

# EFFICIENT PATENT POOLS\*

Josh Lerner<sup>†</sup>      Jean Tirole<sup>‡</sup>

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## Abstract

The paper builds a tractable model of a patent pool, an agreement among patent owners to license a set of their patents to one another or to third parties. It first provides a necessary and sufficient condition for a patent pool to enhance welfare. It shows that requiring pool members to be able to independently license patents matters if and only if the pool is otherwise welfare reducing, a property that allows the antitrust authorities to use this requirement to screen out unattractive pools.

The paper then undertakes a number of extensions: cases where patents differ in importance, where asymmetric blocking patterns exist, and where licensors are also licensees. We also undertake some initial explorations of the impact of pools on innovation. We conclude by showing that the analysis has broader applicability than pools, as it is also relevant to a number of co-marketing arrangements.

*Keywords:* Intellectual property, open and closed pools, essential patents, independent licensing.

*JEL numbers:* K11, L41, M2.

## 1 Introduction

A patent pool is an agreement among patent owners to license a set of their patents to one another or to third parties. Patent pools have played an important role in industry since the 1856 sewing machine pool, although their number and importance considerably subsided in a hostile antitrust environment after World War II. Patent pools have been making a comeback in the last few years, and many believe that pools are bound to be as important or more important in the new economy as they were in traditional sectors. Innovations in computer hardware,

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<sup>†</sup>Harvard University and NBER.

<sup>‡</sup>IDEI and GREMAQ (UMR 5604 CNRS), Toulouse, CERAS (URA 2036 CNRS), Paris, and MIT.

software, and biotechnology often build on a number of other innovations owned by a diverse set of owners and as a result “patent thicket” problems - overlapping patent claims that preclude the adoption of new technologies - can be severe.<sup>1</sup>

There is now widespread agreement among policymakers and economists that patent pools may benefit both intellectual property owners and consumers, provided that the pools include patents that are complementary or blocking. It is perhaps puzzling that so few pools have been formed in the recent past despite the favorable treatment the US Department of Justice (DOJ) and the US Federal Trade Commission (FTC) have given to pools. At least in part, the reluctance to form pools may be due to the ambiguities surrounding the manner in which proposed pools will be evaluated.

This paper analyzes the strategic incentives to form a pool in the presence of innovations that either compete with or are complementary to the patents in the pool. A second focus of our analysis is the process through which competition authorities examine patent pools. A recent doctrine is that only “essential patents” be included in pools. In a number of cases, an independent expert has been assigned the role of ensuring that only essential inventions are added to the pool and removing patents that are no longer essential in the future. In the context of a pool defining a DVD-ROM and video standard, Assistant Attorney General Joel Klein defined essentiality in the following way:<sup>2</sup>

*“Essential patents, by definition, have no substitutes; one needs licenses to each of them in order to comply with the standard.”*

One may wonder whether such requirements are too strict or too lenient.

Another feature of interest in the recent pools approved by American antitrust authorities is that patent owners retain a right to license their invention separately from the pool. More generally, 44% of the 63 pools included in the sample in Lerner et al. (2002) allow pool members to

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<sup>1</sup>See Carlson (1999) and Gilbert (2002) for excellent historical perspectives on patent pools.

<sup>2</sup>Letter of Joel I. Klein to R. Carey Ramos, Esq., June 10, 1999, <http://www.usdoj.gov:80/atr/public/busreview/2485.wpd>.

offer independent licenses outside the pool. When is the independent-licensing provision beneficial to the members of the pool? Is it a (presumably cheap) way of accommodating the concerns of antitrust authorities?

Shapiro (2001) uses Cournot (1838)'s analysis to point out that patent pools raise welfare when patents are perfect complements and harm welfare when they are perfect substitutes. While this is a useful first step in the antitrust analysis of patent pools, patents are rarely perfect complements or perfect substitutes. Indeed, antitrust authorities often wonder whether they are complements or substitutes. Furthermore, patents that are currently complements may in the future become substitutes as they enable new products that compete on the downstream markets. Last, many interesting policy issues do not arise in a world of perfect complementarity. For example, with perfect complements, all patents would necessarily be equally important; the provision of independent licenses by patent owners would be meaningless; and a pool would always reduce price, encourage innovation, and reduce the intensity of patent holders' foreclosure of downstream product market rivals.

The goal of this paper is to develop a richer model, in which we can analyze existing institutional features and antitrust policy. The paper is organized as follows. Section 2 builds a model that allows the full range between the two polar cases of perfectly substitutable and perfectly complementary patents, and yet is tractable. It notes that except in the two polar cases, whether patents are substitutes or complements depends on the level of licensing fees. Section 3 provides a necessary and sufficient condition for pools to be pro-competitive in the absence of independent licenses. Section 4 shows that independent licenses can be used by competition authorities as a screening device. Section 5 considers asymmetric patents. Section 6 generalizes the analysis to the case in which licensors are also licensees. Section 7 analyzes the impact of the prospect of pool formation on the incentive to innovate. And Section 8 discusses more general co-marketing agreements for which the analysis in this paper should be relevant.

## 2 Model

### *Intellectual property rights.*

There are  $n$  owners, each of whom has a patent on one innovation. For the moment, we assume that (a) patents are symmetrical in importance, (b) patent owners are not downstream users and therefore not potential licensees, and (c) the formation of the pool has no effect on future innovation in the industry. We will relax these assumptions in sections 5 through 7. We further assume that all parties are symmetrically informed.<sup>3</sup>

### *Demand for licenses.*

Licensing involves no transaction or other costs. There is a continuum of potential “users” or “licensees”. Users are heterogeneous and are indexed by parameter  $\theta \in [\underline{\theta}, \bar{\theta}]$ . User  $\theta$ ’s gross surplus from using  $m$ ,  $1 \leq m \leq n$ , innovations is

$$\theta + V(m).$$

The non user-idiosyncratic component  $V(\cdot)$  is strictly increasing, unless otherwise specified (we will occasionally consider “limit cases” in which  $V(m) = V(m - 1)$  for some  $m$ ). Thus, it is possible to employ the technology with a subset of patents, but the use of the technology is optimized by combining as many patents as possible.

The idiosyncratic parameter  $\theta$  reflects the heterogeneity in a) the fixed cost for the licensee to adopt the overall technology on which the patents are based, b) her opportunity cost of choosing this technology over an alternative one and c) the benefits she derives from the technology (the technology may enable the user to produce, to reap network externalities, or else to boost its research capability in the area). Letting  $F$  denote the cumulative distribution of  $\theta$ , the demand for the bundle of the  $n$  innovations licensed at price  $P$  is

$$D(P - V(n)) = \Pr(\theta + V(n) \geq P) = 1 - F(P - V(n)).$$

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<sup>3</sup>Under asymmetric information, results might differ. For example, pools are less likely to form when the owners of intellectual property have different information about such topics as the value or the effective duration of individual innovations. Bargaining inefficiencies are then bound to arise. For instance, such information problems might lead the holders of patents that are substitutes to not realize that there are potential gains from trade and to fail to form an anti-competitive pool. In addition, when substantial decisions have to be made after the pool is formed, it may be difficult to design a proper governance structure: i.e., to align the interest of pool members.

We assume that the hazard rate  $f\hat{A}[1 - F]$  is strictly increasing (which is satisfied by almost all familiar distributions). Because  $f\hat{A}[1 - F] = -D'/D$ , this assumption ensures the strict quasi-concavity of the pool's and other maximization programs.<sup>4</sup> We will further assume for conciseness that the support  $[\underline{\theta}, \bar{\theta}]$  is sufficiently wide so as to guarantee interior solutions. In particular,  $\bar{\theta} + V(n) > 0$  (otherwise, the technology would never be used).

While we must consider the details of these assumptions in more detail, it is worth pausing to consider the pool's maximization problem here. Let  $P^*$  denote the optimal price charged by the pool when patent owners cannot issue independent licenses:

$$P^* = \arg \max_P \{PD(P - V(n))\}. \quad (1)$$

There are several motivations for focusing on separable user preferences:

- First, this structure simplifies the analysis and exposition, as it implies that all licensees select the same basket of licenses in the absence of a pool.
- Second, and relatedly, the additive structure implies that it is optimal for a pool to offer solely a package license.<sup>5</sup> In other words, a pool cannot screen the user's type by offering, for example, a choice between the package license and licenses for subsets of patents.<sup>6</sup> A preference structure in which the user's type affects the marginal willingness to pay for patents would induce the pool to offer a menu of options. While such menus of options are interesting in their own right, they would add a distracting complication for the purposes of this paper.

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<sup>4</sup>Consider for example the pool's maximization problem ((1) below). At the optimal price  $PD' + D = 0$ , and so the second derivative of the profit function  $PD'' + 2D'$  has the same sign at that point as  $DD'' - 2(D')^2$ . The monotone hazard rate condition implies that  $DD'' < (D')^2$ , and so the profit function is quasi-concave.

<sup>5</sup>Which they usually do: only 12% of the pools in the Lerner et al. (2002) sample offered menus of patents. To be sure, the absence of a menu may have other motivations than that given here. The MPEG pool, for instance, considered offering menus, and ultimately rejected it. One major concern was the bargaining complexities that would be introduced, given the uncertainty about the valuation of many of the patents and the private information that many of the parties had about particular technologies.

<sup>6</sup>It can further be shown that the pool does not benefit from using stochastic schemes, in which the number of patents received and the price paid by the licensee are random functions of the licensee's announcement.

