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Abstract

My thesis comprises four distinct chapters, which focus primarily on studying competitive markets under adverse selection. The core of my research lies in analyzing competition and incentives within asymmetric information economies. Chapter 1 examines how different market structures impact market outcomes under adverse selection. In Chapter 2, I demonstrate that an upper-bound trade restriction can resolve the issue of equilibrium nonexistence under a nonexclusive environment with a menu game. In Chapter 3, I show that the entry-proof allocation in the static model can extend to multiple periods with a straightforward cross-period policy. In Chapter 4, I analyze the entry-proof allocation in situations where entrants can only offer linear contracts. My research yields novel findings that differ from the existing literature.

The first chapter adopts a unified perspective on multi-contracting in competitive markets plagued by adverse selection. We subsume the two polar cases of exclusive and nonexclusive competition by introducing the concept of a *market structure*, i.e., a trading rule that specifies the subset of sellers with whom buyers can jointly trade. The existing literature shows that the market structure matters greatly in shaping competitive allocations, allowing for either separating allocations (as shown by Rothschild-Stiglitz) or layered pooling (Jaynes-Hellwig-Glosten) allocations. We prove the existence of intermediate "Pooling + Separating" equilibria that allow for simultaneous pooling and low-risk buyer separation. Crucially, those allocations alleviate at the same time the concern of excessive rationing under separation of and cross-subsidies paid by low-risk buyers. They oftentimes Pareto dominate the Rothschild-Stiglitz separating allocation. Our analysis singles out the "1+1" market structure where sellers are separated into two subgroups so that buyers can trade with at most one seller from each subgroup. Any "Pooling + Separating" allocation is an equilibrium here. Finally, we prove that "Pooling + Separating" allocations satisfy a notion of stability that we call serendipitous-aftermarket-proofness.

The second chapter considers fully nonexclusive competitive markets, where privately informed buyers can trade with arbitrarily many sellers at the same time. Attar, Mariotti, and Salanié (2014) show the unique equilibrium candidate is the JHG allocation: low and high-risk buyers pool for a basic layer of insurance, and high-risk buyers purchase an additional layer of insurance. However, they also show that this allocation equilibrium never exists. Here I show that an equilibrium exists if we impose an upper bound on the number of sellers that buyers can jointly trade with. Equilibrium existence requires a strong assumption frequently made in the literature in the context of quadratic or CARA utility functions: translated indifference curves must have an identical shape across types. My findings shed light on the disconnection between continuous type models (associated with existence results) and discrete type models (associated with the absence thereof). It suggests that the choice of parametrization of utility functions is critical.

The third chapter studies the multiple-period nonexclusive competition in annuity mar-

kets. The annuity market represents a special segment of the insurance industry, where buyers acquire contracts during an initial period and receive payments as long as they are alive in subsequent periods. This market is characterized by nonexclusive trade and multiple periods, with considerable evidence pointing to the presence of adverse selection. In their research, Attar, Mariotti, and Salanié investigated nonexclusive trade in the insurance market within a static context, identifying the JHG tariff as entry-proof and budget balanced. Our study reveals that a JHG-similar tariff remains entry-proof when the annuity contract indemnity is consistent across the market. However, without indemnity restrictions, the demand for annuity contracts in later periods may crow out the high-risk demand from earlier periods, causing low-risk individuals to consume more annuities in the initial period and rendering the JHG-similar tariff non-entry-proof. To address this issue, we introduce an optional cross-period policy for planners to consider. By integrating the JHG-similar tariff with this policy, we demonstrate that the tariff becomes entry-proof once again, and the resulting allocation is budget balanced. These findings prove to be robust in any multi-period environment.

The fourth chapter examines entry-proof allocations under linear contract entrants. While the JHG allocation is the unique entry-proof allocation for arbitrary tariff entrants, this paper shows that there are multiple feasible entry-proof allocations, such as the Pauly line allocation and two-part zero-profit allocations. The study provides conditions for the existence of non-Entry-Proof-Pareto-Dominated (EPPD) allocations from a planner's perspective and demonstrates that the absence of a planner in a linear pricing market is less efficient than in a market with the planner's intervention. However, the paper finds that linear side trading can often destroy efficiency in the convex tariff setting, even with non-convex tariffs and restrictive linear side trading. Furthermore, the Non-EPPD allocation can coincide with the second-best allocation under non-convex tariffs in some cases. The paper also shows that any entry-proof efficient allocation should provide an average unit price of c to L type buyers if there is no second-best allocation that coincides with the Non-EPPD allocation.

Chapter 1: Market Structure and Adverse Selection

Joint with Christopher Sandmann

Abstract

This paper adopts a unified perspective on multi-contracting in competitive markets plagued by adverse selection. We subsume the two polar cases of exclusive and nonexclusive competition by introducing the concept of a *market structure*, i.e., a trading rule that specifies the subset of sellers with whom buyers can jointly trade. The existing literature shows that the market structure matters greatly in shaping competitive allocations, allowing for either separating allocations (as shown by Rothschild-Stiglitz) or layered pooling (Jaynes-Hellwig-Glosten) allocations. We prove the existence of intermediate "Pooling + Separating" equilibria that allow for simultaneous pooling and low-risk buyer separation. Crucially, those allocations alleviate at the same time the concern of excessive rationing under separation of and cross-subsidies paid by low-risk buyers. They oftentimes Pareto dominate the Rothschild-Stiglitz separating allocation. Our analysis singles out the "1+1" market structure where sellers are separated into two subgroups so that buyers can trade with at most one seller from each subgroup. Any "Pooling + Separating" allocation is an equilibrium here. Finally, we prove that "Pooling + Separating" allocations satisfy a notion of stability that we call serendipitous-aftermarket-proofness.

1 Introduction

Since Akerlof (1970) first showed that markets plagued by adverse selection[1] can unravel, a large literature has examined the impact of information asymmetries on market outcomes. The conclusions drawn from the theory are negative and emphasize the need for regulatory oversight and intervention.[2] In contrast to the theory's emphasis on market failures, empirical studies document that many active markets where one may suspect adverse selection function well.[3] This has led some authors to question whether adverse selection occurs in the first place. We suggest a different possibility: even in the presence of adverse selection, benevolent planners or competitive markets can self-regulate by imposing, prior to the contracting stage, adequate exclusivity constraints. Our analysis identifies intuitive exclusivity constraints that generate new equilibrium allocations and successfully navigate many of the issues raised by the theory.

Our current theoretical understanding of competitive markets is based on Rothschild and Stiglitz (1976).[4] Sellers compete by posting *exclusive* price-quantity contracts so that each buyer who has private information can trade with at most one seller. As is well-known, a (pure strategy) equilibrium need not exist.[5] If it does, it gives rise to a unique separating allocation that often involves the rationing of low-risk buyers. More recently, Attar et al. (2014) and Attar et al. (2022), dispensed with the assumption that sellers propose exclusive contracts. They show that if buyers can purchase arbitrarily many contracts from different sellers, a paradigm referred to as *nonexclusive* competition, the Jaynes-Hellwig-Glosten allocation emerges as the unique equilibrium candidate: Sellers actively trade pooling and high type separating contracts so that each contract is fairly priced given the consumer types who purchase it.[6] Note that the allocation forecloses the possibility of separating contracts only traded by low types; active low-risk buyers always cross-subsidize high-risk buyers.

In practice, we observe many trading rules that are neither fully exclusive nor nonexclusive.[7] Their prevalence suggests possible improvements to the identified problems of rationing and cross-subsidies. In this paper, we adopt a unified perspective on multi-contracting. We

[1]A market is adversely selected if the least desirable informed trading partners are also those most eager to trade.

[2]low-risk agents must be incentivized or compelled to cross-subsidize high-risk agents through subsidies or mandates Einav et al. (2010); costly pre-market trades by the planner must change the prevailing risk profile of the market Tirole (2012), or costly verification must first restore the symmetry of information for trade to resume.

[3]There is mixed evidence on whether contracts are adversely selected and the welfare loss due to rationing is estimated to be small (see Einav et al. (2010) and references therein).

[4]A version of their model allowing for withdrawal of contracts was concurrently studied by Wilson (1977)

[5]Azevedo and Gottlieb (2017) restore equilibrium existence under exclusive competition by reducing the space of admissible deviations. See also Ania et al. (2002).

[6]The JHG allocation is based on earlier work by Jaynes (1978), Hellwig (1983), and Glosten (1994).

[7]In the publicly regulated health insurance market in France, for instance, buyers purchase one basic insurance contract from one group of sellers and complement it with another contract "Mutuelle" from another group of sellers. In corporate finance, multiple banking relationships are the norm (see Detragiache et al. (2000)), but potential lenders can monitor the previous trades by a bank, imposing essential limits on the number of debt contracts a lender can sign. Life insurance contracts are per se nonexclusive, but insurance companies will not insure a combined level of cover they deem to be excessive.

view *exclusive* and *nonexclusive* competition as the two polar cases within the larger set of *market structures*. The concept of a market structure has a precise definition in our paper and determines which sellers a buyer can jointly trade with. Examples include trading rules where buyers face a restriction on the number of policies they can sign, or a classification of sellers into those that offer exclusive contracts and those that do not. The only restriction we make is to require that the market structure is *competitive*: given a set of sellers whose policies can be purchased jointly, each seller must be replaceable by both competing sellers (potentially offering different contracts) and the null trade. By contrast, we do not impose any restriction on the contract offered by an individual seller.

On a high level, our analysis identifies a binary partition of the set of competitive market structures: those that permit some sellers to propose exclusive contracts (e.g., exclusive competition), and those where no seller can prohibit further side-trading (e.g., nonexclusive competition). As a preliminary result (see Claims 1 and 2) we show that the unique equilibrium candidate for any competitive partially exclusive market structure is the Rothschild-Stiglitz allocation. In particular and as is well-known, cream-skimming deviations uniquely targeting low-risk buyers destabilize any equilibrium that involves pooling. By contrast, fully separating allocations can never be sustained in equilibrium under a never exclusive market structure. The reason is that the low price offered to low-risk buyers is also attractive to high-risk buyers if these buyers can complement the low-risk buyers' contract with additional coverage. What remains to be seen—both from a positive and a normative planner's point of view—is a characterization of equilibria that occur under a never exclusive market structure.

Our first main result presents four necessary conditions that any equilibrium allocation in a never exclusive market must satisfy (see Theorem 1): incentive compatibility, competitive pricing, large pooling, and conditional efficiency. As under nonexclusive competition, these conditions imply that in equilibrium both buyer types must trade a pooling contract; the layered pooling JHG allocation emerges as the special case that satisfies these. What is novel about our result is that we do not rule out the possibility of low-risk separating contracts. In fact, our necessary conditions single out a set of "Pooling + Separating" aggregate trades where, in addition to pooling, low-risk buyer types almost always trade a separating contract. For fixed unit prices, equilibrium candidates are described by a continuum: trading greater aggregate pooling quantities entails less separating quantities for both types (see Proposition 2). Perhaps no less surprising, incentive constraints will typically be slack. Finally, low-risk separating contracts can be profitable to the seller.

The derivation of the set of candidate equilibria relies on probing active trades with three classes of unilateral deviations: undercutting, pivoting, and efficiency-improvements. Undercutting is familiar from Bertrand's competition. Efficiency-improvements occur when a seller offers a (from the buyers' point of view) more desirable quantity at the same marginal price. Pivoting warrants a discussion. A *pivoting* contract entails a quantity that, if traded jointly with the low type's separating contract, matches the high type's initial quantity allocation at an incrementally lower total price. Clearly, any buyer will prefer the combination of the two over the pricier high type separating contract. In fully nonexclusive markets, this argument is sufficient to ensure that no low type separating contract is actively traded. Given an arbitrarily never exclusive market structure, by contrast, the availability of both a low type separating and a pivoting contract does not guarantee that both policies can be purchased jointly. As will become clear shortly, exclusivity restrictions that precede the contracting stage can sometimes shield sellers that exclusively serve low type buyers from being targeted by another seller that offers a pivoting contract.

To sustain a "Pooling + Separating" allocation as an equilibrium of the multiple-contracting game, our analysis singles out the "1+1" market structure. Here, sellers are separated into two subgroups so that buyers can trade with at most one seller from each group. In effect, there is exclusive competition within groups and nonexclusive competition between groups. A natural division of labor suggests that sellers in group 1 actively trade pooling contracts whereas sellers in group 2 actively trade separating contracts only. What is key here is that group 2 sellers that serve low-risk buyers are somewhat shielded from pivoting that would also attract high-risk buyers: exclusivity of each group means that any pivoting contract that is to be traded in conjunction with the low-risk separating contract must be offered in group 1. This implies that when selecting the pivoting contract high type buyers can no longer trade the pooling contract. Since the presence of low-risk buyers improves this latter contract's break-even price, not selecting the pooling contract entails foregoing low type buyers' cross-subsidies. Consequently, for sufficiently large pooling quantities (which was one of our necessary conditions) a pivoting contract that manages to attract high-risk buyers must make a loss.

Our second main result (see Theorem 2) establishes that any allocation that satisfies the four necessary conditions identified by Theorem 1 can occur as an equilibrium under the "1+1" market structure. For this result to hold we require, as in Attar et al. (2022), the additional *flatter curvature assumption* that imposes restrictions on the curvature of indifference curves across types. Moreover, as in their work, equilibrium existence hinges on so-called latent contracts, i.e., contracts that are not traded actively in equilibrium but that play a role to deter cream-skimming deviations uniquely targeting low-risk buyers. As in their work, we identify a principal latent contract that can be derived via the following thought experiment: suppose that some inactive sellers were to offer a cream-skimming contract uniquely targeting low-risk buyers. Since low-risk buyers cross-subsidize high-risk buyers when purchasing the pooling contract, cream-skimming can exploit potential gains from trade (to the detriment of sellers offering pooling contracts). The flatter curvature assumption posits that there always exists an additional, latent contract so that the latent contract and the cream-skimming contract taken together achieve greater high type utility than the high type's initial allocation. This ensures that large cream-skimming deviations are never profitable. The assumption can be more easily depicted graphically. We require that the translated low type upper contour sets are included in the high type's upper contour set. As a corollary, it follows that the thus identified latent contract is unique. Reassuringly, Attar et al. (2022) show that quadratic and constant-absolute-risk-aversion (CARA) preferences under the assumption that low-risk buyers are weakly more risk-averse than high-risk buyers satisfy the flatter curvature assumption.

Our analysis has normative implications. Instead of or in addition to considering more direct interventions in insurance markets (such as mandates, costly verification of types, or taxes and subsidies) the choice of market structure itself can be seen as an optimization problem given some utilitarian welfare objective. If the planner controls the platform through which insurees sign up for insurance, the cost of enforcing any choice of market structure is arguably low. Our existence result suggests that the planner's problem reduces to the binary choice between the fully exclusive "1 or 1" and the never exclusive "1+1" market structure. A comparison between equilibria under both market structures is therefore required. We show that "Pooling + Separating" allocations that occur under the "1+1" market structure Pareto dominate the Rothschild-Stiglitz allocation when adverse selection and therefore rationing of low-risk buyers is severe. By contrast, the Rothschild-Stiglitz allocation never Pareto dominates because high-risk buyers benefit from the cross-subsidies implied by pooling contracts. Within the set of "Pooling + Separating" allocations, high and low-risk buyers rank more and less pooling in opposite directions. Finally, we would like to mention that the "1+1" market structure does not render obsolete but complements more direct regulation on contracts. In fact, preliminary results (hopefully to be included in the next draft) suggest that a minimum quantity requirement (frequently observed in insurance markets) in group 1 can further improve upon welfare under the "1+1" market structure in that it eliminates possible cream-skimming deviations and thereby tightens the slackness of incentive constraints.

