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# Self-Employment as a Signal: Career Concerns with Hidden Firm Performance*

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**Abstract**

We study a stationary labour market in which risk-averse workers privately know their permanent talent and choose, period by period, between risky self-employment, whose outcomes become part of a portable public record, and firm employment, which pays a competitive wage but keeps individual performance hidden from the outside market. Because each worker decides whether to generate another public outcome or to apply to a firm, both the population holding a given record and the pool of applicants at that record are endogenous. Market beliefs are therefore constructed in two stages: first from the stationary flow of types through all histories leading to—and retaining—each record, and only then by conditioning on the current application decision. It is shown that, when the effective continuation factor is below one half, a stationary sequential competitive equilibrium exists and occupational choice follows a talent cutoff at every record; the equilibrium need not be unique, and an explicit example with two distinct equilibria is provided. Firm employment persists whenever it is strictly optimal for a given type at a given record. At any on-path record where both occupations are chosen, higher-talent workers select into self-employment, and the applicant wage lies below mean talent among holders of that record; this discount decomposes exactly into the self-employed share and the talent gap between the two groups. The model yields within-record predictions for occupational choice, wages, subsequent performance, and the duration of opaque employment spells.

**Keywords:** Career concerns, reputation, signalling, self-employment, transparency, labour market learning

**JEL:** J24, D82, L26, D83, J31

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# 1 Introduction

Labour markets see some of what workers do and miss the rest. An open-source developer's commits are public; a freelancer's platform ratings follow her from client to client; a researcher's preprints and contest entries travel across institutions; yet most of the day-to-day output produced inside a firm never reaches the outside market.[1] A worker therefore chooses not only how to earn an income, but also whether her performance will become public: self-employment creates portable evidence at the price of output risk, while firm employment offers a stable wage behind a veil of opacity. This raises the question at the centre of the paper: how should the market read a public record that workers themselves decide whether to extend—and what does the very act of applying for opaque employment reveal about the applicant?

We study this question in a stationary labour market. Workers privately know their permanent talent and choose, at each public record, between self-employment and firm employment. Self-employment produces a public success or failure and extends the record; firm employment pays a competitive wage, produces no public outcome, and so leaves the record unchanged. Firms observe the record and the worker's current application, but neither talent nor career age nor the dates at which earlier outcomes were produced.

The central difficulty is that the record is endogenous, and in two distinct ways. First, the population holding a record is selected by every occupational path that can lead to it. A blank record pools new entrants with older workers who have quietly retained it through years of opaque employment; a long record, by contrast, can be held only by workers who survived and repeatedly chose to keep producing public outcomes; and workers with the same counts of successes and failures may have arrived through different orderings of earlier choices. Second, the current application adds a further layer of selection: firms hire from the workers who apply, not from everyone holding the same record. Treating the observed outcomes as an exogenous sequence misses the first layer; pricing all record holders misses the second.[2]

The present paper tries to fill this gap. We track the stationary composition of workers at every record: entry, public outcomes, survival, and decisions to retain a record through firm employment jointly determine who is present there. Employers update once more when a worker applies, and competition sets the wage of the resulting applicant pool. Occupational choices thus shape beliefs and wages, while wages feed back into occupational choices; solving this feedback is the main equilibrium problem of the paper.

Our first result is that occupational choice has a simple structure. Under a sufficient short-horizon condition, workers below a record-specific talent threshold choose firm employment and workers above it continue in self-employment, and a stationary equilibrium exists. This is less immediate than it may appear: common sense would suggest that a success should always improve future opportunities relative to a failure, but once wages price endogenously selected

[1] We interpret self-employment broadly, as any project or market activity whose individual-level outcomes are externally visible—platform work, contracting, open-source contributions, contests, entrepreneurship—in contrast to work whose output remains inside a firm. What matters for the analysis is the visibility of the outcome, not the legal form of the occupation.

[2] The first layer is easy to overlook precisely because it operates through what is *not* observed. In the reviewer's illustrative cutoff example, a blank record disproportionately contains older types below one half who have never chosen public production, whereas a worker with many failures and no successes must be a type above one half with poor luck. A posterior built from the outcome counts alone, however cleverly truncated by the current applicant set, misses both inferences.

pools no such ordering of continuation values is available, and the cutoff must be established without it.[3] The existence argument does not, however, select a unique equilibrium—nor could it: we provide an explicit specification with two distinct stationary equilibria, in which a low blank-record cutoff sustains a low applicant wage and conversely (Remark 1).

The model then yields three qualitative implications. First, firm employment is self-confirming: when it is strictly preferred, the job changes neither the public record nor the stationary wage, so a surviving worker faces the same comparison again and makes the same choice—opaque employment persists until exogenous exit.[4] Second, at any record where both occupations are chosen, higher-talent workers select into public production and lower-talent workers apply to firms; the competitive applicant wage therefore lies below mean talent among all holders of the record, and the discount decomposes exactly into the self-employed share and the talent gap between the two groups. Third, occupational choices determine the flow of new public successes and failures, and hence how much information the market receives at all.

These implications are within-record comparisons: conditional on the same public record, an application for opaque employment should predict lower performance on a comparable later measure, applicant wages should display a record-specific selection discount, and moves into opaque employment should persist while the worker remains in the market. The model deliberately does not imply that an additional success, failure, or unit of record length raises or lowers wages—such comparisons change the composition of the record along with occupational choices, and their signs are equilibrium outcomes rather than primitives.

The baseline is deliberately narrow. Talent is fixed and known by the worker, individual performance inside firms is hidden from the outside market, and wages and strategies are stationary. Private learning, observable career age, dated histories, public in-firm signals, richer contracts, hidden effort, and welfare analysis all change the state space or the game played by firms, and require additional structure; Section 6 explains what would have to be reconstructed in each case.

**Roadmap.** Section 2 relates the paper to the literature. Section 3 presents the model and the formation of beliefs and wages. Section 4 establishes equilibrium existence, talent thresholds, persistence, and within-record selection. Section 5 develops the empirical implications, and Section 6 discusses scope and extensions. Section 7 concludes. Appendix A contains the technical details, and the Appendix contains the equilibrium-multiplicity construction.

# 2 Related literature

The paper stands at the intersection of three literatures: career concerns and employer learning; dynamic reputation with endogenous records; and labour markets in which performance is

[3] The argument runs through a uniform single-crossing bound: when the effective continuation factor is below one half, the value of producing one more public outcome rises in talent faster than any continuation-value reversal can undo. The bound is uniform over records and wage schedules, which is what permits an existence proof for the entire wage schedule at once.

[4] Persistence attaches to a state–type pair rather than to a record. At one and the same record, lower types can be absorbed into firm employment while higher types keep producing public outcomes, so persistence and continued public-information production coexist within a record cell.

portable. All three study how observed outcomes affect beliefs and later rewards. Our focus is on the decision that comes *before* the outcome—whether to generate another public signal at all, or to enter an applicant pool instead—which makes both the population at a record and the set of applicants endogenous objects.

The career-concerns literature shows how market learning from performance creates implicit incentives: Holmström (1999) develops the central dynamic mechanism, Dewatripont et al. (1999a,b) emphasise the role of the information structure, and Gibbons and Murphy (1992) study the interaction between career concerns and explicit contracts. Employer-learning models examine how wages respond as information about productivity accumulates (Farber and Gibbons, 1996; Altonji and Pierret, 2001), while models of internal labour markets connect learning to wage and promotion dynamics (Gibbons and Waldman, 1999, 2006; Waldman, 2013). We retain the link between performance and future opportunities, but change the worker's decision margin: the worker chooses whether to produce portable information in the first place, and the application for opaque employment is itself informative.

The model also belongs to the broader literature on dynamic reputation, experimentation, and career signalling, in which current choices or evolving track records shape future beliefs and payoffs (e.g., Board and Meyer-ter-Vehn, 2013; Thomas, 2019; Kaniel and Orlov, 2025). Onuchic and Ray (2023) study reputational incentives in the choice between solo and collaborative projects when joint production obscures individual credit. An especially close connection is Pei (2026), who studies repeated interaction when agents can add or remove signals from their records; in our setting, workers likewise choose whether their records change. The equilibrium object is different, however. Occupational choices generate a stationary distribution of talent at every record, and competitive wages depend on the selected workers who apply there: beliefs must first aggregate all paths leading to a record, including retention through firm employment, and only then condition on the current application.

Online labour markets provide a natural empirical analogue, and the evidence there suggests that portable public histories carry first-order weight in hiring.[5] Our mechanism is distinct from certification or reputation intermediation: firms observe the portable outcomes produced during self-employment, and the current application further selects the applicant pool. The model also connects to talent discovery. Terviö (2009) studies the market failure that arises when an early employer cannot capture the full return from discovering an inexperienced worker's talent; our baseline characterises private occupational choice and applicant wages, but does not determine whether the market produces too much or too little information. Open-source production offers a related setting in which current work becomes a portable signal: Lerner and Tirole (2002) identify career incentives and labour-market signalling as important motives for open-source participation, Abou El-Komboz and Goldbeck (2025) study career concerns and signalling in open-source software development, and Belenzon and Schankerman (2015) examine motivation and the sorting of human capital into open innovation. More generally, reputation systems use public histories to guide later transactions (Bar-Isaac and Tadelis, 2008).

[5] Pallais (2014) shows experimentally that detailed public evaluations of inexperienced workers improve later employment outcomes; Stanton and Thomas (2016) study outsourcing agencies that signal worker quality; and Leung (2018) documents how employers' earlier experiences with workers from particular countries affect later hiring from those countries. Related evidence indicates that a spell of self-employment can itself be read as a signal when a worker returns to paid employment (Koellinger et al., 2015).

Relative to these literatures, the contribution of the present paper is a belief system constructed from the endogenous stationary flow of workers through records, together with the equilibrium analysis this construction supports: existence and talent-threshold behaviour without assuming that successes and failures order continuation values, persistence in opaque employment, positive selection into public production, and a record-specific applicant-wage discount with an exact decomposition. These are conditional, within-record implications; the model does not claim a general ordering of records or a globally signed effect of greater transparency.

# 3 Environment and endogenous public records

This section sets out the environment and, no less importantly, the order in which its objects are determined: first the demographic flows that decide who holds each public record; then the beliefs and wages that competitive firms attach to records and to current applications; and only then the worker's dynamic problem, whose solution feeds back into the flows. The order is the point at which the analysis departs from treating the record as an exogenous sequence of signals.

