U'(q^*) = x^* = \gamma y^*$$

As soon as some limits in the interconnection capacities emerge, it is not optimal anymore to equalize the social marginal surplus. More production occurs in the domestic country and only a fraction of the excess supply is transferred to the foreign country. The most striking effects lie in the organization of production. It is not possible anymore to ensure inter-country productive efficiency. In fact, the marginal costs of production are only equalized within each country. Moreover, the lack of transmission capacity leads the domestic country to reduce its energy production, both for the RES and for the NRE, and the foreign country to increase its own to compensate partially the decrease in the import of energy.

Concerning the level of consumption, the energy consumption in the home country increases as soon as the constraint becomes tighter whereas, abroad, having transmission capacity constraint leads to a decrease in the energy consumption with respect to the first-best case.

As we see, the lack of transmission capacity hinders the implementation of the first-best solution. As long as a central planner can control the production and consumption decisions, increasing transmission capacity raises gross social welfare[7].

## 4.4 Decentralized equilibrium without transmission constraint

In what follows, we assume that there is unlimited transmission capacity. This means that the transmission constraints never bind and that the energy can flow freely from one country to the other.

### 4.4.1 Promoting RES without coordination

This section aims at analyzing the impact of national non-coordinated policies for promoting RES. Even if there are different tools to support RES, we focus here on feed-in tariffs[8].

In order to promote the production of RES, regulators choose a subvention to align private and public incentives. This does not directly affect the prices faced by the consumers for the different types of energy but constitutes an additional revenue for the producers[9]. It is therefore as if the RES producers were now facing a higher price than the price actually paid by the consumers. As the marginal benefit of producing RES in the home country is $\beta$, the subsidized prices for this energy, $\overline{p}$ for the domestic country and $\overline{p}^*$ for the foreign country, are such that

$$\overline{p} = p + s \text{ and } \overline{p}^* = p^* + s^*$$
$$\text{with} \qquad s = \beta \text{ and } s^* = 0$$

Since there are now two types of energy that can be traded, it is necessary to look at the price differential in both goods to know which one will be traded.

What is the market equilibrium of the uncoordinated game? Since some electricity

[7]The net-benefit of increasing transmission capacity on the social welfare depends on the cost of installing new capacities between countries.

[8]As long as there is no uncertainty, Weitzman [1974] shows that price-based instruments, such as feed-in tariffs or fiscal incentives, are equivalent to quantity-based instruments, as quotas or green certificates.

[9]Feed-in tariffs are financed thanks to taxes. For instance, in France, the subsidizing of RSE production is covered thanks the "contribution aux services publics de l'électricité" (CSPE) that appears on consumers' bill whatever their retailers. This does not affect the market prices of electricity.

must be available in each country, and using the assumption that $\beta > \beta^* = 0$, we must have $p \leqslant p^* \leqslant p + \beta$.

In a market equilibrium without any constraint, producers will choose to provide their energy in the country where the price is the highest. Two regimes must be considered: i) $p + \beta > p^* = p$; and ii) $p + \beta \geqslant p^* > p$:

i) $p + \beta > p^* = p$

We start by considering the situation where the NRE prices are the same in both markets while the RES price (the price paid to the producers) is greater in home country than in the foreign country. In this case, all the RES are consumed in the home country since the market price for this type of energy is higher in this country, i.e. $y^E = 0$ and $y^{*E} = y^*$. Meanwhile the production of NRE is shared between countries, since market prices are the same. The equilibrium production is defined by the following expressions:

$$U'(q) = x = \gamma y - \beta = p = p^* = U'(q^*) = x^* = \gamma y^* - \beta$$
$$q = y + y^* + x - x^E + x^{*E} \text{ and } q^* = x^* + x^E - x^{*E}.$$

Since $p = p^*$, it is easy to see that $q = q^*$ and therefore

$$x^E - x^{*E} = y^{*E} = (p + \beta)/\gamma > 0$$

Note that the higher $\beta$, the higher the net export of NRE from the home to the foreign country is. However, those exports are bounded above by the production in the home country, i.e. $x$. Therefore, this case holds as long as

$$x^E - x^{*E} < x \Leftrightarrow \frac{p + \beta}{\gamma} < p$$
$$\Leftrightarrow p > \beta\left(\frac{1}{\gamma - 1}\right)$$

Using the expressions of the equilibrium, one can show that the equilibrium price $p$ is a decreasing function of $\beta$. Therefore, this situation is only valid for small values of the externality $\beta$.

ii) $p + \beta \geqslant p^* > p$

When $\beta$ increases, the structural difference (linked to externality) between countries increases. Then, the total energy production becomes too high in the home country to maintain the same price than in the foreign country, so $p$ decreases. As soon as $p$ becomes smaller than $p^*$, all the production of NRE is consumed in the foreign country while all the RES is still consumed in the home country. The competitive equilibrium is then defined by

$$U'(q) = \gamma y - \beta = \gamma y^* - \beta = p < p^* = U'(q^*) = x^* = x$$
$$q = y + y^* \text{ and } q^* = x^* + x.$$

Using the above equations, we find that the equilibrium prices $(p, p^*)$ are such that:

$$U'(2p^*) = p^* \text{ and } U'\left[\left(p+\beta\right)\left(\frac{2}{\gamma}\right)\right] = p$$

It is clear that $p^*$ is now independent of $\beta$ and that $p$ still decreases with $\beta$. This regime is valid as long as

$$p \leqslant \beta\left(\frac{1}{\gamma - 1}\right).$$

One can show that the case where $p + \beta = p^*$ never happens. Indeed, when $p + \beta = p^*$, the quantity of RES that the foreign country exports is not sufficient to have $q > q^*$. So it is impossible to have simultaneously $p + \beta = p^* > p$ and $q > q^*$.

In the first regime (when $\beta$ is small), as $y^E = 0$ and $x^E - x^{*E} = y^{*E}$, we have $q = x + y$. The energy price (which is the same for both countries) is then implicitly defined by

$$U'\left(p + \frac{(p+\beta)}{\gamma}\right) = p$$

From the previous equation, it comes that $p$ decreases with $\beta$ while $p+\beta$ increase with $\beta$.

In the second regime, the prices of energy are different between countries and each country consumes only one type of energy. While $p^*$ is not affected by $\beta$, $p$ depends on this parameter since it is implicitly defined by

$$U'\left((p+\beta)\frac{2}{\gamma}\right) = p$$

As before, $p$ decreases with $\beta$ and $p + \beta$ increases with $\beta$. After computation, we find that $dp/d\beta > -1$.

Note that with $0 < \beta^* < \beta$, one could have $p + \beta = p^* + \beta^*$. In this case, part of the RES could be consumed in the foreign country. In this case, we still have that $dp/d\beta > -1$. So the total price received by the producer is increasing with $\beta$.

### 4.4.2 Promoting RES with coordination

Let us now consider the situation in which a central agency controls the level of subsidies prevailing in both countries. We assume that the central agency is bound to propose the same level for both countries. This common level of subsidies is denoted by $h \in [0, \beta]$.

As the subsidies are equal across countries, the equilibrium energy prices $p$ and $p^*$ should also be equal. Otherwise, the country with a lower price would receive no energy. The common price is then simply denoted by $p_c$.

Facing a price $p_c$ for the NRE and a price $p_c + h$ for the RES, energy producers will choose $(x, x^*, y, y^*)$ such that

$$x = x^* = p_c \text{ and } y = y^* = \frac{p_c + h}{\gamma}$$

Knowing that prices are determined by the same competitive process in both countries, the quantity consumed at home is the same as the one consumed in the foreign country and the price is such that

$$U'\left[p_c + \frac{p_c + h}{\gamma}\right] = p_c \tag{4.4.1}$$

$p_c$ is a decreasing function of $h$ while $p_c + h$ increases with $h$. The welfare function, incorporating the equilibrium behavior of firms can be written

$$W(h) = U\left[p_c + \frac{p_c + h}{\gamma}\right] + \beta\left[\frac{p_c + h}{\gamma}\right] + U\left[p_c + \frac{p_c + h}{\gamma}\right] - \frac{p_c^2}{2} - \frac{p_c^2}{2} - \frac{\gamma}{2}\left[\frac{p_c + h}{\gamma}\right]^2 - \frac{\gamma}{2}\left[\frac{p_c + h}{\gamma}\right]^2 \tag{4.4.2}$$

Using the envelop theorem, we have:

$$\frac{dW(h)}{dh} = \left(\frac{\partial p_c}{\partial h} + 1\right)\left[\frac{\beta}{\gamma} - \frac{2h}{\gamma}\right].$$

We know that $(\frac{\partial p_c}{\partial h} + 1) > 0$ since $p_c + h$ increases with $h$. So $W(h)$ is maximized when $h = \frac{\beta}{2}$.

Incorporating this value, equation (4.4.1) writes as:

$$U'\left[p_c + \frac{2p_c + \beta}{2\gamma}\right] = p_c \tag{4.4.3}$$

One can see that the equilibrium quantities in this case corresponds to the first-best quantities. Indeed, we have: $x = x^* = U'(x + \frac{2x+\beta}{2\gamma})$ and concerning the RES:

$$y = \frac{2x+\beta}{2\gamma} < y^{FB} = \frac{x+\beta}{\gamma} \text{ and } y^* = \frac{2x+\beta}{2\gamma} > y^{*FB} = \frac{x}{\gamma}$$

Therefore, the first result of harmonization is to provide the global level of production than at first best even if the quantity of RES produced within each country is distorted.

### 4.4.3 Welfare comparisons

We now compare this centralized solution with the two previous non-coordinated solutions. When $\beta$ is small, the solution without coordination induces $p = p^*$. All firms consider the highest subsidy to determine their production of RES. This leads to an excess production of energy. More precisely, there is underproduction of NRE but it is more than compensated by the overproduction of RES. In the coordinated solution, the global quantity of RES is optimal even if there is too few production at home and too much in the foreign country. It is easy to compare the welfare levels in those two situations. Indeed, the non-coordinated situation corresponds to the case where $h = \beta$, which is less beneficial than when $h$ takes its coordinated value, $\frac{\beta}{2}$.

When $\beta$ is large, there is still overproduction in the home country. Indeed, the consumption is given by $q = \left(\frac{2(p+\beta)}{\gamma}\right)$ which is greater than the first-best value as long as $p < \frac{3\beta}{2(\gamma-1)}$. From the previous section, we know that, in the intermediate case, $p$ is such that:

$$p < \beta\left(\frac{1}{\gamma-1}\right) < \frac{3\beta}{2(\gamma-1)}$$

Hence, in this case, the level of consumption at home is always greater than the first-best value. On the other hand, in the foreign country, it is not clear whether there will be overproduction or underproduction. One can show that if $p^* < \frac{\beta}{2(\gamma-1)}$, there is underproduction in the foreign country and overproduction otherwise.

