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City Risk and Industrial Policy

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Abstract

We explore the diversification of an urban economy where the labor specialization choices of its residents determine the city's exposure to sectoral shocks. The presence of demand-driven externalities introduces the possibility of city-wide coordination failures. Residents, when making their specialization choices, do not account for the costs of these coordination failures, and as a result, the equilibrium level of diversification is inefficient. The optimal policies that address these externalities depend on the city's economic condition, with prosperous urban economies deriving a greater benefit from fostering diversification. Thus, the paper rationalizes the widespread industrial policies that in some cases promote diversifying, while in others, specializing a city's economy.

Keywords: City diversification, industrial policy, city risk, coordination failures.

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1 Introduction

Should urban policymakers promote economic diversification? While the benefits of developing industry clusters are often discussed by both policymakers and academics, the potential risk-reduction benefits of industrial diversification have received little attention in the economics literature, despite being frequently discussed by policymakers.[1] For example, in a 2021 New York Times opinion article, former New York City Mayor Michael Bloomberg argued that "[W]hatever policies the next mayor pursues, the crucial idea is that putting a city back on its feet economically requires more than aiding existing businesses.It requires creating the conditions for new ones to open and expand, further diversifying the economy."[2] Along a similar line, a study by EY, commissioned by the Greater Austin Economic Development Corporation, made the case for "[A] diversified industry base that can help the region withstand a downturn in any one key industry while providing multiple opportunities across sectors for innovation-based growth and investment."[3]

This paper studies the costs and benefits of policies that favor industrial diversification. To explore these issues, we model a city which is a small part of a system of cities that constitute the aggregate economy. Location plays a role in this economy for two reasons. The first is that workers cannot rapidly respond to sector-specific shocks by moving to more favorable locations. The second is that in addition to traded goods, that are transported freely between cities, city residents produce and consume non-traded goods, such as restaurant meals, entertainment, education, and health services, which must be produced in the city where they live. Shocks to traded-good prices create uncertainty in the model. These sectoral shocks, which affect the city's revenue from producing traded goods, are transmitted to non-traded-good prices, e.g., the price of real estate in a city increases when the prices of the traded goods it produces increases. Because of the transmissions of these shocks, city residents are exposed to risks

[1] It is noteworthy that, Duranton and Venables (2021) and Juhasz, Lane and Rodrik (2023), two recent articles discussing the motivations for place-based and industrial policies, do not mention diversification.

[2] In a 2017 document, the Office of the New York City Comptroller also expressed concern about the city's economy overexposure to the finance industry. See ttps://comptroller.nyc.gov/wp-content/uploads/documents/NYC_Economy_Diversified.pdf.

[3] See https://opportunityaustin.com/wp-content/uploads/2024/02/OA-5.0-Strategy.pdf

that we assume cannot be hedged. Specifically, markets are incomplete in that city residents cannot trade claims on either traded goods or real estate with residents in other cities.

The set of traded-good industries in the city –the city's industrial base– is determined by the labor choices of its residents. We assume that each city has a comparative advantage in the production of one particular traded-good, which provides an exogenous force towards specialization. Since traded goods can be transported freely between cities, production may be more efficient if each city specializes in producing the traded good in which it has a comparative advantage. There is, however, an offsetting benefit to diversification, even when the city's residents are risk-neutral. The benefit to diversification arises because some of the non-traded goods, such as real estate, are in fixed supply or more generally, they have a low supply elasticity. Intuitively, if a city produces or acquires 10% more traded goods, the utility of a representative risk-neutral resident increases less than 10% because the consumption of non-traded-goods cannot be proportionally adjusted. This implies that the residents of a city receive higher utility on average when the city's income from traded goods is less volatile, that is, city residents benefit from diversification.

This benefit of diversification does not by itself justify policy interventions that favor specific industries. Indeed, in our benchmark setting, where non-traded-goods are in fixed supply, the equilibrium industrial base is socially optimal. In this equilibrium, some city residents are willing to work in sectors that generate less income on average because by doing so, they benefit from having more income when the price of real estate and other non-traded goods in the city are lower.

When we extend the model by allowing the supply of non-traded goods to adjust to changing market conditions, the equilibrium level of diversification may no longer be socially optimal. Specifically, we depart from the benchmark setting by assuming that non-traded goods are produced by entrepreneurial city residents who increase production by exerting costly effort when the demand for non-traded goods is sufficiently high. In this setting, demand-driven complementarities exist in the production of non-traded goods – the incentives of entrepreneurs to produce and consume non-traded goods increases when other entrepreneurs produce and consume more. These complementarities create the possibility of multiple equilibria and coordination failures, wherein all non-traded-good entrepreneurs produce at inefficiently low levels.[4]

To overcome the indeterminacy stemming from multiple equilibria, we apply the global games refinement introduced in Carlsson and Van Damme (1993), which shows that introducing incomplete information in games with strategic complementarities can lead to a unique equilibrium. In our case, the equilibrium that survives the global games refinement depends on the city's income from its traded-good sector. Specifically, coordination failure states are triggered when the income from the traded-good sector is sufficiently low. Intuitively, in these failure states, the decrease in demand for non-traded goods, generated by a decline in income from the traded-good sector, is amplified by an endogenous drop in the demand generated from within the non-traded sector. For example, if the price of software produced in Seattle experiences a large decline, it can lead to the closure of some restaurants that programmers frequent, causing both restaurateurs and programmers to attend fewer movies. This, in turn, results in the closure of cinemas, further reducing the demand for restaurants, and so forth.[5]

Because workers fail to internalize the effect that their specialization choices have on the possibility of such a city-wide coordination failure, the equilibrium industrial base is not socially optimal. The optimal policy addressing this externality depends on the city's economic condition. Specifically, a city with a profitable traded-good sector that supports a "vibrant" non-traded-good sector, characterized by high-effort entrepreneurs, may benefit from subsidizing diversification. This subsidy reduces the likelihood of a negative shock that triggers a coordination failure, thereby preserving the vibrancy of the city's non-traded-good sector. Conversely, a city with a failing traded-good sector, already suffering from a coordination failure, derives fewer benefits from industrial diversification. In such cases, policy choices

[4] See Diamond (1982), Hart (1982), Weitzman (1982), Kiyotaki (1988), and Cooper and John (1988) for seminal papers in the literature that studies the possibility of economy-wide coordination failures.

[5] Our description of the relationship between income generated in a city's traded-good sector and activity in its non-traded goods sector is consistent with what some urban economists describe as the local multiplier effect that arises from the creation of traded-good jobs. For instance, Moretti (2010) finds that adding one skilled manufacturing job in a city creates 2.5 jobs in the city's non-traded-good sector.

that encourage a more focused industrial base may be preferred. Intuitively, such a city can potentially benefit from promoting one traded-good sector that can spark the revitalization of its economy.

While there are a number of literatures that address related issues, we believe we are the first to explicitly model the risk-reduction benefits of urban diversification. The urban economics literature typically stresses economies of scale and scope driving a wedge between equilibrium outcomes and social optima to justify policies that promote either specialization or diversification.[6] For instance, externalities from clustering (see Henderson, 1974 and Abdel-Rahman and Fujita, 1990) can create a rationale for subsidizing specialization, while externalities from economies of scope (see Abdel-Rahman, 1990 and Abdel-Rahman and Fujita, 1993) can create a rationale subsidizing diversification. In this literature, however, city risk is notably absent, despite the fact that promoting diversification as a strategy to mitigate risk tends to be a key objective in the industrial policies of municipalities and regions, e.g., Figueiredo, Honiden, and Schumann (2018).

There is also an urban planning and economic geography literature that describes higher risk as an unfortunate consequence of an increase in urban focus. Borrowing from the finance literature, this literature views industrial diversification as a portfolio choice, which trades off risk and return. (See Conroy, 1974, for an early example of this approach.) The literature has also documented a relation between industrial diversity and economic resilience (see Brown and Greenbaum, 2017, for a recent example). As we show, risk reduction, by itself, does not rationalize the need for industrial policy, but we provide micro-foundations for a market failure based on demand externalities that does.

While we provide a new role for urban diversification, it is related to what Krugman (1991) characterizes as the benefits of labor pooling. This is the idea that firms that are exposed to different shocks may benefit from locating close to each other since the laid off workers at a firm experiencing a negative shock may have opportunities to work at a neighboring firm

[6]For overviews of this extensive literature see, for example, Fujita, Krugman and Venables (1999), Duranton and Puga (2000), Fujita and Thisse (2002), Henderson and Thisse (2004), and Duranton, Henderson and Strange (2015).

that may have experienced a positive shock. Our model abstracts from the benefits of labor pooling, since we assume that workers do not change jobs. However, non-traded-good workers realize a benefit from what we would characterize as effective labor pooling. In particular, although restaurant workers do not change jobs, they benefit from being located in a more diversified economy that allows them to serve more meals to programmers when the demand for software is higher and more meals to auto workers when the demand for autos is higher.

While our focus is on the diversification of cities, our analysis is also related to papers that explore the importance of systematic sources of risk at more aggregate levels. For example, Acemoglu and Zilibotti (1997) studies the link between the diversification of countries and growth. They argue that because of indivisibilities and capital scarcity, the competitive equilibrium is inefficient, in that too few risky high-expected-return projects are undertaken. Specifically, the opening of an additional sector creates a positive pecuniary externality on other potential projects since it allows consumers to bear less risk when they buy securities. Our paper also generates a link between diversification and growth since the coordination failures in our model can affect economic growth. However, unlike Acemoglu and Zilibotti (1997), in which diversification always creates a positive pecuniary externality, in our case, the city can feature excessive specialization or excessive diversification, depending on its economic condition. Our channel, which does not rely on risk aversion, is also distinctly different: individuals do not internalize that their combined specialization choices determine the city's aggregate risk, and hence the probability of the city's economy falling into an equilibrium with a depressed level of economy activity.

Finally, our paper relates to current debate in the trade literature about supply chain resilience and the benefits of trade diversification, e.g., Grossman, Helpman and Lhuillier (2023). Our model can be repositioned to explore the costs and benefits of diversifying the suppliers to a city's industrial base. Specifically, the logic of our model can be applied to a setting where shocks to suppliers to a city's traded goods sector can be transferred to the city's non-traded-good sector, triggering the type of coordination failure illustrated in our model.

2 Model

We consider a city that is a small part of a system of cities that constitute the aggregate economy. City residents are characterized by the goods that they produce. A portion of the residents produce traded consumption goods, like computers or automobiles, that can be consumed within the city or transported to be sold and consumed by residents in other cities. The city residents that produce traded goods choose the particular traded-good sector in which to specialize their labor and after making that choice, devote an exogenous unit of labor to the activity. Another portion of the residents produce non-traded goods, like restaurant meals, that can only be consumed within the city. In contrast to workers that produce traded goods, workers that produce non-traded goods do not face a specialization decision. While a city can specialize in the production of a few traded goods and trade to consume other traded goods, all non-traded goods consumed in a city need to be produced by city residents. Finally, there is a portion of residents endowed with real estate that is consumed by local residents. Next we describe in detail each type of city residents and the sectors that they work in.

2.1 Traded-Good Sectors

There is a unit interval of traded goods indexed by $x_t \in [0, 1]$. These goods are produced and consumed across the economy. Since the city we consider is small with respect to the whole economy, we take the prices of traded goods, $p_t(x_t)$ for $x_t \in [0, 1]$, as exogenous. Specifically, traded goods prices are independent and uniformly distributed on $(0, \overline{p})$, and their fluctuations are the sole source of uncertainty in the model. Since there is a continuum of traded-good sectors with i.i.d. prices, a basket with one unit of each traded good has no price uncertainty. We take this basket as the economy's numeraire, and normalize the upper bound $\overline{p}$ to be equal to 2.

The city has a comparative advantage in the production of traded good $x_t = 1$: one unit of labor in the city generates $\alpha + \delta$ units of traded-good 1, with $\alpha > 0$ and $\delta > 0$. By contrast, in each of the other traded-good sectors $x_t \in X_N$, one unit of labor generates δ units of the traded good, where $X_N \subset [0, 1)$ is the set of N traded-good sectors available for production in the city, in addition to traded-good sector 1.[7] The city's greater productivity in the production of traded-good 1 captures the idea that cities have unique characteristics which provide them with comparative advantages in the production of certain goods. These comparative advantages can originate from their locations (e.g., by a river), their natural resources (e.g., oil) or their access to specific types of knowledge and human capital (e.g., being next to a major university). Note also that the model's production technology rules out economies of scale at the industry level or economies of scope across industries that generate externalities that workers fail to internalize, as there is already an extensive literature exploring the importance of these externalities for industrial policy. Instead, our analysis will focus on externalities that work through the city's aggregate risk, that is, through the city's exposure to sectoral shocks.

The city is populated by a mass $\overline{L}$ of workers with the skills to produce traded goods.[8] These workers are endowed with one unit of labor, which they supply inelastically, and decide ex-ante (before the traded-good prices are realized) the particular traded-good sector $x_t \in X_N \cup \{1\}$ in which to specialize their labor. We let $L(x_t)$ be the mass of workers that choose to specialize in the production of traded good x_t and refer to $\{L(x_t)\}_{x_t \in X_N \cup \{1\}}$ as the city's industrial base, that is, the portfolio of traded goods that the city produces.[9] The industrial base determines the city's income from the production of traded goods:

$$Y_t = (\delta + \alpha)L(1)p_t(1) + \sum_{x_t \in X_N} \delta L(x_t)p_t(x_t). \tag{1}$$

Note that Y_t depends on the realization of the traded-goods prices $\{p_t(x_t)\}_{x_t \in X_N \cup \{1\}}$ and therefore, the city's industrial base governs the city's exposure to these sector-specific shocks. For example, if $L(x_t) = \overline{L}$, the city is fully specialized and only exposed to shocks to traded-good sector x_t.

[7] We introduce the subset X_N where N can be an arbitrarily larger number for technical reasons: having a finite set of traded-good sectors from which workers can choose keeps their specialization problem well-defined.

[8] See Section 5.2 for a dicussion of how to endogenize $\overline{L}$, and more generally, for the role of migration in the context of the model.

[9] While the number of traded-good sectors in the city is at most $N+1$, the mass of workers specialized in each of the $N+1$ sectors is endogenously determined. Therefore, the city may end up producing fewer than $N+1$ different traded goods, that is, in equilibrium, some sectors in X_N can have $L(x_t) = 0$.

2.2 Non-Traded-Good Sectors and Real Estate

In addition to the traded goods, a separate group of workers produce a unit interval of non-traded goods, indexed by $x_{nt} \in [0, 1]$. Unlike the traded-good sectors, the non-traded-good sectors need not differ across cities, e.g., restaurants in Cleveland and Boston can potentially be identical. However, workers can make choices that generate endogenous differences in the non-traded sectors across cities.

Each non-traded good sector is populated with one of two possible types workers. The first type always produces a fixed amount, while the second type is more ambitious and can potentially increase production. To simplify our analysis, we assume there are exactly two workers in each non-traded-good sector x_{nt}, one is constrained to produce $\frac{q_0}{2} > 0$ units of the non-traded good and the other can also produce $\frac{q_0}{2}$ units of the non-traded good, but has the option to produce an additional $q_1 > 0$ units of the good by incurring a non-pecuniary fixed cost c. Hereafter, we refer to workers that can increase production by incurring a non-pecuniary fixed cost c as entrepreneurs, therefore, each non-traded-good sector has one worker and one entrepreneur.[10] The entrepreneurs' production decisions are made after the traded-good prices are realized, and therefore, after the city's income from the production of traded goods Y_t becomes known.[11]

While the demand for traded goods is determined at the economy-wide level, the demand for non-traded goods is entirely determined at the city level. Consequently, the prices of non-traded goods, $\{p_{nt}(x_{nt})\}_{x_{nt}\in[0,1]}$, are endogenously determined and will depend on the city's industrial base, $\{L(x_t)\}_{x_t\in X_N\cup\{1\}}$, the realized prices of the traded goods, $\{p_t(x_t)\}_{x_t\in[0,1]}$, and the entrepreneurs' production decisions.

Note that there are differences in our modelling of workers in the traded- and non-traded good sectors. This asymmetry is in part motivated by the need to keep the model tractable, but also captures fundamental differences between the two of types of goods. First, we endogenize

[10] As it will become clear below, the only role in the model of the worker that produces a fixed amount $\frac{q_0}{2}$ of no-traded-good x_{nt} is to make the production decision of the entrepreneur in that sector x_{nt} a meaningful one.

[11] Even though the production of traded goods is entirely determined by the ex-ante specialization choices, all production (for traded and for non-traded goods) takes places after the traded-good prices are realized.

workers' specialization decisions across traded-good sectors, but assume that workers and entrepreneurs are uniformly distributed throughout the non-traded-good sectors. Underlying this assumption is the idea that a city can specialize in the production of a few traded goods and trade to consume the other traded goods, while all non-traded goods consumed in the city need to be produced locally. Second, non-traded-good entrepreneurs make endogenous production choices that affect the level of output, while traded-good workers do not.[12] As we will show, the fact that the demand for non-traded goods is entirely determined at the city level generates strategic complementarities in the production of non-traded-goods that can lead to multiple equilibria and city-wide coordination failures. These strategic complementarities, however, do not arise in the production of traded-goods since the demand for these goods (and therefore, their prices) are exogenous to the city.[13]

Finally, real estate is a special case of a non-traded good. In contrast to the other non-traded goods, the supply of real estate is fixed. Specifically, we assume there is a mass R of real estate owners who are each endowed with one unit of real estate. After the realization of the traded-good prices, real estate R is competitively traded within the city at a market-clearing price p_r.

2.3 Consumption and Timing

Each consumer i (i.e., workers in the traded-good sectors, workers and entrepreneurs in the non-traded-good sectors, and real estate owners) have a utility function

$$U_i = \exp\left[\int_0^1 \gamma_t \ln c_{i,t}(x_t)dx_t + \int_0^1 \gamma_{nt} \ln c_{i,nt}(x_{nt})dx_{nt} + \gamma_r \ln c_{i,r}\right] \quad (2)$$

where $c_{i,t}(x_t)$ is the consumption of traded good $x_t \in [0,1]$ by consumer i, $c_{i,nt}(x_{nt})$ is the consumption of non-traded good $x_{nt} \in [0,1]$, and $c_{i,r}$ is the consumption of real estate. We assume that $\gamma_t + \gamma_{nt} + \gamma_r = 1$, so that the utility function is homogenous of degree one, and

[12] Traded-good workers make ex-ante (before prices are realized) specialization decisions, but do not make ex-post (after prices are realized) production decisions, that is, they devote their one unit of labor to produce at $t = 1$ the traded-good in which they have specialized at $t = 0$.

[13] If traded-good-workers were to make ex-post (after prices are realized) production decisions that affect their level of output, while non-traded-good entrepreneurs did not, these ex-post production decisions would be strategic substitutes rather than complements, and hence, there would be no scope for coordination failures.

the indirect utility function is a linear function of the consumer's income.

Figure 1 below illustrates the timing of events. At $t = 0$, workers in the traded-good sectors choose how to specialize their labor. At $t = 1$, after the traded-good prices are realized, production, trading, and consumption takes place.

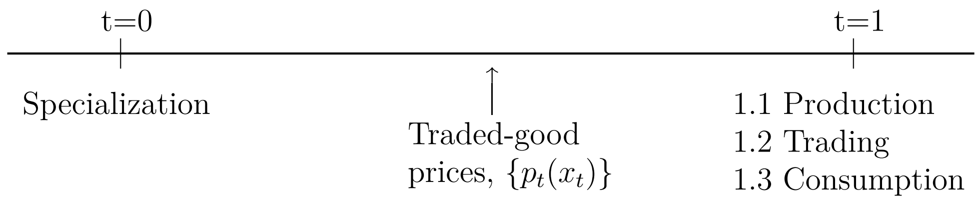

Figure 1. Timing of events.

3 Benchmark Case: Real Estate as the only non-Traded Good.

This section considers a benchmark case in which the only non-traded good is real estate (i.e., $\gamma_{nt} = 0$). This case allows us to study a city's diversification across traded-good sectors without introducing the coordination failures in the production of non-traded goods sectors that we consider in Section 4.