## 3.1 Population, talent, and survival

Time is discrete. In every period there is a unit mass of active workers. Each worker has permanent talent $\theta \in [0,1]$, privately known to the worker. New entrants draw talent from a distribution $F_0$ with a continuous density $f_0$ that is strictly positive on $[0,1]$. An incumbent survives to the next period with probability $\rho \in (0,1)$ and exits with probability

$$\lambda \equiv 1 - \rho.$$

Survival is independent of talent, the public record, and the worker's action. An exiting worker receives no continuation utility. The mass $\lambda$ of exiting workers is replaced by new entrants, so the population is stationary. Firms observe neither a worker's calendar age nor the dates on which earlier public outcomes were produced; past spells of firm employment are not separately recorded.[6]

Workers discount future utility by $\delta \in (0,1)$ conditional on survival. It is useful to write

$$\gamma \equiv \delta\rho \tag{1}$$

for the effective continuation factor.[7]

[6] Unobserved age is what gives the blank record its bite: a worker with no outcomes may be a new entrant or an older worker who has repeatedly chosen opacity, and the market cannot tell the two apart. If career age were public, beliefs would instead be indexed by the pair of record and age, and firm employment would no longer leave the full public state unchanged; Section 6 discusses this extension.

[7] Survival and discounting have the same effect on the worker's continuation value through $\gamma$, but $\rho$ also affects the stationary cross-sectional composition of workers at each public record.

## 3.2 Actions and public records

The public record is a pair

$$s = (g, b) \in \mathcal{S} \equiv \mathbb{N}_0^2,$$

where $g$ and $b$ are, respectively, the numbers of publicly observed successes and failures generated by self-employment. Every entrant starts at the blank record $s_0 = (0, 0)$. We write

$$s^+ = (g + 1, b) \qquad \text{and} \qquad s^- = (g, b + 1).$$

At the beginning of a period, an active worker observes $s$ and chooses between self-employment ($S$) and firm employment ($E$). Self-employment produces publicly observed output $y \in \{0, 1\}$, which is also the worker's current income, with

$$\Pr(y = 1 \mid \theta, S) = \theta.$$

Conditional on survival, the record becomes $s^+$ after a success and $s^-$ after a failure. Firm employment pays a flat wage $w_s$ and produces no publicly observed signal of individual performance; conditional on survival, it therefore leaves the record at $s$. Firms observe the record and the worker's current application for firm employment, but not talent.

The worker has Bernoulli utility $u : [0, 1] \to [0, 1]$, which is continuous, increasing, and concave, with $u(0) = 0$ and $u(1) = 1$. The current expected utility from self-employment is therefore $\theta$, whereas current utility from firm employment is $u(w_s)$. Concavity captures the insurance value of a deterministic wage.[8]

## 3.3 Stationary type–record distributions and wages

How should the market price a worker who holds record $s$ and asks for a job? The answer requires knowing who holds the record in the first place—a question about equilibrium flows rather than about the record alone. We therefore begin with the stationary demography induced by an arbitrary strategy.

Let $\sigma_s(\theta) \in [0, 1]$ denote the probability that a worker of type $\theta$ chooses self-employment at record $s$, and let $m_s(\theta)$ denote the stationary density of active workers with type $\theta$ and record $s$ at the beginning of a period. For a fixed strategy $\sigma$, these densities are determined recursively.

Let $a_s(\theta)$ denote inflow into record $s$ from entrants or from self-employment at a predecessor, excluding workers already at $s$ who retained it through firm employment. At the blank record,

$$a_{0,0}(\theta) = \lambda f_0(\theta). \tag{2}$$

For $g + b \geq 1$, the inflow is

$$a_{g,b}(\theta) = \rho\Big[\mathbf{1}\{g \geq 1\}\theta\sigma_{g-1,b}(\theta)m_{g-1,b}(\theta) + \mathbf{1}\{b \geq 1\}(1 - \theta)\sigma_{g,b-1}(\theta)m_{g,b-1}(\theta)\Big]. \tag{3}$$

[8] The cutoff and existence results below use continuity and monotonicity; concavity is not needed for those results.

A worker at $s$ who chooses firm employment and survives remains at $s$. Stationarity therefore requires

$$m_s(\theta) = a_s(\theta) + \rho\Big(1 - \sigma_s(\theta)\Big)m_s(\theta),$$

or, equivalently,

$$m_s(\theta) = \frac{a_s(\theta)}{\lambda + \rho\sigma_s(\theta)}. \tag{4}$$

Equations (2)–(4) determine the stationary type density recursively by record length.

These equations condition on the endogenous process that generates a record. For example,

$$m_{0,0}(\theta) = \frac{\lambda f_0(\theta)}{\lambda + \rho\sigma_{0,0}(\theta)}, \qquad a_{0,1}(\theta) = \rho(1-\theta)\sigma_{0,0}(\theta)m_{0,0}(\theta).$$

Thus, even the absence of a public outcome is informative when some types remain at the blank record through firm employment.[9] At the mixed-count record $(1,1)$,

$$a_{1,1}(\theta) = \rho\Big[\theta\sigma_{0,1}(\theta)m_{0,1}(\theta) + (1-\theta)\sigma_{1,0}(\theta)m_{1,0}(\theta)\Big],$$

which aggregates both success-after-failure and failure-after-success histories. Each predecessor density already incorporates every earlier occupational choice along the histories leading to that predecessor. Equation (4) then incorporates earlier decisions to retain the current record through firm employment.

Let

$$M_s = \int_0^1 m_s(\theta)\, d\theta$$

be the stationary mass at record $s$. Whenever $M_s > 0$, the market's public belief before observing the current employment decision is

$$G_s(d\theta) = \frac{m_s(\theta)\, d\theta}{M_s}. \tag{5}$$

The object $G_s$ is the worker's reputation at record $s$. It depends on equilibrium behaviour at every predecessor record that can lead to $s$ and on earlier retention decisions at $s$ itself.

In every period, a competitive fringe of risk-neutral, short-lived firms posts a wage for applicants at each observable record.[10] A firm employing a worker of talent $\theta$ obtains expected output $\theta$. The applicant measure and its total mass at record $s$ are

$$A_s(d\theta) = \Big(1 - \sigma_s(\theta)\Big)m_s(\theta)\, d\theta, \tag{6}$$

$$D_s = \int_0^1 \Big(1 - \sigma_s(\theta)\Big)m_s(\theta)\, d\theta. \tag{7}$$

[9] The logic is the familiar one from survival analysis—who is present carries information beyond what they display—with the difference that survivorship here is chosen rather than mechanical: a worker retains the blank record because, given the wages she faces, she prefers to. The inference and the behaviour must accordingly be solved together.

[10] Short-lived firms rule out dynamic contracts and back-loaded wages by assumption; the fringe is the standard device that makes wages Bayesian prices rather than strategic offers. Richer contracting, in which the contract itself becomes informative, is discussed in Section 6.

If $D_s > 0$, Bayes' rule gives the applicant belief

$$\nu_s(d\theta) = \frac{\big(1-\sigma_s(\theta)\big)m_s(\theta)\,d\theta}{D_s}. \tag{8}$$

Competition and free entry then imply the zero-profit wage

$$w_s = \int_0^1 \theta\,\nu_s(d\theta) = \frac{\displaystyle\int_0^1 \theta\big(1-\sigma_s(\theta)\big)m_s(\theta)\,d\theta}{\displaystyle\int_0^1 \big(1-\sigma_s(\theta)\big)m_s(\theta)\,d\theta}. \tag{9}$$

When $D_s = 0$, Bayes' rule does not directly determine an applicant belief. The equilibrium definition below requires such an off-path belief to be a limit of Bayesian applicant beliefs generated by completely mixed worker strategies; it cannot be chosen arbitrarily.

## 3.4 Worker problem and equilibrium

Fix a stationary wage schedule $w = (w_s)_{s\in\mathcal{S}}$. Let $V_s(\theta)$ denote the expected discounted utility of a worker with talent $\theta$ at record $s$. The Bellman equation is

$$\begin{aligned} V_s(\theta) = \max\Big\{ & u(w_s) + \gamma V_s(\theta), \\ & \theta + \gamma\Big[\theta V_{s^+}(\theta) + (1-\theta)V_{s^-}(\theta)\Big]\Big\}. \end{aligned} \tag{10}$$

The first term is the value of firm employment. It contains $V_s$ because firm employment leaves the public record unchanged. Equation (10) has the equivalent optimal-stopping representation

$$V_s(\theta) = \max\left\{\frac{u(w_s)}{1-\gamma}, \theta + \gamma\Big[\theta V_{s^+}(\theta) + (1-\theta)V_{s^-}(\theta)\Big]\right\}. \tag{11}$$

For the analysis it is convenient to normalise lifetime utility by defining

$$U_s(\theta) = (1-\gamma)V_s(\theta). \tag{12}$$

Let the normalised value of choosing self-employment at $s$ be

$$Q_s(\theta) = (1-\gamma)\theta + \gamma\Big[\theta U_{s^+}(\theta) + (1-\theta)U_{s^-}(\theta)\Big]. \tag{13}$$

The worker's problem can then be written as

$$U_s(\theta) = \max\Big\{u(w_s), Q_s(\theta)\Big\}. \tag{14}$$

For every wage schedule satisfying $w_s \in [0,1]$ at each record, the operator on the right-hand side of (14) is a contraction with modulus $\gamma$. It therefore has a unique bounded solution, and $0 \le U_s(\theta) \le 1$ at every $(s,\theta)$.

The stationary action rule must satisfy

$$\sigma_s(\theta) \in \begin{cases} \{1\}, & Q_s(\theta) > u(w_s), \\ \{0\}, & Q_s(\theta) < u(w_s), \\ [0,1], & Q_s(\theta) = u(w_s). \end{cases} \tag{15}$$

Recall that $\sigma_s(\theta) = 1$ denotes self-employment. Mixing is therefore permitted only for a type that is indifferent between the two actions.

Our main analytical results use the following sufficient restriction.

**Assumption 1.** *The effective continuation factor satisfies* $\gamma < 1/2$.

Assumption 1 will imply that $Q_s(\theta)$ is strictly increasing in talent, uniformly over records and wage schedules.

Since the restriction may appear severe—it asks the entire discounted future to carry less weight than the current period—it is worth being precise about what it does and does not do. Its sole role in the paper is Lemma 1: it delivers single crossing of the action gap in the worst case over *all* wage schedules with values in $[0, 1]$, and this uniformity is exactly what the existence argument of Section 4.2 needs, because the candidate wage schedules encountered along a fixed-point iteration need not resemble equilibrium ones. Nothing else depends on it: the flow equations, the two-stage belief construction, the equilibrium definition, and the persistence result (Proposition 1) are all stated and proved without it. The proof allows the most adverse possible ordering of successor values: the continuation value after a failure may exceed that after a success by as much as one. The current-payoff gain still dominates this worst-case reversal when $\gamma < 1/2$. The bound is therefore sufficient rather than necessary—a particular wage schedule with a smaller reversal can support single crossing for a larger continuation factor—and the multiplicity construction of Remark 1 operates comfortably inside it. Finally, the calibration question is softened by the interpretation of a period: $\gamma = \delta\rho$ compounds discounting with exit from the market, and a period is the production and revelation of one public outcome—a project, an engagement, a release cycle—rather than a calendar year, so effective continuation factors well below one are not implausible in the applications we have in mind.[11]

**Definition 1.** *A stationary sequential competitive equilibrium is a tuple* $(U, \sigma, m, G, \nu, w)$ *satisfying the following conditions.*

1. *The value function* $U$ *solves* (13)–(14)*, and the action rule* $\sigma$ *satisfies* (15) *at every record and for almost every talent type.*

2. *The type–record densities* $m$ *satisfy* (2)–(4) *and are normalised so that*

$$\sum_{s \in \mathcal{S}} \int_0^1 m_s(\theta)\, d\theta = 1.$$

[11] The restriction is sufficient, but not necessary, for cutoff behaviour. It plays no role in the demographic or belief-formation parts of the model.

