Energy taxes and oil price shock[1]

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Abstract

This paper examines if an energy price shock should be compensated by a reduction in energy taxes to mitigate its impact on consumer prices. Such an adjustment is often debated and advocated for redistributive reasons. Our investigation is based on a model that characterizes second-best optimal taxes in the presence of an externality generated by energy consumption. Energy is used by households as a consumption good and by the productive sector as an input. We calibrate this model on US data and proceed to simulations of this empirical model. We assume that energy prices are subject to an exogenous shock. For different levels of this shock, we calculate the optimal tax mix including income, commodity and energy taxes. We show that optimal energy taxes are affected by redistributive consideration and that optimal energy tax is *less* than the Pigouvian tax (marginal social damage). The difference is an implicit subsidy representing roughly 10% of the Pigouvian price. Interestingly, the simulations show that an variation in the energy price only has an almost negligible effect on this percentage. In other words, even a very large oil price increase will only have a small effect on the optimal tax on energy. Nevertheless, it appears that the energy tax is used to mitigate the impact of the energy shock. However, this result is not explained by redistributive consideration but by the fact that the Pigouvian tax (rate) decreases as the price of energy increases. This is a purely arithmetic adjustment due to the fact that the marginal social dammage does not change. Consequently, the marginal dammage as a percentage of the energy price (which defines the Pigouvian tax rate) decreases as the price increases.
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1 Introduction

As energy is heavily taxed in most industrialized countries, an "oil shock" (a sudden and significant increase in energy prices) often leads to political pressure, with various interest groups asking for tax reductions. This issue has been debated during that 2008 presidential campaign in the US, where both candidates made proposals for reducing the impact of the crude oil price increase on consumer prices. In France there has been a similar debate, with many interest groups (truck and fishery industry, agriculture, etc.) asking for an energy tax relief to mitigate the impact of the price increase.

Among the possible justifications of energy tax reductions is the idea that consumers have difficulties to adjust to strong and sudden price shocks. This is because existing technologies and equipment limit the substitution possibilities in the short run. However, this argument would at best lead to a temporary reduction to smooth the transition. Another argument is based on the alleged regressive character of energy consumption and taxes. The share of energy consumption in total spending tends to decrease with income. Consequently, low income individuals are affected more heavily by an oil price shock than the high income people and redistributive concern may then plead for an energy tax reduction.

This paper studies the validity of this redistributive argument. Our investigation is based on the model of optimal emission taxation developed by Cremer *et al.* (1998, 2003 and 2010). It allows us to derive second-best optimal taxes in the presence of an externality generated by energy consumption. Energy is used by households as a consumption good and by the productive sector as an input. With some modification this model can be adapted to study the impact of an exogenous shock in the before tax price of energy. We calibrate this model on US data and proceed to simulations of this empirical model. We assume that energy prices are subject to an exogenous shock. For different levels of this shock, we calculate the optimal tax mix including income, commodity and energy taxes. We show that optimal energy taxes are indeed affected by redistributive consideration and that optimal energy tax is *less* than the Pigouvian tax (marginal social damage). The difference is an implicit subsidy representing roughly 10% of the Pigouvian price. Interestingly, the simulations show that an variation in the energy price only has an almost negligible effect on this percentage. In other words, even a very large oil price increase will only have a small effect on the optimal tax on energy. Nevertheless, it appears that the energy tax is used to mitigate the impact of the energy shock. However, this result is not explained by redistributive consideration but by the fact that the Pigouvian tax (rate) decreases as the price of energy increases. This is a purely arithmetic adjustment due to the fact that the marginal social damage does not change. Consequently, the marginal damage as a percentage of the energy price (which defines the Pigouvian tax rate) decreases as the price increases.

2 The model

Consider an open economy which uses "energy" both as a consumption good and as input. Whether used as a consumption good or as a factor input, energy, which is imported from overseas, is polluting. Apart from energy, consumers consume a nonpolluting good as well. This latter good is produced in the economy using capital and labor (in addition to energy). Capital services are also rented from outside. Labor is the only factor of production which is supplied domestically. Labor is heterogeneous with different groups of individuals having different productivity levels and different tastes. Denote a person's type by j, his productivity factor by n^j, and the proportion of people of type j in the economy by π^j (where the population size is normalized at one). Preferences of a j-type person depend on his consumption of non-polluting goods, x^j, consumption of polluting goods, y^j, labor supply, L^j, and the total level of emissions in the atmosphere, E.

This construct is a slight variation of Cremer *et al.*'s (2010) model. To make this paper self-contained, we first review its main features.[1]

2.1 Preferences

Consumers' preferences are nested CES, first in goods and labor supply and then in the two categories of consumer goods. All consumer types have identical elasticities of substitution between leisure and non-leisure goods, ρ, and between polluting and non-polluting goods, ω. Differences in tastes are captured by differences in other parameter values of the posited utility function (a^j and b^j in equations (2)–(3) below). Assume further that emissions enter the utility function linearly. The preferences for a person of type j can then be represented by

$$\mho^j = \boldsymbol{U}(x, y, L^j; \theta^j) - \phi E, \qquad j = 1, 2, 3, 4, \tag{1}$$

where θ^j reflects the "taste parameter" and

$$\boldsymbol{U}(x, y, L^j, \theta^j) = \left(b^j Q^{j\,\frac{\rho-1}{\rho}} + (1 - b^j)(1 - L^j)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}, \tag{2}$$

$$Q^j = \left(a^j x^{\frac{\omega-1}{\omega}} + (1 - a^j) y^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}}. \tag{3}$$

Consumers choose their consumption bundles by maximizing (1)–(3) subject to their budget constraints. These will be nonlinear functions when the income tax schedule is nonlinear. However, for the purpose of uniformity in exposition, we characterize the consumers' choices, as the solution to an optimization problem in which each person faces a (type-specific) *linearized and possibly truncated* budget constraint. To do this, introduce a "virtual income," G^j, into each type's budget constraint. Denote the j-type's net of tax wage by w_n^j. We can then write j's budget constraint as

$$px^j + qy^j = G^j + M^j + w_n^j L^j, \tag{4}$$

where p and q are the consumer prices of x and y, G^j is the income adjustment term (virtual income) needed for linearizing the budget constraint (or the lump-sum rebate if

