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ABSTRACT

Working-Time Regulation, Firm Heterogeneity, and Efficiency*

A labour-matching economy with *ex post* heterogeneous firms is presented. When bargaining over the wage, firms and workers do not know the level of product demand. Once demand is realized, hours of work are chosen. We show that the existence of a legal workweek may enhance efficiency with respect to laissez-faire: while laissez-faire is good at allocating hours across firms, regulation may be better at reproducing optimal hours. Shortening the legal workweek raises employment and is Pareto-improving if and only if the demand faced by low-demand firms and/or the overtime premium are small enough.

JEL Classification: E24, J22, J23, J30 and J41
Keywords: efficiency, unemployment, work-sharing and working-time

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1 Introduction

A statutory working time exists in 28 out of 30 OECD countries (ILO, 1995). In most cases, the legislation establishes a normal weekly duration and an overtime premium for each hour exceeding the normal duration. A simple way to understand the importance of this institution consists in looking at the distribution of the *effective* number of weekly hours performed by employees. In countries characterised by the existence of a statutory workweek, the hour distribution delivers a remarkable spike in correspondance of the statutory workweek. For example, in the United States, where employees covered by the Fair Labor Standards Act (FLSA) of 1938 are paid time and a half for weekly hours beyond 40, more than one third of employees are currently performing exactly 40 hours weekly, with no other sizeable spike in the hour distribution (see Fig. 1). In France, where the legal workweek was 39 hours in 1999, 43% of the employees performed 39 hours weekly (Fig. 2). In contrast, in a country without a legal working time, such as the United Kingdom in 1991, the highest peak (at 40 hours) concentrates only 18% of the employees, and 4 other peaks above 10% can be detected (see Fig. 3).

The objective of this paper is two-fold. First, it explores whether there is an *efficiency rationale* for regulating working time. In the past, the existence of a legal workweek was essentially aimed at improving working conditions. It is less clear however whether such an institution is useful nowadays in developed countries. In other words, why should the government constrain the choices of workers and employers concerning hours of work? Here we develop a second best framework in which the institution of a legal workweek may be the only way of reproducing *average* efficient hours. However, in our analysis, this institution is not good *per se*, and its ability to improve efficiency depends on the *level* at which the legal working time is set. In other words, modifying the duration of the legal workweek has efficiency consequences. This brings us to the second objective of this paper, which is assessing the efficiency effect of so-called "work-sharing" policies, consisting in shortening the legal workweek. This issue corresponds to an important debate in Europe, as proponents of work-sharing have argued that a reduction of the legal working time could be used as a means of fighting unemployment. This policy has been implemented in a series of countries, the most recent example being France, in which the legal workweek has been shortened from 39 to 35 hours in 2000-2002.

Our economy consists of a fixed number of workers and an endogenous number of firms brought together into productive matches by a standard matching function à la Pissarides (2000). Firms are heterogeneous in their product demand levels. When a firm and a worker meet, hourly wages are negotiated. At this stage, the prospective level of product demand is unknown, but its distribution is known. Once the demand level is realised and all uncertainty is resolved, firms choose the number of hours.

The *laissez-faire* allocation reproduces one of the features of the constrained optimum, namely that the number of hours is strictly increasing in the firm demand level. However, in the absence of contingent contracts that specify a wage rate for each state of demand, the number of hours worked for *each* demand level is inefficiently low. Thus, laissez-faire cannot match the efficient *average* working time.

Under *regulation*, the government establishes a statutory working time and an overtime premium for hours beyond it. Firms and workers bargain first the hourly wage corresponding to the statutory working time, and can adjust hours after demand is realised. The equilibrium distribution of hours is characterised by the existence of a spike at the legal duration, i.e., a set of firms with different demand levels choose exactly the legal working time. Therefore, hours are not always strictly increasing in demand, and the distribution of hours across firms is not efficient. However, since low-demand firms choose longer hours than efficient and high-demand firms shorter hours than efficient, regulation may be good at reproducing the optimal *average* working time.

If firm heterogeneity is low, the loss associated to imposing a homogeneous working time pattern to all firms is smaller. In this case, regulation may perform better than laissez faire, since its ability to match optimal average hours becomes the key feature for getting closer to efficiency[1].

We then characterise the conditions under which work-sharing boosts employment. We show that employment rises if and only if low-demand firms experience "small enough" demand levels and/or the wage rate associated with overtime is relatively small. If the former is true, the existence of a legal working time constrains many low-demand firms to employ their workers for longer hours than efficient. In other words, the legal workweek plays the role of a *downward* working time rigidity for low-demand firms. Thus, this rigidity matters more, the lower the demand level of the least 'successful' firm. In this scenario, reducing the legal workweek allows many low-demand firms to choose shorter hours and boosts employment, as the resources formerly devoted to longer hours in low-demand firms can now be translated into newly created jobs. By contrast, if overtime is expensive, the legal workweek constrains many high-demand firms to choose shorter hours than efficient. Firms in the right end of the demand distribution are therefore *upward* constrained. In this case, work-sharing increases the number of high-demand firms who need to pay overtime if they wish to keep long hours. This lowers profitability and worsens employment.

We show that the whenever work-sharing boosts employment, it also enhances efficiency and leads to a Pareto-improvement. These results are robust to the existence of an endogenous overtime premium and to the introduction of part-time labour constracts.

Whether working-time regulations may be desirable in modern economies is an issue

[1] This is a result à la Lipsey and Lancaster (1956): introducing an additional departure from the competitive equilibrium in a second-best economy may improve efficiency.

that has been recently addressed in the theoretical literature. Marimon and Zilibotti (2000) build a matching model with homogeneous firms and workers to compare the properties of the laissez-faire equilibrium (bargaining over wages and hours) with those of a regulated economy, in which hours are exogenously set by the government at a lower level than that resulting in laissez-faire. They show that this type of regulation may improve total surplus. However, this policy is not Pareto-improving, as firms are worse-off. In a matching model with worker moral hazard, Rocheteau (2002) shows that working time regulation defined à la Marimon and Zilibotti (2000) is Pareto-improving in some cases. Our way of comparing laissez-faire and regulation is different from that of Marimon and Zilibotti (2000) and Rocheteau (2002), since in our set-up firms are heterogeneous and therefore we do not need to impose *ex ante* that the equilibrium expected number of hours in laissez-faire be higher than under regulation. Additionally, and unlike Marimon and Zilibotti (2000), overtime regulation is not neutral in our framework, in the sense that the imposition of an overtime premium is not undone through adjustments of the straight time wage. This result is in line with empirical analyses for the U.S which have studied the impact of FLSA overtime premium provisions. Although a consensus has not formed yet,[2] Trejo (1991) and Costa (2000) show that the hourly wage for straight hours does not adjust to the extent of keeping total wage income constant. In other words, working time regulation may affect the equilibrium outcome.

The issue of work-sharing has been studied extensively, although neither the empirical nor the theoretical literature has reached a consensus on its employment effects. As for the former, e.g. Faini and Schiantarelli (1985) and Franz and König (1986) find positive effects, while Brunello (1989), Hunt (1999), Kramarz and Crépon (1999) and Hernanz *et al.* (1999) find negative effects. Theoretical work has identified a series of effects going in opposite directions. Calmfors (1985) and Booth and Schiantarelli (1987) have studied work-sharing in a monopoly union setting, and Calmfors and Hoel (1988) in a competitive framework. Askenazy (2000) studies the impact of the French 35-hours law and argues that the law favours restructuring of firms that boosts job creation. The papers closer in spirit are Marimon and Zilibotti (2000) and Rocheteau (2002), but their definition of work-sharing is different. In their papers, work-sharing is interpreted as the change from laissez-faire to regulation. In our paper, it is viewed as the shift between two regulation equilibria with different number of statutory hours.

[2] Trejo (2001) finds that the expansions in overtime pay coverage (1970-1989) in the U.S. did not have an impact on the number of overtime hours, implying that the straight-time hourly wages adjusted downwards when firms started to be regulated.

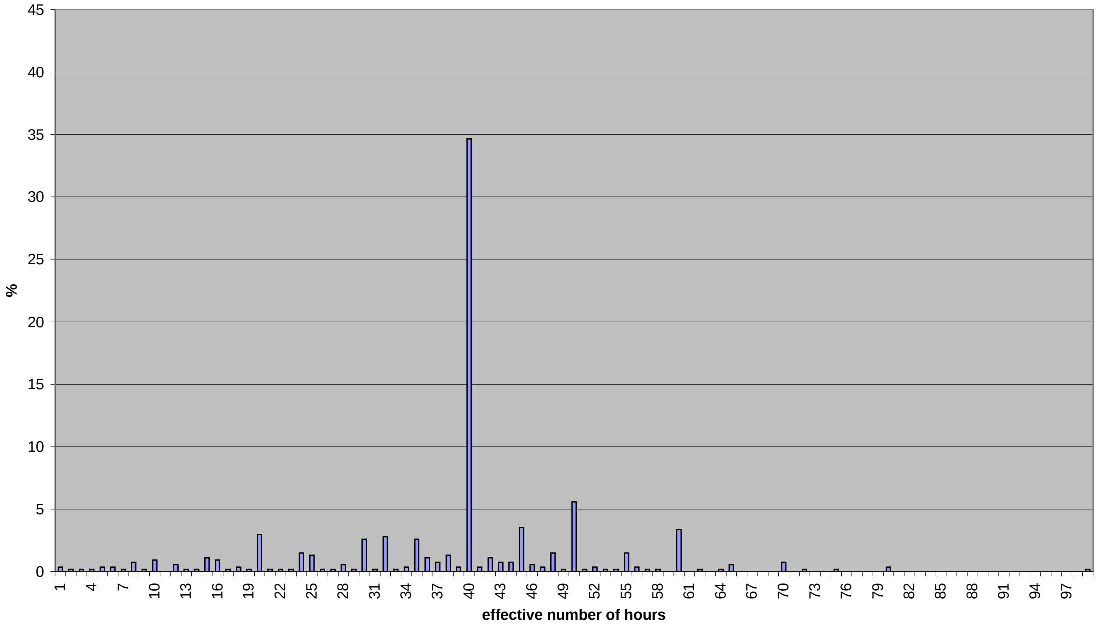

Figure 1: U.S., hours actually worked last week, all jobs, CPS, March 2002.

