

Merger Control and Investment: the Role of Efficiency Assessment*

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Abstract

We study how the precision of merger review affects firms' incentives to generate merger-specific efficiencies. Before the review, merging parties can invest in a project combining complementary assets; the project can be implemented only if the merger is approved. The merger control authority observes a noisy signal about these efficiencies and approves if the evidence satisfies a consumer-surplus standard. In a binary-investment benchmark with exogenous signal threshold, greater signal precision fosters investment incentives. The same holds locally with continuous investment and holds as well with Bayesian approval. Better information can therefore strengthen, rather than weaken, incentives to generate merger efficiencies.

Keywords: merger control; investment; innovation; efficiencies; information; antitrust.
JEL codes: L13; L41; O31; D82.

1 Introduction

The current debate on EU merger policy places unusual emphasis on investment, innovation and competitiveness. The Draghi Report stresses Europe's need for greater investment and innovation, whereas the European Commission's ongoing review of the Merger Guidelines expressly considers how merger analysis should account for innovation and investment effects

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(Draghi, 2024; European Commission, 2026). A recurrent concern is that demanding merger control may discourage transactions that could support productive investment. This note points to a different channel: if the assessment of claimed efficiencies becomes less rigorous, firms may have weaker incentives to generate those efficiencies.

The mechanism is simple. Before the agency decides on a proposed merger, the parties can invest in a project which generates merger-specific efficiencies. The agency only observes a noisy signal of these efficiencies, and it authorizes the transaction if the evidence is sufficiently favorable. The precision of that evidence therefore determines how strongly merger approval rewards investment in the project. We show that better information precision can strengthen the incentives of the parties to generate merger efficiencies. The logic is closely related to Holmström’s (1979) Informativeness Principle.¹

Related literature. The paper contributes to the literature on endogenous efficiencies and information in merger control. Banal-Estañol, Macho-Stadler and Seldeslachts (2008) examine how investment and internal organization determine merger efficiencies, whereas Cosnita-Langlais and Tropeano (2012) study how efficiency defenses and remedies affect merger design. Lagerlöf and Heidhues (2005), Dertwinkel-Kalt and Wey (2021), and Bisceglia, Piccolo and Tarantino (2023) analyze incentives to produce or acquire information about efficiencies. Nocke and Whinston (2010) establish conditions under which myopic consumer-surplus-based merger control is dynamically optimal. Their setting features endogenous merger proposals but exogenous efficiencies. We consider instead endogenous efficiencies resulting from unobserved investment and study how information precision affects its return; this provides an ex ante incentive rationale for rigorous assessment.

Roadmap. We first present the key intuition in a baseline setting with a discrete investment decision and exogenous efficiency threshold (Section 2). We then show how the analysis extends to continuous investment choice (Section 3) and Bayesian approval (Section 4).

2 Baseline setting

Two firms contemplate a horizontal merger, subject to approval by an antitrust agency. Before the merger control, they can invest in a project designed to exploit asset complementarities; let $\delta \in \{0, 1\}$ denote the investment decision. Investing ($\delta = 1$) costs C , drawn from a continuously differentiable distribution $F(C)$ with positive density over $\mathbb{R}_+$, and delivers a given efficiency gain $G > 0$; absent investment ($\delta = 0$), no cost is incurred and no efficiency is generated. By construction, the investment cost is sunk before the merger review, and the resulting benefit is merger-specific: the resulting efficiency gain can be realized only if the merger is implemented.

The merger enhances consumer surplus only if the firms invest in the project. The agency, however, does not observe the investment decision δ or the resulting efficiency gain $g \in \{0, G\}$

¹Holmström (1979) studies how information can improve incentives through the design of optimal contracts. Here, authorization follows a specified merger-review rule rather than an optimally chosen incentive contract.

directly; it observes instead

$$s \equiv g + \frac{\varepsilon}{\tau},$$

where ε is a normally distributed noise – i.e., $\varepsilon \sim N(0, 1)$, with c.d.f. $\Phi(\varepsilon)$ and density $\phi(\varepsilon)$ – and $\tau > 0$ measures the precision of the signal observed by the agency. In this baseline setting, the agency applies a consumer-standard test based on a fixed threshold, $\bar{g}$;² that is, it approves the merger if and only if $s \geq \bar{g}$. We focus on the relevant case in which

$$0 < \bar{g} < G.$$

Hence, given the investment decision $\delta \in \{0, 1\}$, the merger is accepted with probability

$$P_\delta(\tau) \equiv \Pr(s \geq \bar{g} \mid \delta) = \Pr(\delta G + \frac{\varepsilon}{\tau} \geq \bar{g}) = 1 - \Phi(\tau(\bar{g} - \delta G)).$$