**Proposition 4.1.**

*In the case of unconstrained connections, the harmonized system of feed-in tariff must* *be encouraged. This system leads to the optimal global level of production, even if the way this production is shared between the countries is not optimal. In the non-cooperative cases, there will be not only an overproduction of RES, but also an excessive production of energy and, as in the cooperative case, a non-optimal locational choice of production. Therefore, in a well-interconnected market, there is a clear need for some coordination of policies aiming at the promotion of renewable energy.*

## 4.5 Decentralized equilibrium under limited connection

Depending on hours, days and seasons, the connection between countries may be limited in capacity and countries cannot trade as much as they want. The aim of this section is to compare subsidization policies, coordinated or not, depending on the capacity of the line linking the two countries.

### 4.5.1 Behavior of the private agents

Let us first look at the behavior of the different agents when the countries decide to implement a coordinated policy. In this case, the prices are: $p = p^*$ for the NRE and $p + h$ for the RES. As the prices and the cost of both types of energies are the same in the two countries, the objectives of the industry are the identical and lead to:

$$x = x^* = p \text{ and } y = y^* = \frac{p+h}{\gamma}$$

We have seen in the previous section that the optimal level of subsidy under a coordinated policy is: $h = \beta/2$. Two remarks come here. First, under coordinated policy, both countries produce and consume the same amount of NRE and RES. There is no trade among countries to equalize the consumption. As there is no energy flow between the domestic and foreign countries, the capacity of the line has no influence on the level of production and consumption. Then, the social welfare associated to the coordinated policy does not depend on the capacity of the line $K$. In this case, investing to increase the link between the two countries would not affect the social welfare.

Without coordination, each country will fix a subsidy equal to the externality generated by the production of RES. Hence, in the home country, the subsidy is equal to

$\beta$, whereas in the foreign country, as $\beta^* = 0$, there is no subsidy.
To focus on the impact of different policies, we assume that the initial prices are close. That is, we assume that: $p < p^*$. Thanks to the independent policies, the prices of the RES are such that: $p + \beta > p^*$.
Knowing that we have a difference in price for both types of energy among countries, the inter-country trade will concern NRE and RES. From the previous section, the constraints are binding as long as:

$$K \leqslant \overline{K} = \max\left\{p^*, \frac{p+\beta}{\gamma}\right\} \text{ with } p = U'(2(p+\beta)/\gamma) \text{ and } p^* = U'(2p^*)$$

The level of trade is determined by the behavior of the agents. Let us first consider the home country. The program of the producers is

$$\max p(x - x^E) + p^* x^E + (p+\beta)(y - y^E) + p^* y^E - \frac{x^2}{2} - \gamma\frac{y^2}{2}$$

under the constraint (CT1) and the positivity constraint of all variables. The constraint (CT2) has no impact here because it only concerns flows of energy from the foreign country to the home one. The maximization of this program leads to:

- $y = (p+\beta)/\gamma$ and $y^E = 0$.
- the levels of NRE production and NRE export depend on the value of $K$:
  - if $p < K < p^*$, then $x = x^E = K$;
  - if $K < p$, then the constraint is tight and $x = p$ and $x^E = K$.

Hence, in the home country, only NRE is exported. The volume of production and export of NRE depend on the capacity of the line.
Similarly, we can define the behavior of the industry in the foreign country. We know that in this case the RES is traded as long as $p+\beta > p^*$. The program of the industry in the foreign country is:

$$\max p^*(x^* - x^{*E}) + p x^{*E} + p^*(y^* - y^{*E}) + (p+\beta)y^{*E} - \frac{x^{*2}}{2} - \gamma\frac{y^{*E}}{2}$$

under the positivity constraint of all variables and the constraint of capacity (CT2). The first-order conditions of this program lead to:

- $x^* = p^*$ and $x^{*E} = 0$.

- As for the NRE in the home country, the levels of production and of export of RES depend on the value of $K$:
  - if $p^*/\gamma < K < (p+\beta)/\gamma$, then $y^* = y^{*E} = K$;
  - if $K < p^*/\gamma$, then $y^{*E} = K$ et $y^* = \frac{p^*}{\gamma}$.

As before, the value of the capacity determines two different intervals associated to different levels of production and export of RES.

To have a global view of the trade between the two countries, we need to combine the two optimization programs. Thus, we can define four cases depending on the value of the capacity of the line $K$.
The first situation is the one resulting from a very low capacity of the transmission line. This situation is close to the autarky case. Countries export all what they can and the production level in each country is determined by the national prices. This zone (let us call it Zone 1) is characterized by: $K < \min\left\{p, p^*/\gamma\right\}$, $x = p$ and $y^* = p^*/\gamma$ and the volumes of exports are $x^E = y^{*E} = K$.
In Zone 2, when $p^*/\gamma < K < \min\left\{p, (p+\beta)/\gamma\right\}$, the situation is such that the capacity of the line is too low to export all the NRE production of the home country. However all the RES production of the foreign country is exported because the marginal value of the export is greater than the marginal value of a RES unit sold in the foreign country.
Subsequently, in Zone 3 where $K$ is such that $p < K < p^*$ and $p^*/\gamma < K < (p+\beta)/\gamma$, it becomes profitable for the home country to export all his NRE production. In this case, all the RES produced in the foreign country is exported. The production and trade levels in this situation are then such that: $x = x^E = K$ and $y^* = y^{*E} = K$.
In the last zone (hence Zone 4) where $K \geqslant \overline{K} = \max\left\{p^*, \dfrac{p+\beta}{\gamma}\right\}$, the capacity of the line is so large that it does not constrain the trade between countries. Hence, the levels of production are given by: $p = U'(q) = \gamma y - \beta = \gamma y^* - \beta$ and $p^* = U'(q^*) = x^* = x$.

### 4.5.2 Comparison between policies under limited connection

The goal of this section is to determine whether, in case of limited connections, the policies promoting RES should be decentralized (following the “subsidiarity principle”) or, on the contrary, fully coordinated (often by a central authority) such that the same policy is applied in both countries.

First, when $K$ is very large (Zone 4), i.e. $K > \overline{K}$, we have seen in the previous section that the promotion of RES under cooperation dominates decentralized policies. The social welfare, here, does not depend on the value of $K$, knowing that the constraints are not binding. So, without any transmission barriers, one should have one single environmental policy.
When instead $K$ is small (Zone 1), i.e. $K < \min \left\{ p, p^*/\gamma \right\}$, the volume of energy traded is very low. Therefore, both countries prefer to decentralize the decision of promoting RES. From section 4.3.1, we know that the competitive equilibrium corresponds to the decision of having independent policy of promotion. The subsidiarity principle strictly applies.
To determine the optimal policy in the other zone, we need to rely on the following lemma.

**Lemma 4.1.**
*When $p^*/\gamma < K < (p+\beta)/\gamma$, the welfare of both countries when they decide to implement independent policies is a decreasing function of $K$. Then, we have*

$$\frac{dW^{nc}}{dK} < 0$$

*Proof.* See the appendix D.1. □

When the installed capacity of the line increases, countries may trade more and more and having national non-coordinated policies is less desirable. The agents may benefit from the "opening" of their country. To sum up, in Zone 1, the link between countries is so weak that both of them act as if they were in autarky, i.e. they prefer to have different policy for promoting RES. As $K$ increases, the social welfare associated to the non-coordinated case decreases until $K = \overline{K}$. Beyond this point (so in Zone 4), both countries are better off when they implement a coordinated policy. Hence, we can establish the following proposition:

**Proposition 4.2.**
*There exists $\widehat{K} < \overline{K}$ such that coordination is socially optimal if and only if $K > \widehat{K}$.*

This can be illustrated on figure 4.1.
Let us study the evolution of $\widehat{K}$ defined by $W^c(\widehat{K}) = W^{nc}(\widehat{K})$ when $\beta$ varies. Note that the shape of the function $W^{nc}(.)$ depends on the zone considered. Note also that $\widehat{K}$ may be in Zone 2 or Zone 3.

**Corollary 4.1.**
*If $\widehat{K}$ is in Zone 2, that is if $p^* < \gamma\widehat{K} < p+\beta$, then $\widehat{K}$ is an increasing function of $\beta$.*

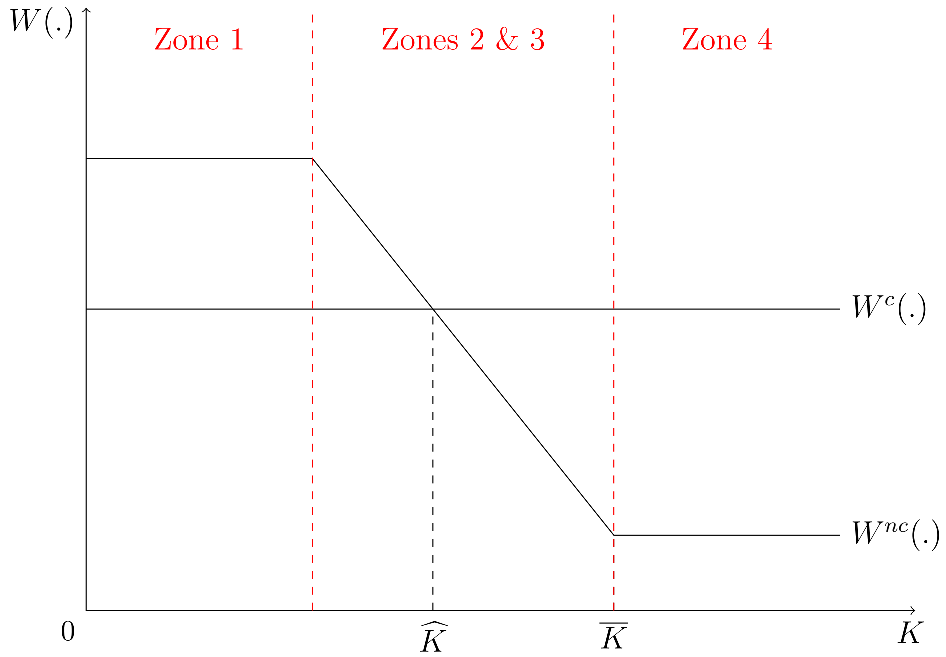



Figure 4.1: Optimal policies

*Proof.* See the appendix D.2. □

This corollary is rather counterintuitive as one may think that an increase in $\beta$ - therefore a higher level of externality - would imply more coordination. However, this increase also induces a higher structural difference among countries which increases the implicit cost of the coordination (compared to decentralized policies). It turns out this latter effect dominates which explains the positive relationship between $\beta$ and $\widehat{K}$.