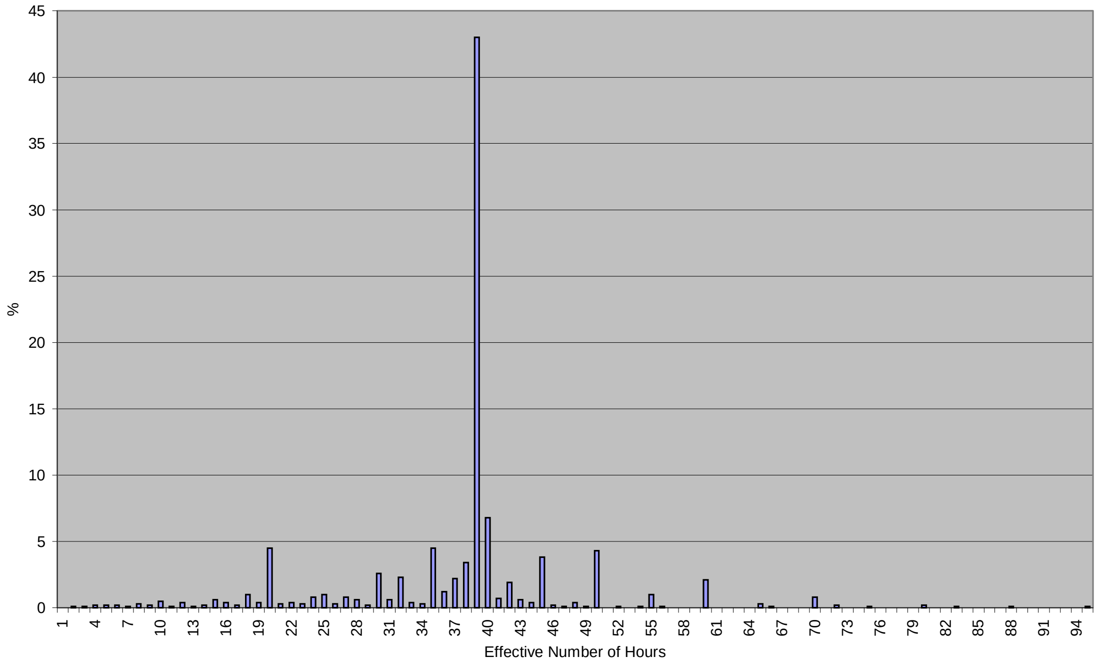

Figure 2: France, Effective Number of Weekly Hours, Employment Survey, March 1999.

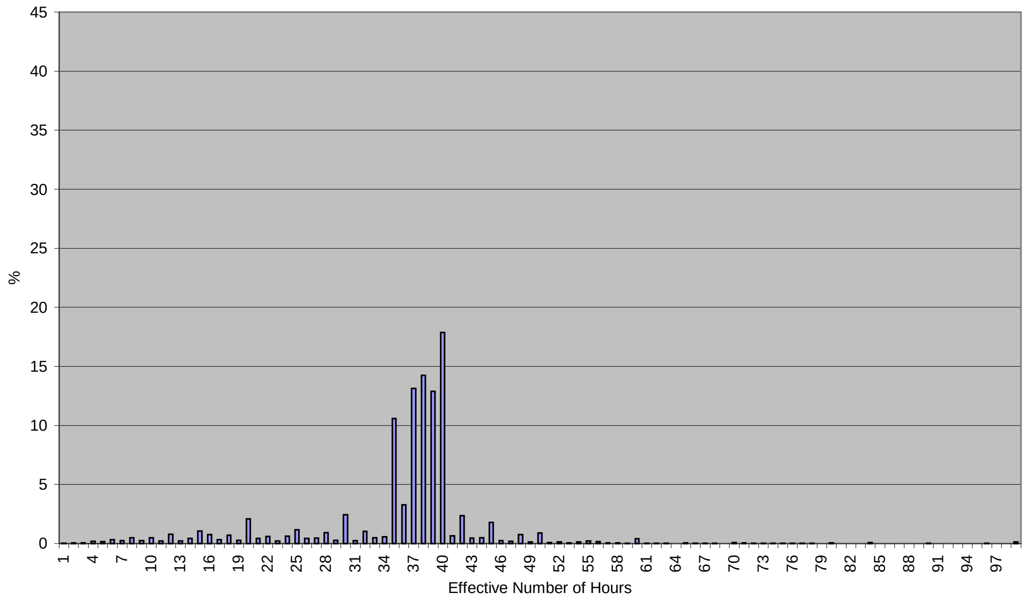

Figure 3: UK, Effective Number of Weekly Hours. Labour Force Survey 1991.

2 The model

2.1 Environment

The economy consists of a fixed number of workers, normalised to 1, and an endogenous number of one-job firms. Time is continuous and lasts forever. The entire analysis is carried out in steady state. Firms post vacancies, and meetings between individuals and firms are ruled by a CRS matching function à la Pissarides (2000):

$$m = m(v, u) \tag{1}$$

where m is the number of job matches, v the number of vacancies and u the number of unemployed. Given CRS and the standard random matching assumption, a vacancy is matched to a worker at Poisson rate $q(\theta) \equiv \frac{m(v,u)}{v} = m\left(1, \frac{1}{\theta}\right)$ and a worker is matched to a vacancy with Poisson rate $\frac{m}{u} = \frac{m(v,u)}{u} = m(\theta, 1) = \theta q(\theta)$ with $\theta \equiv \frac{v}{u}$ denoting market tightness. Existing job matches separate at an exogenous rate s. The dynamic equation governing employment L is $\frac{dL}{dt} = \theta q(\theta)u - sL$. In steady-state, employment depends positively on labour market tightness (θ):

$$L = \frac{\theta q(\theta)}{\theta q(\theta) + s}. \tag{2}$$

When matched to a worker, each firm j produces a homogeneous good and faces a demand level y_j, which is a random draw from a uniform distribution in the interval $[y_l, y_m]$. When a vacancy is posted, the prospective demand level is unknown. At this stage, only its distribution is known.

The production technology available to each firm is:

$$G(h_j) = \frac{h_j^\eta}{\eta}, \tag{3}$$

where h_j is the number of hours of work when the demand level is y_j and $0 < \eta < 1$.

The disutility from work (cost of effort) for each worker is:

$$c(h_j) = h_j^\mu, \tag{4}$$

where $\mu \geq 1$.[3]

2.2 Constrained efficient allocation

We first characterise the efficient allocation of this economy, which will serve as a benchmark for decentralised equilibria. The constrained efficient allocation maximises expected social output (expected production net of expected disutility of work and search costs) under the matching constraint, i.e.,

$$\begin{aligned} \underset{\theta,\ h_j}{MAX}\ W &= (1-u)\int_{y_l}^{y_m} [G(h_j)y_j - c(h_j)]\, dF(y_j) - u\theta\gamma \\ s.t.\ \ u &= \frac{s}{s+\theta^\alpha}, \end{aligned} \tag{5}$$

where we have assumed a Cobb-Douglas matching function with parameter α on v. The F.O.C. to problem (5) are:

$$\frac{\partial W}{\partial \theta} = 0 \Leftrightarrow \alpha \int_{y_l}^{y_m} [G(h_j)y_j - c(h_j)]\, dF(y_j) - s\gamma\theta^{*1-\alpha} - (1-\alpha)\gamma\theta^* = 0; \tag{6}$$

$$\frac{\partial W}{\partial h_j} = 0 \Leftrightarrow h_j^* = \left(\frac{y_j}{\mu}\right)^{\frac{1}{\mu-\eta}}. \tag{7}$$

Efficiency here has two dimensions. First, for a given value of $E[Gy] - Ec$,[4] the planner chooses the number of vacancies taking into account the matching externalities. This is given by (6), and corresponds to the usual condition in the literature (see Pissarides, 2000).

[3] In the simulations, we will assume a slightly more general production technology $G(h_j) = \frac{Ah_j^\eta}{\eta}$ with $A > 0$ and disutility from work, $c(h_j) = Dh_j^\mu$ with $D > 0$.

[4] In the rest of the paper, we simplify the notation by eliminating firm indexes j when taking expectations.

Second, condition (7) guarantees that the allocation of hours *across* firms is efficient, i.e. that the marginal product is equal to the marginal disutility of work in each firm, which implies that hours should increase in product demand. Replacing (7) into (6), the optimal labour market tightness θ^* satisfies:

$$\alpha\left(E[Gy]^* - Ec^*\right) - s\gamma{\theta^*}^{1-\alpha} - (1-\alpha)\gamma\theta^* = 0. \tag{8}$$

Using (7) and the uniform distribution of y, we can derive the expressions for the expected value of production ($E[Gy]^*$), the expected disutility of work (Ec^*), and the expected number of hours (Eh^*) (see appendix (i)):

$$E[Gy]^* = \frac{\mu-\eta}{\eta(2\mu-\eta)}\mu^{\frac{-\eta}{\mu-\eta}}\frac{y_m^{\frac{2\mu-\eta}{\mu-\eta}} - y_l^{\frac{2\mu-\eta}{\mu-\eta}}}{y_m - y_l}; \tag{9}$$

$$Ec^* = \frac{\mu-\eta}{2\mu-\eta}\mu^{\frac{-\mu}{\mu-\eta}}\frac{y_m^{\frac{2\mu-\eta}{\mu-\eta}} - y_l^{\frac{2\mu-\eta}{\mu-\eta}}}{y_m - y_l}; \tag{10}$$

$$Eh^* = \frac{\mu-\eta}{1+\mu-\eta}\mu^{\frac{-1}{\mu-\eta}}\frac{y_m^{\frac{1+\mu-\eta}{\mu-\eta}} - y_l^{\frac{1+\mu-\eta}{\mu-\eta}}}{y_m - y_l}. \tag{11}$$

2.3 Laissez-Faire Economy

The laissez-faire (LF) economy is characterised by two *departures* from the competitive equilibrium. First, there are matching frictions represented by (1). Second, we assume that contracts are incomplete, in the sense that firms and workers cannot sign contingent contracts specifying the hourly wage and the number of hours for each state of demand, i.e. $\{w(y_j), h(y_j)\}_{\forall y_j \in [y_l, y_m]}$. Such contingent contracts are typically not used in actual economies. Indeed, it may be difficult for the employee to observe the actual state of demand.[5] We will consider instead a situation in which firms and workers bargain over wages before knowing the state of demand, and choose hours once the demand is realised. The set of available contracts is therefore $\{w, h(y_j)\}_{\forall y_j \in [y_l, y_m]}$.

