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Mergers and Investments in New Products*

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This paper examines how horizontal mergers affect firms' incentives to invest in R&D leading to the development of new products. We characterize the impact of a merger to monopoly and a 3-to-2 merger on equilibrium innovation efforts and consumer surplus, absent efficiency gains and spillovers. We show that a 3-to-2 merger directly alters the outsider's innovation incentives by shifting its best-response function upward, and we analyze how this mechanism affects merger outcomes for innovation and consumer surplus. Finally, we examine how efficiency gains and remedies modify post-merger innovation efforts.

Keywords: Horizontal Mergers, Product Innovation, R&D Investments.
JEL Classification: K21, L13, L40.

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1 Introduction

Concerns that mergers may weaken firms' incentives to innovate now feature prominently in merger control. In the EU, several mergers in the pharmaceutical, medical-device, and agrochemical sectors were cleared only subject to remedies targeting innovation concerns. Notable examples include *Medtronic/Covidien (2014)*, *Novartis/GSK (2015)*, *Pfizer/Hospira (2015)*, *Dow/DuPont (2017)*, and *Bayer/Monsanto (2018)*. In the US, the Federal Trade Commission (FTC) and the Department of Justice (DoJ) have challenged or conditioned a number of mergers partly on the grounds that they could reduce innovation. For instance, the FTC required significant divestitures in the *Pfizer/Wyeth (2009)* merger to preserve competition and innovation incentives, particularly in specific animal health pharmaceutical and vaccine markets. It also required divestitures in *Thermo Fisher/Life Technologies (2014)* to maintain competition, including competition in innovation, in key life science research products. More recently, in 2024, the FTC sued to block the acquisition of Surmodics (a medical-device-coatings manufacturer) by private-equity firm GTCR, arguing that the merger would not only raise prices, but also reduce innovation.[1]

In this paper, we study how mergers affect innovation aimed at developing "new products", that is, products that create new markets, such as first-in-class gene therapies in pharmaceuticals, novel AI-based platforms in tech, or breakthrough diagnostic tools in life sciences. A key feature of such products is that they do not cannibalize existing products. As a result, the Arrow replacement effect (Tirole, 1988) does not arise in our pre-merger analysis.

We first consider a benchmark duopoly setting in which two symmetric firms invest in R&D to develop new products. Each firm owns a research lab, and competition in the product market arises only if both firms successfully innovate. Firms simultaneously choose their innovation efforts, modeled as probabilities of successful innovation. If only one firm succeeds, it enjoys the monopoly profit from marketing the new product. If both firms succeed, they earn symmetric duopoly profits. We study a merger between the two duopolists. In our baseline model, we assume that the merger does not induce any efficiency gains: the merged entity can coordinate innovation efforts across the two labs and set product-market prices, but the merger does not reduce R&D or production costs.

A horizontal merger allows the merging parties to internalize the negative externalities that each firm's innovation efforts impose on its rival. All else equal, this reduces the merged entity's incentives to innovate. However, absent efficiency gains, a merger also

[1] However, a federal court denied the FTC's request for a preliminary injunction, allowing the merger to proceed based on the merging parties' commitment to a partial divestiture.

leads to higher prices, which increases the expected return from successful innovation and therefore raises the merging firms' innovation efforts. Our first result is that a merger to monopoly can lead to either more or less innovation. More precisely, such a merger increases equilibrium innovation efforts if and only if the merged entity's incremental profit from a second successful innovation exceeds the profit earned by a firm when the two firms are independent and both innovate. Applying this result to a Salop model with quadratic transportation costs, we find that a merger between duopolists raises innovation if products are not too differentiated. In this case, the merger-induced increase in prices (when both firms are successful) is relatively large, and the resulting positive effect on incentives to innovate outweighs the negative effect stemming from direct internalization of innovation externalities. The reverse holds when products are sufficiently differentiated and, as a result, the merger leads to lower innovation.

We next turn to the effect of a merger to monopoly on consumer surplus. Two forces are at work. First, the merger raises prices if there are two successful innovations, thereby reducing consumer surplus. Second, the merger can increase or decrease innovation efforts, thus changing the probability that new products will hit the market. If the merger reduces innovation, consumers are clearly harmed. When the merger raises innovation, there is a trade-off between more frequent product innovation and higher prices. In this case, we show that for some R&D cost functions, the merger leads to an increase in consumer surplus. In the Salop model with two symmetric firms and quadratic transport costs, our simulations show that this result occurs in a small region of the parameter space.

We then consider a richer setting where a merger does not lead to the creation of a monopoly, that is, there are both insiders (merging firms) and outsiders (non-merging firms). For the sake of exposition, we consider an industry with three firms, and investigate the effects of a merger between two firms.[2] We establish that the outsider's innovation best-response function always shifts upward following the merger due to reduced product-market competition in states of the world where the merged entity supplies two products, which increases outsiders' profits conditional on success. Instead, the insiders' innovation best-response function can shift either upward or downward. We refer to these shifts as the *outsider innovation effect* and the *insider innovation effect*, respectively. When the insider innovation effect is negative, the merging firms' equilibrium efforts decrease whereas the outsider's equilibrium effort increases because the efforts of the merged entity and the outsider are strategic substitutes. When both effects are positive, the post-merger equilibrium efforts depend on their relative magnitudes: innovation may increase for the insiders, for the outsider, or for both.

[2]However, we show in the Appendix that our model can be extended to account for any number of symmetric outsiders.

A key insight from our analysis is that the presence (and behavior) of non-merging competitors may have more significant effects than in a traditional setting with no innovation. The reason is that in a setting where firms compete in the product market after observing their rivals' innovation outcomes, a merger affects outsiders directly, by altering their innovation best-response function. Consequently, the impact of the merger on outsiders' equilibrium behavior does not operate solely through their reaction to merger-induced changes in the insiders' strategies. This stands in sharp contrast with standard price- or quantity-setting environments (with no innovation choices), where the effect on outsiders arises only indirectly, through their optimal response to changes in the merged entity's equilibrium price or quantity.

We explore some implications of this insight using a Salop model with three symmetric firms. First, we show that the insiders' equilibrium innovation efforts can decrease despite the insider innovation effect being positive. In such cases, the presence of an outsider reverses the merger's impact on the insiders' incentives relative to the duopoly environment. Second, we find that a 3-to-2 merger reduces consumer surplus in all our simulations. This contrasts with the duopoly Salop benchmark, where our simulations indicate that a merger to monopoly can increase consumer surplus for some parameter values. Third, when a merger to monopoly harms consumers, we find that the loss in consumer surplus may be larger in a 3-to-2 merger than in a merger to monopoly under some conditions.

We also use our framework to study how different forms of merger-induced efficiencies modify post-merger innovation incentives. We distinguish between two sources of efficiencies: efficiency gains in R&D, captured as a reduction in the marginal cost of innovation efforts, and efficiency gains in production, arising from a reduction in the marginal cost of production. Efficiency gains in R&D shift only the merged entity's best-response function upward. By contrast, efficiency gains in production change the innovation incentives of all firms: they raise the insiders' best-response function and depress that of the outsider. Although the underlying mechanisms differ, both forms of efficiency gains lead to qualitatively similar results: they induce an increase in the insiders' innovation efforts and a decrease in the outsider's innovation effort. However, these effects may be stronger when efficiencies arise in production, because they affect all firms' innovation payoffs rather than only those of the merged entity.

Finally, we discuss how various merger remedies affect innovation incentives for both the merged entity and the outsider. We consider three classes of remedies: licensing innovation inputs (e.g., access to data, APIs, R&D know-how), licensing innovation outputs (e.g., mandatory licensing of algorithms), and divesting innovation capabilities (e.g., divestiture of labs or staff). We find that input licensing leads to an increase in the outsider's innovation effort, but reduces insiders' innovation efforts. Output licensing leads to a decrease in either the insiders' or the outsider's innovation efforts, or both. Finally, structural capability transfers lead to higher outsider effort and lower insider effort.

The remainder of the paper is organized as follows. In Section 2 we discuss our contribution to the literature. In Section 3, we analyze the effects of a merger to monopoly on innovation and consumer surplus. In Section 4, we examine the effects of a merger when the merging firms face competition from an outsider. In Section 5 we investigate the effects of efficiency gains on firms' equilibrium innovation efforts. In Section 6 we discuss the impact of potential remedies on innovation efforts. Section 7 concludes.

2 Contribution to the literature

Our paper relates to the recent theoretical literature on the impact of mergers on innovation, and more broadly, to the long-standing literature on the effect of competition on innovation building on Schumpeter (1942) and Arrow (1962).[3]

The closest paper to ours is Federico, Langus and Valletti (2017). They also consider the effect of a horizontal merger on the incentives to develop a new product in a model where investments in R&D affect the probability of success, but not the value of innovation. A key difference, however, is that they assume that products are homogeneous, whereas we allow products to be differentiated. This implies that, in our model, the merged entity's profit when both merging firms innovate can be higher than its profit when a single firm innovates.[4] This explains why a merger between two firms can, under some circumstances, increase the merging firms' incentives to innovate in our setting while this does not happen in the environment considered by Federico, Langus and Valletti (2017). This difference also has implications for the outsiders' incentives to innovate. In particular, while Federico, Langus and Valletti (2017) find that a merger always leads to a decrease in insiders' equilibrium investments and an increase in outsiders' equilibrium investments, we find a richer set of equilibrium outcomes. Finally, contrary to Federico, Langus and Valletti (2017), we analyze the effects of merger-induced efficiency gains and remedies.

Denicolò and Polo (2018) also consider innovation leading to the development of new products, and show that a merger to monopoly can lead to more innovation. However, the mechanism they rely on is fundamentally different from ours. Specifically, they show that a merger spurs innovation when the returns to R&D decrease moderately, so that the

[3]See Lefouili and Madio (2026) for a survey of the literature on mergers and investment (with a particular focus on R&D investment).

[4]Another important difference is that Federico, Langus and Valletti (2017) assume that, when (only) two independent firms are successful in developing a new product, they are able to coordinate their pricing (i.e. collude), which ensures that they make non-zero profits.

merged entity's investment levels differ across the merging firms' research labs, while we consider a setting in which post-merger investment levels in the two labs are symmetric. Another key difference between that paper and ours is that the former focuses on a merger to monopoly while we also study a merger in a setting with outsiders.

Our paper is also related to Moraga-González and Motchenkova (2026), who offer a unified framework to study the effects of mergers to monopoly on incentives to innovate and consumer surplus. They consider general R&D investments (as well as general product-market competition). By contrast, we focus on R&D investments that generate new products. However, we consider a richer setting along another dimension by allowing for the presence of outsiders. This, in particular, enables us to compare 3–to-2 mergers with mergers to monopoly.

In an environment where firms invest in innovation that enhances the quality of existing products—rather than creating new ones— Federico, Langus and Valletti (2018) find that non-merging firms respond to a merger by increasing innovation efforts. However, they also find that the merging parties reduce their innovation efforts absent efficiencies and spillovers. We also investigate the effects of a merger on both insiders' and outsiders' innovation efforts. While the result in Federico, Langus and Valletti (2018)—a decrease in insiders' innovation and an increase in outsiders' innovation— can arise in our setting too, we show that a merger can also induce an increase in insiders' innovation, together with either an increase or a decrease in outsiders' innovation.

Haucap, Rasch and Stiebale (2019) also find that there are circumstances under which a merger has a positive effect on insiders' innovation, and that the impact of the merger on outsiders' innovation can be either positive or negative.[5] However, their setting differs from ours in two important respects. First, whereas our baseline model considers a symmetric environment with general reduced-form payoffs in the product-market stage, they study an asymmetric triopoly with a specific demand system in which one firm is less efficient in R&D than the other two. Second, they assume that the merger takes place between one of the two efficient firms and the inefficient firm, thereby generating efficiency gains through the post-merger adoption of the efficient technology by the formerly inefficient firm (provided it is not shut down).

Motta and Tarantino (2021) primarily analyze a one-stage model in which firms choose prices and investment levels simultaneously, but they also consider an extension with a two-stage game where investments by rivals are observed by firms before they compete in the product market, albeit in a setting of incremental innovation rather than one where innovation generates new products. They show that, in both the Shubik–Levitan and Salop models, a merger reduces the insiders' investments, increases the outsiders'

[5] They also show that a merger can lead to a decrease in both insiders' and outsiders' innovation incentives—an outcome that cannot arise in our setting.

investments, and harms consumers. By contrast, we show that a merger may lead to an increase in the insiders' investments, together with either an increase or decrease in the outsiders' investments. However, our simulation results regarding the effect of a 3-to-2 merger on consumer surplus in the presence of outsiders are consistent with theirs.[6]

Our paper also connects to the literature on acquisitions of potential entrants by incumbents (Cunningham, Ederer and Ma, 2021; Fumagalli, Motta and Tarantino, 2023; Letina, Schmutzler and Seibel, 2024; Dijk, Moraga-González and Motchenkova, 2024, 2025; Bedre-Defolie, Biglaiser and Jullien, 2025; Teh, Banerjee and Wang, 2026). We also consider R&D for entry, but we focus instead on the acquisition of one potential entrant by another in a market without an incumbent.[7]

Finally, our work is related to Chen and Schwartz (2013) who show in a (non-merger) duopoly setting with deterministic R&D that the gain from bringing a new product to the market can be larger for a monopolist than for a firm that faces competition from independent sellers of the old product.[8] Their result relies on the idea that an incumbent monopolist can coordinate the prices of the new product and the old one. Instead, in our setting, the merged entity coordinates the prices of two new products (if both merging firms innovate).