- Third, this structure will enable us to offer a clean description of the two constraints faced by an independent licensor in the absence of a pool. When contemplating a licensing fee increase, the independent licensor will worry either about her patent being excluded from the basket of patents selected by licensees, or, when retained in this basket, about the reduction in the overall demand for the basket. That is, the independent licensor may be constrained by either of two margins: the *competition margin* and the *demand margin*. Thus, the demand margin is said to bind in equilibrium for patent  $i$  if licensor  $i$  could individually raise her license price without triggering an exclusion of her individual license from the basket of patents selected by the licensees; otherwise it is the competition margin that binds for patent  $i$ .

*Substitutes and complements.*

Let

$$w(m) \equiv V(m) - V(m-1) > 0$$

denote the users' willingness to pay for an  $m^{\text{th}}$  patent when already having access to  $m-1$  patents.<sup>7</sup> Because  $V$  is strictly increasing, this marginal willingness to pay is strictly positive.

**Definition 1:** *The surplus function is concave if  $w$  is decreasing in  $m$  and convex if  $w$  is increasing in  $m$ .*

Unless otherwise specified, we will not impose specific restrictions such as convexity or concavity on the surplus function. First, the surplus function may be neither concave nor convex.<sup>8</sup> Second, while there is some connection between concavity and substitutability, and between convexity and complementarity, the degree of complementarity cannot be defined solely on this basis, as we will see.

**Definition 2:** *Patents are perfect substitutes if  $V(n) = V(n-1)$ , and so licensors compete à la Bertrand in the absence of pool, and perfect complements (Cournot-Shapiro case) if  $\bar{\theta} + V(n-1) \leq$*

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<sup>7</sup>For example, for  $V(m) = (m/n)^\alpha$  with  $\alpha < 1$ ,  $w(n) = 1 - ((n-1)/n)^\alpha$ . It converges to 0 as  $\alpha$  converges to 0.

<sup>8</sup>For example, implementing the technology may require a minimum number  $m_0$  of patents, but patents become competitors beyond that level ( $w(m)$  small for  $m \geq m_0$ ).

0, so no licensee benefits from (even free) access to less than the full set of patents.

The traditional definition of substitutability (respectively, complementarity) is that two goods are substitutes (complements) if increasing the price of one raises (lowers) the demand for the other. Except in the non-generic cases of perfect complements or substitutes, however, *patents are complements at low prices and substitutes at high prices*: When  $V(\cdot)$  is strictly increasing and the prices are low, users strictly prefer to use all patents conditionally on adopting the technology; thus a decrease in the price of one patent attracts new users to the technology and boosts the demand for the others. By contrast, with high prices, users may want to use a subset of patents and thus the patents compete with each other.<sup>9</sup>

*Remark (users' market power)*: In some settings, patent users are corporations with non-negligible market power. While, in the presentation of the model, users were described as end-users (or perfectly competitive firms), our analysis does not hinge on this interpretation. All that is needed throughout the paper is that downstream demand depends on the net price  $P - V(n)$  (or the equivalent for a subset of patents).<sup>10</sup> In the presence of downstream market power, our welfare results are unchanged as long as a decrease in the price  $P$  of the package of patents raises total welfare (equal to licensors' and downstream firms' profit, plus the welfare of final users served

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<sup>9</sup>For example, with two patents, if  $p_1 = p_2 = p < V(2) - V(1)$ , the two patents are complements (the demand for each is equal to  $1 - F(p_1 + p_2 - V(2))$ ); if  $p_1$  and  $p_2$  both exceed  $V(2) - V(1)$  (but are smaller than  $\bar{\theta} + V(1)$ ), then the two patents are perfect substitutes in that licensees all select the cheapest patent if they adopt the technology.

The general assertion for an arbitrary number of patents corresponds to Proposition 1.

<sup>10</sup>Let us sketch how a downstream imperfectly competitive market can be subsumed in our framework. Suppose that there are  $K$  potential downstream competitors. In the case of a pool without independent licensing (in the no-pool and pool-with-independent-licensing cases, stage a) is modified appropriately), the timing goes as follows: a) the pool sets price  $P$ ; b) the  $K$  firms each select whether to buy the technology at price  $P$ ; c) firms wage competition in the downstream market. One can think of firms that do not buy the technology as staying out, or more generally as being weak competitors. Let  $x$  denote the equilibrium probability that a given firm buys the technology, and  $\pi^k + \theta + V(n) - P$  as firm's net benefit from buying the technology when  $(k - 1)$  other firms buy it.  $[\theta + V(n)]$  can be thought of as (minus) the fixed cost (or reduction therefore) associated with the adoption of the technology, and the variable profit  $\pi^k$  is decreasing in  $k$ . As above,  $\theta$  is distributed independently across users according to some distribution  $F$ . Let  $\pi(x) \equiv \sum_{k=0}^{K-1} \binom{K-1}{k} x^k (1-x)^{K-1-k} \pi^{k+1}$ , with  $\pi' < 0$ . The stage b) (technology adoption) equilibrium is unique and given by

$$x = 1 - F(P - V(n) - \pi(x)).$$

Hence, the pool's demand ( $Kx$ ) depends only on  $P - V(n)$ ; and similarly when licensors license their patents individually.

by these downstream firms), a most reasonable assumption. This generality comes from the fact that we look at whether a pool, with or without licensing, raises or lowers price, and so our analysis does not depend on the exact expression of welfare.

### 3 When is a pool pro-competitive?

This section proceeds as follows: first, it demonstrates the existence of and characterizes the (unique) symmetric equilibrium (3.1). Focusing on this symmetric equilibrium, it then studies whether the formation of a pool is pro-competitive (3.2). Finally, it investigates the existence of asymmetric equilibria (3.3).

In the absence of a pool, the timing is as follows: a) The licensors choose their individual prices  $p_i$  simultaneously and non-cooperatively. Suppose, thus, that the  $n$  licensors charge prices  $\mathcal{P} \equiv (p_1, \dots, p_n)$ , where without loss of generality,  $p_1 \leq p_2 \leq \dots \leq p_n$ . b) Users select how many and which licenses to buy. A user's licensing decision can be decomposed into two steps. First, the user solves

$$\mathcal{V}(\mathcal{P}) = \max_{m \leq n} \{V(m) - (p_1 + \dots + p_m)\}.$$

We will assume that the users purchase the maximum number of patents in the optimal set whenever this program has multiple solutions.<sup>11</sup> Second, the user adopts the technology if and only if  $\theta + \mathcal{V}(\mathcal{P}) \geq 0$ .

We analyze the pure-strategy equilibria of this game. Note that, if there are licenses with positive sales in equilibrium, then all licensors have positive sales.<sup>12</sup>

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<sup>11</sup>This assumption is required to guarantee the existence of equilibrium when there is no pool or when the pool members can license independently. If the users did not purchase the maximum number of patents when indifferent, an "openness problem" would arise in which, when the competition margin binds, each licensee would want to charge "almost as much" as what keeps her patent in the basket selected by the users, but would not find it optimal to make the consumer indifferent.

<sup>12</sup>Let  $m(\mathcal{P})$  denote the number of licenses with positive sales for price configuration  $\mathcal{P}$  (with  $p_1 \leq p_2 \leq \dots \leq p_n$ ). For equilibrium prices,  $m(\mathcal{P}) = n$ . If this were not the case, licensor  $n$  (the highest price licensor by assumption) would make no profit and so would gain by charging any price exceeding 0 but smaller than  $w(m(\mathcal{P}) + 1)$ , as such a price would induce users to license her technology.



### 3.1 Existence

We first demonstrate the existence of a symmetric equilibrium.

a) *Demand margin binds.*

Recall that the demand margin binds when an individual licensor can raise her price slightly without triggering an eviction of her license from the basket of patents selected by users. Then, individual licensors choose their license price  $p_i = \hat{p}$  by solving:

$$\hat{p} = \arg \max_{p_i} \{p_i D(p_i + (n-1)\hat{p} - V(n))\}. \quad (2)$$

And so:  $\hat{p} D'(n\hat{p} - V(n)) + D(n\hat{p} - V(n)) = 0$ . The monotone hazard rate condition implies that there is a unique such  $\hat{p}$ .

Instead of setting price  $p_i$  for her patent, and given that in equilibrium the  $(n-1)$  other licensors charge  $\hat{p}$  each, licensor  $i$  can be viewed as setting total price  $P$  for the basket of the  $n$  patents and keeping  $p_i = P - (n-1)\hat{p}$  for herself. Thus, rewriting (2), note that  $\hat{P} = n\hat{p}$  satisfies

$$\hat{P} = \arg \max_P \{[P - (n-1)\hat{p}] D(P - V(n))\}, \quad (3)$$

and so, by revealed preference,<sup>13</sup>

$$\hat{P} \geq P^*. \quad (4)$$

Under noncoordinated pricing, each licensor does not internalize the increase in the other licensors' profits when demand for the package is increased by a reduction in her price. This result generalizes the Cournot-Shapiro argument: If the demand margin binds in the absence of pool, then a pool reduces the price paid by users.

<sup>13</sup>Note that (1) and (3) are part of a broader family of maximization programs:

$\max_P \{\pi(P, c) \equiv (P - c) D(P - V(n))\}$ , where  $c = c^* = 0$  in program (1) and  $c = \hat{c} = (n-1)\hat{p}$  in program (3).