[1] For more details, see Cremer *et al.* (1998, 2003 and 2010).

the tax function is linear), and M^j is the individual's exogenous income. The first-order conditions for a j-type's optimization problem are

$$\frac{1-a^j}{a^j}\left(\frac{x^j}{y^j}\right)^{\frac{1}{\omega}} = \frac{q}{p}, \tag{5}$$

$$\frac{(1-b^j)\left(x^j/(1-L^j)\right)^{\frac{1}{\rho}}}{a^j b^j \left[a^j + (1-a^j)(x^j/y^j)^{\frac{1-\omega}{\omega}}\right]^{\frac{\omega-\rho}{\rho(1-\omega)}}} = \frac{w_n^j}{p}. \tag{6}$$

Equations (4)–(6) determine x^j, y^j and L^j as functions of p, q, w_n^j and $G^j + M^j$.

2.2 Production technology

The production process uses three inputs: capital, K, labor, L and energy, D. The technology of production is represented by a nested CES,

$$O = \boldsymbol{O}(L,K,D) = B\left[(1-\beta)\,L^{\frac{\sigma-1}{\sigma}} + \beta\Gamma^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{7}$$

$$\Gamma = A\left[\alpha K^{\frac{\delta-1}{\delta}} + (1-\alpha)\,D^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta}{\delta-1}}, \tag{8}$$

where A and B are constants, σ and δ represent the elasticities of substitution between L and Γ and between K and D (given Γ) respectively. Substituting (8) in (7) yields,

$$O = B\left[(1-\beta)\,L^{\frac{\sigma-1}{\sigma}} + \beta A^{\frac{\sigma-1}{\sigma}}\left[\alpha K^{\frac{\delta-1}{\delta}} + (1-\delta)\,D^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta(\sigma-1)}{\sigma(\delta-1)}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{9}$$

Aggregate output, O, is the numeraire and the units of x and y are chosen such that their producer prices are equal to one.

Capital services and energy inputs are imported at constant world prices of r and p_E where the units of D is chosen such that initially $p_E = 1$. Let w denotes the price of one unit of *effective labor*, τ_D denotes the tax on energy input, and assume that there are no *producer taxes* on labor and capital.[2] The first-order conditions for the firms'

[2] Taxation of capital in a setting like ours will serve no purpose except to violate production efficiency.

input-hiring decisions are, assuming competitive markets,

$$\boldsymbol{O}_L(L,K,D) = w, \tag{10}$$

$$\boldsymbol{O}_K(L,K,D) = r, \tag{11}$$

$$\boldsymbol{O}_D(L,K,D) = p_E(1+\tau_D). \tag{12}$$

Equations (9)–(12) determine the equilibrium values of O, L, K and D as functions of w, r and $p_E(1+\tau_D)$ [where r and p_E are determined according to world prices].

As different types of people have different productivities, labor is an heterogeneous factor of production. When a j-type person with productivity n^j works for L^j hours, his effective labor is n^jL^j resulting in aggregate supply $\sum_{j=1}^{4}\pi^jn^jL^j$. Equating this with aggregate demand gives,

$$L = \sum_{j=1}^{4}\pi^jn^jL^j$$

Total emissions are given by,

$$E = \sum_{j=1}^{4}\pi^jy^j + D$$

where π^j is the proportion of people of type j in the economy.[3]

2.3 Optimal tax policy

The optimal tax policy maximizes an iso-elastic social welfare function

$$W = \frac{1}{1-\eta}\sum_{j=1}^{4}\pi^j(\mho^j)^{1-\eta} \qquad \eta \neq 1 \quad \text{and} \quad 0 \leq \eta < \infty, \tag{13}$$

where η is the "inequality aversion index". The value of η dictates the desired degree of redistribution in the economy: The higher is η the more the society cares about equality,

[3] The population size is normalized to 1, consequently $\sum_{j=1}^{4}\pi^jy^j$ represents total households' energy consumption.

here we retain a relatively low value, $\eta = 0.1$.[4]

The feasibility of tax instruments depends on information available to the tax administration. Generally, this information allows for linear commodity taxes and non-linear income tax. This is why we restrict our analysis to this case even if other possibilities could be considered.[5] Under linear commodity taxation, all consumers face the same commodity prices. The social welfare function (13) must thus be written as a function of the prices of goods.

Denote c^j the after-tax income (outlay) of a j-type household. Maximizing, the utility function (1) with respect to the budget constraint

$$px^j + qy^j = c^j,$$

we obtain the demand functions for x^j and y^j as $x^j = \boldsymbol{x}\left(p, q, c^j; \theta^j\right)$ and $y^j = \boldsymbol{y}\left(p, q, c^j; \theta^j\right)$. Substituting these equations in the j-type person utility function (1), we have

$$\boldsymbol{V}\left(p, q, c^j, \frac{I^j}{wn^j}; \theta^j\right) = \boldsymbol{U}\left(\boldsymbol{x}\left(p, q, c^j; \theta^j\right), \boldsymbol{y}\left(p, q, c^j; \theta^j\right), \frac{I^j}{wn^j}, \theta^j\right)$$

where

$$I^j \equiv wn^j L^j$$

We have four feasible tax instruments in our model: two commodity taxes, an input tax and an income tax. As the demand functions for goods and the labor supply function are all homogeneous of degree zero, there is no loss of generality when setting one tax rate to zero. Since energy consumption creates an externality we choose to impose a zero tax on non-energy goods.

[4] As is well-known, $\eta = 0$ implies a utilitarian social welfare function and $\eta \rightarrow \infty$ a Rawlsian. The value we use is chosen according to the observed degree of redistribution of existing tax systems; see Bourguignon and Spadaro (2000).

[5] In Cremer and al. (2008) differents possibilities are examined including the case all taxes are non-linear.