**Corollary 4.2.**
*If $\widehat{K}$ belongs to Zone 3, that is if $p < \widehat{K} < p^*$ and $p^*/\gamma < \widehat{K} < (p+\beta)/\gamma$, and if $U''(.)$ constant and "not too small", then $\widehat{K}$ is an increasing function of $\beta$.*

*Proof.* See the appendix D.3. □

Hence, if the utility function is not too concave, the level of capacity $\widehat{K}$ is an increasing function of $\beta$. As in Zone 2, this result is explained by an increase in the structural difference in both countries which induces a higher cost of the cooperation. This effect dominates the pure impact of increasing $\beta$ which tends to increase the interest of having a cooperated policy among countries.

## 4.6 Conclusion

This paper has undertaken an analysis of the pros and cons of cooperation between countries, or equivalently questioned the relevance of the subsidiarity principle in a context of promotion of renewable energy. In this perspective, a particular attention has been put on the influence of limited transmission capacity. In a framework with two countries and two sources of energy (RES and NRE), we have derived the market equilibria regarding the transmission constraint and the choice of feed-in tariffs. In particular, we have shown that the policies aiming at promoting RES should be coordinated if and only if the transmission capacity is large enough. This paper thus provides useful guidelines on the relevance of coordination in a context of externality and (potentially) binding transmission constraints.

From this initial approach, it may be interesting to enlarge the perspective and one may look at different connected topics. First, we have assumed that each government, when choosing the subsidy to promote the RES, was neglecting the impact on the current account balance. Taking into account the latter would probably decrease the difference between the policies chosen by each country. Second, we have assumed that the industries were competitive in both countries. It could be interesting to consider the polar case of imperfect competition, in particular a monopoly in one country. Lastly, we could introduce some environment-sensitive consumers that would value differently the two types of energy RES and NSE. We leave these potentially interesting developments for future research.

# Appendix A

# Pertes d'énergie dans les réseaux de distribution d'électricité*

Les pertes d'énergie dans les réseaux électriques, et dans les réseaux de distribution en particulier, représentent pour les gestionnaires de réseaux un enjeu important. Le premier rapport d'activité de la société ERDF, filiale à 100% de EDF, insiste sur l'importance de l'amélioration de la performance énergétique des réseaux de distribution. En améliorant cette performance, la société de distribution réduit sa consommation d'énergie ce qui diminue l'empreinte carbone de son entreprise et permet de réduire les coûts liés au rachat d'énergie destiné à la couverture des pertes. Les pertes représentent sur le réseau de distribution pas moins de 5% de l'énergie consommée. Par ailleurs, sachant que la longueur des lignes et les conditions climatiques ont un impact sur les pertes d'énergie, ces dernières jouent aussi un rôle dans la manière dont les réseaux de distribution doivent être conçus et installés. Cette note traite des aspects techniques et économiques des pertes d'énergie dans les réseaux de distribution d'électricité. Je présenterai d'abord les facteurs techniques influençant les pertes d'énergie dans un réseau de distribution d'électricité puis la manière dont ERDF estime et comptabilise ces pertes. Enfin, je présenterai les solutions qui peuvent être apportées pour les réduire.

*Je tiens à remercier tout particulièrement Philippe Loevenbruck pour ses explications concernant le phénomène des pertes d'énergie dans les réseaux de distribution d'électricité et sur les méthodes de comptabilisation utilisées par Electricité Réseaux Distribution France.

## A.1 Déterminants des pertes dans un réseau de distribution

Les pertes d'énergie dans un réseau de distribution d'électricité sont de deux types: les pertes techniques et les pertes non-techniques. Les pertes non-techniques proviennent de consommations d'énergie non enregistrées. Ces pertes résultent de vols d'énergie ou d'erreurs de comptage et/ou de profilage. Dans certains pays en voie de développement, ces pertes peuvent représenter jusqu'à 50% de la quantité d'électricité injectée dans le réseau notamment en raison de problèmes de corruption[1].
Les pertes techniques ont, elles aussi, des origines diverses. Elles peuvent provenir de pertes en ligne (voir plus bas) mais aussi de pertes liées à la transformation haute tension (HTB)/moyenne tension (HTA) et à la transformation moyenne tension (HTA)/basse tension (BT). Ces pertes apparaissent dans les transformateurs et proviennent de pertes par effet Joule et de "pertes fer". Les "pertes fer" dépendent de la tension et de la fréquence d'alimentation, des matériaux utilisés et sont décomposées en pertes par courants de Foucault et pertes par hystérésis[2]. ERDF mène actuellement une politique de remplacement de certains de ses transformateurs afin d'améliorer la performance énergétique de ses réseaux de distribution. Fin 2010, 9000 transformateurs à haut rendement devaient remplacer les transformateurs contenant du PCB (polychlorobiphényle) pour permettre à ERDF de diminuer ses pertes électriques annuelles de 21GWh, soit la consommation d'une ville de 5000 habitants[3].

Comme les pertes présentes dans les transformateurs, les pertes en ligne sont à mettre en relation avec des phénomènes physiques des réseaux de distribution et plus particulièrement avec les pertes thermiques dues à l'effet Joule. Il convient de noter ici que les pertes liées à la distribution d'électricité sont plus importantes que celles constatées lors du transport de l'électricité. En effet, la tension est plus faible dans un réseau de distribution, or le phénomène de pertes thermiques est une fonction décroissante de la tension à laquelle le courant circule. D'autre part, le fait d'utiliser un courant alternatif dans les réseaux de distribution engendre plus de pertes par effet Joule que si les distributeurs utilisaient du courant continu[4]. De plus, le courant alternatif permet

[1]Selon un rapport sur le contrôle des pertes non-techniques, publié par l'IEPF et disponible à l'adresse: http://www.iepf.org/ressources/ressources-pub-desc.php?id=239

[2]Pour plus de détails voir: http://stielec.ac-aix-marseille.fr/cours/bonnet/transformateur.htm.

[3]Ces chiffres sont données dans le rapport de développement durable 2008 d'ERDF, disponible à l'adresse: `http://www.erdfdistribution.fr/electricite-reseau-distribution-france/distribution-d-electricite-130321.html`.

[4]Le courant continu est plus contraignant quant à son utilisation par les particuliers.

une utilisation par les consommateurs finals plus sûrs grâce notamment à l'utilisation de disjoncteurs rendue possible par ce type de courant.

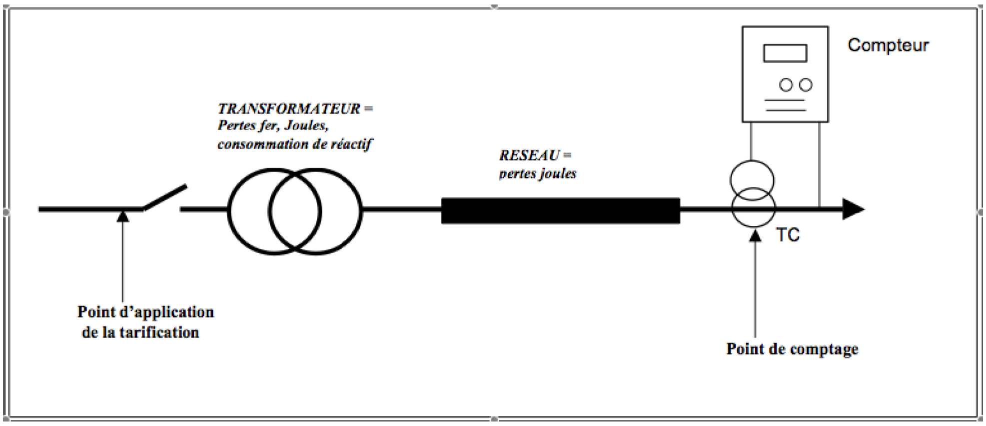



Figure A.1: Schéma des pertes (Source : site ERDF)

Les pertes générées par le réseau de distribution dépendent des quantités injectées au niveau des postes sources, c'est-à-dire au niveau des transformateurs HTB/HTA, entre le réseau de transport (exploité par RTE) et le réseau de distribution (exploité par ERDF). La longueur des lignes installées peut aussi être source de pertes en augmentant la résistance des lignes. Par conséquent, les pertes créées par la distance ne s'ajoutent pas de manière linéaire aux pertes liées à la quantité d'électricité soustirée mais se multiplient avec ces dernières. Pour synthétiser, la résistance d'un réseau $\rho$ peut se décomposer de la manière suivante:

$$\rho = \mu * l$$

où $l$ est la longueur de ligne installée dans le réseau et $\mu$ est la résistance par mètre de ligne installée exprimée en Ohm par mètre ($\Omega/m$) par exemple. Cette résistance dépend de la nature du câble utilisé (matériau utilisé pour la section de câble, diamètre) et du type d'ouvrage (aérien ou sous-terrain).

## A.2 Estimation des pertes en ligne

Selon la loi du 10 février 2000 qui donne aux gestionnaires de réseaux la responsabilité de l'achat d'électricité pour compenser les pertes, ERDF est responsable de la couverture des pertes qui apparaissent dans son réseau de distribution.

En physique, les pertes sur une ligne ou dans un réseau électrique sont calculées grâce à l'équation de l'effet Joule, c'est-à-dire:

$$L_{th} = \rho * I^2$$

où $\rho$ représente la résistance de la section de ligne considérée et $I$ l'intensité du courant dans ce réseau.

Les opérateurs de réseau évaluent les pertes ex-post en fonction de la courbe de charge des injections issues du réseau RTE. Concrètement, ERDF réalise une régression statistique. ERDF considère un panel de réseau pour déterminer le profil moyen d'un réseau de distribution sur le territoire français. L'équation résultant de cette estimation n'est pas simplement le carré de l'intensité multiplié par la résistance mais un polynôme de degré 2. Compte tenu des données utilisées, il existe en fait deux polynômes de degré 2, un permettant d'évaluer les pertes en semaine et l'autre permettant d'évaluer les pertes le week-end et les jours fériés. Ce dédoublement s'explique par le fait que les courbes de charge des injections sont différentes pour les jours de semaine d'une part et d'autre part les week-ends et jours fériés. Cela permet d'avoir une estimation plus juste des pertes effectives sur le réseau. Ces polynômes sont de la forme:

$$L = aP^2 + bP + c \Leftrightarrow \frac{L}{P} = aP + b + \frac{c}{P} \tag{A.2.1}$$

où $L$ (les pertes) et $P$ (les injections de RTE dans le réseau ERDF) sont exprimées en $kW$. Les coefficients des régressions sont tels que $a > 0, b > 0, c > 0$ pour les week-ends et jours fériés et $a > 0, b < 0, c > 0$ pour les jours en semaine. La constante $c$ permet de comptabiliser les pertes qui sont indépendantes de la puissance. La partie linéaire (coefficient $b$) dépend du degré de synchronisme entre les charges. Le polynôme permettant d'évaluer les pertes en semaine[5] est:

$$L = P^2 * 1,08 * 10^{-9} - P * 1,09 * 10^{-2} + 9,64 * 10^5$$

Concernant le week-end, le polynôme prend la forme:

$$L = P^2 * 9,09 * 10^{-10} + P * 1,48 * 10^{-2} + 5,02 * 10^5$$

[5]Source ERDF: `http://www.erdfdistribution.fr`.