2.3.1 Firms

Firms post vacancies, which are filled with probability $q(\theta)$. The value of an unfilled vacancy is:

$$rV = -\gamma + q(\theta)(J^e - V), \tag{12}$$

where γ is the flow cost of posting a vacancy, and J^e is the expected value of a filled job, as the state of demand is still unknown when a vacancy is posted.

[5] We study below the consequences of the existence of such type of contracts.

When a firm and a worker meet, the timing is as follows. First, the hourly wage w is bargained. Second, the state of product demand is revealed, and the firm chooses the working time. The value of a job in a firm with product demand y_j and hours h_j is therefore:

$$rJ_j = G(h_j)y_j - wh_j + s(V - J_j). \tag{13}$$

Firms open vacancies up to the point where the expected value of posting a further vacancy is zero ($V = 0$). Then, from (12):

$$J^e = \frac{\gamma}{q(\theta)}, \tag{14}$$

stating that, in equilibrium, the expected income from a filled vacancy must equal the total costs of posting it.

2.3.2 Workers

Unemployed workers receive no unemployment income (with no loss of generality) and find jobs at rate $\theta q(\theta)$. The asset value of unemployment is given by:

$$rU = \theta q(\theta)(E^e - U), \tag{15}$$

where E^e is the expected value of employment. Once a worker meets a firm with demand level y_j, her employment value (E_j) is given by her wage minus the disutility of work and the loss incurred in the event of a separation:

$$rE_j = wh_j - c(h_j) + s(U - E_j). \tag{16}$$

2.3.3 Wages

Wages are the outcome of a bilateral Nash bargain between each firm and worker. When bargaining over the wage, the state of demand is unknown. The worker receives expected employment value E^e if an agreement is reached and her threat point is U. The expected value of the firm in case of succesful agreement is J^e and its threat point is V. The wage w solves:

$$\underset{w}{MAX}\ (E^e - U)^{\beta}(J^e - V)^{1-\beta}, \tag{17}$$

where β is the worker's bargaining power. The optimality condition of this program is:

$$\beta(J^e - V)\frac{\partial E^e}{\partial w} + (E^e - U)(1-\beta)\frac{\partial J^e}{\partial w} = 0. \tag{18}$$

Once the level of demand has been realised, the number of hours chosen solves:

$$\underset{h_j}{MAX}\ G(h_j)y_j - wh_j \qquad (19)$$
$$s.t.\ w \geq c'(h_j),$$

i.e. the firm picks the number of hours that maximises its profits, subject to worker acceptance. We will consider situations in which this constraint is *ex post* verified. The solution to the hours' problem is given by:

$$h_j^{LF} = \left(\frac{y_j}{w}\right)^{\frac{1}{1-\eta}}, \qquad (20)$$

with the number of hours increasing in the demand level. From (20) and the uniform distribution of y_j, the values of $E[Gy]^{LF}$, Ec^{LF} and Eh^{LF} can be derived:

$$E[Gy]^{LF} = \frac{1-\eta}{\eta(2-\eta)} \frac{y_m^{\frac{2-\eta}{1-\eta}} - y_l^{\frac{2-\eta}{1-\eta}}}{y_m - y_l} w^{\frac{-\eta}{1-\eta}} \qquad (21)$$

$$Ec^{LF} = \frac{1-\eta}{1-\eta+\mu} \frac{y_m^{\frac{1-\eta+\mu}{1-\eta}} - y_l^{\frac{1-\eta+\mu}{1-\eta}}}{y_m - y_l} w^{\frac{-\mu}{1-\eta}} \qquad (22)$$

$$Eh^{LF} = \frac{1-\eta}{2-\eta} \frac{y_m^{\frac{2-\eta}{1-\eta}} - y_l^{\frac{2-\eta}{1-\eta}}}{y_m - y_l} w^{\frac{-1}{1-\eta}}. \qquad (23)$$

Finally, using these expressions in the FOC to the Nash bargain (18), we obtain the wage [see equation (a1)].

2.3.4 Efficiency properties of laissez-faire

Comparing (7) with (20), it clearly appears that the number of hours in the LF allocation differs from the constrained-efficient number of hours unless $\mu = 1$ and the equilibrium wage happens to be equal to 1. However, the following proposition can be stated:

Proposition 1 *If the marginal disutility of work is constant ($\mu = 1$), the equilibrium wage is strictly greater than 1. Therefore, the laissez-faire number of hours in each firm is inefficiently low. In addition, the relative number of hours allocated to each firm is efficient i.e.* $\frac{h^{LF}(y_1)}{h^{LF}(y_2)} = \frac{h^*(y_1)}{h^*(y_2)}\ \forall y_1, y_2$.

Proof. *Imposing* $\mu = 1$ *in (a1),*$w^{\frac{\eta}{1-\eta}}[\beta(1-\eta)+\eta(1-w)]+\frac{(1-\eta)^2}{\eta(2-\eta)}\frac{\beta}{\gamma(r+s)^2}\frac{y_m^{\frac{2-\eta}{1-\eta}}-y_l^{\frac{2-\eta}{1-\eta}}}{y_m-y_l}(1-\eta w) = 0$. *Assume that* $w \leq 1$. *Then,*$\beta(1-\eta)+\eta(1-w) > 0$ *and* $1-\eta w > 0$, *leading to a contradiction.* $w > 1$ *implies then that* $h_j^{LF} < h_j^*\ \forall y_j$ *.If* $\mu = 1$, $\frac{h^{LF}(y_1)}{h^{LF}(y_2)} = \frac{h^*(y_1)}{h^*(y_2)}\forall y_1, y_2$ *[see (7) and(20)].* ■

Further properties of LF when $\mu = 1$ are stated in Proposition 2:

Proposition 2 *If the marginal disutility of work is constant ($\mu = 1$), $E[Gy]^{LF}$, Eh^{LF} and $E[Gy]^{LF} - Ec^{LF}$ are always inefficiently too low. The laissez-faire labour market tightness is higher or lower than its efficient value depending on parameter values.*
Proof. *see appendix (ii)* ■

As the number of hours for each demand level is inefficiently low, expected hours and the expected value of production are also inefficiently low. $E[Gy]^{LF} - Ec^{LF}$ is smaller than $E[Gy]^{*} - Ec^{*}$ since the allocation of hours across firms in LF is inefficient. This means that there exists a reallocation of hours across firms such that $E[Gy]$ can be increased while keeping Ec constant.[6]

In the presence of contingent contracts $\{w(y_j), h(y_j)\}_{\forall y_j \in [y_l, y_m]}$, the following proposition holds:

Proposition 3 *If firms and workers can sign contingent contracts $\{w(y_j), h(y_j)\}_{\forall y_j \in [y_l, y_m]}$, the laissez-faire allocation is constrained-efficient if and only if $\beta = 1 - \alpha$.*
Proof. *see appendix (iii)* ■

With complete contracts, the only departure from the competitive equilibrium arises from the matching structure. The matching externality is internalised if and only if the bargaining power of the worker equals the elasticity of the number of matches with respect to the number of unemployed, as stated by Hosios (1989).

2.4 Regulated Economy

We now characterise a decentralised economy in which working time is legally regulated. The law establishes a legal workweek ($\overline{h}$ hours) and a premium λ ($\lambda > 1$) for each additional hour over $\overline{h}$. We also allow for part-time labour contracts (i.e. $h_j < \overline{h}$).

2.4.1 Firms and workers

The value of a vacancy to a firm is unchanged from the LF economy (12). However, the value of an occupied job is affected by the working time regulation in the following way:

$$rJ_j = G(h_j)y_j - w\overline{h} - \lambda w h_j^{+} + w h_j^{-} + s(V - J_j). \tag{24}$$

The firm pays an hourly wage w for each of the $\overline{h}$ legal hours, λw for every overtime hour h_j^{+}, if any, and saves wh_j^{-} if it employs the worker on a part-time basis (for $\overline{h} - h_j^{-}$ hours). Note that production $G(h_j)$ simply depends on the total number of hours worked (h_j) irrespectively of the prevailing worktime regime.

[6] Or Ec lowered keeping $E[Gy]$ constant.

For workers, the value of unemployment (U) is given as before by (15). The corresponding value for an employed worker (E_j) in a firm with demand y_j becomes:

$$rE_j = w\overline{h} + \lambda w h_j^+ - w h_j^- - c(h_j) + s(U - E_j), \tag{25}$$

where account is taken that the employee may work for exactly the legal working time, do overtime hours or be employed part-time.

2.4.2 Wages

The FOC of the Nash bargain is still given by (18) but now, from (24) and (25),

$$(r+s)\frac{\partial E^e}{\partial w} = \overline{h} - Eh^- - w.\frac{\partial Eh^-}{\partial w} + \lambda Eh^+ + \lambda w.\frac{\partial Eh^+}{\partial w} - \frac{\partial Ec}{\partial w}; \tag{26}$$

$$(r+s)\frac{\partial J^e}{\partial w} = \frac{\partial E[Gy]}{\partial w} - \overline{h} + Eh^- + w.\frac{\partial Eh^-}{\partial w} - \lambda Eh^+ - \lambda w.\frac{\partial Eh^+}{\partial w}, \tag{27}$$

so the negotiated hourly wage depends in general on the expected number of hours.