The optimal tax structure is derived as the solution to

$$\max_{q,c^j,I^j,K,D,w} \frac{1}{1-\eta}\sum_{j=1}^{4}\pi^j\left[\boldsymbol{V}\left(p,q,c^j,\frac{I^j}{wn^j};\theta^j\right)-\phi\sum_{j=1}^{4}\pi^j\boldsymbol{y}\left(p,q,c^j;\theta^j\right)-\phi D\right]^{1-\eta} \tag{14}$$

under the resource constraint,

$$\boldsymbol{O}(L,K,D)-\sum_{j=1}^{4}\pi^j\left(\boldsymbol{x}p,q,c^j;\theta^j\right)-rK-p_E\left[\sum_{j=1}^{4}\pi^j\boldsymbol{y}\left(p,q,c^j;\theta^j\right)+D\right]-\bar{R}\geq 0, \tag{15}$$

the incentive compatibility constraints,

$$\mathbf{V}\left(p,q,c^j,\frac{I^j}{wn^j};\theta^j\right)\geq\mathbf{V}\left(p,q,c^k,\frac{I^k}{wn^j};\theta^j\right) \tag{16}$$

the endogeneity of wage condition,

$$w-\boldsymbol{O}_L\left(L,K,D\right)=0 \tag{17}$$

with

$$L=\sum_{j=1}^{4}\pi^j n^j L^j=\sum_{j=1}^{4}\pi^j\frac{I^j}{w}$$

The analytical results of Cremer *et al.* (2010) can easily be extended to show that the optimal tax on energy inputs (τ_D) is Pigouvian and equal to its marginal social damage of emissions. The optimal tax on the *consumption* of energy, on the other hand, is generally different from its Pigouvian level. See the Appendix

3 Data and calibration

To solve our model numerically, one must know the values of the parameters of the utility functions (ρ, ω, a^j, b^j, a^j, ϕ), and the values of the parameters of the production function (σ, δ, α, β, A, B). The data sources are the PSID (Panel Study of Income Dynamics, web site: http://psidonline.isr.umich.edu), US Bureau of Labor Statistics (web site: http://www.bls.gov/) and the US Bureau of Economic Analysis. The two

	Managers & professionals (Type 1)	Technical sales & clerical workers (Type 2)	Service workers, operators, fabricators & laborers (Type 3)	Construction workers & mechanics (Type 4)
π	35.18 %	28.90 %	28.86 %	7.06 %
I	68712	40147	31887	44111
px	51134	34742	29155	37498
qy	3051	2612	2520	3100
n	1.33620	0.90094	0.71472	0.88815
L	0.50731	0.43961	0.44015	0.48998
t	28.0 %	15.0 %	15.0 %	15.0 %
G	9797	2195	2280	2363
M	-5085	1034	2290	741
a	0.99997	0.99993	0.99989	0.99991
b	0.53201	0.39970	0.39438	0.46747
Type-independent figures				
	$\sum_j \pi^j n^j L^j = 0.47446$	$K = 3169954$	$D = 490364$	$O = 1$
	$p_O = 1.0$	$w = 101364$	$r = 4.2$ %	$p_D = 1.0$
	$p = 1.00000$	$q = 1.00000$	$\sigma = 0.8$	$\delta = 0.42141$
	$\rho = 0.66490$	$\omega = 0.26892$	$\alpha = 0.98662$	$\beta = 0.54242$
	$A = 1.28395$	$B = 0.74215$		

Table 1: Calibration: main parameters. Monetary figures are in USD

first one gives data on households' consumption, income and labor. The latter reports macroeconomic data from the EUKLEMS data base on capital, labor and energy. The calibration process follows the one we have used in our previous paper (see Cremer and al. (2008)). The data allows us to identify four types of households, "managers and professionals" (type 1), "technical sales and clerical workers" (type 2), "service workers, operators, fabricators and laborers" (type 3) and "construction workers and mechanics" (type 4),

Table 1 provides a summary of the data and parameter values. Finally, our optimal tax calculations are based on the assumption that the government's external revenue requirement (share in GDP of expenditures on non-transfer payments) remains unchanged.

4 Results I

Optimal energy taxes/subsidies in different scenarios are determined by solving the calibrated version of the model. There are two forces at work. One is Pigouvian in nature. To correct for the marginal social damage of emissions, one wants to impose a correcting tax on energy. In case of energy inputs, this is the only force at work. Another reason comes into play in case of energy consumption goods. This arises because of the distributional considerations. Because the share of energy expenditures tends to decrease with one's income, one may want to subsidize energy consumption goods to offset this regressive bias. It is true that an optimally designed income tax mitigates this regressive bias, but in a world of asymmetric information (where first best lump-sum taxes are unavailable), it cannot eliminate it completely (as long as Atkinson and Stiglitz Theorem does not apply). There still remains a role for energy subsidies; see Cremer *et al.* (1998), (2003) and (2010).

For the time being, we assume that the environmental damage (cost of carbon emissions) is represented by $\phi = 0.24$. This is the level used by Cremer *et al.* (2003) and corresponds to a Pigouvian tax of 50%.

In the case of inputs, the optimal energy tax, expressed in units of the numeraire output, is equal to

$$\tau^{pig} = \left[\boldsymbol{V}\left(p, q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \boldsymbol{y}\left(p, q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \frac{\phi}{\mu},$$

where the term on the right hand side of equation (18) is the Pigouvian tax (the marginal social damage of emission).[6] When expressed as a percentage of the energy price, this tax rate decreases with the price of energy. Specifically, we have

[6] This is an implicit expression. Setting the tax at its Pigouvian level affects all the other taxes and the entire allocation. To deal with this in a consistent way we calculate the Pigouvian tax by solving our general problem to which we add (18) as a constraint.

p_E	τ^{pig}	$\frac{\tau^{pig}}{p_E}$
1.0	0.4823	48,23%
1.1	0.4769	43,35%
1.2	0.4718	39,32%
1.3	0.4668	35,91%
1.4	0.4622	33,01%
1.5	0.4577	30,51%
1.6	0.4534	28,34%
1.7	0.4493	26,43%
1.8	0.4454	24,74%
1.9	0.4416	23,24%
2.0	0.4380	21,90%

