Journal of Quantitative Economics (2023) 21:803–815
https://doi.org/10.1007/s40953-023-00361-y

ORIGINAL ARTICLE

Mobility and Income Distribution

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Accepted: 21 June 2023 / Published online: 6 July 2023

Abstract
This article presents a model for income distribution among factors of production in the context of a globalised economy. Previous models are most often static and do not take into account the geographical location of the factors of production nor the mobility costs that result. We have created a dynamic Nash bargaining model that integrates the geographical distance between companies and the mobility costs for each production factor. The main result of this model is that income distribution closely depends on mobility costs: production factors with low mobility costs are those whose incomes increase most rapidly.

Keywords Factors of production · Income · Mobility costs · Regulation

JEL Classification E25 · F20 · F40

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Introduction

The objective of this paper is to propose an explanatory model for the distribution of wealth among the different factors of production. We take up here the point of view of Colletis (2008, 2012): far from being remunerated according to their marginal contribution to production, the capacity of factors to capture part of the wealth created would depend on their mobility cost.

From the point of view of the theory of wealth distribution, two statistical observations were the object of particular attention: Arthur Bowley's (1937) study of the United Kingdom and Paul Douglas' (1934) study of the United States, both of which attest to the remarkable long-term constancy of the sharing of value added.

To explain this stylized fact, known as "Bowley's Law", two main approaches are developed. In the first approach, economists have analyzed the effects of imperfections in competition on the short term determinants of the sharing of value added. In the second approach, papers based on the Cobb–Douglas function have focused on the long-term determinants of income distribution.

Harrod (1934), Dunlop (1938), Tarshis (1939) and Kalecki (1938), have renewed the analysis of the short-term determinants of income distribution by emphasizing the role of imperfect competition. Different conceptions of the distribution of value added are compatible with imperfect competition hypotheses: they allow increasing returns, or they allow the mark-up rate to vary in the medium term with the volume of activity.

In the long term, the Cobb–Douglas-type production function has made it possible to explain the constancy of the wage share. When the elasticity of capital/labor substitution is unitary, the "quantity effect" exactly counterbalances the "price effect". If there are no rigidities in the markets for goods and labour, a production function with a unitary elasticity of substitution thus gives a constant share to each of the factors.

In this paper, we wish to completely renew the analysis by providing a different point of view. Thus, we rely on the thesis of Colletis (2008): he defends the hypothesis that the remuneration of the different factors must depend on the costs associated with their movement, because the latter give factors a bargaining power (Godechot 2008). The production factor with zero mobility costs is thus endowed with the capacity to acquire a large share of the wealth created. Conversely, factors that incur high mobility costs are unable to negotiate high remuneration.

To construct a formalized representation of the distribution of wealth, we rely on Nash's well-known bargaining model (1950, 1953)[1]: if we consider K agents who must share a certain amount of good V, then the remuneration of the participants in the negotiation is the only solution of the program:

$$\left(W^1, ..., W^K\right) = Arg\max \prod_{k=1}^{K} \left(W^k - \theta^k\right) \quad (1)$$

under the constraints: Individual rationality: $W^k > \theta^k$ for every $k \in \{1, ..., K\}$. The sum of the remunerations and the volume of the value added to be shared are equal, that is to say: $\sum_{k=1}^{K} W^k = V$ where V (the volume of the good to be shared) and θ^k (the external option) are both exogenous.

We will explain the distribution of income among economic agents linked to firms located at different points in space. We wish to analyze the impact of differences in mobility costs on the distribution of the value added to be shared V.

In a first section, we present the characteristics of our spatial bargaining model with endogenous external options, then its dynamics in a second section. In a third section, we present the results of our simulations, then we discuss them in a fourth and last section.

A Spatial Bargaining Model with Endogenous External Options

We consider an economy composed of several firms distant from each other on a discrete space $\left\{X_1, ..., X_N\right\} \in \Re^N$. This location is exogenous to the model. For the sake of simplification, we assume that all firms are aligned, but that they may belong to different sectors of activity.

