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"All-receive procurement auctions"

Simon Loertscher, Leslie M. Marx and Patrick Rey

Toulouse School of Economics

All-receive procurement auctions*

Simon Loertscher[†] Leslie M. Marx[‡] Patrick Rey[§]

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Abstract

We develop the procurement analogue to an all-pay auction for an independent private values model with identical distributions. In this *all-receive procurement auction* (ARPA), suppliers simultaneously submit bids. Suppliers with bids below (above) the reserve are paid their bids (are paid and produce nothing). The supplier with the largest bid below the reserve produces the good. With appropriately chosen reserves, which decrease in the number of suppliers, the ARPA is efficient and, given increasing virtual costs, implements the optimal procurement. Appropriately adjusted, ARPAs implement the optimal procurement in general. ARPAs can render supply chains resilient to nonanticipated liquidity shocks.

Keywords: Resilience, liquidity shocks, all-pay auctions, multiple-receive procurement auctions

JEL Classification: D44, D82, L41

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[†]University of Melbourne

[‡]Duke University

[§]Toulouse School of Economics, University of Toulouse Capitole

1 Introduction

The pandemic and ongoing geopolitical tensions have led policy makers and business leaders to call for more *resilient* supply chains. Typically, reducing reliance on a single, low-cost, but often distant, supplier involves trading off higher costs against a lower risk that the supply chain breaks down. This raises the question of how sustainable efforts to increase domestic production and on-shoring will be. To address this question, we develop procurement auction formats that involve no tradeoff in terms of expected payoffs, yet increase the resilience of the supply chain. Of particular interest is the procurement analogue of the all-pay auction, which we call *all-receive procurement auction* (ARPA).

We study a standard procurement model with independent private values in which a buyer with known value wants to procure one unit from a set of suppliers who draw their costs independently from a common, known distribution. In the ARPA, suppliers submit bids simultaneously. *All* suppliers with bids not exceeding the reserve set by the procurer are paid their bids, and the supplier with the *largest* bid below the reserve is asked to produce the good. Suppliers with bids above the reserve are paid nothing and are never asked to produce. Like first-price and second-price procurement auctions (FPPA and SPPA), with appropriately chosen reserves, the ARPA is efficient and, if the cost distribution exhibits increasing virtual types, an optimal procurement auction. By the payoff equivalence theorem, the ARPA, FPPA and SPPA thus induce the same interim and thus the same ex ante expected payoffs for the suppliers and the same expected payoff for the buyer.

Another feature of the ARPA is that the buyer's total payment is spread across all suppliers with bids below the reserve. Consequently, relative to other two formats, the ARPA increases suppliers' liquidity. As we show by considering liquidity shocks, occurring at the end of the game and not anticipated in the auction, this has the advantage of allowing more suppliers to survive a liquidity shock than the FPPA and SPPA. Specifically, there are conditions under which a liquidity shock eliminates all suppliers in the FPPA and SPPA, while in the ARPA it only eliminates the supplier that is selected to produce. Under other conditions, either all or all but the selected producer survive in the ARPA and only the winner in the FPPA and SPPA survives the liquidity shock. With the ARPA, the buyer can thus make the supply chain more resilient—and in particular preserve competition among suppliers—without any sacrifice in expected payoffs.

We moreover show that, with appropriate adjustments of the rules to allow ironing (Myerson, 1981), the payoff equivalence and resilience properties of the ARPA extend to the case in which the virtual cost is not globally monotone; in the process, we also show how to adjust the rules of the FPPA and SPPA to permit ironing. However, the ARPA and its adjusted variant may degrade the liquidity of the supplier selected to produce. A generalization of the ARPA that mitigates this concern, at the expense of some resilience, would be multiple-receive procurement auctions, in which only a subset of suppliers are paid their bids (with the ARPA and the FPPA being extremal special cases). Interestingly, as we show in an extension, multiple-receive procurement auctions do not necessarily exhibit a monotone equilibrium and can therefore fail to be payoff equivalent to more standard formats. In further extensions, we derive the direct mechanism that is maximally resilient without ever degrading suppliers' liquidity, and address possible concerns about the susceptibility of the ARPA to collusion—showing in particular that the ARPA can actually be less susceptible to collusion than the other formats.

The auction literature is, evidently, too vast to review here; see, for example, Krishna (2010) for an authoritative overview. Revenue equivalence between different auction formats was first observed by Vickrey (1961, 1962) with the driving force behind it, and the deeper result of payoff equivalence, uncovered by Riley and Samuelson (1981) and Myerson (1981). We develop the procurement equivalent to the all-pay auction, the ARPA, and show that, with an appropriate reserve, it is payoff equivalent to standard formats like the FPPA and SPPA. While we do *not* analyze the design of (optimal) mechanisms in the presence of liquidity constraints, which is the subject of papers such as Ganuza (2007) and Arve and Martimort (2024), we analyze the robustness of fixed and otherwise equivalent auction formats to unanticipated liquidity shocks.

In what follows, in Section 2, we introduce the setup and the formal definition of the ARPA. In Section 3, we derive the suppliers' equilibrium behavior in the ARPA. Implementation of the optimal procurement mechanism via the ARPA is studied in Section 4. Section 5 introduces and analyzes liquidity shocks, and Section 6 provides additional discussion and concludes the paper.

2 Setup

There is one buyer, with single-unit demand and value $v > \underline{c}$, and $n \geq 2$ suppliers, each drawing its cost independently from a continuous distribution F with density f and support $\mathcal{C} \equiv [\underline{c}, \overline{c}]$.[1] The buyer and the suppliers are all risk neutral; they have quasilinear utility, and the value of their outside option is 0.

In an *all-receive procurement auction* (ARPA) with reserve R, all suppliers simultaneously

[1]A supplier's production cost c can be interpreted either as the opportunity cost of selling an existing asset or as the financial cost of producing an asset or service. For the analysis in Sections 3 and 4, these two interpretations are equivalent. However, for some of the analysis and results in Section 5 on resilience to liquidity shocks, which specification is used matters.

submit bids.[2] If all bids (strictly) exceed R, the procurement is unsuccessful and there are no payments or production. Otherwise, each supplier i with a bid $B_i \leq R$ is paid B_i, and the supplier with the highest bid produces. If ties occur among the highest bidders below R, then the supplier that has to produce is selected uniformly randomly from among these bidders. This format inherits from the all-pay auction that each agent's transaction price is what it bids. There is however a notable difference between the sale and the procurement contexts. For sale auctions, all-pay and first-price formats have in common that the highest bidder wins. In the procurement context, the ARPA selects the *highest* bidder as supplier, whereas a FPPA or SPPA selects instead the *lowest* bidder. Because incentive compatibility requires an agent's selection probability to be decreasing in its cost, this in turn implies that, in the ARPA, equilibrium bids are *decreasing* in costs.

3 Suppliers' response

We first show that the ARPA elicits a unique symmetric response from the suppliers. For any $c \in \mathcal{C}$, let

$$P_n(c) \equiv [1 - F(c)]^{n-1}$$

denote the probability that $n-1$ suppliers have costs higher than c and, for any $r \in (\underline{c}, \overline{c}]$ and $c \in [\underline{c}, r]$, define

$$B_n\left(c; r\right) \equiv \mathbb{E}_{\mathbf{c}_{-i}}[\min\{\mathbf{c}_{-i}, r\} \mid \min \mathbf{c}_{-i} > c] P_n(c) = c P_n\left(c\right) + \int_c^r P_n\left(x\right) dx, \tag{1}$$

and

$$R_n\left(r\right) \equiv \mathbb{E}_c[\min\{c, r\}] = B_n\left(\underline{c}; r\right) = \underline{c} + \int_{\underline{c}}^r P_n(x) dx. \tag{2}$$

Observe that $R_n(r) < r$ because $\underline{c} + \int_{\underline{c}}^r [1 - F(x)]^{n-1} dx < \underline{c} + \int_{\underline{c}}^r dx = r$ and that $R_n(r)$ is decreasing in n because the integral decreases in n.

We have:

Proposition 1. *For any $r \in (\underline{c}, \overline{c}]$, the ARPA with reserve $R = R_n\left(r\right)$ induces a unique symmetric Bayesian Nash equilibrium, in which every supplier with cost $c \leq r$ bids $B_n\left(c; r\right)$ and every supplier with cost $c > r$ refrains from bidding. This equilibrium replicates the allocation of the FPPA and SPPA with reserve r.*

[2] Without a reserve, the ARPA would have no equilibrium, as any bidder would find it optimal to submit an infinite bid. But this is not different from an all-pay auction in which, without a reserve (typically taken to be zero), bidders would optimally submit an infinitely negative bid.

Proof. Take $r \in (\underline{c}, \overline{c}]$ as given and consider an ARPA with reserve $R_n(r)$. Standard arguments imply that in any symmetric equilibrium, (i) suppliers produce with positive probability if and only if their cost does not exceed a cost threshold, which we denote by $\hat{r} \in [\underline{c}, \overline{c}]$, and (ii) for a supplier with cost $c \leq \hat{r}$, the probability of being selected is $P_n(c)$ and the bid function $B(c)$ is continuous and differentiable. For $c \leq \hat{r}$, we thus have, by incentive compatibility, $c = \arg\max_{\hat{c}} B(\hat{c}) - cP_n(\hat{c})$. The first-order condition, evaluated at $c = \hat{c}$, yields $0 = B'(c) - cP_n'(c)$, leading to

$$B(c) = k + cP_n(c) + \int_c^{\hat{r}} P_n(x)dx = k + B_n(c; r),$$

for some constant k. Furthermore, because the lowest type produces with probability 1, it cannot be induced to submit any other bid than the maximal one, $R_n(r)$. Thus, $B(\underline{c}) = R_n(r) = B_n(\underline{c}; r)$, implying that $k = 0$. The threshold type $\hat{r}$ must moreover be indifferent between bidding and not, $B_n(\hat{r}; r) - \hat{r}P_n(\hat{r}) = 0$, which implies that $\hat{r} = r$. It follows that a supplier with cost $c \in [\underline{c}, r]$ submits a bid $B(c) = B_n(c; r) \leq R_n(r)$.

To establish sufficiency of the first-order condition, denote by $U(c, \hat{c})$ the utility of a type c that submits the bid $B_n(\hat{c}; r)$. We have:

$$U(c, \hat{c}) = B_n(\hat{c}; r) - cP_n(\hat{c})$$

and

$$\frac{\partial U(c, \hat{c})}{\partial \hat{c}} = (\hat{c} - c)P_n'(\hat{c}),$$

which is positive for $\hat{c} < c$ and negative for $\hat{c} > c$ because $P_n' < 0$. Thus, the first-order condition characterizes the unique maximum.

To conclude the proof, recall that a FPPA or SPPA with reserve r induces suppliers with costs less than or equal to r to submit bids that are increasing in their types and less than or equal to r, ensuring that the lowest-type supplier produces whenever its cost is less than or equal to r, and no suppliers produce when the lowest type is greater than r, generating the same allocation as the ARPA with reserve $R_n(r)$. ■

The equilibrium bidding strategy is *decreasing* in the supplier's cost: $B_n'(c; r) = P_n'(c)c < 0$. This occurs because a higher-cost supplier is less likely to be selected, and is therefore willing to accept a lower (unconditional) payment. (For an illustration, see Supplemental Appendix Figure C.1(a).) Furthermore, a supplier with cost r bids $B_n(r; r) = P_n(r)r$ and thus obtains a zero payoff, as it is asked to produce with probability $P_n(r)$, in which case it incurs the cost r. Let

$$U_n(c;r) \equiv B_n(c;r) - cP_n(c)$$

denote the resulting expected payoff for a supplier with cost $c \in [\underline{c}, r]$, which thus satisfies $U_n(r;r) = 0$. Because the ARPA bid function is decreasing in c, the highest bid is submitted by the supplier with the lowest cost: $\max_c B_n(c;r) = B_n(\underline{c};r)$. Hence, the reserve R in the ARPA that corresponds to a reserve r in a FPPA or SPPA is $R = B_n(\underline{c};r) = R_n(r)$, which satisfies $R_n(r) = r$ for $r = \underline{c}$ and $R_n(r) \in (\underline{c}, r)$ for $r \in (\underline{c}, \overline{c}]$.

Consider now the relation between bidding strategies. In a FPPA with reserve $r \in \mathcal{C}$, a supplier with cost $c \leq r$ bids

$$b_n^{FPPA}(c;r) \equiv c + \frac{\int_c^r P_n(x)dx}{P_n(c)}.$$

Consequently, we have

$$B_n(c;r) = P_n(c) b_n^{FPPA}(c;r). \tag{3}$$

This is the same relationship as between an all-pay and a first-price auction, where, as is well known, the bid in the all-pay auction is equal to the bid in the first-price auction multiplied by the probability of winning. For example, for F uniform on $[0,1]$ and $r = 1$, we have $b_n^{FPPA}(c;1) = c + \frac{1-c}{n}$ and $B_n(c;1) = (1-c)^{n-1}(c + \frac{1-c}{n})$.

In particular, as a supplier with cost $\underline{c}$ is selected with probability 1, it bids the same amount in the ARPA and the FPPA; hence, $R_n(r) = B_n(\underline{c};r) = b_n^{FPPA}(\underline{c};r)$. As the latter is decreasing in n, the same applies to $R_n(r)$ (for an illustration, see Figure C.1(b) in the Supplemental Appendix). Furthermore, as bids are decreasing in c in the ARPA and increasing in the FPPA, all bids in the FPPA weakly exceed all bids in the ARPA, as illustrated in Supplemental Appendix Figure C.2.

Finally, by construction, for any $r \in (\underline{c}, \overline{c}]$, $B_n(c;r)$ is continuous and decreasing in c and satisfies $B_n(\underline{c};r) = R_n(r) > \underline{c}$ and $B_n(r;r) = P_n(r)r < r$. Hence, there exists a unique $\tilde{c}_n(r) \in (\underline{c}, r)$ such that

$$B_n(c;r) \gtreqqless c \text{ if and only if } c \lesseqqgtr \tilde{c}_n(r). \tag{4}$$

This means that suppliers with costs below $\tilde{c}_n(r)$ make a profit even if called upon to produce, whereas suppliers with costs above $\tilde{c}_n(r)$ make a profit only if they do not have to produce (and make a loss otherwise). Because $B_n(c;r)$ is decreasing in n and increasing in r, $\tilde{c}_n(r)$ is also decreasing in n and increasing in r (see Supplemental Appendix Figure C.3).