Table 2: Energy input tax as a percentage of the world price of energy ($\phi = 0.24$)

In the case of energy consumption goods, both forces are at work. To measure the relative importance of Pigouvian and redistributive considerations in the calculation of optimal energy taxes/subsidies, as the world price of energy changes, we start by defining the "Pigouvian" price. Using Cremer *et al.*'s (1998) definition of the Pigouvian tax, this is defined by

$$q^{pig} = p_E + \tau^{pig}. \tag{18}$$

As shown in the Appendix, the optimal energy tax is then given by

$$q = q^{pig} + \frac{\sum_{j=1}^{4} \sum_{k \neq j} \lambda^{kj} \left\{ \mathbf{V}_c \left(q, c^j, \frac{I^j}{wn^k}; \theta^k \right) \left[\mathbf{y} \left(q, c^j; \theta^j \right) - \mathbf{y} \left(q, c^j; \theta^k \right) \right] \right\}}{\mu \sum_{j=1}^{4} \pi^j \widetilde{\mathbf{y}}_q \left(q, c^j; \theta^j \right)}.$$

We have

Finally, we have

$$\begin{aligned} \frac{q - p_E}{p_E} &= \frac{\left(q - q^{pig}\right) + \left(q^{pig} - p_E\right)}{p_E} \\ &= \frac{q - q^{pig}}{p_E} + \frac{\tau^{pig}}{p_E}, \end{aligned}$$

with the following calculations for $\left(q - p_E\right)/p_E$.

p_E	q^{pig}	q	$q - q^{pig}$	$(q - q^{pig})/p_E$
1.0	1.4823	1.3359	-0.1464	-14.64%
1.1	1.5769	1.4210	-0.1559	-14.17%
1.2	1.6718	1.5064	-0.1654	-13.78%
1.3	1.7668	1.5920	-0.1748	-13.45%
1.4	1.8622	1.6777	-0.1845	-13.18%
1.5	1.9577	1.7637	-0.1940	-12.93%
1.6	2.0534	1.8499	-0.2035	-12.72%
1.7	2.1493	1.9362	-0.2131	-12.54%
1.8	2.2454	2.0226	-0.2228	-12.38%
1.9	2.3416	2.1092	-0.2324	-12.23%
2.0	2.4380	2.1960	-0.2420	-12.10%

Table 3: Redistributive subsidy on households' energy price when $\phi = 0.24$

5 Results II

In the absence of redistributive considerations, the energy tax should be Pigouvian. The difference between the Pigouvian tax and the optimal tax is thus the implicit subsidy needed when consumers have heterogeneous preferences and productivities. Table 2, gives the optimal consumer price for energy (q) when there is no externality ($\phi = 0$) so that the Pigouvian tax is zero. This price is calculated for different values of the world price of energy (p_E), with the benchmark price normalized at one so that a price of 2 corresponds to an oil price shock of 100%. The difference between p_E and q is the implicit subsidy on energy. These results are also represented in Figure 1.

The difference between these two prices is the implicit subsidy allowed by the government for redistributive purposes. This subsidy is shown to be (roughly) equal to 10% of p_E whatever the level of this price. In other words, redistributive considerations call for a subsidy that is not increasing at the same rate as the world price of energy. Consequently an exogenous shock on the international price of energy, let us say an oil shock, is *not* compensated by an increase of the implicit subsidy (see also figure 1).

Table 3 presents the result with a positive externality ($\phi = 0.05$). As Figure 2 shows,

p_E	q	$p_E - q$	Implicit subsidy
1.0	0.8993	0.1007	10.07 %
1.1	0.9892	0.1108	10.07 %
1.2	1.0791	0.1209	10.07 %
1.3	1.1690	0.1310	10.07 %
1.4	1.2589	0.1411	10.08 %
1.5	1.3488	0.1512	10.08 %
1.6	1.4387	0.1613	10.08 %
1.7	1.5287	0.1713	10.08 %
1.8	1.6186	0.1814	10.08 %
1.9	1.7085	0.1915	10.08 %
2.0	1.7984	0.2016	10.08 %