Each firm "combine" different "factors of production", each type of which is noted $k \in \{1, \dots K\}$. Each firm is located in X_i and has a budget constraint V_i; the different factors enter into a Nash negotiation procedure to set their income. For all $i \in \{1, \dots N\}$, the k-type factors get an income W_i^k, which is the result of the following maximization program:

[1] The applications of this model are so numerous that it would be impossible to give an exhaustive account of them here. However, let us give some significant examples. This model has been used in a number of studies on wage and employment determination. The "right to manage" model (Nickell and Andrews 1983) shows how employee wages are negotiated between a union and the employer, with the employer retaining the prerogative of hiring. This approach was taken up by Layard, Nickell and Jackman (1991) to construct the WS PS model, which is well known in macroeconomics. In addition, the "Nash bargaining" model has been used extensively to account for households' family decisions (Manser and Brown 1980, McElroy and Horney 1990) or household decisions on labour supply (Bargain and Moreau 2005, Donni 2003, Clark, Couprie and Sofer 2004). More theoretically, Nash's model presented above has had many extensions: static models with incomplete information (Rubinstein 1985), dynamic models with complete information (Rubinstein 1982, Binmore, Rubinstein and Wolinsky 1986), dynamic models with incomplete information (Rubinstein and Wolinsky 1985, see also Rochet 1988).

$$\left(W_i^1, \ldots, W_i^K\right) = Arg \max \prod_{k=1}^{K} \left(W_i^k - \theta_i^k\right) \tag{2}$$

The most obvious extension of the model presented above consists in endogenizing the external option θ_i^k. Indeed, we wish to introduce the effect of competition between firms with regard to the unit remuneration of factors of production (which takes productivity into account). This is why we hypothesize that there are several firms in the model. It is this latter hypothesis that allows us to introduce the external option. We consider that the external option θ_i^k depends on the unit remuneration of the factor of the same type in all other firms, but also on the cost of mobility that the factor must undergo in order to move from one firm to another. The external option for a factor of type k at the point X_i is given by:

$$\theta_i^k = Max\left\{W_{i'}^k - c^k\left|X_i, X_{i'}\right|\right\}$$
$$i\prime \in \{1, \ldots, N\}$$

where $\left|X_i, X_{i\prime}\right|$ measures the distance between X_i and $X_{i'}$ and $c^k\left|X_i, X_{i\prime}\right|$ is the cost for a k-factor to move from X_i to $X_{i'}$.

For simplicity, the functions c^k are assumed to be linear. Thus, we have: $c^k\left(\left|X_i, X_{i\prime}\right|\right) = c^k.\left|X_i - X_{i\prime}\right|$ where c^k is a constant that depends only on the type of