Our analysis also has positive implications. We argue that the "1+1" market structure shares common features with many regulated insurance markets. In particular, the interaction between public and private insurance sectors implies that many buyers supplement public policies with private complementary coverage. In the US, Medicare and employer-sponsored retiree health plans are a case in point; in France, workers must complement public insurance policy with additional coverage called Mutuelle where different policies vary greatly in benefits and fees. Our analysis helps explain the allocations observed in these markets. From a theoretical point of view, we emphasize the perspective that the "1+1" market structure can arise as an informal industry agreement. In particular, we show (see Proposition 3) that for any "Pooling + Separating" allocation and once the trade has taken place, no seller can deviate from the pre-assigned identities and propose profitable additional trades to the buyers. We refer to this stability concept as serendipitous-aftermarket-proofness in that the aftermarket occurs unexpectedly. Notably, the Rothschild-Stiglitz allocation need not be serendipitous-aftermarket-proof, even if it is an equilibrium. This means that an incentive remains to sell further contracts to low-risk buyers once initial contracting has taken place.

An important drawback of our model is that we require that each seller offers a single contract only. This is at odds with the traditional focus on menu games in the literature. The problem with menu pricing is that it relaxes the requirement that each contract must earn a nonnegative profit. What is concerning are double deviations: offer one loss-making contract to high-risk buyers in group 2 and a cream-skimming deviation to low-risk buyers in group 1. The loss-making contract in group 2 ensures that latent contracts no longer render the cream-skimming contract attractive to high-risk buyers. The deviation is however profitable in the aggregate because the risk-profile (and hence the profitability) of the cream-skimming contract has improved considerably vis-a-vis the pooling contract initially traded in group 1. Since our necessary conditions (Theorem 1) continue to apply when firms post menus, it follows that partially loss-making double deviations destabilize the market and imply that a pure strategy equilibrium fails to exist under the "1+1" market structure.[8]

From a practical point of view it is unsatisfying to require that firms offer unique policies only, because insurance providers are not in infinite supply. It is therefore worth noting that our equilibrium analysis and existence result would also hold if firms were heavily penalized for incurring losses on some contracts. In fact, Attar et al. (2022) (refer especially to their Section 5) argue that this regulation plays a key role in stabilizing fully nonexclusive insurance markets plagued by adverse selection. Our analysis renders their proposal all the more prescient in that we show that their proposed regulation equally helps stabilize the market if it is governed by the "1+1" market structure. In practice, cross-subsidies between profitable and loss-making contracts are effectively banned in several countries through cost-sharing mechanisms. These pool and redistribute costs among sellers of a standardized basic-coverage contract but are also more far-reaching in that they regulate contract characteristics.[9]

Related Literature

Our paper relates to the literature on adverse selection in competitive markets. The dominant framework is exclusive competition, introduced by Rothschild and Stiglitz (1976) with an important recent contribution by Azevedo and Gottlieb (2017). A more recent, burgeoning literature considers nonexclusive competition instead. An early paper is Pauly (1974) who restricts attention to linear contracts. The JHG allocation was derived by Jaynes (1978), Hellwig (1988), Glosten (1994). Our paper is most closely related to Attar et al. (2022), who share our focus on single contracts as opposed to menus, and more broadly their research agenda on nonexclusive competition beginning with Attar et al. (2011). Attar et al. (2014) first singled out the JHG allocation as the unique equilibrium candidate in competitive insurance markets. Attar et al. (2021) propose a sequential auction mechanism to sustain it as an equilibrium. In a model that lacks the restriction on single contracts as in Attar et al. (2022), Stiglitz et al.

[8] In a companion paper, Huang (2022), studies the menu pricing game in an otherwise identical insurance economy. He shows that, given any never exclusive market structure, the unique equilibrium candidate is the JHG allocation. An equilibrium only exists however under a perfect translation property of indifference curves (e.g., quadratic or CARA utility with identical risk preferences) which is stronger than requiring the flatter curvature assumption.

[9] As noted by Attar et al. (2022), besides Germany and Switzerland, other countries using such schemes include Australia, Ireland, the Netherlands, and Slovenia. Glazer and McGuire (2006) offer a more detailed description of risk-adjustments and offer a discussion of the "sickness fund" system in Germany.

(2020) argue that consumer disclosure and asymmetric equilibrium information flows can alternatively support the JHG allocation as an equilibrium. In a related environment, Asriyan and Vanasco (2022) investigate security design when competition is nonexclusive. Here the finiteness of collateral allows the security originator to effectively commit to exclusive trading. Auster et al. (2022) consider an otherwise standard directed search model where agents can apply for several contracts. In relation to the literature on nonexclusive competition, they show that agents' indirect utilities conditional on already made applications can feature a reversal of single-crossing.

It is a priori not clear whether multiple contracting is pro-competitive or anti-competitive. On the one hand, and as Attar et al. (2011) argue, nonexclusive competition allows for more possible deviations for sellers, in particular pivoting on other sellers' contracts. On the other hand, sellers have more tools at their disposal to block deviations through the issuance of latent contracts. Latent contracts were first introduced in the context of competing mechanisms in the seminal paper by Peters (2001). Since, latent contracts have mainly appeared under multiple contracting in the literature on moral hazard. Hellwig (1983) argues that latent contracts can deter entry into the insurance market when agents' effort decisions are not contractible. This can result in positive equilibrium profits. In Attar et al. (2019) latent contracts collectively sustain the monopoly profit for the sellers. Ours is the first paper on multiple contracting in the context of adverse selection where latent contracts allow some sellers to sustain positive profits.

There is an extensive empirical literature that tests whether adverse selection occurs. Finkelstein and Poterba (2002) and Finkelstein and Poterba (2004) study the UK annuities market and find a positive correlation between buyers' ex-post risk (i.e., longevity) and the coverage level purchased. Bauer et al. (2020) find evidence for adverse selection in the secondary market for settlements of life insurance policies. Other studies point towards the absence of adverse selection. Cawley and Philipson (1999) finds no evidence that adverse selection exists in the primary life insurance market. Chiappori and Salanie (2000) use data on the French car insurance market on contracts and accidents and finds no evidence for adverse selection, even when considering senior drivers only. As pointed out by Attar et al. (2022), multiple contracting or nonexclusive competition likely reverses the predictions on the correlation between risk and coverage on a *per-contract* basis. Mimra and Waibel (2021) conduct an experiment to assess whether the predictions of the theory on contracting under adverse selection hold up in a laboratory environment. They show that in the context of menu-pricing and both exclusive and nonexclusive competition the theory's predictions match the observed behavior. Their study design invites further investigation into the "1+1" market structure; it would equally be helpful to see whether double-deviations that are feasible under menu-pricing play as destructive a role in a laboratory environment as the theory suggests.

Our paper suggests that markets can perform better under segmentation—indeed, the "1+1" market structure proposes such segmentation for policies that could a priori be sold as a single policy. Up until now, similar positive results on market segmentation were only reported in the context of private values: Malamud and Rostek (2017) show that equilibrium utilities in a decentralized market can be strictly higher in the Pareto sense than in a centralized market with the same traders and assets. Chen and Duffie (2021) argue that fragmentation induces agents to trade more aggressively; any degree of fragmentation is welfare-superior to a centralized financial market. And Rostek and Yoon (2021) find that multiple trading protocols that clear independently can be designed to be at least as efficient as joint market clearing for all assets.

This paper is organized as follows. Section 2 introduces the model and proposes the concept of a market structure. Section 3 partitions the set of competitive market structures into partially exclusive and never exclusive market structures that subsume exclusive and nonexclusive competition as two (polar) cases. Sections 4 and 5 present our main results: necessary conditions that any equilibrium candidate under a never exclusive competitive market structure must satisfy, and an equilibrium existence result. In particular, we prove the existence of "Pooling + Separating" equilibrium allocations that have never been studied before. Finally, Section 6 explores normative and positive implications.

2 Set-up

We here introduce a model of strategic price-setting in a competitive market plagued by adverse selection. The common interpretation given to this model is that of an insurance economy in which buyers with an exogenously given high and low-risk profile purchase coverage in exchange for a premium.

Our description of the economy (preferences, cost and the space of admissible contracts) is identical to that in Attar et al. (2021), Attar et al. (2022) and encompasses the classical set-up in Rothschild and Stiglitz (1976). We innovate in that we introduce the concept of a *market structure*, i.e., a trading rule that specifies the subsets of sellers whom the buyers can jointly with. The definition admits as special cases the market structures where each buyer can trade with at most one (exclusive competition) and with arbitrarily many (nonexclusive competition) sellers.

2.1 The Contracting Environment

We consider a finite set of sellers $\mathcal{K} = \{1, ..., K\}$ and a continuum of buyers that are characterized by their type $\theta \in \{L, H\}$. Sellers compete by proposing contracts $(q^k, t^k) \in \mathbb{R}^2_+$ specifying a quantity and a transfer. Types are non-contractible so that all buyers can select identical trades if they so wish.

Preferences. Buyers view sellers as perfect substitutes, so that their preferences can be represented over aggregate trades: Denote $M \subset \{1, ..., K\}$ a subset of sellers that buyer type θ trades with and $Q = \sum_{k \in M} q^k$ and $T = \sum_{k \in M} t^k$ the corresponding aggregate quantity and transfer. Preferences over aggregate trades are represented by a utility function $U_\theta(Q,T)$ that is increasing in Q and decreasing in T and that satisfies the following assumptions:

First, we impose regularity conditions so that the buyer's demand is well-behaved.[10]

Assumption 1 (quasi-concavity)**.** *$U_\theta(q,t)$ is strictly quasi-concave, i.e. $\forall \alpha \in (0,1)$, and $(q_1,t_1) \neq (q_2,t_2)$ it holds that $U_\theta(\alpha(q_1,t_1)+(1-\alpha)(q_2,t_2)) > \min\{U_\theta(q_1,t_1), U_\theta(q_2,t_2)\}$.*

Assumption 2 (finite demand)**.** *$\arg\max_{Q\geq 0} U_\theta(Q, Qc_x)$ is finite $\forall\, c_x > 0$ and $\theta \in \{L,H\}$.*

Notice that strict quasi-concavity implies that $\arg\max_{Q\geq 0} U_\theta(Q,Qc_x)$ is a singleton.

Second, we assume that types are ordered so that high buyer types' demand exceeds low buyer types' demand:[11]

Assumption 3 (single-crossing)**.** *For all (q,t) and (q',t') so that $q' > q$ it holds that $U_L(q',t') \geq U_L(q,t) \quad \Rightarrow \quad U_H(q',t') > U_H(q,t)$.*

Cost. Trading a contract (q,t) with a buyer type θ earns the seller an expected profit $t - c_\theta q$. Here c_θ denotes the marginal cost of serving type θ. In line with a model of adverse selection, we assume that those buyer types most eager to trade, i.e., high types H, are also the most costly to serve.

Assumption 4 (Adverse Selection)**.** *$c_H > c_L$.*

Finally, denote m_H the proportion of type H and m_L the proportion of type L buyers so that the average marginal cost is $c = c_H m_H + c_L m_L$.

2.2 Market Structure

The key innovation of our framework is the concept of a market structure. A market structure specifies which sellers a buyer can jointly trade with.

Definition 1. *A market structure $\mathcal{M}$ is a (non-empty) collection of subsets of sellers with whom a buyer can jointly trade: $\mathcal{M} \subseteq \mathcal{P}(\{1,...,K\}) \equiv \mathcal{P}(\{\text{all sellers}\})$.*

The two polar cases considered in the literature are defined as follows:

Example 1. (i) *Exclusive competition $\mathcal{M} = \big\{\emptyset, \{1\}, \{2\}, ..., \{K\}\big\}$.*

(ii) *Nonexclusive competition : $\mathcal{M} = \mathcal{P}(\{1,...,K\})$*

Here $\mathcal{P}(\{1,...,K\})$ denotes the power set, i.e., the set of all subsets of $\{1,...,K\}$.

In line with the two polar cases of exclusive and nonexclusive market structures our focus is on competitive market structures. We therefore require that the sellers' offers can be declined by the buyers, and that each seller is replaceable.

[10] In Section 5 we introduce further regularity conditions that ensure the existence of an equilibrium.

[11] Provided that utility is differentiable, this is equivalent to assuming that the slope of the indifference curve $\tau_H(Q,T) = -\frac{\partial_1 U_\theta(Q,T)}{\partial_2 U_\theta(Q,T)}$ is greater for higher types, i.e., $\tau_H(Q,T) > \tau_L(Q,T)$.

Definition 2. *A market structure $\mathcal{M}$ is competitive if*

- *buyers can trade with any subset of a feasible set of trading partners, i.e., for all $M \in \mathcal{M}$ and $j \in \mathcal{K}$, if $j \in M$, then also $M \setminus \{j\} \in \mathcal{M}$;*
- *each seller is twice replaceable, i.e. for all $M \in \mathcal{M}$ and $j \in \mathcal{K}$, if $j \in M$, then there exist distinct $k_1, k_2 \in \mathcal{K} \setminus M$ so that $M \cup \{k_1\} \setminus \{j\} \in \mathcal{M}$ and $M \cup \{k_2\} \setminus \{j\} \in \mathcal{M}$.*

The ability to decline offers allows the buyers to play some sellers against others by threatening to accept only a subset of the offers they receive. Bernheim and Whinston (1986) call this arrangement delegated common agency. Less intuitively, our definition insists on sellers to be twice replaceable. This arrangement ensures that an active seller is always competing with an inactive seller and preserves undercutting incentives.[12]

2.3 Equilibrium

One can imagine a benevolent planner that decides on the jointly feasible trades before the market opens. For now we shall take the (competitive) market structure as given and focus our analysis on the ensuing equilibria. Welfare considerations that can inform the selection of different market structures will be discussed in Section 6.

The Simultaneous Move Game. We consider a competitive screening game in which firms compete by each posting a single contract.[13] Given a fixed market structure $\mathcal{M} \in \mathcal{P}(\{1, ..., K\})$, the game unfolds as follows:

- Stage 1: Each seller k proposes a contract $(q^k, t^k) \in \mathbb{R}^2_+$.
- Stage 2: Each buyer learns her type, selects some $M \in \mathcal{M}$ and derives utility $U_\theta\big(\sum_{k \in M} q^k, \sum_{k \in M} t^k\big)$.

Our equilibrium concept is standard:

Definition 3 (Equilibrium of the game)**.** *Fix $\mathcal{M}$. A pure strategy perfect Bayesian equilibrium (PBE) is a pair $(\mathcal{C}, S)$ where $\mathcal{C} = ((q^k, t^k))_{k \in \mathcal{K}}$ is the set of trades proposed by the sellers and $S = (S_\theta)_{\theta \in \{L,H\}}$ is the buyers' strategy profile $S_\theta : \prod_{k \in \mathcal{K}} \mathbb{R}^2_+ \to \mathcal{M}$ so that*

- *the buyers' strategy profile S satisfies buyer optimality*

$$S_\theta(\mathcal{C}') \in \arg\max_{M \in \mathcal{M}} U_\theta\Big(\sum_{k \in M} q^k, \sum_{k \in M} t^k\Big) \quad \forall \mathcal{C}' \in \mathbb{R}^{2K}_+;$$

[12] If seller h had only one replacement seller ℓ, it is conceivable that seller ℓ also has only one replacement seller h. This would allow sellers h and ℓ to collude with one seller profitably serving high types and the other seller profitably serving low types.