Because  $\partial^2 \pi / \partial P \partial c > 0$ ,  $\hat{P} \geq P^*$  by revealed preference: If  $P_i$  is optimal for  $c_i$  and  $P_j$  is optimal for  $c_j$ , then  $\pi(P_i, c_i) \geq \pi(P_j, c_i)$ . Applying these two inequalities for  $(P^*, c^*)$  and  $(\hat{P}, \hat{c})$  and adding them up yields

$$\int_{c^*}^{\hat{c}} \int_{P^*}^{\hat{P}} (\partial^2 \pi / \partial P \partial c) dP dc \geq 0,$$

and so  $\hat{P} \geq P^*$  as  $\hat{c} > c^*$ .

b) *Competition margin binds.*

Let us now consider a symmetric equilibrium in which the competition margin binds. In such an equilibrium, each licensor would like to increase her price at least slightly (that is,  $p_i D(p_i + P_{-i} - V(n))$  is locally increasing in  $p_i$  where  $P_{-i} \equiv \sum_{j \neq i} p_j$ ), but cannot do so because her patent would then be evicted from the basket of patents selected by the users. Accordingly, let  $p = z(n)$  denote the unique price  $p$  satisfying:

$$V(n) - np = \max_{m < n} \{V(m) - mp\}. \quad (5)$$

In a symmetric price configuration with price  $z(n)$ , licensors are constrained by the competition margin as long as  $p_i D(p_i + (n-1)z(n) - V(n))$  is locally increasing in  $p_i$  at  $p_i = z(n)$ . Note that  $z(n)$  is independent of the distribution of  $\theta$ . In the concave case

$$z(n) = w(n).$$

More generally

$$z(n) \leq w(n),$$

(since users can select  $n-1$  patents), possibly with strict inequality.<sup>14</sup> As we will see, keeping  $n$  and  $V(n)$  fixed, licensors are constrained to charge (weakly) lower prices when  $z(n)$ , or equivalently  $Z(n) \equiv nz(n)$ , decreases. The following definition thus compares surplus functions  $V(\cdot)$  in the family of functions characterized by a common surplus  $V(n)$ :

**Definition 3.** *For a fixed number  $n$  of patents, consider two surplus functions:  $V_1(\cdot)$  and  $V_2(\cdot)$ , with associated  $Z_1(n) = nz_1(n)$  and  $Z_2(n) = nz_2(n)$ , such that  $V_1(n) = V_2(n)$ . Patents are more substitutable for surplus function  $V_1(\cdot)$  than for surplus function  $V_2(\cdot)$  if*

$$Z_1(n) < Z_2(n).$$

Note also that, fixing  $V(n)$ , the pool price  $P^*$  depends only on the elasticity of the demand curve, and not on the substitutability  $Z(n)$  among patents. Conversely,  $Z(n)$  depends on the

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<sup>14</sup>Suppose  $n = 3$ ,  $\{V(0) = 0, V(1) = 5, V(2) = 5 \text{ and } V(3) = 8\}$  (a possible interpretation is that one patent suffices for a low-quality production, while the full set is necessary for a high-quality one). Then  $w(n) = 3$  while  $z(n) = 1.5$ .

surplus function  $V(\cdot)$ , but not on the elasticity of the demand curve. This means that the competition and the demand margins are conceptually distinct. In many respects, the interaction between the demand and the competition margin is reminiscent of the literature on discrete choice models (for an overview, see Anderson, de Palma, and Thisse (1992)). In models such as the nested multinomial logit, consumers make choices sequentially, first making a choice among the options in cluster of goods and then deciding what cluster to purchase. Like the consumers in a nested model, here the users vary in their preference for the basket of pooled patents relative to the alternative technology. When choosing between the patents in the pool, however, the users are homogenous in their preferences.

c) *Equilibrium.*

Finally, let us put the two possibilities together.

**Proposition 1** (*symmetric equilibrium in the absence of a pool*)

*There exists a unique symmetric equilibrium:*

(i) *If  $z(n) < \hat{p}$  (i.e.,  $z(n) D'(Z(n) - V(n)) + D(Z(n) - V(n)) > 0$ ), then licensors are constrained only by the competition margin and charge equilibrium price  $z(n)$ .*

(ii) *If  $z(n) > \hat{p}$  (i.e.,  $z(n) D'(Z(n) - V(n)) + D(Z(n) - V(n)) < 0$ ), then licensors are constrained only by the demand margin, and charge price  $\hat{p}$ .*

**Proof.** If  $\hat{p} > z(n)$ , then there is no equilibrium in which the demand margin binds, since users would not want to use all patents when licensors charge individual prices  $\hat{p}$ . The licensors then charge  $z(n)$  each; they cannot raise their price without their license being evicted. They do not want to lower their price either because prices are already low.<sup>15</sup>

Conversely, if  $\hat{p} < z(n)$ , then the competition margin cannot bind since licensors would want to reduce their price in order to boost demand. The symmetric equilibrium has all licensors

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<sup>15</sup>Technically, the revenue  $p_i D(p_i + (n-1)z(n) - V(n))$  increases with  $p_i$  at  $p_i = z(n)$ . To see this, recall that industry marginal revenue  $D(P - V(n)) + PD'(P - V(n))$  is decreasing in  $P$  from the monotone hazard rate condition; so is, a fortiori,  $D(P - V(n)) + \frac{P}{n}D'(P - V(n))$ . And so  $z(n) < \hat{p}$  implies

$$D(Z(n) - V(n)) + z(n) D'(Z(n) - V(n)) > D(\hat{P} - V(n)) + \hat{p} D'(\hat{P} - V(n)) = 0.$$

charge  $\hat{p}$ .<sup>16</sup> ■

## 3.2 Welfare analysis

We now compare the outcome of a pool in which members are not allowed to grant independent licenses with that in the absence of a pool. In this section, we analyze the competitive impact of the formation of a pool of existing patents (the *ex post* view), without considering the impact of the formation of the pool on future innovation.<sup>17</sup> This approach substantially simplifies the analysis. It is also consistent with the typical approach of antitrust authorities in assessing proposed pools.

**Proposition 2** (*normative analysis of pool*)

- (i) A pool always increases welfare when the demand margin binds in the absence of pool.
- (ii) A pool may increase or decrease welfare when the competition margin binds in the absence of a pool, depending on whether  $P^* \leq Z(n)$ .

**Proof.** Part (i) of the proposition results from inequality (4). Part (ii) is a direct corollary of the fact that each licensor charges  $z(n)$  when the competition margin binds in the absence of a pool. ■

A simple corollary of Propositions 1 and 2 is

**Proposition 3** (*substitutability among patents*) *As patents become more substitutable (in the sense of Definition 3),*

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<sup>16</sup>Interestingly, it can be shown that the competition margin is more likely to bind when the demand grows. The intuition is that if the demand margin binds, licensors increase their prices when the elasticity decreases. Licensees are then more tempted to do with a limited set of patents. Let us index the distribution of types by a parameter  $\gamma$ :  $F(\theta | \gamma)$ . It is standard to compare distributions through their hazard rates. Parameter  $\gamma_1$  corresponds to a lower demand (and higher elasticity) than parameter  $\gamma_2$  if for all  $\theta$

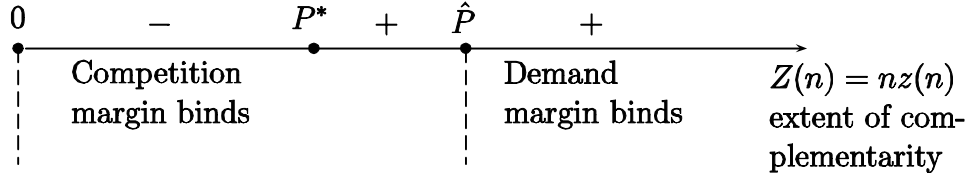
$$\frac{f(\theta | \gamma_2)}{1 - F(\theta | \gamma_2)} < \frac{f(\theta | \gamma_1)}{1 - F(\theta | \gamma_1)}.$$

It is then clear that if the competition margin binds for parameter  $\gamma_1$ , then it binds a fortiori for parameter  $\gamma_2$ .

<sup>17</sup>Section 7 will take the *ex ante* view, accounting for the pre pool-formation incentive to engage in R&D and thus for the impact of the antitrust treatment of pools on the number  $n$  of innovations.

- (i) the competition margin is more likely to bind (since it binds if and only if  $Z(n) \leq \hat{P}$ ),
- (ii) the pool is more likely to decrease welfare (since it does so if and only if  $Z(n) < P^*$ ).

The following figure summarizes the welfare analysis. It depicts pools falling along a spectrum, where zero denotes patents that are perfect substitutes and the complementarity between patents increases along the axis. As noted above, when patents are suitably complementary, the demand margin will bind instead of the competition margin. The pluses and minuses indicate the impact of the pool on social welfare. As is clear from the figure, while all pools where the demand margin binds are socially beneficial, only some of those where the competition margin binds (those where the extent of complementarity is the greatest) are.



**Figure 1**

### 3.3 Uniqueness

The welfare analysis of section 3.2 is predicated on the independent licensors achieving a symmetric equilibrium. While we have shown that there is a unique equilibrium in the class of symmetric equilibria, we have not investigated the possibility of asymmetric ones. We now do so.

Consider an asymmetric equilibrium configuration  $\mathcal{P} = (p_1, \dots, p_n)$ , where without loss of generality,  $p_1 \leq p_2 \leq \dots \leq p_n$ . Let  $P \equiv \sum_j p_j$ ,  $P_{-i} \equiv \sum_{j \neq i} p_j$ ; and for a subset  $J$  of patents, let  $m(J)$  denote the number of patents in that subset and  $P_J \equiv \sum_{j \in J} p_j$ .