All in all, our contribution to the literature can be summarized as follows. First, we provide new insights into the effects of a merger to monopoly on investments in new product development and consumer surplus. Second, we offer a tractable framework with both insiders and outsiders that clarifies how a merger directly affects outsiders' innovation incentives by altering their best-response functions. Third, we examine the implications of outsiders' responses for innovation and consumer surplus by comparing a merger to monopoly with a 3-to-2 merger. Finally, we explore the effects of efficiencies (in R&D and production) and remedies on post-merger innovation.

3 Benchmark: Merger to Monopoly

We start by analyzing a benchmark setting in which merging firms do not face competition from non-merging rivals.

[6] Baranes and Vuong (2021) extend Motta and Tarantino (2021) to a triopoly setting with two symmetric firms facing a more efficient rival and show that a merger between the symmetric firms can increase consumer surplus when the outsider is sufficiently more efficient.

[7] Part of the literature on mergers and innovation considers settings in which firms can adjust their R&D portfolios or, more generally, the direction of innovation post-merger (see, e.g., Gilbert 2019; Moraga-González, Motchenkova and Nevrekar 2022; Dijk, Moraga-González and Motchenkova 2024, 2025). In our setting, we take the direction of innovation as given.

[8] See Greenstein and Ramey (1998) for a related analysis in a model where products are vertically differentiated.

Consider two firms – firm 1 and firm 2 – that invest in product innovation creating a new market. Each firm owns a research lab that aims to develop a new product, and firms compete in the product market if both succeed in innovating. Firms simultaneously choose their innovation efforts, defined as the probabilities of success of their R&D investments. Formally, firm $i = 1, 2$ must incur a cost $C(\lambda_i)$ to achieve a probability λ_i to innovate, where $C(0) = 0, C'(0) = 0, C'(\cdot) > 0$, and $C''(\cdot) > 0$. A firm's success (or failure) is independent of the other firm's innovation effort or outcome.[9]

When a firm succeeds in innovating while the other does not, the sole innovator obtains the single-product monopoly profit from marketing the new product, denoted as Π_1. When both firms succeed in innovating, each of them obtains a duopoly profit, π_2. We assume that $\pi_2 < \Pi_1$. For example, if the product is the same for both firms and firms compete in prices, the value of π_2 is zero. If instead they compete *à la* Cournot, or if there is some differentiation between the firms' products, then π_2 is positive.[10]

Suppose firm i chooses an innovation effort λ_i. Conditional on success, firm i earns profit Π_1 if firm j fails to innovate (which occurs with probability $1 - \lambda_j$) and π_2 if firm j succeeds (which occurs with probability λ_j). This implies that firm i's expected profit is

$$\lambda_i \left[(1 - \lambda_j) \Pi_1 + \lambda_j \pi_2 \right] - C(\lambda_i).$$

Therefore, firm i's best response to firm j's innovation effort λ_j is characterized by the first-order condition

$$(1 - \lambda_j) \Pi_1 + \lambda_j \pi_2 = C'(\lambda_i).$$

We assume that a unique equilibrium exists and that it is interior. In this equilibrium, both firms choose the same innovation effort λ^{pre}, which is the unique solution to the following equation:[11]

$$(1 - \lambda^{pre}) \Pi_1 + \lambda^{pre} \pi_2 = C'(\lambda^{pre}). \tag{1}$$

Merger to monopoly. Let us now consider what happens if the two firms merge, thus becoming a monopolist. We assume that there are no efficiency gains in either R&D or production, so that the merged entity can only coordinate R&D investments across the two firms' labs and prices in the product market.[12] The merged entity chooses the probabilities of success λ_1 and λ_2 for the labs of firms 1 and 2, respectively. The profit of a two-product monopolist is denoted by Π_2. We make the following natural assumption

[9] In particular, there are no R&D spillovers.

[10] The profit π_2 can be positive in a homogeneous product setting if firms are able to collude (see e.g., Federico, Langus and Valletti 2017).

[11] Note that, in our symmetric setting, a unique equilibrium must be symmetric, since any asymmetric equilibrium would necessarily come in pairs.

[12] We maintain this assumption throughout the paper, except in Section 5.

about the profits of a two-product monopolist and a one-product monopolist:

Assumption 1. $\Pi_2 \geq \Pi_1$.

The above assumption covers both the case of homogeneous products and the case of differentiated products. If the products are homogeneous, then $\Pi_2 = \Pi_1$. By contrast, if the products are differentiated, the profit with two products is greater than that with one product, i.e., $\Pi_2 > \Pi_1$.

The merged entity's expected profit is

$$\lambda_1 (1 - \lambda_2) \Pi_1 + \lambda_2 (1 - \lambda_1) \Pi_1 + \lambda_1 \lambda_2 \Pi_2 - C(\lambda_1) - C(\lambda_2),$$

which we assume to be concave in (λ_1, λ_2).[13] Then, it is optimal for the merged entity to choose symmetric innovation efforts for the merging firms ($\lambda_1 = \lambda_2$), and the optimal efforts are equal to

$$\lambda^{post} = \arg\max_{\lambda} \quad 2\lambda (1 - \lambda) \Pi_1 + \lambda^2 \Pi_2 - 2C(\lambda).$$

The corresponding first-order condition is

$$\left(1 - \lambda^{post}\right) \Pi_1 + \lambda^{post} (\Pi_2 - \Pi_1) = C'(\lambda^{post}). \tag{2}$$

The comparison of (1) and (2) yields the following result.

Proposition 1. *A merger to monopoly leads to an increase (resp., decrease) in innovation efforts if the merged entity's incremental gain* $\Pi_2 - \Pi_1$ *from a second innovation is greater (resp., smaller) than the profit* π_2 *obtained by each firm when both firms innovate in the pre-merger scenario.*

The intuition behind this result is as follows. The marginal benefit of increasing a given firm's innovation effort differs between the no-merger and merger scenarios only when the other firm successfully innovates. Therefore, only the comparison of marginal benefits in this state of the world matters for comparing innovation efforts across the two scenarios.

Consumer surplus. We now examine the impact of the merger on consumer surplus. Let CS_1 denote the consumer surplus in the case of a one-product monopoly, CS_2^M the consumer surplus in the case of a two-product monopoly, and CS_2 the consumer surplus in the symmetric duopoly. We assume that $CS_1 < CS_2$ and $CS_2^M < CS_2$, that

[13] A sufficient condition for this is $C''(\lambda) > |2\Pi_1 - \Pi_2|$.

is, consumers benefit from competition. Thus, (expected) consumer surplus in the pre-merger equilibrium is

$$CS^{pre} = 2\lambda^{pre}(1-\lambda^{pre})CS_1 + (\lambda^{pre})^2 CS_2,$$

whereas consumer surplus in the post-merger equilibrium is

$$CS^{post} = 2\lambda^{post}(1-\lambda^{post})CS_1 + (\lambda^{post})^2 CS_2^M.$$

We decompose the effect of the merger on consumer surplus into two effects:

$$\begin{aligned} &CS^{post} - CS^{pre} \\ &= \underbrace{\left\{\left[2\lambda^{post}(1-\lambda^{post})CS_1 + (\lambda^{post})^2 CS_2\right] - \left[2\lambda^{pre}(1-\lambda^{pre})CS_1 + (\lambda^{pre})^2 CS_2\right]\right\}}_{\text{innovation effect (-/+)}} + \\ &+ \underbrace{(\lambda^{post})^2[CS_2^M - CS_2]}_{\text{market power effect (-)}}. \end{aligned} \tag{3}$$

The first term in (3) captures an *innovation effect*, which reflects the impact of the merger on innovation efforts. This effect has the same sign as $\lambda^{post} - \lambda^{pre}$ because the assumption $CS_1 < CS_2$ implies that an increase in (symmetric) innovation efforts always increases expected consumer surplus. Formally, the function $2\lambda(1-\lambda)CS_1 + \lambda^2 CS_2$ is increasing in λ over $[0,1]$.[14] The second term in the decomposition in (3) captures a *market power effect*, which is negative because the merged entity coordinates prices if both firms innovate, thereby reducing consumer surplus.

If $\lambda^{post} \leq \lambda^{pre}$, the innovation effect is (weakly) negative, which implies that $CS^{pre} > CS^{post}$. Instead, if $\lambda^{post} > \lambda^{pre}$, there is a trade-off between the (positive) innovation effect and the (negative) market power effect.

To explore this trade-off, it is useful to first discuss how consumer surplus depends on innovation efforts in the pre- and post-merger settings. As noted above, in the pre-merger scenario consumer surplus is always increasing in firms' (symmetric) innovation efforts. A similar property holds in the post-merger equilibrium provided that the function $2\lambda(1-\lambda)CS_1 + \lambda^2 CS_2^M$ is increasing in λ over the entire interval $[0,1]$. This, however, requires that $CS_1 < 2CS_2^M$. While this condition is satisfied in settings where consumers prefer a two-product monopolist to a single-product monopolist (i.e., $CS_1 < CS_2^M$), it is less universal than the assumptions $CS_1 < CS_2$ and $CS_2^M < CS_2$.[15] To avoid imposing

[14] To see why, note that $2[(1-2\lambda)CS_1 + \lambda CS_2] > 2[(1-2\lambda)CS_1 + \lambda CS_1] > 0$ for any $\lambda < 1$.

[15] For example, in the Hotelling model with linear transportation costs and full market coverage, Chen and Schwartz (2013) show that $CS_1 > CS_2^M$. The same occurs for some parameter ranges in our application to a Salop model with quadratic transportation costs.

it, we instead make an assumption on the innovation cost function, which ensures that $\lambda^{post} \in [0, 1/2]$, an interval over which $2\lambda(1-\lambda)CS_1 + \lambda^2 CS_2^M$ is always increasing. Specifically, we assume that $C'(1/2) \geq \Pi_2/2$. Under this assumption, it is possible to derive conditions on equilibrium innovation efforts under which the effect of a merger to monopoly on consumer surplus can be signed. This corresponds to part (i) of the following proposition. Part (ii) of the proposition considers a specific class of R&D cost functions and provides sufficient conditions on the model's primitives under which a merger to monopoly raises consumer surplus.[16]

Proposition 2. *Consider the effect of a merger to monopoly on consumer surplus.*

(i) Assume that $C'(1/2) \geq \Pi_2/2$, so that $\lambda^{post} \leq 1/2$. There exist thresholds $\tilde{\lambda}^{pre} \in (0, 1/2)$ and $\tilde{\lambda}^{post} \in (0, 1/2)$ such that the following holds:

  (a) If $\lambda^{pre} > \tilde{\lambda}^{pre}$, the merger reduces consumer surplus.

  (b) If $\lambda^{pre} < \tilde{\lambda}^{pre}$, the merger reduces (resp. increases) consumer surplus when $\lambda^{post} < \tilde{\lambda}^{post}$ (resp. $\lambda^{post} > \tilde{\lambda}^{post}$).

(ii) Moreover, if $\Pi_2 - \Pi_1 > \pi_2$, then within the class of R&D cost functions of the form $C(\lambda) = \frac{\beta}{1+\alpha}\lambda^{1+\alpha}$, the merger increases consumer surplus if $\alpha < \frac{\Pi_2-\Pi_1-\pi_2}{CS_2-CS_2^M}\frac{2CS_1}{\Pi_1}$ and β is sufficiently large.

Part (i) provides sufficient conditions under which a merger to monopoly either reduces or increases consumer surplus. The intuition behind this result is that, when pre-merger innovation efforts are relatively high, even if the merger increases innovation, this increase is insufficient to offset the negative market power effect, so the merger reduces consumer surplus. However, when pre-merger innovation is sufficiently low and the merger raises innovation, the resulting innovation benefit may outweigh the harm from the merger-induced increase in market power.

Part (ii) shows that, when the benefit a monopolist obtains from developing a second product exceeds the benefit an independent competitor obtains from developing that same product (and hence the merger increases innovation by Proposition 1), there exist innovation cost functions for which the merger has a positive impact on consumer surplus. A necessary condition that the cost functions considered in the proposition need to meet for this to happen is that their curvature must be sufficiently small. Note that, while the above proposition focuses on the effect of a merger on consumer surplus, Moraga-González and Motchenkova (2026) show in a more general setting that the pre-merger level of innovation and, therefore, the shape of the innovation cost function, may matter not

[16]The proof of the Proposition is provided in the Appendix.

only for the effect of a merger on consumer surplus but also for its impact on innovation incentives.

We now examine a specific setting that sheds additional light on these general insights.

Example. Consider a symmetric Salop model with quadratic transportation costs. Assume that consumers are uniformly distributed along the circle $[0, 1]$ and have a unit demand. Firm 1 is located at $x = 0$ whereas firm 2 is at $x = 1/2$. Let u be the (gross) value a consumer derives from consuming a product and td^2 the transportation costs she incurs, where d is the distance to this product.

For ease of exposition, we assume that marginal production costs are equal to zero. We also assume that the merging firms do not relocate on the Salop circle, which implies that the firms do not adjust their portfolio of projects after the merger.