Le tableau ci-dessous résume l'évolution des coefficients semaine et week-end entre 2005 et 2009.

| Period of validity | Working days | Week-ends and holidays |
|---|---|---|
| from June 2005 to December 2006 | $a_s = 9,52.10^{-10}\ kW^{-1}$<br>$b_s = -8,28.10^{-3}$<br>$c_s = 8,70.10^5\ kW$ | $a_w = 7,76.10^{-10}\ kW^{-1}$<br>$b_w = 1,53.10^{-2}$<br>$c_w = 4,56.10^5\ kW$ |
| from January 2007 to June 2009 | $a_s = 9,72.10^{-10}\ kW^{-1}$<br>$b_s = -4,38.10^{-3}$<br>$c_s = 8,10.10^5\ kW$ | $a_w = 8,22.10^{-10}\ kW^{-1}$<br>$b_w = 1,88.10^{-2}$<br>$c_w = 4,13.10^5\ kW$ |
| from July 2009 to June 2010 | $a_s = 1,08.10^{-9}\ kW^{-1}$<br>$b_s = -1,09.10^{-2}$<br>$c_s = 9,64.10^5\ kW$ | $a_w = 9,09.10^{-10}\ kW^{-1}$<br>$b_w = 1,48.10^{-2}$<br>$c_w = 5,02.10^5\ kW$ |
| from July 2010 to June 2011 | $a_s = 1,07.10^{-9}\ kW^{-1}$<br>$b_s = -3,03.10^{-3}$<br>$c_s = 7,12.10^5\ kW$ | $a_w = 7,28.10^{-10}\ kW^{-1}$<br>$b_w = 3,73.10^{-2}$<br>$c_w = 5,88.10^4\ kW$ |
| from July 2011 to June 2012 | $a_s = 8,18.10^{-10}\ kW^{-1}$<br>$b_s = 1,78.10^{-2}$<br>$c_s = 4,75.10^5\ kW$ | $a_w = 5,25.10^{-10}\ kW^{-1}$<br>$b_w = 5,25.10^{-2}$<br>$c_w = -5,10.10^4\ kW$ |
| from July 2012 | $a_s = 7,72.10^{-10}\ kW^{-1}$<br>$b_s = 2,12.10^{-2}$<br>$c_s = 4,00.10^5\ kW$ | $a_w = 4,70.10^{-10}\ kW^{-1}$<br>$b_w = 5,61.10^{-2}$<br>$c_w = -1,15.10^5\ kW$ |

Table A.1: Periodic coefficients in French electricity distribution (Source: site ERDF)

En réalité, le polynôme servant à estimer les pertes sur le réseau de distribution devrait être un polynôme de degré 4. En effet, “des pertes sont transportées sur des pertes” puisqu'il faut y ajouter les pertes dues au transport de l'électricité (réseau RTE). En fait, il faut transporter plus d'électricité afin de couvrir les pertes qui apparaissent sur le réseau, mais cette augmentation du volume d'électricité transportée entraîne une augmentation des pertes du réseau[6].

En 2011, les pertes d'énergie sur les réseaux de distribution d'ERDF (y compris les pertes non techniques) représentent 25 TWh, soit environ 6, 3% du volume d'électricité injectée dans le réseau de distribution (394, 6 TWh)[7].

[6]De même dans le transport aérien, du kérosène est transporté pour couvrir la surconsommation de kérosène liée au transport du combustible.

[7]Ce volume correspond à la consommation totale (478, 2 TWh) moins les pertes sur le réseau de transport (10 TWh) et la consommation des clients connectés au réseau de transport (73, 6 TWh).

## A.3 Traitement des pertes

Au niveau comptable, en 2011, les pertes ($1,5$ milliard d'euros) représentent environ 12% du chiffre d'affaires ($12,3$ milliard d'euros) réalisé par ERDF. Les pertes sont rémunérées grâce à un prix moyen estimé à 60€/MWh mais qui varie beaucoup puisque l'électricité servant à couvrir les pertes est achetée sur le marché de gros. Les fluctuations du marché de gros ne sont pas supportées par ERDF grâce à un compte de régularisation pour couverture de pertes. Si le coût effectif lié à la couverture des pertes est supérieur au coût prévu, la différence est payée par la CRE. De même, en cas de situation favorable sur le marché de gros, le surplus réalisé par ERDF est placé dans ce compte de régularisation.

Selon le TURPE 3[8], l'activité liée à la couverture des pertes est incluse dans les coûts calculés par la CRE. Le tarif en vigueur permet de payer les coûts de manière générale ainsi que de dégager une marge calculée de telle sorte qu'elle rémunère le réseau installé. Il apparaît donc que la logique de tarification dans la couverture des pertes est une logique de coût moyen et non une logique de coût marginal.

## A.4 Réduction des pertes

Il y a essentiellement deux moyens pour diminuer les pertes sur le réseau de distribution: l'un est économique, l'autre est technique. Sur le plan technique, ERDF peut investir pour:

1. diminuer $\mu$: en agissant sur la résistance au mètre (nature de l'alliage utilisé, diamètre du câble...). Cette solution ne peut être considérée que pour les nouvelles installations et s'avère trop onéreuse pour des lignes installées;

2. modifier les paliers dans les transformateurs: cela permettrait de diminuer les "pertes fer" des transformateurs;

3. optimiser le réseau: c'est la seule solution pour les ouvrages existants; elle consiste à équilibrer les volumes de charge dans les transformateurs ou à "court-circuiter" certains transformateurs inutiles l'été tout en préservant la sécurité du réseau;

4. investir en recherche et développement: développement de compteurs intelligents comme ceux proposés en Californie par PG&E (San Francisco) et Southern Cal-

[8]Tarif d'Utilisation des Réseaux Publics d'Electricité, entré en vigueur au $1^{er}$ août 2009.

ifornia Edison (Los Angeles)[9] qui permettent une meilleure anticipation et une meilleure gestion de la consommation d'électricité tant au niveau des consommateurs qu'au niveau des exploitants. C'est aussi l'un des objectifs des projets Linky en France et Grid4EU en Europe.

L'autre voie de recherche pour réduire les pertes est de transmettre des signaux de rareté marginaux pour que les utilisateurs du réseau internalisent l'effet produit par leurs actions sur la charge alors qu'un prix moyen ne transmet pas l'information nécessaire à des choix décentralisés efficients. Pour que ce mécanisme fonctionne, il faut que les compteurs intelligents transmettent non seulement la valeur de l'énergie sur le marché de gros mais aussi les pertes provoquées sur les réseaux de transport et de distribution.
Enfin, il y a une solution radicale au traitement des pertes, c'est la production décentralisée. L'un des avantages de l'éolien, du photovoltaïque ou de la petite hydraulique par exemple est d'être produits sur les lieux de consommation réduisant à zéro la longueur des lignes d'acheminement et donc les pertes créées par celles-ci.

[9]Les Echos du 27/04/09, p12: "Etats-Unis: un réseau électrique sobre et intelligent".

# Appendix B

# Pricing the connection to a distribution network

## B.1 Electrification rates

**SOURCE: IEA, World Energy Outlook 2011**

*Table T:* **8d85(rf5f(yyr 558ssyfgy5008yy88gf0gr dyr ggr8gr (8s**

| | Population without electricityy<br>millions | ElEctrfffcr tfonyrr tE<br>% | Urbr n<br>UlUctrfffcr tfonyrr tU<br>% | Rcrr l<br>RlRctrfffcr tfonyrr tR<br>% |
|---|---|---|---|---|
| Afrfca | 587 | 41,1 | 61,1 | 22,0 |
| *North Africa* | *2* | *99,0* | *99,6* | *98,6* |
| *Sub-Sahaban Afbina* | *5u5* | *3- b3* | *33b3* | *1- b1* |
| Devevapepg Dsea | DvD | eev0 | 99v0 | v3v9 |
| *Chis a & Eas- &sia* | *CuC* | *C- bu* | *C- b* | *u- b* |
| *Cuu-h &sia* | *- CC* | *- ubC* | *uCbC* | *CCbC* |
| Lat3p L L 3t3ca | 33 | L 3tL | L 3t3 | t 3tL |
| M3ddd3 EaEd | M3 | 3Md0 | M3dM | d3d3 |
| **DDvDlovfggycocgtrfDs** | **1y314** | **74,7** | **90,6** | **2, ,2** |
| **W0rdy(fg2d2dWyOWCWyr gdyWs(WgyW2r0, W/W2rr sfr )** | **Wy WW** | **80,5** | **8, ,8** | **58,0** |

## B.2 Restriction of the shape of the function $v(p_e, K)$

Having an interior solution for the social surplus maximization at first best requires a social surplus function concave in K. Denoting $\Delta v \overset{def}{=} v(p_e, K) - v(p_a)$, the surplus function is

$$S = \int_0^{\widetilde{\theta}} \left[\Delta v - \frac{\theta c}{f(\theta)}\right] dF(\theta) - rK$$

Then,

$$\frac{\partial S}{\partial K} = \left(\Delta v f(\widetilde{\theta}) - c\widetilde{\theta}\right) \frac{\partial \widetilde{\theta}}{\partial K} - r$$

and

$$\frac{\partial^2 S}{\partial K^2} = \left(\Delta v f(\widetilde{\theta}) - c\widetilde{\theta}\right) \frac{\partial^2 \widetilde{\theta}}{\partial K^2} + \frac{\partial \widetilde{\theta}}{\partial K} \left(\Delta v f'(\widetilde{\theta}) - c\frac{\partial \widetilde{\theta}}{\partial K}\right)$$

We know that $\frac{\partial \widetilde{\theta}}{\partial K} = \frac{f(\widetilde{\theta})}{c} \frac{\partial v(p_e, K)}{\partial K} > 0$ and $\frac{\partial^2 \widetilde{\theta}}{\partial K^2} = \frac{f(\widetilde{\theta})}{c} \frac{\partial^2 v(p_e, K)}{\partial K^2} < 0$. Consequently, $f' < 0$ (the "valley model") or $f' = 0$ (the "suburb model") is sufficient for $\frac{\partial^2 S}{\partial K^2} < 0$. But if we want $\frac{\partial S}{\partial K} = 0$ to be reached at $\widetilde{\theta}(c, K) < \overline{\theta}$, we need an additional condition.