The highest price,  $z_i$ , that patent holder  $i$  can charge without her patent being evicted from the basket of patents is now licensor-specific. It is uniquely defined by:

$$V(n) - P_{-i} - z_i = \max_{J \not\ni i} \{V(m(J)) - P_J\}. \quad (6)$$

The left-hand side of (6) is the net surplus when the user buys all patents. The right-hand side of (6) is the user’s net surplus obtained by evicting patent  $i$  from the basket.<sup>18</sup>

The following proposition (proved in the Appendix) contains this section’s main insights:

**Proposition 4** (*asymmetric equilibria*)

(i) *An asymmetric equilibrium, if it exists, is characterized by:*

*Either licensor  $i$  is constrained by the demand margin ( $i \in D$ ) and then  $p_i = \hat{p} > z_i$ , where  $\hat{p}D'(P - V(n)) + D(P - V(n)) = 0$ ; or licensor  $i$  is constrained by the competition margin ( $i \in C$ ) and then  $p_i = z_i \leq \hat{p}$ .*

(ii) *There exists no asymmetric equilibrium (and therefore the symmetric equilibrium is unique) when the surplus function  $V(\cdot)$  is concave.*

(iii) *There may exist asymmetric equilibria for non-concave surplus functions.*

(iv) *In case of multiple equilibria, the symmetric equilibrium is also the highest industry-profit equilibrium.*

The intuition for part (iv) is that for a given total price, the users’ option to buy a subset of patents only is more attractive when there is price dispersion. In this sense licensors feel more “competitive pressure” in an asymmetric than in a symmetric equilibrium. In the following (except of course when we consider asymmetric surplus functions in section 5), we will focus on the symmetric equilibrium.

## 4 Independent licenses as a screening mechanism

Patent owners who request a statement of the Department of Justice’s antitrust enforcement intentions with respect to a proposed pool arrangement usually include the provision that the individual patents that are part of the pool may still be licensed from the original patents’

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<sup>18</sup>Actually, one could restrict attention to the subset  $J$  of patents with prices equal to or smaller than  $p_i$ : if patent  $i$  is evicted from the basket, so will any strictly more expensive patent  $k$  ( $p_k > p_i$ ), since otherwise the users would be better off keeping patent  $i$  and dispensing with patent  $k$ .

owners.<sup>19</sup> In contrast with a merger, thus, the patent owners (the counterparts of the merging parties) still act independently and keep marketing their own intellectual property. They just agree to market a jointly produced “good” – the package license — at some pre-agreed price — (the package price).

This common provision raises two related questions: First, what is the cost for pool members of including this provision (given that the pool administrator could offer individual patent licenses and not only the package license<sup>20</sup>)? Second, would it be optimal for antitrust authorities to insist on this provision?

We assume that pool members share royalties equally and consider a two-stage game following the constitution of the pool:

- (i) The pool chooses a price  $P$  for its bundle.
- (ii) Owners non-cooperatively and simultaneously set license prices  $(p_1, p_2, \dots, p_n)$  for their individual patents. Users then choose among a) not buying at all, b) buying the package from the pool, c) buying some or all the individual licences.

To break ties, we assume that if  $V(n) - P \geq \max_J \{V(m(J)) - P_J\}$ , users don't buy individual licenses. Fixing  $P$ , an equilibrium of stage (ii) is called a “continuation equilibrium”. The stage (i) choice of  $P$  maximizes the representative member's profit, anticipating the subsequent stage (ii) equilibrium. Regarding the latter continuation equilibrium, it may be unique (and then symmetric if stake holdings in the pool are equal) or there may also exist asymmetric equilibria, depending on the technology. Similarly, the potential multiplicity of equilibria in the absence of a pool (Proposition 4) may render the welfare analysis of a pool ambiguous. This section focuses on *symmetric* equilibria (both in the absence of a pool, or with a pool allowing independent

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<sup>19</sup>The independent licensing provision is by no means specific to the recent pools that have obtained review letters from the Department of Justice. As noted above, nearly half the pools in the Lerner et al (2002) sample allow independent licensing.

<sup>20</sup>For different specifications of user preferences, the pool might want to issue sublicenses; but recall that we have chosen licensees' preferences so that it is optimal for the pool to offer only the package license.

licensing). Proposition 6 (iv) below, applied to the symmetric case, provides insights that are similar, but not based on symmetric equilibrium selections.

**Definition 4:** *Consider a pool with symmetric stake holdings. The pool is strongly stable to independent licensing if, when the pool charges  $P^*$  (the pool-profit maximizing price when there is no independent licensing), in all symmetric pure-strategy equilibria of stage (ii) users buy solely from the pool. The pool is weakly unstable to independent licensing if whenever the pool charges a price exceeding the total price charged in the absence of a pool, there exists an equilibrium in which the patent owners all charge  $z(n)$  and users buy individual licenses. The pool is strongly unstable if whenever the pool charges a price exceeding the total price charged in the absence of a pool, patent owners all charge  $z(n)$  and users buy individual licenses in all symmetric equilibria of stage (ii).*

In words (and focusing again on symmetric equilibria), (i) independent licensing is irrelevant for a strongly stable pool, (ii) independent licensing undoes pool formation in the case of a strongly unstable pool, and (iii) there exists an equilibrium that undoes the pool in the case of a weakly unstable pool.

**Proposition 5** *(independent licensing by pool members)*

*Consider a pool with symmetric stake holdings.*

*(i) A welfare-enhancing pool is strongly stable to independent licensing by pool members.*

*(ii) A welfare-decreasing pool is weakly unstable for all  $n$ , and is strongly unstable for  $n = 2$ .*

*Furthermore, the pool cannot avoid this instability through asymmetric stake holdings: For any stake holdings distribution, and for any pool price  $P > Z(n)$ , there exists an equilibrium in which all licensors charge  $z(n)$ , and this equilibrium is the unique equilibrium for  $n = 2$ .*

*Proof:* (i) Consider, first, a welfare-enhancing pool:

$$P^* < \tilde{P} \equiv \min \left( Z(n), \hat{P} \right).$$



*Existence:* Let all patent owners charge  $\tilde{p} = \tilde{P}/n$  at stage (ii). And so all users buy from the pool. To raise profit, an individual licensor must reduce its price sufficiently so that the total price  $P$  falls below  $P^*$ . Because an individual price decrease never leads to the eviction of patents from the package, this licensor's profit is then

$$\begin{aligned} [P - (n-1)\tilde{p}] D(P - V(n)) &< \left[ P - (n-1)\frac{P}{n} \right] D(P - V(n)) \\ &< \frac{P^*}{n} D(P^* - V(n)) \end{aligned}$$

since, by definition,  $P^*$  maximizes  $D(P - V(n))$ . So, licensors cannot gain by changing their price.

*Uniqueness:* Suppose that users purchase individual licenses rather than from the pool (they then buy all of them, since a patent holder whose patent was not included in the basket would reduce her price sufficiently so as to have it included). Furthermore, all licenses sell at the same price in a symmetric continuation equilibrium. Let  $P$  denote the price of the basket of all individual licenses, with  $P < P^*$ . Because  $P < P^* < Z(n)$ ,

$$V(n) - P > \max_m \left\{ V(m) - m \frac{P}{n} \right\}.$$

And because  $P < P^* < \hat{P}$ ,  $PD^0(P - V(n)) + D(P - V(n)) > 0$ , and so, a fortiori:

$$\frac{P}{n} D'(P - V(n)) + D(P - V(n)) > 0.$$

Hence, each licensor would like to increase the price of her individual license and can do so because she is unconstrained by the competition margin, a contradiction.

(ii) Consider now a welfare-decreasing pool ( $Z(n) < P^*$ ).

*Existence:* Consider a pool package price  $P > Z(n)$ , and let all patent owners charge  $z(n)$  each, and so users buy individual licenses rather than from the pool. We know from section 3 that no one would want to deviate by charging less than  $z(n)$  (section 3 applies because the pool's

package offer remains irrelevant when one owner reduces her price). Conversely, if owner  $i$  charges  $p_i > z(n)$ , patent  $i$  is evicted from the basket of individual licences. Furthermore, users still buy individual licenses since, by definition,

$$\max_{m < n} \{V(m) - m z(n)\} = V(n) - Z(n) > V(n) - P.$$

*Uniqueness (for  $n = 2$ ):* Suppose that the pool charges  $P > Z(2)$ , and that users buy from the pool. Then the payoff of an owner with (at most) a 50% ownership stake is (at most):

$$\frac{P}{2} D(P - V(2)).$$

Alternatively, the owner could contemplate selling an individual license at price  $p$  (slightly below the level) given by:

$$V(1) - p = V(2) - P.$$

It then makes

$$p D(p - V(1)) = [P - Z(2)] D(p - V(2)) > \frac{P}{2} D(P - V(2)).$$

Hence the pool is strongly unstable. ■

This claim is borne out by the analysis in Lerner et al. (2002). This paper estimates a set of simultaneous equations to predict the structure of pools formed before the mid-1990s. One equation estimates a latent variable, an index of the extent that the patents in the pool are substitutes or complements (independent variables include the date of pool formation and an indicator if the pool was subsequently identified as violating antitrust laws). This measure proves to be highly significant in explaining the use of independent licensing: pools whose patents are more likely to be complements are more likely to allow licensing by individual firms.