Letting $\Pi_0 > 0$ denote the profit generated by the merger absent investment, and $\Pi_1 > \Pi_0$ the profit in case of investment (gross of investment costs), the firms have an incentive to invest when they face a cost $C < V(\tau)$, where

$$V(\tau) \equiv P_1(\tau)\Pi_1 - P_0(\tau)\Pi_0 \tag{1}$$

denotes the (expected) investment return. The investment probability is therefore

$$I(\tau) \equiv F(V(\tau)).$$

We have:

Proposition 1 (baseline). *With a fixed approval threshold, greater information precision strictly increases the investment return and the investment probability:*

$$V'(\tau) > 0 \text{ and } I'(\tau) > 0.$$

Proof. See Appendix A.1. ■

The result captures the incentive effect of evidentiary precision. When τ is small, approval probabilities are similar under the two investment choices, as the agency has little ability to distinguish mergers that satisfy the consumer-surplus standard from those that do not – indeed, as $\tau \rightarrow 0$, both approval probabilities converge to 1/2, and so investing has no impact on the probability of approval. As precision rises, $P_1(\tau)$ increases and $P_0(\tau)$ falls. Approval therefore becomes more sensitive to investment, which in turn fosters investment incentives.

²We relax this assumption in Section 4, where the test is endogenously based on the agency's beliefs, at the time of the merger review.

3 Continuous investment

The same mechanism extends locally to continuous investment choice. Let $x \geq 0$ denote investment, $C(x) > 0$ the associated sunk cost, $G(x) > 0$ the prospective merger-specific efficiency gain, and $\Pi(x) > 0$ the profit generated by the merger if implemented, gross of investment cost. Assume that C , G and Π are twice continuously differentiable, with $C(0) = G(0) = \Pi(0) = 0$, $\lim_{x \rightarrow +\infty} G(x) = +\infty$, and

$$C''(x) > 0, \quad G''(x) > 0, \quad \Pi'(x) > 0.$$

We focus on investments that meet the consumer-surplus standard and exceed it by at most one standard deviation of the signal noise; that is, $G(x) \in [\bar{g}, \bar{g} + 1/\tau]$, or:³

$$x \in \mathcal{X}(\tau) \equiv \left\{ x \in \mathbb{R}_+ \mid G(x) \in \left[\bar{g}, \bar{g} + \frac{1}{\tau} \right] \right\}.$$

The approval probability is

$$P(x; \tau) \equiv 1 - \Phi(\tau[\bar{g} - G(x)]),$$

and the net expected value from investing x is $V(x; \tau) - C(x)$, where

$$V(x; \tau) \equiv P(x; \tau) \Pi(x).$$

The marginal return from investment is therefore:

$$\frac{\partial V(x; \tau)}{\partial x} = P(x; \tau) \Pi'(x) + \frac{\partial P(x; \tau)}{\partial x} \Pi(x). \quad (2)$$

Building on this leads to:

Proposition 2 (continuous investment). *In the range $\mathcal{X}(\tau)$, greater signal precision strictly increases the marginal return to investment:*

$$\frac{\partial^2 V(x; \tau)}{\partial x \partial \tau} > 0 \text{ for any } x \in \mathcal{X}(\tau).$$

Consequently, if $x^*(\tau)$ is a strict investment response lying in $\mathcal{X}(\tau)$, then

$$\frac{dx^*(\tau)}{d\tau} > 0.$$

Proof. See Appendix A.2. ■

Proposition 2 extends the insight from the baseline analysis to continuous investments: precision strengthens investment incentives when the project is close enough to the approval

³That is, $\mathcal{X}(\tau) = [\underline{x}, \bar{x}(\tau)] \equiv [G^{-1}(\bar{g}), G^{-1}(\bar{g} + \frac{1}{\tau})]$, where $\bar{x}(\tau)$ decreases from $+\infty$ to $\underline{x}$ as τ goes from 0 to infinity.