In particular, in the case where $f' = 0$,

$$\frac{\partial^2 S}{\partial K^2} < 0 \;\Leftrightarrow\; \frac{\partial v}{\partial K}\frac{1}{c\bar{\theta}^2}\frac{\partial v}{\partial K} + \frac{\Delta v}{c\bar{\theta}^2}\frac{\partial^2 v}{\partial K^2} < 0 \;\Leftrightarrow\; \frac{\partial^2 v}{\partial K^2} < -\frac{1}{\Delta v}\left[\frac{\partial v}{\partial K}\right]^2$$

It shows that to have a positive interior solution, we need a function $v(p_e, K)$ strongly concave in $K$.

# B.3 The optimal network under budget constraint

## B.3.1 Determination of the optimal price

From the budget constraint, we know that the optimal price is such that:

$$\int_0^{\widehat{\theta}(p,K)} \left(p - \frac{c}{f(\theta)}\right) \theta dF(\theta) - rK = 0$$

When $\theta$ is uniformly distributed between 0 and $\bar{\theta}$, the previous equation is equal to:

$$\begin{aligned}
&\left(\frac{p}{\bar{\theta}} - c\right)\left[\frac{\theta^2}{2}\right]_0^{\frac{\Delta v}{p}} - rK = 0 \\
\Leftrightarrow\; &\frac{(\Delta v)^2}{2\bar{\theta}p} - c\frac{(\Delta v)^2}{2p^2} - rK = 0 \\
\Leftrightarrow\; &2rKp^2 - \frac{(\Delta v)^2}{\bar{\theta}}p + c(\Delta v)^2 = 0 \qquad \text{(B.3.1)}
\end{aligned}$$

Since social welfare is a decreasing function of the price, $p^{SB}$ is the smallest price compatible with equation (B.3.1). Consequently, $p^{SB}$ is equal to:

$$p^{SB} = \frac{\frac{(\Delta v)^2}{\bar{\theta}} - \sqrt{\frac{(\Delta v)^4}{\bar{\theta}^2} - 8rKc(\Delta v)^2}}{4rK}$$

Let us define:

$$A = \frac{2rK\bar{\theta}}{[v(p_e, K) - v(p_a)]^2} \tag{B.3.2}$$

Hence, we can write:

$$p^{SB} = \frac{1 - \sqrt{1 - 4\bar{\theta}Ac}}{2A}$$

It is easy to check that $p^{SB}$ exists if and only if $A \in [0, \frac{1}{4\bar{\theta}c}]$.

### B.3.2 Example

Consider the example where $v(p_a) = 0$ and $v(p_e, K) = \frac{K^n}{p_e}$ with $n > 0$ and $p_e = 1$. Thus, we have:

$$\frac{\partial v}{\partial K} = nK^{n-1} \; and \; \frac{\partial^2 v}{\partial K^2} = n(n-1)K^{n-2} \tag{B.3.3}$$

To have an interior solution for the maximization of the social welfare under balancing budget, the second order condition leads to $0 < n < 1$. Let us assume that $n < 1/2$, in this case, we have a maximum where $A = 2r\bar{\theta}K^{1-2n}$. For such $n$, we see easily that $A$ increases with respect to K. Moreover, $p^{SB}$ is an increasing function of $A$:

$$\begin{aligned}
\frac{\partial p^{SB}}{\partial A} &= \frac{2A\frac{4\bar{\theta}c}{\sqrt{1-4\bar{\theta}Ac}} - 2\left(1 - \sqrt{1-4\bar{\theta}Ac}\right)}{(2A)^2} \\
&= \frac{8\bar{\theta}Ac - 2\left(\sqrt{1-4\bar{\theta}Ac} - (1 - 4\bar{\theta}Ac)\right)}{\left(\sqrt{1-4\bar{\theta}Ac}\right)(2A)^2} \\
&= \frac{2\left(1 - \sqrt{1-4\bar{\theta}Ac}\right)}{\left(\sqrt{1-4\bar{\theta}Ac}\right)(2A)^2} > 0
\end{aligned}$$

Therefore, with this specification, the price is an increasing function of the level of installed capital.

### B.3.3 Optimal level of capital

The optimal level of capital comes from the maximization of the social welfare

$$\max_K \int_0^{\widehat{\theta}(p^{SB},K)} \left[ v(p_e, K) - v(p_a) - \frac{\theta c}{f(\theta)} \right] dF(\theta) - rK$$

that is

$$\max_K \frac{\Delta v}{\bar{\theta}} \Big( \widehat{\theta}(p^{SB}, K) \Big) - \frac{c}{2} \Big( \widehat{\theta}(p^{SB}, K) \Big)^2 - rK$$

for the uniform distribution of $\theta$.

We know that the last customer that should be connected is defined by:

$$\widehat{\theta}(p^{SB}, K) = \frac{v(p_e, K) - v(p_a)}{p^{SB}} \tag{B.3.4}$$

Hence, the previous program becomes:

$$\max_K \left( \frac{p^{SB}}{\bar{\theta}} - \frac{c}{2} \right) \Big( \widehat{\theta}(p^{SB}, K) \Big)^2 - rK$$

The first-order condition is:

$$\frac{\partial p^{SB}}{\partial K} \left( \frac{\widehat{\theta}^2}{\bar{\theta}} + 2\widehat{\theta} \frac{\partial \widehat{\theta}}{\partial p^{SB}} \left( \frac{p^{SB}}{\bar{\theta}} - \frac{c}{2} \right) \right) + 2\widehat{\theta} \left( \frac{p^{SB}}{\bar{\theta}} - \frac{c}{2} \right) \left( \frac{\partial \widehat{\theta}}{\partial K} \right) = r$$

The first derivative of $\widehat{\theta}$ with respect to the price $p^{SB}$ is:

$$\frac{\partial \widehat{\theta}}{\partial p^{SB}} = -\frac{\Delta v}{(p^{SB})^2} = -\frac{\widehat{\theta}}{p^{SB}} \tag{B.3.5}$$

Then, the first-order condition becomes:

$$-\frac{\partial p^{SB}}{\partial K} \frac{\widehat{\theta}^2}{\bar{\theta}} \left[ \frac{p^{SB} - \bar{\theta} c}{p^{SB}} \right] + 2\widehat{\theta} \left( \frac{p^{SB}}{\bar{\theta}} - \frac{c}{2} \right) \left( \frac{\partial \widehat{\theta}}{\partial K} \right) = r$$

Besides, we have:

$$\frac{\partial \widehat{\theta}}{\partial K} = \frac{1}{p^{SB}} \frac{\partial v(p_e, K)}{\partial K} \tag{B.3.6}$$

Consequently, the first-order condition can be rewritten as follows:

$$-\frac{\partial p^{SB}}{\partial K} \frac{\widehat{\theta}^2}{\bar{\theta}} \left[ \frac{p^{SB} - \bar{\theta} c}{p^{SB}} \right] + \frac{v(p_e, K) - v(p_a)}{p^{SB}} \frac{\big(2p^{SB} - \bar{\theta} c\big)}{p^{SB} \bar{\theta}} \frac{\partial v(p_e, K^{SB})}{\partial K} = r$$

## B.4 Intra-zone price adjustment

### B.4.1 Determination of the fixed part of the second-best two-part tariff

When it comes to implement first best with a two-part tariff, the relation between the fixed part and the variable part of the tariff is given by $\breve{\theta}(p_f, p, K^*) = \widetilde{\theta}(c, K^*)$. Using the zero-profit condition $\Pi = 0$, we obtain:

$$\breve{p}_f F\big(\widetilde{\theta}\big) + \int_0^{\widetilde{\theta}} \left(\breve{p} - \frac{c}{f(\theta)}\right)\theta dF(\theta) - rK^* = 0 \tag{B.4.1}$$

In the uniform distribution case, it gives

$$\begin{aligned}
&\breve{p}_f \frac{\widetilde{\theta}}{\bar{\theta}} + \int_0^{\widetilde{\theta}} \left(\frac{\breve{p}}{\bar{\theta}} - c\right)\frac{\big(\widetilde{\theta}\big)^2}{2} - rK^* = 0 \\
\Leftrightarrow\ &\Delta v\left(1 - \frac{\breve{p}}{\bar{\theta}c}\right)\frac{\Delta v}{\bar{\theta}^2 c} + \left(\frac{\breve{p}}{\bar{\theta}} - c\right)\frac{(\Delta v)^2}{2\bar{\theta}^2 c^2} - rK^* = 0 \\
\Leftrightarrow\ &\frac{(\Delta v)^2}{2\bar{\theta}^2 c}\left(1 - \frac{\breve{p}}{\bar{\theta}c}\right) = rK^* \\
\Leftrightarrow\ &\breve{p} = \bar{\theta}c(1 - A\bar{\theta}c)
\end{aligned}$$

And from the definition of the (first-best) marginal consumer, we can derive the fixed part of the tariff:

$$\breve{p_f} = \Delta v\left(1 - \frac{\breve{p}}{\bar{\theta}c}\right) = \Delta v A\bar{\theta}c \tag{B.4.2}$$

### B.4.2 Determination of the fixed part of the two-part tariff with kilometers priced at marginal cost

If the variable part in the two-part tariff is equal to the marginal cost per kilometer, the fixed part balancing the budget is such that:

$$p_f = \frac{rK}{F\left[\breve{\theta}(p_f, c, K)\right]} \tag{B.4.3}$$

In the case of a uniform distribution:

$$\begin{aligned} p_f &= \frac{rK\bar{\theta}}{\breve{\theta}(p_f, c, K)} \\ \Leftrightarrow p_f &= \frac{rK\bar{\theta}^2 c}{\Delta v - p_f} \\ \Leftrightarrow (p_f)^2 &- (\Delta v)p_f + rK\bar{\theta}^2 c = 0 \end{aligned}$$

The aim of the distributor is not to make profit, so it fixes the smallest fixed part for which the budget constraint is balanced, that is:

$$\begin{aligned} p_f &= \frac{\Delta v - \sqrt{(\Delta v)^2 - 4rK\bar{\theta}^2 c}}{2} = \frac{\Delta v}{2}\left(1 - \sqrt{1 - 2\bar{\theta}c\frac{rK\bar{\theta}}{(\Delta v)^2}}\right) \\ &= \frac{\Delta v}{2}\left(1 - \sqrt{1 - 2\bar{\theta}Ac}\right) \end{aligned}$$

The reasoning to determine the level of capital that should be installed is the same as the one developed in appendix B.3.3.

