Consensus building: How to persuade a group

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Abstract

Many decisions in private and public organizations are made by groups. The paper explores strategies that the sponsor of a proposal may employ to convince a qualified majority of group members to approve the proposal. Adopting a mechanism design approach to communication, it emphasizes the need to distill information selectively to key members of the group and to engineer persuasion cascades in which members who are brought on board sway the opinion of others. The paper shows that higher congruence among group members benefits the sponsor. The paper also unveils the factors, such as the extent of congruence between the group and the sponsor, the size and governance of the group, that condition the sponsor's ability to maneuver and get his project approved.

Keywords Group decision-making, selective communication, persuasion cascade, internal and external congruence.

1 Introduction

Many decisions in private and public organizations are made by groups. For example, in western democracies Congressional committees command substantial influence on legislative outcomes through their superior information and their gatekeeping power. Academic appointments are made by committees or departments; corporate governance is run by boards of directors; firms' strategic choices are often made internally by groups of managers; and decisions within families or groups of friends usually require consensus-building.

"Group decision-making" is also relevant in situations in which an economic agent's project requires adhesion by several parties as in joint ventures, standard setting organizations, coalition governments, or complementary investments by several other agents. For example, an entrepreneur may need to convince a financier to fund his project and a supplier to make a specific investment. Similarly, for its wide-body aircraft A380, Airbus had to convince its board and relevant governments, and must now convince airlines to buy the planes and airports to make investments in order to accommodate them. Finally, a summer school organizer may need to convince both a theorist and an empiricist to accept teaching the chosen topic.

While the economics literature has studied in detail whether the sponsor of an idea or a project can persuade a single decision-maker to endorse his proposal, surprisingly little has been written on group persuasion. Yet group decision-making provides for a rich layer of additional persuasion strategies, including (a) "selective communication", in which the sender distills information selectively by choosing whom to talk to, and (b) "persuasion cascades", in which the sender "levers support", namely approaches group members sequentially and builds on one's gained adhesion to the project to convince another either to take a careful look or to rubberstamp altogether.

Persuasion cascades are relied upon early in life as when a child tries to strategically convince one of his parents with the hope that this will then trigger acceptation by the other. Lobbyists in Congress engage in so-called "legislator targeting"; and organizations such as the Democracy Center provide them with advice on how to proceed.[1] Supporters of an academic appointment trying to convince the department to vote for an offer to the candidate, or corporate executives hoping to get a merger or an investment project approved by the board, know that success relies on convincing key players (whose identity depends on the particular decision), who are then likely to win the adhesion of others.

[1]The reader interested in a pragmatic approach to these questions in the political context is invited to refer to the Democracy Center's website at http://www.democracyctr.org/.

The paper builds a sender/multi-receiver model of persuasion. The receivers (group members) adopt the sender's (sponsor's) project if all or a qualified majority of them are in favor at the end of the communication process. Unlike existing models of communication with multiple receivers, which focus on soft information ("recommendations"), the sender can transmit hard information (evidence, reports, material proofs) to a receiver, who can then use it to assess her payoff from the project. While the sender has information that bears on the receivers' payoffs, we assume for simplicity that he does not know the latter (and check the robustness of the analysis to this assumption). Communication is costly in that receivers who are selected to receive this hard information must incur a private cost in order to assimilate it. Thus, convincing a group member to "take a serious look at the evidence" may be part of the challenge faced by the sponsor.

The introduction of hard information in sender/multi-receiver modeling underlies the possibility of persuasion cascade, in which one member is persuaded to endorse the project or at least to take a serious look at it when she is aware that some other member with at least some alignment in objectives has already investigated the matter and came out supportive of the project. Hard information also provides a foundation for strategies involving selective communication. We for example give formal content to the notion of "key member" or "member with string pulling ability" as one of "informational pivot", namely a member who has enough credibility within the group to sway the vote of (a qualified majority of) other members.

Another departure from the communication literature is that we adopt a mechanism design approach: The sender builds a mechanism (à la Roger Myerson 1982) involving a sequential disclosure of hard and soft information between the various parties as well as receivers' investigation of hard information. This approach can be motivated in two ways. First, it yields an upper bound on what the sponsor can achieve. Second, and more descriptively, it gives content to the pro-active role played by sponsors in group decision-making. Indeed, we show how both selective communication and persuasion cascades are in equilibrium engineered by the sponsor.

The sponsor's ability to maneuver and get his project approved depends on how congruent members are among themselves ("internal congruence") and how congruent they are with the sponsor ("external congruence"). For example, for a symmetric distribution, external congruence refers to the prior probability that a given member benefits from the sponsor's project. Under the unanimity rule, the proper notion of internal congruence refers to the vector of probabilities that a given member benefits from the project given that other members benefit; put differently, and fixing the level of external congruence, one distribution exhibits more internal congruence than another if its hazard rates are smaller. We show that under the unanimity rule an increase in internal congruence, keeping external congruence fixed, makes the sponsor's better off and reduces communication between the sponsor and the committee in equilibrium. Surprisingly, an increase in external congruence may hurt the sponsor when members are asymmetric.

We also relate the sponsor's ability to get his project approved to the size of the group and its decision-rule. Interestingly, increasing the number of veto powers may make it easier for the sponsor to have his project adopted even when all members are a priori reluctant to adopt it. Finally, we show that it may be optimal for the sponsor to create some ambiguity for each member as to whether other members are already on board.

The paper is organized as follows: Section 2 sets up the sender/multi-receiver model. Section 3, in the context of a two-member group, develops a mechanism-design approach and derives the optimal deterministic mechanism. Section 4 studies its properties and demonstrates its robustness to the sender's inability to control communication channels among members and to his holding private information on the members' benefits. Section 5 allows stochastic mechanisms and shows that ambiguity may benefit the sender. Section 6 extends the analysis to N members. Finally, Section 7 summarizes the main insights and discusses alleys for future research.

Relationship to the literature

Our paper is related to and borrows from a number of literatures. Of obvious relevance is the large single-sender/single-receiver literature initiated by Vincent Crawford and Joel Sobel (1982)'s seminal paper on the transmission of soft information, and by the work of Sanford J. Grossman (1981), Grossman and Oliver Hart (1980), and Paul R. Milgrom (1981) on the disclosure of hard information. Much of this work has assumed that communication is costless, although possibly limited. By contrast, Mathias Dewatripont and Jean Tirole (2005) emphasize sender and receiver moral hazard in communication.

Following Milgrom (1981), the literature on persuasion games with hard information investigates optimal mechanisms for the receiver, focusing on the sender's discretion in selectively communicating evidence (e.g. Michael J. Fishman and Kathleen M. Hagerty 1990) and on the receiver's verification strategies (e.g. Jacob Glazer and Ariel Rubinstein 2004). In comparison, the sender's optimal persuasion strategy in our paper relies on communication of all evidence to a selectively chosen subset among several receivers. Our model also relates to the formal mechanism design approach with hard evidence of Jess Bull and Joel Watson (2006).

Our paper is also related to a large literature on committees, that addresses issues relative to the composition, the internal decision rule and the size of committees. Most of this literature assumes that group members have exogenous information and that communication is soft. Committees are viewed as a collection of informed experts in multi-sender models where the receiver is the unique decision maker who optimally designs the rules of communication with the committee.[2] The focus is also on the aggregation of dispersed information through debate within a decision making committee, where efficiency is con-

[2]See among others Thomas W. Gilligan and Keith Krebhiel (1989), Keith Krebhiel (1990), David Austen-Smith (1993a & b), Marco Ottaviani and Peter Normal Sorensen (2001), Vijay Krishna and John Morgan (2001), Klaas Beniers and Otto Swank (2004).

sidered from the committee point of view.[3] Closer to our contribution, Joe Farrell and Robert Gibbons (1989)'s model of cheap talk with multiple audiences addresses the problem of selective communication to several receivers. Besides the sole focus on cheap talk, which precludes persuasion cascades in our model, a key difference with our framework is that the members of the audience do not form a single decision-making body and so no group persuasion strategies emerge in their paper.

A recent strand in the literature extends the analysis of committees by explicitly recognizing, as our paper does, that members have to acquire information before making a decision and that information acquisition is costly. Nicola Persico (2004), Hao Li (2001) and Dino Gerardi and Leeat Yariv (2006) consider homogenous committees with simultaneous acquisition of information and focus on the tension between ex post efficient aggregation of information through the choice of a decision rule and ex ante efficient acquisition of information. They characterize the optimal design, in terms of decision rule or in terms of size, from the committee's perspective. Hongbin Cai (2003) introduces heterogenous preferences in a similar model with a given decision rule and analyzes the socially optimal size of the committee. Like us, Alex Gershkov and Balazs Szentes (2004) adopt a mechanism design approach to characterize the optimal game form of information acquisition by committee members and show that information acquisition is sequential; they derive the stopping rule that is optimal from the committee's perspective, while we focus on the optimal persuasion strategy by an external agent who controls the members' access to information.

Finally, we rule out the possibility of targeting resources or paying bribes to committee members unlike in some of the literature on lobbying (e.g., Timothy Groseclose and Jim Snyder 1996 or Assar Lindbeck and Jorgen Weibull 1987).

[3]See e.g. David Spector (2000) and Hao Li, Sherwin Rosen and Wing Suen (2001).

2 Model

We consider a sender (S) / multi-receiver (R_i) communication game. An N-member committee $(R_1, R_2, ..., R_N)$ must decide on whether to endorse a project submitted by a sponsor S. Committee members simultaneously vote in favor of or against the project. The decision rule defines an aggregation procedure: under the unanimity rule, all committee members must approve the project and so the sponsor needs to build a consensus;[4] under the more general K-majority rule, no abstention is allowed and the project is adopted whenever at least K members vote in favor of it.

The project yields benefits $s > 0$ to S and r_i to committee member R_i. The status quo yields 0 to all parties. The sponsor's benefit s is common knowledge[5] and his objective is to maximize the expected probability that the project is approved.

R_i's benefit r_i is a priori unknown to anyone[6] and the question for R_i is whether her benefit from the project is positive or negative. A simple binary model captures this dilemma; r_i can a priori take two values, $r_i \in \{-L, G)$, with $0 < L, G$. The realization of r_i in case the project is implemented is not verifiable.

Committee member R_i can simply accept or reject the project on the basis of her prior $p_i \equiv \Pr\{r_i = G\}$. Alternatively, she can learn the exact value of her individual benefit r_i by spending time and effort investigating a detailed report about the project if provided by the sponsor: the sponsor is an information gatekeeper.[7] Investigation is not verifiable, and so is subject to moral hazard. The personal cost of investigation is denoted c and

[4] In the unanimity case (and ruling out weakly dominated strategies), each member can as well assume that all other members vote in favor of the project.

[5] More generally, receivers infer that the sponsor benefits from the observation that he proposes the project.

[6] We later allow S to have a private signal about the distribution of the members' benefits. See also Dewatripont-Tirole (2005) for a comparison, in the single-receiver case, of equilibrium behaviors when the sender knows and does not know the receiver's payoff.

[7] The report does not directly contain information about r_i. It provides details and data that enable R_i to figure out the consequences of the project for her own welfare, provided that she devotes the necessary time and effort.

is identical across committee members. There are several possible interpretations for the "report". It can be a written document handed over by the sponsor. Alternatively, it could be a "tutorial" (face–to-face communication) supplied by the sponsor. Its content could be "issue-relevant" (examine the characteristics of the project) or "issue-irrelevant" (provide the member with track-record information about the sponsor concerning his competency or trustworthiness). Committee member R_i can also try to infer information from the opinion of another member who has investigated. That is, committee member R_i may use the correlation structure of benefits $\{r_i\}_{i=1}^N$ to extract information from R_j's having investigated and decided to approve the project.

The dictator case

Let $u^I(p) \equiv pG - c$ denote the expected benefit from investigation for a single decision-maker (a "dictator"), when her prior is $\Pr\{r = G\} = p$, and let $u^R(p) \equiv pG - (1-p)L$ denote her expected benefit when granting approval without investigation, i.e. when rubberstamping S's proposal.

The dictator prefers rubberstamping to rejecting the project without investigation if:[8]

$$u^R(p) \geq 0 \Longleftrightarrow p \geq p_0 \equiv \frac{L}{G+L}.$$

Similarly, when asked to investigate, she prefers investigating and approving whenever $r = G$ to rejecting without investigation if

$$u^I(p) \geq 0 \Longleftrightarrow p \geq p_- \equiv \frac{c}{G}.$$

And she prefers rubberstamping to investigating and approving whenever $r = G$ if

$$u^R(p) \geq u^I(p) \Longleftrightarrow p \geq p_+ \equiv 1 - \frac{c}{L}.$$

These thresholds play a central role in the analysis.

[8] In the analysis, we neglect boundary cases and always assume that when indifferent, a committee member decides in the sponsor's best interest.

Assumption 1. $c < \frac{GL}{G+L}$.

If c were too large, i.e. violated Assumption 1, a committee member would never investigate as a dictator, and a fortiori as a member of a multi-member committee. The dictator's behavior is summarized in Lemma 1 and depicted in Figure 1.