2.4.3 The equilibrium distribution of hours

Once product demand is realised, the firm and the worker decide whether to stick to the legal working time ($\overline{h}$), whether to prolong it through overtime (h_j^+) or whether to choose a part-time regime ($\overline{h} - h_j^-$ hours of work).

The firm chooses overtime hours $h_j^+ \geq 0$ so as to maximise operating profits. Using (24), this can be written as:

$$\underset{h_j^+}{MAX}\ G(\overline{h} + h_j^+)y_j - \lambda w h_j^+ \tag{28}$$
$$s.t.\ \lambda w \geq c'(\overline{h} + h_j^+).$$

Again, the constraint requires that the individual be willing to do an additional overtime hour. We study situations in which this inequality is verified *ex post*. Using (3), the FOC to this problem are:

$$h_j^+ = 0 \ \ if \ \ y_j \leq y^* = \lambda w \overline{h}^{1-\eta} \tag{29}$$

$$h_j^+ = \left(\frac{y_j}{\lambda w}\right)^{\frac{1}{1-\eta}} - \overline{h} \ \ if \ \ y_j \geq y^*. \tag{30}$$

Equation (29) states that there exists a threshold level for product demand, y^*, below which overtime is not chosen. According to (30), above y^* the amount of overtime performed increases with the level of demand. From (29), (30) and the distribution of y_j, we obtain the expected number of overtime hours as a function of the wage [see (a12)]. It can be shown that $\frac{\partial Eh^+}{\partial w} = \frac{-\lambda(h_m^{2-\eta} - \overline{h}^{2-\eta})}{(y_m - y_l)(2-\eta)} \leq 0$, i.e. that the number of overtime hours decreases with the wage.

Analogously, some firms may wish their employees work part-time ($0 < h_j^- < \overline{h}$):

$$\underset{h_j^-}{MAX}\ G(\overline{h} - h_j^-)y_j - wh_j^-$$
$$s.t.\ \ w \leq c'(\overline{h} - h_j^-).$$

The FOC are:

$$h_j^- \ = \ 0 \ \ if \ \ y_j \geq \widetilde{y} \ = w\overline{h}^{1-\eta} \tag{31}$$
$$h_j^- \ = \ \overline{h} - \ \left(\frac{y_j}{w}\right)^{\frac{1}{1-\eta}} \ \ if \ \ y_j < \widetilde{y}. \tag{32}$$

From appendix (iv), $\frac{\partial Eh^-}{\partial w} = \frac{\overline{h}^{2-\eta} - h_l^{2-\eta}}{(y_m - y_l)(2-\eta)} \geq 0$. The equilibrium distribution of hours for a given wage w is represented in Fig. 4. Thus, a nice feature of the model is that it reproduces the observed spike in the equilibrium distribution of hours, even when the underlying demand distribution has no mass points.

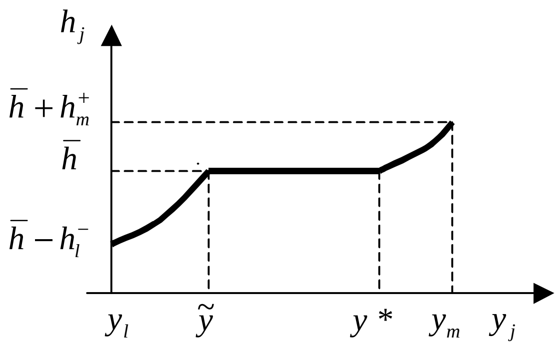

Figure 4: Equilibrium distribution of hours (hours performed in each firm) with a legal workweek

2.4.4 Solving the model

The expected value of a filled vacancy for a firm depends on the average wage it expects to pay to its workers. Taking expectations in (24) and using $V = 0$:

$$J^e = \frac{E[Gy] - w\overline{h} - \lambda wEh^+ + wEh^-}{r+s}. \tag{33}$$

Equalising (33) with (14), and imposing the Cobb-Douglas assumption in (1) with $\alpha = 0.5$, labour market tightness is given by

$$\theta = \left[\frac{E[Gy] - w\overline{h} - \lambda wEh^+ + wEh^-}{\gamma(r+s)}\right]^2. \tag{34}$$

Using (a3), the FOC condition of the Nash bargain (18) can be written as :

$$\beta J^e \frac{\partial E^e}{\partial w} + (1-\beta) E^e \frac{\partial J^e}{\partial w} + \beta\gamma\theta \frac{\partial E^e}{\partial w} = 0. \tag{35}$$

Using (26), (27), (a12), (a13), (a14), and (34), (35) gives the equilibrium value of w.

3 Efficiency properties of working time regulation

In this section we compare the efficiency performances of workweek regulation and laissez-faire, and show that imposing a legal workweek enhances efficiency within some plausible range of parameters. As an analytical solution for the wage under LF cannot be obtained [see (a1)], we must resort to numerical simulations. In all the simulations throughout the paper, we impose $\beta = 1 - \alpha = 0.5$ (Hosios condition), $\lambda = 1.5$,[7] $\eta = 0.75$, $\mu = 1.25$, and $r = 0.01$.[8] Fig. 5 graphs the equilibrium distribution of hours resulting in a regulated economy (LEG), characterised by a 40-hours legal workweek and a 50% overtime premium, for some given heterogeneity of product demand across firms and disutility of work.[9] Such distribution is compared with that resulting under LF and with the constrained-efficient distribution (OPT). While in the LF allocation the number of hours worked is inefficiently low at all levels of demand, the regulated economy imposes an inefficiently high number of hours in low-demand firm, and an inefficiently low number of hours in high-demand firms. Then, a 40-hour regulation is better at reproducing the efficient *average* workweek than LF.[10] This feature of the equilibrium distribution of hours implies that a 40-hour workweek improves efficiency with respect to the LF allocation. With the parameter values used, the LF surplus corresponds to 58.3% of the constrained-efficient, while regulation accounts for 59.6%.

This is not however the only possible outcome. Consider next a situation in which firm heterogeneity and/or the disutility of work are higher, as for the parameters underlying the curves of Fig. 6.[11] In this case, a 40-hour regulation *underperforms* LF.[12] This happens because, on the one hand, as a result of the higher disutility of work, a 40-hour workweek imposes an inefficiently high number of hours to 77% of firms in Fig. 6, and to only 53%

[7] According to ILO (1995), 50 countries out of 122 fix the premium at 50 per cent above the usual wage. The next most common premium is 25 per cent.

[8] When comparing LF or regulation to the efficient allocation, we impose $r = 0$, since the planner does not discount time.

[9] Here, $y_l = 1$, $y_m = 40$, $A = 5.5$, $D = 15$, $\gamma = 200$, and $s = 0.4$.

[10] More precisely, in this example, the efficient average number of hours is $Eh^* = 47.1$. LF leads to a very low number of hours $Eh^{LF} = 23.9$, while a 40-hour workweek performs much better ($Eh = 40.1$).

[11] Heterogeneity is now higher than in Fig. 5 since $y_m = 52$ instead of 40 and the common demand component A is 5.5 instead of 4.5. The disutility from work is also higher ($D = 23$). Both cases share the rest of the parameters.

[12] The 40-hour regulation surplus amounts only to 17.68% of the constrained-efficient surplus, while LF guarantees 61.2%.

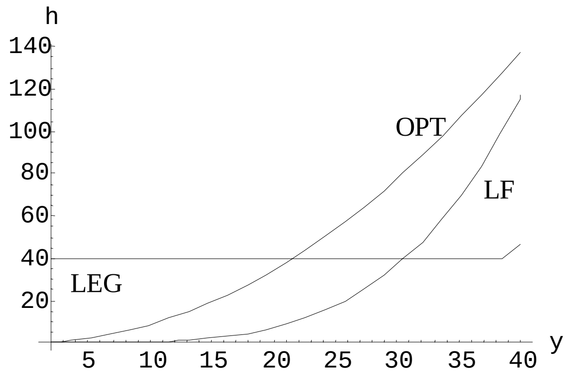

Figure 5: Constrained-efficient allocation of hours and equilibrium distributions under regulation (LEG) and LF. LEG leading to higher efficiency than LF.

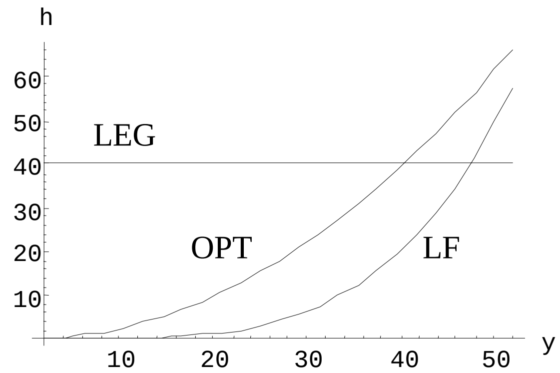

Figure 6: 40-hours legal workweek leading to lower efficiency than LF. Distribution of hours.

of firms in Fig. 5.[13] And, imposing a homogeneous 40-hour workweek cannot improve efficiency when three quarters of firms produce more than in OPT, and only one quarter produce less than in OPT. On the other hand, as firm heterogeneity is higher in Fig. 6,[14] the loss associated to a homogenous working time is larger. Indeed, under regulation, marginal products are not equalised to marginal disutilities across firms, and this departure from efficiency becomes more severe the higher heterogeneity. In contrast, LF reproduces a feature of the constrained-efficient allocation of hours, namely that hours are strictly increasing in product demand, so the wedge between marginal products and marginal disutilities is smaller than under regulation.

[13]These figures can be computed using the p.d.f. of h_j.

[14]Since $y_m - y_l$ is bigger and the common demand component A is smaller than in the economy of Fig. 5.

