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# "The Incentive Virtues of Performance-Based Trade Allowances and Loss Leading"

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# The Incentive Virtues of Performance-Based Trade Allowances and Loss Leading*

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**Abstract**

A retailer can boost demand for a manufacturer's product through non-verifiable activities. Performance-based trade allowances—rebates conditional on successful retailer's sales efforts—help mitigate this moral hazard problem. In equilibrium, the wholesale contract includes a retail price set below cost, complemented by a rebate for incremental units purchased when efforts successfully increase sales. Loss leading thus emerges as an incentive mechanism, rather than a practice driven by anti-competitive or exploitative intent. A ban on below-cost pricing leads to higher retail prices and reduced promotional efforts.

KEYWORDS: vertical restraints; moral hazard; loss leading; performance-based allowances; below-cost pricing.

JEL CODE: L11, L42, L81.

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## 1. Introduction

Manufacturers, distributors, and retailers exchange billions of dollars every year through trade allowances. These payments have become essential to modern retail strategies. In the 1990s, trade allowances in the U.S. totaled $1 billion annually. By 2015, this figure had jumped to $18 billion, and by 2022, trade allowances represented 16% of manufacturers' gross revenue, up from 12% in 2015.[1] The literature has extensively studied *placement and slotting fees*, defined as fixed upfront payments to secure shelf space or introduce new products (Lariviere and Padmanabhan, 1997; Marx and Shaffer, 2010),[2] as well as *price promotions* that aim to incentivize retailers to reduce retail prices (Varian, 1980; Gerstner and Hess, 1991; Inderst and Obradovits, 2024). However, *performance-based trade allowances* represent a distinct and increasingly critical category. Unlike fixed fees and retail price reductions, these are variable payments contingent on sales exceeding a reference level.[3] To illustrate the importance of these compensations, Elberg and Noton (2025), relying on a unique dataset from the retail industry in Chile, report that while 71% of suppliers pay fixed slotting fees, 52% engage in variable, performance-based allowances.[4]

This shift toward performance-based incentives is driven by the increasing ability of manufacturers to monitor sales via real-time scan data.[5] By linking payments to verifiable sales, these allowances allow manufacturers to mitigate moral hazard and reward distributors for their efforts in demand-enhancing activities.[6]

In practice, the legal environment contrains the implementation of these contracts. In several jurisdictions, regulations prohibit retailers from selling products below the wholesale price. A case in point is the French "Loi Galland" (1996), which effectively fixed the resale-below-cost threshold at the level of the invoice price (i.e., not including the "off-invoice rebates" or "backward margins"). In this paper, we analyze how such a constraint may interact with the design of incentive contracts.

This paper investigates the economics of trade allowances by introducing two key features of manufacturer-retailer relationships. First, retailers promote manufacturers'

[1]NielsenIQ (2022) and The Economist (2015).

[2]Slotting fees have also been shown to have either pro-competitive (Chu, 1992; Foros et al., 2009) or anti-competitive (Shaffer, 1991; Marx and Shaffer, 2007; Miklós-Thal et al., 2011; Piccolo and Miklós-Thal, 2012) effects, depending on the context.

[3]Elberg and Noton (2025) propose to classify trade allowances into four broad categories which all fall under the umbrella of non-price incentives. Those categories are i) "marketing and merchandising incentives" (payments to remunerate the retailer for specific promotional efforts and services executed in the store or in media); "cooperative advertising" (payments to have products featured in the retailer's weekly circulars and newspaper inserts), ii) "display allowances" (fees for special displays), iii) "quota incentives" (payments conditional on the achievement of sales targets), iv) "spot contracts" (negotiated during the year for specific, unforeseen promotional campaigns and tied to the execution of specific short-term marketing initiatives).

[4]For major manufacturers like PepsiCo, these trade spends can represent up to 30% of net revenue, a significant portion of which is linked to merchandising performance rather than simple price cuts (Allen et al., 2011).

[5]While only 30% of traditional price promotions can be accurately tracked, 85% of manufacturers can now use real-time scan data to monitor non-price allowances (McKinsey, 2021).

[6]The emergence of such contracts can also be interpreted as a reaction to the historical opacity of trade negotiations. In the past, discretionary allowances often led to accounting scandals and regulatory investigations, such as the high-profile bankruptcy of Fleming Companies in the early 2000s (Kaikati and Kaikati, 2006; Allen et al., 2011).

products through various activities. While some of these activities can be specified contractually, others remain by and large non-verifiable—a source of moral hazard. Conditional rebates tied to the success of efforts in boosting demand serve to mitigate this moral hazard issue. Second, assuming manufacturers cannot rely on lump-sum transfers, the optimal wholesale contract must combine wholesale prices and conditional rebates to jointly manage incentives and profit distribution.[7]

Our analysis offers several new insights. First, below-cost pricing combined with rebates emerges as an efficiency-enhancing tool to align retailers' incentives with manufacturers' goals—specifically to induce optimal effort—without anti-competitive or exploitative intent. Second, bans on below-cost pricing can unintentionally raise retail prices and reduce demand-enhancing effort.

Below-Cost Pricing. A critical issue in this context is how manufacturers balance their desire to extract retailers' profits with the need to incentivize those retailers' efforts. Our analysis reveals that optimal wholesale contracts often induce below-cost pricing at the retail level. This is achieved through high wholesale prices that raise retailers' marginal costs, coupled with significant rebates contingent on promotional success.

The reasoning is intuitive. Below-cost pricing acts as a bonding mechanism: Retailers incur initial losses, motivating them to exert sufficient effort to recoup these losses through rebates. Manufacturers, in turn, adopt a "stick and carrot" approach. The stick is the threat of negative profits if effort fails to boost demand, while the carrot is the reward of rebates when effort succeeds. Such dynamics resemble marketing the product in two distinct markets. On the "base market," the retailer incurs losses, while, on the "extra market," rebates generate profits. Hence, our analysis contributes also to the economics of loss leading by emphasizing its incentive role.

Sales-Below-Cost Laws. Sales-below-cost laws, often introduced to protect small producers, significantly alter such dynamics. These laws have a long history worldwide, with mixed assessments of their impact. Several European Union Member States and U.S. states prohibit below-cost pricing, though the scope and nature of these bans vary.[8]

When manufacturers lack bargaining power, wholesale prices are set at marginal cost, rendering bans on below-cost pricing irrelevant. However, when manufacturers possess significant market power, such bans impose constraints on retail pricing. Contrary to naive stances, these laws may reduce efficiency by raising retail prices and limiting demand-enhancing activities. Manufacturers can no longer rely on the stick of negative base profits to motivate retailers. Instead, they must increase rebates and grant retailers a moral hazard rent to incentivize efforts. Echoing a fundamental principle of contract theory,[9] manufacturers are willing to reduce this rent, which, in turn, leads to lower efforts from retailers. Additionally, bans on below-cost pricing limit manufacturers'

[7]Allen et al. (2011) explain that many non-price incentives are not unconditional lump sums but are structured as sales-based (rather than units-bought) "bill-backs" or "scan-backs." Relatedly, "failure fees" may be charged if a new product does not meet expected sales goals, effectively making the net payment conditional on success (Bloom et al., 2000; and Klein and Wright, 2007, for a related argument). Last, Elberg and Noton (2025) provide evidence that payments on established products (like placement fees) can be volume-based rather than lump-sum.

[8]For example, Ireland's "Groceries Order" banned sales below net invoice prices, while France's Loi Galland prevents retailers from passing on anticipated rebates to consumers. Both regulations have been linked to higher food prices (Irish Competition Agency, 2005; Biscourp, Boutin, and Vergé, 2013).

[9]See Laffont and Martimort (2002, Chapter 4).

ability to capture downstream profits. Under weak specifications about preferences and technologies, we show that such bans result in higher retail prices and reduced efforts.

Extensions and Applications. Vertical control through resale price maintenance (RPM) is another common practice in manufacturer-retailer relationships. Under RPM, manufacturers dictate both wholesale and retail prices but do not use incentive rebates tied to promotional success. We compare RPM and trade allowances and delineate the conditions under which incentive rebates outperform RPM agreements. We also extend our model to the case of complementary products sold by the retailer. We show that our insights naturally extend to these scenarios.

Finally, our framework is applied to antitrust analysis. In 2014, several manufacturers of household and hygiene products were sanctioned by the French Competition Authority for collusion aimed at maintaining high retail prices and limiting payments to retailers. Using our analysis, we outline how to estimate the profit loss suffered by affected retailers under such conditions.

Literature Review. That retailers can increase the sales of manufacturers' products is a central tenet of the literature on vertical relationships. With simple contracts between manufacturers and retailers, and when retailers cannot fully appropriate the benefits of their efforts because of free-riding or spillovers, vertical restraints find a possible pro-competitive rationale. That argument, first made by Telser (1960), has been extended to more general settings by Mathewson and Winter (1984), Rey and Tirole (1986), Krishnan and Winter (2007), Kastl, Piccolo and Martimort (2011), and Hunold and Muthers (2017). Lømo and Ulsaker (2016, 2021) view promotional allowances as fixed payments used discretionarily, in addition to two-part tariffs, by manufacturers in a relational contracting framework. We do not consider externalities across retailers and focus, as in Winter (1993), on a vertical externality between the retailer and the manufacturer. Our main point of departure is that we assume that the retailer's effort has some observable, but random, impact on the demand for the manufacturers' product. Incentive payments, such as rebates conditional on performance, can thus be used.

Several explanations have been pushed forward to explain below-cost pricing. Loss leading emerges as an advertising strategy in Ellison (2005). Bliss (1988) views loss leading as a cross-subsidization strategy between products with different demand elasticities, an idea further developed in Beard and Stern (2008) and Ambrus and Weinstein (2008). Chen and Rey (2012, 2019) show that loss leading facilitates screening of consumers according to their shopping costs. Loss leading is a response to vertical opportunism in Allain and Chambolle (2011). Finally, Inderst and Obradovits (2024) examine how retailers' discounts on branded products, used as loss leaders, can shift consumer focus from quality to price, thereby eroding brand value. In stark contrast with all these papers, loss leading has no strategic role hereafter. Instead, it emerges in our simpler setting (a single retailer, a single product, perfectly informed consumers) as an incentive device to solve a simple (but overlooked) moral hazard problem on the retailer's side.

A distinct strand of the literature has analyzed all-unit quantity discounts (AUDs), where a lower wholesale price applies to all units purchased once a threshold is reached. Among other rationales,[10] AUDs have been shown to be used as a screening device when

[10] Conlon and Mortimer (2013, 2021) argue that such tariffs may foreclose upstream competition while simultaneously helping upstream manufacturers secure retailers' ancillary services. Feess and Wohlschlegel (2010) contend that all-unit discounts primarily shift rent from entrants, while Ide, Mon-

retailers possess private information about demand as in Kolay, Shaffer, and Ordover (2004). Trade allowances in our context are significantly different. Performance-based contracts (an incremental rebate) are hereafter designed not to screen hidden information, but to provide incentives for hidden action.

ORGANIZATION OF THE PAPER. Section 2 introduces the model and examines several useful benchmarks. Section 3 explores how the manufacturer uses rebates and wholesale payments to incentivize the retailer. Section 4 investigates the impact of banning below-cost pricing. Section 5 compares incentive allowances with resale price maintenance agreements. Section 6 extends the basic framework to a more complex scenario where the retailer sells other complementary products in the final market. Section 7 applies the analysis to compute the damages suffered by retailers due to upstream cartels between manufacturers. Finally, Section 8 concludes. All proofs are provided in the Appendix and the Online Appendix.

## 2. MODEL AND BENCHMARKS

MODEL. We consider the bilateral relationship between an upstream manufacturer $M$, who produces at marginal cost $c$, and a downstream retailer $R$, whose cost is normalized to 0 without loss of generality. Given a retail price $p$, the demand for the good is denoted by $D(p)$, with $D'(p) < 0$ for all price $p$ such that $D(p) > 0$.