# Appendix C

# Alternative designs for the management of electric thermal losses

## C.1 Analysis of $\xi(K)$

The function $\xi(K) = -\rho'(K)\dfrac{K}{\rho(K)}$ is the elasticity of the resistance factor to changes in the invested equipment. It is equal to the product of the marginal resistance corrected for the sign times the inverse of the average resistance of the network for a given level of capital. This function is used to determine whether the break-even constraint is binding or not and if the first-best outcome can be achieved thanks to a public monopoly decentralization. The first derivative is

$$\begin{aligned}\frac{d\xi}{dK} &= -\left(\rho"(K)\frac{K}{\rho(K)} + \rho'(K)\frac{\rho(K) - K\rho'(K)}{(\rho(K))^2}\right) \\ &= -\frac{1}{\rho(K)}\Big[\rho"(K)K + \rho'(K)\left(1 + \xi(K)\right)\Big]\end{aligned}$$

For small values of $K$, $\rho"(K)K$ has little impact on the sign of the first derivative of $\xi$. Knowing that $-1/\rho < 0$ and that $\rho'(K)\left(1 + \xi(K)\right) < 0$, it is easy to see that $\xi$ is an increasing function of $K$. However, when $K$ becomes larger, the first term into brackets influences more and more the sign of the bracket so that the bracket becomes positive and, then, the first derivative becomes negative. Then, for large values of $K$, $\xi$ is a decreasing function of $K$.

## C.2 Comparison of $\widehat{p}_c$ and $p_c^m$

**Non decreasing price elasticity**

• When $\eta$ is a constant function of $p_c$, $p_c^m > \widehat{p}_c$ is an obvious result of the comparison of equations (3.4.2) and (3.5.2).

• We prove by contradiction that $p_c^m > \widehat{p}_c$ is also true when the demand function has an elasticity increasing with the price. From equations (3.4.2) and (3.5.2), we know that $\frac{p_c^m - c}{p_c^m} > 1/\eta^m$ and $\frac{\widehat{p}_c - c}{\widehat{p}_c} < 1/\widehat{\eta}$. Assume that $p_c^m < \widehat{p}_c$, then:

$$1 - \frac{c}{p_c^m} < 1 - \frac{c}{\widehat{p}_c} \Leftrightarrow \frac{1}{\eta^m} < \frac{1}{\widehat{\eta}}$$

which is a contradiction when $\eta$ is increasing in $p_c$.

**Decreasing price elasticity**

When $\eta$ is decreasing in $p_c$, one cannot rank $p_c^m$ and $\widehat{p}_c$ by using equations (3.4.2) and (3.5.2).

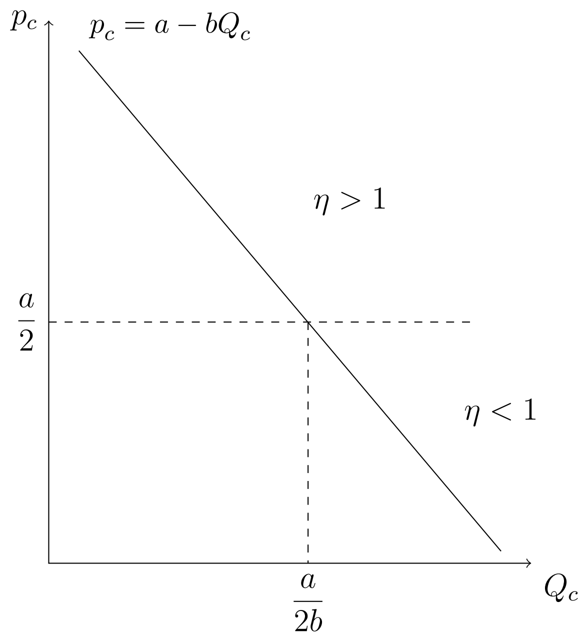



Figure C.1: Price elasticity for a linear demand function

## C.3 Comparison of $\widehat{p}_c$ and $p_c^{sb}$

### C.3.1 Condition to have $\frac{p_c^{sb} - c}{p_c^{sb}} > 1/\eta^{sb}$

From equation (3.4.5), we know that:

$$\frac{p_c(Q_c^{sb}) - c}{p_c(Q_c^{sb})} = \frac{\lambda}{1+\lambda}\left[\frac{1}{\eta^{sb}} + \frac{2c\rho Q_c^{sb}}{p_c^{sb}}\right]$$

Then, having $\frac{p_c^{sb} - c}{p_c^{sb}} > 1/\eta^{sb}$ is equivalent to:

$$\begin{aligned} & \lambda\frac{2c\rho Q_c^{sb}}{p_c^{sb}} > \frac{1}{\eta^{sb}} \\ \Leftrightarrow \quad & \lambda 2c\rho Q_c^{sb} > -Q_c^{sb}(p_c^{sb})' \\ \Leftrightarrow \quad & \lambda > \frac{-(p_c^{sb})'}{2c\rho} \end{aligned} \tag{C.3.1}$$

The last inequality is satisfied as long as the marginal price is low, that is the price is not sensible to variations in quantity consumed. During off-peak period, the production capacities are sufficient and increasing the quantity consumed induces little variations of the price. However, during on-peak period, a small variation of the quantity consumed has an important impact on the price and thus, $(p_c^{sb})'$ is large. Moreover, condition (C.3.1) requires to have $\rho$ and c large. $\rho$ is large when the level of installed capital is quite low, then for small values of $K^{sb}$ the condition can be satisfied. Finally, as for the level of the marginal price, the level of the marginal cost, $c$, depends on the period. Off-peak, the plants used are the cheapest one, so $c$ is low but, on-peak, the marginal plant called is more and more expensive and then $c$ is large.

## C.4 Optimal level investment when the producer is in charge

### C.4.1 First-order condition when $L < \overline{L}$

The first-order condition when the producer/retailer has to pay for thermal losses whereas $L < \overline{L}$ is:

$$t\,\frac{\partial Q_c^m}{\partial p_c^m}\,\frac{dp_c^m}{d\rho(K)}\,\rho'(K) = r$$

The first derivative of the price with respect to the resistance factor comes from the expression of $p_c^m$. This price is given by equation (3.5.8) which can be written as follows:

$$p_c^m = -(p_c^m)' Q_c^m + 2c\rho(K)Q_c + c$$

Using the implicit function theorem, we can write that:

$$\frac{dp_c^m}{d\rho(K)} = \frac{2cQ_c^m}{1 - (Q_c^m)' \Big[ - (p_c^m)' + 2c\rho(K) \Big]} \tag{C.4.1}$$

Moreover, it easy to see that:

$$\frac{\partial Q_c^m}{\partial p_c^m} = -\eta^m \frac{Q_c^m}{p_c^m} \tag{C.4.2}$$

Then, using equations (C.4.1) and (C.4.2), the first-order condition becomes:

$$-c\rho'(K)\,[Q_c^m]^2 \;\; \frac{2t\eta^m}{p_c^m\,[1 - (Q_c^m)'(-(p_c^m)' + 2c\rho)]} = r \tag{C.4.3}$$

where we denote $A = \dfrac{2t\eta^m}{p_c^m\,[1 - (Q_c^m)'(-(p_c^m)' + 2c\rho)]}$. Except for this factor $A$, the decision rule is the same as the one of the private monopoly. In the next paragraph, we study the condition to have $A < 1$.

### C.4.2 Value of $A$

As defined above, we know that:

$$A = \frac{2t\eta^m}{p_c^m\,[1 - (Q_c^m)'(-(p_c^m)' + 2c\rho)]}$$

Then, having $A < 1$ requires to have:

$$2t\eta^m < p_c^m \left[1 - (Q_c^m)'((p_c^m)' + 2c\rho)\right] \tag{C.4.4}$$

$$\Leftrightarrow 2t\eta^m < p_c^m \left[1 + (Q_c^m)'(p_c^m)' - 2c\rho(Q_c^m)'\right]$$

$$\Leftrightarrow \qquad t < \frac{p_c^m}{2\eta^m} \left[1 + (Q_c^m)'(p_c^m)' - 2c\rho(Q_c^m)'\right] \tag{C.4.5}$$

The term into bracket is greater than 1 as $(Q_c^m)'$ and $(p_c^m)'$ are negative. Moreover, from different studies, it appears that $\eta^m$ is lower than 1, so it follows that: $p_c^m < \frac{p_c^m}{2\eta^m}\left[1 + (Q_c^m)'(p_c^m)' - 2c\rho(Q_c^m)'\right]$. So a sufficient condition to satisfy inequality (C.4.5) is to have $t$ such that: $t < p_c^m$.

### C.4.3 The effect of investment on $\mathring{p}_c$

The level of investment affects the price fixed by the producer through the volume of thermal losses and the threshold $\tau(\mathring{p}_c, \overline{L}, K)$. The effect of $K$ on the price $\mathring{p}_c$ is given by equation (3.5.20). Let us denote the left-hand side of equation (3.5.20), $Z(\mathring{p}_c, K)$. Using the implicit function theorem, we have:

$$\frac{d\mathring{p}_c}{dK} = -\frac{\dfrac{\partial Z(.)}{\partial K}}{\dfrac{\partial Z(.)}{\partial \mathring{p}_c}} \tag{C.4.6}$$

where:

