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Alex Bao, Philippe Bontems, Jean-Marie Cardebat and Raphael Chiappini

Toulouse School of Economics

Exporters' behaviour in the face of climate volatility *

Alex Bao† Philippe Bontems‡ Jean-Marie Cardebat§ Raphaël Chiappini¶

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Abstract

This paper investigates how exporters adjust trade margins to demand shocks and climate-induced production volatility. Combining French wine export data (113 PDOs, 49 destinations, 2001–2020) with high-resolution weather records, we estimate theory-consistent structural gravity models using instrumental variables that exploit exogenous variation in extreme temperatures. Production volatility significantly reduces export volumes, while demand shocks shape market allocation. A heterogeneity shows that exporters absorb production shocks by reallocating supply across markets, protecting volumes in core destinations while curtailing shipments to less profitable peripheral ones. A theoretical model of risk-averse heterogeneous firms rationalizes these findings through scale, redeployment, and selection effects driving export reallocation under climate risk.

JEL: F12, F18, Q17, Q56.

Key-words: Climate Change, Cost shocks, Demand shocks, Gravity Model, Heterogeneous Firms, Risk Averse Firms.

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†Univ. Bordeaux, CNRS, INRAE, BxSE, UMR 6060, UMR 1441, F-33600 Pessac, France. Email: alex.bao@u-bordeaux.fr.

‡Toulouse School of Economics, INRAE, University of Toulouse Capitole, France. I acknowledge funding from ANR under grant ANR-17-EURE-0010 (Investissements d'Avenir program) and from the SUDOE VINCI program (SOE3/P2/F0917). Email:philippe.bontems@tse-fr.eu. Corresponding Author.

§Univ. Bordeaux, CNRS, INRAE, BxSE, UMR 6060, UMR 1441, F-33600 Pessac, France. Email: jean-marie.cardebat@u-bordeaux.fr.

¶Univ. Bordeaux, CNRS, INRAE, BxSE, UMR 6060, UMR 1441, F-33600 Pessac, France. Email: raphael.chiappini@u-bordeaux.fr.

1 Introduction

Climate-induced volatility poses significant challenges for firms, particularly through weather shocks such as rising temperatures, altered precipitation patterns, and extreme weather events. These changes can disrupt supply chains, increase resource scarcity, and heighten exposure to natural disasters, necessitating adaptive strategies. Increasing evidence highlights the economic impacts of such shocks, including effects on incomes and economic growth (e.g. Dell et al., 2009, 2012), as well as alterations in trade patterns (e.g. Jones and Olken, 2010; Costinot et al., 2016; Dallmann, 2019). This study investigates, both empirically and theoretically, how exporters when confronted to the uncertainties brought about by climate-induced volatility make export decisions.

Previous studies highlight that the agricultural sector is the most sensitive to weather variations (Jones and Olken, 2010; Dell et al., 2014; Costinot et al., 2016; Zappala, 2024). Wine is an ideal sector to study the impact of weather on international trade. Like all agricultural goods, wine production is highly sensitive to weather conditions (Ashenfelter and Storchmann, 2016). However, wine stands out among agricultural products due to the significant impact of weather on its quality, encapsulated in the concept of vintage, where each year's quality varies due to weather conditions (Van Leeuwen and Darriet, 2016; Jones et al., 2005; Lecocq and Visser, 2006; Ashenfelter, 2008; Ashenfelter and Storchmann, 2010). These dual channels of quantity and quality directly influence wine prices and, consequently, the dynamics of international wine trade. Furthermore, the wine trade can be influenced by strategic decisions made by exporters in response to weather disruptions. In France, wineries produce a diverse range of wines, which are particularly differentiated by quality and geographical production areas, making them likely to be sorted by quality (Crozet et al., 2012; Emlinger and Lamani, 2020). These quality discrepancies among exported products, driven by heterogeneous weather variations, may lead to both quantity and price discrimination strategies (Bastos and Silva, 2010; Martin, 2012; Fontaine et al., 2020). Exporting countries of high-end products, such as France with its fine wines, can adjust their selling prices based on the geographical distance or the attractiveness of the importing market (Manova and Zhang, 2012). Studying the impact of weather on French wine exports is therefore a pertinent choice for exploring the influence of weather on international trade.

Our empirical investigation relies on an original dataset provided by the French Federation of Wine and Spirits Exporters (Fédération des Exportateurs de Vins et Spiritueux de France, FEVS). It allows us to have access to the universe of wine shipments of France between 2001 and 2020. It comprises 113 Protected Denomination of Origin (PDO) from the different wine regions (e.g. Bordeaux, Burgundy, Rhone Valley, etc.) exported to 49 countries during the studied period.[1] We merge these data with high-

[1] In the rest of the paper, we will refer to PDO as appellations.

resolution meteorological information from SAFRAN, available on an 8 km × 8 km grid that covers all of France. We employ a two-step empirical strategy: first, we instrument appellation-level yield volatility with exogenous weather shocks, and second, we estimate gravity models with high-dimensional fixed effects. This IV–gravity approach allows us to identify the causal role of production volatility in shaping exporters' intensive and extensive margins, wine prices, and perceived wine quality.

Our empirical findings can be summarized as follows. We show that production volatility, instrumented by extreme temperature events, significantly reduces export volumes and raises the probability of market exit, while also exerting a negative effect on perceived wine quality.[2] The heterogeneity analysis reveals sharp differences across destinations: in core markets, export volumes remain largely insulated from production volatility, but the likelihood of exit increases, whereas in peripheral markets volatility primarily reduces volumes without systematically driving exit. This pattern stands in contrast with the effects of demand volatility. Excessive fluctuations in wine consumption in destination countries disproportionately reduce exports to core markets, consistent with the evidence reported by De Sousa et al. (2020). Finally, we show that production volatility also depresses export prices overall, with the effect being substantially stronger in core markets than in peripheral markets. This underscores the role of competitive pressure in shaping both the extensive margin of trade and exporters' pricing strategies under climate-induced production risk.

We then propose a theoretical explanation of the mechanisms that may underlie these observations. To this end, we consider monopolistic competition between firms in the spirit of Melitz (2003) and Chaney (2008) and develop a theory where firms possess an early and specific prior on their future total productivity and have to invest into marketing effort to reach consumers on markets of interest, before learning their actual productivity as a production shock occurs afterwards. More precisely, risk averse managers have to select the destination markets through some endogenous fixed cost of marketing like in Arkolakis (2010), before actually knowing their precise production possibilities. The premise is that firm owners are unable to diversify their risk and to hedge risks through financial markets, so that idiosyncratic production and demand risks matter for decisions. This is consistent with Esposito (2022) and De Sousa et al. (2020) who study how risk averse managers react to demand shocks.[3]

In the present model, we build on Esposito (2022) to include idiosyncratic production shocks as well as demand shocks. The presence of production shocks allows to take account the effects of weather uncertainty on production possibilities as well as quality outcomes. Furthermore, the effect of climate change on wine production could be well

[2] Perceived quality at the appellation–destination level is inferred using the approach developed by Khandelwal et al. (2013).

[3] Juvenal and Santos Monteiro (2023) on the contrary assume complete markets to study how aggregate risks impacts the trade equilibrium.

represented through an increase in the volatility of this shock. We show that firms are then subject to hybrid or composite shocks that mix demand and production shocks. Even if demand shocks are uncorrelated as we assume, the presence of production shocks ensures some correlation between composite shocks and makes the different problems of choosing destinations and quantities intertwined. Overall, the volatility of the production shock commands the correlation between composite shocks on profits made on each destination markets.

The model indicates that production risk and demand risk exert opposing effects on trade through their respective impacts on marketing strategies. Specifically, increased volatility in production risk, driven by climate change, prompts firms to focus their marketing efforts on more attractive destination markets. The attractiveness of these markets is rigorously defined by factors such as access costs, national income, and other relevant characteristics. Conversely, heightened demand volatility incentivizes firms to diversify their marketing efforts, distributing them more evenly across various destination countries.

We also derive the gravity equation corresponding to the modeling and show how uncertainty on demand and production cost shock influences bilateral exports. More precisely, the impact of increased production volatility on firm's level investment decisions and aggregate export value at the industry's level can be decomposed into a scale, a redeployment and a selection effect. Firstly, because an increased production volatility makes the world riskier by increasing the correlation between profits made on each market, it reduces the interest of diversification and this leads all firms to reduce their investment level on all markets (scale effect). This also contributes to decrease aggregate export values. Secondly, the redeployment of investments within the portfolio of destination markets has effects on sales on each market that depend on the composition of the optimal portfolio. More precisely, a firm tends to increase (decrease) its investments to reach consumers in a given market if this market is more (less) attractive than the average in its portfolio, the average being understood as weighted by the relative risk of demand. Overall, the impact of the redeployment effect on aggregate export value remains largely an empirical question.

Last, the selection effect reflects the fact when the volatility of production increases then a greater productivity is required to include a given market in one's portfolio, so the number of exporters to that market decreases. This also contributes to lower exports in value terms. Overall, the theoretical findings are consistent with the empirical results.

Related literature. Our paper contributes to several strands of the literature. First, it contributes to the literature on the impact of climate change on trade flows. Most analyses in this field are empirical, focusing either on the impact of temperatures or precipitation on export flows at the country level (Jones and Olken, 2010; Dallmann, 2019; Martínez-Martínez et al., 2023) and at the city level (Li et al., 2015), or on specific natural disasters (Gassebner et al., 2010; Volpe Martincus and Blyde, 2013; Friedt, 2021; Boehm et al., 2019; Freund et al., 2022). The literature has demonstrated that natural disasters affect trade directly through human casualties and the destruction of human capital (Gassebner et al., 2010) and indirectly through the destruction of transportation infrastructure (Gassebner et al., 2010; Volpe Martincus and Blyde, 2013) and through disruptions in Global Value Chains (GVCs) (Boehm et al., 2019; Freund et al., 2022). Nevertheless, there is a limited knowledge concerning the manner in which exporters respond to natural disasters when selecting destination markets and the means by which they differentiate between markets in accordance with their relative attractiveness. The question of destination market selection is of great importance for exporters, as it affects the sustainability and expansion of their operations. The choice of destination markets directly influences a number of key factors, including market access, pricing, distribution channels, and overall competitiveness. This paper addresses this gap both empirically and theoretically.

Second, we add to a literature (De Sousa et al., 2020; Esposito, 2022), considering risk averse exporters facing demand risks and where financial markets are absent so that international trade can be viewed as a tool of diversification.[4] Here, we introduce production risks that impact quantity and quality in addition to demand uncertainty and show how production volatility changes impact the diversification strategy of heterogeneous firms. Given our assumption of independent demand risks, the production risk is the only source of correlation across profits made on destination markets.

Third, this paper is also related to the determinants of trade literature. While it is common to assume that exporters make independent entry decisions for each destination market (e.g. Melitz, 2003; Chaney, 2008), here market entry depends on the portfolio composition and thus the diversification strategy of the firm. This difficult problem is related to the class of combinatorial discrete choice problems as defined by Arkolakis et al. (2023).[5] In this paper, we actually solve a relaxed problem where the ex ante decision in terms of marketing investment is continuous and where the specification (mean-variance preferences, production risk as the only source of correlation) made it possible to solve the model explicitly, without the need to use the squeezing and branching procedures proposed by Arkolakis et al. (2023).

Last, while the agricultural and wine economics literatures have extensively examined the effects of climate on yields, expert ratings, and prices (Ashenfelter et al., 2009), the role of climate as an identifying source of production risk in international trade has received

[4]It has long been recognized that the incompleteness of financial markets has an impact on international trade under production uncertainty (Pomery, 1980; Newbery and Stiglitz, 1984; Helpman and Razin, 2014; Kucheryavyy, 2014).

[5]See also e.g. Antràs et al. (2017), Antràs and De Gortari (2020) and Huppertz (2024) for related studies.

little attention. This study addresses this gap by exploiting high-resolution weather data as instruments for yield volatility, thereby making three contributions to the literature on wine and trade. First, we provide causal evidence that climate-induced volatility is a key determinant of export outcomes. Previous studies only captured weather indirectly through time fixed effects or treat it as part of unobserved "dark trade costs" (Bargain et al., 2023). Our IV design reveals its distinct and systematic impact on both intensive and extensive margins. Second, our theoretical and econometric analyses highlight exporters' strategic behavior in response to production risk, complementing existing work on quality sorting and market prioritization (e.g. Crozet et al. (2012)) by showing that volatility shapes not only quality choices but also quantities and market selection. Third, our empirical framework demonstrates how high-frequency meteorological data can be used in trade analysis, offering a roadmap for future research linking climate variability, product quality, and export strategies.

The rest of the paper is organized as follows. Section 2 presents the data, the empirical methodology and the identification strategy used in this paper. Section 3 displays estimation results. Section 4 presents the theoretical model, while section 5 concludes.

2 Data and empirical strategy

2.1 Data

Trade data. We exploit an original dataset provided by the French Federation of Wine and Spirits Exporters (Fédération des Exportateurs de Vins et Spiritueux de France, FEVS) on shipments of French bottled wines between 2001 and 2020. We focus on bottled wine of 113 appellations[6] exported to 49 countries[7]. These importing markets represent more than 90% of French wine imports over the period. Furthermore, the largest part of wine trade with these countries concern bottled wine, which accounts for 90% of total imports. The database includes a wide variety of wines across the different producing regions and *terroirs*, while distinguishing between the colour of the wines. Consequently, there exists not only a lot of variation in export destination across the different wine appellations, but also a great diversity in terms of quality and prices of exported wines.

Among the 49 importing countries retained in this analysis[8], five represents 51% of the value of French exports in 2020[9]. This subsample of importing countries, namely the U.S., the U.K., Germany, Japan and China represents the "Core" markets, as they have a strategic importance for French wine exporters. Among this subsample, four

[6]Online Appendix OA1 reports the full list of appellations (PDOs) included in the analysis.

[7]We restrict our analysis to importing countries that exhibit strictly positive wine consumption, as such consumption is necessary to accurately compute demand volatility.

[8]See Table A1.

[9]They also represent 55% of the volume of French exports in 2020.

countries are "historical" markets, and one represents the "new dynamic market", namely China[10]. The other subsample is composed of the rest of importers, which seem to be less important from a strategic point of view for French wine exporters. We call this subsample "Peripheral" markets. Note that among the seventh main importing countries, Belgium and Hong Kong play an important role. Nevertheless, these two economies are considered as re-export platforms and that is why, at first, we do not consider them in the "Core" market subsample. Figure A1 in the Appendix displays the total amount of export volumes (a) and values (b) going to "Core " markets and "Peripheral" countries. We can remark that since 2008, there exists an increasing gap between volumes exported to core and peripheral markets. This is mainly due to the rise of Chinese imports of French wines. Figure A1 also seems to illustrate a strategy of quantity-based discrimination between markets. The available quantities are directed towards core markets, particularly China, at the expense of peripheral markets. The year 1998 is particularly indicative of this trend. Export volumes to peripheral countries were markedly low, while exports to core markets remained robust. This trend was particularly evident following the poor harvest of 1997, which was precipitated by adverse weather conditions. The observed export patterns imply a strategic prioritization of key markets when available quantities are constrained. However, an analysis of export values reveals a more nuanced picture. Both core and peripheral markets experienced a notable increase in the value of imports. This suggests that, beyond mere quantity discrimination, there may also be mechanisms of price discrimination and/or quality sorting at play across different markets.

Appellation data vs. firm-level data. It is important to note that the specificity of the French wine industry does not allow for the acquisition of firm-level data. In France, wineries rarely export wine themselves but instead rely on intermediaries (Crozet et al., 2012; Bargain et al., 2023); the only exception being Champagne[11]. However, for the purpose of this study, Champagne data would not have been useful either. The unique characteristic of French 'Maison de Champagne' is that they purchase grapes from different vineyards after the harvest. Thus, linking weather conditions to different exporting firms would have posed a significant challenge.

Weather indicators. Meteorological data are drawn from the SAFRAN database, which reports weather conditions on an 8 km $\times$ 8 km grid covering the entirety of France. Each of the 113 appellations is mapped to a single grid cell; however, multiple appellations may fall within the same cell. The database provides daily observations from 1995 through 2020,

[10]Exported volumes and values to China has begun to strongly increase after 2007. Indeed, in 2007, China only represents 1% of French wine exports in volume, while it accounts for 12%, ten years later.

[11]Firm-level exports of Champagne have been used in Crozet et al. (2012).

including minimum and maximum temperatures as well as cumulative precipitation[12].

We derive two extreme weather indicators to capture conditions that are expected to impact overall production, thereby modifying the composition of export flows (Jones et al., 2005; Roberts et al., 2013; Keane and Neal, 2020), namely the Killing Degree Days (KDD) and the Freezing Degree Days (FDD). KDD captures the cumulative number of hours with temperatures above a critical threshold 35°C, reflecting heat stress that damages grape physiology (Hochberg et al., 2014; Pagay and Collins, 2017), while FDD records the cumulative hours below 0°C, indicating frost exposure that can destroy buds and reduce yields[13]. Figures A2 and A3 depicts the dynamics of the total harmful temperatures (in hours) over time for main French wine regions, while Figures A4 and A5 reports the dynamics of total freezing temperatures (in hours). Initially, we observe heterogeneity between wine regions, with some experiencing fewer extreme temperature events than others. Particularly, Bordeaux and Beaujolais appear to be more susceptible to extreme temperatures compared to Burgundy, Champagne, and the Rhône Valley. The Beaujolais region stands out as the most affected by both harmful and freezing temperatures. Additionally, we identify three major weather shocks recorded in French regions between 1995 and 2020. Notably, the calculation of the indicator for 2003 accurately reflects the occurrence of heatwaves, indicative of the significant drought experienced in France during that year.

Wine consumption. In addition to trade and weather data, we incorporate information on wine consumption in destination countries sourced from the International Organisation of Vine and Wine (OIV). This data enables us to directly measure the consumption expenditure variable R of each destination country, in contrast to De Sousa et al. (2020), who inferred it using production and trade data.

2.2 Identification strategy

Our objective is to discern the causal impact of demand uncertainty and production shocks (yield volatility) on wine exporters.

Demand uncertainty. The identification of the causal effect of demand uncertainty could present a challenge, as reverse causality stemming from trade to demand uncertainty may arise. To tackle this issue, we adopt the identification approach developed in De Sousa et al. (2020). Consequently, our dependent variables (export volumes, probability of exiting the market, unit values, and perceived quality) are measured at the appellation level, while

[12]It is worth noting that additional years are included compared to the export data, as the four preceding years of weather indicators will be used in the analysis to account for the time lag between wine production, harvest, and exports to various destinations.

[13]The presentation of all calculated indicators with their formulas is provided in Table A2 in the Appendix.

the central moments of the consumption expenditure distribution are computed at the importing country level. It is, thus, reasonable to assume that shipments of a particular appellation do not affect the total wine consumption expenditure distribution.

Yield volatility. A central challenge in estimating the impact of production volatility on export volumes is endogeneity. Yield volatility at the appellation–year level is not exogenous: it may be correlated with unobserved supply shocks (for instance disease outbreaks like the mildew, or vineyard management practices) that simultaneously influence exports, or it may suffer from measurement error given its construction over rolling 5-year windows. Both concerns imply that OLS estimates of the elasticity of exports with respect to yield volatility would be biased and inconsistent.

To address this issue, we exploit the fact that weather shocks provide plausibly exogenous variation in the volatility of production. Specifically, we construct an instrument based on the KDD, measured at an 8×8 km grid-cell resolution and aggregated to the appellation level. This variable capture the frequency of extreme heat events between January and September, i.e. over the full growing season. This type of shock is well-documented to reduce yields through vine damage, fruit abortion, and reduced berry set, thereby increasing the volatility of harvest outcomes over time.

Formally, in the first stage we regress the 5-year rolling standard deviation of yields on 5-year aggregates mean of KDD, as follows:

$$Volatility_{kt} = \alpha + \pi_1 Z_{kt-4:t} + \gamma_k + \tau_t + u_{kt} \tag{1}$$

where $Volatility_{kt}$ is the rolling standard deviation (5-years) of yields for appellation k in year t, $Z_{kt-4:t}$ is the 5-year moving average of KDD between January and September, γ_k and τ_t are appellation and year fixed effects, respectively. Standard errors in this first stage are clustered at the grid-cell level to account for the fact that weather shocks are spatially correlated within cells.

In the second stage, to empirically evaluate the impact of both demand uncertainty and production shocks on intensive and extensive margins of trade, and export unit values, we use a theory-consistent estimation of the gravity model of trade Anderson and van Wincoop (2003). Therefore, we estimate structural gravity models at the appellation level, as described in Equation 2:[14]

$$\begin{aligned} y_{jkt} = {} & \mu_1 ln\mathbb{E}_t(R_{jt}) + \mu_2 Higher * ln\left(\mathbb{V}_t(R_{jt}) - \mathbb{V}_t(R_{Ft})\right) + \mu_2 Lower * ln\left(|\mathbb{V}_t(R_{jt} - \mathbb{V}_t(R_{Ft})|\right) \\ & + \mathbb{S}_t(R_{jt}) + \beta \widehat{Volatility}_{kt} + \mu_k + \lambda_j + \tau_t + \epsilon_{kjt} \end{aligned} \tag{2}$$

[14]Equation (2) is estimated using a theory-consistent structural gravity framework à la Anderson and van Wincoop (2003), where appellation, destination, and year fixed effects respectively capture exporter and importer multilateral resistance terms as well as global shocks. Our specification thus corresponds to a structural gravity model augmented with measures of demand and supply uncertainty.

The subscripts k, j and t denote appellation, destination country and year, respectively. We consider as dependent variables (y_{kjt}): (i) the volume of exports (in logarithm) of appellation k, to destination country j, in year t; (ii) a dummy variable that equals one if appellation k, exported in $t-1$ to destination j, exits the market in year t and 0 otherwise.

In Equation 2, $\mathbb{E}_t(R_{jt})$ represents the expected value of wine consumption expenditure in year t, computed as the mean of wine consumption expenditure R over the previous 5 years. This allows us to capture the market size effect on trade. $Higher * \ln\left(\mathbb{V}_t(R_{jt}) - \mathbb{V}_t(R_{Ft})\right)$ represents the excess volatility of consumption expenditure in the destination market compared to the French market F. To compute volatility, we follow De Sousa et al. (2020) and calculate the yearly growth rates of wine consumption over rolling 6-year periods. Then, volatility is simply the standard deviation of these yearly growth rates[15]. $Lower * \ln\left(|\mathbb{V}_t(R_{jt} - \mathbb{V}_t(R_{Ft})|\right)$ is computed in a similar way but represents the lower volatility of the destination market compared to the French market. $\mathbb{S}_t(R_{jt})$ represents the third moment of the consumption expenditure distribution and is measured as the unbiased skewness.

The identifying assumption is that, conditional on this rich set of fixed effects, weather shocks affect exports only through their impact on the volatility of yields, not through any other channel. Since the dependent variable is export volumes rather than quality exported, this assumption is particularly plausible: while extreme weather may influence wine quality, such changes do not mechanically alter the quantity exported, conditional on available production.

Unit values and quality exported. The identification strategy for export quality and unit values differs from that for export volumes. While extreme weather events such as KDD over the entire January–September growing season plausibly affect only production quantities, they are also known to directly alter grape composition during ripening, modifying sugar concentration, acidity, and phenolic content, which are the key determinants of wine quality. Using the same full-season weather shocks as instruments would therefore violate the exclusion restriction in the quality specification, since weather would affect quality both indirectly through production volatility and directly through grape composition.

To isolate the causal impact of yield volatility on export quality (and unit values), we focus the instrument set on pre-ripening weather shocks, specifically FDD measured between January and March[16]. These conditions strongly influence vine survival, bud development, and fruit set, thereby driving variability in yields over time, but they do not directly determine the chemical properties of grapes at harvest. This separation allows us to satisfy the exclusion restriction: pre-ripening weather shocks influence export quality

[15] As a robustness check, we also use a 5-year rolling period.

[16] We do not use KDD here as it is almost always equal to zero during this period of low temperatures.

only through their effect on production volatility, conditional on fixed effects.

Note that in this paper, we rely on the method proposed by Khandelwal et al. (2013) to infer quality. Justification and description of the method is provided in Appendix B. Finally, the definition and sources of all variables are detailed in Table A3, while Table A4 provides the summary statistics.

3 Empirical evidence

This section presents our estimations of the intensive (export volumes) and extensive (probability of exiting) margins, export prices (unit values) and perceived quality.

3.1 Intensive margin of trade

Table 1 presents estimation results for the intensive margin. All estimations are conducted on 113 appellations exported to 49 countries during the 2001-2020 period. The standard errors are clustered at the destination-appellation level. Column (1), (3) and (5) are estimated using the two-stage least square estimator (2SLS), while column (2) and (4) rely on the OLS. In columns (2) and (4), we include appellation-year fixed effects to control for all production shocks on exports, while columns (3) and (5) display estimation results including destination-year fixed effects to control for all demand shocks and more rigorously for the multilateral resistance and reduce the omitted variable bias. Note that results of the first-stage estimation linking extreme weather temperatures and volatility of production is displayed in Table D1 in the Appendix. The results confirm that extreme temperature events are strong predictors of yield volatility. The coefficient on average KDD ranges from 0.0037 to 0.0044 and is highly significant at the 1% level, indicating that an additional hour of extreme heat exposure increases the predicted volatility of wine yields by about 0.004 units. Given a mean volatility of 0.213 and a standard deviation of 0.201, this effect corresponds to roughly a 1.8–2.0% increase in yield volatility per additional hour of extreme heat. Similarly, the coefficient on average FDD during the pre-ripening season is 0.00098, also significant at the 1% level, implying that more frequent or intense frost events between January and March heighten production instability. These results are consistent with agronomic evidence that both heat stress and frost damage contribute to greater variability in grape yields. The precision of the estimates indicate that the instruments are both statistically strong and economically meaningful, validating their use in the second-stage IV estimations. It is also confirmed by the different F-statistics reported in Tables 1 to 4.

Demand factors. The results of columns (1) and (2) confirm the significant positive impact of the first moment of wine consumption expenditure distribution on export volumes. They also indicate that excess volatility of wine consumption in the destination country

Table 1: Demand uncertainty, production shocks and the intensive margin

Dependent variable:	Export volumes: $\ln(y_{jkt})$				
	(1)	(2)	(3)	(4)	(5)
$Ln\ Cons.\ Expenditure_{jt-1}$	0.346***	0.350***		0.353***	
	(0.0397)	(0.0354)		(0.0355)	
$Higher * Ln\ Exp.\ Volatility_{jt}$	-0.0135**	-0.0103**			
	(0.00599)	(0.00518)			
$Lower * Exp.\ Volatility_{jt}$	0.0336***	0.0346***		0.0334***	
	(0.00712)	(0.00615)		(0.00609)	
$Cons.Expenditure\ Skewness_{jt}$	0.00346	0.00355		0.00380	
	(0.00570)	(0.00490)		(0.00490)	
$\widehat{Volatility}_{kt}$	-2.301***		-2.382***		
	(0.346)		(0.317)		
$Core * Higher * Ln\ Exp.\ Volatility_{jt}$				-0.0268***	
				(0.00761)	
$NoCore * Higher * Ln\ Exp.\ Volatility_{jt}$				-0.00707	
				(0.00542)	
$Core * \widehat{Volatility}_{kt}$					0.231
					(1.354)
$NoCore * \widehat{Volatility}_{kt}$					-2.807***
					(0.395)
F-test of excluded instruments	813.514***		794.339***		245.166***
Kleibergen-Paap rk LM statistic	583.159***		574.682***		272.210***
Observations	66,289	66,289	66,287	66,289	66,287
Destination FE	YES	YES	NO	YES	NO
Appellation FE	YES	NO	YES	NO	YES
Year FE	YES	YES	YES	YES	YES
Destination-year FE	NO	NO	YES	NO	YES
Appellation-year FE	NO	YES	NO	YES	NO

Note: Dependent variable is the logarithm of exported volumes.
Robust standard errors, clustered at destination-appellation level, in parentheses.
Regressions include a constant which is not reported in the Table.
Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

compared to France negatively affects wine exports to this economy. Thus, when the risk in the destination market is higher than that in the domestic market, wine exporters appear to react by reallocating exports to other markets. This confirms the previous findings of De Sousa et al. (2020) within a specific sector. Conversely, when the volatility of consumption expenditure is lower in a destination country than in France, exporters tend to favor exports to this specific economy. Nevertheless, the impact of the third moment (skewness) is not significant in our specifications.