4 Optimal procurement auctions

Let
$$\gamma(c) \equiv c + \frac{F(c)}{f(c)}.$$
denote the *virtual cost* of a supplier with cost c. We begin with the case in which γ is increasing (the *regular* case hereafter), before addressing the case in which it is not.

4.1 The regular case

When γ is increasing (e.g., when the reverse hazard rate F/f is itself increasing), we can define
$$r^* \equiv \gamma^{-1}(\min\{v, \bar{c}\}).$$
As is well known, in this regular case, the mechanism that maximizes the buyer's expected payoff (the optimal mechanism hereafter) selects a supplier with cost c with probability $P_n(c)$ if $c \leq r^*$, and probability 0 otherwise. Moreover, it can be implemented using an FPPA or SPPA with reserve r^* (see Supplemental Appendix A for details). It follows from Proposition 1 that it can also be implemented using an ARPA with an appropriate reserve:

Corollary 1. *If $\gamma(c)$ is increasing in c, then the optimal mechanism can be implemented using an ARPA with reserve $R_n^* \equiv R_n(r^*)$.*

In the case of ARPA, the optimal reserve is therefore decreasing in the number of suppliers. This comparative static contrasts with the case of a FPPA or SPPA, where the optimal reserve is independent of the number of suppliers, but it is the same as for the efficient and the optimal all-pay selling auction.[3]

4.2 The nonregular case

Consider now the case in which γ is not strictly monotone in c.[4] The optimal mechanism then selects agents on the basis of the *ironed* virtual cost function $\overline{\gamma}$, which is continuous, nondecreasing, and differs from γ only on a set of so-called ironing intervals.[5] Specifically,

[3]To see the latter, consider for instance the case in which n buyers draw their values independently from F with support $[0, 1]$ and density f, and the auctioneer's opportunity cost of selling the good is $c \in (0, 1)$. The reserve of the efficient all-pay auction is then $F(c)^{n-1}c$, whereas the reserve of the optimal all-pay auction, assuming increasing virtual values, is $F(\tilde{r})^{n-1}\tilde{r}$, where $\tilde{r}$ is such that $\tilde{r} - \frac{1-F(\tilde{r})}{f(\tilde{r})} = c$. Both reserves are decreasing in n.

[4]The virtual cost function is locally increasing at $\underline{c}$ because $\gamma'(c)|_{c=\underline{c}} = 2$. Hence, it is either increasing everywhere (the regular case) or not strictly monotone (the nonregular case).

[5]This is the procurement analogue of the optimal selling mechanism; see Myerson (1981). For a practical implementation of the optimal selling mechanism, see Condorelli (2012).

there exists $K \equiv \{1, \ldots, K\}$ and $\{\underline{c}_k, \overline{c}_k\}_{k \in \mathcal{K}}$, satisfying $\underline{c}_1 < \overline{c}_1 < \cdots < \underline{c}_K < \overline{c}_K$, such that $\overline{\gamma}(c) = \overline{\gamma}_k \equiv \mathbb{E}_c[\gamma(c) \mid c \in \mathcal{I}_k]$ for $c \in \mathcal{I}_k \equiv [\underline{c}_k, \overline{c}_k]$, and $\overline{\gamma}(c) = \gamma(c)$ for $c \notin \cup_{k \in \mathcal{K}} \mathcal{I}_k$. Note that the ironing intervals, and hence $\overline{\gamma}(\cdot)$, do not depend on the number of bidders. We illustrate the ironing of a nonmonotone virtual cost function in Figure 1.

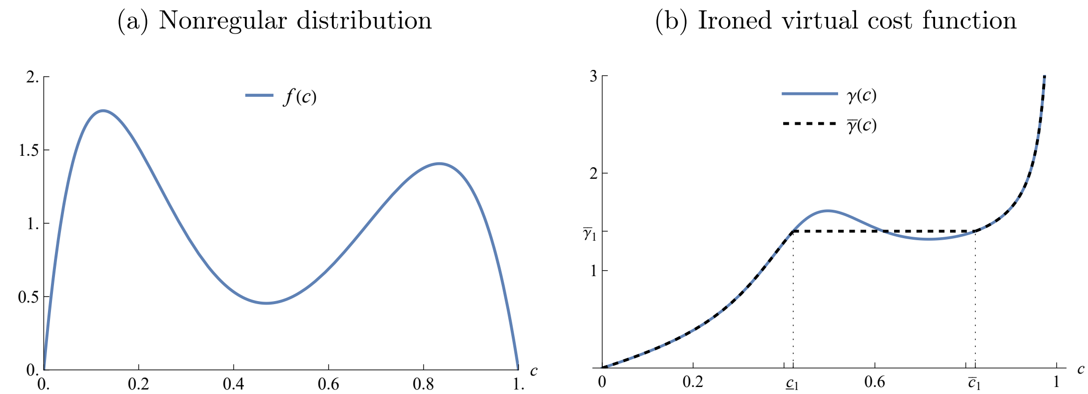

Figure 1: Nonregular cost distribution and its ironed virtual cost function. Assumes $f(c) = \text{Beta}(c; 2, 8)/2 + \text{Beta}(c; 6, 2)/2$, which has one ironing interval $[\underline{c}_1, \overline{c}_1] = [0.420, 0.821]$ and one ironing parameter $\overline{\gamma}_1 = 1.404$.

The optimal reserve price may not be unique if $v = \overline{\gamma}_k$ for some $k \in \mathcal{K}$, as the designer is then indifferent about including or not suppliers with $c \in \mathcal{I}_k$.[6] Without loss of generality, we break ties in favor of social surplus and include these suppliers; we thus choose

$$r^* = \max\{c \in [\underline{c}, \overline{c}] \mid \overline{\gamma}(c) \leq v\},$$

illustrated in Figure 2(a). Let $\mathcal{K}(v) \equiv \{k \in \mathcal{K} \mid \overline{c}_k \leq r^*\}$ and $\mathcal{I}(v) \equiv \cup_{k \in \mathcal{K}(v)} \mathcal{I}_k$ refer to the included ironing intervals.

For $k \in \mathcal{K}(v)$ and $c_i \in \mathcal{I}_k$, the probability of selecting supplier i is positive but must not vary with c_i. In addition, if multiple agents have the lowest ironed virtual cost, then, without loss of generality, we uniformly randomly select one of these suppliers.[7] We show in Supplemental Appendix B.1 that the probability of selecting supplier i is then equal to

$$\overline{P}_n(k) \equiv \mathbb{E}_c\left[P_n(c) \mid c \in \mathcal{I}_k\right].$$

[6] This is the procurement analogue to the observation made by Bulow and Roberts (1989) for the problem faced by a monopoly producing with constant marginal costs.

[7] Because they have the same ironed virtual cost, any other cost-independent tie-breaking rule, such as prioritizing agents on the basis of their names or identities, would work as well. Breaking ties uniformly randomly paves the way towards existence of a symmetric equilibrium.

The selection probability for a supplier with cost c is therefore:

$$P_n^*(c) \equiv \begin{cases} P_n(c) & \text{if } c \in [\underline{c}, r^*] \setminus \mathcal{I}(v), \\ \overline{P}_n(k) & \text{if } c \in \mathcal{I}_k \text{ for } k \in \mathcal{K}(v), \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

as illustrated in Supplemental Appendix Figure C.4.

By the payoff equivalence theorem, the expected payoff of a supplier with cost c, denoted $U_n^*(c)$, is, using $U_n^*(\overline{c}) = 0$,

$$U_n^*(c) = \int_c^{r^*} P_n^*(x)dx.$$

We amend the ARPA to implement the optimal procurement in the nonregular case, referring to the amended mechanism as the ARPA*. By construction, the ARPA* must induce a supplier with cost $c < r^*$ to bid $\beta_n^*(c) \equiv P_n^*(c) + U_n^*(c)$. Like the selection probability, the bid function is constant over any ironing interval, and jumps down at both ends of the interval: for $k \in \mathcal{K}(v)$,

$$\beta_n^-(k) \equiv \lim_{c \to \underline{c}_k^-} \beta_n^*(c) > \overline{\beta}_n(k) \equiv \beta_n^*(\underline{c}_k) = \beta_n^*(\overline{c}_k) > \beta_n^+(k) \equiv \lim_{c \to \overline{c}_k^+} \beta_n^*(c),$$

as illustrated in Figure 2(b).

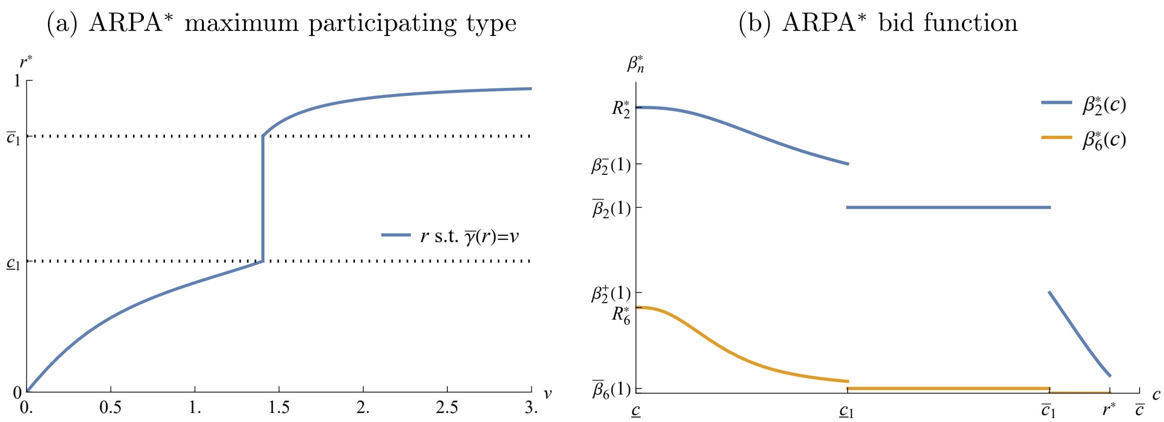

Figure 2: Maximum participating type in the ARPA* and the ARPA* bid function for a nonregular cost distribution. Panel (b) assumes $v = 2$, which implies $r^* = 0.941$, and $n \in \{2, 6\}$. Both panels assume $f(c) = \text{Beta}(c; 2, 8)/2 + \text{Beta}(c; 6, 2)/2$, which is shown in Figure 1(a).

The rules of the ARPA* are the same as in the ARPA with reserve $R_n^* \equiv \beta_n^*(\underline{c})$, with the caveat that, for selection purposes, any (off-path) bid, i.e. any b between $\overline{\beta}_n(k)$ and $\beta_n^+(k)$, for $k \in \mathcal{K}(v)$, is treated as $b' = \overline{\beta}_n(k)$. With this adjustment, we have:

Proposition 2. *In the ARPA* with reserve R_n^*, there exists a symmetric Bayesian Nash equilibrium, in which every supplier with cost $c \leq r^*$ bids $\overline{\beta}_n^*(c)$ and every supplier with cost $c > r^*$ refrains from bidding. This equilibrium implements the optimal procurement mechanism.*

Proof. The incentive compatibility of the optimal mechanism ensures that, for any $c \in \mathcal{C}$, bidding $\beta_n^*(c)$ (weakly) dominates any other bid in $\beta_n^*(\mathcal{C})$. It remains to show that doing so also dominates any unexpected bid, that is, any bid in $[-\infty, \beta_n^*(r))$ or in $[\beta_n^+(k), \beta_n^-(k)] \setminus \{\overline{\beta}_n(k)\}$, for $k \in \mathcal{K}(v)$. To see this, it suffices to note that: (i) any bid $b \in [-\infty, \beta_n^*(r))$ (resp., $b \in [\beta_n^+(k), \overline{\beta}_n(k))$) is dominated by $b' = \beta_n^*(r)$ (resp., $b' = \overline{\beta}_n(k)$), which yields the same selection probability, namely, 1 (resp., $\overline{P}_n(k)$, thanks to the adjustment brought to the ARPA* selection rule) but a larger payment; (ii) likewise, any bid $b \in (\overline{\beta}_n(k), \beta_n^-(k))$ is dominated by $b' = \beta_n^-(k)$, which again yields the same selection probability (namely, $P_n(\underline{c}_n(k))$) but a larger payment; and (iii) bidding $b = \beta_n^-(k)$ yields approximately the same payoff as bidding slightly above $\beta_n^-(k)$. ■

5 Resilience to liquidity shocks

The question of why a procurer should use an ARPA or ARPA* rather than a standard format such as the FPPA or SPPA (or the appropriately adjusted formats FPPA* and SPPA* defined below for the nonregular case) is a natural one. To address it, we extend the model and assume that suppliers are liquidity constrained and exposed to liquidity shocks that are not anticipated. We provide conditions under which the ARPA and ARPA* afford resilience to liquidity shocks in the sense that they permit a competitive supply structure to survive where none of the other formats does. In this thought experiment, we keep fixed the direct mechanism that a given auction format implements—efficient or optimal procurement—and merely ask which format is more resilient in the sense just defined. In particular, this is *not* an exercise in designing optimal mechanisms in the face of liquidity constraints and liquidity shocks.