$$\begin{aligned}
\frac{\partial Z(.)}{\partial K} = & -2c\rho'(K)\int_0^{\tau_{max}} \mathring{Q}_c \frac{\partial \mathring{Q}_c}{\partial \mathring{p}_c} f(\tau)d\tau + \rho'(K)\int_{\tau(\overline{L},K)}^{\tau_{max}} \mathring{Q}_c^2 f(\tau)d\tau \\
& + 2\mathring{p}_c\rho'(K)\int_{\tau(\overline{L},K)}^{\tau_{max}} \mathring{Q}_c \frac{\partial \mathring{Q}_c}{\partial \mathring{p}_c} f(\tau)d\tau \\
& - 2\mathring{p}_c\rho(K)\left[\mathring{Q}_c(\mathring{p}_c, \tau(\overline{L},K))\frac{\partial \mathring{Q}_c(\mathring{p}_c, \tau(\mathring{p}_c, \overline{L}, K))}{\partial \mathring{p}_c} f(\tau(\mathring{p}_c, \overline{L}, K))\right]\frac{\partial \tau(\mathring{p}_c, \overline{L}, K)}{\partial K}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial Z(.)}{\partial \mathring{p}_c} = & \quad 2E\left(\frac{\partial \mathring{Q}_c}{\partial \mathring{p}_c}\right) + (\mathring{p}_c - c)E\left(\frac{\partial^2 \mathring{Q}_c}{\partial {\mathring{p}_c}^2}\right) + 4\rho(K)\int_{\tau(\overline{L},K)}^{\tau_{max}} \mathring{Q}_c \frac{\partial \mathring{Q}_c}{\partial \mathring{p}_c} f(\tau)d\tau \\
& - 2c\rho(K)\int_0^{\tau_{max}} \left[\mathring{Q}_c \frac{\partial^2 \mathring{Q}_c}{\partial {\mathring{p}_c}^2} + \left(\frac{\partial \mathring{Q}_c}{\partial \mathring{p}_c}\right)^2\right] f(\tau)d\tau \\
& + 2\mathring{p}_c\rho(K)\int_{\tau(\overline{L},K)}^{\tau_{max}} \left[\mathring{Q}_c \frac{\partial^2 \mathring{Q}_c}{\partial {\mathring{p}_c}^2} + \left(\frac{\partial \mathring{Q}_c}{\partial \mathring{p}_c}\right)^2\right] f(\tau)d\tau \\
& - 2\mathring{p}_c\rho(K)\left[\mathring{Q}_c(\mathring{p}_c, \tau(\overline{L},K))\frac{\partial \mathring{Q}_c(\mathring{p}_c, \tau(\mathring{p}_c, \overline{L}, K))}{\partial \mathring{p}_c} f(\tau(\mathring{p}_c, \overline{L}, K))\right]\frac{\partial \tau(\mathring{p}_c, \overline{L}, K)}{\partial \mathring{p}_c}
\end{aligned}$$

There is no clear answer concerning the impact of the level of investment on the price fixed by the producer. Increasing the level of investment may either increase or decrease the price of electricity determined by the monopoly.

# Appendix D

# Promoting renewable energy in a common market with transmission constraint

## D.1 Variation of $W^{nc}$

***Proof of Lemma 4.1.***

i) Let us consider the case of Zone 2, where $x = p$, $x^E = K$, $y = \dfrac{p+\beta}{\gamma}$ and $y^* = y^{*E} = K$. The social welfare function is:

$$W^{nc}(K) = U\left(p + \frac{p+\beta}{\gamma}\right) + U(p^* + K) + \beta\frac{p+\beta}{\gamma} - \frac{p^2}{2} - \frac{p^{*2}}{2} - \frac{\gamma}{2}\left(\frac{p+\beta}{\gamma}\right)^2 - \frac{\gamma}{2}K^2$$

Note that the price $p$ is such that $p = U'(q)$ where

$$q = x - x^E + y + y^{*E} = x + y = p + \frac{p+\beta}{\gamma}$$

So the price $p$ does not depend on the value of $K$. Therefore, the derivative of the welfare function with respect to $K$ is given by:

$$\frac{dW^{nc}(K)}{dK} = \left(1 + \frac{dp^*}{dK}\right)U'(q^*) - K - \frac{dp^*}{dK}p^* - \gamma K$$

$$\Leftrightarrow \frac{dW^{nc}(K)}{dK} = U'(q^*) - (1+\gamma)K + \frac{dp^*}{dK}(U'(q^*) - p^*)$$

Since we are in the Zone 2, $p^* = U'(q^*) < \gamma K$. It is then direct to see that the first derivative of $W^{nc}(.)$ with respect to $K$ is negative.

ii) Consider now the case where $\max\{p, p^*/\gamma\} < K < \min\{p^*, (p+\beta)/\gamma\}$, which corresponds to the Zone 3 with $x = x^E = y^* = y^{*E} = K$. Therefore, the levels of consumption in each country are:

$$q = K + \frac{p+\beta}{\gamma} \text{ and } q^* = p^* + K$$

The social welfare level is characterized by:

$$W^{nc}(K) = U\Big(K + \frac{p+\beta}{\gamma}\Big) + U(p^* + K) + \beta\frac{p+\beta}{\gamma} - \frac{K^2}{2} - \frac{p^{*2}}{2} - \frac{\gamma}{2}\Big(\frac{p+\beta}{\gamma}\Big)^2 - \frac{\gamma}{2}K^2 \tag{D.1.1}$$

The first derivative with respect to the capacity of the line $K$ is:

$$\frac{dW^{nc}(K)}{dK} = U'(q) - K + U'(q^*) - \gamma K + \frac{dp}{dK}\Big(\frac{U'(q) - p}{\gamma}\Big) + \frac{dp^*}{dK}(U'(q^*) - p^*)$$

To determine the impact of $K$ on $W^{nc}(.)$, we have to determine the sign of: $U'(q) - K$ and $U'(q^*) - \gamma K$. But, as $\max\{p, p^*/\gamma\} < K < \min\{p^*, (p+\beta)/\gamma\}$, and knowing that $U'(q) = p$ and $U'(q^*) = p^*$, we can write that:

$$U'(q) - K = p - K < 0 \text{ and } U'(q^*) - \gamma K = p^* - \gamma K < 0$$

Hence, when $\max\{p, p^*/\gamma\} < K < \min\{p^*, (p+\beta)/\gamma\}$, the social welfare function is a decreasing function of the capacity of the line $K$.

We have proved that, for Zone 2 and Zone 3, $\dfrac{dW^{nc}}{dK} < 0$. □

## D.2 Variation of $\widehat{K}$ in Zone 2

***Proof of Corollary 4.1.***
From the definition of $\widehat{K}$, we find that:

$$\frac{d\widehat{K}}{d\beta} = \frac{\frac{2U'(q_c)+\beta}{2\gamma} - \frac{U'(q)+\beta}{\gamma}}{U'(q^*) - \gamma\widehat{K}} \tag{D.2.1}$$

As $U'(q^*) - \gamma\widehat{K}$ is negative (see proof of lemma 1), the first derivative of $\widehat{K}$ with respect to $\beta$ is positive if and only if:

$$\frac{2U'(q_c)+\beta}{2\gamma} - \frac{U'(q)+\beta}{\gamma} < 0 \Leftrightarrow U'(q_c) - U'(q) < \frac{\beta}{2}$$
$$\Leftrightarrow p_c - p < \frac{\beta}{2}$$

In the coordinated case, the price $p_c$ is defined by

$$U'\left(p_c + \frac{p_c + \beta/2}{\gamma}\right) = p_c$$

whereas $p$ is defined by:

$$U'\left(p + \frac{p+\beta}{\gamma}\right) = p$$

We remark that the two formula are similar except for the subsidy that the producer receives. In section 4.4.1, we have seen that the price is a decreasing function of the subsidy, so we can conclude that $p_c < p$. This relation implies that the total quantities consumed in each case are such that $q_c - q < 0$. Let us look at the value of this difference.

$$q_c - q < 0 \Leftrightarrow p_c + \frac{p_c + \beta/2}{\gamma} - p - \frac{p+\beta}{\gamma} < 0$$
$$\Leftrightarrow p_c - p < \frac{\beta}{2(\gamma+1)}$$

By assumptions, we know that $\gamma > 1$, thus we can write that $p_c - p < \beta/2$. We just have proved that $\frac{2U'(q_c)+\beta}{2\gamma} - \frac{U'(q)+\beta}{\gamma} < 0$, and thus that

$$\frac{d\widehat{K}}{d\beta} > 0$$

□

## D.3 Variation of $\widehat{K}$ in Zone 3

***Proof of Corollary 4.2.***

The value of the first derivative of $\widehat{K}$ with respect to $\beta$ is:

$$\frac{d\widehat{K}}{d\beta} = \frac{\frac{U'(q_c)+\beta/2}{\gamma} - \frac{U'(q)+\beta}{\gamma}}{U'(q) - \widehat{K} + U'(q^*) - \gamma\widehat{K}} \tag{D.3.1}$$

From section 4.5.1, we know that in Zone 3,

$$U'(q) - \widehat{K} + U'(q^*) - \gamma\widehat{K} < 0$$

Then, it appears that $\widehat{K}$ is an increasing function of $\beta$ if and only if the numerator of equation (D.3.1) is negative which is equivalent to:

$$p_c - p < \frac{\beta}{2}$$

The two prices of the zone are defined by:

$$\begin{cases} U'(p_c + \dfrac{p_c + \beta/2}{\gamma} - p_c = 0 \\ \\ U'\left(\widehat{K}(p, p_c) + \dfrac{p + \beta}{\gamma}\right) - p = 0 \end{cases}$$

We know that at the beginning of the zone, by continuity, we have $p_c - p < \frac{\beta}{2}$. The idea is to see how $p_c - p < \frac{\beta}{2}$ evolves when $\beta$ changes in Zone 3. Thus, we will determine the expressions of the first derivative of the prices with respect to $\beta$. Let us first compute $dp_c/d\beta$:

$$\frac{dp_c}{d\beta} = -\frac{\frac{U''(q_c)}{2\gamma}}{(1 + 1/\gamma)U''(q_c) - 1}$$

which is negative by $U'' < 0$. The computation of $dp/d\beta$ is trickier since $p$ depends on $\widehat{K}$ which depends on $p_c$ and $p$. So, the computation leads to:

$$\frac{dp}{d\beta} = -\frac{U''(\widehat{q})\left[\frac{\partial\widehat{K}}{\partial p_c}\frac{dp_c}{d\beta} + \frac{1}{\gamma}\right]}{U''(\widehat{q})\left[\frac{\partial\widehat{K}}{\partial p} + \frac{1}{\gamma}\right] - 1}$$

We know that $\widehat{K}$ comes from $W^c(\widehat{K}) - W^{nc}(\widehat{K}) = 0$. Let us compute $\dfrac{\partial\widehat{K}}{\partial p_c}$ and $\dfrac{\partial\widehat{K}}{\partial p}$.

$$\begin{cases} \dfrac{\partial \widehat{K}}{\partial p_c} = -\dfrac{\dfrac{dW^c}{dp_c} - \dfrac{dW^{nc}}{dp_c}}{\dfrac{dW^c}{d\widehat{K}} - \dfrac{dW^{nc}}{d\widehat{K}}} \\ \\ \dfrac{\partial \widehat{K}}{\partial p} = -\dfrac{\dfrac{dW^c}{dp} - \dfrac{dW^{nc}}{dp}}{\dfrac{dW^c}{d\widehat{K}} - \dfrac{dW^{nc}}{d\widehat{K}}} \end{cases}$$