Table 4: Implicit subsidy on households' energy price when $\phi = 0$

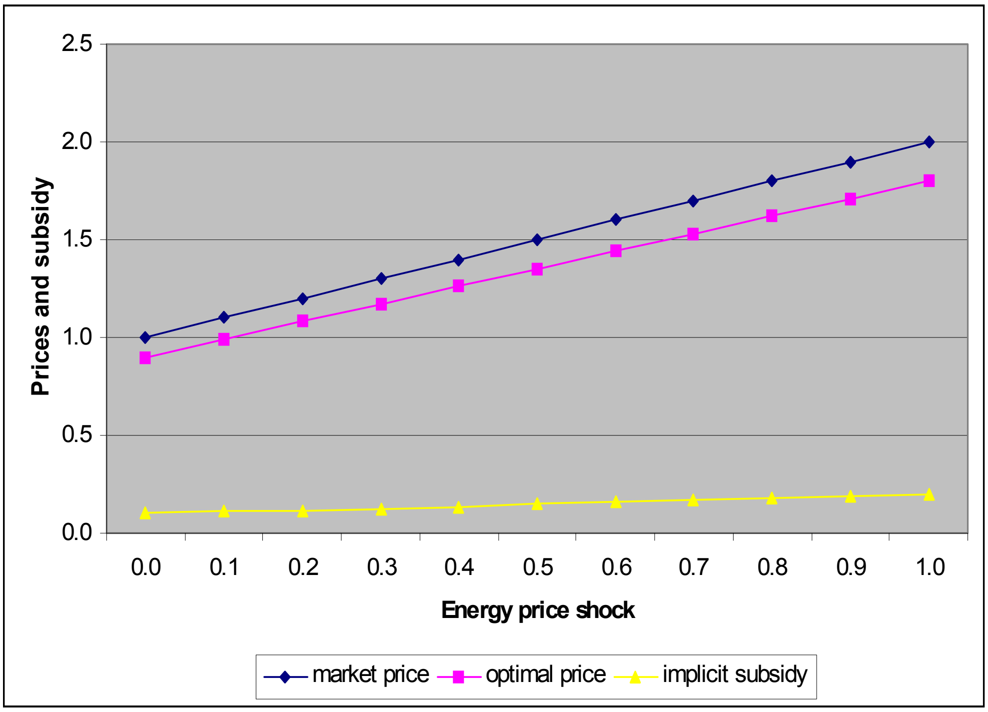

Figure 1: Prices and subsidy when $\phi = 0$

p_E	q^{pig}	q	τ^{pig}	$q - p_E$	$\frac{q-p_E}{p_E}$	$\frac{\Delta q}{q_0}$	Implicit subsidy
1.0	1.4823	1.3359	0.4823	0.3359	33.59 %		9.88%
1.1	1.5769	1.4210	0.4769	0.3210	29.19 %	6,37%	9.88%
1.2	1.6718	1.5064	0.4718	0.3064	25.53 %	12,76%	9.89%
1.3	1.7668	1.5920	0.4668	0.2920	22.46 %	19,17%	9.90%
1.4	1.8622	1.6777	0.4622	0.2777	19.84 %	25,59%	9.90%
1.5	1.9577	1.7637	0.4577	0.2637	17.58 %	32,02%	9.91%
1.6	2.0534	1.8499	0.4534	0.2499	15.62 %	38,48%	9.91%
1.7	2.1493	1.9362	0.4493	0.2362	13.89 %	44,94%	9.92%
1.8	2.2454	2.0226	0.4454	0.2226	12.37 %	51,40%	9.92%
1.9	2.3416	2.1092	0.4416	0.2092	11.01 %	57,89%	9.92%
2.0	2.4380	2.1960	0.4380	0.1960	9.80 %	64,38%	9.93%

Table 5: Implicit subsidy on households' energy price when $\phi = 0.05$

we have a very similar result regarding the implicit subsidy which remains at a level of about 10% of the Pigouvian price q^{pig}.

However, the Table also points to a number of new and very interesting results. First, the consumer price of energy increase at a significantly slower speed than the producer price (64% vs. 100%). Consequently, we do get the result that the tax is used to mitigate the impact of the energy price increase on consumers. Second, the per-unit tax decreases only very slightly. Third, and most interestingly the driving force in the adjustment appears to be the *decrease* in the Pigouvian tax

$$\tau^{pig} = \frac{q^{pig} - p_E}{p_E} = \frac{(p_E + \varphi'/\mu) - p_E}{p_E} = \frac{\varphi'/\mu}{p_E}.$$

This makes a lot of sense. The marginal social damage of the externality φ'/μ does not directly depend on the price of energy. Consequently, as p_E increase the Pigouvian tax *rate* quite naturally decreases.

To sum up, we find support for the argument that energy taxes should be used to alleviate the impact of an oil shock on consumers. However, the underlying reason is not of redistributive nature (the redistributive term remain more or less constant).

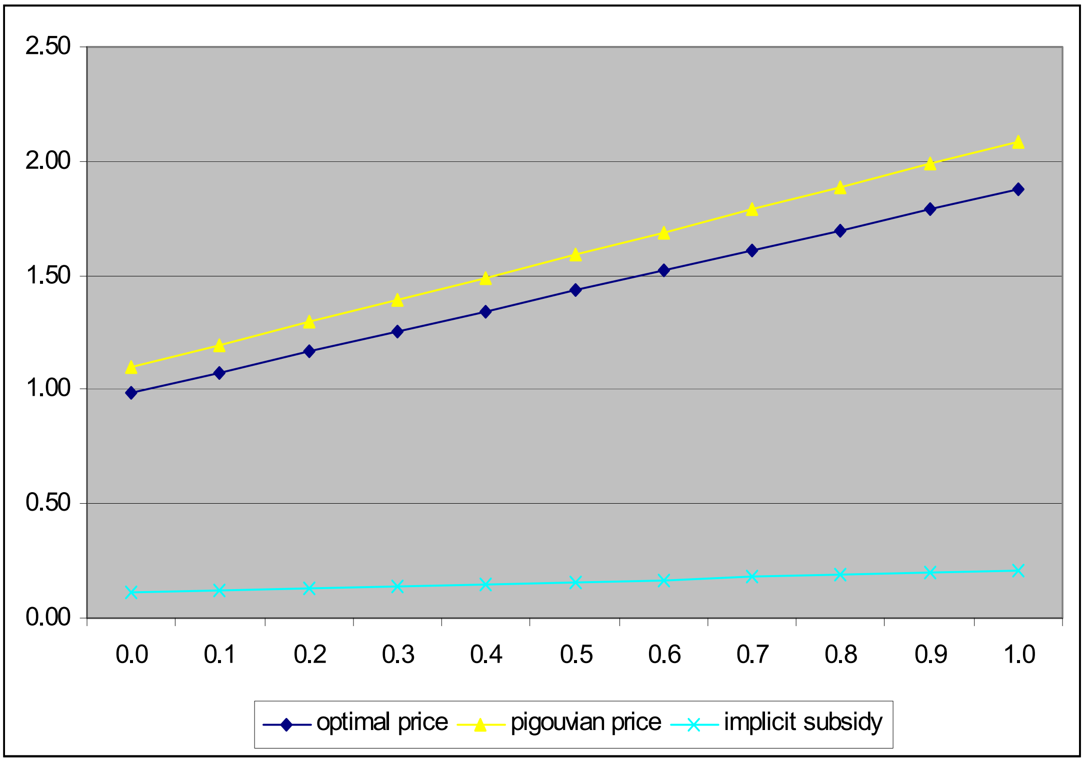

Figure 2: Prices and subsidy when $\phi = 0.24$

The crucial element is an essentially purely arithmetic adjustment of the Pigouvian tax *rate*. Still, at the end of the day it remains that energy taxes ought to be adjusted so that consumers do not face the full impact of the world price increase in energy prices. Observe that producers who face a Pigouvian tax rate benefit from a similar relief.