Lemma 1. *In the absence of a report, the dictator rubberstamps whenever $p \geq p_0$. When provided with a report, she rubberstamps the project whenever $p \geq p_+$, investigates whenever $p_- \leq p < p_+$, and turns down the project whenever $p < p_-$. Under Assumption 1, $p_- < p_0 < p_+$.*

The following terminology, borrowed and adapted from the one used on the Democracy Center website,[9] may help grasp the meaning of the three thresholds. Based on her prior, a committee member is said to be a *hard-core opponent* if $p < p_-$, a *mellow opponent* if $p_- \leq p < p_0$, an *ally* if $p_0 \leq p$; an ally is a *champion* for the project if $p \geq p_+$. The lemma simply says that only a *moderate* ($p_- \leq p < p_+$) investigates when she has the opportunity to do so, while an *extremist*, i.e. either a hard-core opponent or a champion, does not bother to gather further information by investigating.

FIGURE 1 HERE

Faced with a dictator, the sponsor has two options: present a detailed report to the dictator and thereby allow her to investigate, or ask her to rubberstamp the project (these two strategies are equivalent when $p \geq p_+$ since the dictator rubberstamps anyway, and when $p < p_-$, as the dictator always rejects the project).

Proposition 1. (The dictator case) *When $p \geq p_0$, the sponsor asks for rubberstamping and thereby obtains approval with probability 1; when $p_- \leq p < p_0$, the sponsor lets the dictator investigate and obtains approval whenever $r = G$, that is with probability p.*

[9] See http://www.democracyctr.org/resources/lobbying.html for details.

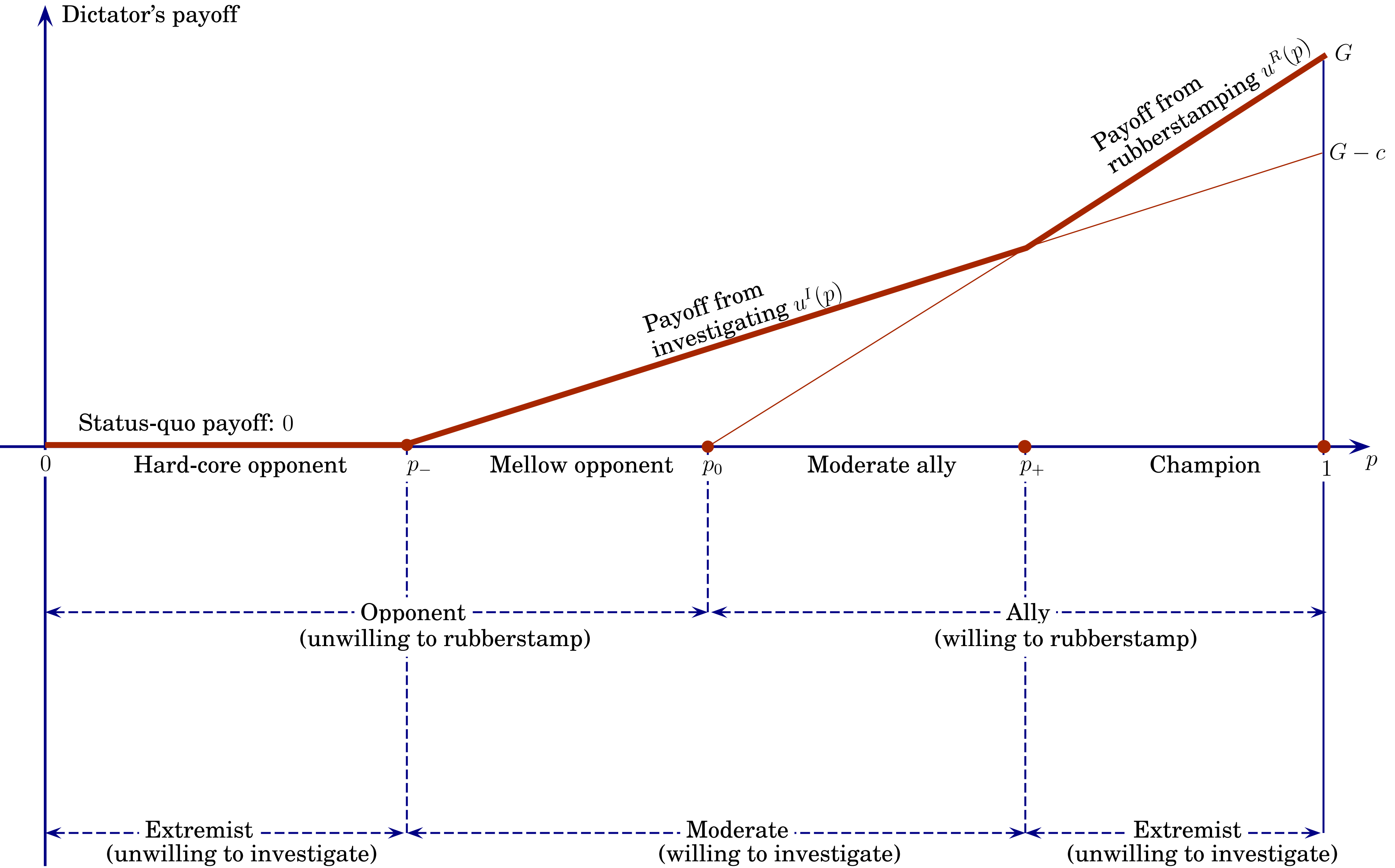
Dictator's payoff
Payoff from rubberstamping $u^R(p)$
G
$G-c$
Payoff from investigating $u^I(p)$
Status-quo payoff: 0
0
Hard-core opponent
p_-
Mellow opponent
p_0
Moderate ally
p_+
Champion
1
p
Opponent
(unwilling to rubberstamp)
Ally
(willing to rubberstamp)
Extremist
(unwilling to investigate)
Moderate
(willing to investigate)
Extremist
(unwilling to investigate)

Figure 1

It is optimal for S to let the dictator investigate only when the latter is a mellow opponent; in all other instances, the decision is taken without any information exchange. A moderate ally, in particular, would prefer to investigate if she had the chance to, but she feels confident enough not to oppose the project in the absence of investigation; S therefore has real authority in this situation.[10]

3 Optimal deterministic mechanism

For a two-member committee, let $P \equiv \Pr\{r_1 = r_2 = G\}$ denote the joint probability that both benefit from the project. The Bayesian update of the prior on r_i conditional on the other member's benefiting from the project is: $\hat{p}_i \equiv \Pr\{r_i = G \mid r_j = G\} = P/p_j$. We assume that committee members' benefits are affiliated for $i = 1, 2$, $\hat{p}_i \geq p_i$.[11] This stochastic structure is common knowledge and we label committee members so that R_1 is a priori more favorable to the project than R_2; that is, $p_1 \geq p_2$.

We characterize the sponsor's optimal strategy to obtain approval from the committee under the unanimity rule.[12] S chooses which committee members to provide the report to, in which order, and what information he should disclose in the process. The specification of the game form to be played by S and committee members is part of S's optimization problem. Therefore we follow a mechanism design approach (see Myerson 1982), where S is the mechanism designer, to obtain an upper bound on S's payoff without specifying a game form; we will later show that this upper bound can be simply implemented. The formal analysis is relegated to Appendix 1 (and to section 5 for general mechanisms); we provide here only an intuitive presentation of the *deterministic* mechanism design

[10] Here, we follow the terminology in Philippe Aghion and Tirole (1997).

[11] See Proposition 4 for the case of negative correlation in two-member committees.

[12] The sponsor takes the voting rule as a given. This optimization focuses on his communication strategy. Note also that we do not consider governance mechanisms in which e.g. voting ties are broken by a randomizing device (for example the project is adopted with probability 1/2 if it receives only one vote).

problem. While focusing on deterministic mechanisms is restrictive, we are able to obtain a complete characterization of the optimum and to provide intuition for our main results.

A deterministic mechanism maps (r_1, r_2) (the state of nature) into the set of members who investigate and the final decision. From the Revelation Principle, we know that we can restrict attention to obedient and truthful mechanisms: given the information provided by S, a member must have an incentive to comply with the investigation recommendation and to report truthfully the value of her benefit to S whenever she investigates. Given the unanimity decision rule, the optimal obedient and truthful mechanism maximizes Q, the expected probability that the project is implemented, under these incentive constraints, under individual rationality constraints and under measurability constraints.

Individual rationality constraints refer to the members' veto power in the committee: The project can be approved only if each member expects a non-negative benefit from the project given (if relevant) her own information from investigation. Measurability constraints refer to the fact that the outcome cannot depend upon information that is unknown to all receivers. For instance, a mechanism cannot lead to no one investigating for some state of nature and R_i investigating in another state of nature, since the recommendation is necessarily made under complete ignorance of the state of nature. Similarly, the final decision cannot depend on the value of r_j if R_j does not investigate.

Working out all the constraints, Appendix 1 shows that one can restrict attention to only three types of deterministic mechanisms that yield a positive probability of implementing the project, provided they are incentive compatible: (a) the no-investigation mechanism in which S asks members to vote on the project without letting them investigate, (b) mechanisms with investigation by R_i, $i = 1$ or 2, in which S provides only R_i with a report and asks R_j to rubberstamp, and (c) mechanisms with two sequential investigations, in which S lets R_i investigate, approve or reject the project, and then lets

R_j investigate if $r_i = G$.[13]

Ignoring incentive compatibility, S has a clear *pecking order* over these mechanisms. He prefers the no-investigation mechanism, yielding $Q = 1$; his next best choice is investigation by R_1 only, yielding $Q = p_1$, and then investigation by R_2 only, yielding $Q = p_2$; finally, his last choice is to have both committee members investigate, with $Q = P$. We therefore simply move down S's pecking order and characterize when a mechanism is incentive compatible while all preferred ones (absent incentive constraints) are not.

Note first that if both committee members are allies of the sponsor, i.e. if $p_1 \geq p_2 \geq p_0$, members are willing to rubberstamp without investigation and so the project is implemented with probability $Q = 1$. This outcome is similar to the one obtained in the dictator case. The committee is reduced to a mere rubberstamping function even though moderate allies ($p_0 \leq p_i < p_+$) would prefer to have a closer look at the project if given the chance to. Note also that if both committee members are hard-core opponents, i.e. if $p_2 \leq p_1 < p_-$, the project is never implemented.[14] We therefore restrict attention to the constellation of parameters for which at least one member is not an ally ($p_2 \leq p_0$), and at least one member is not a hard-core opponent ($p_1 \geq p_-$).

We focus first on the case where R_1 is a champion for the project ($p_1 > p_+$), while R_2 is an opponent ($p_2 < p_0$). There is no way to induce the champion to investigate; she always prefers rubberstamping to paying the investigation cost. Referring to S's pecking order, the only way to get the project approved is to let R_2 investigate and decide.

Proposition 2. *If R_1 is a champion ($p_1 > p_+$) and R_2 a mellow opponent ($p_- \leq p_2 < p_0$),*

[13] Three comments must be made here. (1) This restriction rests on the more general Lemma 3 (see Appendix 3): one can restrict attention to no-wasteful-investigation mechanisms. (2) To avoid the standard multiplicity of Nash equilibria in the voting subgame, we assume that committee members never play weakly dominated strategies in this voting subgame. (3) We do not need to consider truthful revelation constraints. When R_i has investigated and $r_i = -L$, she vetoes the project and so her utility is not affected by her report of r_i to S. And when $r_i = G$, lying can only hurt R_i in any of mechanisms described here.

[14] As we will later see, this no longer holds when stochastic mechanisms are allowed.

the project is implemented with probability $Q = p_2$. If R_1 is a champion and R_2 a hard-core opponent, there is no way to have the project approved ($Q = 0$)

The proposition formalizes the idea that *too strong a support is no useful support.* S's problem here is to convince the opponent R_2. Without any further information, this opponent will simply reject the proposal. To get R_2's approval, it is therefore necessary to gather good news about the project. Investigation by R_1 could deliver such good news, but committee member R_1 is so enthusiastic about the project that she will never bother to investigate.[15] The sponsor has no choice but to let the opponent investigate. R_2 de facto is a dictator and R_1 is of no use for the sponsor's cause.

Finally assume that $p_- \leq p_1 \leq p_+$ and $p_2 < p_0$. In this region, we move down S's pecking order, given that having both members rubberstamp is not incentive compatible. The following proposition, proved in Appendix 1, characterizes the optimal scheme.

Proposition 3. *Suppose the committee consists of a moderate and an opponent, i.e. $p_- \leq p_1 \leq p_+$ and $p_2 < p_0$;*

- *if $\hat{p}_2 \geq p_0$, the optimal mechanism lets the most favorable member R_1 investigate and decide: the project is implemented with probability $Q = p_1$;*
- *if $\hat{p}_2 < p_0$ and $\hat{p}_1 \geq p_0$, the optimal mechanism lets R_2 investigate and decide (so, $Q = p_2$) if $p_2 \geq p_-$; the project cannot be implemented if $p_2 < p_-$;*
- *if $\hat{p}_i < p_0$ for $i = 1, 2$, the optimal mechanism lets both members investigate provided $P \geq p_-$, in which case $Q = P$; the status quo prevails if $P < p_-$.*[16]

[15] Why is a champion R_1 part of the committee if she always rubberstamps ? Although it may appear ex post that a champion for the project has a conflict of interest, the distribution of the member's benefit or her position on the specific policy might not have been known ex ante, when she was appointed in the committee, or else her appointment might result from successful lobbying by S.

[16] The order of investigation does not matter here. Had we introduced a possibly small cost of communication on the sponsor's side, the optimal mechanism with double investigation would start with an investigation by R_2, so as to save on communication costs (R_2 is more likely to reject than R_1).