We provide below a summary of the main payoffs, focusing on the case in which the market is fully covered with two products, and partially covered with one-product (i.e., $5/16 \leq u/t < 3/4$):[17]

$$\Pi_1 = \frac{4u}{3}\sqrt{\frac{u}{3t}}, \qquad \Pi_2 = u - \frac{t}{16}, \qquad \pi_2 = \frac{t}{8}. \tag{4}$$

We now apply the result in Proposition 1. We show in the Appendix that there exists a critical value $k^* \approx 0.34$ such that

$$\Pi_2 - \Pi_1 \begin{cases} < \pi_2, & \dfrac{u}{t} \in \left[\dfrac{5}{16}, k^*\right), \\ > \pi_2, & \dfrac{u}{t} \in \left(k^*, \dfrac{3}{4}\right). \end{cases}$$

This leads to the following result.[18]

Corollary 1. *Consider a duopoly Salop model with quadratic transportation costs and assume that $5/16 \leq u/t < 3/4$. Then a merger to monopoly reduces (resp., increases) innovation efforts if $t > u/k^*$ (resp., $t < u/k^*$).*

This result is driven by two opposite effects of the merger on innovation incentives. First, the merged entity internalizes the negative externality that each firm's innovation exerts on the other firm. More specifically, the profit of a successful firm is lower if the other firm also succeeds in innovating. After the merger this effect is internalized, reducing the merged entity's incremental return to increasing the probability of an additional success.

[17] Details are provided in the Appendix.
[18] The proof is provided in the Appendix.

This *business-stealing effect* decreases innovation effort. Second, when both firms are successful, the merged entity controls two products and can set their prices jointly. This *price-coordination effect* raises the profitability of success in the state where two products are available, thereby increasing innovation effort. In the context of our Salop application, the magnitudes of both effects decrease with product differentiation and, therefore, it is a priori unclear how product differentiation affects their relative strength. However, it turns out that, in this setting, the price coordination effect dominates the business-stealing effect for sufficiently low values of product differentiation, whereas the reverse holds for sufficiently high values of product differentiation.

The effect of the merger on consumer surplus is not obvious a priori. As discussed before, if innovation decreases post-merger, consumer surplus necessarily falls. However, if innovation increases post-merger, the outcome becomes ambiguous.

To determine the effect of the merger on consumer surplus, we compute it explicitly in each of the possible outcomes:

$$CS_1 = \frac{4\sqrt{3}u^{3/2}}{27\sqrt{t}}, \qquad CS_2 = u - \frac{13t}{48}, \qquad CS_2^M = \frac{t}{24}. \tag{5}$$

We consider an innovation cost function of the form $C(\lambda) = \beta\lambda^{\alpha+1}/(\alpha+1)$, with $\beta > 0$ and $\alpha > 0$.

Figure 1 presents a region plot in (β, t) depicting the difference in consumer surplus between the post- and pre-merger scenarios for different values of α (i.e., $\alpha = \{0.1, 0.13, 0.16\}$) and $u = 0.6$.[19] The dashed line indicates the area below (respectively, above) which innovation increases (respectively, decreases) post-merger, i.e., $t \approx 1.76$.

Figure 1 shows that, for sufficiently low values of α, consumer surplus can increase following the merger (blue region, left and middle panel). In this case, the effect of the merger on consumer surplus is non-monotonic in the transportation costs t. This pattern reflects the interplay of two potentially opposing forces: while a higher t reduces the adverse price effect of a merger, it has an ambiguous impact on its innovation effect. In particular, the difference between the merged entity's marginal benefit from a second innovation and that of an independent firm (i.e., $\Pi_2 - \Pi_1 - \pi_2$) is inverted-U shaped in t.[20]

Figure 1 also shows that, as α increases, the parameter range in which consumer surplus increases following the merger fades out as the market power effect prevails over

[19] For comparison with the 3-to-2 merger, we focus on the relevant parameter range for the transport cost $t \in [0.8, 1.8]$. See also the Appendix.

[20] In the same vein, Das, Mayskaya and Nikandrova (2025) find that, in a dynamic setting with sequential innovation, the effect of a merger on consumer surplus may be non-monotonic in the degree of substitutability between the first and second innovations.

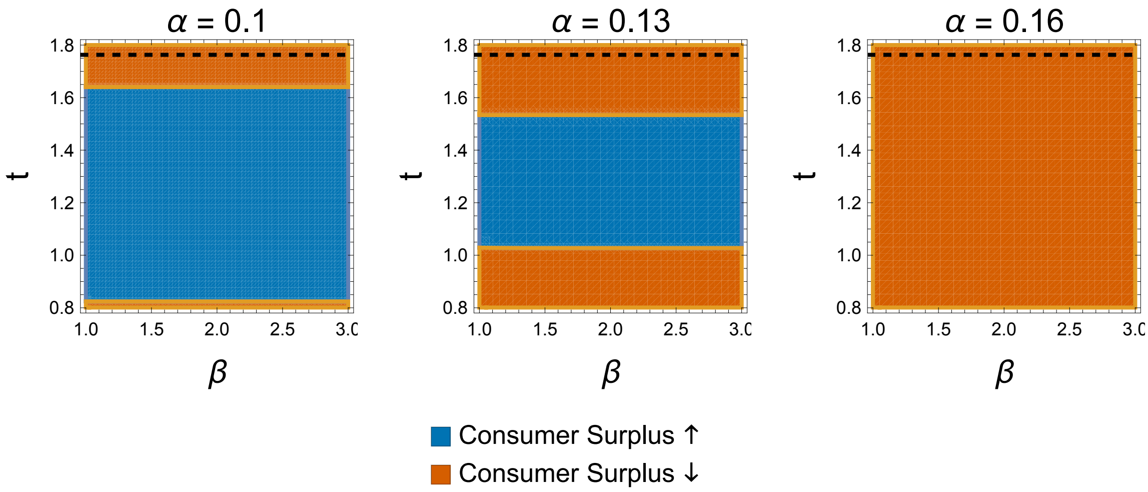


Figure 1: Post-merger innovation and consumer surplus in a Salop duopoly with quadratic transportation cost. Parameter: $u = 0.6$. The area below (respectively, above) the dashed line identifies the area where innovation increases (respectively, decreases) in a 2-to-1 merger.

the innovation effect. For $\alpha \geq 0.16$, the effect of the merger on consumer surplus is always negative in our simulations. This implies that, within the considered class of cost functions, there is only a small region of parameters in which the effect of the merger on consumer surplus is positive.

These results resemble and complement those derived in Moraga-González and Motchenkova (2026) in a Hotelling model where firms invest in increasing the quality of existing products (rather than in a new product). They show that a merger to monopoly can increase consumer surplus when the R&D cost function is very steep, but leads to a decrease in consumer surplus for a quadratic R&D cost function.

4 Merger in the presence of outsiders

We now study the impact of a merger between two firms in a richer environment with both merging firms (insiders) and non-merging rivals (outsiders). For the sake of exposition, we consider a setting with a single outsider (i.e., a triopoly). However, we show in the Appendix that our model can be extended to account for any number of symmetric outsiders.

We consider a potential market populated by three firms, indexed by $i = 1, 2, 3$, each owning a research lab. Each firm invests in a product innovation that possibly creates a new product. As in the benchmark duopoly model, we assume that a firm's success (or failure) is independent of the other firms' innovation efforts or outcomes.

Each firm i chooses an innovation effort (i.e., a probability of success) λ_i. When a firm succeeds in innovating while the others do not, the sole innovator obtains the single-product monopoly profit Π_1 from marketing the new product. When two independent firms succeed in innovating, each firm's profit is π_2 (duopoly profit), whereas when the three firms succeed in innovating and are independent, each firm's profit is π_3 (triopoly profit).

If a merger between two firms, say 1 and 2, occurs, the merged entity chooses the innovation efforts λ_1 and λ_2 for the labs of firm 1 and firm 2, respectively, while the outsider (firm 3) chooses the innovation effort λ_3. When only one lab of the merged entity is successful whereas the outsider is not, the merged entity obtains the single-product monopoly profit Π_1 and the outsider obtains 0. When only one lab of the merged entity is successful and the outsider is also successful, each of them obtains the duopoly profit π_2.

When both labs of the merged entity are successful, the merged entity coordinates the marketing of the two innovations. This allows it to obtain the two-product monopoly profit Π_2 if the outsider has not been successful. Instead, if the two-product merged entity faces competition from a single-product outsider, its profit is $\tilde{\Pi}_3$. The outsider instead obtains 0 if its lab has not been successful, whereas it obtains $\tilde{\pi}_3$ if its lab is successful and it faces competition from a two-product firm.

We make the following assumptions:

Assumption 2.

(i) $\Pi_1 > \pi_2 \geq \pi_3$*;*

(ii) $\tilde{\pi}_3 > \pi_3$*;*

(iii) $\tilde{\pi}_3 < \pi_2$*;*

(iv) $\tilde{\Pi}_3 - \pi_2 < \Pi_2 - \Pi_1$*.*

Part (i) means that the single-product monopoly profit is larger than the duopoly profit, which is weakly larger than the triopoly profit. For example, if the products are homogeneous and firms compete in prices, then π_2 and π_3 are equal to zero. If firms compete *à la* Cournot or if there is some differentiation between the firms' products, then π_2 and π_3 are positive and different.

Part (ii) means that the outsider benefits from the merger, which is natural under the assumption that the merger does not induce efficiency gains in production. Part (iii) states that the benefit from entering the market is lower when there is a two-product monopolist than when there is a single-product monopolist. Finally, Part (iv) means that the benefit for a single-product firm to develop a second product is lower if it faces competition than if it does not. As we show below, Parts (iii) and (iv) guarantee that the strategic substitutability property of the pre-merger game extends to the post-merger game.[21]

To study the impact of a merger between firms 1 and 2 on the insiders' and the outsider's innovation efforts, we now characterize the pre- and post-merger best-response functions and the corresponding equilibria.

Pre-merger equilibrium. Firm i's expected profit is given by

$$\lambda_i \left\{ (1-\lambda_j)(1-\lambda_k)\Pi_1 + \left[\lambda_j (1-\lambda_k) + \lambda_k (1-\lambda_j)\right]\pi_2 + \lambda_j \lambda_k \pi_3 \right\} - C(\lambda_i),$$

where $j, k \neq i$, and $j \neq k$. Therefore, firm i's best-response function is characterized by the first-order condition

$$(1-\lambda_j)(1-\lambda_k)\Pi_1 + \left[\lambda_j (1-\lambda_k) + \lambda_k (1-\lambda_j)\right]\pi_2 + \lambda_j \lambda_k \pi_3 = C'(\lambda_i).$$

Note that the left-hand side of the above equation is decreasing in λ_j and λ_k.[22] This implies that the pre-merger best-response function of firm i is decreasing in the innovation probabilities chosen by its rivals. In other words, innovation efforts are strategic substitutes in the absence of a merger.

To compare the pre- and post-merger equilibria, it is useful to define the pre-merger aggregate best-response function of firms 1 and 2, obtained by considering the (symmetric) equilibrium of the two-player game derived from the three-player game by fixing the strategy of firm 3.[23] In other words, the pre-merger aggregate best-response function of firms 1 and 2, which we denote $R_{12}^{pre}(\lambda_3)$, is the solution in λ_{12} of

$$(1-\lambda_3)\left[(1-\lambda_{12})\Pi_1 + \lambda_{12}\pi_2\right] + \lambda_3\left[(1-\lambda_{12})\pi_2 + \lambda_{12}\pi_3\right] = C'(\lambda_{12}). \tag{6}$$

Furthermore, the pre-merger best-response function of firm 3 when $\lambda_1 = \lambda_2 = \lambda_{12}$, which we denote $R_3^{pre}(\lambda_{12})$, is the solution in λ_3 of

$$(1-\lambda_{12})^2\Pi_1 + 2\lambda_{12}(1-\lambda_{12})\pi_2 + \lambda_{12}^2\pi_3 = C'(\lambda_3). \tag{7}$$

As discussed before, $R_3^{pre}(\lambda_{12})$ is decreasing. Note also that the aggregate best-response function $R_{12}^{pre}(\lambda_3)$ is also decreasing (because the left-hand side of (6) is decreasing in λ_3).

[21] For example, these assumptions hold in a symmetric Salop model with quadratic costs in the parameter region where there is full coverage when there are at least two firms and partial coverage when there is a single-product monopolist. Note that Part (iv) may not be satisfied outside this parameter region.

[22] The derivative of the left-hand side with respect to λ_j is given by $(1-\lambda_k)(\pi_2 - \Pi_1) + \lambda_k(\pi_3 - \pi_2)$, which is negative by Assumption 2 (i).

[23] We assume that the equilibrium of this two-player game is unique.

We assume that a unique pre-merger equilibrium exists and that this equilibrium is interior. Let λ^{pre} denote each firm's equilibrium effort in the pre-merger situation.[24] We also suppose that the slopes of the best-response functions satisfy $(dR_{12}^{pre}/d\lambda_3)(dR_3^{pre}/d\lambda_{12}) < 1$, which guarantees stability of the equilibrium.

Post-merger equilibrium. Consider now a scenario in which firms 1 and 2 merge, while firm 3 remains independent. Assume that the merged entity's expected profit is concave in (λ_1, λ_2). Combined with the symmetry of the profit with respect to (λ_1, λ_2), this assumption implies that the merged entity chooses the same innovation effort for each of the insiders. When $\lambda_1 = \lambda_2 = \lambda_{12}$, the merged entity's expected profit is given by

$$(1-\lambda_3)\left(2\lambda_{12}\left(1-\lambda_{12}\right)\Pi_1 + \lambda_{12}^2\Pi_2\right) + \lambda_3\left(2\lambda_{12}\left(1-\lambda_{12}\right)\pi_2 + \lambda_{12}^2\tilde{\Pi}_3\right) - 2C(\lambda_{12}).$$

The merged entity's best-response function $R_{12}^{post}(\lambda_3)$ solves the following first-order condition with respect to λ_{12}:

$$(1-\lambda_3)\left[(1-\lambda_{12})\Pi_1 + \lambda_{12}(\Pi_2 - \Pi_1)\right] + \lambda_3\left[(1-\lambda_{12})\pi_2 + \lambda_{12}\left(\tilde{\Pi}_3 - \pi_2\right)\right] = C'(\lambda_{12}). \quad (8)$$

Consider now firm 3. The only change in its best-response function when firms 1 and 2 merge is that π_3 needs to be replaced with $\tilde{\pi}_3$. Thus, firm 3's post-merger best-response function $R_3^{post}(\lambda_{12})$ is the solution in λ_3 of

$$(1-\lambda_{12})^2\Pi_1 + 2\lambda_{12}(1-\lambda_{12})\pi_2 + \lambda_{12}^2\tilde{\pi}_3 = C'(\lambda_3). \quad (9)$$

Note that under Assumption 2 (iii), firm 3's best-response function $R_3^{post}(\lambda_{12})$ is decreasing. Moreover, under Assumption 2 (iv), we have that $R_{12}^{post}(\lambda_3)$ is also decreasing. Thus, under these two assumptions, the post-merger innovation efforts are strategic substitutes.