margin, as better information makes the approval probability more responsive to investment. The restriction to $x \in \mathcal{X}(\tau)$ is a sufficient condition for this monotonicity: the condition $G(x) \geq \bar{g}$ ensures that the approval probability increases with τ (i.e., $\partial P / \partial \tau \geq 0$), and in this range, the condition $G(x) \leq \bar{g} + 1/\tau$ ensures instead that the impact of a marginal investment on the approval probability also increases in τ (i.e., $\partial^2 P / \partial x \partial \tau > 0$). Outside the range $\mathcal{X}(\tau)$, stronger shape restrictions would be needed for a global result.⁴

4 Bayesian approval

Considering an exogenous signal threshold provides a transparent benchmark. The insight survives, however, if the agency explicitly bases its decision on its belief, given the information it has at the time of the merger. To see this, let us return to the discrete investment choice of the baseline setting, but assume that the agency approves the merger if, according to its information, consumer surplus is not expected to decrease. Let $-L < 0$ and $S > 0$ denote, respectively, the impact on consumer surplus absent and with the efficiency gain. If the agency believes that investment has taken place with probability ρ , it approves if and only if $\rho S - (1 - \rho)L \geq 0$, which amounts to

$$\rho \geq \bar{\rho} \equiv \frac{L}{L + S}.$$

Equivalently, expected efficiency must satisfy

$$\rho G \geq \bar{g} \equiv \frac{LG}{L + S} \in (0, G).$$

The posterior probability ρ is endogenous: it depends on the prior probability of investment, which we will denote by r , as well as on the information technology. Specifically, upon observing s , using Bayes' rule the agency will update its belief from $r \in [0, 1]$ to

$$\rho(s; r, \tau) \equiv \frac{r \phi(\tau(s - G))}{r \phi(\tau(s - G)) + (1 - r) \phi(\tau s)}.$$

There always exists a coordination-breakdown equilibrium in which the firms never invest (hence, $r = 0$) and the agency blocks the merger (as $\rho(\cdot; 0, \tau) = 0$). We now look for an interior equilibrium, in which the firms invest with positive probability. Using $\phi(\varepsilon) = [\exp(-\varepsilon^2/2)] / \sqrt{2\pi}$, the approval test $\rho(s; r, \tau) \geq \bar{\rho}$ is equivalent to

$$s \geq \bar{s}(r, \tau) \equiv \frac{G}{2} + \frac{1}{\tau^2 G} \log \frac{(1 - r) L}{r S}.$$

Investing raises the approval probability from $\Pr[\varepsilon/\tau \geq \bar{s}(r; \tau)]$ to $\Pr[G + \varepsilon/\tau \geq \bar{s}(r; \tau)]$; hence, the return on investment is

$$V(r; \tau) \equiv \Pi_1[1 - \Phi(\tau[\bar{s}(r; \tau) - G])] - \Pi_0[1 - \Phi(\tau \bar{s}(r; \tau))]. \quad (3)$$

⁴A weaker condition for $\partial^2 V / \partial x \partial \tau > 0$ is $\Pi'(x) / [G'(x) \Pi(x)] > \tau^2 [G(x) - \bar{g}] - 1 / [G(x) - \bar{g}]$.

For this extension, we adopt the following assumption:

Assumption A:

$$F(\Pi_1 - \Pi_0) < \bar{\rho} = \frac{L}{L + S}.$$

If approval were guaranteed, firms would invest only to obtain the additional profit $\Pi_1 - \Pi_0$. Assumption A asserts that this would generate too little investment to justify unconditional approval based on the corresponding prior. Merger control has therefore a substantive role: expectations of private investment alone do not adequately protect consumers.

Given the threshold $\bar{s}(r, \tau)$ induced by the agency's prior belief r , firms invest if they face a cost $C \leq V(r; \tau)$, leading to $r = F(C)$; hence, in equilibrium $r = \hat{r}(\tau)$ satisfies

$$\hat{r}(\tau) = F(V(\hat{r}(\tau), \tau)). \quad (4)$$

Building on this leads to:⁵

Proposition 3. *Under Assumption A, greater precision strictly raises the investment return and thus fosters investment at every stable interior equilibrium.*

Proof. See Appendix A.3. ■

Proposition 3 shows that the basic mechanism extends to does not rely on the evidentiary cutoff being an exogenous. As in the baseline setting, greater precision fosters investment incentive. The only difference is that, with Bayesian decision-making this induces the agency to become more optimistic, which may have an ambiguous impact on investment incentives. However, as long as the equilibrium is stable, the direct effect prevails and greater precision therefore induces higher investment.