[13] Our focus on single contracts avoids that on-path some sellers offers loss-making contracts. Alternatively, we can consider a regulated insurance market (as in Attar et al. (2022)) in which sellers are heavily fined if some contracts on their menu incurs a loss. The restriction that contracts must not be loss-making is critical in our environment. In a companion paper, Huang (2022) shows that if firms can post menus instead of single contracts, there exists a unique equilibrium allocation and this is the JHG allocation studied in the next section.

- *and the trades proposed satisfy seller optimality*

$$(q^k, t^k) \in \underset{(q'^k, t'^k)\in\mathbb{R}^2_+}{\arg\max} \sum_{\theta\in\{L,H\}} \big[t'^k - c_\theta\, q'^k\big] m_\theta\, 1\Big\{k \in S_\theta\big(\{(q^\ell, t^\ell), (q'^k, t'^k)\}_{\ell\neq k}\big)\Big\}.$$

An immediate consequence of our focus on PBE is that along the equilibrium path we can distinguish between active sellers and inactive sellers. Active sellers propose contracts that are actively traded, inactive sellers may propose so called latent contracts. As we shall see, latent contracts play an important role to sustain an equilibrium (if it exists).

3 Partially and Never Exclusive Market Structures

As a precursor to our main results, we here introduce a partition of the set of competitive market structures into two disjoint subsets. We show that if the market structure permits exclusive trades, then the Rothschild-Stiglitz separating allocation is the unique equilibrium candidate allocation. Conversely, we show that if the market structure prohibits exclusive trades, then any admissible equilibrium candidate allocation is at least partially pooling. This serves as a precursor to our subsequent analysis which will be concerned with characterizing the set of non-separating equilibria when the market structure is never exclusive.

3.1 Partially Exclusive Market Structures

The most famous equilibrium candidate discussed in the literature is the Rotschild-Stiglitz separating allocation. It is defined as follows:

Definition 4 (Rotschild-Stiglitz (RS))**.** *The RS allocation is the separating allocation* (Q_L^{RS}, T_L^{RS}) *and* (Q_H^{RS}, T_H^{RS}) *where*

$$\begin{aligned} Q_H^{RS} &= \underset{Q_H\geq 0}{\arg\max}\, U_H(Q_H, c_H Q_H), \quad T_H^{RS} = c_H Q_H^{RS} \\ Q_L^{RS} &= \underset{Q_L\geq 0}{\arg\max}\, U_L(Q_L, c_L Q_L), \quad T_L^{RS} = c_L Q_H^{RS} \\ &\textit{subject to } U_H(Q_H^{RS}, T_H^{RS}) \geq U_H(Q_L^{RS}, T_L^{RS}). \end{aligned}$$

This allocation usually entails rationing (as depicted in Figure 1): low type buyers would like to purchase more quantity at the low unit price c_L offered. The sellers, by contrast, refuse to provide more coverage because they anticipate that doing so would also attract high type buyers who are more costly to serve. Rothschild and Stiglitz (1976) show that the RS separating allocation is the unique equilibrium candidate when the market structure is exclusive.

We extend this result and introduce the largest class of market structures for which the RS allocation is the unique equilibrium candidate. This class, rather than imposing that all sellers can offer exclusive trades, only requires that some seller can offer exclusive contracts. (Since

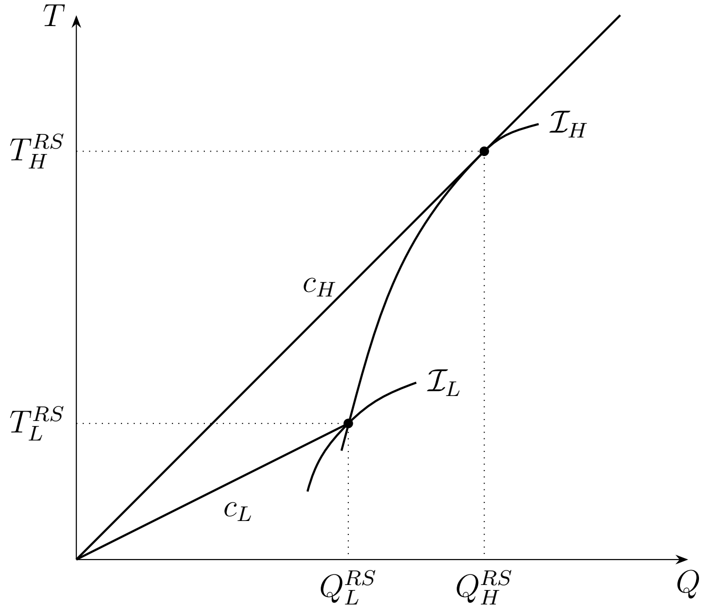


Figure 1: Exclusive Competition:RS Allocation

each seller is twice replaceable, the presence of one exclusive seller implies that there are at least three sellers that offer exclusive contracts.) If a market structure satisfies this property, we say that the market structure is partially exclusive.

Definition 5. *A market structure $\mathcal{M}$ is partially exclusive if there exists a seller that has the right to exclusively trade with buyers, i.e.,* $\max_{M \in \mathcal{M}: k \in M} |M| = 1$ *for some* $k \in \mathcal{K}$.

The right to offer an exclusive contract gives sellers the ability to destabilize any partially pooling equilibrium via cream-skimming deviations, i.e., deviations that uniquely target low type (and low cost) buyers. It follows that any equilibrium allocation must be fully separated. Efficiency arguments then imply that the separating equilibrium candidate is uniquely defined.

Claim 1. *Posit Assumptions 3 and 4 . The RS separating allocation is the unique equilibrium candidate allocation under a partially exclusive and competitive market structure.*

3.2 Never Exclusive Market Structures

We now consider the complement of partially exclusive market structures: no seller can offer an exclusive contract that prohibits further trade with other sellers. If so, we say that the market structure is never exclusive.

Definition 6. *A market structure $\mathcal{M}$ is never-exclusive if no seller has the right to exclusively trade with buyers, i.e.,* $\max_{M \in \mathcal{M}: k \in M} |M| \neq 1$ *for all* $k \in \mathcal{K}$.

Nonexclusive competition is an example of a never exclusive market structure. Another example of a never exclusive market structure —indeed the key example —is the following:

Example 2 ("1+1" market structure)**.** *$\mathcal{M}$ is a "1+1" market structure if the sellers $\mathcal{K} = \{1, ..., K\}$ can be partitioned into two disjoint subgroups $\mathcal{K}_1$ and $\mathcal{K}_2$ so that buyers can never trade with two sellers from the same subgroup at the same time:*

$$\mathcal{M} = \big\{\{j, k\} : j \in \mathcal{K}_1 \cup \{\emptyset\},\, k \in \mathcal{K}_2 \cup \{\emptyset\}\big\}.$$

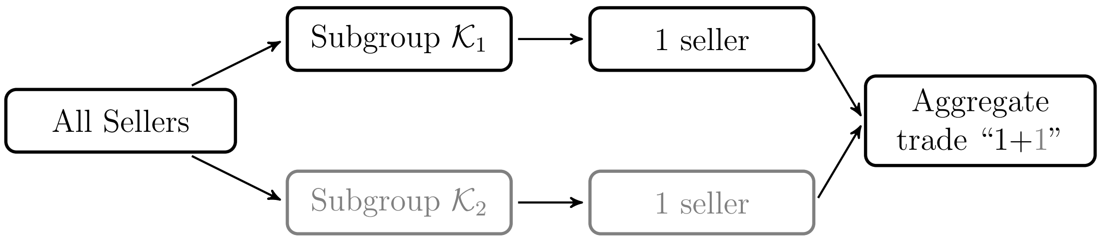


Figure 2: The "1+1" market structure. Buyers select at most one seller from each subgroup.

In contrast to partially exclusive market structures, never exclusive market structures never admit fully separating equilibria. This means that if a (non-trivial) equilibrium exists, some sellers must actively trade a pooling contract with both buyer types.

Claim 2. *Posit Assumptions 3 and 4. No allocation that is fully-separating can occur as an equilibrium allocation under a never exclusive and competitive market structure.*

This claim is due to a pivoting argument: Suppose that some seller exclusively trades with low type buyers. Since the separating contract has low unit cost, there always exists a profitable complementary contract so that both the separating and the complementary contract taken together match the high type's quantity allocation at a lower total price. In particular, following the introduction of the complementary contract, the initially separating contract is pooling and loss-making.

If not fully separating, what will be the equilibrium? One candidate is given by the Jaynes-Hellwig-Glosten allocation. This allocation consists of two competitively priced layers: a basic pooling layer and an additional layer purchased only by high type buyers. In a series of recent contributions, Attar et al. (2022) show that the JHG allocation is the unique equilibrium candidate when the market structure is nonexclusive. As the next section shows, the JHG allocation remains a viable equilibrium candidate when considering the larger class of never exclusive market structures. The more surprising insight is that many other equilibrium candidates become viable, too!

Definition 7 (Jaynes-Hellwig-Glosten (JHG))**.** *The JHG allocation is the partially pooling*

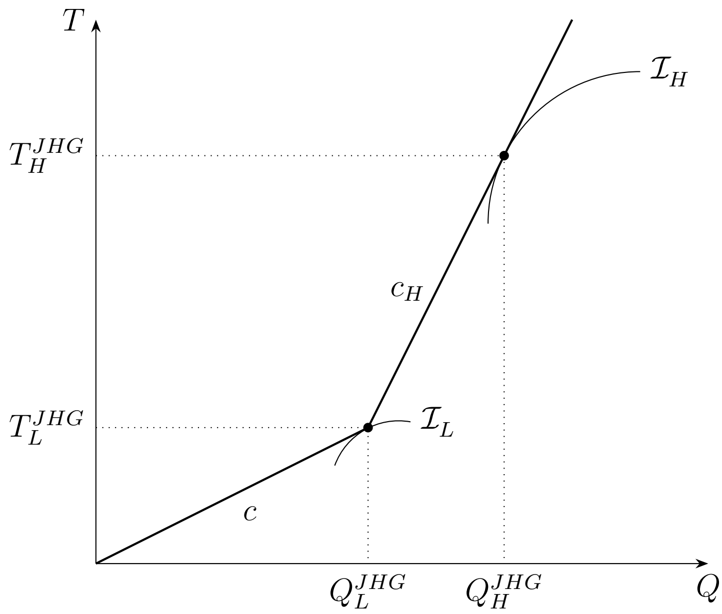


Figure 3: Nonexclusive Competition:JHG Allocation

allocation (Q_L^P, T_L^P) *and* (Q_H^P, T_H^P) *where*

$$\begin{aligned} Q_L^P &= \arg\max_{Q \geq 0} U_L(Q, cQ), \\ T_L^P &= cQ_L^P \\ Q_H^P - Q_L^P &= \arg\max_{Q \geq 0} U_H(Q_L^P + Q, T_L^P + c_H Q) \\ T_H^P - T_H^P &= c_H(Q_H^P - Q_L^P). \end{aligned}$$

4 Non-Fully-Separating Equilibrium Candidates

The preceding section shows: if an equilibrium exists, the ensuing allocation will be non-fully-separating if and only if the market structure is never exclusive. But what are the possible partially pooling allocations that can occur in equilibrium? A characterization extending beyond the example of the JHG allocation is still missing. We here consider arbitrary competitive (and in light of Claim 1 necessarily never exclusive) market structures. Our objective in this section is to present four necessary conditions that any non-fully-separating equilibrium allocation must satisfy.

An allocation consists of the sum of trades of the low type, (Q_L, T_L), and the sum of trades of the high type, (Q_H, T_H). Clearly, equilibrium allocations must be incentive compatible, that is the high (low) type prefers her allocation over the low (high) type's allocation.

Condition 1 (Incentive Compatibility)**.** *An allocation* (Q_L, T_L) *and* (Q_H, T_H) *is incentive compatible if* $U_L(Q_L, T_L) \geq U_L(Q_H, T_H)$ *and* $U_H(Q_H, T_H) \geq U_H(Q_L, T_L)$.

In general, thinking in terms of allocations is imprecise. Since high buyer types are more costly to serve than low types, we must distinguish between contracts purchased by low, high and both buyer types. Refer to these as the aggregate active trades $(q_L, t_L), (q_H, t_H)$ and (Q^P, T^P). Then $(Q_L, T_L) = (Q^P + q_L, T^P + t_L)$ and $(Q_H, T_H) = (Q^P + q_H, T^P + t_H)$.

4.1 Necessary Conditions

As is commonly the case in competitive equilibrium we identify the set of equilibrium candidates via possible seller and buyer one-shot deviations.

4.1.1 Undercutting and Pivoting

Much insight can be won by probing a candidate equilibrium with two kinds of seller deviations only: undercutting, i.e., sell the same quantity of an existing active contract for less, and pivoting, ask the buyer to combine an existing contract with a deviating contract without changing the aggregate quantity traded. Undercutting deviations are well-understood in the context of Bertrand competition. Pivoting deviations have been extensively explored in the context of non-exclusive competition but may in general be less well-known. What is important to observe is that neither deviation requires actual knowledge of the buyers' preferences. This makes necessary conditions derived from this class of deviations particularly robust.

Condition 2. *Active trades are single-seller separating and competitively priced if*

(i) *each component is competitively priced, i.e.,*

$$t^\ell \in q^\ell[c_L, c], \qquad t^h = q^h c_H \qquad \text{and} \qquad t^p = q^p c$$

for all $\ell \in M_L \backslash M_H$ *and* $h \in M_H \backslash M_L$ *and* $p \in M_L \cap M_H$*;*

(ii) *the low type separating component, if non-zero, is actively traded by a single seller only, i.e.,* $|M_L \backslash M_H| = 1$.

Figure 4 illustrates the content of Condition 2.

Proposition 1. *Posit Assumptions 3 and 4 . Active trades that occur in a non-fully-separating equilibrium under a competitive market structure are competitively priced and single-seller separating.*

The idea that all contracts must be competitively priced is reminiscent of Bertrand competition. What is concerning is that it does not apply to the low type contract. Serving the low type is potentially profitable. The reason is that by undercutting a contract exclusively targeting low type buyers one may also attract high type buyers, thereby greatly increasing the cost of said contract.