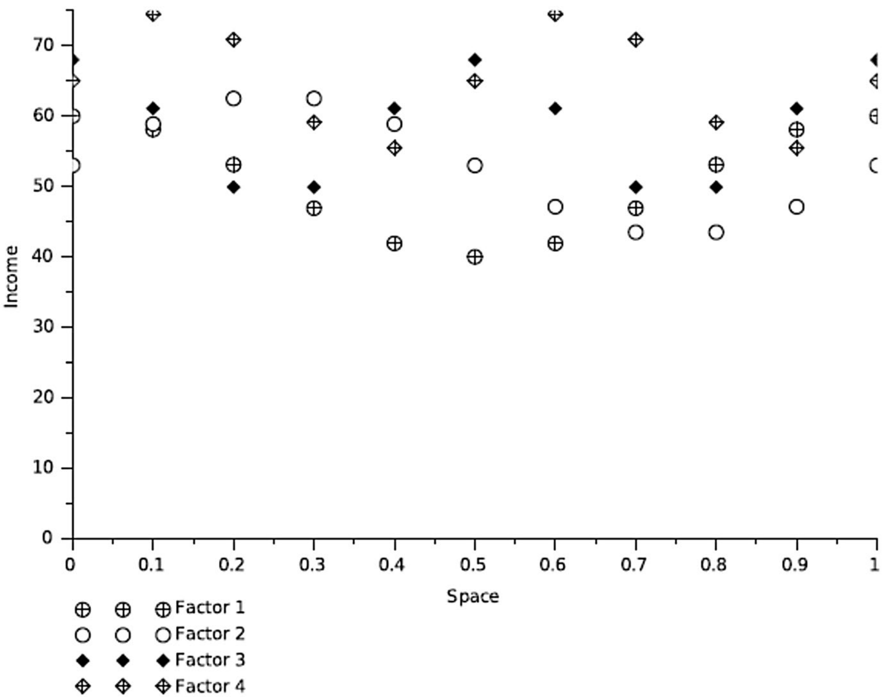

Fig. 1 Initial factors incomes

production factor considered. This formalizes the idea that the cost of mobility is different from one type of factor to another. Without losing the generality of the assumption that: $c^1(=0) < c^2 < c^3 < c^4$.

The Dynamics of the Model

In this article, we are interested in the dynamics of income for each type of production factor during negotiation.

Let $W_i^k(t)$ the remuneration obtained by the factor of type k localized in X_i at time t. The initial remuneration $W_i^k(0)$ of all the factors in different positions in space is given, as well as $V_i = \sum_{k=1}^{K} W_i^k(0)$, representing the added value of the firm located in X_i. This added value is created thanks to the productive contribution of the factors of production that enter into the negotiation. In each firm, their remuneration is therefore complementary. At each stage of bargaining t, Nash's negotiation takes place simultaneously in each firm for all i:

$$W_i^1(t+1), ..., W_i^K\left((t+1) = Arg\max \prod_{k=1}^{K} \left(W_i^k - \theta_i^k(t)\right)\right) \quad (3)$$

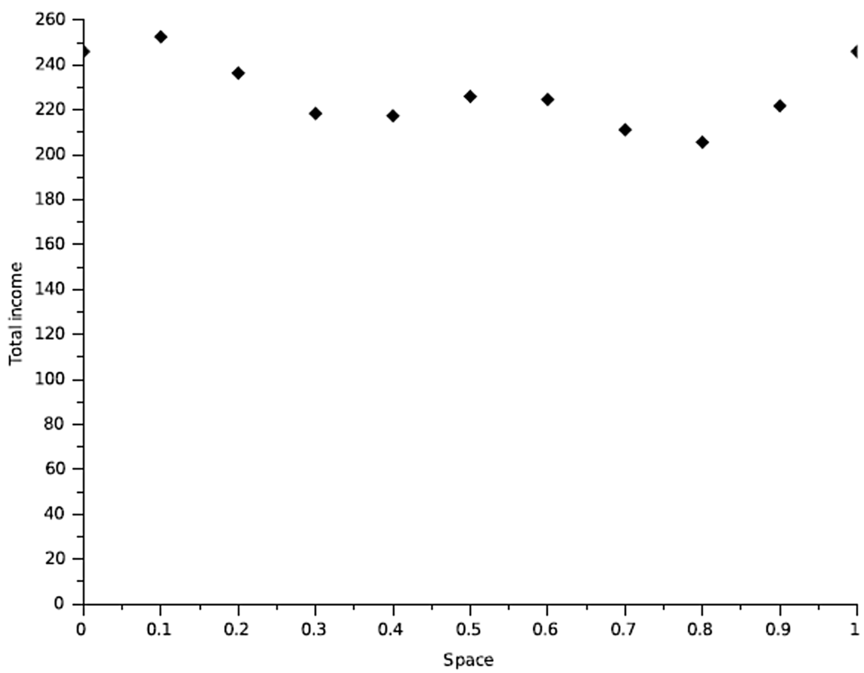

Fig. 2 Distribution of value added in each firm (total revenue is constant over time)

