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Monsieur Mudit DHAKAR

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Thèse dirigée par *Monsieur Renato GOMES et Monsieur Doh-Shin JEON*

Composition du jury

Rapporteur : M. Marc BOURREAU
Rapporteure : Mme Senay SOKULLU
Co-directeur de thèse : M. Renato GOMES
Co-directeur de thèse : M. Doh-Shin JEON

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Mudit Dhakar

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Introduction

This thesis consists of three chapters in the field of Industrial Organisation. The first two chapters address topics in digital economics, while the third chapter focuses on sustainability in aviation. Each chapter is motivated by the contemporary policy debates relevant at the time of writing.

The first chapter investigates the effects of mandating interoperability between messaging services. We examine a situation in which a high-privacy paid niche app competes with a low-privacy free app with a large installed user base. Consumers differ in their privacy concerns and can multi-home across different apps. We find that niche app users may be worse off under partial privacy-preserving interoperability compared to no interoperability. Moreover, full interoperability may not necessarily promote market contestability, especially when privacy concerns significantly outweigh network benefits. We identify a misalignment between the low-privacy app's incentives for interoperability and the welfare of niche app users, highlighting the importance of mandating a carefully calibrated level of interoperability. These results hold even when interoperability leads to privacy loss for users who opt into the policy.

The second chapter examines the impacts of a data sharing policy in innovative markets. We consider products with two-dimensional quality: one driven by data-enabled learning and the other by standalone innovation. By allowing firms to compete on these dimensions, we study the implications of differences in data collected across firms on the incentives to invest in standalone quality in a two-period model and show that the investment by a data-lagging firm increases as data dominance shrinks. We then explore the use of a deterministic full data sharing policy to reduce the data dominance. While such a policy always improves upon the innovation level, it can reduce consumer surplus in some cases, and we provide a sufficient condition for cases where consumer surplus improves.

The third chapter addresses the design of tax instruments to reduce emissions from the aviation sector. We compare taxes on airport charges with more conventional instruments such as fuel taxes, focusing on their effectiveness in discouraging air travel. We find that, while long-haul flight passengers have a higher tax burden with fuel taxes compared to short-haul passengers, taxes on airport charges affect all types of passengers equally and are more effective at discouraging short-haul traffic, which tends to be more polluting and more substitutable (with other transport modes) compared to long-haul. In addition, we find that a tax on airport charges can be effectively used to compensate airports for losses from reduced traffic.

Contents

Chapter 1

Interoperability & Privacy: The Case of Messaging Apps

Mudit Dhakar

Jun Yan
School of Economics and Wang Yanan Institute for Studies in Economics
Xiamen University

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Abstract

This paper investigates mandated interoperability between messaging services. We consider a situation in which a high-privacy paid niche app competes with a low-privacy free app with a large installed user base. Consumers vary in privacy concerns and can multi-home across different apps. We find that niche app users may be worse off under partial privacy-preserving interoperability compared to no interoperability. Moreover, full interoperability may not necessarily promote market contestability, especially when privacy concerns significantly outweigh network benefits. We identify misalignment between the low-privacy app's incentives for interoperability and the welfare of niche app users, highlighting the importance of mandating a carefully calibrated level of interoperability. These results hold even when interoperability leads to privacy loss for users who opt into the policy.

Keywords: Interoperability, Compatibility, Multi-homing, Messaging Apps, Network Effects, Privacy, Regulation, Digital Markets Act

JEL Clasification: L15, L51, L96

1.1 Introduction

The messaging apps market features strong network effects. The utility consumers derive from using a messaging app increases with the number of people they can contact through it. These economies of scale on the consumer side make the market prone to tipping. As the incumbent enjoys the advantage of a large installed user base, it is hard for small entrants to compete for the market, even if they offer higher-quality services such as unique features or better privacy protection. For instance, privacy-focused apps like Signal and Threema together boast 50 million users as of 2022, a fraction compared to the billions on WhatsApp.

To promote contestability in markets with strong network effects, policymakers and scholars worldwide are considering mandating interoperability for big incumbent firms with other players in the market (Digital Market Act, 2022; Fletcher et al., 2021). The rationale is to enforce the incumbent apps to share their direct network effects, thereby levelling the playing field for small apps. For instance, interoperable messaging apps allow a Threema user to communicate with WhatsApp users without concern that using a smaller niche app will result in losing connection with friends who exclusively use larger platforms like WhatsApp.

However, from an economic perspective, the effects of mandating interoperability could be ambiguous for several reasons. First, some users are multi-homers in the messaging apps market. As interoperability and multi-homing are substitutes, mandating interoperability reduces the incentives of users to multi-home, which may adversely affect the viability of small apps. Just as Julia Wess, the spokesperson of Threema, argues: "If existing users of free messenger A with bad privacy practices could communicate with users of privacy-conscious paid messenger B, they will not pay money for messenger B, effectively depriving it of its only source of revenue."[1] Second, app developers will adjust their strategic decisions, such as pricing, in response to the obligation of interoperability. In the regime of interoperability, small niche apps may find it more profitable to target consumers who highly value their products rather than appealing to multi-homers. In this case, the users of niche apps can be worse off due to higher prices.

In this paper, we explore the effect of mandating interoperability on the contestability of the messaging app market and its welfare implications. To address this regulatory question, we build a simple model of two competing messaging apps, a low-privacy incumbent app W and a high-privacy niche app T. The incumbent app W is free and has a large installed user base, whereas the small niche app T has no installed user base and generates revenue through user payments. There are two groups of consumers: the captive consumers constitute the installed user base of W, i.e. they are assumed to single-home on the incumbent app; the non-captive consumers have heterogeneous privacy costs and can use either app or both apps. That is, the non-captive users are allowed to multi-home. In the baseline model, we focus on the case where the niche app T is the only strategic player that sets its own price, and interoperability is privacy-preserving, meaning that being interoperable with W does not compromise the privacy of T's single-homers. In the extensions, we allow the incumbent app W to strategically choose the level of interoperability and consider the scenario where being interoperable with the low privacy incumbent app T can partly lead to privacy loss of T's single-homers, depending on the level of interoperability.

The key aspect of our model is to consider partial interoperability[2]. The degree of interoperability will determine the benefit a user obtains, such as the ease of communication or the ability to access certain features, when they use an app i to communicate with a user on j.

We find that in the absence of interoperability, partial multi-homing occurs, where low-privacy users utilise both apps and high-privacy users stick to single-homing on the high-privacy app when the captive base is large.[3]. However, with the introduction of interoperability, multi-homing incentives diminish. At the same time, the network effects on the niche app improve. We show that when interoperability is limited to a few features and the

[1] "Europe's Digital Markets Act Takes a Hammer to Big Tech", Morgan Meaker, Wired, 2022

[2] This occurs when not all features work across messaging apps, which is relevant to policy since the Digital Markets Act mandates only basic functionalities to be interoperable.

[3] A captive base of at least two-thirds of all consumers is a sufficient condition for this result

network effect is weak, the niche app is forced to reduce its price as the prior effect dominates and the niche app's profit shrinks. On the contrary, this ends up benefiting its single-homers.

As interoperability improves, there comes a tipping point where multi-homing incentives diminish to such an extent that, rather than maintaining a very low price to retain multi-homers, raising prices and focusing solely on single-homers becomes more advantageous. In such a scenario, while the high-privacy app has a smaller market share, it can increase its price significantly. Under specific scenarios, these prices can get so high that single-homers on high-privacy apps might get a zero net surplus, contrasting with their positive net surplus in the absence of interoperability. This highlights how imperfect interoperability can have adverse effects. The rationale behind these high prices being optimal lies in the fact that the low-privacy firm has a large captive base, which makes it very hard for a smaller high-privacy firm to attract more consumers by reducing prices. The latter, therefore, focuses on attracting only very high-privacy consumers and, in turn, can charge large prices. So, if interoperability is supposed to be helpful to people who are on the niche app with access to a limited network size, then interoperability ends up doing just the very opposite by hurting them.

Further improvements in interoperability can significantly reduce the captive advantages of the incumbent app. Consequently, changes in market share become responsive to price adjustments by the high-privacy app. When the gap between privacy and the marginal network effect is narrow, the high-privacy app is incentivised to maintain competitive prices to expand its market share, resulting in strictly positive surpluses for its users. So, users can benefit from interoperability only when it approaches perfection.

However, situations can be constructed where privacy concerns outweigh network benefits to such an extent that even full interoperability may not enhance market contestability. Therefore, a meticulous examination of the trade-off between privacy loss and network gain is essential to comprehend the implications of mandated interoperability.

In Section 1.6, we extend our model to study the private incentive of the low-privacy incumbent app in choosing the interoperability level. We find that when the network effect is not too large, the choice of interoperability leads to high-privacy users on the niche app getting fully extracted. The intuition is that the low-privacy app will choose a level of interoperability such that it leads to single-homing. The high-privacy app, then, becomes sufficiently bottlenecked in attracting consumers due to the captive advantages of the competitor. It, therefore, focuses on charging a high price rather than competing for market share. This maximises market share for the dominant low-privacy app, and high-privacy users are worse off. This implies that if the welfare of high-privacy concerned consumers using niche apps matters, then the interoperability level must be mandated by the regulator. In another extension, we consider a crowdfunded high-privacy niche app instead of a profit-maximising one to model the situation with messaging apps like Signal. We show that our results on imperfection in interoperability hurting consumers of the niche app remain robust.

In Section 1.7, we consider the privacy-costing interoperability. This happens when a user of the high-privacy app opts in to interoperate with users of the low-privacy app and in the process, ends up losing a part of their privacy. This loss can occur if using interoperability features requires personal information like name, phone number, etc. Under this type of interoperability, our numerical results show that our baseline observations hold true. In addition, we find that privacy loss incurred due to opting into interoperability should be lower than the interoperability level; otherwise, no high-privacy type consumers would opt in to interoperate with low-privacy app consumers, rendering such a policy useless. This simply follows from the observation that interoperability now has a privacy cost, which needs to be smaller than the network benefit it brings for high-privacy users to opt into it. This suggests that when a mandated interoperability policy allows the low-privacy app to collect information from high-privacy users, it should also specify how much information can be shared between apps to ensure that the policy remains effective.

1.1.1 Related Literature

There is a huge body of literature that studies compatibility/standardisation in markets with direct network effects. Crémer, Rey, and Tirole 2000 extends the work by Katz and Shapiro 1985 and shows that the larger player prefers a lower quality of interconnection to the smaller one in a Cournot setting with homogeneous consumers. Farrell and Saloner 1986 shows that standardisation has an additional cost via reducing product variety. Matutes and Regibeau 1988, on the contrary, argues that it increases the range of consumers' choices. Our results are in line with these, which show that compatibility can increase prices and reduce consumer surplus. We differentiate ourselves by focusing on heterogeneous privacy-focused consumers who face a trade-off between preserving privacy by single-homing on a high-privacy platform or losing it, but increasing the network effect via multi-homing. This allows us to derive new insights on the effects of interoperability in the presence of privacy concerns.

Doganoglu and Wright 2006 look at the interaction of multi-homing and compatibility on network effects. They find that multi-homing can be a poor substitute for compatibility and can cause an increase in prices. While they consider horizontal differentiation, we show that under vertical differentiation, multi-homing has the opposite effect. It leads to much lower prices compared to when only single-homing is possible.

Bourreau and Krämer 2022 is the most closely related paper. Both of the papers are interested in the effects of imperfectly mandated interoperability when users can multi-home. They explore a price-free competition of a dominant platform with a high-quality entrant and show that mandated interoperability can impede the ability of this more efficient entrant to compete. This happens in their model due to interoperability and multi-homing being substitutes. An increasing interoperability reduces multi-homing incentives, which in turn can shrink the market share of the entrant. In this paper, we instead look at the contestability between platforms, which are differentiated in terms of their privacy levels and study how the price (of the high privacy app) will react to the changes in interoperability. We show that, as interoperability reduces multi-homing, the market share of niche apps shrinks, and it increases prices. On the other hand, interoperability also improves the network effect. The prior can dominate the latter, making consumers on niche apps worse off.

Most of the literature on interoperability has focused on the competition between services/platforms with similar business models, such as price-based, free and ad-based, free, and data-collecting.

Bourreau, Raizonville, and Thébaudin 2023 look at interoperability in ad-financed platforms and argue that interoperability can be too high or low from a welfare point of view and that mandating interoperability between asymmetric platforms (based on captivity advantages) is not always socially optimal. The key trade-off in their model is that while interoperability reduces the number of multi-homers, which are relatively less valuable to advertisers, it increases the number of single-homers, which in turn increases firms' market power over advertisers.

Samranchit 2022 models competition with two price-based services horizontally differentiated and shows that interoperability affects market structure when users single-home, i.e., whether multiple firms can co-exist and that welfare under duopoly with full interoperability can be lower than monopoly without interoperability. We add to this by showing that in vertical differentiation, consumers can be hurt by interoperability compared to no interoperability.

A major difference in our paper is that we consider a face-off between apps with different business models. One is free and potentially earns revenue from the data it collects, while the other platform is committed to high privacy and, in turn, collects minimal data. The latter can only monetise its services by directly charging for its product. The specification explicitly allows us to deal with the question of sustainability of high-privacy firms. In addition, we also allow for multi-homing to study pricing-related issues, which has seen lesser focus.

The interaction between interoperability and privacy has been explored in lesser detail. We are only aware of one paper by Rasch, Shekhar, and Wenzel 2023, which explores this theme by assuming privacy loss due to data spillover during interoperability. They argue that mandated full interoperability reduces consumer surplus if these privacy spillovers are large. We instead focus on privacy-preserving mandated interoperability and demonstrate that this policy can still harm some consumers.

1.2 Discussion on Messaging Apps and associated privacy concerns

In general, messaging apps tend to be very secure[4]. WhatsApp, for instance, offers end-to-end encryption on chats. So this means that the message content users tend to share is protected, but the issue of privacy can arise. Such apps can still learn about whom we are talking to, the kind of groups one is part of, the intensity/length of interactions, etc., and since they have collected personal information like phone number, photograph, date of birth, email IDs, etc during the sign-up process, they can generate a social graph[5]. This is a significant concern for certain users, as is evident from the observation that WhatsApp's privacy policy changed in 2021, which caused a massive user backlash and significant consumer withdrawal. [6].

We do see more secure apps like Signal and Threema. They are both privacy-focused, albeit they both remain small compared to WhatsApp [7]. Telegram is another big messaging app which claims to be privacy-focused, but it doesn't offer end-to-end encryption in group chats. By default, it doesn't enable individual chats. So, we are not sure how to rank them in terms of their privacy level.

We model this privacy as the cost of participation if the user decides to use a low-privacy product. This type of privacy modelling is known as intrinsic privacy, which originates from the inherent preferences of consumers. See Tang 2019, Lin 2022 and Prince and Wallsten 2022 for empirical evidence on intrinsic privacy.

Business Models of Messaging Apps:
Users' sensitivity to privacy concerns seems to have given rise to a rich class of business models of messaging apps. We are broadly aware of four such types: 1) Free like WhatsApp 2) Ad-based like Telegram 3) Paid like Threema 4) Crowdfunded by Signal

For services like WhatsApp, less privacy allows Meta to increase its value of advertisements on Instagram and Facebook. [8]. For example, Facebook ads have links that direct users to a business's WhatsApp account, where they can chat directly. And Facebook can identify the companies with which one wants to interact via WhatsApp's Metadata.[9]

[4]Secure Messaging Scorecard, EFF
[5]"The truth about WhatsApp's and Apple's privacy promises", Washington Post, 2023
[6]"WhatsApp loses millions of users after terms update", The Guardian, 2021
[7]In 2022, Threema had around 11 million, and Signal had around 40 million (Business of Apps)
[8]"Relaxing Privacy Vow, WhatsApp Will Share Some Data with Facebook", New York Times, 2016
[9]"Why Metadata Matters", Surveillance Self-Defense, 2019

Other messaging apps, devoid of such ecosystem benefits, seem to have adopted alternatives to make money. Additionally, high privacy-focused apps, by design, commit to not using data and hence cannot monetise it. See Bian, Ma, and Tang 2021 for empirical evidence on how higher privacy policies by the hosting platform reduce revenues of apps operating on it.

Telegram uses an ad-based model. They claim to offer privacy in the sense that their ads are not targeted.

In this paper, we will examine the contestability between a free app with a captive base and a niche, high-privacy paid app that maximises its profit. Signal, on the other hand, is a free high-privacy app, but they do have expenses that are expected to reach 50 million euros. [10] Signal crowd-sources this cost; in essence, people pay them indirectly. In our extension, we tweak our model to accommodate this by having a budget-balanced paid high-privacy app instead, which charges prices to earn revenue to match its fixed cost of operations.

1.3 Model Setup

There are two messaging apps. W is the low-privacy dominant app with an incumbency advantage due to the presence of some captive consumers, and T is the high-privacy app that lacks a captivity advantage. W is free to use, while T charges a price of p.

On the consumer side, there is a unit mass of them. Consumers are either captives or non-captives. $\phi > 0$ is the fraction of non-captive consumers, and the rest $1 - \phi > 0$ are captives on W. The captives are the consumers who have been associated with W for a long time and are simply not interested in shifting to an alternative app or even in multi-homing. These facts could be micro-founded by arguing that these consumers have high switching and high multi-homing costs, albeit we don't explicitly model these costs in the model. Hence, we assume that they are always single-homing on W . Non-captives, on the other hand, are the set of consumers actively thinking about their choice of messaging apps. We assume these consumers can multi-home and are heterogeneous on privacy cost $\alpha \sim U[0, 1]$. All consumers are assumed to have a homogeneous level of network effect $\beta > 0$.

We make assumptions about consumers to model what we see today in messaging apps without interoperability. The two assumptions about α and β allow us to focus on only one dimension of vertical differentiation (privacy) and help make the model more tractable. We assume heterogeneity in privacy as this is an essential element to explain why the high-privacy apps manage to sustain themselves despite having a minimal user base (and, in turn, a feeble network effect). The captivity setup allows us to capture that high-privacy apps (in

[10] "Running Signal Will Soon Cost $50 Million a Year", Wired, 2023

turn being of higher quality) continue to have a small user base compared to low-privacy apps.

We normalise the outside option to zero. Some examples of such options include using SMS services or calling. The additional utility brought by messaging apps over the outside option in our model can be justified because they are feature-packed, improving interaction quality.

Let N_W and N_T denote the total number of users on W and T, respectively, and N_W^s and N_T^s be their number of single homers. N_m denotes number of multi homers. Let $\tau \in [0, 1]$ be the level of interoperability between the two messaging apps. Then, the net utility of α-type consumer:

- Single-homers on W : $u_W^s \equiv \beta(N_W + \tau N_T^s) - \alpha$
- Single-homers on T: $u_T^s \equiv \beta(N_T + \tau N_W^s) - p$
- Multi-homers: $u_m \equiv \beta - \alpha - p$

There are two components to net utility. First is the network effect. Part of it comes from directly interacting with users on the app one uses. These users are single-homers of the app and multi-homers. If the consumer is a single-homer, then the other part comes from using interoperability to talk to single-homers of the other app. If a consumer is multi-homer, then they talk directly to everyone and get the full network effect (i.e. they are assumed not to be dependent on interoperability at all). We also assume that multi-homers don't make double gains from talking to the same people twice on two different apps[11].

The second part of the net utility is the cost. On W, being a low-privacy app, users are assumed to lose their privacy via incurring a cost of α. On the other hand, T is a high-privacy app, and users are assumed not to lose privacy as long as they single-home on it. However, they end up with a cost by paying the price p of the app.

Multi-homers are assumed to incur both the cost of privacy (due to participation in the low-privacy app W) and the high-privacy app T's price p. We assume that the loss of privacy of multi-homers is similar to that of single-homers. We do this since we model privacy loss as a fixed cost of participation, which does not depend on the level of interaction. So, we don't have a clear direction if we wanted to model multi-homers losing less privacy as they might share lesser info of their social graph to W since a part of their social graph lies with T.[12]

We also make a tie-breaking assumption: for a given price and other consumers' response, if users are indifferent between multi-homing and single-homing, we assume they choose

[11] This is in line with Bakos and Halaburda 2020

[12] A reduced form way is assume only a fractional loss of privacy α. While we abstract away from it, this feature can be easily incorporated into our model without affecting much of the analysis.

multi-homing. Finally, we make some assumptions on the range of parameters. Firstly, we assume $\beta < 1$. This implies that the highest level of privacy cost $\alpha = 1$ exceeds even a full network coverage advantage ($\beta * 1$). This allows the high-privacy app T to have a non-zero market share if its price is zero and, in turn, rules out cases where it cannot compete with W. This is important as we observe the sustenance of high privacy-focused messaging apps today. Secondly, we assume that $\phi < 1/3$. This means that captive consumers are at least twice as many as non-captive consumers and is in line with our motivation to capture the real-life equivalence of a huge entrenched market share of incumbent low-privacy messaging apps. The specific choice of 1/3 is determined to keep certain parts of the analysis simple.

Timing of the game:

- Stage 1: T sets its price p to maximise its profit [13]
- Stage 2: Non-captive consumers form rational expectations about network size and decide whether to single home or multi-home and on which app.

Privacy-preserving interoperability[14] - In the baseline mode, we abstract from privacy loss due to interoperability. This means every single homer on T has the same net utility. Therefore, we focus only on complete market coverage in our analysis (as incomplete market coverage can only happen when T has no consumers)

1.3.1 Discussion of modelling choices

Value of Data & Choice of Business Model

W has access to make money via data collection, while T does not. W is a dominant player and likely has alternative venues to utilise the data it has collected without requiring any sale to a third party. T is likely a niche player and does not have access to making money directly out of data. Without dominance and under no significant horizontal differentiation, its only way of competing is to offer a high-privacy product, which prevents it from collecting any data. Hence, the only way to make money is to price its product (or raise funds via donations).

W's monetisation strategy: On one hand, it can take advantage of its dominance and captive user base to start charging a positive price at any time. On the other hand, if the gains from data are large enough, it might want to increase data collection while offering a negative price as a means to subsidise privacy losses. However, because negative pricing could

[13]we assume that W is non-strategic, and so, T is the only strategic player.

[14]Privacy-preserving interoperability in messaging apps would be possible when the identity of a consumer stays hidden. For instance, a numeric key is used to identify each consumer whose real identity (think of attributes like name, phone number, etc) is only known to the hosting app. So, consumers of high-privacy apps will find it difficult to track by low-privacy apps.

potentially be exploited by opportunistic users, it ends up charging a zero price. Supporting evidence for this can be seen in WhatsApp's declining privacy standards over the years as it became more dominant. This objective can be added to the current model by allowing W to endogenously determine its privacy level.

Digital Privacy Paradox

While we assume that W or the incumbent low privacy firm has a large captive base, this assumption can be visualised in an alternative way where there are no captives but a point mass of consumers with a privacy cost of zero. This would quantitatively produce similar results and is justified by the phenomenon known as the Digital Privacy Paradox. Surveys have shown that while many people claim to value privacy, they have zero willingness to pay for it. Since our model involves a price for keeping privacy, a point mass at $\alpha = 0$ captures these people with zero willingness to pay. See Goldfarb and Que 2023 for a detailed discussion on this.

Heterogeneous network effects

In this paper, we make a strong assumption of the homogeneous network effect. While assuming heterogeneity in network effects might sound natural, we abstract away from this to keep the model tractable. With heterogeneity, we will be in a world with a strictly positive set of consumers who are single-homers on W and T and multi-homers. The mechanism of how interoperability affects one's multi-homing decision and, in turn, the pricing decision of T remains the same. Interoperability remains a substitute for multi-homing, and a higher level of it will discourage multi-homing, forcing T to focus on single-homers. So, we hypothesise that most of the results should pass through in this case.

Local network effects

We assume global network effects here. However, there might be reasons to believe that people have local network effects in the sense that they care only about a subset of users and are, hence, sensitive to where these are located. This is a very different set-up and beyond the scope of this paper.

Heterogeneous privacy interactions

People might have some interactions that demand high privacy while others do not so much. For instance, official interactions might be sensitive, while unofficial interactions (say, ones with friends and family) might not demand high privacy. In this, the story of usage of privacy-focused apps might differ, where the multi-homers will carry out privacy-demanding interactions on high-privacy apps while still talking to friends and family on low-privacy apps. This is, again, a different research dimension, and we leave it for future research.

Notion of Privacy & independence from network size
The privacy concern stems from losing social graph information. Regardless of one's level of social activity, individuals may equally value the confidentiality of their social connections. Therefore, in our context, privacy cost is assumed to be network size-independent, i.e. we assume only loss of social graph matters for privacy, but not the size of it. So, a multi-homer with a larger network compared to a single-homer will have a similar privacy loss.

1.4 Equilibrium Analysis

1.4.1 Demand Configurations

We first look at a consumer's participation decisions. Given their expectations over the total number of users of each app, N_W^e and N_T^e, their expectations over the number of single-homing users on each app and the number of multi-homing users are respectively: $N_W^{s,e} = 1 - N_T^e, N_T^{s,e} = 1 - N_W^e, N_m = N_W^e + N_T^e - 1$.

Lemma 1.1. *If it is optimal for a consumer to multi-home, that is,* $u_m \geq \max\{u_T^s, u_W^s, 0\}$*, then they must also obtain positive utilities when single-homing on either messaging app, i.e.,* $u_W^s > 0$ *and* $u_T^s > 0$.

Proof. The extra benefit a consumer enjoys by multi-homing on W in addition to T is $u_m - u_T^s = \beta(1-\tau)N_W^{s,e} - \alpha$, which stems from enhanced interactions with single-homing users on W. As it is optimal for the consumer to multi-home, we have $\beta(1-\tau)N_W^{s,e} - \alpha \geq 0$. Then, this implies that: $u_W^s = \beta(N_W^e + \tau N_{T,e}^s) - \alpha = \beta(N_W^{s,e} + N_m^e + \tau N_T^{s,e}) - \alpha > \beta(1-\tau)N_W^{s,e} - \alpha > 0$. By the same logic, we also have $u_W^s > 0$. □

This lemma implies that a consumer multi-homes only if single-homing on either messaging app generates positive utilities. Therefore, when making the participation decision, a consumer will first evaluate the single-homing decision. Then, they multi-home if the extra benefits are positive.

In the model, as consumers have a homogeneous network effect β if T ever wants to make any sales, its price must satisfy $p \leq \beta(N_T^s + \tau N_W^s)$. This suggests that it pays off, at least, for a consumer to single-home on T.

Let $\bar{\alpha}$ denote the cutoff such that $u_W^s \equiv \beta(N_W^e + \tau N_T^{s,e}) - \bar{\alpha} = 0$. According to the lemma, for consumers with $\alpha > \bar{\alpha}$ such that $u_W^s < 0$, they will just single-home on T. For consumers with $\alpha \leq \bar{\alpha}$ such that $u_W^s > 0$ and $u_T^s > 0$, they multi-home if and only if $u_m - u_W^s \geq 0$ and $u_m - u_T^s \geq 0$. Otherwise, they will just single-home on the app that delivers greater utility.

The two multi-homing conditions can be rewritten as:

$$p \leq \beta(1-\tau)N_T^{s,e}$$
$$\alpha \leq \beta(1-\tau)N_W^{s,e} \equiv \hat{\alpha}$$

$\hat{\alpha}$ is the cutoff of $u_m - u_T^s = 0$. As $u_W^s \geq u_m - u_T^s$, we have $\hat{\alpha} \leq \bar{\alpha}$.

As a result, taking T's price p as given, there exist two possible demand equilibria for non-captive consumers: partial multi-homing and single-homing. Next, we derive the equilibrium number of users in each configuration.

Configuration 1: partial multi-homing.
The demand configuration of partial multi-homing arises (Figure 1.1) when $p \leq \beta(1-\tau)N_T^{s,e}$, where $N_T^{s,e} = 1 - N_W^e$. In this case, T sets a sufficiently low price to make multi-homing on its app alongside W appealing. Hence, all non-captive consumers will purchase the service of T. More specifically, consumers with $0 \leq \alpha \leq \hat{\alpha}$ multi-home on both apps, while consumers with $\hat{\alpha} < \alpha \leq 1$ single-home on T.

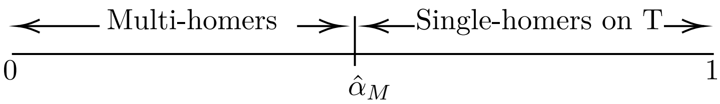

Figure 1.1: Partial multi-homing

Therefore, we have $N_W^s = 1 - \phi$, $N_T^s = (1-\hat{\alpha})\phi$, $N_m = \hat{\alpha}\phi$, where $\hat{\alpha} = \beta(1-\tau)N_w^{s,e}$.

In equilibrium, consumers have rational expectations over the apps' number of users and optimally make decisions about whether to multi-home or not. Therefore, the equilibrium conditions are:

$$\begin{cases} N_T^{s,e} = N_T^s \\ N_W^{s,e} = N_W^s \\ p \leq \beta(1-\tau)N_T^{s,e} \end{cases}$$

Solving these conditions, we obtain:

$$\begin{cases} \hat{\alpha} = \beta(1-\tau)(1-\phi) \\ p \leq \beta(1-\tau)\phi[1-\beta(1-\tau)(1-\phi)] \end{cases}$$

The upper constraint on price comes from what we call Direct Network Benefit, which is an improvement in network quality when a user switch to multi-homing from single-homing, as interoperability is imperfect. For multi-homing to be preferable over single-homing, a user must see the price equal or lower to the direct network benefit. In sum, when $p \leq \beta\phi(1-\tau)[1-\beta(1-\tau)(1-\phi)] \equiv \beta\phi(1-\tau)(1-\hat{\alpha})$, there exists a partial multi-homing demand equilibrium, in which the total number of users on T is:

$$N_T = \phi$$

Configuration 2: prefect single-homing.