6 Conclusion

This paper examines if an energy price shock should be compensated by a reduction in energy taxes to mitigate its impact on consumer prices. Such an adjustment is often debated and advocated for redistributive reasons. Our investigation is based on a model on the model of optimal emission taxation developed by Cremer *et al.* (1998, 2003 and 2010). It characterizes second-best optimal taxes in the presence of an externality generated by energy consumption. Energy is used by households as a consumption good and by the productive sector as an input. We have shown that with some modification this model can be adapted to study the impact of an exogenous shock in the before tax price of energy. We have calibrated this model on US data and proceed with simulations of this empirical model. We have assumed that energy prices are subject to an exogenous shock. For different levels of this shock, we have calculated the optimal tax mix including income, commodity and energy taxes. We show that optimal energy taxes are indeed affected by redistributive consideration and that optimal energy tax is *less* than the Pigouvian tax (marginal social damage). The difference is an implicit subsidy representing roughly 10% of the Pigouvian price. Interestingly, the simulations show that an variation in the energy price only has an almost negligible effect on this percentage. In other words, even a very large oil price increase will only have a small effect on the optimal tax on energy and the consumer price ought to increase (roughly) at the same rate as the world market producer price. Nevertheless, it appears that the energy tax is used to mitigate the impact of the energy shock. Specifically, when the world price of energy doubles, the consumer price (including taxes) only increases by

64%. However, this result is not explained by redistributive consideration but by the fact that the Pigouvian tax (rate) decreases as the price of energy increases. This is a purely arithmetic adjustment due to the fact that the marginal social damage does not change. Consequently, the marginal damage as a percentage of the energy price (which defines the Pigouvian tax rate) decreases as the price increases.

Appendix

A1 General income plus linear commodity taxes

The Lagrangian for the second-best problem is (where p is set equal to 1),

$$\pounds = \frac{1}{1-\eta}\sum_{j=1}^{4}\pi^{j}\left[\mathbf{V}\left(q,c^{j},\frac{I^{j}}{wn^{j}};\theta^{j}\right)-\phi\sum_{j=1}^{4}\pi^{j}\mathbf{y}\left(q,c^{j};\theta^{j}\right)-\phi D\right]^{1-\eta}+$$
$$\mu\left\{\mathbf{O}\left(K,L,D\right)-\sum_{j=1}^{4}\pi^{j}\mathbf{x}\left(q,c^{j};\theta^{j}\right)-rK-p_{E}\left[\sum_{j=1}^{4}\pi^{j}\mathbf{y}\left(q,c^{j};\theta^{j}\right)+D\right]-\overline{R}\right\}+$$
$$\sum_{j}\sum_{k\neq j}\lambda^{jk}\left[\mathbf{V}\left(q,c^{j},\frac{I^{j}}{wn^{j}};\theta^{j}\right)-\mathbf{V}\left(q,c^{k},\frac{I^{k}}{wn^{j}};\theta^{j}\right)\right]+\gamma\left[w-\mathbf{O}_{L}\left(K,D,L\right)\right]. \tag{A1}$$

where μ, λ^{jk} and γ are the multipliers associated respectively with the resource constraints, the incentive constraint and the endogenous wage condition. The first-order conditions are, for $j = 1, 2, 3, 4$,

$$\begin{aligned}
\frac{\partial \pounds}{\partial q} &= \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \times \\
&\quad \left[\mathbf{V}_q\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}_q\left(q, c^j; \theta^j\right) \right] - \mu \sum_{j=1}^{4} \pi^j \left[\mathbf{x}_q\left(q, c^j; \theta^j\right) + p_E \mathbf{y}_q\left(q, c^j; \theta^j\right) \right] + \\
&\quad \sum_{j=1}^{4} \sum_{k \neq j} \lambda^{jk} \left[\mathbf{V}_q\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \mathbf{V}_q\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right) \right] = 0, \qquad \text{(A2)}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pounds}{\partial c^j} &= \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \\
&\quad \phi \pi^j \mathbf{y}_c\left(q, c^j; \theta^j\right) \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} - \\
&\quad \mu \pi^j \left[\mathbf{x}_c\left(q, c^j; \theta^j\right) + p_E \mathbf{y}_c\left(q, c^j; \theta^j\right) \right] + \sum_{k \neq j} \lambda^{jk} \mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \\
&\quad \sum_{k \neq j} \lambda^{kj} \mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) = 0, \qquad \text{(A3)}
\end{aligned}$$

$$\frac{\partial \pounds}{\partial I^j} = \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \frac{1}{wn^j} \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) +$$
$$\mu \mathbf{O}_L(L,K,D)\frac{\pi^j}{w} + \sum_{k\neq j} \lambda^{jk} \frac{1}{wn^j} \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) -$$
$$\sum_{k\neq j} \lambda^{kj} \frac{1}{wn^k} \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) - \gamma \frac{\pi^j}{w} \mathbf{O}_{LL}(L,K,D) = 0, \quad \text{(A4)}$$
$$\frac{\partial \pounds}{\partial D} = -\sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \phi +$$
$$\mu \left[\mathbf{O}_D(L,K,D) - p_E \right] - \gamma \mathbf{O}_{LD}(L,K,D) = 0, \quad \text{(A5)}$$
$$\frac{\partial \pounds}{\partial K} = \mu \left[\mathbf{O}_K(L,K,D) - r \right] - \gamma \mathbf{O}_{LK}(L,K,D) = 0, \quad \text{(A6)}$$
$$\frac{\partial \pounds}{\partial w} = \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \left(\frac{-I^j}{n^j w^2}\right) \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) +$$
$$\mu \mathbf{O}_L(L,K,D)\frac{-1}{w^2} \sum_{j=1}^{4} \pi^j I^j + \sum_{j=1}^{4} \sum_{k\neq j} \lambda^{jk} \left(\frac{-I^j}{n^j w^2}\right) \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) +$$
$$\sum_{j=1}^{4} \sum_{k\neq j} \lambda^{jk} \left(\frac{I^k}{n^j w^2}\right) \mathbf{V}_L\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right) + \gamma \left[1 + \frac{1}{w^2} \sum_{j=1}^{4} \pi^j I^j \mathbf{O}_{LL}(L,K,D) \right] = 0. \quad \text{(A7)}$$