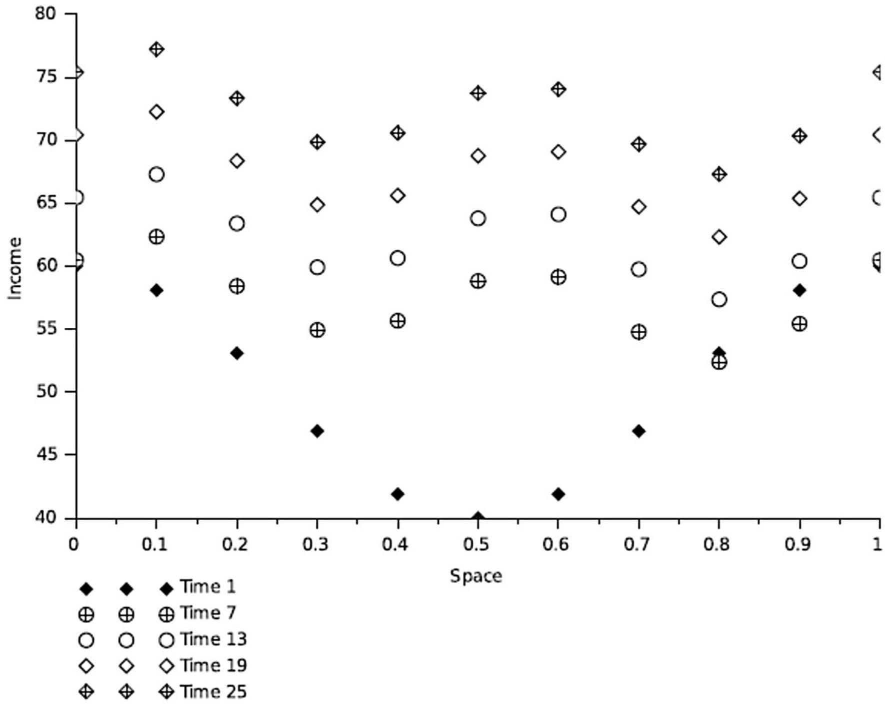

Fig. 3 Factor 1 income in different firms distributed over space [0.1] at times 1, 7, 13, 19 and 25

With: $\theta_i^k(t) = \max_{i' \in \{1,...,N\}\setminus\{i\}} \left\{ W_{i'}^k(t) - c^k(|X_i - X_{i'}|) \right\}$.

It should be noted that within the budget constraint, the added value *Vi* and the factor productivity remain constant over time. In this article, we choose to set the overall volume of income to be shared in each firm. It is therefore exogenous. The reason that justifies this choice is that making the global income variable risks making the results difficult to read: the sharing of income would then depend not only on the costs of mobility but also on the differentiated economic growth of the firms and on differentiated growth in factor productivity. It would then be impossible to attribute income inequalities to one or other factor in a specific way. It could also be assumed that the growth of the income to be shared would be identical in all firms. In this case, the results would be exactly the same as those we obtain. We therefore do not think it is necessary to further complicate the model here. On the other hand, it is not said that the *Vi* are identical in all firms. The added values are heterogeneous.

It is easy to see that at each period, our problem (3) does not always have a solution.

This is the case when the constraint $V_i = \sum_{k=1}^{K} W_i^k(t)$ cannot be satisfied.

For example, let us assume two production factors 1 and 2 (k = 2) and two locations (i = 2) that are distant by 1. In *i = 1*, factor (1) and factor (2) both have an income of 1 (so $V1=2$). In $i=2$, both factors have an income of 10. The constraint of the problem imposes that the factors in $i=1$ will both want an income of more than 8 (10–2), which is impossible with a $V1=2$.

It is impossible in this case to find an analytical solution, which is common in this type of complex model. One then seeks to analyze the qualitative properties of the solutions obtained by simulation, which is carried out in the following section.