The demand configuration of perfect single homing arises (Figure 1.2) when $\beta(1-\tau)N_T^{s,e} < p \leq \beta(N_T^e + \tau N_W^{s,e})$. In this case, T only appeals to users who value privacy a lot, such as those willing to single-home on T. In other words, it doesn't pay off for any consumer to use T alongside W. Let $\tilde{\alpha}$ denote the cutoff such that $u_W^s = u_T^s$, i.e. $\tilde{\alpha} = \beta(1-\tau)(N_W^{s,e} - N_T^{s,e}) + p$. Therefore, consumers with $\alpha \leq \tilde{\alpha}$ single-home on W, while consumers with $\alpha \geq \tilde{\alpha}$ single home on T.

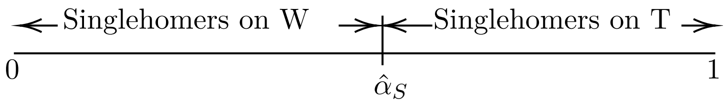

Figure 1.2: Perfect single-homing

Hence $N_W^s = 1 - \phi + \tilde{\alpha}\phi, N_T^s = (1-\tilde{\alpha})\phi, N_m = 0$.

Solving the equilibrium conditions :

$$\begin{cases} N_T^{s,e} = N_T^s \\ N_W^{s,e} = N_W^s \\ \beta(1-\tau)N_T^{s,e} < p \leq \beta(N_T^e + \tau N_W^{s,e}) \end{cases}$$

We obtain:

$$\begin{cases} \tilde{\alpha} = \dfrac{\beta(1-\tau)(1-2\phi)}{1-2\beta(1-\tau)\phi} + \dfrac{p}{1-2\beta(1-\tau)\phi} \equiv \tilde{\alpha}(p) \\ \\ \dfrac{\beta(1-\tau)\phi(1-\beta+\beta\tau)}{1-\beta(1-\tau)\phi} < p \leq \dfrac{\beta\tau + \beta(1-\tau)\phi[1-\beta(1+\tau)]}{1-\beta(1-\tau)\phi} \end{cases}$$

We show in the appendix that in the equilibrium, we must have $0 < \tilde{\alpha} < 1$.

In sum, when $\frac{\beta\phi(1-\tau)(1-\beta(1-\tau))}{1-\beta(1-\tau)\phi} < p \leq \frac{\beta\tau+\beta(1-\tau)\phi[1-\beta(1+\tau)]}{1-\beta(1-\tau)\phi}$, there exists a perfect single-homing demand equilibrium, in which the total number of users on T is:

$$N_T = \Big(\frac{1-\beta(1-\tau)-p}{1-2\beta(1-\tau)\phi}\Big)\phi$$

As a result, when $\frac{\beta(1-\tau)\phi(1-\beta(1-\tau))}{1-\beta(1-\tau)\phi} < p \leq \beta(1-\tau)\phi[1-\beta(1-\tau)(1-\phi)]$, both demand equilibria exist [15].

Next, we use two alternative selection criteria to solve for the full equilibrium. The first criterion assumes that consumers believe that each one of them will choose multi-homing whenever possible. It favours entrants (as it has a higher market share) and also favours higher levels of interaction between non-captive consumers. So it can be interpreted as an equilibrium with *highest aggregate network effects within non-captive consumers.*

The second selection criterion assumes that consumers believe each one of them will choose single-homing. This can be thought of as a loose interpretation of *focal equilibrium* (Halaburda, Jullien, and Yehezkel 2020). Under focal equilibrium, when multiple equilibria are possible, all consumers move to the focal platform (In literature, the incumbent or dominant platform is often assumed to be the focal one.). In our case, since all consumers cannot move to one platform due to heterogeneity among them, we interpret focality as consumers choosing the equilibrium that favours the focal platform by maximising its market share. This equilibrium can also be considered the worst-case equilibrium for firm T. We will see later that this results in a lower profit than the first type.

If firm T has the ability to somehow steer consumers to its desired equilibrium, then the selection of the first type of equilibrium makes sense. Alternatively, if firm T doesn't have this ability and if the firm is ambiguity averse, then it might want to consider its worst-case equilibrium. This justifies the second selection rule.

We characterise the equilibrium for the first selection rule in the main text. The qualitative results derived for the first selection stay robust to the second selection and are discussed in the appendix 1.C.

1.4.2 The app T's pricing decision

We start with the first equilibrium selection rule, which states that consumers believe each other will choose multi-homing whenever possible. Therefore, the demand for T is

$$N_T = \begin{cases} \phi, & p \leq \beta(1-\tau)[1-\beta(1-\tau)(1-\phi)]\phi \\ \left(\frac{1-\beta(1-\tau)-p}{1-2\beta(1-\tau)\phi}\right)\phi & \text{otherwise} \end{cases}$$

We split the further analysis into partial multi-homing and single-homing demand configurations.

[15] The inequality is established as

$$\frac{\beta\phi(1-\tau)(1-\beta(1-\tau))}{1-\beta\phi(1-\tau)} < \beta\phi(1-\tau)(1-\beta(1-\tau)) < \beta(1-\tau)[1-\beta(1-\tau)(1-\phi)]\phi$$

as $0 < \phi < 1/3$

Profit under configuration 1 - Partial multi-homing

If T wants to appeal to multi-homers to reach full market coverage, then its profit is given by

$$\pi^{configM} = \max_p \phi p \quad \text{with} \quad p \leq \beta(1-\tau)[1-\beta(1-\tau)(1-\phi)]\phi$$

So, T's optimal price is to set the highest possible price which meets the constraint, i.e. the constraint binds and $p^{configM} = \beta(1-\tau)\phi(1-\beta(1-\tau)(1-\phi))$
The profit is given by :

$$\pi^{configM} = \underbrace{\beta(1-\tau)\phi}_{\text{Effect 2 (-)}} \phi \underbrace{(1-\beta(1-\tau)(1-\phi))}_{\text{Effect 1 (+)}}$$

Observe that the profit expression is concave in τ, the interoperability level. This happens because there are two counter-effects. Firstly, as the level of interoperability improves, T's captivity disadvantage shrinks, making it a more attractive option to consumers. The larger the network effect β, the higher the rate of increase of this effect. This increases T's partial multi-homing profit. The second effect is the reduction in multi-homing benefits, which arises as interoperability improves. This means to incentivise multi-homing, T's price needs to be reduced. Hence, the second effect runs in the opposite direction, reducing profit. Which effect dominates determines the evolution of profits as interoperability changes and is also affected by the values of β and τ. We provide more details later in the equilibrium profit section.

Profit under configuration 2 - Single-homing

If T wants to only appeal to single-homers, then it needs to set a price

$$p > \beta(1-\tau)[1-\beta(1-\tau)(1-\phi)]\phi$$

So, the profit function of T under this configuration is given by

$$\pi^{configS} = \max_p \phi\Big(\frac{1-\beta(1-\tau)-p}{1-2\beta(1-\tau)\phi}\Big)p$$
$$\text{s.t. } \beta(1-\tau)[1-\beta(1-\tau)(1-\phi)]\phi < p \leq \frac{\beta\tau+\beta\phi(1-\tau)[1-\beta(1+\tau)]}{1-\beta\phi(1-\tau)}$$

We can, however, ignore the lower bound and solve the relaxed profit maximisation problem of the single-homing configuration. The reason is that if optimal prices from the relaxed problem fall below this lower bound, then in equilibrium, T's response is to set the optimal price as per configuration 1 (as it yields higher prices with a higher market share). Such prices are, therefore, off-equilibrium quantities and can be ignored.

Now, to solve this relaxed maximisation problem, we solve the unconstrained program and then check if the optimal price violates the upper constraint or not. If the constraint is violated, then it must bind, and that sets the optimal price. Define the upper bound on p as

$$p_{max}^{configS} \equiv \frac{\beta\tau + \beta\phi(1-\tau)[1-\beta(1+\tau)]}{1-\beta\phi(1-\tau)}$$

Unconstrained maximisation :

$$\pi_{uncons}^{configS} = \max_p \phi(\frac{1-\beta(1-\tau)-p}{1-2\beta(1-\tau)\phi})p$$

This yields a price :

$$p_{uncons}^{configS} = \frac{1-\beta(1-\tau)}{2}$$

By comparing $p_{uncons}^{configS}$ and $p_{max}^{configS}$, we can establish the range of parameters where the upper constraint bound. We now establish the optimal prices in this configuration

Lemma 1.2. *The prices under the single-homing configuration are:*

- *If* $\beta \leq \frac{1}{2+\phi}$ *, then optimal* $p^{configS} = p_{max}^{configS} = \frac{\beta(\tau+(1-\tau)\phi-(1-\tau^2)\beta\phi)}{1-\beta(1-\tau)\phi}$
- *If* $\beta > \frac{1}{2+\phi}$*, then* $\exists \tau_{cons-uncons}^{configS} = \frac{-(1-3\phi+2\beta\phi)+\sqrt{1+9\phi^2+16\beta^2\phi^2-2\phi-24\beta\phi^2}}{2\beta\phi}$ *such that* $0 < \tau_{cons-uncons}^{configS} < 1$ *and optimal price p is given by*

$$p^{configS} = \begin{cases} p_{max}^{configS} = \frac{\beta(\tau+(1-\tau)\phi-(1-\tau^2)\beta\phi)}{1-\beta(1-\tau)\phi} & \tau \leq \tau_{cons-uncons}^{configS} \\ p_{uncons}^{configS} = \frac{1-\beta(1-\tau)}{2} & otherwise \end{cases}$$

The lemma shows that, under single-homing, for low beta ranges, firm T prices its product high enough that its users' net utility is zero. It's equivalent to users having the choice of using firm W's product and giving up on their privacy vs. being out of the market (effectively, T brings on user benefits).

However, for intermediate and large values of β, there is a cut-off level of interoperability, after which the upper constraint of price equals gross utility is slack, and users of T get a strictly positive surplus.

The intuition for this is that in the low range of β, the products of T and W are sufficiently differentiated. T's main offering of high-privacy products carries much value, while W offers large network effects (which will shrink with rising interoperability). This reduces incentives for T to reduce its price to make its product more attractive to consumers. As β rises, the "competition" like situation arises where the privacy benefits T can offer are no longer very large compared to the network effect. T has stronger incentives to reduce its price to gain a larger market share. For low ranges of interoperability, the problem always remains constrained, and that is because of the large captivity advantage of W, which is that any price reduction by T results in a minimal gain in market share. At a higher level of interoperability, the gap between the network effects of the two platforms shrinks, which causes a reduction in the differentiation of products (the competitive edge of W erodes), which, in turn, causes T to consider reducing its price.

For these given prices, it is straightforward to get profit expressions.
Under $p = p_{max}^{configS}$, we have $\tilde{\alpha} = \tilde{\alpha}_{max} = \dfrac{\beta - \beta(1-\tau)\phi}{1-\beta(1-\tau)\phi}$ and $\pi_{cons}^{configS} = \phi(1-\tilde{\alpha}_{max})p_{max}^{configS}$

Under $p = p_{uncons}^{configS}$, we have $\tilde{\alpha} = \dfrac{p + \beta(1-\tau)(1-2\phi)}{1-2\beta(1-\tau)\phi}$ and
$\pi_{uncons}^{configS} = \phi(\dfrac{1-\beta(1-\tau) - p_{uncons}^{configS}}{1-2\beta(1-\tau)\phi})p_{uncons}^{configS}$

Corollary 1.1. *The profits under the single-homing configuration are :*

$$\pi^{configS} = \begin{cases} \pi_{cons}^{configS} = \frac{\beta(1-\beta)\phi(\tau+(1-\tau)\phi-(1-\tau^2)\beta\phi)}{(1-\beta(1-\tau)\phi)^2} & \textit{if either } (\beta \leq \frac{1}{2+\phi}) \textit{ or } (\beta > \frac{1}{2+\phi} \ \& \ \tau \leq \tau_{cons-uncons}^{configS}) \\ \pi_{uncons}^{configS} = \frac{\phi(1-\beta(1-\tau))^2}{4(1-2\beta(1-\tau)\phi)} & \textit{otherwise} \end{cases}$$

Equilibrium profits of T

Since we have solved for profits under two configurations, we evaluate which is the maximum and will be the equilibrium profit of T.
We have,

$$\pi_T = \begin{cases} max\{\pi^{configM}, \pi_{cons}^{configS}\} & \text{if either } (\beta \leq \frac{1}{2+\phi}) \text{ or } (\beta > \frac{1}{2+\phi} \ \& \ \tau \leq \tau_{cons-uncons}^{configS}) \\ max\{\pi^{configM}, \pi_{uncons}^{configS}\} & \text{otherwise} \end{cases}$$

We first establish that without interoperability, multi-homing is always the preferred outcome.

Lemma 1.3. *In equilibrium under selection 1, T chooses configuration 1 at $\tau = 0$, i.e., partial multi-homing always occurs in the absence of interoperability.*

This captures the spirit of privacy-focused messaging apps in the present scenario, where there is no interoperability: they believe that many of their users are multi-homers. (See Threema's spokesperson's statement).

Now, we move to cases when there is a strictly positive level of interoperability. Comparing profits across the two configurations, we get the following lemmas :

Lemma 1.4. *$\exists$ a unique $\tau_{cons}^{configM-S}$, with $0 < \tau_{cons}^{configM-S} < 1$, such that*

$$\begin{cases} \pi^{configM} \geq \pi_{cons}^{configS} & \text{if } \tau \leq \tau_{cons}^{configM-S} \\ \pi^{configM} < \pi_{cons}^{configS} & \text{otherwise} \end{cases}$$

Lemma 1.5. *$\exists$ a unique $\tau_{uncons}^{configM-S}$, with $0 \leq \tau_{uncons}^{configM-S} < 1$ such that*

- $\pi^{configM} \leq \pi_{uncons}^{configS}\ \forall\tau$ *if* $\tau_{uncons}^{configM-S} = 0$
- *for* $\tau_{uncons}^{configM-S} > 0$

$$\begin{cases} \pi^{configM} \geq \pi_{uncons}^{configS} & \text{if } \tau \leq \tau_{uncons}^{configM-S} \\ \pi^{configM} < \pi_{uncons}^{configS} & \text{otherwise} \end{cases}$$

So, we have established three relevant thresholds, i.e., $\tau_{cons-uncons}^{configS}$, $\tau_{cons}^{configM-S}$ and $\tau_{uncons}^{configM-S}$. Since we cannot explicitly solve for the last two, we will describe their properties instead. By definition, we know that $\pi_{uncons}^{configS} \geq \pi_{cons}^{configS}$, so $\tau_{uncons}^{configM-S} \leq \tau_{cons}^{configM-S}$

Lemma 1.6. *We can never have $\tau_{uncons}^{configM-S} < \tau_{cons-uncons}^{configS} < \tau_{cons}^{configM-S}$*

So either $\tau_{cons-uncons}^{configS} < \tau_{uncons}^{configM-S}$ or $\tau_{cons}^{configM-S} < \tau_{cons-uncons}^{configS}$

Define

$$\tau^* \equiv \begin{cases} \tau_{uncons}^{configM-S} & \text{if } \tau_{cons-uncons}^{configS} < \tau_{uncons}^{configM-S} \\ \tau_{cons}^{configM-S} & \text{if } \tau_{cons}^{configM-S} < \tau_{cons-uncons}^{configS} \end{cases}$$

τ^* is one of the two thresholds where the profits from configuration 2 start exceeding configuration 1. The choice among these two thresholds is decided by whether we have an unconstrained or relaxed problem of configuration 2 when it starts to yield higher profits than configuration 1. Therefore, for interoperability levels below (above) τ^*, configuration 1 (2) arises.

Proposition 1.1. *In equilibrium under selection 1, for $\tau < \tau^*$, configuration 1 arises, i.e. there is partial multi-homing, while for $\tau > \tau^*$, configuration 2 arises, i.e. there are only single-homers.*

This proposition extends the results in Lemma 1.3. We have partial multi-homing for zero and lower levels of interoperability. Since we know that interoperability and multi-homing are substitutes, increasing interoperability will affect multi-homing. This is exactly what we observe here. At high levels, the price for keeping multi-homers around is too low (Recall that the price of T in the case of multi-homing is the additional benefit T offers to multi-homers over single-homing with interoperability, and this shrinks with better interoperability). T finds it more optimal to switch to single-homers only. It reduces T's market share but allows a much larger price gain.

Finally, we can characterise the profits. Above τ^*, we are in the partial multi-homing configuration, but we can have either a constrained or an unconstrained problem. This can be solved by a simple observation - if $\tau^* < \tau^{configS}_{cons-uncons}$, then configuration 2's problem is constrained when it starts to yield higher profits than configuration 1. So we move from configuration 1's profit to configuration 2's constrained profit as τ increases. Eventually, for $\tau > \tau^{configS}_{cons-uncons}$, the problem becomes unconstrained, and we have configuration 2's unconstrained profits. However, $\tau^* > \tau^{configS}_{cons-uncons}$, then configuration 2's problem is already unconstrained when it starts to exceed configuration 1's profit, and so we shift from configuration 1's profit to configuration 2's unconstrained profit directly.

Proposition 1.2. *In equilibrium under selection 1,*

When $\beta \leq \frac{1}{2+\phi}$

$$\pi_T = \begin{cases} \pi^{configM} & for\ \tau \leq \tau^* = \tau^{configM-S}_{cons} \\ \pi^{configS}_{cons} & otherwise \end{cases}$$

When $\beta > \frac{1}{2+\phi}$

- *When* $\tau^* = \tau^{configM-S}_{cons}$ *(i.e.,* $\tau^* < \tau^{configS}_{cons-uncons}$*)*

$$\pi_T = \begin{cases} \pi^{configM} & for\ \tau \leq \tau^* \\ \pi^{configS}_{cons} & for\ \tau^{configS}_{cons-uncons} > \tau > \tau^* \\ \pi^{configS}_{uncons} & otherwise \end{cases}$$

- *When* $\tau^* = \tau^{configM-S}_{uncons}$ *(i.e.,* $\tau^* > \tau^{configS}_{cons-uncons}$*)*

$$\pi_T = \begin{cases} \pi^{configM} & for\ \tau \leq \tau^* \\ \pi^{configS}_{uncons} & otherwise \end{cases}$$

So, as interoperability increases, the equilibrium shifts from a multi-homing set-up to a niche market set-up, where the network effects play an increasingly weaker role later. The choice for consumers increasingly looks like choosing between a high-privacy premium app and a low-privacy free app. Very high levels of interoperability are similar to competition between high and low-quality products when consumers are vertically differentiated. Another similar setting would be ad-based free apps and ad-free premium apps, where consumers are differentiated on their ad nuisance costs.

We provide a discussion on changes in the profit functions as interoperability changes in the appendix 1.A. Recall the discussion of concavity in τ of the partial multi-homing profit. When β is low, the effect 1 is weak compared to effect 2, giving us the following results :

Corollary 1.2. $\beta(1-\phi) < 0.5$ *is a sufficient condition under which*

$$\frac{d\pi_T}{d\tau} < 0 \quad \textit{for } \tau < \tau^*$$

When $\tau < \tau^*$, partial multi-homing occurs and under weak network effect (β), profits of T are always hurt by increasing levels of interoperability.

However, when $\tau > \tau^*$, single-homing configuration occurs and in this case, profits of T always increase with increasing levels of interoperability.

1.5 Consumer Welfare

We focus on the net surplus of single-homers on T as these consumers are using niche platforms, and any interoperability setting in messaging apps will likely give these consumers access to talk to far more consumers. Recall that the net utility of a single-homer on T is $\beta\tau + \beta(1-\tau)(N_T^s + N_m) - p$; every single-homer on T has the same net utility.

When the multi-homing occurs, the price of T is set at the extra quality it offers to these multi-homers over single-homing with interoperability. This implies that single-homers on T make a strictly positive net surplus since their surplus on T comes from interaction between all multi-homers and single-homers. That is, with $p^{configM} = \beta(1-\tau)\phi(1-\beta(1-\tau)(1-\phi))$ and network effect of a single-homer on T = reaching captives of W via interoperability plus reaching all users on T (all non-captives) directly= $\beta\tau + \beta(1-\tau)\phi$. We have,

$$u_T^{s,configM} = \beta[\tau + \beta(1-\tau)^2(1-\phi)\phi]$$

When single-homing occurs, the net utility of these single-homers depends on whether the τ is below or above $\tau_{cons-uncons}^{configS}$. If the τ is below the threshold, it implies that the price on T is set to gross surplus on T, and therefore, users of T are fully extracted.

$$u_T^{s,configS-cons} = 0$$

When τ is above the threshold, the price is strictly below the gross surplus of T and users on T can expect a strictly net positive surplus. With $p_{uncons}^{configS} = \frac{1-\beta(1-\tau)}{2}$ and the network effect of a single-homer on T = reaching users of W via interoperability plus users on T directly= $\beta\tau + \beta(1-\tau)\phi(1-\tilde{\alpha}) = \beta\tau + \beta(1-\tau)\phi\Big(\frac{1-\beta(1-\tau)}{2(1-2\beta(1-\tau))\phi}\Big)$. We have,

$$u_T^{s,configS-uncons} = \beta\tau + \Big(\frac{1-\beta(1-\tau)}{2}\Big)\Big(\frac{3\beta(1-\tau)\phi - 1}{1-2\beta(1-\tau)\phi}\Big)$$

Combining the above expressions with proposition 1.2, we get

Lemma 1.7. *In equilibrium under selection 1, the net surplus of single-homers on T is given by*

When $\beta \leq \frac{1}{2+\phi}$

$$u_T^s = \begin{cases} u_T^{s,configM} & \text{for } \tau \leq \tau^* = \tau_{cons}^{configM-S} \\ 0 & \text{otherwise} \end{cases}$$

When $\beta > \frac{1}{2+\phi}$

- *When* $\tau^* = \tau_{cons}^{configM-S}$ *(i.e.,* $\tau^* < \tau_{cons-uncons}^{configS}$*)*

$$u_T^s = \begin{cases} u_T^{s,configM} & \text{for } \tau \leq \tau^* \\ 0 & \text{for } \tau_{cons-uncons}^{configS} > \tau > \tau^*, \pi = u_T^{s,configS-cons} \\ u_T^{s,configS-uncons} & \text{otherwise} \end{cases}$$

- *When* $\tau^* = \tau_{uncons}^{configM-S}$ *(i.e.,* $\tau^* > \tau_{cons-uncons}^{configS}$*)*

$$u_T^s = \begin{cases} u_T^{s,configM} & \text{for } \tau \leq \tau^* \\ u_T^{s,configS-uncons} & \text{otherwise} \end{cases}$$

Fixing β and ϕ, we report these values for different values of τ in the following proposition:

Proposition 1.3. *The net surplus of single-homing users on T varies non-linearly with interoperability.*
It is a strictly positive value at no interoperability and increases till τ^* *and sees an abrupt fall at* τ^**. It can fall all the way to zero if* $\tau^* < \tau_{cons-uncons}^{configS}$*. It starts to rise again after* $\tau > max\{\tau_{cons-uncons}^{configS}, \tau^*\}$

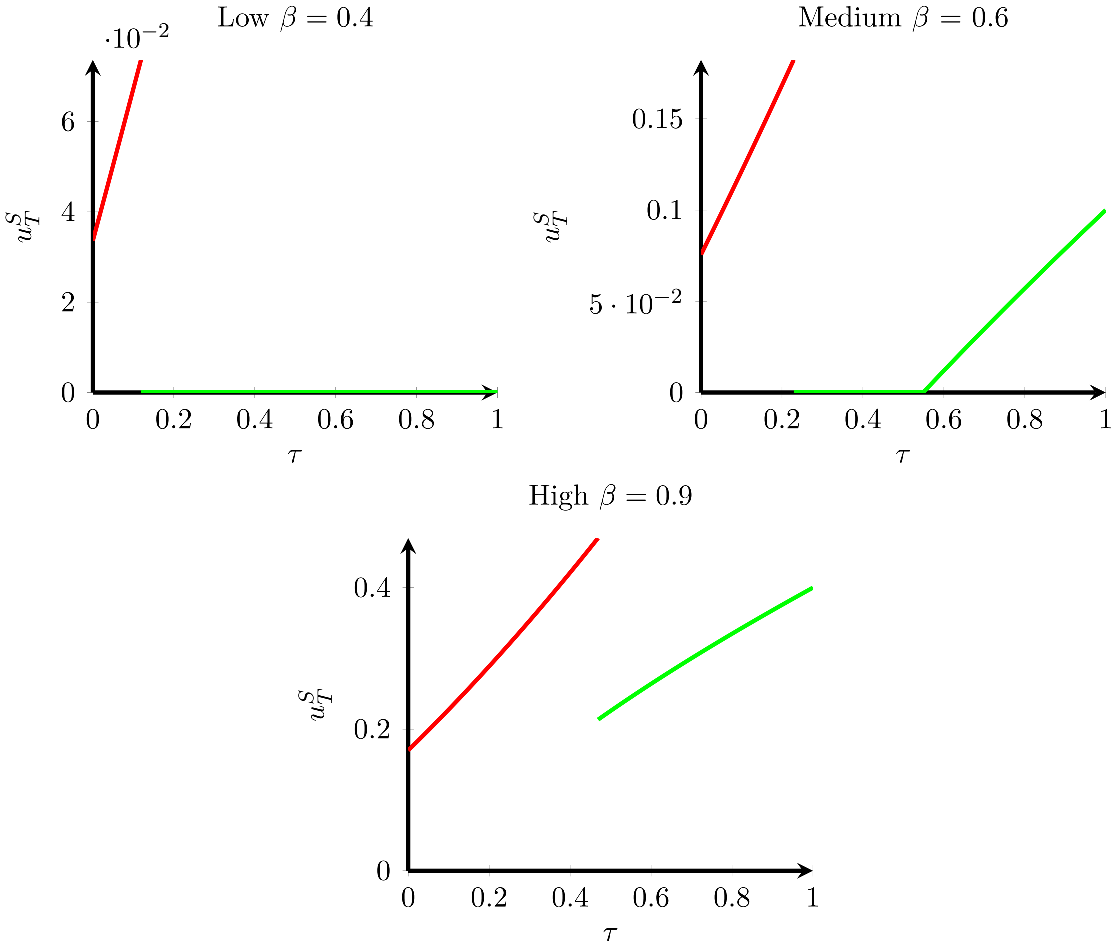

Figure 1.3: This figure demonstrates lemma 1.7 and proposition 1.3. The red lines correspond to partial multi-homing configuration and the green lines correspond to single-homing. Fixing non-captive share $\phi = 0.3$, three figures correspond to low, medium and high values of network effect (β). The x-axis represents the level of interoperability and the y-axis denotes the net surplus of a single-homer on T

Figure (1.3) demonstrates this proposition. Interestingly, we can observe that for the values for which we do plots, we can find cases where high-privacy users of the niche app are strictly better at the highest imperfect level of interoperability, which allows for multi-homing, compared to full interoperability.

1.6 Extensions

1.6.1 Strategic W which maximises its market share

So far, we have focused on only one strategic player, where the high-privacy firm T maximises its profit. We extend this model by making the low-privacy firm W also strategic. We do this by assuming that firm W's objective is to maximise its market share over the set of possible interoperability levels. This choice is inspired by the fact that the low-privacy app can use the information on one's social graph to monetise it elsewhere, for example, by improving targeted advertisement, etc and would therefore like to attract as many people as possible to its platform. In addition, this allows us to study the effect of W's incentives on the consumers of the smaller app.

The timing of the game stays as is in the benchmark with the addition of stage 0 where firm W sets its level of $\tau \in [0,1]$ which it will offer to the users of high-privacy app T.

Proposition 1.4. *When $\beta \leq \frac{1}{2+\phi}$ or $\tau^* = \tau_{cons}^{configM-S}$, W's optimal choice of $\tau = \tau_{cons-uncons}^{configS}$, otherwise W chooses $\tau = \tau_{uncons}^{configM-S}$*

The intuition is that compared to the multi-homing configuration, the single-homing configuration has worse network effects which firm T can offer. Hence, T can attract fewer consumers, increasing the number of consumers on W. Within the single-homing configuration, the optimal choice of τ depends on whether there are some values of τ, where firm T fully extracts its consumers or not. If there are such values of τ then firm W will find it optimal to set the level of interoperability at the highest τ value where T will still fully extract the consumer. If not, W's optimal choice is the lowest value of τ which generates a single-homing configuration.

The intuition is that within the single-homing configuration, if T offers a net utility of zero, the market share of W increases with τ as it offers improved network benefits. However, as soon as T starts offering strictly positive net utility, it does so as it wants to offer a cheaper price to compete with W on market share. This competition effect becomes stronger as τ improves, as T's network gains from interoperability are much higher than W due to the presence of W's large captive base. Therefore, W's market share starts to shrink as τ increases.

In the prior case, single-homers on T make zero net utility and are worse off than with no interoperability. In the latter, these consumers may or may not be better off than no interoperability, but still see one of the lowest values of net utilities among the values possible of the entire range of interoperability.