We assume that a unique post-merger equilibrium exists and that this equilibrium is interior. Denote λ_{12}^{post} the equilibrium post-merger effort of each of the insiders and λ_3^{post} the equilibrium post-merger effort of the outsider. As before, we assume that the slopes of the best-response functions satisfy $(dR_{12}^{post}/d\lambda_3)(dR_3^{post}/d\lambda_{12}) < 1$, which guarantees stability of the equilibrium.

Comparison. We now compare the pre- and post-merger scenarios. We denote λ^{pre} the (symmetric) pre-merger equilibrium innovation effort, λ_{12}^{post} the merged entity's post-merger equilibrium innovation effort (in each lab), and λ_3^{post} the outsider's post-merger equilibrium innovation effort.

[24] A unique equilibrium must be symmetric, as any asymmetric equilibrium would come in triplets due to the symmetry of the setting.

First, for any $\lambda_{12} \in [0,1]$, we have

$$R_3^{post}(\lambda_{12}) > R_3^{pre}(\lambda_{12}),$$

where the right-hand side and left-hand side of this inequality are implicitly defined by (7) and (9), respectively. In other words, the merger leads to an upward shift in the outsider's best-response function: because the outsider's post-innovation profit is higher in the merger scenario (conditional on all R&D investments being successful), the outsider chooses a higher innovation effort than in the no-merger scenario, for given innovation efforts by firms 1 and 2. We call this the *outsider innovation effect*. It is important to note that, in our setting, the merger has a *direct* effect on the outsider: it affects its equilibrium innovation effort through a change in its best-response function and not merely through its reaction to changes in the insiders' innovation efforts.

Second, it is straightforward that $R_{12}^{post}(\lambda_3) > R_{12}^{pre}(\lambda_3)$ if

$$\Pi_2 - \Pi_1 > \pi_2 \text{ and } \tilde{\Pi}_3 - \pi_2 > \pi_3,$$

while $R_{12}^{post}(\lambda_3) < R_{12}^{pre}(\lambda_3)$ if

$$\Pi_2 - \Pi_1 < \pi_2 \text{ and } \tilde{\Pi}_3 - \pi_2 < \pi_3.$$

This implies that the merger can lead to either an upward or downward shift in the aggregate best-response function of the insiders. We call this the *insider innovation effect.*

The following lemma summarizes the above discussion.

Lemma 1. *The outsider innovation effect is always positive, whereas the insider innovation effect is positive if* $\Pi_2 - \Pi_1 > \pi_2$ *and* $\tilde{\Pi}_3 - \pi_2 > \pi_3$, *and negative if* $\Pi_2 - \Pi_1 < \pi_2$ *and* $\tilde{\Pi}_3 - \pi_2 < \pi_3$.

Using the above lemma and the strategic substitutability of innovation efforts, we establish the following result.[25]

Proposition 3. *(i) If* $\Pi_2 - \Pi_1 < \pi_2$ *and* $\tilde{\Pi}_3 - \pi_2 < \pi_3$, *then the merger leads to a decrease in the insiders' innovation efforts and an increase in the outsider's innovation effort.*
(ii) If $\Pi_2 - \Pi_1 > \pi_2$ *and* $\tilde{\Pi}_3 - \pi_2 > \pi_3$, *then the merger leads to an increase in the innovation efforts of either the insiders, or the outsider, or both.*

[25] The proof is provided in the Appendix.

The above proposition shows that the only outcome that cannot arise is the one in which the merger leads to a decrease in both insiders' and outsider's innovation efforts. We now provide a graphical illustration of each of the possible outcomes.

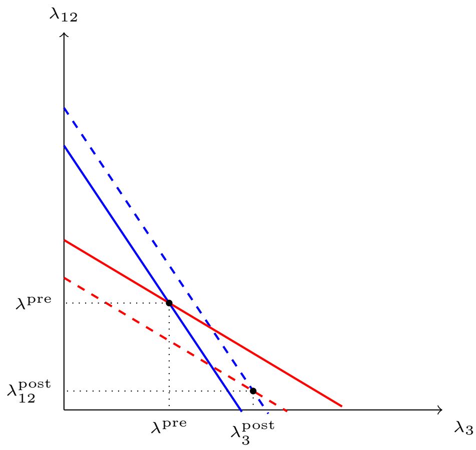
Consider first the scenario where $\Pi_2 - \Pi_1 < \pi_2$ and $\tilde{\Pi}_3 - \pi_2 < \pi_3$, under which the insider innovation effect is negative. In this case, the merger shifts the insiders' aggregate best-response function downward and the outsider's best-response function upward. Figure 2a shows that, in this case, the merger leads to a decrease in the insiders' equilibrium innovation efforts and an increase in the outsider's innovation effort. Note that the effect of the merger on the outsider's innovation incentives amplifies its direct negative effect on the insiders' innovation incentives.

Let us now turn to the scenario where $\Pi_2 - \Pi_1 > \pi_2$ and $\tilde{\Pi}_3 - \pi_2 > \pi_3$, under which the insider innovation effect is positive. In this case, the merger shifts upward both the insiders' aggregate best-response function and the outsider's best-response function. Equilibrium innovation efforts then depend on the relative magnitude of these two shifts, that is, the relative strength of the insider innovation effect and the outsider innovation effect. If the insider innovation effect is sufficiently weak (relative to the outsider innovation effect), the merger reduces the insiders' innovation efforts and increases the outsider's (see Figure 2b). If the insider innovation effect is of intermediate magnitude, then the merger increases all firms' innovation efforts (see Figure 2c). Finally, if the insider innovation effect is sufficiently strong then the merger raises the insiders' innovation efforts and lowers the outsider's (see Figure 2d).

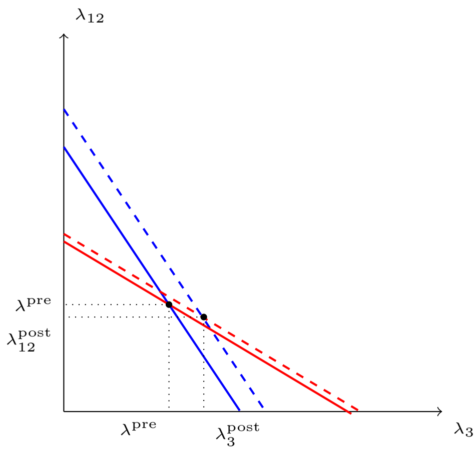
To illustrate the implications of the *direct* effect of a merger on an outsider's best-response function, consider the special case in which the merger does not affect the insiders' equilibrium innovation efforts (i.e., when $\Pi_2 - \Pi_1 = \pi_2$ and $\tilde{\Pi}_3 - \pi_2 = \pi_3$). In this case, the merger nonetheless leads to an increase in the outsider's innovation effort as the merger changes the outsider's payoff from innovating: when the insiders merge, competition is softened in states of the world where the merged entity supplies two products. This stands in sharp contrast to standard one-stage price- or quantity-setting games, where the equilibrium actions of outsiders cannot be affected by a merger unless those of insiders are as well.

We now turn to the effect of the merger on (expected) consumer surplus. We denote by CS_1 the consumer surplus in the case of a one-product monopoly, CS_2^M the consumer surplus in the case of a two-product monopoly, CS_2 and CS_3 the consumer surplus in the symmetric duopoly and triopoly, respectively, and CS_3^M the consumer surplus in the case of a two-product firm competing with a one-product firm. We assume the following (natural) ranking:

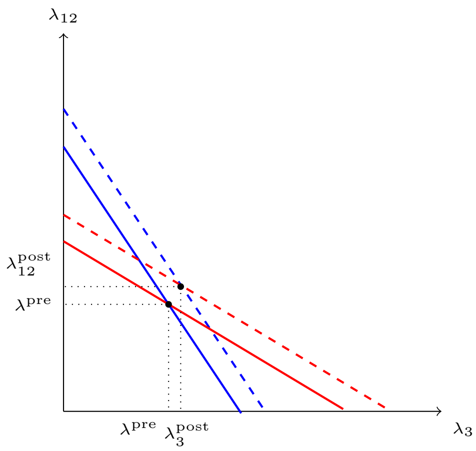
$$CS_1 < CS_2 < CS_3, \qquad CS_2^M < CS_3^M.$$



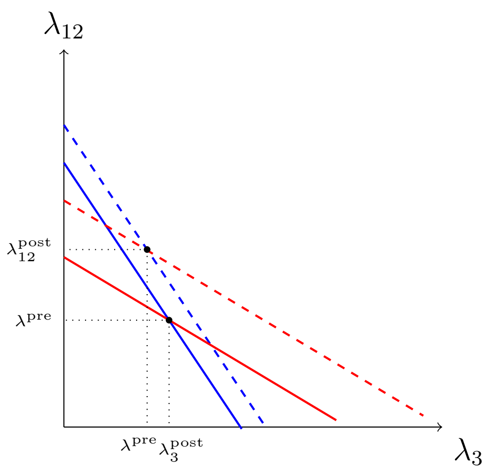
(a) $\Pi_2 - \Pi_1 < \pi_2$ and $\tilde{\Pi}_3 - \pi_2 < \pi_3$



(b) $\Pi_2 - \Pi_1 > \pi_2$ and $\tilde{\Pi}_3 - \pi_2 > \pi_3$



(c) $\Pi_2 - \Pi_1 > \pi_2$ and $\tilde{\Pi}_3 - \pi_2 > \pi_3$



(d) $\Pi_2 - \Pi_1 > \pi_2$ and $\tilde{\Pi}_3 - \pi_2 > \pi_3$

Figure 2: Aggregate best-response function of Firms 1 and 2 and best-response function of Firm 3. Solid lines depict pre-merger best responses, whereas dashed lines show post-merger best responses. Red curves correspond to the insiders and blue curves to the outsider.

Pre-merger consumer surplus is given by

$$CS^{pre} = 3\lambda^{pre}(1-\lambda^{pre})^2 CS_1 + 3(\lambda^{pre})^2(1-\lambda^{pre})CS_2 + (\lambda^{pre})^3 CS_3,$$

whereas post-merger consumer surplus is

$$\begin{aligned} CS^{post} =& \Big[2\lambda_{12}^{post}(1-\lambda_{12}^{post})(1-\lambda_3^{post}) + \lambda_3^{post}(1-\lambda_{12}^{post})^2\Big] CS_1 \\ &+(\lambda_{12}^{post})^2(1-\lambda_3^{post})CS_2^M + 2\lambda_{12}^{post}(1-\lambda_{12}^{post})\lambda_3^{post} CS_2 \\ &+(\lambda_{12}^{post})^2\lambda_3^{post} CS_3^M. \end{aligned}$$

Therefore, the difference between post-merger and pre-merger consumer surplus is given by

$$\begin{aligned} CS^{post} - CS^{pre} =& \Big[2\lambda_{12}^{post}(1-\lambda_{12}^{post})CS_1 + (\lambda_{12}^{post})^2 CS_2^M\Big] \\ &-\Big[2\lambda^{pre}(1-\lambda^{pre})CS_1 + (\lambda^{pre})^2 CS_2\Big] \\ &+\lambda_3^{post}\Big[(1-\lambda_{12}^{post})^2 CS_1 + 2\lambda_{12}^{post}(1-\lambda_{12}^{post})(CS_2 - CS_1) + (\lambda_{12}^{post})^2(CS_3^M - CS_2^M)\Big] \\ &-\lambda^{pre}\Big[(1-\lambda^{pre})^2 CS_1 + 2\lambda^{pre}(1-\lambda^{pre})(CS_2 - CS_1) + (\lambda^{pre})^2(CS_3 - CS_2)\Big]. \end{aligned} \tag{10}$$

The effect of the merger on consumer surplus is a priori more complex to determine than in a merger-to-monopoly case due to the strategic response of the outsider. The first two lines in (10) are equivalent to those in the benchmark merger-to-monopoly model (see equation (3)), and refer to the potential trade-off between the market power effect and the innovation effect in the absence (or for a given innovation effort) of a rival firm. Thus, the sign of this term depends on the pre- and post-merger innovation efforts of the insiders. The third and fourth lines capture the incremental effect of outsider success on expected consumer surplus, weighted by the probability that the outsider innovates (post- versus pre-merger). The term $(1-\lambda_{12}^{post})^2 CS_1$ corresponds to the state where only the outsider succeeds (one product on the market). The term $2\lambda_{12}^{post}(1-\lambda_{12}^{post})(CS_2 - CS_1)$ corresponds to the state where the outsider succeeds and exactly one insider lab succeeds, so that two independent products are available and consumer surplus rises from CS_1 to CS_2 relative to the case where only the outsider succeeds. Finally, the term $(\lambda_{12}^{post})^2(CS_3^M - CS_2^M)$ corresponds to the state where the outsider succeeds and both insider labs succeed: relative to outsider failure (where consumers face a two-product monopoly and surplus is CS_2^M), outsider success adds a third product and the market structure becomes a two-product firm competing with a single-product rival, yielding surplus CS_3^M. It is a priori unclear whether $CS_3 - CS_2$ is larger or smaller than $CS_3^M - CS_2^M$. Therefore, the effect of the merger on consumer surplus is a priori ambiguous.