3. *At every record with $M_s > 0$, the public belief $G_s$ is given by (5). At every record with $D_s > 0$, the applicant belief $\nu_s$ is given by (8). At every record, including one with no applicants, the wage satisfies*

$$w_s = \int_0^1 \theta \, \nu_s(d\theta).$$

4. *There exists a sequence of completely mixed stationary worker strategies $\{\sigma^n\}_{n\geq 1}$, with $0 < \sigma_s^n(\theta) < 1$, that converges to $\sigma_s(\theta)$ for almost every $\theta$ at every record. If $G_s^n$ and $\nu_s^n$ are the Bayesian beliefs generated by $\sigma^n$ and its induced stationary type–record densities, then*

$$G_s^n \Rightarrow G_s \qquad \text{and} \qquad \nu_s^n \Rightarrow \nu_s$$

*at every record, where $\Rightarrow$ denotes weak convergence.*

The consistency condition disciplines beliefs and wages when a record or an applicant pool has zero equilibrium mass.[12] Every firm is Bayesian: its wage incorporates both the public record and the selection conveyed by the current application.

## 3.5 Persistence of strict firm employment

For a record $s$, define the strict-employment set

$$\mathcal{E}_s^\circ = \big\{\theta \in [0,1] : u(w_s) > Q_s(\theta)\big\}. \tag{16}$$

This is a set of state–type pairs: a given record can support firm employment for some types and self-employment for others.

**Proposition 1.** *In any stationary sequential competitive equilibrium, fix a record $s$ and a type $\theta \in \mathcal{E}_s^\circ$. The worker chooses firm employment at $(s, \theta)$. After every surviving period of firm employment, the worker again chooses firm employment at the same record. Thus, firm employment is absorbing for this state–type pair, conditional on the worker remaining in the market.*

*Proof.* By (16), $u(w_s) > Q_s(\theta)$, so worker optimality in (15) requires $\sigma_s(\theta) = 0$. Firm employment generates no public outcome and hence leaves the record at $s$. Conditional on survival, the worker therefore faces the same record, the same stationary wage $w_s$, and the same continuation values in the next period. The strict inequality $u(w_s) > Q_s(\theta)$ is unchanged, so firm employment is again uniquely optimal. Repeating this argument proves the result. □

The proposition requires neither Assumption 1 nor a cutoff strategy. Its force comes from stationarity and from firm employment freezing the public record—the worker who prefers opacity today is handed, tomorrow, exactly the situation in which she preferred it.[13]

[12] The definition also makes explicit the population-flow and zero-profit conditions that would be hidden by referring only to a stationary perfect Bayesian equilibrium.

[13] Strictness matters: if $u(w_s) = Q_s(\theta)$, both actions are optimal, and persistence depends on the equilibrium tie-breaking rule.

This result should not be interpreted as an absorbing region of public records in which every type chooses firm employment. It applies to a state–type pair. Once cutoff behaviour is established, the strict-employment set is an interval of lower types at each record, while higher types can continue to generate public outcomes at the same record. Conditional on entering strict firm employment, only exogenous exit ends the employment spell; its duration is geometrically distributed with exit probability $\lambda$.

The argument also relies on talent remaining fixed and on firm employment changing neither the public record nor any private state relevant to the worker's future choice. Private learning about talent or a public signal generated inside the firm changes the state and requires a separate analysis; persistence is not an immediate corollary in those extensions.

# 4 Equilibrium and cutoff behaviour

Two questions organise this section. Is occupational choice at each record governed by a talent threshold, even though nothing guarantees that a success leads to better prospects than a failure? And does an equilibrium of the joint system—strategies, stationary stocks, beliefs, and the entire wage schedule—exist at all? It is shown that the answer to both is yes under Assumption 1. The section then closes with the implications that survive without equilibrium uniqueness—which, by Remark 1, cannot be taken for granted.

## 4.1 Uniform single crossing and cutoff behaviour

Common sense would suggest that a success improves a worker's prospects relative to a failure, and that cutoff behaviour should simply follow from this ordering. Once wages price endogenously selected pools, however, no such ordering is available—nothing rules out that a failure leads to a record whose applicant pool commands the higher wage—so single crossing has to be obtained without it. The following uniform bound does exactly that.

**Lemma 1.** *Fix any stationary wage schedule satisfying $w_s \in [0,1]$ at every record. Under Assumption 1, $U_s(\theta)$ is non-decreasing in $\theta$ at every record. Moreover, for every $y > x$,*

$$Q_s(y) - Q_s(x) \geq (1-2\gamma)(y-x) > 0. \tag{17}$$

*The bound is uniform over records and wage schedules.*

*Proof.* Consider value iteration for (14), initialised with $U_s^0(\theta) = u(w_s)$. The initial function is constant, and hence non-decreasing, in talent. Suppose that the two successor values at some iteration are non-decreasing and take values in $[0,1]$. Write

$$H_s(\theta) = \theta U_{s^+}(\theta) + (1-\theta)U_{s^-}(\theta).$$

For $y > x$,

$$\begin{aligned} H_s(y) - H_s(x) = {} & y\Big[U_{s^+}(y) - U_{s^+}(x)\Big] \\ & + (1-y)\Big[U_{s^-}(y) - U_{s^-}(x)\Big] \\ & + (y-x)\Big[U_{s^+}(x) - U_{s^-}(x)\Big]. \end{aligned}$$

The first two terms are non-negative. Because both successor values lie in $[0,1]$, the last term is bounded below by $-(y-x)$. It follows from (13) that

$$Q_s(y) - Q_s(x) \geq (1-\gamma)(y-x) - \gamma(y-x) = (1-2\gamma)(y-x).$$

Taking the maximum of the constant $u(w_s)$ and the increasing function $Q_s$ preserves monotonicity. Induction therefore shows that every value iterate is non-decreasing in talent. The Bellman operator is a contraction, so the iterates converge uniformly to $U$. Passing to the limit gives the monotonicity of $U$ and the bound in (17). □

The same value-iteration argument, starting from continuous functions, shows that $U_s$ and $Q_s$ are continuous in talent. Hence the action gap

$$h_s(\theta) = Q_s(\theta) - u(w_s) \tag{18}$$

is continuous and strictly increasing.

**Proposition 2.** *Under Assumption 1, at every record $s$ there is a cutoff $c_s \in [0,1]$ such that firm employment is optimal for $\theta < c_s$ and self-employment is optimal for $\theta > c_s$. If $c_s \in (0,1)$, the cutoff type satisfies*

$$Q_s(c_s) = u(w_s). \tag{19}$$

*Mixing can occur only at the cutoff type and therefore has zero population mass.*

*Proof.* By Lemma 1, the gap $h_s$ in (18) is continuous and strictly increasing. Its strict sublevel set is therefore a lower interval and its strict superlevel set is an upper interval. Worker optimality in (15) gives the claimed cutoff rule. An interior boundary must satisfy (19); strict monotonicity permits equality at no more than one type. Since the stationary type–record distributions have densities, a singleton has zero population mass. □

The proposition includes the corner cases in which all types choose the same action. It also implies that the strict-employment set $\mathcal{E}_s^\circ$ from (16) is a lower interval. It does not imply that cutoffs are ordered across records.

One conditional comparative static is immediate. Holding the continuation value $Q_s$ fixed, an increase in the current wage weakly raises the cutoff and expands the employment set. This is a partial-equilibrium statement. Changing patience, survival, preferences, or the record generally changes the entire endogenous wage schedule and stationary population. Moreover, the existence argument below does not establish equilibrium uniqueness. We therefore do not assert global comparative statics in those primitives without an equilibrium-selection rule.

## 4.2 Existence with endogenous applicant beliefs

Why can the equilibrium wage not be determined record by record? Because the applicant distribution at $s$ is generated by the stationary stock $m_s$, which depends on worker behaviour at every predecessor record and on retention at $s$, while continuation values link the worker's choice at $s$ to wages at successor records: information flows into a record from its past,

and incentives flow into it from its future. Existence must therefore be established for the complete wage schedule and stationary population jointly.

We begin with an auxiliary finite-record economy. For an integer $K \geq 1$, let $\mathcal{S}_K = \{s \in \mathcal{S} : |s| \leq K\}$, where $|(g,b)| = g + b$. At records with $|s| < K$, the worker has the same two actions as in the original economy. At a boundary record with $|s| = K$, only firm employment is available, so the public record remains fixed until exit. This boundary condition is used only to obtain a finite approximation; it disappears as $K$ tends to infinity.

Fix a wage vector $w \in [0,1]^{\mathcal{S}_K}$. Let $U^{K,w}$ be the unique solution of the capped worker problem. At the boundary,

$$U_s^{K,w}(\theta) = u(w_s), \qquad |s| = K, \tag{20}$$

and at every interior record it satisfies (13)–(14), with $U_{s^+}^{K,w}$ and $U_{s^-}^{K,w}$ used at the successor records. Denote the resulting self-employment value by $Q_s^{K,w}(\theta)$.

For $\eta \in (0, 1/2)$, write

$$L(z) = \frac{1}{1+e^{-z}}$$

and define the completely mixed response at an interior record by

$$\sigma_s^{K,w,\eta}(\theta) = \eta + (1-2\eta) L\left(\frac{Q_s^{K,w}(\theta) - u(w_s)}{\eta}\right), \qquad |s| < K. \tag{21}$$

Both actions therefore receive probability strictly between zero and one. Let $m^{K,w,\eta}$ be the stationary densities generated by this response. The inflow equations are (2)–(3), and the stock equation is

$$m_s^{K,w,\eta}(\theta) = \begin{cases} \dfrac{a_s^{K,w,\eta}(\theta)}{\lambda + \rho\sigma_s^{K,w,\eta}(\theta)}, & |s| < K, \\ \dfrac{a_s^{K,w,\eta}(\theta)}{\lambda}, & |s| = K. \end{cases} \tag{22}$$

The second line reflects that every surviving worker at a boundary record remains there. Set

$$e_s^{K,w,\eta}(\theta) = \begin{cases} 1 - \sigma_s^{K,w,\eta}(\theta), & |s| < K, \\ 1, & |s| = K, \end{cases}$$

and define the finite-economy wage map

$$\Phi_s^{K,\eta}(w) = \frac{\displaystyle\int_0^1 \theta e_s^{K,w,\eta}(\theta) m_s^{K,w,\eta}(\theta)\, d\theta}{\displaystyle\int_0^1 e_s^{K,w,\eta}(\theta) m_s^{K,w,\eta}(\theta)\, d\theta}. \tag{23}$$

**Lemma 2.** *For every $K \geq 1$ and $\eta \in (0, 1/2)$, the map $\Phi^{K,\eta}$ has a fixed point in $[0,1]^{\mathcal{S}_K}$.*

*Proof.* The capped Bellman operator is a contraction on the finite state space, so $U^{K,w}$ is unique and continuous in $w$. Equation (21) then makes the interior action probabilities continuous in $(w, \theta)$. The inflow and stock equations determine the density at a record recursively from densities at shorter records and are continuous in $w$.