3.1 Consumption at $t = 1$

For a given city's industrial base $\{L(x_t)\}_{x_t \in X_N \cup \{1\}}$ chosen at $t = 0$ and traded-good prices, $\{p_t(x_t)\}_{x_t \in [0,1]}$ realized at $t = 1$, consumer i with income y_i maximizes her utility in eq. 2, which for $\gamma_{nt} = 0$ can be written as

$$U_i = \exp\left[\int_0^1 \gamma_t \ln c_{i,t}(x_t) dx_t + \gamma_r \ln c_{i,r}\right] \quad \text{with} \quad \gamma_t + \gamma_r = 1, \tag{3}$$

subject to budget constraint

$$y_i = \int_0^1 p_t(x_t) c_{i,t}(x_t) dx_t + p_r c_{i,r}. \tag{4}$$

From this optimization problem and following standard derivations, consumer i has indirect utility function

$$v(y_i) = \frac{\gamma_t^{\gamma_t}\gamma_r^{\gamma_r} \times y_i}{P_t^{\gamma_t} p_r^{\gamma_r}}, \tag{5}$$

where the geometric average $P_t \equiv \exp\left[\int_0^1 \ln p_t(x_t) dx_{nt}\right]$ is a constant as prices $\{p_t(x_t)\}_{x_t \in [0,1]}$ are i.i.d. Note that consumers are risk-neutral since $v(y_i)$ is linear in y_i for any given set of prices $\{\{p_t(x_t)\}_{q_t \in [0,1]}, p_r\}$.

Let Y be the city's income, that is, the sum of the income from the traded-good sectors, Y_t (see eq. 1) and from the real estate sector, $Y_r = R \times p_r$. The Cobb-Douglas utility function implies that the city's expenditure in the consumption of traded goods and real estate are $\gamma_t Y$ and $\gamma_r Y$, respectively. Since the income in the real estate sector, $Y_r = R \times p_r$, must be equal to the city's expenditure in the consumption of real estate, $\gamma_r(Y_t + Y_r)$, it follows that

$$R \times p_r = \frac{\gamma_r}{\gamma_t} Y_t \Rightarrow p_r = \frac{\gamma_r}{\gamma_t}\frac{Y_t}{R}. \tag{6}$$

In words, the price of real estate at $t = 1$ increases in the city's income from the production of traded goods, Y_t, and in the share of income spent on the consumption of real estate, γ_r, and decreases in the city's endowment of real estate, R, and in the share of income spent on the consumption of traded goods, γ_t.

3.2 Equilibrium Industrial Base at $t = 0$

Consider now workers' decisions to specialize in the production of traded goods at $t = 0$. For each worker i, let $l_i(x_t)$ be an indicator function defined on $x_t \in X_N \cup \{1\}$ such that $l_i(x_t') = 1$ if the worker chooses to specialize in the production of traded good x_i', and $l_i(x_t') = 0$, if the worker chooses to specialize in the production of traded good $x_t \neq x_t'$. Therefore, the measure of workers that choose to specialize in the production of traded good x_t' is $L_i(x_t') = \int_0^{\overline{L}} l_i(x_t') di$.

For a given specialization choice $l_i(x_t)$, worker i obtains income

$$y_i = (\alpha + \delta)\, l_i(1) p_t(1) + \sum_{x_t \in X_N} \delta l_i(x_t) p_t(x_t) \tag{7}$$

and hence, from eq. 5, has indirect utility

$$v(y_i) = \gamma_t \frac{(\alpha+\delta)\, l_i(1)p_t(1) + \sum_{x_t \in X_N} \delta l_i(x_t)p_t(x_t)}{P_t^{\gamma_t} \left(\frac{Y_t}{R}\right)^{\gamma_r}}. \tag{8}$$

Note that the indirect utility in eq. 8 depends on Y_t, and therefore on the city's industrial base $\{L(x_t)\}_{x_t \in X_N \cup \{1\}}$. While the city's industrial base is the by-product of all the workers' specialization decisions, workers, being small with respect to the whole city, ignore the combined impact of their individual specialization decisions on the city's industrial base. More specifically, worker i solves the following optimization problem

$$\max_{\{l_i(x_t)\}_{x_t \in X_N \cup \{1\}}} \mathbb{E}[v(y_i)] \tag{9}$$

where $l_i(x_t) \in \{0,1\}$ and $\sum_{x_t \in X_N \cup \{1\}} l_i(x_t) = 1$, and the expectation is taken with respect to the traded-good prices, $\{p_t(x_t)\}_{x_t \in [0,1]}$. The next proposition characterizes the city's equilibrium industrial base and income from the production of traded goods.

Proposition 1 *There exists $\overline{\alpha}_N > 0$, such that*

1. *if $\alpha \geq \overline{\alpha}_N$, the city fully specializes in sector 1:*

$$L^*(1) = \overline{L} \text{ and } Y_t = (\alpha+\delta)\, p_t(1)\overline{L}.$$

2. *if $\alpha < \overline{\alpha}_N$, the city diversifies into sectors $x_t \in X_N$:*

$$L^*(1) \in \left(\frac{\overline{L}}{N+1}, \overline{L}\right) \text{ and } Y_t = (\alpha+\delta)\, p_t(1)L^*(1) + \sum_{x_t \in X_N} \delta p_t(x_t)\frac{\overline{L} - L^*(1)}{N}.$$

Part (1) of Proposition 1 is straightforward: if the productivity gap α between traded-good sector 1 and the other traded-good sectors $x_t \in X_N$ is large enough, all traded-good workers specialize in the production of traded-good 1. Part (2) shows that diversification can arise in equilibrium if the productivity gap α is not too large, yet strictly positive. In that case, the equilibrium features a mass $L^*(1) < \overline{L}$ of workers in traded-good sector 1 and an equal mass of workers $L^*(x_t) = \frac{\overline{L} - L^*(1)}{N}$ in each other traded-good sector $x_t \in X_N$. In this equilibrium, each worker is indifferent between specializing in traded-good sector 1 or in any other traded-good sector $x_t \in X_N$,

$$(\alpha + \delta)\,\mathbb{E}[v(p_t(1))] = \delta\mathbb{E}[v(p_t(x_t))], \tag{10}$$

despite sector 1 generating a higher expected income, i.e., $(\alpha + \delta)\,\mathbb{E}[p_t(1)] > \delta\mathbb{E}[p_t(x_t)]$. Intuitively, the income from traded-good sector 1 is more correlated with the city's income from producing traded-goods, Y_t, than the income from any other traded-good sector. This is both because sector 1 is more productive and because a larger share of the workforce works in sector 1 than in any other sector. As a result, real estate prices are also more correlated with the income in traded-good sector 1 than with the income in other traded-good sectors. This makes traded-good sectors other than sector 1 attractive as workers are more likely to take advantage of low real estate prices when the city's income is low.

From Proposition 1, if $\alpha < \overline{\alpha}_N$, the city is exposed to shocks to all traded-good sectors in X_N, but the labor devoted to the production of these traded-good sectors, $\overline{L} - L^*(1)$, is well-diversified, that is, an equal mass of workers $L^*(x_t) = \frac{\overline{L}-L^*(1)}{N}$ choose to specialize in each traded-good sector $x_t \in X_N$. Therefore, as N increases, the city becomes less exposed to these other traded-good sectors $x_t \in X_N$, and, at the limit when $N \to +\infty$, the city becomes only exposed to shocks to traded-good sector 1.

Corollary 1 *As $N \to +\infty$, there is an $\overline{\alpha}_\infty > 0$, such that $Y_t = (\alpha + \delta)\,p_t(1)L^*(1) + \delta(\overline{L} - L^*(1))$ with $L^*(1) = \overline{L}$ for $\alpha \geq \overline{\alpha}_\infty$, and $L^*(1) \in (0, \overline{L})$ for $\alpha < \overline{\alpha}_\infty$.*

3.3 Welfare

We next examine the social optimality of the equilibrium industrial base in Proposition 1. Since the indirect utility function is linear in y_i, from eq. 5 and eq. 6, the sum of the utility of all the city's residents can be written as

$$v_s(Y) = \frac{(\gamma_t Y)^{\gamma_t}\,R^{1-\gamma_t}}{P_t^{\gamma_t}}. \tag{11}$$

where we have used the fact that $Y_t = \gamma_t Y$. While each consumer's indirect utility $v(y_i)$ is linear in her income y_i (see eq. 6), the sum of the utility of all the city residents $v_s(Y)$ is concave in the city's income Y, i.e., $v_s''(Y) < 0$. At the city level, an increase in the city's income from the production of traded goods Y_t (and hence, in the city's consumption of traded goods) cannot be met with a proportional increase in the consumption of real estate, because real estate is in fixed supply. Therefore, because the supply of traded goods is perfectly elastic (the city is small with respect to the aggregate economy), while real estate is in fixed supply (or more generally, in less elastic supply), the city's social welfare function is concave with respect to the city's traded-good-sector income.

Even though each consumer's indirect utility function $v(y_i)$ is linear in y_i, workers internalize the social cost of specialization. Specifically, the positive correlation between the price of estate price and the city's income decreases the marginal utility of a worker's income in those states in which the city's income is high. In equilibrium, unless $\alpha > \overline{\alpha}_N$, in which case we have a corner solution with full city specialization, workers are indifferent between specializing in traded-good sector 1 and specializing in any other traded-good sector. These equilibrium specialization choices are socially optimal, as the next proposition shows.

Proposition 2 *The equilibrium city's industrial base in Proposition 1 is socially optimal.*

While numerous studies have documented a relation between industrial diversification and economic resilience, e.g., Brown and Greenbaum (2017), Proposition 2 illustrates that having a benefit associated to diversification does not imply the need for active industrial policy, that is, the equilibrium industrial base can be socially optimal. The next section, however, shows that, when the city produces non-traded as well as traded goods, individual choices lead to an equilibrium industrial base that is no longer socially optimal, even under risk-neutrality. Specifically, depending on the city's economic conditions, the equilibrium industrial can feature excessive specialization or diversification.

4 City Risk and Coordination Failures

Section 3 has one non-traded good (real estate) in fixed supply. This section introduces other non-traded goods whose supply can adjust to an increase in demand. We show that production decisions in these non-traded good sectors are strategic complements, which creates scope for coordination failures. We also show that shocks to traded-good sectors influence production decisions in non-traded good sectors, and therefore, also influence the likelihood of a coordination failure. Finally, we analyze how the potential for coordination failures affect the optimality of the equilibrium industrial base.

Concretely, we first examine the production and consumption decisions at $t = 1$, conditioned on the specialization decisions at $t = 0$. We then consider the $t = 0$ equilibrium specialization decisions and assess their optimality.

4.1 Consumption at $t = 1$

The consumer's problem is similar to the one in the benchmark case of Section 3.1. For any given realization of the traded-good prices, $\{p_t(x_t)\}_{x_t \in [0,1]}$, consider consumption decisions at $t = 1$. Consumer i with income y_i maximizes her utility in eq. 2 subject to the following budget constraint:

$$y_i = \int_0^1 p_t(x_t) c_{i,t}(x_t) dq_t + \int_0^1 p_{nt}(x_{nt}) c_{i,nt}(x_{nt}) dx_{nt} + p_r c_{i,r}. \tag{12}$$

From this optimization problem, consumer i's indirect utility function can be expressed as

$$v(y_i) = \frac{\gamma_t^{\gamma_t} \gamma_{nt}^{\gamma_{nt}} \gamma_r^{\gamma_r} \times y_i}{P_t^{\gamma_t} \exp\left[\gamma_{nt} \int_0^1 \ln p_{nt}(x_{nt}) dx_{nt}\right] p_r^{\gamma_r}}, \tag{13}$$

which, as in the benchmark case, is increasing and linear in y_i.

Let Y be the city's income, that is, the sum of the income from the production of traded goods Y_t, from the production and endowment of non-traded goods Y_{nt}, and from real estate Y_r. The Cobb-Douglas utility function implies that the city's expenditure in the consumption of traded goods, non-traded-goods, and real estate, E_t, E_{nt} and E_r, respectively are

$$E_t = \gamma_t Y \ , \ \ E_{nt} = \gamma_{nt} Y \ \text{ and } E_r = \gamma_r Y. \tag{14}$$

Since the city's income from the production of traded goods must be equal to the city's consumption in traded goods, i.e., $Y_t = E_t$, from eq. 14, it follows that the market-clearing prices of non-traded good x_{nt} and real estate are

$$p_{nt}(x_{nt}) = \frac{\gamma_{nt}}{\gamma_t} \frac{Y_t}{Q_{nt}(x_{nt})} \tag{15}$$

and

$$p_r = \frac{\gamma_r}{\gamma_t} \frac{Y_t}{R}. \tag{16}$$

where $Q_{nt}(x_{nt})$ is the production of non-traded good x_{nt} in the city. While the prices of traded goods are taken as exogenous, the prices of non-traded goods and real estate are endogenously determined and depend on the city's income from the production of traded goods, Y_t, as well as by the production choices of the entrepreneurs supplying the non-traded goods. These production choices are considered in the next section.

4.2 Production at $t = 1$.

At $t = 1$, the production of traded goods in the city is determined by its industrial base, which, in turn, is influenced by the specialization decisions of the city's traded-good workers at $t = 0$. Hence, $L_i(x_t) = \int_0^{\overline{L}} l_i(x_t) di$ units of labor are devoted to the production of traded-good $x_t \in X_N \cup \{1\}$ at $t = 1$, and Y_t, as expressed in eq. 1, is the city's income from the production of traded goods.

Regarding the production of non-traded-goods, there is one worker and one entrepreneur in each non-traded-good sector x_{nt}. The worker and the entrepreneur both produce $\frac{q_0}{2} > 0$ units of the non-traded good, but the entrepreneur has the option to produce $q_1 > 0$ additional units of the good by incurring a non-pecuniary fixed cost c. The total quantity produced is thus $Q_{nt}(x_{nt}) = q_0 + q_1$ if the entrepreneur in non-traded-good sector x_{nt} produces q_1 additional units of the good, and $Q_{nt}(x_{nt}) = q_0$ if she does not.

The above specification contributes to the tractability of our model in a few ways. First, the assumption that $q_0 > 0$ guarantees that every non-traded good is available in the market, and therefore, consumed whatever the entrepreneurs' production decisions. This implies that the marginal utility of income is always positive, even when $Q_{nt}(x_{nt}) = q_0$. Second, the Cobb-Douglas utility function makes the elasticity of demand of each consumption good equal to

1, which implies that a single monopolist-entrepreneur would never incur a non-pecuniary fixed cost c to produce q_1 additional units of the good. Therefore, having one worker and one entrepreneur for each non-traded good who compete to gain market share makes the entrepreneur's production decision a meaningful one. Finally, having only one worker and one entrepreneur, rather than two entrepreneurs, allows us to avoid solving a Cournot game (under asymmetric information in the case of the Global Games refinement) for each non-traded good market, and it just simplifies the analysis.

Consider the production choice of the entrepreneur in non-traded-good sector x_{nt}. From eq. 15, the price of non-traded good x_{nt} is

$$p_{nt}(x_{nt}) = \frac{\gamma_{nt}}{\gamma_t} \frac{Y_t}{Q_{nt}(x_{nt})}, \tag{17}$$

implying that the entrepreneur's income, as a function of the total quantity of non-traded-good x_{nt} produced is

$$y(Q_{nt}(x_{nt})) = \frac{\gamma_{nt}}{\gamma_t} \frac{Y_t}{Q_{nt}(x_{nt})} \left(Q_{nt}(x_{nt}) - \frac{q_0}{2} \right). \tag{18}$$

Entrepreneur x_{nt} produces q_1 additional units when the utility of the extra income from doing so provides sufficient compensation for the non-pecuniary fixed cost c, that is, when $v\left[y(q_0+q_1)\right] - v\left[y(q_0)\right] \geq c$. From eq. 18, this condition can be expressed as

$$v\left[\frac{\gamma_{nt}}{\gamma_t} \frac{\frac{1}{2}q_0 + q_1}{q_0 + q_1} Y_t\right] - v\left[\frac{\gamma_{nt}}{\gamma_t} \frac{Y_t}{2}\right] \geq c\,. \tag{19}$$

Using eq. 13 and eq. 16, the condition in eq. 19 can be written as

$$\frac{\gamma_t^{\gamma_t} \gamma_{nt}^{\gamma_{nt}} \gamma_r^{\gamma_r} \times \frac{\gamma_{nt}}{\gamma_t} \frac{q_1}{q_0+q_1} \frac{Y_t}{2}}{P_t^{\gamma_t} \exp\left[\gamma_{nt} \int_0^1 \ln p_{nt}(x_{nt}) dx_{nt}\right] \left[\frac{\gamma_r}{\gamma_t} \frac{Y_t}{R}\right]^{\gamma_r}} \geq c, \tag{20}$$

where $P_t \equiv \exp\left[\int_0^1 \ln p_t(x_{nt}) dx_{nt}\right]$. According to eq. 20, the production decision of entrepreneur in sector x_{nt} depends on the city's income from the traded-good sector Y_t (and hence, on the city's industrial base), and also on the prices of other non-traded goods in the city $\int_0^1 \ln p_{nt}(x_{nt}) dx_{nt}$ (and hence, on the production decisions of entrepreneurs in all the other non-traded-good sectors $x'_{nt} \in [0,1] \backslash \{x_{nt}\}$).

Assume that all entrepreneurs $x'_{nt} \in [0,1] \backslash \{x_{nt}\}$ produce the same amount Q_{nt} of non-traded good x'_{nt}. Then, for all $x'_{nt} \neq x_{nt}$, $p_{nt}(x'_{nt}) = \frac{\gamma_{nt}}{\gamma_t}\frac{Y_t}{Q_{nt}}$, where $Q_{nt} \in \{q_0, q_0 + q_1\}$. The condition in eq. 20 can then be expressed as

$$\left[\frac{Y_t}{P_t}\right]^{\gamma_t} \geq \frac{2c}{\gamma_{nt} Q_{nt}^{\gamma_{nt}} R^{\gamma_r}} \left(1 + \frac{q_0}{q_1}\right). \tag{21}$$

According to eq. 21, an entrepreneur in sector x_{nt} is more likely to produce q_1 additional units when entrepreneurs in other non-traded sectors also produce q_1 additional units, that is, when $Q_{nt} = q_0 + q_1$. Intuitively, an increase in the supply of other non-traded goods in the city lowers their prices, which increases the amount of goods that the entrepreneur in sector x_{nt} can consume if she decides to produce. This implies that entrepreneurs' production decisions are strategic complements. This strategic complementarity can lead to multiple equilibria in the production decisions at $t = 1$, as we show next.

Assume that all entrepreneurs $x'_{nt} \in [0,1]\backslash\{x_{nt}\}$ produce q_1 additional units so that $Q_{nt} = q_o + q_1$. From eq. 21, the entrepreneur in sector x_{nt} also produces if

$$\left[\frac{Y_t}{P_t}\right]^{\gamma_t} \geq \frac{2c}{\gamma_{nt} R^{\gamma_r}} \frac{1 + \frac{q_0}{q_1}}{(q_0 + q_1)^{\gamma_{nt}}} \tag{22}$$

Alternatively, assume that all entrepreneurs $x'_{nt} \in [0,1]\backslash\{x_{nt}\}$ do not produce q_1 additional units so that $Q_{nt} = q_0$. From eq. 21, the entrepreneur in sector x_{nt} does not produce either if

$$\left[\frac{Y_t}{P_t}\right]^{\gamma_t} < \frac{2c}{\gamma_{nt} R^{\gamma_r}} \frac{1 + \frac{q_0}{q_1}}{q_0^{\gamma_{nt}}} \tag{23}$$

Combining eq. 22 and eq. 23, we obtain the following lemma.