As mentioned in footnote 1, production costs can alternatively be interpreted as (i) opportunity costs of selling an existing asset or as (ii) the financial cost of producing an asset or service. To fix ideas, for the remainder of this section, we stick to interpretation (ii), that is, the liquidity of a supplier decreases when it is selected to produce. Specifically, we stipulate that suppliers are initially cash constrained and can moreover be hit by an industry-wide liquidity shock $\varepsilon > 0$, which occurs after procurement and production. To ensure that bidding is not affected by the possibility of a liquidity shock, we assume that the probability of the shock occurring is 0, to be interpreted as the limit of a very small shock probability. The eventual cash holding, once the procurement and liquidity shocks have been realized is $p_i - c_i - \tilde{\varepsilon}$ for an agent with liquidity shock $\tilde{\varepsilon} \in \{0, \varepsilon\}$ who is paid p_i and has to produce the good at cost c_i, and $p_i - \tilde{\varepsilon}$ for the same agent if it does not have to produce. Finally, suppliers who end up with negative liquidity go bankrupt.

We will say that an auction format *preserves competition* if the event that two or more suppliers survive the liquidity shock occurs with positive probability.

5.1 Survival in efficient procurement auctions

We begin by considering the case of efficient procurement auctions, in which in equilibrium the supplier with the lowest cost produces with probability 1, provided this cost is below v. The FPPA and SPPA with reserve $r = \min\{v, \bar{c}\}$ are efficient, as is the ARPA with reserve $R_n(r)$, and this holds independently of whether γ is increasing.

Given r, if $\varepsilon \leq R_n(r) - \underline{c}$, there exists a unique $\tilde{c}_\varepsilon(r)$, defined by $B_n(\tilde{c}_\varepsilon(r); r) - \tilde{c}_\varepsilon = \varepsilon$, such that the supplier selected to produce in the ARPA survives after a liquidity shock if and only if its cost is less than or equal to $\tilde{c}_\varepsilon$. If, instead, $\varepsilon > R_n(r) - \underline{c}$, then the producer never survives a liquidity shock in the ARPA (nor in the FPPA or SPPA, as we will see). Similarly, if $\varepsilon \leq R_n(r)$, then a supplier that does *not* produce survives a liquidity shock in the ARPA if and only if its cost is less than or equal to $\tilde{c}_\varepsilon(r) \equiv B_n^{-1}(\varepsilon; r)$, with the inverse being taken with respect to the first argument of $B_n(c; r)$.

In contrast, in the FPPA and SPPA, only the winning supplier has a chance to survive a liquidity shock, and it moreover does so only if its cost is low enough. In particular, for the FPPA, if $\varepsilon \in (R_n(r) - \underline{c}, R_n(r))$, then a liquidity shock kills all suppliers in the FPPA,[8] but in the ARPA it kills only the supplier that produces and the suppliers with costs above $B_n^{-1}(\varepsilon; r)$. For smaller liquidity shocks, the supplier that produces in the FPPA survives only if its cost is less than $\tilde{c}'_\varepsilon(r)$, defined by $b_n^{FPPA}(\tilde{c}'_\varepsilon(r); r) - \tilde{c}'_\varepsilon = \varepsilon$. Because $B_n(c; r) < b_n^{FPPA}(c; r)$ holds for any $c > \underline{c}$ (see (3)), we have for any $\varepsilon \in (0, R_n(r) - \underline{c})$, $\tilde{c}'(r) > \tilde{c}(r)$. This implies that for liquidity shocks of this form, the supplier that is selected to produce is more likely to survive in the FPPA than in the ARPA. However, because suppliers that do not produce are paid nothing in the FPPA or, for that matter, the SPPA, only the ARPA has the potential to preserve competition.

Summarizing, we have:

Proposition 3. *Assume $r = \min\{v, \bar{c}\}$ and let the liquidity shock ε tend to 0. In the FPPA and SPPA with reserve r, only one supplier—the winner—survives a liquidity shock. In*

[8] Recall that $R_n(r) = b_n^{FPPA}(\underline{c}; r)$; hence, if $\varepsilon > R_n(r) - \underline{c}$, a liquidity shock kills the winning supplier even if it has the lowest possible cost, $\underline{c}$.

the ARPA with reserve $R_n(r)$, the supplier that produces survives with probability $1-[1-F(\tilde{c}_n(r))]^n$, and every other supplier with a cost $c<r$ survives.

Obviously, if $v<\bar{c}$, then suppliers with $c>v$ are excluded, regardless of the auction format. If, instead, $v\geq\bar{c}$, then a liquidity shock kills all non-selected suppliers in a FPPA or SPPA, and none of them in the ARPA.

Because $\tilde{c}_n(r)$ is decreasing in n, it is not a priori clear whether the probability that the producing supplier survives the liquidity shock when $\varepsilon\to 0$ increases or decreases in n because, for a fixed c, $1-[1-F(c)]^n$ increases with n. As shown in Supplemental Appendix Figure C.5, if costs are uniformly distributed, then the probability that the producing supplier survives such a liquidity shock decreases in n. However, numerical results suggest that it is bounded from below by 1/2.

5.2 Survival in optimal procurement auctions

We now extend the resilience analysis to optimal procurement auctions. In the regular case, Proposition 3 extends directly to optimal procurement auctions with r replaced by $r^*=\gamma^{-1}(\min\{v,\bar{c}\})$. For the nonregular case, we first need to identify variants of the FPPA and SPPA, referred to as FPPA* and SPPA*, that implement the optimal mechanism characterized in Section 4.2. In doing so, we hold constant the central features of these formats, namely that the winning supplier is paid its bid in a FPPA and that an SPPA endow suppliers with dominant strategies to bid truthfully.

For the FPPA*, we seek to generate a symmetric Bayesian Nash equilibrium in which a supplier with cost $c<r^*$ bids $\beta_n^{\mathrm{FPPA}^*}(c)$, equal to $\beta_n^*(c)/P_n^*(c)$; contrary to $\beta_n^*(c)$, this bid function jumps *up* at both ends of any relevant ironing interval, as illustrated in Supplemental Appendix Figure C.6. The rule of the FPPA* is the same as that of a FPPA with reserve r^*, except that for selection purposes, any (off-path) bid $b\in[\lim_{c\to\underline{c}_k^-}\beta_n^{\mathrm{FPPA}^*}(c),\beta_n^{\mathrm{FPPA}^*}(\underline{c}_k))$, for $k\in\mathcal{K}(v)$, is treated as $b'=\beta_n^{\mathrm{FPPA}^*}(\underline{c}_k)$.[9]

Similarly, the rule of the SPPA* is the same as that of the SPPA, unless the second-lowest bid is in $\mathcal{I}_k$, for some $k\in\mathcal{K}(v)$. In that case, if the lowest cost is also in $\mathcal{I}_k$, then one of the bidders in $\mathcal{I}_k$ is selected uniformly randomly and paid $\bar{c}_k$. If, instead, the lowest bid is less than $\underline{c}_k$, then the lowest bidder is selected and paid

$$p_m(k)\equiv\frac{1}{m+1}\bar{c}_k+\frac{m}{m+1}\underline{c}_k,$$

[9]The argument establishing that $\beta_n^{\mathrm{FPPA}^*}(c)$ constitutes a symmetric response to the FPPA* is similar to that for the ARPA*.

where m is the number of bids in $\mathcal{I}_k$. As in the usual second-price auction, truthful bidding is a dominant strategy in the SPPA*.[10] It then follows from the description of the SPPA* that it implements the optimal selection probabilities. Furthermore, incentive compatibility, together with the fact that a supplier with $c > r^*$ obtains zero payoff, ensures that a supplier with cost c obtains $U_n^*(c)$.

With these adjusted formats in hand, we have:

Proposition 4. *Assume $r = r^*$ and let the liquidity shock ε tend to 0. Among the auction formats SPPA, FPPA, and ARPA, only the ARPA preserves competition, and among the auction formats SPPA*, FPPA*, and ARPA*, only the ARPA* preserves competition.*

Proof. In the SPPA and FPPA, as well as in the SPPA* and FPPA*, at most one bidder receives a payment, and so only that bidder has a positive probability of surviving a liquidity shock. In the ARPA, two bidders survive, for example, whenever there are two bidders with costs below $\tilde{c}_n(r)$. Similarly, in the ARPA*, two bidders survive, for example, whenever there are two bidders with costs c such that $c < \beta_n^*(c)$, which occurs with positive probability. ■

As this proposition indicates, when suppliers have low liquidity, if the buyer wants to preserve competition, then the ARPA (or ARPA*) is a forced move because neither first-price nor second-price formats can "save" competition.

Although the propositions in this section apply for $\varepsilon \to 0$, for larger values of ε, it is still the case that in the FPPA and SPPA (and FPPA* and SPPA*), at most one supplier survives. In contrast, in the ARPA, all suppliers survive a shock of size ε whenever all suppliers' costs c satisfy $\varepsilon \leq B_n(c; r) \leq R_n(r)$ and the lowest cost is less than $\tilde{c}_\varepsilon$ (and analogously for the ARPA*). In that sense, $\varepsilon \to 0$ is a best-case scenario for the first-price and second-price formats.

6 Discussion

In this section, we discuss alternative auction formats, address collusion concerns, and provide concluding remarks.

6.1 Multiple-receive procurement auctions and maximal resilience

With the FPPA, the symmetric equilibrium bids are increasing in and exceed costs, implying that no supplier ever regrets ex post being selected to produce; in contrast, with the ARPA the equilibrium bids are decreasing in and eventually fall below costs, implying that

[10] See Supplemental Appendix B.2 for details.

the individual rationality constraint can be violated ex post for the selected supplier.[11] On the other hand, the ARPA has desirable resilience properties. To balance ex post participation and resilience concerns, one avenue is a *multiple-receive procurement auction*, where in equilibrium the $m \in \{1, \ldots, n\}$ bidders with the lowest costs are paid their bids and the lowest-cost bidder is selected to produce—the FPPA corresponding to $m = 1$, and the ARPA to $m = n$.

The above description of the auction rule is unorthodox, in that it relies on agents' types rather than bids. However, without knowing whether the equilibrium bidding strategy is increasing (as for the FPPA) or decreasing (as for the ARPA), this unorthodoxy cannot be avoided. If the bidding strategy is increasing (resp., decreasing), then the m lowest (resp., highest) bids will be paid and the lowest (resp. highest) bidder asked to produce. Interestingly, and maybe unfortunately, we show in Supplemental Appendix B.3 that a monotone equilibrium bidding strategy need not exist for $m \in \{2, \ldots, n-1\}$.

Alternatively, any given nonincreasing allocation (or selection) rule $\tilde{P}_n(c)$, together with associated informational rents, can be implemented in a way that compensates the selected supplier for the cost of production. Specifically, this implementation asks every supplier to report its cost; any bidder with a reported cost c is paid $\int_c^{\overline{c}} \tilde{P}_n(x)dx$ and, if selected to produce, is additionally paid c. To see that this induces truthful reporting in a Bayesian Nash equilibrium, observe that the expected payoff of a bidder with cost c that reports $\hat{c}$ is

$$\int_{\hat{c}}^{\overline{c}} \tilde{P}_n(x)dx + \tilde{P}_n(\hat{c})(\hat{c} - c),$$

whose derivative with respect to $\hat{c}$ is $\tilde{P}_n'(\hat{c})(\hat{c} - c)$. Thus, the first-order and second-order conditions are satisfied at $\hat{c} = c$. This implementation clearly *maximizes resilience*, as it always satisfies the individual rationality constraints ex post and pays every bidder the maximum amount possible, subject to this constraint and subject to implementing this selection rule.

6.2 Susceptibility to collusion

The susceptibility of various auction formats to collusion has been a frequent topic in the literature. The general view is that second-price formats are more susceptible to collusion than first-price formats (see, e.g., Kovacic et al., 2006). In the case of the ARPA, a collusive scheme might attempt to have the lowest-cost supplier submit the highest admissible bid,

[11] This partly mirrors a property of the all-pay auction, where ex post individual rationality is always violated for all but the winning bidder.

with all others submitting bids slightly below that, thereby securing the highest payment for all bidders. However, a number of factors mitigate the vulnerability of the ARPA to collusion, including the fact that the reserve is there substantially lower, and a strong incentive for the supplier designated to produce to deviate to a lower bid in order to avoid incurring the production cost.

To make things precise, consider the setup of Iossa et al. (2024), in which, in each period over an infinite horizon: (i) a buyer with value v seeks to purchase in two identical markets, in which two suppliers draw costs independently (across suppliers, markets, and time); (ii) all agents discount the future according to a common discount factor $\delta \in [0, 1)$; and (iii) bids submitted in one period are observed prior to the next period. Collusion takes the form of a market allocation in which, for each market $i = 1, 2$, supplier i is the designated supplier and the other supplier bids slightly less aggressively—detected deviations being punished with reversion to competitive bidding in both markets forever.

If the auction format is a FPPA with reserve r, then the designated supplier bids slightly below the reserve if its cost lies below the reserve and at cost otherwise, whereas the non-designated supplier bids the reserve if its cost lies below it and at cost otherwise. If the auction format is a SPPA with reserve r, then the designated supplier bids its cost and the nondesignated supplier bids the reserve if its cost lies below it, and at cost otherwise. As shown in Iossa et al. (2024), consistent with intuition, collusion is profitable in both the FPPA and SPPA, and the SPPA is more susceptible to collusion in the sense of having a lower critical discount factor such that collusion is sustainable.

For the ARPA, at least two collusive schemes are potentially relevant. In the first one, given the reserve $R_2(r)$, the designated bidder bids $R_2(r)$ and the nondesignated bidder bids slightly below $R_2(r)$, regardless of their respective costs; both bidders are then paid $R_2(r)$, and the designated bidder incurs the production cost. In the second scheme, the designated bidder bids $R_2(r)$ when its cost is less than $R_2(r)$ and bids its cost otherwise, whereas the nondesignated bidder bids slightly below $R_2(r)$ when its cost is less than

$$\hat{c}(r) \equiv \frac{R_2(r)}{1 - F(R_2(r))},$$

and bids its cost otherwise.[12] In the second scheme, the designated bidder with cost c receives $\max\{0, R_2(r) - c\}$, the nondesignated bidder with cost $c < \hat{c}(r)$ has an expected payoff of $R_2(r) - [1 - F(R_2(r))]c$, and the nondesignated bidder with a higher cost receives zero.