Production volatility. Columns (1) and (3) underscore that yield volatility is a pivotal factor driving wine trade. The estimated coefficient of −2.282 indicates that yield volatility significantly reduces export volumes. A one standard deviation increase in volatility (0.201)[17] lowers export volumes by about 0.46 log points, which corresponds to a decline of approximately 37%. This effect is economically large: appellations facing greater production instability export substantially less wine, underscoring the importance of stable yields for sustaining international market presence.

[17]The standard deviation is 0.201 when restricting the sample to strictly positive export volumes as in Table 1, and 0.225 for the full sample, as shown in Table A4.

Core vs. peripheral markets. In column (4), we investigate whether the negative impact of excess volatility in consumption expenditure on export volumes varies regarding market potential, as in De Sousa et al. (2020). In column (5), we explore the heterogeneous impact of yield volatility on exports regarding the importing markets. To examine these phenomena, we create a dummy variable capturing the core importing markets, representing the main importing countries of French wines in 2020[18]. Initially, we exclude Belgium and Hong Kong from core markets as they are considered re-export platforms[19]. We then interact this dummy variable with the variable capturing excess volatility in the destination market (column 4) and with the yield production volatility variable (column 5).

First, in column (4), we observe that the impact of excess volatility in consumption expenditure is more pronounced for core markets than for peripheral ones. Higher expenditure uncertainty tends to attenuate the positive impact of market potential. Thus, the greater the market potential in a destination market, the higher the exports, and consequently, the higher the risk at the margin. This corroborates the findings of De Sousa et al. (2020). Second, the results in column (5) provide evidence that in peripheral destinations, yield volatility has a large and statistically significant negative effect: a one standard deviation increase in volatility reduces exports by approximately 43%. By contrast, exports to core markets are unaffected by volatility, with the interaction term close to zero and statistically insignificant. These results suggest that producers prioritize stable relationships with core markets during periods of production instability, while peripheral markets absorb the adjustment.

3.2 Extensive margin of trade

We define the extensive margin as the probability that appellation k exported destination j in year $t-1$, exits the market in year t. Then, we analyze the impact of demand and production risks on the likelihood that a given appellation exits a given destination country employing a linear probability model (LPM). This modeling approach circumvents the incidental parameter concern inherent in probit or logit models when incorporating fixed effects. Additionally, the coefficients derived from the LPM offer straightforward interpretability.

Table 2 presents the summary of estimation results. Mean consumption expenditure significantly decreases the likelihood of an appellation exiting a given destination j, as indicated in columns (1), (2), and (4). As for the intensive margin, the skewness of the change in consumption expenditure is not significant.

Lower volatility in the destination market compared to France markedly reduces the

[18] The following five importing countries are considered here: China, Germany, Japan, the United Kingdom, and the United States.

[19] In Table D8 in the Appendix, we test the sensitivity of our results to the inclusion of these two countries in core markets.

Table 2: Demand uncertainty, production shocks and the extensive margin

Dependent variable:	Probability of exiting: $Prob(y_{jkt} = 0 \mid y_{jkt-1} = 1)$				
	(1)	(2)	(3)	(4)	(5)
$Ln\ Cons.\ Expenditure_{jt-1}$	-0.00867**	-0.00853***		-0.00845***	
	(0.00342)	(0.00327)		(0.00327)	
$Higher * Ln\ Exp.\ Volatility_{jt}$	-0.000658	-0.000656			
	(0.000628)	(0.000604)			
$Lower * Exp.\ Volatility_{jt}$	-0.00159**	-0.00156**		-0.00162**	
	(0.000655)	(0.000633)		(0.000637)	
$Cons.Expenditure\ Skewness_{jt}$	0.000198	0.000157		0.000171	
	(0.000670)	(0.000646)		(0.000646)	
$\widehat{Volatility}_{kt}$	0.0535*		0.0530*		
	(0.0314)		(0.0313)		
$Core * Higher * Ln\ Exp.\ Volatility_{jt}$				-0.00155**	
				(0.000688)	
$NoCore * Higher * Ln\ Exp.\ Volatility_{jt}$				-0.000512	
				(0.000629)	
$Core * \widehat{Volatility}_{kt}$					0.349***
					(0.0688)
$NoCore * \widehat{Volatility}_{kt}$					0.0164
					(0.0340)
F-test of excluded instruments	1465.749***		1452.370***		39.174***
Kleibergen-Paap rk LM statistic	946.613***		946.309***		52.081***
Observations	102,401	102,401	102,401	102,401	102,401
Destination FE	YES	YES	NO	YES	NO
Appellation FE	YES	NO	YES	NO	YES
Year FE	YES	YES	YES	YES	YES
Destination-year FE	NO	NO	YES	NO	YES
Appellation-year FE	NO	YES	NO	YES	NO

Note: Dependent variable is the probability of exiting a given market.
Robust standard errors, clustered at destination-appellation level, in parentheses.
Regressions include a constant which is not reported in the Table.
Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

probability of exiting a destination. Our results also suggest that production volatility significantly increases the risk of market exit. Indeed, we find that a one-unit increase in volatility raises the probability of exiting a destination by 53 percentage points. More economically relevant, a one–standard deviation increase in volatility (0.225) increases the probability of exit by about 12 percentage points. This effect is substantial, suggesting that appellations with unstable yields are considerably more likely to withdraw from foreign markets, highlighting the importance of production stability for sustaining the extensive margin of trade. Furthermore, the results show that production volatility has heterogeneous effects on the probability of market exit across destinations. In core markets, a one–standard deviation increase in volatility raises the probability of exit by about 7 percentage points, while the effect in peripheral markets is small and statistically insignificant. This pattern can be rationalized by differences in competitive pressure: core markets are highly attractive but also more contested, so instability in supply undermines an appellation's reliability and increases the likelihood of being displaced by alternative suppliers. By contrast, peripheral markets are less competitive and already characterized by sporadic trade relationships, meaning that volatility primarily affects export volumes when trade continues but does not systematically translate into higher exit probabilities.

3.3 Export prices

The estimations related to export prices are reported in Table 3. Columns (1), (3), and (5) use FDD during the pre-ripening season (January–March) as instrument. The results for the first and third moments of wine consumption expenditure are consistent with De Sousa et al. (2020). In line with their findings, we also show that higher consumption volatility in destination markets raises export prices, particularly in peripheral markets. This asymmetry reflects differences in competitive pressure: core markets are highly contested and constrained by reputational concerns, which limit exporters' pricing flexibility, whereas peripheral markets are less competitive, allowing producers to adjust prices more freely in response to volatile demand.

Table 3: Demand uncertainty, weather shocks and export prices

Dependent variable:	Export prices: $Ln(p_{jkrt})$				
	(1)	(2)	(3)	(4)	(5)
$Ln\ Cons.\ Expenditure_{jt-1}$	-0.0148	-0.0181		-0.0182	
	(0.0168)	(0.0141)		(0.0141)	
$Higher * Ln\ Exp.\ Volatility_{jt}$	0.00274	0.00410**			
	(0.00248)	(0.00194)			
$Lower * Ln\ Exp.\ Volatility_{jt}$	0.000377	0.000789		0.000682	
	(0.00256)	(0.00203)		(0.00204)	
$Cons.Expenditure\ Skewness_{jt}$	-0.00359	-0.00347*		-0.00344*	
	(0.00238)	(0.00189)		(0.00189)	
$\widehat{Volatility}_{kt}$	-1.213***		-1.147***		
	(0.310)		(0.301)		
$Core * Higher * Ln\ Exp.\ Volatility_{jt}$				0.00269	
				(0.00248)	
$NoCore * Higher * Ln\ Exp.\ Volatility_{jt}$				0.00437**	
				(0.00199)	
$Core * \widehat{Volatility}_{kt}$					-1.316***
					(0.285)
$NoCore * \widehat{Volatility}_{kt}$					-0.714***
					(0.138)
F-test of excluded instruments	113.235***		114.329***		245.166***
Kleibergen-Paap rk LM statistic	110.939***		113.369***		272.210***
Observations	66,084	66,084	66,287	66,084	66,287
Destination FE	YES	YES	NO	YES	NO
Appellation FE	YES	NO	YES	NO	YES
Year FE	YES	YES	YES	YES	YES
Destination-year FE	NO	NO	YES	NO	YES
Appellation-year FE	NO	YES	NO	YES	NO

Note: Dependent variable is the logarithm of unit values.
Robust standard errors, clustered at destination-appellation level, in parentheses.
Regressions include a constant which is not reported in the Table.
Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Regarding the influence of production shocks, our results mirror those obtained for the intensive margin. Yield volatility exerts substantial downward pressure on export prices, with the effect being more pronounced in core markets than in peripheral ones. At first glance, this may appear counterintuitive, since exported volumes to core markets remain stable following extreme weather shocks (see Table 1). Two mechanisms may explain this result. First, consumers in core markets are more discerning and sensitive to quality differentials; as adverse weather conditions degrade perceived quality, demand in these markets declines more sharply, leading to stronger price reductions. Second, pricing-to-market behavior may also play a role: exporters may partially absorb production shocks by lowering markups to preserve competitiveness in highly contested destinations. The results reported in Table 4 on perceived quality are consistent with the latter interpretation, suggesting that strategic pricing contribute to the observed price differentials between core and peripheral markets.

3.4 Perceived quality

It is pertinent to underscore that we estimate Equation 2 without incorporating demand variables for perceived quality. Notably, in the methodology advanced by Khandelwal et al. (2013), quality is derived from an equation (referred to as Equation B1) that encompasses destination-year fixed effects, thereby directly capturing demand components[20]. Table 4 reports estimation results for the impact of production shocks on perceived quality.

Table 4: Production shocks and inferred quality

Dependent variable:	Inferred quality: $\widehat{\lambda}_{jkrt}$	
	(1)	(2)
$\widehat{Volatility}_{kt}$	-0.841**	
	(0.377)	
$Core * \widehat{Volatility}_{kt}$		-0.337
		(1.101)
$NoCore * \widehat{Volatility}_{kt}$		-0.901**
		(0.401)
F-test of excluded instruments	114.329***	4.753***
Kleibergen-Paap rk LM statistic	110.939***	10.203***
Observations	66,287	66,287
Appellation FE	YES	YES
Destination-Year FE	YES	YES

Note: Dependent variable is the inferred quality using the method of Khandelwal et al. (2013).
Robust standard errors, clustered at destination-appellation level, in parentheses.
Regressions include a constant which is not reported in the Table.
Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

The results highlight that yield volatility is a key determinant of perceived wine quality. Production shocks caused by extreme weather events significantly deteriorate the quality of wines exported abroad. Moreover, the adverse effect of production volatility on perceived quality is stronger in peripheral markets. This asymmetry suggests that producers engage in quality discrimination across destinations: when faced with production instability, they allocate their best-quality wines to core markets, which are more competitive and populated by consumers who can better assess quality differences.[21]

[20]For further explanations, see Appendix B.

[21]Core markets are characterized by the early emergence of consumer-oriented wine publications such as Vine and Decanter (both founded in 1975 in the U.K.) or Wine Spectator (1976 in the U.S.). For further details on the evolution of wine quality evaluation, see Dubois (2021).

3.5 Robustness checks

This section examines the robustness of the aforementioned results. Initially, sensitivity tests were conducted concerning the findings pertaining to demand uncertainty. Specifically, two tests were undertaken: (i) Estimations excluding skewness, and (ii) estimations employing alternative measures for expenditure moments based on log differences. The results, presented in Tables D2 and D3 in the Appendix, confirm the robustness of our primary conclusions regarding the influence of excess volatility on both intensive and extensive margins.

We next conduct a series of robustness checks related to production shocks. First, we perform sensitivity analyses to assess whether the results are affected by changes in the definition of extreme temperature events. Specifically, we recompute the KDD index using an alternative temperature threshold of 36°C instead of 35°C, and we apply the nonlinear approach proposed by Schlenker and Roberts (2009)[22]. Second, we construct our instruments, the average KDD and the average FDD, using four-year moving averages that exclude the current year, to verify that our findings are not driven by contemporaneous weather conditions.

Tables D5 and D6 confirm that adopting a different temperature threshold or an alternative method to compute the KDD does not alter our results. Furthermore, Table D4 shows that excluding the current year when constructing the four-year moving average for the instruments does not change our previous findings.

Lastly, specific sensitivity tests were conducted by (i) estimating effects using the value of exports rather than volumes, (ii) adjusting the scope of the core market variable to include re-export platform countries such as Belgium and Hong Kong, and (iii) using an alternative value for the elasticity of substitution in the computation of inferred quality[23]. Table D7 demonstrates that substituting export values for volumes does not affect our findings, while Table D8 indicates that our results remain robust even with the inclusion of re-export platforms to core markets. Finally, Table D9 reveals that altering the elasticity of substitution does not alter our primary results.

4 A theoretical analysis

This section presents the principal assumptions and notations of the model, as outlined in Section 4.1, and establishes a preliminary analysis of the optimal decisions of each firm in terms of marketing investments and prices, as detailed in Section 4.2. Subsequently, we present the optimal investment rule for a given portfolio as a function of the demand and

[22]See Appendix C for a detailed description of this methodology.

[23]We adopt $\sigma = 3.085$ as per Emlinger and Lamani (2020). This value corresponds to the elasticity estimate associated with spirits produced by distilling grape wine or marc, as provided by Kee et al. (2008).

production risks' characteristics, and furthermore determine the optimal portfolio as a function of productivity (Section 4.3). Finally, in Section 4.4, we derive the implications of climate-induced volatility on the trade equilibrium.

4.1 Assumptions and notations

Preferences and demand risk. Let us consider N countries that produce and trade wines and where an origin country is indexed by i and a destination country by j. Let us also denote $\mathcal{N} = \{1, ..., N\}$ as the index set of all countries. In each country j, there is a mass $\tilde{L}_j$ of (immobile) workers and M_j of winery owners who derive utility from consuming a continuum of differentiated varieties, indexed by ω. Thus the total mass of consumers is $L_j = \tilde{L}_j + M_j$. Preferences for an agent indexed by ι, whether it is a worker or a winery owner, for the differentiated good are given by a CES function:

$$u_j = \left(\sum_i \int_{\omega \in \Omega_{ij}} \alpha_j^{\frac{1}{\sigma}}(\omega) \left[\eta_i(\omega) q_{ij}(\omega, \iota) \right]^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}.$$

where Ω_{ij} is the set of varieties from origin country i available on market j and $q_{ij}(\omega, \iota)$ is the quantity of variety ω consumed by agent ι. Moreover, α_j is a firm-specific and exogenous demand shock in market j, whereas η_i is a firm-specific and exogenous production shock whose natural interpretation is in terms of the quality of the wine.[24] A high quality wine could thus be represented here by a high value of η_i. Furthermore, the elasticity of substitution $\sigma > 1$ measures the intensity of horizontal differentiation in the destination market. The budget constraint is

$$\sum_i \int_{\omega \in \Omega_{ij}} p_{ij}(\omega) q_{ij}(\omega, \iota) d\omega \leq y_j(\iota)$$

where p_{ij} is the price and y_j is the income (and expenditure) of the agent ι. Workers earn the same non stochastic wage w_j by working inelastically for the winery owners. Winery owners get their income from the profits they obtain on the markets.

The demand risk α_j can be interpreted as a shock to tastes or to regulation, and is independent from the quality shock η_i. Denoting $\alpha(\omega)$ as the vector of shocks on all markets, we assume like Esposito (2022) that demand shocks are drawn independently across varieties from a multivariate distribution characterized by a vector of means $\overline{\alpha}$ and a variance-covariance matrix.

Assumption 1. *The vector $\alpha(\omega)$ is i.i.d. accross ω with $\overline{\alpha}$ denoting the vector of means and where the variance-covariance matrix is assumed to be diagonal.*

To focus on production risk as the source of correlation between market outcomes, we assume like De Sousa et al. (2020) and unlike Esposito (2022) that the variance-covariance

[24] More precisely, η_i could be interpreted as a function mapping quality into a quantity equivalent as in Crozet et al. (2012).

matrix is diagonal in that $\mathbb{C}ov(\alpha_j, \alpha_k) = 0$ for all $j \neq k$. We denote thus σ^2_α the vector of variances. The distribution of the quality shock $\eta_i(\omega)$ will precised later.

Supply side and production risk. As is usual in the literature (Melitz (2003)), labor is the only factor of production and is inelastically supplied in a competitive market in each country. Entrepreneurs are the only owners and managers of their winery and produce a unique variety using labor, with a productivity φ drawn from a distribution $G_i(\varphi)$ on the set $\Phi_i = \left[\underline{\varphi}_i, \infty\right)$ in origin country i. Importantly, φ is drawn independently from other firms and demand shocks and also from production shocks. Since each firm with a type φ produces a unique variety ω, we identify a variety with φ. Simultaneously with the quality shock $\eta_i(\varphi)$, another exogenous and firm-specific production shock occurs, after marketing and distribution investments in destination markets and before production, affecting the marginal cost of production. Let us denote it by $\theta_i(\varphi)$ and its distribution will also be specified later.[25]

Assumption 2. *The quality shock $\eta_i(\varphi)$ and the cost shock $\theta_i(\varphi)$ are drawn independently from other firms, from productivity and from demand shocks.*

Denoting $\pi_i(\varphi) = \sum_j \pi_{ij}(\varphi)$ the net profit of a firm that produces in country i, the winery owner maximises its indirect utility in real income:

$$\max V_i = \mathbb{E}\left(\frac{\pi_i(\varphi)}{P_i}\right) - \frac{\gamma}{2}\mathbb{V}\left(\frac{\pi_i(\varphi)}{P_i}\right)$$

which follows a mean-variance specification and where γ is the degree of risk aversion.[26] P_i denotes the Dixit-Stiglitz price index and its expression is given below. The assumption of risk averse managers appears recently in the international trade literature (Esposito (2022), De Sousa et al. (2020), Juvenal and Santos Monteiro (2023)). There is empirical evidence that managers are risk averse and care about demand and production shocks. This is particularly important for wineries where the cash-flow volatility can be a source of financial distress and where owner's wealth is highly tied to the value of the winery, exposing them to firm-specific risks that are difficult to diversify.

As in Esposito (2022), production takes place in two stages. Once productivity is known, but before demand and production shocks are known, firms choose destination markets and invest into marketing and distribution activities like in Arkolakis (2010). These decisions make it possible to reach a certain proportion of consumers in each market, depending on the efforts made. These decisions are assumed to be irreversible and can no longer be changed once the demand and production shocks have been drawn. Firms can

[25] A possible interpretation of the cost shock is that it is related to the quality shock. A larger quality shock would then be the source of a larger marginal cost like in Crozet et al. (2012).

[26] As in Esposito (2022), De Sousa et al. (2020) or Ingersoll (1987), the mean-variance specification can be derived from a second-order Taylor approximation of the expectation of a CARA utility in real income.

only adjust the quantity produced or, equivalently, the price to adapt to the particular conditions of production and demand on the destination markets. This modeling is a short-cut for a more complex dynamic model of investments over time (see Alessandria et al. (2021) for an excellent discussion of these issues). As it will be clear below, the model is tractable enough to study how a larger volatility in the production shock, due to e.g. climate change, influences equilibrium decisions of winery owners with respect to their marketing and distribution activities.

Let us denote $n_{ij}(\varphi) \in (0, 1)$ the marketing effort on market j. It denotes the fraction of consumers that can be reached on market j through some costly ads. If $n_{ij}(\varphi) = 0$ then market j is not served by the firm. Assuming that ads are sent independently accross firms and destinations and denoting Y_j as the income devoted in market j to consumption (which originates from the wages of workers and the profits of winery owners), the aggregate demand $q_{ij}(\varphi)$ for a given variety φ depends negatively on its price p_{ij} and positively on n_{ij}:

$$q_{ij}(\varphi) = \alpha_j(\varphi)\eta_i^{\sigma-1}(\varphi)p_{ij}^{-\sigma}(\varphi)A_j n_{ij}(\varphi) \tag{3}$$

as well as on destination market characteristics summarized by the demand shifter $A_j = P_j^{\sigma-1}Y_j$ where P_j is the Dixit-Stiglitz price index given by:

$$P_j = \left(\sum_i M_i \int_0^\infty n_{ij}(\varphi)\mathbb{E}\left[\alpha_j(\varphi)\eta_i^{\sigma-1}(\varphi)p_{ij}^{1-\sigma}(\varphi)\right] dG_i(\varphi) \right)^{\frac{1}{1-\sigma}}, \tag{4}$$

which measures the intensity of competition on market j. Furthermore, the aggregate income Y_j in (3) is the sum of labor wages for workers and the sum of profits in the destination country:

$$Y_j = w_j\tilde{L}_j + \Pi_j.$$

Each firm may produce only one variety under constant return to scale, using labor. The expenditures in terms of labor from the origin country needed to produce $q_{ij}(\varphi)$ is:

$$w_i l_{ij}(\varphi) = \theta_i(\varphi)\frac{w_i\tau_{ij}}{\varphi}q_{ij}(\varphi) \tag{5}$$

where l_{ij} is the quantity of labor, $\tau_{ij} \geq 1$ is the variable trade cost, w_i is the price of labor that prevails in country i and θ_i a production shock. There is also an endogenous trade and marketing cost that writes:

$$f_{ij}(\varphi) = w_j f_j L_j n_{ij}(\varphi) \geq 0 \tag{6}$$

where w_j is the labor price that prevails in the destination country and $f_j > 0$ is a parameter. This fixed cost is proportional to the effort n_{ij} put to reach consumers in the destination country.

The timing of decisions is as follows. Productivity is drawn according to $G_i(\varphi)$. The winery owner first decides how much marketing effort $n_{ij} \in (0, 1)$ to deploy on each destination market. Then, each winery owner learns its production and demand shocks and decides whether to stay on each destination market and, if he stays, he chooses the price of the variety.

Formally, the first stage problem consists of choosing n_{ij} to maximize:

$$\max_{\{n_{ij}\}} \sum_j \mathbb{E}\left(\frac{\pi_{ij}(\varphi)}{P_i}\right) - \frac{\gamma}{2}\sum_j\sum_k \mathbb{C}ov\left(\frac{\pi_{ij}(\varphi)}{P_i}; \frac{\pi_{ik}(\varphi)}{P_i}\right) \quad (7)$$
$$\text{s.t. } 0 \leq n_{ij} \leq 1$$

where $\pi_{ij}(\varphi)$ is the net profit obtained from the destination market j:

$$\pi_{ij}(\varphi) = p_{ij}(\varphi)q_{ij}(\varphi) - \theta_i(\varphi)\frac{w_i\tau_{ij}}{\varphi}q_{ij}(\varphi) - f_{ij}, \quad (8)$$

with $q_{ij}(\varphi)$ given by (3) and f_{ij} given by (6). The second stage is to choose $p_{ij}(\varphi)$ that maximizes (8) given n_{ij} determined as a solution of maximization problem (7).

Contrary to Esposito (2022), the destination-variety specific shocks on demand are not correlated between countries (Assumption 1). As we will explain in the following analysis, *in the absence of production shocks*, this would lead winery owners to seek maximum diversification by investing in all profitable markets in expectation, the set of these profitable markets in expectation depending on the productivity of each of them. The problem of choosing n_{ij} on market j is then separable from choosing n_{ik} on some other market k. However, in the more realistic situation where production shocks occur, we will see that the presence of origin-variety specific production shocks is sufficient to make all the risk-averse manager's export decisions interdependent, both in terms of extensive margin (where to export?) and intensive margin (how much to invest in marketing?).

4.2 Preliminary analysis

In the rest of the paper, we take a partial equilibrium perspective by taking the price indexes, the national incomes and the wage rates in both countries as fixed.[27] We will also concentrate on the equilibrium outcomes at one particular origin country, say i, as we can deduce straightforwardly the equilibrium outcomes in any other country.

Once demand and production shocks are drawn, it is straightforward to show that the optimal price for any producer is given by:

$$p_{ij} = \frac{\sigma}{\sigma-1}\theta_i(\varphi)\frac{w_i\tau_{ij}}{\varphi},$$

that is a constant mark-up over marginal cost, thanks to the CES assumption. This allows to rewrite the profit given by (8) as follows:

$$\pi_{ij}(\varphi) = \alpha_j(\varphi)\beta_i(\varphi)n_{ij}\left(\frac{\tau_{ij}}{\varphi}\right)^{1-\sigma}\frac{A_j}{\delta_i} - f_{ij}. \quad (9)$$

[27]For the sake of completeness, we describe the general trade equilibrium corresponding to the framework developed in Online Appendix OB.

where δ_i is a rescaling of the wage w_i :

$$\delta_i = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}w_i\right)^{\sigma-1},$$

and where we denote

$$\beta_i(\varphi) = \left(\frac{\theta_i(\varphi)}{\eta_i(\varphi)}\right)^{1-\sigma}$$

as the production shock that results from the quality and cost shocks (also appropriately rescaled using σ). Hence, a high quality can compensate at least partially a large marginal cost from the profit viewpoint. Let us denote the mean of β_i by $\bar{\beta}_i$ and its variance by $\mathbb{V}(\beta_i)$. This change of variable reveals that profit (9) is a function of an hybrid or composite shock denoted $\varepsilon_{ij}(\varphi) \equiv \alpha_j(\varphi)\beta_i(\varphi)$, made of demand and production shocks that are independent. The hybrid shock ε_{ij} is distributed with law such that vector of means is $\bar{\varepsilon}_i = (\bar{\varepsilon}_{i1}, ..., \bar{\varepsilon}_{ij}, ..., \bar{\varepsilon}_{iN})$ and a matrix of variance covariance Σ_i with element $\Sigma_{i,jk} = \mathbb{C}ov\left(\varepsilon_{ij}, \varepsilon_{ik}\right)$. Note that given our independence assumptions, we have:

$$\bar{\varepsilon}_{ij} = \bar{\alpha}_j\bar{\beta}_i,$$

and

$$\begin{aligned}\mathbb{C}ov\left(\varepsilon_{ij}, \varepsilon_{ik}\right) &= \mathbb{C}ov\left(\alpha_j\beta_i, \alpha_k\beta_i\right) = \mathbb{E}\beta_i^2\alpha_j\alpha_k - \mathbb{E}\beta_i\alpha_j\mathbb{E}\beta_i\alpha_k \\ &= \bar{\alpha}_j\bar{\alpha}_k\mathbb{V}(\beta_i),\end{aligned}$$

and finally

$$\mathbb{V}(\varepsilon_{ij}) = \mathbb{E}\beta_i^2\alpha_j^2 - \left(\bar{\beta}_i\bar{\alpha}_j\right)^2 = \left(\mathbb{V}(\beta_i) + \bar{\beta}_i^2\right)\mathbb{V}(\alpha_j) + \mathbb{V}(\beta_i)\bar{\alpha}_j^2.$$

From (9), we get (dropping the dependence on φ for simplicity):

$$\mathbb{E}\left(\frac{\pi_{ij}(\varphi)}{P_i}\right) = \bar{\varepsilon}_{ij}n_{ij}r_{ij} - \frac{f_{ij}}{P_i} \qquad (10)$$

where we denote r_{ij} as the variable profit on market j gross of shocks and per unit of marketing effort n_{ij} :

$$r_{ij} = \left(\frac{\tau_{ij}}{\varphi}\right)^{1-\sigma}\frac{A_j}{\delta_i P_i}.$$

Also the term in covariance yields:

$$\mathbb{C}ov\left(\frac{\pi_{ij}(\varphi)}{P_i}; \frac{\pi_{ik}(\varphi)}{P_i}\right) = n_{ij}r_{ij}n_{ik}r_{ik}\mathbb{C}ov\left(\varepsilon_{ij}, \varepsilon_{ik}\right).$$

Let us posit the Lagrangean corresponding to the problem (7):

$$\begin{aligned}\mathcal{L} &= \sum_j \mathbb{E}\left(\frac{\pi_{ij}(\varphi)}{P_i}\right) - \frac{\gamma}{2}\sum_j\sum_k \mathbb{C}ov\left(\frac{\pi_{ij}(\varphi)}{P_i}; \frac{\pi_{ik}(\varphi)}{P_i}\right) - \sum_j \bar{\nu}_{ij}(n_{ij}-1) + \sum_j \underline{\nu}_{ij}n_{ij} \\ &= \sum_j\left(\bar{\varepsilon}_{ij}n_{ij}r_{ij} - \frac{n_{ij}w_jf_jL_j}{P_i}\right) - \frac{\gamma}{2}\sum_j\sum_k n_{ij}r_{ij}n_{ik}r_{ik}\mathbb{C}ov\left(\varepsilon_{ij}, \varepsilon_{ik}\right) - \sum_j \bar{\nu}_{ij}(n_{ij}-1) + \sum_j \underline{\nu}_{ij}n_{ij}\end{aligned}$$

where $\bar{\nu}_{ij}$ and $\underline{\nu}_{ij}$ are the multipliers corresponding to the constraints on marketing efforts.