In what follows we use a Salop model to illustrate some of the above analysis and derive further results on the effects of a merger on innovation efforts and consumer surplus.

Example. Consider a symmetric Salop model with consumers uniformly distributed in $x \in [0, 1]$, firms located at $x = \{0, 1/3, 2/3\}$, and quadratic transportation costs td^2. We focus on the case where $4u/3 < t \leq 3u$, to guarantee that the market is fully covered in all scenarios but in the single-product monopolist case. Moreover, this condition ensures that Assumption 2 holds.

Pre- and post-merger profits are as follows:

$$\Pi_1 = \frac{4u\sqrt{u}}{3\sqrt{3t}}, \qquad \Pi_2 = u - \frac{t}{9}, \qquad \pi_2 = \frac{t}{9}, \qquad \pi_3 = \frac{t}{27}, \qquad \tilde{\pi}_3 = \frac{16t}{243}, \qquad \tilde{\Pi}_3 = \frac{25t}{243}.$$

Consumer surplus is as follows:

$$CS_1 = \frac{4u\sqrt{u}}{9\sqrt{3t}}, \qquad CS_2^M = \frac{t}{12}, \qquad CS_2 = u - \frac{t}{4}, \qquad CS_3 = u - \frac{13t}{108}, \qquad CS_3^M = u - \frac{271t}{972}.$$

Note that Π_1 and CS_1 are the same as in the benchmark duopoly case.

First, we provide numerical simulations where we fix $u = 0.6$ and focus on the parameter range where $t \in [0.8, 1.8]$, which guarantees that Assumption 2 holds, and $\alpha = 1$.[26] The insider innovation effect is positive (i.e., $\Pi_2 - \Pi_1 > \pi_2$ and $\tilde{\Pi}_3 - \pi_2 > \pi_3$) for any $t \leq 1.27$, and negative otherwise. This is represented by the dashed horizontal line in Figure 3a.[27] Therefore, for any $t > 1.27$, the insider innovation effect is negative while the outsider innovation effect is positive. As a result, the merger leads to a higher equilibrium effort for the outsider and a lower effort for the insiders (orange area).

For $t < 1.27$, three possible scenarios arise. For descriptive purposes, consider when t is sufficiently low. In this case, for a given transportation cost, a higher β yields a situation in which the equilibrium innovation effort increases post-merger for the insiders and decreases for the outsider (green area); for intermediate values of β both innovation efforts increase post-merger (red area); for very low values of β, the equilibrium innovation effort is higher for the outsiders and lower for the insider (orange area).

An interesting observation concerns the orange region below the dashed line. In this area, the merger shifts the insider's best-response function upward (as in the Salop duopoly setting). However, it also shifts the outsider's best-response function upward, which

[26] For this value of α, innovation efforts are always interior both pre-merger and post-merger (both in a merger to monopoly and a 3-to-2 merger), and the effect of a merger to monopoly on consumer surplus is always negative.

[27] Note that the merger is profitable in the entire parameter range considered in the figure.

exerts a negative indirect effect on the insiders' innovation efforts. Notably, when β is sufficiently small, the indirect effect dominates the direct effect: the insider's equilibrium innovation effort decreases post-merger.

To examine the effect of a 3-to-2 merger on consumer surplus, we run a more exhaustive set of simulations. In addition to the simulations described above, we also perform simulations in which we fix $\beta = 5$ and let α vary in the interval $[1, 3]$.[28] Importantly, across our entire set of simulations, the effect on consumer surplus is always negative—even in cases where the merger increases all firms' innovation efforts. This result is striking, given that in the merger-to-monopoly case there exists a small parameter region in which consumer surplus rises post-merger. The intuition is that the presence of an outsider mitigates the merger-induced increase in market power, which in turn limits the potential innovation benefits generated by the merger. This example illustrates that this limitation can dominate the difference in adverse price effects between a merger to monopoly and a 3-to-2 merger, potentially making a merger involving an outsider more harmful for consumers than a merger to monopoly. To further explore this issue, Figure 3b compares the merger-induced change in consumer surplus under duopoly (i.e., merger to monopoly) and triopoly (i.e., 3-to-2 merger) over a parameter range under which the effect of the merger on consumer surplus is always negative, under both market structures. It suggests that the loss in consumer surplus resulting from the merger, $|\Delta CS|$, tends to be more severe in the case of a 3-to-2 merger (i.e., $|\Delta CS_{2to1}| < |\Delta CS_{3to2}|$) when competition is relatively weak (i.e., high t), whereas consumers are harmed more in a 2-to-1 merger than in a 3-to-2 merger when competition is relatively strong (i.e., low t).[29]

5 Efficiency gains

In this section, we allow for the possibility that the merger generates efficiency gains, either in R&D or in production.[30]

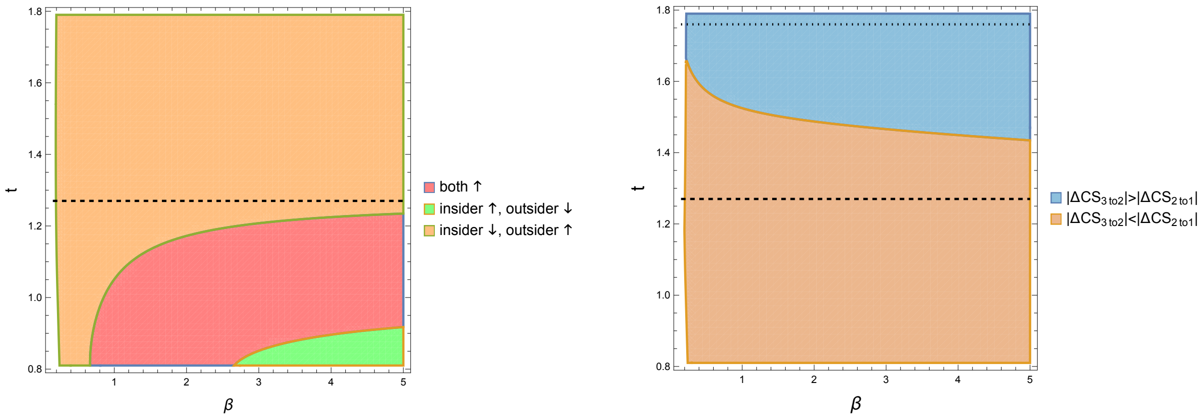
5.1 Efficiency gains in R&D

We begin by assuming that the merger generates R&D efficiency gains, modeled as a reduction in the marginal cost of innovation effort.

[28] We restrict attention to values of α greater than or equal to 1 to avoid non-interior equilibria.

[29] Note that in the area in which consumers are harmed more in a 3-to-2 merger, the insider innovation effect is always negative in the presence of an outsider, but can be positive in a merger to monopoly (i.e., the area below the dotted line and above the dashed line in Figure 3b).

[30] See Régibeau and Rockett (2019) for a discussion of merger-induced efficiencies in R&D.



(a) Equilibrium innovation levels pre- and post-merger.

(b) Difference in the consumer surplus pre- and post-merger

Figure 3: (a) Innovation effects in a 3-to-2 merger and (b) comparison of merger effects on consumer surplus. Parameters: $u = 0.6, \alpha = 1$. The area below (respectively, above) the dashed line identifies the area where the insider innovation effect is positive (respectively, negative) in a 3-to-2 merger. The area below (respectively, above) the dotted line identifies the area where innovation increases (respectively, decreases) in a 2-to-1 merger.

We illustrate a way to interpret R&D efficiency gains with several examples. Consider two pharmaceutical companies, each operating independent research labs working on related therapeutic areas. Suppose that prior to the merger, the firms duplicate several components of their research process, such as pre-clinical toxicology, screening platforms, data infrastructure, and regulatory expertise. The merger allows the combined entity to integrate these activities and eliminate redundancies. For instance, the merged entity may centralize early-stage discovery in a single research unit, consolidate clinical-trial management teams, or pool proprietary datasets that improve the precision of pre-clinical predictions. Each of these changes effectively lowers the marginal cost of raising the probability of successful discovery, thereby generating R&D efficiency gains.

Also, consider production of semiconductors, where the combination of complementary design architectures and fabrication know-how can reduce the marginal cost of incremental design improvements. In the chemical sector, mergers may allow the integration of R&D platforms (e.g., catalysts or polymerization processes) that reduce the cost of developing new formulations. In digital markets, shared access to training data, model-development infrastructure, or software-engineering resources can similarly reduce the marginal cost of algorithmic improvements.

It is straightforward that such efficiency gains shift the merged entity's best-response function upward, while leaving the outsider's best-response function unchanged. Given strategic substitutability, this leads to a higher equilibrium innovation effort by the merged entity and a lower effort by the outsider, relative to the post-merger outcome without efficiency gains.

Figure 4a illustrates this case. The solid lines depict the best-response functions of the merged entity and the outsider in the absence of efficiencies. The dashed line represents the merged entity's best-response function in the presence of R&D efficiencies. The new equilibrium is identified by $(\lambda_{12}^{post,R}, \lambda_3^{post,R})$, where $\lambda_{12}^{post,R} > \lambda_{12}^{post,}$ and $\lambda_3^{post,R} < \lambda_3^{post}$.

5.2 Efficiency gains in production

We now consider the possibility that the merger increases production efficiency. For instance, production efficiencies may arise from improved coordination along the supply chain. When two manufacturers procure similar inputs (e.g., specialized chemicals, components, data-processing services, or manufacturing intermediates), the merged entity may be able to standardize input specifications, harmonize quality controls, or streamline logistics. Such integration reduces the effective marginal cost of expanding output, particularly when supply-chain bottlenecks or quality variability are relevant. Similarly, in industries with substantial process automation or digital workflow integration, a merger may allow the use of shared production software, common monitoring systems, or unified process-optimization algorithms. These changes can lower variable operating costs by reducing downtime, minimizing waste, or improving yield rates.

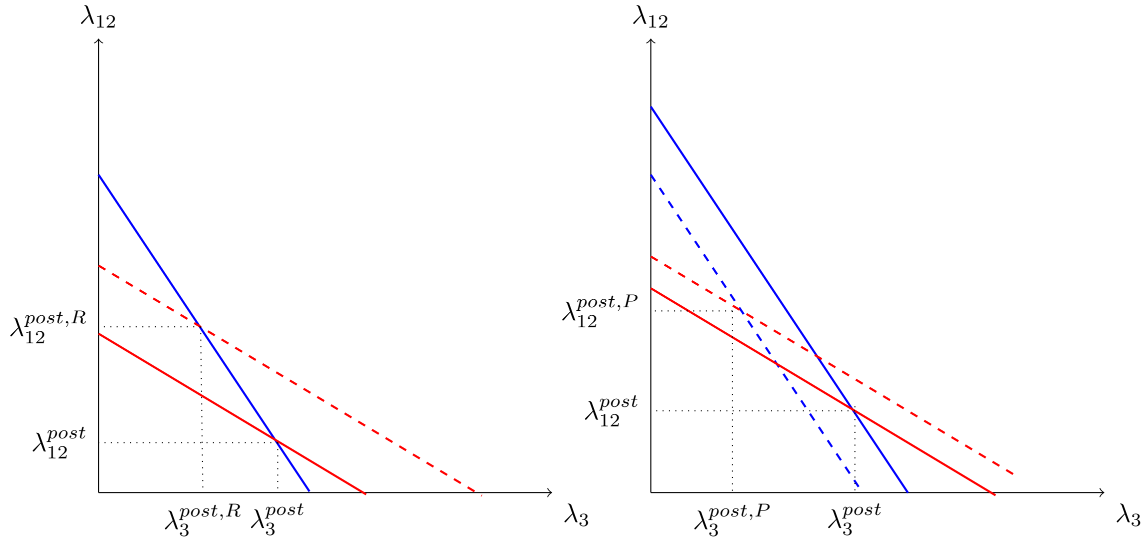
Formally, we assume that the post-merger marginal production cost of products 1 and 2 is lower than their pre-merger marginal cost of production.

Since the merged entity benefits from efficiency gains in production, it is natural to assume that both its profit when it does not face competition and its profit when it does are increasing with the efficiency gains in production. It is also natural to assume that the outsider's profit when it faces competition from the merged entity decreases with the merged entity's efficiency gains.

Efficiency gains in production affect firms' post-merger profits in each innovation state and therefore their marginal benefit from investing in innovation. Under the above assumptions, it follows directly from the first-order conditions defining the post-merger innovation best-response functions that the outsider's best-response function shifts downward with efficiency gains in production, whereas the merged entity's best-response function shifts upward. Given strategic substitutability, it then follows that the merged entity's equilibrium innovation effort increases with efficiency gains in production, while the outsider's equilibrium effort decreases.

Interestingly, the effects of efficiency gains in production on equilibrium innovation efforts are qualitatively similar to those of efficiency gains in R&D. However, perhaps counterintuitively, they are more pronounced for efficiency gains in production, as these affect the innovation best-response functions of all firms, whereas efficiency gains in R&D only affect the insiders' innovation best-response function. In particular, in a situation where efficiency gains in production would lead to the same upward shift in the merged entity's best-response function than efficiency gains in R&D, the insiders (respectively, outsider) would increase (respectively, decrease) more their innovation efforts with efficiency gains in production than with efficiency gains in R&D.