[12] In a third variant, the designated bidder bids $R_2(r)$ when its cost is less than $R_2(r)$ and bids its cost otherwise, and the nondesignated bidder bids slightly below $R_2(r)$ regardless of its cost. Our simulations suggest that this variant is dominated in terms of profitability and sustainability by at least one of the other variants.

There are a number of ways in which the ARPA is less vulnerable to collusion than the FPPA and SPPA (see Supplemental Appendix D for details). First, if the hazard rate is monotone, collusion is always profitable in the FPPA and SPPA (Iossa et al., 2024, Lemma 1); in contrast, it is not necessarily profitable in the ARPA. For example, if $F(c) = c^s$ on $[0, 1]$ with $s \in (0, 1/2)$, and $r = 1$, then ARPA collusion is not profitable under either variant. This is because the payment $R_2(r)$ that the colluding bidders receive approaches zero as s approaches zero, and so collusive payments are outweighed by the inefficiency associated with collusion.

Second, collusion in the ARPA can, in some settings, be harder to sustain than in the other formats. Indeed, to be sustainable, the short-term gain from deviation should be sufficiently lower than the long-term loss from reverting to competition. With the ARPA, the short-term gain from deviation can be quite high, and the long-term loss quite low. For instance, in the first scheme, a designated bidder with cost $\overline{c}$ has a short-term gain of $\overline{c}$ because it avoids production by deviating;[13] and for uniformly distributed costs on $[0, 1]$, the long-term loss for the second scheme is always below that of the FPPA and SPPA.[14] Thus, the ARPA can be less susceptible to collusion than the other formats—for specific settings, however, it can be more susceptible (see Supplemental Appendix D.5).

Finally, while market allocations in the FPPA and SPPA ensure that trade occurs whenever it would occur under competition, i.e., if and only if at least one supplier has a cost below the reserve r, this is not the case in the ARPA for $r < \overline{c}$. In variant 1, trade always occurs, which means that there is trade even when both suppliers have costs in $(r, \overline{c}]$ and so there would be no trade under competition. In variant 2, there is trade when both costs are in $(r, \overline{c}]$ and the nondesignated supplier's cost is less than $\hat{c}(r)$, and there is no trade when the designated supplier's cost is in $(R_2(r), r)$ and the nondesignated supplier's cost is in $(\hat{c}(r), \overline{c})$, even though in that case there is a supplier with cost below r.

Thus, while the ARPA, like other auction formats, is potentially vulnerable to collusion, it does not appear to be significantly more vulnerable than other commonly used auction formats.

6.3 Concluding remarks

Motivated by the recent upsurge of interest in supply chain resilience, this paper proposes a new family of auction formats, called all-receive procurement auctions, in which all suppliers with bids below the reserve are paid their bids and the highest bidder is selected to produce.

[13]In the second scheme, the most profitable deviation is by a designated supplier with cost $R(c)$ that deviates to a bid below that of the nondesignated supplier, for a gain of $F(\hat{c}(r))R(r)$.

[14]See supplemental appendix D.

These formats are payoff equivalent to standard formats such as the first-price and second-price procurement auction and permit implementing both efficient and optimal procurement auctions. Because in equilibrium more suppliers are paid than in the standard formats, the liquidity holdings of the suppliers that are not selected to produce are larger than in the standard formats, making them more resilient to certain forms of nonanticipated liquidity shocks. Because they are payoff equivalent, all-receive procurement auctions may afford resilience without any cost to the procurer.

Rather than paying multiple suppliers while having a single supplier produce, a procurer might consider spreading production across multiple suppliers. This would allow the buyer to obtain parts of the input even if a subset of the suppliers fail to deliver, for example, because of a breakdown in their production facility or problems associated with shipment. Notwithstanding the intuitive appeal, multisourcing of this form is inefficient in independent private values models and not optimal in the regular case because, with probability one, a single supplier has the lowest (virtual) cost. Consequently, under these conditions, multisourcing comes at a cost to the procurer. However, without regularity, multisourcing becomes optimal without any cost to the procurer because rather than breaking ties randomly among suppliers with the lowest ironed virtual cost to select a single supplier to produce, ties can equivalently be broken to have each of the suppliers involved in the tie produce the same fraction of the total. While away from efficiency, our focus has been on profit-maximizing auctions, in general (i.e., without sufficiently strong regularity) ironing and tie-breaking remain part of the procurement mechanisms that maximize a convex combination of social surplus and the buyer's profit, provided that the weight on the latter is positive. Thus, multisourcing for the purpose of increasing supply chain resilience can be optimal quite generally.

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Supplemental Appendix

to accompany

"All-receive procurement auctions"

by

Simon Loertscher, Leslie M. Marx, and Patrick Rey

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This Supplemental Appendix contains supplemental details and illustrations. Appendix A provides mechanism design foundations for the optimal auction. Appendix B provides supplemental details for some points made in the paper. Appendix C provides supplemental illustrations. Appendix D provides supplemental details for the analysis of collusion.

A Optimal auction design

We characterize here the mechanism that maximizes the buyer's expected payoff. Let $P_i(c_i)$ denote the expected probability that supplier i is selected as supplier and $T_i(c_i)$ denote its expected payment when its cost is c_i. Supplier i's expected gain can then be expressed as

$$G_i(c_i) = T_i(c_i) - c_i P_i(c_i).$$

Standard arguments imply that the mechanism is incentive-compatible if and only if, for every supplier i, $P_i(\cdot)$ is continuous and nonincreasing and, for every $c_i \in \mathcal{C}$,

$$G_i(c_i) = G_i(\bar{c}) + \int_{c_i}^{\bar{c}} P_i(c)\, dc. \tag{A.1}$$

The buyer's expected payoff U can be expressed as the total expected surplus, $\sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} (v - c_i) P_i(c_i)\, dF(c_i)$, minus the suppliers' total expected gains, $\sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} G_i(c_i)\, dF(c_i)$:

$$\begin{aligned} U &= \sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} \left[(v - c_i) P_i(c_i) - G_i(c_i)\right] dF(c_i) \\ &= \sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} \left[(v - c_i) P_i(c_i) - \int_{c_i}^{\bar{c}} P_i(c)\, dc\right] dF(c_i) - \sum_{i=1}^{n} G_i(\bar{c}). \end{aligned}$$

Obviously, it is optimal for the buyer to set $G_i(\bar{c}) = 0$ for every $i \in \{1, \ldots, n\}$ to minimize the suppliers' expected gains. Using Fubini's theorem, we can then rewrite the buyer's expected payoff as

$$\begin{aligned}
U &= \sum_{i=1}^{n} \Big(\int_{\underline{c}}^{\bar{c}} (v - c_i) P_i(c_i) f(c_i) dc_i - \int_{\underline{c}}^{\bar{c}} \int_{c_i}^{\bar{c}} P_i(x) f(c_i) dx dc_i \Big) \\
&= \sum_{i=1}^{n} \Big(\int_{\underline{c}}^{\bar{c}} (v - c_i) P_i(c_i) f(c_i) dc_i - \int_{\underline{c}}^{\bar{c}} P_i(x) \, F(x) \, dx \Big) \\
&= \sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} \Big(v - c_i - \frac{F(c_i)}{f(c_i)} \Big) P_i(c_i) dF(c_i) .
\end{aligned}$$

Finally, for $\mathbf{c} \in \mathcal{C}^n$, let $\hat{P}_i(\mathbf{c})$ denote supplier i's probability of being selected, as a function of *all* suppliers' costs, $\mathbf{c} = (\mathbf{c}_{-i}, c_i) = (c_1, ..., c_n)$. We thus have:

$$P_i(c_i) = \int_{\underline{c}}^{\bar{c}} \cdots \int_{\underline{c}}^{\bar{c}} \hat{P}_i(\mathbf{c}_{-i}; c_i) \, d\mathbf{F}_{-i}(\mathbf{c}_{-i}),$$

and the buyer's expected payoff can be expressed as:

$$\begin{aligned}
U &= \sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} \Big(v - c_i - \frac{F(c_i)}{f(c_i)} \Big) P_i(c_i) dF(c_i) \\
&= \sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} \Big(v - c_i - \frac{F(c_i)}{f(c_i)} \Big) \Big[\int_{\underline{c}}^{\bar{c}} \cdots \int_{\underline{c}}^{\bar{c}} \hat{P}_i(\mathbf{c}_{-i}; c_i) \, d\mathbf{F}_{-i}(\mathbf{c}_{-i}) \Big] dF(c_i) \\
&= \sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} \cdots \int_{\underline{c}}^{\bar{c}} \Big(v - c_i - \frac{F(c_i)}{f(c_i)} \Big) \hat{P}_i(\mathbf{c}) \, dF(c_1) \cdots dF(c_n) \\
&= \int_{\underline{c}}^{\bar{c}} \cdots \int_{\underline{c}}^{\bar{c}} \sum_{i=1}^{n} \Big(v - c_i - \frac{F(c_i)}{f(c_i)} \Big) \hat{P}_i(\mathbf{c}) \, dF(c_1) \cdots dF(c_n) \\
&= \mathbb{E}_{\mathbf{c}} \big[\sum_{i=1}^{n} (v - \gamma(c_i)) \hat{P}_i(\mathbf{c}) \big],
\end{aligned}$$

where the final equality uses

$$\gamma(c) \equiv c + \frac{F(c)}{f(c)}.$$

Hence, ideally the auction designer would like to maximize, for every $\mathbf{c} \in \mathcal{C}^n$,

$$\sum_{i=1}^{n} (v - \gamma(c_i)) \hat{P}_i(\mathbf{c}) .$$

The probabilities $\hat{P}_i(\cdot)$ must satisfy the feasibility constraints $\hat{P}_i(\cdot) \geq 0$ and $\sum_{i=1}^n \hat{P}_i(\cdot) \leq 1$. Ignoring incentive constraints, the buyer would therefore not procure if $v < \min_{i\in\mathcal{N}} \gamma(c_i)$, and otherwise would like to select the supplier with the lowest $\gamma(c_i)$. Let

$$\hat{P}_i^*(\mathbf{c}) = \begin{cases} 1 \text{ if } \gamma(c_i) < \min\{\min_{j\neq i} \gamma(c_j), v\}, \\ 0 \text{ otherwise,} \end{cases}$$

denote this unconstrained solution. Finally, for every supplier i, let

$$P_i^*(c_i) \equiv \int_{\underline{c}}^{\bar{c}} \cdots \int_{\underline{c}}^{\bar{c}} \hat{P}_i^*(\mathbf{c}_i; c_i)\, d\mathbf{F}_{-i}(\mathbf{c}_{-i})$$

denote the resulting interim expected selection probability.

Related to Corollary 1, building on the analysis above, if $\gamma(\cdot)$ is increasing, the unconstrained solution selects the lowest-cost supplier if its cost lies below $r^* \equiv \min\{\gamma^{-1}(v), \bar{c}\}$, and does not select any supplier otherwise:

$$\hat{P}_i^*(\mathbf{c}) \equiv \begin{cases} 1 \text{ if } c_i < \min\{\min_{j\neq i} c_j, r^*\}, \\ 0 \text{ otherwise.} \end{cases}$$

The resulting expected selection probability is then symmetric and given by

$$P_i^*(c_i) = P^*(c_i) \equiv \begin{cases} P_n(c_i) \text{ if } c_i < r^*, \\ 0 \text{ otherwise.} \end{cases}$$

If instead $\gamma(\cdot)$ is not everywhere increasing, then the unconstrained solution does not have the monotonicity property required for incentive compatibility. The optimal allocation then relies on an "ironed" version of the virtual cost, $\bar{\gamma}(c)$, which is nondecreasing, continuous, and coincides with $\gamma(c)$ whenever $\bar{\gamma}(c)$ is increasing (see Myerson, 1981). Specifically, there exists a sequence of $K = |\mathcal{K}|$ "bunching" ranges (or "ironing intervals") $\mathcal{C}_k \equiv [\underline{c}(k), \bar{c}(k)]$, for $k \in \mathcal{K}$ satisfying $\{\underline{c}_k, \bar{c}_k\}_{k\in\mathcal{K}}$ with $\underline{c} \leq \underline{c}_1 < \bar{c}_1 < \underline{c}_2 < \cdots < \bar{c}_{K-1} < \underline{c}_K < \bar{c}_K \leq \bar{c}$, such that:

- $\bar{\gamma}(c) = \gamma(c)$ for $c \notin \cup_k \{\mathcal{C}_k\}$;
- $\bar{\gamma}(c) = \gamma(\underline{c}_k) = \gamma(\bar{c}_k) \equiv \overline{\gamma}_k$ for $c \in \mathcal{C}_k$; and
- $\int_{\underline{c}_k}^{\bar{c}_k} [\bar{\gamma}(c) - \gamma(c)]\, dF(c) = 0$.

The optimal mechanism is then such that:

$$\hat{P}_i^{**}(\mathbf{c}) = \begin{cases} 1 \text{ if } \bar{\gamma}(c_i) < \min\{\min_{j\neq i} \bar{\gamma}(c_j), v\}. \\ 0 \text{ otherwise.} \end{cases}$$

implying:[1]

$$P_i^{**}(c_i) = P^{**}(c_i) \equiv \int_{\underline{c}}^{\bar{c}} \cdots \int_{\underline{c}}^{\bar{c}} \hat{P}_i^{**}(\mathbf{c}_{-i}; c_i)\, d\mathbf{F}_{-i}(\mathbf{c}_{-i}),$$

and:

$$T_i^{**}(c_i) = T^{**}(c_i) \equiv c_i P^{**}(c_i) + \int_{c_i}^{\bar{c}} P^{**}(c)\, dc,$$

By construction, $\hat{P}_i^{**}(\cdot; c_i)$ and $P_i^{**}(c_i)$ are constant in any range $c_i \in \mathcal{C}_k$; from incentive-compatibility, $T_i^{**}(c_i)$ is thus also constant in any such range. By contrast, outside these ranges, $\hat{P}_i^{**}(\cdot; c_i)$, $P_i^{**}(c_i)$, and $T_i^{**}(c_i)$ are all decreasing in c_i.