Corollary 1.3. *When $\beta \leq \frac{1}{2+\phi}$ or $\tau^* = \tau_{cons}^{configM-S}$, W's optimal choice of interoperability level makes single-homing users on T worse off than no interoperability at all.*

So firm W's incentives are misaligned with the interests of high privacy consumers, which single-home on firm T. This calls for mandating interoperability level to be essential if an interoperability as a policy is to benefit users in the niche app.

1.6.2 "Free" high privacy niche app

In this extension, we model a free high-privacy niche app (instead of a free one in the baseline) to analyse the situation we see with apps like Signal while keeping the low-privacy app with the captive base as is.

While apps like Signal have no participation fee, they have significant expenses to cover their operational costs. Signal, for instance, has a running cost of around 50 million US dollars (See section 1.2) and crowdfunds its operations. We tweak our benchmark model where we model this as a zero-profit or budget-balanced firm which charges a price such that its revenue can pay for its fixed cost.

There is a fixed cost of operation F. We assume that $F < \beta\phi^2(1 - \beta(1 - \phi))$ to ensure that there is partial multi-homing in case of no interoperability to match our benchmark set-up.

For partial multi-homing to occur, since the market share of firm T is ϕ, it will charge a price of $\frac{F}{\phi}$. The users will find it beneficial to multi-home as long as the prices are below the direct communication benefit, which comes from multi-homing over single-homing. As direct communication benefit shrinks with increasing levels of interoperability, this price will become too high, and users will start to only single-home. However, in this case, the market share of firm T is strictly lower now as only a fraction of non-captives are single-homing on it. So, in order to meet its cost of operations, it now raises the price to $\frac{F}{\phi N_T^s}$. The single-homers on T now face a higher price and lower network quality due to the loss of multi-homers on T, making them worse off. However, as the interoperability improves further, the improvement in network quality in T's product is larger than W due to W's large captive base becoming more accessible on T. This in turn also increases the incentives to single-home on T. So market share of T expands bring prices down while the quality of product improves as well. This makes single-homers on T better off.

Proposition 1.5. *In the presence of a zero-profit high-privacy niche app, $\exists\ \tau^{**} < 1$, such that $\tau < \tau^{**}$, partial multi-homing occurs with price $p = F/\phi$ and for $\tau \geq \tau^{**}$, single-homing occurs with price $p > F/\phi$.*

So we get a similar story as in the benchmark, where high-privacy users on the niche app are worse in the intermediate levels of interoperability.

1.7 Privacy loss due to Interoperability

Now, we relax the assumption of privacy-preserving interoperability. We assume that participating in interoperability can lead to a partial loss of privacy. An example could be if interoperating from a high-privacy platform to a low one results in the sharing of personal details to a low-privacy platform, then a low-privacy platform can essentially build a social graph, leading to a loss in privacy.

We assume the loss to be partial, denoted by $\nu \in (0,1)$. This means that a single-homer of type α on the high-privacy app loses $\nu\alpha$ level of privacy when they interoperate with the low-privacy app.

We think that the relevant privacy loss is due to the loss of the social graph in our case. Therefore, this loss will simply depend on the participation decision of whether to use interoperability or not. So, we assume that ν is independent of the level of interoperability τ.

Single-homers on T no longer get equal net utility from interoperability as in the benchmark. For users with very high privacy costs, it might be optimal to never participate in interoperability due to associated privacy loss. We, therefore, allow users to choose whether they would opt-in or opt-out of interoperability. This setting is in line with the Digital Market Act, which says that the decision to use interoperable features, if any, rests with the users.

Observe that for any non-captive user of W, it is weakly dominant for them to opt-in to the interoperability as they have already lost their privacy. So we have four situations:

We make an additional assumption to keep the model tractable that the outside options yield a large negative utility for users, such that in equilibrium, they end up using at least one of the two apps. For a given β & ϕ, we assume $u_{\text{outside option}} < -1 + (1-\phi)\beta$ which ensures that in equilibrium, everyone will participate in at least one of the apps (as $\min_{\tau,\alpha} u_W^s > 0$ [16])

- Single-homers on T with opt-in: $u_T^{s,opt-in} \equiv \beta(N_T + \tau N_W^s) - \nu\alpha - p$
- Single-homers on T with opt-out : $u_T^{s,opt-out} \equiv \beta N_T - p$
- Single-homers on W (always opt-in): $u_W^s \equiv \beta(N_W + \tau N_T^{s,opt-in}) - \alpha$
- Multi-homers: $u_m \equiv \beta - \alpha - p$

where $N_T^{s,opt-in}$ stands for the number of single-homers on T opt-in to interoperability.

Comment on why we don't assume ν level of loss of privacy for multi-homers. :

[16] The least possible net utility of a single-homer on W is made by the highest privacy type $\alpha = 1$ who will get at least $\beta(1-\phi)$ from networking with captives on W but pay the privacy cost of 1.

Multi-homing is different from interoperability. The former involves signing up for both services, implying that you share more of your information with the low-privacy app. A signing-up process on an app instead might involve sharing other things like email IDs, date of birth, location, one's photograph, etc. In the latter case, single-homers only sign up with one app. People who only signed up with the high-privacy app will unlikely to be required to reveal all their info to the low-privacy app during interoperability, as it involves sharing of limited info like an identifier (say a phone number or a numeric ID, and may or may not involve name sharing). Hence, we assume that interoperability results in less information sharing and, in turn, a strictly lower loss of privacy compared with full information sharing when multi-homing.

Timing of the game:

- Stage 1: T sets its price p to maximise its profit
- Stage 2: Non-captive consumers form rational expectations about network size and decide among the four choices of Single-home on T with opt-in, Single-home on T with opt-out, Single-home on W with opt-in or Multi-home.

Observe that $\nu = 0$ is the baseline/benchmark case. Additionally, $\tau = 0$ involves no privacy spillovers and, therefore, the multi-homing configuration must occur as in the benchmark. So, we focus on $\nu > 0$ and $\tau > 1$ for the rest of the discussion.

Lemma 1.8. *When $\nu = 1$, any $\tau < 1$ is an ineffective interoperability policy. The equilibrium is equivalent to that of the $\tau = 0$ setting.*

Under this setting, it's a strictly dominant strategy to multi-home rather than single-home on T while opting into interoperability. This happens as the privacy loss cost, as well as the price of T associated with both multi-homing and single-homing on T, are the same, the multi-homing results in a strictly higher level of network effect, as interoperability is imperfect. This implies that at $\nu = 1$, we have a special case, where there will be zero opt-in by single-homers of T. (i.e. no single-homer on T uses interoperability), making the interoperability policy of any interoperable level ineffective. Ensuring effectiveness thus requires $\nu < 1$.

So we restrict our attention to $0 < \nu < 1$ and $\tau > 0$ cases and assume the equilibrium selection 1 for the rest of the analysis. We first fix the price p and solve for stage 2.

1.7.1 Demand Configurations

As in the benchmark case, there are two possible configurations: Partial multi-homing and single-homing.

Configuration 1: Partial multi-homing analysis

The split of non-captive users is depicted in Figure 1.4. Unlike in the baseline case, very high privacy-concerned consumers may now find it optimal to opt out of interoperability.

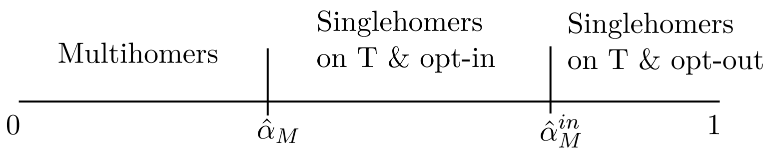

Figure 1.4: Partial multi-homing under privacy costing interoperability

For the given price p, we can get market shares by solving for $\underline{\hat{\alpha}}$ and $\hat{\hat{\alpha}}$

People who will multi-home: $u_m \geq max\{u_T^{s,opt-in}, u_T^{s,opt-out}\} \implies \alpha \leq min\{\frac{\beta(1-\tau)N_W^s}{1-\nu}, \beta N_W^s\}$

Solving for $u_T^{s,opt-out} \geq u_T^{s,opt-in}$, we get $\alpha \geq \frac{\beta\tau N_W^s}{\nu}$

Finally, $N_W^s = 1 - \phi$ as all non-captives on W are multi-homers and so only single-homers on W are the captives.

It's easily deduced that

$$\underline{\hat{\alpha}} = \begin{cases} \beta(1-\phi) & \tau \leq \nu \\ \frac{\beta(1-\tau)(1-\phi)}{1-\nu} & \tau > \nu \end{cases}$$

Observe that $\underline{\hat{\alpha}} < 1$

For $\hat{\hat{\alpha}}$ observe that if solving $u_m = u_T^{s,opt-in}$ gives $\alpha > \frac{\beta\tau N_W^s}{\nu}$, then no single-homer on T will choose the opt-in option and $\hat{\hat{\alpha}} = \underline{\hat{\alpha}}$. We also need to ensure that $\hat{\hat{\alpha}} \leq 1$ which means when $\frac{\beta\tau N_W^s}{\nu} > 1$, $\hat{\hat{\alpha}} = 1$, i.e., no single-homer on T chooses the opt-out option.

$$\hat{\hat{\alpha}} = \begin{cases} \underline{\hat{\alpha}} = \beta(1-\phi) & \tau \leq \nu \\ \frac{\beta\tau(1-\phi)}{\nu} & min\{\frac{\nu}{\beta(1-\phi)}, 1\} \geq \tau > \nu \\ 1 & \text{otherwise} \end{cases}$$

$N_T^{s,opt-in} = \phi(\hat{\hat{\alpha}} - \underline{\hat{\alpha}})$ and $N_T^{s,opt-out} = \phi(1 - \hat{\hat{\alpha}})$; the latter stands for the number of single-homers on T opting out.

Observe that a high-privacy single-homing user would opt-in to interoperability when the gains in the network benefit from it exceed the privacy loss associated with it. When the interoperability level is weak compared to privacy loss, all high-privacy users single-homing on T opt-out. Once the interoperability improves, we will start seeing strictly positive opt-in numbers among these consumers. At a very high level of interoperability and privacy loss level is not very high, we can see another extreme of all high-privacy users opting-in to interoperability.

The gain from multi-homing over single-homing on W $= \beta((1-\tau)N_T^{s,opt-in} + N_T^{s,opt-out}) - p$ sets the upper constraint on the price which will generate partial multi-homing: $p \leq \beta((1-\tau)N_T^{s,opt-in} + N_T^{s,opt-out})$. Based on the market share calculations, we get

$$p \leq \begin{cases} \beta\phi(1-\beta(1-\phi)) & \tau \leq \nu \\ \beta\phi[1 - \frac{\beta\tau^2(1-\phi)}{\nu} - \frac{\beta(1-\tau)^2(1-\phi)}{1-\nu}] & min\{\frac{\nu}{\beta(1-\phi)}, 1\} \geq \tau > \nu \\ \beta(1-\tau)\phi(1 - \frac{\beta(1-\tau)(1-\phi)}{1-\nu}) & \text{otherwise} \end{cases} \tag{1.1}$$

Configuration 2: Single-homing analysis

If the price exceeds the upper bound on price for partial multi-homing, single homing will occur as in the baseline. We once again have three types of consumers

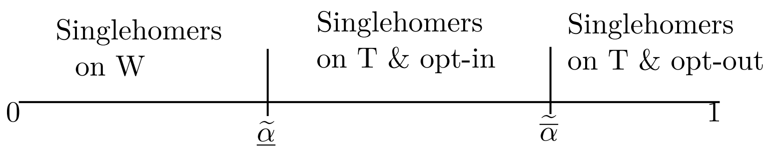

Figure 1.5: Partial multi-homing under privacy costing interoperability

Solving for $\underline{\widetilde{\alpha}}$ and $\widetilde{\widetilde{\alpha}}$ given the price p

People who will single-home on W: $u_W^s \geq max\{u_T^{s,opt-in}, u_T^{s,opt-out}\}$

As in earlier configuration, solving $u_T^{s,opt-out} \geq u_T^{s,opt-in}$, we get $\alpha \geq \frac{\beta\tau N_W^s}{\nu}$

Observe that $N_W = N_W^s = 1 - \phi + \phi\underline{\widetilde{\alpha}}$

There are three possibilities:

Case 1: $\underline{\widetilde{\alpha}} = \widetilde{\widetilde{\alpha}} < 1$: Only opt-outs on T

Case 2: $\underline{\widetilde{\alpha}} < \widetilde{\widetilde{\alpha}} < 1$: Mix of opt-ins and opt-outs on T

Case 3: $\underline{\widetilde{\alpha}} < \widetilde{\widetilde{\alpha}} = 1$: Only opt-ins on T

We show in Appendix 1.D that there are two price thresholds p_I & p_{II} and a condition (1.3) which depends on parameters (β, τ, ϕ, ν) such that

τ Range	Case 1	Case 2	Case 3
$\tau \leq \frac{\nu}{\beta}$	$p > p_I$	$p \leq p_I$ and Condition (1.3)	-
$\frac{\nu}{\beta} < \tau < \frac{\nu}{\beta(1-\phi)}$	$p_{II} > p > p_I$	$p \leq \min\{p_I, p_{II}\}$ and Condition (1.3)	$p_{II} < p < p_I$
$\tau \geq \frac{\nu}{\beta(1-\phi)}$	-	-	$p > p_{II}$

Table 1.1: Relevancy of cases

Observe that the relevancy of cases follows a logic similar to one discussed in the partial multi-homing configuration, that the level of interoperability, to that of the associated privacy loss, decides how many high-privacy single-homers on T opt-in and opt-out. When the former is low (high) relative to the latter, we expect more opt-outs (opt-ins).

1.7.2 T's maximisation problem

In partial multi-homing :

Since firm T's market share is fixed at ϕ as all non-captives use its product, its profit is maximised by setting the highest possible price, i.e., all the above constraints bind in equilibrium and we get the following profits :

$$\pi^{configM} = \begin{cases} \beta\phi^2(1-\beta(1-\phi)) & \tau \le \nu \\ \beta\phi^2[1-\frac{\beta\tau^2(1-\phi)}{\nu} - \frac{\beta(1-\tau)^2(1-\phi)}{1-\nu}] & min\{\frac{\nu}{\beta(1-\phi)},1\} \ge \tau > \nu \\ \beta(1-\tau)\phi^2(1-\frac{\beta(1-\tau)(1-\phi)}{1-\nu}) & \text{otherwise} \end{cases}$$

In single-homing :

$\pi^{configS} = (1-\underset{\sim}{\alpha})\phi p$ s.t. condition (1.1) on price constraint to generate partial multi-homing is violated.

It's straightforward to derive profit expressions for the three cases based on $\underset{\sim}{\alpha}$ expressions derived before.

Case 1:

$$\pi^{case1}_{configS} = \left(1 - \frac{\beta(1-2\phi)+p}{1-2\beta\phi}\right)\phi p$$

Case 2:

$$\pi^{case2}_{configS} = \left(1 - \frac{\beta[(1-\tau)(1-\phi)-\phi+\frac{\beta\tau^2\phi(1-\phi)}{\nu}]+p}{1-\nu-2\beta(1-\tau)\phi-\frac{\beta^2\tau^2\phi^2}{\nu}}\right)\phi p$$

Case 3:

$$\pi^{case3}_{configS} = \left(1 - \frac{\beta(1-\tau)(1-2\phi)+p}{1-\nu-2\beta(1-\tau)\phi}\right)\phi p$$

Using the profit expressions and the table (1.1), we get:

When $\tau \le \frac{\nu}{\beta}$

$$\pi_{configS} = \max \mathbb{1}_{p>p_I}\pi^{case1}_{configS} + \mathbb{1}_{\text{Condition (1.3)}}\mathbb{1}_{p\le p_I}\pi^{case2}_{configS}$$

When $\frac{\nu}{\beta} < \tau < \frac{\nu}{\beta(1-\phi)}$

$$\pi_{configS} = \begin{cases} \max_p \mathbb{1}_{p_I > p > p_{II}} \pi^{case3}_{configS} + \mathbb{1}_{\text{Condition (1.3)}} \mathbb{1}_{p \leq p_{II}} \pi^{case2}_{configS} & p_I > p_{II} \\ \max_p \mathbb{1}_{p_{II} > p > p_I} \pi^{case1}_{configS} + \mathbb{1}_{\text{Condition (1.3)}} \mathbb{1}_{p \leq p_I} \pi^{case2}_{configS} & p_I < p_{II} \\ \max_{p < p_I = p_{II}} \mathbb{1}_{\text{Condition (1.3)}} \pi^{case2}_{configS} & p_I = p_{II} \end{cases}$$

When $\tau \geq \frac{\nu}{\beta(1-\phi)}$

$$\pi_{configS} = \max \mathbb{1}_{p > p_{II}} \pi^{case3}_{configS}$$

Equilibrium profits of T

As in the benchmark, since we have the profits under two configurations, we can evaluate which is the maximum and will be the equilibrium profit of T.

$$\pi_T = max\{\pi^{configM}, \pi^{configS}\}$$

1.7.3 Numerical analysis

Due to analytical intractability in solving T's profit maximisation problem, we evaluate it numerically to determine the equilibrium outcomes.

We keep $\phi = 0.3$ for all, carried for low, medium and high β, i.e. $\beta = 0.4, 0.6$ and 0.9 for the range of ν and τ between $(0, 1)$

All the details and the graphs for this analysis are in Appendix 1.E

Observations from numerical results:

1. As in the benchmark, the results about intermediate values of interoperability hurting single-homers on T carry over.

2. Profits of T: For lower values of ν, profits are highest around full interoperability as in the benchmark. However, under high ν, T's profits are lowest in this region. This happens as when ν is higher than τ, a lot of users on T tend to opt-out of interoperability, so the direct communication benefit from multi-homing over single-homing stays high even as the interoperability level improves. This results in T finding it optimal to stick to the partial multi-homing configuration. However, the direct communication benefit is still shrinking as τ improves for opt-ins on T, causing its market share to shrink for a given price. So its profits go down as interoperability improves. *So a near-perfect interoperability policy with badly designed privacy protection can hurt the smaller firm when there is a large privacy loss for users by opting into interoperability.*

3. *We need τ to be lower than ν, otherwise no consumers of the high privacy app opt into interoperability and the policy is ineffective.*

4. The above two points call for a strong focus on designing interoperability with high privacy protection.

5. The result of misalignment in W's incentive to set interoperability level with that of the welfare of high privacy consumers in the benchmark also extends to this setting. W's incentive to choose a value of (τ, ν) results in low values of aggregate non-captives' consumer surplus compared to other parameter regions. Therefore, we advocate for an interoperability policy mandatorily setting the levels of both interoperability and privacy protection.

1.8 Policy Implications

In this section, we discuss the policy concerns associated with mandated interoperability.

Policy Implication 1 : *Imperfection in interoperability can backfire for privacy-focused users.*
As we saw under imperfect levels of interoperability, consumers of the niche app, who tend to be high privacy types, will end up with zero net surpluses compared to strictly positive levels without interoperability. Interoperability "kills" multi-homing and shrinks the market share of the niche app, and this causes it to increase its price. Users on the niche app can no longer talk to multi-homers directly and only expect low-quality interaction via imperfect interoperability, but still face high prices.

Policy Implication 2 : *In case privacy concerns are significantly high compared to network effects, then implementing even perfect interoperability can backfire for privacy-focused users.*
When $\beta < \frac{1}{2+\phi}$, as soon as interoperability is high enough to "kill" multi-homing, the single-homers on the high privacy app face a price which equals their gross surplus and make a net surplus of zero. This happens as at lower levels of β, a high privacy app has a very differentiated product compared to a low privacy-weak network effect product of a rival, and this situation is similar to a weak competition scenario. This causes the high-privacy app to charge large prices, which is sustained even for large levels of interoperability.

Policy Implication 3 : *The optimal interoperability policy for privacy-focused users of the niche app seems to be either the maximum interoperability level that still results in multi-homing or one that maximises interoperability to the highest technically feasible level.*
The highest interoperability, which still allows for multi-homing, might sound contradictory.

However, it helps consumers by forcing the niche app to reduce its price as multi-homing incentives shrink with higher interoperability. This, in turn, can benefit consumers.

When network effects are not very weak, i.e., $\beta > \frac{1}{2+\phi}$, the net surplus of users of the niche app increases at high levels of interoperability. Although technical obstacles may hinder the attainment of near-perfect interoperability, the goal should remain to strive for the highest feasible level.

Policy Implication 4 : *If the welfare of privacy-conscious consumers using niche apps is a priority, then the regulator must mandate the interoperability level.*
This follows from W's choice of an interoperability level that maximises its market share, which occurs when consumers single-home, and single-homers on T are either fully extracted or left with a very low surplus.

Policy Implication 5 : *When interoperability involves a privacy cost, the associated privacy loss must be kept at a low level for the policy to encourage participation in interoperability.*
As we saw from the numerical results, when the privacy loss from interoperability (ν) exceeds the level of interoperability, no high-privacy niche app users will opt-in, and the policy will have no impact on all players.

1.9 Conclusion

In this paper, we study how a high-privacy paid app adjusts to varying levels of mandated interoperability while facing a low-privacy free app with a large captive advantage. We focus on a privacy-preserving form of interoperability in the benchmark case and show that our results extend to privacy-costing interoperability as well.

We focus on paid apps because we observe that a truly high-privacy app must have a source of revenue to remain sustainable in the long run, and therefore, a truly high-privacy app cannot be free. In practice, we see high-privacy apps either employ non-personalised advertisement, crowd-sourcing or directly charge a fee to the consumers. Thus, consumers have to bear some form of cost, whether through advertising nuisance, direct payment or donation, to use these apps. Our benchmark contains a profit-maximising high-privacy app, but we also show that our results extend to a crowdfunded high-privacy app that targets zero profit.

This paper builds on the idea that interoperability and multi-homing are substitutes. In the presence of heterogeneous privacy costs among users, we show that interoperability can have non-trivial effects for high-privacy users who tend to use the niche app in our model. Their net surplus can be strictly lower with a positive level of interoperability than without it, and very high levels of interoperability may be required to make users better off. Hence, we argue for an interoperability policy design that either maintains low interoperability to preserve multi-homing incentives or implements interoperability at the highest possible level to significantly reduce the captive advantages of the dominant platform in the market (resulting in lower prices for consumers). However, a caveat is that this only works when network effects are strong. When these are weak, even high levels of interoperability may not be enough to overcome the captivity advantage, as the competition stays weak due to the key issue becoming more about differentiation in offering the privacy-focused product.

We also establish a misalignment between the low-privacy app's incentives for setting interoperability levels and the surplus of niche app users. Therefore, if the welfare of high-privacy users who use the niche app is of concern for regulators mandating interoperability policy, then specifying a particular level of interoperability is essential in such a policy.

Finally, we show that if privacy loss due to opting into interoperability is a concern for high-privacy users, then the interoperability policy should also mandate how much information must be shared across apps when opting for interoperability features in order to remain effective.

In the paper, while we assume a homogeneous network effect for tractability, studying heterogeneity in network effects remains an interesting possibility for the future. Another potential area to study is to analyse firms' optimal choice of privacy offered to consumers.

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Appendix for Chapter 1

Appendix 1.A Properties of Equilibrium Profit Candidates of T

For $\pi^{configM}$:

$$\pi^{configM} = \beta(1-\tau)\phi^2(1-\beta(1-\tau)(1-\phi))$$
$$\frac{d\pi^{configM}}{d\tau} = \beta\phi^2[-1+2\beta(1-\tau)(1-\phi)]$$
$$\implies \frac{d\pi^{configM}}{d\tau} > 0 \iff \tau < \frac{1-\phi-\frac{1}{2\beta}}{1-\phi}$$

A sufficient condition for the derivative to be negative for all τ can be obtained from the observation that the derivative is reducing in τ and takes a maximum value at zero.

$$\text{i.e.,} \left.\frac{d\pi^{configM}}{d\tau}\right|_{\tau=0} < 0 \implies \tau < \frac{d\pi^{configM}}{d\tau} < 0 \;\forall\; \tau$$

We get that $\beta(1-\phi) < 0.5$ is a sufficient condition to ensure that profits in partial multi-homing decline for all levels of interoperability. Since $\phi < 1/3$ by assumption, the condition implies that for low ranges of β, the profits will decline. However, when the sufficient condition is violated, we will have a range of τ(s) for which profits will increase.

$$\frac{d^2\pi^{configM}}{d\tau^2} = -2\beta^2\phi^2(1-\phi) < 0$$

Explains that while for higher β we may have a strictly positive first derivative, the second derivative is also more negative. First-order derivative ultimately turns negative and profit falls to zero as τ reaches 1.

For $\pi^{configS}_{cons}$:

$$\pi^{configS}_{cons} = \frac{\beta(1-\beta)\phi(\tau+(1-\tau)\phi-(1-\tau^2)\beta\phi)}{(1-\beta(1-\tau)\phi)^2}$$
$$\frac{d\pi^{configS}_{cons}}{d\tau} = \underbrace{\frac{\beta(1-\beta)\phi}{(1-\beta(1-\tau)\phi)^3}}_{>0}[1-\phi+(1-\tau)\beta\phi(2\beta\phi-1-\phi)]$$

Observe that $2\beta\phi-1-\phi < \frac{2}{3}\beta-1-\phi < 0$ as $\phi < \frac{1}{3}$

Therefore, $1-\phi+(1-\tau)\beta\phi(2\beta\phi-1-\phi) \geq 1-\phi+\beta\phi(2\beta\phi-1-\phi)$ as $(\tau \geq 0)$

$= 1+\phi\underbrace{[-1+\beta(2\beta\phi-1-\phi)]}_{<0} > 1+\frac{1}{3}[-1+\beta(2\beta\phi-1-\phi)]$ as $(\phi < \frac{1}{3})$

$= \frac{1}{3}[2+2\beta^2\phi-\beta-\beta\phi] > 0$ as $(\beta+\beta\phi < 2)$

So, we get that

$$\frac{d\pi_{cons}^{configS}}{d\tau} > 0 \ \forall \ \tau$$

For $\pi_{uncons}^{configS}$ **:**

$$\pi_{uncons}^{configS} = \frac{\phi(1-\beta(1-\tau))^2}{4(1-2\beta(1-\tau)\phi)}$$

$$\frac{d\pi_{uncons}^{configS}}{d\tau} = \underbrace{\frac{\phi\beta(1-\beta(1-\tau))}{2(1-2\beta(1-\tau)\phi)^2}}_{>0}[1 - \underbrace{\phi(1+\beta(1-\tau))}_{<1 \text{ as } \phi<1/3 \ \& \ \beta<1}] > 0 \ \forall \ \tau$$

Appendix 1.B Proofs

Lemma : $1 > \tilde{\alpha} \geq 0$

Proof. Since

$$\tilde{\alpha} = \frac{p + \beta(1-\tau)(1-2\phi)}{1-2\beta(1-\tau)\phi}$$

We see that $\tilde{\alpha} = 0$ only when $p = 0$ and $\tau = 1$. For any $\tau < 1$ and $p \geq 0$, $\tilde{\alpha} > 0$

To show the upper limit, observe that the maximum possible market share for W happens when the net utility on T = 0 as T charges the highest possible price. Maximum share of W implies that maximum value of $\tilde{\alpha}$.