## 5 Asymmetric patents

Patents may be asymmetric in two related ways. First, a patent may improve on a pre-existing one, but its value added is obtained only if the two patents are combined. The former, called a “subservient patent,” is then valueless on a stand-alone basis. The latter, called the “dominant

patent,” can by contrast be licensed on a stand-alone basis. User  $\theta$  then obtains gross surplus  $\theta + V(2)$  from the two patents combined,  $\theta + V(1)$  from the dominant patent only, and 0 from the subservient patent only.<sup>21</sup>

Second, patents’ contributions to total surplus may be different. Some may be minor patents while others are key to the technology. We formalize this in our set-up by assigning contribution  $n_i \in [0, n]$  to patent  $i$ , with the normalization

$$\sum_{i=1}^n n_i = n.$$

So, in the symmetric case,  $n_i = 1$  for all  $i$ . Let  $x_i = 1$  if user  $\theta$  obtains a license for patent  $i$  and  $x_i = 0$  otherwise. User  $\theta$ ’s gross surplus is then:<sup>22</sup>

$$\theta + V\left(\sum_{i=1}^n x_i n_i\right).$$

Because asymmetric patterns in general require asymmetric ownership stakes in the pool in order for a pool allowing independent licensing to be viable, we let  $\alpha_i$  denote a patent owner  $i$ ’s share of the royalties collected by the pool, with

$$\sum_{i=1}^n \alpha_i = 1.$$

The following proposition, proved in the appendix, generalizes some of the analysis of sections 3 and 4 to the case in which contributions are unequal, but, for all  $i$ , patent  $i$ ’s contribution  $n_i$  can be reaped regardless of the combination of patents:

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<sup>21</sup>The user always has the choice of not using the subservient patent (free disposal).

<sup>22</sup>For  $n = 2$  (or more generally, in the context covered by the corollary below), the former form of asymmetry can be subsumed in the latter form by assigning to the dominant and subservient patents contributions  $n_1$  and  $n_2$  with  $n_1 > n_2$  and redefining the surplus function:

$\tilde{V}(2) = V(2)$ ,  $\tilde{V}(n_1) = V(1)$ , and  $\tilde{V}(n_i) = -\infty$  for  $n_i < n_1$  (together with free disposal). This may no longer be so with more than two patents. For example, patent 3 may add as much to patent 1 and to patent 2, but may be subservient to patent 1 (i.e., build on its design), but be autonomous with respect to patent 2.

**Proposition 6** (*asymmetric importance*)

Suppose that the gross surplus of a type  $\theta$  user from adopting the technology is  $\theta + V\left(\sum_{i=1}^n x_i n_i\right)$ , where  $x_i = 1$  if the user obtains a license for patent  $i$  and  $x_i = 0$  otherwise,  $\sum_{i=1}^n n_i = n$ , and  $n_1 \leq n_2 \leq \dots \leq n_n$ .

(i) In the absence of a pool, there exists  $m$ ,  $1 \leq m \leq n$ , such that licensors 1 through  $m$  are constrained by the competition margin and charge their (licensor-specific) price  $p_i = z_i$  above which their patent is evicted from the basket, whereas the most important patents  $m + 1$  through  $n$  are constrained by the demand margin and all charge the same price  $\hat{p}$  (with  $z_i < \hat{p}$  for all  $i \leq m$ ).

(ii) If in (i),  $m < n$  (so at least one licensor is constrained by the demand margin in the absence of pool), a pool is welfare-enhancing.

(iii) When the surplus function is concave, the equilibrium in the absence of a pool is unique, and more valuable patents are more expensive ( $p_1 \leq p_2 \leq \dots \leq p_n$ ).

(iv) If all equilibria in the absence of a pool yield total price  $P > P^*$ , where  $P^* D^0(P^* - V(n)) + D(P^* - V(n)) = 0$ , (and so a pool is welfare enhancing), then for appropriate stakes in the pool the continuation equilibrium is unique and has users buy from the pool (independent licensing is irrelevant).

If all equilibria in the absence of a pool yield total price  $P < P^*$  (and so a pool is welfare decreasing), then, regardless of the distribution of stake holdings, whenever the pool attempts to charge a price above the maximum of these equilibrium prices, there exists a continuation equilibrium in which users purchase only individual licenses; for  $n = 2$ , users purchase only individual licences in any continuation equilibrium.

Regarding the impact of independent licensing, Proposition 6 (iv) is stronger than Proposition 5 if the equilibrium in the absence of a pool is unique (e.g., in the concave-surplus-function case). In general, though, it is neither weaker nor stronger: While it does not presume a focus on symmetric equilibria, it assumes that equilibria in the absence of a pool all yield a higher or a lower price than the pool.

*Application to dominant / subservient patents.*

Let us assume that patent 1 is dominant, and patents 2,  $\dots$ ,  $n$  subservient (that is, valueless unless combined with patent 1). Using parts ii) and iv) of Proposition 6, the antitrust implications of pools are rather straightforward in this case:

**Corollary:** *With the asymmetric, dominant / subservient pattern,*

*(i) independent licenses are irrelevant (the pool is strongly stable) when members can freely bargain over their respective stakes, and*

*(ii) pools unambiguously enhance welfare.*

The key to understanding why pools are always welfare enhancing here is to note that, by assumption, the subservient patents are valueless on a stand-alone basis, and so in the absence of a pool *the demand margin always binds for the dominant patent*; this property creates a potential double marginalization, and thereby a potential social gain to the formation of a pool.<sup>23</sup>

## 6 Licensors are also licensees

We have until now analyzed pools whose members are upstream patent owners and license to third-party downstream users. Let us now allow licensors to be also licensees. Two competition-policy issues then arise:

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<sup>23</sup>In equilibrium, patent owners grant the dominant patent owner a sufficient stake  $\alpha_1$  in the pool so as not to encourage her to license her property independently. One can show that

$$\alpha_1 \geq \alpha_1^* \equiv \frac{P^* - [V(n) - V(1)]}{P^*},$$

where  $P^*$ , as earlier, maximizes pool profit  $PD(V(n))$ .

Gilbert (2002) looks at a two-patent situation in which the first patent is dominant and the second patent is subservient. As in part (ii) of the corollary, a pool increases welfare. Gilbert shows that that welfare can be further increased by having Firm 1 (the owner of the first patent) license the patent (without royalties) to Firm 2 while still licensing directly to end users. Then Firm 2, which has a superior offering as it combines the two patents, competes against Firm 1, acting as a fringe when licensing the first patent. Gilbert's Firm 2 is then equivalent to a patent pool in which Firm 1 has no stake.

Gilbert's result does not contradict part (i) of the corollary. Indeed, independent licenses are irrelevant as long as the members can choose the stakes held by each member of the pool. The pool members choose a large enough stake for the dominant patent's owner so as to deter her from competing with the pool's offer.

More generally, stake ownership must match patent importance in order for the pool to be stable to independent licensing, which is a fairly intuitive requirement.

- *foreclosure*: the licensors may not want to create their own competition downstream by licensing to third parties;
- *raising each other's cost*: licensors may want to raise each other's cost by charging royalties in cross-licensing or pool agreements.

The two concerns are distinct. The possibility of foreclosure arises quite generally, whether or not licensors can charge royalties on the usage of the technology. The latter question does not arise when licensors can charge only fixed fees in licensing contracts. We study the two issues in sequence, each time focusing on only one issue for a clean analysis. Our study of foreclosure assumes that only fixed-fee licenses are possible (the extent of usage of the technology is not metered), in order to eliminate the raise-each-other's-cost strategy. The study of the latter strategy by way of contrast assumes that there are no third-party downstream competitors, and so openness of access to the technology is not an issue.

The contribution of this section is of course not the identification of these two (standard) issues, but rather the study of what they imply as to the desirability of pools.

## 6.1 Foreclosure

Our theory generalizes easily to the introduction of downstream competition as long as licensing contracts take the form of fixed fees and the technology helps reduce downstream users' fixed cost (e.g., the cost of writing software). Consider the following simple situation (as the reader will see, both the insights and the approach extend to more complex environments). There are, as in sections 2 through 4,  $n$  symmetric owners of intellectual property. These  $n$  owners are each in one of  $n$  separate downstream markets, and initially command a monopoly position in that market. In each market, there is one potential entrant. The entrant cannot enter without access to the technology, and if it enters, obtains gross duopoly profit

$$\pi^d + \theta + V(m)$$

when using  $m \leq n$  patents. So,  $\pi^d$  can be viewed as the ex post duopoly profit and  $\theta + V(m)$  as (minus) the entry cost. The market  $i$  entrant's draw  $\theta$  from distribution  $F(\theta)$  is private information to the entrant.

An incumbent's gross profit is  $\pi^m$  if it remains a monopolist and  $\pi^d < \pi^m$  if it becomes a duopolist.<sup>24</sup> The case in which the licensors are not integrated downstream can be obtained as a special case of our analysis, by setting  $\pi^m = \pi^d$  in the equations below (so there is no profit loss for the licensors in the downstream market in case of entry). We assume that the final consumers' welfare is higher under entry.

In the absence of pool, the game proceeds as follows:

- (i) Licensors set non-discriminatory prices  $\{p_i\}$  for their individual licenses.
- (ii) Entrants 1 through  $n$  decide whether to enter their respective markets, and, if they do, which licenses to purchase.
- (iii) Product market competition in the  $n$  markets occurs.

Stages (i) and (ii) of the timing for the pool and pool-with-independent licensing are modified appropriately.<sup>25</sup> We focus on symmetric equilibria.

**Proposition 7** (*vertical integration*)

*(i) In the absence of pool, when the competition margin is binding, prices are the same ( $z(n)$  for individual licenses,  $Z(n) = nz(n)$  in total) as when the licensors are not vertically integrated with the incumbent monopolies. They are higher than under non-integration when the demand*

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<sup>24</sup> $2\pi^d$  may exceed  $\pi^m$ , that is, entry may be profitable for the industry. But we will not need to make this assumption. Even a pool may want to offer licenses when  $\pi^m > 2\pi^d$ , because the entrants may benefit from the technology in other (undescribed) markets.

Note also that we have assumed the patent-owner's entry cost is sunk; or, equivalently, if the patent-owners need access to technology covered by others' patents, they will cross-license for free with each other (as it is optimal for them to do so).