B Supplemental details

B.1 Derivation of $\overline{P}_n(k)$

$$\begin{aligned}
\overline{P}_n(k) &= \sum_{k=0}^{n-1} \binom{n-1}{k} [1 - F(\bar{c}_k)]^{n-1-k} \frac{[F(\bar{c}_k) - F(\underline{c}_k)]^k}{k+1} \\
&= \sum_{k=0}^{n-1} \frac{(n-1)!}{k!\,(n-1-k)!} [1 - F(\bar{c}_k)]^{n-1-k} \frac{[F(\bar{c}_k) - F(\underline{c}_k)]^k}{k+1} \\
&= \frac{1}{n\,[F(\bar{c}_k) - F(\underline{c}_k)]} \sum_{k=0}^{n-1} \frac{n!}{(k+1)!\,[n-(k+1)]!} [1 - F(\bar{c}_k)]^{n-(k+1)} [F(\bar{c}_k) - F(\underline{c}_k)]^{k+1} \\
&= \frac{1}{n\,[F(\bar{c}_k) - F(\underline{c}_k)]} \sum_{k=1}^{n} \frac{n!}{k!\,(n-k)!} [1 - F(\bar{c}_k)]^{n-k} [F(\bar{c}_k) - F(\underline{c}_k)]^k \\
&= \frac{1}{n\,[F(\bar{c}_k) - F(\underline{c}_k)]} \left(\sum_{k=0}^{n} \binom{n}{k} [1 - F(\bar{c}_k)]^{n-k} [F(\bar{c}_k) - F(\underline{c}_k)]^k - [1 - F(\bar{c}_k)]^n \right) \\
&= \frac{[1 - F(\underline{c}_k)]^n - [1 - F(\bar{c}_k)]^n}{n\,[F(\bar{c}_k) - F(\underline{c}_k)]}.
\end{aligned}$$

[1] By construction, $\hat{P}_i^{**}(\mathbf{c})$ is symmetric, implying that $P_i^{**}(c_i)$ is also symmetric: $\hat{P}_i^{**}((\mathbf{c}_{-i,j}, c_j); c_i) = \hat{P}_j^{**}((\mathbf{c}_{-i,j}, c_i); c_j)$ and $P_i^{**}(c) = P_j^{**}(c)$ for any $i \neq j \in \{1, \ldots, n\}$.

Further, we have:

$$\begin{aligned}\mathbb{E}_c\left[P_n(c)\mid c\in\mathcal{I}_k\right] &= \int_{\underline{c}_k}^{\overline{c}_k}\left[1-F(c)\right]^{n-1}\frac{f(c)}{F(\overline{c}_k)-F(\underline{c}_k)}dc\\ &= \frac{\left[1-F(\underline{c}_k)\right]^n-\left[1-F(\overline{c}_k)\right]^n}{n\left[F(\overline{c}_k)-F(\underline{c}_k)\right]}\\ &= \overline{P}_n(k).\end{aligned}$$

B.2 Dominant strategies in the adjusted SPPA

As noted, truthful bidding is a dominant strategy in the SPPA*. The usual reasoning ensures that, compared with bidding at cost, no supplier can benefit from bidding below cost, or from bidding above cost when the second-lowest cost does not lie in $\mathcal{I}(v)$.[2] Hence, to establish dominant strategy incentive compatibility (DIC), we only need to check that a supplier with cost c cannot benefit from bidding $b>c$ when $m>0$ next-lowest costs lie in $\mathcal{I}_k$, for some $k\in\mathcal{K}_v$. If $c\in\mathcal{I}_k$, then any $b\in\mathcal{I}_k$ yields the same payoff as bidding c, and any $b>\overline{c}_k$ would prevent the supplier from being selected, and is thus dominated by bidding c. Consider now a bidder with cost $c\leq\underline{c}_k$ that is paid p when bidding below $\underline{c}_k$. By bidding in $\mathcal{I}_k$, the supplier would be selected with probability $1/(m+1)$ and paid in that case $\overline{c}_k$. DIC thus requires $p-c\geq\frac{\overline{c}_k-c}{m+1}$. This constraint is tightest at $c=\underline{c}_k$ because the derivative with respect to c of the right-hand side, $-1/(m+1)$, exceeds that of the left-hand side, -1. Setting $p=p_m(k)$ thus satisfies DIC with equality for the cost $\underline{c}_k$ and with a strict inequality for all lower costs.

B.3 Details for multiple-receive auctions

In this appendix, we now briefly elaborate on the scope and challenges for using multiple-receive procurement auctions. For this purpose, we focus on ex post efficiency and assume that $v\geq\overline{c}$.

For there to be an equilibrium in which paying the m lowest or highest bidders means paying the m suppliers with the lowest costs, the equilibrium bid function needs to be strictly monotone. While this is, evidently, the case for $m\in\{1,n\}$ regardless of the distribution, whether it holds for any $m\in\{2,\ldots,n-1\}$ depends on the distribution, as we show next. This suggests that the scope for using multiple-receive procurement auctions to implement the efficient allocation, while striking a different balance between ex post participation constraints and resilience, comes with a caveat.

Even if equilibrium bidding is monotone for all m, because it is increasing for $m=1$ and

[2] Recall that $\mathcal{I}_k$ is the k-th ironing interval, and we define $\mathcal{K}(v)\equiv\{k\in\mathcal{K}\mid\overline{c}_k\leq r^*\}$ and $\mathcal{I}(v)\equiv\cup_{k\in\mathcal{K}(v)}\mathcal{I}_k$.

decreasing for $m = n$, to determine the rules of the auction, the designer will need to know for which values of m the bid function is increasing (decreasing). The approach we take is, therefore, akin to reverse auction theory—we first assume that a monotone equilibrium exists in which either the m lowest or m highest bidders are paid, and then we verify whether the assumption is correct and, if so, determine the appropriate auction rules. For $m \in \{1, \dots, n\}$ and $c \in [\underline{c}, \overline{c}]$, let

$$q_{m,n}(c) \equiv \sum_{i=0}^{m-1} \binom{n-1}{i} F(c)^i [1 - F(c)]^{n-1-i}$$

denote the probability that no more than $m - 1$ draws among $n - 1$ are less than c. Notice that $q_{1,n}(c) = [1 - F(c)]^{n-1} = P_n(c)$ and $q_{n,n}(c) = 1$. Moreover, we have $q_{m+1,n}(c) > q_{m,n}(c)$ for any $m < n$ and $c \in (\underline{c}, \overline{c})$.

In a symmetric equilibrium with bid function $\beta_{m,n}(c)$, a supplier with cost c maximizes

$$\beta_{m,n}(\hat{c}) q_{m,n}(\hat{c}) - P_n(\hat{c}) c.$$

As usual, this has to be maximized at $\hat{c} = c$, yielding

$$\beta_{m,n}(c) = \frac{c P_n(c) + \int_c^{\overline{c}} P_n(x) dx}{q_{m,n}(c)}.$$

Because $q_{1,n}(c) = P_n(c)$, we have $\beta_{1,n}(c) = c + \frac{\int_c^{\overline{c}} P_n(x) dx}{P_n(c)}$, which is increasing, as it should be, because it is the equilibrium bid function in a FPPA. Similarly, because $q_{n,n}(c) = 1$, $\beta_{n,n}(c) = c P_n(c) + \int_c^{\overline{c}} P_n(x) dx$, which is the (decreasing) ARPA bid function. Moreover, because $q_{m,n}(c)$ decreases in m, $\beta_{m,n}(c)$ decreases in m, in line with the hypothesis that decreasing m reduces the scope that a selected supplier's participation constraint is violated ex post.

Lemma B.1. *For $n \geq 2$ and $m \in \{1, \dots, n\}$,*

$$\beta'_{2,n}(c) = \frac{(n-1)(1 - F(c))^{n-3} f(c)}{(q_{2,n}(c))^2} \left(-c(1 - F(c))^{n-1} + (n-2) F(c) \int_c^{\overline{c}} (1 - F(x))^{n-1} dx \right).$$

Proof. We have

$$\frac{P_n(c)}{q_{m,n}(c)} = \frac{1}{\sum_{i=0}^{m-1} \binom{n-1}{i} \left[\frac{F(c)}{1-F(c)} \right]^i},$$

which is decreasing in c for $m > 1$. The derivative of $\beta_{m,n}$ is

$$\beta'_{m,n}(c) = c\left[\frac{P_n(c)}{q_{m,n}(c)}\right]' - \frac{q'_{m,n}(c)}{q_{m,n}(c)}\frac{\int_c^{\bar{c}} P_n(x)dx}{q_{m,n}(c)}.$$

The first term is negative and the second is negative, which with the negative sign makes it positive. Focusing on the case of $m = 2$, $q_{2,n}(c) = (1 - F(c))^{n-2}(1 - F(c) + (n-1)F(c))$, $q'_{2,n}(c) = -(n-1)(n-2)F(c)(1-F(c))^{n-3}f(c)$, and $\left[\frac{P_n(c)}{q_{2,n}(c)}\right]' = \frac{-(n-1)(1-F(c))^{2n-4}f(c)}{(q_{2,n}(c))^2}$, so

$$\begin{aligned}
\beta'_{2,n}(c) &= c\frac{-(n-1)(1-F(c))^{2n-4}f(c)}{(q_{2,n}(c))^2} - \frac{q'_{2,n}(c)}{q_{2,n}(c)}\frac{\int_c^{\bar{c}} P_n(x)dx}{q_{2,n}(c)} \\
&= \frac{(n-1)(1-F(c))^{n-3}f(c)}{(q_{2,n}(c))^2}\left(-c(1-F(c))^{n-1} + (n-2)F(c)\int_c^{\bar{c}}(1-F(x))^{n-1}dx\right),
\end{aligned}$$

which completes the proof. ■

Using Lemma B.1, for uniformly distributed costs, $\beta_{2,n}(c)$ is decreasing. This means that for uniformly distributed costs, the multiple-receive procurement auction that pays the 2 highest bidders and selects the highest bidder to produce has a monotone equilibrium.

However, there are distributions such that this is not the case. For example, for $F(c) = \sqrt{c}$ with support $[0,1]$, $\beta_{2,n}(c)$ is nonmonotone for $n \geq 3$. To see this, note that using Lemma B.1, for the case of $F(c) = \sqrt{c}$, we have

$$\beta'_{2,n}(c) = -\frac{(n-1)\left(-\sqrt{c}\left(n^2 - 7n + 4\right) + 2c(n-2)n - 2n + 4\right)}{2n(n+1)\left(\sqrt{c}(n-2)+1\right)^2},$$

where $\beta'_{2,n}(0) = \frac{2-3n+n^2}{n+n^2}$, which is positive for $n > 2$, and $\beta'_{2,n}(1) = \frac{-1}{2(n-1)}$, which is negative for $n \geq 2$, establishing that $\beta_{2,n}(c)$ is not monotone for this case. This is illustrated in Figure B.1.

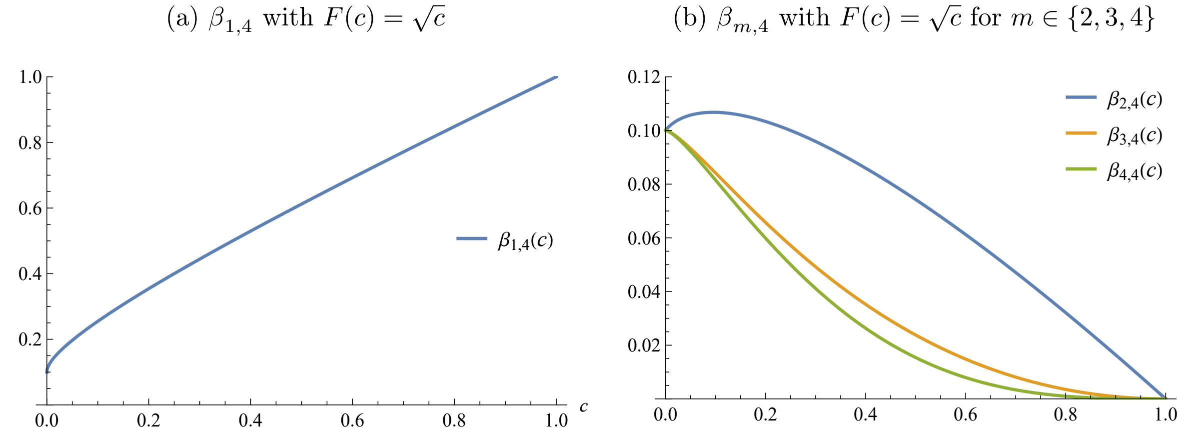


Figure B.1: Bid functions $\beta_{m,n}(c)$ for $n = 4$ and $m \in \{1, 2, 3, 4\}$ assuming $F(c) = \sqrt{c}$ with support $[0, 1]$.

One can also show that the expression for $\beta'_{2,n}(c)$ in Lemma B.1 is positive for $F(c) = \sqrt{c}$ with support $[0, 1]$ and $n \in \{3, \dots\}$ if $c = 0$, but negative if $c = 1$. Thus, for $F(c) = \sqrt{c}$ and $n = 4$, we require m of at least 3 to have a monotone decreasing bid function.

C Supplemental illustrations

C.1 Illustration of the ARPA bid function and reserve.

In Figure C.1, will illustrate the ARPA bid function and reserve.