Every record has positive mass: the entry density is strictly positive, both actions occur with positive probability at every interior record, and both binary outcomes have positive probability for almost every type. The denominator in (23) is therefore positive. Hence $\Phi^{K,\eta}$ is a continuous self-map of the non-empty compact convex set $[0,1]^{\mathcal{S}_K}$. Brouwer's fixed-point theorem gives the result. □

The perturbed fixed points provide the approximating economies from which equilibrium and consistent off-path beliefs are obtained.

**Proposition 3.** *Under Assumption 1, a stationary sequential competitive equilibrium exists. In any such equilibrium, worker behaviour has the cutoff form in Proposition 2.*

*Proof.* Choose $K_j \uparrow \infty$ and $\eta_j \downarrow 0$. For each $j$, select a fixed point $w^j$ supplied by Lemma 2, and let $U^j$ and $\sigma^j$ denote the corresponding capped values and interior responses. Extend $w^j$ arbitrarily to records outside $\mathcal{S}_{K_j}$. Extend $\sigma^j$ to a completely mixed strategy on the full record space by, for example, assigning probability $1/2$ to self-employment at records with $|s| \geq K_j$. Let $\widetilde{m}^j$, $G^j$, and $\nu^j$ be the stationary densities and Bayesian beliefs generated by this full strategy using (2)–(8).

Fix a record $s$. Once $K_j > |s|$, its density and applicant belief depend only on behaviour at $s$ and at predecessor records, all of which are interior to the capped economy. Consequently, at that record the full-economy objects $\widetilde{m}^j_s$ and $\nu^j_s$ coincide with the objects used in the fixed-point wage equation (23). In particular,

$$w^j_s = \int_0^1 \theta \, \nu^j_s(d\theta) \qquad \text{for all sufficiently large } j. \tag{24}$$

Because wages lie in $[0,1]$ and probability measures on $[0,1]$ are weakly compact, a diagonal subsequence can be chosen such that, at every record,

$$w^j_s \longrightarrow w_s, \qquad G^j_s \Rightarrow G_s, \qquad \nu^j_s \Rightarrow \nu_s.$$

The capped value functions converge at every fixed record to the unique bounded solution of the infinite-record Bellman equation associated with $w$. To see this, fix a continuation depth $H$. For all sufficiently large $j$, the first $H$ successor layers of a fixed record are interior to $\mathcal{S}_{K_j}$, and wages on this finite collection of records converge. The effect of continuation values beyond those layers is bounded by $\gamma^H$, uniformly in talent. First letting $j$ and then $H$ tend to infinity gives

$$U^j_s(\theta) \longrightarrow U_s(\theta) \quad \text{uniformly in } \theta \tag{25}$$

at every fixed record. Lemma 5 gives the detailed finite-depth argument.

Let $h_s(\theta) = Q_s(\theta) - u(w_s)$. By Lemma 1, $h_s$ is strictly increasing and therefore vanishes for at most one talent type. Equations (21) and (25) imply, for almost every $\theta$,

$$\sigma^j_s(\theta) \longrightarrow \sigma_s(\theta) = \begin{cases} 1, & h_s(\theta) > 0, \\ 0, & h_s(\theta) < 0. \end{cases}$$

The action at a possible indifferent cutoff may be assigned arbitrarily. Thus, the limiting strategy is optimal and has the cutoff form.

For each fixed record, the stationary-stock recursion gives $\widetilde{m}_s^j(\theta) \to m_s(\theta)$ for almost every $\theta$. A strategy-independent integrable envelope then permits dominated convergence, as formalised in Lemma 4. The limiting densities satisfy (2)–(4). No mass escapes to records of unbounded length. Indeed, a worker at a record of length at least $L$ must have survived at least $L$ periods, and hence

$$\sum_{|s|\geq L} \int_0^1 \widetilde{m}_s^j(\theta)\, d\theta \leq \sum_{a=L}^{\infty} \lambda \rho^a = \rho^L. \tag{26}$$

Together with convergence at every fixed record, this uniform tail bound implies that the limiting stationary densities have total mass one.

At every record with positive mass, and for every applicant pool with positive mass, Lemma 4 gives the Bayesian formulas (5) and (8). At a zero-mass record or applicant pool, the weak limits $G_s$ and $\nu_s$ provide consistent beliefs. Passing to the limit in (24) gives

$$w_s = \int_0^1 \theta\, \nu_s(d\theta)$$

at every record. Finally, the extended strategies $\sigma^j$ are completely mixed and converge to $\sigma$ almost everywhere at every record. They and their induced beliefs therefore supply the consistency sequence required by Definition 1. All equilibrium conditions follow. □

The proposition solves wages, strategies, and the stationary distribution jointly across records. The argument does not establish equilibrium uniqueness, and none is assumed below. Uniqueness in fact fails, and not merely at knife-edge parameters.

**Remark 1.** *Suppose that entry talent is uniform on $[0,1]$, that $u(w) = \sqrt{w}$, and that $\gamma < 1/9$. Then the model has at least two distinct stationary sequential competitive equilibria in pure cutoff strategies: one with blank-record cutoff $c_{0,0} \in (0, 1/10)$ and one with $c_{0,0} \in (1/10, 4/5)$. In both, the blank-record applicant pool has positive mass and both first-outcome records are on path, so the second equilibrium is not a degenerate profile supported by pessimistic off-path beliefs. The construction, given in the Appendix, produces two sequences of capped equilibria whose blank-record cutoffs lie in disjoint regions and shows that their limits are distinct equilibria of the infinite-record model. The economics is a complementarity between wages and cutoffs: a low blank-record cutoff produces a low-talent applicant pool and hence a low wage, which in turn sustains the low cutoff, while a high cutoff sustains a high wage by the same logic.*

## 4.3 Selection and robust equilibrium implications

What can be said without ordering records and without knowing which equilibrium is played? A fair amount, it turns out: the cutoff structure alone delivers several implications. Let

$$R_s = \int_0^1 \sigma_s(\theta) m_s(\theta)\, d\theta \tag{27}$$

be the mass choosing self-employment at record $s$. Recall that $D_s$ is the mass choosing firm employment and $M_s$ is the total mass at the record. At an on-path record, define mean talent

in the full record-specific population by

$$\bar{\theta}_s = \frac{1}{M_s} \int_0^1 \theta m_s(\theta)\, d\theta. \tag{28}$$

Whenever $R_s > 0$, the expected success rate among workers choosing self-employment is

$$p_s^S = \frac{\int_0^1 \theta \sigma_s(\theta) m_s(\theta)\, d\theta}{R_s}. \tag{29}$$

Because the stationary type–record distributions have densities, the action assigned to the single cutoff type has no effect on these objects. The cutoff rule therefore gives the accounting identities

$$D_s = \int_0^{c_s} m_s(\theta)\, d\theta, \qquad R_s = \int_{c_s}^1 m_s(\theta)\, d\theta, \qquad M_s = D_s + R_s. \tag{30}$$

**Proposition 4.** *Fix an on-path record $s$ in a stationary sequential competitive equilibrium.*

1. *If $D_s > 0$, then $c_s > 0$ and the competitive wage is*

$$w_s = \frac{\int_0^{c_s} \theta m_s(\theta)\, d\theta}{\int_0^{c_s} m_s(\theta)\, d\theta} < c_s. \tag{31}$$

2. *If both $D_s > 0$ and $R_s > 0$, then*

$$w_s < c_s < p_s^S \qquad \text{and} \qquad w_s < \bar{\theta}_s < p_s^S. \tag{32}$$

   *Moreover, the difference between the full-population mean at the record and the competitive applicant wage satisfies*

$$\bar{\theta}_s - w_s = \frac{R_s}{M_s}\left(p_s^S - w_s\right) > 0. \tag{33}$$

3. *The mass $R_s$ is the mass of new public outcomes produced at record $s$ in the current period. The expected masses of successes and failures generated there are, respectively, $R_s p_s^S$ and $R_s(1 - p_s^S)$.*

*Proof.* Apart from a zero-mass cutoff type, applicants are precisely the workers with $\theta < c_s$, whereas workers choosing self-employment have $\theta > c_s$. Substituting the first group into the Bayesian wage formula (9) gives (31). A positive-mass distribution that is absolutely continuous and supported below $c_s$ has mean strictly below $c_s$. Similarly, if $R_s > 0$, the mean talent of the self-employed group is strictly above $c_s$. This proves the first ranking in (32).

Using (30), the full-population mean can be written as

$$\bar{\theta}_s = \frac{D_s}{M_s} w_s + \frac{R_s}{M_s} p_s^S. \tag{34}$$

If both groups have positive mass, this is a strict convex combination of $w_s$ and $p_s^S$. The second ranking follows, and subtracting $w_s$ from (34) gives (33). The final claim follows because each worker choosing self-employment produces one Bernoulli public outcome with success probability $\theta$. □

Equation (33) is the relevant adverse-selection comparison. The benchmark $\bar{\theta}_s$ is mean talent among all workers who hold record $s$. Conditional on $D_s > 0$, the discount is strictly positive exactly when $R_s > 0$; if every worker at the record applies, it is zero. Its magnitude is the product of the self-employed share and the talent gap between self-employed workers and applicants.

The cutoff also gives a mechanical comparative static useful for interpretation. Holding the record-specific density $m_s$ fixed, increasing $c_s$ weakly increases the applicant mass and, whenever that pool is non-empty, its mean talent, while reducing the mass that produces public outcomes. This is not a comparative static in a primitive: an equilibrium change in preferences, survival, or patience generally changes $m_s$, $w_s$, and all continuation values at the same time.

Finally, every type with $\theta < c_s$ strictly prefers firm employment. Proposition 1 therefore implies that, conditional on survival, such a worker remains in firm employment at the same record. The model consequently predicts both positive selection into public production within a record and persistence after strict entry into opaque employment. These implications can be taken to data after conditioning on the worker's complete public record.

No general ordering of $c_s$, $w_s$, or $\bar{\theta}_s$ across records follows from the model as stated. In particular, record length $g + b$ is not by itself a posterior-precision parameter once the endogenous paths leading to a record are incorporated. Nor does the existence result select a unique equilibrium; Remark 1 shows that multiplicity actually occurs. We therefore do not assign signs to global changes in $\delta$, $\rho$, risk aversion, or record length without additional restrictions and an equilibrium-selection argument.