The first-order condition writes:

$$\frac{\partial \mathcal{L}}{\partial n_{ij}} = \bar{\varepsilon}_{ij} r_{ij} - \frac{w_j f_j L_j}{P_i} - \gamma \sum_k r_{ij} n_{ik} r_{ik} \mathbb{C}ov\left(\varepsilon_{ij}, \varepsilon_{ik}\right) - \bar{\nu}_{ij} + \underline{\nu}_{ij} = 0, \tag{11}$$

and the system of FOCs can be rewritten in matrix terms:

$$\mathbf{n}_i = \frac{1}{\gamma} \Sigma_i^{-1} \tilde{\mu}_i \tag{12}$$

where $\mathbf{n}_i$ is the vector of n_{ij}, $\tilde{\mu}_i$ is the vector with element $\tilde{\mu}_{ij} = \mu_{ij} - \bar{\nu}_{ij} + \underline{\nu}_{ij}$ where $\mu_{ij} = \bar{\varepsilon}_{ij} r_{ij} - \frac{w_j f_j L_j}{P_i}$ represents the expected real return per unit of n_{ij}, and Σ_i is a $N \times N$ matrix of profits covariance with element $\Sigma_{i,jk} = r_{ij} r_{ik} \mathbb{C}ov\left(\varepsilon_{ij}, \varepsilon_{ik}\right)$ and assumed to be non-singular, i.e. $\det \Sigma_i > 0$. Hence, as shown by Esposito (2022) (Proposition 1), it is optimal to invest in marketing efforts such that the fraction of consumers to be reached on each market is proportional to the inverse of the covariance matrix of real returns, times the vector of expected real returns. Intuitively, risk aversion with $\gamma > 0$ makes the maximization problems with respect to all n_{ij} interelated. If $\gamma = 0$ then problems are separable like in traditional trade models, and it is optimal to choose $n_{ij} = 1$ for all destination markets that are profitable in expectation. The assumption that Σ_i is non-singular is a necessary and sufficient condition to have uniqueness of the optimal solution.[28]

The first-order condition (12) describes the optimal investment rule in a similar way as is it done for a classic problem of mean-variance portfolio selection in financial economics (see e.g. Constantinides and Malliaris (1995) and Ingersoll (1987)). The constraints on n_{ij} are equivalent to what is often imposed in portfolio theory to avoid an unrealistic solution with extreme "short" or "long" positions (see e.g. Jin et al. (2016)). At this level of generality and taking into account the added complexity brought by the constraints on n_{ij}, it is clear that there is no analytical solution, except in some special cases. In the context of demand risks only, Esposito (2022) considers two symmetric countries under autarky and under free trade in which case a closed form solution is available.

However, in our context with demand and production risks, the particular structure of correlation we assume makes it possible to characterize the equilibrium for an arbitrary number of asymmetric countries and under costly trade. This is particularly useful to assess the impact of climate change, through changes in the relative volatility of the production shock, on marketing efforts in all relevant destination markets, as we now show.[29]

[28]As shown by Esposito (2022), the objective is concave and the linear constraints form a convex set. Hence, the solution described by (12) is a global maximum.

[29]In Online Appendix OC, we also explore an alternative timing where production takes place before demand shocks are realized but after production shocks are realized. We show the analysis pursued in the paper is not substantially modified.

4.3 Costly trade between asymmetric countries

At this step of the analysis, it is convenient to normalize all shocks by their means. For this, let us denote $\tilde{\varepsilon}_{ij} = \varepsilon_{ij}/\bar{\varepsilon}_{ij}$ with $\mathbb{E}\tilde{\varepsilon}_{ij} = 1$. We obtain the following result.

Lemma 1. *Denote $SCV_{\beta_i} \equiv \mathbb{V}(\beta_i)/\bar{\beta}_i^2$ as the Squared Coefficient of Variation of production shock β_i and $SCV_{\alpha_i} \equiv \mathbb{V}(\alpha_i)/\bar{\alpha}_j^2$ as the Squared Coefficient of Variation of demand shock α_j.*[30]

(i) *The covariance between normalized shocks affecting profits made on destination countries j and k $(j \neq k)$, from origin country i is given by:*

$$\mathbb{C}ov\left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}\right) = SCV_{\beta_i}. \tag{13}$$

(ii) *The variance of the normalized shock affecting profit made on destination country j from origin country i is given by:*

$$\mathbb{V}(\tilde{\varepsilon}_{ij}) = \left(1 + SCV_{\beta_i}\right) SCV_{\alpha_j} + SCV_{\beta_i}. \tag{14}$$

Proof. Part (i): We have $\mathbb{C}ov\left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}\right) = \mathbb{C}ov\left(\frac{\varepsilon_{ij}}{\bar{\varepsilon}_{ij}}, \frac{\varepsilon_{ik}}{\bar{\varepsilon}_{ik}}\right) = \frac{\bar{\alpha}_j\bar{\alpha}_k\mathbb{V}(\beta_i)}{\bar{\alpha}_j\bar{\alpha}_k\bar{\beta}_i^2} = \frac{\mathbb{V}(\beta_i)}{\bar{\beta}_i^2}$. Also part (ii) follows from $\mathbb{V}(\tilde{\varepsilon}_{ij}) = \frac{\mathbb{V}(\varepsilon_{ij})}{\bar{\varepsilon}_{ij}^2} = \frac{(\mathbb{V}(\beta_i)+\bar{\beta}_i^2)\mathbb{V}(\alpha_j)+\mathbb{V}(\beta_i)\bar{\alpha}_j^2}{\bar{\alpha}_j^2\bar{\beta}_i^2} = \left(1 + \frac{\mathbb{V}(\beta_i)}{\bar{\beta}_i^2}\right)\frac{\mathbb{V}(\alpha_j)}{\bar{\alpha}_j^2} + \frac{\mathbb{V}(\beta_i)}{\bar{\beta}_i^2}$. ■

Hence, according to (13), the covariance between normalized shocks, affecting profits from two destination countries j and k, is determined only by the production shock from the origin country i. More precisely, an increase in the relative volatility of the production shock in origin country i (i.e. an increase in SCV_{β_i}) raises the covariance of composite shocks affecting profits from two destination countries j and k. Moreover, (14) indicates that the variance of the normalized shock $\tilde{\varepsilon}_{ij}$ is an increasing function of both relative volatilities SCV_{β_i} and SCV_{α_j}.

Using these notations, the system of necessary and sufficient first-order conditions (11) can be rewritten as follows, for all i and j:

$$\bar{\varepsilon}_{ij}r_{ij} - \frac{w_j f_j L_j}{P_i} - \gamma\bar{\varepsilon}_{ij}r_{ij}\sum_k n_{ik}\bar{\varepsilon}_{ik}r_{ik}\mathbb{C}ov\left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}\right) - \bar{\nu}_{ij} + \underline{\nu}_{ij} = 0 \tag{15}$$

The system of equations given by (15) can actually be broken into two parts. First, like Esposito (2022), let us concentrate the analysis on settings where it is never profitable to reach all consumers on a given destination market. Intuitively, any firm must be sufficiently risk averse to find optimal not to reach all consumers in each market so that $n_{ij} < 1$ or equivalently $\bar{\nu}_{ij} = 0$ for any j.[31] Moreover, denote $\mathcal{S} \subseteq \mathcal{N}$ as a possible choice in terms of

[30] The squared coefficient of variation is the ratio between the variance and the square of mean and it represents the variance of the random variable normalized by its mean or equivalently its *relative volatility*. The increase in relative volatility may come from a reduction in the mean and/or an increase in the variance.

[31] Like Esposito (2022), we will provide a lower bound on γ to ensure this. Under risk neutrality ($\gamma = 0$), it is optimal for a given firm to choose $n_{ij} = 1$ for all destination markets that are profitable.

the set of destination countries and conditionally on the *portfolio* $\mathcal{S}$ let us now characterize the optimal choices in terms of marketing effort with $1 > n_{ij} > 0$ for all $j \in \mathcal{S}$.

Clearly, for a given origin country i, the system of first-order conditions given by (15) reduces to, for all $j \in \mathcal{S}$:

$$\bar{\varepsilon}_{ij} r_{ij} \left(1 - \gamma \sum_{k \in \mathcal{S}} n_{ik} \bar{\varepsilon}_{ik} r_{ik} \mathbb{C}ov\left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}\right) \right) = \frac{w_j f_j L_j}{P_i} \tag{16}$$

and for all $j \notin \mathcal{S}$,

$$\bar{\varepsilon}_{ij} r_{ij} \left(1 - \gamma \sum_{k \in \mathcal{S}} n_{ik} \bar{\varepsilon}_{ik} r_{ik} \mathbb{C}ov\left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}\right) \right) - \frac{w_j f_j L_j}{P_i} + \underline{\nu}_{ij} = 0. \tag{17}$$

Equation (16) suggests that a correction due to risk aversion must be made when assessing the expected marginal return of n_{ij} to be equated with its marginal cost for an interior solution. Furthermore, note that because $j \notin \mathcal{S}$, the covariance term in (17) is $\mathbb{C}ov\left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}\right) = SCV_{\beta_i}$ for all $k \in \mathcal{S}$ and for all $j \notin \mathcal{S}$. It follows that the system given by (16) can be solved independently from (17) to obtain the equilibrium value of n_{ij} for all $j \in \mathcal{S}$. And then (17) can be used to obtain the multiplier $\underline{\nu}_{ij}$ for all $j \notin \mathcal{S}$. Clearly, assuming independent demand shocks and introducing a correlation between hybrid shocks only through the common production shock allows to break down the system (15) so that the system (16) is autonomous.

In the rest of this section, we discuss first the investment rule for a given portfolio $\mathcal{S}$ composed of destination markets. Then we characterize the optimal portfolio for each productivity φ, before determining its value for the firm.

Optimal investment rule in marketing effort. To pursue further, let us denote $\tilde{\Sigma}_i$ as the variance-covariance matrix based on (13) and (14). Let us also denote $\tilde{\Sigma}_i^{-1}$ its inverse with generic term $\tilde{\Sigma}_{i,jk}^{-1}$ at the intersect of line j and column k. Solving the system (16) yields, for all $j \in \mathcal{S}$:

$$n_{ij} = \frac{1}{\gamma \bar{\varepsilon}_{ij} r_{ij}} \sum_{k \in \mathcal{S}} \frac{\tilde{\Sigma}_{i,jk}^{-1}}{\bar{\varepsilon}_{ik} r_{ik}} \left(\bar{\varepsilon}_{ik} r_{ik} - \frac{w_k f_k L_k}{P_i} \right). \tag{18}$$

To interpret the optimality condition (18) for n_{ij}, let us introduce some additional notations.

Definition 1. *Consider a set of destination markets $\mathcal{S}$ with $|\mathcal{S}| \geq 2$. The diversification index of destination country $j \in \mathcal{S}$ from the perspective of origin country i is denoted D_{ij} and is given by:*

$$D_{ij} = \sum_{k \in \mathcal{S}} \tilde{\Sigma}_{i,jk}^{-1}. \tag{19}$$

The term $\tilde{\Sigma}^{-1}_{i,jk}$ measures the contribution of destination market k to the diversification index D_{ij} and its relative weight is denoted $\omega_{i,jk}$ given by,

$$\omega_{i,jk} = \frac{\tilde{\Sigma}^{-1}_{i,jk}}{D_{ij}} \text{ with } \sum_{k \in \mathcal{S}} \omega_{i,jk} = 1. \tag{20}$$

As will be clear below, the diversification index D_{ij} is an inverse measure of the overall riskiness of destination country j from the perspective of origin country i[32]. Moreover, because all decisions about all markets are intertwined due to risk aversion, the term $\tilde{\Sigma}^{-1}_{i,jk}$ measures the contribution of destination market k to the diversification index D_{ij}. Importantly, the weight $\omega_{i,jk}$ can be positive or negative in which case market k contributes respectively positively or negatively to D_{ij}.

Definition 2. *Consider a set of destination markets $\mathcal{S}$ with $|\mathcal{S}| \geq 2$. The relative profitability index of destination country $j \in \mathcal{S}$ from the perspective of origin country i for a firm with productivity φ is denoted $\mathcal{C}_{ij}(\varphi)$ and is given by:*

$$\mathcal{C}_{ij}(\varphi) = \sum_{k \in \mathcal{S}} \omega_{i,jk} \left(\frac{\bar{\varepsilon}_{ik} r_{ik}(\varphi) - w_k f_k L_k / P_i}{\bar{\varepsilon}_{ik} r_{ik}(\varphi)} \right). \tag{21}$$

The term between brackets in (21) is a *profitability ratio* that divides the net expected profit by the gross expected profit and it measures in relative terms how much is left once the fixed cost of marketing are paid.[33] Hence, expression (21) represents the *weighted sum of (expected) profitability ratios* accross destination countries, and where the weight is the relative contribution $\omega_{i,jk}$ of each market to the diversification index D_{ij}.

The above definitions allow to rewrite the optimality condition (18) for n_{ij} as follows.

Proposition 1. *Consider a set of destination markets $\mathcal{S}$ with $|\mathcal{S}| \geq 2$. A firm with productivity φ in origin country i that finds optimal to reach an interior solution for n_{ij} in some countries, i.e. $0 < n_{ij} < 1$ for all $j \in \mathcal{S}$, invests according to the following rule:*

$$n_{ij}(\varphi) = \frac{D_{ij}}{\gamma \bar{\varepsilon}_{ij} r_{ij}(\varphi)} \mathcal{C}_{ij}(\varphi). \tag{22}$$

Expression (22) reveals that both a larger diversification index D_{ij} and a larger relative profitability index $\mathcal{C}_{ij}$ stimulate marketing investment on market j. Moreover, taking a partial equilibrium perspective (i.e. assuming that price indexes, national incomes and wage rates are fixed), we see that a change in the squared coefficient of variation of

[32]Our definition of the diversification index is consistent with that of Esposito (2022). Indeed, he defines the diversification index as the sum of terms in the appropriate line of the inverse covariance matrix, each term being multiplied by the corresponding expected demand shock. Our definition is similar albeit we work with shocks normalized by their means.

[33]On a given market, say k, the expected real gross profit is $\bar{\varepsilon}_{ik} r_{ik} n_{ik}$ and the expected real net profit is $\bar{\varepsilon}_{ik} r_{ik} n_{ik} - w_k f_k L_k n_{ik} / P_i$. The ratio of the latter over the former measures the rate of expected profitability on market k.

idiosyncratic production shocks in the origin country impacts the marketing choice n_{ij} through two channels. First, a change in SCV_{β_i} impacts directly the diversification index D_{ij}, and second it also impacts the weights $\omega_{i,jk}$ used to compute the relative profitability index $\mathcal{C}_{ij}$. Third, there is the possibility of corner solutions for some markets so that a third channel also appears through changes in the multipliers $\underline{\nu}_{ij}$ for all $j \notin \mathcal{S}$.[34]

Importantly, our analysis departs here from Esposito (2022) in the following way. Esposito (2022) focuses his analysis on the role of the diversification index and shows that a sufficient condition for n_{ij} to grow with D_{ij} is that the variance-covariance matrix of demand shocks has at least a negative correlation. He subsequently suggests that D_{ij} is a sufficient statistic to measure the impact of shocks on the equilibrium. In our context with uncorrelated demand shocks and production shocks, all covariances are necessarily positive because the common underlying production shock makes all composite shocks positively correlated. Furthermore, we are interested in how a change in the (relative) volatility of the production shock affects equilibrium and clearly from the discussion above, not only D_{ij} is impacted but also the relative contributions $\omega_{i,jk}$ of each market to D_{ij}. Because Esposito (2022) focuses on the particular case of free trade with symmetric countries, it appears that the (expected) profitability ratio is uniform across markets so that $\mathcal{C}_{ij}$ no longer depends on either volatility, but only on expected values of demand and production shocks.[35]

By contrast, in our analysis, we are interested in costly trade with potentially asymmetric countries and this makes a huge difference as D_{ij} is no longer a sufficient statistics to measure the impact of volatility on the trade equilibrium. To pursue further the analysis, it is crucial to understand how demand and production shocks impact the diversification index D_{ij} as well as the relative contributions $\omega_{i,jk}$ of each market to D_{ij}. This is the purpose of the following Proposition.

Proposition 2. *For a given portfolio $\mathcal{S} \subseteq \mathcal{N}$ and such that $|\mathcal{S}| \geq 2$, the diversification index D_{ij} is given by:*

$$D_{ij} = \frac{1}{SCV_{\alpha_j}} \frac{1}{1 + SCV_{\beta_i}\left(1 + \sum_{k \in \mathcal{S}} \frac{1}{SCV_{\alpha_k}}\right)}. \tag{23}$$

The weights system used to form the weighted sum of profitability ratios $\mathcal{C}_{ij}(\varphi)$ is:

$$\omega_{i,jk} = \begin{cases} -\frac{SCV_{\beta_i}}{SCV_{\alpha_k}(1+SCV_{\beta_i})} < 0 & \text{for } k \neq j \\ 1 + \frac{SCV_{\beta_i}}{1+SCV_{\beta_i}} \sum_{l \in \mathcal{S}, l \neq j} \frac{1}{SCV_{\alpha_l}} > 1 & \text{for } k = j. \end{cases}$$

Proof. See Appendix E. ∎

[34]And in the general case for $\bar{\nu}_{ij}$.

[35]Indeed, under free trade with symmetric countries, domestic sales and exports are identical as well as the corresponding marketing efforts, for a firm that is sufficiently productive.

To complete Propositions 1 and 2, in the situation where $|\mathcal{S}| = 1$, it is straightforward to establish from (16) that the optimal marketing effort for the unique destination market j is then given by:

$$n_{ij}(\varphi) = \frac{1/\mathbb{V}(\tilde{\varepsilon}_{ij})}{\gamma\bar{\varepsilon}_{ij}r_{ij}(\varphi)}\left(1 - \frac{w_j f_j L_j/P_i}{\bar{\varepsilon}_{ij}r_{ij}(\varphi)}\right).$$

Everything happens as if $D_{ij} = 1/\mathbb{V}(\tilde{\varepsilon}_{ij})$, $\omega_{i,jk} = 0$ for $k \neq j$ and $\omega_{i,jj} = 1$. As expected, a larger variance of the hybrid shock or a reduced profitability ratio reduces the incentives to invest on market j.

When the firm with productivity φ considers at least two destination markets ($|\mathcal{S}| \geq 2$) then Proposition 2 indicates that both the diversification index D_{ij} and the weights $\omega_{i,jk}$ only depends on relative volatilities of production and demand shocks. To interpret this result, it is convenient to consider first the limit case where wine production is not random. When $SCV_{\beta_i} = 0$, then Propositions 1 and 2 together indicate that $D_{ij} = 1/SCV_{\alpha_j}$, $\omega_{i,jk} = 0$ for $k \neq j$ and $\omega_{i,jj} = 1$, which in turn yields $\mathcal{C}_{ij}(\varphi) = 1 - \frac{w_j f_j L_j/P_i}{\bar{\varepsilon}_{ij}r_{ij}(\varphi)}$ and the optimal $n_{ij}(\varphi)$ only depends on market $j's$ characteristics:

$$n_{ij}(\varphi) = \frac{1/SCV_{\alpha_j}}{\gamma\bar{\varepsilon}_{ij}r_{ij}(\varphi)}\left(1 - \frac{w_j f_j L_j/P_i}{\bar{\varepsilon}_{ij}r_{ij}(\varphi)}\right).$$

Not surprisingly, in the absence of uncertainty with respect to production which implies the absence of correlation between shocks $\tilde{\varepsilon}_{ij}$, the problems of choosing how much to invest in terms of marketing effort on each market are separable.

It is worth noting that this separability result also holds approximately when, for a given relative volatility of production $SCV_{\beta_i} > 0$, the demand is highly volatile everywhere. To see this, let us interpret

$$\sum_{k\in\mathcal{S}} \frac{1}{SCV_{\alpha_k}} \equiv \mathcal{I}(\mathcal{S})$$

as an *index of demand riskiness of the portfolio* $\mathcal{S}$. If demand is highly volatile everywhere, then the index $\mathcal{I}(\mathcal{S})$ is close to zero and consequently, $D_{ij} \approx \frac{1}{SCV_{\alpha_j}}\frac{1}{1+SCV_{\beta_i}}$. It also follows that $\omega_{i,jk} \approx 0$ for $k \neq j$ and $\omega_{i,jj} \approx 1$, so that $\mathcal{C}_{ij}(\varphi) \approx 1 - \frac{w_j f_j L_j/P_i}{\bar{\varepsilon}_{ij}r_{ij}(\varphi)}$. Hence, the problem of choosing n_{ij} almost depends only market $j's$ characteristics.

Now, let us investigate how the relative volatility of production and demand impacts the diversification index D_{ij}. First of all, a raise in D_{ij} increases ceteris paribus the incentives to invest to reach consumers on market j. In other words, the higher the interest in market j in terms of diversification, the higher incentives to invest there. Proposition 2 suggests that a raise in the relative volatility of β_i reduces D_{ij}, while on the contrary, a raise in the relative volatility of demand shock in any destination market except j increases D_{ij}. As shown by (23), the same effect holds for SCV_{α_j} but there is also a direct effect in the opposite direction whereby D_{ij} decreases as the relative volatility of demand on market j increases. In total, we have:[36]

$$\frac{\partial D_{ij}}{\partial SCV_{\alpha_j}} = \frac{D_{ij}}{SCV_{\alpha_j}} \left(SCV_{\beta_i} D_{ij} - 1 \right) < 0. \tag{24}$$

To sum up, first, a larger relative volatility for production makes the world riskier and thereby tends to reduce the incentives to invest everywhere. Second, a larger relative volatility for demand on a given market tends to reduce the incentives to invest there, but tends to increase the incentives to invest elsewhere. Comparing market j and market k in the same portfolio, the ratio of their diversification indexes reflects their respective relative volatility of demand shocks:

$$\frac{D_{ij}}{D_{ik}} = \frac{SCV_{\alpha_k}}{SCV_{\alpha_j}},$$

and consequently, when demand is more volatile on market j relative to market k then the diversification index of market j is lower than the diversification index of market k.

Furthermore, note that increasing the size of the portfolio by adding a new country to $\mathcal{S}$ implies that the index of demand riskiness $\mathcal{I}(\mathcal{S}) = \sum_{k \in \mathcal{S}} \frac{1}{SCV_{\alpha_k}}$ raises and consequently the diversification index D_{ij} for all markets in the portfolio decreases. More generally, suppose that market j belongs to portfolios $\mathcal{S}$ and $\mathcal{S}'$ with $|\mathcal{S}'| > |\mathcal{S}|$ then

$$D_{ij}(\mathcal{S}') - D_{ij}(\mathcal{S}) = \frac{1}{SCV_{\alpha_j}} \frac{SCV_{\beta_i} \left[\mathcal{I}(\mathcal{S}) - \mathcal{I}(\mathcal{S}') \right]}{\left[1 + SCV_{\beta_i} \left(1 + \mathcal{I}(\mathcal{S}) \right) \right] \left[1 + SCV_{\beta_i} (1 + \mathcal{I}(\mathcal{S}')) \right]} < 0.$$

Not surprisingly, increasing the size of the portfolio reduces the interest of each country in terms of diversification, which we refer to as the *dilution of the diversification effect on investment* in the following. However, this dilution effect only appears because of the production risk that makes profits across markets correlated. In the absence of production risk ($SCV_{\beta_i} = 0$), then the diversification index of market j only depends on the relative volatility of its demand, and not on the composition of the portfolio considered.

Finally, let us investigate how the relative volatility of production and demand impact the relative contribution of each market in the portfolio $\mathcal{S}$ to the diversification index D_{ij}. Proposition 2 shows that $\omega_{i,jk} < 0$ for all $k \neq j$. Intuitively, any increase in profitability on a destination market k other than j will reduce $\mathcal{C}_{ij}(\varphi)$ and therefore the incentive to invest in j. And this negative effect is all the stronger when demand on market k is not very volatile and production in i is very volatile. By mirroring effect, $\omega_{i,jj}$ is larger than unity and an increase in j's profitability increases the interest in investing in j, especially when production in origin country i is volatile and demands on other markets are not very volatile. In other words, a raise in the relative volatility of production implies more

[36]To see that (24) holds, let us compute $SCV_{\beta_i} D_{ij} = \frac{SCV_{\beta_i}}{SCV_{\alpha_j}} \frac{1}{1+SCV_{\beta_i}\left(1+\sum_k \frac{1}{SCV_{\alpha_k}}\right)}$ and it is immediate to see that $\frac{SCV_{\beta_i}}{SCV_{\alpha_j}} < 1 + SCV_{\beta_i} \left(1 + \sum_k \frac{1}{SCV_{\alpha_k}} \right)$ holds because $0 < 1 + SCV_{\beta_i} \left(1 + \sum_{k \neq j} \frac{1}{SCV_{\alpha_k}} \right)$. Hence $SCV_{\beta_i} D_{ij} < 1$ and (24) holds.

polarization between market j and other markets when evaluating $\mathcal{C}_{ij}(\varphi)$. Also, comparing $\omega_{i,jk}$ and $\omega_{i,kj}$, we get:

$$\frac{\omega_{i,kj}}{\omega_{i,jk}} = \frac{SCV_{\alpha_k}}{SCV_{\alpha_j}}.$$

Hence, when demand is more volatile on market j relative to market k, $\omega_{i,kj}/\omega_{i,jk} < 1$ and hence $\mathcal{C}_{ij}(\varphi)$ is more sensitive to any changes in the profitability ratio on market k than $\mathcal{C}_{ik}(\varphi)$ is to any changes in the profitability ratio on market j.

Moreover, note that adding a new country to $\mathcal{S}$ implies that $\omega_{i,jj}$ is increasing for all j. In other words, increasing the size of the portfolio implies that investing in j relies more on the profitability of market j, due to the increased polarization between market j and the other markets when evaluating $\mathcal{C}_{ij}(\varphi)$. But this effect is small if the new country has a highly volatile demand.

Characterizing the optimal portfolio for a given productivity. In this section, we determine the equilibrium outcome for domestic firms according to their productivity φ. First, for the clarity of exposition, we concentrate on equilibria where all firms find optimal not to reach all consumers on any market, so that $\bar{\nu}_{ij} = 0$ for i, j. As suggested above, we will check that this situation occurs when firms are sufficiently risk averse, i.e. γ is sufficiently large. Second, a given set $\mathcal{S}$ of destination countries is said *admissible* to a firm with productivity φ if and only if the two following conditions are met:

$$n_{ij}(\varphi) > 0 \text{ for all } j \in \mathcal{S} \quad (25)$$

$$\underline{\nu}_{ij}(\varphi) > 0 \text{ for all } j \notin \mathcal{S}. \quad (26)$$

These two conditions simply state that the firm φ if considering $\mathcal{S}$ should find optimal to exert some positive marketing effort in all chosen destination countries and should refrain from doing so elsewhere. For a given $\mathcal{S}$, one would like to characterize the set of productivities $\mathcal{D}_i(\mathcal{S})$ that would consider $\mathcal{S}$ as admissible. Intuitively, for φ to belong to $\mathcal{D}_i(\mathcal{S})$, it must be that φ is large enough for the firm to be able to invest into marketing even in the *least attractive* market in $\mathcal{S}$ and at the same time φ has to be low enough for the firm not to be tempted considering the *most attractive* market outside of $\mathcal{S}$. In the following analysis, we will confirm this intuition while making precise our definition of market attractiveness. Once $\mathcal{D}_i(\mathcal{S})$ is defined, one can consider the problem of firm with productivity φ choosing the best set $\mathcal{S}$ in order to maximize its indirect utility of real income, while taking into account that $\mathcal{S}$ has to be admissible to the firm with productivity φ, i.e.,

$$\max_{\mathcal{S}} V_i \equiv V_i(\varphi, \mathcal{S}) \text{ s.t. } \varphi \in \mathcal{D}_i(\mathcal{S}).$$

To characterize $\mathcal{D}_i(\mathcal{S})$, let us first define our notion of market attractiveness.