The retailer exerts promotional effort $e$, which can increase demand for the manufacturer's product. For example, the demand for standard products and services can be boosted by enhancing promotional efforts. For more complex products, the retailer might improve customer information and reduce search costs. The cost of exerting effort is denoted by $\psi(e)$, where $\psi(\cdot)$ is assumed to be strictly increasing and convex ($\psi'(\cdot) > 0$, $\psi''(\cdot) > 0$) with $\psi'''(\cdot) \geq 0$ and $\psi(0) = 0$, ensuring interior solutions to all optimization problems below. Finally, we assume that effort $e$ is non-verifiable, introducing moral hazard into the vertical relationship. For future reference, we also define $R(e) = e\psi'(e) - \psi(e)$ and notice that $R(\cdot)$ is increasing and convex ($R'(\cdot) > 0$, $R''(\cdot) > 0$) with $R(0) = 0$ under the assumptions made above for $\psi(\cdot)$. Readers accustomed to the principal-agent literature will have recognized that $R(e)$ is the so-called *moral hazard rent* that might accrue to the retailer when effort is non-verifiable.[11]

We normalize effort such that $e \in [0, 1]$. This normalization allows us to interpret $e$ as the probability that consumer demand increases from $D(p)$ to $(1 + \theta)D(p)$, where $\theta \geq 0$ is a scale parameter. With the complementary probability $1 - e$, the demand remains at its base level $D(p)$.[12] Since the boost in demand raises revenues, a positive shock in demand is necessarily observable by the manufacturer.

tero, and Figueroa (2016) show that such tariffs cannot be anticompetitive unless supplemented with upfront payments. Chao, Tan, and Wong (2018) demonstrate that all-unit discount tariffs not only enhance the incumbent's profit compared to linear pricing but may also improve overall surplus–albeit at the buyer's expense. Abada et al. (2025) show that key features of AUDs are closely related to the properties of the optimal nonlinear and non-conditional tariff.

[11] Laffont and Martimort (2002, Chapter 4).

[12] A more general structure would consider two different demand functions with possibly different elasticities, $\underline{D}(p) < \overline{D}(p)$, depending on the success of the retailer's effort. The multiplicative structure implied by the scale parameter $\theta$ is chosen for tractability since it keeps elasticity constant as demand is scaled up. Relatedly, we could assume that there is a whole continuum of "additional sales" that could be induced by the retailer's effort. In other words, the scale parameter $\theta$ could be distributed on the positive real line according to a density function conditional on the retailer's effort, say $f(\theta|e)$.

The manufacturer offers a wholesale contract, which consists of a per-unit wholesale price $w$ paid by the retailer and a rebate $z$ paid by the manufacturer on all incremental sales that result from the retailer's promotional effort. Specifically, if demand exceeds its base level, the retailer receives an additional payment of $z\theta D(p)$. The retailer can either accept or reject the contract. Upon acceptance, the retailer exerts promotional effort $e$ and chooses a retail price $p$. When demand is high, the manufacturer pays the rebate $z$ on all incremental units sold.[13]

We now examine a benchmark that provides valuable insights into both the emergence and the role of promotional allowances.

VERTICALLY INTEGRATED OUTCOME. Suppose that the manufacturer is vertically integrated with the retailer. We take the standard short-cut that integration gives access to information and facilitates control.[14] The vertically integrated outcome maximizes the overall industry profit

$$\Pi_I(p,e) = (1+\theta e)\pi(p,c) - \psi(e),$$

where $\pi(p,c) = (p-c)D(p)$ stands as the "base profit" absent effort.

Throughout, we shall assume that the following condition holds.

ASSUMPTION 1.

$$p + \frac{D(p)}{D'(p)} \textit{ is increasing.}$$

Assumption 1 holds for most standard demand specifications (linear, exponential, constant elasticity, etc.). It ensures that $\pi(p,c)$ and $\Pi_I(p,e)$ are both quasi-concave in $p$.

The monopoly outcome $(p^m, e^m)$ that maximizes the profit of the vertically integrated structure is thus readily obtained as follows:[15]

$$p^m - c = -\frac{D(p^m)}{D'(p^m)}, \tag{2.1}$$

$$\psi'(e^m) = \theta\pi^m, \tag{2.2}$$

where $\pi^m = \pi(p^m, c)$ stands for the monopoly base profit. Since the promotional effort boosts demand multiplicatively, the monopoly price always maximizes profit whether

Assuming that the manufacturer could observe the magnitude of incremental sales (i.e., precisely observe the realization of $\theta$), more general performance-based contracts could a priori be designed. For instance, such a contract could entail a reward $z(\theta)D(p)$ where $z(\theta)$ could be an arbitrary function of the realized scale parameter $\theta$. Within this class, "bang-bang" contracts that would entail $z(\theta) = z$ for $\theta \geq \theta^*$ and $z(\theta) = 0$ for $\theta \in [0, \theta^*)$ for some $z$ and some $\theta^*$ would just partition the state space into two regions with respective probability $F(\theta^*|e)$ and $1 - F(\theta^*|e)$ where $F(\theta|e)$ is the cumulative distribution function. The corresponding model would thus be roughly similar to our binary-scale environment.

[13] An equivalent but more abstract formulation would be that the manufacturer offers two different wholesale prices, one when demand remains at its base level $D(p)$ and another one when demand jumps at $(1+\theta)D(p)$. In practice, this solution may be hard to implement because wholesale prices are set once for all before demand realizes. Ex post rebates are more meaningful.

[14] See for instance Arrow (1975) and Riordan (1990).

[15] One simplifying feature of our model is that the monopoly price is independent of the retailer effort, a consequence of the fact that effort impacts multiplicatively demand. This assumption brings tractability and allows us to separate the impacts of promotional allowances on prices and effort in a clean way. Micro-foundations for such a multiplicative functional form can be found in Mathewson and Winter (1984) (when consumers are initially unaware of the product's existence), Bester (1998) (in the case of informative advertising), and in Hunold and Muthers (2017) (when consumers initially do not know which product suits their needs and rely on the retailer's advice).

the demand has been scaled up or not. The optimal effort thus simply trades off the marginal benefit coming from enjoying some extra monopoly profit $\theta\pi^m$ beyond the base level against the retailer's marginal disutility of effort.

It is well known, at least since Dixit (1983) and Mathewson and Winter (1984), that a manufacturer can implement this first-best outcome using a simple two-part tariff $(w, F)$ with no conditional rebate $(z = 0)$. By setting the wholesale price equal to the marginal cost of production $(w = c)$, the manufacturer eliminates the double marginalization problem. The retailer then becomes the residual claimant of the vertical chain's profit and chooses $p$ and $e$ to maximize $\Pi_I(p, e) - F$, leading naturally to the vertically integrated solution $(p^m, e^m)$. In this setup, the fixed fee $F$ plays a dual role: it allows the manufacturer to extract the retailer's surplus, and it acts as a "stick" forcing the retailer to exert effort to cover this fixed cost. Consequently, in an unconstrained environment with two-part tariffs, conditional rebates are redundant.

However, in practice, manufacturers rarely collect fixed payments from retailers. The reliance on such fixed payments may be, moreover, limited because of asymmetric information regarding the retailer's costs, which would require complex screening mechanisms.[16] Furthermore, fixed fees are typically collected ex post. If the retailer is cash-constrained or subject to adverse demand shocks, it may behave opportunistically and renege on the payment. Consequently, unable to use lump-sum transfers to extract surplus, manufacturers may be forced to rely only on per-unit wholesale prices and conditional rebates.[17] This leads us to the analysis of the optimal second-best contract which is undertaken in next section.

## 3. Main Analysis

We now turn to the analysis of the manufacturer's optimal contracting problem.

Linear Wholesale Pricing. To gauge the impact of conditional rebates and the below-cost pricing regulation, suppose the manufacturer uses only a simple linear wholesale price (i.e., $z = 0$). This benchmark establishes the standard vertical externalities—double marginalization and effort under-provision—that the manufacturer seeks to correct.

The retailer would then maximize

$$(p - w)D(p)(1 + \theta e) - \psi(e).$$

The optimal retail price $p$ induced by a wholesale price $w$ follows the familiar pass-through formula in the context of a double-marginalization scenario à la Spengler (1950), namely

$$w = p + \frac{D(p)}{D'(p)}.$$

[16] This point is clearly reminiscent of an argument often found in Regulatory Economics to justify that Ramsey-Boîteux pricing might have some appeal in some regulatory settings; see Laffont and Tirole (1993, Chapter 1).

[17] A slightly more subtle argument is whether the restriction to linear conditional rebates is warranted. We comment on this issue at the end of Section 3.

Charging this wholesale price $w$ leaves the retailer a positive profit worth[18]

$$(1+\theta e)\varphi(p)-\psi(e),$$

where $\varphi(p)=-D^2(p)/D'(p)$ stands for the retail base profit once the retailer has optimally chosen his retail price in response to the wholesale price. When Assumption 1 holds, a higher retail price decreases retail profit (i.e., $\varphi'(\cdot)<0$).[19] The optimal retail price that the manufacturer would like to induce by a convenient choice of $w$ would thus maximize the manufacturer's profit, which can be written as follows

$$(\pi(p,c)-\varphi(p))(1+\theta e).$$

We shall further assume that $\pi(p,c)-\varphi(p)$ is quasi-concave in $p$ which requires a slightly stronger version of Assumption 1.[20]

Assumption 2.

$$p+\frac{D(p)}{D'(p)}-(1-\lambda)\frac{\varphi'(p)}{D'(p)} \text{ is increasing for any } \lambda\in[0,1].$$

The optimal retail price that the manufacturer wants to implement is thus defined as follows

$$\pi_p(\tilde{p},c)=\varphi'(\tilde{p}).$$

From now on, we shall assume that the corresponding retail profit $\varphi(\tilde{p})$ does not suffice to induce the maximal effort level $e=1$ that would best serve the manufacturer's interest.

Assumption 3.

$$(1+\theta)\varphi(\tilde{p})<\psi(1).$$

When Assumption 3 holds, the manufacturer certainly wants to use a rebate to boost the retailer's incentives. Without a rebate, incentives provided by the sole share of the base profit that cannot be appropriated by the manufacturer do not suffice.

The Retailer's Problem. Suppose now that the retailer operates under a wholesale contract $(w,z)$. The retail price $p$ and promotional effort $e$ are optimally chosen so as to maximize its expected profit

$$(p-w)D(p)(1+\theta e)+\theta ezD(p)-\psi(e). \tag{3.1}$$

With more compact notations, the retailer's profit in (3.1) can be rewritten as

$$\pi(p,w)+\theta e\pi(p,w-z)-\psi(e).$$

Focusing on interior solutions, the first-order optimality conditions associated to the

[18]Throughout our analysis, profits are expressed in terms of the retail price and effort, consistently with our approach that highlights retail decisions and not the wholesale contract that induces these decisions.

[19]Indeed, $\varphi'(p)=-D(p)\big(2-\frac{D''(p)D(p)}{(D'(p))^2}\big)=-D(p)\frac{d}{dp}\big(p+\frac{D(p)}{D'(p)}\big)<0$, where the right-hand side inequality follows from Assumption 1.

[20]Again, this assumption, which is supposed to hold throughout, is satisfied for most usual demand specifications.

optimal retail price $p$ and effort $e$ are thus respectively given by[21]

$$\pi_p(p,w) + \theta e \pi_p(p, w - z) = 0, \tag{3.2}$$
$$\theta \pi(p, w - z) = \psi'(e). \tag{3.3}$$

We may develop the first-order condition (3.2) and rewrite it as

$$\frac{1}{1+\theta e}\frac{p-w}{p} + \frac{\theta e}{1+\theta e}\frac{p-w+z}{p} = -\frac{D(p)}{pD'(p)}. \tag{3.4}$$

Condition (3.4) shows that the inverse price-elasticity of demand is actually an average of the retail price-cost markups with and without rebate. As the impact of the promotional effort $e$ on demand increases, the retail price becomes more responsive to the rebate $z$ earned on incremental sales and, as such, decreases.

Consider now the optimality condition (3.3) in terms of effort. Observe that decreasing the wholesale price $w$ and increasing the rebate $z$ increases the retailer's profit from incremental sales (since $\pi(p, w - z)$ is decreasing in its second argument), which boosts incentives to exert effort. Simultaneously, choosing a lower retail price maximizes this profit on incremental sales and also boosts effort.

THE MANUFACTURER'S PROBLEM. The optimality conditions (3.2) and (3.3) allow us to express the contracting variables $(w, z)$ in terms of the pair $(p, e)$ that the manufacturer induces from the retailer through the wholesale contract. This approach is reminiscent of the principal-agent literature where the focus is not necessarily on the contracting instruments used to implement a given effort profile from the agent but, instead, on this effort profile and on the cost for the principal of reaching it. Doing so yields

$$w = p + \frac{D(p)}{D'(p)}(1+\theta e) + \frac{e\psi'(e)}{D(p)}, \tag{3.5}$$
$$z = (1+\theta e)\left(\frac{\psi'(e)}{\theta D(p)} + \frac{D(p)}{D'(p)}\right). \tag{3.6}$$

Equation (3.5) is particularly important in view of our forthcoming analysis of the role of a ban on below-cost pricing. Indeed, this condition delineates the key restriction on implementable pairs $(p, e)$ that ensures a positive margin $p - w$.