The efficiency properties of alternative regimes can be directly assessed in Fig. 7, where the surplus generated under a 40-hour workweek and LF are plotted against levels of heterogeneity (different levels of y_m for a given value of y_l).[15] For low enough heterogeneity values, a legal workweek outperforms the LF allocation.

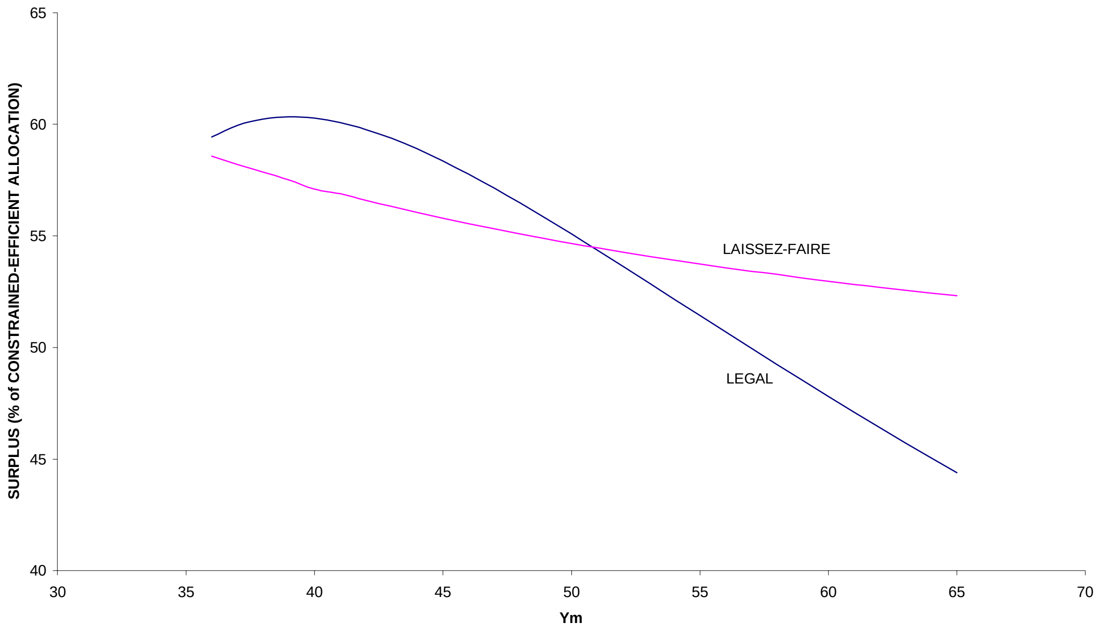

Figure 7: Surplus in the laissez-faire economy and in the economy with legal working time as a function of y_m

The efficiency properties of a legal workweek also depend on the overtime premium in the expected way: the higher the penalty for using overtime, the worse the performance of a legal workweek with respect to LF (see Fig. 8).[16]

4 Work-sharing

4.1 A simple case

We next analyse the effects of work-sharing, which consists in reducing the legal working time from $\overline{h}$ to $\overline{h}' < \overline{h}$. We first derive the impact of this policy on hours, wages and employment, and finally characterise its efficiency implications. In doing this, we start from a simple case, in which part-time work is ruled out, and the overtime wage w_0 is

[15] We compute the equilibrium for $y_m \in (36, 65)$.The values of the rest of the parameters are : $A = 6$, $D = 15$, $y_l = 1$, $\gamma = 200$, $\overline{h} = 40$, and $s = 0.4$.

[16] In this example, $A = 6$, $D = 15$, $y_l = 1$, $y_m = 52$, $\overline{h} = 40$, $\gamma = 200$, and $s = 0.4$.

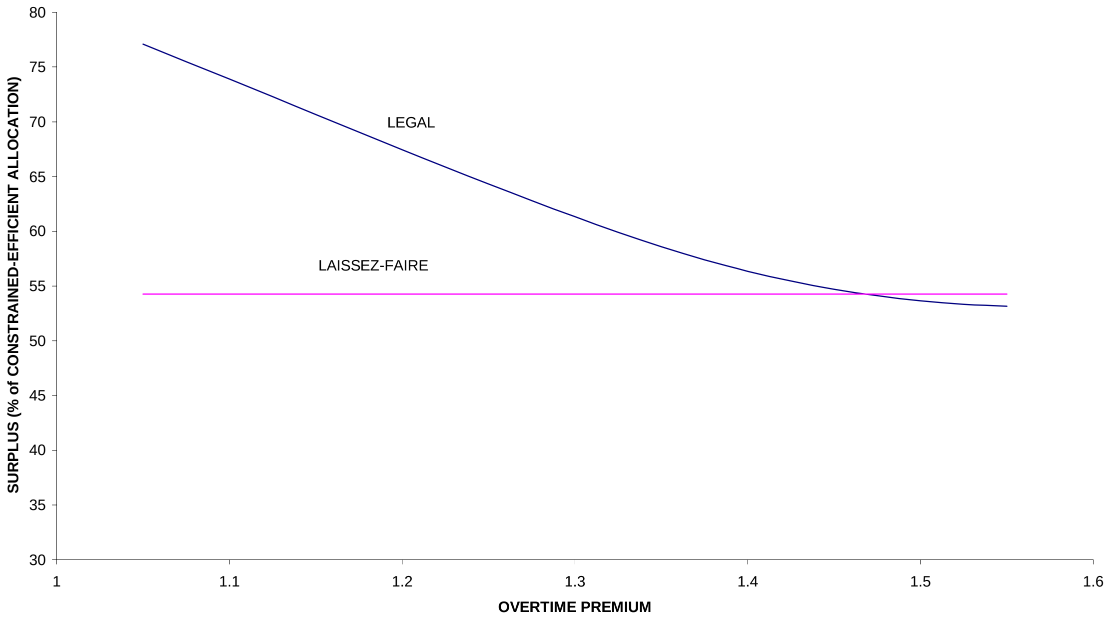

Figure 8: Surplus in the laissez-faire economy and in the economy with legal working time for different values of the overtime premium (λ).

exogenous (with $w_0 > w$ being verified *ex post*). In this case, the choice of hours in the second stage does not depend on w and therefore the wage itself is independent of the expected number of overtime hours, since from (26) and (27), $(r+s)\frac{\partial E^e}{\partial w} = \overline{h} - \frac{\partial Ec}{\partial w}$ and $(r+s)\frac{\partial J^e}{\partial w} = \frac{\partial E[Gy]}{\partial w} - \overline{h}$. Proceeding as in appendix (ii), equilibrium labour market tightness is given by:

$$\theta = \left(\frac{-\gamma(r+s) + \sqrt{\gamma^2(r+s)^2 + 4\gamma(1-\beta)\beta(E[Gy]-Ec)}}{2\gamma\beta} \right)^2 . \qquad (36)$$

Thus, labour market tightness depends positively on the net expected value of production $E[Gy] - Ec$. As employment increases with labour market tightness [see (2)], it also increases with $E[Gy] - Ec$.

Under the above assumptions, the FOC (29) and (30) can be rewritten as :

$$h_j^+ = 0 \;\; if \;\; y_j \leq y^* = w_0 \overline{h}^{1-\eta} \qquad (37)$$

$$h_j^+ = \left(\frac{y_j}{w_0} \right)^{\frac{1}{1-\eta}} - \overline{h} \;\; if \;\; y_j \geq y^*. \qquad (38)$$

The effects of the reduction of the legal workweek from $\overline{h}$ to $\overline{h}'$ are represented graphically in Fig. 9. A first effect of this policy is an increase in the number of firms that choose to be in the overtime regime. Indeed, at the new shorter legal working time, the marginal productivity of labour is higher, implying that overtime is chosen for a smaller level of product demand [i.e. y^* is increasing in $\overline{h}$ from (37)]. Firms whose demand is below the new threshold ($y_i < y^{*\prime}$) choose to employ their workers for exactly the new legal workweek: these firms were (downward) constrained before the policy, and would have chosen (at most) an effective working time $\overline{h}'$, had this been possible. At the other end of the demand distribution, firms that were already in the overtime regime ($y_i > y^*$) keep their effective working time unchanged, as can be seen from (38), though with higher incidence of overtime in the composition of total hours worked. This stems from an exogenous overtime wage, so that the cost of overtime does not respond to changes in the equilibrium wage. Finally, firms whose demand lies in between the two thresholds ($y^{*\prime} < y_i < y^*$) switch from the (old) legal working week to an overtime regime, and in the new equilibrium choose an effective number of hours that is strictly lower than $\overline{h}$ [from the continuity of (38)].

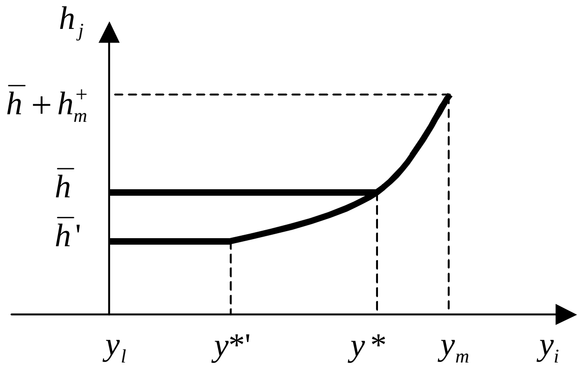

Figure 9: Change in the distribution of hours after a reduction of the legal workweek from $\overline{h}$ to $\overline{h}'$ hours (simple case)

As each firm either reduces or keeps its effective number of hours after the shortening of the legal workweek, work-sharing reduces the expected number of effective hours. The effects of work-sharing can therefore be summarised in the following proposition:

Proposition 4 *Work-sharing reduces the expected number of effective hours, i.e.,* $\frac{\partial Eh}{\partial \overline{h}} > 0$. *It also reduces the expected value of production and the expected disutility of work, i.e.* $\frac{\partial E[Gy]}{\partial \overline{h}} > 0$ *and* $\frac{\partial Ec}{\partial \overline{h}} > 0$.