The key insight here is that sellers can be endogenously separated into two groups, $\mathcal{K}_1$ and $\mathcal{K}_2$, so that along the equilibrium path sellers in group one sell pooling contracts and

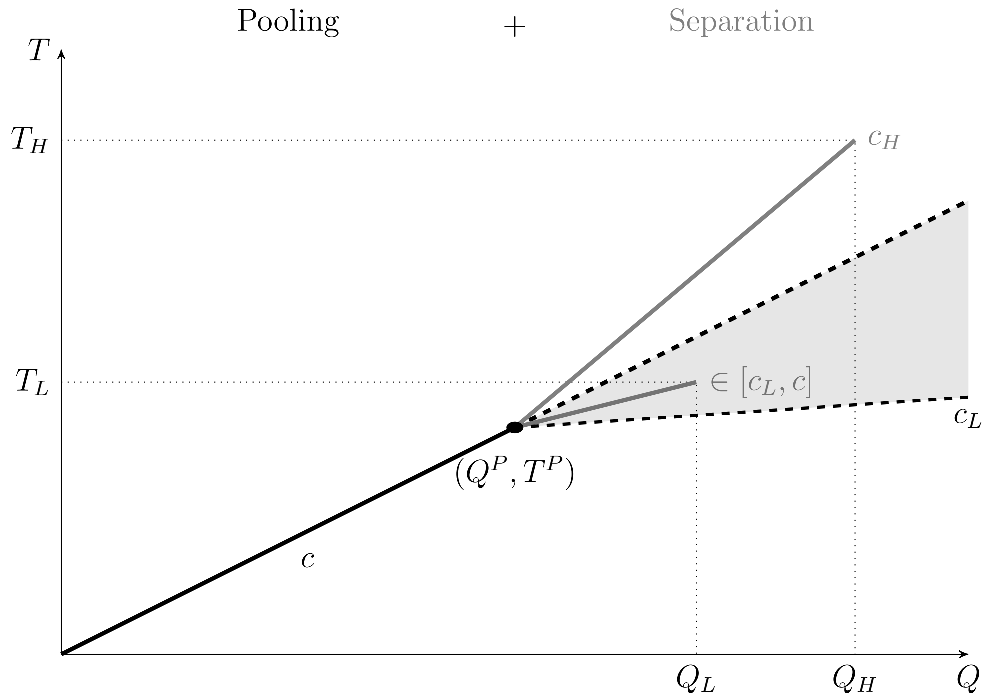


Figure 4: Condition 2 asserts that aggregate trades satisfy $T^P = Q^P c$ and $t_L \in q_L[c_L, c]$, $t_H = q_H c_H$.

sellers in group two sell separating contracts. Denote $(Q^P, T^P) = \sum_{k \in M_L \cap M_H}(q^k, t^k)$ the sum of active pooling contracts and $(q_L, t_L), (q_H, t_H)$ the (sum of) active separating contracts. The parentheses are warranted, for Proposition 1 asserts that (q_L, t_L) is in fact a single contract. Single-seller separation must occur because of possible pivoting deviations. To see why, suppose there were (at least) two sellers that exclusively trade with low type buyers, e.g., $q_L = q_L^1 + q_L^2$. Then possible undercutting deviations ensure that none of them makes a profit. Yet one of the two, say seller one, could pivot on seller two's contract traded in conjunction with the aggregate pooling trade (Q^P, T^P) and propose the quantity $Q_H - q_L^1 - Q^P$ at a unit price slightly exceeding c_H (and thereby be profitable). Due to (q_L^2, t_L^2) being priced competitively (at unit cost c_L), high type buyers must be strictly better off following this deviation.

Proposition 1 opens up the possibility of separation of low type buyers. Whether this can happen is a property of the prevailing market structure. (It does for some). A consequence of another pivoting deviation is however that the separating contract must be sufficiently small vis-à-vis the aggregate pooling quantity.

Condition 3 (large pooling)**.** *Active aggregate trades are largely pooling if*

$$T_H - T_L + T^P \leq c_H(Q_H - Q_L + Q^P).$$

Condition 3 prevents pivoting deviations that target the high type: complement type L's separating contract (q_L, t_L) with a pivoting contract promising quantity $(Q_H - q_L)$. Trading the pivoting contract is potentially attractive, because high type buyers benefit from the lower unit cost of the separating contract (q_L, t_L). Previously, they elected not to trade this contract because it forced them to forfeit some (if not all) of the contracts that made up the aggregate separating trade (q_H, t_H). Yet by complementing the low type's separating contract, high type buyers can now purchase the same aggregate quantity as before. Condition 3 ensures that rendering this deviation incentive compatible is too costly to the seller. Indeed, the pivoting contract comprises quantity $Q_H - q_L = Q_H - Q_L + Q^P$ which includes the pooling segment Q^P. The cost of this segment is higher than before, because the pivoting unlike the pooling contract exclusively targets high type buyers.

Lemma 1. *Posit Assumptions 3 and 4 . Any aggregate active trades that occur in a non-full-separating equilibrium under a competitive market structure are largely pooling.*

Due to competitive pricing asserted by Proposition 1, the lower bound on the pooling quantity can be expressed more explicitly. Competitive pricing, e.g., Condition 2, implies that $T^P = cQ^p$ and $T_H - T^P = c_H(Q_H - Q^P)$. Then Condition 3 re-writes as follows:

$$c_H q_L - t_L \leq (c_H - c)Q^P.$$

4.1.2 Efficiency-improving Deviations

Undercutting and pivoting deviations only require the seller to observe which trades are active. They do not require the seller to know the buyers' preferences. We now consider properties of any equilibrium that must hold due to possible efficiency-improving deviations.

Condition 4 (conditional efficiency)**.** *Active aggregate trades are conditionally efficient if* $Q^P \in \arg\max_{Q\geq 0} U_L(Q + q_L, Qc + t_H)$*, and* $q_H \in \arg\max_{q\geq 0} U_H(Q^P + q, T^P + qc_H)$ *whenever* $q_H > 0$*. If instead* $q_H = q_L = 0$*, it must hold that* $\max_{q\geq 0} U_H(1/2Q^P + q, 1/2Q^P c + qc_H) \leq U_H(Q^P, Q^P c)$*.*

Of course, assuming that indifference curves are continuously differentiable, this is equivalent to requiring that the slope of the indifference curve satisfies $\tau_L(Q^P + q_L, T^P + t_L) = c$ whenever $Q^P > 0$ and $\tau_H(Q^P + q_H, T^P + t_H) = c_H$ whenever $q_H > 0$.

Condition 4 is motivated as follows. Fix as a unit price the lower tail expectation, e.g., the expected unit cost conditional on all buyer types greater than oneself purchasing the same contract. If a buyer wanted to trade at this price to deviate from the candidate allocation, the desire to trade would not go away if the unit price were slightly less favourable to him. Since this price is profitable to the sellers, we should expect that any such gain from trade will be exploited in equilibrium.[14]

[14] The case where $Q_H = Q^P$, yet $\max_{q\geq 0} U_H(Q^P + q, Q^P c + qc_H) > U_H(Q^P, Q^P c)$ is pathological. It allows for situations in which the high type would like to purchase additional coverage at unit price c_H but cannot, because he has exhausted his purchasing options in both groups by choosing pooling contracts. Efficiency would suggest that only one group should offer (the entire) pooling contract $(Q^P, Q^P c)$, to liberate the high buyer type's option of purchasing a separating contract in the other group. This could only arise due to a coordination failure among sellers. We view such coordination failure as implausible: profitable deviations exist if two sellers from both groups could simultaneously deviate.

Lemma 2. *Posit Assumptions 3 and 4 . Any aggregate active trades that occur in a non-fully-separating equilibrium under a competitive market structure are conditionally efficient.*

Review of the Necessary Conditions

We now take stock and summarize in a single theorem the findings of Proposition 1, Lemma 1 and Lemma 2.

Theorem 1. *Posit Assumption 3 and 4 . Active trades that occur in a non-fully-separating equilibrium are single-seller separating. Moreover, aggregate active trades satisfy conditions 1-4, i.e., are incentive compatible, competitively priced, largely pooling and conditionally efficient.*

4.2 Existence of an Equilibrium Candidate

How can we ensure that aggregate active trades satisfying conditions 1-4 do exist? First, observe that the set of equilibrium candidates is non-empty: the JHG allocation is an admissible candidate. Do there exist other? And how large a space do they comprise?

If we fix the low type contract's unit price $c_x \in [c_L, c)$, the equilibrium candidate is uniquely determined by the pooling quantity Q^P. T^P follows from competitive pricing, and $(q_H, q_H c_H)$ and $(q_L, q_L c_x)$ are uniquely determined by conditional efficiency. Visually, one may walk along the separating c_H and c_x-unit cost lines emanating from the pooling allocation until one finds the separating trades that satisfy conditional efficiency.

Uniqueness of q_H and q_L, however, is conditional on the competitively priced pooling allocation $(Q^P, Q^P c)$. And as we now shall see, many pooling quantities Q^P are conceivable. For every low type unit price $c_x \in [c_L, c)$ there exists an open set of equilibrium candidates. This stands in contrast to the unique equilibrium candidates that have been identified for exclusive and nonexclusive market structures. To show this result, we must slightly strengthen our assumptions: we require that utility is strictly quasi-concave and twice continuously differentiable.

Assumption 5 (Twice continuously differentiable)**.** *The utility function of buyers $U_\theta(Q, T)$ is twice continuously and differentiable.*

Proposition 2 (Existence of an open set of equilibrium candidates)**.** *Posit Assumption 1, 3, 4 and 5. Then for all $c_x \in [c_L, c]$ there exist $\underline{Q}_x^P, \overline{Q}_x^P$: $\frac{1}{2}Q_L^{JHG} \leq \underline{Q}_x^P \leq Q_L^{JHG} \leq \overline{Q}_x^P$ where $\overline{Q}_x^P > \underline{Q}_x^P$ so that for all $Q^P \in [\underline{Q}_x^P, \overline{Q}_x^P]$ there exists a unique q_L, q_H so that the aggregate trades $(q_L, q_L c_x), (q_H, q_H c_H)$ and $(Q^P, Q^P c)$ satisfy conditions 1-4.*

Note that typically (but not always) $\overline{Q}_x^P = Q_L^{JHG}$.

5 Existence of Non-Fully-Separating Equilibria

The existence of a pure strategy Bayesian equilibrium is a thorny issue. As Rothschild-Stiglitz and Attar-Mariotti-Salanié show, an equilibrium need not exist for every market structure. In partially exclusive markets, pooling deviations can unravel separating allocations when the proportion of low type buyers is sufficiently large. In nonexclusive markets, Attar et al. (2022) demonstrate that the existence of a PBE can be guaranteed by imposing a further curvature assumption on the buyers' utility functions.

In this section, we establish a sweeping existence result. Maintaining the same curvature assumption as in Attar et al. (2022), we show that any set of aggregate trades that satisfy our necessary conditions can occur in an equilibrium under the "1+1" market structure. In particular, it follows from here that from a planner's point of view this simplest never exclusive market structure is sufficient to implement any equilibrium allocation that occurs for some never exclusive and competitive market structure.

5.1 Cream-Skimming Deviations and Latent Contracts

What can destabilize an equilibrium allocation are cream-skimming deviations: a *cream-skimming deviation* is a contract (q', t') that (the more profitable) low type buyers find attractive, whereas high type buyers prefer their initial allocation (Q_H, T_H). In partially exclusive markets the possibility of cream-skimming deviations alone guarantees that any equilibrium candidate must be fully separating. In never exclusive markets, by contrast, competing sellers have more tools at their disposal to "block" a cream-skimming deviation, i.e., render it attractive to (less profitable) high type buyers also. These tools are contracts that are not traded actively in equilibrium. The literature calls these *latent* contracts.

The purpose of the flatter curvature assumption (Assumption 6) is to identify a principal latent contract that can block large cream-skimming deviations, i.e., contracts (q', t') that (the more profitable) low type buyers find attractive on a stand-alone basis. The latent contract blocks such a cream-skimming deviation if by combining contract (q', t') with the latent contract (q^ℓ, t^ℓ) also high type buyers find it advantageous to purchase contract (q', t'). If so, the unit cost of the deviating contract is c, and so it can never be at the same time profitable and attract the low type.[15]

[15] The terminology principal latent contract is motivated by the fact that further latent contracts are required. Indeed, the presence of the principal latent contract invites the possibility of pivoting: complement the latent contract with a deviating contract proposed by an inactive seller. Such a deviation motivates further, derivative latent contracts that block the pivoting contract. And those derivative latent contract invite even further pivoting deviations. Since, as we shall see, latent contracts are pricey, i.e., $t^\ell > q^\ell c$, this motivates a problem of finite regress only so that finitely many latent contracts suffice to stabilize the equilibrium.

5.2 Flatter Curvature Assumption

We will first introduce the flatter curvature assumption as a property that resembles increasing differences. This representation highlights why the flatter curvature assumption is sufficient to block cream-skimming deviations. We will then discuss geometric implications for the slope of translated indifference curves.[16]

Assumption 6 (flatter-curvature). *$U_L(q,t)$ is of flatter-curvature than $U_H(q,t)$ if for all $(Q_L, T_L), (Q_H, T_H)$ satisfying $U_H(Q_H, T_H) \geq U_H(Q_L, T_L)$ there exists (q^ℓ, t^ℓ) so that*

$$U_L(q', t') > U_L(Q_L, T_L) \quad \Rightarrow \quad U_H(q^\ell + q', t^\ell + t') > U_H(q^\ell + Q_L, t^\ell + T_L) \quad \textit{for all } (q', t')$$

where $U_H(q^\ell + Q_L, t^\ell + T_L) = U_H(Q_H, T_H)$.

It is readily apparent from here that the latent contract (q^ℓ, t^ℓ) does indeed block a large cream-skimming deviation (q', t'). What is surprising initially is the additional requirement that $U_H(q^\ell + Q_L, t^\ell + T_L) = U_H(Q_H, T_H)$. Why not require directly that $U_H(q^\ell + q', t^\ell + t') > U_H(Q_H, T_H)$ for any possible cream-skimming deviation? The reason is the following: latent contracts must not be overly attractive so that buyers prefer a latent contract over their candidate allocation. We claim that if $U_H(Q_H, T_H) < U_H(q^\ell + Q_L, t^\ell + T_L)$ this would happen in equilibrium. The footnote provides more details.[17]

Then consider the geometric interpretation of the flatter curvature assumption. The key observation is that the latent contract is uniquely pinned down by a tangency condition: the high type's utility when trading $(Q_L + q^\ell, T_L + t^\ell)$ must lie on the on same indifference curve as (Q_H, T_H) and be tangent to average unit cost c. Since the indifference curve's slope is uniquely defined, so is the latent contract.

[16]The importance of latent contracts in stabilizing an equilibrium is not a new insight. Assumptions 5 and 6, however presented in a new guise, are equivalent to Assumption C in Attar et al. (2022). They also provide a characterization in the context of the classical preferences over final wealth when types encode the probability of suffering a loss considered by Rothschild-Stiglitz Rothschild and Stiglitz (1976) (also more recently studied by Chade and Schlee (2012)): $U_\theta(Q,T) = p_\theta u_\theta(w - (1-Q)\ell - T) + (1-p_\theta)u(w-T)$. More specifically, they show that if low type consumers are uniformly weakly more risk-averse, i.e., if $\min_w -\frac{v_L''(w)}{v_L'(w)} > \max_w -\frac{v_H''(w)}{v_H'(w)}$, then the flatter curvature assumption holds. CARA utility with a weakly greater coefficient of risk-aversion for low types is one common and admissible example.

[17]Consider a relaxed flatter curvature assumption where for all incentive compatible allocations there exists (q^ℓ, t^ℓ) so that

$$U_L(q', t') > U_L(Q_L, T_L) \quad \Rightarrow \quad U_H(q^\ell + q', t^\ell + t') > U_H(Q_H, T_H) \quad \text{for all } (q', t').$$

Clearly, $U_H(Q_H, T_H) \leq U_H(q^\ell + Q_L, t^\ell + T_L)$ is implied by this relaxed assumption due to the continuity of the utility function (consider $\epsilon > 0$ so that $U_H(Q_H, T_H) > U_H(q^\ell + Q_L, t^\ell + T_L - \epsilon)$). Then suppose that $U_H(Q_H, T_H) < U_H(q^\ell + Q_L, t^\ell + T_L)$. We claim that an equilibrium will fail to exist in such circumstances. Indeed, in our equilibrium construction the aggregate trade $(q^\ell + Q_L, t^\ell + T_L)$ is available to buyers: Why do we need this? Consider a cream-skimming deviation in group 1 that proposes a quantity different from Q^P at a lower unit price than c to exclusively target the low type. We require (see Lemma 5) the latent contract $(q^\ell + q_L, t^\ell + t_L)$ in group 2 to block this deviation, i.e., to ensure that the high type also purchases such a deviation. But then the high type can purchase (Q^P, T^P) in group 1 and the latent contract $(q^\ell + q_L, t^\ell + t_L)$ in group 2, and thereby the aggregate trade $(q^\ell + Q_L, t^\ell + T_L)$. So it must be that $U_H(Q_H, T_H) \geq U_H(q^\ell + Q_L, t^\ell + T_L)$.