5 Policy implication

The paper has a simple policy message: investment and innovation efficiencies do not call for weakening merger control. If the assessment of efficiencies becomes less rigorous, then the approval-related return to producing those efficiencies falls. Thus, rigorous merger control is not only important to screen out mergers that cause a significant loss of competition, it is also important to preserve parties' incentives to invest in generating efficiencies.

This implication is relevant to the current European debate. Competitiveness may require more investment, but the analysis shows that investment incentives can be strengthened by making authorization more responsive to demonstrated merger-specific efficiencies. In this sense, rigorous assessment and investment incentives can be complements rather than substitutes.

⁵Stability refers to the usual tâtonnement process; see Appendix A.3 for a precise definition.

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A Appendix

A.1 Proof of Proposition 1

Using $P'_1(\tau) = (G - \bar{g})\phi(\tau(\bar{g} - G))$ and $P'_0(\tau) = -\bar{g}\phi(\tau\bar{g})$, differentiating (1) yields:

$$V'(\tau) = P'_1(\tau)\Pi_1 - P'_0(\tau)\Pi_0 = (G - \bar{g})\phi(\tau(\bar{g} - G))\Pi_1 + \bar{g}\phi(\tau\bar{g})\Pi_0 > 0,$$

where the inequality stems from $G > \bar{g} > 0$, $\phi(\cdot) > 0$, and $\Pi_\delta > 0$ for $\delta = 0, 1$. Furthermore, as $V(\tau)$ strictly increases with τ , so does the resulting probability of investment, $I(\tau) = F(V(\tau))$.

A.2 Proof of Proposition 2

Differentiating (2) with respect to τ yields:

$$\frac{\partial^2 V(x; \tau)}{\partial x \partial \tau} = \frac{\partial P(x; \tau)}{\partial \tau} \Pi'(x) + \frac{\partial^2 P(x; \tau)}{\partial x \partial \tau} \Pi(x), \quad (\text{A.1})$$

where

$$\begin{aligned} \frac{\partial P(x; \tau)}{\partial \tau} &= -\phi(\tau[\bar{g} - G(x)])[\bar{g} - G(x)], \\ \frac{\partial^2 P(x; \tau)}{\partial x \partial \tau} &= \phi(\tau[\bar{g} - G(x)])G'(x) + \phi'(\tau[\bar{g} - G(x)])\tau[\bar{g} - G(x)]G'(x) \\ &= \{1 - \tau^2[\bar{g} - G(x)]^2\}\phi(\tau[\bar{g} - G(x)])G'(x), \end{aligned}$$

where the last equality uses $\phi'(z) = -z\phi(z)$. In the range $x \in \mathcal{X}(\tau)$, we have $\bar{g} \leq G(x) \leq \bar{g} + 1/\tau$; hence, $\partial P/\partial \tau \geq 0$ and $\partial^2 P/\partial x \partial \tau \geq 0$, implying $\partial^2 V/\partial x \partial \tau \geq 0$ (as $\Pi(x) > 0$ for $x > (G^{-1}(\bar{g})) \geq 0$ and $\Pi'(\cdot) > 0$). Furthermore, the first term in (A.1) is positive for $G(x) > \bar{g}$ and the second term is positive for $G(x) < \bar{g} + 1/\tau$. It follows that $\partial^2 V/\partial x \partial \tau \geq 0$ in the entire range $\mathcal{X}(\tau)$.

If $x^*(\tau)$ is a strict investment response lying in $\mathcal{X}(\tau)$, then, from the implicit function theorem:

$$\frac{dx^*(\tau)}{d\tau} = -\frac{\partial^2 V(x^*(\tau); \tau)}{\partial x \partial \tau} / \left[\frac{\partial^2 V(x^*(\tau); \tau)}{\partial x^2} - C'''(x^*(\tau)) \right] > 0,$$

where the inequality follows from $\partial^2 V/\partial x \partial \tau > 0$ for $x \in \mathcal{X}(\tau)$ and the local second-order condition $\partial^2 V(x^*(\tau); \tau)/\partial x^2 - C'''(x^*(\tau)) < 0$.