We now show that whereas the optimal tax on the polluting good is non-Pigouvian, the optimal tax on polluting input is Pigouvian. Consider first the polluting good tax. We have:

Proposition A1 *The optimal tax on the polluting good is non-Pigouvian.*

Proof. Multiply equation (A3) by $\mathbf{y}\left(q, c^j; \theta^j\right)$, sum over j, and add the resulting equation to (A2). Simplifying, using Roy's identity, results in

$$
\begin{aligned}
& -\phi \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \times \\
& \left[\sum_{j=1}^{4} \pi^j \left[\mathbf{y}_q\left(q, c^j; \theta^j\right) + \mathbf{y}\left(q, c^j; \theta^j\right) \mathbf{y}_c\left(q, c^j; \theta^j\right) \right] \right] \\
& -\mu \sum_{j=1}^{4} \pi^j \left[\mathbf{x}_q\left(q, c^j; \theta^j\right) + \mathbf{y}\left(q, c^j; \theta^j\right) \mathbf{x}_c\left(q, c^j; \theta^j\right) \right] \\
& -\mu p_E \sum_{j=1}^{4} \pi^j \left[\mathbf{y}_q\left(q, c^j; \theta^j\right) + \mathbf{y}\left(q, c^j; \theta^j\right) \mathbf{y}_c\left(q, c^j; \theta^j\right) \right] \\
& -\sum_{j=1}^{4} \sum_{k \neq j} \left[\lambda^{kj} \mathbf{y}\left(q, c^j; \theta^j\right) \mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) + \lambda^{jk} \mathbf{V}_q\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right) \right] = 0 \qquad \text{(A8)}
\end{aligned}
$$

To simplify equation (A8), partially differentiate the j-type individual's budget constraint, $\mathbf{x}\left(q, c^j; \theta^j\right) + q\mathbf{y}\left(q, c^j; \theta^j\right) = c^j$, once with respect to c^j and once with respect to q. This yields

$$
\begin{aligned}
\mathbf{x}_c\left(q, c^j; \theta^j\right) + q\mathbf{y}_c\left(q, c^j; \theta^j\right) &= 1, & \text{(A9)} \\
\mathbf{x}_q\left(q, c^j; \theta^j\right) + q\mathbf{y}_q\left(q, c^j; \theta^j\right) &= -\mathbf{y}\left(q, c^j; \theta^j\right). & \text{(A10)}
\end{aligned}
$$

Multiply equation (A9) by $\mathbf{y}\left(q, c^j; \theta^j\right)$ and add the resulting equation to equation (A10). We get

$$
\begin{aligned}
& \mathbf{x}_q\left(q, c^j; \theta^j\right) + \mathbf{y}\left(q, c^j; \theta^j\right) \mathbf{x}_c\left(q, c^j; \theta^j\right) = \\
& -q \left[\mathbf{y}_q\left(q, c^j; \theta^j\right) + \mathbf{y}\left(q, c^j; \theta^j\right) \mathbf{y}_c\left(q, c^j; \theta^j\right) \right]. \qquad \text{(A11)}
\end{aligned}
$$

Substituting from equation (A11) into (A8), the latter equation is rewritten as

$$
\begin{aligned}
& \sum_{j=1}^{4} \pi^j \left[\mathbf{y}_q\left(q, c^j; \theta^j\right) + \mathbf{y}\left(q, c^j; \theta^j\right) \mathbf{y}_c\left(q, c^j; \theta^j\right) \right] \times \\
& \left\{ \mu\left(q - p_E\right) - \phi \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \right\} \\
& -\sum_{j=1}^{4} \sum_{k \neq j} \left[\lambda^{kj} \mathbf{y}\left(q, c^j; \theta^j\right) \mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) + \lambda^{jk} \mathbf{V}_q\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right) \right] = 0. \qquad \text{(A12)}
\end{aligned}
$$

Next, rewrite the last term on the left-hand side of equation (A12) as

$$\begin{aligned}
\sum_{j=1}^{4}\sum_{k\neq j}\lambda^{jk}\mathbf{V}_q\left(q,c^k,\frac{I^k}{wn^j};\theta^j\right) &= \sum_{j=1}^{4}\sum_{k\neq j}\lambda_q^{kj}\mathbf{V}_q\left(q,c^j,\frac{I^j}{wn^k};\theta^k\right) \\
&= \sum_{j=1}^{4}\sum_{k\neq j}\lambda_c^{kj}\mathbf{V}_c\left(q,c^j,\frac{I^j}{wn^k};\theta^k\right)\mathbf{y}\left(q,c^j;\theta^k\right), \quad \text{(A13)}
\end{aligned}$$

where in going from the second to the last expression, we have made use of Roy's identity. Now substituting from (A13) into (A12) results in

$$\begin{aligned}
&\sum_{j=1}^{4}\pi^j\left[\mathbf{y}_q\left(q,c^j;\theta^j\right)+\mathbf{y}\left(q,c^j;\theta^j\right)\mathbf{y}_c\left(q,c^j;\theta^j\right)\right]\times \\
&\left\{\mu\left(q-p_E\right)-\phi\sum_{j=1}^{4}\pi^j\left[\mathbf{V}\left(q,c^j,\frac{I^j}{wn^j};\theta^j\right)-\phi\sum_{j=1}^{4}\pi^j\mathbf{y}\left(q,c^j;\theta^j\right)-\phi D\right]^{-\eta}\right\} \\
&-\sum_{j=1}^{4}\sum_{k\neq j}\lambda^{kj}\left\{\mathbf{V}_c\left(q,c^j,\frac{I^j}{wn^k};\theta^k\right)\left[\mathbf{y}\left(q,c^j;\theta^j\right)-\mathbf{y}\left(q,c^j;\theta^k\right)\right]\right\}=0.
\end{aligned}$$