Definition 3. *The attractiveness index of the destination market j from the perspective of the origin market i is defined as follows:*

$$\Gamma_{ij} = \frac{w_j f_j L_j}{\tau_{ij}^{1-\sigma} \bar{\varepsilon}_{ij} A_j}.$$

The destination market j is said to be *less attractive* than market k from the perspective of origin country i if and only if $\Gamma_{ij} > \Gamma_{ik}$. Observe that the attractiveness index Γ_{ij} raises in all components of the marginal cost of the marketing effort n_{ij}, i.e. the wage rate in j, the fixed cost f_j and the size of the economy L_j. In addition, Γ_{ij} raises in the variable trade cost τ_{ij}. By contrast, a larger expected shock $\bar{\varepsilon}_{ij}$ or a larger demand shifter A_j increase the attractiveness of the destination market j. Hence, Γ_{ij} gathers parameters of fixed and variable trade costs as well as some characteristics of the demand in the destination market. The attractiveness index Γ_{ij} is linked to the minimum productivity required to obtain a positive real profit in expectations on the market $j : \mathbb{E}\left(\frac{\pi_{ij}(\varphi)}{P_i}\right) \geq 0$ if and only if $\varphi \geq (\delta_i \Gamma_{ij})^{\frac{1}{\sigma-1}}$.[37]

Second, consider condition (25). Using the above definition of market attractiveness, one can obtain an alternative expression of the optimal marketing effort $n_{ij}(\varphi)$ given by Proposition 1 which allows to identify a cutoff productivity $\hat{\varphi}_{ij}$ below which it is not optimal to invest in market j.

Lemma 2. *Given the set of destination markets $\mathcal{S}$, the optimal marketing effort $n_{ij}(\varphi)$ is*

$$n_{ij}(\varphi) = \frac{D_{ij}}{\gamma \bar{\varepsilon}_{ij} r_{ij}(\varphi)} \left(1 - \left(\frac{\hat{\varphi}_{ij}}{\varphi}\right)^{\sigma-1}\right)$$

and is positive if and only if $\varphi \geq \hat{\varphi}_{ij}$ where

$$\hat{\varphi}_{ij} = \left(\delta_i \sum_{k \in \mathcal{S}} \omega_{i,jk} \Gamma_{ik}\right)^{\frac{1}{\sigma-1}}.$$

In addition, $n_{ij}(\varphi) < 1$ provided $\gamma > \underline{\gamma} = \sup_{j \in \mathcal{S}} \frac{D_{ij}}{4\bar{\varepsilon}_{ij} r_{ij}(\hat{\varphi}_{ij})}$.

Proof. See Appendix F ■

It follows that as long as φ is larger than $\max_{j \in \mathcal{S}} \hat{\varphi}_{ij}$ all marketing efforts $n_{ij}(\varphi)$ for all $j \in \mathcal{S}$ are positive. Moreover, note that

$$(\hat{\varphi}_{ij})^{\sigma-1} = \delta_i \sum_{k \in \mathcal{S}} \omega_{i,jk} \Gamma_{ik} = \delta_i \left[\Gamma_{ij} + \frac{SCV_{\beta_i}}{1 + SCV_{\beta_i}} \sum_{k \in \mathcal{S}} \frac{\Gamma_{ij} - \Gamma_{ik}}{SCV_{\alpha_k}}\right] \tag{27}$$

which implies that $\hat{\varphi}_{ij}$ is strictly increasing in δ_i and Γ_{ij} and strictly decreasing in $\frac{\Gamma_{ik}}{SCV_{\alpha_k}}$ for all $k \neq j$. Hence, the cutoff productivity for market j intuitively raises when wage in origin

[37] This can be established directly from (10) by rearranging.

country raises and when market j becomes less attractive. In addition, $\hat{\varphi}_{ij}$ decreases when other markets in $\mathcal{S}$ are less attractive. However, if a market k faces very volatile demand, the effect of a change in its attractiveness index Γ_{ik} on $\hat{\varphi}_{ij}$ will be small. Overall, observe that the country $j \in \mathcal{S}$ which is associated to the largest cutoff $\hat{\varphi}_{ij}$ is characterized by the largest attractiveness index Γ_{ij}. Note that the impact of the portfolio $\mathcal{S}$ on the cutoff $\hat{\varphi}_{ij}$ needed to be active on market j disappears when the source of correlation between markets vanishes, i.e. when $SCV_{\beta_i} = 0$. Indeed, in that case, $(\hat{\varphi}_{ij})^{\sigma-1} = \delta_i \Gamma_{ij}$ and is solely determined by Γ_{ij}. And when $SCV_{\beta_i} > 0$, $(\hat{\varphi}_{ij})^{\sigma-1} > \delta_i \Gamma_{ij}$ if and only if $\sum_{k \in \mathcal{S}} \frac{\Gamma_{ij} - \Gamma_{ik}}{SCV_{\alpha_k}} > 0$ or equivalently,

$$\Gamma_{ij} > \bar{\Gamma}_i(\mathcal{S}) \equiv \sum_{k \in \mathcal{S}} \frac{1/SCV_{\alpha_k}}{\sum_{l \in \mathcal{S}} \frac{1}{SCV_{\alpha_l}}} \Gamma_{ik}.$$

This reflect the fact that if market j is less attractive than on average in the portfolio $\mathcal{S}$, then the cut-off productivity on market j is higher than it would be in the absence of production volatility.

Now, consider condition (26) on $\underline{\nu}_{ij}(\varphi)$ for any $j \notin \mathcal{S}$. Similarly to the above analysis, we derive a cutoff productivity φ^*_{ij} such that $\underline{\nu}_{ij}(\varphi) > 0$ for any $\varphi < \varphi^*_{ij}$.

Lemma 3. *It is never optimal to invest on market $j \notin \mathcal{S}$ if and only if $\varphi < \varphi^*_{ij}$ where*

$$\left(\varphi^*_{ij}\right)^{\sigma-1} = \delta_i \left[\Gamma_{ij} + \frac{SCV_{\beta_i}}{1 + SCV_{\beta_i}} \sum_{k \in \mathcal{S}} \frac{\Gamma_{ij} - \Gamma_{ik}}{SCV_{\alpha_k}}\right]. \tag{28}$$

Proof. See Appendix G. ■

Note that by comparing (28) and (27), the cutoff types $\hat{\varphi}_{ij}$ and φ^*_{ij} share the same expression in function of the indexes of attractiveness, the only difference is that in the former $j \in \mathcal{S}$ while in the latter $j \notin \mathcal{S}$. Hence, φ^*_{ij} is also increasing in Γ_{ij} and thus the lowest value of φ^*_{ij} corresponds to the market with the lowest attractiveness index not in $\mathcal{S}$. It follows that as long as φ is lower than $\min_{j \notin \mathcal{S}} \varphi^*_{ij}$ then all the multipliers $\underline{\nu}_{ij}(\varphi)$ are strictly positive and the firm with productivity φ will never consider investing in a market that does not belong to $\mathcal{S}$.

From Lemmas 2 and 3, one can summarize the range $\mathcal{D}_i(\mathcal{S})$ of φ that makes $\mathcal{S} \subset \mathcal{N}$ admissible as $\varphi \in \Phi_i$ and

$$\max_{j \in \mathcal{S}} \hat{\varphi}_{ij} \leq \varphi \leq \min_{j \notin \mathcal{S}} \varphi^*_{ij}.$$

and for $\mathcal{S} = \mathcal{N}$ the condition defining $\mathcal{D}_i(\mathcal{N})$ is simply $\max_{j \in \mathcal{N}} \hat{\varphi}_{ij} \leq \varphi$. This set, if non empty, defines the range of productivities φ consistent with $\mathcal{S}$ that allows to consider the indirect utility $V_i(\varphi, \mathcal{S})$. Actually, for $\mathcal{S} \subset \mathcal{N}$, $\mathcal{D}_i(\mathcal{S})$ is non empty if and only if $\max_{j \in \mathcal{S}} \Gamma_{ij} < \min_{j \notin \mathcal{S}} \Gamma_{ij}$. Hence, an admissible portfolio of size l necessarily contains the l most attractive markets. It follows that, without loss of generality, we can reindex markets from 1 to N according to their degree of attractiveness from the perspective of the origin country i, so that Γ_{i1} corresponds to the most attractive market and Γ_{iN} to the least attractive market. A firm with productivity φ has only one admissible and thus optimal portfolio whose size is determined by the domain that contains φ. We can thus denote the unique optimal portfolio of firm φ by $\mathcal{S}(\varphi)$ and its value by $V_i^*(\varphi) = V_i(\varphi, \mathcal{S}(\varphi))$. When $\mathcal{S}(\varphi)$ is composed of the l most attractive markets, for any $l = 1...N$, let us denote $\varphi_{il} = \max_{j \in \mathcal{S}(\varphi)} \hat{\varphi}_{ij}$ and $\varphi_{i,l+1} = \min_{j \notin \mathcal{S}(\varphi)} \varphi_{ij}^*$, with the convention that $\varphi_{i,N+1} = \infty$. Using (27) and (28), we can sum up our result in the following Proposition.

Proposition 3. *The unique optimal portfolio $\mathcal{S}(\varphi)$ for a firm with productivity φ is the set of the l most attractive markets when $\varphi_{il} \leq \varphi \leq \varphi_{i,l+1}$ where for all $l = 1, ..., N-1$,*

$$(\varphi_{il})^{\sigma-1} = \delta_i \left[\Gamma_{il} + \frac{SCV_{\beta_i}}{1 + SCV_{\beta_i}} \sum_{k=1}^{l} \frac{\Gamma_{il} - \Gamma_{ik}}{SCV_{\alpha_k}} \right]$$

and $\varphi_{i,N+1} = \infty$.

The value of the optimal portfolio. Let us compute the value $V_i^*(\varphi)$ of the optimal portfolio $\mathcal{S}(\varphi)$ from the viewpoint of a firm with productivity φ in origin country i. Its expression is given in the following Proposition.

Proposition 4. *At the equilibrium, a firm with productivity φ and from origin country i*

(i) *either does not produce when $\varphi \leq \varphi_{i1}$ and gets $V_i^*(\varphi) = 0$,*

(ii) *or produces and sells in the l most attractive markets when $\varphi_{il} \leq \varphi \leq \varphi_{i,l+1}$ and gets*

$$V_i^*(\varphi) = \frac{1}{2\gamma} \sum_{j=1}^{l} D_{ij} \left(1 - \left(\frac{\hat{\varphi}_{ij}}{\varphi} \right)^{\sigma-1} \right) \left(1 - \frac{\delta_i \Gamma_{ij}}{\varphi^{\sigma-1}} \right) > 0$$

where

$$(\hat{\varphi}_{ij})^{\sigma-1} = \delta_i \left[\Gamma_{ij} + \frac{SCV_{\beta_i}}{1 + SCV_{\beta_i}} \sum_{k=1}^{l} \frac{\Gamma_{ij} - \Gamma_{ik}}{SCV_{\alpha_k}} \right].$$

Proof. See Appendix H. ■

As indicated by Proposition 4, the value brought by a given market to the firm depends on its interest in terms of diversification measured through D_{ij}, on its attractiveness index Γ_{ij} as well as the cutoff $\hat{\varphi}_{ij}$ which partly determines how much to invest there (see Lemma 2). While the attractiveness index Γ_{ij} depends only market j's characteristics, both D_{ij} and $\hat{\varphi}_{ij}$ depends in general on the optimal portfolio composition, and this reminds us that the problems of how much to invest on each market are not separable, except in two specific cases that we now review.

- All markets have the same attractiveness index, i.e. $\Gamma_{ij} = \Gamma_i$ for all $j \in \mathcal{N}$. Then from Proposition 4, we deduce that $\varphi_{ij} = \hat{\varphi}_{ij} = (\delta_i \Gamma_i)^{\frac{1}{\sigma-1}}$ for all j. Hence, for $\varphi \geq (\delta_i \Gamma_i)^{\frac{1}{\sigma-1}}$

$$V_i^*(\varphi) = \frac{1}{2\gamma} \left(1 - \frac{\delta_i \Gamma_i}{\varphi^{\sigma-1}} \right)^2 \sum_{j=1}^{N} D_{ij}.$$

Hence, for the firms that are sufficiently productive, i.e. $\varphi \geq (\delta_i \Gamma_i)^{\frac{1}{\sigma-1}}$, we have that $\mathcal{S}(\varphi) = \mathcal{N}$.[38] Intuitively, as all markets share the same attractiveness index, the only factor that differentiate them is the relative volatility of their demand. But as all demand shocks are independent, if a firm is sufficiently productive, it is thus optimal to diversify as much as possible by investing on all markets. The problems of how much to invest on each market are separable and a higher demand volatility on a market translates into less investment.

- When there is no production shock ($SCV_{\beta_i} = 0$), then $D_{ij} = 1/SCV_{\alpha_j}$ and Proposition 4, we deduce once again that $\varphi_{ij} = \hat{\varphi}_{ij} = (\delta_i \Gamma_{ij})^{\frac{1}{\sigma-1}}$ for all j and hence, for $\varphi \geq (\delta_i \Gamma_{ij})^{\frac{1}{\sigma-1}}$

$$V_i^*(\varphi) = \frac{1}{2\gamma} \sum_{j \in \mathcal{S}(\varphi)} \frac{1}{SCV_{\alpha_j}} \left(1 - \frac{\delta_i \Gamma_{ij}}{\varphi^{\sigma-1}}\right)^2.$$

  The firm φ chooses a portfolio with the l most attractive markets when $\varphi_{il} \leq \varphi \leq \varphi_{i,l+1}$. As in the previous case, the problems of how much to invest on each market are separable and a higher demand volatility on a market translates into less investment.

Finally, note that the investment problems remain intertwined even if the relative volatility of demand shocks is the same everywhere, i.e. $SCV_{\alpha_j} = SCV_\alpha$ for all $j \in \mathcal{N}$. Indeed, in that context, the diversification index is uniform across markets in the portfolio because D_{ij} only depends on $\mathcal{S}$ through its cardinal $|\mathcal{S}|$:

$$D_{ij} = D_i(|\mathcal{S}|) = \frac{1}{SCV_\alpha} \frac{1}{1 + SCV_{\beta_i}\left(1 + \frac{|\mathcal{S}|}{SCV_\alpha}\right)}.$$

Nevertheless, the value brought by a given market still depends on the composition of the portfolio through the cutoff $\hat{\varphi}_{ij}$.

4.4 The impact of climate change

This last section is devoted to examine the impacts of climate change, interpreted as a raise in production shock volatility, on the (partial) equilibrium. Let us start with the impacts at the firm's level, before looking at the consequences for the industry.

Implications for the firm's decisions. Consider a firm with productivity φ that belongs to $[\varphi_{il}, \varphi_{i,l+1}]$ and its optimal portfolio $\mathcal{S}$. On the intensive margin, two channels convey the impacts of an increase in the volatility of β_i measured by $\mathbb{V}(\beta_i)$. First, the diversification index D_{ij} decreases whatever j, which means that there are incentives to

[38] And when $\varphi < (\delta_i \Gamma_i)^{\frac{1}{\sigma-1}}$, then $\mathcal{S}(\varphi) = \varnothing$.

invest less in every market in the portfolio ceteris paribus. Moreover, from (23) we get that:

$$\frac{\partial \ln D_{ij}}{\partial \ln SCV_{\beta_i}} = -\frac{SCV_{\beta_i}\left(1+\mathcal{I}(\mathcal{S})\right)}{1+SCV_{\beta_i}\left(1+\mathcal{I}(\mathcal{S})\right)} \tag{29}$$

and hence the elasticity of D_{ij} w.r.t SCV_{β_i} is constant whatever j. In other words, the lower the diversification index or, equivalently, the riskier the demand, the less D_{ij} decreases as a result of increased production volatility. To sum up, the effect of climate change on the diversification index is leading the firm to reduce its marketing investments, as the world is riskier due to increased correlation between profit risks. We refer to this as the *scale effect* of climate change on investment decisions.

Second, $\mathcal{C}_{ij}(\varphi)$, the weighted sum of profitability ratios, changes as the productivity cut-off $\hat{\varphi}_{ij}$ changes. More precisely, we see from (27) that $\hat{\varphi}_{ij}$ increasing in the volatility of β if and only if $\sum_{k\in\mathcal{S}} \frac{\Gamma_{ij}-\Gamma_{ik}}{SCV_{\alpha_k}} > 0$ or equivalently when

$$\Gamma_{ij} > \bar{\Gamma}_i(\mathcal{S}) \equiv \sum_{k=}^{l} \frac{1/SCV_{\alpha_k}}{\mathcal{I}(\mathcal{S})}\Gamma_{ik}.$$

$\bar{\Gamma}_i(\mathcal{S})$ denotes a weighted average of attractiveness indexes in the portfolio $\mathcal{S}$ where each attractiveness index Γ_{ik} is weighted by its *relative* demand riskiness $\frac{1/SCV_{\alpha_k}}{\mathcal{I}(\mathcal{S})}$. When market j is less attractive than average, the productivity threshold $\hat{\varphi}_{ij}$ increases with the volatility of the production shock, while when market j is more attractive than average, $\hat{\varphi}_{ij}$ decreases. All other things being equal, climate change leads the firm to increase its marketing investments in the most attractive markets in its portfolio and reduce them elsewhere. We refer to this as the *redeployment effect* of climate change on investment decisions.

Finally, on the extensive margin, clearly both φ_{il} and $\varphi_{i,l+1}$ increase with the volatility of the production shock. It is therefore possible that the firm considered is no longer productive enough to choose the optimal portfolio $\mathcal{S}$ with l markets, and must therefore abandon the least attractive market to concentrate on the more attractive ones.[39] We refer to this as the *selection effect* of climate change on investment decisions. Overall, the impact on the value $V_i^*(\varphi)$ of the optimal portfolio results from the confrontation of the scale, redeployment and selection effects described above.

Implications for export value and number of exporters at the industry's level. Let us now characterize how climate change impacts the aggregate export value X_{ij} from origin country i to destination country j, and also the number of exporting firms M_{ij}. Starting with the latter, and using Proposition 3, we know that all firms with a productivity larger than φ_{ij} will invest in market j in varying degrees, to reach consumers

[39] This happens when φ is lower than the resulting threshold φ_{il} following the change in production volatility.

there. Therefore, the number of exporting firms from i to j is given by

$$M_{ij} = M_i \sum_{l=j}^{N} \int_{\varphi_{il}}^{\varphi_{i,l+1}} dG_i(\varphi) = M_i \left(1 - G_i(\varphi_{ij})\right)$$

where the cutoff φ_{ij} is given by:

$$(\varphi_{ij})^{\sigma-1} = \delta_i \left[\Gamma_{ij} + \frac{SCV_{\beta_i}}{1 + SCV_{\beta_i}} \sum_{k=1}^{j} \frac{\Gamma_{ij} - \Gamma_{ik}}{SCV_{\alpha_k}}\right]. \tag{30}$$

Without ambiguity, φ_{ij} is increasing in SCV_{β_i} and hence we get the result that climate change by raising the volatility of production decreases the number of exporting firms. Another interesting result is that the probability of exporting, i.e. $1 - G_i(\varphi_{ij})$, depends on the attractiveness indexes as well as the demand riskiness of *all more attractive* markets than market j, as indicated by (30).

Concerning the aggregate export value between i and j, we have, by virtue of the Law of Large Numbers:

$$X_{ij} = M_i \int_0^{\infty} \mathbb{E}\left[p_{ij}(\varphi) q_{ij}(\varphi)\right] dG_i(\varphi) = M_i \sum_{l=j}^{N} \int_{\varphi_{il}}^{\varphi_{i,l+1}} \mathbb{E}\left[p_{ij}(\varphi) q_{ij}(\varphi)\right] dG_i(\varphi) \tag{31}$$

where

$$\mathbb{E}\left[p_{ij}(\varphi) q_{ij}(\varphi)\right] = P_i \frac{D_{ij}(\varphi)}{\gamma} \left(1 - \left(\frac{\hat{\varphi}_{ij}}{\varphi}\right)^{\sigma-1}\right)$$

with

$$\begin{aligned}
D_{ij}(\varphi) &= \frac{1}{SCV_{\alpha_j}} \frac{1}{1 + SCV_{\beta_i}\left(1 + \mathcal{I}\left(\mathcal{S}(\varphi)\right)\right)} \\
\mathcal{I}\left(\mathcal{S}(\varphi)\right) &= \sum_{k=1}^{l} \frac{1}{SCV_{\alpha_k}} \\
\hat{\varphi}_{ij} &= \left(\delta_i \left[\Gamma_{ij} + \frac{SCV_{\beta_i}}{1 + SCV_{\beta_i}} \sum_{k=1}^{l} \frac{\Gamma_{ij} - \Gamma_{ik}}{SCV_{\alpha_k}}\right]\right)^{\frac{1}{\sigma-1}}
\end{aligned}$$

Hence, (31) rewrites as follows:

$$X_{ij} = \frac{M_i P_i}{\gamma} \sum_{l=j}^{N} D_{ij} \int_{\varphi_{il}}^{\varphi_{i,l+1}} \left(1 - \left(\frac{\hat{\varphi}_{ij}}{\varphi}\right)^{\sigma-1}\right) dG_i(\varphi) \tag{32}$$

Not surprisingly given the discussion in the previous section, (32) allows to decompose the impact of increased production volatility on X_{ij} into a scale, a redeployment and a selection effect. Firstly, because an increased production volatility makes the world riskier by increasing the correlation between profits made on each market, it reduces the interest of diversification, i.e. D_{ij} decreases, and this leads all firms exporting to j to reduce their investment level there. As shown by (29), the scale effect is more pronounced when the demand riskiness $\mathcal{I}(\mathcal{S}(\varphi))$ of the portfolio is larger, that is for firms with bigger portfolios and thus larger productivity. Overall, the scale effect contributes to decrease X_{ij} following an increase in production volatility.

Secondly, the redeployment of investments within the portfolio has effects on sales on market j that depend on the composition of the portfolio. More precisely, a firm tends to increase (decrease) its investments to reach consumers in j if this market is more (less) attractive than the average in its portfolio, the average being understood as weighted by the relative risk of demand. Overall, the impact of the redeployment effect on X_{ij} remains largely an empirical question.

Lastly, the selection effect reflects the fact that the bounds φ_{il} and $\varphi_{i,l+1}$ are increasing in the production volatility. In other words, a greater productivity is required to include market j in one's portfolio, so the number of exporters decreases. This contributes to lower exports in value terms.

Importantly, note that because climate change affects product quality, which in turn directly shapes demand, production shocks β_i—whether operating through costs or quality—interact with demand shocks α_j in determining the overall consequences of climate change. This interaction can be clearly seen by considering a related model that is identical in all respects except that demand shocks α_j are absent. In that case, the impact of climate change collapses to a pure scale effect, while the redeployment and selection effects vanish.[40]

5 Conclusion

This paper has examined how firms, confronted with production and demand shocks, navigate marketing investments and export decisions in response to climate-induced volatility, thereby impacting global trade dynamics. While climate change poses multifaceted risks to firms, disrupting production processes across various industries, the wine industry is particularly susceptible. Temperature fluctuations can alter grape cultivation, and changes in precipitation can lead to water stress and increased pest susceptibility. As our empirical analysis demonstrates, wineries must adapt to these climate disruptions by strategically selecting export markets amidst yield uncertainty.

The theoretical analysis provided in this paper elucidate how the volatility of climate shocks, impacting production and quality, influences exports. Firms may reduce marketing investments to reach consumers while reallocating resources to the most attractive markets. Additionally, some firms may find it optimal to streamline their portfolio by exiting less favorable markets. In the analysis, we define precisely what are the attractivity index, the diversification index and the relative profitability index which are key to understand how

[40] Online Appendix OD provides a full derivation of the model's solution in this benchmark case.

risk averse entrepreneurs make export decisions. A natural extension of the present work would be to consider that market penetration decisions and investments are made over time, as in Alessandria et al. (2021), rather than in a static framework as in this paper, but this is left for further research.

References

Alessandria, G., Arkolakis, C., and Ruhl, K. J. (2021). Firm dynamics and trade. *Annual Review of Economics*, 13:253–280.

Anderson, J. E. and van Wincoop, E. (2003). Gravity with gravitas: A solution to the border puzzle. *American Economic Review*, 93(1):170–192.

Antràs, P. and De Gortari, A. (2020). On the geography of global value chains. *Econometrica*, 88(4):1553–1598.

Antràs, P., Fort, T. C., and Tintelnot, F. (2017). The margins of global sourcing: Theory and evidence from us firms. *American Economic Review*, 107(9):2514–2564.

Arkolakis, C. (2010). Market penetration costs and the new consumers margin in international trade. *Journal of Political Economy*, 118(6):1151–1199.

Arkolakis, C., Eckert, F., and Shi, R. (2023). Combinatorial discrete choice: A quantitative model of multinational location decisions. Technical report, National Bureau of Economic Research.

Ashenfelter, O. (2008). Predicting the quality and prices of bordeaux wine. *The Economic Journal*, 118(529):F174–F184.

Ashenfelter, O., D.A., and Lalonde, R. (2009). Bordeaux wine vintage quality and the weather. In Satchell, S., editor, *Collectible Investments for the High Net Worth Investor*, pages 233–244. Academic Press, Boston.

Ashenfelter, O. and Storchmann, K. (2010). Using Hedonic Models of Solar Radiation and Weather to Assess the Economic Effect of Climate Change: The Case of Mosel Valley Vineyards. *The Review of Economics and Statistics*, 92(2):333–349.

Ashenfelter, O. and Storchmann, K. (2016). Climate change and wine: A review of the economic implications. *Journal of Wine Economics*, 11(1):105–138.

Bargain, O., Cardebat, J.-M., and Chiappini, R. (2023). Trade uncorked: Genetic distance and taste-related barriers in wine trade. *American Journal of Agricultural Economics*, 105(2):674–708.

Bastos, P. and Silva, J. (2010). The quality of a firm's exports: Where you export to matters. *Journal of International Economics*, 82(2):99–111.

Boehm, C. E., Flaaen, A., and Pandalai-Nayar, N. (2019). Input Linkages and the Transmission of Shocks: Firm-Level Evidence from the 2011 Tōhoku Earthquake. *The Review of Economics and Statistics*, 101(1):60–75.

Chaney, T. (2008). Distorted gravity: the intensive and extensive margins of international trade. *American Economic Review*, 98(4):1707–1721.

Chen, N. and Juvenal, L. (2016). Quality, trade, and exchange rate pass-through. *Journal of International Economics*, 100:61–80.

Chen, N. and Juvenal, L. (2018). Quality and the great trade collapse. *Journal of Development Economics*, 135:59–76.

Chen, N. and Juvenal, L. (2022). Markups, quality, and trade costs. *Journal of International Economics*, 137:103627.

Constantinides, G. M. and Malliaris, A. G. (1995). Portfolio theory. *Handbooks in Operations Research and Management Science*, 9:1–30.

Costinot, A., Donaldson, D., and Smith, C. (2016). Evolving comparative advantage and the impact of climate change in agricultural markets: Evidence from 1.7 million fields around the world. *Journal of Political Economy*, 124(1):205–248.

Crozet, M., Head, K., and Mayer, T. (2012). Quality Sorting and Trade: Firm-level Evidence for French Wine. *The Review of Economic Studies*, 79(2):609–644.

Dallmann, I. (2019). Weather Variations and International Trade. *Environmental and Resource Economics*, 72:155–206.

De Sousa, J., Disdier, A.-C., and Gaigné, C. (2020). Export decision under risk. *European Economic Review*, 121:103342.

Dell, M., Jones, B., and Olken, B. (2014). What do we learn from the weather? the new climate-economy literature. *Journal of Economic Literature*, 52(3):740–798.

Dell, M., Jones, B. F., and Olken, B. A. (2009). Temperature and income: reconciling new cross-sectional and panel estimates. *American Economic Review*, 99(2):198–204.

Dell, M., Jones, B. F., and Olken, B. A. (2012). Temperature shocks and economic growth: Evidence from the last half century. *American Economic Journal: Macroeconomics*, 4(3):66–95.

Dubois, M. (2021). The market for wine quality evaluation: evolution and future perspectives. Technical Report 261, American Association of Wine Economists.