Proof. : see appendix (iv). ■

Due to shorter hours, total wages fall:[17]

Proposition 5 *A shorter legal working time leads to a reduction in the total wage, i.e.,* $\frac{\partial [w\overline{h} + w_0 Eh^+]}{\partial \overline{h}} > 0$.

[17] The evolution of the hourly wage is ambiguous.

Proof. : see appendix (iv). ■

The effects of work-sharing on labour market tightness θ and employment depend upon $E[Gy] - Ec$ [as from (36)]. If after work-sharing, $E[Gy]$ decreases less (more) than Ec, employment and labour market tightness increase. Proposition 6 states the conditions under which each outcome arises:

Proposition 6 *A decrease in the legal working time leads to an increase in employment if and only if* $y_l < 2\mu\overline{h}^{\mu-\eta} - w_0\overline{h}^{1-\eta}$.

Proof. : see appendix (iv). ■

The intuition for this result is as follows. If the demand faced by low-demand firms is low enough, the existence of a legal working time $\overline{h}$ compels these firms to have their employees working for an inefficiently long number of hours. Had these firms been able to choose freely the duration of the working week, they would have chosen a workweek shorter than $\overline{h}$. In other words, the existence of a statutory working time plays here the role of a downward working time rigidity. The importance of this rigidity in the determination of employment depends then on the number of firms which are effectively constrained. As these firms are situated in the left end of the distribution, the rigidity matters if the demand level of the least 'successful' firms (y_l) is small enough. In this case, a reduction of the legal working time alleviates this problem and improves the expected value of production relative to the disutility of work. As the allocation of hours becomes more efficient, the profitability of a match is enhanced, firms open more vacancies (θ rises), and employment rises.

Clearly, overtime wages w_0 also play a role in determining the employment effects of work-sharing, as they affect costs in high-demand firms: if w_0 is low enough, profits in high-demand firms are affected to a relatively small extent by the reduction in $\overline{h}$.

An analogous condition can be stated concerning efficiency :

Proposition 7 *A decrease in the legal working time leads to an increase in the surplus W and to a Pareto-improvement if and only if* $y_l < 2\mu\overline{h}^{\mu-\eta} - w_0\overline{h}^{1-\eta}$.

Proof. see appendix (iv). ■

Proposition 7 shows that firm and worker gains (or losses) from work-sharing purely correspond to changes in the total surplus, i.e., to variations in the size of the cake that is to be divided between workers and firms. Consider the case in which work-sharing enhances efficiency. Firm profits are higher because work-sharing enables them to better match, on average, the different states of demand: although satisfying the demand in good times becomes more expensive, the cost of producing in bad times is lower. As for workers, their expected value of employment is higher, and so are the employment rate and the value of being unemployed (since both the probability of finding a job and the value of having a job are higher).

4.2 The general case

4.2.1 Overtime wage proportional to the straight time wage

In most countries, the price of overtime work is set as a percentage above the straight time wage. When allowing for a proportional overtime premium, the analysis of our model is considerably burdened, as the outcome of the Nash bargain now also affects the number of overtime hours, and in turn the negotiated wage depends on the length of overtime. To see this, note that from (26),

$$(r+s)\frac{\partial E^e}{\partial w} = \overline{h} - \frac{\partial Ec}{\partial w} + \lambda E h^+ + \lambda w \frac{\partial E h^+}{\partial w}. \tag{39}$$

With respect to the case of exogenous overtime wages, in which $(r+s)\frac{\partial E^e}{\partial w} = \overline{h} - \frac{\partial Ec}{\partial w}$, the last two terms of (39) are new. On the one hand, the term $\lambda E h^+$ indicates that workers take into account that a higher negotiated wage implies a higher income for each overtime hour. On the other hand, the term $\lambda w \frac{\partial E h^+}{\partial w}$ indicates that a higher negotiated wage makes firms less prone to offer overtime hours, thus reducing the expected value of being employed.[18]. As an analytical solution cannot be obtained in this case, we must resort to numerical simulations.

The effects of the shortening of the legal workweek from 39 to 35 hours are represented in Table 1 and the corresponding distribution of hours are plotted in Fig. 10. Column 2 of Table 1 shows that the reduction in the legal workweek is in this case associated with an increase in the equilibrium hourly wage.[19]

$\overline{h}$	w	y^*	θ	u (%)	Eh	$E(Gy)$	Ec	$E(Gy) - Ec$	W/W^* (%)
39	13.93	40.15	8.47	6.43	41.4	715	211	504	36.02
38	14.07	40.3	7.86	6.66	40.22	699	203	496	35.53
37	14.22	40.46	7.26	6.91	39.04	683	196	487	35.04
36	14.38	40.64	6.67	7.19	37.86	666	188	478	34.5
35	14.55	40.84	6.07	7.51	36.63	650	181	469	34

Table 1: Work-Sharing leading to lower efficiency with an endogenous overtime premium

As a consequence of the higher cost of overtime, the total number of hours chosen by high-demand firms falls with $\overline{h}$ (see Fig. 10). For the same reason, the threshold demand level for using overtime (y^*) rises slightly as $\overline{h}$ falls. These are two notable differences with respect to the simple case with exogenous overtime wages, in which the number of effective

[18] The relation between the payoff of the firm and the negociated wage is modified in an analogous way. Indeed, from (27), $(r+s)\frac{\partial J^e}{\partial w} = \frac{\partial E[Gy]}{\partial w} - \overline{h} - \lambda E(h^+) - \lambda w \frac{\partial E(h^+)}{\partial w}$ instead of $(r+s)\frac{\partial J^e}{\partial w} = \frac{\partial E[Gy]}{\partial w} - \overline{h}$ in the simple case.

[19] In this example, $A = 1.3$, $D = 2$, $y_l = 2$, $y_m = 47$, $\gamma = 200$, and $s = 0.2$.

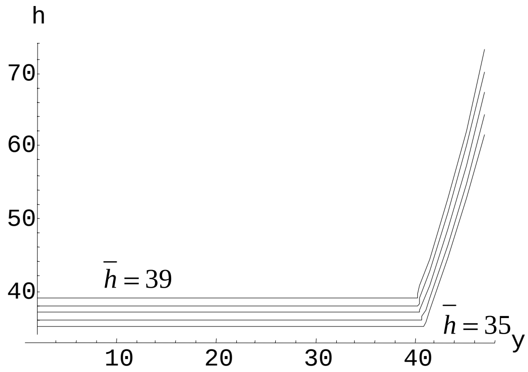

Figure 10: Change in the equilibrium distribution of hours (higher equilibrium wage).

hours worked in high-demand firms was independent of $\overline{h}$, and the threshold demand level for using overtime was actually falling with $\overline{h}$. The impact of work-sharing on the other relevant variables is qualitatively the same as in the simple case. The set of parameter values used implies here a decrease in efficiency: as the value of production falls more than the disutility of work, the surplus W/W^* also falls,[20] and the situation of all agents is worsened.[21]

However, it should be noted that this is not the only configuration possible. We replicate the exercise for an alternative set of parameter values for which heterogeneity is smaller and the disutility of work is higher.[22] Table 2 and Fig. 11 present the equilibrium outcome, in which the hourly wage which actually falls with work-sharing. The price of overtime is thus also lower, in turn implying a more intensive use of overtime in high-demand firms, and a lower threshold demand level for using overtime (y^*). The resulting reallocation of hours from low-demand to high-demand firms reduces unemployment and enhances efficiency by improving the situation of all agents. Thus, the results with an endogenous overtime premium follow qualitatively the patterns of Proposition 6, in the sense that improvements in efficiency associated to work-sharing are more likely the lower firm heterogeneity and the higher the disutility of work.

Note finally that, although work-sharing improves efficiency, this does not imply that regulation is better than LF. Specifically, with this configuration of parameters, LF performs better than any of the legal worweeks under consideration, since $\frac{W^{LF}}{W^*} = 62.54\%$.

[20] Computed for $r = 0$.

[21] U, E^e , $Z \equiv (1-u)E^e + uU$, and J^e fall.

[22] In this example, $A = 2$, $D = 4.5$, $\gamma = 500$, $s = 0.3$, and the rest of the parameters are kept unchanged. Heterogeneity is smaller than in the preceeding case since the common demand component A is bigger. The disutility of work is higher since D is bigger.

$\overline{h}$	w	y^*	θ	u (%)	Eh	$E(Gy)$	Ec	$E(Gy) - Ec$	W/W^* (%)
39	17.62	33.02	6	10.91	54.6	1479	695	784	53.69
38	17.47	32.53	6.28	10.69	54.85	1494	702	792	53.73
37	17.35	32.1	6.51	10.52	55	1506	707	799	53.78
36	17.25	31.68	6.72	10.37	55.1	1517	712	805	53.84
35	17.15	31.29	6.91	10.25	55.15	1526	715	811	53.92

Table 2: Work-Sharing leading to higher efficiency with an endogenous overtime premium

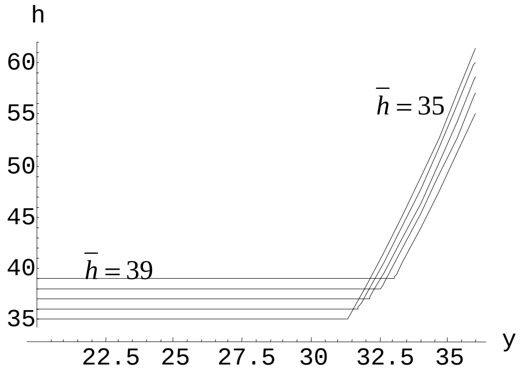

Figure 11: Change in the equilibrium distribution of hours (decrease of the equilibrium hourly wage).

4.2.2 Part-time

Table 3 and Fig. 12 simulate the effects of work-sharing when part-time is allowed, i.e. when firms can employ workers for less than $\overline{h}$.[23] The equilibrium distribution of hours shifts downwards as $\overline{h}$ falls. Also, efficiency first rises and then falls as the workweek is shortened. Comparing the equilibrium distribution of hours under regulation with the optimal distribution (OPT), it appears that the legal workweek plays the role of a downward working time rigidity for low-demand firms even if these firms can sign part-time contracts.