Denote the compensated demand function for y by $\widetilde{\mathbf{y}}\left(q,c^j;\theta^j\right)$. Substituting $\widetilde{\mathbf{y}}_q\left(q,c^j;\theta^j\right)$ for $\mathbf{y}_q\left(q,c^j;\theta^j\right)+\mathbf{y}\left(q,c^j;\theta^j\right)\mathbf{y}_c\left(q,c^j;\theta^j\right)$ in above, dividing the resulting equation by $\mu\sum_{j=1}^{4}\pi^j\widetilde{\mathbf{y}}_q\left(q,c^j;\theta^j\right)$, and rearranging yields

$$\begin{aligned}
q-p_E &= \frac{\phi}{\mu}\sum_{j=1}^{4}\pi^j\left[\mathbf{V}\left(q,c^j,\frac{I^j}{wn^j};\theta^j\right)-\phi\sum_{j=1}^{4}\pi^j\mathbf{y}\left(q,c^j;\theta^j\right)-\phi D\right]^{-\eta}+ \\
&\quad \frac{\sum_{j=1}^{4}\sum_{k\neq j}\lambda^{kj}\left\{\mathbf{V}_c\left(q,c^j,\frac{I^j}{wn^k};\theta^k\right)\left[\mathbf{y}\left(q,c^j;\theta^j\right)-\mathbf{y}\left(q,c^j;\theta^k\right)\right]\right\}}{\mu\sum_{j=1}^{4}\pi^j\widetilde{\mathbf{y}}_q\left(q,c^j;\theta^j\right)}. \quad \text{(A14)}
\end{aligned}$$

This proves that $q-p_E$ is non-Pigouvian unless the polluting good demand depends only on one's income but not on his taste so that the second expression on the right-hand side of (A14) will be zero. ■

Second, we prove that the input tax is Pigouvian regardless of individuals' tastes. The proof is facilitated through the following lemma.

Lemma A1 *In the optimal income tax problem (A1), and characterized by the first-order conditions (A2)–(A7), the Lagrange multiplier associated with the constraint* $w = O_L(K, D, L)$*,* γ*, is equal to zero.*

Proof. Multiply equation (A4) through by I^j/w, sum over j, and simplify to get

$$\frac{1}{w^2}\sum_{j=1}^{4}\frac{\pi^j I^j}{n^j}\left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j};\theta^j\right) - \phi\sum_{j=1}^{4}\pi^j \mathbf{y}\left(q, c^j;\theta^j\right) - \phi D\right]_L^{-\eta}\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j};\theta^j\right) +$$
$$\mu L + \frac{1}{w^2}\sum_j\sum_{k\neq j}\left[\left(\frac{I^j}{n^j}\right)\lambda_L^{jk}\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j};\theta^j\right) - \left(\frac{I^j}{n^k}\right)\lambda_L^{kj}\mathbf{V}\left(q, c^j, \frac{I^j}{wn^k};\theta^k\right)\right] -$$
$$\frac{1}{w^2}\gamma\mathbf{O}_{LL}(L, K, D)(wL) = 0. \tag{A15}$$

Substituting (A15) into (A7) and simplifying, we get

$$\sum_j\sum_{k\neq j}\left(\frac{I^j}{n^k}\right)\lambda_L^{kj}\mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^k};\theta^k\right) = \gamma w^2 + \sum_j\sum_{k\neq j}\left(\frac{I^k}{n^j}\right)\lambda_L^{jk}\mathbf{V}_L\left(q, c^k, \frac{I^k}{wn^j};\theta^j\right). \tag{A16}$$

Then rewrite the left-hand side of (A16) as

$$\sum_j\sum_{k\neq j}\left(\frac{I^j}{n^k}\right)\lambda_L^{kj}\mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^k};\theta^k\right) = \sum_j\sum_{k\neq j}\left(\frac{I^k}{n^j}\right)\lambda_L^{jk}\mathbf{V}_L\left(q, c^k, \frac{I^k}{wn^j};\theta^j\right). \tag{A17}$$

Substituting from (A17) into (A16) implies

$$\gamma = 0.$$

■

Observe that Lemma A1 is in fact an application of the production efficiency result as it tells us that $w = \mathbf{O}_L(K, D, L)$ imposes no constraint on our second-best problem. Using this lemma, we can easily show:

Proposition A2 *The optimal tax on energy input is Pigouvian.*

Proof. Using the result that $\gamma = 0$ in the first-order conditions (A4)–(A7), simplifies them to

$$\frac{\partial £}{\partial I^j} = \pi^j\left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j};\theta^j\right) - \phi\sum_{j=1}^{4}\pi^j \mathbf{y}\left(q, c^j;\theta^j\right) - \phi D\right]^{-\eta}\frac{1}{wn^j}\mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j};\theta^j\right) +$$
$$\mu\pi^j + \sum_{k\neq j}\lambda^{jk}\frac{1}{wn^j}\mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j};\theta^j\right) - \sum_{k\neq j}\lambda^{kj}\frac{1}{wn^k}\mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^k};\theta^k\right) = 0, \tag{A18}$$

$$
\begin{aligned}
\frac{\partial \pounds}{\partial D} =& - \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \phi + \\
& \mu \left[\mathbf{O}_D(L, K, D) - p_E \right] = 0, \qquad \text{(A19)} \\
\frac{\partial \pounds}{\partial K} =& \mu \left[\mathbf{O}_K(L, K, D) - r \right] = 0, \qquad \text{(A20)} \\
\frac{\partial \pounds}{\partial w} =& \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \left(\frac{-I^j}{n^j w^2}\right) \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) + \\
& \mu \mathbf{O}_L(L, K, D) \frac{-1}{w^2} \sum_{j=1}^{4} \pi^j I^j + \sum_{j=1}^{4} \sum_{k \neq j} \lambda^{jk} \left(\frac{-I^j}{n^j w^2}\right) \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) + \\
& \sum_{j=1}^{4} \sum_{k \neq j} \lambda^{jk} \left(\frac{I^k}{n^j w^2}\right) \mathbf{V}_L\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right). \qquad \text{(A21)}
\end{aligned}
$$

That the input tax is Pigouvian follows immediately from equation (A19). ■

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