# 5 Empirical content and testable implications

What would count as evidence for, or against, the model? The sharp empirical restrictions are within-record restrictions. We compare workers in the same stationary environment who hold the same observed public record $s = (g, b)$. This conditioning does not make the record exogenous: the composition of each record cell still reflects all the occupational choices and public outcomes that led workers to it. It simply holds the equilibrium state fixed. The baseline does not, by contrast, order wages, cutoffs, or occupational choices across different records.

## 5.1 Within-record selection into public production

Suppose that both employment modes are used at an on-path record. Apart from the zero-mass cutoff type, workers choosing firm employment have $\theta < c_s$, whereas those choosing self-employment have $\theta > c_s$. Proposition 4 therefore gives

$$w_s < c_s < p_s^S. \tag{35}$$

Here $w_s$ is mean talent in the applicant pool under competitive pricing, and $p_s^S$ is the expected success rate among workers who continue to produce public outcomes.

Conditional on the same public record, workers who remain in self-employment have higher talent than workers who apply for opaque firm employment. Talent need not be

observed directly to test this ranking. Suppose, for example, that the researcher observes a later performance measure for both groups and that its conditional expectation is the same strictly increasing function of talent in both modes. Mean later performance must then be higher among workers who continue in public production. The comparability requirement matters: an outcome that is mechanically affected differently by the two occupations is not, without further structure, a valid proxy for this test.

The relevant comparison conditions on the complete public record used in the model, rather than on a coarser summary of that record. After this conditioning, the current application still contains information: it selects a worker from the lower part of the endogenous talent distribution in that cell. This is a cross-sectional prediction within a record. It does not say that a success or failure causes a worker to change occupation, since the cutoffs at successor records need not be ordered.

## 5.2 Application-based wage discounts

At an on-path record with applicants, define the selection discount by

$$d_s \equiv \bar{\theta}_s - w_s. \tag{36}$$

When both modes have positive mass, Proposition 4 gives the exact decomposition

$$d_s = \frac{R_s}{M_s}\left(p_s^S - w_s\right) > 0. \tag{37}$$

The competitive wage offered to applicants is below mean talent among all workers who hold the same record. Equation (37) separates this discount into the share of workers who remain in self-employment and the talent gap between public producers and applicants. Both margins are endogenous, but together they account exactly for the record-specific discount.

An exact implementation of this decomposition requires productivity and wages to be measured on the common scale represented by $\theta$ in the model. A setting in which individual performance inside the firm is available to the researcher but hidden from the outside market is particularly suitable.[14] Mean performance among all workers in a record cell can then identify $\bar{\theta}_s$, the public success rate identifies $p_s^S$, and zero profit implies that the applicant wage equals mean applicant productivity.

The benchmark $\bar{\theta}_s$ is generated by the stationary population flows, not by the success and failure counts in isolation. Nor does (37) imply that wages rise or fall after a success, a failure, or an increase in record length. Such comparisons change the record-specific density and the cutoff; their signs are equilibrium outcomes.

## 5.3 Persistence of opaque employment

For every type strictly below $c_s$, firm employment is uniquely optimal. Since firm employment changes neither the public record nor the stationary wage, Proposition 1 implies that the same choice remains uniquely optimal in every surviving period. Because the stationary

[14] If the researcher has only a monotone proxy for talent, the within-record ordering remains testable, but the cardinal decomposition in (37) need not be preserved.

type–record distributions have densities, the indifferent cutoff type has zero mass. Thus, whenever the firm-employment pool has positive mass, this strict prediction applies to almost every worker in it.

Conditional on survival and an unchanged environment, a worker who enters opaque firm employment does not return to public production. In the baseline, an opaque-employment spell ends only through exogenous exit and therefore has a geometric duration with exit probability $\lambda$. This is persistence in the employment mode; the model does not imply tenure with the same firm. Systematic returns to public production at an unchanged record would point to a state variable absent from the baseline, such as private learning, a changing contract, or a signal generated inside the firm.

This prediction applies to a state–type pair, not to a record at which every worker becomes absorbed. At the same record, lower types can remain in opaque employment while higher types continue to produce public outcomes. Persistence and continuing public-information production can therefore occur side by side within one record cell.

## 5.4 Public-information flows and implementation

At record $s$, the mass $R_s$ produces one public outcome per worker and period. The expected masses of successes and failures are $R_s p_s^S$ and $R_s(1-p_s^S)$, respectively. The stationary model also supplies an aggregate restriction on long records. Reaching a record of length at least $L$ requires survival for at least $L$ periods, and hence

$$\sum_{|s|\geq L} M_s \leq \rho^L. \tag{38}$$

This cross-sectional bound is independent of equilibrium selection and of the location of the cutoffs.[15]

The most direct application is a panel in which the researcher observes a portable public history, the choice between further public production and an opaque job, the applicant wage, and a comparable measure of later performance. Online labour markets, open-source contribution histories, contests, and professional portfolios approximate these requirements. Evidence that public evaluations and credible intermediaries affect later employment outcomes motivates these settings (Pallais, 2014; Stanton and Thomas, 2016); experience-based hiring in online markets provides a related application (Leung, 2018).

A non-parametric implementation can form cells defined by $(g, b)$ within a common market and period, and compare choices and later outcomes inside each cell. A structural implementation can instead estimate the stationary-flow equations (2)–(4) jointly with the cutoff rules and the wage equation. The latter approach must account for every predecessor route and for retention through firm employment. Treating the observed successes and failures as an exogenous sequence would omit the selection mechanism that determines both the record-specific population and the applicant pool.

The baseline therefore yields four closely related empirical objects: positive selection into public production, a record-specific applicant-wage discount, persistent opaque-employment

[15] It is, in fact, the only restriction of the model that is entirely free of the equilibrium object: it uses survival and the record-length accounting, nothing more, and is therefore the natural first falsification test in any data set with portable histories.

choices, and endogenous flows of public outcomes. Comparative predictions in patience, risk aversion, transparency, or record length require additional restrictions and are not imposed as baseline tests.

# 6 Scope and extensions

Several restrictions do substantive work in the baseline. Talent is fixed and privately known by the worker. The public state retains only the counts of outcomes produced in self-employment; calendar age and the dates of those outcomes are not observed. Firm employment is opaque and leaves that state unchanged. Finally, firms offer a stationary competitive wage after observing the public record and the worker's application. Even under these restrictions, beliefs must incorporate the endogenous process that brings each type to a record and then into its applicant pool. The stationary equilibrium above does so.

The following extensions cannot be handled by changing one parameter in the baseline. Each alters the state space, the population flows, or the game played by firms. We therefore indicate what would have to be reconstructed, without attaching comparative statics to extensions that have not been solved.

## 6.1 Private learning about talent

In the baseline, a worker's private information is summarised by the fixed type $\theta$. If workers do not know their talent and instead learn from privately informative experience, a private posterior becomes part of the state. Even when the market continues to observe only $s$, an application can then reveal information about both the worker's posterior and the underlying talent.

A stationary version of this extension would require a joint distribution of talent, private beliefs, and public records. The applicant belief would be obtained by conditioning this joint distribution on the equilibrium application rule. If a worker privately observes performance inside a firm, firm employment may change the private state although it leaves the public record unchanged. The repeated-comparison argument in Proposition 1 then fails: after one period in the firm, the worker may face a different decision problem.

A cutoff in a scalar posterior may be recovered under additional sufficiency and single-crossing conditions. It is not automatic. In particular, two posterior distributions with the same mean can generate different continuation values. The known-type model is the limiting case in which the worker's posterior is degenerate and does not evolve.

## 6.2 Observable age and richer public histories

The stationary density $m_s$ pools all workers who hold record $s$, irrespective of their unobserved career age or the time elapsed between public outcomes. Thus, at the blank record, a worker may be a new entrant or an older worker who has repeatedly retained the record through firm employment. The stock equation incorporates this difference even though firms do not observe it.

If career age $a$ were public, beliefs and wages would instead be indexed by $(s, a)$. Since age advances after a surviving period of firm employment, that action would no longer leave the full public state fixed; the persistence result would therefore have to be reconsidered. Likewise, if the order or dates of past outcomes were observed, the count state $s$ would have to be replaced by the corresponding richer history. In either case, the distribution at a public state would still be generated by earlier occupational choices. Observing more of the history changes the relevant flow equations; it does not remove application-based selection.

## 6.3 Public signals generated inside firms

Complete opacity gives firm employment its stopping character in the baseline. Suppose instead that firm employment generates a portable public signal with conditional distribution $Q_E(z \mid \theta)$. The public state must then include signals produced in both modes, and beliefs must condition on the endogenous choice of mode that precedes each signal. Wages and stationary type–record distributions must be reconstructed on this enlarged state space.

More informative in-firm signals can help talented workers build a reputation, but they can also alter which types apply and hence feed back into competitive wages. Without restrictions on this selection response, a Blackwell ordering of the signal technology alone is not enough to order cutoffs, applicant wages, or the mass choosing either mode. Once firm employment changes the public state, strict persistence need not survive either. The baseline is the boundary case in which the in-firm signal is degenerate and firm employment produces no public-state transition.

## 6.4 Contracts, disclosure, and hidden effort

Allowing firms to combine wages with different disclosure policies changes the firm side of the model. A contract menu cannot be determined by zero profit alone. The output and disclosure technologies, any cost of visibility, firms' commitment possibilities, and an equilibrium notion for competing menus must also be specified. Since contract choice is informative, firms must form beliefs about the types selecting each on-path menu. Pooling, separation within firm employment, and selection back into self-employment may all arise; the baseline does not rank them.

Hidden effort introduces a related moral-hazard problem. If public success occurs with probability $q(\theta, e)$ and effort $e$ is costly and unobserved, outcomes reflect both talent and the equilibrium effort policy. The success rate among public producers then need no longer equal their mean talent. The transition terms in the stationary-flow equations and the worker's dynamic action gap must both be modified. Cutoff behaviour and within-record positive selection may survive under suitable restrictions on $q$ and effort costs, but they do not follow from the baseline argument.

## 6.5 Taxes, subsidies, and welfare

The baseline is a positive model of occupational choice, belief formation, and competitive wages. It does not specify the production and assignment environment, a social value of information, or a planner's objective. More public information may improve later matching,

but producing it exposes workers to risk and changes selection across modes. The wage equation alone is therefore insufficient for a welfare comparison.

If a tax on self-employment is introduced as a reduction in its current payoff, it weakly reduces the attractiveness of that action when wages, continuation values, and the population distribution are held fixed. This partial-equilibrium observation is neither a welfare result nor a general-equilibrium comparative static. A tax or subsidy changes occupational choices, stationary type–record distributions, applicant beliefs, and wages simultaneously. With possible equilibrium multiplicity, a policy comparison also requires an equilibrium-selection rule.