<sup>25</sup>For a pool: (i) The pool sets price  $P$  for the package. (ii) Entrants 1 through  $n$  decide whether to enter their respective markets by buying the package. For a pool with independent licensing, there is a prior stage in which the pool price  $P$  is set. Then: (i) Licensors set non-discriminatory prices for their individual licenses. (ii) Entrants 1 through  $n$  decide whether to enter their respective markets and, if they do, which licenses to purchase (either a set of individual licenses or the pool's package offer).

margin binds. Hence, vertical integration makes it more likely that the competition margin binds.

(ii) Vertical integration raises the pool's price  $P^*$ .

(iii) As in Proposition 2, a pool increases welfare if the demand margin binds in the absence of a pool. It increases or decreases welfare when the competition margin binds in the absence of a pool, depending on whether  $P^* \gtrless Z(n)$ .

(iv) Proposition 5 (on independent licensing) is still valid.

*Proof.*

*Pool:* Let us start with the case of a pool. The representative (vertically integrated) member's profit is

$$\begin{aligned} & \max_P \{ [1 - D(P - V(n) - \pi^d)] \pi^m + [D(P - V(n) - \pi^d)] [\pi^d + P] \} \\ \iff & \max_P \{ [P - (\pi^m - \pi^d)] D(P - V(n) - \pi^d) \}. \end{aligned}$$

Thus, everything is as if the licensors incurred a marginal licensing cost equal to  $\pi^m - \pi^d$ .

*No pool:* As in section 3, we must consider the two cases in which the demand and competition margins are binding. Suppose, first, that the *demand margin* is binding. Licensor  $i$  solves:

$$\max_{p_i} \left\{ \begin{array}{l} (n-1) p_i D(p_i + P_{-i} - V(n) - \pi^d) \\ + [1 - D(p_i + P_{-i} - V(n) - \pi^d)] \pi^m + [D(p_i + P_{-i} - V(n) - \pi^d)] [\pi^d + p_i] \end{array} \right\}.$$

In words, licensor  $i$  de facto is a pure licensor in  $(n-1)$  markets, and is both a licensor and a licensee in her own market. Licensor  $i$  therefore solves:

$$\max_{p_i} \left\{ \left[ p_i - \left( \frac{\pi^m - \pi^d}{n} \right) \right] D(p_i + P_{-i} - V(n) - \pi^d) \right\}.$$

Comparing this program to that of the pool, note that individual licensing introduces two opposite biases: a) multiple marginalization (as in section 3), and b) non-internalization of the profit



destruction impact of entry for the  $(n - 1)$  other licensors. The former effect however always dominates the latter.<sup>26</sup>

This implies that if the demand margin is binding, a pool reduces the total price. It also increases welfare: a lower price benefits both downstream entrants (the direct users of the technology) and the final consumers (who benefit indirectly through increased entry).

The analysis of the *competition margin* is unchanged relative to section 3.  $z(n)$  is still defined as the maximal possible price  $p$  satisfying:

$$V(n) - np = \max_{m < n} \{V(m) - mp\}.$$

In the absence of a pool, licensors constrained by the competition margin have no ability to lower the users' net surplus (since each, by definition, is indifferent between using all patents and dropping some), and so the fact that the licensors are also downstream users is irrelevant. Put differently, the prices are the same as if the licensors were not vertically integrated with downstream incumbents.

Last, the proof of Proposition 5 carries over to the vertical integration case. ■

## 6.2 Royalties in closed-pool agreements

In order to clearly distinguish the two concerns, we now assume that the pool does not offer licenses to third parties (there are none, say), but can collect royalties. Its  $n$  members form a symmetric  $n$ -firm downstream oligopoly. Members pay royalty rate (or access charge)  $a$  to the pool, whose profit is then redistributed equally among its members. For simplicity, let us assume that  $n = 2$ .

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<sup>26</sup>The first-order condition in the absence of a pool is

$$\left[ \frac{\hat{P} - (\pi^m - \pi^d)}{n} \right] D'(\hat{P} - V(n) - \pi^d) + D(\hat{P} - V(n) - \pi^d) = 0,$$

and for a pool:

$$[P^* - (\pi^m - \pi^d)] D'(P^* - V(n) - \pi^d) + D(P^* - V(n) - \pi^d) = 0.$$

So marginal revenue is smaller in the case of a pool, and because marginal revenues are decreasing,  $\hat{P} > P^*$ .

Patent owners who are also downstream competitors will never want to join a pool, if the pooling of their patents make them undifferentiated. To account for pool formation we therefore assume that the two firms are differentiated in two ways (the following analysis is inspired by the “double differentiation model” in Hausman et al. (2003)).<sup>27</sup> The first dimension of differentiation is technology unrelated; the two firms are located at the two extremes of an Hotelling segment  $[0, 1]$ . Consumers are located uniformly on the segment and incur unit transportation cost  $t$ .

Second, patents 1 and 2 describe two technologies that are differently suited to the needs of the consumers. Namely, patents 1 and 2 are located at the two extremes of an Hotelling segment  $[0, 1]$ , and consumers are uniformly<sup>28</sup> distributed along that segment (independent of their location in the other dimension), with transportation cost  $u$  per unit of distance. Pooling the patents then allows firms to offer a better service to consumers: each can offer the patent 1- and patent 2-enabled versions and so consumers have a better match for their needs. To capture the demand augmentation effect in a tractable way (that is, not interfering with the double-differentiation analysis), let us assume that users are ex ante identical.<sup>29</sup> At “search or set up cost”  $s$ , they adapt their technology to that covered by the two patents, and learn about their own locations in the two spaces. We assume that the hazard rate  $g/G$  is decreasing so as to guarantee the quasi-concavity of profit functions.

The following proposition, proved in the appendix, shows that a per-se rule against royalties charged to pool members may reduce welfare to the extent that it discourages the formation of an otherwise desirable pool:

**Proposition 8** *(i) There exists  $\underline{a}(t,u) \geq 0$  and  $\bar{a}(t,u)$  such that a pool increases patent-owners’ profit if and only if  $a \in [\underline{a}(t, u) , \bar{a}(t, u)]$ .*

*(ii) Firms may have too little incentive to form a pool if  $a$  is constrained to be equal to 0. In*

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<sup>27</sup>Alternatively, we could assume that cross-licenses reduce the firms’ marginal cost from, say,  $c_1$  to  $c_2 < c_1$ . The analysis is then unsurprising (and well-known). As long as the industry profit decreases with marginal cost (a reasonable assumption), the firms are willing to enter royalty-free cross-licensing arrangements. Royalties then unambiguously reduce welfare, unlike here.

<sup>28</sup>The results in Hausman et al. (2003) hold for arbitrary distributions. The assumption of uniform distributions is used here to show that the markup increases with differentiation, and is much stronger than needed.

<sup>29</sup>This simplification is also used in Hausman et al (2003).

*particular, a no-royalty pool never forms if the firms are little differentiated along the non-patent-related dimension.*

*(iii) The socially optimal royalty rate among those that induce the firms to form a pool is  $a = \underline{a}$ .*

## 7 Impact on innovation: the ex ante view

Until now, the welfare analysis has followed antitrust practice in taking the ex post perspective for reviewing the impact of a pool. But the prospect of being able to form a pool also affects the incentive to innovate.<sup>30</sup> So suppose that an innovator can bring about innovation  $n + 1$  at some investment cost  $I$ .<sup>31</sup> Does allowing pools increase innovation and welfare?

The answer to the first question is straightforward: Innovators enter a pool only if this increases their profit. So, the prospect of a pool raises individual profit and thereby encourages innovation. It is only in the extreme case in which the entrant has no bargaining power in the negotiation that the possibility of forming a pool does not boost innovation.

The impact of a pool on the ex ante welfare is more complex. As the industrial organization literature has repeatedly emphasized, markets can deliver too much or too little innovation, and so the impact of pools is likely to be ambiguous. The case of too much innovation in our context can be illustrated by assuming that  $n = 1$  (monopoly licensor initially) and that the entrant's innovation is a perfect substitute for the incumbent's. Hence, when pools are prohibited, the entrant faces the prospect of Bertrand competition and does not enter. Allowing pools then induces wasteful business stealing (or "entry for buyout," in the terminology of Rasmusen (1988)). As long as  $I$  is smaller than half of the monopoly profit ( $I < [\max PD(P - V(1))]/2$ ) and bargaining powers of the two parties are equal, the entrant enters, without any benefit for the consumers. Welfare is just reduced by  $I$ .

There is, however, some hope of being able to overcome the usual ambiguity as to the ex

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<sup>30</sup>Such an approach is in line with Denicolo (2002), who considers sequential innovation in a two-stage patent race model and argues that the prospect of an agreement between the owners of competing, sequential, but non-infringing patents increases investment in the second innovation and may raise welfare.