Proposition 3 shows that for committees that consist of a moderate and an opponent, communication is required to get the project adopted. More importantly, it demonstrates the importance of *persuasion cascades,* in which an opponent R_i ($p_i < p_0$) is induced to rubberstamp if another committee member R_j approves the project after investigation. In a sense, R_i is willing to delegate authority over the decision to R_j, knowing that R_j will endorse the project after investigation only if her benefit is positive ($r_j = G$). R_j is "reliable" for R_i because the information that $r_j = G$ is sufficiently *good news* about r_i that the updated beliefs $\Pr\{r_i = G \mid r_j = G\}$ turn R_i into an ally ($\hat{p}_i \geq p_0$).

Of course, the sponsor prefers to rely on a persuasion cascade triggered by the most favorable committee member, R_1, since the probability that this member benefits from the project is larger than the corresponding probability for the other member. But this strategy is optimal only if news about $r_1 > 0$ carries enough information to induce R_2 to rubberstamp. If not, the next best strategy is to rely on a persuasion cascade triggered by the less favorable committee member R_2. Even though this implies a smaller probability of having the project adopted, this strategy dominates having both members investigate, which leads to approval with probability P.

An illustrative example: the case of nested benefits.

Assume that a project that benefits committee member R_2 necessarily also benefits R_1: $P = p_2$. Committee members are then ranked ex post as well as ex ante in terms of how aligned their objectives are with the sponsor's. Updated beliefs are: $\hat{p}_1 = 1$ and $\hat{p}_2 = p_2/p_1$.

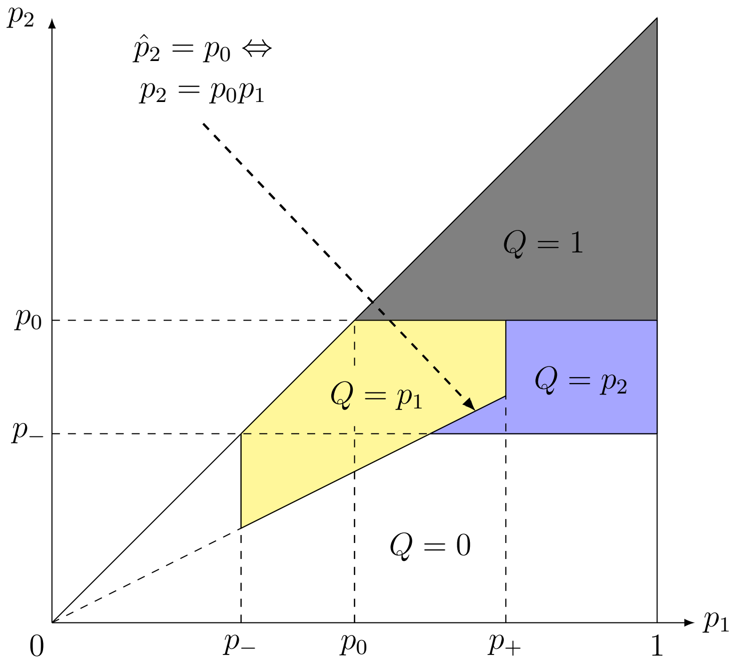

FIGURE 2

Note that R_1 always rubberstamps R_2's informed decision and so persuasion cascades triggered by R_2 are possible provided that $p_2 \geq p_-$. Persuasion cascades triggered by R_1 if feasible are preferred by S since R_1 is a priori more favorable. The optimal mechanism, depicted in Figure 2, can be straightforwardly computed from previous propositions using the fact that $\hat{p}_2 \geq p_0 \Leftrightarrow p_2 \geq p_0 p_1$.

Persuasion cascades rely on the fact that it is good news for one committee member to learn that the other benefits from the project. If committee members' benefits are stochastically independent ($\hat{p}_i = p_i$ for each i), no such cascade can exist. Each committee member is de facto a dictator and the sponsor has no sophisticated persuasion strategy relying on group effects.

The cases of nested and independent benefits are two polar cases; indeed, any joint distribution of benefits can be represented as a mixture of the two:

- with probability $\rho \in [0, 1]$, the two members' benefits are nested: R_1 benefits from the project whenever R_2 does (but the converse does not hold if $p_1 > p_2$);

- with probability $1 - \rho$, the two members' benefits are drawn independently.[17]

This representation enables us to define key concepts of congruence. For a stochastic structure given by (p_1, p_2, ρ), *external congruence* refers to the vector (p_1, p_2) of prior probabilities that the members' interests are aligned with the sponsor's: for a given ρ, (p'_1, p'_2) exhibits more external congruence than (p_1, p_2) if $p'_i \geq p_i$ for all i (with at least one strict inequality). By contrast, *internal congruence* among committee members captures the correlation among the members' benefits: an increase in internal congruence for a given (p_1, p_2) refers to an increase in ρ.

Our results show that the choice of the optimal persuasion strategy does not depend only on the external congruence of the committee, but also on the degree of internal congruence among committee members. The sponsor finds it optimal to trigger a persuasion cascade when facing a committee with high internal congruence, while he must convince both members of a committee with poor internal congruence.

Internal dissonance

When the members' benefits are negatively correlated, $r_i = G$ is bad news for R_j and therefore $\hat{p}_i < p_i$: there is internal *dissonance* within this committee. Focusing on the non-trivial case in which $\hat{p}_2 < p_2 < p_0$, no persuasion cascade can be initiated with R_1 investigating and R_2 rubberstamping. So, whenever $p_2 < p_0$, R_2 must investigate for the project to have a chance of being approved. Then, R_1 knows that the project can be adopted only if $r_2 = G$. In the optimal mechanism, R_1 acts as a dictator conditional on $r_2 = G$ and decides to rubberstamp, investigate or reject the project based on the

[17]Then $P = p_2 [\rho + (1 - \rho)p_1]$. Conversely, for any (p_1, p_2, P) with $p_1 \geq p_2$ and $p_1 p_2 \leq P$ (affiliation), one can find a unique $\rho = \frac{P - p_1 p_2}{p_2 - p_1 p_2}$ that yields an equivalent representation of the stochastic structure. In the symmetric case $(p_1 = p_2)$, nested benefits correspond to perfectly correlated benefits.

posterior $\hat{p}_1$. Since $\hat{p}_1 < p_1$, it is more difficult to get R_1's approval when she is part of a committee with internal *dissonance*. Following similar steps as in the proof of Proposition 3, it is easy to characterize the optimal mechanism under internal dissonance:

Proposition 4. (Internal dissonance) *Assume the committee is characterized by internal dissonance, i.e.* $\hat{p}_i \leq p_i$ *for* $i = 1, 2$*, and that* $p_2 < p_0$ *and* $p_1 > p_-$*:*

- *if* $p_2 < p_-$*, the project cannot be implemented;*
- *if* $p_2 \geq p_-$ *and* $\hat{p}_1 \geq p_0$*, the optimal mechanism is to let* R_2 *investigate and* R_1 *rubberstamp:* $Q = p_2$*;*[18]
- *if* $p_2 \geq p_-$ *and* $\hat{p}_1 < p_0$*, the optimal mechanism is to let both members investigate whenever* $P \geq p_-$*, with* $Q = P$*; if* $P < p_-$ *however, the status quo prevails.*

Members' optimum

For reasons given earlier, we focused on the sponsor's optimal mechanism. It is nonetheless interesting to compare the members' and the sender's preferences regarding communication. One question is whether members, assuming that they design the communication process, can force the sender to communicate. Indeed, receivers have access to a smaller set of mechanisms than the sponsor if the latter, when prompted to transfer information to a receiver, can do so perfunctorily (by explaining negligently) or can overload the receiver with information. When receivers can force the sponsor to communicate, then it is easy to see that their optimum never involves less communication

[18] Note that this does not describe a persuasion cascade: R_1 would be willing to rubberstamp based on her prior and the mechanism exploits the fact that she is still willing to rubberstamp *despite* the bad news $r_2 = G$.

than the sponsor's optimum,[19] and can involve more.[20] The next proposition by contrast assumes that the committee members cannot force the sponsor to communicate, but can cut communication channels.[21]

Proposition 5. *In the symmetric-receiver case or if $G > L$, the members never gain from preventing the sponsor from communicating with one specific receiver (or both).*

Intuitively, members want more communication, not less. If they are asymmetric and the sender communicates with R_1, the more favorable receiver, investigation by R_2 instead might maximize the receivers' average welfare. If $G > L$, though, the negative externality from preventing a receiver from benefiting from the project exceeds that imposed on a receiver who would not like it.

4 Comparative statics and robustness

Stochastic structure

A mere examination of Proposition 3 delivers the following corollary:

Corollary 1. (Benefits from internal congruence) *Fixing priors (p_1, p_2), the probability of having the project approved is (weakly) increasing in the internal congruence parameter ρ (or equivalently in $\hat{p}_1$ and $\hat{p}_2$).*

[19]There may be a conflict on who should investigate, though. When the sponsor's optimum is to let R_1 investigate, members may strictly prefer to let R_2 investigate (under incentive compatibility) if:

$$u^I(p_2) + p_2 u^R(\hat{p}_1) - u^I(p_1) - p_1 u^R(\hat{p}_2) > 0 \Longleftrightarrow L > G \text{ and } p_1 > p_2,$$

that is, if the cost of type I error in adopting the project is larger than the cost of type II error.

[20]E.g. when $p_0 < p_1 = p_2 < p_+$ and $\rho = 1$, which corresponds to the dictator case.

[21]Here a sketch of proof. When the sponsor's optimum is to let R_1 investigate and members consider cutting communication between S and R_1, either investigation by R_2 is incentive compatible and footnote 19 shows that members lose, or the project is never approved and members lose as well. If the sponsor's optimum is to let R_2 investigate, R_1-investigation violates incentive compatibility and cutting any communication channels also leads to reject the project. The conclusion is immediate in other cases.

The sponsor unambiguously benefits from higher internal congruence within the committee. It should be noted that this holds even when only one member investigates (when both members investigate, an increase in ρ mechanically raises the probability P that both favor the project). Proposition 4 further implies that the sponsor benefits from a decrease in internal dissonance[22] under negative correlation.

As for the impact of external congruence, the next corollary shows that an increase in p_1 may hurt S for two reasons: first, if R_1 investigates, her endorsement may no longer be credible enough for R_2 (if $\hat{p}_2 = p_2 \left[\frac{\rho}{p_1} + (1-\rho)\right]$ falls below p_0); second R_1 may even no longer investigate (if p_1 becomes greater than p_+). In either case, an increase in R_1's external congruence prevents a persuasion cascade.

Corollary 2. (Potential costs of external congruence) *Fixing the degree of internal congruence ρ, an increase in p_1 may lead to a smaller probability of having the project approved.*[23]

Payoffs

First, although R_1's prior benefit distribution first-order stochastically dominates R_2's, R_1's expected payoff may be smaller than R_2's, because R_1, but not R_2, may incur the investigation cost.[24] The sponsor's reliance on the member with highest external congruence to win the committee's adhesion imposes an additional burden on this member and she may be worse off than her fellow committee member.[25]

[22] Under dissonance, ρ can be defined as the probability that r_1 and r_2 are drawn independently: $P = \rho p_1 p_2 + (1-\rho)\max\{p_1 + p_2 - 1; 0\}$. A decrease in internal dissonance then corresponds to an increase in ρ.

[23] By contrast, fixing internal congruence, an increase in p_2, the external congruence of the less favorable member, unambiguously increases Q.

[24] In particular, when p_1 is slightly above p_- (while $\hat{p}_2 \geq p_0$), R_1's expected benefit is almost null.

[25] This point is to be contrasted with one in Dewatripont-Tirole (2005), according to which a dictator may be made worse off by an increase in her congruence with the sponsor because she is no longer given the opportunity to investigate (see also Proposition 1). It also suggests that if the a priori support of committee members were unknown to the sponsor, the latter could not rely on voluntary revelation of priors by committee members (the same point also applies to the dictator case).

Second, suppose that the sponsor can modify project characteristics so as to raise the members' benefits or reduce their losses. Such manipulations do not necessarily make it easier to get the project adopted; as in Corollary 2, too strong an ally is useless, and raising an ally's external congruence may decrease the probability of approval.[26]

Corollary 3. *The probability of having the project approved may fall when the potential loss of the more favorable member is reduced (when her potential loss is $L' < L$ instead of L).*[27]

It is moreover immediate to check that reducing the cost of communication for committee members always raises the probability of implementing the project.[28]

Third, *our analysis extends to continuous payoffs.* Intuitively, once a committee member knows that she benefits (or loses) from the project, this member wants the probability that the project is implemented to be maximized (minimized), regardless of the magnitude of her payoff. Thus, with deterministic mechanisms and no side communication, there is no way to elicit anything else but the sign of the payoff.[29]

Do more veto powers jeopardize project adoption?

Intuition suggests that under the unanimity rule, the larger the committee the stronger the status-quo bias. Although our model so far deals only with one- and two-member

[26]Corollary 3 focuses on the interesting case. For completeness, let us summarize the other results: an increase in any member R_i's potential gain $G' > G$, or a decrease in R_2's potential loss, increase the probability of having the project approved.

[27]In that case, the thresholds $p_{0,1} = \frac{L'}{G+L'}$ and $p_{+,1} = 1 - \frac{c}{L'}$ become member-specific and smaller than their counterparts p_0 and p_+ for R_2. A decrease in R_1's potential loss may then turn R_1 into a champion, which prevents a persuasion cascade initiated by R_1.

[28]Monetary transfers from the sponsor or across committee members could also be part of the mechanism design analysis, although they would probably be deemed illegal in many situations, e.g. in congressional committees. Although we have not done the complete analysis, it appears that the basic tradeoff between the number of investigations and the sponsor's objective remains.