Emlinger, C. and Lamani, V. (2020). International trade, quality sorting and trade costs: the case of cognac. *Review of World Economics*, 156:354–385.

Esposito, F. (2022). Demand risk and diversification through international trade. *Journal of International Economics*, 135:103562.

Fontaine, F., Martin, J., and Méjean, I. (2020). Price discrimination within and across emu markets: Evidence from french exporters. *Journal of International Economics*, 124:103300.

Freund, C., Mattoo, A., Mulabdic, A., and Ruta, M. (2022). Natural disasters and the reshaping of global value chains. *IMF Economic Review*, 70:590–623.

Friedt, F. (2021). Natural disasters, aggregate trade resilience, and local disruptions: Evidence from hurricane katrina. *Review of International Economics*, 29(5):1081–1120.

Gassebner, M., Keck, A., and Teh, R. (2010). Shaken, not stirred: The impact of disasters on international trade. *Review of International Economics*, 18(2):351–368.

Hallak, J. and Schott, P. (2011). Estimating Cross-Country Differences in Product Quality. *The Quarterly Journal of Economics*, 126(1):417–474.

Helpman, E. and Razin, A. (2014). *A theory of international trade under uncertainty.* Academic Press.

Hochberg, U., Batushansky, A., Degu, A., Rachmilevitch, S., and Fait, A. (2014). Metabolic and Physiological Responses of Shiraz and Cabernet Sauvignon (Vitis vinifera L.) to Near Optimal Temperatures of 25 and 35° C. *International Journal of Molecular Sciences*, 16(10):24276–24294.

Hummels, D. and Skiba, A. (2004). Shipping the good apples out? an empirical confirmation of the alchian-allen conjecture. *Journal of Political Economy*, 112(6):1384–1402.

Huppertz, M. (2024). Sacking the sales staff: Firm reactions to extreme weather and implications for policy design. *mimeo*.

Ingersoll, J. E. (1987). *Theory of financial decision making*, volume 3. Rowman & Littlefield.

Jin, Y., Qu, R., and Atkin, J. (2016). Constrained portfolio optimisation: The state-of-the-art markowitz models. In *International Conference on Operations Research and Enterprise Systems*, volume 2, pages 388–395. SCITEPRESS.

Jones, B. and Olken, B. (2010). Climate shocks and exports. *American Economic Review*, 100(2):454–459.

Jones, G., White, M., Cooper, O., and Storchmann, K. (2005). Climate change and global wine quality. *Climatic Change*, 73:319–343.

Juvenal, L. and Santos Monteiro, P. (2023). Risky gravity. *Journal of the European Economic Association*, (jvad060).

Keane, M. and Neal, T. (2020). Climate change and u.s. agriculture: Accounting for multidimensional slope heterogeneity in panel data. *Quantitative Economics*, 11(4):1391–1429.

Kee, H. L., Nicita, A., and Olarreaga, M. (2008). Import Demand Elasticities and Trade Distortions. *The Review of Economics and Statistics*, 90(4):666–682.

Khandelwal, A., Schott, P., and Wei, S.-J. (2013). Trade liberalization and embedded institutional reform: Evidence from chinese exporters. *American Economic Review*, 103(6):2169–95.

Kucheryavyy, K. (2014). Comparative advantage and international risk sharing: Together at last.

Lecocq, S. and Visser, M. (2006). Spatial variations in weather conditions and wine prices in bordeaux. *Journal of Wine Economics*, 1(2):114–124.

Li, C., Xiang, X., and Gu, H. (2015). Climate shocks and international trade: Evidence from china. *Economics Letters*, 135:55–57.

Manova, K. and Yu, Z. (2017). Multi-product firms and product quality. *Journal of International Economics*, 109:116–137.

Manova, K. and Zhang, Z. (2012). Export Prices Across Firms and Destinations. *Quarterly Journal of Economics*, 127(1):379–436.

Martin, J. (2012). Markups, quality, and transport costs. *European Economic Review*, 56(4):777–791.

Martínez-Martínez, A., Esteve-Pérez, S., Gil-Pareja, S., and Llorca-Vivero, R. (2023). The impact of climate change on international trade: A gravity model estimation. *The World Economy*, 46(9):2624–2653.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6):1695–1725.

Miller, K. S. (1981). On the inverse of the sum of matrices. *Mathematics magazine*, 54(2):67–72.

Newbery, D. M. G. and Stiglitz, J. E. (1984). Pareto inferior trade. *The Review of Economic Studies*, 51(1):1–12.

Pagay, V. and Collins, C. (2017). Effects of timing and intensity of elevated temperatures on reproductive development of field-grown shiraz grapevines. *OENO One*, 51(4).

Pomery, J. (1980). A theory of international trade under uncertainty. elhanan helpman , assaf razin. *Journal of Political Economy*, 88(5):1061–1064.

Roberts, M., Schlenker, W., and Eyer, J. (2013). Agronomic weather measures in econometric models of crop yield with implications for climate change. *American Journal of Agricultural Economics*, 95(2):236–243.

Schlenker, W. and Roberts, M. (2009). Nonlinear temperature effects indicate severe damages to us crop yields under climate change. *Proceedings of the National Academy of sciences*, 106(37):15594–15598.

Snyder, R. (1985). Hand calculating degree days. *Agricultural and Forest Meteorology*, 35(1):353–358.

Van Leeuwen, C. and Darriet, P. (2016). The impact of climate change on viticulture and wine quality. *Journal of Wine Economics*, 11(1):150–167.

Volpe Martincus, C. and Blyde, J. (2013). Shaky roads and trembling exports: Assessing the trade effects of domestic infrastructure using a natural experiment. *Journal of International Economics*, 90(1):148–161.

Zappala, G. (2024). Estimating sectoral climate impacts in a global production network. *mimeo.*

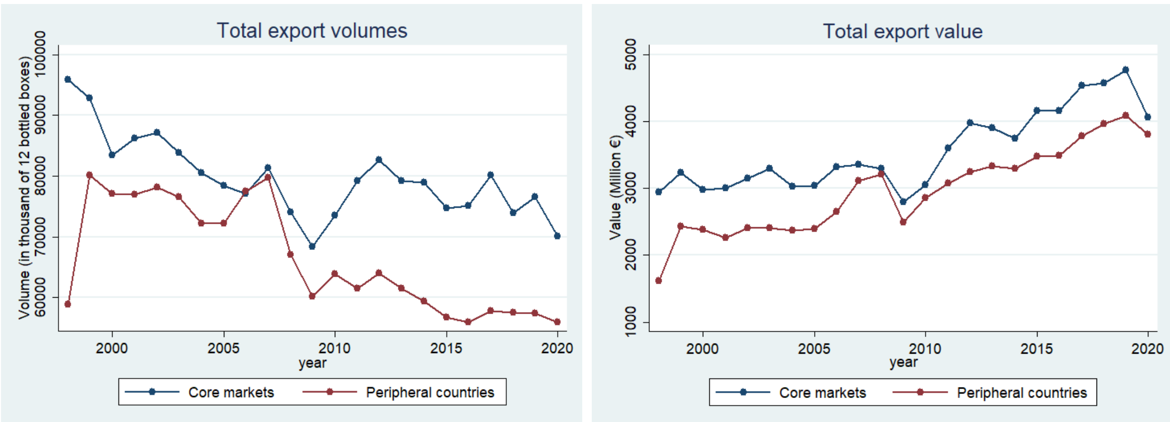
Appendix

A Description of variables and descriptive statistics

Table A1: Sample of importing countries

Argentina	Greece	Norway
Australia	Hong-Kong	Philippines
Austria	Ireland	Poland
Belgium	Israel	Portugal
Brazil	Italy	Germany
Cambodia	Ivory Coast	Russia
Cameroon	Japan	Singapore
Canada	Lebanon	South Africa
Chile	Luxembourg	South Korea
China	Malaysia	Spain
Colombia	Malta	Sweden
Cyprus	Mauritius	Switzerland
Czech Republic	Mexico	Thailand
Denmark	Morocco	Vietnam
Netherlands	United Kingdom	Finland
New Zealand	United States	Gabon
Nigeria		

Figure A1: Export volumes and value to core and peripheral markets



(a) Export volumes

(b) Export value

Table A2: Calculus formulas of weather indicators

Variables name	Descriptions	Sources
KDD/FDD	$\sum_{d=1}^{D} DD_C$, with: $DD_C = \begin{cases} 0 & \text{if } C > T_{Max} \\ T_{avg} - C & \text{if } C < T_{Min} \\ \frac{((T_{avg}-C)cos^{-1}(S)+(T_{Max}-T_{Min})sin\frac{(S)}{2})}{\pi^{-1}} & \text{otherwise} \end{cases}$	Keane and Neal (2020)

Figure A2: Dynamics of KDD (extremely high temperatures) in most impacted regions

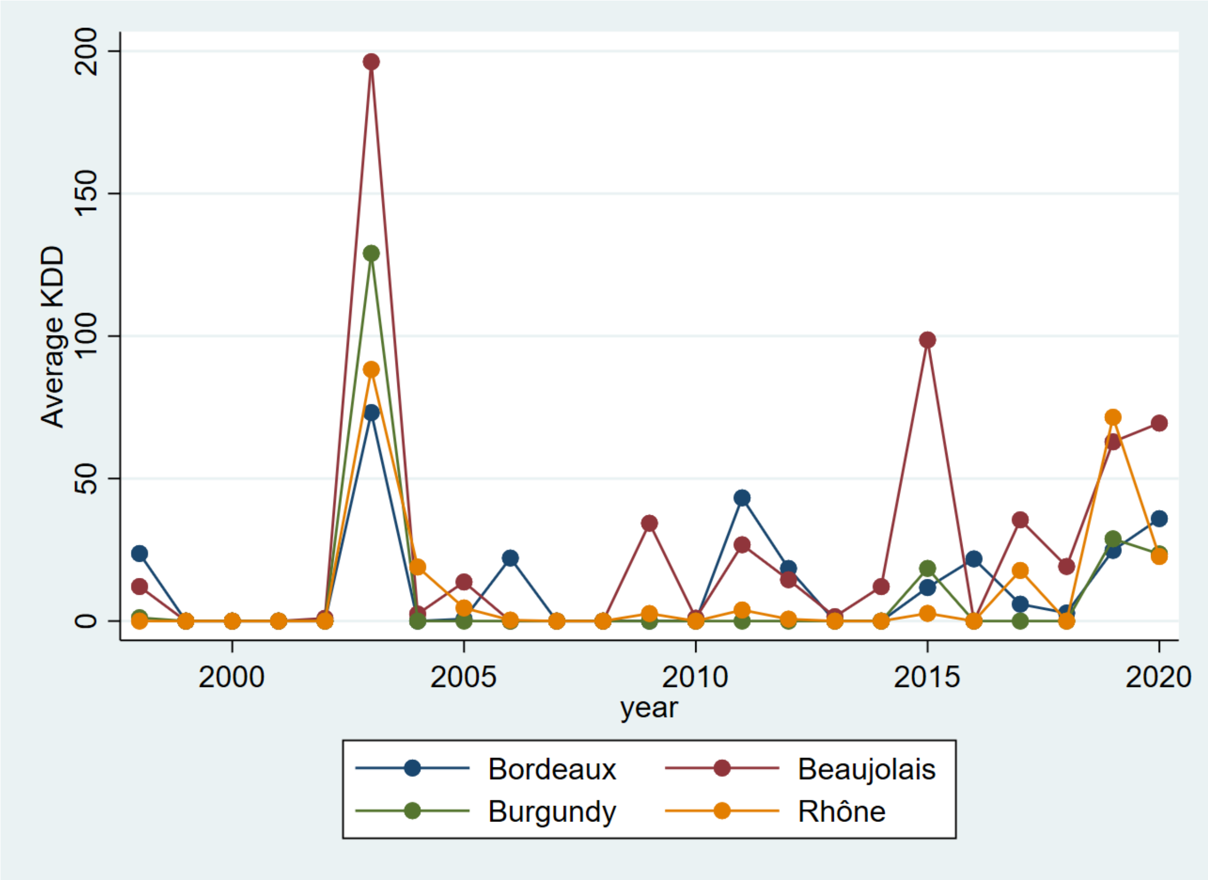


Table A3: Definition of variables

Name	Definition	Source
Mean consumption expenditure	Mean of wine consumption expenditure R over the previous 5 years	De Sousa et al. (2020) and OIV data
Excess of wine consumption expenditure volatility	Positive difference between destination country volatility computed as the standard deviation of yearly growth rates of wine consumption expenditure over 5-year rolling periods and French market volatility	De Sousa et al. (2020) and OIV data
Lower wine consumption expenditure volatility	Negative difference between destination country volatility computed as the standard deviation of yearly growth rates of wine consumption expenditure over 6-year rolling periods and French market volatility	De Sousa et al. (2020) and OIV data
Wine consumption expenditure skewness	Unbiased skewness of the wine consumption expenditure in the destination country	De Sousa et al. (2020) and OIV
Killing degree days (KDD)	Total killing degree days from January to August, base temperature of 35 °C	Keane and Neal (2020) and SAFRAN
Freezing degree days (FDD)	Total freezing degree days from January to March, base temperature of 0°C	Keane and Neal (2020) and SAFRAN
Yield production	Total harvest by PDO (in 1000 Hl)	Direction générale des Douanes et Droits indirects (D.G.D.D.I.)

Figure A3: Dynamics of KDD (extremely high temperatures) in other regions

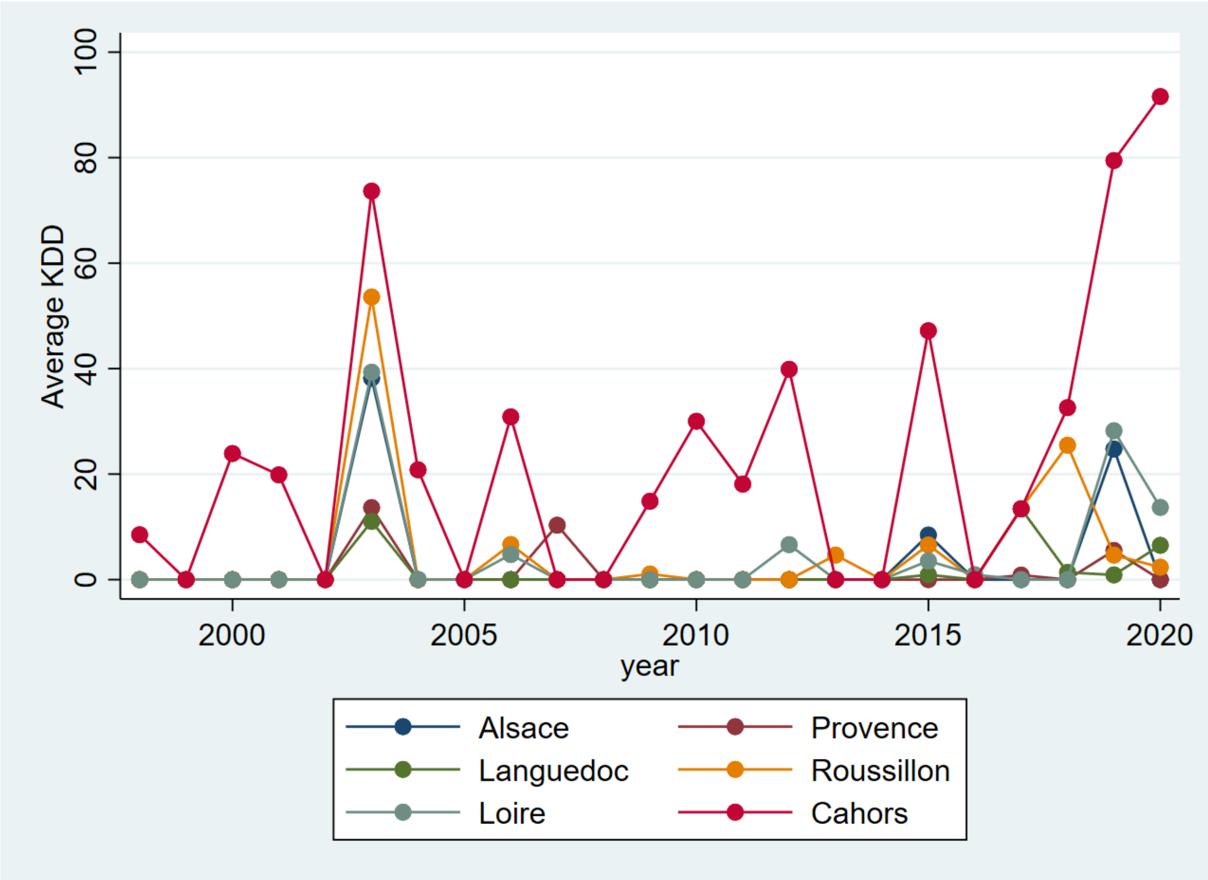


Table A4: Descriptive statistics

Variable	Obs	Mean	Std. dev.	Min	Max
$Ln(Volume_{jkt})$	66,289	6.880576	2.423812	0	15.27363
$Exit_{jkt}$	102,401	.0529194	.2238737	0	1
$Ln(Price_{jkt})$	66,289	-2.76902	.8294963	-6.507278	3.044522
$Ln\ Cons.\ Exp_{jt-1}$	102,401	6.694687	2.088651	1.335001	10.38574
$High * Exp.\ Vol_{jt}$	102,401	-2.422624	1.650582	-9.721178	0
$Low * Exp.\ Vol_{jt}$	102,401	-.7905111	1.77148	-7.572125	0
$Cons.Exp.\ Skew_{jt}$	102,401	.0795997	1.025541	-2.224161	2.20792
$Prod.\ Volatility_{kt}$	102,401	.21301	.2249992	.0089672	3.463856
$Avg.\ KDD_{kt}$	102,401	7.680082	9.44946	0	43.41286
$Avg.\ FDD_{kt}$	102,401	27.86451	25.58392	0	129.2354

Figure A4: Dynamics of FDD (extremely low temperatures) in most impacted regions

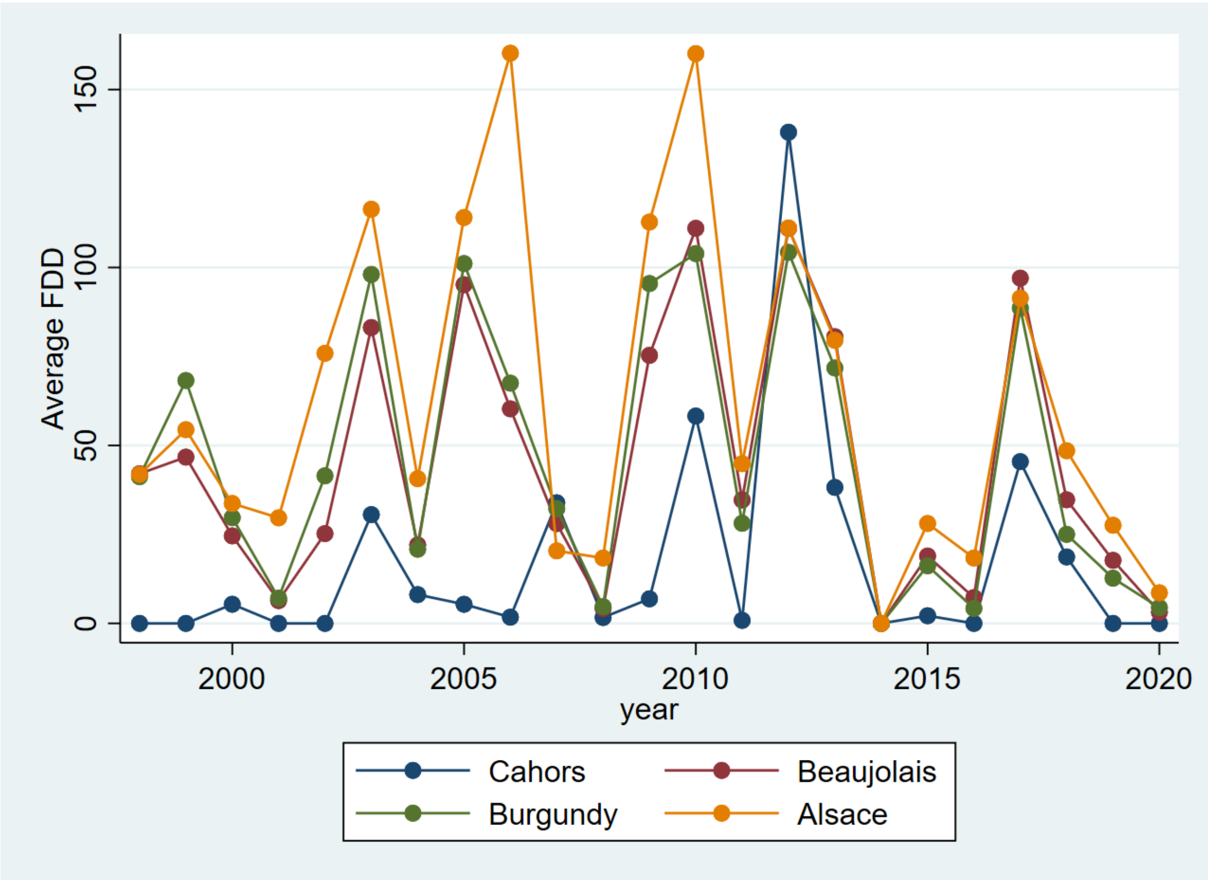


Figure A5: Dynamics of FDD (extremely low temperatures) in other regions

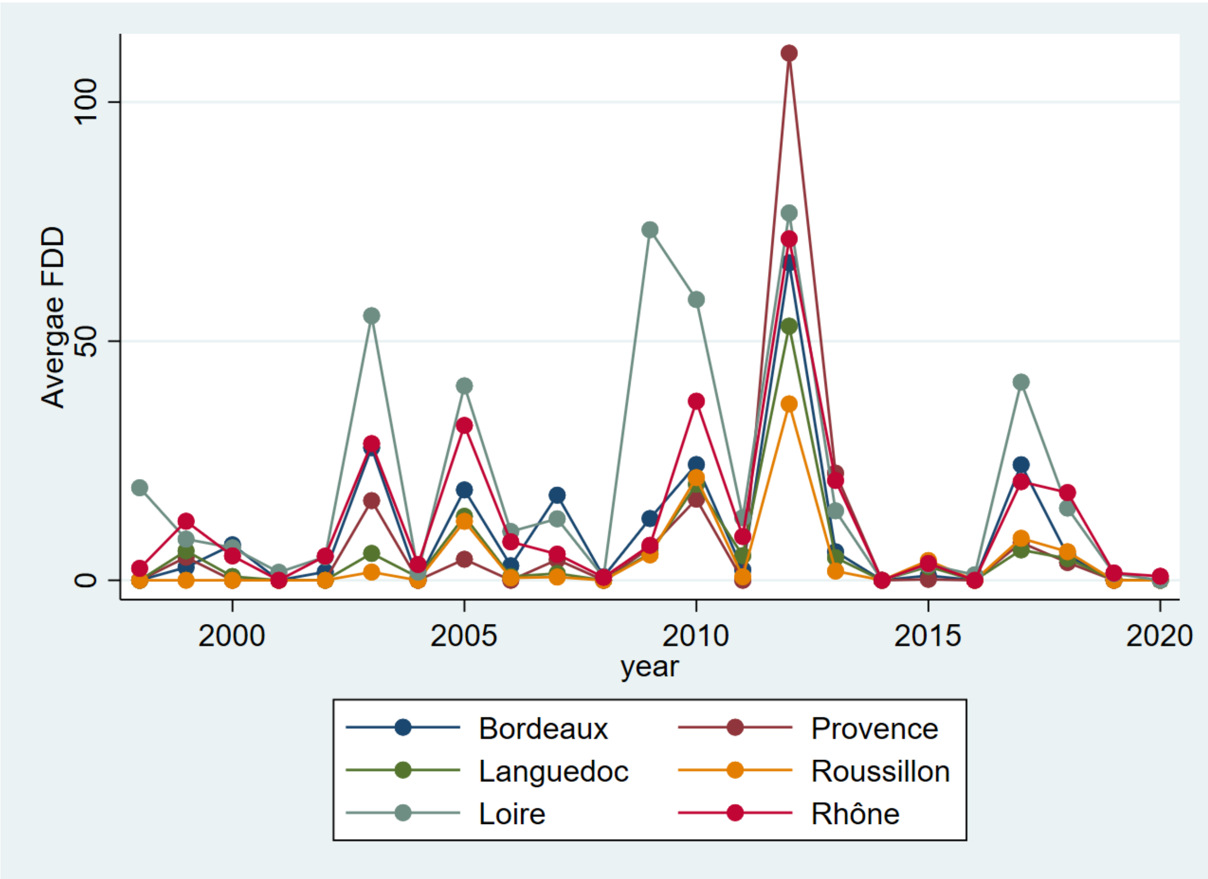


Table A5: Correlation matrix

	$Ln\ Cons.\ Exp_{jt-1}$	$High*\ Exp.\ Vol_{jt}$	$Low*\ Exp.\ Vol_{jt}$	$Cons.Exp.\ Skew_{jt}$	$Prod.\ Volatility_{kt}$	$Avg.\ KDD_{kt}$	$Avg.\ FDD_{kt}$
$Ln\ Cons.\ Exp_{jt-1}$	1.0000						
$High*\ Exp.\ Vol_{jt}$	-0.1724	1.0000					
$Low*\ Exp.\ Vol_{jt}$	-0.2702	-0.6550	1.0000				
$Cons.Exp.\ Skew_{jt}$	-0.0458	0.0862	-0.0291	1.0000			
$Prod.\ Volatilitykt$	0.0132	0.0030	-0.0293	-0.0104	1.0000		
$Avg.\ KDD_{kt}$	-0.0084	0.0207	-0.0264	-0.0058	0.1046	1.0000	
$Avg.\ FDD_{kt}$	0.0036	-0.0898	0.1116	-0.0045	0.0251	0.0286	1.0000

B Description of Khandelwal et al. (2013) method to infer quality

The literature has developed several methods in order to infer wine quality. First, a vast majority of research papers uses export unit values to evaluate product quality (Hummels and Skiba, 2004; Martin, 2012). However, this method is unsatisfactory, as export unit values may vary for other reasons than quality, such as firms' market power, differences in production costs, or differences in exchange rates (Hallak and Schott, 2011). The second method, widely used for the evaluation of wine quality, lies on the use of ratings from experts or guidebooks. For instance, Crozet et al. (2012) rely on Juhlin quality rating to evaluate French Champagne quality, while Chen and Juvenal (2016), Chen and Juvenal (2018) and Chen and Juvenal (2022) focus on the *Wine Spectator* magazine in order to infer quality of Argentinean wines. Bargain et al. (2023) use the scores from Robert Parker attributed to French regions and subregions each year as broad proxies for local quality. However, these ratings do not cover all wine appellations and only include 18 regions or subregions over the period 1998-2020[41]. As we are interested in a measure that is importing-country specific, we cannot follow this method. Consequently, we rely on the third method developed by Khandelwal et al. (2013) that relies on the extrapolation of quality based on the estimation of an empirical demand function. This allows to estimate the perceived quality of French wines and assess if consumers in destination markets distinguish French wine appellations. This method has been implemented, for instance, in Emlinger and Lamani (2020) to infer Cognac quality.

Following the methodology of Khandelwal et al. (2013), the quality of appellation k, exported by France to destination country j at time t is estimated as the residual of the following OLS regression:

$$ln\ Q_{jkt} + \sigma\ ln\ p_{jkt} = \nu_k + \nu_{jt} + \epsilon_{jkt} \tag{B1}$$

Q_{jkt} is the volume of appellation k exported to destination country j at time t, p_{jkt} is the price of the appellation k in market j at time t, ν_k represents appellation fixed effects that capture price and quantity differences between appellations, ν_{jt} represents time-varying destination country fixed effects that capture both the price index and the income level of the destination country and ϵ_{jkt} is the error term. Note that σ represents the elasticity of substitution, with $\sigma > 1$.

Thus, the inferred quality of exported wines is given by $\widehat{\lambda_{jkt}} = \frac{\widehat{\epsilon_{jkt}}}{\sigma-1}$. Following previous empirical studies such as Manova and Yu (2017), we set the value of σ to 5. As a result, we obtain an importer-appellation-year specific quality measure.

[41] See `https://www.robertparker.com/resources/vintage-chart`. for more details on vintage scores from the Parker rating.