A.3 Proof of Proposition 3

Consider an interior equilibrium $r = \hat{r}(\tau) > 0$, which thus satisfies (4). Differentiating with respect to τ yields:

$$\hat{r}'(\tau) = \frac{f(V(r; \tau)) \frac{\partial V(r; \tau)}{\partial \tau}}{1 - f(V(r; \tau)) \frac{\partial V(r; \tau)}{\partial \tau}} \bigg|_{r=\hat{r}(\tau)}.$$

Furthermore, the denominator is positive whenever the equilibrium is stable in the usual sense. To see this, consider a standard tâtonnement process in which, in response to a given agency's prior r_n , the firms invest if $C \leq V(r_n; \tau)$, leading to $r_{n+1} = F(V(r_n; \tau))$. This tâtonnement process converges if $d\hat{r}_{n+1}/d\hat{r}_n < 1$, which yields $1 - f(V(r; \tau)) \partial V(r; \tau) / \partial r|_{r=\hat{r}(r)} > 0$. As $f(\cdot) > 0$, it follows that the sign of $\hat{r}'(\tau)$ is the same as that of $\partial V(r; \tau) / \partial \tau|_{r=\hat{r}(r)}$.

Differentiating (3) with respect to τ yields:

$$\frac{\partial V(r; \tau)}{\partial \tau} = -\Pi_1 \phi(\tau [\bar{s}(r; \tau) - G]) \left[\bar{s}(r; \tau) - G + \tau \frac{\partial \bar{s}(r; \tau)}{\partial \tau} \right] + \Pi_0 \phi(\tau \bar{s}(r; \tau)) \left[\bar{s}(r; \tau) + \tau \frac{\partial \bar{s}(r; \tau)}{\partial \tau} \right],$$

where:

$$\frac{\partial \bar{s}(r; \tau)}{\partial \tau} = -\frac{2}{\tau^3 G} \log \frac{(1-r)L}{rS} = \frac{G - 2\bar{s}(r, \tau)}{\tau}.$$

Combining these expressions yields:

$$\frac{\partial V(r; \tau)}{\partial \tau} = \Pi_1 \phi(\tau [\bar{s}(r; \tau) - G]) \bar{s}(r, \tau) + \Pi_0 \phi(\tau \bar{s}(r; \tau)) [G - \bar{s}(r, \tau)]. \quad (\text{A.2})$$

If $\bar{s}(r; \tau) \in [0, G]$, then the right-hand side is positive (both terms are non-negative, and at least one of them is positive). If instead $\bar{s}(r; \tau) > G$, then:

$$\frac{\partial V(r; \tau)}{\partial \tau} = \Pi_1 \phi(\tau [\bar{s}(r; \tau) - G]) G + \{\Pi_1 \phi(\tau [\bar{s}(r; \tau) - G]) - \Pi_0 \phi(\tau \bar{s}(r; \tau))\} [\bar{s}(r, \tau) - G],$$

where both terms of the right-hand side are positive: this is obvious for the first one, and for the second one, it stems from the working assumption $\bar{s}(r, \tau) > G$ and:

$$\frac{\Pi_1 \phi(\tau [\bar{s}(r; \tau) - G])}{\Pi_0 \phi(\tau \bar{s}(r; \tau))} > \frac{\phi(\tau [\bar{s}(r; \tau) - G])}{\phi(\tau \bar{s}(r; \tau))} = \frac{e^{-\frac{\{\tau[\bar{s}(r; \tau) - G]\}^2}{2}}}{e^{-\frac{[\tau \bar{s}(r; \tau)]^2}{2}}} = e^{\frac{\tau^2 G}{2} [2\bar{s}(r; \tau) - G]} > 1,$$

where the inequality follows again from stems from the working assumption $\bar{s}(r, \tau) > G$.