<sup>31</sup>Alternatively, one could look at  $n$  potential innovators, each considering whether to invest  $I$  to bring about their innovation.

ante welfare impact of policies. Intuitively, such business-stealing or entry-for-buyout excess innovation should be related to cases in which the innovation produces patents that are somewhat substitutable with the existing ones. Our analysis shows that the corresponding pools can be screened out by the requirement of independent licensing by pool members. In particular, *ex post welfare decreasing pools have no impact on innovation once independent licensing is allowed.*

We leave it to future research to provide a general answer to the question of whether ex post welfare enhancing pools are also ex ante welfare increasing. We content ourselves with the much more limited, yet interesting following result (proof available upon request from the authors):

**Proposition 9** (*ex ante view*).

*Suppose that  $n = 1$  and demand is linear (the distribution  $F$  of users' types is uniform). Then allowing a pool with independent licensing (if innovation occurs) never reduces and may increase welfare.*

## 8 Summary and discussion

The paper has built a tractable model of a patent portfolio, that allows for the full range of complementarity/substitutability. In the absence of pool, the demand margin binds if an increase in the license price of a patent leads to a reduction in the demand for the patent basket; the competition margin binds if it leads to the exclusion of the patent from the basket selected by users. Let us first summarize the main insights:

- a) A pool is more likely to be welfare-enhancing if patents are more complementary. That the demand margin binds in the absence of pool is a sufficient, but not a necessary, condition for a pool to be welfare-enhancing.
- b) A pool is never affected by the possibility of independent licensing if and only if the pool is welfare-enhancing. Furthermore, with only two patents, independent licensing always yields the same outcome as in the absence of a pool if the pool is welfare-decreasing in the absence of independent licenses. With more than two patents and a welfare-decreasing pool, there exists an independent-licensing equilibrium with the same outcome as in the absence of a pool.

c) The results generalize to a setting where the patents vary in importance. Furthermore, when one patent is dominant (so that the other patents cannot be used without it), a patent pool unambiguously boosts welfare.

d) When pool members are also downstream users, two additional concerns may emerge: pool members may be reluctant to create competitors by licensing to third parties, and licensors may seek to raise each others' cost by charging royalties. Regarding the first possibility, we show that the results generalize as long as licensing contracts takes the form of fixed fees and the licensed technology reduces downstream users' fixed costs. We consider the impact of royalties in a setting where there are no third-party licensees. We show that if royalties are not allowed, welfare may be harmed as otherwise-desirable pools are discouraged from being formed.

e) Allowing a pool encourages innovation. Determining the impact on ex ante social welfare, however, is likely to be much more difficult: as the literature has emphasized, competition can lead to too much or too little innovation. While we are unable to provide a general answer to this question, we are able to show that under certain conditions, allowing a pool with independent licensing never reduces and may increase ex ante welfare.

This paper is a first step in the analysis of pools comprised of patents that may not be perfect complements or perfect substitutes, and of the criteria that should be employed by competition authorities in their review of pools. Looking forward, our theoretical understanding of patent pools should be deepened in several directions:

- First, our assumption of separability of user preferences, while simplifying the analysis, focused it on package licensing and ruled out price discrimination through menus.
- Second, we have assumed an all-or-nothing pool. In practice, pools may be formed with a subset of the relevant patents, which raises the interesting issue of holdouts.
- Third, pools often seem to reflect equal-treatment preoccupations despite asymmetries in the importance of innovations, in the status of members (licensing and non-licensing owners), or in the ability to clone another member's innovation; theoretical work should

be devoted to the understanding of equal treatment in such circumstances.

- Last, one would want to compare the merits of pools and standard setting processes.

These and many other important questions related to pools lie outside the limited scope of this paper, which we hope will encourage research in these directions.

The phenomena that we have studied here extend well beyond patent pools. There are a large number of arrangements that share important elements with patent pools. In particular, the provision by multiple firms of a bundle of goods or services that are at least partially complementary is commonplace. At the same time, however, none of these arrangements exactly mimic the pool structure. In particular, it is rare to see a new centralized organization established to administer these arrangements. Rather, the arrangements are typically governed through a contract that spells out the key roles and responsibilities of each party.

To cite one example, airlines have undertaken collaborations since the 1960s, which involve the joint sale of seats. Today, these are known as code sharing arrangements. Essentially, a code sharing agreement allows each airline to sell seats on the other's planes. As part of the agreement, there is compensation paid from the one airline to the other, according to a negotiated formula. For instance, a passenger can buy a single ticket from Atlanta to Toulouse from either Delta or Air France, even if the Atlanta-to-Paris leg is on Delta and the Paris-to-Toulouse leg is on Air France. While the passenger could alternatively purchase both segments from respective airlines, the purchase price of the combined ticket is lower. (For a systematic analysis of the impact of these agreements on ticket pricing, see Brueckner and Whalen (2000).) In many cases, the airlines in these agreements have significant number of overlapping routes, making them both substitutes and complements with each other. In some cases, the two parties are free to sell seats at whatever price they see fit, which is similar to the independent licensing case discussed above.

To cite another example, newspapers will frequently offer package-advertising rates. In some cases, the papers are clearly complements: for instance, a number of independent suburban newspapers will allow an individual to place a classified advertisement selling a used car for a fraction of the cost of purchasing the advertisement in each of the papers individually. In

other cases, competing newspapers offer “package rates,” sometimes in conjunction with the establishment of a joint marketing department. For instance, in February 2002, the Mirror Group Newspapers, one of the national newspaper arms of Trinity Mirror Group, announced plans to form a joint advertising venture with the Telegraph Group. The joint venture would allow both groups to cut costs by combining the sales teams of the *Telegraph*, *Sunday Telegraph*, *Daily Mirror*, and *Sunday Mirror*. The group would sell ads both for the individual papers as well as jointly for the two papers. (Thus, there is no analog to independent licensing in this case.) One most common manifestation of such arrangements in the U.S. (with over two dozen formed) are “joint operating agreements” (JOAs) between newspapers that are directly competitive, where advertising is jointly marketed and the profits are split accord to a formula. An example is Seattle’s two newspapers – the Times and the Post-Intelligencer – who since 1983 have published separate newspapers with combined business, advertising and circulation operations. The two papers have separate editorial staffs that produce the news content of the two papers. The Times sells advertising and prints and distributes both papers (again, there is no analogy to independent licensing in this setting). All advertising is apparently sold as a “package,” and allocated according to a formula to the two papers. It is clear that in many of these agreements the newspapers involved are substitutes, though in some cases they are differentiated by the time of day at which the papers are published or by the demographic segments targeted by the papers.

These illustrative examples highlight the extent to which the issues examined in this paper extend beyond the confines of patent pools. It is our hope that our research will stimulate economic analyses of co-marketing agreements more generally.

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# Appendix

## Proof of Proposition 4

(i) Part (i) restates the definition of the demand and competition margins. Just note that all prices in  $D$ , if any, are equal since they must satisfy:

$$p_i D'(P - V(n)) + D(P - V(n)) = 0. \tag{A1}$$

(ii) In any equilibrium,  $p_i \leq w(n)$  for all  $i$ . But when  $V(\cdot)$  is concave,  $V(n) - V(m) \geq (n - m)w(m)$  and so:

$$V(n) - P \geq V(m(J)) - P_J \text{ for all } J.$$

And so a patent holder can charge up to  $w(n)$  without being evicted; hence  $z_i = w(n) = z(n)$  for all  $i$ . Now, use part (i). Let  $\tilde{p}_0$  denote the solution to (A1). Either  $\tilde{p}_0 < z(n)$  and then all licensors charge  $\tilde{p}_0$  (since  $z(n)D'(P - V(n)) + D(P - V(n)) < \tilde{p}_0 D'(P - V(n)) + D(P - V(n)) = 0$ ). Or  $\tilde{p}_0 \geq z(n)$  and then all firms charge  $z(n)$  (since they would be excluded otherwise). So any equilibrium must be symmetric.

(iii) This can be demonstrated by way of an example: Take  $n = 3$ ; licensors 1 and 2 charge  $p_L$  and licensor 3 charges  $p_H > p_L$ , where

$$V(1) - p_L = V(3) - 2p_L - p_H,$$

and

$$V(2) = V(1)$$

This is indeed an asymmetric equilibrium, in which the competition margin binds for all, as long as

$$p_H D'(P - V(3)) + D(P - V(3)) \geq 0$$

(which can be obtained by appropriately choosing the demand function).

(iv) Start from an asymmetric equilibrium with total price  $P$ . If  $D$  is non-empty, let  $\tilde{p}_0$  denote the solution to (A1); raise the prices of licensors constrained by the competition margin and lower those of licensors constrained by the demand margin, keeping the total price constant:

$$\delta p_i = \begin{cases} \Delta & \text{for } i \in C \\ -\frac{\#C}{\#D}\Delta & \text{for } i \in D \end{cases}$$

If no licensor is constrained by the demand margin, define  $D$  as the set of patents with the highest price and  $C$  as the complementary set and perform the same operation of reduction of price dispersion keeping total price constant. Note that for the highest price  $p_n$ ,  $p_n D^0 + D \geq 0$ . Let  $\tilde{p}_0 \equiv p_n$ . In either case, choose  $\Delta$  such that

$$\max_C \{p_i\} + \Delta = \tilde{p}_0 - \frac{\#C}{\#D}\Delta \equiv \tilde{p}_1.$$

Note that the new set of prices,  $\mathcal{P}_1$ , satisfies two properties:

- (a) no patent is evicted from the basket, since on the one hand, the total price  $P$  is unchanged, and on the other hand, the total price of the  $m$  cheapest patents has not decreased, for all  $m$ .
- (b)  $\tilde{p}_1 D^0 (P - V(n)) + D(P - V(n)) > 0$ .

Now iterate the process, with  $D$  augmented (and  $C$  reduced) by the patent holders whose price was previously  $\max_C \{p_i\}$ . One thus obtains sets of prices  $\mathcal{P}_1, \mathcal{P}_2, \dots$  that satisfy properties (a) and (b). In the final iteration, all prices are equal (and equal to  $P/n$ ) and from (a) and (b), satisfy:

$$(a) \quad V(n) - P \geq \max_m \left\{ V(m) - m \frac{P}{n} \right\} \iff Z(n) \geq P,$$

and

$$(b) \quad \frac{P}{n} D' (P - V(n)) + D (P - V(n)) > 0 \iff \hat{P} > P,$$

where  $\hat{P}$  was defined in section 3.