$U_T^s = 0$, the indifferent consumer in config 2, solves

$$U_W^s(\tilde{\alpha}) = 0 \implies \beta\tau + \beta(1-\tau)[1-\phi+\phi\tilde{\alpha}] - \tilde{\alpha} = 0$$

$$\text{Max}_p \ \tilde{\alpha} = \frac{\beta\tau + \beta(1-\tau)(1-\phi)}{1-\beta(1-\tau)\phi} = \frac{\beta - \beta(1-\tau)\phi}{1-\beta(1-\tau)\phi} < 1$$

□

Proof of Lemma 1.2

Define $F \equiv p_{max}^{configS} - p_{uncons}^{configS}$

We have, $F = \frac{\beta(\tau+(1-\tau)\phi-(1-\tau^2)\beta\phi)}{1-\beta(1-\tau)\phi} - \frac{1-\beta(1-\tau)}{2}$

This simplifies to $F = \tau^2\beta^2\phi + \tau\beta(1-3\phi+2\beta\phi) + \beta(1+3\phi(1-\beta)) - 1$

We make an assumption on ϕ to keep our analysis simple :

We now observe some properties of F :

$F(\tau = 0) = \beta(1 + 3\phi(1-\beta) - 1$

Observe that $F(\tau = 0) < 0$: Suppose not $F(\tau = 0) > 0 \implies \beta(1+3\phi(1-\beta) - 1 > 0 \implies 3\beta\phi(1-\beta) > 1-\beta \implies 3\beta\phi > 1$ This violates the assumptions that $\beta < 1$ and $\phi < 1/3$

Next, $\frac{dF}{d\tau} = 2\beta^2\tau\phi + \beta(1-3\phi+2\beta\phi)$. This is again strictly positive as $1 - 3\phi > 0$

Finally, $F(\tau = 1) = \beta(2 + \phi) - 1$ could be positive or negative depending on the values of β and ϕ. It is negative when $\beta < \frac{1}{2+\phi}$. This will happen with low values of β. Since $0 < \tau < 1/3$, we get that upper cap for $\beta = 0.5$ which happens at $\phi = 0$ and when $\phi = 1/3$, upper cap for $\beta = 3/7$ for the problem to stay constrained even at high levels of interoperability. In these cases, T has no incentive to reduce its price to increase its market share. It rather prefers to keep the price at the highest possible level and extract all the surplus from its smaller set of users. Alternatively, T's incentive to reduce the price to increase its market share only kicks in when β is high. A low β, therefore, is reflective of high product differentiation in our case, resulting in high prices for the consumers

This implies that under the assumptions, the problem in config 2 is always constrained at low levels of interoperability (τ) and the problem becomes unconstrained at τ when $F(\tau) = 0$. This will never happen in cases, where $F(\tau = 1) < 0$ or when $\beta < \frac{1}{2+\phi}$
When $\beta > \frac{1}{2+\phi}$, then there exists a τ where $F = 0$.
Solving for this $\tau^{configS}_{cons-uncons}$: $F(\tau^{configS}_{cons-uncons}) = 0$, we get

$$\tau^{configS}_{cons-uncons} = \frac{-(1 - 3\phi + 2\beta\phi) + \sqrt{1 + 9\phi^2 + 16\beta^2\phi^2 - 2\phi - 24\beta\phi^2}}{2\beta\phi}$$

□

Proof of Lemma 1.3

At $\tau = 0$, we already know from config 2 analysis that we are in the constrained case and $p^{config2} = p^{configS}_{max}$.
So all we need to show is at $\tau = 0$ is
$\pi^{configM} > \pi^{configS}_{cons}$
$\iff \beta\phi^2(1 - \beta(1 - \phi)) > \frac{\beta(1-\beta)^2\phi^2}{(1-\beta\phi)^2}$
$\iff (1 - \beta(1 - \phi)) > (\frac{1-\beta}{1-\beta\phi})^2$
This is established by the observation that
$(\frac{1-\beta}{1-\beta\phi})^2 = (1 - \frac{\beta(1-\phi)}{1-\beta\phi})^2$
Since $\beta(1 - \phi) < 1 - \beta\phi$ as $\beta < 1$, we have $1 - \frac{\beta(1-\phi)}{1-\beta\phi} < 1$
$\implies (1 - \frac{\beta(1-\phi)}{1-\beta\phi})^2 < 1 - \frac{\beta(1-\phi)}{1-\beta\phi}$
Since $1 - \beta\phi < 1$, we have $1 - \frac{\beta(1-\phi)}{1-\beta\phi} < 1 - \beta(1 - \phi)$ □

Proof of Lemma 1.4

Define $G \equiv \pi^{configM} - \pi^{configS}_{cons}$
So we have $G = \beta(1 - \tau)\phi^2(1 - \beta(1 - \tau)(1 - \phi)) - \frac{\beta(1-\beta)\phi(\tau+(1-\tau)\phi-(1-\tau^2)\beta\phi)}{(1-\beta(1-\tau)\phi)^2}$
We know from earlier claim that $G(\tau = 0) > 0$.

Observe that at $\tau = 1$, $\pi^{configM} = 0$ and $\pi^{configS}_{cons} = \beta(1-\beta)\phi > 0$.

Therefore $G(\tau = 1) < 0$

The function G is a polynomial of power 4 in τ and, therefore, is tedious to solve analytically for an explicit solution.

We will establish that $\frac{d^2G(\tau)}{d\tau^2} < 0 \;\forall\; \tau$ and this establishes uniqueness of the solution.

$$\frac{d^2\pi^{configS}_{cons}}{d\tau^2} = \frac{2\beta^2(1-\beta)\phi^2}{(1-\beta(1-\tau)\phi)^4}[2\phi - 1 - \beta\phi + (1-\tau)\beta\phi(1+\phi-2\beta\phi)]$$

and using second derivative of $\pi^{configM}$ from appendix 1.A, we get that

$$\frac{d^2G(\tau)}{d\tau^2} = -2\beta^2\phi^2\left((1-\phi) + \frac{(1-\beta)}{(1-\beta(1-\tau)\phi)^4}[2\phi - 1 - \beta\phi + (1-\tau)\beta\phi(1+\phi-2\beta\phi)]\right)$$

$$= \underbrace{\frac{-2\beta^2\phi^2}{(1-\beta(1-\tau)\phi)^4}}_{<0}\left(\underbrace{\overbrace{(1-\phi)(1-\beta(1-\tau)\phi)^4 + (1-\beta)(1-\tau)\beta\phi(1+\phi-2\beta\phi)}^{\equiv K} + (1-\beta)[2\phi-1-\beta\phi]}_{\equiv L}\right)$$

Claim: $K(\tau)$ is minimised at 0 for $\tau \in [0,1]$

Proof: Observe that $(1+\phi-2\beta\phi) = (1+\phi(1-2\beta)) > 1 - 1/3 > 0$ (min at $\phi = 1/3$ and $\beta = 1$)

This means all the elements in K are positive, so $K > 0$

$K'(\tau) = \beta\phi[4(1-\phi)(1-\beta(1-\tau)\phi)^3 - (1-\beta)(1+\phi(1-2\beta))]$

$\geq \beta\phi[4(1-\phi)(1-\beta\phi)^3 - (1-\beta)(1+\phi(1-2\beta))]$ (min at $\tau = 0$)

We numerically establish that the above expression minimises at $\beta = \phi = 0$ by taking a value of 0 and is strictly positive for the range of $0 < \beta < 1$ and $0 < \phi < 1/3$

So, we have $K'(\tau) > 0$ and therefore it minimises at $\tau = 0$ □

So we have, $L(\tau) \geq K(\tau = 0) + (1-\beta)[2\phi - 1 - \beta\phi]$

$= (1-\phi)(1-\beta\phi)^4 + (1-\beta)\beta\phi(1+\phi-2\beta\phi) + (1-\beta)[2\phi-1-\beta\phi]$

We numerically establish that the above expression minimises at $\beta = \phi = 0$ by taking a value of 0 and is strictly positive for the range of $0 < \beta < 1$ and $0 < \phi < 1/3$

Therefore, $L > 0$ and we have that $\frac{d^2G(\tau)}{d\tau^2} < 0$ for $\tau \in [0,1]$, $0 < \beta < 1$ and $0 < \phi < 1/3$

□

Proof of Lemma 1.5

We can use a similar approach as above.

Define $H \equiv \pi^{configM} - \pi^{configS}_{uncons}$

So we have $H = \beta(1-\tau)\phi^2(1-\beta(1-\tau)(1-\phi)) - \frac{\phi(1-\beta(1-\tau))^2}{4(1-2\beta(1-\tau)\phi)}$

Observe that at $\tau = 1$, $\pi^{configM} = 0$ and $\pi^{configS}_{cons} = \frac{\phi}{4} > 0$.

Therefore $H(\tau = 1) < 0$

$H(\tau = 0) = \beta\phi^2(1 - \beta(1 - \phi)) - \frac{\phi(1-\beta)^2}{4(1-2\beta\phi)}$

This can be negative or positive depending on the values of β and ϕ. For instance, it is easy to see that it is negative for β approaching 0 with $\phi > 0$ while positive for β approaching 1

The function H is a polynomial of power 3 in τ and, therefore, is tedious to solve analytically for an explicit solution.

So, first we establish that $\frac{d^2H(\tau)}{d\tau^2} < 0 \; \forall \; \tau$

$$\frac{d^2\pi_{uncons}^{configS}}{d\tau^2} = \underbrace{\frac{\beta^2\phi}{2(1 - 2\beta(1 - \tau)\phi)^3}}_{>0}[1 - 4\phi(1 - \phi)]$$

Observe that $1 - 4\phi(1 - \phi)$ is minimised when $\phi = 1/3$, and at which it takes a value of 1/9. Therefore, we have that

$$\frac{d^2\pi_{uncons}^{configS}}{d\tau^2} > 0 \; \forall \; \tau \in [0, 1]$$

Since $\dfrac{d^2\pi^{configM}}{d\tau^2} < 0$ from Appendix 1.A

We have that

$$\frac{d^2H(\tau)}{d\tau^2} = \underbrace{\frac{d^2\pi^{configM}}{d\tau^2}}_{<0} - \underbrace{\frac{d^2\pi_{uncons}^{configS}}{d\tau^2}}_{>0} < 0 \quad \forall\tau, \; 0 < \beta < 1 \;\&\; 0 < \phi < 1/3$$

These observations establish that when $H(\tau = 0) > 0$, there $\exists$ a unique $0 < \tau_{uncons}^{configM-S} < 1$, such that $H(\tau_{uncons}^{configM-S}) = 0$

Next , when $H(\tau = 0) \leq 0$, we show that there is no solution of $H(\tau) = 0$ for $\tau \in (0, 1]$

Firstly, we show that $H'(\tau = 0) \leq 0$

Since,

$$H(\tau = 0) \leq 0 \implies \beta\phi^2(1 - \beta(1 - \phi)) \leq \frac{\phi(1 - \beta)^2}{4(1 - 2\beta\phi)}$$

$$\implies \frac{2\beta\phi(1 - \beta(1 - \phi))}{(1 - \beta)} \leq \frac{(1 - \beta)}{2(1 - 2\beta\phi)} \tag{1.2}$$

$$\frac{dH(\tau)}{d\tau}\Big|_{\tau=0} = \beta\phi^2[-1 + 2\beta(1 - \phi)] - \frac{\phi\beta(1 - \beta)}{2(1 - 2\beta\phi)^2}[1 - \phi(1 + \beta)]$$

$$< \beta\phi^2[-1 + 2\beta(1 - \phi)] - \frac{2\beta^2\phi^2(1 - \beta(1 - \phi))}{(1 - 2\beta\phi)(1 - \beta)}[1 - \phi(1 + \beta)] \quad \text{From equation (1.2)}$$

$$= \underbrace{\frac{\beta\phi^2}{(1 - 2\beta\phi)(1 - \beta)}}_{>0} \underbrace{[-1 - \beta^3\phi^2 - 6\beta^2\phi + 6\beta^3\phi + 2\beta\phi + 6\beta^2\phi^2]}_{\equiv N}$$

We numerically establish that $N < 0$ for the $0 < \beta < 1$ and $0 < \phi < 1/3$

Therefore, we have that $H'(\tau = 0) \leq 0$ and using $H''(\tau) < 0$, we have that $H'(\tau) < 0 \ \forall \ \tau$.
Since, $H(\tau = 0) \leq 0 \implies H(\tau) < 0 \ \forall \tau \in (0, 1]$
Therefore, when $H(\tau = 0) \leq 0$, there is no solution to solution of $H(\tau) = 0$ for $\tau \in (0, 1]$. In this case, define $\tau_{uncons}^{configM-S} = 0$

□

Proof of Lemma 1.6

Suppose not, $\tau_{uncons}^{configM-S} < \tau_{cons-uncons}^{configS} < \tau_{cons}^{configM-S}$,
Since $\tau_{uncons}^{configM-S} < \tau_{cons-uncons}^{configS}$, we have that at $\tau = \tau_{cons-uncons}^{configS}$, $\pi^{configM} \leq \pi_{uncons}^{configS}$
Since, $\tau_{cons-uncons}^{configS} < \tau_{cons}^{configM-S}$, we have that at $\tau = \tau_{cons-uncons}^{configS}$, $\pi_{uncons}^{configS} < \pi^{configM}$
From the above two, we get $\pi_{uncons}^{configS} < \pi_{uncons}^{configS}$
This contradicts the fact at $\tau = \tau_{cons-uncons}^{configS}$, we know that $\pi_{uncons}^{configS} = \pi_{cons}^{configS}$ (from the definition of $\tau_{cons-uncons}^{configS}$

□

Proof of Proposition 1.3

Proposition is establish by Lemma 1.7 and observing that

$$\frac{du_T^{s,configM}}{d\tau} = \frac{d\beta[\tau + \beta(1-\tau)^2(1-\phi)\phi]}{d\tau} > 0$$

$$\frac{du_T^{s,configS-uncons}}{d\tau} = \frac{d\beta\tau + \left(\frac{1-\beta(1-\tau)}{2}\right)\left(\frac{3\beta(1-\tau)\phi-1}{1-2\beta(1-\tau)\phi}\right)}{d\tau} > 0 \quad \text{when } \beta > \frac{1}{2+\phi}$$

□

Proof of Proposition 1.4

Based on the prices and the corresponding market shares derived for partial multi-homing and single-homing configurations in the benchmark case, we have that the non-captive share of W is given by

$$= \begin{cases} \phi\beta(1-\tau)(1-\phi) & \text{Under partial multi-homing} \\ \phi\frac{\beta-\beta(1-\tau)\phi}{1-\beta(1-\tau)\phi} & \text{Under single-homing constrained price} \\ \phi\frac{1+\beta(1-\tau)(1-4\phi)}{2(1-2\beta(1-\tau)\phi)} & \text{Under single-homing unconstrained price} \end{cases}$$

Claim: For a given set of parameters (τ, β, ϕ), W's non-captive market share is higher under single-homing constrained price than partial multi-homing.

Proof: We have to show that

$\phi\beta(1-\tau)(1-\phi) < \phi\frac{\beta-\beta(1-\tau)\phi}{1-\beta(1-\tau)\phi}$

$\iff (1-\tau)(1-\phi) < \frac{1-(1-\tau)\phi}{1-\beta(1-\tau)\phi}$

$\iff (1-\tau)(1-\phi)(1-\beta(1-\tau)\phi) < 1-(1-\tau)\phi$

Established by $(1-\beta(1-\tau)\phi) < 1$ as $\beta > 0$ and

$(1-\tau)(1-\phi) = 1-\tau-(1-\tau)\phi < 1-(1-\tau)\phi$ □

Claim: For a given set of parameters (τ, β, ϕ), W's non-captive market share is higher under single-homing unconstrained price than partial multi-homing.

Proof: We have to show that

$\phi\beta(1-\tau)(1-\phi) < \phi\frac{1+\beta(1-\tau)(1-4\phi)}{2(1-2\beta(1-\tau)\phi)}$

$\iff \beta(1-\tau)(1+\phi[2-4\beta(1-\tau)+4\beta(1-\tau)\phi]) < 1$

Since the left-hand side (LHS) is increasing in ϕ, we get its max value by setting $\phi = 1/3$ (the upper constraint assumed on non-captive base),

LHS $< \frac{\beta(1-\tau)(15-8\beta(1-\tau))}{9}\ \forall\phi$

Right-hand side in the above expression is quadratic in $\beta(1-\phi)$ and maximises at $\beta(1-\phi) = \frac{15}{16}$

We get, LHS $< \frac{15.15}{18.16} < 1\ \forall\phi, \beta, \tau$ as required. □

So the above two claims establish that W would always prefer a τ which results in single-homing compared to partial multi-homing.

Also, observe that only market share under constrained price increases in τ while the other two decrease.

This implies that when $\exists\tau$ such that single-homing happens under the constrained price in the equilibrium (When $\beta \leq \frac{1}{2+\phi}$ or $\tau^* = \tau_{cons}^{configM-S}$) and since the market share increases in τ under this, W chooses the highest level of τ where this situation can be sustained in the equilibrium which is given by $\tau_{cons-uncons}^{configS}$ where the equilibrium shifts from constrained to unconstrained price in single-homing configuration.

When $\nexists$ such a τ, the equilibrium has only an unconstrained price solution under single-homing. Since the market share of W falls with τ under such a case, W chooses the lowest level of $\tau = \tau = \tau_{uncons}^{configM-S}$, which shifts the solution from partial multi-homing to single-homing under unconstrained prices.

□

Proof of Proposition 1.5

Under multi-homing, the market share $=\phi$ and therefore a price of $\frac{F}{\phi}$ results in zero profit. However, to ensure partial multi-homing, firm T needs to meet the upper constraint on price, so that the direct benefit from multi-homing is greater than that from single-homing. i.e.

$p = \frac{F}{\phi} < \beta(1-\tau)\phi(1-\beta(1-\tau)(1-\phi))$

The assumption on F ensures that this equation is satisfied at $\tau = 0$

The right-hand side of the above condition is quadratic in τ and it straightforward to show that it $\exists \tau^{**} = \frac{2\beta(1-\phi)-1+\sqrt{1-\frac{4(1--\phi)F}{\phi^2}}}{2\beta(1-\phi)} < 1$ such that the above condition holds for $\forall\ \tau < \tau^{**}$.

However, as higher τ, single-configuration has to arise as partial multi-homing prices are too low to cover the fixed cost of firm T.

Using the market share calculation for a given price p in the single-homing configuration unconstrained case, p can be calculated from the following:

$$\pi = p\Big(\frac{1-\beta(1-\tau)-p}{1-2\beta(1-\tau)\phi}\Big)\phi - F = 0$$

This gives us the $p > F/\phi$ as the above expression would be negative at prices lower than this.

□

Appendix 1.C Analysis for the Second equilibrium selection

We extend the analysis carried out for the first equilibrium selection to the second equilibrium selection rule, which states that consumers believe each other will choose single-homing whenever possible. Therefore, the demand for T is

$$N_T = \begin{cases} \phi, & p \leq \frac{\beta(1-\tau)\phi(1-\beta(1-\tau))}{1-\beta(1-\tau)\phi} \\ \frac{\phi[1-\beta(1-\tau)]}{1-2\beta(1-\tau)\phi} - \frac{\phi}{1-2\beta(1-\tau)\phi}p, & \text{otherwise} \end{cases}$$

This selection rule limits the price range for which partial multi-homing can occur compared to selection 1. We repeat the analysis as carried out for our first equilibrium selection rule.

1.C.1 The app T's pricing decision

Profit under configuration 1 - Partial multi-homing

$$\pi_{SS}^{configM} = \max_p \phi p \quad \text{with} \quad p \leq \frac{\beta(1-\tau)\phi(1-\beta(1-\tau))}{1-\beta(1-\tau)\phi}$$

where SS denotes the Second equilibrium Selection

So, T's optimal price is to set the highest possible price which meets the constraint, i.e. the constraint binds and $p_{SS}^{configM} = \frac{\beta(1-\tau)\phi(1-\beta(1-\tau))}{1-\beta(1-\tau)\phi}$

The profit is given by :

$$\pi_{SS}^{configM} = \frac{\beta(1-\tau)\phi^2(1-\beta(1-\tau))}{1-\beta(1-\tau)\phi}$$

Profit under configuration 2 - Single-homing

If T wants to only appeal to single-homers, then it needs to set a price $p > \frac{\beta(1-\tau)\phi^2(1-\beta(1-\tau))}{1-\beta(1-\tau)\phi}$. However, we can ignore the lower bound (as we did in the first selection type analysis) because optimal prices from the relaxed problem fall below this lower bound then in equilibrium, T's response is to set the optimal price as per configuration 1 (as it yields higher prices with a higher market share). Such prices are, therefore, off-equilibrium quantities and can be ignored.

Now, the relaxed problem is the same as the one solved for the first equilibrium selection, and so all the results for this configuration can be used from the earlier analysis.

Equilibrium profits of T

So, we have

$$\pi_T = \begin{cases} max\{\pi_{SS}^{configM}, \pi_{cons}^{configS}\} & \text{if either } (\beta \leq \frac{1}{2+\phi}) \text{ or } (\beta > \frac{1}{2+\phi} \text{ \& } \tau \leq \tau_{cons-uncons}^{configS}) \\ max\{\pi_{SS}^{configM}, \pi_{uncons}^{configS}\} & \text{otherwise} \end{cases}$$

We will now show that this equilibrium selection is qualitatively similar to the first equilibrium selection.

Lemma 1.9. *In equilibrium under selection 2, T chooses configuration 1 at $\tau = 0$, i.e., partial multi-homing always occurs in the absence of interoperability.*

Proof. At $\tau = 0$, we already know from config 2 analysis that we are in the constrained case and $p^{config2} = p_{max}^{configS}$.

So all we need to show is that at $\tau = 0$ is

$\pi_{SS}^{configM} > \pi_{cons}^{configS}$

$\iff \frac{\beta\phi^2(1-\beta)}{1-\beta\phi} > \frac{\beta(1-\beta)^2\phi^2}{(1-\beta\phi)^2}$

$\iff 1 > \frac{1-\beta}{1-\beta\phi}$

$\iff 1 > \phi$

which is true as $\phi < 1/3$ by assumption □

Lemma 1.10. *∃ a unique* $\tau^{configM-S}_{cons,SS}$ *, with* $0 < \tau^{configM-S}_{cons,SS} < 1$*, such that*

$$\begin{cases} \pi^{configM}_{SS} \geq \pi^{configS}_{cons} & \text{if } \tau \leq \tau^{configM-S}_{cons,SS} \\ \pi^{configM}_{SS} < \pi^{configS}_{cons} & \text{otherwise} \end{cases}$$

Proof. T solves $max\{\pi^{configM}_{SS}, \pi^{configS}_{cons}\}$
Define $G_{SS} \equiv \pi^{configM}_{SS} - \pi^{configS}_{cons}$
So we have $G_{SS} = \frac{\beta(1-\tau)\phi^2(1-\beta(1-\tau))}{1-\beta(1-\tau)\phi} - \frac{\beta(1-\beta)\phi(\tau+(1-\tau)\phi-(1-\tau^2)\beta\phi)}{(1-\beta(1-\tau)\phi)^2}$
We know from earlier claim that $G_{SS}(\tau = 0) > 0$.
Observe that at $\tau = 1$, $\pi^{configM}_{SS} = 0$ and $\pi^{configS}_{cons} = \beta(1-\beta)\phi > 0$.
Therefore $G_{SS}(\tau = 1) < 0$
The function G_{SS} is a polynomial of power 4 in τ and, therefore, is tedious to solve analytically for an explicit solution.
These statements can be established by following a similar logic used in the proof of the lemma 1.4. □

Lemma 1.11. *∃ a unique* $\tau^{configM-S}_{uncons,SS}$ *, with* $0 \leq \tau^{configM-S}_{uncons,SS} < 1$ *such that*

- $\pi^{configM}_{SS} \leq \pi^{configS}_{uncons}$ $\forall\tau$ *if* $\tau^{configM-S}_{uncons,SS} = 0$
- *for* $\tau^{configM-S}_{uncons,SS} > 0$

$$\begin{cases} \pi^{configM}_{SS} \geq \pi^{configS}_{uncons} & \text{if } \tau \leq \tau^{configM-S}_{uncons,SS} \\ \pi^{configM}_{SS} < \pi^{configS}_{uncons} & \text{otherwise} \end{cases}$$

Proof. We can use a similar approach as above.
T solves $max\{\pi^{configM}_{SS}, \pi^{configS}_{uncons}\}$
Define $H_{SS} \equiv \pi^{configM}_{SS} - \pi^{configS}_{uncons}$
So we have $H_{SS} = \frac{\beta(1-\tau)\phi^2(1-\beta(1-\tau))}{1-\beta(1-\tau)\phi} - \frac{\phi(1-\beta(1-\tau))^2}{4(1-2\beta(1-\tau)\phi)}$
Observe that at $\tau = 1$, $\pi^{configM}_{SS} = 0$ and $\pi^{configS}_{cons} = \frac{\phi}{4} > 0$.
Therefore $H_{SS}(\tau = 1) < 0$

$$H_{SS}(\tau = 0) = \frac{\beta\phi^2(1-\beta)}{1-\beta\phi} - \frac{\phi(1-\beta)^2}{4(1-2\beta\phi)}$$

This can be negative or positive depending on the values of β and ϕ. For instance, it is easy to see that it is negative for $\beta = 0$ with $\phi > 0$ while positive for $\beta = 1$
The function H is a polynomial of power 3 in τ and, therefore, is tedious to solve analytically for an explicit solution.
If $H_{SS}(\tau = 0) \leq 0$, then there is no solution of $H_{SS}(\tau) = 0$ for $\tau \in (0, 1)$.

When $H_{SS}(\tau = 0) \leq 0$, Define $\tau^{configM-S}_{uncons,SS} = 0$

If $H_{SS}(\tau = 0) > 0$, then $\exists$ a unique $0 < \tau^{configM-S}_{uncons,SS} < 1$, such that $H_{SS}(\tau^{configM-S}_{uncons,SS}) = 0$

These statements can be established by following a similar logic used in the proof of the lemma 1.5 □

By definition, we know that $\pi^{configS}_{uncons} \geq \pi^{configS}_{cons}$, so $\tau^{configM-S}_{uncons,SS} \leq \tau^{configM-S}_{cons,SS}$

Lemma 1.12. *We can never have* $\tau^{configM-S}_{uncons,SS} < \tau^{configS}_{cons-uncons} < \tau^{configM-S}_{cons,SS}$

Proof. Established via arguments in the lemma 1.6 □

So either $\tau^{configS}_{cons-uncons} < \tau^{configM-S}_{uncons,SS}$ or $\tau^{configM-S}_{cons,SS} < \tau^{configS}_{cons-uncons}$

Define

$$\tau^*_{SS} \equiv \begin{cases} \tau^{configM-S}_{uncons,SS} & \text{if } \tau^{configS}_{cons-uncons} < \tau^{configM-S}_{uncons,SS} \\ \tau^{configM-S}_{cons,SS} & \text{if } \tau^{configM-S}_{cons,SS} < \tau^{configS}_{cons-uncons} \end{cases}$$

Proposition 1.6. *In equilibrium under selection 2, for* $\tau < \tau^*_{SS}$*, configuration 1 arises, i.e. there is partial multi-homing, while for* $\tau > \tau^*_{SS}$*, configuration 2 arises, i.e. there are only single-homers.*

Proposition 1.7. *In equilibrium under selection 2,*

When $\beta \leq \frac{1}{2+\phi}$

$$\pi_{T,SS} = \begin{cases} \pi^{configM}_{SS} & \text{for } \tau \leq \tau^*_{SS} \\ \pi^{configS}_{cons} & \text{otherwise} \end{cases}$$

When $\beta > \frac{1}{2+\phi}$

- *When* $\tau^*_{SS} = \tau^{configM-S}_{cons,SS}$ *(i.e.,* $\tau^*_{SS} < \tau^{configS}_{cons-uncons}$*)*

$$\pi_{T,SS} = \begin{cases} \pi^{configM}_{SS} & \text{for } \tau \leq \tau^*_{SS} \\ \pi^{configS}_{cons} & \text{for } \tau^{configS}_{cons-uncons} > \tau > \tau^*_{SS} \\ \pi^{configS}_{uncons} & \text{otherwise} \end{cases}$$

- *When* $\tau^*_{SS} = \tau^{configM-S}_{uncons,SS}$ *(i.e.,* $\tau^* > \tau^{configS}_{cons-uncons}$*)*

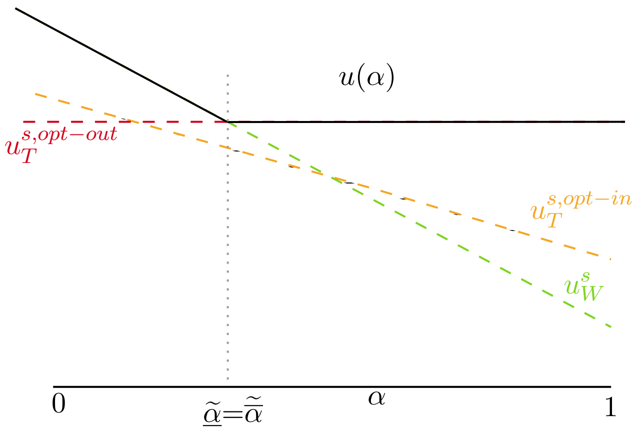
$$\pi_{T,SS} = \begin{cases} \pi^{configM}_{SS} & \text{for } \tau \leq \tau^*_{SS} \\ \pi^{configS}_{uncons} & \text{otherwise} \end{cases}$$

So, the second equilibrium selection affects prices and profits under partial multi-homing configuration, and this in turn affects the cut-off points at which the change from partial multi-homing to single-homing occurs as levels of interoperability change (keeping β and ϕ fixed). So qualitatively we have a very similar setting, and all the discussion carried out for the first equilibrium selection should pass through this selection as well. Hence, the discussions in the papers are robust to this equilibrium selection.

Appendix 1.D Single-homing possibilities in privacy loss due to interoperability setting

Under the single-homing configuration, three cases are possible depending upon the relative levels of interoperability τ with respect to ν. In this appendix, we identify when each of these cases happens and solve for the corresponding market shares.