Figure 4b illustrates the effect of efficiency gains in production on post-merger equilibrium innovation efforts. The solid lines depict the best-response functions of the merged entity and the outsider in the absence of efficiencies. The dashed line represents the merged entity's best-response function in the presence of production efficiencies. The new equilibrium is identified by $(\lambda_{12}^{post,P}, \lambda_3^{post,P})$, where $\lambda_{12}^{post,P} > \lambda_{12}^{post}$ and $\lambda_3^{post,P} < \lambda_3^{post}$.



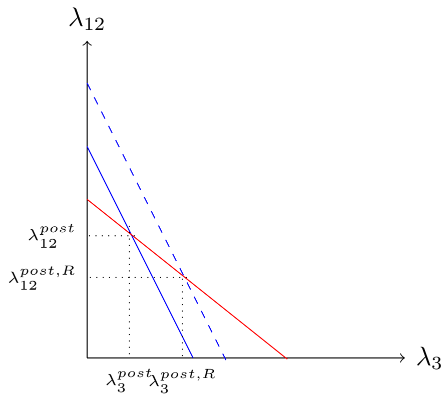
(a) R&D Efficiencies.

(b) Production Efficiencies.

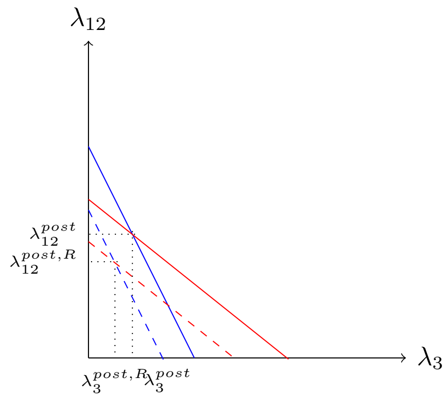
Figure 4: R&D and Production Efficiencies. Dashed line indicates the post-merger best response in the presence of efficiency gains whereas the solid line indicates the post-merger best response absent efficiency gains. Red curves correspond to the merged entity and blue curves to the outsider.

6 Remedies

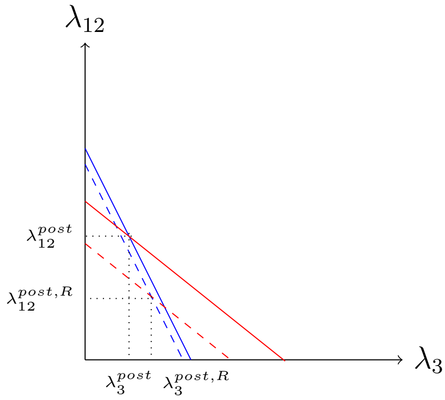
A common way to address potentially harmful mergers is to require the merging firms to implement structural or behavioral remedies. In this section, we study the equilibrium effects of three potential remedies.



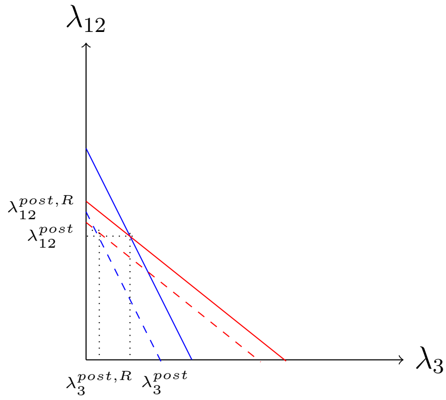
(a) Licensing innovation input



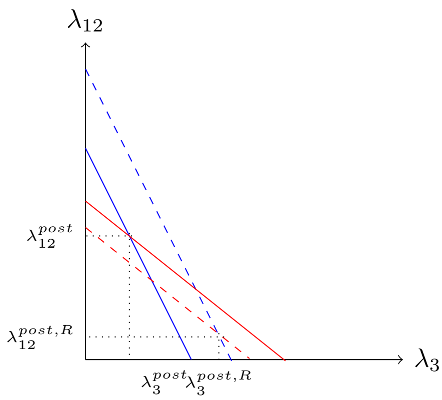
(b) Licensing innovation output (i)



(c) Licensing innovation output (ii)



(d) Licensing innovation output (iii)



(e) Transferring innovation capabilities

Figure 5: Remedies. Solid lines depict pre-remedy best-response functions; dashed lines show post-remedy best responses. Red curves correspond to the merged entity, and blue curves correspond to the outsider.

6.1 Licensing innovation input

We first consider a remedy requiring the merged entity to license innovation inputs to the non-merging rival. In practice, this covers arrangements whereby the merged entity must provide access to R&D know-how or key data that enter the innovation process.

Examples include sharing interim results from clinical trials in pharmaceutical R&D and providing access to training data in the context of AI development.

In digital and data-driven markets, recent EU decisions have imposed behavioral commitments that can be interpreted as licensing innovation inputs. In *Google/Fitbit*, the Commission required Google to maintain free third-party access to Fitbit users' health and fitness data via the Fitbit Web API, for at least ten years, and to continue licensing core Android APIs needed for interoperability of wearable devices. In *Meta/Kustomer*, Meta committed to provide non-discriminatory, free access to its messaging APIs (Messenger, Instagram and WhatsApp) to competing CRM providers and to maintain feature parity over time, effectively ensuring ongoing access to data streams that are crucial inputs for developing AI-based customer-service tools.

In our model, such remedies imply a reduction in the outsider's marginal cost of R&D, which shifts its innovation best-response function upward. This upward shift increases the outsider's equilibrium innovation effort relative to the no-remedy post-merger benchmark. Because innovation efforts are strategic substitutes, the insiders' optimal investment decreases in response. Figure 5a summarizes these effects. The blue (solid) line denotes the outsider's post-merger best response and the red (solid) line the insiders'. After the remedy, the outsider's best response shifts upward (blue dashed), generating a new equilibrium—labelled with an additional superscript "R"—in which the outsider innovates more and the merged entity innovates less.

6.2 Licensing innovation output

Second, we consider a remedy that mandates the merged entity to license innovation output to the outsider. In this case the merged entity is required to provide access to the discoveries and innovations underlying or embedded in final products (for instance, an active pharmaceutical ingredient in the context of over-the-counter or prescription drugs, or critical algorithms in the context of AI startups). A concrete example can be found in the pharmaceutical sector: in numerous EU decisions the merging parties offered to commit to supply rival firms (or divested buyers) with pipeline compounds or license terms enabling them to compete effectively. According to the European Commission's 2024 Report on Competition Enforcement in the Pharmaceutical Sector, remedies in merger cases have increasingly targeted access to essential R&D pipelines and licensing of innovation output (EC, 2024).

Such output-licensing remedies reduce the appropriability of returns from innovation for the insiders and, therefore, lead to a downward shift in their innovation best-response function. They also lead to a downward shift in the outsider's best-response function as its marginal benefit from investing in innovation also decreases because of direct access to insiders' innovation.

Given strategic substitutability of efforts, the overall equilibrium effect is not a priori clear. However, we can conclude that innovation cannot increase at equilibrium for both the merged entity and the outsider. Depending on the relative magnitude of the shifts in the merged entity's and outsider's best-response functions, one of three outcomes may result: both firms reduce innovation, the outsider increases innovation while the merged entity reduces innovation, or the merged entity increases innovation while the outsider reduces innovation.

In Figure 5b, the post-remedy best responses (dashed) both shift downward, in a way that leads to lower equilibrium innovation for all firms. In Figure 5c, the outsider's best response shifts downward significantly less than the merged entity's one. Consequently, the outsider innovates more, whereas the merged entity innovates less. In Figure 5d, the opposite occurs: the merged entity's best response shifts downward significantly less than the outsider's one, so the merged entity's innovation effort rises whereas the outsider's innovation effort falls.

The above analysis suggests that this remedy may significantly reduce innovation efforts, thereby harming consumers. However, for any given innovation efforts, the remedy also increases the diffusion of innovation, which benefits consumers. The desirability of the remedy therefore depends on which of the two effects dominates.

6.3 Transferring innovation capabilities

A third remedy we consider is a structural remedy which mandates the merging party to divest part of its innovation capabilities to an existing rival. For example, this can be a partial divestiture of a research lab (e.g., staff members, machinery) from the merged entity to an outsider. For example, in *Dow/DuPont*, the European Commission cleared the merger subject to a divestment of DuPont's global crop-protection R&D organization, including its laboratories, staff, pipeline projects and associated know-how and data, to preserve rival firms' ability to discover new active ingredients. Similarly, in *Bayer/Monsanto*, Bayer was required to divest its crop-seeds and traits R&D activities and digital agriculture assets to BASF, together with licenses and technical information (the "Digital Agriculture License"), so that the buyer could replicate the underlying innovation capabilities.

In our model, this structural transfer is equivalent to raising the merged entity's marginal cost of R&D (as it loses some capability) and reducing the outsider's marginal R&D cost (as it gains capability). As a result, the merged entity's best-response function shifts downward while the outsider's best-response function shifts upward. In equilibrium the outcome is that the outsider now invests more in innovation, and the merged entity invests less. Figure 5e illustrates this effect: the dashed curves show post-remedy best responses, and the resulting equilibrium yields higher innovation effort for the outsider and lower innovation effort for the merged entity.

7 Conclusion

This paper develops a tractable framework to study how horizontal mergers affect incentives to develop new products when R&D success is stochastic. First, we consider a setting in which two firms merge to monopoly. We show that, absent spillovers and efficiency gains, the merger raises innovation if and only if the merged entity's incremental gain from a second successful innovation exceeds the individual duopoly profit earned when both firms successfully innovate. In a Salop model with quadratic transportation costs, this condition holds when products are not too differentiated. We also examine the trade-off between higher innovation and higher prices from the consumer perspective when such a trade-off arises. In particular, focusing on a specific class of cost functions, we provide conditions on their shape under which a merger increases consumer surplus. In the simulations using a Salop application, this occurs for a small range of parameter values.

We then extend our analysis to a triopoly setting and investigate the effects of a 3-to-2 merger on the insiders' and outsider's innovation efforts. Unlike in standard one-stage price- or quantity-setting models, the merger has a *direct* impact on the outsider's incentives: it affects the outsider's return from innovation and, therefore, its innovation effort, even when the insiders' innovation efforts are held fixed. As a result, equilibrium innovation may increase for the merged entity, for the outsider, or for both, but it cannot decrease for both. Importantly, the presence of an outsider may overturn the merger's direct impact on the insiders (which is the only impact in the merger-to-monopoly benchmark): the insiders' equilibrium innovation can fall even when their return from innovation, for a given outsider innovation effort, increases. Related to this, we find that the effect of the merger on consumer surplus is always negative in our simulations in the Salop model. Moreover, in some circumstances, 3-to-2 mergers can harm consumers more than mergers to monopoly when the latter also reduce consumer surplus.

While one should be cautious in drawing strong policy implications from the results of our Salop application, our findings indicate that mergers with outsiders may, in some cases, be qualitatively different from mergers to monopoly. They also highlight the need for a better understanding of how outsiders' reactions to mergers influence their effects on innovation and consumer surplus.

Several avenues for future research emerge from our analysis. A first natural extension would be to allow for asymmetries between firms, for instance in their R&D efficiency levels. In particular, such asymmetries would enable us to gain a better understanding of the effects of mergers between laggards and of mergers between a leader and a laggard. A second important extension would be to introduce R&D spillovers. In a merger-to-monopoly setting, the internalization of spillovers has a positive impact on the merging firms' innovation efforts. In a setting with outsiders, the effects are more subtle: the merger leads to the internalization of spillovers among insiders, but also to the internalization of the negative externality each merging firm exerts on the other through spillovers to rivals. Exploring how firm asymmetries and R&D spillovers modify our results would help assess the robustness of our findings and further inform the analysis of mergers in innovation-intensive industries.

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Appendix

Proof of Proposition 2

Consider Part *(i)*. Since $2\lambda(1-\lambda)CS_1 + \lambda^2 CS_2^M$ is increasing in λ over $[0,1/2]$ and $\lambda^{post} \in (0,1/2]$ then $CS^{post} \leq CS_1/2 + CS_2^M/4$, where the left-hand side is the value of CS^{post} when $\lambda^{post} = 1/2$.

Since $2\lambda(1-\lambda)CS_1 + \lambda^2 CS_2$ is increasing in λ over $[0,1/2]$ and $CS_1/2 + CS_2^M/4 < CS_1/2 + CS_2/4$, where the left-hand side is the value of CS^{pre} when $\lambda^{pre} = 1/2$, then there exists a threshold $\tilde{\lambda}^{pre} \in (0,1/2)$ such that CS^{pre} is greater (resp., smaller) than $CS_1/2 + CS_2^M/4$ if λ^{pre} is greater (resp., smaller) than $\tilde{\lambda}^{pre}$.

This implies that, if $\lambda^{pre} > \tilde{\lambda}^{pre}$, then $CS^{post} < CS^{pre}$. However, if $\lambda^{pre} < \tilde{\lambda}^{pre}$ then the following equation

$$2\lambda(1-\lambda)CS_1 + \lambda^2 CS_2^M = CS^{pre} \tag{A-1}$$

has a unique solution in $(0,1/2)$, which we denote $\tilde{\lambda}^{post}$, and CS^{post} is greater (resp, smaller) than CS^{pre} if λ^{post} is greater (resp., smaller) than $\tilde{\lambda}^{post}$.