[23] Here $A = 1.3$, $y_l = 20.2$, $y_m = 43$, $\gamma = 20$, $s = 0.2$ and $D = 5.5$. We have checked that the equilibrium wage *ex post* satisfies $\frac{u'(\overline{h}+h_j^+)}{\lambda} \leq w \leq u'(\overline{h} - h_j^-)$.

$\overline{h}$	w	$\widetilde{y}$	y^*	θ	u (%)	Eh	$E(Gy)$	Ec	$E(Gy) - Ec$	W/W^* (%)
39	13.08	25.16	37.73	15.1	4.89	56.1	880.7	543.8	336.9	97.24
38	13.16	25.14	37.71	15.4	4.85	54.7	864.4	527.1	337.3	97.36
37	13.24	25.13	37.69	15.7	4.81	53.3	848	510.5	337.5	97.41
36	13.33	25.11	37.67	16	4.76	51.9	831.4	494	337.4	97.39
35	13.41	25.09	37.64	16.3	4.72	50.5	814.8	477.6	337.2	97.31

Table 3: Work-Sharing with part-time and an endogenous overtime premium: effiency follows an inverted U shape

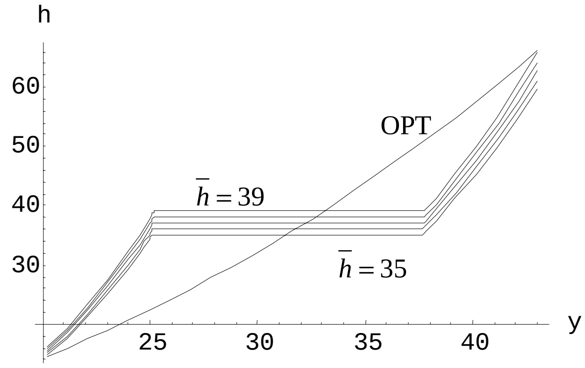

Figure 12: Optimal distribution of hours and change in the equilibrium distribution of hours after a reduction in the legal workweek (from 39 to 35) when firms can offer part-time contracts.

5 Conclusion

This paper proposes an efficiency rationale for the existence of a legal working time and argues that efficiency issues should be taken into account when evaluating the impact of work-sharing policies.

The novelty of the analysis is based on the explicit consideration of firm heterogeneity. We believe firm heterogeneity to be a key element in the study of this type of regulation. Indeed, by its very nature, the existence of a legal workweek tends to impose a common working time pattern *across firms.* In a first-best setting, this can never improve efficiency, since the decentralised economy can attain the efficient allocation. However, this view implicitly assumes that agents are able and willing to sign contracts contingent on the state of demand. In a second best world à la Lispsey and Lancaster (1956), the institution of a legal workweek may be the only way of reproducing optimal *average* hours, by imposing an inefficiently low number of hours to high-demand firms and an inefficiently high number of hours to low-demand firms. This institution is however not good nor harmful *per se* and its impact on efficiency does depend on the *level* at which the legal workweek is set.

By explicitly recognising the efficiency consequences of a legal working time, this paper has shown that widely discussed worksharing policies affect the total surplus of the economy, and not simply its distribution. Specifically, we have identified the conditions under which work-sharing brings the equilibrium distribution of hours closer to the constrained-efficient allocation or further away from it.

Appendix

(i) Constrained efficient allocation:

From the uniform distribution of y_j and (7), $f(h_j) = \frac{\mu(\mu-\eta)}{(y_m-y_l)} h_j^{\mu-\eta-1}$ is the p.d.f. of hours. Then, $Eh^* = \frac{\mu-\eta}{1+\mu-\eta}\mu^{\frac{-1}{\mu-\eta}} \frac{y_m^{\frac{1+\mu-\eta}{\mu-\eta}} - y_l^{\frac{1+\mu-\eta}{\mu-\eta}}}{y_m-y_l}$. (9) and (10) are analogously derived.

(ii) Laissez-faire

Wages In LF, $(r+s)\frac{\partial E^e}{\partial w} = Eh + w\frac{\partial Eh}{\partial w} - \frac{\partial Ec}{\partial w}$ and $(r+s)\frac{\partial J^e}{\partial w} = \frac{\partial E[Gy]}{\partial w} - Eh - w\frac{\partial Eh}{\partial w}$. From (13) and using $Eh^{LF} = \eta w^{-1} E[Gy]^{LF}$ [see (21) and (23)], $\theta = \frac{E[Gy]^2(1-\eta)^2}{\gamma^2(r+s)^2}$. Using the expressions for $\frac{\partial E^e}{\partial w}$, $\frac{\partial J^e}{\partial w}$, θ, $E[Gy]^{LF}$, Ec^{LF}, and Eh^{LF} [(21) to (23)] into the FOC of the Nash bargain (35) yields the equation determining the equilibrium wage in LF:

$$-\eta w^{\frac{\mu}{1-\eta}} + \frac{2-\eta}{1-\eta+\mu} \frac{y_m^{\frac{1-\eta+\mu}{1-\eta}} - y_l^{\frac{1-\eta+\mu}{1-\eta}}}{y_m^{\frac{2-\eta}{1-\eta}} - y_l^{\frac{2-\eta}{1-\eta}}} [\beta\mu + \eta(1-\beta)] w^{\frac{\eta}{1-\eta}} \tag{a1}$$

$$-\frac{(1-\eta)^2}{2-\eta} \frac{\beta}{\gamma(r+s)^2} \frac{y_m^{\frac{2-\eta}{1-\eta}} - y_l^{\frac{2-\eta}{1-\eta}}}{y_m - y_l} w^{\frac{\mu-\eta}{1-\eta}} + \frac{(1-\eta)^2}{\eta(1-\eta+\mu)} \frac{\beta\mu}{\gamma(r+s)^2} \frac{y_m^{\frac{1-\eta+\mu}{1-\eta}} - y_l^{\frac{1-\eta+\mu}{1-\eta}}}{y_m - y_l} = 0.$$

Proof of Proposition 2 If $\mu = 1$, from (9), (10), (21), and (22), $E[Gy]^{LF} = E[Gy]^* w^{\frac{-\eta}{1-\eta}}$, $Ec^{LF} = Ec^* w^{\frac{-1}{1-\eta}}$, and $E[Gy]^{LF} - Ec^{LF} = (E[Gy]^* - Ec^*)\ \frac{w-\eta}{1-\eta} w^{\frac{-1}{1-\eta}}$. As $w > 1$, $E[Gy]^{LF} < E[Gy]^*$ and $Ec^{LF} < Ec^*$. As $\frac{w-\eta}{1-\eta} w^{\frac{-1}{1-\eta}}$ is decreasing in w and $\lim_{w\to 1} \frac{w-\eta}{1-\eta} w^{\frac{-1}{1-\eta}} = 1$, $E[Gy]^{LF} - Ec^{LF} < E[Gy]^* - Ec^*$.

To derive labour market tightness, note that from (18), (13), and (16),

$$wEh\left[-\beta\frac{\partial E^e}{\partial w} + (1-\beta)\frac{\partial J^e}{\partial w}\right] = -\beta E[Gy]\frac{\partial E^e}{\partial w} + (1-\beta)Ec\frac{\partial J^e}{\partial w} + (1-\beta)rU\frac{\partial J^e}{\partial w}. \tag{a2}$$

Using (14), (15), and the zero-profit condition for firms $V_i = 0$ in (18),

$$(1-\beta)rU\frac{\partial J^e}{\partial w} = -\beta\gamma\theta\frac{\partial E^e}{\partial w}. \tag{a3}$$

From (13), $J^e = \frac{E[Gy]-wEh}{r+s}$. Equalising this expression with (14), $E[Gy] - wEh - (r + s)\gamma\theta^{1-\alpha} = 0$. Replacing (a2) and (a3) in this equation and imposing $r = 0$ yields:

$$-(1-\beta)\left(E[Gy]-Ec\right)\frac{\partial J^e}{\partial w} + s\left[-\beta\frac{\partial E^e}{\partial w} + (1-\beta)\frac{\partial J^e}{\partial w}\right]\gamma\theta^{1-\alpha} - \beta\gamma\theta\frac{\partial E^e}{\partial w} = 0. \quad \text{(a4)}$$

Using the expressions for $\frac{\partial E^e}{\partial w}$, $\frac{\partial J^e}{\partial w}$, $E[Gy]$, Ec, and Eh [(21) to (23)] in (a4),

$$(1-\beta)(1-\eta)w(E[Gy]-Ec) - \beta\gamma\theta(1-\eta w) - s\left[\beta(1-w) + w(1-\eta)\right]\gamma\theta^{1-\alpha} = 0. \quad \text{(a5)}$$

Replacing $E[Gy]^{LF} - Ec^{LF} = (E[Gy]^* - Ec^*)\ \frac{w-\eta}{1-\eta}w^{\frac{-1}{1-\eta}}$ in (a5),

$$(1-\beta)\left(\frac{w-\eta}{1-\eta w}\right)w^{\frac{-1}{1-\eta}}(E[Gy]^* - Ec^*) - \beta\gamma\theta - s\left(\frac{\beta(1-w)+w(1-\eta)}{1-\eta w}\right)\gamma\theta^{1-\alpha} = 0. \quad \text{(a6)}$$

Imposing $\mu = 1$ in (a1), it can be shown that $w \in \left(1 + \frac{\beta(1-\eta)}{\eta}, \frac{1}{\eta}\right)$ since otherwise all the terms in (a1) are of the same sign. Then, $\left(\frac{w-\eta}{1-\eta w}\right)w^{\frac{-1}{1-\eta}} > 1$ and $\frac{\beta(1-w)+w(1-\eta)}{1-\eta w} > 1$. Recall that in the constrained-efficient allocation, the equation determining θ^* is $(1-\beta)(E[Gy]^* - Ec^*) - \beta\gamma\theta^* - s\gamma\theta^{*1-\alpha} = 0$. Comparing it with (a6), $\theta^{LF} \lesseqgtr \theta^*$ depending on parameter values. ■