From the above, $\partial V(r; \tau) / \partial \tau > 0$ as long as $\bar{s}(\hat{r}(\tau); \tau) \geq 0$. Consider the case $\bar{s}(\hat{r}(\tau); \tau) < 0$. We first note that this implies $\hat{r}(\tau) > \bar{\rho}$, as the agency would then approve the merger upon observing $s = 0 (> \bar{s}(\hat{r}(\tau); \tau))$, implying:

$$\bar{\rho} < \rho(0; \hat{r}(\tau), \tau) = \frac{\hat{r}(\tau) \phi(-\tau G)}{\hat{r}(\tau) \phi(-\tau G) + (1 - \hat{r}(\tau)) \phi(0)} = \frac{\hat{r}(\tau)}{\hat{r}(\tau) + (1 - \hat{r}(\tau)) \frac{\phi(0)}{\phi(-\tau G)}} < \hat{r}(\tau),$$

where the inequality stems from $\phi(-\tau G) = \phi(\tau G) < \phi(0)$. Furthermore, using $\Phi(-\varepsilon) = 1 - \Phi(\varepsilon)$, $V(r; \tau)$ can be expressed as:

$$V(r; \tau) = \Pi_1 \Phi(\tau [G - \bar{s}(r; \tau)]) - \Pi_0 \Phi(-\tau \bar{s}(r; \tau)).$$

It then follows from Assumption A and $F(V(\hat{r}(\tau); \tau)) = \hat{r}(\tau) > \bar{\rho}$ that

$$F(V(\hat{r}(\tau); \tau)) > F(\Pi_1 - \Pi_0)$$

implying:

$$\begin{aligned}
& \Pi_1 \Phi(\tau [G - \bar{s}(r; \tau)]) - \Pi_0 \Phi(-\tau \bar{s}(r; \tau)) > \Pi_1 - \Pi_0 \\
\iff & \Pi_0 [1 - \Phi(-\tau \bar{s}(r; \tau))] > \Pi_1 [1 - \Phi(\tau [G - \bar{s}(r; \tau)])] \\
\iff & \frac{\Pi_0}{\Pi_1} > \frac{1 - \Phi(\tau [G - \bar{s}(r; \tau)])}{1 - \Phi(-\tau \bar{s}(r; \tau))}.
\end{aligned}$$

Noting that $\varepsilon [1 - \Phi(\varepsilon)] / \phi(\varepsilon)$ is strictly increasing,⁶ the last inequality implies⁷

$$\frac{\Pi_0}{\Pi_1} > \frac{-\tau \bar{s}(r; \tau) \phi(\tau [G - \bar{s}(r; \tau)])}{\tau [G - \bar{s}(r; \tau)] \phi(-\tau \bar{s}(r; \tau))} = \frac{-\bar{s}(r; \tau) \phi(\tau [\bar{s}(r; \tau) - G])}{[G - \bar{s}(r; \tau)] \phi(\tau \bar{s}(r; \tau))},$$

where the last equality uses $\phi(-\varepsilon) = \phi(\varepsilon)$. It follows that the right-hand side of (A.2) is again positive.

Summing-up, Assumption A ensures that $\partial V(r; \tau) / \partial \tau > 0$. Together with equilibrium stability, this implies $\hat{r}'(\tau) > 0$.

⁶ $1 - \Phi(\varepsilon)$ can be expressed as:

$$1 - \Phi(\varepsilon) = \int_{\varepsilon}^{\infty} e^{-\frac{x^2}{2}} dx = \int_0^{\infty} e^{-\frac{1}{2}(\varepsilon + \frac{y}{\varepsilon})^2} \frac{dy}{\varepsilon} = \frac{\phi(\varepsilon)}{\varepsilon} \int_0^{\infty} e^{-y - \frac{y^2}{2\varepsilon^2}} dy,$$

implying

$$\frac{\varepsilon [1 - \Phi(\varepsilon)]}{\phi(\varepsilon)} = \int_0^{\infty} e^{-y - \frac{y^2}{2\varepsilon^2}} dy,$$

where the right-hand side strictly increases with ε .

⁷Using $G > 0$ and the monotonicity of $\varepsilon [1 - \Phi(\varepsilon)] / \phi(\varepsilon)$, we have:

$$\frac{\tau [G - \bar{s}(r; \tau)] [1 - \Phi(\tau [G - \bar{s}(r; \tau)])]}{\phi(\tau [G - \bar{s}(r; \tau)])} > \frac{-\tau \bar{s}(r; \tau) [1 - \Phi(-\tau \bar{s}(r; \tau))]}{\phi(-\tau \bar{s}(r; \tau))}.$$

The conclusion follows.