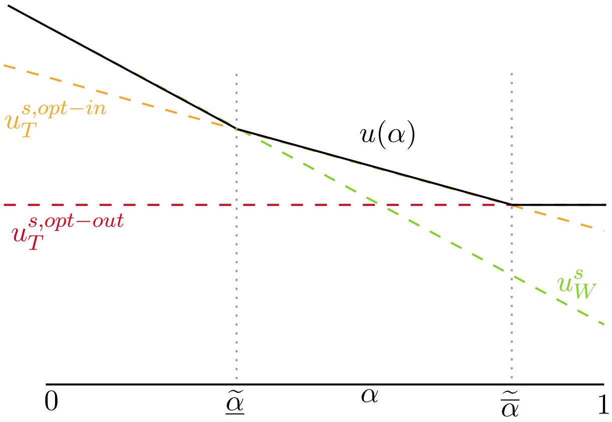
Case 1: $\underset{\sim}{\alpha} = \widetilde{\widetilde{\alpha}} < 1$: Only opt-outs on T



$\underset{\sim}{\alpha} = \widetilde{\widetilde{\alpha}}$ solves $u_W^s = u_T^{s,opt-out}$

$$\underset{\sim}{\alpha} = \widetilde{\widetilde{\alpha}} = \frac{\beta(1-2\phi)+p}{1-2\beta\phi}$$

Case 2: $\underset{\sim}{\alpha} < \widetilde{\widetilde{\alpha}} < 1$: Mix of opt-ins and opt-outs on T

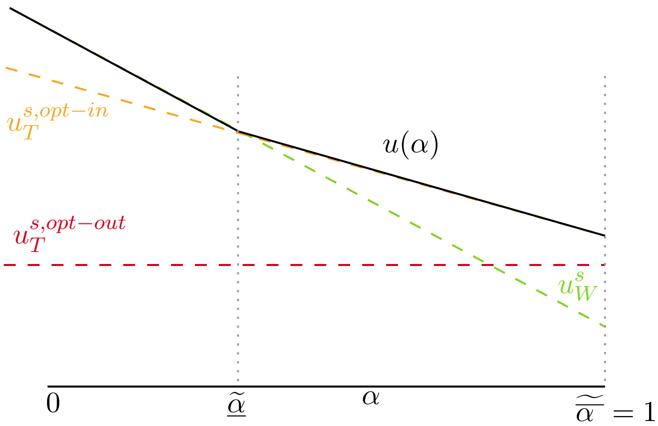


$\underline{\widetilde{\alpha}}$ solves $u_W^s = u_T^{s,opt-in}$ and $\widetilde{\widetilde{\alpha}} = \frac{\beta\tau N_W^s}{\nu}$

$$\underline{\widetilde{\alpha}} = \frac{\beta[(1-\tau)(1-\phi) - \phi + \frac{\beta\tau^2\phi(1-\phi)}{\nu}] + p}{1 - \nu - 2\beta(1-\tau)\phi - \frac{\beta^2\tau^2\phi^2}{\nu}}$$

$$\widetilde{\widetilde{\alpha}} = \frac{\beta\tau(1-\phi+\phi\underline{\widetilde{\alpha}})}{\nu} = \frac{\beta\tau(1-\phi)}{\nu} + \frac{\beta\tau\phi}{\nu}\Big[\frac{\beta[(1-\tau)(1-\phi) - \phi + \frac{\beta\tau^2\phi(1-\phi)}{\nu}] + p}{1 - \nu - 2\beta(1-\tau)\phi - \frac{\beta^2\tau^2\phi^2}{\nu}}\Big]$$

Case 3: $\widetilde{\widetilde{\alpha}} = 1$: Only opt-ins on T



$\underline{\widetilde{\alpha}}$ solves $u_W^s = u_T^{s,opt-in}$

$$\underline{\widetilde{\alpha}} = \frac{\beta(1-\tau)(1-2\phi) + p}{1 - \nu - 2\beta(1-\tau)\phi}$$

Necessary conditions corresponding to the above cases:

Case 1 : A necessary condition is that at $\alpha = \underline{\widetilde{\alpha}} = \widetilde{\widetilde{\alpha}}$, $u_T^{s,opt-out} = u_W^s > u_T^{s,opt-in}$
$\underline{\widetilde{\alpha}} = \widetilde{\widetilde{\alpha}} > \frac{\beta\tau N_W^s}{\nu}$.

$\implies \underset{\sim}{\widetilde{\alpha}}[\nu - \beta\tau\phi] > \beta\tau(1-\phi)$

$\implies \frac{\beta(1-2\phi)+p}{1-2\beta\phi}[\nu - \beta\tau\phi] > \beta\tau(1-\phi)$

$$\implies p > \underbrace{\frac{\beta\tau(1-\phi)(1-2\beta\phi)}{\nu - \beta\tau\phi} - \beta(1-2\phi)}_{\equiv p_I} \quad \text{if } \nu - \beta\tau\phi > 0$$

if $\nu - \beta\tau\phi \leq 0$, then this condition fails for all p.

In addition, it requires $\frac{\beta\tau N_W^s}{\nu} < 1$

Since $\frac{\beta\tau(1-\phi)}{\nu} < \frac{\beta\tau N_W^s}{\nu}$, we have that $\tau < \frac{\nu}{\beta(1-\phi)}$ is another necessary condition . [17]

Case 3: A necessary condition is that $u_T^{s,opt-in} = u_T^{s,opt-out}$ is solved by $\alpha > 1$

$\implies \frac{\beta\tau N_W^s}{\nu} > 1$

Firstly, $\frac{\beta\tau}{\nu} > \frac{\beta\tau N_W^s}{\nu} > 1$

$\implies \tau > \frac{\nu}{\beta}$ is a necessary condition.

Claim: $\tau > \frac{\nu}{\beta}$ is sufficient for $\underline{\hat{\alpha}} > 0$, i.e., $1 - \nu - 2\beta(1-\tau)\phi > 0$

Proof: Since $\tau > \frac{\nu}{\beta}$

We have that, $1 - \nu - 2\beta(1-\tau)\phi > 1 - \nu - 2\beta(1-\frac{\nu}{\beta})\phi = 1 - 2\beta\phi + \nu(2\phi - 1)$

$1 - 2\beta\phi + \nu(2\phi - 1)$ is minimised w.r.t ν at $\nu = 1$ as $2\phi - 1 < 0$.

So, $\min_{\nu}(1 - 2\beta\phi + \nu(2\phi - 1)) = 2\phi(1-\beta) > 0$ □

Secondly, another necessary condition from $\frac{\beta\tau N_W^s}{\nu} > 1$

$\implies \frac{\beta\tau(1-\phi+\phi\underset{\sim}{\widetilde{\alpha}}))}{\nu} > 1 \implies 1 - \phi + \phi\Big[\frac{\beta(1-\tau)(1-2\phi)+p}{1-\nu-2\beta(1-\tau)\phi}\Big] > \frac{\nu}{\beta\tau}$

$$\implies p > \underbrace{\frac{(1-\nu-2\beta(1-\tau)\phi)}{\phi}\Big[\frac{\nu}{\beta\tau} - 1 + \phi\Big] - \beta(1-\tau)(1-2\phi)}_{p_{II}}$$

Case 2: A necessary condition is that $\underset{\sim}{\widetilde{\alpha}} > 0$ i.e.,

$$1 - \nu - 2\beta(1-\tau)\phi - \frac{\beta^2\tau^2\phi^2}{\nu} > 0 \tag{1.3}$$

In addition, observe that this case requires case 1's price condition to fail $p \leq p_I$ when $\tau < \frac{\nu}{\beta(1-\phi)}$ and case 3's price condition to fail $p \leq p_{II}$ when $\tau > \frac{\nu}{\beta}$

Finally, $\widetilde{\widetilde{\alpha}} < 1 \implies \frac{\beta\tau N_W^s}{\nu} < 1$ Since, $\frac{\beta\tau(1-\phi)}{\nu} < \frac{\beta\tau N_W^s}{\nu}$, we get that $\tau < \frac{\nu}{\beta(1-\phi)}$

To summarise, we have the following necessary conditions:

- Case 1 : $p > p_I$ and $\tau < \frac{\nu}{\beta(1-\phi)}$
- Case 2: Condition (1.3) : $1 - \nu - 2\beta(1-\tau)\phi - \frac{\beta^2\tau^2\phi^2}{\nu} > 0$ and $p \leq p_I$ when $\tau < \frac{\nu}{\beta(1-\phi)}$ and $p \leq p_{II}$ when $\tau > \frac{\nu}{\beta}$

[17] Note that $\frac{\nu}{\beta(1-\phi)} < \frac{\nu}{\beta\phi}$ as $\phi < 1/3$

- Case 3: $p > p_{II}$ and $\tau > \frac{\nu}{\beta}$

When $\frac{\nu}{\beta} \leq \tau < \frac{\nu}{\beta\phi}$, both case 1 and 3's necessary condition on τ are met. Now $p_I \lesseqgtr p_{II}$ depending upon parameters. Given a price p, note that $p_{II} < p_I < p$ or $p_I < p_{II} < p$ will not occur in equilibrium as they imply that $\underset{\sim}{\alpha} = \tilde{\tilde{\alpha}} = 1$, so $N_T = 0$ and the profit of firm T is zero. So we can ignore them. Therefore when $p_I < p_{II}$: Case 3 becomes irrelevant and when $p_I > p_{II}$: case 1 becomes irrelevant.

Appendix 1.E Numerical results for privacy loss due to interoperability setting

We set the non-captive number to $\phi = 0.3$ throughout the analysis and evaluate results for three values of β corresponding to a low, medium and high network effect. ($\beta = 0.4,\ 0.6$ and 0.9) .

To derive values, we vary ν and τ between $(0.01, 0.99)$ in intervals of 0.05 for each. So, we end up with $197 * 197$ simulated values. We carry out the analysis in the R programming language.

For each β, we generate five values. For a given τ, ν, these values describe the following:
1) Configuration: This variable describes firstly, the configuration type - Whether we have partial-multi-homing (M) or single-homing (S) configuration and secondly, the type of mixture of opt-ins and opt-outs. "Mix of both" implies strictly positive levels of both opt-ins and opt-outs.
2) Profit of firm T
3) Average net U_T^s or net utility of single-homers on T: We evaluate this by summing up the net utility of all single-homers on T and dividing them by the number of single-homers on T.

$$\text{Avg Net } U_T^s = \begin{cases} \dfrac{\int_{\underline{\hat{\alpha}}}^{\hat{\hat{\alpha}}} u_T^{s,opt-in}\, d\alpha + \int_{\hat{\hat{\alpha}}}^{1} u_T^{s,opt-out}\, d\alpha}{\phi(1-\underline{\hat{\alpha}})} & \text{Under partial multi-homing} \\ \dfrac{\int_{\underline{\tilde{\alpha}}}^{\tilde{\tilde{\alpha}}} u_T^{s,opt-in}\, d\alpha + \int_{\tilde{\tilde{\alpha}}}^{1} u_T^{s,opt-out}\, d\alpha}{\phi(1-\underline{\tilde{\alpha}})} & \text{Under single-homing} \end{cases}$$

4) CS (consumer surplus) non-captives: We evaluate this by summing up the net utility of all single-homers on T as well as non-captives on W and dividing them by the number of non-captive consumers.

$$\text{CS non-captives} = \begin{cases} \dfrac{\int_0^{\hat{\underline{\alpha}}} u_m \, d\alpha + \int_{\hat{\underline{\alpha}}}^{\hat{\overline{\alpha}}} u_T^{s,opt-in} \, d\alpha + \int_{\hat{\overline{\alpha}}}^{1} u_T^{s,opt-out} \, d\alpha}{\phi} & \text{Under partial multi-homing} \\ \dfrac{\int_0^{\widetilde{\underline{\alpha}}} u_w^s \, d\alpha \int_{\widetilde{\underline{\alpha}}}^{\widetilde{\overline{\alpha}}} u_T^{s,opt-in} \, d\alpha + \int_{\widetilde{\overline{\alpha}}}^{1} u_T^{s,opt-out} \, d\alpha}{\phi} & \text{Under single-homing} \end{cases}$$

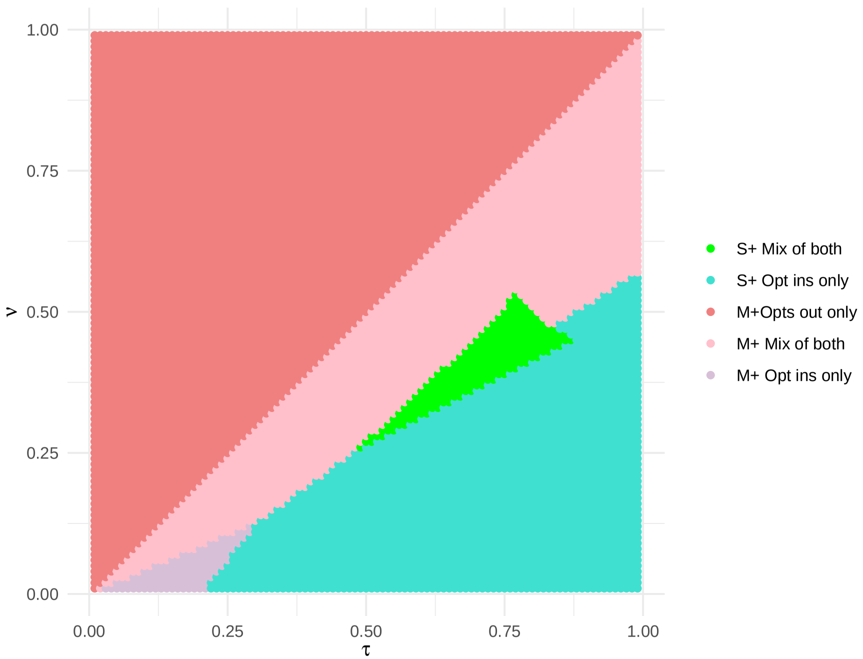
5) Market share of firm W

$$N_W = \begin{cases} 1 - \phi + \phi \hat{\underline{\alpha}} & \text{Under partial multi-homing} \\ 1 - \phi + \phi \widetilde{\underline{\alpha}} & \text{Under single-homing} \end{cases}$$

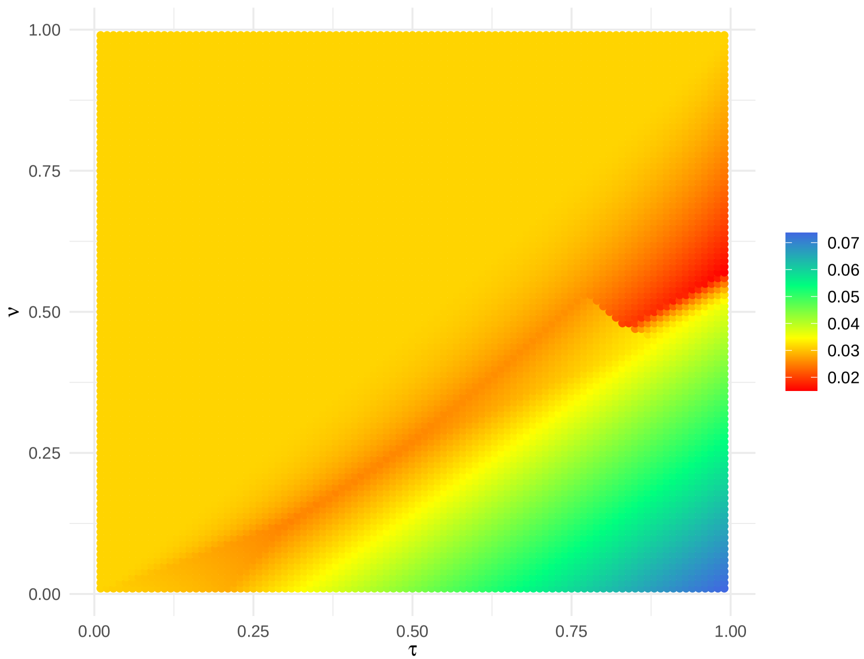
We plot all these five variables in heat maps where the x-axis is the interoperability level τ and the y-axis is the privacy loss level due to opting into interoperability ν.

The first four variables are shown in the figures (1.6), (1.7) and (1.8) corresponding to medium, low and high values of network effect β. The last variable is plotted in Figure (1.9) for all three values of network effect β.

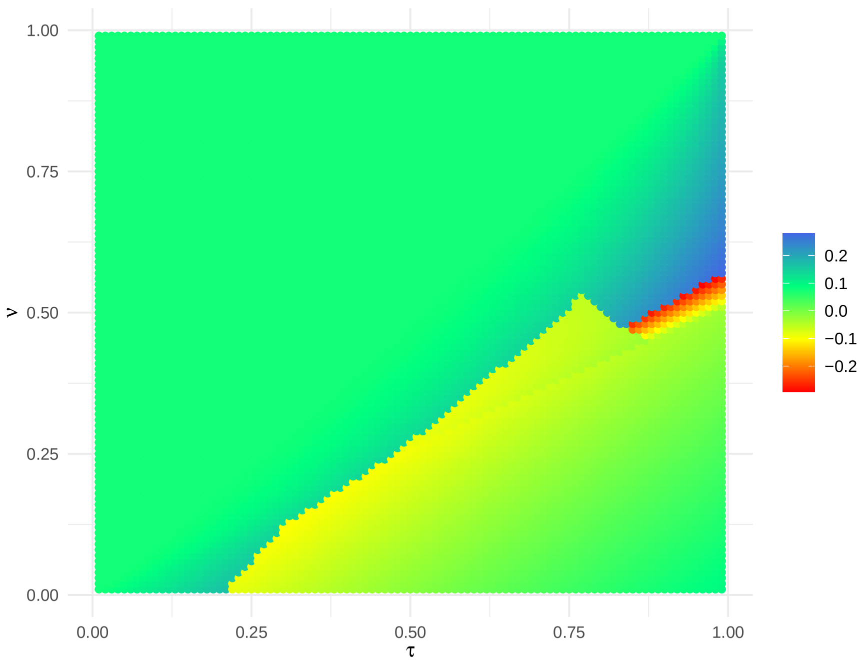
The code for this analysis is accessible at
https://drive.google.com/file/d/1q-MaZib1edPX5ZEUNFaZtdwA3bzRlvrg/view