C Description of the methodology of Schlenker and Roberts (2009)

Schlenker and Roberts (2009) measure heat exposure in degree days by quantifying the accumulation of heat above a specified temperature threshold. This agronomic unit is determined by implementing a sinusoidal function to capture daily temperature exposure above the specified threshold C (Snyder, 1985). The degree days are thereby calculated for each region r and each day d of the growing season t by applying the following formula:

$$DD_{r,d,t,C} = \begin{cases} 0 & \text{if } C \geq T_{Max} \\ T_{avg} - C & \text{if } C \leq T_{Min} \\ \frac{((T_{avg}-C)S+(T_{Max}-T_{Min})sin\frac{(S)}{2})}{\pi} & \text{otherwise} \end{cases}$$

where T_{Min} and T_{Max} are respectively the minimum and maximum temperature for each region r and day d of the growing season t, $T_{avg} = \frac{T_{Max}+T_{Min}}{2}$ and $S = cos^{-1}(\frac{2C-T_{Max}-T_{Min}}{T_{Max}-T_{Min}})$.

Once the degree days are obtained, we can determine the Growing Degree Days (GDD), the total accumulation of heat above the specified threshold C until reaching the bound of harmful temperature for crop development, whose accumulation of heat from it corresponds to Killing Degree Days (KDD). In our study, the base temperature enabling vine growth is 10°C, while the threshold separating GDD and KDD indicators is 35°C. This leads us to compute daily GDD and KDD using these formulas:

$$GDD_{r,d,t} = DD_{r,d,t,10} - DD_{r,d,t,35} \tag{D1}$$

$$KDD_{r,d,t} = DD_{r,d,t,35} \tag{D2}$$

Then, we aggregate the indicators to annual values by summing the daily values of the growing season from January 1^{st} to August 31^{st} to obtain $GDD_{r,t}$ and $KDD_{r,t}$.

D Robustness checks

Table D1: Results from the first-stage estimation

	$Volatility_{kt}$		
	(1)	(2)	(3)
$KD\bar{D}_{kt-4:t}$	0.00369***	0.00441***	
	(0.000148)	(0.000137)	
$FD\bar{D}_{kt-4:t}$			0.000978***
			(9.86e-05)
Intercept	0.184***	0.179***	0.186***
	(0.00118)	(0.00107)	(0.00283)
Observations	66,287	102,401	66,287
Appellations FE	YES	YES	YES
Destination-year FE	YES	YES	YES
Year FE	YES	YES	YES
R-squared	0.411	0.383	0.406

Robust standard errors, clustered at destination-grid level, in parentheses.

Column (1) reports the corresponding first-stage for the log export volumes in column (3) of Table 1.

Column (2) reports the corresponding first-stage for the probability of market exit in column (3) of Table 2.

Column (3) reports the first-stage for the regression of export prices in column (3) of Table 3.

Column (3) also reports the first-stage for the regression of inferred quality in column (1) of Table 4.

Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table D2: Results without including the skewness of wine consumption expenditure

Dependent variable:	Export volumes: $Ln(y_{jkt})$		Probability of exiting: $Prob(y_{jkt}=0 \mid y_{jkt-1}=1)$		Export prices: $Ln(p_{jkt})$	
	(1)	(2)	(3)	(4)	(5)	(6)
$Ln\ Cons.\ Expenditure_{jt-1}$	0.349***	0.352***	-0.00857***	-0.00849***	-0.0206	-0.0206
	(0.0354)	(0.0355)	(0.00327)	(0.00327)	(0.0140)	(0.0140)
$Higher * Ln\ Exp.\ Volatility_{jt}$	-0.0101*		-0.000646		0.00390**	
	(0.00517)		(0.000604)		(0.00194)	
$Lower * Ln\ Exp.\ Volatility_{jt}$	0.0346***	0.0334***	-0.00156**	-0.00162**	0.000755	0.000642
	(0.00615)	(0.00609)	(0.000633)	(0.000637)	(0.00203)	(0.00205)
$NoCore * Higher * Ln\ Exp.\ Volatility_{jt}$		-0.00688		-0.000502		0.00418**
		(0.00541)		(0.000630)		(0.00199)
$Core * Higher * Ln\ Exp.\ Volatility_{jt}$		-0.0265***		-0.00153**		0.00241
		(0.00762)		(0.000689)		(0.00248)
Observations	66,289	66,289	102,401	102,401	66,084	66,084
Destination FE	YES	YES	YES	YES	YES	YES
Appellation-Year FE	YES	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES	YES

Robust standard errors, clustered at destination-appellation level, in parentheses.

Regressions include a constant which is not reported in the Table.

Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table D3: Results using log differences to compute expenditure moments

Dependent variable:	Export volumes: $Ln(y_{jkt})$		Probability of exiting: $Prob(y_{jkt}=0 \mid y_{jkt-1}=1)$		Export prices: $Ln(p_{jkt})$	
	(1)	(2)	(3)	(4)	(5)	(6)
$Ln\ Cons.\ Expenditure_{jt-1}$	0.344***	0.349***	0.0523***	0.0523***	-0.0175	-0.0175
	(0.0353)	(0.0355)	(0.00670)	(0.00671)	(0.0142)	(0.0142)
$Higher * Ln\ Exp.\ Volatility_{jt}$	-0.0139***		-0.00409***		0.00435**	
	(0.00504)		(0.00101)		(0.00194)	
$Lower * Ln\ Exp.\ Volatility_{jt}$	0.0248***	0.0235***	0.000379	0.000350	-0.000169	-0.000185
	(0.00581)	(0.00576)	(0.00106)	(0.00106)	(0.00193)	(0.00194)
$Cons.ExpenditureSkweness_{jt}$	-0.000388	-2.72e-05	0.00110	0.00111	-0.00290	-0.00289
	(0.00489)	(0.00488)	(0.000997)	(0.000997)	(0.00188)	(0.00188)
$NoCore * Higher * Ln\ Exp.\ Volatility_{jt}$		-0.00961*		-0.00401***		0.00440**
		(0.00526)		(0.00104)		(0.00201)
$Core * Higher * Ln\ Exp.\ Volatility_{jt}$		-0.0350***		-0.00459***		0.00409*
		(0.00752)		(0.00126)		(0.00243)
Observations	66,289	66,289	102,401	102,401	66,084	66,084
Destination FE	YES	YES	YES	YES	YES	YES
Appellation-Year FE	YES	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES	YES

Robust standard errors, clustered at destination-appellation level, in parentheses.

Regressions include a constant which is not reported in the Table.

Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table D4: Results using the four-year moving average of extreme weather variables (excluding current year)

Dependent variable:	Export volumes: $Ln(y_{jkt})$		Probability of exiting: $Prob(y_{jkt}=0\vert y_{jkt-1}=1)$		Export prices: $Ln(p_{jkt})$		Inferred quality: $\widehat{\lambda_{jkt}}$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\widehat{Volatilitykt}$	-3.124***		0.0741*		-1.100***		-0.911**	
	(0.375)		(0.0379)		(0.319)		(0.390)	
$Core * \widehat{Volatilitykt}$		-0.390		0.384***		2.945		-0.333
		(1.376)		(0.0738)		(1.869)		(1.184)
$NoCore * \widehat{Volatilitykt}$		-3.539***		0.0363		-1.473***		-0.964**
		(0.444)		(0.0408)		(0.389)		(0.404)
F-test of excluded instruments	659.585***	236.344***	1400.418***	699.334***	73.554***	4.555***	73.554***	9.607***
Kleibergen-Paap rk LM statistic	474.712***	305.672***	851.860***	847.148***	73.600***	9.607***	73.600***	4.555***
Observations	66,287	66,287	102,401	102,401	66,287	66,287	66,287	66,287
Destination-year FE	YES	YES	YES	YES	YES	YES	YES	YES
Appellation FE	YES	YES	YES	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES	YES	YES	YES

In columns (1) to (4), the four-year moving average of KDD is used as instrument, while in columns (5) to (8), the four-year moving average of FDD is used as instrument.
Robust standard errors, clustered at destination-appellation level, in parentheses.
Regressions include a constant which is not reported in the Table.
Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table D5: Results using the threshold 36°C for the computation of the KDD

Dependent variable:	Export volumes: $Ln(y_{jkrt})$		Probability of exiting: $Prob(y_{jkt} = 0 \mid y_{jkt-1} = 1)$	
	(1)	(2)	(3)	(4)
$\widehat{Volatility}kt$	-3.924***		0.0729	
	(1.018)		(0.0781)	
$Core * \widehat{Volatility}kt$		4.326		0.243
		(13.69)		(0.188)
$NoCore * \widehat{Volatility}kt$		-5.209**		0.0523
		(2.352)		(0.0887)
F-test of excluded instrument	98.785***	0.323	284.927***	3.013
Kleibergen-Paap rk LM statistic	108.566***	0.698	298.663***	7.023***
Observations	66,287	66,287	102,401	102,401
Appellation FE	YES	YES	YES	YES
Destination-Year FE	YES	YES	YES	YES
Year FE	YES	YES	YES	YES

Robust standard errors, clustered at destination-appellation level, in parentheses.
Regressions include a constant which is not reported in the Table.
Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table D6: Results using the method of Schlenker and Roberts (2009) to compute the KDD indicator

Dependent variable:	Export volumes: $Ln(y_{jkrt})$		Probability of exiting: $Prob(y_{jkt} = 0 \mid y_{jkt-1} = 1)$	
	(1)	(2)	(3)	(4)
$\widehat{Volatility}kt$	-3.054**		0.0464	
	(1.186)		(0.0830)	
$Core * \widehat{Volatility}kt$		-4.998		-0.643
		(4.173)		(0.785)
$NoCore * \widehat{Volatility}kt$		-2.818**		0.129
		(1.307)		(0.121)
F-test of excluded instrument	55.148***	2.390**	200.659***	0.476
Kleibergen-Paap rk LM statistic	54.354***	4.616**	189.074***	0.942
Observations	66,287	66,287	102,401	102,401
Appellation FE	YES	YES	YES	YES
Destination-Year FE	YES	YES	YES	YES
Year FE	YES	YES	YES	YES

Robust standard errors, clustered at destination-appellation level, in parentheses.
Regressions include a constant which is not reported in the Table.
Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table D7: Demand uncertainty, production shocks and the intensive margin (using export values)

Dependent variable:	Export values: $\ln(v_{jkt})$				
	(1)	(2)	(3)	(4)	(5)
$LnCons.Expenditure_{jt-1}$	0.376***	0.379***		0.382***	
	(0.0420)	(0.0354)		(0.0355)	
$Higher * Ln\ Exp.\ Volatility_{jt}$	-0.00966	-0.00566			
	(0.00627)	(0.00498)			
$Lower * Ln\ Exp.\ Volatility_{jt}$	0.0348***	0.0360***		0.0347***	
	(0.00736)	(0.00586)		(0.00578)	
$Cons.ExpenditureSkweness_{jt}$	-7.00e-05	-3.82e-07		0.000276	
	(0.00594)	(0.00463)		(0.00462)	
$\widehat{Volatility}kt$	-3.112***		-3.171***		
	(0.351)		(0.321)		
$Core * Higher * Ln\ Exp.\ Volatility_{jt}$				-0.0239***	
				(0.00702)	
$NoCore * Higher * Ln\ Exp.\ Volatility_{jt}$				-0.00213	
				(0.00524)	
$Core * widehatVolatilitykt$					-1.092
					(1.287)
$NoCore * \widehat{Volatility}kt$					-3.509***
					(0.389)
F-test of excluded instruments	814.274***		795.030***		244.958***
Kleibergen-Paap rk LM statistic	583.757***		575.291***		272.079***
Observations	66,317	66,317	66,316	66,317	66,316
Destination FE	YES	YES	NO	YES	NO
Appellation FE	YES	NO	YES	NO	YES
Year FE	YES	YES	YES	YES	YES
Destination-year FE	NO	NO	YES	NO	YES
Appellation-year FE	NO	YES	NO	YES	NO

Note: Dependent variable is the logarithm of exported volumes.
Robust standard errors, clustered at destination-appellation level, in parentheses.
Regressions include a constant which is not reported in the Table.
Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table D8: Results using an alternative definition of core markets (including re-export platforms)

Dependent variable:	Export volumes: $Ln(y_{jkt})$		Probability of exiting: $Prob(y_{jkt}=0 \mid y_{jkt-1}=1)$		Export prices: $Ln(p_{jkt})$		Inferred quality: $\widehat{\lambda_{jkt}}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$Ln\ Cons.\ Expenditure_{jt-1}$	0.350***		-0.00854***		-0.0173		
	(0.0354)		(0.00327)		(0.0141)		
$Lower * Ln\ Exp.\ Volatility_{jt}$	0.0342***		-0.00159**		0.000606		
	(0.00612)		(0.000634)		(0.00203)		
$Cons.\ Expenditure\ Skewness_{jt}$	0.00375		0.000168		-0.00339*		
	(0.00490)		(0.000646)		(0.00189)		
$Core * Higher * Ln\ Exp.\ Volatility_{jt}$	-0.0236***		-0.00134**		-0.00181		
	(0.00663)		(0.000637)		(0.00250)		
$NoCore * Higher * Ln\ Exp.\ Volatility_{jt}$	-0.00628		-0.000489		0.00589***		
	(0.00557)		(0.000646)		(0.00200)		
$Core * \widehat{Volatility}kt$		-1.637		0.368***		2.688	-0.323
		(1.183)		(0.0648)		(1.698)	(1.128)
$NoCore * \widehat{Volatility}kt$		-2.530***		0.00626		-1.722***	-0.918**
		(0.401)		(0.0345)		(0.414)	(0.414)
F-test of excluded instruments		301.278***		51.424***		4.080***	4.080***
Kleibergen-Paap rk LM statistic		351.585***		69.190***		8.753***	8.753***
Observations	66,289	66,287	102,401	102,401	66,084	66,287	66,287
Destination FE	YES	NO	YES	NO	YES	NO	NO
Year FE	YES	YES	YES	YES	YES	YES	YES
Appellation FE	NO	YES	NO	YES	NO	YES	YES
Destination-Year FE	NO	YES	NO	YES	NO	YES	YES
Appellation-Year FE	YES	NO	YES	NO	YES	NO	NO

In columns (2) and (4), the four-year moving average of KDD is used as instrument, while in columns (6) and (7), the four-year moving average of FDD is used as instrument.
Robust standard errors, clustered at destination-appellation level, in parentheses.
Regressions include a constant which is not reported in the Table.
Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table D9: Results for perceived quality using an alternative level of elasticity of substitution

Dependent variable:	Inferred quality: $\widehat{\lambda_{jkt}}$ (1)	(2)
$Core * \widehat{Volatility}_{kt}$		-0.337
		(1.101)
$NoCore * \widehat{Volatility}_{kt}$		-0.901**
		(0.401)
$\widehat{Volatility}_{kt}$	-0.841**	
	(0.377)	
F-test of excluded instruments	114.329***	4.753***
Kleibergen-Paap rk LM statistic	113.369***	10.203***
Observations	66,287	66,287
Appellation FE	YES	YES
Destination-Year FE	YES	YES
Year FE	YES	YES

Robust standard errors, clustered at destination-appellation level, in parentheses.

Regressions include a constant which is not reported in the Table.

Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

E Proof of Proposition 2

The proof makes use of a result obtained by Miller (1981) which allows to invert the sum of two arbitrary non singular square matrices of the same dimension. For the simplicity of exposition, the proof considers only the case where the set of destination markets is $\mathcal{S} = \mathcal{N}$. The extension of the result to any other set of destination markets $\mathcal{S} \subseteq \mathcal{N}$ is straightforward. Consider the variance-covariance matrix $\tilde{\Sigma}_i$ given by:

$$\tilde{\Sigma}_i = \begin{pmatrix} a_{i1} & b_i & ... & ... & b_i \\ b_i & ... & b_i & ... & ... \\ ... & b_i & a_{ij} & b_i & ... \\ ... & ... & b_i & ... & b_i \\ b_i & ... & ... & b_i & a_{iN} \end{pmatrix}$$

where in order to simplify the notations, we denote the generic term of the diagonal as $a_{ij} \equiv \mathbb{V}(\tilde{\varepsilon}_{ij})$ given by (14) and all terms outside the diagonal, that actually share the same value, as $b_i \equiv \mathbb{C}ov\left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}\right)$ given by (13). Let us decompose $\tilde{\Sigma}_i$ into the sum of two matrices M_{i1} and M_{i2} as follows:

$$\tilde{\Sigma}_i = M_{i1} + M_{i2}$$

where

$$M_{i1} = \begin{pmatrix} a_{i1} - b_i & 0 & ... & ... & 0 \\ 0 & ... & 0 & ... & ... \\ ... & 0 & a_{ij} - b_i & 0 & ... \\ ... & ... & 0 & ... & 0 \\ 0 & ... & ... & 0 & a_{iN} - b_i \end{pmatrix} \text{ and } M_{i2} = b_i \begin{pmatrix} 1 & ... & ... & ... & 1 \\ 1 & ... & ... & ... & 1 \\ 1 & ... & ... & ... & 1 \\ 1 & ... & ... & ... & 1 \\ 1 & ... & ... & ... & 1 \end{pmatrix}.$$

Clearly, M_{i2} has rank 1. Moreover, as M_{i1} is diagonal, its inverse can be obtained straightforwardly. The following Lemma indicates how to obtain $\tilde{\Sigma}_i^{-1}$, using the above decomposition.

Lemma 4 (Miller (1981))**.** *Let M_{i1} and $\tilde{\Sigma}_i = M_{i1} + M_{i2}$ be non singular matrices where M_{i2} has rank* 1*. Then $TrM_{i2}M_{i1}^{-1} \neq -1$ and the inverse of $\tilde{\Sigma}_i = M_{i1} + M_{i2}$ is given by:*

$$\tilde{\Sigma}_i^{-1} = M_{i1}^{-1} - \frac{1}{1 + TrM_{i2}M_{i1}^{-1}} M_{i1}^{-1} M_{i2} M_{i1}^{-1}.$$

It follows that first M_{i1}^{-1} is given by:

$$M_{i1}^{-1} = \begin{pmatrix} \frac{1}{a_{i1} - b_i} & 0 & ... & ... & 0 \\ 0 & ... & 0 & ... & ... \\ ... & 0 & \frac{1}{a_{ij} - b_i} & 0 & ... \\ ... & ... & 0 & ... & 0 \\ 0 & ... & ... & 0 & \frac{1}{a_{iN} - b_i} \end{pmatrix}.$$

Moreover, the trace of $M_{i2}M_{i1}^{-1}$ is given by:

$$
\begin{aligned}
TrM_{i2}M_{i1}^{-1} &= b_i Tr \begin{pmatrix} 1 & ... & ... & ... & 1 \\ 1 & ... & ... & ... & 1 \\ 1 & ... & ... & ... & 1 \\ 1 & ... & ... & ... & 1 \\ 1 & ... & ... & ... & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{a_{i1}-b_i} & 0 & ... & ... & 0 \\ 0 & ... & 0 & ... & ... \\ ... & 0 & \frac{1}{a_{ij}-b_i} & 0 & ... \\ ... & ... & 0 & ... & 0 \\ 0 & ... & ... & 0 & \frac{1}{a_{iN}-b_i} \end{pmatrix} \\
&= b_i Tr \begin{pmatrix} \frac{1}{a_{i1}-b_i} & ... & \frac{1}{a_{ij}-b_i} & ... & \frac{1}{a_{iN}-b_i} \\ \frac{1}{a_{i1}-b_i} & ... & \frac{1}{a_{ij}-b_i} & ... & \frac{1}{a_{iN}-b_i} \\ \frac{1}{a_{i1}-b_i} & ... & \frac{1}{a_{ij}-b_i} & ... & \frac{1}{a_{iN}-b_i} \\ \frac{1}{a_{i1}-b_i} & ... & \frac{1}{a_{ij}-b_i} & ... & \frac{1}{a_{iN}-b_i} \\ \frac{1}{a_{i1}-b_i} & ... & \frac{1}{a_{ij}-b_i} & ... & \frac{1}{a_{iN}-b_i} \end{pmatrix} \\
&= b_i \sum_j \frac{1}{a_{ij}-b_i}.
\end{aligned}
$$

Furthermore, we have

$$
\begin{aligned}
\tilde{\Sigma}_i^{-1} &= M_{i1}^{-1} - \frac{1}{1+TrM_{i2}M_{i1}^{-1}} M_{i1}^{-1}M_{i2}M_{i1}^{-1} \\
&= \begin{pmatrix} \frac{1}{a_{i1}-b_i} & 0 & ... & ... & 0 \\ 0 & ... & 0 & ... & ... \\ ... & 0 & \frac{1}{a_{ij}-b_i} & 0 & ... \\ ... & ... & 0 & ... & 0 \\ 0 & ... & ... & 0 & \frac{1}{a_{iN}-b_i} \end{pmatrix} \\
&\quad - \frac{b_i}{1+b_i\sum_j \frac{1}{a_{ij}-b_i}} \begin{pmatrix} \frac{1}{a_{i1}-b_i} & 0 & ... & ... & 0 \\ 0 & ... & 0 & ... & ... \\ ... & 0 & \frac{1}{a_{ij}-b_i} & 0 & ... \\ ... & ... & 0 & ... & 0 \\ 0 & ... & ... & 0 & \frac{1}{a_{iN}-b_i} \end{pmatrix} \begin{pmatrix} \frac{1}{a_{i1}-b_i} & ... & \frac{1}{a_{ij}-b_i} & ... & \frac{1}{a_{iN}-b_i} \\ \frac{1}{a_{i1}-b_i} & ... & \frac{1}{a_{ij}-b_i} & ... & \frac{1}{a_{iN}-b_i} \\ \frac{1}{a_{i1}-b_i} & ... & \frac{1}{a_{ij}-b_i} & ... & \frac{1}{a_{iN}-b_i} \\ \frac{1}{a_{i1}-b_i} & ... & \frac{1}{a_{ij}-b_i} & ... & \frac{1}{a_{iN}-b_i} \\ \frac{1}{a_{i1}-b_i} & ... & \frac{1}{a_{ij}-b_i} & ... & \frac{1}{a_{iN}-b_i} \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{a_{i1}-b_i} & 0 & ... & ... & 0 \\ 0 & ... & 0 & ... & ... \\ ... & 0 & \frac{1}{a_{ij}-b_i} & 0 & ... \\ ... & ... & 0 & ... & 0 \\ 0 & ... & ... & 0 & \frac{1}{a_{iN}-b_i} \end{pmatrix} \\
&\quad - c_i \begin{pmatrix} \frac{1}{(a_{i1}-b_i)^2} & ... & \frac{1}{(a_{i1}-b_i)(a_{ij}-b_i)} & ... & \frac{1}{(a_{i1}-b_i)(a_{iN}-b_i)} \\ ... & ... & ... & ... & ... \\ \frac{1}{(a_{i1}-b_i)(a_{ij}-b_i)} & ... & \frac{1}{(a_{ij}-b_i)^2} & ... & \frac{1}{(a_{iN}-b_i)(a_{ij}-b_i)} \\ & ... & ... & ... & ... \\ \frac{1}{(a_{i1}-b_i)(a_{iN}-b_i)} & ... & \frac{1}{(a_{iN}-b_i)(a_{ij}-b_i)} & ... & \frac{1}{(a_{iN}-b_i)^2} \end{pmatrix}
\end{aligned}
$$

where

$$c_i = \frac{b_i}{1+b_i\sum_j \frac{1}{a_{ij}-b_i}}$$

It follows that $\tilde{\Sigma}_i^{-1}$ is symmetric and given by:

$$\tilde{\Sigma}_i^{-1} = \begin{pmatrix} \frac{1}{a_{i1}-b_i}\left(1-\frac{c_i}{a_{i1}-b_i}\right) & \dots & -\frac{c_i}{(a_{i1}-b_i)(a_{ij}-b_i)} & \dots & -\frac{c_i}{(a_{i1}-b_i)(a_{iN}-b_i)} \\ \dots & \dots & \dots & \dots & \dots \\ -\frac{c_i}{(a_{i1}-b_i)(a_{ij}-b_i)} & \dots & \frac{1}{a_{ij}-b_i}\left(1-\frac{c_i}{a_{ij}-b_i}\right) & \dots & -\frac{c_i}{(a_{iN}-b_i)(a_{ij}-b_i)} \\ & \dots & \dots & \dots & \dots \\ -\frac{c_i}{(a_{i1}-b_i)(a_{iN}-b_i)} & \dots & -\frac{c_i}{(a_{iN}-b_i)(a_{ij}-b_i)} & \dots & \frac{1}{a_{iN}-b_i}\left(1-\frac{c_i}{a_{iN}-b_i}\right) \end{pmatrix}.$$

In the particular case where all demand shocks follow the same distribution, that is for all j, $\bar{\alpha}_j = \bar{\alpha}$ and $\mathbb{V}(\alpha_j) = \sigma_\alpha^2$, then for all j, $a_{ij} = \mathbb{V}(\tilde{\varepsilon}_{ij}) = \left(1 + \frac{\mathbb{V}(\beta_i)}{\bar{\beta}_i^2}\right)\frac{\sigma_\alpha^2}{\bar{\alpha}^2} + \frac{\mathbb{V}(\beta_i)}{\bar{\beta}_i^2} \equiv a_i$ and thus $c_i = \frac{b_i}{1+\frac{Nb_i}{a_i-b_i}}$. In that case, all diagonal terms of $\tilde{\Sigma}_i^{-1}$ are equal to $\frac{1}{a_i-b_i}\left(1-\frac{c_i}{a_i-b_i}\right) = \frac{1}{a_i-b_i}\left(\frac{a_i+(N-2)b_i}{a_i+(N-1)b_i}\right)$ while all off diagonal terms are equal to $-\frac{c_i}{(a_i-b_i)^2}$.

The diversification index D_{ij} is the sum of all terms in line j in $\tilde{\Sigma}_i^{-1}$:

$$\begin{aligned} D_{ij} &= \sum_{k\neq j}\left(-\frac{c_i}{(a_{ik}-b_i)(a_{ij}-b_i)}\right) + \frac{1}{a_{ij}-b_i}\left(1-\frac{c_i}{a_{ij}-b_i}\right) \\ &= \frac{1}{a_{ij}-b_i}\left(1 - c_i\sum_k \frac{1}{a_{ik}-b_i}\right) \\ &= \frac{1}{a_{ij}-b_i}\left(1 - \frac{b_i}{1+b_i\sum_j\frac{1}{a_{ij}-b_i}}\sum_k\frac{1}{a_{ik}-b_i}\right) \\ &= \frac{1}{a_{ij}-b_i}\left(\frac{1}{1+b_i\sum_j\frac{1}{a_{ij}-b_i}}\right) \\ D_{ij} &= \frac{c_i}{(a_{ij}-b_i)\,b_i}. \end{aligned}$$

And the weight $\omega_{i,jk}$ represents the relative contribution of market k to D_{ij} : for $j \neq k$,

$$\omega_{i,jk} = \frac{\tilde{\Sigma}_{i,jk}^{-1}}{D_{ij}} = \frac{-\frac{c_i}{(a_{ik}-b_i)(a_{ij}-b_i)}}{\frac{c_i}{(a_{ij}-b_i)b_i}} = -\frac{b_i}{a_{ik}-b_i} < 0$$

and for $j = k$,

$$\omega_{i,jj} = \frac{\tilde{\Sigma}_{i,jj}^{-1}}{D_{ij}} = 1 + \sum_{l\neq j}\frac{b_i}{a_{il}-b_i} > 1$$

Using (14) and (13), we get:

$$\begin{aligned} D_{ij} &= \frac{1}{SCV_{\alpha_j}}\frac{1}{1+SCV_{\beta_i}\left(1+\sum_k\frac{1}{SCV_{\alpha_k}}\right)} \\ \omega_{i,jk} &= \begin{cases} -\frac{SCV_{\beta_i}}{SCV_{\alpha_k}(1+SCV_{\beta_i})} < 0 & \text{for } j \neq k \\ 1+\frac{SCV_{\beta_i}}{1+SCV_{\beta_i}}\sum_{l\neq j}\frac{1}{SCV_{\alpha_l}} > 1 & \text{for } j = k \end{cases} \end{aligned}$$

F Proof of Lemma 2

From Proposition 1, we have $n_{ij}(\varphi) = \frac{D_{ij}}{\gamma \bar{\varepsilon}_{ij} r_{ij}(\varphi)} \mathcal{C}_{ij}(\varphi)$ with

$$\mathcal{C}_{ij}(\varphi) = \sum_{k \in \mathcal{S}} \omega_{i,jk} \left(\frac{\bar{\varepsilon}_{ik} r_{ik}(\varphi) - w_k f_k L_k / P_i}{\bar{\varepsilon}_{ik} r_{ik}(\varphi)} \right).$$

Using $r_{ij}(\varphi) = \frac{1}{\sigma} \left(\frac{\sigma}{\sigma-1} \frac{w_i \tau_{ij}}{\varphi} \right)^{1-\sigma} \frac{A_j}{P_i} = \frac{A_j \tau_{ij}^{1-\sigma}}{\delta_i P_i} \varphi^{\sigma-1}$ where $\delta_i = \frac{1}{\sigma} \left(\frac{\sigma}{\sigma-1} w_i \right)^{\sigma-1}$ and replacing in the expression of $\mathcal{C}_{ij}(\varphi)$, we obtain:

$$\mathcal{C}_{ij}(\varphi) = 1 - \sum_{k \in \mathcal{S}} \omega_{i,jk} \left(\frac{w_k f_k L_k / P_i}{\bar{\varepsilon}_{ik} \frac{A_k \tau_{ik}^{1-\sigma}}{\delta_i P_i} \varphi^{\sigma-1}} \right) = 1 - \sum_{k \in \mathcal{S}} \omega_{i,jk} \left(\delta_i \frac{\Gamma_{ik}}{\varphi^{\sigma-1}} \right)$$

where the last expression follows from using Definition 3. Defining $\hat{\varphi}_{ij} = \left(\delta_i \sum_{k \in \mathcal{S}} \omega_{i,jk} \Gamma_{ik} \right)^{\frac{1}{\sigma-1}}$, it follows that $\mathcal{C}_{ij}(\varphi) = 1 - \left(\frac{\hat{\varphi}_{ij}}{\varphi} \right)^{\sigma-1}$. Hence, $\mathcal{C}_{ij}(\varphi) \geq 0$ and thus $n_{ij}(\varphi) \geq 0$ if and only if $\varphi \geq \hat{\varphi}_{ij}$.