Using Equations (3.5) and (3.6), we may express the manufacturer's and the retailer's profits, still in terms of the pair $(p, e)$ to be implemented, respectively as follows

$$\Pi_M(p,e) = (\pi(p,c) - \varphi(p))(1+\theta e), \tag{3.7}$$

and

$$\Pi_R(p,e) = (1+\theta e)\varphi(p) - \psi(e). \tag{3.8}$$

OPTIMAL WHOLESALE CONTRACT. This contract must maximize the manufacturer's

[21]The so-called first-order approach is standard in Principal-Agent problems under moral hazard. We discuss its validity in our setting in the Online Appendix.

profit $\Pi_M(p,e)$ subject to the retailer's participation condition

$$\Pi_R(p,e) \geq 0. \tag{3.9}$$

We shall assume that this problem is quasi-concave in $(p,e)$ and denote by $(w^u, z^u)$ the optimal wholesale contract for the manufacturer. Let also $(p^u, e^u)$ be the corresponding retail price and effort level induced by such contract.

PROPOSITION 1. *The optimal wholesale contract $(w^u, z^u)$ satisfies the following properties.*

1. *The retailer makes zero profit*

$$\Pi_R(p^u, e^u) = (1+\theta e^u)\varphi(p^u) - \psi(e^u) = 0. \tag{3.10}$$

2. *There is below-cost pricing when demand remains at its base level*

$$p^u - w^u < 0. \tag{3.11}$$

3. *Rebates are strictly positive*

$$z^u > 0. \tag{3.12}$$

The manufacturer would ideally like to implement the highest possible effort and induce the heavily distorted retail price $\tilde{p}$ corresponding to the double-marginalization scenario. Unfortunately, doing so would induce the retailer to make negative profits when Assumption 3 holds.[22] The optimal contract moves along the retailer's break-even condition towards a point that maximizes the manufacturer's profit. The wholesale price is used to extract profit whereas the rebate serves to induce effort. More precisely, using Equation (3.5) and the binding participation constraint (3.9), the retailer's base profit can be expressed as follows

$$(p^u - w^u)D(p^u) = -R(e^u) < 0. \tag{3.13}$$

Equation (3.13) implies below-cost pricing as stated in (3.11) and a negative base profit; the stick side of the mechanism. Instead, the rebate provides the moral hazard rent $R(e^u)$ needed to induce effort; the carrot side. At the optimal wholesale contract, the retailer would not recover the loss on the base profit, $(p^u - w^u)D(p^u) < 0$ without exerting at least effort $e^u$. The loss on base profit thus acts as a bonding device that induces the demand-expanding effort.

We now turn to an important comparison.

PROPOSITION 2. *In comparison with the vertically-integrated outcome, the retail price and the promotional effort both increase*

$$p^u > p^m \text{ and } e^u > e^m.$$

[22] Although the manufacturer cannot use fixed fees, the wholesale price $w$ serves as a rent-extraction device. In equilibrium, the manufacturer raises $w$ to extract surplus until the retailer is indifferent between accepting and rejecting the contract.

Even though fixed fees are no longer available, the manufacturer still needs to incentivize effort on the one hand and extract the retailer's downstream profit on the other hand. Without a fixed fee, this extraction is incomplete but the residual $\varphi(p)$ left to the retailer can also play an incentive role. Increasing $p$ beyond the monopoly outcome increases the retailer's profit and thus relaxes the break-even condition.

The upward distortion of effort is more subtle. To see why, it is useful to take for granted the result of Proposition 1 and denote by $E(p)$ the decreasing function implicitly defined through the binding break-even condition (3.9). The optimality condition on effort can then be rewritten in terms of $p$ only as follows

$$\psi'(e^u) = \theta\pi(p^u, c) + (1 + \theta e^u)\frac{\pi_p(p^u, c)}{E'(p^u)}.$$

There are two forces that determine the effort distortion. On the one hand, the first term on the right-hand side captures how increasing the retail price from $p^m$ to $p^u$ decreases overall profit ($\pi(p^u, c) < \pi(p^m, c)$) and this tends to reduce the optimal effort. On the other hand, increasing the retail price along the break-even condition (3.9) also raises effort. This effect is captured in the second term above which is negative (since $\pi_p(p^u, c) < 0$ for $p^u > p^m$ and $E'(p^u) < 0$). Proposition 2 shows that this second effect always dominates.

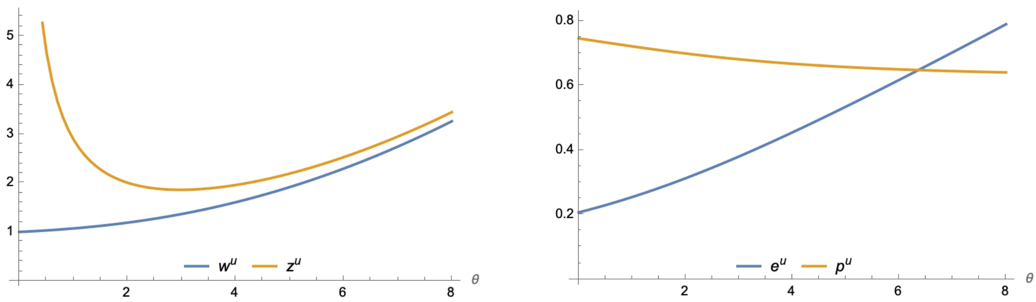
ANALYTICAL ILLUSTRATION AND COMPARATIVE STATICS. To illustrate the mechanics of the optimal contract and provide intuition for the results established in Propositions 1 and 2, we consider the specific case of a linear demand $D(p) = a - bp$ (with $a > bc$) and a quadratic cost of effort $\psi(e) = \mu e^2/2$. Under this specification, the manufacturer's optimal contract $(w^u, z^u)$ induces a retail price $p^u$ that can be expressed explicitly as a function of the Lagrange multiplier $\lambda^u \in (0, 1)$ associated with the binding participation constraint:

$$p^u = \underbrace{\frac{a + bc}{2b}}_{p^m} + \underbrace{\frac{\lambda^u(a - bc)}{2b(2 - \lambda^u)}}_{\text{Distortion} > 0}.$$

This formula clearly showcases the upward price distortion relative to the integrated monopoly price $p^m$. The magnitude of this distortion increases with the shadow cost of the participation constraint $\lambda^u$. Furthermore, the optimal wholesale price $w^u$ is set such that $(p^u - w^u)(a - bp^u) < 0$. The retailer thus incurs a structural loss on base demand (the "stick"), which must be exactly offset by the expected rent from the rebate $z^u$ on incremental sales (the "carrot") to satisfy the retailer's break-even condition.

We now explore how these distortions evolve with the efficiency of the effort. We determine numerically the optimal contract for $\theta \in [.1, 8]$ and vary $\theta$ by an increment of .1 for a total of 80 simulations.[23] Figure 1 represents the optimal contract for the manufacturer, and the corresponding price and effort level chosen by the retailer.

[23]Simulations are performed using Mathematica and can be found on the second author's webpage.




(a) Wholesale price and rebate. (b) Price and effort level.

Figure 1: With a linear demand and a quadratic disutility of effort ($D(p) = 1 - p$, $c = 0$, and $\psi(e) = \frac{3}{2}e^2$), the optimal contract for the manufacturer $(w^u, z^u)$ is represented in Panel (a) and the optimal price and effort level $(p^u, e^u)$ are depicted in Panel (b).

As the benefit of effort increases (i.e., $\theta$ increases), the wholesale price $w^u$ and the bonus $z^u$ become more similar (Figure 1 (a)). This result is intuitive. Under those circumstances, there is less need to enlarge the gap between the wholesale price and the rebate to induce effort. More subtle is the fact that the rebate $z^u$ is non-monotonic in the scale parameter. Indeed, there are potentially two effects from raising that rebate. First, it boosts efforts and this first effect is more significant when effort is more valuable. Second, raising the bonus also decreases the retail price, increases demand and thus increases the residual profit left to the retailer. Figure 1 (a) shows that this second effect may dominate when effort has a relatively small marginal value.

An intuitive consequence of an increase in the benefits of effort is that the equilibrium effort also increases (Figure 1 (b)). In turn, the fact that $E'(p) < 0$ implies that the retail price decreases under those circumstances.

More General Conditional Rebates. So far, we have assumed that a conditional rebate depends only linearly on all the additional sales which go beyond the base level. In this respect, we should notice that this modeling certainly fits real world practices. Indeed, there is ample evidence that trade allowances are most often conditional on the achievement of sales targets, and that conditional fixed fees may play a limited role as it has been pointed out by Elberg and Noton (2025). On the theoretical side, more general nonlinear contracts, say of the form $Z(\theta D(p))$ with $Z(\cdot)$ being non-negative and contingent on the realization of additional sales, might be attractive.

To illustrate, consider a two-part conditional contract of the form $Z(\theta D(p)) = A + z\theta D(p)$. With such scheme, the retailer now gets an expected profit worth

$$\pi(p, w) + e(\theta\pi(p, w - z) + A) - \psi(e).$$

Focusing again on interior solutions, the first-order optimality conditions associated to the optimal retail price $p$ and effort $e$ are thus respectively given by (3.2) and now

$$\theta\pi(p, w - z) + A = \psi'(e). \tag{3.14}$$

The conditional fee $A$ can now be used to boost effort incentives while those direct incentives may substitute to the profit incentives that a per-unit rebate $z$ would induce.

Observe that, given (3.14), the retailer's profit writes now as

$$\pi(p, w) + R(e). \tag{3.15}$$

Interestingly, there is a simple way for the manufacturer to achieve the vertically integrated outcome in this setting.[24] To see how, first notice that the manufacturer can choose a wholesale price $w^* > p^m$ to extract all of the retailer's profit when

$$\pi(p^m, w^*) = -R(e^m) < 0.$$

This formula is similar to (3.13) above, except that it applies to the vertically integrated retail price and effort. By also setting a positive per-unit rebate $z^* = w^* - p^m$ that compensates for the negative margin $p^m - w^*$, the contract ensures that no profits are ever made on additional sales, $\pi(p^m, w^* - z^*) = 0$. From (3.14), the retailer is then given the right incentives to exert effort from the vertical structure's viewpoint with a conditional fee which is worth

$$A^* = \psi'(e^m) > 0. \tag{3.16}$$

Several conclusions follow. The first one is that the wholesale price $w^*$ is still above the monopoly retail price $p^m$. That the retailer thus makes negative profits when the base level of demand realizes is thus a robust finding even in this environment with an enlarged set of contracts. These negative profits still act as a bonding device. The retailer will break even only if it exerts effort $e^m$.

The second robust insight is that the vertically integrated outcome can here also be achieved with a positive per-unit conditional rebate which now compensates the retailer for the negative margin that applies to the base level of demand. Lastly, the conditional fixed fee $A^*$ needed to provide incentives is now positive; which means that part of the incentives for effort can be advantageously provided with that tool without distorting pricing.

Of course, the usual caveat applies for the use of such conditional fixed fee. To optimally decouple incentives and pricing, the fee would have to be precisely tailored to information on the disutility of effort that might be hard to get for the manufacturer. Invoking again asymmetric information (admittedly without modeling it) and the difficulty in tailoring those conditional fees to information privately held by the retailer again provides a justification for our focus on pure linear conditional rebates.

[24] There could exist other ways to implement the vertically integrated outcome. The implementation chosen here is attractive because, first, it shows that a fixed payment from the manufacturer to the retailer emerges even in a bilateral monopoly environment and, second, it highlights the robustness of our main findings.

## 4. Impact of Sales-Below-Cost Laws

We assume now that, by regulation, retail prices cannot be set below the retailer's cost. The following non-negativity constraint must thus always hold[25]

$$p - w \geq 0. \tag{4.1}$$

Because the retail price margin satisfies the moral hazard incentive constraint (3.5), profits must at least cover the moral hazard rent needed to induce effort. The non-negativity constraint (4.1) becomes in fact a non-negative lower bound on the retailer's profit, or

$$\Pi_R(p, e) \geq R(e), \tag{4.2}$$

where the retailer's profit $\Pi_R(p, e)$ is still given by (3.8).

Since the retailer's profit may also be written as $(p-w)D(p)+R(e)$ (as we did to derive (3.13)), a first consequence of a ban on below-cost pricing appears immediately. The base profit $(p-w)D(p)$ that accrues to the retailer must be non-negative and, therefore, can no longer be used to extract the retailer's overall profit. Put differently, the retailer can no longer run a loss if effort fails to increase demand. Sticks are no longer available and only carrots can be used. Implementing a large rebate becomes the only channel to reward effort, and this is obviously costly for the manufacturer. The constraint on retail price imposed by regulation refers to an argument familiar from the moral hazard literature with limited liability.[26] The optimum is characterized in the next proposition.