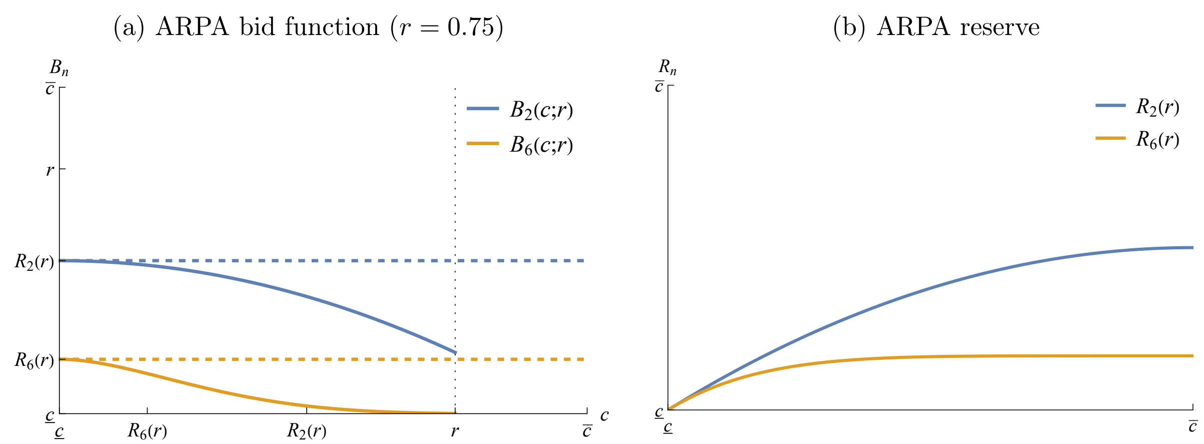


Figure C.1: Illustration of the ARPA bid function and reserve. Panel (a) shows the ARPA equilibrum bid strategy for $r = 0.75$ and $n \in \{2, 6\}$. Panel (b) shows the ARPA reserve corresponding to a second-price or first-price procurement reserve of r. Assumes uniformly distributed costs on $[0, 1]$.

C.2 Illustration of the comparison between FPPA and ARPA bid functions

A supplier with cost $\underline{c}$ bids the same amount in the ARPA and in the FPPA. In the ARPA, bids are decreasing in c, but in the FPPA are increasing in c. As a result, all bids in the FPPA weakly exceed all bids in the ARPA. This is illustrated in Figure C.2.

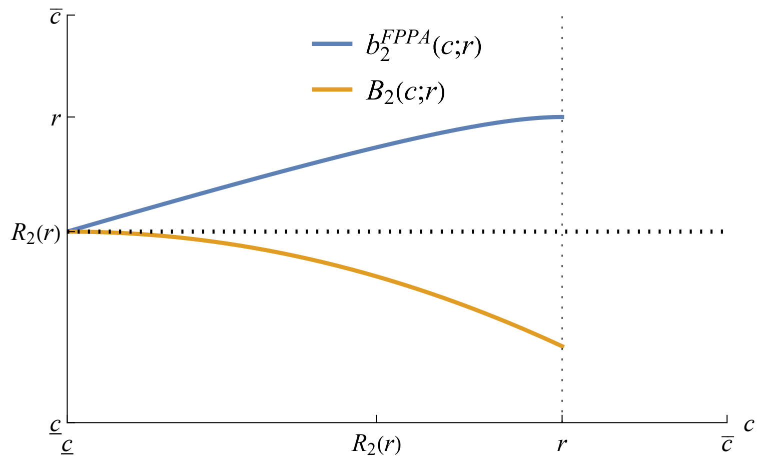


Figure C.2: Comparison of the ARPA and FPPA bid functions. Assumes $r = 0.75$ and uniformly distributed costs on $[0, 1]$.

C.3 Illustration of the interim expected payoffs and threshold type in the ARPA

As discussed in Section 3, there exists a unique $\tilde{c}_n(r) \in (\underline{c}, r)$ such that $B_n(c; r) \gtreqless c$ if and only if $c \lesseqgtr \tilde{c}_n(r)$. This means that suppliers with costs below $\tilde{c}_n(r)$ make a profit even if called upon to produce, whereas suppliers with costs above $\tilde{c}_n(r)$ make a profit only if they do not have to produce (and make a loss otherwise). Because $B_n(c; r)$ is decreasing in n and increasing in r, $\tilde{c}_n(r)$ is also decreasing in n and increasing in r. This is illustrated in Figure C.3.

(a) ARPA interim expected payoffs ($r = 0.75$) (b) Threshold type for payoff sign

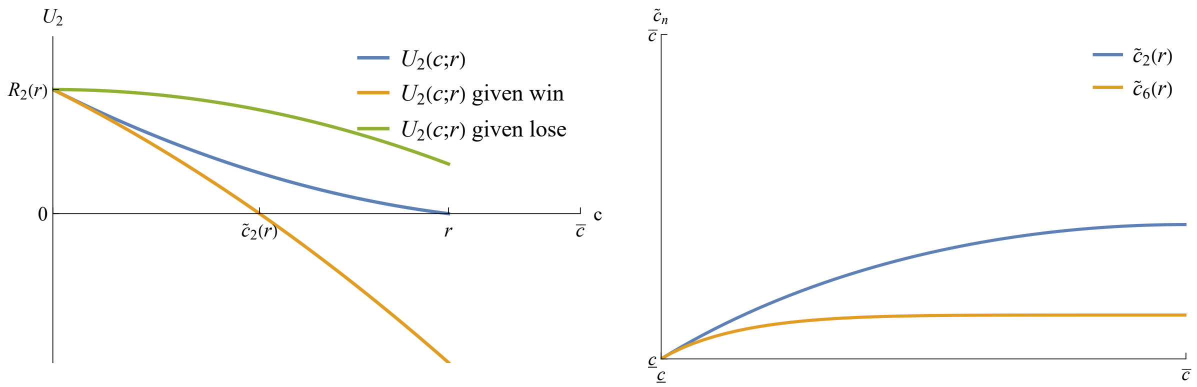


Figure C.3: Panel (a): interim expected payoffs in the ARPA for $r = 0.75$ and $n = 2$; Panel (b): threshold type $\tilde{c}_n(r)$ for $n \in \{2, 6\}$ such that, conditional on producing the good, interim expected payoffs are positive for lower types and negative for higher types. Assumes uniformly distributed costs on $[0, 1]$.

C.4 Illustration of the selection probability in the optimal mechanism

As defined in equation (5), $P_n^*(c)$ is the selection probability for a supplier with cost c in the optimal mechanism with uniform random tie-breaking. It is illustrated in Figure C.4.

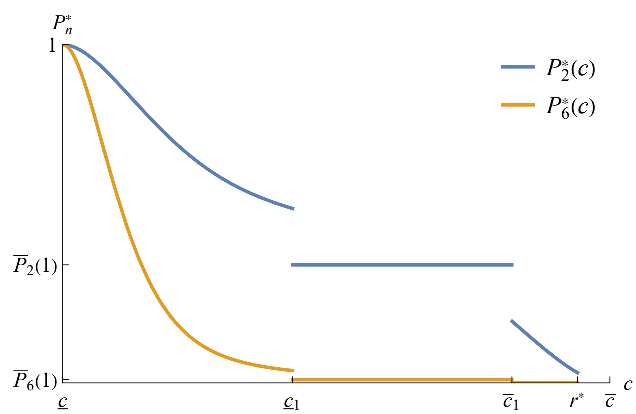


Figure C.4: Selection probability $P_n^*(c)$ in the optimal mechanism for a nonregular cost distribution. Assumes $f(c) = \text{Beta}(c; 2, 8)/2 + \text{Beta}(c; 6, 2)/2$, which is shown in Figure 1(a), and $r^* = 0.9409$, which is optimal for $v = 2$.

C.5 Illustration of the survival rate in the ARPA

In Figure C.5, we illustrate the survival rate in the ARPA, which is discussed in Section 5.1.

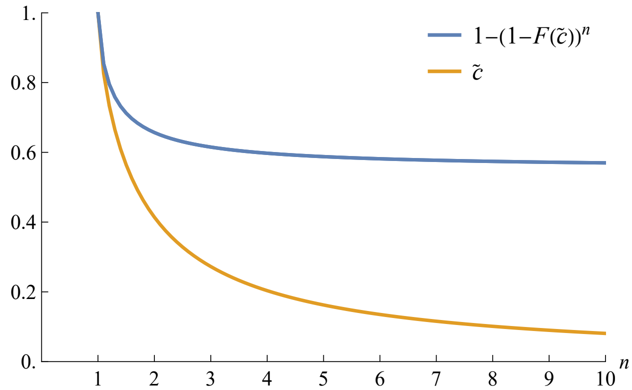


Figure C.5: Threshold type $\tilde{c}$ such that $B_n(\tilde{c};\overline{c}) - \tilde{c} = 0$ and the probability, $1 - (1 - F(\tilde{c}))^n$, that the producing supplier survives a small liquidity shock in the ARPA. Assumes uniformly distributed costs on $[0, 1]$ and $r = \overline{c}$.

C.6 Illustration of the adjusted FPPA

In Figure C.6, we illustrate the equilibrium bid functions in the adjusted FPPA, which is defined in Section 5.2.

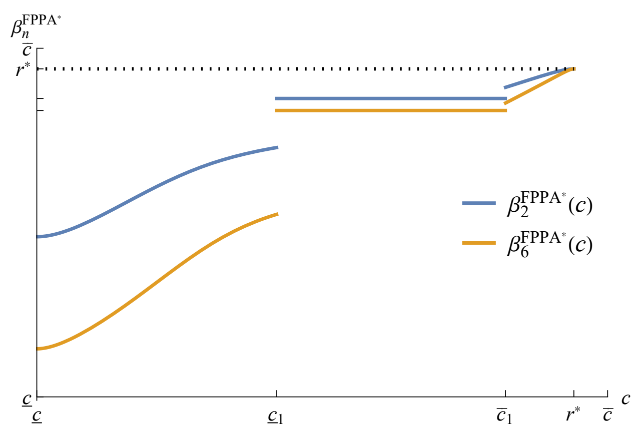


Figure C.6: FPPA* bid function assuming $f(c) = \text{Beta}(c; 2, 8)/2 + \text{Beta}(c; 6, 2)/2$, which is shown in Figure 1(a), and $r^* = 0.9409$, which is optimal for $v = 2$.

D Details for analysis of collusion

D.1 Framework

We consider the framework of Iossa et al. (2024). There are two identical markets. A buyer operates in both markets and has value v for one unit of the good in each market in each period over time. There are two suppliers 1 and 2, with costs drawn from distribution F with density f over the support $[\underline{c}, \overline{c}]$ and increasing reversed hazard rate $F(c)/f(c)$. Cost draws are independent across suppliers, markets, and time. All agents are risk neutral with quasi-linear utility, and discount the future according to the common discount factor $\delta \in [0,1)$. Bids are observed at the end of each period.

We consider three auction formats: FPPA or SPPA with reserve $r \in (\underline{c}, \overline{c}]$, and ARPA with reserve $R(r)$, where $R(r)$ is defined to be the same as $R_2(r)$ as defined in equation (2). Thus, we have

$$R(r) \equiv \underline{c} + \int_{\underline{c}}^{r} [1 - F(c)]\, dc = r - \Phi(r), \tag{D.2}$$

where

$$\Phi(r) \equiv \int_{\underline{c}}^{r} F(c)\, dc$$

is the primitive of $F(\cdot)$ satisfying $\Phi(\underline{c}) = 0$. By construction, $R(r) \in (\underline{c}, r)$ and is strictly increasing in r in the range $r \in (\underline{c}, \overline{c}]$. Note that:[3]

$$R(r) > [1 - F(r)]\, r. \tag{D.3}$$

We begin by defining the competitive profit. Under a SPPA, a supplier with cost c obtains an expected profit equal to:

$$\pi^c(c;r) \equiv \mathbb{E}_{\tilde{c}}\left[\max\{0, \hat{\pi}^c(c;r) \min\{r, \tilde{c}\} - c\}\right] = \mathbf{1}_{c<r} \times \hat{\pi}^c(c;r),$$

where (using $n = 2$)

$$\begin{aligned}
\hat{\pi}^c(c;r) &\equiv \int_{c}^{r} (\tilde{c} - c)\, dF(\tilde{c}) + [1 - F(r)](r - c) \\
&= [(\tilde{c} - c) F(\tilde{c})]_c^r - \int_{c}^{r} F(\tilde{c})\, d\tilde{c} + [1 - F(r)](r - c) \\
&= (r - c) F(r) - [\Phi(r) - \Phi(c)] + [1 - F(r)](r - c) \\
&= r - c - [\Phi(r) - \Phi(c)] \\
&= R(r) - R(c).
\end{aligned} \tag{D.4}$$

Using the revenue equivalence theorem, under all three formats, the expected competitive profit is:

$$\overline{\pi}^c(r) \equiv \mathbb{E}_c[\pi^c(c;r)] = \int_{\underline{c}}^{r} F(c)[1 - F(c)]\, dc = \Phi(r) - \Phi_2(r),$$

[3] Indeed, for $r = \underline{c}$ we have $R(\underline{c}) = [1 - F(\underline{c})]\, \underline{c}\, (= \underline{c})$ and, for $r > \underline{c}$:

$$\frac{d}{dr}\left(R(r) - [1 - F(r)]\, r\right) = [1 - F(r)] - ([1 - F(r)] - f(r)\, r) = f(r)\, r > 0.$$

where

$$\Phi_2(r) \equiv \int_{\underline{c}}^{r} F^2(c)dc = [F(c)\Phi(c)]_{\underline{c}}^{r} - \int_{\underline{c}}^{r} \Phi(c)\,dF(c) = F(r)\left(\Phi(r) - \mathbb{E}_c[\Phi(c) \mid c \leq r]\right).$$

Total industry profit is then

$$\overline{\Pi}_C(r) \equiv 2\pi^c(r) = 2[\Phi(r) - \Phi_2(r)].$$

D.2 Market allocation

We consider collusion that takes the form of a market allocation. The general idea is that in each market a designated supplier is selected as provider; the other bids slightly less aggressively. In a FPPA with reserve r, the designated supplier bids slightly below the reserve if its cost lies below it, and at cost otherwise; nondesignated supplier bids the reserve if its cost lies below it, and at cost otherwise. In an SPPA with reserve r, the designated supplier bids its cost; the nondesignated supplier bids the reserve if its cost lies below it, and at cost otherwise.

We now turn to the ARPA with reserve $R(r)$. We look for a collusive mechanism that has the following features:

- each supplier is either active (bids weakly below the reserve) or inactive (bids strictly above the reserve);
- active suppliers obtain (almost) $R(r)$;
- when both suppliers are active, one of them (the designated supplier, hereafter) is selected to be the provider;
- this is achieved by requiring the designated supplier to bid $R(r)$ whenever active, and the nondesignated supplier to bid slightly below $R(r)$ whenever active.