[29]General rubberstamping is optimal when $\mathbb{E}[r_i] \geq 0$ for $i = 1, 2$; if not, R_i-investigation with R_j rubberstamping is optimal and yields $Q = \Pr\{r_i \geq 0\}$ provided $\mathbb{E}[\max\{r_i, 0\}] \geq c$ and $\mathbb{E}[r_j \mid r_i \geq 0] \geq 0$; if no other mechanism is feasible, sequential investigation by R_i and then R_j yields $Q = \Pr\{\min\{r_1, r_2\} \geq 0\}$ provided $\mathbb{E}[r_i \mid \min\{r_1, r_2\} \geq 0] \Pr\{\min\{r_1, r_2\} \geq 0\} \geq c$ and $\mathbb{E}[\max\{r_j, 0\} \mid r_i \geq 0] \geq c$.

committees, it may shed a new light on this idea and enrich our understanding of bureaucracies. The conjecture that larger communities are more likely to vote against change misses the main point about the use of persuasion cascades to persuade a group. When internal congruence within the committee is high enough so that $\hat{p}_2 \geq p_0$, it is possible to win R_2's adhesion to the project even though she started as an hard-core opponent ($p_2 < p_-$). Adoption would not be possible with a hard-core opponent dictator.

Suppose that committees are formed by randomly selecting members within a given population of potential members with ex ante unknown support for S's project. For a one-member committee (a dictator) the probability of implementing the project is based merely on external congruence with S; two-member committees may compensate poor external congruence of some of its members by high internal congruence among its members and therefore lead, ex ante, to a higher probability of implementing S's project.

Proposition 6. *A randomly drawn two-member committee may approve the project more often than a randomly drawn dictator: a two-member committee is not necessarily more prone to the status-quo bias than a one-member committee.*

Proof. The proof is by way of an example. A randomly-drawn member is a mellow opponent with probability β (has congruence $p = p_H$, where $p_- < p_H < p_0$), and a hard-core opponent with probability $1 - \beta$ (has congruence $p_L < p_-$). Assume that the hard-core opponent rubberstamps if the mellow opponent investigates and favors the project. The optimal organization of a two-member committee that turns out to be composed of at least one mellow opponent is to let a mellow opponent investigate and the other rubberstamp. The ex ante probability that a randomly drawn two-member committee approves the project is larger than for a random dictator:

$$\mathbf{E}[Q] = \beta^2 p_H + 2\beta(1-\beta)p_H = \beta(2-\beta)p_H > \beta p_H. \qquad \blacksquare$$

Side communication

We have assumed that communication can only take place between the sponsor and committee members. There may be uncontrolled channels of communication among members, though. First, members may exchange soft information about their preferences and about whether they have been asked to investigate. Second, an investigator may, in the absence of a confidentiality requirement imposed by the sponsor, forward the file to the other committee member. It is therefore interesting to question the robustness of our results to the possibility of side communication between committee members. To this purpose, we exhibit implementation procedures in which the equilibrium that delivers the optimal outcome is robust to the possibility of side communication, whether the latter involves cheap talk or file transfer among members.[30]

Obviously, side communication has no impact when both members rubberstamp, as they then have no information. Intuitively, it also does not matter under sequential investigation, because the sponsor both reveals the first investigator's preferences and hands over the file to the second investigator. Under a single investigation and rubberstamping, the member who rubberstamps can as well presume that the investigator liked the project (otherwise her vote is irrelevant); furthermore, conditional on liking the project, the investigator is perfectly congruent with the sponsor and has no more interest than the sponsor in having the second member investigate rather than rubberstamp. Appendix 2 makes this reasoning more rigorous and also looks at side communication following out-of-equilibrium moves.

Proposition 7. (Robustness to side communication) *The sponsor can obtain the same expected utility even when he does not control communication channels among members.*

We should however acknowledge that this robustness result is fragile. First, there may

[30] So, we focus on a weak form of robustness; there exist other equilibria that do not implement the optimal outcome, if only because of the voting procedure.

exist other equilibria in which side communication matters. Second, as seen in section 5, the proposition depends on our focusing on deterministic mechanisms. And third, side communication could matter if investigation imperfectly revealed to a member her payoff, since the latter might want double-checking by the other committee member and would then transmit the file, even when S does not want double investigation.

Informed sponsor

While the sponsor may not know how the description of the project will map into receivers' taste for it, one can think of cases where he has a private signal about external or internal congruence (or both). The following proposition shows that (i) there exists a pooling equilibrium in which the sponsor behaves as if he had no more information about the receivers' payoffs than they do, and (ii) there may exist multiple equilibria, but the pooling equilibrium is Pareto dominant for all sponsor's types. Intuitively, the sponsor wants to minimize the number of investigations, regardless of her information. The equilibrium in which the sender behaves as if he had no private information is dominant, but "suspicion equilibria", in which the committee members become pessimistic if the sponsor does not let them investigate, may co-exist and involve higher amounts of investigation.

More precisely, and focusing on the symmetric case, let the sponsor's type $t = (p, \rho)$ reflect the knowledge he has about the members' benefits. Assume t is distributed on $[0, 1]^2$ with full support and let $p^a \equiv \mathbb{E}\left[p(t)\right]$ and $\hat{p}^a \equiv \mathbb{E}\left[\rho(t) + (1 - \rho(t))p(t)\right]$.

Proposition 8. *Assume the committee consists of two symmetrical members and focus on deterministic mechanisms.*

i) The equilibrium for the symmetric information situation in which it is common knowledge that $p = p^a$ and $\hat{p} = \hat{p}^a$ is also a pooling equilibrium of the informed-sponsor game.

ii) This equilibrium is Pareto-dominant for all types of sender.

Proof. We content ourselves with a sketch of the proof.

i) Suppose that the sponsor offers the best deterministic mechanism for the fictitious symmetric information case with p^a and $\hat{p}^a$, whatever his true type. Suppose that beliefs are equal to the prior on and off the equilibrium path. By definition of the best mechanism, a deviation mechanism cannot generate fewer investigations before approval and so all types weakly prefer the pooling outcome.

ii) In any equilibrium, the number of investigations must be the same; so any equilibrium is a full pooling equilibrium. By definition of the optimal mechanism for $(p^a, \hat{p}^a)$, the number of investigations cannot be smaller than in the equilibrium of i).[31] ■

5 Stochastic mechanisms

The restriction to deterministic mechanisms involves some loss of generality, as we now show. In this section, we consider a symmetric two-member committee ($p_1 = p_2 = p$, $\hat{p}_1 = \hat{p}_2 = \hat{p}$), and we investigate whether S can increase the probability of project approval by using (symmetric) stochastic mechanisms.

Assume that $p_- < p < p_0 < \hat{p}$, so that the optimal deterministic mechanism consists in a persuasion cascade where R_1, say, investigates and R_2 rubberstamps. In this deterministic mechanism, R_2 knows that R_1 investigates and, given this, she strictly prefers rubberstamping to rejecting the project: $u^R(\hat{p}) > 0 = u^R(p_0)$. If R_2 knew that R_1 does not investigate, however, she would not rubberstamp since $u^R(p) < 0$. Suppose now that R_1 may or may not investigate; R_2 is then willing to rubberstamp provided she is confident enough that R_1 investigates. So, when $p < p_0 < \hat{p}$, it is not necessary to have R_1 investigate with probability 1 to get R_2's approval.

[31] There may indeed exist Pareto-inferior equilibria. For example, suppose $p_- < p^a < p_0 < \hat{p}^a$. The pooling in Proposition 8 involves one investigation. Provided that $(p^a)^2 > p_-$, there exists a pooling, two-investigation equilibrium in which the receivers believe that $p = p^a$ and $\rho = 0$ in any other mechanism is offered: S is suspected to know that internal congruence is low when he refuses to let both members investigate.

Intuitively, the incentive constraint corresponding to rubberstamping is slack in the deterministic mechanism. Stochastic mechanisms are lotteries over deterministic mechanisms, the realization of which is not observed by committee members. So, they may be designed to induce appropriate beliefs from the committee members: we say that stochastic mechanisms exhibit *constructive ambiguity*. Constructive ambiguity enables S to reduce the risk that one member gets evidence that she would lose from the project, and thereby increases the overall probability of having the project adopted.

In Appendix 3, we develop the general mechanism design approach (without the symmetry assumption). We prove that we can restrict attention to the simple class of no-wasteful-investigation mechanisms and we fully characterize the optimal stochastic mechanism in the symmetric setting. We here summarize the main implications of this latter characterization.

Proposition 9. *In the symmetric two-member committee, stochastic mechanisms strictly improve the probability of the project being approved in the following cases:*

i) when $p_- < p < p_0 < \hat{p}$, the following mechanism is optimal and yields $Q > p$: with probability $\theta^* \in (0, \frac{1}{2})$, S asks R_1 to investigate and R_2 to rubberstamp, with probability θ^*, S asks R_2 to investigate and R_1 to rubberstamp, and with probability $\gamma = 1 - 2\theta^*$, S asks both members to rubberstamp;

ii) when $p < p_- < p_0 < \hat{p}$ and $p_0 > \frac{1+p_-}{2}$, the optimal mechanism yields $Q > 0$ provided p is close enough to p_-.

Part i) formalizes the intuition provided before. Part ii) shows that, using stochastic mechanisms, S may obtain approval even when facing two hard-core opponents. The intuition for this particularly striking result is quite similar to that for the previous result. Suppose the committee consists of two hard-core opponents ($p < p_-$) with strong internal congruence ($\hat{p} > p_0$). S's problem is to induce one member to investigate. If a committee member thought with sufficiently high probability that she is asked to investigate after her fellow committee member has investigated and discovered that her own benefits are positive, she would be willing to investigate herself. Hence, there is room again for constructive ambiguity: S can simply randomize the order in which he asks members to investigate, without revealing the order that is actually followed.[32] This approach is similar to the practice that consists in getting two major speakers interested in attending a conference by mentioning the fact that the other speaker will likely attend the conference herself. If each is sufficiently confident that the other one is seriously considering to attend, she might indeed be induced to look closely at the program of the conference and investigate whether she can move her other commitments.

The one-investigation stochastic mechanism described in the first result of Proposition 9 can be easily implemented; the sponsor simply commits to secretly approach one of the members before the final vote. By contrast, implementing the random sequential investigation mechanism discussed above may be involved if S can approach only one committee member at a time and communication requires time.[33] Note also that Proposition 9 yields only a weak implementation result: the random order mechanism for example admits another equilibrium where both members simply refuse to investigate and reject the project. Stochastic mechanisms may therefore come at a cost in terms of realism.

[32] This mechanism is not optimal; the optimal stochastic mechanism is characterized in Appendix 3.

[33] To illustrate the difficulty, suppose that there is a date $t = 1$ at which the committee must vote and that with probability $\frac{1}{2}$ the sponsor presents R_1 with a detailed report at time $t = 1/3$ and, if $r_1 = G$, he transfers the report to R_2 at time $t = 2/3$; with probability $\frac{1}{2}$ the order is reversed. When being approached at $t = 1/3$, R_i then knows for sure that she is the first to investigate and constructive ambiguity collapses. Implementing constructive ambiguity requires a more elaborate type of commitment. In addition to randomizing the order, the sponsor must also commit to draw a random time $t \in (0, 1)$, according to some probability distribution, at which he will present the first member with a report; if presenting the report lasts Δ, at $t + \Delta$ he should (conditionally) present the other member with the report. The distribution of t must be fixed so that when being approached, a committee member draws an asymptotically zero-power test on the hypothesis that she is the first to be approached, when Δ goes to 0.

Finally, let us discuss the robustness of stochastic mechanisms to side communication. While the mechanism exhibited in the first result of Proposition 9 is robust to file transfers (for the now-usual reason that a member who knows she will benefit from the project does not want to jeopardize the other member's assent), it is not robust to soft communication before voting. Indeed it is Pareto optimal and incentive compatible for the members to communicate to each other when they have been asked to rubberstamp. This prevents them from foolishly engaging in collective rubberstamping.

By contrast, it can be argued that the random order mechanism is robust to side communication. It is obviously robust to soft communication before voting as both know that they benefit from the project when they actually end up adopting it. Similarly, file transfers are irrelevant. Furthermore the equilibrium outcome is under some conditions Pareto optimal for the members and remains an equilibrium under soft communication as to the order of investigation .

6 Internal congruence and selective communication in N-member committees

Finally, we turn to N-member committees and generalize some of the insights presented in previous sections. First, in the symmetric case, we propose a more general view on internal congruence and of how much communication is required in order to obtain approval from a N-member committee. Second, we discuss selective communication and the role of the committee's decision rule in the case of nested preferences.

6.1 Congruence in a symmetric N-member committee

The stochastic structure is symmetric; for any numbering $k = 1, 2, ..., N$ of committee members, let $P_k \equiv \Pr\{r_1 = r_2 = ... = r_k = G\}$, $P_1 = p$ and $P_0 = 1$. Note that P_k is non-increasing in k. It follows that for all $k \in \{1, ..., N-1\}$,

$$\Pr\{r_{k+1} = G \mid r_1 = r_2 = ... = r_k = G\} = \frac{P_{k+1}}{P_k}.$$

Affiliation implies that $\frac{P_{k+1}}{P_k}$ is non-decreasing in k.