Proposition 3. *Suppose there is a ban on below-cost pricing.*

1. *The retail price is equal to the wholesale price,*

$$p^b = w^b.$$

2. *The retailer makes a strictly positive profit equal to the moral hazard rent,*

$$\Pi_R(p^b, e^b) = R(e^b). \tag{4.3}$$

With a ban on below-cost pricing, the manufacturer's ability to extract the retailer's profit through the wholesale price is reduced because the base profit can no longer be negative. A moral hazard rent must be given up to the retailer.

Satisfying constraint (4.2) is thus clearly more demanding than satisfying the break-even condition (3.9) that prevails absent such a ban. A first intuition would then go as follows. Imposing a ban on below-cost pricing is akin to replacing the cost of effort $\psi(e)$ by a virtual disutility of effort $e\psi'(e) = \psi(e) + R(e)$ which is more costly. This calls for lowering effort. Moreover, lowering the retail price towards the monopoly level $p^m$ also contributes to relaxing constraint (4.2). These are the direct effects of a ban on below-cost pricing.

[25] Retrospective payments given at the end of the accounting year are typically not accounted for to evaluate whether the retailer sells below its cost. The retailer is thus said to sell below cost when $p - w < 0$.

[26] See Laffont and Martimort (2002, Chapter 4).

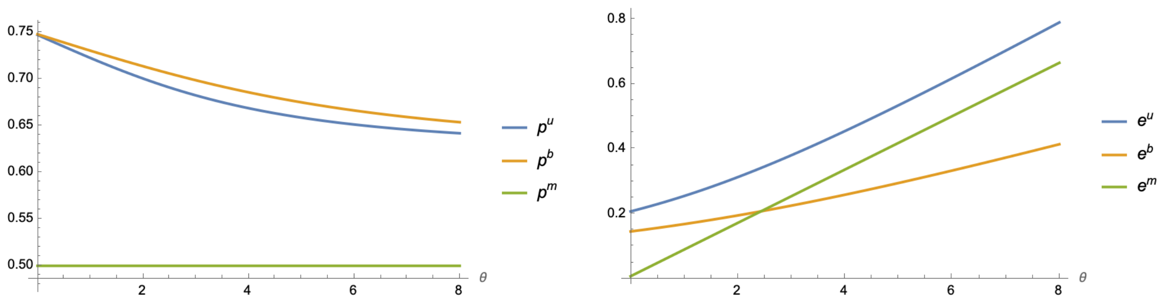
Indirect effects come from the fact that reducing effort affects marginal incentives to change the retail price, and vice versa. More formally, the value of the Lagrange multiplier associated to the retailer's participation constraint changes as one moves from one institutional environment to the other. A priori, one could think that replacing the break-even condition (3.9) by the more stringent condition (4.2) calls for increasing the value of the Lagrange multiplier. This intuition may be misleading, though, since the value of the multiplier depends on the equilibrium choices of price and effort. These indirect effects make the comparison between $(p^u, e^u)$ and $(p^b, e^b)$ difficult. Comparative statics can nevertheless be further explored in some specific environments.

Proposition 4. *Suppose that demand is given by* $D(p) = (a - bp)^{\frac{1}{\delta}}$ *(with* $\delta \geq 1$*)*[27] *and the disutility of effort is quadratic (*$\psi(e) = \frac{\mu}{2}e^2$, $\mu > 0$*). Imposing a ban on below-cost pricing has the following consequences:*

1. *The effort decreases:* $e^b < e^u$.
2. *The retail price increases:* $p^b > p^u$.
3. *The value of the Lagrange multiplier of the retailer's participation constraint decreases:* $\lambda^b < \lambda^u$.

Given that a linear demand ($\delta = 1$) and a quadratic disutility of effort may be viewed as first-order approximations of more complex specifications of preferences and technologies, Proposition 4 strongly suggests that the retail price should increase following a ban on below-cost pricing even with more general specifications. Far from promoting competition in the hypothetical scenario where below-cost pricing would be used for predatory purposes, such a law may well harm consumers and reduce overall welfare.

Figure 2 provides a comparison of price and effort as function of the demand shock $\theta$. It shows in particular that the effort level under a ban on below-cost pricing ($e^b$) may be below or above the vertically integrated level ($e^m$). Keeping this in mind will be useful later on.




(a) Prices. (b) Effort levels.

Figure 2: With a linear demand and a quadratic disutility of effort ($D(p) = 1 - p$, $c = 0$, and $\psi(e) = \frac{3}{2}e^2$), comparison of the optimal price (Panel (a)) and effort level (Panel (b)) with no ban on below-cost pricing ($p^u$ and $e^u$), with a ban on below-cost pricing ($p^b$ and $e^b$), and under integration ($p^m$ and $e^m$).

[27]This class of demand, first used by Bulow and Pfleiderer (1983), has the property that the cost pass-through is constant and given by $1/(1 + \delta)$.

## 5. Resale Price Maintenance vs. Incentive Rebates

Resale Price Maintenance (RPM) has long been identified in the literature as a canonical solution to induce retailer service when effort is non-contractible.[28] In these frameworks, the manufacturer cannot reward effort directly and instead relies on RPM to protect the retailer's margin, thereby creating an indirect incentive to exert effort.

However, while effort itself may remain difficult to verify directly, the widespread availability of real-time scan data and digital tracking solutions has made its outcome—sales volume—increasingly easy to observe and contract upon. This technological shift motivates the comparison between RPM and conditional rebates. We ask whether a performance-based contract (conditional rebates), which targets the *outcome* of effort, performs better or worse than a price-based constraint (RPM), which targets the *incentive structure* of effort. This comparison is particularly relevant given that RPM remains subject to strict antitrust scrutiny (and is considered a hard-core restriction in jurisdictions like the EU), whereas volume-based rebates are generally viewed more leniently.

Observe first that the joint use of a RPM agreement together with incentive rebates allows the manufacturer to implement the integrated outcome $(p^m, e^m)$ and to capture the whole industry profit.[29] These two instruments provide the manufacturer with enough control over the retailer to overcome the moral hazard problem at no cost.

We study now the situation where the manufacturer uses only a RPM agreement, i.e., the manufacturer controls both the wholesale price $w$ and the retail price $p$ of the product. The retailer's only decision is to choose the level of promotion effort so as to maximize $(p-w)D(p)(1+\theta e)-\psi(e)$. The manufacturer's problem is to maximize $(w-c)D(p)(1+\theta e)$ subject to the retailer's participation constraint, and anticipating the choice of effort by the retailer.

Proposition 5. *Under a RPM agreement, the retail price is always set at the monopoly base level, or $p^{rpm} = p^m$. Moreover:*

- *If $\frac{\theta^2 \pi^m}{\psi''(0)} < 1$, the retailer exerts no effort and makes no profit.*
- *Otherwise, the retailer exerts a strictly positive effort $e^{rpm} < e^m$ and earns a strictly positive profit $\frac{\psi'(e^{rpm})}{\theta} + R(e^{rpm})$.*

*Furthermore, assuming a linear demand and a quadratic cost of effort:*

- *With no ban on below-cost pricing, the manufacturer prefers a RPM agreement over incentive rebates if and only if $\frac{\theta^2\pi^m}{\psi''(0)}$ is sufficiently small, whereas the retailer always prefers incentive rebates.*
- *With a ban on below-cost pricing, the manufacturer always prefers a RPM agreement, whereas the retailer always prefers incentive rebates.*

[28] Telser (1960) and Matthewson and Winter (1984). See also Dertwinkel-Kalt and Wey (2024) for a general analysis of RPM agreements and Martimort and Piccolo (2007) for a comparison between RPM and quantity fixing agreements under asymmetric information.

[29] A wholesale price $w = p^m + R(p^m)/D(p^m)$ together with an incentive rebate $z = (\theta R(e^m) + \psi'(e^m))/(\theta D(p^m))$ lead the retailer to choose $e = e^m$ and the manufacturer to set $p = p^m$. The retailer makes no profit, the industry profit is maximized and fully captured by the manufacturer.

From the manufacturer's perspective, the benefit of a RPM is to limit distortions on the retail price, which always coincides with the monopoly price $p^m$. The downside is that there is only one pricing instrument, namely the wholesale price $w$, to pursue two conflicting objectives, namely extracting the retailer's profit and providing incentives to effort. As a result, the promotion effort is below the vertically integrated level (and sometimes null). Intuitively, when the benefits associated with the effort are small (for instance, when $\theta$ is small), the manufacturer should prefer a RPM agreement over a system of incentive rebates. By contrast, when these benefits are large, the manufacturer should prefer to give up the control of the retail price to the retailer and implement a system of incentive rebates. Observe that the retailer always prefers a system of incentive rebates over a RPM agreement. Provided that effort has a sufficiently strong impact on demand, both the manufacturer and the retailer view rebates as superior to a RPM agreement.

However, the presence of a ban on below-cost pricing impacts that conclusion. In that case, there is a strong disagreement between the manufacturer and the retailer on which sort of vertical control (RPM or incentive rebates) ought to be implemented. Key to this disagreement is the fact that providing incentives is costly for the manufacturer when below-cost pricing is not allowed, which in turn provides a substantial moral hazard rent to the retailer.

## 6. MULTI-PRODUCT RETAILER WITH DEMAND COMPLEMENTARITIES

We now evaluate the importance of below-cost pricing in the more traditional multi-product context where below-cost pricing has already been proved useful. To illustrate, suppose that the retailer can also sell another good, say good 2. For notational simplicity, we assume that good 2 is also produced at marginal cost $c$. Good 1, whose demand is still given by $D(p_1)$ as before, is a loss leader in the following sense. Consumers may be eager to obtain the loss leader and, when doing so, they may also express a demand for good 2. Such complementarity is captured by assuming a simple multiplicative structure for the demand for good 2[30]

$$D_2(p_2, p_1) = D(p_1)\tilde{D}(p_2).$$

In this setting, the non-integrated structure stands ready to make a loss on the sales of good 1 if doing so sufficiently raises demand and profit on good 2.

The retailer's profit on good 2, conditional on selling good 1, is given by

$$\tilde{\pi}(p_2, c) = (p_2 - c)\tilde{D}(p_2).$$

This profit is maximized at a monopoly price, independent of $p_1$, which is given by

$$\tilde{p}^m = \arg\max_{p_2} (p_2 - c)\tilde{D}(p_2).$$

Denoting by $\tilde{\pi}^m$ the corresponding monopoly profit, we may rewrite the retailer's overall

[30]For a model also featuring such a multiplicative demand structure, see Martimort, Pommey and Pouyet (2022) who develop a model of conditional sales. More complex demand structures could be entertained with no additional insight. The benefit of this multiplicative structure is that it allows us to import mutatis mutandis insights and techniques from our previous analysis.

downstream profit as follows

$$(p_1 - w_1 + \tilde{\pi}^m)D(p_1)(1 + \theta e) + \theta e z_1 D(p_1) - \psi(e). \tag{6.1}$$

OPTIMAL WHOLESALE CONTRACT. The expression of the retailer's profits in (6.1) highlights that the retail price and effort level that are chosen by the retailer only depend on the *net* wholesale price

$$\tilde{w}_1 = w_1 - \tilde{\pi}^m,$$

which is the opportunity cost for the retailer of buying one extra unit of good 1. When not buying one extra unit of good 1, the retailer also forgoes the monopoly profit $\tilde{\pi}^m$ made on good 2. This extra benefit can in fine be captured by the manufacturer whose perceived cost of producing good 1 is now

$$\tilde{c} = c - \tilde{\pi}^m.$$

From there, our previous analysis can be replicated mutatis mutandis. First, we need to modify Assumption 3 to account for this change of cost.

ASSUMPTION 4.

$$(1 + \theta)\varphi(\tilde{p}(\tilde{\pi}^m)) < \psi(1),$$

*where* $\tilde{p}(\tilde{\pi}^m)$ *is now defined as* $\pi_p(\tilde{p}(\tilde{\pi}^m), \tilde{c}) = \varphi'(\tilde{p}(\tilde{\pi}^m))$.