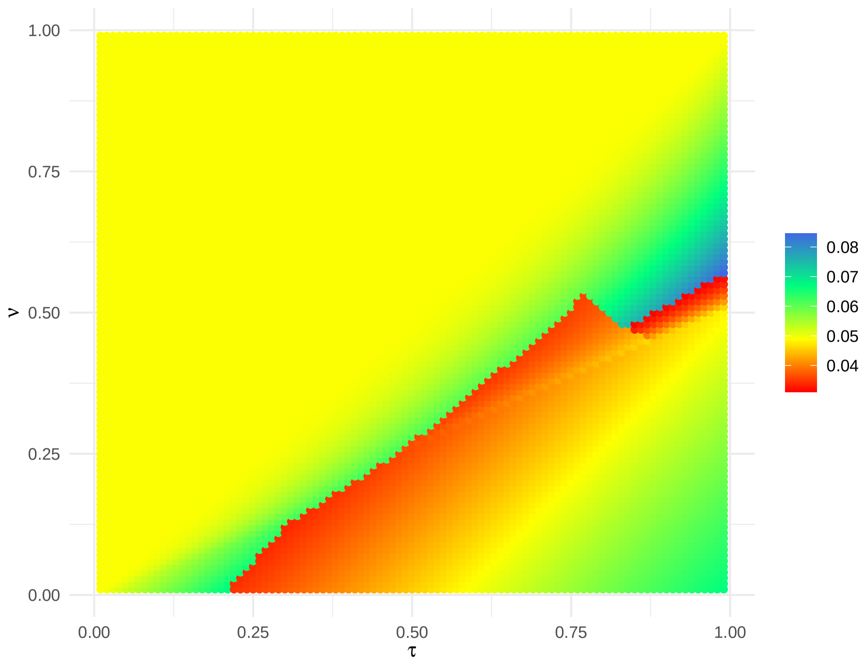
(a) Configuration



(b) Profit of T

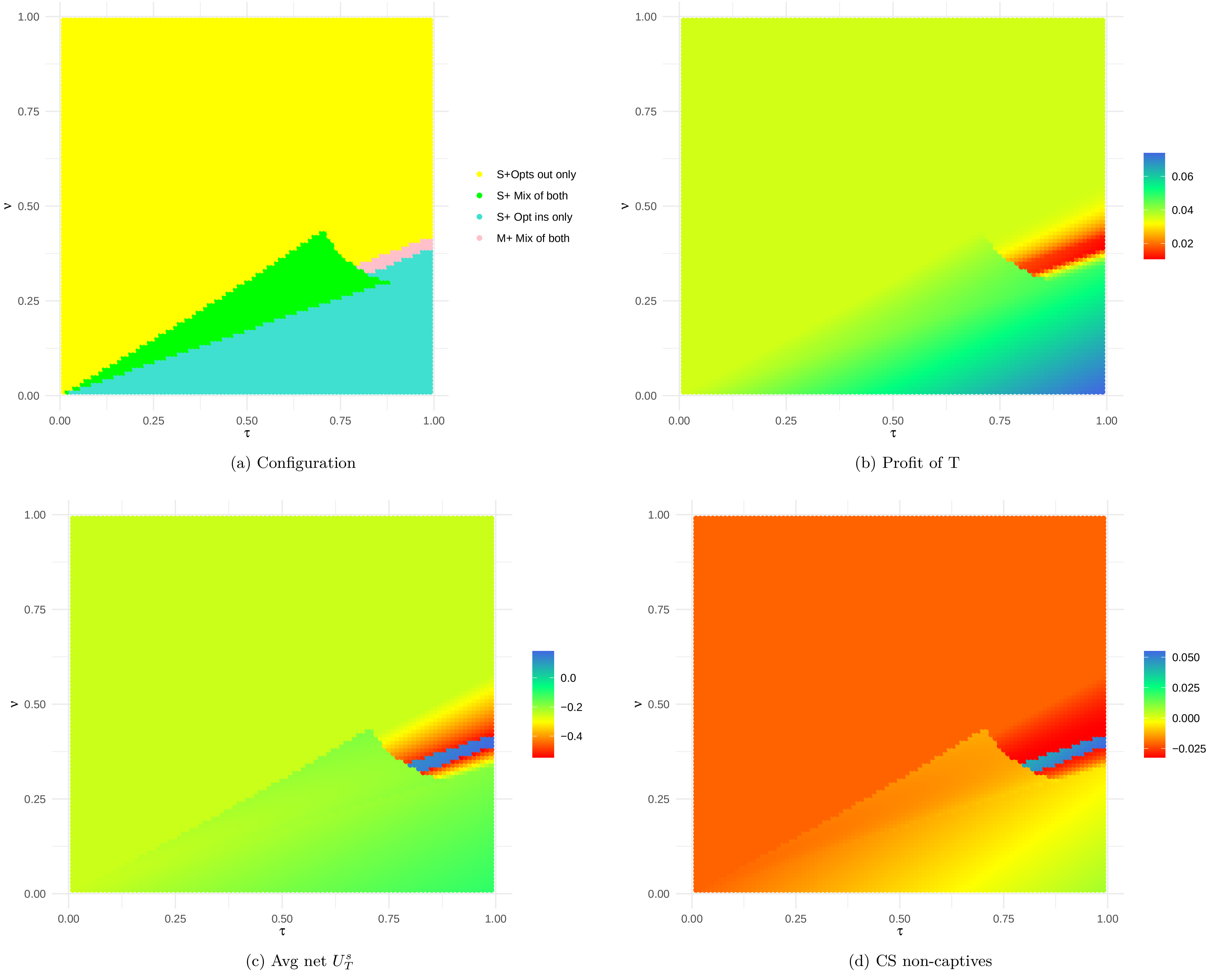


(c) Avg net U_T^s



(d) CS non-captives

Figure 1.6: Medium $\beta = 0.6$



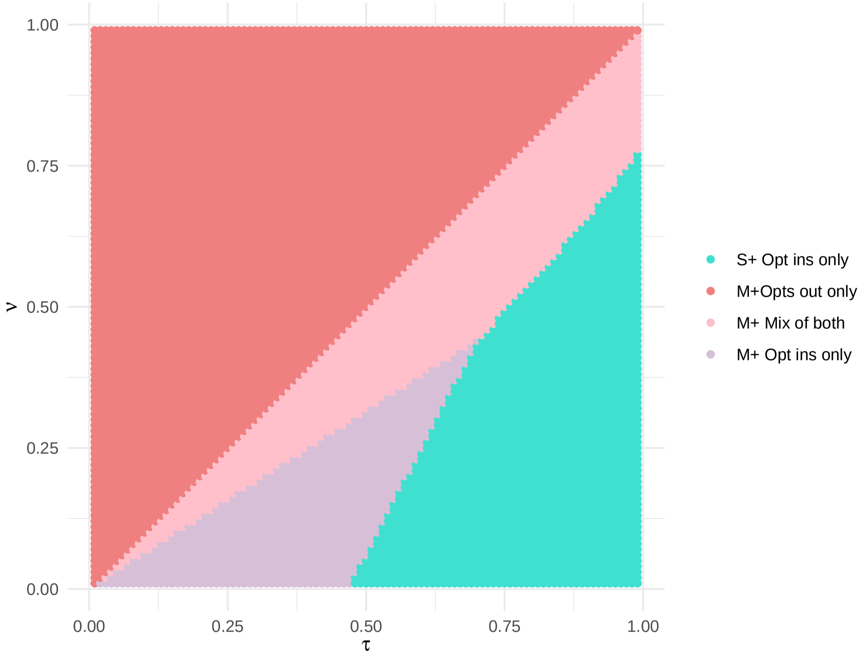
(a) Configuration

(b) Profit of T

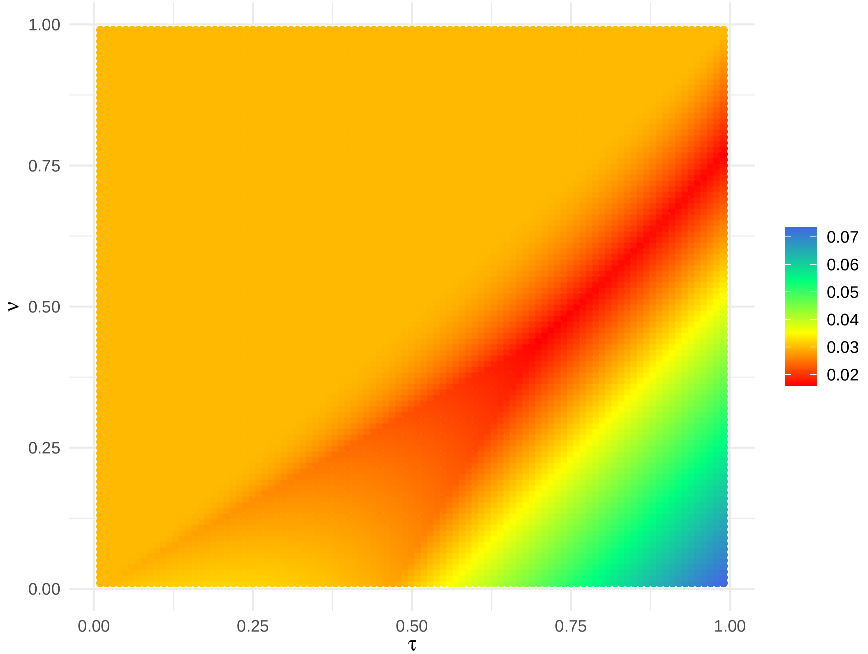
(c) Avg net U_T^s

(d) CS non-captives

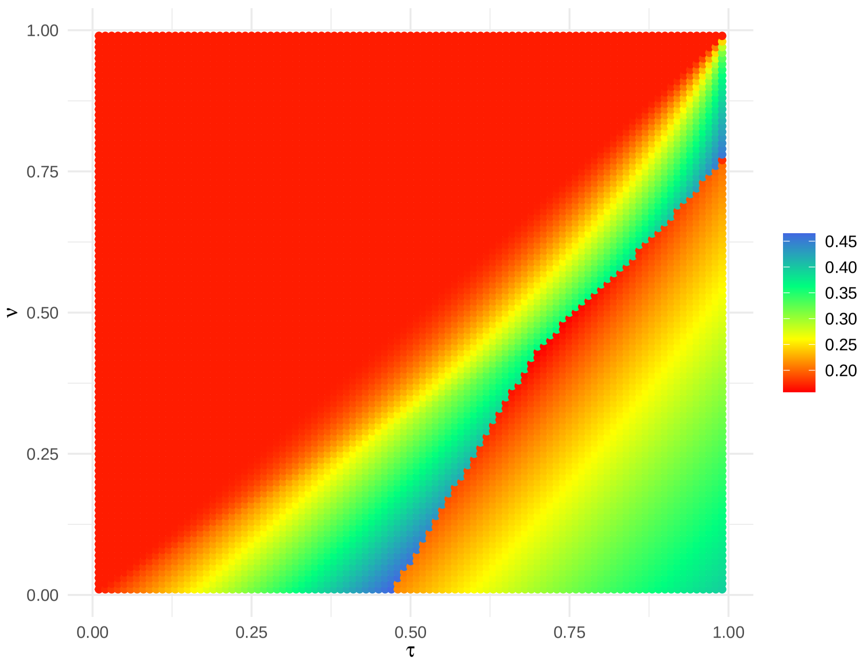
Figure 1.7: Low $\beta = 0.4$



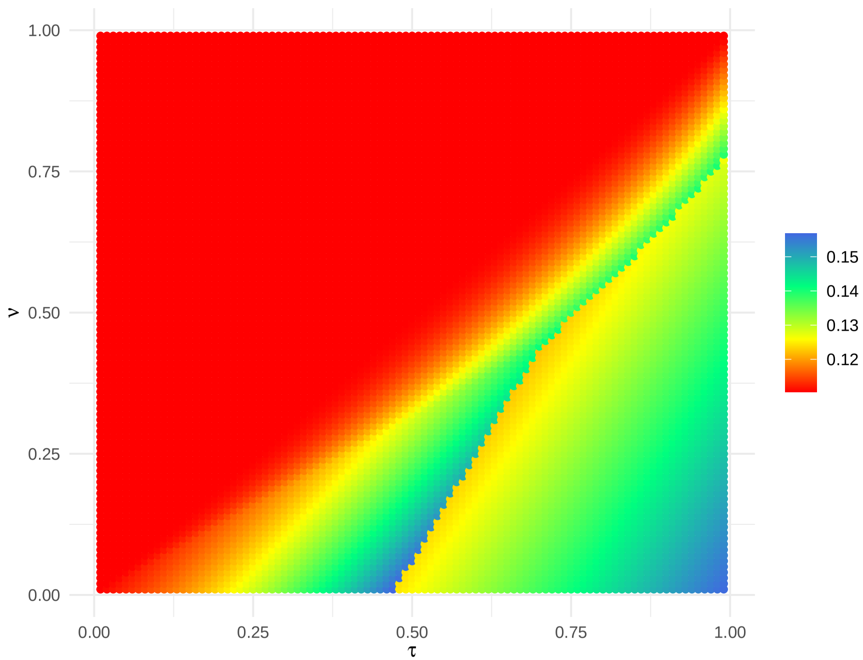
(a) Configuration



(b) Profit of T

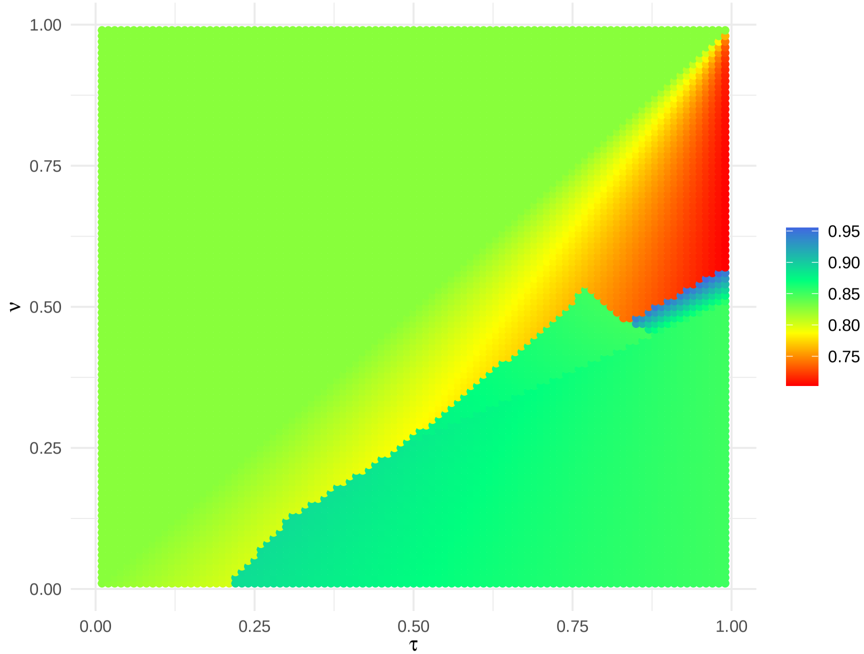


(c) Avg net U_T^s

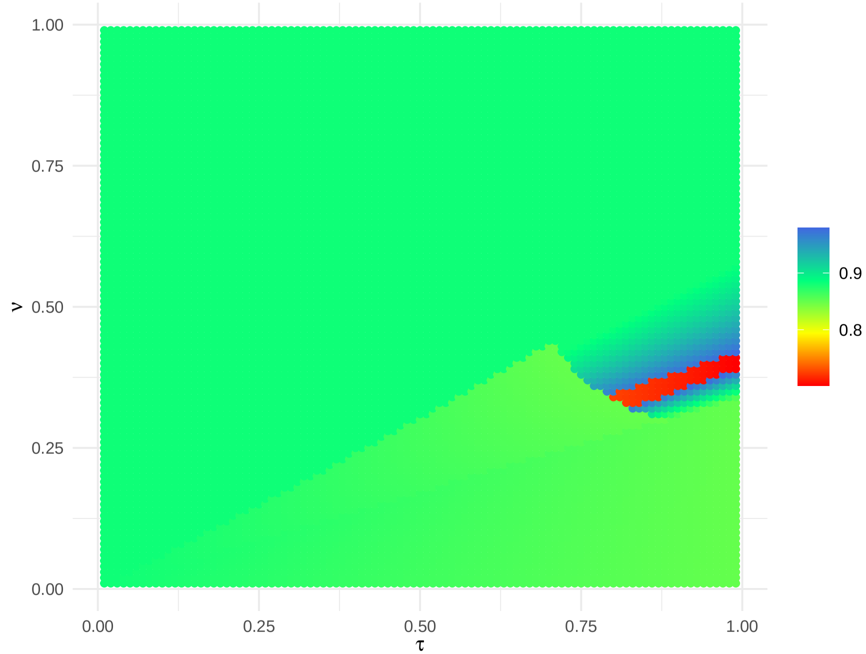


(d) CS non-captives

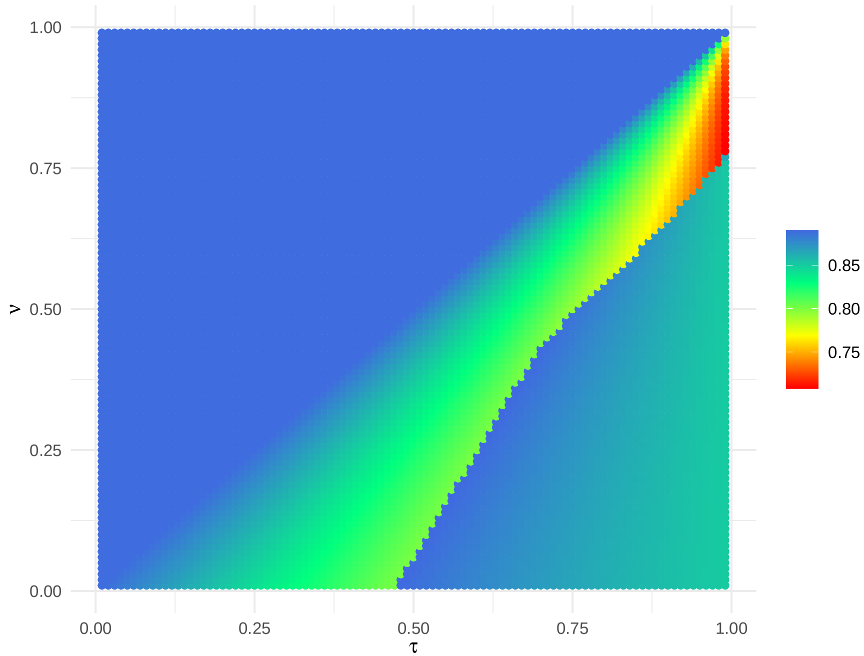
Figure 1.8: High $\beta = 0.9$



(a) Market share of W for $\beta = 0.6$



(b) Market share of W for $\beta = 0.4$



(c) Market share of W for $\beta = 0.9$

Figure 1.9: Market share of W is highest in the regions of low consumer surplus of non-captive consumers

Chapter 2

Data Sharing In Innovative Markets

Mudit Dhakar

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Abstract

We consider products with two-dimensional quality: one derived from data-enabled learning and another based on the level of standalone innovation. By allowing firms to compete on these dimensions, we study the implications of differences in data collected across firms on the incentives to invest in standalone quality in a two-period model and show that the investment by a data-lagging firm increases as data dominance shrinks. We then explore the possibility of using a non-stochastic full data sharing policy to reduce the data dominance. While such a policy always improves upon the innovation level, it can reduce consumer surplus in certain cases, and we provide a sufficient condition for cases where consumer surplus improves.

Keywords: Data Sharing, Data-enabled learning, Innovation, Regulation, Digital Markets Act

JEL Clasification: L15, L51,L86

2.1 Introduction

There are many products where qualities can be improved from the data which has been collected from earlier consumer usage. Think of search engines which can improve their algorithms as more and more users search on them. This can result in a self-reinforcing cycle and has been referred to in the literature as data-enabled learning (Hagiu and Wright 2023). These effects can potentially result in market tipping, and the policy-makers have, therefore, considered regulating such markets with data-sharing policies. An example of this is the European Union's Digital Market Act. The act designates entrenched firms as Gatekeepers and its Article 6(11) says that "The gatekeeper shall provide to any third-party undertaking providing online search engines, at its request, with access on fair, reasonable and non-discriminatory terms to ranking, query, click and view data in relation to free and paid search generated by end users on its online search engines. Any such query, click and view data that constitutes personal data shall be anonymised."

However, search engines fall into the category of products where qualities are not only determined by data but also non-data aspects. Schaefer, Sapi, and Lorincz 2018 shows that their quality is dependent on the past searches (data), and the general quality of the algorithm (non-data). This raises the question that while a data sharing policy might help smaller 3rd party search engines to improve their product on the data-enabled learning, how will this affect the incentives to innovate on non-data aspects? Will the benefits of data sharing policies be passed on to the consumers?

To look into these questions, we consider products where qualities are simultaneously determined by two distinct dimensions: a standalone quality and a data-based quality. We consider data as the cumulative number of sales a firm has made in the past. The product quality increases with higher data level (i.e. there are data-driven indirect effects). This allows us to capture the incumbency/first mover benefits as these firms have managed to collect a lot more data, as they had a longer time to sell their product compared to entrant firms. We model this data-enabled learning as per Hagiu and Wright (2020), by considering exogenously specified learning functions based on data which are strictly increasing. We consider a model where one player wins in each period, i.e. "winner-takes-all" competition. The data is then interpreted as the number of times a firm has won in the past.

The standalone (non-data-based) quality is modelled by assigning only two technology states, which are solely based on the level of standalone quality (and therefore, independent of data): technology leader and technology laggard. The gap in standalone quality between the two types is kept fixed. This is in line with Cabral 2018, Segal and Whinston 2007, which argue that IP protection is often limited and imitation is possible. Unlike the data-enabled learning functions, which are assumed to be exogenous, we let firms innovate on the standalone technology. For this paper, we consider an Arrow 1962's replacement effect style setting, where only the data-lagging firm can invest in innovation because we are interested in analysing the data-lacking firm's responses to different levels of data disadvantage it faces compared to the data-dominant firm.

These two dimensions of quality essentially provide firms with two separate tools to compete in the market. The only difference between the two is that while non-data quality can be improved quickly with investment, data quality takes time to improve due to the inherent need to collect data over time. Therefore, the non-data component is an effective tool for a data-lacking firm to overcome the obstacle of data dominance it faces in the market.

We study this model under a two-period set-up while assuming the data and non-data components to be independent of each other. We consider one firm to have dominance over both the data and non-data components in the starting period, and then analyse the innovation incentives of the data-lagging firm. We show that the level of innovation of the data lagging firm is decreasing in data dominance but increasing in standalone tech lead value. We then consider a data sharing policy where the data-dominant firm has to always share its data in the latter period. We show that such a policy always benefits a data-lacking firm, but it can increase or decrease the consumer surplus. It increases consumer surplus when the level of innovation is on the lower side. However, if the investment levels are high, such a policy can potentially end up doing more harm than benefit to the consumer. So, if consumer welfare is a priority for policymakers, we show that a data-sharing policy intervention is only recommended when there are high levels of data-based learning differences, which discourage data-lagging firms from investing.

This paper is limited in scope by not being suitable to answer any questions related to products which have interaction between data and non-data components. In addition, it will not be able to capture the dynamics seen in long-run models.

2.2 Related Literature

The effect of dominance on standalone innovation has been widely studied. Two of the studies are Cabral 2018, Segal and Whinston 2007 . They both assume a fixed pre-existing incumbency advantage. Both papers are concerned with the implications of dominance on innovation. The former studies the effort of technology sharing, and the latter focuses on antitrust policies. While they allow the process of becoming dominant to be endogenous (based on a successful innovation), the dominance level itself is unchanged, which we relax in this paper by allowing a level of innovation that is large enough to overcome data dominance.

This paper adds to the discussion on data sharing policies for products with data-enabled learning. Hagiu and Wright 2023 argue that when the standalone quality of the entrant is lower than that of the incumbent, which is also data dominant, then consumer surplus improves on data sharing. They derive this by assuming fixed values of the standalone qualities of the products of both firms. By allowing standalone quality to be an endogenously determined variable, we show that even when the incumbent is dominant in data as well as non-data dimensions, the consumer surplus can decrease upon implementing data sharing. This happens in certain cases when the innovation levels on standalone quality are often high under the no data sharing case.

De Corniere and Taylor 2020 is a complementary paper to ours, where they provide a broader discussion on when data sharing policies can backfire for consumers. They model competition in utilities and show that when access to higher data leads a firm to unilaterally increase the utility it offers to the consumers, then in equilibrium, consumer surplus goes up. On the other hand, if a firm wants to unilaterally decrease the utility with an increase in data and provided that offering extra utility is costly to the firm, consumer surplus goes down in equilibrium with higher data.

Prüfer and Schottmüller 2021 also considers innovation in data-driven markets. However, they focus on cost reduction based on an increase in the amount of data. Unlike them, we abstract away from any cost associated with the data-component and focus on costly innovation solely based on the data-independent component.

There are also empirical papers that have looked into data-sharing policies. Allcott et al. 2025 finds that if Bing has access to Google's data, its click-through rate of the top link will go up by around 24%. Their results support the idea that data-enabled quality improves with access to larger data.

2.3 The model

There are two firms, referred to as incumbent I and entrant E, which compete over two periods. They sell a product with two dimensions of quality: one standalone and one extracted from learning based on the data collected in the past.

We set up the date-enabled learning quality as per Hagiu and Wright 2023. In each period, there is a unit (indivisible) demand. Hence, in each period, the winner takes up the entire market. Data collected in the past is calculated by the number of wins in the past. Here, we let the initial data collection levels be N_I, N_E for firm I and E, respectively. The firm which wins in period 1 sees a unit increase in its data level and hence ends up with an improved product on the data dimension in period 2. The firms can process the data based on their exogenous specified data learning function at no additional cost. These functions are denoted by $f_I(.)$ and $f_E(.)$ respectively and are assumed to be strictly increasing and positive.

On standalone quality, in each period, there is a technology leader, denoted by the subscript T and a technology laggard, denoted by the subscript L. This assumes that the leader can be only one step ahead (denoted by λ), in terms of quality offered, than the laggard in any period [1]. For now, we are interested in studying the entrant firm's response to different levels of data disadvantage faced compared to the incumbent firm and, therefore, restrict our model to where only firm E will invest in innovation[2]. The firm E chooses the level of innovation $\phi \in [0, 1]$ with the cost of $c(\phi)$, which results in the probability of ϕ with which firm E will become the technology leader in period 2. The probabilistic nature of innovation captures the idea that not all efforts will lead to successful projects, and therefore, an entrant is facing an underlying uncertainty when it decides to innovate. Reducing uncertainty, by potentially carrying out multiple improvements simultaneously to improve the chances of success, is costly.

In any period, a firm $k \in \{I, E\}$ with price p_k and data level of N_k offers a utility of $\lambda + f_k(N_K) - p_k$ when it is tech leader, and otherwise it offers a utility of $f_k(N_K) - p_k$. We

[1] This is based on the idea that IP protection is often limited in markets like search, see Segal and Whinston 2007 for this type of modelling

[2] We present a discussion of when both firms can invest in innovation in appendix (2.B) and the results remain qualitatively similar.

allow the prices to be negative as well [3]. We assume the data and the non-data components of the quality to be independent and that there are no direct interactive effects between these two. The firms and consumers have perfect and complete information within each period, except for the realisation of whether an innovation is successful or not in period 1. This becomes known at the beginning of period 2.

The firms compete in asymmetric Bertrand competition in each period, and we focus on subgame perfect equilibrium. We assume that each firm maximises its present discount value (to be called PDV from now on) in period 1 with the flexibility of offering negative prices in period 1. The discount factor is common for both the firms as well as consumers, and is denoted by δ.

We focus on a setting where the incumbent has an initial advantage in both components of the quality. We assume that the data advantage is so large that one additional period of data will not be enough for firm E to overcome firm I's advantage. This can be seen as having an entrant that is severely disadvantaged, inspired by the situation we see in the search engine market today. To do this, we set (1) $f_I(N_I) > f_E(N_E + 1)$ and (2) that Firm I is the technology leader in period 1.

We denote the data-based learning difference of firm I over firm E: $\Delta(N_I, N_E) \equiv f_I(N_I) - f_E(N_E)$ and use the following notations for the rest of the papers:
(i) $\Delta_o \equiv \Delta(N_I, N_E)$ denotes the data learning advantage firm I has over firm E in period 1.
(ii) $\Delta_I \equiv \Delta(N_I + 1, N_E)$ denotes the data learning advantage firm I has over firm E in period 2 if firm I wins in period 1.
(iii) $\Delta_E \equiv \Delta(N_I, N_E + 1)$ denotes the data learning advantage firm I has over firm E in period 2 if firm E wins in period 1.

Assumption 2.1. *(a)* $\Delta_I < \lambda$
(b) Cost function $c(\cdot) : c(0) = 0, c'(0) = 0, c'' > 0$
(c) $c'(1) > \delta(\lambda - \Delta_E)$

The assumption (a) implies that when firm I has the highest level of data dominance in period 2 (via winning in period 1), firm E still has the potential to overcome this and win in period 2 when an innovation is successfully realised. This allows us to focus on cases where firm E has incentives to innovate. [4] (b) is a standard restriction on the behaviour of cost function. (c) ensures that if firm E were to win in period 1, which would allow it to charge

[3] In case of the search engines, which might seem to have an explicit zero price, there might be implicit prices. A positive price could be associated with the advertisement-based disutility. A negative price could be generated by a reward program. Bing Rewards is an example of this; it awards its users points based on product usage, which can then be used to get rebates on other products.

[4] Otherwise, there are no incentives to innovate, and that is beyond the scope of this paper, which is focused on innovative markets

higher prices, it still doesn't find it profitable to invest in innovation at a level where it would make the outcome of innovation deterministic (i.e. $\phi = 1$). This allows us to capture the idea that there are always risks for firm E when it invests in research.

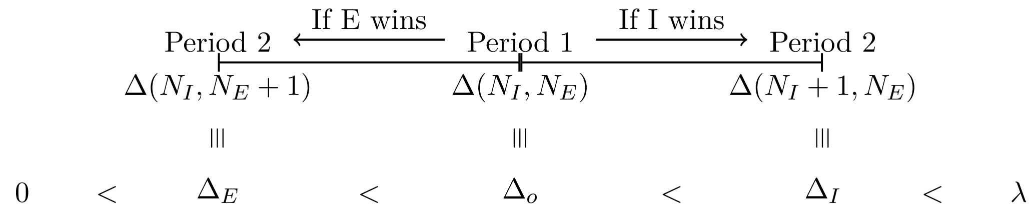

Figure 2.1: Shows the possible data-enabled learning quality advantage of firm I over firm E when going from period 1 to 2, along with assumptions on their values

The timing of the game is as follows:

Period 1: Firm I decides its price while firm E decides its price as well as the level of innovation. These decisions are taken simultaneously. The firms compete in asymmetric Bertrand competition.

Period 2: Firms and consumers learn about whether the innovation is successful or not. If successful, Firm E becomes the tech lead; otherwise, Firm I stays as the tech lead. The firms set the price simultaneously, again with no further innovation, and compete again in asymmetric Bertrand competition. This is the termination stage.

When consumers are indifferent between two firms, we assume that the tie is broken in favour of the tech-leading firm.

Equilibrium analysis:

Now, we can express the profit functions of both firms in period 1, expressed in net PDV, as follows :

$$V_{IT}^{(1)}(N_I, N_E) = \begin{cases} P_I^{(1)} + \delta[\phi V_{IL}^{(2)}(N_I+1, N_E) + (1-\phi)V_{IT}^{(2)}(N_I+1, N_E)] & \text{if I wins period 1} \\ \delta[\phi V_{IL}^{(2)}(N_I, N_E+1) + (1-\phi)V_{IT}^{(2)}(N_I, N_E+1)] & \text{otherwise} \end{cases}$$

$$V_{EL}^{(1)}(N_I, N_E) = \begin{cases} P_E^{(1)} + \delta[\phi V_{ET}^{(2)}(N_I, N_E+1) + (1-\phi)V_{EL}^{(2)}(N_I, N_E+1)] - c(\phi) & \text{if E wins period 1} \\ \\ \delta[\phi V_{ET}^{(2)}(N_I+1, N_E) + (1-\phi)V_{EL}^{(2)}(N_I+1, N_E)] - c(\phi) & \text{otherwise} \end{cases}$$

$V^{(1)}$ and $V^{(2)}$ represent the profits of period 1 (in PDV sense) and period 2, respectively. The subscript stands for the concerned firm and its tech status. Due to double dominance in period 1, the firm I is the tech leader and firm E is the laggard and hence only the value functions concerned with these states are relevant for period 1. For period 2, the firms weigh in on their probability of being tech leaders or laggards. Firm E is in the lead with ϕ probability while firm I is with $1 - \phi$. Since only firm E is allowed to invest, the cost of innovation gets reflected only in its profit function.

We solve this using backward induction. Under *Assumption 1*, λ is big enough to counter any disadvantages from the data component. For instance, the worst case of data dominance happens when firm I wins in period 1 and firm E loses. Still, firm E can win and make a profit of $\lambda - \Delta_I$, which is positive by *Assumption 1(a)*, by offering a price to exactly match firm I 's utility level of $f_I(N_I + 1)$. This ensures that firm E always wins in period 2 when it's the tech lead. Trivially, Firm I wins when it's the tech lead, as it has more to offer to consumers due to its advantage on both data and non-data components over Firm E. This means that the tech leader in period 2 will always become the winner. This generates the incentives for firm E to always invest a strictly positive amount in innovation in period 1. So, the standalone quality in this model is an effective tool to overcome the data dominance effect.

Firms internalise this behaviour of period 2 in period 1. And we have the following result :

Proposition 2.1. *The firm I always wins in period 1. The technologically advanced firm wins in period 2. The optimal level of investment ϕ^* is positive and solves : $c'(\phi^*) = (\lambda - \Delta_I)\delta$*

Due to the assumed double dominance, firm E will only offer a huge subsidy to win in period 1 if gains from data learning improvements in period 2 are massive. However, under the assumption of very high levels of data advantage of firm I over firm E, firm E will remain data disadvantaged in period 2, even if it wins in period 1 (i.e. $f_I(N_I) > f_E(N_E + 1)$). Firm E, therefore, will not find it profitable to offer this subsidy in period 1. So the firm E can only make some positive profit in period 2 if it were to become the tech leader. It sets its optimal level of investment in innovation to maximise this profit, and therefore, the level of investment is only a function of the profit of firm E in period 2, where firm I has won in period 1.

Corollary 2.1. *The optimal level of investment ϕ^* is increasing in the standalone quality gap λ and decreasing in firm I's data dominance in the second period, Δ_I.*

This simply follows from the fact that a higher gap in quality between firm E and firm I in period 2, when firm E is tech lead, allows it to charge higher prices and, in turn, have higher profits when the innovation is successful. Therefore, there are higher incentives for firm E to invest in innovation with a higher quality gap, which increases with λ and decreases with the data advantage firm I has over firm E.

So, we see that a reduction in data dominance helps in innovation incentives. Policies like the European Union's Digital Market Act are aimed at achieving this reduction in dominance via data sharing from data-dominant firms to data-disadvantaged firms. But the question of whether these benefits pass on to consumers remains. We look at this in the following section.

2.4 Data Sharing

In order to introduce a data sharing policy, we make some modifications to the baseline model. We restrict the data learning function to be similar for both the firms, i.e. $f_I = f_E \equiv f$ and assume $N_I \geq N_E + 2$. The prior allows a full data sharing policy to completely kill the data dominance advantage, while the latter is essentially an adaptation of an earlier assumption of $f_I(N_I) > f_E(N_E + 1)$ for the same learning function. In this context, it implies that even if firm E were to win in period 1, it would stay disadvantaged from the lack of data compared to firm I and any data sharing policy will present itself with a scope of improving firm E's product.

While data sharing policies may manifest in many forms, we consider a full non-stochastic data sharing policy: Firm I has to share its entire data with Firm E at the beginning of period 2. While restrictive, this simple policy is sufficient to illustrate non-monotonic changes in consumer surplus.[5].

For the sake of simplicity, we assume that firm E doesn't originally possess any unique dataset [6]. So, under this setting, in period 2, both firms have access to the same data and, in turn, the same data-enabled learning quality. Therefore, the tech leader in period 2 will win and will charge λ as the price. Notice that firm E now has no incentive to win in period 1, as it gets to free ride on firm I's data in period 2. It, therefore, offers a subsidy of zero in period 1. Firm I wins in period 1 by charging a price equal to the difference between the product qualities of both firms. The incentive to put money in innovation is still determined by the incentive to become the tech leader in order to make positive profits in period 2.

Proposition 2.2. *1) Firm I will always win in period 1 under a non-stochastic full data sharing policy. The technology leader wins in period 2.*
2) The optimal innovation level under data sharing $\phi_s^* = min\{1, \hat{\phi}\}$ *where* $\hat{\phi}$ *solves* $c'(\hat{\phi}) = \lambda\delta$

[5]The discussion below will naturally extend to a stochastic data sharing policy where firm I has to share its data with some probability only, and the results will be robust to such a specification.

[6]In case of search engines, this assumption can be justified by the observations in Klein et al. 2023 that a small search engine can produce non-personalised results of similar quality to the dominant firm's but struggle with search related to rare search queries. A data sharing policy would give Firm E access to these additional, rarely searched queries. These rare queries form 74% of the observations in Klein et al. 2023 and hence are of significant nature.

. The level of investment in innovation by firm E is strictly higher with a data sharing policy and is independent of the disadvantage due to the data dominance of firm I.

So, the data sharing policy is beneficial to firm E as it can price higher when it wins (it no longer has to compensate consumers due to its product being inferior on the data dimension in the baseline case). The benefit of the price increase gets further magnified as firm E also finds it profitable to invest more in innovation to increase its chances of becoming a tech leader and, in turn, increase its expected PDV in period 1.

We now look at the consumer surplus. To analyse this, we consider the present discounted value of surplus for consumers in period 1. When a data sharing policy is implemented, it generates two opposite effects on the consumer surplus. Firstly, data sharing makes the market more competitive in period 2 as there is lower product differentiation compared to the baseline. This causes firm I, on winning, to offer a net surplus of $f(N_E)$ without data sharing and $f(N_I + 1)$ with data sharing. Therefore, consumers benefit when firm I is the tech lead and wins in period 2. Firm E's net surplus offer stays the same (as its competitor's quality of $f(N_I+1)$ stays the same with or without the data sharing), and the consumers are indifferent when firm E is tech lead compared to the baseline case. In expectation in period 1, consumers are better off. Secondly, in period 1, since firm E no longer offers any subsidy (i.e. charges a zero price), firm I in response increases its price. Since firm I is always the winner in period 1, consumers are worse off in this period with data sharing. The following result illustrates a cut-off across which one effect dominates the other.

Proposition 2.3. *1) A non-stochastic full data sharing policy will increase the price of products by both firms in period 1. It increases the price of firm E's product while keeping the net surplus unchanged in period 2. Firm I's product price declines in period 2.*
2) If $\delta(\lambda - \Delta_I) < c'\left(\frac{1}{2 - \frac{\Delta_E}{\Delta_I}}\right)$*, then the PDV of consumer surplus increases under data sharing, and it reduces otherwise.*

The intuition is as follows: Under data sharing, firm I's price reduces by Δ_I as it faces a stronger competitor in period 2. The higher the Δ_I, the higher will be the gain in the consumer surplus, and the more likely the above inequality will be satisfied as the left-hand side reduces in value. The other effect of data sharing is the removal of the subsidy in period 1 by firm E. This subsidy is determined by the gap $(\Delta_E - \Delta_I)$ as this reflects the gain in data-enabled learning product quality in period 2 that firm E can make over firm I, if firm E were to win in period 1 instead of firm I. The smaller this gap, the smaller the subsidy in the baseline case, the smaller the negative effect on consumer surplus with data sharing. A smaller gap also means higher $\frac{\Delta_E}{\Delta_I}$, and this increases the right-hand side of the equality.