Lemma 3. *Posit Assumptions 5 and Condition 4. Then for given* (Q_L, T_L) *and* (Q_H, T_H) *the principal latent contract as given by Assumption 5 is unique, and for this contract it holds that*

$$\max_q U_H(Q_L + q^\ell + q, T_L + t^\ell + q\,c) = U_H(Q_L + q^\ell, T_L + t^\ell).$$

In particular, it follows that the low type's upper contour set at (Q_L, T_L) translated to $(Q_L + q^\ell, T_L + t^\ell)$ is a subset of the high type's upper contour set at $(Q_L + q^\ell, T_L + t^\ell)$. Figure 5 illustrates.

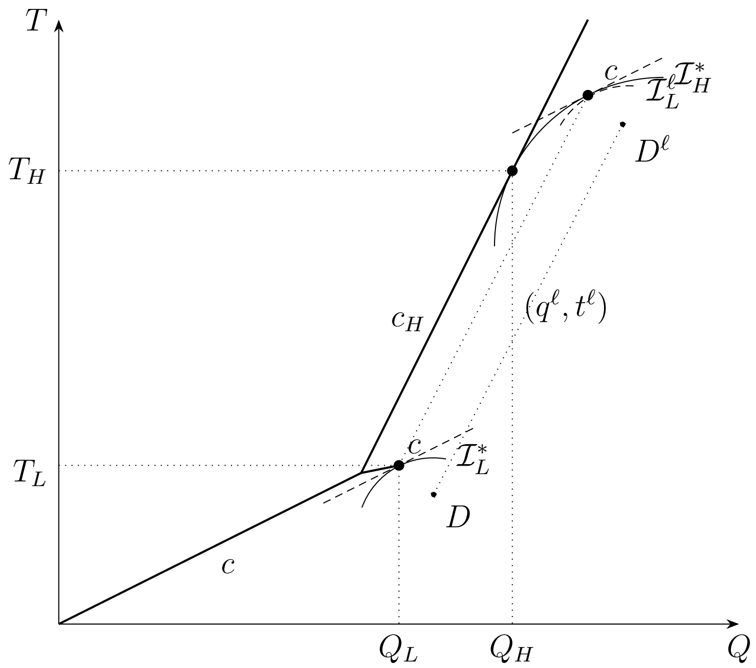


Figure 5: Cream-Skimming Deviation and Latent Contract

One final point is noteworthy: the latent contract has a unit price strictly greater than average serving cost for both types, i.e., $t^\ell > q^\ell c$ (and the bound appears to be rather loose). Indeed,

$$\begin{aligned} U_H(Q_H, Q_H\,c) > U_H(Q_H, T_H) &= \max_q U_H(Q_L + q^\ell + q, T_L + t^\ell + q\,c) \\ &\geq U_H(Q_H, T_L + t^\ell + (Q_H - Q_L - q^\ell)c) \geq U_H(Q_H, cQ_H + t^\ell - q^\ell c) \end{aligned}$$

where the second inequality owes to the fact that $T_L \leq cQ_L$ due to conditional efficiency (see Condition 2). Since utility is decreasing in transfers, it thereby follows that $Q_H c \leq Q_H c + t^\ell - q^\ell c$. As we shall see (see Lemma 4), this guarantees that no type can be tempted to purchase the latent contract on-path.

5.3 The Equilibrium Existence Theorem

Equipped with the flatter curvature assumption, we can state our main existence result.

Theorem 2. *Posit assumptions 1, 3, 4, 6. Any non-fully-separating aggregate active trades that satisfy conditions 1-4 can occur in an equilibrium under the "1+1" market structure.*

This is a positive result: a PBE may fail to exist for many market structures. Yet under the stated assumptions a PBE always exists under the "1+1" market structure. Moreover, if a PBE exists for some competitive market structure, we need not look further than at the more familiar "1+1" or exlusive "1 or 1" market structures, for the same allocation can also occur as an equilibrium here.

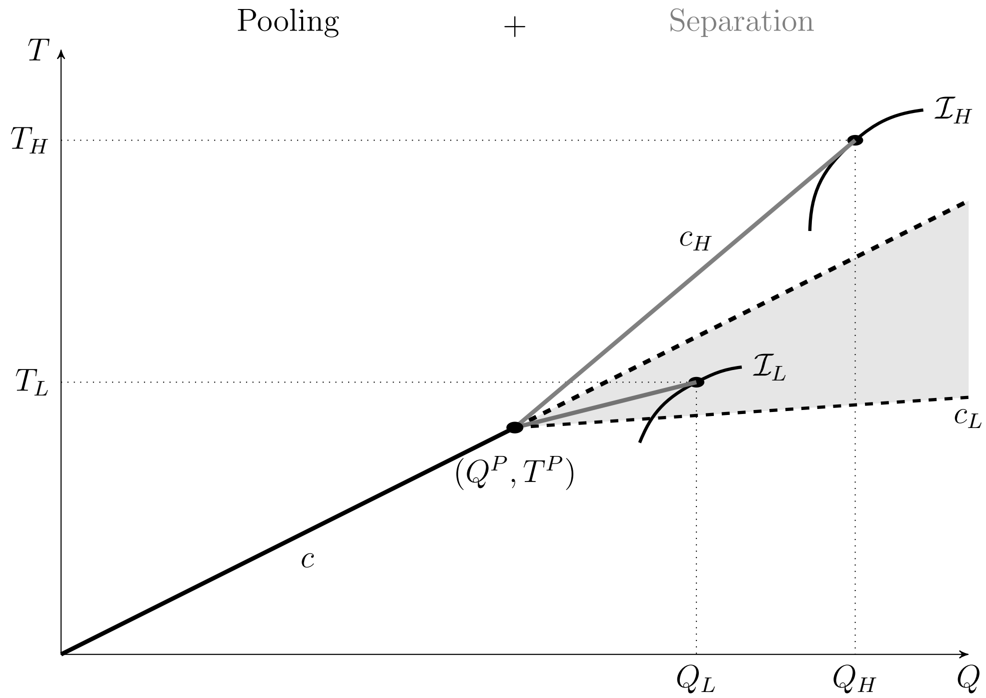


Figure 6: The equilibrium in "1+1" market structure

Corollary 1. *Posit assumptions 1,2,3,4. Any equilibrium allocation that occurs under a never exclusive competitive market structure is also an equilibrium allocation under the "1+1" market structure. Any equilibrium allocation that occurs under a partially exclusive competitive market structure is also an equilibrium allocation under the "1 or 1" market structure.*

The first part of the corollary is an immediate consequence of Theorem 2 because Assumptions 1,2 guarantee that any non-fully separating equilibrium trades must satisfy conditions 1-4 (see Proposition 1, Lemma 1 and Lemma 2). And the second part of the corollary is a reminder of Claim 1.

5.4 Proof of the Equilibrium Existence Theorem

We distinguish between two possible classes of equilibrium. In one set of equilibrium candidates, sellers in both groups actively trade pooling contracts. Due to its inefficiency—there exist additional incentive compatible separating contracts that are profitable—Footnote 14 and our subsequent Proposition 3 argue that this case is pathological. In another set of equilibrium candidates, actively traded contracts in group $\mathcal{K}_1$ are pooling, and actively traded contracts in group $\mathcal{K}_2$ are separating. We will consider this case first.

"Pooling + Separating" Equilibria

We here construct an equilibrium where pooling contracts are only traded with sellers in group $\mathcal{K}_1$. Any active aggregate trades (Q^P, T^P) and $(q_L, t_L), (q_H, t_H)$ that satisfy conditions 1,2,3 and 4 with the restriction that the separating quantity q_H satisfies $q_H \in \arg\max_{q \geq 0} U_H(Q^P + q, T^P + qc_H)$ are admissible. This includes the possibility that $q_H = 0$. Sellers in group $\mathcal{K}_1$ offer contract (Q^P, T^P) or latent contracts, sellers in group $\mathcal{K}_2$ offer contracts (q_L, t_L), (q_H, t_H) or latent contracts. Latent contracts are as follows: The principal latent contract (q^ℓ, t^ℓ) is offered by inactive sellers in both groups $\mathcal{K}_1$ and $\mathcal{K}_2$, with the principal contract as defined in Lemma 3. For reasons that shall become apparent shortly, we also introduce a larger class of derivative latent contracts. Denote $(q^{n\ell}, t^{n\ell}) = (nq^\ell, nt^\ell)$. And let $N \in \mathbb{N}$ be the smallest N such that $q^{N\ell} > Q_H$. The set of latent contracts offered in group $\mathcal{K}_1$ is

$$\left\{(q^{n\ell}, t^{n\ell}), (Q^P + q^{n\ell}, T^P + t^{n\ell}) : 1 \leq n \leq N\right\}.$$

The set of latent contracts offered in group $\mathcal{K}_2$ is

$$\left\{(q^{n\ell}, t^{n\ell}), (q_L + q^{n\ell}, t_L + t^{n\ell}), (q_H + q^{n\ell}, t_H + t^{n\ell}) : 1 \leq n \leq N\right\}.$$

Figure 7 summarizes the entire set of contracts offered.

	Subgroup $\mathcal{K}_1$	Subgroup $\mathcal{K}_2$
On-path sellers (actively trade)	(Q^P, T^P)	(q_L, t_L), (q_H, t_H)
Off-path sellers (latent contracts)	$(q^{n\ell}, t^{n\ell})$ $(Q^P + q^{n\ell}, T^P + t^{n\ell})$	$(q^{n\ell}, t^{n\ell})$ $(q_L + q^{n\ell}, t_L + t^{n\ell})$ $(q_H + q^{n\ell}, t_H + t^{n\ell})$

Figure 7: The on-path and off-path contracts in equilibrium

The proof of Theorem 2 under partial pooling is the consequence of three lemmata, each of which maintains identical assumptions and conditions as the main theorem. We first ensure that the latent contracts are not destabilizing. That is to say we verify that on-path no agent has an incentive to purchase latent contracts.

Lemma 4 (the principal latent contract is off-path)**.** *No buyer is better off when actively trading at least one latent contract.*

We then show that for every cream-skimming deviation there exists a latent contract that blocks it.

Lemma 5 (cream-skimming deviations are not profitable)**.** *No seller can offer a profitable deviating contract that only attracts low type buyers.*

We finally show that pivoting on latent contracts to attract high type buyers cannot be profitable. This concludes the proof of Theorem 2, for the set of relevant one-shot seller deviations, comprises (i) undercutting, (ii) pivoting on the on-path contracts to attract high types, (iii) efficiency-improving deviations, (iv) pivoting on the on-path contracts to attract low types, (v) pivoting on the off-path latent contracts to attract low types, (vi) pivoting on the off-path contracts to attract high types. (i) Undercutting was not profitable due to Condition 2, (ii) pivoting on the on-path contracts to attract high types was not profitable due to Condition 3, (iii) efficiency-improving deviations do not exist due to Condition 4, and so called cream-skimming deviations (iv) and (v) were not profitable due to the preceding lemma.

Lemma 6 (No profitable deviation only attracts type H)**.** *No seller can offer a profitable deviating contract that only attracts high type buyers.*

"Pooling + Pooling" Equilibria

We now construct an equilibrium where sellers from both groups actively trade pooling contracts. Any active aggregate trades (Q^P, T^P) and $(q_L, t_L), (q_H, t_H)$ that satisfy conditions 1,2,3 and 4 with the restriction that the separating quantities q_H, q_L are zero are admissible. In the aggregate, both types purchase the conditionally efficient quantity for the low type. For comparison, this is the low type's JHG allocation. To keep the analysis disjoint from the preceding, further assume that $0 \notin \arg\max_{q \geq 0} U_H(Q^P + q, T^P + qc_H)$, i.e., that the active aggregate trades are payoff-dominated by a more efficient equilibrium that occurs when there is "Pooling + Separation".

To construct the equilibrium allocation of "pooling+pooling", sellers in each group propose contracts $(\frac{1}{2}Q^P, \frac{1}{2}T^P) = \frac{1}{2}(Q_L^{JHG}, T_L^{JHG})$ or latent contracts. The principal latent contract (q^ℓ, t^ℓ) is still as given by Lemma 3. As before, denote $(q^{n\ell}, t^{n\ell}) = (nq^\ell, nt^\ell)$. And let $N \in \mathbb{N}$ be the smallest N such that $q^{N\ell} > Q_H$. The set of latent contracts offered in group $\mathcal{K}_1$ and $\mathcal{K}_2$ is

$$\big\{(q^{n\ell}, t^{n\ell}), (\frac{1}{2}Q^P + q^{n\ell}, \frac{1}{2}T^P + t^{n\ell}) : 1 \leq n \leq N\big\}.$$

The set of contracts that sustain the equilibrium is depicted in figure 8.

To see that this is an equilibrium in which both types actively trade pooling contracts in both groups, one must consider the same set of deviations as in the analysis under "Pooling + Separation". Inspection of the proofs of Lemma 4 and Lemma 5 reveals that both results hold for identical reasons as before. It remains to verify that no profitable deviation only attracts high type buyers, i.e., Lemma, 6. In addition to the deviations considered in the proof in the

	Subgroup $\mathcal{K}_1$	Subgroup $\mathcal{K}_2$
On-path sellers (actively trade)	$(\frac{1}{2}Q^P, \frac{1}{2}T^P)$	$(\frac{1}{2}Q^P, \frac{1}{2}T^P)$
Off-path sellers (latent contracts)	$(q^{n\ell}, t^{n\ell})$ $(\frac{1}{2}Q^P + q^{n\ell}, \frac{1}{2}T^P + t^{n\ell})$	$(q^{n\ell}, t^{n\ell})$ $(\frac{1}{2}Q^P + q^{n\ell}, \frac{1}{2}T^P + t^{n\ell})$

Figure 8: The equilibrium contracts of "pooling +pooling "

Appendix, buyers in either group can now pivot on $(\frac{1}{2}Q^P, \frac{1}{2}T^P)$ to attract type H buyers. Doing so is not feasible due to Condition 4: $\max_{q\geq 0} U_H(1/2Q^P+q, 1/2Q^Pc+qc_H) \leq U_H(Q^P, Q^Pc)$ ensures that no profitable deviation can attract high type buyers by pivoting on $(\frac{1}{2}Q^P, \frac{1}{2}T^P)$. In terms of pivoting on the latent contract, we use the same argument as in Lemma 6: $U_H(Q_H, T_H) = \max_q U_H(q_L + q^\ell + q, t_L + t^\ell + q\,c) \geq \max_q U_H(q^\ell + q, t^\ell + q\,c)$, so that any pivoting contract that attracts the high type must have a unit price lower than c.

6 Normative and Positive Implications

In the introduction, we had emphasized two viewpoints that one can adopt vis-a-vis the concept of a market structure. The positive view was to take the market structure as given and analyze its variation across markets. We shall adopt this first view when assessing the plausibility of our equilibrium predictions. To that end, we introduce an equilibrium refinement in competitive markets. The normative view was to consider the market design problem where the planner selects the market structure as part of an optimization problem with a social objective in mind. We shall adopt this second view when comparing the welfare properties of equilibria across different market structures.