Lemma 1 *At $t = 1$, the production subgame among the entrepreneurs that produce non-traded goods is such that*

1. *If $\left[\frac{Y_t}{P_t}\right]^{\gamma_t} < \frac{2c}{\gamma_{nt} R^{\gamma_r}} \frac{1+\frac{q_0}{q_1}}{(q_o+q_1)^{\gamma_{nt}}}$, producing $\frac{q_0}{2}$ is a strictly dominant strategy;*
2. *If $\left[\frac{Y_t}{P_t}\right]^{\gamma_t} > \frac{2c}{\gamma_{nt} R^{\gamma_r}} \frac{1+\frac{q_0}{q_1}}{(q_o+q_1)^{\gamma_{nt}}}$, producing $\frac{q_0}{2} + q_1$ is a strictly dominant strategy.*
3. *If $\left[\frac{Y_t}{P_t}\right]^{\gamma_t} \in \left[\frac{2c}{\gamma_{nt} R^{\gamma_r}} \frac{1+\frac{q_0}{q_1}}{(q_o+q_1)^{\gamma_{nt}}}, \frac{2c}{\gamma_{nt} R^{\gamma_r}} \frac{1+\frac{q_0}{q_1}}{q_0^{\gamma_{nt}}}\right]$, an equilibrium where all entrepreneurs produce $\frac{q_0}{2}$ coexists with an equilibrium where all entrepreneurs produce $\frac{q_0}{2} + q_1$;*

Note that Lemma 1 establishes the existence of demand spillovers not only across non-traded goods, as discussed earlier, but also between traded and non-traded goods. Indeed, an equilibrium in which the city is vibrant, that is, in which the production of non-traded goods is high, i.e., $Q_{nt} = q_0 + q_1$, is more likely when the income from the production of traded goods in the city, Y_t, is also high. Intuitively, when the income from traded goods is high, so is the demand for non-traded goods in the city, and hence, the price of non-traded-goods is high relative to the price of traded-goods. This increase in the relative price of non-traded-goods makes it more profitable for entrepreneurs to produce q_1 additional units of any given non-traded good.

Lemma 1 also confirms the familiar intuition that strategic complementarities across non-traded goods can generate multiple equilibria when

$$\left[\frac{Y_t}{P_t}\right]^{\gamma_t} \in \left[\frac{2c}{\gamma_{nt} R^{\gamma_r}} \frac{1+\frac{q_0}{q_1}}{(q_0+q_1)^{\gamma_{nt}}}, \frac{2c}{\gamma_{nt} R^{\gamma_r}} \frac{1+\frac{q_0}{q_1}}{q_0^{\gamma_{nt}}}\right]. \tag{24}$$

The multiplicity of equilibria arises because an entrepreneur is more likely to produce q_1 additional units when entrepreneurs in other non-traded sectors also produce q_1 additional units. A self-fulfilling coordination failure can thus exist in which entrepreneurs do not produce q_1 additional units because they expect other entrepreneurs to also not produce. In such a case, the city is trapped in a low level of economic activity.[14]

4.3 Unique Equilibrium in Production at $t = 1$.

The multiplicity of equilibria at the production stage $t = 1$, arising from the strategic complementarities in the production of non-traded goods, creates an indeterminacy that precludes the analysis of the workers' specialization decisions at $t = 0$. To resolve this indeterminacy, we apply a global games approach (Carlsson and Van Damme, 1993), which introduces dispersed information, breaking the ability of agents to perfectly coordinate and

[14] The presence of multiple equilibria and the importance of aggregate demand is in line with the Keynesian narrative. For example, Blanchard and Quah (1989) use structural VARs to provide evidence in support of the idea that business cycles are driven by shifts in aggregate demand. The same idea is corroborated by recent work that exploits the regional variation in business cycles, such as Mian and Sufi (2014) and Beraja, Hurst and Ospina (2019).

generating a unique equilibrium. A well-known property of global games is that equilibrium uniqueness carries over to the asymptotic case in which agents' private signals become infinitely precise and the information structure is arbitrarily close to common knowledge. This limit case, which highlights that equilibrium multiplicity is a by-product of the common knowledge assumption, will be our focus in the rest of the paper.

Formally, we enrich the production game by assuming that at $t = 1$, prior to making their production choices, the entrepreneur in each sector $x_{nt} \in [0,1]$ observes a private noisy signal of the city's income from the production of traded goods, $s_{x_{nt}} = Y_t + \varepsilon_{x_{nt}}$, where $\varepsilon_{x_{nt}}$ is uniformly distributed in the interval $[-\underline{\varepsilon}, \underline{\varepsilon}]$ and independent across entrepreneurs. In other words, information is dispersed in the sense that each of the entrepreneurs observe a slightly different signal of the same fundamental. We also make the following parametric assumption:

$$\left[\frac{2\delta\overline{L}}{P_t}\right]^{\gamma_t} > \frac{2c}{\gamma_{nt}R^{\gamma_r}} \frac{1+\frac{q_0}{q_1}}{q_0^{\gamma_{nt}}}. \tag{25}$$

The assumption in eq. 25, which follows directly from Case 2 in Lemma 1 if we set $Y_t = 2\delta\overline{L}$, implies that there exist realizations of the traded-good income Y_t sufficiently large to make producing $\frac{q_0}{2} + q_1$ a dominant strategy.[15] Given that realized traded-good prices are positive but can be potentially very low, there also exist realizations of the Y_t that are small enough to make producing $\frac{q_0}{2}$ a dominant strategy (Case 1 in Lemma 1). The existence of these upper and lower dominance regions is a standard requirement in global games for equilibrium uniqueness, but the probability that Y_t falls in either of these regions can be arbitrarily small.

The equilibrium derivation is a direct application of global games and left to the proof of Proposition 3 in the Appendix. In this derivation, we first show that for $\underline{\varepsilon}$ small enough, the unique equilibrium strategy is for the entrepreneur in sector x_{nt} to produce if and only if her signal $s_{x_{nt}}$ is higher than a threshold $\hat{s}$. We then take the limit when $\underline{\varepsilon}$ tends to 0 to recover an information structure arbitrarily close to common knowledge while retaining equilibrium uniqueness.

[15] Producing $\frac{q_0}{2} + q_1$ is a dominant strategy when the realized prices of the traded goods produced in the city are sufficiently high. The assumption in eq. 25 guarantees that prices need not be higher than upper bound $\overline{p} = 2$ for $\frac{q_0}{2} + q_1$ to become a dominant strategy.

Proposition 3 . *When* $\underline{\varepsilon} \to 0$,

(i) All entrepreneurs produce $\frac{q_0}{2} + q_1$ *units at* $t = 1$ *if* $Y_t \geq Y_t^T$ *and produce only* $\frac{q_0}{2}$ *units if* $Y_t < Y_t^T$ *where* Y_t^T *is implicitly defined as the solution to the identity*

$$\left(\frac{Y_t^T}{P_t}\right)^{\gamma_t} \equiv \frac{\ln(q_0 + q_1) - \ln q_0}{[q_0 + q_1]^{\gamma_{nt}} - q_0^{\gamma_{nt}}} \frac{q_0 + q_1}{q_1} \frac{2c}{\gamma_{nt} R^{\gamma_r}}; \tag{26}$$

(ii) At the threshold Y_t^T, *welfare is strictly higher if entrepreneurs produce* $\frac{q_0}{2} + q_1$ *units than if they produce* $\frac{q_0}{2}$, *as expressed in the following inequality*

$$\left(\frac{Y_t^T}{P_t}\right)^{\gamma_t} (q_0 + q_1)^{\gamma_{nt}} R^{\gamma_r} - c > \left(\frac{Y_t^T}{P_t}\right)^{\gamma_t} q_0^{\gamma_{nt}} R^{\gamma_r}. \tag{27}$$

Part (i) of Proposition 3 shows that there is a unique equilibrium level of production at $t = 1$, which depends on the city's traded-good income Y_t. Specifically, entrepreneurs coordinate on the high-output equilibrium for high realizations of the traded-good income (i.e., $Y_t \geq Y_t^T$) and on the low-output equilibrium for low realizations (i.e., $Y_t < Y_t^T$). Moreover, the production threshold Y_t^T in eq. 26 belongs to the multiple equilibrium region under common knowledge, i.e.,

$$\left[\frac{Y_t^T}{P_t}\right]^{\gamma_t} \in \left(\frac{2c}{\gamma_{nt} R^{\gamma_r}} \frac{1 + \frac{q_0}{q_1}}{(q_o + q_1)^{\gamma_{nt}}}, \frac{2c}{\gamma_{nt} R^{\gamma_r}} \frac{1 + \frac{q_0}{q_1}}{q_0^{\gamma_{nt}}}\right). \tag{28}$$

Unique threshold equilibria are standard in global games settings. In our case, the threshold nature of the equilibrium emanates from the fact that the entrepreneurs' benefit from producing more is higher when the traded-good income is high and when other entrepreneurs also produce more. Consequently, when entrepreneurs observe a high signal of the traded-good income, they make two type of inferences. First, they expect the traded-good income to be high, which increases the benefit of exerting effort and producing and second, they expect other entrepreneurs to have observed a high signal. Due to the fact that entrepreneurs benefit more from production when other entrepreneurs produce, higher-order beliefs generate a sharp transition in the production of non-traded goods when traded-good income becomes sufficiently high. Intuitively, for a high enough signal, entrepreneurs expect other entrepreneurs to produce, entrepreneurs expect other entrepreneurs to expect other entrepreneurs to produce, and so on.

As explained in detail in Morris and Shin (2003), an iterative dominance argument implies that the threshold equilibrium is the unique equilibrium. Lastly, the impact of dispersed information on higher-order beliefs and the corresponding equilibrium characteristics persist when $\underline{\varepsilon}$ approaches zero and the model converges towards an information structure arbitrarily close to common knowledge.

Part (ii) of Proposition 3 shows that the production threshold Y_t^T lies within a region where it is efficient for entrepreneurs to produce additional units. Part (ii) thus implies that the negative effect imposed on other entrepreneurs from increased production within their own non-traded-good sector is smaller than the benefit from the overall increase in production within the city. In essence, our setting is one in which the equilibrium at $t = 1$ exhibits a coordination failure: there exists a region below Y_t^T where welfare is diminished relative to one in which entrepreneurs collectively decide to produce at the high level.

Overall, Proposition 3 illustrates the importance of strategic uncertainty and higher-order beliefs in determining the city's level of economic activity. The importance of strategic uncertainty is consistent with a recent literature in macroeconomics that uses models of incomplete information to introduce coordination frictions to shed light on the dynamics of business cycles and economic crises.[16] For instance, Schaal and Taschereau-Dumouchel (2019) show that a large transitory shock may push the economy into a quasi-permanent recession, helping explain the slow recovery and other salient features of the Great Recession. In the context of our model, Proposition 3 shows that a shock to the city's income from the production of traded goods Y_t can generate a sharp transition between low– and high–levels of economic activity. Indeed, there are numerous examples of cities such as Detroit whose economic fate has mirrored the fate of the sector in which the city was specialized in. Importantly, there is also evidence of sectoral shocks having important local multiplier effects. For instance, Moretti (2010) finds that for each additional job in manufacturing in a given city, 1.6 jobs are created in the non-traded sector in the same city.[17]

[16] See Angeletos and Lian (2016) for an overview of this literature.

[17] See also Foerster, Sarte and Watson (2011) and Di Giovanni, Levchenko and Mejean (2014) for additional evidence of sectoral shocks having a large multiplying effect in the rest of the economy.

4.4 Equilibrium Industrial Base at $t = 0$.

This section builds on the production equilibrium in Proposition 3 to study traded-good workers' specialization choices at $t = 0$. We focus our attention on a particular type of specialization equilibrium and prove that such an equilibrium exists. Specifically, we prove that there always exists an equilibrium industrial base that puts more weight on the favored traded-good sector –traded-good sector 1– than in another traded-good sector. We will then provide conditions under which a more diversified industrial base improves welfare relative to this equilibrium allocation. In Section 4.6.2, we will consider the possibility of other equilibria that put less weight on the favored traded-good sector and assess their welfare.

As in Section 3, let $l_i(x_t)$ be an indicator function defined on $x_t \in X_N \cup \{1\}$ such that $l_i(x_t') = 1$ if worker i specializes in the production of traded good x_i', and $l_i(x_t') = 0$ otherwise. Therefore, the mass of workers that choose to specialize in the production of traded good $x_t \in X_N \cup \{1\}$ is $L_i(x_t) = \int_0^{\overline{L}} l_i(x_t) di$. For a given specialization choice $l_i(x_t)$, worker i obtains income

$$y_i = (\alpha + \delta)\, l_i(1) p_t(1) + \sum_{x_t \in X_N} \delta l_i(x_t) p_t(x_t) \tag{29}$$

and from eq. 13, his indirect utility function can be written as

$$v(y_i) = \frac{\gamma_t Q_{nt}^{\gamma_{nr}} R^{\gamma_r} \times y_i}{P_t^{\gamma_t} Y_t^{\gamma_{nt}+\gamma_r}} \tag{30}$$

where, from Proposition 3, $Q_{nt} = q_0$ if $Y_t < Y_t^T$ and $Q_{nt} = q_0 + q_1$ if $Y_t \geq Y_t^T$. Therefore, at $t = 0$, worker i solves:

$$\max_{\{l_i(x_t)\}_{x_t \in X_N \cup \{1\}}} \mathbb{E}[v(y_i)]. \tag{31}$$

To simplify the analysis, in the rest of this section, we assume that $N = 1$, so that workers either specialize in traded-good sector 1 or in some other traded-good sector $x_t \in X_1$. (Section 4.6.3 below considers the case in which N is arbitrarily large.) The next proposition shows that there exists an equilibrium in which the city's industrial base has more weight in the production of the traded-good in which the city has a productivity advantage, i.e., traded-good 1.

Proposition 4 *For $\alpha > 0$, there exists an equilibrium such that $L(1)^* > L(x_t)^*$.*

Proposition 4 states that there always exists an equilibrium in which the mass of workers specialized in the production of traded-good 1 is greater than the mass of workers specialized in the production of traded-good x_t. Intuitively, at $L(1) = L(x_t)$, if $\alpha = 0$, a worker would be indifferent between specializing in traded-good sector 1 and x_t, and if $\alpha > 0$, a worker strictly prefers specializing in traded-good sector 1. However, as the next proposition shows, this does not imply that the city ends up fully specialized in traded-good sector 1.

Proposition 5 *There is an $\tilde{\alpha} > 0$ such that if $\alpha \in (0, \tilde{\alpha})$, there exists an equilibrium with $L(1)^* > L(x_t)^* > 0$.*

Proposition 5 provides conditions for the existence of an interior equilibrium in which the city produces both traded-goods. In any interior equilibrium, $L(1)^* > L(x_t)^* > 0$, a worker must be indifferent between specializing in traded-good sectors 1 and x_t, that is,

$$\mathbb{E}\left[v((\alpha + \delta)p_t(1))\right] = \mathbb{E}\left[v(\delta p_t(x_t))\right], \tag{32}$$

where $v(y_i)$ is defined as in eq. 30. While the productivity advantage of traded-good sector 1 induces workers to specialize in this sector, if this productivity advantage is not large enough (if $\alpha < \tilde{\alpha}$), the only equilibrium with $L(1)^* > L(x_t)^*$ is one in which the city does not fully specialize in traded-good sector 1, i.e., $L(1)^* < \overline{L}$. In his specialization decision, a worker considers the expected income (i.e., $(\alpha + \delta)\mathbb{E}\left[p_t(1)\right]$ vs. $\delta\mathbb{E}\left[p_t(x_t)\right]$) as well as the correlations between his income and the prices of real estate and of non-traded goods. These prices depend on the income from the production of traded-goods Y_t: In the case of real estate, Y_t determines the demand for real estate by the city residents, as we saw in Section 3; In the case of non-traded goods, Y_t not only determines the demand for non-traded-goods, but also its supply Q_{nt}. As we see next, the effect of Y_t on Q_{nt} has welfare implications for the equilibrium industrial base.

4.5 Welfare

This section examines the welfare properties of the equilibrium industrial base in Proposition 5. Our concept of social optimum is *constrained* efficiency: The social planner chooses the industrial base at $t = 0$, but cannot avoid coordination failures at $t = 1$ when $Y_t < Y_t^T$. From eq. 30, the sum of the indirect utility of all the city's residents can be written as

$$v_s(Y) = \frac{Q_{nt}^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{P_t^{\gamma_t}} - c\mathbb{1}_{Y_t \geq Y_t^T}, \tag{33}$$

where we have used the fact that $Y_t = \gamma_t Y$, and $\mathbb{1}_{Y_t \geq Y_t^T}$ is an indicator function that takes a value of 1 if $Y_t \geq Y_t^T$.

The following proposition evaluates the welfare of any interior equilibrium industrial base, i.e. $L(1)^* > L(x_t)^* > 0$.

Proposition 6 *Consider an interior equilibrium such that $L(1)^* > L(x_t)^* > 0$. If $Y_t^T < \delta\overline{L}$, the social planner can increase welfare by decreasing $L(1)$, and if $Y_t^T > \delta\overline{L}$, the social planner can increase welfare by increasing $L(1)$.*

Proposition 6 shows that when coordination failures are possible, the equilibrium industrial base is not socially optimal. To better understand the source of the inefficiency, note that a worker's specializing choice affects the city's overall production along two dimensions. First, it directly affects the income from the production of traded goods, Y_t. The direct impact of Y_t on overall welfare is proportional to

$$\mathbb{E}\left[v((\alpha + \delta)p_t(1))\right] - \mathbb{E}[v(\delta p_t(x_t))], \tag{34}$$

that is, to the difference in expected utility from receiving income in sector 1 rather than in sector x_t. In an interior equilibrium, workers are indifferent between specializing in traded-good sector 1 and x_t and hence, eq. 34 is equal to zero. In other words, workers internalize the direct effect of their sectoral choices on Y_t.

Second, a worker's sectoral choice indirectly influences the production of non-traded goods by affecting the likelihood that Y_t exceeds the threshold Y_t^T. However, when choosing the sector in which to specialize, workers take this probability as given, thereby resulting in a socially suboptimal equilibrium industrial base. Specifically, when $Y_t^T < \delta\overline{L}$, excessive specialization occurs in equilibrium, and the social planner can enhance welfare by reducing $L(1)$. Conversely, when $Y_t^T > \delta\overline{L}$, insufficient specialization arises. Intuitively, in scenarios where the city's productivity (as measured by δ) is high and the size of the traded-good sector (as measured by $\overline{L}$) is large, the demand for non-traded goods is expected to be high, and the city is likely to be in the high-output equilibrium. When this is the case, there is a social benefit to increasing diversification, thereby reducing the possibility of a negative shock to any one traded-good sector triggering a coordination failure. In contrast, in scenarios where the city is likely to be in a low-output equilibrium, increasing specialization can have the social benefit of increasing the possibility of escaping the low-output equilibrium when there is a positive shock to traded-good sector 1.

The analysis in this section is consistent with cities, when compared with countries, being highly specialized economies dependent on one or a few traded-good sectors. It provides a rationale for numerous instances in which industrial policy aims to reduce a city's exposure to sectoral shocks by diversifying its industrial base. Indeed, there is a literature that emphasizes the importance of diversification for economic resilience. For instance, Brown and Greenbaum (2017) examines the influence of industrial diversity on unemployment rate stability in Ohio counties between 1977 and 2011 finds that while more concentrated counties had lower unemployment rates when times were good, counties with more diverse industry structures fared better during times of local employment shocks.

As we noted in the introduction, the objective of reducing exposure to sectoral shocks through diversification has been a driver of urban policy. Nonetheless, policymakers have sometimes endorsed a Porter-type cluster strategy (see Porter, 1990). This type of strategy aims to concentrate resources in related industries, with the underlying notion that firms can benefit from the activities of neighboring firms within the same or related industries, an idea that goes back to Marshall (1890). In the context of our model, a cluster strategy entails increasing the city's exposure to sectoral shocks, something that according to our analysis is optimal for less productive regions (low δ) and for regions with scarce resources (low $\overline{L}$).

4.6 Equilibrium Industrial Base at $t = 0$: Robustness.

Section 4.4 assumes that workers choose between two traded-good sectors (i.e., sector 1 and sector x_t) and focuses on equilibria in which the mass of workers specialized in the production of traded-good 1 is greater than the mass of workers specialized in the production of traded-good x_t. This section explores the possibility of other type of equilibria and studies the case in which workers choose between more than two traded-good sectors, i.e., $N > 1$.

4.6.1 Corner Equilibrium

Proposition 6 considers the welfare properties of an interior equilibrium, $\overline{L} > L(1)^* > L(x_t)^* > 0$, which is likely to be the most common case, as most cities produce more than one traded-good. For completeness, however, we next assess the welfare of a corner equilibrium with $\overline{L} = L^*(1)$, which can also exist provided that α is large enough.