In cases where the solution exists, the analytical solution (see appendix) is explicitly given by:

$$W_i^k(t+1) = \frac{1}{K}\left[V(j+1) - \sum_{k'=1}^{K} \theta_i^{k'}(j)\right] + \theta_i^k(t) \tag{4}$$

Application of the Model

We consider four types of production factors. Each type of factor is characterized by a particular mobility cost. We assume an equilibrium distribution of agents in the firms $\left\{X_{i\in[1,N]}\right\}$ that are aligned. We consider that the cost for a factor $k \in \{1, \dots K\}$ to move from X_i to $X_{i'}$ is equal to: $c^k\left(\left|X_i, X_{i'}\right|\right) = c^k.\left|X_i - X_{i'}\right|$, where $0 = c^1 < c^2 < \dots < c^K$.

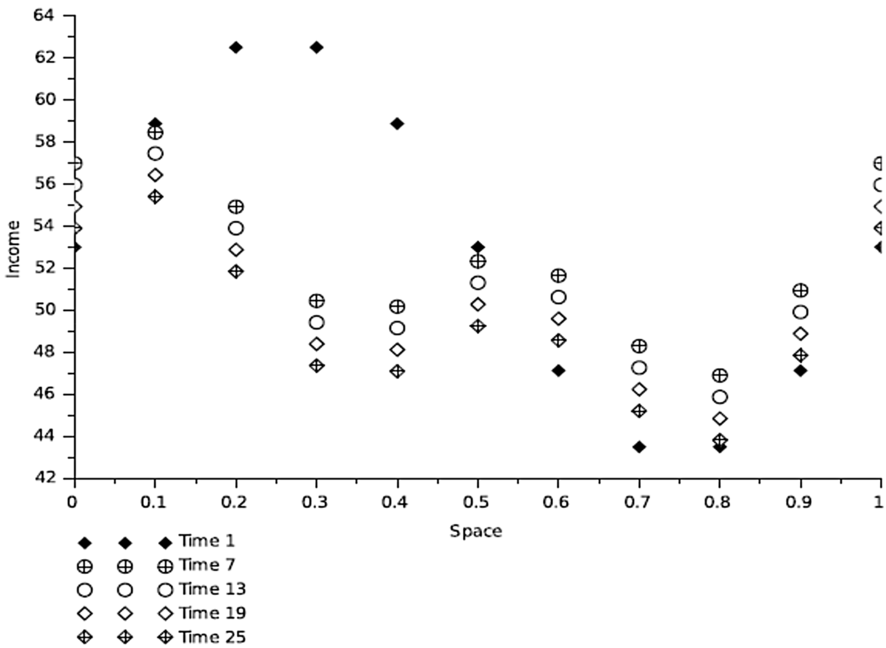

Fig. 4 Factor 2 income in different firms distributed over space [0.1] at times 1, 7, 13, 19 and 25