Under the unanimity rule, we restrict again the analysis to deterministic mechanisms that involve k sequential investigations such that R_j investigates for $j \leq k$ only if all R_i with $i < j$ have investigated and $r_i = G$.[34] External congruence simply refers to the marginal probability p that an arbitrary member benefits from the project. We assume $p < p_0$; otherwise general rubberstamping is obviously the optimal mechanism for the sponsor. Under this assumption, some communication is required for the project to be approved and the following proposition characterizes the optimal number of (sequential) investigations.

Proposition 10. *Consider a symmetric N-committee; if there exists k^*, solving*

$$k^* = \min\{k \in \{1, 2, ..., N\}, \frac{P_{k+1}}{P_k} \geq p_0 \text{ and } P_k \geq p_-\},$$

the optimal deterministic mechanism consists in k^* sequential investigations and $Q = P_{k^*}$. If k^* does not exist, the project cannot be approved.

Proof. A k-investigation mechanism is implementable if the following holds:

- Any non-investigating member is willing to rubberstamp, i.e. $\frac{P_{k+1}}{P_k} \geq p_0$.
- Any investigating member j prefers to investigate after $j-1$ other members, which leads to approval with probability $\Pr\{r_j = r_{j+1} = ... = r_k = G \mid r_1 = r_2 = ... = r_{j-1} = G\} = \frac{P_k}{P_{j-1}}$, rather than to veto the project, i.e. for any $j \leq k$, $\frac{P_k}{P_{j-1}}G - c \geq 0$. These inequalities are equivalent to: $P_k \geq p_-$, since the first investigator (without any signal about the project) is the most reluctant one.

[34] Following the same approach as in Appendix 1, it can be proved that there is no loss in generality in restricting attention to these conditional mechanisms in the class of deterministic mechanisms.

- Any investigating member j prefers to investigate rather than to rubberstamp, which would lead to approval with probability $\Pr\{r_{j+1} = ... = r_k = G \mid r_1 = r_2 = ... = r_{j-1} = G\} = \frac{P_{k-1}}{P_{j-1}}$, i.e. for any $j \leq k$,

$$\frac{P_k}{P_{j-1}}G - c \geq \frac{P_{k-1}}{P_{j-1}}\left[\frac{P_k}{P_{k-1}}G - (1 - \frac{P_k}{P_{k-1}})L\right] \Longleftrightarrow \frac{P_{k-1}}{P_{j-1}}(1 - \frac{P_k}{P_{k-1}})L \geq c.$$

  This set of inequalities is equivalent to: $P_{k-1} - P_k \geq 1 - p_+$.

Suppose k^* exists. Since $P_{k^*-1} \geq P_{k^*} \geq p_-$, it must be that $\frac{P_{k^*}}{P_{k^*-1}} < p_0$. But then,

$$P_{k^*-1} - P_{k^*} > \frac{1-p_0}{p_0}P_{k^*} \geq \frac{1-p_0}{p_0}p_- = 1 - p_+.$$

Therefore, the last implementability condition above is slack and a mechanism with k^* investigations is implementable. Since no mechanism with fewer investigations meets all incentive compatibility constraints, the conclusion follows. ■

The project can be approved after investigation by a subset of k members if having all members in this subset benefit from the project is sufficiently good news for other members to be willing to rubberstamp. But when k grows, it becomes less likely that all members in the group benefit and so, that the project be implemented. Investigating members may then be reluctant to spend the cost of investigation.

Proposition 10 suggests a natural definition of internal congruence in this symmetric N-member setting.

Definition 1. *Fix external congruence p; a committee with stochastic structure $\mathcal{P} = \{P_k\}_{k=1}^N$ with $P_1 = p$ exhibits higher internal congruence than a committee with stochastic structure $\mathcal{P}' = \{P'_k\}_{k=1}^N$ with $P'_1 = p$ if for all $k \in \{1, 2, ..., N-1\}$, $\frac{P_{k+1}}{P_k} \geq \frac{P'_{k+1}}{P'_k}$.*

Higher internal congruence therefore coincides with uniformly smaller hazard rates, as:

$$\Pr\{r_{k+1} = -L \mid r_1 = r_2 = ... = r_k = G\} = \frac{P_k - P_{k+1}}{P_k}.$$

Fixing external congruence ($P_1 = P'_1$), Definition 1 implies that for all k, $P_k \geq P'_k$. Consequently, the sponsor benefits from an increase in internal congruence, for a given external congruence (this result generalizes Corollary 1).

Corollary 4. *Fix external congruence p and consider two stochastic structures $\mathcal{P} = \{P_k\}_{k=1}^N$ and $\mathcal{P}' = \{P'_k\}_{k=1}^N$, with $P_1 = P'_1 = p$, such that $\mathcal{P}$ exhibits more internal congruence than $\mathcal{P}'$; then, under the optimal deterministic mechanism, $Q \geq Q'$.*

Remark. While internal congruence is properly apprehended by the (partial) order associated with hazard rates under the unanimity rule, it could for more general voting rules be captured by resorting to the theory of copulas, which in particular formalizes the dependence (internal congruence) between random variables with given marginal distributions (external congruence): see e.g. Roger Nelsen (2006).

6.2 Selective communication in a N-committee with nested preferences

This subsection focuses on nested preferences. Let $p_i = \Pr\{r_i = G\}$ and suppose committee members are ranked with respect to their degree of external congruence with the sponsor, R_1 being the most supportive and R_N the most radical opponent:

$$0 \leq p_N \leq p_{N-1} \leq ... \leq p_2 \leq p_1 \leq 1.$$

Projects that benefit a given member also benefit all members who are a priori more supportive of (or less opposed to) the project. That is, for any j, k such that $j > k$, $r_j = G \Longrightarrow r_k = G$.

The committee makes its decision according to a K-majority rule, with $K \leq N$ ($K = N$ corresponds to the unanimity rule). So, S needs to build a consensus among (at least) K members of the committee to get the project approved. As before, we focus on the interesting case in which general rubberstamping is not feasible: $p_K < p_0$.

If the *voting pivot* R_K is not a priori willing to rubberstamp ($p_K < p_0$), some information must be passed on inside the committee to obtain approval. Let us define the *informational pivot* R_{i^*} as the member who is the most externally congruent member within the set of committee members R_j who are not champions and whose internal congruence with R_K is sufficiently high to sway the voting pivot's opinion: $\Pr\{r_K = G \mid r_j = G\} \geq p_0$. Formally, in the nested structure:

$$i^* = \min\{j \mid p_0 p_j \leq p_K \text{ and } p_j \leq p_+\}.$$

Clearly, i^* exists and $i^* \leq K$.

When rubberstamping by K members is not feasible, the sponsor has to let at least one committee member investigate. The sponsor has an obvious pecking order if he chooses to let exactly one member investigate: he prefers to approach the most favorable member among those who have the right incentives to investigate and whose endorsement convinces a majority of K members to vote in favor of the project. The informational pivot is then a natural target for selective communication of the report by S. In the unanimity case, it is indeed possible to prove that the optimal *deterministic* mechanism is the informational-pivot mechanism where R_{i^*} investigates and all other members rubberstamp. But it is even possible to characterize the optimal *stochastic* mechanism.

Moreover, the approach extends to the more general setting with a K-majority rule, provided that a "coalition walk-away" option is available to committee members at the voting stage.[35] This option is formally defined as follows: if a mechanism yields negative expected utility ex ante to at least $N - K + 1$ committee members, these members can

[35] Alternatively, we can extend the result that the optimal deterministic mechanism is the informational-pivot mechanism in the case of K-majority under an additional assumption of *transparency*: under transparency, all committee members observe which members have investigated and there is a stage of public communication within the committee before the final vote. Then, it is a dominant strategy for informed members to publicly and truthfully disclose the value of their benefits at the communication stage. Transparency is restrictive as it assumes that investigation by a member who is presented with a report is observable by other members, or else that it is costless.

coordinate, refuse to communicate with S and (rationally) vote against the project. Introducing this option imposes that mechanisms be ex ante rational for any voting coalition.[36]

Proposition 11. (Informational-pivot mechanisms) *Suppose that the information structure is nested, that the committee follows either a K-majority rule with the coalition walk-away option or the unanimity rule, and that $p_K < p_0$;*

- *if $i^* > 1$, $p_- \leq p_{i^*} \leq p_{i^*-1} \leq p_+$ and $p_{i^*} \geq p_0 p_{i^*-1}$, the optimal stochastic mechanism involves a single investigation and $Q = \frac{p_K}{p_0}$, the investigating member is random, equal to either R_{i^*} or R_{i^*-1}, and her benefit from the project is disclosed but not her identity;*
- *if $i^* = 1$ and $p_0 \leq p_1$, the optimal stochastic mechanism involves zero or one investigation by R_1 and similarly relies on constructive ambiguity, yielding $Q = \frac{p_K}{p_0}$.*

The proposition identifies the committee member whom the sponsor should try to convince.[37] The informational pivot R_{i^*}, or the next more favorable member R_{i^*-1}, is asked to investigate and members vote under ambiguity about the investigating member's identity. S only reveals that the investigating member benefits from the project, which generates a strong enough persuasion cascade that member R_K, and a fortiori all more favorable ones, approve the project without further investigation.

In general, the informational pivot differs from the voting pivot R_K. That is, the best strategy of persuasion is usually not to convince the least enthusiastic member. A better approach is to generate a persuasion cascade that reaches R_K. With nested preferences, this persuasion cascade involves at most a single investigation under unanimity as well as under a more general K-majority rule. The choice of the informational pivot reflects

[36]We conjecture that this condition is much stronger than needed.

[37]The proposition is proved in Appendix 4.

the trade-off between internal congruence with R_K and external congruence with S.[38] Selective communication therefore is a key dimension in the sponsor's optimal strategy; irrespective of the rules governing decision-making in the committee, it leads to a strong distinction between the voting and the informational pivot.[39]

7 Conclusion

Many decisions in private and public organizations are made by groups. The economic literature on organizations has devoted surprisingly little attention to how sponsors of ideas or projects should design their strategies to obtain favorable group decisions. This paper has attempted to start filling this gap. Taking a mechanism design approach to communication, it shows that the sponsor should distill information selectively to key members of the group and engineer persuasion cascades in which members who are brought on board sway the opinion of others. The paper unveils the factors, such as the extent of congruence among group members ("internal congruence") and between them and the sponsor ("external congruence"), and the size and governance of the group, that condition the sponsor's ability to maneuver and get his project approved. While external congruence has received much attention in the literature on a single decision-maker, the key role of internal congruence and its beneficial effect for the sponsor (for a given external congruence) is novel.

This work gives content not only to the pro-active role played by sponsors in group decision-making, but also to the notion of "key member", whose endorsement is looked after. A key member turns out to be an "informational pivot", namely the member who is most aligned with the sponsor while having enough credibility within the group to

[38]Note that the existence of a *single* informational pivot relies on the nestedness of the information structure (see the previous subsection).

[39]Interestingly, the Democracy Center's website makes a similar distinction between "decision-influencer" and "decision-maker".

sway the vote of (a qualified majority of) other members; a key member in general is not voting-pivotal and may initially oppose the project.

Even in the bare-bones model of this paper, the study of group persuasion unveils a rich set of insights, confirming some intuitions and invalidating others. On the latter front, we showed that adding veto powers may actually help the sponsor while an increase in external congruence may hurt him; that a more congruent group member may be worse off than an a priori more dissonant member; and that, provided that he can control channels of communication, the sponsor may gain from creating ambiguity as to whether other members really are on board. Finally, an increase in internal congruence always benefits the sponsor.

Needless to say, our work leaves many questions open. Let us just mention three obvious ones:

Multiple sponsors: Sponsors of alternative projects or mere opponents of the existing one may also be endowed with information, and themselves engage in targeted lobbying and the building of persuasion cascades. Developing such a theory of competing advocates faces the serious challenge of building an equilibrium-mechanism-design methodology for studying the pro-active role of the sponsors.

Size and composition of groups: We have taken group composition and size as given (although we performed comparative static exercises on these variables). Although this is perhaps a fine assumption in groups like families or departments, the size and composition of committees, boards and most other groups in work environments are primarily driven by the executive function that they exert. As committees and boards are meant to serve organizational goals rather than lobbyists' interests, it would thus make sense to move one step back and use the results of analyses such as the one proposed here to answer the more normative question of group size and composition.[40]

[40]In the context of a committee model with exogenous signals and no communication among members, Philip Bond and Hülya Eraslan (2006) makes substantial progress in characterizing the optimal majority

Two-tier persuasion cascades: even though their informational superiority and gate-keeping privileges endow them with substantial influence on the final decision, committees in departments, Congress or boards must still defer to a "higher principal" or "ultimate decision-maker" (department; full House or, because of pandering concerns, the public at large; general assembly). Sponsors must then use selective communication and persuasion building at two levels. For example, lobbying manuals discuss both "inside lobbying" and "outside lobbying" (meetings with and provision of analysis to legislators, selective media and grassroot activities).

We leave these and many other fascinating questions related to group persuasion for future research.

rule for members (taken behind the veil of ignorance).

References

Aghion, Philippe, and Jean Tirole. 1997. "Formal and Real Authority in Organizations." *Journal of Political Economy*, 105: 1–29.

David Austen-Smith. 1993a. "Information and Influence: Lobbying for Agendas and Votes." *American Journal of Political Science*, 37(3), 799-833.

David Austen-Smith. 1993b. "Interested Experts and Policy Advice: Multiple Referrals under Open Rule." *Games and Economic Behavior*, 5: 3-44.

Beniers, Klaas J. and Otto H. Swank (2004) "On the Composition of Committees." *Journal of Law, Economics and Organizations*, 20(2): 353–79.