Second, while the retailer's profit, when expressed in terms of the retail price and effort $(p_1, e)$ implemented by the retailer, is kept unchanged as in (3.8), we observe that the manufacturer's profit accounts for the perceived cost of producing good 1 and can be written as follows

$$\Pi_M(p_1, e) = (\pi(p_1, \tilde{c}) - \varphi(p_1))(1 + \theta e). \tag{6.2}$$

The characterization of the optimal wholesale contract $(w^u(\tilde{\pi}^m), z^u(\tilde{\pi}^m))$ is thus readily obtained by applying Proposition 1 for the perceived cost $\tilde{c}$. In particular, a positive rebate for expanding demand on good 1 is again warranted. With respect to the single-good scenario, because the perceived cost of good 1 is now reduced, the manufacturer offers a lower wholesale price which induces in fine a lower retail price. A lower retail price increases the retailer's profit which boosts effort.

BAN ON BELOW-COST PRICING. The analysis here also mimics our earlier findings. A ban on below-cost pricing imposes the condition

$$p_1 - w_1 \geq 0.$$

It in turn implies that the retailer's break-even condition writes now as

$$\Pi_R(p_1, e) \geq \tilde{\pi}^m D(p_1) + R(e). \tag{6.3}$$

The monopoly manufacturer now maximizes its profit $\Pi_M(p_1, e_1)$ as given in (6.2) subject to (6.3). Again, the retailer's participation constraint (6.3) is hardened and Assumption 4 suffices to ensure that this constraint is binding at the optimum. With respect to the single-good scenario, the main difference is that the rent that must be given up to the retailer depends directly both on the effort (through the moral hazard rent $R(e)$ in (6.3)) and on the price of the loss leader (the term $\tilde{\pi}^m D(p_1)$ in (6.3)).

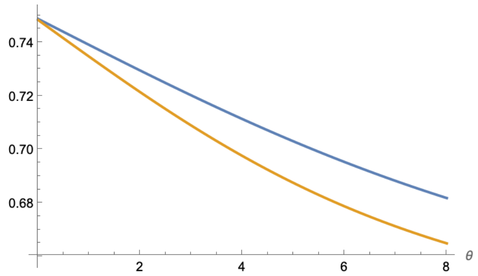
From the analysis of the single-good scenario performed in Section 4, we know that the comparison between price and effort levels with and without a ban on below-cost pricing is a priori ambiguous. That ambiguity remains with complementary products. The new effect is that the price of the loss leader might be further increased in order to reduce the term $\tilde{\pi}^m D(p_1)$ in (6.3).

To illustrate, let us perform some numerical simulations, using $D(p) = 1 - p$, $c = 0$ and $\psi(e) = 5e^2$. There are two parameters of interest, $\theta$ and $\tilde{\pi}^m$. The profit on the complementary good 2, $\tilde{\pi}^m$, takes four values: 0, 0.25, 0.5 and 0.75. As before, $\theta$ varies between 0.1 and 8 by increments of .1. We report in Figure 3 the corresponding simulations that help to understand the impact of a ban on below-cost pricing with complement goods.

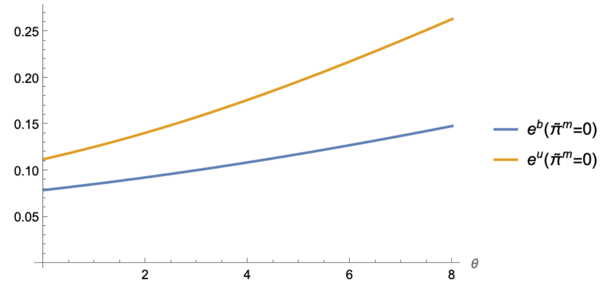
When the profit on the complementary good, $\tilde{\pi}^m$, is zero, we recover the results of the single-good scenario. Regardless of whether below-cost pricing is permitted, the price decreases, and the effort increases as the promotional effort becomes more effective (i.e., as $\theta$ increases). However, following a ban on below-cost pricing, the price increases, and the effort decreases.

As the profit from the complementary good, $\tilde{\pi}^m$, increases, the same comparative statics hold for the promotion effort. A closer examination of the graphs reveals that the difference between the effort under no ban and the effort under a ban on below-cost pricing also widens as $\tilde{\pi}^m$ increases. This is due to the participation constraint becoming more challenging to satisfy (see (6.3)), which indirectly necessitates a further downward distortion of the effort when a ban on below-cost pricing is enforced.

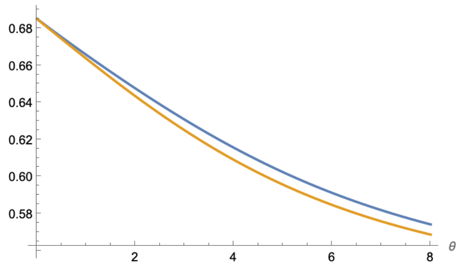
Turning on to the price of good 1, the comparative statics are qualitatively impacted by the presence of a complementary good. When $\tilde{\pi}^m$ is small, imposing a ban still results in a higher price for the loss leader. However, as $\tilde{\pi}^m$ increases, the price of the loss leader under a ban on below-cost pricing may fall below the price with no ban. This arises because constraint (6.3) is more difficult to satisfy. This is an interesting consequence of a ban on below-cost pricing: Such a ban may lead to a lower price for the loss leader when the product generates significant profits from complementary goods. In these cases, consumer surplus could increase following a ban on below-cost pricing, depending on how consumers value the price reduction on the loss leader relative to the reduction in promotional effort.



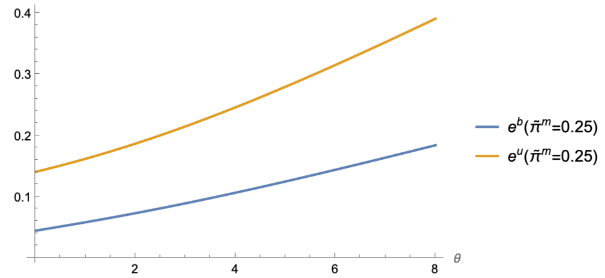

(a) Prices for $\tilde{\pi}^m = 0$.



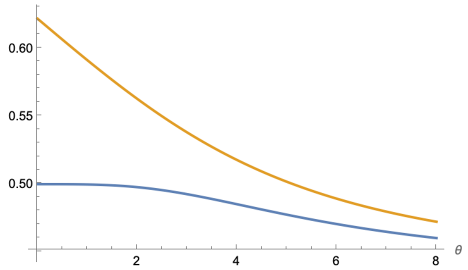

(b) Efforts for $\tilde{\pi}^m = 0$.



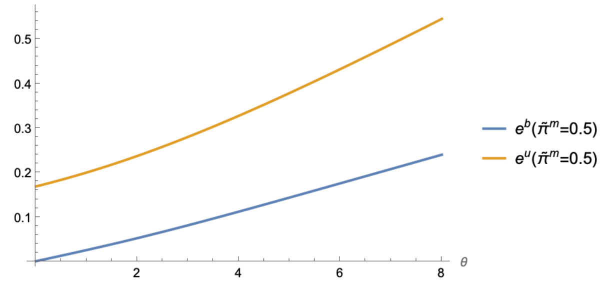

(c) Prices for $\tilde{\pi}^m = 0.25$.



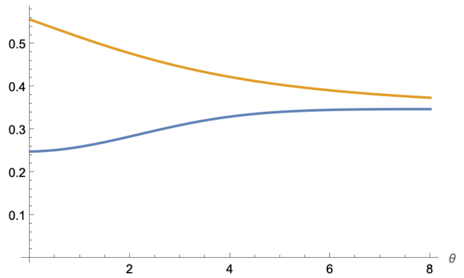

(d) Efforts for $\tilde{\pi}^m = 0.25$.



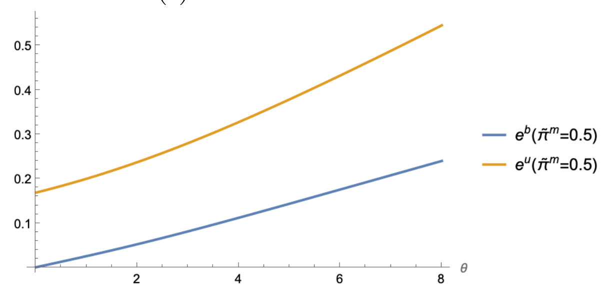

(e) Prices for $\tilde{\pi}^m = 0.5$.




(f) Efforts for $\tilde{\pi}^m = 0.5$.




(g) Prices for $\tilde{\pi}^m = 0.75$.




(h) Efforts for $\tilde{\pi}^m = 0.75$.

Figure 3: Price of the loss leader good and effort level with no ban ($p_1^u$ and $e^u$) and with a ban on below-cost pricing ($p_1^b$ and $e^b$) as function of $\theta$ and for different values of $\tilde{\pi}^m$.

## 7. QUANTIFYING DAMAGES CAUSED BY UPSTREAM CARTELS

In 2014, the French Competition Authority sanctioned a cartel of manufacturers in the home and personal care sector (Decision 14-D-19).[31] This case offers a striking illustration of the mechanism analyzed in this paper. As documented in the decision, the regulatory

[31]See "Décision n° 14-D-19 du 18 décembre 2014 relative à des pratiques mises en œuvre dans le secteur des produits d'entretien et des insecticides et dans le secteur des produits d'hygiène et de soins pour le corps," available at https://www.autoritedelaconcurrence.fr/fr/decision/relative-des-pratiques-mises-en-oeuvre-dans-le-secteur-des-produits-dentretien-et-des. Two sanctions were pronounced: €345.2M for manufacturers of cleansing products and €605.9M for manufacturers of hygiene products.

environment (the Loi Galland) defined the loss-leading threshold based on the invoice price, effectively neutralizing retail price competition (paras. 182–185). Consequently, negotiations shifted toward "back margins" or "commercial cooperation" services. The decision confirms that these transfers were invoiced "most often as a percentage of the total turnover" rather than as lump-sum payments (para. 142). Manufacturers coordinated to freeze the annual growth—known as the "drift"—of these rebate rates (paras. 152–153). By agreeing to "stop the drifts" (paras. 364, 398), manufacturers effectively capped the variable component of the contract, thereby reducing the retailer's marginal return on effort compared to a competitive benchmark.

To quantify the damages resulting from such collusive practices, we must compare the cartel equilibrium derived in the previous sections with a competitive benchmark.[32] We characterize this counterfactual by assuming that, in the absence of collusion, the upstream market would be perfectly competitive. Head-to-head competition in wholesale contracts between manufacturers is akin to shifting all the bargaining power towards the retailer. Competition thus drives the manufacturers' profit to zero, or

$$((w - c)(1 + \theta e) - \theta e z)\, D(p) = 0. \tag{7.1}$$

It is straightforward to see that the simple wholesale contract with a wholesale price that just covers the manufacturer's marginal cost and no rebate (i.e., $w^d = c$ and $z^d = 0$) ensures that manufacturers make zero profit.[33,34] Moreover, this solution clearly aligns the retailer's objectives with those of the vertically integrated structure so that the retail price and effort coincide with integrated outcome, or $p^d = p^m$ and $e^d = e^m$.

Absent collusion, the retailer appropriates the whole profit of the vertically integrated structure while, with collusion and a ban on below-cost pricing, the retailer only appropriates the corresponding moral hazard rent. Formally, the retailer's loss from the collusion between manufacturers $\Delta$ is worth

$$\Delta \equiv \underbrace{(1 + \theta e^m)\pi^m - \psi(e^m)}_{\substack{\text{Retailer's profit} \\ \text{with upstream competition}}} \quad - \quad \underbrace{R(e^b),}_{\substack{\text{Moral hazard rent under manufacturers'} \\ \text{collusion and no below-cost pricing}}}$$

[32] Our measure of damages therefore relies on a competitive counterfactual. If the upstream market were oligopolistic in the absence of collusion, wholesale prices would be higher, and our calculated damages would represent an upper bound of the actual harm.

[33] See the Online Appendix. Superscript "$d$" refers to the fact that with competitive manufacturers everything happens as if the downstream retailer had all the bargaining power.

[34] In its decision, the French Competition Authority argues that by increasing transparency on retail markets, the Loi Galland neutralized retail price competition and shifted the core of negotiations toward these rebates (paras. 152, 187), a situation qualified as "tacit collusion" before "explicit collusion" took place (paras. 194–196) after several regulatory reforms ("Circular Dutreil" and "Sarkozy Commitment") reintroduced uncertainty in the negotiations between manufacturers and retailers (paras. 212–221, 227, 231 and 352). In our framework, this "tacit collusion" counterfactual could be represented by assuming that producers collude on the wholesale price (and set $w = p$) but compete head-to-head on the rebate and set $z = (p-c)\frac{1+\theta e}{\theta e}$ such that (7.1) is binding. The retailer would then still choose $p = p^m$ and $e = e^m$, leading to the same profit $(1+\theta e^m)\pi^m - \psi(e^m)$. Producers' profit from colluding on the wholesale price is fully dissipated by their competition on the rebate.

which may be rewritten as follows (using Equation (2.2))

$$\Delta = \underbrace{\pi^m}_{\substack{\text{Monopoly} \\ \text{base profit}}} + \underbrace{R(e^m) - R(e^b).}_{\substack{\text{Difference in moral hazard rents} \\ \text{with and without collusion}}}$$

Damages are thus the sum of two terms. The first one is the monopoly level of the base profit that accrues to the retailer if, absent collusion, no effort was undertaken. The second term is the incremental value of the moral hazard rent as collusion would be deterred and effort would change from $e^b$ to $e^m$, which may be positive or negative as illustrated in Figure 2.