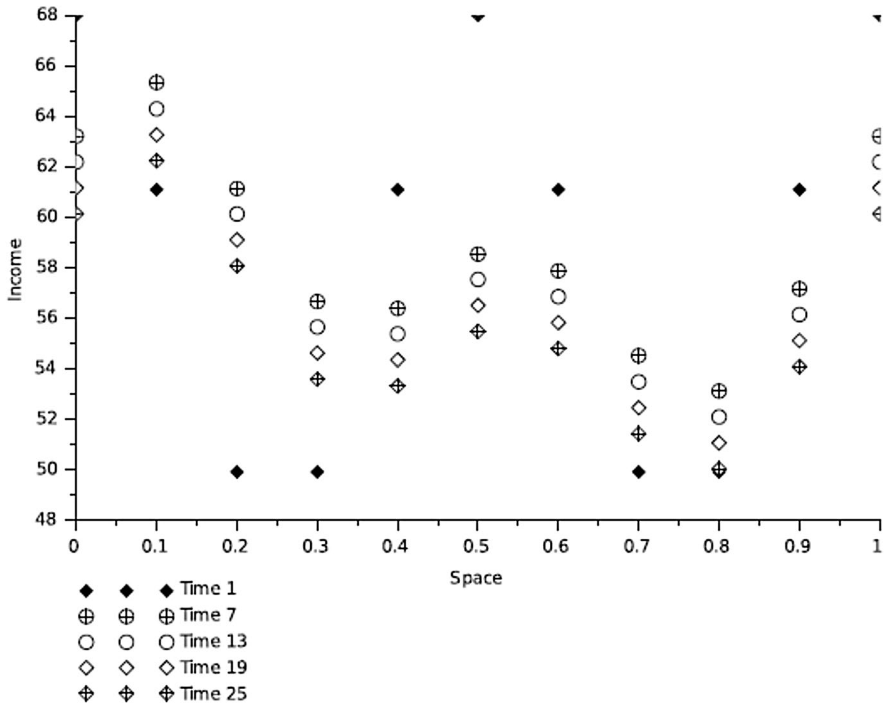

Fig. 5 Factor 3 income in different firms distributed over space [0.1] at times 1, 7, 13, 19 and 25

We consider that the factors are differentiated only by their cost of mobility. Their weight in negotiation is identical. This simplification is carried out in order to isolate the particular effect of the cost of mobility on the distribution.

The cost of mobility is assumed to be 5 for factor (1), 15 for factor (2), 17 for factor (3) and 25 for factor (4). The agents' initial income is shown in Fig. 1. The total value added in the firms is represented in Fig. 2.

The evolution over time of the incomes of the different factors of production are shown in Figs. 3, 4, 5, 6. As mentioned above, there are initial conditions for which there is no solution to the problem (3). Here we have made a choice of initial conditions that ensure the existence of a solution at each iteration, for a sufficiently large total number of iterations.

Discussion

We can see that factor (1) income, which has a very low mobility cost, increases while total value added remains constant over time (Figs. 2 and 3). It is also observed that the income level of this factor is relatively homogeneous in space, whereas initial incomes are disparate (Fig. 3). Symmetrically, the income of factor (4), which has a very high mobility cost, decreases with each period regardless of

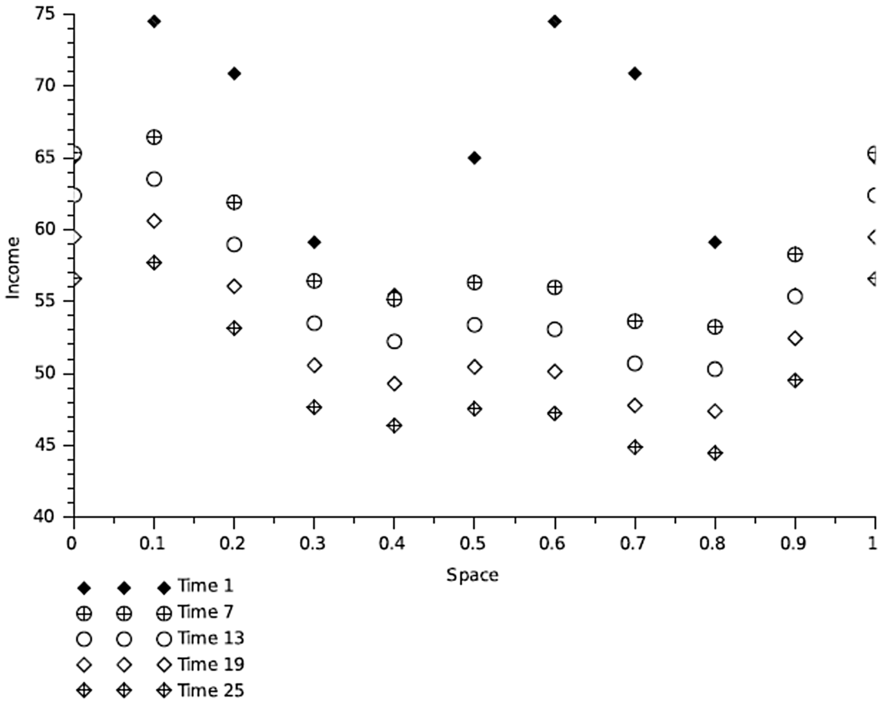

Fig. 6 Factor 4 income in different firms distributed over space [0.1] at times 1, 7, 13, 19 and 25

the firm. This decrease is all the more pronounced the lower the firm's value added (Figs. 2 and 6). For factor 3, the same phenomenon occurs but is less pronounced (Fig. 5). The income of factor 2 increases slightly over time (Fig. 4). It should be noted that the model generates very different incomes depending on the location of the factors in space.