Bond, Philip, and Hülya Evaslan. 2006. "Bargaining Collectively." The Wharton School, University of Pennsylvania, mimeo.

Bull, Jesse and Joel Watson. 2007. "Hard Evidence and Mechanism Design." *Games and Economic Behavior*, 58(1): 75-93.

Cai, Hongbin. 2003. "Costly Participation and Heterogenous Preferences in Informational Committees." UCLA WP N°257.

Crawford, Vincent, and Joel Sobel. 1982. "Strategic Information Transmission." *Econometrica*, 50(6): 1431–1451.

Dewatripont, Mathias and Jean Tirole. 2005. "Modes of Communication." *Journal of Political Economy*, 113(6): 1217–1238.

Farrell, Joe, and Robert Gibbons. 1989. "Cheap Talk with Two Audiences." *American Economic Review*, 79(5): 1214-1223.

Fishman, Michael J., and Kathleen M. Hagerty. 1990. "The Optimal Amount of Discretion to Allow in Disclosure." *Quarterly Journal of Economics*, 105: 427-444.

Gerardi, Dino, and Leeat Yariv. 2006. "Information Acquisition in Committees." Yale University, mimeo.

Gershkov, Alex, and Balazs Szentes. 2004. "Optimal Voting Scheme with Costly Information Acquisition." Hebrew University of Jerusalem, mimeo.

Gilligan, Thomas W., and Keith Krehbiel. 1989. "Asymmetric Information and Legislative Rules with A Heterogenous Committee." *American Journal of Political Science*, 33: 459-490.

Glazer, Jacob, and Ariel Rubinstein. 2004. "On Optimal Rules of Persuasion." *Econometrica,* 72(6): 1715-1736.

Groseclose, Timothy J. and Jim Snyder. 1996. "Buying Supermajorities." *American Political Science Review*, 90: 303–315.

Grossman, Sanford J.. 1981. "The Informational Role of Warranties and Private Disclosure about Product Quality." *Journal of Law and Economics*, 24(3): 461–483.

Grossman, Sanford J., and Oliver Hart. 1980. "Disclosure Laws and Take over Bids." *Journal of Finance*, 35: 323–334.

Krehbiel, Keith. 1990. "Are Congressional Committees composed of Preference Outliers." *American Political Science Review*, 84(1): 149–163.

Krishna, Vijay, and John Morgan. 2001. "A Model of Expertise." *Quarterly Journal of Economics*, 116: 747–775.

Li, Hao. 2001. "A Theory of Conservatism." *Journal of Political Economy,* 109(3): 617-636.

Li, Hao, Sherwin Rosen and Wing Suen. 2001. "Conflicts and Common Interests in Committees." *American Economic Review*, 91(5): 1478–1497.

Lindbeck, Assar, and Jorgen Weibull. 1987. "Balanced-Budget Redistribution as the Outcome of Political Competition." *Public Choice*, 52: 273–297.

Milgrom, Paul R.. 1981. "Good News and Bad News: Representation Theorems and Applications." *Bell Journal of Economics*, 12: 380–391.

Myerson, Roger. 1982. "Optimal Coordination Mechanisms in Generalized Principal-

Agent Problems." *Journal of Mathematical Economics* 10: 67–81.

Nelsen, Roger. 2006. *An Introduction to Copulas.* Springer Verlag, 2nd edition.

Persico, Nicola. 2004. "Committee Design with Endogenous Information." *Review of Economic Studies,* 71(1): 165-191.

Ottaviani, Marco, and Peter Norman Sorensen. 2001. "Information Aggregation in Debate: Who Should Speak First?" *Journal of Public Economics*, 81: 393-421.

Spector, David. 2000. "Rationale Debate and One-Dimensional Conflict." *Quarterly Journal of Economics*, 115(1): 181–200.

Appendix 1: The optimal deterministic mechanism.

We consider a two-member committee with the unanimity rule.

Preliminaries on measurability constraints.

Let $\omega_0 \equiv (-L,-L)$, $\omega_1 = (G,-L)$, $\omega_2 = (-L,G)$, and $\omega_3 = (G,G)$ be the states of nature, with probabilities of occurrence $\pi(\omega_h), h \in \{0,1,2,3\}$. A payoff-relevant outcome consists of a list $I \in 2^{\{1,2\}}$ of investigating members and a decision $d \in \{0,1\}$. Let $X((I,d) \mid \omega)$ denote the probability that the outcome is (I,d) when the state of nature is $\omega \in \Omega$.

Definition 2. (Measurability) *A mechanism is said to be measurable if for any (I,d), $X((I,d) \mid \omega)$ is measurable with respect to the partition of Ω induced by $\{r_i(.), i \in I\}$; that is:*

$$X((I,d) \mid \omega) \neq X((I,d) \mid \omega') \Longrightarrow \exists i \in I, \text{ such that } r_i(\omega) \neq r_i(\omega').$$

Restricting attention to deterministic mechanisms, the definition implies:

Lemma 2. *Consider a deterministic and measurable mechanism $(I(.), d(.))$; if $(I(\omega), d(\omega)) \neq (I(\omega'), d(\omega'))$ for $\omega \neq \omega'$, there exists $i \in I(\omega) \cap I(\omega')$ such that $r_i(\omega) \neq r_i(\omega')$.*

Proof. Consider ω and ω' such that $\omega \neq \omega'$ and $(I(\omega), d(\omega)) \neq (I(\omega'), d(\omega'))$. Suppose that $I(\omega) \cap I(\omega')$ is empty or that any $i \in I(\omega) \cap I(\omega')$ satisfies $r_i(\omega) = r_i(\omega')$. Then, there exists a state of nature ω" such that for any $i \in I(\omega)$, $r_i(\omega") = r_i(\omega)$ and for any $i \in I(\omega')$, $r_i(\omega") = r_i(\omega')$. If $(I(\omega"), d(\omega")) \neq (I(\omega), d(\omega))$, the outcome $(I(\omega), d(\omega))$ has probability 1 in state of nature ω and 0 in state of nature ω". So, measurability implies that there exists $j \in I(\omega)$ such that $r_j(\omega) \neq r_j(\omega")$, which contradicts the definition of ω". So, $(I(\omega"), d(\omega")) = (I(\omega), d(\omega))$. Similarly, $(I(\omega"), d(\omega")) = (I(\omega'), d(\omega'))$; we obtain $(I(\omega), d(\omega)) = (I(\omega'), d(\omega'))$, a contradiction. ■

In a two-member committee, this lemma implies first that if $I(\omega) = \varnothing$ for some state ω, then $I(\omega) = \varnothing$ for all states in Ω, which corresponds to the no-investigation mechanism. Second, if $I(\omega) = \{i\}$ for some ω, then $\{i\} \subset I(\omega)$ for all ω in Ω; so, either i is the sole investigator ever, or there exists some state ω for which both members investigate.

Deterministic mechanisms: no-wasteful-investigation and optimum.

Within the class of measurable deterministic mechanisms, individual rationality implies that if $i \in I(\omega)$ and $r_i(\omega) = -L$, then $d(\omega) = 0$.

If $d(\omega) = 0$ for all ω such that $i \in I(\omega)$, then R_i's expected utility conditional on being asked to investigate is equal to $-c$ and R_i prefers to veto without investigation; the mechanism would not be obedient. This implies if $I(\omega_i) = I(\omega_3) = \{i\}$, then $d(\omega_i) = d(\omega_3) = 1$, and if $I(\omega_3) = \{1, 2\}$, then $d(\omega_3) = 1$. In words, it is not incentive compatible to recommend investigation by R_i if the value of r_i does not impact the final decision; we say that the mechanism implies no-wasteful investigation.

We end up with three types of mechanisms, that lead to $Q > 0$:

- The no-investigation mechanism where $Q = 1$; incentive compatibility requires:

$$u^R(p_i) \geq 0 \Longleftrightarrow p_i \geq p_0.$$

- Two mechanisms in which only R_i investigates while R_j rubberstamps (for $i = 1, 2$ and $j \neq i$); then, $Q = p_i$ and incentive compatibility requires:

$$\begin{aligned} u^I(p_i) &\geq \max\{u^R(p_i), 0\} \Longleftrightarrow p_- \leq p_i \leq p_+ \\ u^R(\hat{p}_j) &\geq 0 \Longleftrightarrow \hat{p}_j \geq p_0. \end{aligned}$$

- Mechanisms with two investigations, so that $Q = P$: two conditional-investigation mechanisms where R_i investigates first and then R_j if $r_i = G$, for $i = 1, 2$, for which incentive constraints are:

$$\begin{aligned} u^I(P) &\geq \max\{p_j u^R(\hat{p}_i), 0\} \Longleftrightarrow P \geq p_- \text{ and } p_j - P \geq 1 - p_+ \\ u^I(\hat{p}_j) &\geq \max\{u^R(\hat{p}_j), 0\} \Longleftrightarrow p_- \leq \hat{p}_j \leq p_+, \end{aligned}$$

and one mechanism where both investigate (e.g. simultaneously), for which the incentive constraints are, for all i and $j \neq i$:

$$u^I(P) \geq \max\{p_i u^R(\hat{p}_j), 0\} \Longleftrightarrow P \geq p_- \text{ and } p_i - P \geq 1 - p_+.$$

Note that $P \geq p_-$ implies that $\hat{p}_j \geq p_-$ and

$$p_i - P \geq 1 - p_+ \Longleftrightarrow \hat{p}_j \leq 1 - \frac{1-p_+}{p_i} = p_+ - (1-p_+)\frac{(1-p_i)}{p_i} < p_+.$$

Therefore, if the simultaneous investigation mechanism is incentive compatible, so are both conditional investigation mechanisms; since all yield the same probability of approval Q, one can restrict attention to the class described in the text. Propositions 2 and 3 follow straightforwardly from these incentive constraints.

Appendix 2: Proof of Proposition 7.

Assume first, that $p_2 < p_0$, $p_- \leq p_1 \leq p_+$ and $\hat{p}_2 \geq p_0$, so that the optimal mechanism is to let R_1 investigate and R_2 rubberstamp.[41] Consider the following game form Γ:

- S presents R_1 with a report; R_1 investigates (or not) and reports r_1 publicly;
- R_1 may communicate with R_2, that is, she may send R_2 a message or transfer her the file, in which case R_2 may investigate;
- R_1 and R_2 can exchange information;

[41] The case where R_2 investigates and R_1 rubberstamps is similar.

- finally members vote on the project.

Game form Γ has an equilibrium that implements the optimal mechanism and in which no side communication takes place on the equilibrium path: R_1 investigates in the first stage, does not transfer the file, and reports truthfully; R_2 approves the project, R_2 always believes that R_1 has investigated, regardless of whether R_1 hands over the file, and R_2 never investigates in case R_1 transfers the file (R_2's beliefs over the value of r_1 in case of file transfer are irrelevant). R_1 is a de facto dictator.

Let us now assume that $p_- \leq P < p_1 \leq p_+$ and for all $i = 1, 2$, $\hat{p}_i < p_0$, in which case the optimal mechanism is to have both members investigate, with say R_2 investigating conditionally on $r_1 = G$. Consider the following game form Γ':

- S presents R_1 with a report; R_1 investigates (or not), and reports r_1 publicly;
- R_1 may send R_2 a message or transfer her the file;
- S presents R_2 with a report if R_1 has announced that $r_1 = G$;
- R_2 and R_1 may exchange information;
- finally members vote on the project.

Game form Γ' has an equilibrium that implements the optimal mechanism and in which no side communication takes place on the equilibrium path: R_1 investigates and reports truthfully; if R_1 reports she favors the project, R_2 investigates; if R_1 reports that $r_1 = -L$, R_2 does not investigate even if R_1 hands over the file; at the voting stage, both vote according to their benefit.

Appendix 3: Stochastic mechanisms

General approach and no-wasteful-investigation mechanisms.

Let $\omega_0 \equiv (-L, -L)$, $\omega_1 = (G, -L)$, $\omega_2 = (-L, G)$ and $\omega_3 = (G, G)$. Let us introduce the following notation: for each state ω_h, with probability

- $\gamma_h \geq 0$, no-one investigates and the project is implemented;
- $\eta_h \geq 0$, no-one investigates and the project is NOT implemented;
- $\theta_h^i \geq 0$, only R_i investigates and the project is implemented;
- $\mu_h^i \geq 0$, only R_i investigates and the project is NOT implemented;
- $\xi_h \geq 0$, both investigate and the project is implemented;
- $\nu_h \geq 0$, both investigate and the project is NOT implemented.

Measurability. γ_h and η_h must be constant across ω, equal to γ and η; moreover, θ_h^i and μ_h^i can only depend on r_i and we define: $\theta_i^i = \theta_3^i \equiv \theta_i$, $\mu_i^i = \mu_3^i \equiv \mu_i$ and $\mu_j^i = \mu_0^i = \bar{\mu}_i$.

Interim individual rationality. A member vetoes the project when she knows she loses from it: $\theta_j^i = \theta_0^i = 0$, $\xi_0 = \xi_1 = \xi_2 = 0$.