These effects are illustrated in the figure 2.2, where both types of effects on consumer surplus are shown by performing comparative statics with Δ_I and Δ_E. The first figure illustrates only the subsidy effect by fixing Δ_I. As Δ_E increases, the difference between the two values Δ_I and Δ_E reduces, and the subsidy level in period 1 gets smaller in the baseline case. Therefore, the negative effect of data sharing continues to shrink while the positive effect of reduced price by firm I in period 2 stays fixed (as Δ_I is fixed). Hence, the change in consumer surplus with the data sharing eventually shifts from negative to positive as Δ_E increases. The second illustrates both effects by fixing Δ_E. As Δ_I increases, the positive effect of reduced prices by firm I in period 2 in the baseline increases and this increases gains in consumer surplus associated with data sharing. There is, however, a subsidy effect too, which is getting stronger with the increasing gap between Δ_I and Δ_E, but this effect is much weaker than the price reduction effect. Therefore, the consumer surplus changes with data sharing shifts from positive to negative as Δ_I increases.
Observe that in the inequality in proposition 2.3, the left-hand side also dictates the level of innovation in the baseline case and therefore, can also be read in the following way :

Corollary 2.2. *1) The consumer surplus increases under data sharing compared to the baseline case when the innovation level by firm E (denoted by ϕ^*) under the no data sharing case satisfies* $\phi^* < \dfrac{1}{2 - \frac{\Delta_E}{\Delta_I}}$
2) $\delta(\lambda - \Delta_I) < c'(0.5)$ *i.e.,* $\phi^* < \frac{1}{2}$ *is a sufficient condition for increasing consumer surplus on implementation of data sharing policy*

Therefore, we have that *when the level of investment in innovation is low in the baseline, a data sharing policy will always improve consumer surplus. Alternatively, when the level of innovation is high in the baseline, such a policy could potentially be detrimental to consumers' welfare.* When consumers are exactly worse off under a high innovation level depends on the level of subsidy by firm E in period 1 in the baseline. If the subsidy level is high, the data sharing policy hurts consumers, while if it's low, the data sharing policy will still be beneficial. A (smaller) larger subsidy effect could be seen in cases where the returns of data are (small) large, as that would generate a (smaller) larger gap between Δ_I and Δ_E. *Hence, when there is low data dominance or high innovation quality gain (i.e. low Δ_I or high λ causing higher innovation level in baseline) and higher returns to the data (resulting in high subsidies in baseline), the data sharing policy will backfire for the consumers.*

Special Case of logarithmic learning function and the maturity of the industry
An example to illustrate proposition 2.3 is to consider a special case of logarithmic learning, which Allcott et al. 2025 finds to be valid in the case of the search engines. Since, we know,

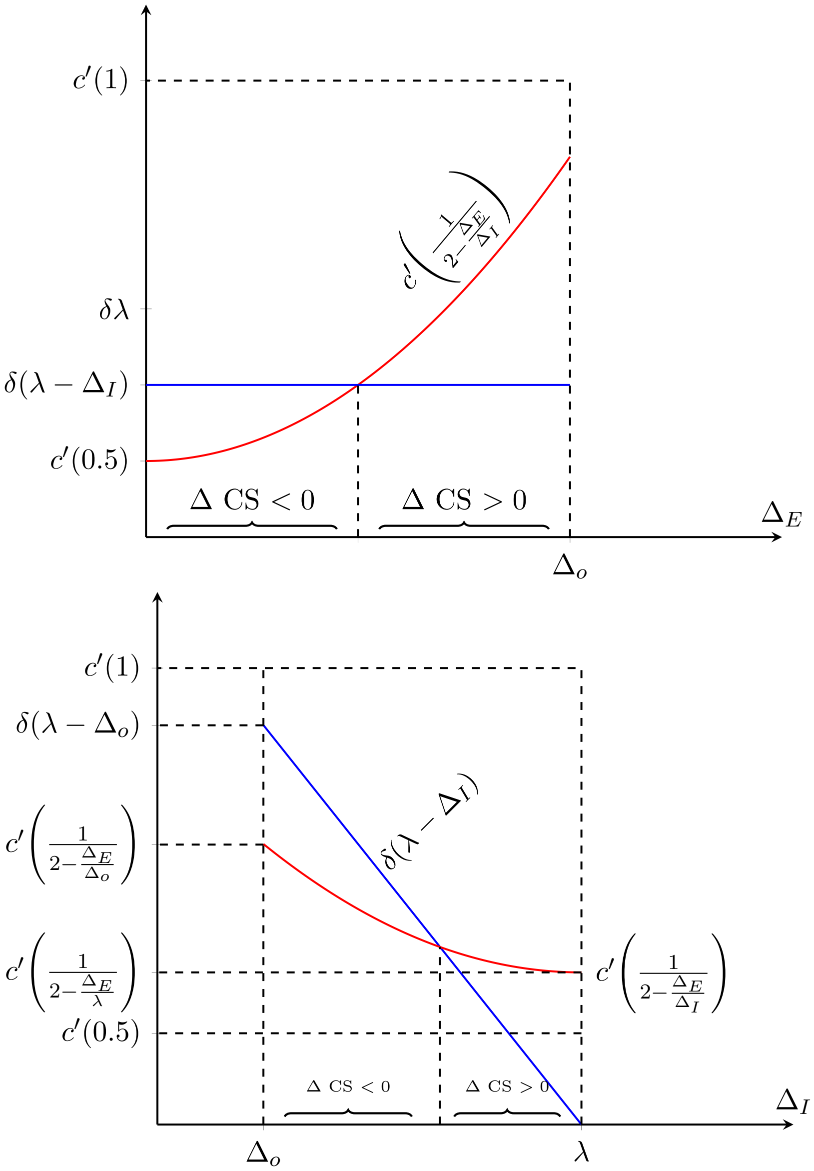

Figure 2.2: The figure illustrates cases of where CS declines post data sharing. $\Delta CS \equiv$ CS under sharing- CS under baseline case. The first figure fixes Δ_I such that $\delta(\lambda + \Delta_I) > c'(0.5)$ and varies Δ_E on x-axis. The second figure fixes Δ_E and varies Δ_I on x-axis.

that very high levels of data dominance can lead to low levels of innovation in the baseline case, which is then associated with an improvement in consumer surplus with a data sharing policy; we focus on the interesting case of setting data dominance to as low as possible, i.e, by setting just one period gap of data levels between firm I and E: $N_I = N_E + 1$. This implies that $\Delta_E = 0$, $\Delta_I = f(N_E + 2) - f(N_E)$. The inequality in proposition 2.3 is then reduced to

$$\lambda - (f(N_E + 2) - f(N_E)) < \frac{1}{\delta} c' \left(\frac{1}{2}\right)$$

When the marginal returns to data are (low) high, despite having a small data difference, the data-enabled learning difference between firm I and E, $f(N_E + 2) - f(N_E)$, is (small) large. This results in (high) low levels of innovation in the baseline, and the consumer surplus (falls) improves with a data sharing policy. This effect is illustrated in the figure 2.3. In the logarithmic curve, lower(higher) levels of data are associated with higher(lower) marginal gains in learning from additional data. *The consumer surplus with data sharing, thus, improves at lower levels of data (i.e., a young market) while it decreases at higher levels of data (i.e., a mature/established market)*

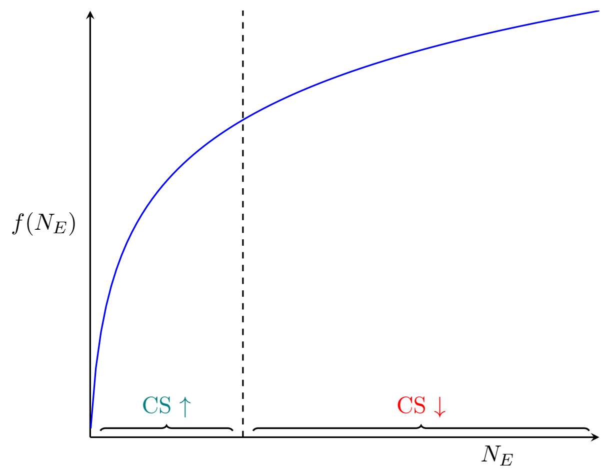

Figure 2.3: Under limited data dominance and when the data-enabled learning is a logarithmic function, the implications of change in consumer surplus with a data sharing policy will depend on the age of the industry (captured by the level of periods of data observations firm E has, N_E)

2.5 Conclusion

We consider the products with two-dimensional quality: one based on data and the other a standalone component. By letting companies innovate to improve the standalone component, under an exogenous data learning function specification, we study the effect of data sharing policy on innovation and, in turn, consumer surplus in a two-period model.

We show that under a deterministic full data sharing policy, which always forces a firm with data dominance to share its data, the innovation level of the data-lagging firm always increases. However, the effects of data sharing are less clear on its effect on consumer surplus. While a policy leads to reduced pricing by a data-dominant firm in the latter period without affecting the behaviour of the data-lagging firm, it also causes an increase in prices by both firms in the period just before its implementation. Whichever effect among the aforementioned two dominates gets to decide the change in consumer surplus on the implementation of data sharing.

We show that when the level of investment in innovation is on the lower side in the case with no data sharing, then it's beneficial to implement data sharing as it increases consumer surplus. However, if the investment levels are high, a data sharing policy can potentially end up doing more harm than benefit to the consumer. The data-lagging firm always benefits from a data-sharing policy. So, if consumer welfare is a priority for policymakers, a policy intervention is only recommended when there are high levels of data-based learning differences, which discourage data-lagging firms from investing.

While we have considered a simplistic model to illustrate that data-sharing policies in data-dominant markets might not always benefit consumers, several interesting future prospects remain to be studied. Some of these include understanding data trading incentives in the model, extending the model to a finite horizon, designing a stochastic as well as partial data sharing policy and a dominance threshold-based data sharing policy (only kicks in when a certain data dominance level is exceeded).

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Appendix for Chapter 2

Appendix 2.A Proofs

Proof of Proposition 2.1

The firm I always wins in period 1. The technologically advanced firm wins in period 2. The optimal level of investment ϕ^* is positive and solves : $c'(\phi^*) = (\lambda - \Delta_I)\delta$

Solving period 2's subgame,

There are four possibilities depending on which firm won in period 1 and which firm becomes the tech leader in period 2.

Winner in (1)	Tech Leader in (2)	Gross Utility of I	Gross utility of E	Difference between I and E
I	E	$f_I(N_I+1)$	$\lambda + f_E(N_E)$	$\Delta_I - \lambda$
I	I	$\lambda + f_I(N_I+1)$	$f_E(N_E)$	$\Delta_I + \lambda$
E	E	$f_I(N_I)$	$\lambda + f_E(N_E+1)$	$\Delta_E - \lambda$
E	I	$\lambda + f_I(N_I)$	$f_E(N_E+1)$	$\Delta_E + \lambda$

From Assumption (2.1) we have $\lambda > \Delta_I$, it follows that the difference between I and E's quality is only positive(negative) when I(E) is the tech lead.

So, it follows that the tech laggard firm sets a price of zero, and the lead firm sets the price such that it offers competitors gross utility, as this makes consumers indifferent, and they choose the tech leading firm's product.

Winner in (1)	Tech Leader in (2)	$P_I^{(2)}$	$P_E^{(2)}$	Net utility in (2)
I	E	0	$\lambda - \Delta_I$	$f_I(N_I+1)$
I	I	$\lambda + \Delta_I$	0	$f_E(N_E)$
E	E	0	$\lambda - \Delta_E$	$f_I(N_I)$
E	I	$\lambda + \Delta_E$	0	$f_E(N_E+1)$

Next, solving for period 1's:

The updated PDV are as follows:

$$V_{IT}^{(1)}(N_I, N_E) = \begin{cases} P_I^{(1)} + \delta(1-\phi)(\lambda + \Delta_I) & \text{if I wins period 1} \\ \delta(1-\phi)(\lambda + \Delta_E) & \text{otherwise} \end{cases}$$

$$V_{EL}^{(1)}(N_I, N_E) = \begin{cases} P_E^{(1)} + \delta\phi(\lambda - \Delta_E) - c(\phi) & \text{if E wins period 1} \\ \delta\phi(\lambda - \Delta_I) - c(\phi) & \text{otherwise} \end{cases}$$

Firm E prefers winning in the first period, if

$$P_E^{(1)} + \delta\phi(\lambda - \Delta_E) - c(\phi) \geq \delta\phi(\lambda - \Delta_I) - c(\phi)$$

$$\implies P_E^{(1)} \geq \delta\phi(\Delta_E - \Delta_I)$$

This inequality shows the minimum price (or maximum subsidy) at which firm E finds it optimal to win in period 1.

At this minimum price, consumers' net utility from E in period 1 $= f_E(N_E) - \delta\phi(\Delta_E - \Delta_I)$

Similarly, for firm I,

$$P_I^{(1)} \geq \delta(1-\phi)(\Delta_E - \Delta_I)$$

At this minimum price, consumers' net utility from I in period 1 $= \lambda + f_I(N_I) - \delta(1-\phi)(\Delta_E - \Delta_I)$

If both firms were to charge the minimum price, consumers would be better off with firm I when

$$\lambda + f_I(N_I) - \delta(1-\phi)(\Delta_E - \Delta_I) > f_E(N_E) - \delta\phi(\Delta_E - \Delta_I)$$

$$\iff \lambda + \Delta_o > \delta(1-2\phi)(\Delta_E - \Delta_I)$$

which is true $\forall\phi$ as $\lambda > \Delta_I > \delta(\Delta_I - \Delta_E) > \delta(1-2\phi)(\Delta_I - \Delta_E)$

So, in equilibrium, firm E will charge its minimum price while firm I's pricing strategy is to make consumers indifferent so that consumers choose firm I due to it being the tech leader.

$$P_E^{(1)} = \delta\phi(\Delta_E - \Delta_I)$$

and $P_I^{(1)}$ solves

$$\lambda + f_I(N_I) - P_I^{(1)} = f_E(N_E) - \delta\phi(\Delta_E - \Delta_I)$$

$$\implies P_I^{(1)} = \lambda + \Delta_o + \delta\phi(\Delta_E - \Delta_I)$$

Firm E's problem is to

$$\max_{\phi} \delta\phi(\lambda - \Delta_I) - c(\phi)$$

which is maximised at

$$c'(\phi^*) = \delta(\lambda - \Delta_I)$$

□

Proof of Proposition 2.2

Proof is similar to that of proposition 2.1, we have four possibilities in stage 2 :

Winner in (1)	Tech Leader in (2)	Gross Utility of I	Gross utility of E	Difference between I and E
I	E	$f(N_I+1)$	$\lambda+f(N_I+1)$	$-\lambda$
I	I	$\lambda+f(N_I+1)$	$f(N_I+1)$	λ
E	E	$f(N_I)$	$\lambda+f(N_I)$	$-\lambda$
E	I	$\lambda+f(N_I)$	$f(N_I)$	λ

Since the level of data-enabled learning quality is the same across firm I and E in period 2, it's straightforward to see that the tech leader wins in period 2.

Winner in (1)	Tech Leader in (2)	$P_I^{(2)}$	$P_E^{(2)}$	Net utility in (2)
I	E	0	λ	$f(N_I+1)$
I	I	λ	0	$f(N_I+1)$
E	E	0	λ	$f(N_I)$
E	I	λ	0	$f(N_I)$

Next, solving for period 1's:

The updated PDV are as follows:

$$V_{IT}^{(1)}(N_I,N_E)=\begin{cases} P_I^{(1)}+\delta(1-\phi)\lambda & \text{if I wins period 1} \\ \delta(1-\phi)\lambda & \text{otherwise} \end{cases}$$

$$V_{EL}^{(1)}(N_I,N_E)=\begin{cases} P_E^{(1)}+\delta\phi\lambda-c(\phi) & \text{if E wins period 1} \\ \delta\phi\lambda-c(\phi) & \text{otherwise} \end{cases}$$

It follows that the minimum price at which firms win is zero. Given firm I's double dominance in period 1, we have that, in equilibrium, firm E will charge 0 while firm I's pricing strategy, again, is to make consumers indifferent so that consumers choose firm I due to it being the tech leader.

$$P_E^{(1)}=0$$

and $P_I^{(1)}$ solves

$$\lambda+f(N_I)-P_I^{(1)}=f(N_E)$$

$$\implies P_I^{(1)}=\lambda+\Delta_o$$

Firm E's problem is to

$$\max_{\phi}\delta\phi\lambda-c(\phi)$$

which is maximised at

$$\begin{cases} c'(\phi_s^*) = \delta\lambda \text{ if } c'^{-1}(\delta\lambda) < 1 \\ \phi_s^* = 1 \text{ otherwise} \end{cases}$$

The level of innovation increases under data sharing compared to the baseline as $\delta(\lambda - \Delta_I) < \delta\lambda \implies c'(\delta(\lambda - \Delta_I)) < c'(\delta\lambda) \implies \phi^* < \phi_s^*$

□

Proof of Proposition 2.3

The first part of the proposition is established by comparing prices derived in the proof of propositions 2.1 and 2.2.

Consumer surplus = Net surplus in (1) + $\delta[\phi$ Net utility in (2) when E is tech lead+ $(1-\phi)$ Net utility in (2) when I is the tech lead]

Under the baseline case, we have ‘

$$\text{CS}_o = f(N_E) - \delta\phi^*(\Delta_E - \Delta_I) + \delta[\phi^*(f(N_I+1)) + (1-\phi^*)(f(N_E))]$$

Under the data sharing case, we have

$$\text{CS}_{DS} = f(N_E) + \delta f(N_I + 1)$$

The consumer surplus increases under data sharing if

$$\Delta\text{CS} = \text{CS}_{DS} - \text{CS}_o \geq 0$$

$$\iff \delta f(N_I+1) + \delta\phi^*(\Delta_E - \Delta_I) - \delta[\phi^*(f(N_I+1)) + (1-\phi^*)(f(N_E))] > 0$$

$$\iff \phi^*(\Delta_E - \Delta_I) - [-f(N_I+1) + \phi^*(f(N_I+1)) + (1-\phi^*)(f(N_E))] > 0$$

$$\iff \phi^*(\Delta_E - \Delta_I) + (1-\phi^*)[f(N_I+1) - f(N_E)] > 0$$

$$\iff \frac{1}{2 - \dfrac{\Delta_E}{\Delta_I}} > \phi^* \quad \text{using } \Delta_I = f(N_I+1) - f(N_E)$$

Above equation implies that

$$\iff c'\left(\frac{1}{2 - \frac{\Delta_E}{\Delta_I}}\right) > c'(\phi^*) \quad \text{as} \quad c''(.) > 0$$

Since $c'(\phi^*) = \delta(\lambda - \Delta_I)$, we have :

$$\iff c'\left(\frac{1}{2 - \frac{\Delta_E}{\Delta_I}}\right) > \delta(\lambda - \Delta_I)$$

□

Appendix 2.B Double Innovation

We extend the model to a setting where firm I can also innovate. Let the level of innovation by firm I and E be ϕ_I and ϕ_E respectively. Based on the success of innovation, the new tech leader is determined as follows :

I's innovation	E's Innovation	Tech leader
Success	Success	I with the probability of retention $0 < r < 1$, E otherwise
Success	Failure	I
Failure	Success	E
Failure	Failure	I

So, the probability of firm E becoming tech leader is given by

$$\phi_E(1 - \phi_I) + \phi_E(\phi_I)(1 - r) = \phi_E(1 - \phi_I r) \equiv \tilde{\phi}$$

We can simply follow the proof of proposition 2.1 replacing ϕ with $\tilde{\phi}$ as probability of firm E becoming tech lead.
The new PDV of profit are:
For firm E:

$$\max_{\phi_E} \ \delta\tilde{\phi}(\lambda - \Delta_I) - c(\phi_E)$$

For fi'rm I:

$$\max_{\phi_I} \ \lambda + \Delta_o + \delta\tilde{\phi}(\Delta_E - \Delta_I) + \delta(1 - \tilde{\phi})(\lambda + \Delta_I) - c(\phi_I)$$

First order conditions:

$$c'(\phi_E) = \delta(1 - \phi_I r)(\lambda - \Delta_I)$$

$$\begin{cases} c'(\phi_I) = \delta\phi_E r[\lambda + 2\Delta_I - \Delta_E] \text{ when } c'(1) > \delta\phi_E r[\lambda + 2\Delta_I - \Delta_E] \\ \phi_I = 1 \quad \text{otherwise} \end{cases}$$

The existence of a solution to these two equations can be established graphically as seen in the figure below under the assumption 2.1.
Consumers surplus from the proof of proposition 2.3 can also be written using $\tilde{\phi}$ under

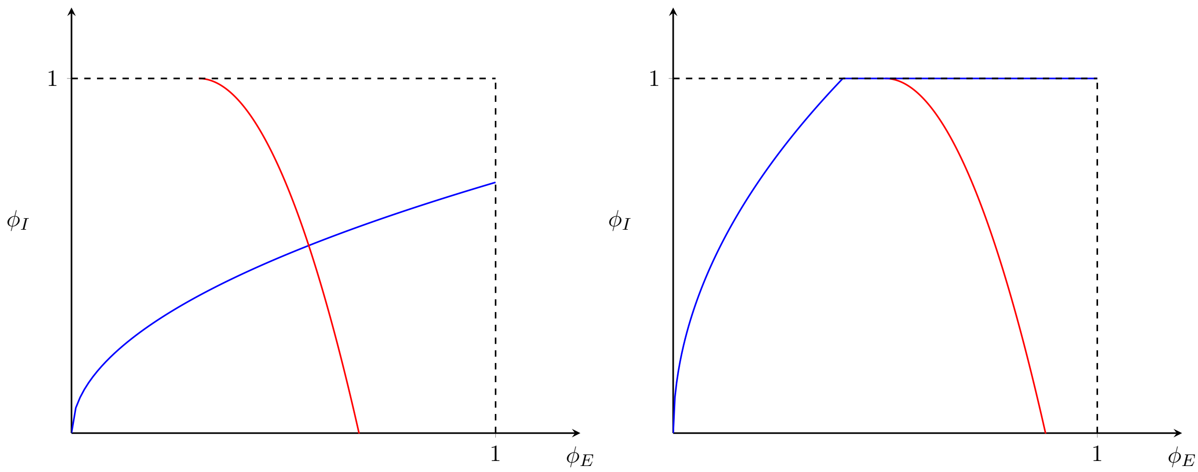

Figure 2.4: The figure shows plots of the two first-order conditions of firm I and E depicting all possibilities. Red (blue) corresponds to firm E (I)'s first order condition.

baseline as '

$$\text{CS}_o = f(N_E) - \delta\tilde{\phi}(\Delta_E - \Delta_I) + \delta[\tilde{\phi}(f(N_I + 1)) + (1 - \tilde{\phi})(f(N_E))]$$

While it remains unchanged under the data sharing case:

$$\text{CS}_{DS} = f(N_E) + \delta f(N_I + 1)$$

As in proposition 2.3, we get

$$\frac{1}{2 - \dfrac{\Delta_E}{\Delta_I}} > \tilde{\phi} = \phi_E(1 - \phi_I r)$$

So, when there is a low innovation level by firm E and/or a high innovation by firm I without data sharing, a data sharing policy is beneficial for consumers.

Chapter 3

Decarbonising Aviation by Taxes

Mudit Dhakar

Marc Ivaldi
Toulouse School of Economics

Miguel Urdanoz
Toulouse Business School

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Abstract

We compare the taxes on airport charges with those of conventional taxes like fuel tax as tools to discourage flying and, in turn, reduce carbon emissions. We find that, unlike fuel taxes, where long-haul flight passengers have a higher tax burden compared to short-haul passengers, airport charge taxes affect all types of passengers equally and are more effective at discouraging short-haul traffic, which tends to be more polluting and more substitutable compared to long-haul. In addition, we find that a tax on airport charges can be effectively used to compensate the airports for their losses due to a reduction in traffic.

3.1 Introduction

Aviation faces a major environmental challenge. It is up to seven times more polluting than a high-speed rail in emissions per seat-km (Strauss, Li, and Cui 2021). It accounts for 3% of global CO2 emissions. Further impacts on climate happen via the release of nitrogen oxides, water vapour, sulphate and soot particles at high altitudes. In addition, it's a high-growth sector, and as a result, international aviation emissions are forecasted by ICAO to triple by 2050 compared with 2015 under no technological and operational improvements. [1].

To address this trend. Various technological approaches are under consideration, such as sustainable aviation fuels, improvements in airplane and engine designs, etc. However, these solutions remain many years away. For immediate solutions, policy can turn out to be an effective tool. There are broadly two types of policy solutions. The first ones are of restriction type. Some examples include curfew times for airports, banning short-distance flights where good alternative low-emission transportation modes are available and promoting sobriety to encourage reducing air travel as much as possible. The second types are market-based measures. They include taxes on fuel and carbon emissions, floor prices on tickets and airport charges. This paper is aimed at understanding the performance of these different taxes with the objective of introducing research on airport charge taxes in addition to more conventional taxes in aviation. These airport charge taxes have been gaining the attention of researchers as of late, but there has been limited work in this area, as per our knowledge.

An advantage of taxes on airport charges is that they are proportional to the level of activity at the airport. Any tax will increase the ticket price for consumers, and this will reduce airport revenues. The proportionality to activity level makes these taxes a potential tool to compensate the airport. Fuel taxes, on the other hand, are not so much about the activity at an airport as they are about the location of the airport. If an airport is far off from

[1] https://www.icao.int/environmental-protection/Pages/ClimateChange_Trends.aspx

other major airports, its passengers are on average going to take longer flights and, in turn, get penalised more from fuel taxes. So, such far-off airports will lose more revenue simply due to their location and compensating them might require a more complicated design. Airport charges, which are independent of an airport's location, remove such disadvantages for passengers of these far-off airports. In this paper, we specifically look at the airport charge taxes, which are paid back to the airport fully as a means of compensation due to a reduction in passengers.

A key issue in studying airport charges has been the lack of standardisation of these across various airports. We contribute by compiling a list of airport charges across major US airports and standardising them in per-passenger and per-flight types. [2]

We then derive passengers' demand for flights in a discrete choice model with logit specification and airlines' unobserved marginal costs under Nash-Bertrand equilibrium. This allows us to then run counterfactuals, where we implement taxes, which in turn can change these estimated marginal costs or airport charges.

We run three counterfactuals: taxes on airport charges, both on per passenger and per flight, taxes on fuel and finally, a combination of both. We find that fuel taxes can be effective at reducing overall flying distance, but they come at a heavy cost to airports. On the other side, taxes on airport charges, designed to be paid back to the airport to recoup some of the losses due to reduced activity, are so effective that the airport's profit increases. When we look at the difference between taxes on short vs long-haul flights, we find that fuel taxes, by design, penalise long-haul flyers, and airport charge taxes penalise all passengers in a similar fashion. As short-haul flights tend to be more polluting than long-haul flights (for example, see Graver, Zhang, and Rutherford 2018), due to take off and landing being a significant part of CO2 emission generation compared to the flying part, an airport charge tax might be more effective at reducing emissions. Another reason to prefer airport charge tax is that long-haul flights are way harder to substitute compared to short-haul flights with other transport forms, like buses, etc. A fuel tax might not penalise short-haul passengers enough to switch to these other means of transport. Finally, we find that if all types of flight travel are to be discouraged via taxation, a combination of two taxes has the potential to achieve this while maintaining the airport's revenue.

3.1.1 Related Literature

Doi 2022 is the closest paper to ours and looks at per-passenger and per-flight airport charges in a structural model (these affect prices and frequency decisions) in Japan. Socially preferred

[2] We only at a subset of airport charges in this paper for which we have enough data to evaluate them. Airports have charges for baggage, retail space, parking, etc., which we ignore as we do not have data on the number of baggages, retail space, and parking spaces, respectively, to evaluate them.

types of charges differ across routes, with a per-passenger preference on larger routes. This paper is limited to only direct routes, as this suits the Japanese market well. We add to this by including connecting routes as well.

We add to the literature on charges in the aviation sector. A common type of them is taxes, and some papers on this include the ones by Doi, Kono, and Suzaki 2023, and Cardoso 2025. We contribute by thinking of the taxes on airport charges in addition to taxes on airlines or consumers.

There is some literature on tax salience. Bradley and Feldman 2020 identify tax salience in the US via difference-in-difference from a policy change that made all taxes visible to consumers right next to the prices. The policy change made the fraction of taxes that airlines passed onto consumers fall by roughly 75 cents for every dollar of tax. Chuang 2019 analyses data between 2012-17 in the US and uses the tax variation across different routes and changes over time; IV estimation identifies tax salience with tax elasticity being 1.5 times the price elasticity. Since we instead focus on airport charges, which will be indirectly transferred to consumers, unlike directly observable taxes, we believe that tax salience is not a significant issue for us.

For estimation techniques, we use the structural approach seen in Berry and Jia 2010, which also deals with the aviation sector.

3.2 Data

3.2.1 Data Sources

We focus on the US domestic passenger aviation market between 2016 and 2019. The reason to choose this period is that before 2016, there was a series of mergers, and after 2019 was the COVID period.

The data related to ticket prices and tickets sold are public in the US, and they come from the Bureau of Transportation Statistics. We specifically use DB1B, which contains information about 10% of the tickets sold in the US, and the T-100 domestic segment, which contains monthly data on seats, frequencies and aircraft types. We also collect aerial distances between airports.

For the supply side, we purchase the data from RDC Aviation. This contains information on airport charges like landing fees, infrastructure fees, gate access fees, etc. We also take aircraft characteristics from them.

We use another public data source, which comes from the Federal Aviation Administration and the Transportation Security Administration, to collect taxation data. Based on Chuang (2019), we collect information about the four most common types of taxes on consumers in domestic travel: the Ticket Tax, the Passenger Facility Charge, the Segment Fee, and the September 11 Security Fee. We exclude the Alaska/Hawaii Ticket Tax (Travel Facilities Tax) as these airports will not be considered in the analysis, as discussed later in this section. In addition, we also collect information on fuel tax from the same source.

We collect data on fuel prices from the US Energy Information Administration and use U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Prices FOB.

To estimate market size and to control for income effects due to variations in economic prosperity across the US, we collect public data from the US Census Bureau on population and income in the US cities.

3.2.2 Markets considered

The US passenger aviation sector is a huge market. In general, it is common in the literature to focus on the top 100 metropolitan areas (127 airports) (see Bet 2021) or medium to large metropolitan areas. Berry and Jia 2010 consider all metro areas with a population of at least $850,000$. The choice depends on the research question and the complexity of estimation, as well as data availability. The smaller airports are generally always excluded due to limited flights and, in turn, limited data.

In our case, the key issue is that airport charges are not standardised. This requires us to collect relevant charges at each airport. Due to this challenge, we restrict the number of airports we consider. We do so by considering the top 200 routes in the US and then collecting the list of all the airports involved in these routes. This gives us a list of 33 airports which we consider in this paper. We consider all the flights which are direct and connecting involving these airports and further filter out certain markets for which we don't have the data on the plane type used (this is essential to estimate airport charges). These give us a total of 2840 combinations of origin-destination-year, which will be the markets considered.

3.2.3 Data Cleaning

On the demand side, we follow commonly adopted practices in the literature and ignore tickets below 50 dollars, purchased in bulk, zero-dollar, and incredibly small markets (like 90 passengers, for example). Additionally, we drop code-sharing tickets as they are beyond the scope of this paper. On the supply side, we ignore certain charges like baggage-related, counter-related, etc. These charges are based on the number of bags used, area used, etc, for which we don't have any data. In addition, we also abstract away from parking charges because of a lack of information on how long planes are parked.