Since the symmetric equilibrium has total price  $\min \{ \hat{P}, Z(n) \}$ , and

$$P \leq \min \left\{ \widehat{P}, Z(n) \right\},$$

the asymmetric equilibrium is preferred by users to the symmetric one. ■

## Proof of Proposition 6

(i) Let  $P$  denote the total price in the absence of pool. Licensors unconstrained by the competition margin charge  $p_i$  satisfying

$$p_i D'(P - V(n)) + D(P - V(n)) = 0,$$

and so all such licensors charge the same price,  $\widehat{p}$  say. The others are constrained by the competition margin: They would like to raise their prices (in that  $p_i < \widehat{p}$ ), but cannot because their patent would be evicted from the basket chosen by users. They therefore charge  $z_i$  given by

$$V(n) - P = \max_{\{x \mid x_i = 0\}} \{V(\sum_j x_j n_j) - \sum_j x_j p_j\},$$

where  $x \equiv (x_1, \dots, x_n)$ .

(ii) If in the absence of pool,

$$p_n D'(P - V(n)) + D(P - V(n)) = 0,$$

then

$$(P - \sum_{j < n} p_j) D'(P - V(n)) + D(P - V(n)) = 0,$$

and hence

$$P D'(P - V(n)) + D(P - V(n)) < 0,$$

implying that  $P > P^*$ . So, a pool is welfare enhancing.

(iii) If  $V(\cdot)$  is concave, then by the same proof as in Proposition 4:

$$z_i = V(n) - V(n_{-i}),$$

where  $n_{-i} \equiv \sum_{j \neq i} n_j$ .

Because  $n_{-i}$  is (weakly) decreasing,  $z_i$  is (weakly) increasing. Together with (i), this implies that the most valuable patents command higher prices ( $p_1 \leq p_2 \leq \dots \leq p_n$ ).

To demonstrate uniqueness, define

$$MR(\hat{p}) \equiv \hat{p}D'(P(\hat{p}) - V(n)) + D(P(\hat{p}) - V(n)),$$

where

$$P(\hat{p}) \equiv \sum_{i=1}^{m(\hat{p})} z_i + [n - m(\hat{p})]\hat{p},$$

and  $m(\hat{p})$  is uniquely defined by

$$z_{m(\hat{p})} \leq \hat{p} < z_{m(\hat{p})+1} \quad (\text{and } m(\hat{p}) = 0 \text{ if } \hat{p} < z_1, \quad m(\hat{p}) = n \text{ if } \hat{p} > z_n).$$

$P(\hat{p})$  is continuously increasing and  $MR(\hat{p})$  continuously decreasing in  $\hat{p}$ . Hence, if  $MR(z_1) \leq 0$ , all licensors are constrained by the demand margin and their common price is given by  $MR(\hat{p}) = 0$  and satisfies  $\hat{p} < z_1 \leq z_2 \leq \dots \leq z_n$ . If  $MR(z_n) \geq 0$ , all licensors are constrained by the competition margin and  $p_i = z_i$  for all  $i$ . Last, if  $MR(z_1) > 0 > MR(z_n)$ , there is a unique  $\hat{p}$  in  $(z_1, z_n)$  satisfying  $MR(\hat{p}) = 0$ . Licensors  $i \leq m(\hat{p})$  are constrained by the competition margin and charge  $z_i$ , and the others charge  $\hat{p}$ .

(iv) Let  $P = \sum_i p_i > P^*$  denote an equilibrium total price in the absence of pool, and pick shares  $\alpha_i$  in the pool satisfying:

$$\alpha_i \geq \frac{p_i - (P - P^*)}{P^*}.$$

[This is doable as  $\Sigma_i \left( \frac{p_i - (P - P^*)}{P^*} \right) = \frac{nP^* - (n - 1)P}{P^*} < 1$ .]

First, note that  $\{p_i\}$  (the no-pool equilibrium) is a pure-strategy equilibrium of the independent licensing stage, in which independent licenses aren't sold. By definition,  $p_i$  is a best response to the other licensors' price vector  $p_{-i}$  in the absence of the pool's package offer. To upset this offer, licensor  $i$  would have to lower her price to  $p_i - (P - P^*)$  or below. She does not want to go below this level from the concavity of her objective function in the absence of pool. Licensor  $i$ 's therefore does not want to deviate as long as

$$[p_i - (P - P^*)] D(P^* - V(n)) \leq \alpha_i P^* D(P^* - V(n)),$$

which is guaranteed by the choice of stakes. Second, uniqueness (for such a choice of stakes) is guaranteed by the fact that a pure-strategy equilibrium in which  $P < P^*$ , and so the pool is upset, must, from the local optimality conditions, also be an equilibrium of the no-pool game. By assumption, all such equilibria involve total price above  $P^*$ , a contradiction.

Consider next the highest total price  $\bar{P} = \Sigma_i \bar{p}_i$  among equilibria in the absence of pool. And suppose  $\bar{P} < P^*$  and that the pool is trying to sustain  $P > \bar{P}$ . Then, we know from part (ii) that every licensor is competition constrained in (any) equilibrium in the absence of pool. Hence suppose that at the independent licensing substage each licensor  $i$  charges  $\bar{p}_i$ . As in the proof of Proposition 5 noone can raise its price and induce users to buy from the pool (and so stakes in the pool are irrelevant). Hence price  $P$  cannot be sustained and the pool is "weakly unstable". Last, for  $n = 2$ , there is a unique equilibrium; in this equilibrium, the licensors are constrained by the competition margin:

$$\bar{p}_i = z_i = V(2) - V(n_j).$$

Suppose the pool charges  $P > \bar{P}$ , and that the users buy from the pool. Then, licensor  $i$  obtains  $\alpha_i P D(P - V(2))$ . By charging (slightly below)  $p_i = P - (V(2) - V(n_j))$ , she could obtain  $p_i D(P - V(2))$ . Because  $z_1 + z_2 < P$ , one cannot find stakes  $\alpha_1$  and  $\alpha_2$  such that:

$$\alpha_i P D(P - V(2)) \geq [P - (V(2) - V(n_j))] D(P - V(2))$$

for  $i = 1, 2$ . Hence the pool is strongly unstable. ■

## Proof of Proposition 8

(a) *No pool.*

If  $x$  and  $y$  denote the locations of a consumer in the “natural” differentiation space and the technology space, and  $p_1$  and  $p_2$  denote the prices charged by the firms in the absence of pool, then the consumer selects firm 1 if and only if

$$p_1 + tx + u y \leq p_2 + t(1 - x) + u(1 - y).$$

The outcome is the Hotelling outcome  $(p^*, p^*)$  for marginal cost 0 and a differentiation whose distribution is equal to the convolution of the two distributions.<sup>32</sup> One has  $p^* > t$  (unless  $u = 0$ , in which case  $p^* = t$ ).

(b) *Pool.*

In case of a pool with royalty rate  $a$ , the opportunity cost of stealing a customer from one’s rival is equal to  $a$  (given that the dividend  $a/2$  accrues to the firm regardless of who serves the consumer). Each firm offers the patent 1- and patent 2-enabled versions and the unique price equilibrium is  $a + t$  (see Hausman et al. 2003). [This assumes that  $a$  is “not too large”, so that the two firms do compete for consumers. The analysis below extends when the royalty rate is large as it relies on the prices’ increasing in  $a$ .] Each firm charges a fee to consumers equal to the opportunity cost of acquiring the consumer plus the differentiation markup, and lets the consumers select the version that best suits them by not charging different prices for different versions.<sup>33</sup>

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<sup>32</sup>The resulting distribution can be represented by a variable  $X \in [0, 1 + \frac{u}{t}]$  and a transportation parameter  $t$ . The variable  $X$  has distribution given by  $L(X) \equiv \Pr(x + \frac{u}{t}y < X) = \int_0^1 K(X - \frac{u}{t}y) h(y) dy$ , where  $K$  and  $H$  denote the cumulative distributions (here, the identity on  $[0, 1]$ ) of variables  $x$  and  $y$ .

<sup>33</sup>The result is obvious when the two patents are incorporated in the good (say, a software) manufactured by the firms, which then do not offer multiple versions. The discussion above refers to the versioning case. Please see Hausman et al (2003) for a discussion of the intuition behind this result.

Letting  $v$  denote the gross surplus, a user spends the search cost  $s$  if and only if  $s \leq s^*$ , where

$$s_P^*(a+t) = v - E_{(x,y)} \left\{ \min_{\{i,j\}} \{a+t+t | x-x_i | + u | y-y_j | \} \right\}$$

under a pool, and

$$s_{NP}^*(p^*) = v - E_{(x,y)} \left\{ \min_{\{i\}} \{p^*+t | x-x_i | + u | y-y_i | \} \right\}$$

in the absence of a pool. The distribution of  $s$  in the population is given by the cumulative  $G(s)$ , and so total demand is  $G(s_P^*)$  under a pool and  $G(s_{NP}^*)$  in the absence of a pool. A pool creates a better fit and, keeping prices constant, increases demand. The per-firm profit is

$$\pi_P(a) = \frac{G(s_P^*(a+t))}{2} (a+t)$$

under a pool, and

$$\pi_{NP} = \frac{G(s_{NP}^*(p^*))}{2} p^*$$

in the absence of a pool.

The monotone hazard rate condition together with the linearity of  $s_P^*(\cdot)$  imply that  $\pi_P$  is concave. Last, note that  $s_P^*(p) > s_{NP}^*(p)$  for all  $p$  (a pool allows for better quality offers). ■