Considering the specification $D(p) = (a - bp)^{\frac{1}{\delta}}$ and $\psi(e) = \frac{\mu}{2}e^2$, it is remarkable that damages can be expressed only in terms of values of the demand function before and after collusion has been deterred, namely $D(p^b)$ and $D(p^m)$. Indeed, the percentage of after-collusion profits that should be paid as damages amounts to

$$\frac{\Delta}{\pi^m} = 1 + \frac{4(\lambda^b)^2(1+\delta(1-\lambda^b))^{\delta} - (1+\delta(1-\lambda^b))^{-\delta}}{2(1+\delta-(2+\delta)\lambda^b)} \tag{7.2}$$

where the Lagrange multiplier of (4.2) $\lambda^b$ is given by $(1+\delta(1-\lambda^b))^{\frac{1}{\delta}} = \frac{D(p^m)}{D(p^b)}$. In other words, the evaluation of damages, a task often viewed as informationally demanding, only requires the pre- and post-collusion demand levels if one is ready to adopt the above functional forms.

Adopting the same linear-quadratic specification as in Section 3, we can quantify the damages suffered by the retailer from upstream collusion between manufacturers using (7.2). These damages are obviously always strictly positive, but, perhaps less intuitively, non-monotonic with the demand shock $\theta$. This comes from the fact that, first, damages depend on the difference $R(e^m) - R(e^b)$ as shown above and, second, that the difference $e^m - e^b$ is positive for low demand shocks but negative for large ones.

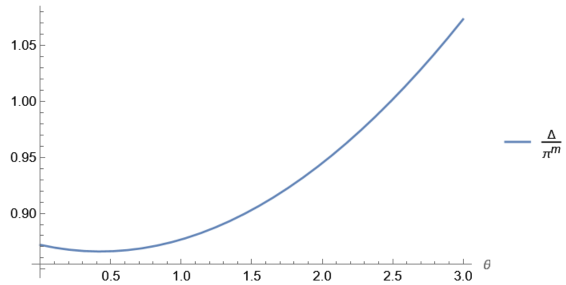



Figure 4: Damages $\frac{\Delta}{\pi^m}$ as function of the demand shock $\theta$ (with $D(p) = 1 - p$, $\psi(e) = \frac{3}{2}e^2$, $c = 0$).

## 8. Conclusion

This paper has analyzed the interplay between vertical contracting and retail regulations in the presence of moral hazard. Our central insight is that trade allowances and loss leading—selling below the wholesale price—can emerge endogenously as efficient incentive devices. By setting a high wholesale price (acting as a "stick") combined with conditional rebates (acting as a "carrot"), the manufacturer effectively forces the retailer to internalize the marginal benefit of promotional efforts.

Our analysis highlights the unintended consequences of banning resale below cost. While such regulations are often intended to protect smaller retailers or dampen downstream competition, we show that they restrict the set of feasible incentive contracts. Specifically, the ban forces manufacturers to lower wholesale prices and reduce the power of rebate schemes to ensure retailer participation without violating the law. The result is a "chilling effect" on promotional activity: equilibrium effort levels decrease, and consumer welfare is harmed not by high prices per se, but by the under-provision of demand-enhancing services.

From an antitrust perspective, these findings offer a nuanced view on the treatment of conditional rebates. Following the *Velux* case (COMP/39.451), European competition policy has tended to view incremental rebates more favorably than retroactive schemes, considering them less likely to foreclose competitors (Albaek and Claici, 2009).[35] Our analysis suggests that this leniency should be qualified when below-cost pricing laws are present. While incremental rebates are indeed efficiency-enhancing in an unregulated environment, their interaction with price floor regulations can generate allocative inefficiencies. The harm to consumers arises from the constraint the regulation imposes on the manufacturer's ability to solve the agency problem.

Our framework relies on specific assumptions and relaxing those assumptions would open avenues for future research. First, we assumed that fixed fees are unavailable. A natural extension would be to study situations where slotting fees can be used but cannot perfectly capture the retailer's rents, perhaps due to private information on demand parameters. In such environments, there would be a complex interplay between incentivizing effort and screening for information. Second, we abstracted away from horizontal competition between manufacturers. Introducing inter-brand competition might limit a manufacturer's ability to use high wholesale prices as an incentive device, as retailers could delist products that expose them to excessive risk. Exploring how these competitive forces interact with the regulatory constraint remains a fruitful direction for future work.

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# APPENDIX

PROOF OF PROPOSITIONS 1 AND 2. The monopoly manufacturer's optimization problem can be written as follows:

$$\max_{(p,e)} \Pi_M(p,e) \equiv (\pi(p,c) - \varphi(p))(1+\theta e) \text{ subject to (3.9)}.$$

Let $\lambda^u$ denote the non-negative Lagrange multiplier of the retailer's break-even condition (3.9). The corresponding Lagrangian for this problem writes as follows:

$$\mathcal{L}(p,e,\lambda^u) = (\pi(p,c) - \varphi(p))(1+\theta e) + \lambda^u (\varphi(p)(1+\theta e) - \psi(e)).$$

The Karush-Kühn-Tucker first-order necessary conditions for optimality with respect to $p$ and $e$ (for an interior solution) write respectively as

$$\pi_p(p^u,c) - (1-\lambda^u)\varphi'(p^u) = 0, \tag{A.1}$$

$$\theta(\pi(p^u,c) - (1-\lambda^u)\varphi(p^u)) = \lambda^u \psi'(e^u), \tag{A.2}$$

while the complementary slackness condition is

$$\lambda^u ((1+\theta e^u)\varphi(p^u) - \psi(e^u)) = 0. \tag{A.3}$$

We now establish several properties of the optimum.

CLAIM 1. $\lambda^u > 0$ *and thus* (3.9) *is binding:*

$$(1+\theta e^u)\varphi(p^u) = \psi(e^u). \tag{A.4}$$

*Proof.* Suppose instead that $\lambda^u = 0$. Then (A.1) leads to $p^u = \tilde{p}$. Because $\pi(\tilde{p},c) > \varphi(\tilde{p})$, the optimal effort choice that the manufacturer would like to implement would be $e^u = 1$. The retailer's profit would then be equal to $(1+\theta)\varphi(\tilde{p}) - \psi(1)$, which is strictly negative when Assumption 3 holds; a contradiction. Therefore, at the optimum, it must be $\lambda^u > 0$. Condition (A.4) then follows from (A.3). □

CLAIM 2. $\lambda^u < 1$.

*Proof.* Suppose $\lambda^u = 1$. Then, from (A.1) and (A.2), we obtain $p^u = p^m$ and $e^u = e^m$. Observe now that, by definition of the monopoly price $p^m$, $\varphi(p^m) = \pi^m$, and $e^m$ is such that $\pi^m = \psi'(e^m)/\theta$. Thus,

$$(1+\theta e^m)\varphi(p^m) - \psi(e^m) = (1+\theta e^m)\pi^m - \psi(e^m) = \pi^m + R(e^m) > 0. \tag{A.5}$$

(A.5) shows the retailer's participation constraint is strictly satisfied; a contradiction with Claim 1. Hence, by continuity, it must be that $\lambda^u \in (0,1)$. □

Claim 2 together with Assumption 2 implies that the Lagrangian is quasi-concave in $p$, and thus in $(p,e)$ given the convexity of $\psi$ and Claim 1.

CLAIM 3. $\tilde{p} > p^u > p^m$.

*Proof.* Together with Claim 2, (A.1) implies

$$\pi_p(p^u, c) = (1 - \lambda^u)\varphi'(p^u) < 0.$$

Because $\pi(p, c)$ is quasi-concave in $p$ when Assumption 1 holds, we have

$$p^u > p^m.$$

Similarly, we have

$$\pi_p(p^u, c) - \varphi'(p^u) = -\lambda^u \varphi'(p^u) > 0.$$

Because $\pi(p, c) - \varphi(p)$ is also quasi-concave in $p$ when Assumption 2 holds, we have $p^u < \tilde{p}$. □

CLAIM 4.

$$z^u > 0 \tag{A.6}$$

*and*

$$p^u - w^u = -R(e^u) < 0. \tag{A.7}$$

*Proof.* Let us rewrite the first-order condition (3.3) for the optimal solution $(p^u, e^u)$ in a more explicit manner as

$$\theta(p^u - w^u + z^u)D(p^u) = \psi'(e^u). \tag{A.8}$$

From (A.8), we obtain:

$$p^u - w^u = \frac{\psi'(e^u)}{\theta D(p^u)} - z^u.$$

Inserting into (3.4) for the optimal solution $(p^u, e^u)$ yields:

$$z^u \frac{D(p^u)}{1 + \theta e^u} = \frac{\psi'(e^u)}{\theta} - \varphi(p^u). \tag{A.9}$$

Using (A.1) and (A.2), we get:

$$\psi'(e^u) = \theta f(p^u) \tag{A.10}$$

where

$$f(p) = \pi(p, c) + \frac{\pi_p(p, c)}{\varphi'(p) - \pi_p(p, c)}(\pi(p, c) - \varphi(p)).$$

Using (A.10), (A.9) can be rewritten as follows:

$$z^u \frac{D(p^u)}{1 + \theta e^u} = \frac{\varphi'(p^u)(\pi(p^u, c) - \varphi(p^u))}{\varphi'(p^u) - \pi_p(p^u, c)}.$$

From Claim 1 and Equation (A.1), we get $\varphi'(p^u) - \pi_p(p^u, c) < 0$. Therefore, we have also $\varphi'(p^u) < \pi_p(p^u, c) < 0$ since $p^u > p^m$. Moreover, notice that $\pi(p, c) > \varphi(p) \Leftrightarrow p - c > -\frac{D(p)}{D'(p)} \Leftrightarrow p > p^m$. Since $p^u > p^m$, we have $\pi(p^u, c) > \varphi(p^u)$. Hence, the right-hand side of (A.10) is positive and (A.6) holds.

Inserting now (A.9) into (3.4), we obtain

$$(p^u - w^u)D(p^u) = (1 + \theta e^u)\varphi(p^u) - e^u \psi'(e^u).$$

Using the binding participation constraint (3.9) and $R(e) = e\psi'(e) - \psi(e)$, we obtain (A.7). □

CLAIM 5.

$$e^u > e^m. \tag{A.11}$$

*Proof.* Observe that

$$\psi'(e^m) = \theta f(p^m). \tag{A.12}$$

Taken together with (A.10), Condition (A.11) will be proved (thanks to the convexity of $\psi$) if $f(p^u) > f(p^m)$. We thus compute

$$f'(p) = (\pi(p,c) - \varphi(p))\frac{d}{dp}\left(\frac{\pi_p(p,c)}{\varphi'(p) - \pi_p(p,c)}\right).$$

We may write

$$\frac{\pi_p(p,c)}{\varphi'(p) - \pi_p(p,c)} = \frac{p + \frac{D(p)}{D'(p)} - c}{-\left(p + \frac{D(p)}{D'(p)} - \frac{\varphi'(p)}{D'(p)} - c\right)}.$$

Assumption 1 (resp. Assumption 2) ensures that the numerator (resp. denominator) is non-decreasing (resp. non-increasing). Hence, $f'(p) \geq 0$ whenever $\pi(p,c) - \varphi(p) \geq 0$, which holds for $p \geq p^m$ (with a strict inequality for $p > p^m$). Since $p^u > p^m$, we then obtain $f(p^u) > f(p^m)$ and thus $e^u > e^m$. □

This concludes the proof of Propositions 1 and 2. □

PROOF OF PROPOSITION 3. With a ban on below-cost pricing, the manufacturer's problem is to maximize $\Pi_M(p,e)$ subject to (4.2). We rewrite (4.2) as follows:

$$(1+\theta e)\varphi(p) - e\psi'(e) \geq 0, \tag{A.13}$$

so that the manufacturer's problem may be expressed in a more compact form as follows:

$$\max_{(p,e)} (\pi(p,c) - \varphi(p))(1+\theta e) \text{ subject to (A.13).}$$

Denote by $\lambda^b$ the Lagrange multiplier for constraint (A.13). The Lagrangian writes as follows:

$$\mathcal{L}(p,e,\lambda^b) = (\pi(p,c) - \varphi(p))(1+\theta e) + \lambda^b\left((1+\theta e)\varphi(p) - e\psi'(e)\right).$$

Assuming concavity of this Lagrangian in $(p,e)$ and optimizing with respect to $p$ and $e$ respectively yields the following Karush-Khün-Tucker first-order necessary conditions:

$$\pi_p(p^b) - (1-\lambda^b)\varphi'(p^b) = 0, \tag{A.14}$$

$$\theta\left(\pi(p^b) - (1-\lambda^b)\varphi(p^b)\right) = \lambda^b\left(\psi'(e^b) + e^b\psi''(e^b)\right). \tag{A.15}$$

We now prove that (A.13) is binding. We proceed by contradiction. Suppose not, that is, $\lambda^b = 0$. Then, we would have $p^b = \tilde{p}$ and $e^b = 1$. These values do not satisfy the break-even condition (3.9) and, *a fortiori*, the more demanding constraint (4.2) when Assumption 4 holds. Therefore, (4.2), or equivalently (4.1), is binding at the optimum.