Feasibility constraints. For all h, probabilities add up to one. Using previous results, feasibility constraints for $h = 0, 1, 2$ and 3 can be written as:

$$\begin{aligned}
\gamma + \eta + \bar{\mu}_1 + \bar{\mu}_2 + \nu_0 &= 1, \\
\gamma + \eta + \theta_1 + \mu_1 + \bar{\mu}_2 + \nu_1 &= 1, \\
\gamma + \eta + \theta_2 + \bar{\mu}_1 + \mu_2 + \nu_2 &= 1, \\
\gamma + \eta + \theta_1 + \theta_2 + \mu_1 + \mu_2 + \xi_3 + \nu_3 &= 1.
\end{aligned}$$

Incentive constraints. When R_i is supposed to investigate, not investigating and playing as if $r_i = G$ must be an unprofitable deviation. By pretending that $r_i = G$, R_i induces a distribution over outcomes corresponding to $\omega_{g_i(h)}$ in state of nature ω_h where

$g_i(0) = g_i(i) = i$ and $g_i(j) = g_i(3) = 3$, so that the incentive constraints can be written as: for all i,

$$\sum_{h=0}^{3} \pi(\omega_h) \left[\left(\theta_h^i + \xi_h\right) r_i(\omega_h) - \left(\theta_h^i + \mu_h^i + \xi_h + \nu_h\right) c \right] \geq \sum_{h=0}^{3} \pi(\omega_h) \left(\theta_{g_i(h)}^i + \xi_{g_i(h)}\right) r_i(\omega_h). \tag{1}$$

When R_i is supposed to investigate, not investigating and playing as if $r_i = -L$ must be an unprofitable deviation, too. Pretending that $r_i = -L$ amounts to vetoing the project, so that this incentive constraint can be simply written as: for all i,

$$\sum_{h=0}^{3} \pi(\omega_h) \left[\left(\theta_h^i + \xi_h\right) r_i(\omega_h) - \left(\theta_h^i + \mu_h^i + \xi_h + \nu_h\right) c \right] \geq 0. \tag{2}$$

Finally, when the project is supposed to be implemented without R_i's investigation, R_i must not prefer vetoing the project (which can also be viewed as ex ante individual rationality): for $i = 1, 2$ and $j \neq i$,

$$\sum_{h=0}^{3} \pi(\omega_h) \left[\gamma_h + \theta_h^j\right] r_i(\omega_h) \geq 0. \tag{3}$$

Sponsor's objectives. The sponsor maximizes the probability of approval Q, given by:

$$Q = \sum_{h=0}^{3} \pi(\omega_h) \left[\gamma_h + \theta_h^1 + \theta_h^2 + \xi_h\right],$$

under all the constraints presented above.

We first state a central lemma that allows us to restrict attention to the class of *no-wasteful-investigation mechanisms*. The proof is available in a supplementary document on the journal webpage.

Lemma 3. (No wasteful investigation) *There is no loss of generality in looking for the optimal mechanism within the class of no-wasteful-investigation mechanisms, that is the class of mechanisms described by an element of the 5-simplex $(\gamma, \theta_1, \theta_2, \lambda_1, \lambda_2)$, with $\lambda_i = \bar{\mu}_i - \theta_i$, such that:*

- *with probability γ, both R_1 and R_2 rubberstamp the project;*
- *with probability θ_i, R_i investigates, and R_j rubberstamps;*
- *with probability λ_i, R_i investigates; R_j investigates if R_i benefits from the project;*
- *with probability $1-\gamma-\Sigma_i\theta_i-\Sigma_i\lambda_i$, there is no investigation and the status quo prevails.*

Optimal stochastic mechanisms in the symmetric setting.

Focusing now on the symmetric setting, the next proposition completely characterizes the optimal (stochastic) mechanism; only part of it is used in Proposition 9, and the proof is available in the supplementary document on the journal webpage.

Proposition 12. *The following (symmetric) mechanism is optimal:*

- *If $p \geq p_0$, members are asked to rubberstamp ($\gamma = 1$) and $Q = 1$;*
- *If $p_- \leq p < p_0 < \hat{p}$, each member is asked to investigate with some probability smaller than 1/2 and to rubberstamp otherwise: $\lambda_i = 0$, $\theta_i = \frac{1-\gamma}{2} = \theta^*(p,\hat{p}) \equiv \frac{p_0-p}{2(p_0-p)+p(\hat{p}-p_0)}$, leading to $Q = 1-2\theta^*(p,\hat{p})(1-p) > p$;*
- *If $p < p_- < p_0 \leq \hat{p}$, the optimal mechanism has full support with $\theta_i = \frac{1-\gamma}{2} = \theta^{**} \equiv \left(\frac{2}{\frac{(2P-(1+p)p_-)}{p_- - p}} + \frac{1}{\theta^*}\right)^{-1} > 0$, $\lambda_i = \frac{(p_- - p)}{(2P-(1+p)p_-)}\theta^{**} > 0$, and $Q > 0$ provided:*

$$\hat{p} \geq \max\left\{\frac{(1+p)p_-}{2p}, \frac{(1+p)}{2} + \frac{(1-p)(p_- - p)}{2p(p_+ - p_-)}\right\}; \tag{4}$$

  if (4) does not hold, the project cannot be implemented and $Q = 0$.
- *If $\hat{p} < p_0$, the optimum is $\lambda_i = \frac{1}{2}$ provided $P \geq p_-$, otherwise $Q = 0$.*

In a left neighborhood of p_-, both terms in the maximum in (4) tends to $\frac{1+p_-}{2} < 1$; therefore, the domain for which $Q > 0$ is not empty. Moreover, $\frac{1+p_-}{2} < p_0$ is a sufficient condition that guarantees that for p close enough to p_-, there exists $\hat{p}$ close to and above p_0 that satisfies the condition. It is routine calculation to prove that the random order mechanism that corresponds to $\lambda_i = \frac{1}{2}$ and that is discussed in the text is implementable under this sufficient condition.

Appendix 4: Proof of Proposition 11.

In the nested case with N members, there are $N+1$ states of nature: ω_0 by convention denotes the state in which no-one benefits from the project and, for $h \in \{1, 2, ...N\}$, ω_h denotes the state in which all $j \leq h$ benefit from the project and all $j > h$ suffer from it. The probability of state ω_h for $h \in \{1, 2, ...N\}$ is $\pi(\omega_h) = p_h - p_{h+1}$, with $p_{N+1} = 0$, and the probability of state ω_0 is $\pi(\omega_0) = 1 - p_1$.

Consider a (stochastic) mechanism and let $x(\omega_h)$ denote the probability that the project is approved in state ω_h, for $h = 0, 1, ...N$. Let U_i denote member R_i's expected benefit under this mechanism, not taking into account the cost of possible investigation:

$$U_i \equiv \sum_{h=0}^{N} \pi(\omega_h)x(\omega_h)r_i(\omega_h) = G\sum_{h=i}^{N} \pi(\omega_h)x(\omega_h) - L\sum_{h=0}^{i-1} \pi(\omega_h)x(\omega_h).$$

Suppose that $U_K < 0$. Then, for any $i > K$, $U_i < 0$ so that all members in the coalition $\{R_i, i = K, K+1, ..., N\}$ lose from the project ex ante. Under unanimity, R_N then simply vetoes the project. Under K-majority, the coalition blocks the project by voting against it, under the coalition walk-away option. In both cases, the mechanism induces rejection of the project. Therefore, any mechanism that yields ex ante strictly positive probability of approval must necessarily satisfy: $U_K \geq 0$.

It is therefore possible to find an upper bound on the ex ante probability of approval of the project in the optimal mechanism:

$$Q \leq \bar{Q} \equiv \quad \begin{array}{l} \max_{x(.)} \sum_{h=0}^{N} \pi(\omega_h) x(\omega_h) \\ \text{s.t. } U_K \geq 0. \end{array}$$

In this program, it is immediate that $x(\omega_h) = 1$ for all $h = K, K+1, ...N$ and that the constraint $U_K \geq 0$ is binding. Therefore,

$$\bar{Q} \leq \sum_{h=0}^{K-1} \pi(\omega_h) x(\omega_h) + p_K = \frac{G}{L} p_K + p_K = \frac{p_K}{p_0}.$$

Consider the following stochastic mechanism: S lets R_{i^*} investigate with probability z and R_{i^*-1} with probability $1-z$, with z such that $\frac{p_K}{p_0} = z p_{i^*} + (1-z) p_{i^*-1}$; S discloses the outcome of investigation but not the identity of the investigating member before the vote.

Suppose first that $i^* > 1$. By definition, $p_{i^*} \leq \frac{p_K}{p_0} < p_{i^*-1}$ so that z is uniquely defined. If $p_- \leq p_{i^*} \leq p_{i^*-1} < p_+$, R_{i^*} and R_{i^*-1} are actually willing to investigate when asked to. Conditional on the investigator benefiting from the project, R_j's posterior probability of benefiting from it is $\min\left\{\frac{p_j}{z p_{i^*} + (1-z) p_{i^*-1}}; 1\right\} = \min\left\{p_j \frac{p_0}{p_K}; 1\right\}$ for $j \neq i^*$ and $j \neq i^* - 1$; it is $\frac{p_{i^*}}{p_{i^*-1}} \geq p_0$ for R_{i^*} if R_{i^*-1} is the investigating member, and it is equal to 1 for R_{i^*-1} if R_{i^*} is the investigator. So, each member R_j, $j = 1, 2, ...K$, is willing to vote in favor of the project. This mechanism is incentive compatible and it generates a maximal ex ante probability of approval: $Q = z p_{i^*} + (1-z) p_{i^*-1} = \frac{p_K}{p_0} = \bar{Q}$; it is therefore optimal.

Suppose then that $i^* = 1$. Then p_{i^*-1} should be replaced by 1 and the mechanism is to be interpreted as follows: R_1 is asked to investigate with probability z and no one is asked to investigate with probability $1-z$, where z is uniquely defined by: $\frac{p_K}{p_0} = z p_{i^*} + (1-z)$, given that $p_{i^*} = p_1 \leq \frac{p_K}{p_0} < 1$; then, before the vote, S discloses the outcome of R_1's investigation only if she loses from the project. The analysis is then similar.

Supplementary material

Proof of Lemma 3

Referring to Appendix 4 and using feasibility constraints, note that a mechanism is alternatively given by $(\gamma, \theta_i, \mu_i, \bar{\mu}_i, \nu_0, \nu_3)$ and:

$$\begin{aligned} \eta &= 1 - \gamma - \bar{\mu}_1 - \bar{\mu}_2 - \nu_0 \geq 0, & (5) \\ \nu_1 &= \nu_0 - \theta_1 - \mu_1 + \bar{\mu}_1 \geq 0, & (6) \\ \nu_2 &= \nu_0 - \theta_2 - \mu_2 + \bar{\mu}_2 \geq 0, & (7) \\ \xi_3 &= \nu_0 - \theta_1 - \theta_2 - \mu_1 - \mu_2 + \bar{\mu}_1 + \bar{\mu}_2 - \nu_3 \geq 0. & (8) \end{aligned}$$

Using measurability and individual rationality, the expected probability that the project is implemented is given by:

$$Q = \gamma + p_1\theta_1 + p_2\theta_2 + P\xi_3.$$

Plugging in the value of ξ_3 from (8), we find:

$$Q = \gamma + (p_1 - P)\theta_1 + (p_2 - P)\theta_2 + P\nu_0 - P\mu_1 - P\mu_2 + P\bar{\mu}_1 + P\bar{\mu}_2 - P\nu_3. \quad (9)$$

We now write incentive constraints using measurability, individual rationality and feasibility constraints. (1) can be written as:

$$\begin{aligned} & \theta_i p_i(G - c) - (\mu_i p_i + \bar{\mu}_i(1 - p_i))c + \xi_3 P(G - c) \\ & -(\nu_0(1 + P - p_1 - p_2) + \nu_1(p_1 - P) + \nu_2(p_2 - P) + \nu_3 P)c \\ \geq \ & \theta_i p_i G - \theta_i(1 - p_i)L + \xi_3 PG - \xi_3(p_j - P)L. \end{aligned} \quad (10)$$

Given previous results and using the expressions for ξ_3 and ν_i, this constraint can be written as: for $i = 1, 2$ and $j \neq i$,

$$\begin{aligned} & \theta_i(1 - p_i)L + (\nu_0 - \theta_1 - \theta_2 - \mu_1 - \mu_2 + \bar{\mu}_1 + \bar{\mu}_2 - \nu_3)(p_j - P)L \\ \geq \ & c[\nu_0 + \bar{\mu}_i + p_j(\bar{\mu}_j - \theta_j - \mu_j)]. \end{aligned} \quad (11)$$

(2) can be written as:

$$\begin{aligned} 0 &\leq \theta_i p_i(G-c) - (\mu_i p_i + \bar{\mu}_i(1-p_i))c + \xi_3 P(G-c) \\ &\quad -(\nu_0(1+P-p_1-p_2) + \nu_1(p_1-P) + \nu_2(p_2-P) + \nu_3 P)c. \end{aligned} \tag{12}$$

Using the same manipulations as above, the latter inequality becomes: for $i = 1, 2$ and $j \neq i$,

$$\begin{aligned} &\theta_i p_i G + (\nu_0 - \theta_1 - \theta_2 - \mu_1 - \mu_2 + \bar{\mu}_1 + \bar{\mu}_2 - \nu_3)\, PG \\ \geq\ & c[\nu_0 + \bar{\mu}_i + p_j\,(\bar{\mu}_j - \theta_j - \mu_j)]. \end{aligned} \tag{13}$$

Finally, we write (3) as follows: for $i = 1, 2$ and $j \neq i$,

$$\gamma u^R(p_i) + \theta_j p_j u^R(\hat{p}_i) \geq 0. \tag{14}$$

The program is to maximize (9) under the constraints (5)-(6)-(7)-(8), (11), (13) and (14).