A welfare analysis would consequently have to specify output and assignment, the value of the information produced, fiscal incidence and any resource costs, and the treatment of tax revenue. The resulting equilibrium belief system would then have to be solved again. Without this additional structure, neither the welfare effect nor the change in the total production of public outcomes is signed.

These qualifications preserve the scope of the baseline results. They establish cutoff behaviour, persistence after strict entry into opaque employment, and within-record selection for the stated public state and stationary environment. Once private learning, richer histories, in-firm information, contracts, effort, or policy changes that state or environment, beliefs must again be constructed from the induced paths and current selection, and the substantive results must be proved anew.

# 7 Conclusion

We have studied a labour market in which workers choose between self-employment, which makes performance public and portable, and firm employment, which offers a stable competitive wage while keeping individual output hidden. Because workers decide when to generate a public outcome and whether to apply for opaque employment, both the observed record and the applicant pool are selected objects—and the key modelling step is to respect this selection twice over: first by constructing each record-specific population from the stationary flow through all predecessor records, including the periods in which the record is quietly retained inside firms, and only then by conditioning on the current application.

The analysis delivers four conclusions. When future career rewards are not too strong, a stationary equilibrium exists and occupational choice follows a record-specific talent threshold; the equilibrium need not be unique, and we exhibit a specification with two distinct equilibria sustained by the complementarity between cutoffs and applicant wages. Where both modes are used at a record, higher-talent workers continue in public production while lower-talent workers apply to firms, so the applicant wage lies below mean talent among all holders of the record, with a discount that decomposes exactly into the self-employed share and the talent gap between the two groups. Firm employment is persistent whenever it is strictly preferred, because it leaves both the record and the stationary wage unchanged—the choice is self-confirming. And occupational choices determine the flow of new public outcomes, and hence how much the market ever learns. These are within-record conclusions: they neither order different records nor imply that a success or failure raises or lowers wages.

The natural empirical settings are panels that combine portable histories, occupational

choices, wages, and comparable later outcomes—online labour markets, open-source contribution records, contests, and professional portfolios. Conditional on the public record, applicants for opaque employment should perform worse on a common later measure, applicant wages should display a record-specific selection discount, and opaque-employment choices should persist. Private learning, observable career age, dated histories, in-firm signals, richer contracts, hidden effort, and policy interventions would each change the state space or the equilibrium game, and require the belief system to be rebuilt from the induced paths. The broader lesson of the paper is easily stated: beliefs must account not only for the outcomes a market observes, but also for the choices that generated the record and selected the applicant pool—including the choice to produce no outcome at all.

# Appendix A Technical details for stationary equilibrium

This appendix supplies the population-flow, continuity, and limiting details used in the equilibrium construction. All objects are those defined in the main text. In particular, the public record is $s = (g, b)$, its length is $|s| = g + b$, and $\sigma_s(\theta)$ is the probability of self-employment at that record.

## Appendix A.1 Stationary stocks induced by a strategy

The stationary recursion in (2)–(4) can be derived directly from the age distribution of the population. The derivation also provides the domination and record-tail bounds used below.

**Lemma 3.** *Fix any measurable stationary strategy $\sigma = (\sigma_s)_{s\in\mathcal{S}}$. Then:*

1. *Equations (2)–(4) determine a unique collection of nonnegative integrable densities $(m_s)_{s\in\mathcal{S}}$.*

2. *For every fixed record $s$, there is a finite constant $C_s$, independent of $\sigma$, such that*

$$0 \le m_s(\theta) \le C_s f_0(\theta) \quad \text{for almost every } \theta. \tag{A.1}$$

3. *The densities have total mass one and satisfy*

$$\sum_{|s|\ge L} \int_0^1 m_s(\theta)\, d\theta \le \rho^L \qquad \text{for every } L \ge 0. \tag{A.2}$$

4. *If the strategy is completely mixed, then $M_s > 0$ and $D_s > 0$ at every record.*

*Proof.* For each non-negative integer $a$, let $q_s^a(\theta)$ be the joint density of talent and record among workers of age $a$, conditional on having survived to that age. At entry,

$$q_s^0(\theta) = f_0(\theta)\mathbf{1}\{s = s_0\}. \tag{A.3}$$

Conditional on survival for one more period, the strategy and the outcome technology imply

$$\begin{aligned} q_{g,b}^{a+1}(\theta) = \Big(1-\sigma_{g,b}(\theta)\Big) q_{g,b}^{a}(\theta) \\ + \mathbf{1}\{g \geq 1\}\theta\sigma_{g-1,b}(\theta) q_{g-1,b}^{a}(\theta) \\ + \mathbf{1}\{b \geq 1\}(1-\theta)\sigma_{g,b-1}(\theta) q_{g,b-1}^{a}(\theta). \end{aligned} \tag{A.4}$$

The transition probabilities from any record sum to one. Consequently,

$$\sum_{s\in\mathcal{S}} \int_0^1 q_s^a(\theta)\, d\theta = 1 \qquad \text{for every } a.$$

The stationary population is a mixture of these cohorts with age weights $\lambda\rho^a$:

$$\widehat{m}_s(\theta) = \lambda \sum_{a=0}^{\infty} \rho^a q_s^a(\theta). \tag{A.5}$$

Using (A.3)–(A.4) in (A.5) shows that $\widehat{m}_s$ satisfies

$$\begin{aligned} \widehat{m}_{g,b}(\theta) = \lambda f_0(\theta)\mathbf{1}\{(g,b)=(0,0)\} \\ + \rho\Big(1-\sigma_{g,b}(\theta)\Big)\widehat{m}_{g,b}(\theta) \\ + \rho\mathbf{1}\{g \geq 1\}\theta\sigma_{g-1,b}(\theta)\widehat{m}_{g-1,b}(\theta) \\ + \rho\mathbf{1}\{b \geq 1\}(1-\theta)\sigma_{g,b-1}(\theta)\widehat{m}_{g,b-1}(\theta). \end{aligned}$$

This is precisely the balance equation underlying (2)–(4). Conversely, the stock equation has a denominator $\lambda + \rho\sigma_s(\theta) \geq \lambda > 0$, and its numerator depends only on densities at shorter records. Therefore, it uniquely determines the densities by induction on $|s|$. Hence, $\widehat{m} = m$.

For the domination claim, $m_{0,0} \leq f_0$. If envelopes are obtained from the predecessor records, then

$$m_{g,b}(\theta) \leq \frac{\rho}{\lambda}\left[\mathbf{1}\{g \geq 1\}m_{g-1,b}(\theta) + \mathbf{1}\{b \geq 1\}m_{g,b-1}(\theta)\right].$$

Thus, (A.1) follows by induction, for example, with $C_{0,0} = 1$ and

$$C_{g,b} = \frac{\rho}{\lambda}\left[\mathbf{1}\{g \geq 1\}C_{g-1,b} + \mathbf{1}\{b \geq 1\}C_{g,b-1}\right].$$

Summing (A.5) over records and integrating over talent yields

$$\sum_s M_s = \lambda \sum_{a=0}^{\infty} \rho^a = 1.$$

A worker of age $a$ can have generated at most $a$ public outcomes, so $q_s^a = 0$ whenever $|s| > a$. Therefore

$$\sum_{|s|\geq L} M_s \leq \lambda \sum_{a=L}^{\infty} \rho^a = \rho^L,$$

which proves (A.2).

Finally, suppose that the strategy is completely mixed. At every interior talent $\theta \in (0,1)$, each finite sequence of self-employment choices and binary outcomes has a strictly positive probability. Since $f_0$ is strictly positive, induction on record length gives $m_s(\theta) > 0$ for every finite record and almost every talent. Hence $M_s > 0$. Moreover, $1 - \sigma_s(\theta) > 0$, so the applicant density has a positive integral and $D_s > 0$. □

The next result justifies the dominated convergence steps in the proof of Proposition 3.

**Lemma 4.** *Let $\sigma^n$ and $\sigma$ be measurable stationary strategies satisfying*

$$\sigma_s^n(\theta) \longrightarrow \sigma_s(\theta) \quad \text{for almost every } \theta$$

*at every fixed record. Let $m^n$ and $m$ be their induced stationary density systems. Then, for every fixed record $s$,*

$$\int_0^1 |m_s^n(\theta) - m_s(\theta)|\, d\theta \longrightarrow 0. \tag{A.6}$$

*If $M_s > 0$, the induced public beliefs converge to $G_s$ in total variation. If $D_s > 0$, the induced applicant beliefs converge to $\nu_s$ in total variation. The corresponding masses and first moments converge as well.*

*Proof.* We prove almost-everywhere convergence and $L^1$ convergence simultaneously. At the blank record,

$$m_{0,0}^n(\theta) = \frac{\lambda f_0(\theta)}{\lambda + \rho\sigma_{0,0}^n(\theta)} \longrightarrow \frac{\lambda f_0(\theta)}{\lambda + \rho\sigma_{0,0}(\theta)} = m_{0,0}(\theta)$$

for almost every talent. The envelope in (A.1) and dominated convergence give (A.6) at $s_0$.

Suppose that both convergence properties hold at all records of length less than $|s|$. The inflow formula (3) then converges almost everywhere at $s$. The stock formula has a denominator bounded below by $\lambda$, so $m_s^n \to m_s$ almost everywhere. The same strategy-independent envelope again yields $L^1$ convergence. Induction establishes (A.6) at every fixed record.

It follows immediately that $M_s^n \to M_s$ and that $\int \theta m_s^n(\theta)\, d\theta$ converges to the corresponding limiting moment. If $M_s > 0$, normalising the densities therefore gives total-variation convergence of the public beliefs. Likewise,

$$\Big(1 - \sigma_s^n(\theta)\Big) m_s^n(\theta) \longrightarrow \Big(1 - \sigma_s(\theta)\Big) m_s(\theta)$$

almost everywhere and is dominated by $C_s f_0$. Its mass and first moment converge. If the limiting mass $D_s$ is positive, normalisation gives total-variation convergence of the applicant beliefs. □

## Appendix A.2 Worker values and finite-record caps

The Bellman operator associated with (14) acts on the space of bounded functions on $\mathcal{S} \times [0,1]$, equipped with the supremum norm. The maximum operator is nonexpansive, and the continuation term is multiplied by $\gamma$, so the operator is a contraction with modulus $\gamma$. This proves existence and uniqueness of the bounded value function. Starting value iteration from a function taking values in $[0,1]$ also shows that the fixed point remains in $[0,1]$.

The following finite-depth argument supplies the convergence assertion in (25) without requiring the uniform convergence of the entire infinite wage schedule.