6.1 The Positive View: Serendipitous-Aftermarket-Proofness

Theorems 1 and 2 provided us with a set of aggregative active trades that occur in equilibrium. This set can be divided into two subsets: a continuum of allocations ("Pooling + Separation") and two isolated points (Rothschild-Stiglitz and full pooling). Is there a sense in which one prediction is more plausible than the other? We now propose an equilibrium refinement that will argue that "Pooling + Separating" allocations are more plausible than either a fully pooling or a fully separating allocation. This refinement is motivated by the conspicuous absence of dynamics from our model thus far. We now introduce dynamics, albeit in a very crude way.

Fix an allocation (Q_L, T_L) and (Q_H, T_H) and define indirect utility functions

$$V_L(q,t) = U_L(Q_L + q, T_L + t) \qquad \text{and} \qquad V_H(q,t) = U_H(Q_H + q, T_H + t).$$

Our equilibrium refinement requires that conditional on the initial allocation any additional deviation contracts is loss-making. This implies that the market is in a form of rest. Future side-trading is impossible. What is crude about this definition is that ex-post trading opportunities are not anticipated.

Definition 8. *An allocation (Q_L, T_L) and (Q_H, T_H) is serendipitous-aftermarket-proof if in the aftermarket economy $((V_L, V_H), (c_L, c_H), (m_L, m_H))$ there do not exist profitable seller deviations.*

We view this definition complementary to Hendren (2013). Whereas Hendren adopts an ex-ante perspective and asks when adverse selection shuts down the market, serendipitous-aftermarket-proofness adopts an ex-post perspective: once trading has taken place, will the market remain inactive going forward?

Proposition 3. *Under Assumption 6, it holds that:*

1. *Full pooling is serendipitous-aftermarket-proof if and only if full pooling is conditionally efficient, i.e.,* $0 = \arg\max_{q\geq 0} U_H(Q^P + q, T^P + qc_H)$ *and* $0 = \arg\max_{q} U_L(Q^P + Q, T^P + qc)$.
2. *Any "Pooling + Separating" allocation satisfying conditions 1-4 is serendipitous-aftermarket-proof.*
3. *The RS allocation is serendipitous-aftermarket-proof if and only if low type buyers do not wish to trade additional, competitively priced pooling contracts, i.e.,* $0 = \arg\max_{q\geq 0} U_L(Q_L + q, T_L + qc)$.

With regard to Claim 1 note that whenever an equilibrium fails to exist under an exclusive market structure, the Rotschild-Stiglitz allocation is not serendipitous-aftermarket-proof. Conversely, the Rotschild-Stiglitz allocation may occur in equilibrium under the exclusive market structure, yet not be serendiptious-aftermarket-proof. All in all, we believe that this result weakens the plausibility of the Rothschild-Stiglitz allocation as an equilibrium prediction in competitive markets plagued by adverse selection.

Discussion

Two implications of Proposition 3 are likely relevant for empirical work:

Market Structures as Industry Agreements

First, Proposition 3 speaks directly to the origin of market structures. Existing theoretical analysis takes the market structure as exogenously given, e.g., because it is superimposed by a planner. While we will study the welfare implications of this view in the next subsection, a more natural (and certainly more optimistic) view may be to think of the market structure as an informal industry agreement between the sellers.

To formalize this view, take a dynamic perspective and imagine that every period a new cohort of previously uninsured buyers purchases insurance policies. In a steady state, there will exist a huge population of policy holders. Some policy holders will exogenously exit from this population, e.g., due to death or expiry of existing policies. And there will be a continuous inflow of entrants who are yet to make their first purchase. Then we can ask: What would happen in such a dynamic market if the sellers had agreed, e.g., through the appropriate labeling of their policies in red or blue, or core and complementary policies, that buyers should only be able to trade according to the '1+1" market structure? Would any seller be able to approach (or rather poach) the existing policy holders and profitably offer an additional contract? Such a trading offer would be an unexpected, serendipitous opportunity for the buyer. Based on the belief that no further offers are to follow, Proposition 3 asserts that no such offer can be made if the initial allocation is serendipitous-aftermarket-proof, e.g., satisfies conditions 1-4. This gives credence to the view that the '1+1" market structure can endogenously arise as an industry agreement.

What is positive about this perspective is that it allows us to discard the prediction of the Rothschild-Stiglitz allocation whenever this allocation is not in the interest of low type buyers and in particular not an equilibrium. (It is always Pareto-dominated by some 1+1 allocation for high type buyers.)

Testing for Adverse Selection

Second, it is a common empirical finding that in many markets where one may suspect adverse selection, the contracts actually traded are not always adversely selected (see Einav et al. (2010)). The applied literature has always interpreted this finding from the point of view of the Rothschild-Stiglitz allocation. If this is the only allocation that can occur in equilibrium, the absence of adversely selected contracts can only suggest that adverse selection is not prevalent in the market, e.g., because those individuals less costly to insure are also more risk-averse.

Our theory, based on the view that equilibrium predictions must be serendipitous-aftermarket-proof, suggests another interpretation. Proposition 3 Claim 3, implies that the Rothschild-Stiglitz allocation is not serendipitous-aftermarket-proof when preferences are largely similar yet marginal cost greatly differ across different risk profiles. And "Pooling + Separating" allocations (satisfying conditions 1-4) are likely to feature no separation at all. So the conditionally efficient pooling allocation can plausibly be the unique equilibrium allocation (moreover serendipitous-aftermarket-proof due to Claim 1) under the 1+1 market structure. Following this line of thought, the absence of adversely selected contracts in the data does not contradict the hypothesis that the market is plagued by severe adverse selection.

6.2 The Normative View: Welfare and the Planner's Problem

We now consider a benevolent planner who can select any competitive market structure. On the face of it, a large number of admissible market structures make this look like a formidable task. Corollary 1 shows that it is not: any equilibrium allocation that occurs under an arbitrary competitive market structure is also an equilibrium under either the "1+1" or the fully exclusive "1 or 1" market structure. It follows that even if the planner could select any competitive market structure, she could just as well consider the binary choice between the "1+1" or the "1 or 1" market structure. But which market structure should she select? And how does such a choice differ for different welfare objectives?

The most common objectives enunciated in policy debates revolve around decreasing transfers paid by those unfortunate enough to require a great amount of health care, and raising the amount of coverage purchased by low-risk buyers. Early on, it had been recognized that both objectives go hand in hand if insurance provision entails the pooling of low and high types. Since our analysis reveals that equilibrium can involve both pooling and separating trades, this coincidence does not always hold. By selecting an equilibrium that increases cross-subsidies to high types, one may concurrently decrease the amount of coverage purchased by low types.

Since these conflicts can and usually do arise, but are hard to pinpoint precisely, we organize them as themes. To understand how these are organized, recall the characterization of the "Pooling + Separating" allocations by Proposition 2: for fixed unit cost there exists an interval of possible pooling trades so that each pooling trade uniquely pins down the separating trades.

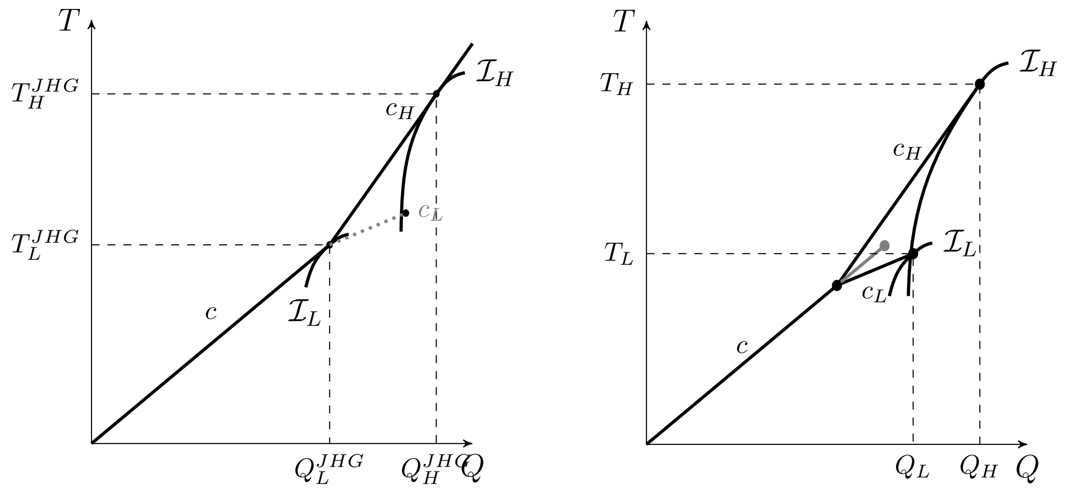
Theme 1. *High and low types (usually) rank "Pooling + Separating" allocations with more or less pooling in opposite directions.*

It is unambiguously true that high type buyers prefer more pooling over less: more pooling entails more cross-subsidies from low types, while conditional efficiency ensures that the last layer of coverage bought is always the most desirable amount of coverage at unit price c_H. As to low types, more pooling is (usually) offset by a decrease in the separating quantity.[18]

It is also instructive to compare the JHG allocation to the second-best allocation.[19] Then

[18] One can construct the odd example where the greatest admissible pooling contract exceeds the JHG pooling contract. If so, more pooling is actually Pareto improving. A necessary and sufficient condition for this to occur is that there exists $c_x > c_L$ so that the derivative in direction $(1, c_x)$ is zero, i.e., $D_Q\tau_L(Q_L^{JHG}, T_L^{JHG}) + D_T\tau_L(Q_L^{JHG}, T_L^{JHG})c_x = 0$. If so, there exists an indifference curve improving upon the JHG allocation whose slope is c (thereby satisfying conditional efficiency) for some (Q, T) above the separating zero-profit line emanating from the JHG allocation. In this situation, there is exists a " Pooling + Separating " allocation that Pareto dominates the JHG allocation.

[19] As in Attar et al. (2020), we adopt the definition of the second-best allocation due to Crocker and Snow (1985). It suggests that in any second-best allocation, if the MRS of each type is not equal to the marginal cost for serving this type, then the zero profit condition should be satisfied and the incentive compatibility condition should be binding.



a. JHG allocation: Inefficient Quantity to low types

b. "Pooling+Separating" Allocation: Less Inefficient Quantity to low types

Figure 9

note that the JHG allocation is Pareto dominated by one allocation which satisfies the necessary condition of second-best allocation because the incentive constraints will be slack as shown in the left panel of Figure 9. However, the "Pooling + Separating" allocation can alleviate this inefficiency by reducing the quantity of the pooling contract and increasing the quantity of the separating contract. This in turn reduces the slackness of incentive constraints. Sometimes there even exists a "Pooling + Separating" allocation that coincides with the second-best allocation[20], as shown on the right side of Figure 9.

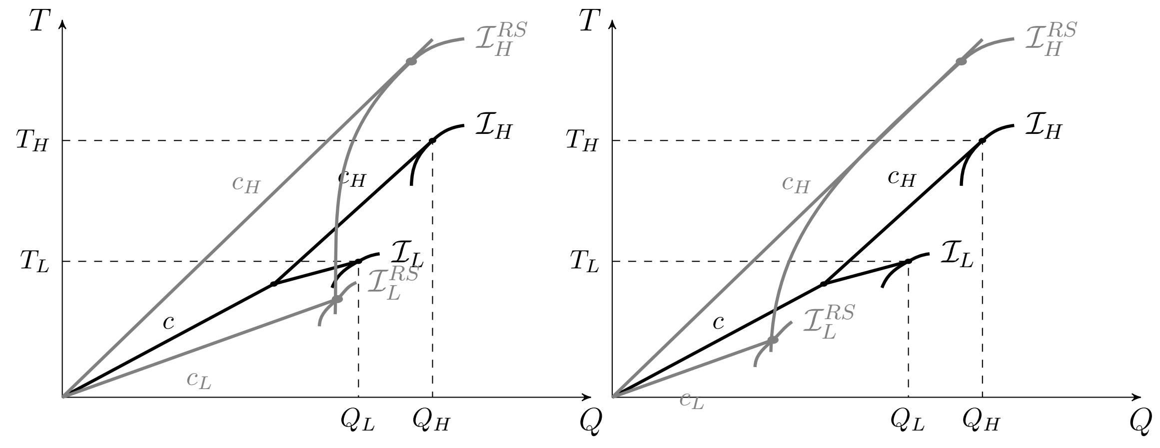
Theme 2. *High types prefer any "Pooling + Separating" allocation over the separating Rothschild-Stiglitz allocation, and so do low types if rationing under the latter is severe.*

For identical reasons as before, high-risk buyers are always better off under any "Pooling + Separating" equilibrium allocation $(Q_H^{\text{“1+1”}}, T_L^{\text{“1+1”}})$ than under the Rothschild-Stiglitz allocation that occurs when competition is exclusive: this holds due to cross-subsidies from low to high-risk buyers that render the high type's allocation cheaper without affecting conditional efficiency: $U_H(Q_H^{\text{“1+1”}}, T_L^{\text{“1+1”}}) \geq \max_Q U_H(Q^P + Q, Q^P\, c + Q\, c_H) > \max_Q U_H(Q, Q\, c_H) = U_H(Q_H^{RS}, T_H^{RS})$. As to low-risk buyers, the comparison is ambiguous. Low types prefer the Rothschild-Stiglitz allocation when incentive constraints are slack. If instead, adverse selection is severe so that the Rothschild-Stiglitz allocation entails a lot of low-risk type rationing, the low type is better off when purchasing a greater amount of coverage at a higher unit price in a "Pooling + Separating" equilibrium.

Matters are more straightforward when the buyers' utility is quasi-linear.

Example 3 (Quasi-Linear Utility)**.** *Suppose that $U_\theta(Q, T) = U_\theta(Q) - T$ for both types. This means that the amount of transfer paid does not affect preferences over coverage and is an appro-*

[20] This equilibrium allocation coincides with the Miyazaki-Wilson equilibrium if it exists.



(a) Rationing is not severe; low types prefer the Rothschild-Stiglitz allocation.

(b) A "Pooling + Separating" allocation Pareto dominates Rothschild-Stiglitz.

Figure 10

priate assumption when T is small, e.g., because rare risks are being insured. Then the aggregate quantities trades Q_L and Q_H are uniquely defined due to conditional efficiency. Transfers are not unique however,

7 Conclusion

This paper revisits the canonical model of competitive markets plagued by adverse selection. We propose the concept of a market structure and identify new, potentially welfare improving equilibrium allocations when competition is neither fully exclusive nor nonexclusive.

Appendix

.1 Proof of Claim 1

Proof. Fix a partially exclusive and competitive market structure and consider an equilibrium allocation (Q_L, T_L) and (Q_H, T_H).

Step 1. we show that $T_L = Q_L\, c_L$. First, $T_L \geq Q_L\, c_L$. Otherwise, there exists at least one seller who makes a negative profit. Then this seller can choose to be inactive instead. Second, $T_L \leq Q_L\, c_L$. Otherwise, suppose that $T_L > Q_L c_L + \epsilon$. Since the market structure is competitive and partially exclusive, there exist at least three exclusive sellers, i.e., sellers k for whom $\max_{M \in \mathcal{M}: k \in M} |M| = 1$. Then at least one of these three is inactive and makes zero profit on-path. A profitable deviation for an exclusive seller consists in offering a cream-skimming contract (q', t') so that $U_L(q', t') > U_L(Q_L, T_L)$ and $U_H(q', t') < U_H(Q_L, T_L)$. Following standard arguments (using single-crossing and continuity of the utility function), such a contract (q', t') always exists and can be chosen to be arbitrarily close to (Q_L, T_L). In effect, one can choose a (q', t') that is profitable conditional on trading with low type buyers only: $q' < Q_L + \frac{\epsilon}{2}$ and $t' > T_L - \frac{\epsilon}{2} > Q_L c_L + \frac{\epsilon}{2} > q' c_L + \frac{\epsilon}{2}$. And since $U_H(Q_L, T_L) \leq U_H(Q_H, T_H)$ due to incentive compatibility, the exclusive contract (q', t') only attracts low type buyers.