Proposition 7 *Consider a corner equilibrium, with $L^*(1) = \overline{L}$. If $Y_t^T < \delta\overline{L}$, a marginal decrease in $L(1)$ from $\overline{L}$ may increase or decrease welfare, and if $Y_t^T > \delta\overline{L}$, a marginal decrease in $L(1)$ from $\overline{L}$ decreases welfare.*

Similar to the interior equilibrium case analyzed in Section 4.5, in a corner equilibrium, workers do not internalize the effect of their specialization choices on the probability of a coordination failure. However, unlike an interior equilibrium, in a corner equilibrium, workers strictly prefer specializing in traded-good sector 1, i.e., $\mathbb{E}\left[v((\alpha+\delta)p_t(1))\right] > \mathbb{E}\left[v(\delta p_t(x_t))\right]$, which implies that there is now an additional cost of reducing $L(1)$ from $\overline{L}$. When $Y_t^T < \delta\overline{L}$, reducing $L(1)$ from $\overline{L}$ still increases welfare by reducing the probability of coordination failure, but there is now a cost to reducing $L(1)$. If the difference $\mathbb{E}\left[v((\alpha+\delta)p_t(1))\right] - \mathbb{E}\left[v(\delta p_t(x_t))\right]$ is small, the effect of reducing $L(1)$ on the probability of a coordination failure dominates, and hence, a decrease in $L(1)$ from $\overline{L}$ increases welfare, as it did in Proposition 6. Conversely, when this difference is large enough, a decrease in $L(1)$ from $\overline{L}$ decreases welfare. In contrast, when $Y_t^T > \delta\overline{L}$, a decrease in $L(1)$ from $\overline{L}$ decreases welfare, not only because it increases the probability of a coordination failure (as in the interior equilibrium case), but now also because workers prefer specializing in traded-good sector 1.

4.6.2 Other Equilibria

Proposition 4 guarantees the existence of an equilibrium industrial base that puts more weight on the production of traded-good 1. Section 4.4 focuses on these type of equilibria because the city enjoys a productivity advantage in traded-good sector 1. Nonetheless, Proposition 4 does not rule out the possibility of equilibria in which the city puts more weight on traded-good sector x_t than on sector 1, i.e., $L(x_t)^* > L(1)^*$. We explore these other equilibria next.

In the absence of non-traded goods other than real estate, Section 3 shows that there is a unique equilibrium industrial base in which $L^*(1) > L^*(x_t)$, and that this equilibrium is socially optimal. Two forces shape this unique equilibrium. First, the productivity advantage of traded-good sector 1 induces workers to specialize in sector 1. Second, the positive correlation between the income in traded-good sector 1 and Y_t –correlation that increases in $L(1)$– can induce some workers to specialize in other traded-good sectors to take advantage of the low real estate price when the city's income Y_t is low, provided that α is not too large. These two forces are also present in Section 4.4, but the inclusion of non-traded goods and the possibility of a coordination failure generates an additional force: the correlation between Y_t and the prices of non-traded goods.

An increase in Y_t increases the demand for non-traded goods which, ceteris paribus, increases their prices, as it did for the price of real estate. However, Y_t also affects the prices of non-traded goods because it impacts their supply. The supply of non-traded goods depends on the resources available in the city to produce these goods (i.e., the entrepreneurs in the case of our model) and on the utilization of these resources (i.e., whether the city is in a low- or high-activity equilibrium). Around threshold Y_t^T, the production of non-traded goods rapidly increase as the city's economy shifts from a low- to a high-production equilibrium. This rapid increase in the supply of traded goods can motivate workers to specialize in the same traded-good sector as other workers. To gain intuition, consider a case in which $q_0 = 0$, so there is no production of traded-goods when $Y_t < Y_t^T$, which makes the marginal utility of income zero in the low-production equilibrium. In such case, there is less incentives to diversify away from traded-good sector 1 because, when there is a negative shock to traded-good sector 1, the city is likely to be in the low-production equilibrium and hence, the marginal utility of income is likely to be zero.

While the rapid increase in the supply of non-traded goods when the economy shifts from a low- to a high-production equilibrium generates strategic complementarities in workers' specialization choices, within the low- and high-activity equilibria, production is fixed at q_0 and $q_0 + q_1$, respectively. Again, to gain intuition, assume now that $q_1 = 0$, so that non-traded goods are in fixed supply. Such a case is essentially identical to the benchmark (real estate is just a non-traded in fixed supply) and, as discussed in Section 3, when the supply of traded-goods is inelastic, workers benefit from diversification, that is, from receiving higher income when other workers do not. This effect generates strategic substitutabilities in workers' specialization choices.

In summary, the resources available to produce non-traded goods limits their supply and tends to make workers' specialization choices strategic substitutes, while the rapid change in the supply of non-traded goods when the city's economy transitions between the low- and high-activity equilibria tends to make workers' specialization choices strategic complements. When these strategic complementarities are strong enough, multiple equilibria may arise, and in some of these equilibria, the mass of workers specialized in the production of traded-good x_t could be greater than the mass of workers specialized in the production of traded-good 1.

If there are equilibria in which the mass of workers specialized in the production of traded-good x_t is greater than the mass of workers specialized in the production of traded-good 1, this type of equilibria will be socially inefficient. First, any equilibrium industrial base $\{L^*(1), L^*(x_t)\}$ in which $L^*(1) < L^*(x_t)$ is going to be dominated from the social point of view by an industrial base $\{L'(1), L'(x_t)\}$ such that $L'(1) = L^*(x_t) > L^{'}(x_t) = L^*(x_t)$. This inefficiency does not speak to the city's degree of specialization per se, but to the city specializing in the "wrong" (lower productivity) traded-sector x_t. Second, similar to Section 4.4, workers do not internalize that their combined specialization choices determine the city's income from the production of traded and hence, the probability of a coordination failure as the next proposition shows.

Proposition 8 *(i) Consider an interior equilibrium, with* $0 < L^*(1) < L^*(x_t) < \overline{L}$*. If* $Y_t^T < \delta\overline{L}$*, the social planner can increase welfare by decreasing* $L(x_t)$*, and if* $Y_t^T < \delta\overline{L}$*, the social planner can increase welfare by increasing* $L(x_t)$*. (ii) Consider a corner equilibrium, with* $L^*(x_t) = \overline{L}$*. If* $Y_t^T < \delta\overline{L}$*, a marginal decrease in* $L(x_t)$ *from* $\overline{L}$ *may increase or decrease welfare, and if* $Y_t^T > \delta\overline{L}$*, a marginal decrease in* $L(x)$ *from* $\overline{L}$ *decreases welfare provided that* α *is small enough.*

The message of Proposition 8 is similar to the message of Proposition 6 and Proposition 7 Combining the three propositions, it follows that for any interior equilibrium (i.e., $L^*(1) > 0$ and $L^*(x_t) > 0$), when $Y_t^T < \delta\overline{L}$, an increase in diversification (i.e., a decrease in $L(1)$ when $L^*(1) > L(x_t)$ or a decrease in $L(x_t)$ when $L^*(x_t) > L^*(1)$) increases welfare. Alternatively when $Y_t^T > \delta\overline{L}$, an increase in specialization (i.e., an increase in $L(1)$ when $L^*(1) > L(x_t)$ or an increase in $L(x_t)$ when $L^*(x_t) > L(1)$ provided that α is small enough) decreases welfare.[18] For any corner equilibrium, workers also do not internalize the probability that their specialization choices have on the probability of a coordination failure. However, as explained after Proposition 7, this effect has to be weighed against the fact that in a corner equilibrium, workers prefer to specialize in traded-good sector 1 when $L^*(1) = \overline{L}$ and in traded-good sector x_t when $L^*(x_t) = \overline{L}$. In summary, while equilibria in which $L^*(1) > L^*(x_t)$ always exits and are probably more economically relevant than equilibria in which $L^*(x_t) > L^*(1)$, the welfare implications in terms of diversification of equilibria $L^*(1) > L^*(x_t)$ carry to equilibria $L^*(x_t) > L^*(1)$.

[18] For $L^*(x_t) > L(1)$ and $Y_t^T > \delta\overline{L}$, the assumption that α is not too large guarantees that marginal increase in $L(x_t)$ does not reduce the probability of a coordination failure. Note however, that if α is large an equilibrium with $L^*(x_t) > L^*(1)$ is less likely to exist in the first place.

4.6.3 N Arbitrarily Large

In previous sections, we have assumed that workers either specialize in traded-good sector 1 or in some other traded-good sector $x_t \in X_1$. We next consider the case with $N > 1$, so the set of traded-good sectors in which a worker can specialize, $X_N \cup \{1\}$, is greater than 2. The next proposition assesses the existence and welfare of a symmetric equilibrium industrial base (an equilibrium in which all sectors in X_N have the same size) when $N \to +\infty$. This symmetric equilibrium can be viewed as the combination of traded-good-sector 1 with $L^*(1)$ workers and a composite traded-good sector with $\overline{L} - L^*(1)$ workers, in which all traded-good sectors in X_N have equal weight.

Proposition 9 *(i) If there is an* $\tilde{\alpha} > 0$ *such that if* $\alpha \in (0, \tilde{\alpha})$ *and* $N \to +\infty$*, there exists an equilibrium with* $L(1)^* \in (0, \overline{L})$ *workers specialized in the production of traded-good* 1 *and* $Y_t = (\alpha + \delta)\, p_t(1) L^*(1) + \delta(\overline{L} - L^*(1))$*; (ii) In any such equilibrium, if* $0 < \frac{Y_t^T - \delta\left(\overline{L} - L^*(1)\right)}{(\alpha+\delta) L^*(1)} < 2$*, the social planner can increase welfare by decreasing* $L(1)$ *when* $Y_t^T < \delta \overline{L}$*, and can increase welfare by increasing* $L(1)$ *when* $Y_t^T > \delta \overline{L}$*; and (iii) In any such equilibrium, if* $\frac{Y_t^T - \delta\left(\overline{L} - L^*(1)\right)}{(\alpha+\delta) L^*(1)} \notin (0, 2)$*, a marginal change in* $L(1)$ *does not affect welfare.*

When $N \to +\infty$, city risk could be fully diversified, but provided that $\alpha > 0$, traded-good workers will choose not to do so. In fact, the message of Proposition 9 is again similar to the messages in Proposition 5 and Proposition 6. An interior symmetric equilibrium exists provided that the productivity advantage of traded-good sector 1 is not too large. In this interior equilibrium, if the city can shift between high– and low–activity states (i.e., between $Q_{nt} = q_0 + q_1$ and $Q_{nt} = q_0$) following a shock to traded-good sector 1, that is, if $0 < \frac{Y_t^T - \delta\left(\overline{L} - L^*(1)\right)}{(\alpha+\delta) L^*(1)} < 2$, the equilibrium is not socially optimal because workers do not internalize the effect that their specialization choices have on the probability of the city ending up in a high– or low–activity state. More specifically, as in Proposition 6, an increase in diversification increases welfare if $Y_t^T < \delta \overline{L}$, while an increase in specialization increases welfare if $Y_t^T > \delta \overline{L}$. In Proposition 9 such an increase in diversification (*specialization*) can be achieved by shifting weight from traded-good sector 1 towards (*away*) the composite traded-good sector.

5 Discussion and Extensions

Up to this point we have made a number of assumptions that simplify our analysis. In this section we discuss two of these assumptions – specifically, the ruled out cross-city hedging and city-to-city migration – and consider possible extensions with the assumptions relaxed. As we discuss first, in our setting, markets that allow cross-city hedging can improve the allocation of resources, even though individuals are all risk neutral. Regarding migration, ex-ante migration (migration at $t = 0$, before workers specialized) does not materially affect our analysis, but allowing ex-post migration (migration at $t = 1$, after the realization of the sectoral shocks) can in some situations reduce the possibility of a coordination failure and in others make coordination failures more likely. Finally we discuss the possibility of direct externalities in the consumption of non-traded goods, and how these externalities can reinforce the demand-driven complementarities that this paper studies.

5.1 Incomplete Markets

The agents in our model are assumed to be risk neutral, which simplifies our analysis, and perhaps more importantly, illustrates that diversification in our model plays a role that does not arise because of risk aversion, per se. Given our assumption of risk neutrality, the fact that we also preclude risk sharing across cities appears to be innocuous at first glance. However, because of the inelastic supply of real estate, individuals can in fact gain from hedging city level risk.[19] Indeed, if we extend our model to allow individuals to hedge city risk, we obtain a unique equilibrium in the benchmark case described in Section 3 that has the property that all traded-good workers specialize in the production of traded-good 1, that is, in the traded good that can be most efficiently produced in the city. In this equilibrium, workers choose to hedge their city risk, and as a result, the city's (hedged) income from the production of traded-goods is $E[Y_t] = (\alpha + \delta)\overline{L}$ with probability one. This unique equilibrium, which exhibits efficient production as well as constant consumption of traded goods, is socially optimal.

[19] As previously discussed, although individual indirect utility functions $v(y_i)$ in eq. 13 are linear in y_i, the social indirect utility function $v_s(Y)$ in eq. 33 is concave in the city's aggregate income Y, which is a consequence of the inelastic supply of real estate.

In the general case described in Section 4, in which the city produces non-traded as well as traded goods, there can also be a socially optimal equilibrium in which all traded-good workers specialize in the production of traded-good 1. Specifically, if $(\delta+\alpha)\overline{L} > Y_t^T$, residents hedge their income (i.e., each worker exchanges $(\delta+\alpha)p_t(1)$ for its expectation $\delta+\alpha$), and the production of traded goods is $Q_{nt} = q_o + q_1$ with probability one.[20] Overall, hedging across cities allows the city to benefit from the productivity advantage of a specialized industrial base, while avoiding the more volatile aggregate income Y_t.

There are, of course, a number of reasons why financial markets do not provide the kind of cross-city hedging that eliminates the benefits of a diversified industrial base. As the financial markets develop, however, these impediments to efficient risk sharing are reduced, which in theory, contributes to economic growth. It is noteworthy that our model suggests a novel channel through which financial market development promotes economic growth. Specifically, the model predicts that as financial markets develop, the overall economy benefits as industries migrate to locations in which they have a comparative advantage. This positive relation between financial development and growth is consistent with an extensive literature that studies the benefits of financial market development, see, e.g., Greenwood and Jovanovic (1990), Obstfeld (1994), and Acemoglu and Zilibotti (1997), and Levine (2005) for an overview of the literature. In our case, however, the benefits of hedging do not stem from risk aversion, but from the inelasticity of the supply of real estate and other non-traded goods.

5.2 The Effect of Migration

Another aspect of our analysis that merits discussion is our assumption that the labor force $\overline{L}$ is exogenous and fixed. This is in contrast to existing models of systems of cities that consider migration between cities (see, for example, Rauch, 1993). Since our analysis holds for any $\overline{L}$, it also holds if we allow frictionless ex-ante migration. In particular, one can allow workers to freely migrate in the initial period so that they achieve the same reservation utilities in each location.

[20] If $(\delta+\alpha)\overline{L} < Y_t^T$, it may not be optimal to hedge city risk, as the city would end up in the low-activity equilibrium with probability one.

Our assumption that individuals cannot migrate ex-post (after the realization of a shock) is, however, not innocuous, and warrants further discussion. Intuitively, this assumption captures the enduring nature of a city's industrial base. Indeed, if we allow frictionless ex-post migration, cities would respond to a productivity shock by swiftly changing their industrial composition. For example, migration could allow Detroit to rapidly transform itself from an auto- to a software-manufacturing cluster following a negative shock to the auto industry.

While our model is designed to capture the numerous frictions that inhibit migration, e.g., individuals enjoy social networks in the locations where they have long lived, the types of shocks explored in this paper are likely to trigger at least some migration. If some workers move out of a city following a negative shock, this could amplify the decline in the demand for the city's non-traded goods, which could in turn, increase the probability of a coordination failure. Moreover, there might be strategic complementarities associated with migration choices. For example, the popular press has recently described what is referred to as a "doom-loop" where office workers move out of central business districts because of increased crime and the deteriorating quality of restaurants and other services, and how this, in turn, triggers further deteriorations, accelerating the exit.[21] It should be noted that there can also be offsetting effects that arises because the price of real estate in a city declines when individuals leave the city, which can make the city less expensive for those that stay. In other words, a channel exists that make individual migration choices strategic substitutes that may at least partially offset those that make migration choices strategic complements. While the casual evidence seems to be more consistent with the complementarities, i.e., the "doom-loop" effect, this is a topic that warrants future research.

5.3 Urban Vibrancy

Our model assumes that each city exhibits a productivity advantage for one particular traded-good, and this productivity advantage is fully captured by the traded good workers. Our model, however, abstracts from productivity externalities, such as economies of scale at the

[21] https://www.brookings.edu/articles/breaking-the-urban-doom-loop-the-future-of-downtowns-is-shared-prosperity/

industry level or economies of scope across industries. We do this to isolate the effect of externalities that arise directly from the level of city risk. That is, from the relation between the city's exposure to sectoral shocks and the probability of the city being trapped in an equilibrium where the production of non-traded goods are inefficiently low.

One might characterize the coordination failure described in our model as a loss in "vibrancy" that arises when the production of non-traded goods in a city declines during a downturn. We believe that this notion of vibrancy can be further analyzed in an extension that captures the potential externalities that arise from the consumption of non-traded goods. For instance, externalities emanating from social interactions at restaurants, bars, entertainment venues, social clubs, etc. These vibrancy effects could amplify the link between traded-good shocks and the probability of a coordination failure in a couple of ways. First, to the extent that these social interactions facilitate the transmission of knowledge and ideas, they may directly affect the productivity of the city's industrial base. In addition, because these interaction benefits generate direct strategic complementarities in the consumption of non-traded goods, they can reinforce the demand-driven complementarities in the production of non-traded goods that this paper studies.

6 Conclusion

There is a substantial and varied literature that studies the trade-offs between the productivity advantages of a specialized regional economy and the risk reduction benefits of diversification. For the most part, this literature focuses on the gains associated with specialization, e.g., knowledge spillovers that arise when firms co-locate with industry peers, and takes as given the inherent costs of having a less diversified economy. In this paper, we do the opposite. By assuming that regional economies are endowed with comparative advantages in specific industries, we consider a setting with an exogenous benefit of specialization. However, our modeling of the provision of non-traded goods provides micro-foundations for the benefits of regional diversification.

As we show, there are benefits of having a diversified industrial base even when the residents of a region are all risk neutral. These benefits are related to the idea of labor pooling, as introduced by Krugman (1991). In our case, instead of workers responding to exogenous shocks by shifting between jobs, the workers providing non-traded goods experience a shift in their clienteles. For example, restaurant workers serve more meals to auto workers when the auto market does well and to software programmers when the tech business does well. We have an analogous notion of land pooling that arises from the fact that workers consume more land when they are more prosperous, which plays the same role.

These diversification benefits do not imply that an activist industrial policy that taxes or subsidizes different industries necessarily improves welfare. In particular, when workers supply labor inelastically, cities optimally diversify without interventions. This, however, is not the case in a setting in which the entrepreneurs who provide non-traded goods have the flexibility to work somewhat less when the demand for their services decline. For example, a restaurant may cut back its hours during a downturn. As we show, a city in this setting can experience a coordination failure when the demand for the traded goods produced in the city declines. The coordination failure arises because entrepreneurs providing non-traded goods cut production in bad times. This is partly due to less demand from the traded goods workers, but the effect is amplified because of reduced demand from other non-traded goods entrepreneurs who also scale back both their production and their consumption.

As discussed in detail in Section 5, the model can be extended in a number of interesting directions. Since our focus is primarily on risk, the analysis of financial market developments and the hedging of city risk is clearly warranted. For instance, policy makers may consider subsidies for financial market development as a substitute for subsidies that attract diversifying industries. We also discussed the importance of thinking more carefully about migration and urban vibrancy, and how these factors can amplify the coordination failures that this paper has considered. Each of these topics are likely to be fruitful areas for future research.

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Appendix

Proof of Proposition 1

Let X_N be a subset of $[0,1)$ with N sectors. $X_N \cup \{1\}$ defines the set of sectors available for work in the city. Worker's i indirect utility function (8) can be written as

$$v(y_i) = \gamma_t \frac{(\alpha+\delta)\, l_i(1)p_t(1) + \sum_{x_t \in X_N} \delta l_i(x_t)p_t(x_t)}{N_t^{\gamma_t}\left(\frac{Y_t}{R}\right)^{\gamma_r}}. \tag{A.1}$$

In what follows, recall that workers are atomistic and therefore, each worker's individual sector choice does not affect Y_t.