The last item in the proposition follows from observing that, when $e^b > 0$, we have $(1+\theta e^b)\varphi(p^b) = e^b\psi'(e^b) > \psi(e^b)$. □

PROOF OF PROPOSITION 4. With $D(p) = (a - bp)^{\frac{1}{\delta}}$, (3.5) and (3.6) can respectively be expressed as

$$w = p + e^2\mu(a - bp)^{-\frac{1}{\delta}} - \frac{(a - bp)\delta(1 + \theta e)}{b}, \tag{A.16}$$

$$z = (1 + \theta e)\left(\frac{e\mu(a - bp)^{-\frac{1}{\delta}}}{\theta} - \frac{(a - bp)\delta}{b}\right). \tag{A.17}$$

From this, we can express the manufacturer's and the retailer's profits as function of $(p, e)$ respectively as follows:

$$\Pi_M(p, e) = \frac{(1 + \theta e)(a - bp)^{\frac{1}{\delta}}(b(-c + \delta p + p) - a\delta)}{b}, \tag{A.18}$$

$$\Pi_R(p, e) = \frac{\delta(1 + \theta e)(a - bp)^{\frac{1}{\delta}+1}}{b} - \frac{\mu}{2}e^2. \tag{A.19}$$

Optimizing the Lagrangian with respect to $p$ and $e$ and solving yields the expressions of the price and the effort level as function of the multiplier $\lambda^u$, which we denote by $P^u(\lambda^u)$ and $E^u(\lambda^u)$:

$$P^u(\lambda^u) = \frac{a}{b} - \frac{a - bc}{b(1 + \delta)(1 + \delta(1 - \lambda^u))}, \tag{A.20}$$

$$E^u(\lambda^u) = \frac{\delta\theta}{b\mu}\left(\frac{a - bc}{\delta + 1}\right)^{\frac{1}{\delta}+1}\frac{1}{\lambda^u}\left(\frac{1}{1 + \delta(1 - \lambda^u)}\right)^{\frac{1}{\delta}}. \tag{A.21}$$

Plugging these expressions into the binding retailer's participation constraint, we obtain that $\Pi_R(P^u(\lambda^u), E^u(\lambda^u)) = 0$ amounts to

$$\frac{\delta\theta^2}{2b\mu}\left(\frac{a - bc}{\delta + 1}\right)^{\frac{1}{\delta}+1} = \frac{(\lambda^u)^2(1 + \delta(1 - \lambda^u))^{\frac{1}{\delta}}}{1 + \delta - \lambda^u(2 + \delta)}. \tag{A.22}$$

The left-hand side in (A.22) is strictly positive. It can easily be shown that, for $\lambda^u \in [0, \frac{1+\delta}{2+\delta})$, the right-hand side is strictly increasing in $\lambda^u$ and takes values in $[0, +\infty)$. Therefore, there exists a unique $\lambda^u \in [0, \frac{1+\delta}{2+\delta})$ which satisfies (A.22).

Consider now that there is a ban on below-cost pricing. $(w, z)$ as functions of $(p, e)$ are still given by (A.16) and (A.17). Profits of the retailer and the manufacturer are still given by (A.18) and (A.19).

The Karush-Khün-Tucker first-order condition with respect to price is the same as in the case with no ban on below-cost pricing. Therefore,

$$P^b(\lambda^b) = P^u(\lambda^b), \tag{A.23}$$

where $P^u$ is given by (A.20). The Karush-Khün-Tucker first-order condition with respect to effort leads to $E^b(\lambda^b) = \frac{1}{2}E^u(\lambda^b)$ where $E^u$ is given by (A.21). Replacing in the participation constraint (A.13), which must hold as an equality at the optimum, characterizes the Lagrange multiplier $\lambda^b$:

$$\frac{\delta\theta^2}{2b\mu}\left(\frac{a - bc}{\delta + 1}\right)^{\frac{1}{\delta}+1} = 2\frac{(\lambda^b)^2(1 + \delta(1 - \lambda^b))^{\frac{1}{\delta}}}{1 + \delta(1 - \lambda^b) - 2\lambda^b}. \tag{A.24}$$

The left-hand side in (A.24) is strictly positive. The right-hand side in (A.24) is, for $\lambda^b \in [0, \frac{1+\delta}{4+\delta})$, strictly increasing and takes values in $[0, +\infty)$. Hence, there exists a unique $\lambda^b \in (0, \frac{1+\delta}{4+\delta})$ such that (A.24) holds.

Comparing (A.22) and (A.24), it comes immediately that $\lambda^u > \lambda^b$. From (A.20), we obtain that $(P^u)'(\lambda) < 0$. Therefore, we have that: $p^b = P^b(\lambda^b) = P^u(\lambda^b) > P^u(\lambda^u) = p^u$.

It remains to compare effort levels. Recall that $e^u = E^u(\lambda^u)$ and $e^b = \frac{1}{2}E^u(\lambda^b)$, where $E^u(\cdot)$ is defined by (A.21). Since $\lambda^u$ is implicitly defined by (A.22), we obtain the following simplification:

$$E^u(\lambda^u) = \frac{1}{\theta} \frac{2\lambda^u}{(1+\delta) - (2+\delta)\lambda^u}. \tag{A.25}$$

Similarly, since $\lambda^b$ is implicitly defined by (A.24), we obtain:

$$E^b(\lambda^b) = \frac{1}{2}E^u(\lambda^b) = \frac{1}{\theta} \frac{2\lambda^b}{(1+\delta) - (2+\delta)\lambda^b}. \tag{A.26}$$

Since $\frac{2x}{(1+\delta)-(2+\delta)x}$ is strictly increasing in $x$ and $\lambda^u > \lambda^b$, we have $e^u > e^b$. □

PROOF OF PROPOSITION 5. The retailer's profit is given by $(p-w)D(p)(1+\theta e) - \psi(e)$, which is strictly concave in $e$. Under a RPM agreement, given the wholesale contract $(w, p)$, the retailer chooses an effort level such that

$$\theta(p - w)D(p) = \psi'(e). \tag{A.27}$$

The manufacturer's profit is given by $(w - c)D(p)(1 + \theta e)$. Under a RPM agreement, the manufacturer's problem can be written as follows:

$$\max_{(p,w,e)} (w - c)D(p)(1 + \theta e)$$

subject to the incentive contraint (A.27) and the retailer's participation constraint

$$(p - w)D(p)(1 + \theta e) - \psi(e) \geq 0. \tag{A.28}$$

Denote by $\mu$ and $\lambda$ the multipliers associated to (A.27) and (A.28) respectively. The Karush-Kühn-Tucker first-order necessary conditions for optimality write as follows

$$(w - c)(1 + \theta e)D'(p) + (\lambda(1 + \theta e) + \mu\theta)\big(D(p) + (p - w)D'(p)\big) = 0, \tag{A.29}$$

$$(1 + \theta e)(1 - \lambda) - \mu\theta = 0, \tag{A.30}$$

$$\theta(w - c)D(p) - \mu\psi''(e) = 0. \tag{A.31}$$

Using (A.29) and (A.30) leads to $D(p) + (p - c)D'(p) = 0$, or $p = p^m$. Two cases must then be considered depending on whether the participation constraint binds at the optimum.

Assume that the participation constraint is binding at the optimum, or $\lambda > 0$. Using (A.27), $(p - w)D(p)(1 + \theta e) = \psi(e)$ rewrites as $\psi'(e) + \theta(e\psi'(e) - \psi(e)) = 0$, or $e = 0$. We also immediately deduce that $w = p^m$ and $\lambda = 1 - \frac{\theta^2\pi^m}{\psi''(0)}$. Therefore, this case arises when $1 > \frac{\theta^2\pi^m}{\psi''(0)}$.

If the participation constraint is not binding at the optimum, then $\lambda = 0$. The promotion effort level is given by $\theta\pi^m = \psi'(e) + \frac{(1+\theta e)\psi''(e)}{\theta}$, which implies that $0 < e < e^m$. The optimal wholesale price is then given by $p^m - w = \frac{\psi'(e)}{\theta D(p^m)}$. The retailer makes a strictly positive profit given by $\frac{\psi'(e)}{\theta} + R(e)$.

COMPARISON OF PROFITS. We now compare profits assuming that $D(p) = a - bp$ and $\psi(e) = \frac{\mu}{2}e^2$. Let $\pi^m = \max_p (p - c)D(p) = \frac{1}{b}\big(\frac{a-bc}{2}\big)^2$. With a RPM agreement:

- If $\mu > \theta^2\pi^m$, then $\Pi_R^{rpm} = 0$ and $\Pi_M^{rpm} = \pi^m$.

- If $\mu \leq \theta^2\pi^m$, then $\Pi_R^{rpm} = \frac{1}{8\mu\theta^2}(\theta^2\pi^m - \mu)(\theta^2\pi^m + 3\mu)$ and $\Pi_M^{rpm} = \frac{(\theta^2\pi^m+\mu)^2}{4\mu\theta^2}$

*No Ban on Below-Cost Pricing.* Using the results derived in the Proof of Proposition 4, we obtain that with rebates and no ban on below-cost pricing, profits are given by

$$\Pi_R^u = 0,$$
$$\Pi_M^u = \frac{2(1-\lambda^u)\pi^m\left(\lambda^u\mu(2-\lambda^u)+\theta^2\pi^m\right)}{\lambda^u\mu(2-\lambda^u)^3},$$

where the multiplier $\lambda^u \in [0, \frac{2}{3})$ satisfies the following condition

$$\frac{\theta^2\pi^m}{\mu} = \frac{2(\lambda^u)^2(2-\lambda^u)}{2-3\lambda^u} \equiv h(\lambda^u).$$

$h(\lambda^u)$ is strictly increasing in $\lambda^u$ over the relevant range. Therefore, $\lambda^u = h^{-1}(\frac{\theta^2\pi^m}{\mu})$ increases with $\frac{\theta^2\pi^m}{\mu}$. Moreover, $h^{-1}(1) \approx .454$.

Assume that $\frac{\theta^2\pi^m}{\mu} < 1$. Simple manipulations lead to

$$\Pi_M^u - \Pi_M^{rpm} \geq 0 \Leftrightarrow \frac{-2+3\lambda^u(2-\lambda^u)}{(2-\lambda^u)(2-3\lambda^u)}\pi^m \geq 0,$$

or $\lambda^u \geq 1 - 1/\sqrt{3} \approx .422$. Therefore, for $\frac{\theta^2\pi^m}{\mu} \leq h(.422) \approx .7698$, we have $\Pi_M^u \leq \Pi_M^{rpm}$ and for $1 > \frac{\theta^2\pi^m}{\mu} \geq .7698$, we have $\Pi_M^u \geq \Pi_M^{rpm}$.

Assume now that $\frac{\theta^2\pi^m}{\mu} \geq 1$ or $\lambda^u \geq .454$. Simple manipulations lead to

$$\Pi_M^u - \Pi_M^{rpm} \geq 0 \Leftrightarrow \frac{-4(\lambda^u)^6+16(\lambda^u)^5-28(\lambda^u)^4+16(\lambda^u)^3-9(\lambda^u)^2+12\lambda^u-4}{8(2-\lambda^u)(\lambda^u)^2(2-3\lambda^u)}\pi^m \geq 0,$$

or $\lambda^u \geq .426$ (numerical approximation). Therefore, we always have $\Pi_M^u > \Pi_M^{rpm}$ in this case.