Consider now part *(ii)*. Equation (A-1) can be rewritten as

$$2\left(\lambda^{pre}(1-\lambda^{pre}) - \tilde{\lambda}^{post}(1-\tilde{\lambda}^{post})\right) - \left((\lambda^{pre})^2 - \left(\tilde{\lambda}^{post}\right)^2\right)\frac{CS_2^M}{CS_1} = (\lambda^{pre})^2\,\frac{CS_2 - CS_2^M}{CS_1}.$$

or

$$\left(\lambda^{pre} - \tilde{\lambda}^{post}\right)\left(2\left(\lambda^{pre} + \tilde{\lambda}^{post} - 1\right) - \left(\lambda^{pre} + \tilde{\lambda}^{post}\right)\frac{CS_2^M}{CS_1}\right) = (\lambda^{pre})^2\,\frac{CS_2 - CS_2^M}{CS_1}.$$

Defining $\Delta = \frac{CS_2 - CS_2^M}{2CS_1}$, the above equation is equivalent to

$$\frac{\tilde{\lambda}^{post}}{\lambda^{pre}} = 1 + \lambda^{pre}\Delta + \frac{\left(2 - \frac{CS_2^M}{CS_1}\right)\left(\lambda^{pre} + \tilde{\lambda}^{post}\right)\lambda^{pre}}{2 + \left(\lambda^{pre} + \tilde{\lambda}^{post}\right)\left(\frac{CS_2^M}{CS_1} - 2\right)}.$$

The merger leads to strictly higher consumer surplus if and only if $\tilde{\lambda}^{post} < \lambda^{post}$. This holds if and only if the net marginal gain from innovation for the merged entity at $\tilde{\lambda}^{post}$ is strictly positive, that is, if and only if $C'\left(\tilde{\lambda}^{post}\right) < \Pi_1 + \tilde{\lambda}^{post}\left(\Pi_2 - 2\Pi_1\right)$. Using the pre-merger equilibrium condition $C'\left(\lambda^{pre}\right) = \Pi_1 + \lambda^{pre}\left(\pi_2 - \Pi_1\right)$, we can rewrite this condition as

$$\frac{C'\left(\tilde{\lambda}^{post}\right)}{C'\left(\lambda^{pre}\right)} < \frac{\Pi_1 + \tilde{\lambda}^{post}\left(\Pi_2 - 2\Pi_1\right)}{\Pi_1 + \lambda^{pre}\left(\pi_2 - \Pi_1\right)}. \tag{A-2}$$

For the class of cost functions $C(\lambda) = \frac{\beta}{1+\alpha}\lambda^{1+\alpha}$, condition (A-2) can be written as

$$\left(\frac{\tilde{\lambda}^{post}}{\lambda^{pre}}\right)^{\alpha} < \frac{\Pi_1 + \tilde{\lambda}^{post}(\Pi_2 - 2\Pi_1)}{\Pi_1 + \lambda^{pre}(\pi_2 - \Pi_1)}.$$

or, equivalently, as

$$\alpha \ln\left(\frac{\tilde{\lambda}^{post}}{\lambda^{pre}}\right) < \ln\left(\frac{\Pi_1 + \tilde{\lambda}^{post}(\Pi_2 - 2\Pi_1)}{\Pi_1 + \lambda^{pre}(\pi_2 - \Pi_1)}\right).$$

Moreover, λ^{pre} and $\tilde{\lambda}^{post}$ go to zero when β goes to infinity. Therefore, the condition above holds for sufficiently large β if

$$\alpha < \lim_{\beta \to +\infty} \frac{\ln\left(\frac{\Pi_1+\tilde{\lambda}^{post}(\Pi_2-2\Pi_1)}{\Pi_1+\lambda^{pre}(\pi_2-\Pi_1)}\right)}{\ln\left(\frac{\tilde{\lambda}^{post}}{\lambda^{pre}}\right)} = \frac{\Pi_2 - \Pi_1 - \pi_2}{\Delta\Pi_1}.$$

To see why the above equality holds, note that

$$\frac{\ln\left(\frac{\Pi_1+\tilde{\lambda}^{post}(\Pi_2-2\Pi_1)}{\Pi_1+\lambda^{pre}(\pi_2-\Pi_1)}\right)}{\ln\left(\frac{\tilde{\lambda}^{post}}{\lambda^{pre}}\right)} = \frac{\ln\left(1+\lambda^{pre}\left[\frac{\Pi_2-\Pi_1-\pi_2}{\Pi_1+\lambda^{pre}(\pi_2-\Pi_1)} + \frac{\left(\frac{\tilde{\lambda}^{post}}{\lambda^{pre}}-1\right)(\Pi_2-2\Pi_1)}{\Pi_1+\lambda^{pre}(\pi_2-\Pi_1)}\right]\right)}{\ln\left(1+\lambda^{pre}\left[\Delta + \frac{\left(2-\frac{CS_2^M}{CS_1}\right)\left(\lambda^{pre}+\tilde{\lambda}^{post}\right)}{2+\left(\lambda^{pre}+\tilde{\lambda}^{post}\right)\left(\frac{CS_2^M}{CS_1}-2\right)}\right]\right)}$$

where the square bracket terms go to $\frac{\Pi_2-\Pi_1-\pi_2}{\Pi_1}$ and Δ, respectively, as β goes to infinity. The equality then follows immediately from the application of the L'Hôpital Rule.

Proof of Corollary 1

Consider a unit mass of consumers uniformly distributed along a circle with circumference 1. Firm 1 is located at $x = 0$ and firm 2 is located at $x = 1/2$. The utility of a consumer located at a distance d from a firm is

$$u_i = u - p_i - td^2,$$

where u is the gross utility from consuming any variety, $t > 0$ is the transportation cost, d the distance from the chosen product, and p_i the price charged by firm i.

Duopoly profit π_2. Consider a model where firms are independent and compete on price. We consider a symmetric equilibrium with full-coverage. The equilibrium is such that $p_1 = p_2 = t/4$, and $q_1 = q_2 = 1/2$. Hence, per-firm duopoly profit is $\pi_2 = t/8$.

Consumer surplus is $CS_2 = u - 13t/48$. Full coverage of the duopoly requires $u \geq 5t/16$.[31]

Single-product monopoly profit Π_1. Consider the case where only a single firm innovates and is located at point 0. A consumer buys if and only if she has a positive utility from consuming. Hence, all consumers such that $x \leq x^* := \sqrt{(u-p)/t}$ buy. In the main text, we focus on the case where there is partial coverage, i.e., not all consumers buy the monopolist's product. Demand is $q(p) = 2\sqrt{(u-p)/t}$ and $\Pi_1(p) = 2p\sqrt{(u-p)/t}$. Maximization yields $p = 2u/3$ and $\Pi_1 = (4u\sqrt{u})/(3\sqrt{3t})$. Consumer surplus is $CS_1 = (4u\sqrt{u})/(9\sqrt{3t})$. Partial coverage occurs whenever $q(p) < 1$, i.e., if $u < 3t/4$.

Two-product monopoly profit Π_2. Consider the case where both innovations are successful and a single firm owns the two products. In a symmetric equilibrium, the firm charges the same price p for both products. Full coverage requires that the indifferent consumers between the two products have non-negative utility. This occurs for $p \leq u - t/16$. Under full coverage, the optimal choice is full coverage at $p = u - t/16$. Profits are $\Pi_2 = p = u - t/16$. Consumer surplus is $CS_2^M = t/24$. Full coverage occurs for $u \geq t/16$.

Verifying when $\Pi_2 - \Pi_1 > \pi_2$. For ease of exposition, we define $k := u/t$. We rewrite $\pi_2 = t/8$, $\Pi_2 = t\Big(k - 1/16\Big)$ and $\Pi_1 = (4k^{3/2}t\sqrt{3})/9$. The relevant parameter region is $k < 3/4$ (partial-coverage Π_1). Here, $\Pi_1 = (4k^{3/2}t\sqrt{3})/9$ so

$$\Pi_2 - \Pi_1 - \pi_2 = t\left[\left(k - \frac{1}{16}\right) - \frac{4\sqrt{3}}{9}k^{3/2} - \frac{1}{8}\right].$$

Define

$$F(k) := \left(k - \frac{1}{16}\right) - \frac{4\sqrt{3}}{9}k^{3/2} - \frac{1}{8}.$$

Then $\Pi_2 - \Pi_1 > \pi_2$ if and only if $F(k) > 0$. The function $F(.)$ is continuous, strictly decreasing for small k and eventually strictly increasing, and it has a unique root $k^* \in (0, \infty)$, that can numerically be evaluated at $k^* \approx 0.34035$.

Moreover, $F(k) < 0$ for $k < k^*$ and $F(k) > 0$ for $k > k^*$. Imposing the duopoly

[31] All consumers buy if the utility of the indifferent consumers (located at $x = 1/4$ and $x = 3/4$) are non-negative.

full-coverage condition $k \geq 5/16 \approx 0.3125$, we obtain:[32]

$$\Pi_2 - \Pi_1 \begin{cases} < \pi_2, & k \in \left[\frac{5}{16}, k^*\right), \\ > \pi_2, & k \in \left(k^*, \frac{3}{4}\right). \end{cases}$$

Proof of Proposition 3

Note first that the assumption that the pre-merger equilibrium is unique and stable implies that

$$R_3^{pre}(\lambda_{12}) < (R_{12}^{pre})^{-1}(\lambda_{12})$$

for any $\lambda_{12} < \lambda^{pre}$, and

$$R_3^{pre}(\lambda_{12}) \geq (R_{12}^{pre})^{-1}(\lambda_{12})$$

for any $\lambda_{12} \geq \lambda^{pre}$. Graphically, this means that the best-response curve of firm 3 crosses the aggregate best-response curve of firms 1 and 2 "from above". Since $R_{12}^{pre}(.)$ is decreasing, the above inequalities can be rewritten as:

$$R_{12}^{pre} o R_3^{pre}(\lambda_{12}) > \lambda_{12}$$

for any $\lambda_{12} < \lambda^{pre}$, and

$$R_{12}^{pre} o R_3^{pre}(\lambda_{12}) \leq \lambda_{12}$$

for any $\lambda_{12} \geq \lambda^{pre}$.

(i) Assume that $\Pi_2 - \Pi_1 < \pi_2$ and $\tilde{\Pi}_3 - \pi_2 < \pi_3$. By Lemma 1, this implies that

$$R_{12}^{post}(\lambda_3) < R_{12}^{pre}(\lambda_3)$$

for any λ_3. Let us show that $\lambda_{12}^{post} < \lambda^{pre}$. To do so, let us assume that the opposite holds, that is, $\lambda_{12}^{post} \geq \lambda^{pre}$. Then it follows from the above analysis that

$$R_{12}^{pre} o R_3^{pre}(\lambda_{12}^{post}) \leq \lambda_{12}^{post}. \tag{A-3}$$

Since $R_3^{post}(\lambda_{12}) > R_3^{pre}(\lambda_{12})$ and $R_{12}^{post}(\lambda_3) < R_{12}^{pre}(\lambda_3)$ for any λ_{12} and λ_3, and $R_{12}^{post}(.)$ is decreasing, the following inequalities hold:

$$R_{12}^{post} o R_3^{post}(\lambda_{12}) < R_{12}^{post} o R_3^{pre}(\lambda_{12}) < R_{12}^{pre} o R_3^{pre}(\lambda_{12}).$$

[32] We have considered the case where there is partial coverage under monopoly with one product. Under full coverage, the one-product monopolist gains $\Pi_1 = u - t/4$. The condition $\Pi_2 - \Pi_1 > \pi_2$ holds for all $k \geq 3/4$.

Applying this to λ_{12}^{post} and using (A-3) yields

$$R_{12}^{post} o R_3^{post}(\lambda_{12}^{post}) < R_{12}^{pre} o R_3^{pre}(\lambda_{12}^{post}) \leq \lambda_{12}^{post},$$

which yields a contradiction because $R_{12}^{post} o R_3^{post}(\lambda_{12}^{post}) = \lambda_{12}^{post}$ by definition of λ_{12}^{post} as the post-merger innovation equilibrium effort of the insiders. Therefore, it must hold that

$$\lambda_{12}^{post} < \lambda^{pre}.$$

Combining this with the fact that $R_3^{post}(.)$ is decreasing and that $R_3^{post}(\lambda_{12}) > R_3^{pre}(\lambda_{12})$ for any λ_{12} leads to

$$\lambda_3^{post} = R_3^{post}(\lambda_{12}^{post}) > R_3^{post}(\lambda^{pre}) > R_3^{pre}(\lambda^{pre}) = \lambda^{pre}.$$

This completes the proof of (i).

(ii) Assume that $\Pi_2 - \Pi_1 > \pi_2$ and $\tilde{\Pi}_3 - \pi_2 > \pi_3$. By Lemma 1, this implies that

$$R_{12}^{post}(\lambda_3) > R_{12}^{pre}(\lambda_3)$$

for any λ_3.

A scenario in which $\lambda_{12}^{post} \leq \lambda^{pre}$ and $\lambda_3^{post} \leq \lambda^{pre}$ cannot occur. To see why assume that $\lambda_{12}^{post} \leq \lambda^{pre}$. Then from the fact that $R_3^{post}(.)$ is decreasing and that $R_3^{post}(\lambda_{12}) > R_3^{pre}(\lambda_{12})$ for any λ_{12}, it follows that

$$\lambda_3^{post} = R_3^{post}(\lambda_{12}^{post}) \geq R_3^{post}(\lambda^{pre}) > R_3^{pre}(\lambda^{pre}) = \lambda^{pre}.$$

Therefore, it must be the case that at least one of the following inequalities holds: $\lambda_{12}^{post} > \lambda^{pre}$ or $\lambda_3^{post} > \lambda^{pre}$. In words, this means that the merger leads to an increase in the innovation efforts of either the insiders, the outsider, or both. The analysis under the Salop model shows that each of these three scenarios can indeed arise.