Claim 1 *In any equilibrium, all sectors in X_N have the same mass of workers.*

Proof. The proof is by contradiction. Suppose there exists $\{x_t, x_t'\} \in X_N^2$ such that the mass of workers in x_t is strictly larger than the mass of workers in x_t': $L(x_t) > L(x_t')$. Let

$$\hat{Y}_t \equiv Y_t - L(x_t)\delta p(x_t) - L(x_t')\delta p(x_t') \tag{A.2}$$

be the production of the city without sectors x_t and x_t'. From (A.1), a worker's net expected utility from working x_t' rather than in sector x_t is

$$\frac{\gamma_t R^{\gamma_r}\delta}{4N_t^{\gamma_t}}\mathbb{E}\left[\int_0^2\int_0^2 \frac{p(x_t')-p(x_t)}{(\hat{Y}_t + L(x_t)\delta p(x_t) + L(x_t')\delta p(x_t'))^{\gamma_r}}dp(x_t')dp(x_t)\right], \tag{A.3}$$

where expectation is with respect to prices in sectors other than $\{x_t, x_t'\}$: $\{p(\hat{x}_t)\}$. Therefore, a worker strictly prefers working in x_t' than in x_t if

$$\int_0^2\int_0^2 \frac{p(x_t')-p(x_t)}{(\hat{Y}_t + L(x_t)\delta p(x_t) + L(x_t')\delta p(x_t'))^{\gamma_r}}dp(x_t')dp(x_t) > 0 \tag{A.4}$$

$$\Leftrightarrow \int_0^2\int_{p(x_t)}^2 \frac{p(x_t')-p(x_t)}{(\hat{Y}_t + L(x_t)\delta p(x_t) + L(x_t')\delta p(x_t'))^{\gamma_r}}dp(x_t')dp(x_t) > \tag{A.5}$$

$$\int_0^2\int_{p(x_t')}^2 \frac{p(x_t)-p(x_t')}{(\hat{Y}_t + L(x_t)\delta p(x_t) + L(x_t')\delta p(x_t'))^{\gamma_r}}dp(x_t)dp(x_t')$$

$$\Leftrightarrow \int_0^2\int_0^{2-p(x_t)} \frac{z}{(\hat{Y}_t + (L(x_t)+L(x_t'))\delta p(x_t) + L(x_t')\delta z)^{\gamma_r}}dz dp(x_t) > \tag{A.6}$$

$$\int_0^2\int_0^{2-p(x_t')} \frac{z}{(\hat{Y}_t + (L(x_t)+L(x_t'))\delta p(x_t') + L(x_t)\delta z)^{\gamma_r}}dz dp(x_t'),$$

(A.6) holds if $L(x_t') < L(x_t)$. ■

Claim 2 *There exists a unique $\overline{\alpha}_N > 0$ such that $L(1) = \overline{L}$ is an equilibrium if and only if $\alpha \geq \overline{\alpha}_N$.*

Proof. If $\overline{L}$ workers work in traded-good sector 1, then $Y_t = \overline{L}(\delta+\alpha)p(1)$. Then from (A.1), the worker net utility from working in traded-good sector 1 rather than in $x_t \neq 1$ has the sign of

$$\mathbb{E}\left[\frac{(\delta+\alpha)p(1)-\delta p(x_t)}{(\overline{L}(\delta+\alpha)p(1))^{\gamma_r}}\right] = \frac{(\delta+\alpha)\mathbb{E}\left[p(1)^{1-\gamma_r}\right]-\delta\mathbb{E}\left[p(1)^{-\gamma_r}\right]}{(\overline{L}(\delta+\alpha))^{\gamma_r}}, \tag{A.7}$$

using that $p(x_t)$ is independent from $p(1)$ and has mean 1. The concavity of $p(1)^{1-\gamma_r}$ implies

$$\mathbb{E}\left[p(1)^{1-\gamma_r}\right] < \left(\mathbb{E}\left[p(x_t)\right]\right)^{1-\gamma_r} = 1. \tag{A.8}$$

The convexity of $p(1)^{-\gamma_r}$ implies

$$\mathbb{E}\left[p(x_t)^{-\gamma_r}\right] > \left(\mathbb{E}\left[p(x_t)\right]\right)^{-\gamma_r} = 1. \tag{A.9}$$

It follows that if $\alpha = 0$, (A.7) is strictly negative. Then since (A.7) is strictly increasing in α and tends to $+\infty$ as α tends to $+\infty$, there is a unique $\overline{\alpha}_N > 0$ such that (A.7) equals zero. (A.7) is positive, i.e., $L(1) = \overline{L}$ is an equilibrium if and only if $\alpha \geq \overline{\alpha}_N$. ■

Claim 3 *There exists an equilibrium such that $L(1) < \overline{L}$ and for all $x_t \in X_N$, $L(x_t) = \frac{\overline{L}-L(1)}{N}$ if and only if $\alpha < \overline{\alpha}_N$. $L(1)$ is then unique.*

Proof. In the candidate equilibrium, workers' net utility from working in traded-good sector 1 rather than in $\hat{x}_t \in X_N$ has the sign of

$$\mathbb{E}\left[\frac{(\delta+\alpha)p(1)-\delta p(\hat{x}_t)}{Y_t^{\gamma_r}}\right] \tag{A.10}$$

where $Y_t = (\delta+\alpha)L(1)p(1) + \frac{\overline{L}-L(1)}{N}\sum_{x_t\in X_N}\delta p(x_t)$. If $L(1)=0$, (A.10) becomes

$$\mathbb{E}\left[\frac{(\delta+\alpha)p(1)-\delta p(\hat{x}_t)}{\left(\frac{\overline{L}}{N}\sum_{x_t\in X_N}\delta p(x_t)\right)^{\gamma_r}}\right] > \frac{\delta\mathbb{E}_{p(x_t)\neq p(\hat{x}_t)}\mathbb{E}_{p(\hat{x}_t)}\left[(1-p(\hat{x}_t))\left(\sum_{x_t\in X_N}p(x_t)\right)^{-\gamma_r}\right]}{\left(\delta\frac{\overline{L}}{N}\right)^{\gamma_r}}, \tag{A.11}$$

as $p(1)$ is independent from $\{p(x_t)\}_{x_t\in X_N}$ and of mean 1. The LHS of (A.11) is strictly increasing in α and $\alpha > 0$. Note that $\mathbb{E}_{p(\hat{x}_t)}\left[p(\hat{x}_t)\left(\sum_{x_t\in X_N}p(x_t)\right)^{-\gamma_r}\right]$ is concave in $p(\hat{x}_t)$ so

$$\mathbb{E}_{p(\hat{x}_t)}\left[p(\hat{x}_t)\left(\sum_{x_t\in X_N}p(x_t)\right)^{-\gamma_r}\right] < \left(1+\sum_{x_t\in X_N\backslash\hat{x}_t}p(x_t)\right)^{-\gamma_r}, \tag{A.12}$$

that $\mathbb{E}_{p(\hat{x}_t)}\left[\left(\sum_{x_t\in X_N}p(x_t)\right)^{-\gamma_r}\right]$ is concave in $p(\hat{x}_t)$ so

$$\mathbb{E}_{p(\hat{x}_t)}\left[\left(\sum_{x_t\in X_N}p(x_t)\right)^{-\gamma_r}\right] > \left(1+\sum_{x_t\in X_N\backslash\hat{x}_t}p(x_t)\right)^{-\gamma_r}. \tag{A.13}$$

It follows that $\mathbb{E}_{p(\hat{x}_t)}\left[(1-p(\hat{x}_t))\left(\sum_{x_t\in X_N}p(x_t)\right)^{-\gamma_r}\right] > 0$ and, therefore, that (A.10), i.e., workers' net utility from working in traded-good sector 1 rather than in $\hat{x}_t$, is strictly positive for $L(1) = 0$. We also know from the proof of Claim 2 that if $\alpha < \overline{\alpha}_N$, (A.10) is strictly negative for $L(1) = \overline{L}$. Therefore if $\alpha < \overline{\alpha}_N$, there exists $L(1) \in (0, \overline{L})$ such that (A.10) equals 0. Finally, the symmetry and independence of all traded-good sectors in X_N implies that if (A.10) equals 0 for some $\hat{x}_t \in X_N$, then it equals 0 for any $x_t \in X_N$. This shows existence of the equilibrium in Claim 3 if $\alpha < \overline{\alpha}_N$.

For sufficiency and uniqueness, we use the equilibrium indifference condition that (A.10) equals 0 for all $\hat{x}_t \in X_N$. This implies a necessary equilibrium condition for $L(1)$:

$$\sum_{\hat{x}_t \in X_N} \frac{1}{N} \mathbb{E}\left[\frac{(\delta+\alpha)p(1) - \delta p(\hat{x}_t)}{\left((\delta+\alpha)L(1)p(1) + \frac{\overline{L}-L(1)}{N}\sum_{x_t \in X_N} \delta p(x_t)\right)^{\gamma_r}}\right] = 0 \tag{A.14}$$

Then the derivative of the LHS of (A.14) with respect to $L(1)$ is

$$-\gamma_r \mathbb{E}\left[\frac{\left((\delta+\alpha)p(1) - \frac{1}{N}\sum_{x_t \in X_N} \delta p(x_t)\right)^2}{\left((\delta+\alpha)L(1)p(1) + \frac{\overline{L}-L(1)}{N}\sum_{x_t \in X_N} \delta p(x_t)\right)^{1+\gamma_r}}\right] < 0, \tag{A.15}$$

which implies (A.14) has at most one solution. Furthermore, since we have just shown that the LHS of (A.14) is strictly positive for $L(1) = 0$ and since Claim 2 implies the LHS of (A.14) is positive for $L(1) = \overline{L}$ if $\alpha \geq \overline{\alpha}_N$ and strictly positive if $\alpha > \overline{\alpha}_N$, (A.14) has no solution in $(0, \overline{L})$ if $\alpha \geq \overline{\alpha}_N$. This shows existence of the equilibrium in Claim 3 only if $\alpha < \overline{\alpha}_N$. ■

To conclude, if $\alpha \geq \overline{\alpha}$, we have shown the existence of a (corner) equilibrium described in Claim 2. If $\alpha < \overline{\alpha}_N$, we have shown that there is a unique (interior) equilibrium of the form described in Claim 3. From Claim 1, there is no other possible equilibrium. *QED*

Proof of Corollary 1

$\lim_{N \to +\infty} L(x_t) = \frac{\overline{L}-L(1)}{N} = 0$, which means that for all traded-good sectors $x_t \in X_{+\infty}$, there is a zero measure of workers devoted to the production of the traded good. Therefore, if there is a measure $L(1)$ of workers devoted to the production of traded good 1 and a measure $\overline{L} - L(1)$ of workers devoted to the production of traded goods other than traded-good 1 (with a zero measure devoted to the production of any one of these other traded goods), the city's income from the production of traded goods can be written as

$$Y_t = (\alpha+\delta)p_t(1)L(1) + \delta\left(\overline{L} - L(1)\right), \tag{A.16}$$

an expression that makes use of the fact that $\mathbb{E}[p_t(x_t)] = 1$ and that prices are i.i.d.. In the candidate equilibrium, workers' net utility from working in traded-good sector 1 rather than in $\hat{x}_t \in X_{+\infty}$ has the sign of

$$\mathbb{E}\left[\frac{(\delta+\alpha)p_t(1)-\delta p(\hat{x}_t)}{Y_t^{\gamma_r}}\right]=\mathbb{E}\left[\frac{(\delta+\alpha)p_t(1)-\delta}{\left[(\alpha+\delta)p_t(1)L(1)+\delta\left(\overline{L}-L(1)\right)\right]^{\gamma_r}}\right] \tag{A.17}$$

For $L(1)=\overline{L}$, (A.17) can be written as

$$\mathbb{E}\left[\frac{(\delta+\alpha)p(1)-\delta}{\left[(\alpha+\delta)p_t(1)\overline{L}\right]^{\gamma_r}}\right] \tag{A.18}$$

and following similar steps as in the proof of Claim 2, it follows that there is a unique $\overline{\alpha}_{+\infty}>0$ such that (A.18) equals zero and that $L(1)=\overline{L}$ is an equilibrium if and only if $\alpha\geq\overline{\alpha}_{+\infty}$. For $L(1)=0$ and $\alpha>0$, (A.17) is strictly positive. Following similar steps as in the proof of Claim 3, it follows that there exists an equilibrium such that $L(1)<\overline{L}$ if and only if $\alpha<\overline{\alpha}_{+\infty}$, and that $L(1)$ is unique. *QED*

Proof of Proposition 2

From (11), and since $Y_t=\gamma_t Y$, the social utility function can be written as

$$v_s=\frac{R^{\gamma_r}Y_t^{\gamma_t}}{N_t^{\gamma_t}} \tag{A.19}$$

Therefore, the social planner solves

$$\max_{\{L(x_t)\}_{x_t\in X_N\cup\{1\}}} \mathbb{E}(Y_t^{\gamma_t}) \tag{A.20}$$

$$\text{s.t.}\quad Y_t=(\delta+\alpha)L(1)p_t(1)+\sum_{x_t\in X_N}\delta L(x_t)p(x_t), \tag{A.21}$$

$$\sum_{x_t\in X_N}L(x_t)+L(1)=\overline{L}, \tag{A.22}$$

$$L(x_t)\geq 0\ \text{ for } x_t\in X_N\cup\{1\}. \tag{A.23}$$

First-order conditions with respect to $L(1)$ and $\{L(x_t)\}_{x_t\in X_N}$ can be written as

$$\gamma_t(\delta+\alpha)\mathbb{E}\left[\frac{p_t(1)}{Y_t^{1-\gamma_t}}\right]-\mu+\lambda(1)=0 \tag{A.24}$$

$$\gamma_t\delta\mathbb{E}\left[\frac{p_t(x_t)}{Y_t^{1-\gamma_t}}\right]-\mu+\lambda(x_t)=0 \tag{A.25}$$

where μ and $\lambda(x_t)$ are the non-negative Lagrange multipliers associated with the total labor constraint (A.22) and the non-negativity constraints (A.23).

Claim 4 *At the social optimum, all sectors in X_n have the same amount of workers.*

Proof. By contradiction: suppose there exists $\{x_t,x_t'\}\in X_n^2$ such that $L(x_t)>L(x_t')$. This implies $L(x_t) > 0$, therefore $\lambda(x_t') \geq \lambda(x_t) = 0$. Thus (A.25) imply

$$\mathbb{E}\left[\frac{p_t(x_t') - p_t(x_t)}{Y_t^{\gamma_r}}\right] \leq 0. \tag{A.26}$$

We have shown in the proof of Claim 1 that if $L(x_t) > L(x_t')$, this expression (which has the sign of (A.3)) is strictly positive. ■

Claim 4 implies the social planer's problem reduces to

$$\max_{L(1)\in[0,\overline{L}]} \mathbb{E}\left[\left((\delta+\alpha)L(1)p_t(1) + \frac{\overline{L}-L(1)}{N}\sum_{x_t\in X_N}\delta p(x_t)\right)^{\gamma_t}\right] \tag{A.27}$$

The first-order derivative of this objective function with respect to $L(1)$ is

$$\gamma_t\mathbb{E}\left[\frac{(\delta+\alpha)p(1) - \frac{1}{N}\sum_{x_t\in X_N}\delta p(x_t)}{\left((\delta+\alpha)L(1)p_t(1) + \frac{\overline{L}-L(1)}{N}\sum_{x_t\in X_N}\delta p(x_t)\right)^{\gamma_r}}\right], \tag{A.28}$$

which is, to the factor γ_t, the LHS of the equilibrium condition (A.14). Then the proof of Claim 3 implies that the social planer's objective is strictly concave and that if $\alpha < \overline{\alpha}_N$, it admits an interior maximum $L(1) \in (0,1)$ given by the equilibrium condition (A.14). Similarly, if $\alpha \geq \overline{\alpha}_N$, (A.28) is strictly positive for $L(1) \in (0,1)$, therefore the social optimum is $L(1) = \overline{L}$, and also coincides with equilibrium. *QED*

Proof of Lemma 1

Lemma 1 follows directly from (22) and (23).

Proof of Proposition 3

The derivation of the unique equilibrium of the production subgame follows Morris and Shin (2003). Let θ be the proportion of entrepreneurs that produce $\frac{q_0}{2} + q_1$, while $1-\theta$ produce $\frac{q_0}{2}$. Using (20), let

$$\Delta(\theta) \equiv \frac{\gamma_t^{\gamma_t}\gamma_{nt}^{\gamma_{nt}}\gamma_r^{\gamma_r} \times \frac{\gamma_{nt}}{\gamma_t}\frac{q_1}{q_0+q_1}\frac{Y_t}{2}}{P_t^{\gamma_t}\left[\left(\frac{1}{q_0+q_1}\right)^{\theta}\left(\frac{1}{q_0}\right)^{1-\theta}\frac{\gamma_{nt}}{\gamma_t}Y_t\right]^{\gamma_{nt}}\left[\frac{\gamma_r}{\gamma_t}\frac{Y_t}{R}\right]^{\gamma_r}} - c \tag{A.29}$$

be the net payoff for an entrepreneur of producing $\frac{q_0}{2} + q_1$ given that a proportion θ of entrepreneurs produce $\frac{q_0}{2} + q_1$. Note that $\Delta'(.) > 0$, i.e., entrepreneurs' decision to produce exhibit global strategic complementarities. Then the existence of upper and lower dominance regions (see the assumption in eq. 25 and the discussion that follows) implies that if a unique equilibrium in threshold strategies exist, then it also is the unique equilibrium (Morris and Shin, 2003).

Assume that entrepreneur in sector x_{nt} produces $\frac{q_0}{2} + q_1$ when her signal $s_{x_{nt}}$ is above a threshold s^T. Then, for a given Y_t, the mass of entrepreneurs that produce $\frac{q_0}{2}+q_1$ is

$$\theta(Y_t) \equiv \begin{cases} 1 & \text{if } s^T < Y_t - \underline{\varepsilon} \\ \frac{Y_t+\underline{\varepsilon}-s^T}{2\underline{\varepsilon}} & \text{if } s^T \in [Y_t-\underline{\varepsilon}, Y_t+\underline{\varepsilon}] \\ 0 & \text{if } s^T > Y_t+\underline{\varepsilon} \end{cases} . \tag{A.30}$$

Let $h(.)$ be the probability density function (pdf) of Y_t (recall that $h(.)$ depends on workers' ex-ante specialization choices). For an entrepreneur who observes s_t, the pdf of the posterior distribution of Y_t is

$$\frac{\frac{1}{2\underline{\varepsilon}}h(Y_t)}{\int_{s_t-\underline{\varepsilon}}^{s_t+\underline{\varepsilon}}\frac{1}{2\underline{\varepsilon}}h(z)\,dz} = \frac{h(Y_t)}{\int_{s_t-\underline{\varepsilon}}^{s_t+\underline{\varepsilon}}h(z)\,dz} \tag{A.31}$$

At the threshold s^T, entrepreneur x_{nt} must be indifferent between producing and not producing:

$$\int_{s^T-\underline{\varepsilon}}^{s^T+\underline{\varepsilon}} \Delta(\theta(Y_t))\frac{h(Y_t)}{\int_{s_t-\underline{\varepsilon}}^{s_t+\underline{\varepsilon}}h(z)\,dz}\,dY_t = 0 \tag{A.32}$$

The existence of upper- and lower-dominance region implies that that the LHS of (A.32) is strictly positive for s^T large enough and strictly negative for s^T low enough. It remains to study the monotonicity of the LHS of (A.32) for uniqueness.