(iii) Contingent contracts: proof of Proposition 3

Agents bargain over wages and hours contingent on each state of demand. The program is $\underset{w_j,\ h_j}{MAX}(E^e - U)^\beta(J^e - V)^{1-\beta}$, with FOC

$$\frac{\partial Z}{\partial w_j} = 0 \Leftrightarrow \beta(J^e - V) = (1-\beta)(E^e - U); \quad \text{(a7)}$$

$$\frac{\partial Z}{\partial h_j} = 0 \Leftrightarrow \beta(J^e - V)(w_j - c'(h_j)) + (1-\beta)(E^e - U)(G'(h_j)y_j - w_j) = 0. \quad \text{(a8)}$$

Replacing (a7) in (a8), $h_j = \left(\frac{y_j}{\mu}\right)^{\frac{1}{\mu-\eta}}$. From (7), $h_j = h_j^*\ \forall y$, the expected value of production is $E[Gy]^*$, and the expected disutility of work Ec^*. Following the same steps as in (iii),

$$E[w_j h_j] = \beta E[Gy]^* + (1-\beta)Ec^* + (1-\beta)rU, \quad \text{(a9)}$$

and

$$(1-\beta)rU = \beta\gamma\theta. \quad \text{(a10)}$$

As $J^e = \frac{E[Gy]^* - E[w_j h_j]}{r+s} = \frac{\gamma}{q(\theta)}$, $E[Gy]^* - E[w_j h_j] - (r+s)\gamma\theta^{1-\alpha} = 0$. Replacing (a9) and (a10) in this equation and imposing $r = 0$ yields:

$$(1-\beta)\left(E[Gy]^{*}-Ec^{*}\right)-s\gamma\theta^{1-\alpha}-\beta\gamma\theta=0. \tag{a11}$$

Comparing (a11) and (8), the LF allocation with complete contracts is constrained-efficient if and only if $\beta=1-\alpha$ [Hosios (1989)]. ■

(iv) Regulated economy:

From (30) and the density distribution of y_j, $E[h_j\ /y_j\geq y^*]=\frac{1-\eta}{2-\eta}\frac{h_m^{2-\eta}-\overline{h}^{2-\eta}}{h_m^{1-\eta}-\overline{h}^{1-\eta}}$. From (29) and (30),

$$Eh^{+}=\frac{1}{(y_m-y_l)(2-\eta)}\left[(1-\eta)y_m^{\frac{2-\eta}{1-\eta}}(\lambda w)^{\frac{-1}{1-\eta}}-\overline{h}(2-\eta)y_m+\lambda w\overline{h}^{2-\eta}\right]. \tag{a12}$$

Similarly, from (31) and (32),

$$Eh^{-}=\frac{1}{(y_m-y_l)(2-\eta)}\left[(1-\eta)y_l^{\frac{2-\eta}{1-\eta}}w^{\frac{-1}{1-\eta}}-\overline{h}(2-\eta)y_l+w\overline{h}^{2-\eta}\right]. \tag{a13}$$

As for the value of production,

$E[Gy]=\frac{G(\overline{h})y^*+G(\overline{h})\widetilde{y}}{2}\left(\frac{y^*-\widetilde{y}}{y_m-y_l}\right)+E[Gy\ /y_j\leq y^*]\left(\frac{\widetilde{y}-y_l}{y_m-y_l}\right)+E[Gy\ /y_j\geq y^*]\left(\frac{y_m-y^*}{y_m-y_l}\right).$

Then,

$$E[Gy]=\frac{1}{(y_m-y_l)(2-\eta)}\left[\frac{\overline{h}^{2-\eta}w^2(\lambda^2-1)}{2}+\frac{1-\eta}{\eta}w^{\frac{-\eta}{1-\eta}}\left(y_m^{\frac{2-\eta}{1-\eta}}\lambda^{\frac{-\eta}{1-\eta}}-y_l^{\frac{2-\eta}{1-\eta}}\right)\right]. \tag{a14}$$

(v) Work-sharing: a simple case

Proof of Proposition 4 Proceeding as in (iv), we obtain the expressions for Eh, $E[Gy]$, and Ec when there is no part-time and the overtime wage is w_0. Then, $\frac{\partial Eh}{\partial\overline{h}}=\frac{y^*-y_l}{y_m-y_l}>0$, $\frac{\partial E[Gy]}{\partial\overline{h}}=\frac{y^{*2}-y_l^2}{2\overline{h}^{1-\eta}(y_m-y_l)}>0$, and $\frac{\partial Ec}{\partial\overline{h}}=\frac{\overline{h}^{\mu-1}\mu(y^*-y_l)}{(y_m-y_l)}>0$. ■

Proof of Proposition 5 Proceeding as in (iii), $w\overline{h}+w_0Eh^+=\beta E[Gy]+(1-\beta)Ec+\beta\gamma\theta$. The sign of $\frac{\partial[w\overline{h}+w_0Eh_j^+]}{\partial\overline{h}}$ is the same as that of $2(1-\beta)\frac{\partial Ec}{\partial\overline{h}}\left[(r+s)^2\gamma(1+B)+2\beta(1-\beta)X\right]$ $+\frac{\partial E[Gy]}{\partial\overline{h}}\gamma\left(4\beta(1-\beta)^2X+(r+s)^2B(1+B)(2\beta+B-1)\right)$ where $B\equiv\sqrt{1+4\gamma^{-1}(r+s)^{-2}(1-\beta)\beta X}>1$ and $X\equiv E(Gy)-Ec$. This expression is positive since $\frac{\partial E[Gy]}{\partial\overline{h}}>0$, $\frac{\partial Ec}{\partial\overline{h}}>0$, $B>1$ and $X>0$. ■

Proof of Proposition 6 From (36), $\frac{\partial(1-u)}{\partial\overline{h}}>0\Leftrightarrow\frac{\partial\theta}{\partial\overline{h}}>0\Leftrightarrow\frac{\partial(E[Gy]-Ec)}{\partial\overline{h}}>0$. $\frac{\partial(E[Gy]-Ec)}{\partial\overline{h}}=\frac{y^*-y_l}{2(y_m-y_l)\overline{h}^{1-\eta}}\left(w_0\overline{h}^{1-\eta}+y_l-2\mu\overline{h}^{\mu-\eta}\right)$. As $\frac{y^*-y_l}{2(y_m-y_l)\overline{h}^{1-\eta}}>0$, $\frac{\partial(E[Gy]-Ec)}{\partial\overline{h}}>0\Leftrightarrow w_0\overline{h}^{1-\eta}+y_l-2\mu\overline{h}^{\mu-\eta}>0$. ■

Proof of Proposition 7 Proceeding as in (ii) with $r = 0$, $(1-\beta)X - s\gamma\theta^{1-\alpha} - \beta\gamma\theta = 0$ where $X \equiv E(Gy) - Ec$. Substituting in the definition of W, $W = \beta(1-u)(X+\theta\gamma)$. Then, $\frac{\partial W}{\partial \overline{h}} > 0 \Leftrightarrow w_0\overline{h}^{1-\eta} + y_l - 2\mu\overline{h}^{\mu-\eta} > 0$ since $\frac{\partial(1-u)}{\partial \overline{h}} > 0 \Leftrightarrow \frac{\partial \theta}{\partial \overline{h}} > 0 \Leftrightarrow \frac{\partial X}{\partial \overline{h}} > 0 \Leftrightarrow w_0\overline{h}^{1-\eta} + y_l - 2\mu\overline{h}^{\mu-\eta} > 0$. After some computations, $rU(\theta) = \frac{-X+XB}{1+B}$. Then, the sign of $\frac{\partial rU}{\partial \overline{h}}$ is that of $\frac{\partial X}{\partial \overline{h}} X \left(4\gamma^{-1}(r+s)^{-2}(1-\beta)\beta\right)\left(1+B^{-1}\right)$. As $\left(4\gamma^{-1}(r+s)^{-2}(1-\beta)\beta\right)\left(1+B^{-1}\right)X > 0$, $\frac{\partial U(\theta)}{\partial \overline{h}} > 0 \Longleftrightarrow \frac{\partial X}{\partial \overline{h}} > 0$. Similarly, $rE^e = \frac{\beta[r+\theta^\alpha]}{r+s+\beta\theta^\alpha}$. It is then easy to show that the sign of $\frac{\partial rE^e}{\partial \overline{h}}$ is that of $\frac{\partial X}{\partial \overline{h}}[r+\theta^\alpha]\left[r+s+\beta\theta^\alpha\right] + \frac{\partial \theta^\alpha}{\partial \overline{h}} X[s+(1-\beta)r]$. As $X[s+(1-\beta)r] > 0$, $[r+\theta^\alpha]\left[r+s+\beta\theta^\alpha\right] > 0$, and $\frac{\partial X}{\partial \overline{h}} > 0 \Leftrightarrow \frac{\partial \theta^\alpha}{\partial \overline{h}} > 0$, then $\frac{\partial rE^e}{\partial \overline{h}} > 0 \Leftrightarrow \frac{\partial(E[Gy]-Ec)}{\partial \overline{h}} > 0$. Let $Z \equiv (1-u)E^e + uU$. As $\frac{\partial(1-u)}{\partial \overline{h}} > 0 \Leftrightarrow \frac{\partial(E[Gy]-Ec)}{\partial \overline{h}} > 0$, $\frac{\partial Z(\theta)}{\partial \overline{h}} > 0 \Leftrightarrow \frac{\partial(E[Gy]-Ec)}{\partial \overline{h}} > 0$. As for firms, $J^e = \gamma\theta^{1-\alpha}$ and therefore $\frac{\partial J^e}{\partial \overline{h}} > 0 \Leftrightarrow \frac{\partial(E[Gy]-Ec)}{\partial \overline{h}} > 0$. ■

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