Analytical derivation of the merger with outsiders example

In this section, we extend the model to three firms located symmetrically at $(0, 1/3, 2/3)$. We focus on the case in which there is partial market coverage under a single-product monopolist and full market coverage otherwise.

One-product monopolist This is the same as in the duopoly case.

Two-product monopolist. Consider a merged entity with two products located (without loss of generality) at 0 and 1/3, and that the independent firm has not innovated.

Assume full coverage occurs. In the symmetric equilibrium, the monopolist sets the same price for the two products. Consumers located in $[0, 1/6] \cup [2/3, 1]$ buy the product located at 0, and the consumers located in $[1/6, 2/3]$ buy the product located at 1/3. Full coverage requires that the further consumer from each product (located at 2/3) gets non-negative utility. Hence, the firm sets the price to extract all the surplus of this consumer, that is $p = u - t/9$.

Since everyone buys under full market coverage and the merged entity serves all consumers, then

$$\Pi_2 = u - \frac{t}{9}, \qquad CS_2^M = \frac{t}{12}.$$

The feasible region is such that $u \geq t/9$.

Two-product firm vs. single product firm We are interested now in the case in which a two-product firm jointly controls products located in 0 and 1/3, and competes against an independent outsider located in 2/3. We denote the profit of the merged entity as $\tilde{\Pi}_3$ and that of the outsider as $\tilde{\pi}_3$.

Under full coverage, demands are (for $\{i, j, k\} = \{1, 2, 3\}$)

$$q_i = \frac{9(p_j + p_k - 2p_i) + 2t}{6t},$$

Firm 3 chooses p_3 to maximize $\tilde{\pi}_3 = p_3 q_3$, while the merged entity chooses (p_1, p_2) to maximize $\tilde{\Pi}_3 = p_1 q_1 + p_2 q_2$. From the first-order conditions, we obtain the following equilibrium prices $p_1 = p_2 = 5t/27, p_3 = 4t/27$.

Therefore, replacing into quantities and profits, we obtain the following profits and consumer surplus:

$$\tilde{\Pi}_3 = \frac{25t}{243}, \qquad \tilde{\pi}_3 = \frac{16t}{243}, \qquad CS_3^M = u - \frac{271t}{972}.$$

The feasible region is such that the indifferent consumer with the lowest utility (located at 1/6 distance) has non-negative utility, that is $u \geq 23t/108$.

Symmetric triopoly $\boldsymbol{\pi_3}$. Consider now three independent single-product firms located at 0, 1/3, and 2/3. We focus on the case in which the market is fully covered. The demands are (for $\{i, j, l\} = \{1, 2, 3\}$, with $i \neq j \neq l$)

$$q_i = \frac{9(p_j + p_l - 2p_i) + 2t}{6t},$$

Firm i chooses p_i to maximize $\pi_i = p_i q_i$. In a symmetric Nash equilibrium $p_1 = p_2 = p_3 = t/9$ and $q_i = 1/3$. Hence, the profit of each firm is

$$\pi_3 = \frac{t}{27}.$$

Consumer surplus is

$$CS_3 = u - \frac{13t}{108}.$$

The feasible region is such that the indifferent consumer between any firm has non-negative utility, that is $u \geq 5t/36$.

Single-product duopolist $\boldsymbol{\pi_2}$ Consider two single-product firms located at 0 and 1/3 on the unit circle. We focus on symmetric outcomes and full coverage.

The symmetric Nash price is $p = 2t/9$, with each firm serving $q_i^D = 1/2$. Firm's profit is

$$\pi_2 = \frac{t}{9},$$

and consumer surplus is

$$CS_2 = u - \frac{t}{4}.$$

The feasible region is such that the furthest indifferent consumer (located in 2/3) has non-negative utility, that is $u \geq t/3$.

Parameter region for the working example

We collect here the minimal constraints ensuring that all market configurations used in the triopoly example admit full coverage (except the single-product monopolist, for which we impose partial coverage):

1. **Single-product monopoly (partial coverage):**

$$\frac{u}{t} < \frac{3}{4} \qquad \iff \qquad t > \frac{4u}{3}.$$

2. **Two-product monopoly (full coverage):**

$$u \geq \frac{t}{9} \qquad \iff \qquad t \leq 9u.$$

3. **Two-product vs. single-product firm (full coverage):**

$$u \geq \frac{23}{108}t \qquad \Longleftrightarrow \qquad t \leq \frac{108}{23}u.$$

4. **Symmetric triopoly (full coverage):**

$$u \geq \frac{5}{36}t \qquad \Longleftrightarrow \qquad t \leq \frac{36}{5}u.$$

5. **Symmetric duopoly at 0 and 1/3 (full coverage):**

$$u \geq \frac{t}{3} \qquad \Longleftrightarrow \qquad t \leq 3u.$$

The binding upper bound among (2)–(5) is $t \leq 3u$. Combining this with the partial-coverage requirement $t > 4u/3$ from (1) yields the admissible interval $4u/3 < t \leq 3u$. For the parameter value $u = 0.6$, this becomes $0.8 < t \leq 1.8$, which is precisely the range employed in Figure 3.

Extension to any number of outsiders

In this section, we extend the model by allowing for multiple outsiders. We define the aggregate best-response of insiders and outsiders, and generalize conditions under which they are both decreasing. We then determine whether the merger shifts each aggregate best-response upward or downward.

Consider a setting with $N = 2+n$ firms: 2 insiders (denoted $i = 1, 2$) and $n \geq 1$ outsiders (denoted $i = 3, ..., N$).

Let π_k denote the (symmetric) oligopoly profit obtained by each successful firm when $k \geq 2$ independent products are on the market. Moreover, for the sake of exposition, we denote $\pi_1 = \Pi_1$ the monopoly profit obtained if there is a single product on the market. We assume that $\pi_1 > \pi_2 \geq \pi_3 \geq ... \geq \pi_N$.

Let $\tilde{\Pi}_k$ and $\tilde{\pi}_k$ denote, respectively, the profit of the merged entity and the post-merger profit of a successful outsider, when both labs of the merged entity are successful and $k - 2 \leq n$ outsiders are successful (notice that $\tilde{\Pi}_2 = \Pi_2$) .

Let λ_i denote the innovation effort of firm i. In any equilibrium, firms 1 and 2 choose the innovation effort $\lambda_1 = \lambda_2 = \lambda_{12}$, and firms $i = 3, ...N$ choose the same innovation effort, which we denote λ_3. Following the analysis in the scenario with a single outsider, we compare the pre- and post-merger aggregate best-response functions of firms 1 and 2, as well as those of firms 3 to N.

Pre-merger equilibrium : In the absence of a merger, firms 1 and 2 operate independently and choose their innovation effort separately. Their aggregate best response $R_{12}^{pre}(\lambda_3)$ is the value of λ_{12} that solves

$$\lambda_{12} = \arg\max_{\lambda} \lambda \sum_{j=0}^{n} \binom{n}{j} \lambda_3^j (1-\lambda_3)^{n-j} \left[(1-\lambda_{12})\pi_{j+1} + \lambda_{12}\pi_{j+2}\right] - C(\lambda),$$

where the term in the summation is the expected profit generated by a successful product 1 (or 2), conditional on having $j = 0, 1, ..., n$ successful firms among firms 3 to N.

The aggregate best response $R_3^{pre}(\lambda_{12})$ of firms 3 to N is the value of λ_3 that solves

$$\lambda_3 = \arg\max_{\lambda} \lambda \sum_{l=0}^{n-1} \binom{n-1}{l} \lambda_3^l (1-\lambda_3)^{n-1-l} \left[(1-\lambda_{12})^2 \pi_{l+1} + 2(1-\lambda_{12})\lambda_{12}\pi_{l+2} + \lambda_{12}^2\pi_{l+3}\right] - C(\lambda),$$

where $l = 0, 1, ..., n-1$ denotes the number of successful innovators among firms 3 to N other than the one considered.

The first-order condition defining $R_{12}^{pre}(\lambda_3)$ is

$$\sum_{j=0}^{n} \binom{n}{j} \lambda_3^j (1-\lambda_3)^{n-j} \left[(1-\lambda_{12})\pi_{j+1} + \lambda_{12}\pi_{j+2}\right] = C'(\lambda_{12}).$$

$R_{12}^{pre}(\lambda_3)$ is uniquely defined because the left-hand side of the equation above is decreasing in λ_{12}. Moreover, using the identities

$$j\binom{n}{j} = n\binom{n-1}{j-1}, \qquad (n-j)\binom{n}{j} = n\binom{n-1}{j},$$

the derivative of the left-hand side with respect to λ_3 is

$$n\sum_{j=0}^{n-1} \binom{n-1}{j} \lambda^j (1-\lambda)^{n-1-j} [(1-\lambda_{12})(\pi_{j+2}-\pi_{j+1}) + \lambda_{12}(\pi_{j+3}-\pi_{j+2})],$$

which is negative. This implies that $R_{12}^{pre}(\lambda_3)$ is decreasing in λ_3.

By similar arguments, $R_3^{pre}(\lambda_{12})$ is also uniquely defined and decreasing in λ_{12}.

Post-merger equilibrium. Assume now that firms 1 and 2 merge while the remaining firms remain independent. Then, firms 1 and 2 act as a single entity controlling two innovation projects. The merged entity's best response $R_{12}^{post}(\lambda_3)$ is the value of λ_{12} that solves

$$\lambda_{12} = \arg\max_{\lambda} \sum_{j=0}^{n} \binom{n}{j} \lambda_3^j (1-\lambda_3)^{n-j} \left[2\lambda(1-\lambda)\pi_{j+1} + \lambda^2\tilde{\Pi}_{j+2}\right] - 2C(\lambda).$$

The first-order condition defining $R_{12}^{post}(\lambda_3)$ is

$$\sum_{j=0}^{n}\binom{n}{j}\lambda_3^j\left(1-\lambda_3\right)^{n-j}\left[\left(1-\lambda_{12}\right)\pi_{j+1}+\lambda_{12}\left(\tilde{\Pi}_{j+2}-\pi_{j+1}\right)\right]=C'\left(\lambda_{12}\right).$$

$R_{12}^{post}(\lambda_3)$ is uniquely defined if the left-hand side is decreasing in λ_{12}, which is the case if

$$\sum_{j=0,n}\binom{n}{j}\lambda_3^j\left(1-\lambda_3\right)^{n-j}\left[\tilde{\Pi}_{j+2}-2\pi_{j+1}\right]<0,$$

that is, if the merged entity's expected incremental profit of an innovation is lower for the second innovation than for the first innovation, where the expectation is over the number of independent products.

Moreover, $R_{12}^{post}(\lambda_3)$ is decreasing in λ_3 if $\left(1-\lambda_{12}\right)\pi_{j+1}+\lambda_{12}\left(\tilde{\Pi}_{j+2}-\pi_{j+1}\right)$ is decreasing in j. Given that π_{j+1} is (weakly) decreasing in j, a sufficient condition for this is that $\tilde{\Pi}_{j+2}-\pi_{j+1}$ is decreasing in j. That is, it is sufficient that the incremental profit generated for the merged entity by a second innovation decreases with the number of products on the market.

The outsiders' aggregate best response $R_3^{post}(\lambda_{12})$ is the value of λ_3 that solves

$$\lambda_3=\arg\max_{\lambda}\lambda\sum_{l=0}^{n-1}\binom{n-1}{l}\lambda_3^l\left(1-\lambda_3\right)^{n-1-l}\left[\left(1-\lambda_{12}\right)^2\pi_{l+1}+2\left(1-\lambda_{12}\right)\lambda_{12}\pi_{l+2}+\lambda_{12}^2\tilde{\pi}_{l+3}\right]-C\left(\lambda\right)$$

The first-order condition defining $R_3^{post}(\lambda_{12})$ is

$$\sum_{l=0}^{n-1}\binom{n-1}{l}\lambda_3^l\left(1-\lambda_3\right)^{n-1-l}\left[\left(1-\lambda_{12}\right)^2\pi_{l+1}+2\left(1-\lambda_{12}\right)\lambda_{12}\pi_{l+2}+\lambda_{12}^2\tilde{\pi}_{l+3}\right]=C'\left(\lambda_3\right)$$

$R_3^{post}(\lambda_{12})$ is uniquely defined because the terms between brackets decrease in l, implying that the left-hand side is decreasing in λ_{12}. Moreover, $R_3^{post}(\lambda_{12})$ is decreasing in λ_3 if $\tilde{\pi}_{l+3}<\pi_{l+2}$ for all $l=0,...,n-1$.

To sum up, the conditions that generalize those in Assumption 2 to an arbitrary number of outsiders are the following:

(i) $\pi_1>\pi_2\geq\pi_3\geq...\geq\pi_N$

(ii) $\tilde{\pi}_k>\pi_k$ for $k=3,...,N$

(iii) $\tilde{\pi}_k<\pi_{k-1}$ for $k=3,...,N$

(iv) $\tilde{\Pi}_{k-1}-\pi_{k-2}>\tilde{\Pi}_k-\pi_{k-1}$ for $k=3,....,N$.

Under these conditions, the aggregate best response of the outsiders shifts upward because of condition (ii).

The best response of the insiders (strictly) shifts upward if and only if

$$\sum_{j=0}^{n} \binom{n}{j} \lambda_3^j \left(1-\lambda_3\right)^{n-j} \left[\tilde{\Pi}_{j+2} - \pi_{j+1} - \pi_{j+2}\right] > 0,$$

that is, if the expected incremental profit of a second innovation of the merged entity is larger than the expected profit of success of an independent innovator, where the expectation is taken with respect to the number of independent products on the market.