For $Y_t \in [s^T-\underline{\varepsilon}, s^T+\underline{\varepsilon}]$, $\theta(Y_t) = \frac{Y_t+\underline{\varepsilon}-s^T}{2\underline{\varepsilon}}$ and hence using (A.29), (A.32) can be written as

$$\int_{s^T-\underline{\varepsilon}}^{s^T+\underline{\varepsilon}} \left[(q_0+q_1)^{\frac{Y_t+\underline{\varepsilon}-s^T}{2\underline{\varepsilon}}} q_0^{1-\frac{Y_t+\underline{\varepsilon}-s^T}{2\underline{\varepsilon}}}\right]^{\gamma_{nt}} Y_t^{\gamma_t}\frac{h(Y_t)}{\int_{s^T-\underline{\varepsilon}}^{s^T+\underline{\varepsilon}}h(z)\,dz}dY_t = \frac{q_0+q_1}{q_1}\frac{2cP_t^{\gamma_t}}{\gamma_{nt}R^{\gamma_r}} \tag{A.33}$$

Changing variables, $\theta = \frac{Y_t+\underline{\varepsilon}-s^T}{2\underline{\varepsilon}}$ and $\upsilon = \frac{z+\underline{\varepsilon}-s^T}{2\underline{\varepsilon}}$, (A.33) can be written as

$$\begin{aligned} &\int_0^1 \left[(q_0+q_1)^{\theta} q_0^{1-\theta}\right]^{\gamma_{nt}} (\underline{\varepsilon}(2\theta-1)+s^T)^{\gamma_t}\frac{h(\underline{\varepsilon}(2\theta-1)+s^T)}{\int_0^1 h(\underline{\varepsilon}(2\upsilon-1)+s^T)\,d\upsilon}\,d\theta \\ = \;& \frac{q_0+q_1}{q_1}\frac{2cP_t^{\gamma_t}}{\gamma_{nt}R^{\gamma_r}} \end{aligned} \tag{A.34}$$

Note that h is the density of a linear combination of uniform random variable, hence is differentiable almost everywhere. At any point of differentiability, the derivative of integrand of the LHS of (A.34) with respect to S^T has the sign of

$$\begin{aligned} &\gamma_t(\underline{\varepsilon}(2\theta-1)+s^T)^{\gamma_t-1}\frac{h(\underline{\varepsilon}(2\theta-1)+s^T)}{\int_0^1 h(\underline{\varepsilon}(2\upsilon-1)+s^T)\,d\upsilon} \\ + \;& (\underline{\varepsilon}(2\theta-1)+s^T)^{\gamma_t}\times \\ &\frac{h'(\underline{\varepsilon}(2\theta-1)+s^T)\int_0^1 h(\underline{\varepsilon}(2\upsilon-1)+s^T)\,d\upsilon - h(\underline{\varepsilon}(2\theta-1)+s^T)\int_0^1 h'(\underline{\varepsilon}(2\upsilon-1)+s^T)\,d\upsilon}{\left(\int_0^1 h(\underline{\varepsilon}(2\upsilon-1)+s^T)\,d\upsilon\right)^2} \end{aligned} \tag{A.35}$$

which tends to $\gamma_t(s^T)^{\gamma_t-1} > 0$ as $\underline{\varepsilon}$ tends to 0. It follows that for $\underline{\varepsilon}$ sufficiently small, (A.32) has a unique solution.

Taking the limit of (A.34) as $\underline{\varepsilon} \to 0$, we obtain

$$\int_0^1 (q_0+q_1)^{\gamma_{nt}\theta}\, q_0^{\gamma_{nt}(1-\theta)} \left(Y_t^T\right)^{\gamma_t} d\theta = \frac{q_0+q_1}{q_1}\frac{2cP_t^{\gamma_t}}{\gamma_{nt}R^{\gamma_r}} \tag{A.36}$$

where $Y_t^T \equiv \lim_{\underline{\varepsilon}\to 0} s^T(\underline{\varepsilon})$. (A.36) can be written as

$$\frac{(Y_t^T)^{\gamma_t}}{\gamma_{nt}}\left[\frac{(q_0+q_1)^{\gamma_{nt}\theta}\, q_0^{\gamma_{nt}(1-\theta)}}{\ln(q_0+q_1)-\ln q_0}\right]_0^1 = \frac{q_0+q_1}{q_1}\frac{2cP_t^{\gamma_t}}{\gamma_{nt}R^{\gamma_r}}, \tag{A.37}$$

which in turn can be written as

$$\left(\frac{Y_t^T}{P_t}\right)^{\gamma_t} = \frac{\ln(q_0+q_1)-\ln q_0}{(q_0+q_1)^{\gamma_{nt}}-q_0^{\gamma_{nt}}}\frac{q_0+q_1}{q_1}\frac{2c}{R^{\gamma_r}}. \tag{A.38}$$

Finally, to show that (27) holds, rewrite (A.38) as

$$c = \frac{\left[(q_0+q_1)^{\gamma_{nt}}-q_0^{\gamma_{nt}}\right] R^{\gamma_r}\left(Y_t^T\right)^{\gamma_t}}{N_t^{\gamma_t}}\frac{1}{2}\frac{\ln(\frac{q_0}{q_1}+1)}{\frac{q_0}{q_1}+1} \tag{A.39}$$

and substitute c in (27) to obtain after simplification

$$1-\frac{1}{2}\left(\ln(\frac{q_1}{q_0}+1)(\frac{q_0}{q_1}+1)\right)^{-1} > 0, \tag{A.40}$$

which holds because $f(x) \equiv (\frac{1}{x}+1)\}(x+1) > 1$ for $x > 0$ since $\lim_{x-0} f(x) = 1$ and $f'(x) > 0$ for $x > 0$. *QED*

Proof of Proposition 4

A worker's net benefit from specializing in traded-good sector 1 rather than in traded-good sector x_t is

$$\mathbb{E}_{p_t(1),p_t(x_t)}\left[\frac{Q_{nt}^{\gamma_{nt}}\left((\delta+\alpha)p_t(1)-\delta p_t(x_t)\right)}{(L(1)(\alpha+\delta)p_t(1)+(\overline{L}-L(1))\delta p_t(x_t))^{1-\gamma_t}}\right] \tag{A.41}$$

Suppose (A.41) is strictly positive when evaluated at $L(1) = \frac{\overline{L}}{2}$. Then either (A.41) is always strictly positive for any $L(1) > \frac{\overline{L}}{2}$ and then $L(1) = \overline{L}$ is a (corner) equilibrium, or (A.41) crosses 0 for some $L(1)$ strictly greater than $\frac{\overline{L}}{2}$, which is then an equilibrium.

To show Proposition 4, it is therefore sufficient to show (A.41) is strictly positive for $L(1) = \frac{\overline{L}}{2}$, which is equivalent to

$$\mathbb{E}_{p_t(1),p_t(x_t)}\left[\frac{Q_{nt}^{\gamma_{nt}}\left((\delta+\alpha)p_t(1)-\delta p_t(x_t)\right)}{((\alpha+\delta)p_t(1)+\delta p_t(x_t))^{1-\gamma_t}}\right] > 0 \tag{A.42}$$

We consider three cases: *(i)* $Y_t^T < \delta\overline{L}$, *(ii)* $\delta\overline{L} < Y_t^T < (\alpha+\delta)\,\overline{L}$, and *(iii)* $(\alpha+\delta)\,\overline{L} < Y_t^T < 2\delta\overline{L}$. (Note: The assumption in eq. 25 implies $Y_t^T < 2\delta\overline{L}$.)

Case (i): $Y_t^T < \delta\overline{L}$

Given $Y_t^T < \delta\overline{L}$ and $L(1) = \frac{\overline{L}}{2}$,

- if $p_t(x_t) \in [\frac{2Y_t^T}{\overline{L}\delta}, 2)$, $Y_t \geq Y_t^T$ for all $p_t(1) \in (0, 2)$,
- if $p_t(x_t) \in \left(0, \frac{2Y_t^T}{\overline{L}\delta}\right)$, $Y_t \geq Y_t^T$ for $p_t(1) \in \left(\frac{Y_t^T - \frac{\overline{L}}{2}\delta p_t(x_t)}{(\alpha+\delta)\frac{\overline{L}}{2}}, 2\right)$.[A1]

Therefore, the LHS of (A.42) (the worker's net benefit from specializing in sector 1) has the sign of

$$\int_{\frac{2Y_t^T}{\overline{L}\delta}}^{2} \left[\int_0^2 (q_0+q_1)^{\gamma_{nt}} \frac{(\alpha+\delta)p_t(1) - \delta p_t(x_t)}{((\alpha+\delta)p_t(1)+\delta p_t(x_t))^{1-\gamma_t}}\, dp_t(1)\right] dp_t(x_t)$$
$$+\int_0^{\frac{2Y_t^T}{\overline{L}\delta}} \left[\begin{array}{l} \int_{\frac{Y_t^T - \frac{\overline{L}}{2}\delta p_t(x_t)}{(\alpha+\delta)\frac{\overline{L}}{2}}}^{2} (q_0+q_1)^{\gamma_{nt}} \frac{(\alpha+\delta)p_t(1)-\delta p_t(x_t)}{((\alpha+\delta)p_t(1)+\delta p_t(x_t))^{1-\gamma_t}}\, dp_t(1) + \\ \quad + \int_0^{\frac{Y_t^T - \frac{\overline{L}}{2}\delta p_t(x_t)}{(\alpha+\delta)\frac{\overline{L}}{2}}} q_0^{\gamma_{nt}} \frac{(\alpha+\delta)p_t(1)-\delta p_t(x_t)}{((\alpha+\delta)p_t(1)+\delta p_t(x_t))^{1-\gamma_t}} dp_t(1) \end{array}\right] dp_t(x_t) \tag{A.43}$$

Note first that if $\alpha = 0$, then (A.43) is equal to 0: if both sectors have the same productivity and the same number of workers, then each worker is indifferent between working in sector 1 or x_t. Then to show (A.43) is strictly positive for $\alpha > 0$, it is sufficient to show that (A.43) is strictly increasing in α for $\alpha > 0$, which is what we do next.

A marginal increase in α affects both the integrands in (A.43) and the boundaries of the integrals in $p_t(1)$.[A2] For any $p_t(1)$ and $p_t(x_t)$,

$$\frac{\partial}{\partial\alpha}\frac{(\alpha+\delta)p_t(1) - \delta p_t(x_t)}{((\alpha+\delta)p_t(1)+\delta p_t(x_t))^{1-\gamma_t}} = \frac{p_t(1)(\gamma_t(\alpha+\delta)p_t(1) + (2-\gamma_t)\delta p_t(x_t))}{((\alpha+\delta)p_t(1)+\delta p_t(x_t))^{2-\gamma_t}} > 0, \tag{A.44}$$

that is, the effect on the integrands is strictly positive. The effect of a marginal increase in α

[A1] Note: For $Y_t^T < \delta\overline{L}$ and $L(1) = \frac{\overline{L}}{2}$, $\frac{Y_t^T - \frac{\overline{L}}{2}\delta p_t(x_t)}{(\alpha+\delta)\frac{\overline{L}}{2}} \in (0, 2)$ for all $p_t(x_t) \in (0, \frac{2Y_t^T}{\overline{L}\delta})$.

[A2] The boundaries of the integral in $p_t(x_t)$ are independent from α as Y_t^T is independent from α (see (26)).

on the boundaries of the integral in $p_t(1)$ has the sign of

$$\int_0^{\frac{2Y_t^T}{\overline{L}\delta}} \left(Y_t^T - \delta\overline{L}p_t(x_t)\right)\left(Y_t^T - \delta\frac{\overline{L}}{2}p_t(x_t)\right) dp_t(x_t) \tag{A.45}$$

$$= \left[(Y_t^T)^2 p_t(x_t) - \frac{3}{4}Y_t^T\delta\overline{L}(p_t(x_t))^2 + \frac{1}{6}\delta^2\overline{L}^2(p_t(x_t))^3\right]_0^{\frac{2Y_t^T}{\overline{L}\delta}} \tag{A.46}$$

$$= (Y_t^T)^2\frac{2Y_t^T}{\overline{L}\delta} - 3\frac{\left(Y_t^T\right)^3}{\overline{L}\delta} + \frac{4}{3}\frac{\left(Y_t^T\right)^3}{\overline{L}\delta} \tag{A.47}$$

$$= \frac{\left(Y_t^T\right)^3}{\overline{L}\delta}\left[2 - 3 + \frac{4}{3}\right] > 0 \tag{A.48}$$

To sum up, (A.43) equals 0 for $\alpha = 0$, is strictly increasing in α and is therefore strictly positive for any $\alpha > 0$.

Case (ii): $\delta\overline{L} < Y_t^T < (\alpha+\delta)\,\overline{L}$

For $\delta\overline{L} < Y_t^T < (\alpha+\delta)\,\overline{L}$ and $L(1) = \frac{\overline{L}}{2}$, if $p_t(x_t) \in (0,2)$, $Y_t < Y_t^T$ for $p_t(1) \in \left(0, \frac{Y_t^T - \frac{\overline{L}}{2}\delta p_t(x_t)}{(\alpha+\delta)\frac{\overline{L}}{2}}\right)$. Therefore the LHS of (A.42) has the sign of

$$\int_0^2 \left[\begin{array}{l} \int_{\frac{Y_t^T - \frac{\overline{L}}{2}\delta p_t(x_t)}{(\alpha+\delta)\frac{\overline{L}}{2}}}^2 (q_0+q_1)^{\gamma_{nt}} \frac{(\alpha+\delta)p_t(1) - \delta p_t(x_t)}{((\alpha+\delta)p_t(1)+\delta p_t(x_t))^{1-\gamma_t}}\, dp_t(1) + \\ + \int_0^{\frac{Y_t^T - \frac{\overline{L}}{2}\delta p_t(x_t)}{(\alpha+\delta)\frac{\overline{L}}{2}}} q_0^{\gamma_{nt}} \frac{(\alpha+\delta)p_t(1) - \delta p_t(x_t)}{((\alpha+\delta)p_t(1)+\delta p_t(x_t))^{1-\gamma_t}}\, dp_t(1) \end{array}\right] dp_t(x_t) \tag{A.49}$$

Similarly to *Case (i)*, (A.49) is equal to 0 if $\alpha = 0$ and the effect of a marginal increase in α in (A.49) is strictly positive if the effect through the boundaries of the integral in $p_t(1)$ is positive. This effect has the sign of

$$\int_{\frac{2(Y_t^T - \overline{L}(\alpha+\delta))}{\overline{L}\delta}}^2 \left(Y_t^T - \delta\overline{L}p_t(x_t)\right)\left(Y_t^T - \delta\frac{\overline{L}}{2}p_t(x_t)\right) dp_t(x_t) \tag{A.50}$$

$$= \left[(Y_t^T)^2 p_t(x_t) - \frac{3}{4}Y_t^T\delta\overline{L}(p_t(x_t))^2 + \frac{1}{6}\delta^2\overline{L}^2(p_t(x_t))^3\right]_0^2 \tag{A.51}$$

$$= 2\left(Y_t^T\right)^2 - 3Y_t^T\delta\overline{L} + \frac{4}{3}\delta^2\overline{L}^2 \tag{A.52}$$

At $Y_t^T = \delta\overline{L}$, (A.52) is positive (i.e., $\delta^2\overline{L}^2(2 - 3 + \frac{4}{3}) > 0$). The derivative w.r.t. Y_t^T of (A.52) (i.e, Y_t^T, $4Y_t^T - 3\delta\overline{L}$) is positive for $Y_t^T > \delta\overline{L}$, and therefore, (A.52) is positive for $\delta\overline{L} < Y_t^T < (\alpha+\delta)\,\overline{L}$.

Case (iii): $(\alpha+\delta)\,\overline{L} < Y_t^T < 2\delta\overline{L}$**.**

Given $(\alpha+\delta)\,\overline{L} < Y_t^T < 2\delta\overline{L}$ and $L(1) = \frac{\overline{L}}{2}$,

- if $p_t(x_t) > \frac{2(Y_t^T - \overline{L}(\alpha+\delta))}{\overline{L}\delta}$, $Y_t < Y_t^T$ for $p_t(1) \in \left(0, \frac{Y_t^T - \frac{\overline{L}}{2}\delta p_t(x_t)}{(\alpha+\delta)\frac{\overline{L}}{2}}\right)$,

- if $p_t(x_t) < \frac{2(Y_t^T - \overline{L}(\alpha+\delta))}{\overline{L}\delta}$, $Y_t < Y_t^T$ for $p_t(1) \in (0, 2)$.

Therefore, for $Y_t^T > \overline{L}(\alpha + \delta)$, the LHS of (A.42) has the sign of

$$\int_{\frac{2(Y_t^T - \overline{L}(\alpha+\delta))}{\overline{L}\delta}}^{2} \left[\begin{array}{c} \int_{\frac{Y_t^T - \frac{\overline{L}}{2}\delta p_t(x_t)}{(\alpha+\delta)\frac{\overline{L}}{2}}}^{2} (q_0 + q_1)^{\gamma_{nt}} \frac{(\alpha+\delta)p_t(1) - \delta p_t(x_t)}{((\alpha+\delta)p_t(1) + \delta p_t(x_t))^{1-\gamma_t}} \, dp_t(1) + \\ + \int_0^{\frac{Y_t^T - \frac{\overline{L}}{2}\delta p_t(x_t)}{(\alpha+\delta)\frac{\overline{L}}{2}}} q_0^{\gamma_{nt}} \frac{(\alpha+\delta)p_t(1) - \delta p_t(x_t)}{((\alpha+\delta)p_t(1) + \delta p_t(x_t))^{1-\gamma_t}} \, dp_t(1) \end{array} \right] dp_t(x_t) \quad \text{(A.53)}$$
$$+ \int_0^{\frac{2(Y_t^T - \overline{L}(\alpha+\delta))}{\overline{L}\delta}} \left[\int_0^2 q_0^{\gamma_{nt}} \frac{(\alpha+\delta)p_t(1) - \delta p_t(x_t)}{((\alpha+\delta)p_t(1) + \delta p_t(x_t))^{1-\gamma_t}} dp_t(1) \right] dp_t(x_t)$$

Similarly to *Case (i)*, (A.53) is equal to 0 if $\alpha = 0$, and the effect of a marginal increase in α in (A.53) is strictly positive if the effect though the boundaries of the integral in $p_t(1)$ is positive.[A3] This effect has the sign of

$$\int_{\frac{2(Y_t^T - \overline{L}(\alpha+\delta))}{\overline{L}\delta}}^{2} \left(Y_t^T - \delta \overline{L} p_t(x_t) \right) \left(Y_t^T - \delta \frac{\overline{L}}{2} p_t(x_t) \right) dp_t(x_t) \quad \text{(A.54)}$$

Equation (A.54) can be written as

$$\left[(Y_t^T)^2 p_t(x_t) - \frac{3}{4} Y_t^T \delta \overline{L} (p_t(x_t))^2 + \frac{1}{6} \delta^2 \overline{L}^2 (p_t(x_t))^3 \right]_{\frac{2(Y_t^T - \overline{L}(\alpha+\delta))}{\overline{L}\delta}}^{2} \quad \text{(A.55)}$$

$$= \quad 2 \left(Y_t^T \right)^2 - 3 Y_t^T \delta \overline{L} + \frac{4}{3} \delta^2 \overline{L}^2 \quad \text{(A.56)}$$
$$- \left[2(Y_t^T)^2 \frac{Y_t^T - \overline{L}(\alpha+\delta)}{\delta \overline{L}} - 3 Y_t^T \delta \overline{L} \left(\frac{Y_t^T - \overline{L}(\alpha+\delta)}{\delta \overline{L}} \right)^2 + \frac{4}{3} \delta^2 \overline{L}^2 \left(\frac{Y_t^T - \overline{L}(\alpha+\delta)}{\delta \overline{L}} \right)^3 \right]$$

[A3] Note that the boundaries of the integral in $p_t(1)$ do depend on α, unlike in case (i). However, at $p_t(1) = \frac{2(Y_t^T - \overline{L}(\alpha+\delta))}{\overline{L}\delta}$, the first and second terms between bracket in (A.53) are equal and therefore the marginal effect of α through the boundaries of the integral in $p_t(1)$ is 0.

The first derivative of (A.56) with respect to Y_t^T can be written as

$$4Y_t^T - 3\delta\overline{L} - \left[4Y_t^T \frac{Y_t^T - \overline{L}(\alpha+\delta)}{\delta\overline{L}} - 3\delta\overline{L}\left(\frac{Y_t^T - \overline{L}(\alpha+\delta)}{\delta\overline{L}}\right)^2\right] \tag{A.57}$$

$$- \left[\frac{2(Y_t^T)^2}{\delta\overline{L}} - 6Y_t^T \frac{Y_t^T - \overline{L}(\alpha+\delta)}{\delta\overline{L}} + 4\delta\overline{L}\left(\frac{Y_t^T - \overline{L}(\alpha+\delta)}{\delta\overline{L}}\right)^2\right]$$

$$= \; 4Y_t^T - 3\delta\overline{L} - \left[\frac{2(Y_t^T)^2}{\delta\overline{L}} - 2Y_t^T \frac{Y_t^T - \overline{L}(\alpha+\delta)}{\delta\overline{L}} + \delta\overline{L}\left(\frac{Y_t^T - \overline{L}(\alpha+\delta)}{\delta\overline{L}}\right)^2\right] \tag{A.58}$$

$$= \; 2Y_t^T \frac{\delta-\alpha}{\delta} - 3\delta\overline{L} - \frac{\left[Y_t^T - \overline{L}(\alpha+\delta)\right]^2}{\delta\overline{L}} \tag{A.59}$$

Note the following four facts:

1. (A.59) evaluated at $Y_t = 2\overline{L}\delta$ is negative:

$$4\overline{L}\delta\frac{\delta-\alpha}{\delta} - 3\delta\overline{L} - \frac{\left[2\overline{L}\delta - \overline{L}(\alpha+\delta)\right]^2}{\delta\overline{L}} = -\overline{L}\alpha\left[2 + \frac{\alpha}{\delta}\right] < 0 \tag{A.60}$$

2. (A.59) evaluated at $Y_t = \overline{L}\delta$ is negative:

$$2\overline{L}\delta\frac{\delta-\alpha}{\delta} - 3\delta\overline{L} - \frac{\left[\overline{L}\delta - \overline{L}(\alpha+\delta)\right]^2}{\delta\overline{L}} = -\overline{L}\left[2\alpha + \delta + \frac{\alpha^2}{\delta}\right] < 0 \tag{A.61}$$

3. The second derivative of (A.56) with respect to Y_t^T is positive:

$$\frac{2}{\delta\overline{L}}\left[2\delta\overline{L} - Y_t^T\right] > 0 \tag{A.62}$$

4. (A.54) evaluated at $Y_t^T = 2\delta\overline{L}$ is strictly positive since the integrand is then strictly positive for any $p_t(x_t) \in (0,2)$.