Step 2. we show that $T_H = Q_H\, c_H$. First, step 1 implies that in equilibrium no pooling contract can be actively traded. Since sellers serving high types cannot make a negative profit, it follows that $T_H \geq Q_H\, c_H$. And due to Bertrand's competition, the unit price for serving high type buyers must be smaller or equal to c_H. As a result, we have that $T_H = Q_H\, c_H$.

Step 3. we observe that the allocation must be efficient. This means that $Q_H = \arg\max_{Q \geq 0} U_H(Q, c_H Q)$, and $Q_L = \arg\max_{Q \geq 0} U_L(Q, c_L Q)$ subject to high type incentive compatibility, i.e., $U_H(Q_H, Q_H c_H) \geq U_H(Q_L, Q_L c_L)$. But this is the Rothschild-Stiglitz allocation so efficiency follows from their familiar arguments.

□

.2 Proof of Claim 2

Proof. Denote (Q_H, T_H) and (Q_L, T_L) the equilibrium allocation for high and low type buyers. Then notice that any fully-separating equilibrium allocation must satisfy $T_H = c_H Q_H$ and $T_L \leq c Q_L$. This follows from probing the equilibrium candidate with undercutting deviations familiar from Bertrand's competition. (Actually, it holds that $T_L = c_L Q_L$, but we do not require this here.) The bounds on profit imply that there exists a seller k who actively trades a contract (q', t') with low type buyers such that $t' \leq cq'$. And since the market structure is never exclusive, there exists another seller j_0 who can jointly trade with seller k. If seller j_0 is inactive, denote $j = j_0$. If this seller is active instead, denote j seller j_0's inactive replacement, i.e., $j \notin M_L$, yet $M_L \cup \{j\}\backslash_{j_0} \in \mathcal{M}$. (Seller j_0's replacement j exists because the market structure is competitive.) Then seller j can propose a contract $(Q_H - q', T_H - t' - \epsilon)$. This deviation attracts high types buyers, because trading jointly with sellers k and j following j's deviation gives high type buyers strictly greater utility than the initial allocation which was preferred among all trades excluding seller j: $U_H(Q_H - q' + q', T_H - t' - \epsilon + t') \geq U_H(Q_H, T_H) = \max_{M \in \mathcal{M}: j \notin M} U_H(\sum_{i \in M} q^i, \sum_{i \in M} t^i)$. And attracting high type buyers suffices to render this deviation profitable; profit is $T_H - t' - \epsilon - c_H(Q_H - q') = c_H q' - t' - \epsilon \geq c_H q' - cq' - \epsilon$ which is positive for ϵ sufficiently small.

□

.3 Proof of Proposition 1

Proof. If (Q_L, T_L) and (Q_H, T_H) is an equilibrium allocation, then there exists an equilibrium in which, first, sellers collectively offer the menu $\{(q^k, t^k)\}_{k \in \mathcal{K}}$ and, second, buyer types L and H trade with sellers M_L and M_H in $\mathcal{M}$ so that

$$(Q_L, T_L) = \sum_{k \in M_L} (q^k, t^k) \quad \text{and} \quad (Q_H, T_H) = \sum_{k \in M_H} (q^k, t^k).$$

Then define the pooling component (Q^P, T^P) and the separating components (q_L, t_L) and (q_H, t_H) as in Condition 2. Item (i) trivially holds.

Next observe that seller j's profit is proportional to

$$t^j - c' q^j \qquad \text{where } c' = \begin{cases} c_L & \text{if } j \in M_L \backslash M_H \\ c_H & \text{if } j \in M_H \backslash M_L \\ c & \text{if } j \in M_H \cap M_L. \end{cases}$$

Since seller k can offer the null trade $(0, 0)$ instead, he cannot make a loss in equilibrium. It follows that the unit price of serving the low type θ is at least c_L, the unit price of serving the high type θ is at least c_H and the unit price of serving both types is at least c.

Then we prove the item (ii) by drawing on undercutting deviations as in Bertrand's competition. For there to be viable competitors that can undercut we will require the item (iii). This is proven by drawing on a pivoting deviation instead.

$$\begin{cases} j \in M_L \cap M_H & \Rightarrow \quad \pi^j = 0 \text{ (step 1)} \\ j \in M_H \setminus M_L & \Rightarrow \quad \begin{cases} \pi^j = 0 & \text{if } |M_H \setminus M_L| > 1 \text{ (step 2)} \\ \pi^j = 0 & \text{if } |M_H \setminus M_L| = 1 \text{ (step 5)} \end{cases} \\ j \in M_L \setminus M_H & \Rightarrow \quad \begin{cases} t^j \leq q^j\, c & \text{if } |M_L \setminus M_H| > 1 \text{ (step 3)} \\ t^j \leq q^j\, c & \text{if } |M_L \setminus M_H| = 1 \text{ (step 6)} \end{cases} \\ |M_L \setminus M_H| = 1 & \text{(step 4).} \end{cases}$$

Figure 11: A roadmap of the proof of Proposition 1

Step 1: We show by contradiction that all sellers in $M_L \cap M_H$ make zero profit. Or, suppose that $j \in M_L \cap M_H$ made a positive profit. Since the market structure is competitive, there exists $k \in \mathcal{K} \setminus \{j\}$ so that $M_L \cup \{k\} \setminus \{j\} \in \mathcal{M}$. In particular, seller k does not actively trade in equilibrium and makes zero profit. If so, seller k could propose the contract $(q^j, t^j - \epsilon)$ so that $t^j - \epsilon > q^j c$. In effect, the deviation is profitable conditional on trading with both buyer types.

Then denote $((\hat{q}^i, \hat{t}^i))_{i \in \mathcal{K}}$ the new menu of contracts following seller k's deviation. Clearly, due to monotonicity of transfers, for both $\theta \in \{L, H\}$

$$\begin{aligned}\max_{\substack{M \in \mathcal{M} \\ k \in M}} U_\theta(\sum_{i \in M} \hat{q}^i, \sum_{i \in M} \hat{t}^i) &\geq U_\theta(\sum_{i \in M_\theta \cup \{k\} \setminus \{j\}} \hat{q}^i, \sum_{i \in M_\theta \cup \{k\} \setminus \{j\}} \hat{t}^i) = U_\theta(\sum_{i \in M_\theta} q^i, -\epsilon + \sum_{i \in M_\theta} t^i) > U_\theta(\sum_{i \in M_\theta} q^i, \sum_{i \in M_\theta} t^i) \\ &= \max_{M \in \mathcal{M}} U_\theta(\sum_{i \in M} q^i, \sum_{i \in M} t^i) \geq \max_{\substack{M \in \mathcal{M} \\ k \notin M}} U_\theta(\sum_{i \in M} q^i, \sum_{i \in M} t^i) = \max_{\substack{M \in \mathcal{M} \\ k \notin M}} U_\theta(\sum_{i \in M} \hat{q}^i, \sum_{i \in M} \hat{t}^i).\end{aligned}$$

So, following the deviation both buyer types trade with seller k, thereby rendering seller k's deviating contract $(q^j, t^j - \epsilon)$ strictly profitable.

Step 2: We show by contradiction that if there is more than one seller only serving high type buyers, then all those sellers make zero profit. Or, suppose that some seller $j \in M_H \setminus M_L$ made a positive profit and $|M_H \setminus M_L| > 1$. Then there exists another seller $k \in M_H \setminus (M_L \cup \{j\})$ that could deviate and offer $(q^j + q^k, t^j - \epsilon + t^k)$ instead where $t^j - \epsilon > q^j c_H$. Since (q^k, t^k) is weakly profitable, the deviation is strictly profitable conditional on continued trade with high type buyers (and even more profitable if it also attracts low type buyers).

To see that high type buyers will trade with seller k following the deviation, denote $((\hat{q}^i, \hat{t}^i))_{i \in \mathcal{K}}$ the new menu of contracts and note that

$$\begin{aligned}\max_{\substack{M \in \mathcal{M} \\ k \in M}} U_H(\sum_{i \in M} \hat{q}^i, \sum_{i \in M} \hat{t}^i) &\geq U_H(\sum_{i \in M_H \setminus \{j\}} \hat{q}^i, \sum_{i \in M_H \setminus \{j\}} \hat{t}^i) = U_H(\sum_{i \in M_H} q^i, -\epsilon + \sum_{i \in M_H} t^i) > U_H(\sum_{i \in M_H} q^i, \sum_{i \in M_H} t^i) \\ &= \max_{M \in \mathcal{M}} U_H(\sum_{i \in M} q^i, \sum_{i \in M} t^i) \geq \max_{\substack{M \in \mathcal{M} \\ k \notin M}} U_H(\sum_{i \in M} q^i, \sum_{i \in M} t^i) = \max_{\substack{M \in \mathcal{M} \\ k \notin M}} U_H(\sum_{i \in M} \hat{q}^i, \sum_{i \in M} \hat{t}^i).\end{aligned}$$

Step 3: We claim that if there is more than one seller only serving low type buyers, then unit profit must be bounded by the cost of serving both types, i.e., $t^j \leq q^j c$ for all $j \in M_L \setminus M_H$. This employs essentially identical arguments as in Step 2 where the undercutting deviation comes from a seller $k \in M_L \setminus M_H$.

Step 4: We show by contradiction that there is at most one seller only serving low type buyers. This proves item (iii) of Proposition 1. Or, suppose that there exist distinct $j_1, j_2 \in M_L \setminus M_H$. Since the market structure is competitive, there exist distinct $k_1, k_2 \in \mathcal{M} \setminus M_L$ so that $M_L \cup \{k_1\} \setminus \{j_1\}$ and $M_L \cup \{k_2\} \setminus \{j_1\}$ in $\mathcal{M}$. Then either at least one of the two sellers k_1, k_2 does not belong to M_H, i.e., one of the two is an inactive trader that makes zero profit. Or they both belong to M_H (but not M_L) in which case they must also make zero profit due to step 2.[21]

Then consider the seller k_1 pivoting deviation: propose the contract $(q_H - q_L^2, t_H - t_L^2 - \epsilon)$ with $t_H - t_L^2 - c_H(q_H - q_L^2) > \epsilon$. By jointly trading with sellers in $(M_L \cap M_H) \cup \{j_2\}$ and the pivoting seller k_1, buyers trade the aggregate quantity $Q_H = Q^P + q_H$ at the lesser price $T_H - \epsilon = t^P + t_H - \epsilon$. Moreover, since $t_L^2 \leq q_L^2\, c$, this deviation is profitable conditional on trading with high type buyers (and even more profitable if it also attracts low type buyers).

We verify that the deviation attracts high type buyers. Observe that $(M_L \cap M_H) \cup \{j_2\} \cup \{k_1\} \subset M_L \cup \{k_1\} \setminus \{j_1\}$. Since the market structure is competitive, $M_L \cup \{k_1\} \setminus \{j_1\} \in \mathcal{M}$ implies that buyers can drop trades and trade with sellers $(M_L \cap M_H) \cup \{j_2\} \cup \{k_1\} \in \mathcal{M}$. Then denote $((\hat{q}^i, \hat{t}^i))_{i \in \mathcal{K}}$ the new menu of contracts following seller k's deviation.

$$\max_{\substack{M \in \mathcal{M} \\ k_1 \in M}} U_H(\sum_{i \in M} \hat{q}^i, \sum_{i \in M} \hat{t}^i) \geq U_H(\sum_{i \in (M_L \cap M_H) \cup \{j_2\} \cup \{k_1\}} \hat{q}^i, \sum_{i \in (M_L \cap M_H) \cup \{j_2\} \cup \{k_1\}} \hat{t}^i) = U_H(\sum_{i \in M_H} q^i, -\epsilon + \sum_{i \in M_H} t^i)$$
$$> U_H(\sum_{i \in M_H} q^i, \sum_{i \in M_H} t^i) = \max_{M \in \mathcal{M}} U_H(\sum_{i \in M} q^i, \sum_{i \in M} t^i) \geq \max_{\substack{M \in \mathcal{M} \\ k_1 \notin M}} U_H(\sum_{i \in M} q^i, \sum_{i \in M} t^i) = \max_{\substack{M \in \mathcal{M} \\ k_1 \notin M}} U_H(\sum_{i \in M} \hat{q}^i, \sum_{i \in M} \hat{t}^i).$$

And so high type buyers are strictly better off by trading with seller k_1 following the deviation.

Step 5: We show by contradiction that if there is exactly one seller only serving high type buyers, then this seller makes zero profit. Or, suppose that $|M_H \setminus M_L| = 1$ and seller $j \in M_H \setminus M_L$ made a positive profit. Since sellers are twice replaceable, there exist distinct sellers $k_1, k_2 \in \mathcal{K} \setminus M_H$ so that $M_H \cup \{k_1\} \setminus \{j\} \in \mathcal{M}$ and $M_H \cup \{k_2\} \setminus \{j\} \in \mathcal{M}$. Since $|M_L \setminus M_H| = 1$ due to step 4, at least one of the two, say k_1, must be an inactive seller in $\mathcal{K} \setminus (M_L \cup M_H)$. Then the deviating contract $(q^j, t^j - \epsilon)$ where $t^j - \epsilon > q^j\, c_H$ is strictly profitable for the inactive seller k_1 conditional on trading with high type buyers (and even more profitable if it also attracts low type buyers). And following analogous arguments as before, trading with sellers $M_H \cup \{k_1\} \setminus \{j\}$ is feasible because the market structure is competitive and gives strictly greater utility to high type buyers than not trading with seller k_1: $\max_{\substack{M \in \mathcal{M} \\ k_1 \in M}} U_H(\sum_{i \in M} \hat{q}^i, \sum_{i \in M} \hat{t}^i) > \max_{\substack{M \in \mathcal{M} \\ k_1 \in M}} U_H(\sum_{i \in M} \hat{q}^i, \sum_{i \notin M} \hat{t}^i)$.

Step 6: We show by contradiction that if there is exactly one seller only serving low type buyers, then this seller's unit price must be bounded by the marginal pooling cost c. Or, suppose that $|M_L \setminus M_H| = 1$ and seller $j \in M_L \setminus M_H$'s contract satisfies $t^j > q^j\, c$. Then, following symmetric arguments as in step 5, an inactive seller can propose an undercutting contract and attract low type buyers:Since sellers are twice replaceable, there exist distinct sellers $k_1, k_2 \in \mathcal{K} \setminus M_L$ so that $M_L \cup \{k_1\} \setminus \{j\} \in \mathcal{M}$ and $M_L \cup \{k_2\} \setminus \{j\} \in \mathcal{M}$. Since $|M_H \setminus M_L| = 1$ due to step 4, at least one of the two, say k_1, must be an inactive seller in $\mathcal{K} \setminus (M_L \cup M_H)$. Then the deviating contract $(q^j, t^j - \epsilon)$ where $t^j - \epsilon > q^j\, c$ is strictly profitable

[21] Step 4 and 5 are the only instance where we use the property from definition 2 that sellers are twice replaceable. If sellers were only once replaceable, we could not rule out that seller $\ell \in M_L \setminus M_H$'s replacement is $h \in M_H \setminus M_L$ and seller H's replacement is ℓ and they both make a profit by serving the low and the high type respectively.

for the inactive seller k_1 conditional on trading with buyers of both types (and even more profitable if it only attracts low type buyers). Finally, trading with sellers $M_L \cup \{k_1\} \setminus \{j\}$ is feasible because the market structure is competitive. And, following analogous arguments as before low type buyers are better off when trading with seller k_1 following the undercutting deviation: $\max_{\substack{M\in\mathcal{M}\\k_1\in M}} U_L(\sum_{i\in M}\hat{q}^i, \sum_{i\in M}\hat{t}^i) > \max_{\substack{M\in\mathcal{M}\\k_1\in M}} U_L(\sum_{i\in M}\hat{q}^i, \sum_{i\notin M}\hat{t}^i)$. □

.4 Proof of Lemma 2

Proof. (1) We first show that $Q^P \in \arg\max_q U_L(Q+q_L, Qc+t_L)$ whenever $Q^P > 0$. Otherwise, a seller k actively trading a pooling contract has a profitable deviation.