*Ban on Below-Cost Pricing.* Using again the results derived in the Proof of Proposition 4, we obtain that with rebates and a ban on below-cost pricing, profits are given by the following conditions

$$\Pi_R^b = \frac{\pi^m\left(8\mu(2-\lambda^b)(\lambda^b)^2 - \theta^2\pi^m(2-5\lambda^b)\right)}{8\mu(\lambda^b)^2(2-\lambda^b)^3},$$
$$\Pi_M^b = \frac{(1-\lambda^b)\pi^m\left(2\lambda^b\mu(2-\lambda^b)+\theta^2\pi^m\right)}{\lambda^b\mu(2-\lambda^b)^3},$$

where the multiplier $\lambda^b \in [0, \frac{2}{5})$ satisfies the following condition

$$\frac{\theta^2\pi^m}{\mu} = \frac{4(\lambda^b)^2(2-\lambda^b)}{2-3\lambda^b} = 2h(\lambda^b).$$

Assume that $\frac{\theta^2\pi^m}{\mu} < 1$. Simple manipulations, and using the fact that $\lambda^b \in [0, 2/5)$, lead to

$$\Pi_R^b - \Pi_R^{rpm} = \frac{\pi^m}{2(2-\lambda^b)(2-3\lambda^b)} \geq 0,$$
$$\Pi_M^b - \Pi_M^{rpm} = \left(\frac{2(1-\lambda^b)}{(2-\lambda^b)(2-3\lambda^b)} - 1\right)\pi^m \leq 0.$$

Assume that $\frac{\theta^2\pi^m}{\mu} \geq 1$. Simple manipulations, and using the fact that $\lambda^b \in [0, 2/5)$, lead to

$$\Pi_R^u - \Pi_R^{rpm} = \frac{-16(\lambda^b)^6 + 64(\lambda^b)^5 - 88(\lambda^b)^4 + 64(\lambda^b)^3 + 11(\lambda^b)^2 - 36\lambda^b + 12}{32(2-\lambda^b)(2-3\lambda^b)(\lambda^b)^2}\pi^m \geq 0.$$
$$\Pi_M^b - \Pi_M^{rpm} = -\frac{(1-2\lambda^b)^2(4(\lambda^b)^4 - 12(\lambda^b)^3 + 9(\lambda^b)^2 + 4\lambda^b + 4)}{16(2-\lambda^b)(2-3\lambda^b)(\lambda^b)^2}\pi^m \leq 0.$$

With a ban on below-cost pricing, the manufacturer always prefers a RPM agreement whereas the retailer favors incentive rebates. □

# Online Appendix

Competitive Manufacturers. The optimization problem with competitive manufacturers writes as follows:

$$\max_{(w,z,p,e)} \pi(p,w)(1+\theta e)+\theta ezD(p)-\psi(e) \text{ subject to (7.1), (3.5), (3.6).}$$

Inserting the values of $(w,z)$ obtained from (7.1) into the maximand allows to restate this maximization problem as follows:

$$\max_{(p,e)} \pi(p,c)(1+\theta e)-\psi(e) \text{ subject to (3.5) and (3.6).}$$

Observe that (3.5) and (3.6) define the wholesale price and the rebate in terms of the final retail price and the effort but the former do not enter the maximand. The maximand is thus maximized for the monopoly outcome $(p^m,e^m)$. Inserting into (3.5) and (3.6) then gives the value of the wholesale price and the rebate that implement this outcome, namely $(w^d=c, z^d=0)$. Finally, thanks to Assumption 1 and our assumption on the convexity of $\psi(e)$, the maximand is quasi-concave in $(p,e)$ so that $(p^m,e^m)$ is a global maximum.

Observe that there are actually other equilibrium wholesale contracts that lead to the same retail price, effort level and allocation of surplus. Any pair $(w,z)$ such that (7.1) is binding leads the retailer to perform effort $e^m$ and to charge a price $p^m$. Introducing a small degree of risk-aversion on the retailer's side selects $(w^d=c, z^d=0)$ as the unique optimum.

Quantifying Antitrust Damages in Upstream Collusion Cases. Simple computations lead to the following expressions: $\pi^m=\frac{\delta}{b}(\frac{a-bc}{1+\delta})^{1+\frac{1}{\delta}}$, $D(p^m)=(\frac{a-bc}{1+\delta})^{\frac{1}{\delta}}$, and $e^m=\frac{\theta}{\mu}\frac{\delta}{b}(\frac{a-bc}{1+\delta})^{1+\frac{1}{\delta}}$. When manufacturers collude under a ban on below-cost pricing, using the proof of Proposition 4, we obtain:

$$p^b=\frac{a}{b}-\frac{a-bc}{b(1+\delta)(1+\delta(1-\lambda^b))},$$

$$D(p^b)=\left(\frac{a-bc}{(1+\delta)(1+\delta(1-\lambda^b))}\right)^{\frac{1}{\delta}},$$

$$e^b=\frac{\delta\theta}{2b\mu}\left(\frac{a-bc}{\delta+1}\right)^{\frac{1}{\delta}+1}\frac{1}{\lambda^b}\left(\frac{1}{1+\delta(1-\lambda^b)}\right)^{\frac{1}{\delta}},$$

where $\lambda^b$ solves (A.24).

Then, the damage can be expressed as a function of $\lambda^b$ only:

$$\frac{\Delta}{\pi^m}=1+\frac{4(\lambda^b)^2(1+\delta(1-\lambda^b))^{\delta}-(1+\delta(1-\lambda^b))^{-\delta}}{2(1+\delta-(2+\delta)\lambda^b)},$$

where $\lambda^b$ depends only on the ratio of the demand with and without collusion:

$$(1+\delta(1-\lambda^b))^{\frac{1}{\delta}}=\frac{D(p^m)}{D(p^b)}.$$

This concludes the proof. □

First-Order Approach. We discuss the validity of the first-order approach used in the main analysis. When Assumption 1 holds, the retailer's profit function $\Pi_R(p,e)=\pi(p,w)+\theta e\pi(p,w-z)-\psi(e)$ is strictly concave in $(p,e)$. The first-order conditions (3.2) and (3.3) characterize the retailer's choices of price and promotion effort, assuming that the promotion effort is strictly positive.

A possible deviation for the retailer is to choose $e = 0$ and to set the corresponding price, namely $\overline{p}(w) = \arg\max_p (p - w)D(p)$. Choosing $e = 0$ and $p = \overline{p}(w)$ provides the retailer with a profit worth $\overline{\Pi}_R(w) = (\overline{p}(w) - w)D(\overline{p}(w))$. Observe that $\overline{p}(w)$ is the solution of $h(\overline{p}(w)) = w$ with $h(\overline{p}(w)) = \overline{p}(w) + \frac{D(\overline{p}(w))}{D'(\overline{p}(w))}$. Under our assumptions on $D(\cdot)$, we have $h'(\cdot) > 0$ and thus $(h^{-1})' = \frac{1}{h'} > 0$. Hence, we have $\overline{p}(w) = h^{-1}(w)$ and $\overline{\Pi}_R(w) = \varphi(h^{-1}(w))$.

In the sequel, we show two results. First, for the class of demand functions $D(p) = (a-bp)^{\frac{1}{\delta}}$ with $\delta > 0$ and $a - bp \geq 0$, such a deviation towards no effort and the corresponding price is never profitable for the retailer if $\delta \geq 1$. Second, for an exponential demand function $D(p) = e^{a-bp}$ (which corresponds to the limit case of $D(p) = (a - bp)^{\frac{1}{\delta}}$ when $\delta$ goes to 0), the deviation towards no effort must be taken into account; but this does not change qualitatively our main results.

Consider that demand is given by $D(p) = (a - bp)^{\frac{1}{\delta}}$. It is immediate to show that $\overline{p}(w) = \frac{a\delta + bw}{b(1+\delta)}$, so that $D(\overline{p}(w)) = (\frac{a-bw}{1+\delta})^{\frac{1}{\delta}}$. Using (A.16)-(A.17) and (A.20)-(A.21), we can reconstruct the optimal wholesale price and rebate as functions of the multiplier $\lambda^u$, which we denote by $W^u(\lambda^u)$ and $Z^u(\lambda^u)$. Therefore, at the manufacturer's optimum, the retailer has no incentives to deviate towards no effort and the corresponding price if and only if $\overline{p}(W^u(\lambda^u)) - W^u(\lambda^u) \leq 0$ (or equivalently $D(\overline{p}(W^u(\lambda^u))) \leq 0$), which can be rewritten as follows:

$$\text{(B.1)} \qquad \frac{1}{1 + \delta(1 - \lambda^u)} \left( a - bc - \frac{\delta^2\theta^2(1 - \lambda^u)((\delta + 1)(1 + \delta(1 - \lambda^u)))^{-\frac{1}{\delta}}(a - bc)^{\frac{1}{\delta}+2}}{b(\delta + 1)(\lambda^u)^2\mu} \right) \leq 0.$$

Since the multiplier $\lambda^u$ is given by (A.22), (B.1) can be simplified to:

$$\text{(B.2)} \qquad \frac{1 - \delta(1 - \lambda^u) - 2\lambda^u}{1 + \delta(\delta + 2)(1 - \lambda^u)^2 - 2\lambda^u} \leq 0.$$

The left-hand side term in (B.2) is strictly decreasing in $\lambda^u$. Hence, a sufficient condition for (B.2) to always hold is that this inequality is satisfied for $\lambda^u = 0$, or $\frac{1-\delta}{(\delta+1)^2} \leq 0$, or $\delta \geq 1$.

Notice that (B.2) is never satisfied for $\delta = 0$. Our demand function $D(p) = (a - bp)^{\frac{1}{\delta}}$ is defined for $\delta > 0$ and gives at the limit case $\delta \to 0$ the log-linear/exponential demand. We now consider this case and assume that demand is given by $D(p) = e^{a-bp}$.

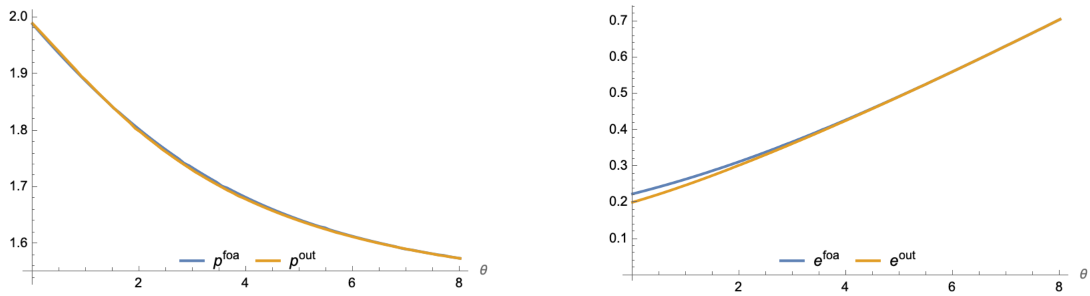
We can then adapt the methodology used in the proof of Propositions 1 and 2 to show that the manufacturer's problem writes as follows:

$$\max_{(p,e)} \left(\pi(p, c) - \varphi(p)\right)(1 + \theta e) \text{ subject to } (1 + \theta e)\varphi(p) - \psi(e) \geq \varphi(\overline{p}(W(p, e))),$$

where $w$ can be expressed as function of the pair $(p, e)$ that the manufacturer wants to implement using (3.5), which leads to $\varphi(\overline{p}(W(p, e))) = \varphi\left(h^{-1}\left(h(p) + \frac{D(p)}{D'(p)}\theta e + \frac{e\psi'(e)}{D(p)}\right)\right)$.

Closed-form solutions of the optimum are not possible. We therefore rely on simulations using the following values of parameters: $a = b = 1$, $c = 0$, $\mu = 15$ and $\theta \in [.1, 8]$. We vary $\theta$ with an increment of .1 to obtain 80 simulations. For each simulation, we determine numerically the optimum using Mathematica.[36] While both the price and effort level are now chosen to limit the retailer's profit if it exerts no effort, those distortions do not appear to change qualitatively our results. Under a ban on below-cost pricing, simulations also show that the possibility to exert no effort and set the corresponding price is never relevant in equilibrium.

[36]The numerical simulations are available on the second author's webpage.




(a) Prices. (b) Effort levels.

Figure B.1: With an exponential demand and a quadratic disutility of effort ($D(p) = e^{1-p}$ and $\psi(e) = \frac{15}{2}e^2$, $c = 0$), comparison of the optimal price (Panel (a)) and effort level (Panel (b)) under the first-order approach ($p^{\text{foa}}$ and $e^{\text{foa}}$) and when the retailer can choose no effort ($p^{\text{out}}$ and $e^{\text{out}}$).