Facts (1), (2) and (3) imply that (A.54) is strictly decreasing in Y_t^T for $Y_t^T \in (\delta\overline{L}, 2\delta\overline{L})$. Together with (4), this implies that (A.54) is strictly positive for $Y_t^T \in (\delta\overline{L}, 2\delta\overline{L})$. *QED*

Proof of Proposition 5

Workers' net utility from specializing in traded-good sector 1 rather than in traded-good sector x_t has the sign of

$$\mathbb{E}\left[\frac{Q_{nt}^{\gamma_{nt}}\left((\delta+\alpha)p_t(1) - \delta p_t(x_t)\right)}{Y_t^{1-\gamma_t}}\right] \tag{A.63}$$

where $Y_t = (\delta+\alpha)L(1)p_t(1) + \delta(\overline{L} - L(1))p_t(x_t)$. If $L(1) = \overline{L}$, (A.63) becomes

$$\mathbb{E}_{p_t(1),p_t(x_t)}\left[\frac{Q_{nt}^{\gamma_{nt}}\left((\delta+\alpha)p_t(1) - \delta p_t(x_t)\right)}{((\delta+\alpha)\overline{L}p_t(1))^{1-\gamma_t}}\right] \tag{A.64}$$

For $L(1) = \overline{L}$, Y_t and $Q_{nt}^{\gamma_{nt}}$ do not depend on $p_t(x_t)$, and hence, (A.64) has the same sign as

$$\mathbb{E}_{p_t(1)}\left[\frac{Q_{nt}^{\gamma_{nt}}\left((\delta+\alpha)p_t(1)-\delta\right)}{p_t(1)^{1-\gamma_t}}\right] \tag{A.65}$$

$$= \frac{1}{2}\int_{\frac{Y_t^T}{(\alpha+\delta)\overline{L}}}^{2}(q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})\frac{(\delta+\alpha)p_t(1)-\delta}{p_t(1)^{1-\gamma_t}}dp_t(1) \tag{A.66}$$

$$+\frac{1}{2}\int_0^{\frac{Y_t^T}{(\alpha+\delta)\overline{L}}} q_0^{\gamma_{nt}}\frac{(\delta+\alpha)p_t(1)-\delta}{p_t(1)^{1-\gamma_t}}dp_t(1)$$

$$= \frac{1}{2}(q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})2^{\gamma_t}\left[\frac{2(\delta+\alpha)}{\gamma_t+1}-\frac{\delta}{\gamma_t}\right] \tag{A.67}$$

$$-\frac{1}{2}\left[(q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})-q_0^{\gamma_{nt}}\right]\left(\frac{Y_t^T}{(\alpha+\delta)\overline{L}}\right)^{\gamma_t}\left[\frac{\frac{Y_t^T}{\overline{L}}}{\gamma_t+1}-\frac{\delta}{\gamma_t}\right]$$

Notice that

$$\frac{\frac{Y_t^T}{\overline{L}}}{\gamma_t+1}-\frac{\delta}{\gamma_t} = \frac{\gamma_t\frac{Y_t^T}{\overline{L}}-(\gamma_t+1)\,\delta}{(\gamma_t+1)\,\gamma_t} < \frac{\frac{Y_t^T}{\overline{L}}-2\delta}{(\gamma_t+1)} < 0 \tag{A.68}$$

which means that (A.65) is increasing in α and tends $+\infty$ as $\alpha \to +\infty$.
Evaluated at $\alpha = 0$, equation (A.67) is negative:

$$= (q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})\delta 2^{\gamma_t}\left[\frac{2}{\gamma_t+1}-\frac{1}{\gamma_t}\right]-\left[(q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})-q_0^{\gamma_{nt}}\right]\delta\left(\frac{Y_t^T}{\delta\overline{L}}\right)^{\gamma_t}\left[\frac{\frac{Y_t^T}{\delta\overline{L}}}{\gamma_t+1}-\frac{1}{\gamma_t}\right] \tag{A.69}$$

$$< (q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})\delta 2^{\gamma_t}\left[\frac{\frac{Y_t^T}{\delta\overline{L}}}{\gamma_t+1}-\frac{1}{\gamma_t}\right]-\left[(q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})-q_0^{\gamma_{nt}}\right]\delta\left(\frac{Y_t^T}{\delta\overline{L}}\right)^{\gamma_t}\left[\frac{\frac{Y_t^T}{\delta\overline{L}}}{\gamma_t+1}-\frac{1}{\gamma_t}\right] \tag{A.70}$$

$$= \left[\frac{\frac{Y_t^T}{\delta\overline{L}}}{\gamma_t+1}-\frac{1}{\gamma_t}\right]\left((q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})\delta 2^{\gamma_t}-\left[(q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})-q_0^{\gamma_{nt}}\right]\delta\left(\frac{Y_t^T}{\delta\overline{L}}\right)^{\gamma_t}\right) < 0, \tag{A.71}$$

Therefore, (A.65) is increasing in α, tends $+\infty$ as $\alpha \to +\infty$, and is negative for $\alpha = 0$. This implies that there is a unique $\alpha^* > 0$ such that $L(1) = \overline{L}$ is an equilibrium if and only if $\alpha > \alpha^*$. From Proposition 4, we know that for $\alpha > 0$ there always exists an equilibrium such that $L(1) > \frac{\overline{L}}{2}$ and therefore, for $\alpha \in (0, \alpha^*)$ there is an interior equilibrium such that $L(1) > \frac{\overline{L}}{2} > L(x_t) > 0$. *QED*

Proof of Proposition 6

From eq. 33, $\mathbb{E}[v_s(y_i)]$, can be written as

$$\mathbb{E}\left[v_s(Y)\right] = \mathbb{E}\left[\frac{Q_{nt}^{\gamma_{nt}}R^{\gamma_r}Y_t^{\gamma_t}}{P_t^{\gamma_t}} - c\mathbb{1}_{Y_t \geq Y_t^T}\right], \tag{A.72}$$

Since

$$Y_t = L(1)(\alpha+\delta)p_t(1) + (\overline{L}-L(1))\delta p_t(x_t), \tag{A.73}$$

the event $Y_t \geq Y_t^T$ can be written as

$$p_t(1) \geq \frac{Y_t^T - (\overline{L} - L(1))\delta p_t(x_t)}{(\alpha + \delta)L(1)} \equiv p_t^T(p_t(x_t)). \tag{A.74}$$

We first consider the case in which $Y_t^T < \delta\overline{L}$ and then, the case in which $Y_t^T > \delta\overline{L}$.

Case ($Y_t^T < \delta\overline{L}$): Note that

$$p_t^T(0) < 2 \Leftrightarrow L(1) > \frac{Y_t^T}{2(\alpha + \delta)} \tag{A.75}$$

which holds if $Y_t^T < \delta\overline{L}$ and $L(1)^* > \frac{\overline{L}}{2}$. This implies that even if $p_t(x_t) = 0$, there exists $p_t(1)$ high enough such that $Y_t > Y_t^T$. On the other hand, for $p_t(x_t)$ high enough, it could be that $Y_t > Y_t^T$ for all $p_t(1) \in (0, 2)$. This is the case if

$$(\overline{L} - L(1))\delta p_t(x_t) > Y_t^T \Leftrightarrow p_t(x_t) > \frac{Y_t^T}{(\overline{L} - L(1))\delta} \tag{A.76}$$

Therefore, (A.72) can be written as

$$\begin{aligned} &\frac{1}{4}\int_{\min\left\{2, \frac{Y_t^T}{(\overline{L}-L(1))\delta}\right\}}^{2} \left[\int_0^2 \frac{(q_0+q_1)^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} - c\, dp_t(1)\right] dp_t(x_t) \qquad \text{(A.77)} \\ &+\frac{1}{4}\int_0^{\min\left\{2, \frac{Y_t^T}{(\overline{L}-L(1))\delta}\right\}} \left[\begin{array}{c} \int_{\frac{Y_t^T - (\overline{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}}^{2} \frac{(q_0+q_1)^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} - c\, dp_t(1) \\ + \int_0^{\frac{Y_t^T - (\overline{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}} \frac{q_0^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}}\, dp_t(1) \end{array}\right] dp_t(x_t). \end{aligned}$$

The derivative with respect to Y_t of the integrands in (A.77) is proportional to

$$\begin{aligned} &\frac{\gamma_t}{4}\int_{\min\left\{2, \frac{Y_t^T}{(\overline{L}-L(1))\delta}\right\}}^{2} \left[\int_0^2 \frac{(q_0+q_1)^{\gamma_{nt}} R^{\gamma_r} \left[(\alpha+\delta)p_t(1) - \delta p_t(x_t)\right]}{N_t^{\gamma_t} Y_t^{1-\gamma_t}}\, dp_t(1)\right] dp_t(x_t) \qquad \text{(A.78)} \\ &+\frac{\gamma_t}{4}\int_0^{\min\left\{2, \frac{Y_t^T}{(\overline{L}-L(1))\delta}\right\}} \left[\begin{array}{c} \int_{\frac{Y_t^T - (\overline{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}}^{2} \frac{(q_0+q_1)^{\gamma_{nt}} R^{\gamma_r} [(\alpha+\delta)p_t(1) - \delta p_t(x_t)]}{N_t^{\gamma_t} Y_t^{\gamma_t}}\, dp_t(1) \\ + \int_0^{\frac{Y_t^T - (\overline{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}} \frac{q_0^{\gamma_{nt}} R^{\gamma_r} [(\alpha+\delta)p_t(1) - \delta p_t(x_t)]}{N_t^{\gamma_t} Y_t^{\gamma_t}}\, dp_t(1) \end{array}\right] dp_t(x_t) \\ &= \gamma_t \left(\mathbb{E}\left[v((\alpha+\delta)p_t(1))\right] - \mathbb{E}\left[v(\delta p_t(x_t))\right]\right) \qquad \text{(A.79)} \end{aligned}$$

where $v(y_i)$ is defined as in eq. 30. At any interior equilibrium $\mathbb{E}\left[v((\alpha+\delta)p_t(1))\right] = \mathbb{E}\left[v(\delta p_t(x_t))\right]$, and threfore, (A.79) equals zero. Therefore, the effect not internalized by the workers in the specialization choices is the effect of $L(1)$ on the boundaries of the integrals in (A.77). Consider first the effect on the integral with respect to $p_t(x_t)$. If $\min\left\{2, \frac{Y_t^T}{(\overline{L}-L(1))\delta}\right\} = 2$ this effect is 0. If $\left\{2, \frac{Y_t^T}{(\overline{L}-L(1))\delta}\right\} = \frac{Y_t^T}{(\overline{L}-L(1))\delta}$, the marginal effect of $L(1)$ on the boundaries of the integrals with respect to $p_t(x_t)$ in (A.77) is

$$\frac{1}{4}\frac{\partial \frac{Y_t^T}{(\alpha+\delta)L(1)}}{\partial L(1)}\left[\begin{array}{c}\int_0^2\left(\frac{(q_0+q_1)^{\gamma_{nt}}R^{\gamma_r}(Y_t^T)^{\gamma_t}}{N_t^{\gamma_t}}-c\right)dp_t(1)\\ -\int_0^2\left(\frac{(q_0+q_1)^{\gamma_{nt}}R^{\gamma_r}(Y_t^T)^{\gamma_t}}{N_t^{\gamma_t}}-c\right)dp_t(1)\end{array}\right]=0. \tag{A.80}$$

Therefore, it follows that the sign of the only effect not internalized by individual workers is driven by the effect of $L(1)$ on the boundaries of the integrals with respect to $p_t(1)$ in (A.77). At the margin, this effect is

$$\frac{1}{4}\left[\frac{[(q_0+q_1)^{\gamma_{nt}}-q_0^{\gamma_{nt}}]\,R^{\gamma_r}\left(Y_t^T\right)^{\gamma_t}}{N_t^{\gamma_t}}-c\right]\int_0^{\min\left\{2,\frac{Y_t^T}{(\overline{L}-L(1))\delta}\right\}}\frac{Y_t^T-\delta\overline{L}p_t(x_t)}{(\alpha+\delta)(L(1))^2}dp_t(x_t) \tag{A.81}$$

From the definition of Y_t^T in Proposition 3

$$\frac{[(q_0+q_1)^{\gamma_{nt}}-q_0^{\gamma_{nt}}]\,R^{\gamma_r}\left(Y_t^T\right)^{\gamma_t}}{N_t^{\gamma_t}}-c>0, \tag{A.82}$$

so the marginal effect of $L(1)$ that workers do not internalize has the same sign as

$$\int_0^{\min\left\{2,\frac{Y_t^T}{(\overline{L}-L(1))\delta}\right\}}\left(Y_t^T-\delta\overline{L}p_t(x_t)\right)dp_t(2) \tag{A.83}$$

$$=\left[p_t(x_t)Y_t^T-\delta\overline{L}\frac{p_t(x_t)^2}{2}\right]_0^{\min\left\{2,\frac{Y_t^T}{(\overline{L}-L(1))\delta}\right\}} \tag{A.84}$$

If $Y_t^T<2(\overline{L}-L(1))\delta$, (A.84) can be written as $2(Y_t^T-\delta\overline{L})$, which is negative since we are considering the case in which $Y_t^T<\delta\overline{L}$. Alternatively, $Y_t^T\geq 2(\overline{L}-L(1))\delta$, (A.83) can be written as,

$$\left(\frac{Y_t^T}{(\overline{L}-L(1))\delta}\right)^2\frac{\delta}{2}\left[\overline{L}-2L(1)\right]<0, \tag{A.85}$$

since $L^*(1)>\frac{\overline{L}}{2}$. Therefore, if $Y_t^T<\delta\overline{L}$, a marginal decrease in $L(1)$ from $L^*(1)$ increases welfare.

Case ($Y_t^T>\delta\overline{L}$): Note that

$$p_t^T(2)>0\Leftrightarrow L(1)>\overline{L}-\frac{Y_t^T}{2\delta} \tag{A.86}$$

which holds if $Y_t^T>\delta\overline{L}$ and $L(1)^*>\frac{\overline{L}}{2}$. This implies that even if $p_t(x_t)=2$, there exists $p_t(1)$ low enough that $Y_t<Y_t^T$. On the other hand, for $p_t(x_t)$ low enough, it could be that $Y_t<Y_t^T$ for all $p_t(1)\in(0,2)$. This is the case if

$$L(1)(\alpha+\delta)2+(\overline{L}-L(1))\delta p_t(x_t)<Y_t^T\Leftrightarrow p_t(x_t)<\frac{Y_t^T-L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta} \tag{A.87}$$

Therefore, (A.72) can be written as

$$\frac{1}{4}\int_{\max\left\{0,\frac{Y_t^T-L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right\}}^{2}\left[\begin{array}{c}\int_{\frac{Y_t^T-(\overline{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}}^{2}\frac{(q_0+q_1)^{\gamma_{nt}}R^{\gamma_r}Y_t^{\gamma_t}}{N_t^{\gamma_t}}-c\,dp_t(1)\\+\int_0^{\frac{Y_t^T-(\overline{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}}\frac{q_0^{\gamma_{nt}}R^{\gamma_r}Y_t^{\gamma_t}}{N_t^{\gamma_t}}\,dp_t(1)\end{array}\right]dp_t(x_t) \tag{A.88}$$
$$+\frac{1}{4}\int_0^{\max\left\{0,\frac{Y_t^T-L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right\}}\left[\int_0^2\frac{q_0^{\gamma_{nt}}R^{\gamma_r}Y_t^{\gamma_t}}{N_t^{\gamma_t}}\,dp_t(1)\right]dp_t(x_t)$$

(Note: If $Y_T^T < 2\overline{L}\delta$ then $\frac{Y_t^T-L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta} < 2$,) Similarly to the previous **Case ($Y_t^T > \delta\overline{L}$)**, the sign of the of the effect not internalized is driven by the effect on the boundaries of the integrals with respect to $L(1)$. From the definition of Y_t^T in Proposition 3

$$\frac{\left[(q_0+q_1)^{\gamma_{nt}}-q_0^{\gamma_{nt}}\right]R^{\gamma_r}\left(Y_t^T\right)^{\gamma_t}}{N_t^{\gamma_t}}-c>0, \tag{A.89}$$

so the marginal effect of $L(1)$ that workers do not internalize has the same sign a

$$\int_{\max\left\{0,\frac{Y_t^T-L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right\}}^{2}\left(Y_t^T-\delta\overline{L}p_t(x_t)\right)\,dp_t(x_t) \tag{A.90}$$

$$=\left[p_t(x_t)Y_t^T-\delta\overline{L}\frac{p_t(x_t)^2}{2}\right]_{\max\left\{0,\frac{Y_t^T-L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right\}}^{2} \tag{A.91}$$

If $Y_t^T < L(1)(\alpha+\delta)2$, (A.91) can be written as $2(Y_t^T-\delta\overline{L})$ which is positive since we are now considering the case in which $Y_t^T > \delta\overline{L}$. Alternatively, if $Y_t^T \geq L(1)(\alpha+\delta)2$, (A.91) can be written as,

$$Y_t^T\left[2-\frac{Y_t^T-L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right]-\frac{\delta\overline{L}}{2}\left[2^2-\left(\frac{Y_t^T-L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right)^2\right], \tag{A.92}$$

whose sign is the same as the sign of

$$Y_t^T-\frac{\delta\overline{L}}{2}\left[2+\frac{Y_t^T-L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right] \tag{A.93}$$

$$=\frac{Y_t^T\left(\frac{\overline{L}}{2}-L(1)\right)+\overline{L}[\delta(2L(1)-\overline{L})+L(1)\alpha]}{\overline{L}-L(1)} \tag{A.94}$$

$$=\frac{Y_t^T\left(L(1)-\frac{\overline{L}}{2}\right)\left(2\overline{L}\delta-Y_t^T\right)+\overline{L}L(1)\alpha}{\overline{L}-L(1)}>0 \tag{A.95}$$

where the inequality follows from $2\overline{L}\delta > Y_t^T$ and $L(1) > \frac{\overline{L}}{2}$. Therefore, if $Y_t^T > \delta\overline{L}$, a marginal increase in $L(1)$ from $L^*(1)$ increases welfare. *QED*

Proof of Proposition 7

The welfare function can be written as in (A.72). We first consider the case in which $Y_t^T < \delta\overline{L}$ and then, the case in which $Y_t^T > \delta\overline{L}$.

Case $(Y_t^T < \delta\overline{L})$: Following similar steps as the ones as in the **Case** $(Y_t^T < \delta\overline{L})$ in the proof of Proposition 6, for $Y_t^T < \delta\overline{L}$, the welfare function (A.72) can be written as in (A.77). The derivative with respect to Y_t of the integrands in (A.77) is proportional to

$$\gamma_t \left(\mathbb{E}\left[v((\alpha+\delta)p_t(1))\right] - \mathbb{E}\left[v(\delta p_t(x_t))\right]\right) \tag{A.96}$$

(see (A.79).) At a corner equilibrium $L^*(1) = \overline{L}$, (A.96) is zero for $\alpha = \alpha^*$ and strictly positive for $\alpha > \alpha^*$. (See proof of Proposition 5.) From **Case** $(Y_t^T < \delta\overline{L})$ in the proof of Proposition 6, the derivative with respect to $L(1)$ on the boundaries of the integrals in (A.77) is negative. Therefore, if $Y_t^T < \delta\overline{L}$, a marginal decrease in $L(1)$ from $L^*(1) = \overline{L}$ increases or decreases welfare depending on whether in (A.77), the effect that works through the integrands or the that works through the boundaries of the integrals dominates.