It is first immediate that $\nu_3 = 0$ at the optimum. With A_i, B_i and C_i the multipliers associated with constraints (11), (13) and (14), and D, E_1, E_2 and F the multipliers associated with (5)-(6)-(7)-(8), one can compute the derivatives of the Lagrangian with respect to $(\mu_1, \mu_2, \bar{\mu}_1, \bar{\mu}_2, \nu_0)$ (omitting the constraints that each of these must lie within $[0, 1]$):

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \mu_i} &= -P - A_i(p_j - P)L - A_j(p_i - P)L + A_j c p_i \\ &\quad - B_i PG - B_j PG + B_j c p_i - (D + E_i) \end{aligned}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{\mu}_i} &= P + A_i(p_j - P)L + A_j(p_i - P)L - A_j c p_i \\ &\quad + B_i PG + B_j PG - B_j c p_i - c(A_i + B_i) + (D + E_i) - F \end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial \nu_0} = P + A_1(p_2 - P)L - cA_1 + A_2(p_1 - P)L - cA_2$$
$$+B_1PG - cB_1 + B_2PG - cB_2 + (D + E_1 + E_2) - F$$

Note that if $(A_j + B_j) = 0$, then $\frac{\partial \mathcal{L}}{\partial \mu_i} < 0$ and so, $\mu_i = 0$.

From the derivatives of the Lagrangian, one can derive useful relationships:

$$\frac{\partial \mathcal{L}}{\partial \mu_i} + \frac{\partial \mathcal{L}}{\partial \bar{\mu}_i} = -F - c(A_i + B_i) \leq 0, \tag{15}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{\mu}_i} + E_j = \frac{\partial \mathcal{L}}{\partial \nu_0} + c(1 - p_i)(A_j + B_j). \tag{16}$$

Claim 1. *The optimum cannot be such that $\nu_0 > 0$, $\mu_1 > 0$ and $\mu_2 > 0$.*

Proof. If $\nu_0 > 0$, $\mu_i > 0$ for $i = 1, 2$, it follows that $\frac{\partial \mathcal{L}}{\partial \nu_0} \geq 0$, $\frac{\partial \mathcal{L}}{\partial \mu_i} \geq 0$. A_1, A_2, B_1 and B_2 must be strictly positive so that $\frac{\partial \mathcal{L}}{\partial \mu_i} + \frac{\partial \mathcal{L}}{\partial \bar{\mu}_i} < 0$. Hence, $\frac{\partial \mathcal{L}}{\partial \bar{\mu}_i} < 0$ and $\bar{\mu}_i = 0$ from (15).

Moreover, (16) implies that $E_j > 0$, which implies $\nu_j = 0$ and so, summing (6) and (7), $\xi_3 = -\nu_0 < 0$, a contradiction. ■

Claim 2. *The optimum is without loss of generality such that for $i = 1, 2$, $\mu_i \bar{\mu}_i = 0$.*

Proof. Fix $\bar{\mu}_i - \mu_i$. A simple examination of Q and of all the constraints reveals that decreasing $\bar{\mu}_i$ only relaxes (5) and (11)-(13). Therefore, if $\bar{\mu}_i - \mu_i \geq 0$, the optimum can be chosen so that $\mu_i = 0$ and if $\bar{\mu}_i - \mu_i \leq 0$, the optimum can be chosen so that $\bar{\mu}_i = 0$. ■

Therefore, we will now focus on optima that satisfy Claim 2.

Claim 3. *An optimum satisfying Claim 2 cannot be such that $\nu_0 = 0$ and $\mu_i > 0$ for some i.*

Proof. Suppose that $\nu_0 = 0$ and there exists i such that $\mu_i > 0$. From Claim 2, the optimum is such that $\bar{\mu}_i = 0$. Then, the constraint that $\nu_i \geq 0$ is violated. ■

Claim 4. *An optimum satisfying Claim 2 cannot be such that $\nu_0 > 0$, $\mu_1 > 0$ and $\mu_2 = 0$.*

Proof. Suppose $\nu_0 > 0$ and $\mu_1 > 0 = \mu_2 = \bar{\mu}_1$. It must be that $\frac{\partial \mathcal{L}}{\partial \nu_0} \geq 0$, $\frac{\partial \mathcal{L}}{\partial \mu_1} \geq 0$, $\frac{\partial \mathcal{L}}{\partial \bar{\mu}_1} \leq 0$ and $A_2 + B_2 > 0$. As in the proof of Claim 1, it follows that $E_2 > 0$, which implies that $\nu_2 = 0$. So, we have:

$$\begin{aligned} 0 &\leq \xi_3 = \nu_0 - \theta_1 - \theta_2 - \mu_1 - \mu_2 + \bar{\mu}_1 + \bar{\mu}_2 \\ &= \nu_2 - \theta_1 - \mu_1 + \bar{\mu}_1 = -\theta_1 - \mu_1 < 0, \end{aligned}$$

a contradiction. ■

Claim 5. *If $\mu_1 = \mu_2 = 0$, the optimum is without loss of generality such that $\nu_0 = 0$.*

Proof. Suppose $\mu_1 = \mu_2 = 0 < \nu_0$, then $\frac{\partial \mathcal{L}}{\partial \nu_0} \geq 0$.

Note first that if there exists i such that $\frac{\partial \mathcal{L}}{\partial \bar{\mu}_i} > 0$, then $\bar{\mu}_i = 1$ and then $\eta < 0$, a contradiction. So, for $i = 1, 2$, $\frac{\partial \mathcal{L}}{\partial \bar{\mu}_i} \leq 0$.

Note also that if $E_i > 0$, then $\nu_i = 0$ so that $\nu_j = \xi_3 + \nu_0 > 0$ and therefore $E_i = 0$. With the previous remark, using (16), this implies that $\frac{\partial \mathcal{L}}{\partial \nu_0} = 0$ and for some i, $A_i = B_i = 0$.

Suppose $A_1 = B_1 = 0 < A_2 + B_2$ and $E_2 > 0 = E_1$. Consider the simplified program where the constraints corresponding to A_1, B_1 and E_1 are omitted. In this program, ν_0 and $\bar{\mu}_2$ enter only through $(\nu_0 + \bar{\mu}_2)$ within $(0, 1]$; and so, there is no loss of generality in looking for the optimum with $\nu_0 = 0$.

The last possibility is such that $A_i = B_i = E_i = 0$ for $i = 1, 2$. Then, the simplified program where all corresponding constraints are omitted only depends upon $\nu_0 + \bar{\mu}_1 + \bar{\mu}_2$, and again, one can set $\nu_0 = 0$ without loss of generality. ■

To summarize, the optimal mechanism is without loss of generality such that $\nu_0 = \mu_1 = \mu_2 = 0$. It is fully characterized by $(\gamma, \theta_1, \theta_2, \bar{\mu}_1, \bar{\mu}_2)$, or, defining $\lambda_i = \bar{\mu}_i - \theta_i$, as in Lemma 3. This completes the proof of Lemma 3.

Proof of Proposition 12

In the symmetric setting, feasibility requires: $\gamma+\theta_1+\theta_2+\lambda_1+\lambda_2 = 1$. Incentive constraints (11), (13) and (14) now become:

$$\theta_i(1-p)L + (\lambda_1+\lambda_2)(p-P)L \geq c[\theta_i+\lambda_i+\lambda_j p], \tag{17}$$

$$\theta_i pG + (\lambda_1+\lambda_2)PG \geq c[\theta_i+\lambda_i+\lambda_j p], \tag{18}$$

$$\gamma(pG-(1-p)L) + \theta_i p(\hat{p}G-(1-\hat{p})L) \geq 0. \tag{19}$$

The sponsor maximizes $Q = \gamma + (\theta_1+\theta_2)p + (\lambda_1+\lambda_2)P$ subject to these constraints.

If $(\gamma, \theta_1, \theta_2, \lambda_1, \lambda_2)$ is an optimal mechanism, $(\gamma, \frac{\theta_1+\theta_2}{2}, \frac{\theta_1+\theta_2}{2}, \frac{\lambda_1+\lambda_2}{2}, \frac{\lambda_1+\lambda_2}{2})$ is a symmetric mechanism that satisfies the feasibility constraints, the incentive constraints, obtained by summing over $i=1$ and 2 the constraints (17), (18) and (19), and that yields the same Q. We will therefore focus wlog on symmetric mechanisms.

For a symmetric mechanism $(\gamma, \theta, \lambda)$, feasibility requires $\gamma+2\theta+2\lambda = 1$ and incentive constraints become:

$$\theta(p_+-p) + \lambda\{p_+(1+p)-(1-p)-2P\} \geq 0, \tag{20}$$

$$\theta(p-p_-) + \lambda(2P-(1+p)p_-) \geq 0, \tag{21}$$

$$\gamma(p-p_0) + \theta p(\hat{p}-p_0) \geq 0. \tag{22}$$

The sponsor maximizes $Q = \gamma + 2\theta p + 2\lambda P$ subject to these constraints. Since for $p \geq p_0$, the unconstrained optimum ($\gamma = 1$) is implementable, we focus on the case where $p < p_0$.

First case: $\hat{p} < p_0$. (22) implies that $\gamma = \theta = 0$. The situation is the symmetric stochastic version of the deterministic situation in which both committee members investigate sequentially. If $P \geq p_-$, the optimum is $\lambda = \frac{1}{2}$ and $Q = P$, while $Q = 0$ if $P < p_-$.

Second case: $p_- \le p < p_0 < \hat{p}$. Consider the relaxed program where (20) and (21) are omitted:

$$\begin{aligned} & \max_{\theta,\lambda \ge 0} \{-2\theta(1-p) - 2\lambda(1-P)\} \\ \text{s.t. } 0 \ & \le \ 1 - 2\theta - 2\lambda \\ 1 \ & \le \ 2\lambda + \theta \frac{[2(p_0-p) + p(\hat{p}-p_0)]}{p_0 - p}. \end{aligned}$$

It is immediate that the solution is $\lambda = 0$ and $\theta = \frac{1-\gamma}{2} = \theta^* \equiv \frac{p_0-p}{2(p_0-p)+p(\hat{p}-p_0)}$. Moreover, since $p - p_- \ge 0$ and $p_+ - p > 0$, this solution satisfies also (20) and (21). Hence, it is the optimal mechanism in this range of parameters.

Third case: $p < p_- < p_0 \le \hat{p}$. As in the previous case, we use variables $(\theta, \lambda) \ge 0$ such that $\gamma = 1 - 2\theta - 2\lambda \ge 0$. The constraints can be written as follows:

$$\lambda \frac{[(1-p) + 2P - p_+(1+p)]}{p_+ - p} \equiv X\lambda \le \theta, \tag{23}$$

$$\theta \le \lambda \frac{(2P - (1+p)p_-)}{p_- - p} \equiv Y\lambda, \tag{24}$$

$$1 \le 2\lambda + \frac{\theta}{\theta^*}. \tag{25}$$

Note first that if $Y \le 0$, then $\theta = \lambda = 0$ necessarily and the set of constraints is empty. Hence $Q = 0$. Suppose now that $Y > 0$. Again, if $X > Y$, then the set of constraints is empty and $Q = 0$. The project can then be implemented with positive probability only if $Y \ge X$ and $Y > 0$. In this last case, consider the relaxed program where the sole constraints are $\theta \ge 0$, $\lambda \ge 0$, (24) and (25):

$$\begin{aligned} & \max_{\theta,\lambda \ge 0} \{-2\theta(1-p) - 2\lambda(1-P)\} \\ \text{s.t. } \theta \ & \le \ Y\lambda \\ 1 \ & \le \ 2\lambda + \frac{\theta}{\theta^*}. \end{aligned}$$

The constraint (25) must necessarily be binding, since otherwise the optimum would be $\theta = \lambda = 0$ which would violate (25). The constraint (24) must also be binding, since otherwise, the optimum would be $\lambda = 0$, $\theta = \theta^*$ and this would violate (24). Hence, the solution is: $\theta = Y\lambda = \theta^{**} \equiv \left(\frac{2}{Y} + \frac{1}{\theta^*}\right)^{-1}$. Moreover, since (24) is binding and $Y \geq X$, (23) is satisfied. For these values,

$$\gamma = 1 - 2\theta^{**}(1 + \frac{1}{Y}) = \frac{\frac{1}{Y} + \frac{1}{\theta^*} - 1}{\frac{2}{Y} + \frac{1}{\theta^*}};$$

since $\theta^* \leq \frac{1}{2}$ and $Y > 0$, $\gamma > 0$.

Therefore, in the range $p < p_- < p_0 \leq \hat{p}$, there exists a stochastic mechanism that yields a positive probability Q if and only if:

$$\begin{aligned} 2P - (1+p)p_- &> 0 \text{ and} \\ \frac{2P - (1+p)p_-}{p_- - p} &\geq \frac{(1-p) + 2P - p_+(1+p)}{p_+ - p} \end{aligned}$$

that is, if and only if:

$$\begin{aligned} 2P &> (1+p)p_- \text{ and} \\ 2P &\geq (1+p)p + (1-p)\frac{p_- - p}{p_+ - p_-}. \end{aligned}$$

The condition for $Q > 0$ is therefore:

$$\hat{p} \geq \max\{\frac{(1+p)p_-}{2p}, \frac{(1+p)}{2} + \frac{(1-p)(p_- - p)}{2p(p_+ - p_-)}\}.$$

In a left neighborhood of p_-, both terms in the supremum tends to $\frac{1+p_-}{2} < 1$; therefore, the domain for which $Q > 0$ is not empty.