**Lemma 5.** *Let $K_n \to \infty$, and for each $n$ let $w^n \in [0,1]^{\mathcal{S}_{K_n}}$. Suppose that for every fixed record $r$, $w_r^n \to w_r$ once $r \in \mathcal{S}_{K_n}$. Let $U^n$ be the capped value function with the boundary*

*condition in (20), and let $U$ be the unique bounded solution of the infinite-record worker problem at $w$. Then, for every fixed record $s$,*

$$\sup_{\theta\in[0,1]} |U_s^n(\theta) - U_s(\theta)| \longrightarrow 0. \tag{A.7}$$

*Proof.* Fix a record $s$ and a continuation depth $H$. For all sufficiently large $n$, every descendant of $s$ at a distance less than $H$ is interior to the cap. The set of descendants is finite. Hence, the continuity of $u$ and pointwise convergence of wages give

$$\varepsilon_{n,H} \equiv \max_{\substack{r \text{ descends from } s \\ \text{by fewer than } H \text{ outcomes}}} |u(w_r^n) - u(w_r)| \longrightarrow 0.$$

At descendants exactly $H$ outcomes from $s$, both the capped and infinite values lie in $[0,1]$, so their difference is at most one. Work backwards through the common Bellman recursion. At each preceding layer,

$$|\max\{a,b\} - \max\{a',b'\}| \le \max\{|a-a'|, |b-b'|\},$$

and the difference between the two continuation terms is at most $\gamma$ times the largest difference at the successor layer. It follows that

$$\sup_{\theta\in[0,1]} |U_s^n(\theta) - U_s(\theta)| \le \max\{\varepsilon_{n,H}, \gamma^H\}$$

for all sufficiently large $n$. First, let $n \to \infty$ and then $H \to \infty$ to obtain (A.7). □

The same backward argument with a fixed cap $K$ shows that $w \mapsto U^{K,w}$ is continuous in the supremum norm. Because the set of records is finite and the logistic function is continuously differentiable, (21) implies that $w \mapsto \sigma^{K,w,\eta}$ is continuous, uniformly in talent.

## Appendix A.3 Continuity of the perturbed wage map

We record the details behind Lemma 2. Fix $K$ and $\eta \in (0, 1/2)$. At every interior record,

$$\eta < \sigma_s^{K,w,\eta}(\theta) < 1 - \eta.$$

Thus every finite record can be reached with positive probability for almost every talent. The capped boundary is reached through completely mixed interior choices and, once reached, all workers apply for firm employment. Consequently, every capped record has positive mass and the applicant denominator in (23) is strictly positive.

Now take $w^n \to w$ in the finite-dimensional cube $[0,1]^{\mathcal{S}_K}$. The preceding subsection gives uniform convergence of the capped values and action probabilities. Applying the stock recursion successively by record length, with denominators bounded below by $\lambda$, gives almost-everywhere convergence of the capped densities. The envelopes constructed in (A.1) apply uniformly in $n$, so all applicant masses and first moments converge by dominated convergence. The ratio defining $\Phi^{K,\eta}$ is therefore continuous at $w$. Since each component is the mean of a probability distribution on $[0,1]$, $\Phi^{K,\eta}$ maps the wage cube into itself. Brouwer's theorem then gives the fixed point asserted in Lemma 2.

## Appendix A.4 Passage to the infinite-record equilibrium

For completeness, we spell out the compactness and identification steps used in Proposition 3. Choose $K_j \uparrow \infty$ and $\eta_j \downarrow 0$, and select a perturbed fixed point at each pair $(K_j, \eta_j)$. Extend its strategy to the full record space by assigning a strictly interior action probability at and beyond the cap, and extend its wage vector arbitrarily outside the cap. The resulting full strategies are completely mixed.

The record space is countably infinite. The wage at each record belongs to the compact interval $[0, 1]$, and the set of probability measures on $[0, 1]$ is compact under weak convergence. A diagonal argument therefore provides a single subsequence along which, simultaneously at every fixed record,

$$w_s^j \to w_s, \qquad G_s^j \Rightarrow G_s, \qquad \nu_s^j \Rightarrow \nu_s. \tag{A.8}$$

Once $K_j > |s|$, the stationary density at $s$ depends only on the strategy at $s$ and its predecessors. All these records are interior to the cap. The full-strategy density and applicant belief at $s$ therefore coincide with those used in the capped fixed-point equation. Hence

$$w_s^j = \int_0^1 \theta \, \nu_s^j(d\theta) \quad \text{for all sufficiently large } j. \tag{A.9}$$

Lemma 5 provides the uniform convergence of values and self-employment values at every fixed record. Let $h_s(\theta) = Q_s(\theta) - u(w_s)$. Under Assumption 1, Lemma 1 implies that $h_s$ is strictly increasing and has at most one zero. Therefore, away from that possible zero, the logistic responses converge to

$$\sigma_s(\theta) = \mathbf{1}\{h_s(\theta) > 0\}. \tag{A.10}$$

Thus convergence holds for almost every talent at every record, and the limit is optimal. The action at an indifferent cutoff can be chosen arbitrarily without affecting any population integral.

Lemma 4 now identifies the limiting stationary density at every fixed record and gives the Bayesian formulas whenever the limiting record or applicant pool has positive mass. The cohort argument in Lemma 3 gives both normalisation and the uniform tail bound

$$\sum_{|s| \geq L} M_s^j \leq \rho^L.$$

Equivalently, for every $L$ the finitely many records with $|s| < L$ contain at least $1 - \rho^L$ of the population. Coordinatewise convergence on this finite set and then $L \to \infty$ rule out loss of mass in the limit.

If a limiting record or applicant pool has zero mass, Bayes' rule does not identify its belief from the limiting density. The weak limits in (A.8) supply exactly the beliefs required by the consistency clause of Definition 1. Finally, because $\theta$ is bounded and continuous, the weak convergence of applicant beliefs allows passage to the limit in (A.9):

$$w_s = \int_0^1 \theta \, \nu_s(d\theta)$$

at every record, including records with no limiting applicants. The same completely mixed full strategies generate convergent beliefs in (A.8), so they constitute the consistency sequence in Definition 1. This completes all the limiting steps in the proof of existence.

# Appendix A The finite-cap environment

An active worker has permanent private talent $\theta \in [0,1]$, uniformly distributed at entry. The public record is $s = (g, b)$, and under cap $K$ the state space is

$$\mathcal{S}_K = \{(g,b) \in \mathbb{N}_0^2 : g + b \le K\}.$$

Firm output equals $\theta$. Utility from a firm wage is $\sqrt{w}$, whereas self-employment has expected flow payoff $\theta$. Write

$$\gamma = \delta\rho$$

for the effective continuation factor and $\lambda = 1 - \rho$ for the exit rate. Firm employment retains the current record. At a nonterminal record, self-employment produces a success with probability $\theta$ and a failure with probability $1-\theta$. At a terminal record, only firm employment is available, so the public record remains fixed until exit.

Let $\sigma_s(\theta)$ be the probability of self-employment and let $m_s(\theta)$ be the stationary type density at record $s$. The flows are

$$a_{0,0}(\theta) = \lambda, \tag{A.1}$$

$$a_{g,b}(\theta) = \rho\Big[\mathbf{1}\{g \ge 1\}\theta\sigma_{g-1,b}(\theta)m_{g-1,b}(\theta) + \mathbf{1}\{b \ge 1\}(1-\theta)\sigma_{g,b-1}(\theta)m_{g,b-1}(\theta)\Big], \tag{A.2}$$

and

$$m_s(\theta) = \begin{cases} a_s(\theta)/(\lambda + \rho\sigma_s(\theta)), & |s| < K, \\ a_s(\theta)/\lambda, & |s| = K. \end{cases} \tag{A.3}$$

For an on-path applicant pool, competition gives

$$w_s = \frac{\int_0^1 \theta(1-\sigma_s(\theta))m_s(\theta)\,d\theta}{\int_0^1 (1-\sigma_s(\theta))m_s(\theta)\,d\theta}. \tag{A.4}$$

Equations (A.1)–(A.3) aggregate all endogenous applications along every ordered history leading to the observed counts.

# Appendix B Normalised values and two elementary wage identities

For a wage vector $w$, let

$$U_s^K(\theta; w) = (1-\gamma)V_s^K(\theta; w)$$

be normalised lifetime utility. At a terminal record,

$$U_s^K(\theta; w) = \sqrt{w_s}. \tag{B.5}$$

At a nonterminal record define the normalised value of choosing self-employment now by

$$Q_s^K(\theta; w) = (1-\gamma)\theta + \gamma\Big[\theta U_{g+1,b}^K(\theta; w) + (1-\theta)U_{g,b+1}^K(\theta; w)\Big]. \tag{B.6}$$

Then

$$U_s^K(\theta; w) = \max\{\sqrt{w_s}, Q_s^K(\theta; w)\}. \tag{B.7}$$

**Lemma 6.** *For every cap, state, wage vector, and type,*

$$0 \le U_s^K(\theta; w) \le 1. \tag{B.8}$$

*At every nonterminal state,*

$$(1-\gamma)\theta \le Q_s^K(\theta; w) \le (1-\gamma)\theta + \gamma. \tag{B.9}$$

*If $\gamma < 1/2$, $Q_s^K(\theta; w)$ is strictly increasing in $\theta$ at every nonterminal state. Consequently, equality $\sqrt{w_s} = Q_s^K(c; w)$ makes $c$ the unique interior cutoff at state $s$.*

*Proof.* The bounds follow by backward induction from (B.5). For single crossing, successor values are nondecreasing and lie in $[0, 1]$. For $y > x$, the change in

$$\theta U_{g+1,b}^K(\theta) + (1-\theta) U_{g,b+1}^K(\theta)$$

is bounded below by $-(y-x)$. Equation (B.6) therefore changes by at least $(1-2\gamma)(y-x) > 0$. □

Two wages can be calculated without making any assumption about later cutoffs. If the blank-record strategy is

$$\sigma_{0,0}(\theta) = \mathbf{1}\{\theta > x\},$$

then blank-record applicants are uniform on $[0, x]$, so

$$w_{0,0} = \frac{x}{2}. \tag{B.10}$$

If the failure-record strategy is also forced to be

$$\sigma_{0,1}(\theta) = \mathbf{1}\{\theta > y\}, \qquad x < y,$$

then the density arriving at $(0, 1)$ is proportional to $(1-\theta)\mathbf{1}\{\theta > x\}$. Hence

$$w_{0,1} = W(x, y) \equiv \frac{\int_x^y \theta(1-\theta)\, d\theta}{\int_x^y (1-\theta)\, d\theta}. \tag{B.11}$$

The survival and stationary-stock factors cancel from both ratios.