Some simplifications are obvious. Indeed, we know that

$$\frac{dW^{nc}}{dp_c} = \frac{dW^c}{d\widehat{K}} = \frac{dW^c}{dp} = 0$$

and that:

$$\frac{dW^{nc}}{d\widehat{K}} \neq 0$$

The rest of the computation is given by:

$$\begin{aligned} \frac{dW^c}{dp_c} &= 2(1+1/\gamma)U'(q_c) + \frac{\beta}{\gamma} - 2p_c - \frac{p_c + \beta/2}{\gamma} \\ &= 2[U'(q_c) - p_c] + \frac{2U'(q_c) + \beta}{\gamma} - \frac{2p_c + \beta}{\gamma} \\ \Leftrightarrow \frac{dW^c}{dp_c} &= 0 \end{aligned}$$

We find the same result for $dW^{nc}/dp$ coming from:

$$\begin{aligned} \frac{dW^{nc}}{dp} &= \frac{U'(q)}{\gamma} + \frac{\beta}{\gamma} - \frac{p + \beta}{\gamma} \\ \Leftrightarrow \frac{dW^n c}{dp} &= 0 \end{aligned}$$

Then, the first derivative of $p$ with respect to $\beta$ is just given by:

$$\frac{dp}{d\beta} = -\frac{U''(\widehat{q})/\gamma}{(U''(\widehat{q})/\gamma) - 1} < 0$$

In order to know how $p_c - p - \frac{\beta}{2}$ evolves with $\beta$, we assume that the function $U''(.)$ is constant, hence we can write that: $U''(q_c) = U''(\widehat{q})$. The question is to know if:

$$\frac{dp_c}{d\beta} - \frac{dp}{d\beta} < 1/2$$

This relation is true if and only if:

$$\begin{aligned}
& -\frac{U''(.)/2\gamma}{(1+1/\gamma)U''(.)-1} + \frac{U''(.)/\gamma}{(U''(.)/\gamma)-1} - \frac{1}{2} < 0 \\
\Leftrightarrow\ & -\frac{U''(.)}{2\gamma}(U''(.)-\gamma) + 2U''(.)(U''(.)(1+1/\gamma)-1) - \frac{1}{2}(U''(.)(1+1/\gamma)-1)(U''(.)-\gamma) < 0 \\
\Leftrightarrow\ & (U''(.))^2 + U''(.)(\gamma+1) - \gamma < 0
\end{aligned}$$

By assumption, $U''(.)$ is constant. We can easily show that the inequality is satisfied if and only if:

$$U''(.) \in \left] -\frac{(\gamma+1)+\sqrt{(\gamma+1)^2+4\gamma}}{2}, 0\right]$$

This is a sufficient condition to have $d\widehat{K}/d\beta > 0$. □

# References


Alzate J.M. and A. Garcia [2010]. "Regulatory design and incentives for renewable energy". *Working Paper.*

Amaral M., S. Saussier and A. Yvrande-Billon [2009]. "Auction procedures and competition in public services: The case of urban public transport in France and London". *Utilities Policy*, vol.17, n°2, pp. 166–175, June.

Aubert C., P. Bontems and F. Salanié [2006]. "Le renouvellement périodique des contrats de concession: Le cas des services de l'eau". *Annals of Public and Cooperative Economics*, vol. 77, n°4, pp. 495–520, December.

Beckmann M. [1976]. "Spatial price policies revisited". *The Bell Journal of Economics*, vol. 7, n° 2, pp. 619–630, Autumn.

Benitez D. A. [2004]. "On Quantity Competition and Transmission Constraints in Electricity Market", *Econometric Society, Latin American Meetings*, n°98, August.

Bernstein M. and J. Griffin [2005]. "Regional Differences in the Price-Elasticity of Demand for Energy". *Santa Monica, CA: RAND Corporation.*

Bessec M. and J. Fouquau [2008]. "The non-linear link between electricity consumption and temperature in Europe: A threshold panel approach". *Energy Economics*, vol. 30, n°5, pp. 2705–2721, September.

Borenstein S., J. Bushnell and S. Stoft [2000]. "The competitive effects of transmission capacity in a deregulated electricity industry", *The RAND Journal of Economics*, vol. 31, n° 2, pp. 294–325, Summer.

Borenstein S., J. Bushnell and F. Wolak [2002]. "Measuring market inefficiencies in California's wholesale electricity industry". *American Economic Review*, vol. 92, n° 5, pp. 1376–1405, Winter.

Commission de Régulation de l'Energie [2010]. "Les Dispositifs de couverture des pertes d'énergie des réseaux publics d'électricité", March.

Commission de Régulation de l'Energie [2010]. "Rapport sur la qualité de l'électricité: Diagnostics et propositions relatives à la continuité de l'alimentation en électricité". October.

Commission de Régulation de l'Energie [2012]. "Consultation publique de la Commission de régulation de l'énergie du 6 novembre 2012 sur les quatrièmes tarifs d'utilisation des réseaux publics d'électricité". November.

Commission de Régulation de l'Electricité et du Gaz [2010]. "Etude relative à la faisabilité de l'instauration d'une tarification progressive de l'électricité en Belgique", June.

Council of European Energy Regulators [2011]. "5th CEER Benchmarking Report on the Quality of Electricity Supply". April.

Coyte P. and C. R. Lindsey [1988]. "Spatial monopoly and spatial monopolistic competition with two-part pricing". *Economica*, vol. 55, n° 220, pp. 461–477, November.

Crampes C. and S. Ambec [2012]. "Electricity provision with intermittent sources of energy". *Resource and Energy Economics*, vol. 34, n°3, pp. 319–336.

Crampes C. and A. Estache [1998]. "Regulatory trade-offs in the design of concession contracts". *Utilities Policy*, vol. 7, issue 1, pp. 1–13, March.

Crampes C. and J.J. Laffont [2001]. "Transport pricing in the electricity industry". *Oxford Review of Economic Policy*, vol. 17, n°3, pp. 313–328.

Crampes C. and M. Moreaux [2010]. "Pumped storage and cost saving", *E*nergy Economics, vol. 32, issue 2, pp. 325-333.

Creti A. and N. Fabra [2007]. "Supply security and short-run capacity markets for electricity". *Energy Economics*, vol. 29, n°2, pp. 259–276, March.

European Regulators' Group of Electricity and Gas [2008]. "Treatment of Losses by Network Operators: ERGEG Position Paper for public consultation". July.

Eurelectric [2008]. "Comments on the ERGEG Position Paper for public consultation on Treatment of Losses by Network Operators". October.

Fabra N., N.-H. M. von der Fehr, and D. Harbord [2006]. "Designing electricity auctions". *RAND Journal of Economics*, vol. 37, n°1, pp. 23–46, Spring.

Faruqui A. [2008]. "Is electricity price-elastic enough for rate designs to matter?". *Fortnightly Magazine* , August.

Glachant J.M. and F. Lévêque [2009]. "Electricity Reform in Europe: Towards a Single Energy Market". *Edward Elgar Pub.*

Green R. and N. Vasilakos [2010]. "Market behaviour with large amounts of intermittent generation". *Energy Policy*, vol. 38, n°7, pp. 3211–3220.

Hart O. [2003]. "Incomplete Contracts and Public Ownership : Remarks, and an Application to Puplic-Private Partnerships". *The Economical Journal*, n°113, pp. C69–C76, March.

Heng, H. Y., Li, F. R. and Wang, X. F. [2009]. "Charging for Network Security Based on Long-Run Incremental Cost Pricing". *IEEE Transactions on Power Systems*, vol. 24, n° 4, pp. 1686–1693.

Hogan W. [1997]. "A Market Power Model with Strategic Interaction in Electricity Networks". *The Energy Journal*, vol.18 , n°4, pp. 107–141.

Jamasb T. and M. Pollitt [2007]. "Incentive regulation of electricity distribution networks: Lessons of experience from Britain". *Energy Policy, Elsevier*, vol. 35, n°12, pp. 6163–6187, December.

Joskow P. and J. Tirole [2000]. "Transmission Rights and Market Power on Electric Power Networks". *The RAND Journal of Economics*, vol. 31, n°3, pp. 450–487, Autumn.

Joskow P. and J. Tirole [2005]. "Merchant Transmission Investment". *Journal of Industrial Economics*, vol. 53, n°2, pp. 233-264.

Joskow P. and J. Tirole [2007]. "Retail electricity competition". *Rand Journal of Economics*, vol. 37, n°4, pp. 799-815, Winter.

Léautier T.O. [2012]. "Is mandating "smart meters" smart?". *IDEI Working Paper Series*, n°747, October.

Leuthold F. and R. Wand [2009]. "Feed-in tariffs for photovoltaics: Learning by doing in Germany?". *Working Paper*, vol. WP-RD-03.

Martimort D. and J. Pouyet [2008]. "To Build or not to build : Normative and positive theories of public-private partnerships". *International Journal of Industrial Organization*, vol. 26, n°2, pp. 393–411.

Maskin E. and J. Tirole [2008]. "Public-Private Partnerships and Government Spending Limits". *International Journal of Industrial Organization*, vol. 26, n°2, pp. 412–420.

Office of Gas and Electricity Markets [2003]. "Electricity Distribution Losses: a consultation document". January.

Oi W. Y. [1971]. "A Disneyland dilemma: Two-part tariffs for a Mickey Mouse monopoly". *Quarterly Journal of Economics*, vol. 85, n°1, pp. 77–96, February.

Rothwell G.S. and T. Gómez [2003]. "Electricity economics: regulation and deregulation". *John Wiley*

Saplacan R. [2008]. "Competition in electricity distribution". *Utilities Policy*, vol. 16, issue 4, pp. 231–237.

Schweppe F.C., M.C. Caramanis, R.D. Tabors and R.E. Bohn [1988]. "Spot Pricing of Electricity". *Boston: Kluwer Academic Press.*

Sénat [2012]. "Rapport fait au nom de la commission d'enquête sur le coût réel de l'électricité afin d'en déterminer l'imputation aux différents agents économiques". *Journal Officiel*,n°667, July.

Spulber D. [1981]. "Spatial Nonlinear Pricing". *American Economic Review*, vol. 71, n°5, pp. 923–933, December.

Stoft S. [1997]. "The effect of the transmission grid on market power". *Berkeley*, LBNL–40479.

Voogt M.H. and M.A. Uyterlinde [2006]. "Cost effects of international trade in meeting EU renewable electricity targets". *Energy Policy*, vol. 34, n°3, pp. 352–364, February.

Weitzman M.L. [1974]. "Prices vs. Quantities". *The Review of Economic Studies*, vol. 41, n°4, pp. 477–491, October.

Willems B. [2002]. "Modeling Cournot competition in an electricity market with transmission constraints". *The Energy Journal*, vol. 23, n°3, pp. 95–125.