Let us now illustrate the revenue dynamics within a given firm (Fig. 7).

Although Factor (1) income is initially very low, it increases significantly over time. For the other factors, the reverse is true: the decline is regular for factors (3) and (4), but it stabilizes for factor (2). The income dynamics of each factor reflect differences in mobility costs. After several iterations, factor incomes are inversely related to mobility costs, even if this is not the case initially (Fig. 5). This conclusion remains valid for a wide variety of initial conditions. In order to illustrate the robustness of this dynamic, we have chosen to present simulations in which the initial conditions are opposite to this trend (with the selected initial factor incomes not being ordered according to their mobility costs).

The difference in mobility costs thus explains the growing inequalities in the distribution of wealth, both in time and space.

The evolution of factor incomes thus depends closely on their respective mobility costs but also on the initial income conditions. The simulation is identical to the one

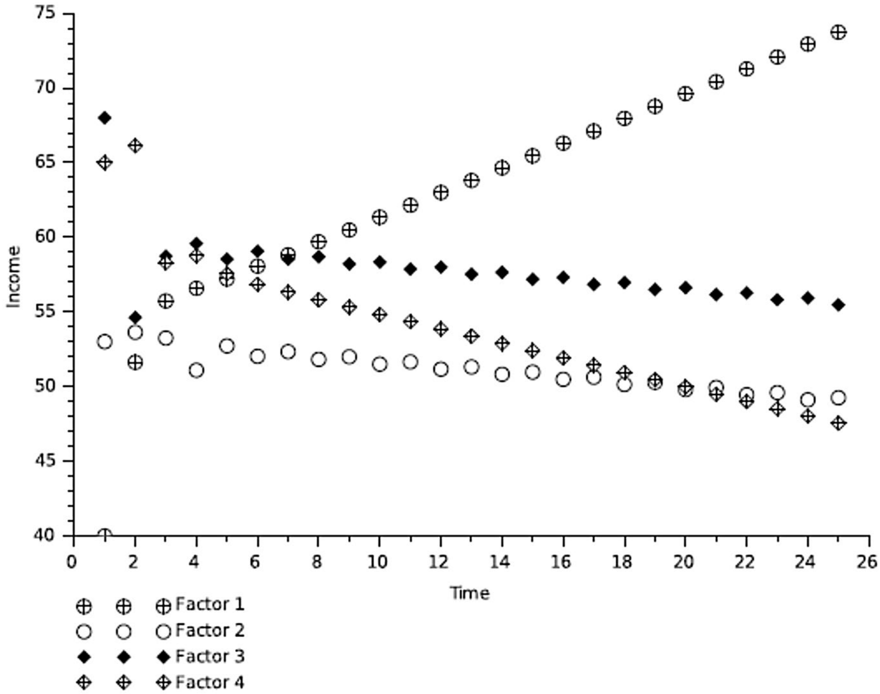

Fig. 7 Income dynamics in the firm located in 0.6

described in Fig. 7 (with identical mobility costs for each factor) but with different initial income conditions (Fig. 8).

Let's add a few remarks:

i. when the cost of mobility is very low, incomes quickly become spatially homogeneous, which illustrates Cassel's law of the unique price (1918).
ii. In firms which have the lowest value added, the incomes of factor (3) and (4) fall more rapidly.

Conclusion

To illustrate the phenomenon of unequal distribution of value added, the model presented in this paper formulates a spatialized and dynamic Nash type negotiation: each type of agent negotiates its income according to its position in space, its mobility costs and the income obtained in the previous period. It is assumed that in each period, bargaining is reproduced. We obtain that a change in the sharing of value added may involve a change in mobility costs for each factor.