Our market definition is based on a directional origin-destination pair; however, for return tickets, we only see the combined price for the whole trip. To break down this price to the market level, we assume an equal split of ticket fare between onward and return directions. In addition, we drop the routes involving two or more connections as they don't occur that often, and so we have limited data on them.

3.3 Model

3.3.1 Demand

We define the market as a direction origin-destination pair. A product is defined by the route taken, the carrier involved and whether the ticket was one-way or return. For return tickets, we split the observations in two to generate the directional nature of products, and since we only observe the total ticket price, we divide the price by half for each way of the trip. To estimate the demand, we use the fixed coefficient nested logit model. The two nests we consider are people taking flights and everyone else, with the idea that people can have a correlation of utilities from within group products, as depicted in Figure 3.1.
Under this structure, based on Berry (1994), we can write the utility of an alternative as

$$V_{ij} = \delta_{\ j} + (1-\sigma)\epsilon_{ij}$$

For consumer I and product j. We define a product as a combination of route of flight, carrier, and round or one-way trip. $\delta_{\ j}$ stands for net quality price trade-off (mean utility), and the remaining terms are the random term /optimisation error, which is added to fit the data to capture unobserved variables for an econometrician. A higher σ indicates higher substitutability in airlines. We set the mean utility as:

$$\delta_j = \alpha p_j + \beta f_j + \gamma X_j + \xi_j$$

where X_j is a set of observable characteristics - Distance, Population, Year fixed effect and Airline fixed effect. f_j stands for the product frequency, i.e., the number of flights on a given route. The utility from the outside option is normalised to zero and ξ_j captures the unobserved product fixed effects.
Under extreme value distribution of the error term ϵ_{ij}, we get

$$\ln(s_j) - \ln(s_o) = \ \delta_j + \sigma \ln\left(s_{j|M}\right)$$

The left-hand side of the equation denotes the odds between taking the product j and not flying. The right-hand side is the sum of the quality-price trade-off and the degree of differentiation between the alternatives. $s_{j|M}$ denotes the product share within the group, i.e. within the sum of all products on the market. We define the market size as the geometric mean of the populations of the origin and destination, which is a common practice in the literature.

3.3.2 Supply - Airlines' Profit Maximisation

We consider a setting where airlines maximise their profits in a Nash-Bertrand equilibrium. An airline f with a product $j \in \mathfrak{J}_f$ on route r maximises:

$$\Sigma_{j \in \mathfrak{J}_f} \max_{p_j^{net}} \pi_j$$

$$\pi_j = \left(p_j^{net} - c_{r,q} - mc_{j,q}\right) q_j(p_j) - c_{r,f}\, nf_j(q_j(p_j))$$

where p_j is the ticket price, p_j^{net} is the net price (i.e., the money an airline gets after excluding taxes from the ticket price), nf_j is the notional frequency of the product, $c_{r,q}$ and $c_{r,f}$ are the airport charges for per-passenger and per-flight, respectively, associated with the route r and $mc_{j,q}$ Is the unobserved marginal cost which has to be estimated.
The notional frequency is defined as the number of flights required to cater for the demand q_j for product j and is given by:

$$nf_j = \frac{q_j}{lf_r \, . \, ps_r}$$

where lf_r is the load factor on route r and ps_r is the average plane size on route r. We use notional frequency instead of product connection, as used in demand, because the later simply tells us about the number of flights between A and B. These flights are used for many products like domestic passengers taking connecting routes and international passengers taking a domestic connection before taking a internation flight (we don't have data on international passengers). Hence, only a fraction of flights between two points are getting used of a product and we have no data on this. In order to estimate the magnitude of airport charges at per flight level, we therefore, create this concept of notional frequency as an approximation.

So, we can rewrite the profit function after substituting the expression for notional frequency as and using $q_j = s_j M$, where M is the market size and s_j is the market share of product j:

$$\pi_{\ j} = \left(p_j^{net} - c_{r,q} - \frac{c_{r,f}}{lf_r\ .\ ps_r} - mc_{j,q}\right) s_j\,(p_j)\,M$$

The relationship between ticket price p_j and net price p_j^{net} is as follows:

$$p_j = (1 + \tau_{tt})\, p_j^{net} + \ \tau_r$$

where τ_r is the lump sum tax of taxes on the route and τ_{tt} is the ad-valorem ticket tax which is fixed in the US at 7.5%.

We get the following first-order conditions as per Berry, Levinsohn, and Pakes 1995

$$s_j + \Sigma_{\{k\in\ \mathfrak{I}_f\}}(p_{net,k} - mc_{k,q})\frac{\partial s_k}{\partial p_{net,j}} \ = 0$$

The derivatives of market share with price are calculated from the demand equation and are given by

$$\frac{\partial s_j}{\partial p_j} = \sigma s_j[(1 - s_j) + \frac{\sigma}{1-\sigma}\ (1 - s_{j|M})]$$

$$\frac{\partial s_k}{\partial p_j} = -\sigma s_k\left[s_j + \frac{\sigma}{1-\sigma}\ s_{j|M}\right] \quad \text{for} \quad k \neq j$$

$$\frac{\partial p_j}{\partial p_{net,j}} \ = \ \tau_{tt}$$

In vector notation, we can rewrite the first-order condition as:

$$s - \Delta\,[p_{net} - mc_q] = 0$$

where

$$\Delta_{jk} = \begin{cases} -\frac{\partial s_k}{\partial p_j}\ .\ \tau_{tt} & if\ k\ and\ j\ are\ produced\ by\ same\ firm \\ 0 & otherwise \end{cases}$$

Inverting Δ , we get the expression to evaluate the marginal cost

$$p_{net} - \Delta^{-1}s = mc_q$$

We posit Berry and Jia (2010)'s version of marginal cost as

$$mc_{j,q} \ = \ w_{jt}\ \psi \ + \ \omega_{jt}$$

where w_{jt} is a set of observable cost shifters like distance, year fixed effect and their interaction, and airline fixed effect and ω_{jt} is an unobserved cost shock.

3.4 Estimation and Results

We estimate demand and supply jointly using GMM to identify their respective parameters $(\alpha, \beta, \gamma, \psi)$ simultaneously.

For demand, we rely on the moment conditions $E[Z.\xi] = 0$ where ξ is the vector of unobservable product characteristics which affect the demand. Due to potential endogeneity concerns with price and frequency, as they might getting decided simultaneously with market share, we use instruments for these two contained in the matrix Z.

The instruments used are based on BLP (1995). They include within a market: the number of rival carriers' products, if a market is a monopoly, the sum of the distance of other products by the same carrier, the sum of the distance of products of rival carriers, the percentage of rival carriers' direct products and the sum of the total number of destinations at the origin of rival carriers.

For supply, we rely on the moment conditions $E[w.\omega] = 0$ where w is the matrix of cost shifters and ω is a vector of unobserved cost shocks.

We then jointly estimate the demand and supply moments.

3.4.1 Results

The results of demand and supply are presented in Tables 3.1 and 3.2 We find that consumers are way more sensitive to the number of connections by a carrier between their origin-destination cities (β=6.144) than to the price (α=−1.536), i.e., one additional product connection is worth a price(gross) increase of 400 dollars. This could capture the fact that higher production frequency would increase the chances of connecting the trip, as when a flight gets cancelled, a high-frequency carrier could easily move consumers to a nearby scheduled flight without much delay for them. We find a U-shaped relationship with distance, as is observed in the literature. Consumers prefer flying as distances grow, but very long flights can create discomfort. σ of 0.645 shows that consumers who fly are indeed more similar than those who don't.

Interestingly, we find that the fixed effect associated with round-trip is negative. This shows that if the prices of one-way flights and similar segment prices of round-trip tickets are the same, then passengers prefer booking one-way flights. Consumers can potentially book different one-way way for the onward and return journey with different carriers and end up with a cheaper overall ticket, when return tickets are not discounted well compared to one-way tickets.

Our own and cross price elasticity estimates are -11.86 and 0.32, respectively. The own-price elasticity estimates are on the higher side than in the literature. These high values could be partly explained by our inclusion of very low market share products, which tend to have higher elasticities, as when we weigh the mean of own-price elasticities with the number of consumers, we get a value of -6.42. Non-connecting passengers with their own price elasticity of -7.3 are less price sensitive than connecting passengers with a value of -12.76.

We find only around 3.44% of the estimated marginal costs to be negative, and the mean value for positive marginal costs to be 150 dollars. Figure 3.2 shows that the marginal costs for Low-Cost Carriers and Southwest (which is also a Low-Cost Carrier, but much bigger than the others) are on the lower side than full-service carriers.

3.5 Counterfactuals

Our objective with the counterfactuals is to understand the difference between the performance of airport charge taxes compared to traditional taxes. To do this, we set up and study three policy changes- imposing taxes on either airport charges, fuel or a combination of the two. We are interested in exploring airport charges taxes as a compensation tool for airports' revenue loss due to taxes, and so we consider that under all scenarios, the revenue from these types of taxes is transferred to airports.

Policy 1: 10% tax on airport charges – both on per passenger and per flight

Policy 2: 1% tax on fuel

Policy 3: Combination: 1% Fuel Tax + taxes on both airport charges set at a level where airport revenues are unchanged.

3.5.1 Algorithm

Once a new tax regime is implemented, it will change the per-passenger cost of the airline. This, in turn, will affect the pricing and, in turn, the product demand.

We proceed by setting all negative estimated marginal costs to zero.

Notation: s_o, p_o and $f_o$be the original market share, price and frequency, respectively.

Define : $delta_o \equiv \log s_o - \log s_{outside\ option} - \sigma \log s_{o|M}$ ($s_{o|M}$ is the nested market share)

We estimate the new price and market share using the following steps:

1. With the original level of market share, we calculate the new price for the first loop and use the iterative new market share for the following loops. Suppose we put a 10% charge on a per-passenger airport charge

$$p_{new} = \left(mc_q^{estimated} + 1.1 * c_q + \frac{c_f}{Load\ factor.\ \text{Average Plane size}} + \Delta^{-1} s \right) * \ \tau_{tt} + \tau$$

2. This new price then goes into calculating new market shares. Note that we keep the product connections fixed. [3]

$$delta_{new} = delta_o + \alpha \left(p_{new} - p_o\right)$$

For a product j,

$$s_{new,j} = \frac{e^{\frac{delta_{new,j}}{1-\sigma}}}{\left[\Sigma_k e^{\frac{delta_{new,k}}{1-\sigma}}\right]^{\sigma} \left[1 + \left[\Sigma_k e^{\frac{delta_{new,k}}{1-\sigma}}\right]^{1-\sigma}\right]}$$

$$s_{new,j|M} = \frac{s_{new,j}}{\Sigma_k \ s_{new,k}}$$

3. Rerun from step 1 again with these new market shares. The loop stops after the old and new market share eventually converge.

$$\left| s_{new}^{n+1} - \ s_{new}^{n} \right| < tolerance$$

Where n and n+1 represent the nth and the following iterations of new market shares

3.5.2 Results

Table 3.3 shows the changes in Consumer surplus, Airline Profits, Airport Revenues (from collecting airport charges only and the taxes on them), Distance Flown and Passengers. Since distance flown can be thought of as a proxy variable for CO2 emissions generated from flying, we can see that while fuel taxes can be effective at reducing overall flying distance, they come at a heavy cost to airports. On the other side, taxes on airport charges, designed to be paid back to the airport to recoup some of the losses due to reduced activity, are so effective that the airport's profit increases.

Short-haul flights tend to be more polluting than long-haul flights (for example, due to take off and landing being a significant part of CO2 emission generation. Graver, Zhang,

[3] Our pricing expression implicitly implies that notional frequency gets updated to cater to the increased/decreased demand. While notional frequency is a subset of product connections, we ignore the changes in the latter. This is because these changes are difficult to evaluate as a product connection between two cities, A and B, caters to multiple products across multiple markets, while our counterfactual algorithm tackles one market at a time. Evaluating effects in changes in product connections due to notional frequency changes across multiple markets would make the problem very computationally taxing.

and Rutherford 2018 show that carbon intensity of medium and long-haul flights varies between 75 to 95 g per passenger kilometre while for short haul flight, this number jumps to 110. Figure 3.3 shows the difference between taxes on short vs long-haul flights. While fuel taxes, by design, penalise long-haul flyers, airport charge taxes penalise all passengers in a similar fashion. This implies that an airport charge tax might be more effective at reducing emissions. Another reason to prefer airport charge tax is that long-haul flights are way harder to substitute compared to short-haul flights with other transport forms, like buses, etc. A fuel tax might not penalise short-haul passengers enough to switch to these other means of transport.

Finally, in the last policy, we looked at the combination of two taxes. If all types of flight travel are to be discouraged via taxation, such a policy has the potential to achieve this while maintaining the airport's revenue.

3.6 Conclusion

This paper analyses the performance of different taxes in aviation to discourage flying by introducing relatively less studied taxes on airport charges to the existing conventional taxes, like the fuel tax. We solve the issue in studying airport charges due to a lack of standardisation by building a compilation of a list of airport charges across major US airports and standardising them in per-passenger and per-flight types.

We find that, unlike fuel taxes, where long-haul flight passengers have a higher tax burden compared to short-haul passengers, airport charge taxes affect all types of passengers equally and are more effective at discouraging short-haul traffic, which tends to be more polluting and more substitutable compared to long-haul. In addition, we find that a tax on airport charges can be effectively used to compensate the airports for their loss due to a reduction in traffic.

In future versions of the paper, we plan on introducing emission calculations which would allow us to calculate taxes which could achieve a certain level of reduction in emissions. It would also allow us to calculate welfare based on carbon prices and, in turn, recommend an iso welfare carbon price. We would also like to study some other types of policy instruments, like taxing only airport charges for narrow-body planes to discourage short-haul flights.

We also plan to add the airport's optimisation problem in future versions as a 2-sided market/platform for passengers and airlines, as per Ivaldi, Sokullu, and Toru 2015. The airport faces a challenge in achieving various objectives when designing both per-flight and per-passenger airport charges. Higher passenger tax reduces demand for flights, and that in turn reduces collection from the per-flight tax and vice versa. This would allow us to explore objectives, such as reducing carbon emissions by a certain percentage point from each airport, while minimising consumer surplus loss, subject to strictly positive airport profits, where airport charge taxes are designed locally at the airport level. Alternatively, we can look at emission-minimising taxes while again maintaining the revenue of the airport. Details of this proposal are in the appendix.

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Appendix for Chapter 3

Appendix 3.A Tables and Figures

Demand Estimates

Ticket Price	−1.536***
	(−0.189)
σ	0.645***
	(−0.052)
Product connections	6.144***
	(−0.54)
Distance	0.554***
	(−0.018)
Distance^2	−0.0001*
	(−0.0001)
Number of carrier destinations at origin airport	0.009***
	(−0.001)
Market size	−0.156***
	(−0.013)
Distance of airport from the city centre	0.013***
	(−0.004)
Roundtrip Dummy	−0.664***
	(−0.103)
Year Fixed Effects	
2017	−0.981***
	(−0.121)
2018	−0.931***
	(−0.122)
2019	−0.978***
	(−0.125)

Carrier Fixed Effects	
American Airlines	−38.960***
	(−2.964)
Alaska Airlines	0.933***
	(−0.092)
JetBlue Airways	0.808***
	(−0.089)
Delta Air Lines	0.029
	(−0.026)
Frontier Airlines	0.638***
	(−0.092)
Allegiant Air	0.439*
	(−0.244)
Spirit Airlines	0.558***
	(−0.093)
Sun Country Airlines	0.593
	(−0.419)
United Airlines	0.165***
	(−0.029)
Virgin America	1.065***
	(−0.146)
Southwest Airlines	0.590***
	(−0.061)

Observations 55,886

Note: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

Table 3.1

Supply Estimates : Marginal Cost

Distance	0.554***
	(−0.018)
Number of carrier destinations	0.009***
at origin airport	(−0.001)
Hub number	0.130***
	(−0.013)
Year Fixed Effects	
2016	−0.956***
	(−0.121)
2017	−0.981***
	(−0.121)
2018	−0.931***
	(−0.122)
2019	−0.978***
	(−0.125)
Distance-Year interaction fixed effects	
Distance*Year 2017	−0.027
	(−0.021)
Distance*Year 2018	−0.049**
	(−0.02)
Distance*Year 2019	−0.013
	(−0.021)

Carrier Fixed Effects	
Alaska Airlines	0.933***
	(−0.092)
JetBlue Airways	0.808***
	(−0.089)
Delta Air Lines	0.029
	(−0.026)
Frontier Airlines	0.638***
	(−0.092)
Allegiant Air	0.439*
	(−0.244)
Spirit Airlines	0.558***
	(−0.093)
Sun Country Airlines	0.593
	(−0.419)
United Airlines	0.165***
	(−0.029)
Virgin America	1.065***
	(−0.146)
Southwest Airlines	0.590***
	(−0.061)

Observations 55,886

Note: *$p < 0.1$; **$p < 0.05$; ***$p < 0.01$

Hub Number = 1 if either origin or destination is a hub for a carrier and = 2 if both ends are hubs.

Table 3.2

Counterfactual – Policy Effects

	10% on airport charges (both) -collection added to airport's revenue	1% on fuel	Combination : 1% Fuel Tax+ 8.15% Airport charge tax paid to airport to generate a zero profit change on airports
Δ Consumer surplus %	−2.07	−5.01	−6.61
Δ Airline Profits %	−1.96	−4.75	−6.28
Δ Airport %	6.70	−5.22	0.00
Δ Distance	−2.16	−6.69	−8.34
Δ Passengers %	−2.05	−4.98	−6.58

Table 3.3

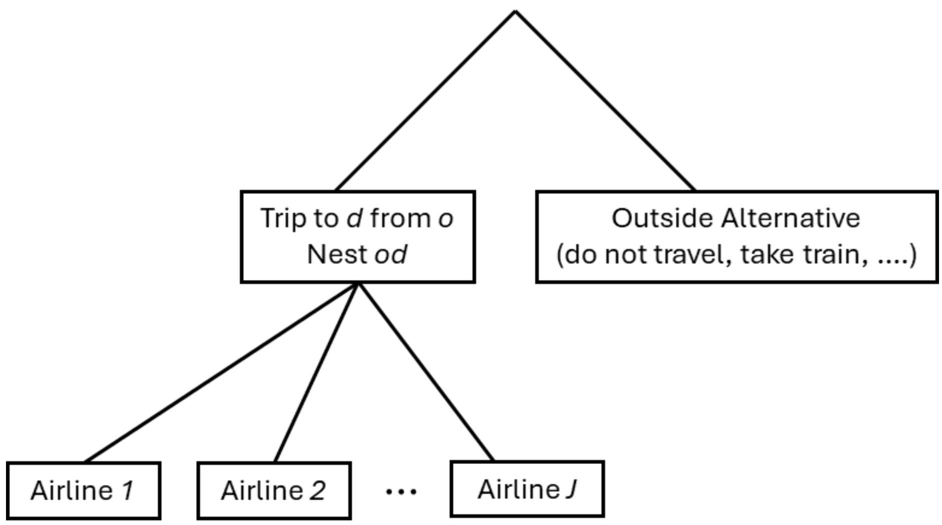

Figure 3.1: Nesting structure of the demand model

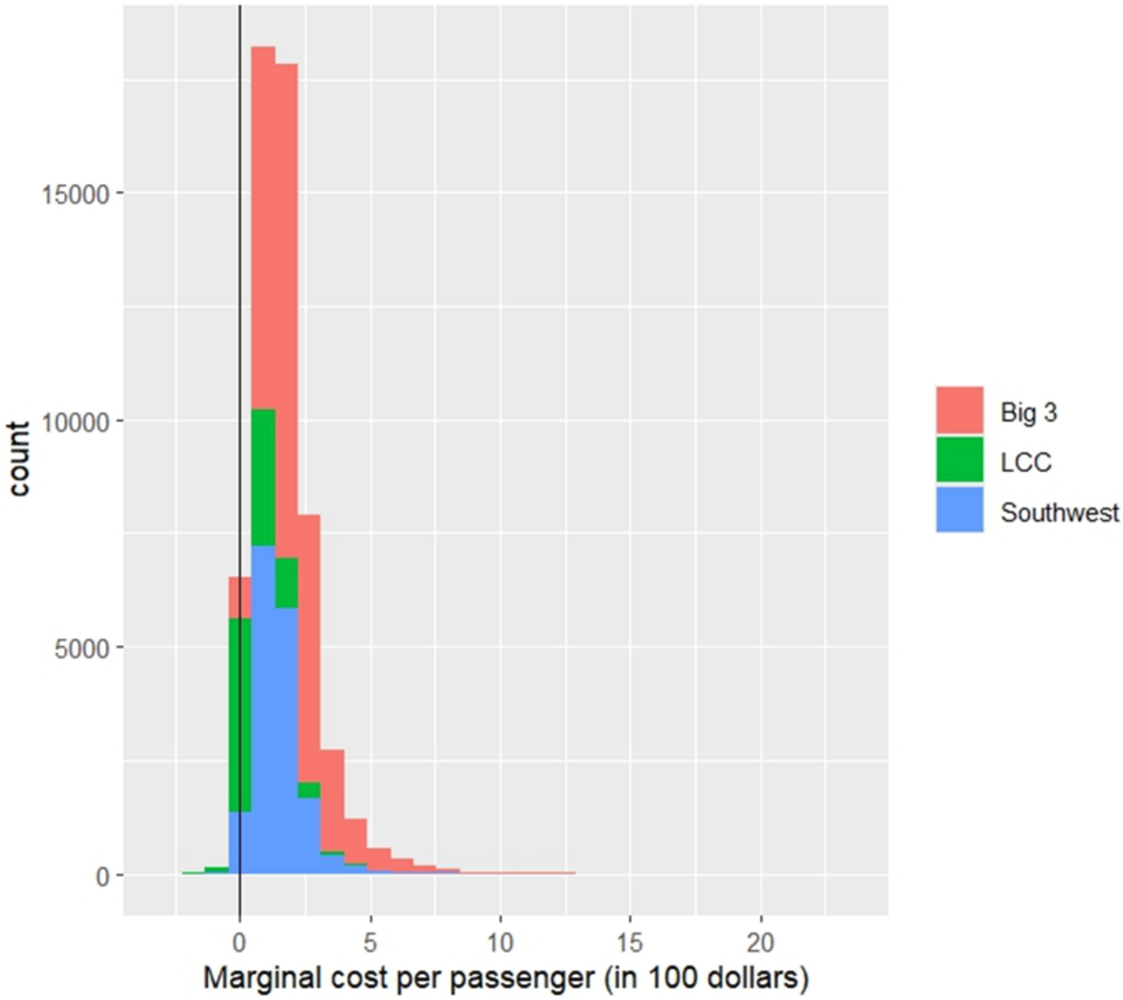

Figure 3.2: Distribution of estimated marginal cost across all products. The Big 3 are Delta Air Lines, American Airlines and United Airlines. LCCs are the low-cost carriers, which include the rest of the airlines other than the Big 3 and Southwest.

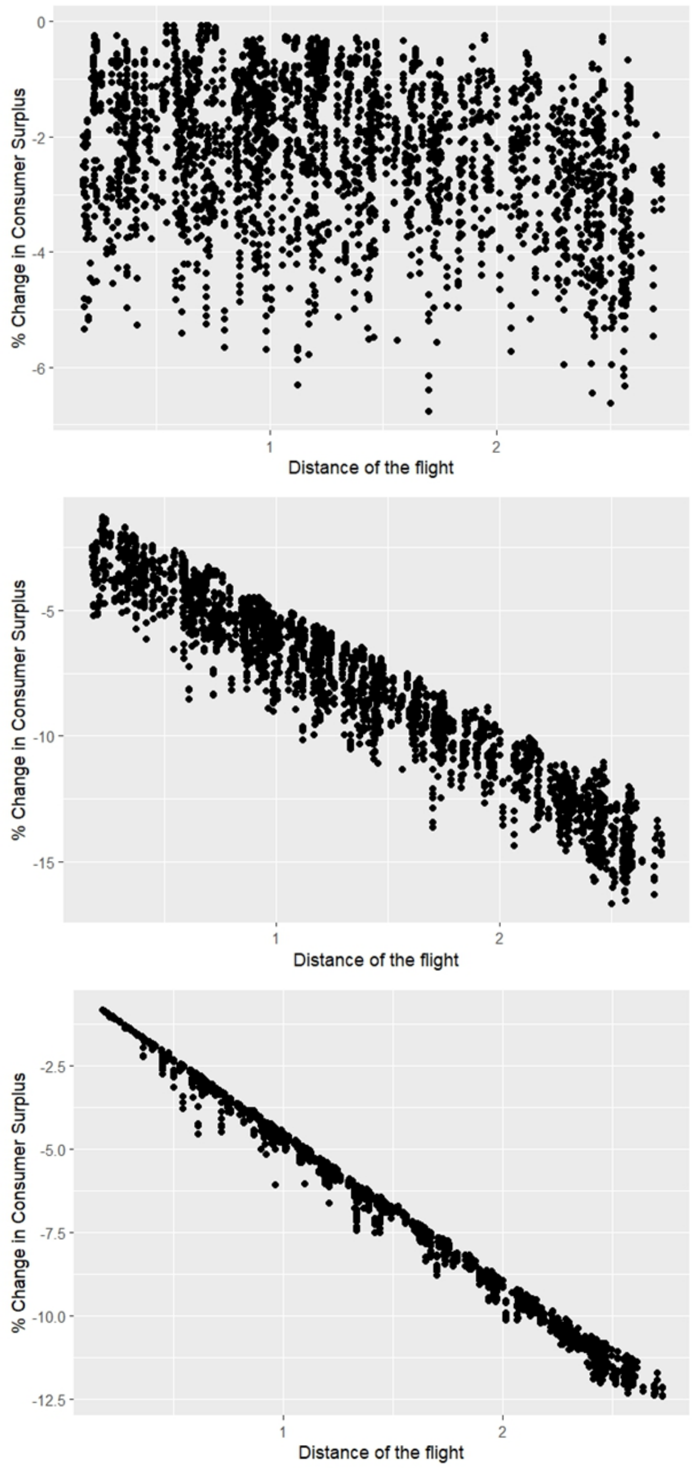

Figure 3.3: Three figures show how the change in consumer surplus upon implementation varies with the distance of flight. The top figure corresponds to 10% airport charge tax, the middle corresponds to 1% fuel tax, and the bottom one corresponds to a combination of the two taxes – 1% fuel tax and 8.15% airport charge tax.

Appendix 3.B Exploring Airport's Problem

We aim to build our two-sided model using techniques seen in the Two-sided platform model approach from Ivaldi, Sokullu and Toru (2015) in future versions of this paper.

We would like to decipher airports' behaviour, where airports have two revenue channels– from passengers and carriers. From passengers: These are generated by collecting per-passenger airport charges. Some examples include gate check-in fees, terminal fees, etc. For an airport charge of p_o^a charged per passenger, the revenue of the airport:

$$\pi_o^a \;=\; (p_o^a - c_o^a)\, q_o - K_o^a$$

where q_o is the number of tickets sold, c_o^a is the marginal cost of the airport per passenger, and K_o^a is the corresponding fixed cost. The associated welfare is given by:

$$Welfare_o^a = \sum_d \pi_o^a \; + \sum_d \sum_j \pi_j + \sum_d Consumer\ Surplus$$

π_j is the profit from product j of airline, and d represents markets.
From carriers: These are generated by collecting per-flight airport charges. Some examples include landing fees, joint use fees, etc.
For an airport charge of p_f^a charged per flight, the revenue of the airport:

$$\pi_f^a \;=\; \left(p_f^a - c_f^a\right) f_o - K_f^a$$

where f_o is the number of flights at the airport, c_f^a is the marginal cost of the airport per flight, and K_f^a is the corresponding fixed cost. The associated welfare is given by:

$$Welfare_f^a = \sum_d \pi_f^a \; + \sum_d \sum_j \pi_j + \sum_d Consumer\ Surplus$$

As the airports in the US are public, their objective might not just be maximising profit. To understand their objectives, we consider multiple scenarios: Firstly, an airport's objective could either be profit maximisation or Welfare Maximisation. For the latter, since airports can't make negative profits, we modify it to Ramsey pricing, which means welfare maximising with a constraint that profits should be non-negative. Secondly, airports can either act as one-sided or two-sided. In the prior case, an airport separately maximises the per-passenger and per-flight charges. In the latter case, the airport internalises the fact that any increase in per-passenger charge will reduce the number of passengers, which in turn then reduces the number of flights and hence the per-flight revenue and vice versa. Therefore, the airport will have a joint profit maximisation or a joint Ramsey pricing problem. So, in total, we have four possibilities:

	Profit-Maximising	Ramsey-Pricing
One-Sided	$\max_{p_o^a} \pi_o^a$ $\max_{p_f^a} \quad \pi_f^a$	$\max_{p_o^a}\ Welfare_o^a\ with\ \pi_o^a\ \geq 0$ $\max_{p_f^a}\ Welfare_f^a\ with\ \pi_f^a\ \geq 0$
Two-Sided	$\max_{p_o^a\ ,\ p_f^a} (\pi_o^a + \pi_f^a\ )$	$\max_{p_o^a\ ,\ p_f^a} \left(Welfare_o^a + Welfare_f^a\ \right)$ $with\ \pi_o^a + \pi_f^a\ \geq 0$

Table 3.4

By estimating the marginal costs under these different scenarios, we should be able to reject scenarios which result in negative values.