Case $(Y_t^T > \delta\overline{L})$: Following similar steps as the ones as in the **Case** $(Y_t^T > \delta\overline{L})$ in the proof of Proposition 6, for $Y_t^T > \delta\overline{L}$, the welfare function (A.72) can be written as in (A.88). The derivative with respect to Y_t of the integrands in (A.88) is again proportional to

$$\gamma_t \left(\mathbb{E}\left[v((\alpha+\delta)p_t(1))\right] - \mathbb{E}\left[v(\delta p_t(x_t))\right]\right), \tag{A.97}$$

and, at corner equilibrium $L^*(1) = \overline{L}$, (A.97) is zero for $\alpha = \alpha^*$ and strictly positive for $\alpha > \alpha^*$. (See proof of Proposition 5.) From **Case** $(Y_t^T > \delta\overline{L})$ in the proof of Proposition 6, the derivative with respect to $L(1)$ on the boundaries of the integrals in (A.77) is also positive. Therefore, if $Y_t^T > \delta\overline{L}$, a marginal decrease in $L(1)$ from $L^*(1) = \overline{L}$ decreases welfare. *QED*

Proof of Proposition 8

From (33), $\mathbb{E}[v_s(y_i)]$, can be written as in (A.72) and the event $Y_t \geq Y_t^T$ can be written as in (A.74):

$$p_t(1) \geq \frac{Y_t^T - (\overline{L} - L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)} \equiv p_t^T(p_t(x_t)). \tag{A.98}$$

In turn we consider (i) $Y_t^T < \delta\overline{L}$ and $0 < L^*(1) < L^*(x_t)$; (ii) $Y_t^T > \delta\overline{L}$ and $0 < L^*(1) < L^*(x_t)$; (iii)$Y_t^T < \delta\overline{L}$ and $L^*(x_t) = \overline{L}$; (iv) $L^*(x_t) = \overline{L}$.

Case (i): $Y_t^T < \delta\overline{L}$ and $0 < L^*(1) < L^*(x_t)$:

$$p_t^T(x_t) < 0 \Leftrightarrow L(1) < \overline{L} - \frac{Y_t^T}{2\delta} \tag{A.99}$$

which holds if $Y_t^T < \delta\overline{L}$ and $L(1)^* < \frac{\overline{L}}{2} < L^*(x_t)$. This implies that for $p_t(x_t)$ high enough

(positive but smaller than 2 given the assumption in eq. 25), $Y_t > Y_t^T$. However for $p_t(x_t) = 0$, there may exist $p_t(1)$ high enough that $Y_t > Y_t^T$.
Therefore, (A.72) can be written as:

$$\int_{\frac{Y_t^T}{(\overline{L}-L(1))\delta}}^{2} \left[\int_0^2 \frac{(q_0+q_1)^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} - c\, dp_t(1)\right] dp_t(x_t) + \tag{A.100}$$

$$+ \int_{\min\left\{0, \frac{Y_t^T - L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right\}}^{\frac{Y_t^T}{(\overline{L}-L(1))\delta}} \left[\begin{array}{c} \int_{\frac{Y_t^T-(\overline{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}}^{2} \frac{(q_0+q_1)^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} - c\, dp_t(1) \\ + \int_0^{\frac{Y_t^T-(\overline{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}} \frac{q_0^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}}\, dp_t(1) \end{array}\right] dp_t(x_t)$$

$$+ \int_0^{\min\left\{0, \frac{Y_t^T - L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right\}} \left[\int_0^2 \frac{q_0^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}}\, dp_t(1)\right] dp_t(x_t)$$

Similarly to the **Case ($Y_t^T > \delta\overline{L}$)** in the Proof of Proposition 6, the effect not internalized by workers'specialization decisions is the effect that $L(1)$ has on the boundaries of the integrals in (A.100). From the definition of Y_t^T in Proposition 3,

$$\frac{\left[(q_0+q_1)^{\gamma_{nt}} - q_0^{\gamma_{nt}}\right] R^{\gamma_r} \left(Y_t^T\right)^{\gamma_t}}{N_t^{\gamma_t}} - c > 0, \tag{A.101}$$

and the marginal effect of $L(1)$ that workers do not internalize has the same has the same sign as

$$\int_{\min\left\{0, \frac{Y_t^T - L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right\}}^{\frac{Y_t^T}{(\overline{L}-L(1))\delta}} \left[Y_t^T - \delta\overline{L} p_t(x_t)\right] dp_t(x_t) \tag{A.102}$$

$$= \left[p_t(x_t) Y_t^T - \delta\overline{L}\frac{(p_t(x_t))^2}{2}\right]_{\min\left\{0, \frac{Y_t^T - L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right\}}^{\frac{Y_t^T}{(\overline{L}-L(1))\delta}} \tag{A.103}$$

If $Y_t^T \leq (\alpha+\delta)L(1)$, (A.103) has the sign of $\overline{L} - 2L(1)$, which is positive for $L^*(x_t) > \frac{\overline{L}}{2}$. Alternatively, if $Y_t^T > (\alpha+\delta)L(1)$, (A.103) has the sign of

$$Y_t^T - \frac{\delta\overline{L}}{2}\left[\frac{Y_t^T}{(\overline{L}-L(1))\delta} + \frac{Y_t^T - L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right] \tag{A.104}$$

$$= \frac{L(1)}{\overline{L}-L(1)}\left((\alpha+\delta)\overline{L} - Y_t^T\right) \tag{A.105}$$

which is positive when $Y_t^T < \delta\overline{L}$.

Case (ii): $Y_t^T > \delta\overline{L}$ and $0 < L^*(1) < L^*(x_t)$:

$$p_t^T(0) > 2 \Leftrightarrow L(1) < \frac{Y_t^T}{2(\alpha+\delta)} \tag{A.106}$$

which holds for some α small enough, yet greater than zero, if $Y_t^T > \delta\overline{L}$ and $L(1)^* < \frac{\overline{L}}{2} < L^*(x_t)$. Which implies that for α and $p_t(2)$ low enough, $Y_t < Y_t^T$. However, when $Y_t^T > \delta\overline{L}$, for $p_t(x_t) = 2$, there may exist $p_t(1)$ low enough such that $Y_t < Y_t^T$.
Therefore, (A.72) can be written as:

$$\int_{\min\left\{\frac{Y_t^T}{(\overline{L}-L(1))\delta},2\right\}}^{2}\left[\int_0^2 \frac{(q_0+q_1)^{\gamma_{nt}}R^{\gamma_r}Y_t^{\gamma_t}}{N_t^{\gamma_t}} - c\,dp_t(1)\right]dp_t(x_t) \tag{A.107}$$
$$+\int_{\max\left\{0,\frac{Y_t^T-L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right\}}^{\min\left\{\frac{Y_t^T}{(\overline{L}-L(1))\delta},2\right\}}\left[\begin{array}{c}\int_{\frac{Y_t^T-(\overline{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}}^{2}\frac{(q_0+q_1)^{\gamma_{nt}}R^{\gamma_r}Y_t^{\gamma_t}}{N_t^{\gamma_t}} - c\,dp_t(1)\\ +\int_0^{\frac{Y_t^T-(\overline{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}}\frac{q_0^{\gamma_{nt}}R^{\gamma_r}Y_t^{\gamma_t}}{N_t^{\gamma_t}}\,dp_t(1)\end{array}\right]dp_t(x_t)$$
$$+\int_0^{\max\left\{0,\frac{Y_t^T-L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right\}}\left[\int_0^2\frac{q_0^{\gamma_{nt}}R^{\gamma_r}Y_t^{\gamma_t}}{N_t^{\gamma_t}}\,dp_t(1)\right]dp_t(x_t)$$

Similarly to the previous **Case** ($Y_t^T > \delta\overline{L}$) in the Proof of Proposition 6, the effect not internalized by workers'specialization decisions is the effect that $L(1)$ has on the boundaries of the integrals in (A.107. From the definition of Y_t^T in Proposition 3,

$$\frac{\left[(q_0+q_1)^{\gamma_{nt}} - q_0^{\gamma_{nt}}\right]R^{\gamma_r}\left(Y_t^T\right)^{\gamma_t}}{N_t^{\gamma_t}} - c > 0, \tag{A.108}$$

so the marginal effect of $L(1)$ that workers do not internalize has the same sign as

$$\int_{\max\left\{0,\frac{Y_t^T-L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right\}}^{\min\left\{\frac{Y_t^T}{(\overline{L}-L(1))\delta},2\right\}}\left(Y_t^T-\delta\overline{L}p_t(2)\right)dp_t(x_t) \tag{A.109}$$
$$= \left[p_t(2)Y_t^T - \delta\overline{L}\frac{p_t(2)^2}{2}\right]_{\max\left\{0,\frac{Y_t^T-L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right\}}^{\min\left\{\frac{Y_t^T}{(\overline{L}-L(1))\delta},2\right\}} \tag{A.110}$$

If $Y_t^T < L(1)(\alpha+\delta)2 < 2\delta(\overline{L}-L(1))$, (A.110) has the sign of

$$\left(Y_t^T - \frac{\delta\overline{L}}{2}\frac{Y_t^T}{(\overline{L}-L(1))\delta}\right) = \frac{Y_t^T}{2((\overline{L}-L(1))}\left((\overline{L}-2L(1)\right) < 0. \tag{A.111}$$

If $L(1)(\alpha+\delta)2 < Y_t^T < 2\delta(\overline{L}-L(1))$, (A.110) has the sign of

$$Y_t^T - \frac{\delta\overline{L}}{2}\left(\frac{Y_t^T}{(\overline{L}-L(1))\delta} + \frac{Y_t^T-L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right) \tag{A.112}$$
$$= L(1)\frac{\overline{L}(\alpha+\delta)-Y_t^T}{\overline{L}-L(1)} \tag{A.113}$$

which is negative for α small when $Y_t^T > \delta\overline{L}$.

If $L(1)(\alpha+\delta)2 < 2\delta(\overline{L}-L(1)) < Y_t^T$, (A.110) has the sign of

$$Y_t^T - \frac{\delta\overline{L}}{2}\left(2+\frac{Y_t^T - L(1)(\alpha+\delta)2}{(\overline{L}-L(1))\delta}\right) \tag{A.114}$$

$$= \frac{(2L(1)-\overline{L})(2\delta\overline{L}-Y_t^T)+2\overline{L}L(1)\alpha}{2(\overline{L}-L(1))}, \tag{A.115}$$

which is negative if for α small, since $2\overline{L}\delta > Y_t^T$ from asumption in eq. 25.

Case (iii): $Y_t^T < \delta\overline{L}$ and $L^*(x_t) = \overline{L}$.

As in **Case (i)**, (A.72) can be written as in (A.100). The proof of this case is similar to the proof of **Case ($Y_t^T < \delta\overline{L}$)** in the Proof of Proposition 7: At a corner equilibrium $L^*(x_t) = \overline{L}$, the derivative of the integrand in (A.100) with respect to $L(1)$ is non-positive and is strictily negative for α small enough. From **Case (i)** in the Proof of Proposition 8, the derivative with respect to $L(1)$ on the boundaries of the integrals in (A.100) is positive. Therefore, if $Y_t^T < \delta\overline{L}$, a marginal increase in $L(1)$ from $L^*(x_t) = \overline{L}$ increases or decreases welfare depending on whether in (A.100), the effect that works through the integrands or the that works through the boundaries of the integrals dominates.

Case (iv): $Y_t^T > \delta\overline{L}$ and $L^*(x_t) = \overline{L}$.

As in **Case (ii)**, (A.72) can be written as in (A.107). The proof of this case is similar to the proof of **Case ($Y_t^T > \delta\overline{L}$)** in the Proof of Proposition 7: At a corner equilibrium $L^*(x_t) = \overline{L}$, the derivative of the integrand in (A.107) with respect to $L(1)$ is non-positive and is strictily negative for α small enough. From **Case (ii)** in the Proof of Proposition 8, the derivative with respect to $L(1)$ on the boundaries of the integrals in (A.100) is negative for α small enough. Therefore, for α small enough, if $Y_t^T > \delta\overline{L}$, a marginal increase in $L(1)$ from $L^*(x_t) = \overline{L}$ decreases welfare. *QED*

Proof of Proposition 9

Part (i)

In any interior equilibrium, a worker must be indifferent between specializing in traded-good sector 1 and in other traded-good sector.

$$\mathbb{E}\left[\frac{Q_{nt}^{\gamma_{nt}}\left((\alpha+\delta)p_t(1)-\delta p_t(x_t)\right)}{Y_t^{1-\gamma_t}}\right] = 0. \tag{A.116}$$

Note

$$\lim_{N\to+\infty} Y_t = (\alpha+\delta)L(1)p_t(1)+\delta(\overline{L}-L(1)) \tag{A.117}$$

and hence when $N \to +\infty$, the LHS of (A.116) is

$$\int_{\max\left\{\min\left\{\frac{Y_t^T-\delta\left(\overline{L}-L(1)\right)}{(\alpha+\delta)L(1)},2\right\},0\right\}}^{2} \frac{(q_1+q_0)^{\gamma_{nt}}\left((\alpha+\delta)p_t(1)-\delta\right)}{\left[(\alpha+\delta)L(1)p_t(1)+\delta(\overline{L}-L(1))\right]^{1-\gamma_t}}dp_t(1) \tag{A.118}$$
$$+\int_0^{\max\left\{\min\left\{\frac{Y_t^T-\delta\left(\overline{L}-L(1)\right)}{(\alpha+\delta)L(1)},2\right\},0\right\}} \frac{q_0^{\gamma_{nt}}\left((\alpha+\delta)p_t(1)-\delta\right)}{\left[(\alpha+\delta)L(1)p_t(1)+\delta(\overline{L}-L(1))\right]^{1-\gamma_t}}dp_t(1),$$

which, if $L(1)=0$, is equal to [A4]

$$\frac{Q_{nt}^{\gamma_{nt}}\alpha}{\left(\delta\overline{L}\right)^{1-\gamma_t}}>0. \tag{A.119}$$

It follows that for N large enough (A.118) is strictly greater than zero and therefore $L^*(1)=0$ cannot be an equilibrium.

For $L(1)=\overline{L}$, Y_t and Q_{nt} do not depend on $p_t(x_t)$, and hence, (A.118) has the same sign as

$$\int_{\frac{Y_t^T}{(\alpha+\delta)\overline{L}}}^{2}(q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})\frac{(\delta+\alpha)p_t(1)-\delta}{p_t(1)^{1-\gamma_t}}dp_t(1) \tag{A.120}$$
$$+\int_0^{\frac{Y_t^T}{(\alpha+\delta)\overline{L}}} q_0^{\gamma_{nt}}\frac{(\delta+\alpha)p_t(1)-\delta}{p_t(1)^{1-\gamma_t}}dp_t(1)$$
$$= (q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})2^{\gamma_t}\left[\frac{2(\delta+\alpha)}{\gamma_t+1}-\frac{\delta}{\gamma_t}\right] \tag{A.121}$$
$$-\left[(q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})-q_0^{\gamma_{nt}}\right]\left(\frac{Y_t^T}{(\alpha+\delta)\overline{L}}\right)^{\gamma_t}\left[\frac{\frac{Y_t^T}{\overline{L}}}{\gamma_t+1}-\frac{\delta}{\gamma_t}\right]$$

Notice that

$$\frac{\frac{Y_t^T}{\overline{L}}}{\gamma_t+1}-\frac{\delta}{\gamma_t}=\frac{\gamma_t\frac{Y_t^T}{\overline{L}}-(\gamma_t+1)\,\delta}{(\gamma_t+1)\,\gamma_t}<\frac{\frac{Y_t^T}{\overline{L}}-2\delta}{(\gamma_t+1)}<0 \tag{A.122}$$

which means that (A.120) is increasing in α and tends $+\infty$ as $\alpha\to+\infty$.

Evaluated at $\alpha=0$, (A.120) is equal to

$$= (q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})\delta 2^{\gamma_t}\left[\frac{2}{\gamma_t+1}-\frac{1}{\gamma_t}\right]-\left[(q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})-q_0^{\gamma_{nt}}\right]\delta\left(\frac{Y_t^T}{\delta\overline{L}}\right)^{\gamma_t}\left[\frac{\frac{Y_t^T}{\delta\overline{L}}}{\gamma_t+1}-\frac{1}{\gamma_t}\right]$$
$$< (q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})\delta 2^{\gamma_t}\left[\frac{\frac{Y_t^T}{\delta\overline{L}}}{\gamma_t+1}-\frac{1}{\gamma_t}\right]-\left[(q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})-q_0^{\gamma_{nt}}\right]\delta\left(\frac{Y_t^T}{\delta\overline{L}}\right)^{\gamma_t}\left[\frac{\frac{Y_t^T}{\delta\overline{L}}}{\gamma_t+1}-\frac{1}{\gamma_t}\right]$$
$$= \left[\frac{\frac{Y_t^T}{\delta\overline{L}}}{\gamma_t+1}-\frac{1}{\gamma_t}\right]\left((q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})\delta 2^{\gamma_t}-\left[(q_0^{\gamma_{nt}}+q_1^{\gamma_{nt}})-q_0^{\gamma_{nt}}\right]\delta\left(\frac{Y_t^T}{\delta\overline{L}}\right)^{\gamma_t}\right)<0, \tag{A.123}$$

[A4] Recall $Q_{nt}=q_1\mathbb{1}_{Y_t^T\geq\delta\overline{L}}+q_0$ where $\mathbb{1}$ is the indicator function.

Therefore, (A.120) is increasing in α, tends $+\infty$ as $\alpha \to +\infty$, and is negative for $\alpha = 0$. This implies that there exist $\tilde{\alpha} > 0$ such that (A.120) is weakly positive if and only if $\alpha \geq \tilde{\alpha}$. If $\tilde{\alpha} < \alpha$, the RHS in (A.118) is negative and bounded away from 0 for $L(1) = \overline{L}$ and positive and bounded away from 0 for $L(1) = 0$. It follows that for N large enough, there exists $\tilde{\alpha}_N$ such that if $\alpha < \tilde{\alpha}_N$, (A.116) is strictly positive for $L(1) = 0$ and strictly negative for $L(1) = \overline{L}$, in which case, any equilibrium of the specialization game $L^*(1)$ is interior ($0 < L^*(1) < \overline{L}$) and if $\alpha \geq \tilde{\alpha}_N$.

Part (ii) and (iii).

From (33), $\mathbb{E}[v_s(y_i)]$, can be written as

$$\mathbb{E}\left[v_s(Y)\right] = \mathbb{E}\left[\frac{Q_{nt}^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{P_t^{\gamma_t}} - c\mathbb{1}_{Y_t \geq Y_t^T}\right] \tag{A.124}$$

When $N \to +\infty$, (A.124) tends to

$$\int_{\max\left\{\min\left\{\frac{Y_t^T - \delta\left(\overline{L} - L(1)\right)}{(\alpha+\delta)L(1)}, 2\right\}, 0\right\}}^{2} \frac{(q_0 + q_1)^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} dp_t(1) \tag{A.125}$$
$$+ \int_0^{\max\left\{\min\left\{\frac{Y_t^T - \delta\left(\overline{L} - L(1)\right)}{(\alpha+\delta)L(1)}, 2\right\}, 0\right\}} \frac{q_0{}^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} dp_t(1)$$

where $Y_t = (\alpha + \delta)L(1)p_t(1) + \delta\left(\overline{L} - L(1)\right)$. For $0 < \frac{Y_t^T - \delta\left(\overline{L} - L^*(1)\right)}{(\alpha+\delta)L^*(1)} < 2$, the derivative of (A.125) with respect to $L(1)$ evaluated at the equilibrium $L(1) = L^*(1)$ has the same sign as $\left(Y_t^T - \delta\overline{L}\right)$. For $\frac{Y_t^T - \delta\left(\overline{L} - L^*(1)\right)}{(\alpha+\delta)L^*(1)} \notin (0, 2)$, the derivative of (A.125) with respect to $L(1)$ evaluated at the equilibrium $L(1) = L^*(1)$ is zero. *QED*