**Lemma 7.** *$W(x, y)$ is strictly increasing in each endpoint on $0 \le x < y < 1$. Moreover,*

$$W\left(0, \frac{1}{5}\right) = \frac{13}{135}, \qquad W\left(\frac{1}{10}, \frac{4}{5}\right) = \frac{62}{165}. \tag{B.12}$$

*Proof.* Writing $D = \int_x^y (1-\theta) d\theta$, differentiation gives

$$W_x = \frac{(1-x)(W-x)}{D} > 0, \qquad W_y = \frac{(1-y)(y-W)}{D} > 0.$$

The two values in (B.12) follow by direct integration. □

# Appendix C A fixed-point device with forced cutoff coordinates

Let

$$L_\eta(z) = \frac{1}{1 + e^{-z/\eta}}.$$

At any nonterminal state whose cutoff is not forced, use the completely mixed rule

$$\sigma_s^\eta(\theta; w) = L_\eta\Big(Q_s^K(\theta; w) - \sqrt{w_s}\Big). \tag{C.13}$$

At every terminal state set $\sigma_s(\theta) = 0$, as required by the capped economy. Given the forced strategies, the terminal action, and (C.13), equations (A.1)–(A.4) define a wage map for all unforced states. For each fixed $\eta > 0$, this map is continuous. Indeed, values are continuous in wages; moving cutoff indicators are continuous in $L^1$; the flow recursion has denominators bounded below by $\lambda$; and every unforced applicant pool is positive under complete mixing.

The rectangles used below keep every forced continuation region of positive length. Thus every public record has positive inflow during the perturbed construction. Every unforced applicant pool is therefore positive. For a forced pool, equations (B.10) and (B.11) also provide the continuous boundary values used by the fixed-point maps; at the interior fixed points established below, those pools have positive mass.

**Lemma 8.** *Suppose a sequence of fixed points of one of the maps constructed below is chosen with $\eta \downarrow 0$. Every convergent subsequence has a limit that is a stationary sequential competitive equilibrium of the capped economy. Its strategy is a pure cutoff almost everywhere at every nonterminal state, and zero-mass applicant beliefs are consistent limits of Bayesian beliefs.*

*Proof.* The state space is finite. Take a convergent subsequence of cutoff coordinates and wages. Backward induction gives uniform convergence of normalised values and action gaps. Lemma 1 implies that each limiting gap has at most one zero, so (C.13) converges almost everywhere to the optimal pure-cutoff action. The forced indicators also converge almost everywhere. Dominated convergence in the recursive flows gives convergence of all stationary densities. Positive limiting applicant pools satisfy (A.4); for a zero-mass pool, take a weak limit of its Bayesian applicant measures. The wage is the mean of that limiting belief.

At a perturbed fixed point, unforced nonterminal choices are completely mixed, while the one or two forced choices are pure. Replace each forced indicator by a logistic cutoff with an auxiliary noise scale $\varepsilon$. For fixed $\eta$, all perturbed applicant denominators are positive and their Bayesian beliefs converge as $\varepsilon \downarrow 0$ to those generated by the forced indicator. Choose $\varepsilon$ sufficiently small along the sequence $\eta \downarrow 0$. This diagonal sequence is completely mixed among the available actions at every state and converges to the same limiting beliefs, establishing consistency. □

# Appendix D A high equilibrium at every finite cap

Fix $K \geq 2$ and $\eta > 0$. On

$$\mathcal{D}_H = \left[\frac{1}{10}, \frac{4}{5}\right] \times [0,1]^{|\mathcal{S}_K|-1},$$

let $x$ be the forced blank cutoff and let the remaining coordinates be candidate wages. Set $w_{0,0} = x/2$, calculate values, impose the blank strategy $\mathbf{1}\{\theta > x\}$, use (C.13) elsewhere, and let $\Phi_H^\eta$ be the resulting Bayesian wage map for the other states. Define

$$h_0(x, w) = \sqrt{\frac{x}{2}} - Q_{0,0}^K(x; w) \tag{D.14}$$

and the continuous self-map

$$T_H^\eta(x, w) = \left(\text{proj}_{[1/10,4/5]}[x + h_0(x, w)], \Phi_H^\eta(x, w)\right). \tag{D.15}$$

Brouwer's theorem gives a fixed point.

The first coordinate cannot lie on either boundary. By (B.9), when $x = 1/10$ and $\gamma < 1/9$,

$$h_0 \geq \sqrt{\frac{1}{20}} - \frac{1-\gamma}{10} - \gamma > \frac{\sqrt{5}}{10} - \frac{1}{5} = \frac{\sqrt{5}-2}{10} > 0. \tag{D.16}$$

When $x = 4/5$,

$$h_0 \leq \sqrt{\frac{2}{5}} - \frac{4(1-\gamma)}{5} < \sqrt{\frac{2}{5}} - \frac{32}{45} < 0, \tag{D.17}$$

where the last inequality follows after squaring. Projection in (D.15) therefore points strictly into the interval at both endpoints. At the fixed point $x$ is interior and $h_0 = 0$. Lemma 1 makes the forced blank strategy optimal. Letting $\eta \downarrow 0$ and using Lemma 3 proves:

**Proposition 5.** *For every $K \geq 2$ and $0 < \gamma < 1/9$, there is an equilibrium with*

$$\frac{1}{10} < c_{0,0} < \frac{4}{5}.$$

# Appendix E A low equilibrium at every finite cap

Now force two cutoff coordinates. On

$$\mathcal{D}_L = \left[0, \frac{1}{10}\right] \times \left[\frac{1}{5}, \frac{4}{5}\right] \times [0,1]^{|\mathcal{S}_K|-2},$$

write $x$ for the blank cutoff and $y$ for the cutoff at $(0,1)$. Set

$$w_{0,0} = \frac{x}{2}, \qquad w_{0,1} = W(x, y),$$

impose the two cutoff strategies, and use (C.13) elsewhere. Let $\Phi_L^\eta$ be the Bayesian wage map for the remaining states. In addition to (D.14), define

$$h_{0,1}(x, y, w) = \sqrt{W(x, y)} - Q_{0,1}^K(y; w). \tag{E.18}$$

Consider

$$T_L^\eta(x, y, w) = \left(\text{proj}_{[0,1/10]}[x - h_0], \text{proj}_{[1/5,4/5]}[y + h_{0,1}], \Phi_L^\eta(x, y, w)\right). \tag{E.19}$$

This is a continuous self-map and therefore has a fixed point.

All four cutoff boundaries point strictly inward. At $x = 0$,

$$h_0 = -\gamma U_{0,1}^K(0; w) \le -\gamma\sqrt{W(0, y)} \le -\gamma\sqrt{\frac{13}{135}} < 0. \tag{E.20}$$

At $x = 1/10$, inequality (D.16) gives $h_0 > 0$. The opposite sign in the first projection of (E.19) is therefore the one that points inward at both endpoints.

At $y = 1/5$, Lemmas 1 and 2 give

$$h_{0,1} \ge \sqrt{\frac{13}{135}} - \frac{1-\gamma}{5} - \gamma > \sqrt{\frac{13}{135}} - \frac{13}{45} > 0. \tag{E.21}$$

The final inequality is exact because

$$\frac{13}{135} - \left(\frac{13}{45}\right)^2 = \frac{26}{2025} > 0.$$

At $y = 4/5$,

$$h_{0,1} \le \sqrt{\frac{62}{165}} - \frac{4(1-\gamma)}{5} < \sqrt{\frac{62}{165}} - \frac{32}{45} < 0, \tag{E.22}$$

where

$$\left(\frac{32}{45}\right)^2 - \frac{62}{165} = \frac{2894}{22275} > 0.$$

Thus the second projection in (E.19) also points inward at both endpoints. Both cutoff coordinates are interior at the fixed point and both indifference equations hold. Lemma 1 makes both forced strategies optimal. Taking $\eta \downarrow 0$ proves:

**Proposition 6.** *For every $K \ge 2$ and $0 < \gamma < 1/9$, there is an equilibrium with*

$$0 < c_{0,0} < \frac{1}{10}, \qquad \frac{1}{5} < c_{0,1} < \frac{4}{5}.$$

**Theorem 1.** *Suppose talent is uniform, $u(w) = \sqrt{w}$, and $0 < \gamma < 1/9$. Every finite cap $K \ge 2$ has at least two distinct stationary sequential competitive equilibria in pure cutoff strategies at every nonterminal record. Their blank-record cutoffs lie on opposite sides of $1/10$.*

# Appendix F Two distinct infinite-record limits

For each $K \ge 2$, select one equilibrium from Proposition 1 and one from Proposition 2. Apply the finite-to-infinite compactness argument separately to the two sequences. The ingredients are uniform: the terminal boundary at cap $K$ affects the value at a fixed state $s$ by at most

$$\gamma^{K-|s|},$$

and stationary population mass at records of length at least $n$ is at most $\rho^n$. A diagonal subsequence therefore converges, state by state, to an equilibrium of the infinite-record model, with the flow recursion of the paper and consistent Bayesian beliefs.

For completeness, consistency can be carried through the same diagonal argument. For each selected capped equilibrium, choose an element sufficiently far along the completely mixed approximating sequence in Lemma 3 that its Bayesian beliefs and belief means are arbitrarily close at every state under the cap, while its strategy is pointwise close away from the finitely many cutoff types. Add a vanishing self-employment tremble at the terminal records and extend the strategy by assigning probability $1/2$ to self-employment beyond the cap. Since each capped state space is finite, these approximations can be chosen to converge along the cap sequence. At every fixed record, the resulting completely mixed strategies converge almost everywhere to the limiting cutoff strategy, and their Bayesian beliefs converge to the beliefs obtained above. They therefore provide the consistency sequence required by the equilibrium definition in the paper.

Let $x_L$ and $y_L$ be the two first cutoffs in a low subsequential limit. They initially lie in the closed rectangle

$$x_L \in \left[0, \frac{1}{10}\right], \qquad y_L \in \left[\frac{1}{5}, \frac{4}{5}\right].$$

The limiting indifference equations and the same strict boundary inequalities (E.20)–(E.22) exclude every boundary. Hence

$$0 < x_L < \frac{1}{10}.$$

The same argument gives $1/5 < y_L < 4/5$. Likewise, the high subsequential limit satisfies

$$\frac{1}{10} < x_H < \frac{4}{5}$$

by (D.16)–(D.17). In particular, the two limits cannot coincide.

**Theorem 2.** *Under the assumptions of Theorem 1, the model of the paper with an unbounded public record has at least two stationary sequential competitive equilibria in pure cutoff strategies. One has $c_{0,0} \in (0, 1/10)$ and another has $c_{0,0} \in (1/10, 4/5)$.*

Because both cutoffs are strictly between zero and one, both infinite-record equilibria have a positive blank-record applicant pool and a positive mass of workers entering self-employment. Success and failure each occur with positive probability, so the two first-signal records are on path. The result is therefore not the degenerate all-self-employment equilibrium supported only by zero-wage off-path beliefs.

# Appendix G Interpretation and remaining limits

The cap no longer drives multiplicity: the same two disjoint cutoff regions contain equilibria at every finite cap $K \geq 2$ and remain disjoint in the limit. No global nesting assumption is used, and every wage is generated by the full stationary-flow recursion.

The result is deliberately an existence and separation theorem. It does not prove uniqueness inside either region, convergence of a particular finite-cap root without subsequences, or

exhaustion of the equilibrium set. The clean bound $\gamma < 1/9$ is sufficient rather than claimed sharp. The proof uses no numerical root or numerical continuation argument.

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