We can distinguish two variants, depending on whether the designated supplier is always active. If the designated supplier is always active, it obtains

$$R(r) - c.$$

The nondesignated supplier then obtains $R(r)$ whenever active; hence, the most profitable scheme has the nondesignated supplier also being always active (Variant 1 hereafter). If instead the designated supplier is not always active, then the on-path incentive constraint (namely deviating from active to inactive) implies that it cannot be active when its cost strictly exceeds $R(r)$; conversely, whenever its cost lies below $R(r)$, it is profitable for it to be active. Hence, the most profitable scheme has the designated supplier being active if and only if its cost lies below $R(r)$. If active, the nondesignated supplier with cost c then obtains

$$R(r) - [1 - F(R(r))]\, c = [1 - F(R(r))]\, [\hat{c}(r) - c],$$

where

$$\hat{c}(r) \equiv \frac{R(r)}{1 - F(R(r))}.$$

Two subcases can therefore be distinguished:

- If $\hat{c}(r) \geq \bar{c}$, then the the most profitable scheme has the nondesignated supplier being always active (Variant $2a$ hereafter).

- If instead $\hat{c}(r) < \bar{c}$, then in addition to Variant $2a$, there exists an alternative variant in which the nondesignated supplier is not always active. The on-path incentive constraint (namely deviating from active to inactive) then implies that it cannot be active when its cost strictly exceeds $\hat{c}(r)$; conversely, whenever its cost lies below $\hat{c}(r)$, it is profitable for it to be active. Hence, the most profitable scheme has the designated supplier being active if and only if its cost lies below $\hat{c}(r)$ (Variant $2b$ hereafter).

Summing-up, there are two relevant variants, and sometimes a third one. In Variant 1, the designated supplier bids $R(r)$ regardless of its cost, and obtains $R(r) - c$; the nondesignated supplier bids slightly below $R(r)$ regardless of its cost, and obtains $R(r)$. In Variant $2a$, the designated supplier bids $R(r)$ if its cost lies below $R(r)$, in which case it obtains $R(r) - c$, and at cost otherwise, in which case it obtains 0; the nondesignated supplier bids slightly below $R(r)$ regardless of its cost, and obtains $R(r) - [1 - F(R(r))]\, c$. In Variant $2b$ (if $\hat{c}(r) < \bar{c}$), the designated supplier acts as in Variant $2a$: it bids $R(r)$ if its cost c lies below $R(r)$, in which case it obtains $R(r) - c$, and at cost otherwise, in which case it obtains 0; the nondesignated supplier bids slightly below $R(r)$ only if its cost lies below $\hat{c}(r)$, in which case it obtains $R(r) - [1 - F(R(r))]\, c$, and at cost otherwise, in which case it obtains 0.

D.3 Costs and benefits of collusion

FPPA and SPPA with reserve r

In the FPPA and SPPA, the designated supplier has expected payoff (with subscript P standing for first/second-Price procurement auction)

$$\pi_P^d(c;r) \equiv \max\{0, r-c\} = \mathbf{1}_{c<r} \times \hat{\pi}_P^d(c;r),$$

where

$$\hat{\pi}_P^d(c;r) \equiv r - c > \hat{\pi}^c(c;r),$$

where the inequality stems from (D.4) and $R'(c) = 1 - F(c) < 1$,[4] and

$$\overline{\pi}_P^d(r) \equiv \mathbb{E}_c\left[\pi_P^d(c)\right] = \Phi(r)\,(> \overline{\pi}^c(r)).$$

The nondesignated supplier has expected payoff

$$\pi_P^n(c;r) \equiv [1 - F(r)]\,\pi_P^d(c;r) = \mathbf{1}_{c<r} \times \hat{\pi}_P^n(c;r),$$

where

$$\hat{\pi}_P^n(c;r) \equiv [1 - F(r)]\,(r - c) < \hat{\pi}^c(c;r),$$

where the inequality stems from

$$\hat{\pi}^c(c;r) - \hat{\pi}_P^n(c;r) = \int_c^r (\tilde{c} - c)\, dF(\tilde{c}) > 0,$$

and

$$\overline{\pi}_P^n(r) \equiv \mathbb{E}_c\left[\pi_P^n(c;r)\right] = [1 - F(r)]\,\overline{\pi}_P^d(r)\,(< \overline{\pi}^c(r)).$$

The net benefit from collusion is constructed as:

$$\begin{aligned}
\overline{\Pi}_P(r) &\equiv \overline{\pi}_P^d(r) + \overline{\pi}_P^n(r) = [2 - F(r)]\,\Phi(r), \\
\overline{\Pi}_C(r) &\equiv 2\overline{\pi}^c(r) = 2\left[\Phi(r) - \Phi_2(r)\right], \\
\Delta_P(r) &\equiv \overline{\Pi}_P(r) - \overline{\Pi}_C(r) = 2\Phi_2(r) - F(r)\,\Phi(r) > 0,
\end{aligned} \tag{D.5}$$

where the inequality stems from the monotonicity of the hazard rate (Iossa et al., 2024,

[4] Using (D.4), we have: $\partial(r - c - \hat{\pi}^c(c;r))/\partial c = R'(c) - 1 < 0$; the conclusion then follows from $r - c = \hat{\pi}^c(c;r) = 0$ for $c = r$.

Lemma 1).[5]

ARPA with reserve $R(r)$

- Variant 1:
  - the designated supplier bids $R(r)$ regardless of its cost, and obtains $R(r) - c$;
  - the nondesignated supplier bids slightly below $R(r)$ regardless of its cost, and obtains $R(r)$.

- Variant $2a$:
  - the designated supplier bids $R(r)$ if its cost lies below $R(r)$, in which case it obtains $R(r) - c$, and at cost otherwise, in which case it obtains 0;
  - the nondesignated supplier bids slightly below $R(r)$ regardless of its cost, and obtains $R(r) - [1 - F(R(r))]\,c$.

- Variant $2b$ (if $\hat{c}(r) < \bar{c}$):
  - the designated supplier acts as in Variant $2a$: it bids $R(r)$ if its cost c lies below $R(r)$, in which case it obtains $R(r) - c$, and at cost otherwise, in which case it obtains 0;
  - the nondesignated supplier bids slightly below $R(r)$ only if its cost lies below $\hat{c}(r)$, in which case it obtains $R(r) - [1 - F(R(r))]\,c$, and at cost otherwise, in which case it obtains 0. [It may therefore still be active even when its cost exceeds the reserve (namely, when its cost lies in $(R(r), \hat{c}(r)]$), as the cost is incurred only if the designated supplier's cost exceeds the reserve; but contrary to Variant 2, the nondesignated supplier is inactive when its cost lies in $(\hat{c}(r), \bar{c}]$.]

In Variant 1, the designated supplier's expected payoff is

$$\pi^d_{R1}(c; r) \equiv R(r) - c < \hat{\pi}^c(c; r),$$

[5] To see this, note that $\Delta_P(\underline{c}) = 0$ and

$$\Delta'_P(r) = F^2(r) - f(r)\Phi(r) = F(r)\int_{\underline{c}}^{r} f(c)dc - \int_{\underline{c}}^{r} f(r)F(c)dc = \int_{\underline{c}}^{r} F(r)F(c)\Big(\frac{f(c)}{F(c)} - \frac{f(r)}{F(r)}\Big)dc > 0.$$

where the inequality stems from $(D.4)$ and $c > R(c)$, and:[6]

$$\overline{\pi}_{R1}^{d}(r) \equiv \mathbb{E}_c\left[\pi_{R1}^{d}(c;r)\right] = R(r) - \int_{\underline{c}}^{\overline{c}} c dF(c) = R(r) - R(\overline{c})\,(\leq 0)\,.$$

The nondesignated supplier's expected payoff is

$$\pi_{R1}^{n}(c;r) \equiv R(r)\,.$$

Thus, the net benefit from collusion is constructed as:

$$\begin{aligned} \overline{\Pi}_{R1}(r) &\equiv \overline{\pi}_{R1}^{d}(r) + \overline{\pi}_{R1}^{n}(r) = 2R(r) - R(\overline{c})\,, \\ \Delta_{R1}(r) &\equiv \overline{\Pi}_{R1}(r) - \overline{\Pi}_{C}(r) = 2R(r) - R(\overline{c}) - 2\left[\Phi(r) - \Phi_2(r)\right]. \end{aligned} \tag{D.6}$$

It follows that collusion in Variant 1 is profitable if and only if $\Delta_{R1}(r) > 0$. This is not always the case. For example, for $F(c) = c^s$ and $s \in (0, 0.5)$, $\Delta_{R1}(\overline{c}) < 0$, in which case collusion is not profitable under the ARPA, although it is profitable under the FPPA and under the SPPA.

In Variant $2a$, the designated supplier has expected payoff

$$\pi_{Ra}^{d}(c;r) \equiv \max\{0, R(r) - c\} = \mathbf{1}_{c<R(r)} \times \pi_{R1}^{d}(c;r),$$

and

$$\overline{\pi}_{Ra}^{d}(r) \equiv \mathbb{E}_c\left[\pi_{Ra}^{d}(c;r)\right] = \int_{\underline{c}}^{R(r)} \left[R(r) - c\right] dF(c) = \Phi(R(r))\,(< \overline{\pi}^{c}(r))\,.$$

The nondesignated supplier has expected payoff

$$\pi_{Ra}^{n}(c;r) \equiv R(r) - \left[1 - F(R(r))\right] c = \left[1 - F(R(r))\right]\left[\hat{c}(r) - c\right],$$

and:

$$\begin{aligned} \overline{\pi}_{Ra}^{n}(r) &\equiv \mathbb{E}_c\left[\pi_{Ra}^{n}(c;r)\right] \\ &= \left[1 - F(R(r))\right] \int_{\underline{c}}^{\overline{c}} \left[\hat{c}(r) - c\right] dF(c) \\ &= \left[1 - F(R(r))\right]\left[\hat{c}(r) - R(\overline{c})\right] \\ &= R(r) - \left[1 - F(R(r))\right] R(\overline{c})\,. \end{aligned}$$

[6] Note that $R(\overline{c}) = \mathbb{E}_c[c]$.

The net benefit from collusion is constructed as:

$$\overline{\Pi}_{Ra}(r) \equiv \overline{\pi}^d_{R2}(r) + \overline{\pi}^n_{Ra}(r) = \Phi(R(r)) + R(r) - [1 - F(R(r))]\, R(\bar{c}),$$

$$\begin{aligned}\Delta_{Ra}(r) &\equiv \overline{\Pi}_{Ra}(r) - \overline{\Pi}_C(r) \\ &= \Phi(R(r)) + R(r) - [1 - F(R(r))]\, R(\bar{c}) - 2[\Phi(r) - \Phi_2(r)]\,. \end{aligned} \tag{D.7}$$

Finally, in Variant $2b$ (if $\hat{c}(r) < \bar{c}$), the designated supplier has the same expected payoff as in Variant $2a$. The nondesignated supplier's expected payoff is

$$\pi^n_{Rb}(c; r) \equiv \max\{0, R(r) - [1 - F(R(r))]\, c\} = \mathbf{1}_{c<\hat{c}(r)} \times \pi^n_{Ra}(c; r),$$

and:

$$\overline{\pi}^n_{Rb}(r) \equiv \mathbb{E}_c[\pi^n_{Rb}(c; r)] = [1 - F(R(r))] \int_{\underline{c}}^{\hat{c}(r)} [\hat{c}(r) - c]\, dF(c) = [1 - F(R(r))]\, \Phi(\hat{c}(r))\,.$$

The net benefit from collusion is then constructed as:

$$\overline{\Pi}_{Rb}(r) \equiv \overline{\pi}^d_{Rb}(r) + \overline{\pi}^n_{Rb}(r) = \Phi(R(r)) + [1 - F(R(r))]\, \Phi(\hat{c}(r)),$$

$$\begin{aligned}\Delta_{Rb}(r) &\equiv \overline{\Pi}_{Rb}(r) - \overline{\Pi}_C(r) \\ &= \Phi(R(r)) + [1 - F(R(r))]\, \Phi(\hat{c}(r)) - 2[\Phi(r) - \Phi_2(r)]\,. \end{aligned} \tag{D.8}$$

D.4 Sustainability

We now consider the sustainability of collusion in the different formats. A market allocation is sustainable in a procurement auction of type $\tau \in \mathcal{T} \equiv \{P, F, S, R1, Ra, Rb\}$ (with F standing for First-price procurement auction, S standing for Second-price procurement auction, and R standing for all-Receive procurement auction), if and only if

$$\frac{\delta}{1-\delta} \geq \lambda_\tau(r) \equiv \frac{SG_\tau(r)}{LL_\tau(r)}, \tag{D.9}$$

where $SG(r)$ denotes the *short-term gain* from a deviation (see below), whereas $LL_\tau(r)$ denotes the (per-period) *long-term loss* of giving up collusion in the future. This condition can equivalently be expressed as:

$$\delta \geq \hat{\delta}_{\tau}(r) \equiv \frac{\lambda_{\tau}(r)}{1+\lambda_{\tau}(r)}$$

In the FPPA and SPPA, the long-term loss is equal to:

$$LL_F(r) = LL_S(r) = \Delta_P(r),$$

where $\Delta_P(r)$ is given by (D.5). Furthermore, the nondesignated supplier is the only one that may be tempted to deviate, and its gain from a deviation is maximal when it has the lowest possible cost, $\underline{c}$.