It remains to check that $n_{ij}(\varphi) < 1$, i.e.

$$\frac{D_{ij}}{\gamma \bar{\varepsilon}_{ij} \frac{A_j \tau_{ij}^{1-\sigma}}{\delta_i P_i}} \frac{1 - \left(\frac{\hat{\varphi}_{ij}}{\varphi} \right)^{\sigma-1}}{\varphi^{\sigma-1}} < 1.$$

Observe that the function $\frac{1 - \left(\frac{\hat{\varphi}_{ij}}{\varphi} \right)^{\sigma-1}}{\varphi^{\sigma-1}}$ is maximized in $\varphi = 2^{\frac{1}{\sigma-1}} \hat{\varphi}_{ij}$ which yields

$$\gamma > \underline{\gamma} = \sup_{j \in \mathcal{S}} \frac{D_{ij}}{4 \bar{\varepsilon}_{ij} r_{ij} \left(\hat{\varphi}_{ij} \right)}.$$

G Proof of Lemma 3

We have

$$\underline{\nu}_{ij} = \frac{w_j f_j L_j}{P_i} - \bar{\varepsilon}_{ij} r_{ij} \left(1 - \gamma H_{ij} \right)$$

where

$$H_{ij} = \sum_{k \in \mathcal{S}} n_{ik} \bar{\varepsilon}_{ik} r_{ik} \mathbb{C}ov \left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik} \right).$$

Let us compute H_{ij} for $j \notin \mathcal{S}$:

$$\begin{aligned} H_{ij} &= \sum_{k \in \mathcal{S}} \frac{D_{ik} \mathcal{C}_{ik}}{\gamma \bar{\varepsilon}_{ik} r_{ik}} \bar{\varepsilon}_{ik} r_{ik} \mathbb{C}ov \left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik} \right) \\ &= \frac{SCV_{\beta_i}}{\gamma} \sum_{k \in \mathcal{S}} D_{ik} \mathcal{C}_{ik} \end{aligned}$$

as $\mathbb{C}ov \left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik} \right) = SCV_{\beta_i}$ for all $j \notin \mathcal{S}$ and for all $k \in \mathcal{S}$ because then $k \neq j$. Plugging this expression of H_{ij} in $\underline{\nu}_{ij}$, we get:

$$\underline{\nu}_{ij} = \frac{w_j f_j L_j}{P_i} - \bar{\varepsilon}_{ij} r_{ij} \left(1 - SCV_{\beta_i} \sum_{k \in \mathcal{S}} D_{ik} \mathcal{C}_{ik} \right)$$

Using $\Gamma_{ij} = \frac{w_j f_j L_j}{\tau_{ij}^{1-\sigma}\bar{\varepsilon}_{ij} A_j}$ and $r_{ij} = \frac{A_j \tau_{ij}^{1-\sigma}}{\delta_i P_i}\varphi^{\sigma-1}$ and replacing, we get

$$\begin{aligned}
\underline{\nu}_{ij} &= \frac{\Gamma_{ij}\tau_{ij}^{1-\sigma}\bar{\varepsilon}_{ij}A_j}{P_i} - \bar{\varepsilon}_{ij}\frac{A_j\tau_{ij}^{1-\sigma}}{\delta_i P_i}\varphi^{\sigma-1}\left(1 - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}\mathcal{C}_{ik}\right) \\
&= \bar{\varepsilon}_{ij}\frac{\tau_{ij}^{1-\sigma}A_j}{\delta_i P_i}\left[\delta_i\Gamma_{ij} - \left(\varphi^{\sigma-1} - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}\left(\varphi^{\sigma-1} - (\hat{\varphi}_{ik})^{\sigma-1}\right)\right)\right] \\
&= \bar{\varepsilon}_{ij}\frac{\tau_{ij}^{1-\sigma}A_j}{\delta_i P_i}\left[\delta_i\Gamma_{ij} - \left(\varphi^{\sigma-1}\left(1 - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}\right) + SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}\hat{\varphi}_{ik}^{\sigma-1}\right)\right]
\end{aligned}$$

Now using $\hat{\varphi}_{ik}^{\sigma-1} = \delta_i \sum_{l\in\mathcal{S}} \omega_{i,kl}\Gamma_{il}$ then

$$\begin{aligned}
\underline{\nu}_{ij} &= \bar{\varepsilon}_{ij}\frac{\tau_{ij}^{1-\sigma}A_j}{\delta_i P_i}\left[\delta_i\Gamma_{ij} - \left(\varphi^{\sigma-1}\left(1 - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}\right) + SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}\delta_i\sum_{l\in\mathcal{S}}\omega_{i,kl}\Gamma_{il}\right)\right] \\
&= \bar{\varepsilon}_{ij}\frac{\tau_{ij}^{1-\sigma}A_j}{\delta_i P_i}\left(1 - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}\right)\left[\delta_i\frac{\Gamma_{ij} - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}\sum_{l\in\mathcal{S}}\omega_{i,kl}\Gamma_{il}}{1 - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}} - \varphi^{\sigma-1}\right].
\end{aligned}$$

Note that

$$1 - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik} = \frac{1 + SCV_{\beta_i}}{1 + SCV_{\beta_i}\left(1 + \sum_{k\in\mathcal{S}}\frac{1}{SCV_{\alpha_k}}\right)} \in (0,1)$$

Let us denote

$$\left(\varphi_{ij}^*\right)^{\sigma-1} = \delta_i\frac{\Gamma_{ij} - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}\sum_{l\in\mathcal{S}}\omega_{i,kl}\Gamma_{il}}{1 - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}}$$

so that

$$\underline{\nu}_{ij} = \bar{\varepsilon}_{ij}\frac{\tau_{ij}^{1-\sigma}A_j}{\delta_i P_i}\left(1 - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}\right)\left[\left(\varphi_{ij}^*\right)^{\sigma-1} - \varphi^{\sigma-1}\right].$$

Clearly, $\underline{\nu}_{ij}$ is a continuous and decreasing function of φ that reaches 0 when $\varphi = \varphi_{ij}^*$.

Finally, we have

$$\begin{aligned}
\left(\varphi_{ij}^*\right)^{\sigma-1} &= \delta_i\frac{\Gamma_{ij} - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}\sum_{l\in\mathcal{S}}\omega_{i,kl}\Gamma_{il}}{1 - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}} \\
&= \delta_i\frac{\Gamma_{ij} - SCV_{\beta_i}\sum_{l\in\mathcal{S}}\Gamma_{il}\sum_{k\in\mathcal{S}} D_{ik}\omega_{i,kl}}{1 - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}}
\end{aligned}$$

by inverting the sum signs over k and over l. Note that

$$\begin{aligned}
\sum_{k\in\mathcal{S}} D_{ik}\omega_{i,kl} &= D_{il}\left(1 + 1 + \frac{SCV_{\beta_i}}{1 + SCV_{\beta_i}}\sum_{k\neq l}\frac{1}{SCV_{\alpha_k}}\right) - \sum_{k\neq l} D_{ik}\frac{SCV_{\beta_i}}{1 + SCV_{\beta_i}}\frac{1}{SCV_{\alpha_l}} \\
&= D_{il} \qquad \text{(G1)}
\end{aligned}$$

using Proposition 4. Hence,

$$\begin{aligned}
\left(\varphi_{ij}^{*}\right)^{\sigma-1} &= \delta_i \frac{\Gamma_{ij} - SCV_{\beta_i}\sum_{l\in\mathcal{S}} D_{il}\Gamma_{il}}{1 - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}} \\
&= \delta_i \frac{\Gamma_{ij}(1 - SCV_{\beta_i}\sum_{l\in\mathcal{S}} D_{il} + SCV_{\beta_i}\sum_{l\in\mathcal{S}} D_{il}) - SCV_{\beta_i}\sum_{l\in\mathcal{S}} D_{il}\Gamma_{il}}{1 - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}} \\
&= \delta_i \left[\Gamma_{ij} + \frac{SCV_{\beta_i}}{1 - SCV_{\beta_i}\sum_{k\in\mathcal{S}} D_{ik}} \sum_{l\in\mathcal{S}} D_{il}(\Gamma_{ij} - \Gamma_{il})\right] \\
&= \delta_i \left[\Gamma_{ij} + \frac{SCV_{\beta_i}}{1 + SCV_{\beta_i}} \sum_{l\in\mathcal{S}} \frac{\Gamma_{ij} - \Gamma_{il}}{SCV_{\alpha_l}}\right]
\end{aligned}$$

where the last line follows from using $D_{ij} = \frac{1}{SCV_{\alpha_j}} \frac{1}{1+SCV_{\beta_i}\left(1+\sum_{k\in\mathcal{S}}\frac{1}{SCV_{\alpha_k}}\right)}$.

H Proof of Proposition 4

We can express $V_i^*(\varphi) = \sum_{j\in\mathcal{S}(\varphi)} V_{ij}(\varphi)$ where,

$$V_{ij}(\varphi) = \mathbb{E}\left(\frac{\pi_{ij}(\varphi)}{P_i}\right) - \frac{\gamma}{2}\sum_{k\in\mathcal{S}(\varphi)} \mathbb{C}ov\left(\frac{\pi_{ij}(\varphi)}{P_i}, \frac{\pi_{ik}(\varphi)}{P_i}\right)$$

is the portion of indirect utility of real income made on market j and where $\mathcal{S}(\varphi)$ is composed of the l most attractive markets, for any $l = 1...N$. For $\varphi \leq \varphi_{i1}$, $\mathcal{S}(\varphi) = \varnothing$ and thus clearly $V_i^*(\varphi) = 0$. For $\varphi > \varphi_{i1}$, dropping the argument for simplicity, we have

$$\begin{aligned}
V_{ij} &= \left(\bar{\varepsilon}_{ij} r_{ij} - \frac{w_j f_j L_j}{P_i}\right) n_{ij} - \frac{\gamma}{2}\bar{\varepsilon}_{ij} r_{ij} n_{ij} \sum_{k\in\mathcal{S}(\varphi)} n_{ik}\bar{\varepsilon}_{ik} r_{ik} \mathbb{C}ov\left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}\right) \\
&= n_{ij}\left[\bar{\varepsilon}_{ij} r_{ij} - \frac{w_j f_j L_j}{P_i} - \frac{\gamma}{2}\bar{\varepsilon}_{ij} r_{ij} H_{ij}\right] \qquad \text{(H1)}
\end{aligned}$$

where H_{ij} for $j \in \mathcal{S}(\varphi)$ is given by:

$$H_{ij} = \sum_{k\in\mathcal{S}(\varphi)} n_{ik}\bar{\varepsilon}_{ik} r_{ik} \mathbb{C}ov\left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}\right). \qquad \text{(H2)}$$

Using $\Gamma_{ij} = \frac{w_j f_j L_j}{\tau_{ij}^{1-\sigma}\bar{\varepsilon}_{ij} A_j}$ and $r_{ij} = \frac{A_j \tau_{ij}^{1-\sigma}}{\delta_i P_i}\varphi^{\sigma-1}$, and replacing in (H1), we get:

$$\begin{aligned}
V_{ij} &= n_{ij}\left[\bar{\varepsilon}_{ij}\frac{A_j\tau_{ij}^{1-\sigma}}{\delta_i P_i}\varphi^{\sigma-1}\left(1 - \frac{\gamma}{2}H_{ij}\right) - \frac{\tau_{ij}^{1-\sigma}\bar{\varepsilon}_{ij} A_j}{P_i}\Gamma_{ij}\right] \\
&= \bar{\varepsilon}_{ij}\frac{A_j\tau_{ij}^{1-\sigma}}{\delta_i P_i} n_{ij}\left[\varphi^{\sigma-1}\left(1 - \frac{\gamma}{2}H_{ij}\right) - \delta_i\Gamma_{ij}\right] \\
&= \frac{1}{\gamma} D_{ij}\left[1 - \left(\frac{\hat{\varphi}_{ij}}{\varphi}\right)^{\sigma-1}\right] \underbrace{\left[1 - \frac{\gamma}{2}H_{ij} - \frac{\delta_i\Gamma_{ij}}{\varphi^{\sigma-1}}\right]}_{(1)} \qquad \text{(H3)}
\end{aligned}$$

where the last line follows from using $n_{ij}(\varphi) = \frac{D_{ij}}{\gamma \bar{\varepsilon}_{ij} r_{ij}(\varphi)} \mathcal{C}_{ij}(\varphi)$ and $\mathcal{C}_{ij}(\varphi) = 1 - \left(\frac{\hat{\varphi}_{ij}}{\varphi}\right)^{\sigma-1}$.

In the rest of the proof, we propose to evaluate separately the term (1) in (H3) to obtain the desired formulation for $V_i^*(\varphi)$. First, using $n_{ij}(\varphi) = \frac{D_{ij}}{\gamma \bar{\varepsilon}_{ij} r_{ij}(\varphi)} \mathcal{C}_{ij}(\varphi)$, (H2) becomes:

$$
\begin{aligned}
H_{ij} &= \frac{1}{\gamma} \sum_{k \in \mathcal{S}(\varphi)} D_{ik} \mathcal{C}_{ik} \mathbb{C}ov\left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}\right) \\
&= \frac{1}{\gamma} D_{ij} \mathcal{C}_{ij} \mathbb{V}\left(\tilde{\varepsilon}_{ij}\right) + \frac{1}{\gamma} \sum_{\substack{k \in \mathcal{S}(\varphi) \\ k \neq j}} D_{ik} \mathcal{C}_{ik} \mathbb{C}ov\left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}\right)
\end{aligned}
$$

Using Lemma 1, we further get:

$$
\begin{aligned}
H_{ij} &= \frac{1}{\gamma} D_{ij} \mathcal{C}_{ij} \left[SCV_{\beta_i} + (1 + SCV_{\beta_i}) SCV_{\alpha_j}\right] + \frac{1}{\gamma} SCV_{\beta_i} \sum_{\substack{k \in \mathcal{S}(\varphi) \\ k \neq j}} D_{ik} \mathcal{C}_{ik} \\
&= \frac{1}{\gamma} D_{ij} \mathcal{C}_{ij} (1 + SCV_{\beta_i}) SCV_{\alpha_j} + \frac{1}{\gamma} SCV_{\beta_i} \sum_{k \in \mathcal{S}(\varphi)} D_{ik} \mathcal{C}_{ik}
\end{aligned}
$$

Hence, term (1) in (H3) becomes:

$$
\begin{aligned}
1 - \frac{\gamma}{2} H_{ij} - \frac{\delta_i \Gamma_{ij}}{\varphi^{\sigma-1}} &= 1 - \frac{\gamma}{2} \left[\frac{1}{\gamma} D_{ij} \mathcal{C}_{ij} (1 + SCV_{\beta_i}) SCV_{\alpha_j} + \frac{1}{\gamma} SCV_{\beta_i} \sum_{k \in \mathcal{S}(\varphi)} D_{ik} \mathcal{C}_{ik}\right] - \frac{\delta_i \Gamma_{ij}}{\varphi^{\sigma-1}} \\
&= 1 - \frac{1}{2} D_{ij} \left(1 - \left(\frac{\hat{\varphi}_{ij}}{\varphi}\right)^{\sigma-1}\right) (1 + SCV_{\beta_i}) SCV_{\alpha_j} \\
&\quad - \frac{1}{2} SCV_{\beta_i} \sum_{k \in \mathcal{S}(\varphi)} D_{ik} \left(1 - \left(\frac{\hat{\varphi}_{ik}}{\varphi}\right)^{\sigma-1}\right) - \frac{\delta_i \Gamma_{ij}}{\varphi^{\sigma-1}} \\
&= \underbrace{1 - \frac{1}{2} D_{ij} (1 + SCV_{\beta_i}) SCV_{\alpha_j} - \frac{1}{2} SCV_{\beta_i} \sum_{k \in \mathcal{S}(\varphi)} D_{ik}}_{(*)} \\
&\quad + \frac{1}{2} D_{ij} \left(\frac{\hat{\varphi}_{ij}}{\varphi}\right)^{\sigma-1} (1 + SCV_{\beta_i}) SCV_{\alpha_j} + \frac{1}{2} SCV_{\beta_i} \sum_{k \in \mathcal{S}(\varphi)} D_{ik} \left(\frac{\hat{\varphi}_{ik}}{\varphi}\right)^{\sigma-1} \\
&\quad - \frac{\delta_i \Gamma_{ij}}{\varphi^{\sigma-1}} \qquad \text{(H4)}
\end{aligned}
$$

Observe that the first term (*) in (H4) reduces simply to:

$$
1 - \frac{1}{2} D_{ij} (1 + SCV_{\beta_i}) SCV_{\alpha_j} - \frac{1}{2} SCV_{\beta_i} \sum_{k \in \mathcal{S}(\varphi)} D_{ik} = \frac{1}{2}
$$

Hence, (H4) becomes

$$\begin{aligned}1-\frac{\gamma}{2}H_{ij}-\frac{\delta_i\Gamma_{ij}}{\varphi^{\sigma-1}} &= \frac{1}{2}+\frac{1}{2\varphi^{\sigma-1}}D_{ij}\left(\hat{\varphi}_{ij}\right)^{\sigma-1}(1+SCV_{\beta_i})SCV_{\alpha_j} \\ &\quad +\frac{1}{2\varphi^{\sigma-1}}SCV_{\beta_i}\sum_{k\in\mathcal{S}(\varphi)}D_{ik}\left(\hat{\varphi}_{ik}\right)^{\sigma-1}-\frac{\delta_i\Gamma_{ij}}{\varphi^{\sigma-1}} \\ &= \frac{1}{2}+\frac{\delta_i}{2\varphi^{\sigma-1}}(D_{ij}(1+SCV_{\beta_i})SCV_{\alpha_j}\sum_{k\in\mathcal{S}(\varphi)}\omega_{i,jk}\Gamma_{ik} \\ &\quad +SCV_{\beta_i}\sum_{k\in\mathcal{S}(\varphi)}D_{ik}\sum_{l\in\mathcal{S}(\varphi)}\omega_{i,kl}\Gamma_{il})-\frac{\delta_i\Gamma_{ij}}{\varphi^{\sigma-1}} \qquad \text{(H5)}\end{aligned}$$

By inverting the sum signs, the term between brackets in (H5) can be rewritten as:

$$\begin{aligned}& D_{ij}(1+SCV_{\beta_i})SCV_{\alpha_j}\sum_{k\in\mathcal{S}(\varphi)}\omega_{i,jk}\Gamma_{ik}+SCV_{\beta_i}\sum_{l\in\mathcal{S}(\varphi)}\Gamma_{il}\sum_{k\in\mathcal{S}(\varphi)}D_{ik}\omega_{i,kl} \\ =\ & D_{ij}(1+SCV_{\beta_i})SCV_{\alpha_j}\sum_{k\in\mathcal{S}(\varphi)}\omega_{i,jk}\Gamma_{ik}+SCV_{\beta_i}\sum_{l\in\mathcal{S}(\varphi)}\Gamma_{il}D_{il} \qquad \text{(H6)}\end{aligned}$$

where the last line follows from using (G1). Using Proposition 2 and rearranging, (H6) further simplifies into:

$$\begin{aligned}& \frac{(1+SCV_{\beta_i})}{1+SCV_{\beta_i}\left(1+\sum_{k\in\mathcal{S}(\varphi)}\frac{1}{SCV_{\alpha_k}}\right)}\left[\Gamma_{ij}+\frac{SCV_{\beta_i}}{1+SCV_{\beta_i}}\sum_{k\in\mathcal{S}(\varphi)}\frac{\Gamma_{ij}-\Gamma_{ik}}{SCV_{\alpha_k}}\right] \\ & +\frac{SCV_{\beta_i}}{1+SCV_{\beta_i}\left(1+\sum_{k\in\mathcal{S}(\varphi)}\frac{1}{SCV_{\alpha_k}}\right)}\sum_{l\in\mathcal{S}(\varphi)}\frac{\Gamma_{il}}{SCV_{\alpha_l}} \\ =\ & \Gamma_{ij}\end{aligned}$$

Hence, (H5) becomes:

$$1-\frac{\gamma}{2}H_{ij}-\frac{\delta_i\Gamma_{ij}}{\varphi^{\sigma-1}}=\frac{1}{2}\left(1-\frac{\delta_i\Gamma_{ij}}{\varphi^{\sigma-1}}\right).$$

Using the above result, (H3) becomes:

$$V_{ij}=\frac{1}{2\gamma}D_{ij}\left(1-\left(\frac{\hat{\varphi}_{ij}}{\varphi}\right)^{\sigma-1}\right)\left(1-\frac{\delta_i\Gamma_{ij}}{\varphi^{\sigma-1}}\right),$$

we obtain the desired formulation for $V_i^*(\varphi)$ as follows:

$$V_i^*(\varphi)=\sum_{j\in\mathcal{S}(\varphi)}V_{ij}=\frac{1}{2\gamma}\sum_{j\in\mathcal{S}(\varphi)}D_{ij}\left(1-\left(\frac{\hat{\varphi}_{ij}}{\varphi}\right)^{\sigma-1}\right)\left(1-\frac{\delta_i\Gamma_{ij}}{\varphi^{\sigma-1}}\right).$$

Observe that for an optimal portfolio with cardinal $|\mathcal{S}(\varphi)|=l$, then $\varphi_{il}=\max_{j\in\mathcal{S}(\varphi)}\hat{\varphi}_{ij}>(\delta_i\Gamma_{ij})^{\frac{1}{\sigma-1}}$ because $\hat{\varphi}_{ij}=\delta_i\left[\Gamma_{ij}+\frac{SCV_{\beta_i}}{1+SCV_{\beta_i}}\sum_{k\in\mathcal{S}(\varphi)}\frac{\Gamma_{ij}-\Gamma_{ik}}{SCV_{\alpha_k}}\right]>\delta_i\Gamma_{ij}$ when $\Gamma_{ij}\geq\Gamma_{ik}$ for all $k\in\mathcal{S}(\varphi)$. As $\varphi\geq\varphi_{il}$ this ensures that $V_i^*(\varphi)$ is strictly positive, because $\varphi\geq\hat{\varphi}_{ij}$ for all $j\in\mathcal{S}(\varphi)$ and $\varphi_{il}>(\delta_i\Gamma_{ij})^{\frac{1}{\sigma-1}}$.

Online Appendix

A List of Appellations (PDOs) in the sample

PDO
Alsace
Alsace
Alsace (Pinot noir rouges et rosés) rb
Alsace Edelzwicker, Gentil et sans mention de cépages bb
Alsace Gewurztraminer bb
Alsace Grand Cru bb
Alsace Pinot blanc bb
Alsace Pinot gris bb
Alsace Riesling bb
Alsace Sylvaner bb
Autres Alsace blancs bb
Crémant d'Alsace
Vendanges tardives et Sélection de grains nobles (Alsace et Grand Cru) bb
Beaujolais
Autres Beaujolais rb
Beaujolais (hors Nouveaux ou Primeurs) et Beaujolais Supérieur, rosés rb
Beaujolais (hors Nouveaux ou Primeurs) et Beaujolais supérieur, rouges rb
Beaujolais et Beaujolais Villages Nouveaux ou Primeurs, rosés rb
Beaujolais et Beaujolais Villages Nouveaux ou Primeurs, rouges rb
Beaujolais Villages (hors Nouveaux ou Primeurs), rosés rb
Beaujolais Villages (hors Nouveaux ou Primeurs), rouges rb
Crus du Beaujolais rb
Bordeaux
Autres blancs doux bb (y compris Graves Supérieures)
Autres Bordeaux bb
Autres Bordeaux rb
Autres Libournais rb
Autres vins blancs secs bb (Côtes de Blaye, ...)
Bergerac et Duras Btl (<2 litres) bb
Bergerac et Duras Btl (<2 litres) rb
Bordeaux avec sucres (5-60g) bb
Bordeaux rb
Bordeaux rosé et Clairet rb
Bordeaux Supérieur rb
Côtes de Bordeaux, Côtes de Bourg, Graves de Vayres rb
Communales du Médoc rb
Entre-deux-mers bb (y.c. Haut Benauge)
Graves et Pessac-Léognan bb (non compris Graves Supérieures)
Graves et Pessac-Léognan rb
Médoc et Haut-Médoc rb
Saint-Emilion et Saint-Emilion Grand Cru rb
Sauternes et Barsac bb
Sud-Ouest Btl (<2 litres) bb
Sud-Ouest Btl (<2 litres) rb
Burgundy
Autres Bourgogne bb
Autres Bourgogne rb
Chablis et Petit Chablis bb
Chablis Grands Crus et Premiers Crus bb
Crémant de Bourgogne
Crus du Mâconnais bb
Grands Crus de la Côte d'Or bb
Grands Crus de la Côte d'Or rb
Régionales Bourgogne bb
Régionales de Bourgogne rb
Régionales Mâcon bb
Régionales Mâcon rb
Villages de l'Auxerrois-Tonnerrois bb
Villages de l'Auxerrois-Tonnerrois rb
Villages et Premiers Crus de la Côte Chalonnaise bb
Villages et Premiers Crus de la Côte Chalonnaise rb
Villages et Premiers Crus de la Côte d'Or bb

PDO
Villages et Premiers Crus de la Côte de Beaune rb
Villages et Premiers Crus de la Côte de Nuits rb
Cahors
Cahors Btl (<2 litres) rb
Languedoc
Autres AOP du Languedoc Roussillon rb
Autres Languedoc Roussillon rb
Corbières rb
Faugères rb
Fitou rb
Languedoc et dénominations blanc (y compris Picpoul de Pinet) bb
Languedoc et dénominations rouge rb
Languedoc Roussillon bb
Roussillon bb
Roussillon rb
Saint-Chinian rb
Loire
Anjou rouge et Anjou Villages rb
Anjou, Saumur et Savennières bb
Autres AOP blancs du Val de Loire bb
Autres AOP rouges et rosés du Val de Loire rb
Autres Val de Loire bb
Bourgueil et Saint Nicolas de Bourgueil rb
Cabernet d'Anjou rb
Chinon rb
Coteaux du Layon et autres vins moelleux bb
Crémant de Loire
Muscadet bb
Rosé d'Anjou rb
Rosé d'Anjou et Cabernet d'Anjou rb
Sancerre bb
Sancerre rb
Saumur
Saumur rouge et Saumur Champigny rb
Touraine (avec ou sans nom de commune) bb
Touraine (avec ou sans nom de commune) rb
Vouvray
Vouvray bb
Provence
Provence rosé (yc Côtes de Provence, Ctx d'Aix, Ctx varois, Bandol, etc) rb
Provence rouge (yc Côtes de Provence, Ctx d'Aix, Ctx varois, Bandol, etc) rb
Rhône
AOP locales septentrionales (Côtes-Rôtie, Cornas, Crozes-Hermitage, Hermitage, Saint-Joseph) rb
Autres AOP locales méridionales (Cairanne, Gigondas, Lirac, Tavel, Vacqueyras, Beaumes de Venise, Vinsobres, Rasteau) rb
Autres AOP Vallée du Rhône bb
Autres Côtes du Rhône rb
Côtes du Rhône (régionales, villages et crus) bb
Côtes du Rhône (régionaux) rb
Côtes du Rhône Villages (Communaux ou non) rb
Châteauneuf-du-Pape rb
Clairette et Crémant de Die
Costières de Nîmes rb
Grignan-les-Adhémar (Ctx Tricastin) rb
Luberon rb
Ventoux rb
Roussillon
Limoux
Muscat de Rivesaltes Btle
Rivesaltes Btle

B Trade equilibrium

In this Appendix, we describe the general trade equilibrium corresponding to the partial equilibrium framework developed in the model. To do this, let us close the model by adding the equations that help to determine the general equilibrium in terms of the price index P_i, the national income Y_i and the wage rate w_i for any country i. Aggregate sales from origin country i to destination country j are:

$$X_{ij} = M_i \int_0^{\infty} \mathbb{E}\left[p_{ij}(\varphi) q_{ij}(\varphi)\right] dG_i(\varphi)$$

and it also represents the total expenditures in country j made on varieties from origin country i. As in Chaney (2008), the mass of firms is fixed and thus there are profits at the equilibrium in the economy given by:

$$\Pi_i = M_i \sum_j \int_0^{\infty} \mathbb{E}\left[\pi_{ij}(\varphi)\right] dG_i(\varphi). \tag{OB1}$$

The current account has to be balanced so that the total expenditures in each country has to be equal to the labor income plus business profits:

$$\sum_k X_{ki} = Y_i = w_i \tilde{L}_i + \Pi_i. \tag{OB2}$$

Finally, the labor market clears and thus the labor supply in origin country i must equal the amount of labor used in domestic production and in marketing (paid by foreign firms employing home workers). Using (5) and (6) yields:

$$M_i \sum_j \int_0^{\infty} \mathbb{E}\left[l_{ij}(\varphi)\right] dG_i(\varphi) + \sum_j M_j \int_0^{\infty} f_i L_i n_{ji}(\varphi) dG_j(\varphi) = \tilde{L}_i. \tag{OB3}$$

The general trade equilibrium is characterized by a vector of wages $\{w_i\}$, a vector of price indexes $\{P_i\}$, and national income $\{Y_i\}$ that solve the system of equations (4), (OB2) and (OB3) where $p_{ij}(\varphi)$ maximizes (8) and $n_{ij}(\varphi)$ is the solution of maximization problem (7).