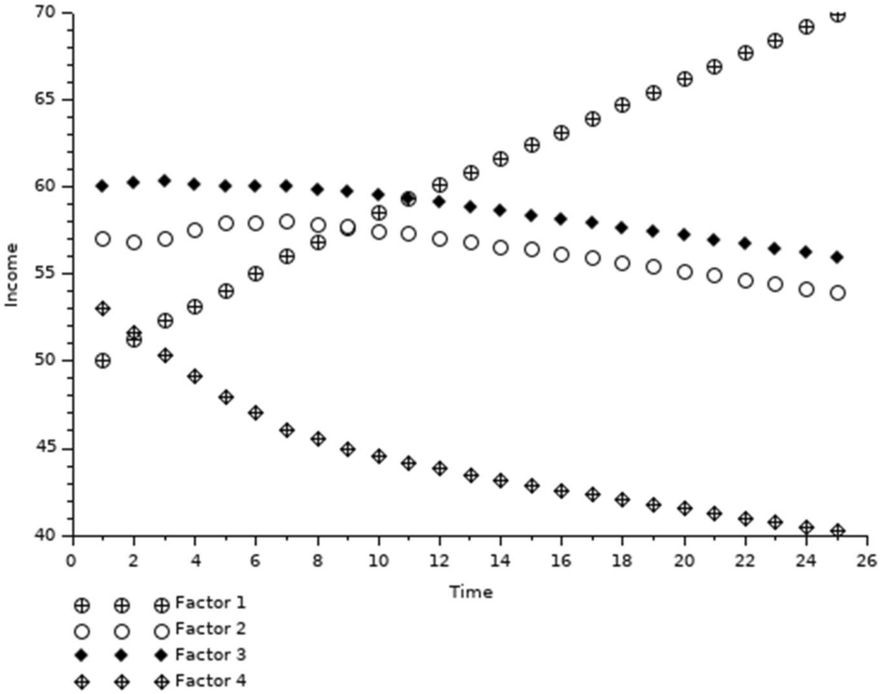

Fig. 8 Income dynamics in the firm located in 0.6 with mobility costs equal to those of Fig. 7 but with different initial income conditions

Appendix (proof of analytical solution)

We have:

$$x^k = \frac{W^k - \theta_i^k(t)}{V_i(t+1) - \sum_{k=1}^{K} \theta_i^k(j)}$$

Problem (3) is equivalent to problem:

$$\left\{x^1, \ldots, x^K\right\} = Arg \max_{\left\{x^k\right\}_k} f\left(x^1, \ldots, x^K\right) \tag{5}$$

where:

$$f(x^k) = \left(1 - \sum_{k=1}^{K-1} x^k\right) \prod_{k=1}^{K-1} x^k$$

Under the constraint $x^k \geq 0$ for all k.

To solve the problem (5), we calculate:$\frac{\partial f}{\partial x_j}(x^1,...,x^K) = -\prod_{k=1}^{K-1} x^k + \left(1 - \sum_{k=1}^{K-1} x^k\right)\prod_{k=1,k\neq j}^{K-1} x^k = \prod_{k=1}^{K-1} x^k \left[\frac{1}{x_j}\left(1 - \sum_{k=1}^{K-1} x^k\right) - 1\right]$.

Then:

$$\vee f = 0 \Leftrightarrow \sum_{k=1,k\neq j}^{K-1} x^k + 2x_j = 1 \Leftrightarrow (I_{K-1} + U_{K-1,K-1})X = U_{K-1,1}$$

where I_{K-1} is the identity matrix of $\mathbb{R}^{K-1}$, $X = (x^1,...,x^K)^T$ and $U_{K-1,h}$ the matrix with K-1 rows and h columns. As $Det(I_{K-1} + U_{K-1,K-1}) \neq 0$,is a Cramer system with a unique solution $x_k = 1/K$ for all k. Returning to the original variable, we find the result to be proven.

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