Turning to the short-term gain, in a FPPA, the best deviation for the nondesignated supplier consists in slightly undercutting the designated supplier's collusive bid, which yields a profit arbitrarily close to $\pi_P^d(\underline{c}, r)$; hence:

$$SG_F(r) \equiv \pi_P^d(\underline{c}, r) - \pi_P^n(\underline{c}, r_j) = F(r)(r - \underline{c}). \tag{D.10}$$

In an SPPA, the best deviation for the nondesignated supplier consists instead in bidding at cost, which yields the competitive profit $\pi^c(\underline{c}, r)$; hence:

$$SG_P(r) \equiv \pi^c(\underline{c}, r) - \pi_P^n(\underline{c}, r_j) = F(r)(r - \underline{c}) - \Phi(r). \tag{D.11}$$

To consider sustainability in the ARPA, we must consider the different variants. Under Variant 1, the long-term loss is equal to:

$$LL_{R1}(r) = \Delta_{R1}(r),$$

where $\Delta_{R1}(r)$ is given by (D.6). By construction, the nondesignated supplier best-responds to the designated supplier's collusive strategy; hence, the designated supplier is the only one that may be tempted to deviate. Furthermore, the best deviation consists in slightly undercutting the nondesignated supplier's bid, so as to get paid (almost) $R(r)$ but let the nondesignated supplier be selected as provider. This gain is maximal when its cost is equal to $\overline{c}$; hence:

$$SG_{R1}(r) \equiv \overline{c}.$$

Under Variant $2a$, the long-term loss is equal to:

$$LL_{Ra}(r) = \Delta_{Ra}(r),$$

where $\Delta_{Ra}(r)$ is given by (D.7). For the designated supplier, the best deviation consists in slightly undercutting the nondesignated supplier's bid, so as to get paid (almost) $R(r)$, but let the nondesignated supplier be selected as provider; the resulting gain from the deviation is as follows:

- for $c \leq R(r)$, the designated supplier was and remains active, but is no longer the provider; the gain from the deviation is therefore equal to c;
- for $c > R(r)$, in the absence of a deviation the designated supplier is inactive and, thus, obtain zero payoff; by deviating, it gets paid (almost) $R(r)$; the gain from the deviation is therefore equal to $R(r)$.

It follows that this short-term gain is maximal for $c = R(r)$, where it is equal to:

$$SG^d_{Ra}(r) \equiv R(r).$$

For the nondesignated supplier, the best deviation consists in bidding above the reserve (e.g., at cost) if its cost c lies above $\hat{c}(r)$, so as to avoid the expected loss $[1 - F(R(r))](\hat{c}(r) - c)$; it follows that the short-term gain is maximal for $c = \overline{c}$, where it is equal to:

$$SG^n_{Ra}(r) \equiv \max\{[1 - F(R(r))](\overline{c} - \hat{c}(r)), 0\}.$$

As $R(r) > (\underline{c} \geq)0$, it follows that the maximal short-term gain is equal to:

$$SG_{Ra}(r) \equiv \max\{R(r), [1 - F(R(r))](\overline{c} - \hat{c}(r))\}.$$

Under Variant 2b, the long-term loss is equal to:

$$LL_{Rb}(r) = \Delta_{Rb}(r),$$

where $\Delta_{Rb}(r)$ is given by (D.8). By construction, the nondesignated supplier best-responds to the designated supplier's collusive strategy; hence, the designated supplier is the only one that may be tempted to deviate. Furthermore, the best deviation consists in slightly undercutting the nondesignated supplier's bid, so as to get paid (almost) $R(r)$, but let the nondesignated supplier be selected as provider whenever active; the resulting gain from the deviation is as follows:

- for $c \leq R(r)$, the designated supplier was and remains active, but with probability $F(\hat{c}(r))$, it is no longer the provider; the gain from the deviation is therefore equal to $F(\hat{c}(r))\, c$, which is increasing in c;

- for $c > R(r)$, in the absence of a deviation the designated supplier is inactive and, thus, obtain zero payoff; by deviating, it gets paid (almost) $R(r)$ and is selected as provider with probability $1 - F(\hat{c}(r))$; the gain from the deviation is therefore $R(r) - [1 - F(\hat{c}(r))]\,c$, which is positive for $c < \hat{c}(r)$ but decreasing in c.

It follows that the short-term gain is again maximal for $c = R(r)$, where it is now equal to:

$$SG_{Rb}(r) \equiv F(\hat{c}(r))\,R(r)\,.$$

To summarize, the question of sustainability depends on a comparison of $\frac{SG_\tau(r)}{LL_\tau(r)}$ to $\frac{\delta}{1-\delta}$, where the long-term loss and short-term gains are:

	LL	SG
FPPA	$2\Phi_2(r) - F(r)\Phi(r)$	$F(r)(r - \underline{c})$
SPPA	$2\Phi_2(r) - F(r)\Phi(r)$	$F(r)(r - \underline{c}) - \Phi(r)$
ARPA-V1	$2R(r) - R(\overline{c}) - 2[\Phi(r) - \Phi_2(r)]$	$\overline{c}$
ARPA-V2a	$\Phi(R(r)) + R(r) - [1 - F(R(r))]\,R(\overline{c}) - 2[\Phi(r) - \Phi_2(r)]$	$\max\{R(r), [1 - F(R(r))]\,[\overline{c} - \hat{c}(r)]\}$
ARPA-V2b	$\Phi(R(r)) + [1 - F(R(r))]\,\Phi(\hat{c}(r)) - 2[\Phi(r) - \Phi_2(r)]$	$F(\hat{c}(r))\,R(r)$

We illustrate in Figure D.7 differences among the ARPA variants in terms of profitability and threshold discount factors. Panel (a) shows that ARPA-V2b is profitable for all r but that the other forms of ARPA collusion require r sufficiently large. For example, ARPA-V1 is profitable only for $r > 0.3700$. Panel (b) shows that the threshold discount factor for sustainability of collusion is lowest for ARPA-V2b for $r < 0.4730$ and lowest for ARPA-V1 for higher r. For sufficiently low r, even with $\delta = 1$, collusion is only sustainable for ARPA-V2b. For $r > 0.3700$, collusion is also sustainable for ARPA-V1 for sufficiently high δ (collusion under format ARPA-V2a is dominated in terms of profitability and sustainability by the other formats when costs are uniformly distributed on $[0, 1]$). As shown in Panel (c), for the case of $F(c) = c^s$, ARPA-V2a is no longer always dominated by ARPA-V2b, but both ARPA-V2a and ARPA-V2b are dominated by ARPA-V1. For $s < 0.5$, collusion is not profitable under the ARPA for any variant.

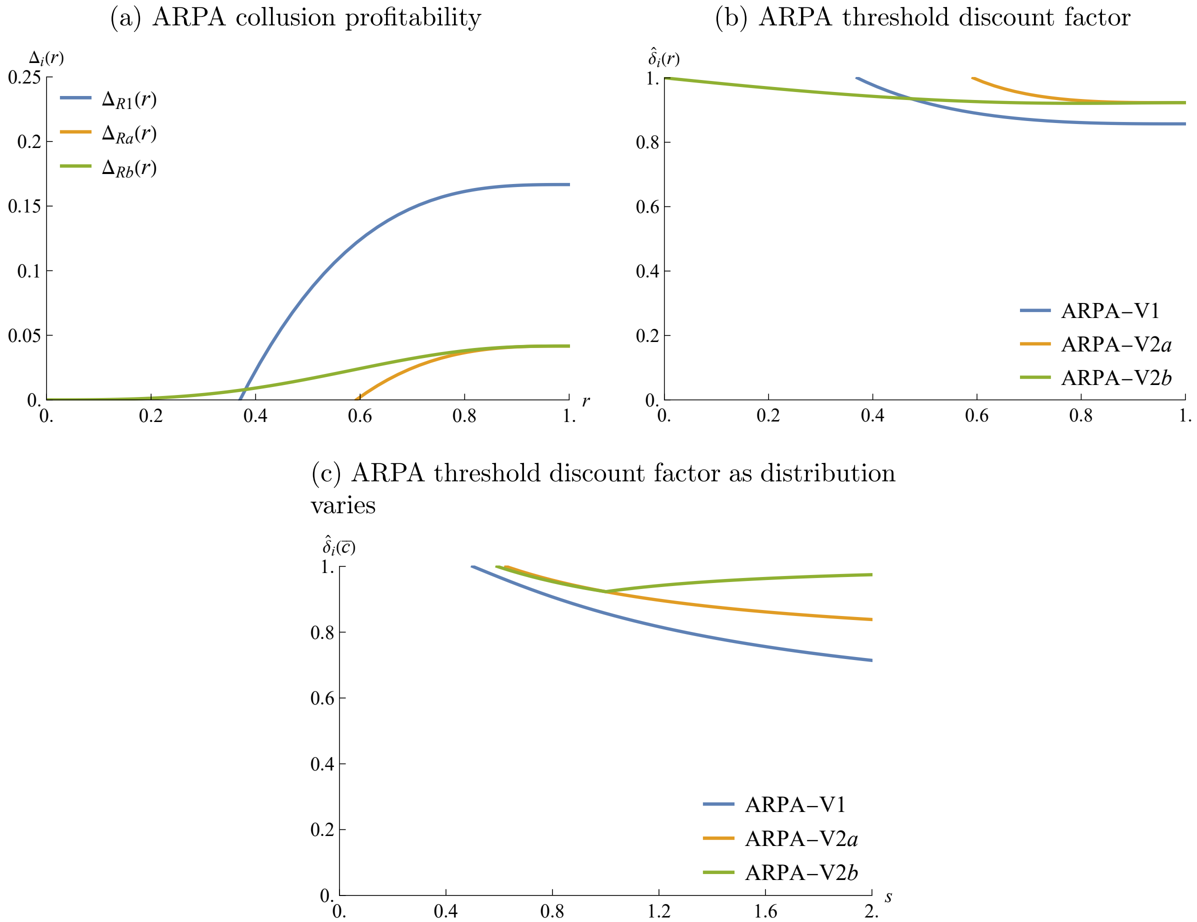


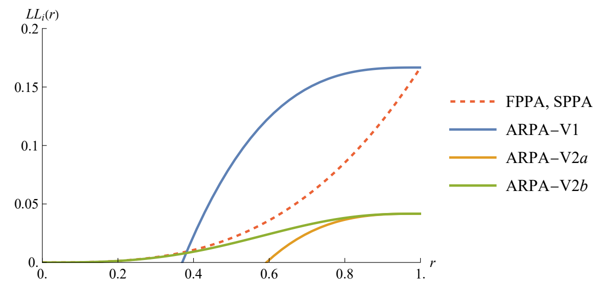
Figure D.7: Panels (a) and (b) assume uniformly distributed costs on $[0, 1]$. Panel (c) assumes that $F(c) = c^s$ on $[0, 1]$ for $s \in (0, 2]$ and $r = 1$.

D.5 Comparisons across auction formats

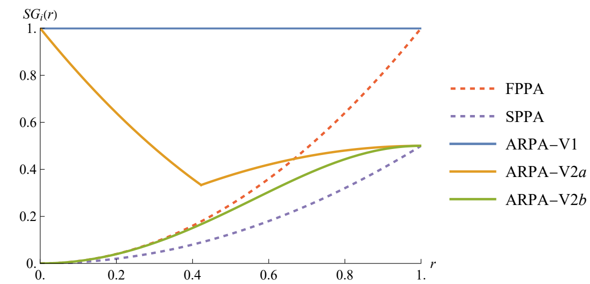
As mentioned in Section 6.2, the ARPA can be more or less susceptible to collusion than the FPPA and SPPA depending on the setup.

As an example, consider the case of uniformly distributed costs on $[0, 1]$. The long-term losses for the ARPA under Variants 2a and 2b are always below that of the FPPA and SPPA, and the long-term loss for the ARPA under Variant 1 is below that of the FPPA and SPPA for $r < 0.3820$ (Figure D.8(a)). The short-term gain for the ARPA under Variant 1 is always above that of the FPPA and SPPA, and the short-term gain for ARPA under Variants 2a and 2b are greater than that of the SPPA (Figure D.8(b)). The result is that under Variants 2a and 2b, the ARPA is always less susceptible to collusion (higher threshold discount factor) than the FPPA and SPPA; and under Variant 1 is always less susceptible to collusion than the SPPA and, for $r < 0.5$, is less susceptible than the FPPA (Figure D.8(c)). Thus, regardless of the variant of ARPA collusion, the ARPA is less susceptible to collusion than the SPPA for all r and less susceptible than the FPPA for $r < 0.5$.

(a) Comparison of long-term losses



(b) Comparison of short-term gains



(c) Comparison of threshold discount factors

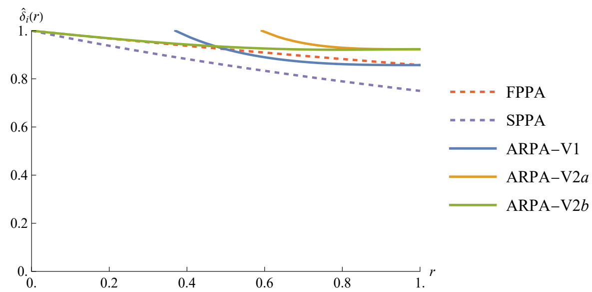


Figure D.8: All panels assume uniformly distributed costs on $[0, 1]$.

As another example, consider $r = \bar{c}$ and $F(c) = c^s$ on $[0, 1]$. In this case, as illustrated in Figure D.9, ARPA collusion Variants 1 and 2 are less susceptible than the other formats for $s \in (0, 1)$, but under Variant 1, the ARPA is even more susceptible to collusion than the SPPA for $s > 1.234$.

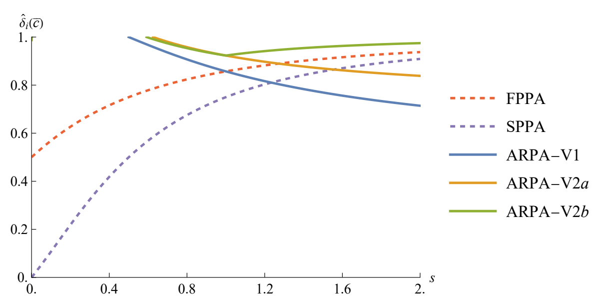


Figure D.9: Comparison of threshold discount factors. Assumes that $F(c) = c^s$ on $[0, 1]$ for $s \in (0, 2]$ and $r = 1$.

References for the Supplemental Appendix

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