C Alternative timing

In this Appendix, we examine an alternative timing where as in De Sousa et al. (2020) production takes place before demand shocks are realized but after production shocks are realized. In the context of wine production, this means that once quality and cost shocks are known at the grapes harvest time, then production takes place while not knowing the demand conditions that will occur later when the wine is ready for selling. We assume that the winery owner commits to send the quantity produced to the destination market as scheduled, but can adjust the price according to the demand conditions. Finally, marketing investments have still to be decided before all shocks as in the baseline model.

Solving the game using backward induction, at the last stage, the price p_{ij} is decided while the quantity produced is already fixed to $\hat{q}_{ij}$ as well as the set of destination countries (and their n_{ij}). It follows that once demand shock is known, the firm adjusts the price p_{ij} to equalize the quantity produced with the quantity realized:

$$q_{ij} = \alpha_j \eta_i^{\sigma-1} n_{ij} A_j p_{ij}^{-\sigma} = \hat{q}_{ij} \tag{OC1}$$

Before the demand shock is known but after η_i and θ_i are known, the firm has to produce to maximize its expected gross profit in each destination:

$$\max_{p_{ij}} \bar{\alpha}_j \eta_i^{\sigma-1} n_{ij} A_j p_{ij}^{-\sigma} (p_{ij} - \theta_i \frac{w_i \tau_{ij}}{\varphi})$$

and this yields $\hat{p}_{ij} = \frac{\sigma}{\sigma-1}\theta_i \frac{w_i \tau_{ij}}{\varphi}$ and hence $\hat{q}_{ij} = \bar{\alpha}_j \eta_i^{\sigma-1} n_{ij} A_j \hat{p}_{ij}^{-\sigma}$. Hence the price realized p_{ij} is given by (OC1) and using the above expression of $\hat{q}_{ij}$ yields:

$$\alpha_j \eta_i^{\sigma-1} n_{ij} A_j p_{ij}^{-\sigma} = \bar{\alpha}_j \eta_i^{\sigma-1} n_{ij} A_j \hat{p}_{ij}^{-\sigma}$$

or equivalently

$$\begin{aligned} p_{ij} &= \hat{p}_{ij} \left(\frac{\bar{\alpha}_j}{\alpha_j}\right)^{-\frac{1}{\sigma}} \\ p_{ij} &= \frac{\sigma}{\sigma-1}\theta_i \frac{w_i \tau_{ij}}{\varphi} \left(\frac{\bar{\alpha}_j}{\alpha_j}\right)^{-\frac{1}{\sigma}} \end{aligned} \tag{OC2}$$

where the last line follows from using the above expression of $\hat{p}_{ij}$. Ex-ante, the profit now writes:

$$\pi_{ij} = \alpha_j \eta_i^{\sigma-1} n_{ij} A_j p_{ij}^{-\sigma} (p_{ij} - \theta_i \frac{w_i \tau_{ij}}{\varphi}) - f_{ij}$$

and using (OC2) and rearranging, we get finally:

$$\pi_{ij} = \tilde{\alpha}_j \beta_i n_{ij} \left(\frac{\tau_{ij}}{\varphi}\right)^{1-\sigma} \frac{A_j}{\delta_i} - f_{ij} \tag{OC3}$$

where $\tilde{\alpha}_j$ is an increasing function of α_j :

$$\tilde{\alpha}_j = \bar{\alpha}_j \left(\sigma \left[\left(\frac{\alpha_j}{\bar{\alpha}_j}\right)^{\frac{1}{\sigma}} - 1\right] + 1\right). \tag{OC4}$$

Comparing the expression of profit in the baseline model given by (9) and expression (OC3), we conclude that, under the alternative timing, the model reaches similar conclusions provided one replaces the original demand shock α_j by its transformation $\tilde{\alpha}_j$ given by (OC4).

D Optimal investment rule and portfolio choice in the absence of demand shocks

In this Appendix, we characterize the optimal investment rule and the optimal portfolio in the absence of demand shocks. We also investigate the impact of a change in volatility of production shocks. The proof is very similar to the general case described in the main text with some adaptations.

Optimal investment rule in a given portfolio. Let us start again from the system of necessary and sufficient first-order conditions given by (15):

$$\bar{\varepsilon}_{ij} r_{ij} - \frac{w_j f_j L_j}{P_i} - \gamma \bar{\varepsilon}_{ij} r_{ij} \sum_k n_{ik} \bar{\varepsilon}_{ik} r_{ik} \mathbb{C}ov\left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}\right) - \bar{\nu}_{ij} + \underline{\nu}_{ij} = 0 \tag{OD1}$$

From Lemma 1, we also have that both the covariances and variances of normalized shocks affecting profits are all equal to the relative volatility of the production shock:

$$\mathbb{C}ov\left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}\right) = \mathbb{V}(\tilde{\varepsilon}_{ij}) = SCV_{\beta_i}.$$

In the absence of demand shocks, the only source of variability in the model is the production shock β.

It follows that (OD1) rewrite

$$\bar{\varepsilon}_{ij}r_{ij} - \frac{w_j f_j L_j}{P_i} - \gamma \bar{\varepsilon}_{ij} r_{ij} SCV_{\beta_i} \sum_k n_{ik}\bar{\varepsilon}_{ik} r_{ik} - \bar{\nu}_{ij} + \underline{\nu}_{ij} = 0$$

As in the general model, this system of equations can be broken into two parts. Once again, let us concentrate the analysis on settings where any firm is sufficiently risk averse to find optimal not to reach all consumers in each market so that $n_{ij} < 1$ or equivalently $\bar{\nu}_{ij} = 0$ for any j. For a given country i and for all countries j that belong to a given portfolio $\mathcal{S}$, we have:

$$\bar{\varepsilon}_{ij}r_{ij}\left(1 - \gamma SCV_{\beta_i} \sum_{k\in\mathcal{S}} n_{ik}\bar{\varepsilon}_{ik} r_{ik}\right) = \frac{w_j f_j L_j}{P_i} \tag{OD2}$$

and for all $j \notin \mathcal{S}$,

$$\bar{\varepsilon}_{ij}r_{ij}\left(1 - \gamma SCV_{\beta_i} \sum_{k\in\mathcal{S}} n_{ik}\bar{\varepsilon}_{ik} r_{ik}\right) - \frac{w_j f_j L_j}{P_i} + \underline{\nu}_{ij} = 0. \tag{OD3}$$

Let us first characterize the optimal investment rule in a given portfolio, from (OD2). In matrix form, the sub-system of equations (OD2) rewrites

$$\begin{pmatrix} SCV_{\beta_i} & ... & ... & ... & SCV_{\beta_i} \\ SCV_{\beta_i} & ... & ... & ... & SCV_{\beta_i} \\ SCV_{\beta_i} & ... & ... & ... & SCV_{\beta_i} \\ SCV_{\beta_i} & ... & ... & ... & SCV_{\beta_i} \\ SCV_{\beta_i} & ... & ... & ... & SCV_{\beta_i} \end{pmatrix} \begin{pmatrix} . \\ . \\ n_{ik}\bar{\varepsilon}_{ik}r_{ik} \\ . \\ . \end{pmatrix} = \frac{1}{\gamma}\begin{pmatrix} . \\ . \\ \frac{\bar{\varepsilon}_{ik}r_{ik} - w_k f_k L_k/P_i}{\bar{\varepsilon}_{ik}r_{ik}} \\ . \\ . \end{pmatrix}.$$

Clearly, this system of linear equations in n_{ik} is overdetermined as the variance-covariance matrix $\tilde{\Sigma}_i$ is here a square matrix of SCV_{β_i} with dimension $|\mathcal{S}|$ that has rank 1 and thus that is singular. However, using its pseudo-inverse (Moore-Penrose inverse) we can force the solution to be given by:

$$\begin{pmatrix} . \\ . \\ n_{ik}\bar{\varepsilon}_{ik}r_{ik} \\ . \\ . \end{pmatrix} = \frac{1}{\gamma}\begin{pmatrix} \frac{1}{|\mathcal{S}|^2 SCV_{\beta_i}} & ... & ... & ... & \frac{1}{|\mathcal{S}|^2 SCV_{\beta_i}} \\ \frac{1}{|\mathcal{S}|^2 SCV_{\beta_i}} & ... & ... & ... & \frac{1}{|\mathcal{S}|^2 SCV_{\beta_i}} \\ \frac{1}{|\mathcal{S}|^2 SCV_{\beta_i}} & ... & ... & ... & \frac{1}{|\mathcal{S}|^2 SCV_{\beta_i}} \\ \frac{1}{|\mathcal{S}|^2 SCV_{\beta_i}} & ... & ... & ... & \frac{1}{|\mathcal{S}|^2 SCV_{\beta_i}} \\ \frac{1}{|\mathcal{S}|^2 SCV_{\beta_i}} & ... & ... & ... & \frac{1}{|\mathcal{S}|^2 SCV_{\beta_i}} \end{pmatrix}\begin{pmatrix} . \\ . \\ \frac{\bar{\varepsilon}_{ik}r_{ik} - w_k f_k L_k/P_i}{\bar{\varepsilon}_{ik}r_{ik}} \\ . \\ . \end{pmatrix}$$

and hence, for any $j \in \mathcal{S}$, we have

$$n_{ij}\bar{\varepsilon}_{ij}r_{ij} = \frac{1}{\gamma\,|\mathcal{S}|^2\, SCV_{\beta_i}} \sum_{k\in\mathcal{S}} \frac{\bar{\varepsilon}_{ik}r_{ik} - w_k f_k L_k/P_i}{\bar{\varepsilon}_{ik}r_{ik}}.$$

By analogy with what have been defined in the general model, let us denote

$$\mathcal{C}_i(\varphi) = \sum_{k\in\mathcal{S}} \frac{1}{|\mathcal{S}|} \frac{\bar{\varepsilon}_{ik}r_{ik}(\varphi) - w_k f_k L_k/P_i}{\bar{\varepsilon}_{ik}r_{ik}(\varphi)}$$

as the average profitability ratio in the portfolio $\mathcal{S}$. Hence, one can write the optimal investment rule for a firm with productivity φ on market j as follows:

$$n_{ij}(\varphi) = \frac{D_i}{\gamma \bar{\varepsilon}_{ij} r_{ij}(\varphi)} \mathcal{C}_i(\varphi) \tag{OD4}$$

where

$$D_i = \frac{1}{|\mathcal{S}|\, SCV_{\beta_i}}$$

is a scale factor that depends on the relative volatility of production and on the size $|\mathcal{S}|$ of the portfolio. To interpret this optimal rule of investment, note that an increase in the average profitability ratio of the portfolio leads to invest more in any market in the portfolio. Also, keeping $\mathcal{C}_i(\varphi)$ constant, increasing the size of the portfolio reduces the marketing investment in each market (dilution effect). A similar effect is obtained by increasing the relative volatility of the production shock. An alternative view is that the optimal investment rule for a firm with productivity φ is such that the expected variable (real) profit $n_{ij}\bar{\varepsilon}_{ij} r_{ij}$ is equalized across markets to an origin-country specific constant, for any $j \in \mathcal{S}$,

$$n_{ij}\bar{\varepsilon}_{ij} r_{ij} = \frac{D_i}{\gamma} \mathcal{C}_i.$$

Let us now rewrite $\mathcal{C}_i(\varphi)$ to exhibit the cut-off productivity:

$$\begin{aligned}
\mathcal{C}_i(\varphi) &= \sum_{k \in \mathcal{S}} \frac{1}{|\mathcal{S}|} \frac{\bar{\varepsilon}_{ik} r_{ik}(\varphi) - w_k f_k L_k / P_i}{\bar{\varepsilon}_{ik} r_{ik}(\varphi)} \\
&= 1 - \sum_{k \in \mathcal{S}} \frac{1}{|\mathcal{S}|} \frac{w_k f_k L_k / P_i}{\bar{\varepsilon}_{ik} A_k \tau_{ik}^{1-\sigma} \varphi^{\sigma-1} / (\delta_i P_i)} \\
&= 1 - \sum_{k \in \mathcal{S}} \delta_i \frac{1}{|\mathcal{S}|} \frac{\Gamma_{ik}}{\varphi^{\sigma-1}} \\
&= 1 - \left(\frac{\hat{\varphi}_i}{\varphi} \right)^{\sigma-1}
\end{aligned}$$

where $\Gamma_{ik} = \frac{w_k f_k L_k}{\bar{\varepsilon}_{ik} A_k \tau_{ik}^{1-\sigma}}$ is the attractivity index of market k and the cut-off productivity is given by:

$$\hat{\varphi}_i = \left(\frac{\delta_i}{|\mathcal{S}|} \sum_{k \in \mathcal{S}} \Gamma_{ik} \right)^{\frac{1}{\sigma-1}}.$$

Here, the marketing investment on any market in the portfolio is positive as long as the firm is sufficiently productive, i.e. $\varphi \geq \hat{\varphi}_i$. This cut-off productivity is the same for all market in the portfolio and depends on the (simple) average attractiveness index.

We can check that $n_{ij} < 1$ as long as the firm is sufficiently risk averse. Indeed,

$$\begin{aligned}
\sup_j n_{ij} &< 1 \Leftrightarrow \sup_j \left[\max_\varphi \frac{D_i}{\gamma \bar{\varepsilon}_{ij} r_{ij}(\varphi)} \mathcal{C}_i(\varphi) \right] < 1 \\
&\Leftrightarrow \sup_j \left[\frac{D_i}{\gamma \bar{\varepsilon}_{ij} A_j \tau_{ij}^{1-\sigma} / (\delta_i P_i)} \max_\varphi \frac{1 - \left(\frac{\hat{\varphi}_i}{\varphi} \right)^{\sigma-1}}{\varphi^{\sigma-1}} \right] < 1
\end{aligned}$$

As $\frac{1 - \left(\frac{\hat{\varphi}_i}{\varphi} \right)^{\sigma-1}}{\varphi^{\sigma-1}}$ is maximized in $\varphi = 2^{\frac{1}{\sigma-1}} \hat{\varphi}_i$, it follows that $n_{ij} < 1$ whenever

$$\gamma > \sup_j \frac{D_i}{4 \bar{\varepsilon}_{ij} r_{ij}(\hat{\varphi}_i)}.$$

Optimal portfolio as a function of productivity. Let us now determine the optimal portfolio for a firm with productivity φ. Recall that the first-order condition for a market j without investment is given by (OD3):

$$\bar{\varepsilon}_{ij} r_{ij} \left(1 - \gamma SCV_{\beta_i} \sum_{k \in \mathcal{S}} n_{ik} \bar{\varepsilon}_{ik} r_{ik}\right) - \frac{w_j f_j L_j}{P_i} + \underline{\nu}_{ij} = 0$$

and using (OD4) we get

$$\begin{aligned}
\underline{\nu}_{ij} &= \frac{w_j f_j L_j}{P_i} - \bar{\varepsilon}_{ij} r_{ij} \left(1 - SCV_{\beta_i} \sum_{k \in \mathcal{S}} D_i \mathcal{C}_i\right) \\
&= \frac{w_j f_j L_j}{P_i} - \bar{\varepsilon}_{ij} r_{ij} \left(1 - \mathcal{C}_i\right) \\
&= \frac{w_j f_j L_j}{P_i} - \bar{\varepsilon}_{ij} r_{ij} \left(\frac{\hat{\varphi}_i}{\varphi}\right)^{\sigma-1} \\
&= \frac{\Gamma_{ij} \bar{\varepsilon}_{ij} A_j \tau_{ij}^{1-\sigma}}{P_i} - \frac{\bar{\varepsilon}_{ij} A_j \tau_{ij}^{1-\sigma}}{\delta_i P_i} \varphi^{\sigma-1} \left(\frac{\hat{\varphi}_i}{\varphi}\right)^{\sigma-1} \\
&= \frac{\bar{\varepsilon}_{ij} A_j \tau_{ij}^{1-\sigma}}{\delta_i P_i} \left(\delta_i \Gamma_{ij} - \hat{\varphi}_i^{\sigma-1}\right) \\
&= \frac{\bar{\varepsilon}_{ij} A_j \tau_{ij}^{1-\sigma}}{P_i} \frac{1}{|\mathcal{S}|} \sum_{k \in \mathcal{S}} \left(\Gamma_{ij} - \Gamma_{ik}\right)
\end{aligned}$$

Hence, a market without investment is characterized by an attractiveness index that has to be larger than the highest attractiveness index of the portfolio.[42] We can thus rank all countries according to their attractiveness index like in the general model, and we obtain the following characterization of the optimal portfolio as follows.

Proposition OD1. *The unique optimal portfolio $\mathcal{S}(\varphi)$ for a firm with productivity φ is the set of the l most attractive markets when $\varphi_{il} \leq \varphi \leq \varphi_{i,l+1}$ where for all $l = 1, ..., N-1$,*

$$(\varphi_{il})^{\sigma-1} = \frac{\delta_i}{l} \sum_{k=1}^{l} \Gamma_{ik}$$

and $\varphi_{i,N+1} = \infty$.

Value of the optimal portfolio. Let us now compute the optimal value of the portfolio with size l. We can express $V_i^*(\varphi) = \sum_{j=1}^{l} V_{ij}(\varphi)$ where,

$$V_{ij}(\varphi) = \mathbb{E}\left(\frac{\pi_{ij}(\varphi)}{P_i}\right) - \frac{\gamma}{2} \sum_{k=1}^{l} \mathbb{C}ov\left(\frac{\pi_{ij}(\varphi)}{P_i}, \frac{\pi_{ik}(\varphi)}{P_i}\right)$$

is the portion of indirect utility of real income made on market j and where $\mathcal{S}(\varphi)$ is composed of the l most attractive markets, for any $l = 1...N$. For $\varphi \leq \varphi_{i1}$, $\mathcal{S}(\varphi) = \varnothing$ and thus clearly $V_i^*(\varphi) = 0$. For $\varphi > \varphi_{i1}$, dropping the argument for simplicity, we have

$$\begin{aligned}
V_{ij} &= \left(\bar{\varepsilon}_{ij} r_{ij} - \frac{w_j f_j L_j}{P_i}\right) n_{ij} - \frac{\gamma}{2} \bar{\varepsilon}_{ij} r_{ij} n_{ij} \sum_{k=1}^{l} n_{ik} \bar{\varepsilon}_{ik} r_{ik} \mathbb{C}ov\left(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}\right) \\
&= n_{ij} \left[\bar{\varepsilon}_{ij} r_{ij} - \frac{w_j f_j L_j}{P_i} - \frac{\gamma}{2} \bar{\varepsilon}_{ij} r_{ij} H_i\right] \qquad \text{(OD5)}
\end{aligned}$$

[42] Clearly, it is not sufficient for Γ_{ij} to be larger than the average index $\frac{1}{|\mathcal{S}|} \sum_{k \in \mathcal{S}} \Gamma_{ik}$ to have $\underline{\nu}_{ij} > 0$ because otherwise there would be a contradiction as country j would be in the portfolio.

where H_i is given by:

$$H_i = SCV_{\beta_i} \sum_{k=1}^{l} n_{ik} \bar{\varepsilon}_{ik} r_{ik}. \tag{OD6}$$

Using $\Gamma_{ij} = \frac{w_j f_j L_j}{\tau_{ij}^{1-\sigma} \bar{\varepsilon}_{ij} A_j}$ and $r_{ij} = \frac{A_j \tau_{ij}^{1-\sigma}}{\delta_i P_i} \varphi^{\sigma-1}$, and replacing in (OD5), we get:

$$\begin{aligned}
V_{ij} &= n_{ij} \left[\bar{\varepsilon}_{ij} \frac{A_j \tau_{ij}^{1-\sigma}}{\delta_i P_i} \varphi^{\sigma-1} \left(1 - \frac{\gamma}{2} H_i\right) - \frac{\tau_{ij}^{1-\sigma} \bar{\varepsilon}_{ij} A_j}{P_i} \Gamma_{ij} \right] \\
&= \bar{\varepsilon}_{ij} \frac{A_j \tau_{ij}^{1-\sigma}}{\delta_i P_i} n_{ij} \left[\varphi^{\sigma-1} \left(1 - \frac{\gamma}{2} H_i\right) - \delta_i \Gamma_{ij} \right] \\
&= \frac{1}{\gamma} D_i \left[1 - \left(\frac{\varphi_{il}}{\varphi}\right)^{\sigma-1} \right] \underbrace{\left[1 - \frac{\gamma}{2} H_i - \frac{\delta_i \Gamma_{ij}}{\varphi^{\sigma-1}} \right]}_{(1)} \qquad \text{(OD7)}
\end{aligned}$$

where the last line follows from using $n_{ij}(\varphi) = \frac{D_i}{\gamma \bar{\varepsilon}_{ij} r_{ij}(\varphi)} \mathcal{C}_i(\varphi)$ and $\mathcal{C}_i(\varphi) = 1 - \left(\frac{\varphi_{il}}{\varphi}\right)^{\sigma-1}$.

In the rest of the proof, we propose to evaluate separately the term (1) in (OD7) to obtain the desired formulation for $V_i^*(\varphi)$. First, using $n_{ij}(\varphi) = \frac{D_i}{\gamma \bar{\varepsilon}_{ij} r_{ij}(\varphi)} \mathcal{C}_i(\varphi)$, (OD6) rewrites:

$$H_i = \frac{SCV_{\beta_i}}{\gamma} \sum_{k=1}^{l} D_i \mathcal{C}_i = \frac{1 - \left(\frac{\varphi_{il}}{\varphi}\right)^{\sigma-1}}{\gamma}$$

Hence, term (1) in (OD7) becomes:

$$\begin{aligned}
1 - \frac{\gamma}{2} H_i - \frac{\delta_i \Gamma_{ij}}{\varphi^{\sigma-1}} &= 1 - \frac{\gamma}{2} \left[\frac{1 - \left(\frac{\varphi_{il}}{\varphi}\right)^{\sigma-1}}{\gamma} \right] - \frac{\delta_i \Gamma_{ij}}{\varphi^{\sigma-1}} \\
&= \frac{1}{2} + \frac{1}{2} \left(\frac{\varphi_{il}}{\varphi}\right)^{\sigma-1} - \frac{\delta_i \Gamma_{ij}}{\varphi^{\sigma-1}} \\
&= \frac{1}{2} + \frac{\delta_i}{2\varphi^{\sigma-1}} \left(\frac{1}{l} \sum_{k=1}^{l} (\Gamma_{ik} - \Gamma_{ij}) - \Gamma_{ij} \right)
\end{aligned}$$

Using the above result, (OD7) becomes:

$$V_{ij} = \frac{1}{2\gamma} D_i \left(1 - \left(\frac{\varphi_{il}}{\varphi}\right)^{\sigma-1} \right) \left(1 + \frac{\delta_i}{\varphi^{\sigma-1}} \left(\frac{1}{l} \sum_{k=1}^{l} (\Gamma_{ik} - \Gamma_{ij}) - \Gamma_{ij} \right) \right),$$

and we obtain the desired formulation for $V_i^*(\varphi)$ as follows:

$$\begin{aligned}
V_i^*(\varphi) &= \sum_{j=1}^{l} V_{ij} = \frac{1}{2\gamma} D_i \left(1 - \left(\frac{\varphi_{il}}{\varphi}\right)^{\sigma-1}\right) \sum_{j=1}^{l} \left(1 + \frac{\delta_i}{\varphi^{\sigma-1}} \left(\frac{1}{l} \sum_{k=1}^{l} (\Gamma_{ik} - \Gamma_{ij}) - \Gamma_{ij}\right)\right) \\
&= .\frac{1}{2\gamma} D_i \left(1 - \left(\frac{\varphi_{il}}{\varphi}\right)^{\sigma-1}\right) \left(l + \frac{\delta_i}{\varphi^{\sigma-1}} \sum_{k=1}^{l} \Gamma_{ik} - \frac{2\delta_i}{\varphi^{\sigma-1}} \sum_{j=1}^{l} \Gamma_{ij}\right) \\
&= \frac{l}{2\gamma} D_i \left(1 - \left(\frac{\varphi_{il}}{\varphi}\right)^{\sigma-1}\right)^2 \\
V_i^*(\varphi) &= \frac{1}{2\gamma SCV_{\beta_i}} \left(1 - \left(\frac{\varphi_{il}}{\varphi}\right)^{\sigma-1}\right)^2
\end{aligned}$$

We sum up these results in the following Proposition.

Proposition OD2. *At the equilibrium, a firm with productivity φ and from origin country i,*

(i) *either does not produce when $\varphi \leq \varphi_{i1}$ and gets $V_i^*(\varphi) = 0$,*

(ii) *or produces and sells in the l most attractive markets when $\varphi_{il} \leq \varphi \leq \varphi_{i,l+1}$ and gets*

$$V_i^*(\varphi) = \frac{1}{2\gamma SCV_{\beta_i}} \left(1 - \left(\frac{\varphi_{il}}{\varphi}\right)^{\sigma-1}\right)^2 > 0$$

where

$$(\varphi_{il})^{\sigma-1} = \frac{\delta_i}{l} \sum_{k=1}^{l} \Gamma_{ik}.$$

Impact of climate change. In the case where demand shocks are absent, the impact of climate change is rather limited as only the scale factor D_i decreases in SCV_{β_i}. In other words, a change in the volatility of β_i only yields a scale effect in investment decisions. The optimal composition of the portfolio for a firm with given productivity does not change and thus there is no selection effect. Moreover, there is no redeployment effect of investments inside the portfolio as well. Finally, the value of the optima portfolio is inversely proportional to the relative volatility of production shocks.