Suppose by contradiction that the pooling quantity $Q^P > 0$ satisfies $U_L(q_L+q', t_L+q'c) > U_L(Q^P+q_L, Q^Pc+t_L)$ for some $q' > 0$. Then there exists $\epsilon > 0$ so that $U_L(q_L+q', t_L+q'c+\epsilon) > U_L(Q_L, T_L)$. And seller k trading a (competitively priced) pooling contract can profitably deviate by proposing the contract $(q', q'c+\epsilon)$ instead of (q^k, q^kc). This deviation is profitable conditional on trading with both buyer types and even more profitable if it only attracts low buyer types. And low type buyers are strictly better off following seller k's deviation and will want to trade the deviating contract (possibly by dropping all other pooling contracts). To see this, denote $((\hat{q}^i, \hat{t}^i))_{i\in\mathcal{K}}$ the new menu of contracts following seller k's deviation. We find that

$$\begin{aligned}\max_{\substack{M\in\mathcal{M}\\k\in M}} U_L(\sum_{i\in M}\hat{q}^i, \sum_{i\in M}\hat{t}^i) &\geq U_L(q'+q_L, q'c+\epsilon+t_L)\\ &> U_L(Q_L, T_L) = \max_{M\in\mathcal{M}} U_L(\sum_{i\in M}q^i, \sum_{i\in M}t^i) \geq \max_{\substack{M\in\mathcal{M}\\k\notin M}} U_L(\sum_{i\in M}q^i, \sum_{i\in M}t^i) = \max_{\substack{M\in\mathcal{M}\\k\notin M}} U_H(\sum_{i\in M}\hat{q}^i, \sum_{i\in M}\hat{t}^i).\end{aligned}$$

(2) Second, we show using analogous arguments that $q_H \in \arg\max_q U_H(Q^P+q, T^P+q\,c_H)$ whenever $q_H > 0$. Otherwise, a seller h actively trading a separating contract with the high type has a profitable deviation.

Suppose by contradiction that there exists $q' > 0$ such that $U_H(Q^P+q', Q^Pc+q'c_H) > U_H(Q^P+q_H, T^P+t_H)$. Then there exists $\epsilon > 0$ so that $U_H(Q^P+q', Q^Pc+q'c_H+\epsilon) > U_H(Q_H, T_H)$. Then a seller h actively trading a (competitively priced) separating contract (q^h, q^hc_H) can profitably deviate by proposing the contract $(q', q'\,c_H+\epsilon)$ instead of (q^h, q^hc_H). This deviation is profitable conditional on trading only with high type buyers and even more profitable if it also attracts low buyer types. And high type buyers are strictly better off the following seller k's deviation and will want to trade the deviating contract (possibly by dropping all other separating contracts). To see this, denote $((\hat{q}^i, \hat{t}^i))_{i\in\mathcal{K}}$ the new menu of contracts following seller k's deviation. We find that

$$\max_{\substack{M\in\mathcal{M}\\k\in M}} U_H(\sum_{i\in M}\hat{q}^i,\sum_{i\in M}\hat{t}^i) \geq U_H(Q^P+q',T^P+q'\,c_H+\epsilon) > U_H(Q_H,T_H)$$
$$= \max_{M\in\mathcal{M}} U_H(\sum_{i\in M}q^i,\sum_{i\in M}t^i) \geq \max_{\substack{M\in\mathcal{M}\\k\notin M}} U_H(\sum_{i\in M}q^i,\sum_{i\in M}t^i) = \max_{\substack{M\in\mathcal{M}\\k\notin M}} U_H(\sum_{i\in M}\hat{q}^i,\sum_{i\in M}\hat{t}^i).$$

(3) Finally, to understand the qualifying condition when $q_H = q_L = 0$ (or equivalently $Q_H = Q^P$), observe that $\max_{q\geq 0} U_H(\frac{1}{2}Q^P+q,\frac{1}{2}Q^Pc+qc_H)$ the utility from purchasing only half the pooling allocation and the most desired separating quantity at unit price c_H is equal to

$$\min_{\alpha\in[0,1]}\max\{\max_{q\geq 0} U_H(\alpha Q^P+q,\alpha Q^Pc+qc_H);\max_{q\geq 0} U_H((1-\alpha)Q^P+q,(1-\alpha)Q^Pc+qc_H)\}$$

due to quasi-concavity. Here α corresponds to the share of the pooling quantity purchased in group one, and $1-\alpha$ to the share purchased in group two. Or, whenever the high type purchases pooling contracts in both groups, the possibility of only purchasing one pooling contract and complementing it with the desired quantity at a unit cost c_H is least attractive when the pooling quantity offered in both groups is identical, i.e., $\alpha = \frac{1}{2}$.

□

.5 Proof of Lemma 3

Proof. Define $\mathcal{T}_H(q),\mathcal{T}_L(q)$ so that $U_H(q,\mathcal{T}_H(q)) = U_H(Q_L+q^\ell,T_L+t^\ell)$ for all q in an open ball around Q_L+q^ℓ and $U_L(q,\mathcal{T}_L(q)) = U_L(Q_L,T_L)$ for all q in an open ball around Q_L. Since $U_H(q,t)$ is increasing in q, decreasing in t and continuously differentiable, $\mathcal{T}_H(q),\mathcal{T}_L(q)$ are well-defined and continuously differentiable due to the implicit function theorem, moreover increasing.

And by construction $U_j(q,\mathcal{T}_j(q))$ is constant in q so that differentiation yields $D_1U_j(q,\mathcal{T}_j(q))+D_2U_j(q,\mathcal{T}_j(q))\mathcal{T}_j'(q) = 0$. Applying the first-order condition to low type conditional efficiency 4 then implies that $D_1U_L(Q_L,T_L)+D_2U_L(Q_L,T_L)c = 0$. Therefore $\mathcal{T}_j'(q) = c$.

Next, observe that for all ϵ in an open ball around zero it must hold that $U_L(Q_L+\epsilon,\mathcal{T}_L(Q_L+\epsilon)-\epsilon^2) > U_L(Q_L,T_L)$. The flatter-curvature assumption 6 thus implies that $U_H(Q_L+q^\ell+\epsilon,\mathcal{T}_L(Q_L+\epsilon)+t^\ell-\epsilon^2) > U_H(Q_L+q^{ell},T_L+t^\ell) = U_H(Q_L+q^\ell+\epsilon,\mathcal{T}_H(Q_L+q^\ell+\epsilon))$. In effect, the function $\epsilon\mapsto U_H(Q_L+q^\ell+\epsilon,\mathcal{T}_L(Q_L+\epsilon)+t^\ell-\epsilon^2) - U_H(Q_L+q^\ell+\epsilon,\mathcal{T}_H(Q_L+q^\ell+\epsilon))$ attains a local minimum at $\epsilon = 0$. Whence differentiating with respect to ϵ and noting that $\mathcal{T}_L(Q_L) = T_L$ and $\mathcal{T}_H(Q_L+q^\ell) = T_L+t^\ell$ establishes that

$$\begin{aligned}0 =& D_1U_H(Q_L+q^\ell,\mathcal{T}_L(Q_L)+t^\ell) - D_1U_H(Q_L+q^\ell,\mathcal{T}_H(Q_L+q^\ell))\\ &+ D_2U_H(Q_L+q^\ell,\mathcal{T}_L(Q_L)+t^\ell)\mathcal{T}_L'(Q_L) - D_2U_H(Q_L+q^\ell,\mathcal{T}_H(Q_L+q^\ell))\mathcal{T}_H'(Q_L+q^\ell),\end{aligned}$$

that is to say that $\mathcal{T}_H'(Q_L+q^\ell) = \mathcal{T}_L'(Q_L) = c$.

Finally, if (q', t') were such that $U_H(Q_L + q^\ell + q', T_L + t^\ell + t') > U_H(Q_L + q^\ell, T_L + t^\ell) = U_H(Q_L + q^\ell + q', \mathcal{T}_H(Q_L + q^\ell + q'))$, it must be that $\mathcal{T}_H(Q_L + q^\ell + q') > T_L + t^\ell + t' = \mathcal{T}_H(Q_L + q^\ell) + t'$. Taking differences gives

$$\frac{\mathcal{T}_H(Q_L + q^\ell + q') - \mathcal{T}_H(Q_L + q^\ell)}{q'} > \frac{t'}{q'}.$$

It remains observed that the left-hand side is less than c. This holds because $U_H(q, t)$ is quasi-concave so that $\mathcal{T}_H(q)$ is concave. In conclusion, $t' < q'\,c$, so that $U_H(Q_L + q^\ell + q', T_L + t^\ell + q'\,c) \leq U_H(Q_L + q^\ell, T_L + t^\ell)$ as claimed. □

.6 Proof of Proposition 2

Proof. Step 1: The JHG allocation $(Q_L^{JHG}, T_L^{JHG}) = (Q^P, T^P) = (Q_L, T_L), (Q_H^{JHG}, T_H^{JHG}) = (Q^P + q_H, T^P + t_H) = (Q_H, T_L)$ (see Definition 7) always satisfies conditions 1-4. By construction, it is single-seller separating, competitively priced, and conditionally efficient. Incentive compatibility follows from conditional efficiency as shown in Section 2. It remains to verify large pooling, i.e., $T_H - T_L + T^P \leq c_H(Q_H - Q_L + Q^P)$. This is always satisfied, for by construction $T_L = T^P$ and $Q_L = Q^P$, and $T_H = T^P + t_H = Q^P c + q_H c_H$.

Step 2: We claim that there exists a regular curve $\gamma(t)$ in an open ball around the JHG allocation (Q_L^{JHG}, T_L^{JHG}) so that $\gamma(0) = (Q_L^{JHG}, T_L^{JHG})$ and $\tau_L(\gamma(t)) = c$ for all $t \in (-1, 1)$.

Case 1: if $cD_{22}^2 U_L(Q_L^{JHG}, T_L^{JHG}) \neq -D_{12}^2 U_L(Q_L^{JHG}, T_L^{JHG})$, the existence of such a curve is a consequence of the implicit function theorem. To see this, define implicitly $\rho(Q)$ so that $\tau_L(Q, \rho(Q)) = c$. Clearly, $\rho(Q_L^{JHG}) = T_L^{JHG}$ by construction of the JHG allocation. We then show that $\rho(Q)$ is well-defined locally around (Q_L^{JHG}, T_L^{JHG}). To apply the implicit function theorem we require that τ is continuously differentiable and that $D_2\tau_L(Q, T) \neq 0$ when evaluated at $(Q, T) = (Q_L^{JHG}, T_L^{JHG})$. $\tau_L(Q, T)$ is continuously differentiable if $U_L(Q, T)$ is twice continuously differentiable. And $D_2\tau_L(Q, T) \neq 0$ at (Q_L^{JHG}, T_L^{JHG}) is equivalent to $D_{22}^2 U_L(Q, T) D_1 U_L(Q, T) \neq D_{12}^2 U_L(Q, T) D_2 U_L(Q, T)$. Then note that by construction of the JHG allocation $c = \tau_L(Q_L^{JHG}, T_L^{JHG}) = -\frac{D_1 U_L(Q_L^{JHG}, T_L^{JHG})}{D_2 U_L(Q_L^{JHG}, T_L^{JHG})}$ So, equivalently, $cD_{22}^2 U_L(Q_L^{JHG}, T_L^{JHG}) \neq -D_{12}^2 U_L(Q_L^{JHG}, T_L^{JHG})$ as we had assumed. And so $\gamma(t)$ is a parametrization of the graph of $\rho(Q)$.

Case 2: if $cD_{12}^2 U_L(Q_L^{JHG}, T_L^{JHG}) \neq -D_{11}^2 U_L(Q_L^{JHG}, T_L^{JHG})$, the existence of such a curve is a consequence of the implicit function theorem. To see this, define implicitly $\sigma(T)$ so that $\tau_L(\sigma(T), Q) = c$ and follow the same steps as before.

Case 3: if both $cD_{22}^2 U_L(Q_L^{JHG}, T_L^{JHG}) = -D_{12}^2 U_L(Q_L^{JHG}, T_L^{JHG})$ and $cD_{12}^2 U_L(Q_L^{JHG}, T_L^{JHG}) = -D_{11}^2 U_L(Q_L^{JHG}, T_L^{JHG})$, then $D_1\tau_L(Q_L^{JHG}, T_L^{JHG}) = D_2\tau_L(Q_L^{JHG}, T_L^{JHG}) = 0$. This means that there exists an open ball around (Q_L^{JHG}, T_L^{JHG}) so that $\tau_L(Q, T) = c$ for all (Q, T) inside.

Step 3: We observe that $Q^P \mapsto \max_q U_H(Q^P + q, Q^P\,c + q\,c_H)$ is continuous. This is a consequence of Berge's maximum theorem: $U_H(Q,T)$ is continuous, moreover $\arg\max_{Q\geq 0} U_H(Q+q, Qc+qc_H) \leq \max\{\arg\max_q U_H(q,qc), \arg\max_q U_H(q,qc_H)\}$ due to quasi-concavity of $U_H(Q,T)$.

Step 4: We show that $U_H(Q_H^{JHG}, T_H^{JHG}) > U_H(Q_L^{JHG}, T_L^{JHG})$. Or, suppose by contradiction that $U_H(Q_H^{JHG}, T_H^{JHG}) = U_H(Q_L^{JHG}, T_L^{JHG})$. Whence, due to strict quasi-concavity, it holds that $U_H(\lambda Q_L^{JHG} + (1-\lambda)Q_L^{JHG}, \lambda T_L^{JHG} + (1-\lambda)T_L^{JHG}) > \min\{U_H(Q_H^{JHG}, T_H^{JHG}), U_H(Q_L^{JHG}, T_L^{JHG})\}$ for all $\lambda \in (0,1)$. Yet, by construction of the JHG allocation, $U_H(Q_H^{JHG}, T_H^{JHG}) = \max_q U_H(Q_L^{JHG} + q, T_L^{JHG} + qc_H)$ and this establishes the desired contradiction.

Conclusion: Fix $\epsilon > 0$ so that $U_H(Q_H^{JHG}, T_H^{JHG}) > U_H(Q_L^{JHG}, T_L^{JHG}) + \epsilon$. Such an ϵ exists due to Step 4. Next, due to step 3, there exists δ_H so that $\max_q U_H(Q^P + q, Q^P c + qc_H) > \max_q U_H(Q_L^{JHG} + q, Q_L^{JHG} c + qc_H) - \frac{\epsilon}{2}$ for all $Q^P : |Q^P - Q_L^{JHG}| < \delta_1$. And due to continuity of U_H there exists δ_L so that $U_H(Q_L^{JHG}, Q_L^{JHG}) + \frac{\epsilon}{2} > U_H(Q^P + q_L, Q^P c + q_L c_x)$ for all (Q^P, q_L) so that $0 < q_L < \delta_2$ and $|Q^P - Q_L^{JHG}| < \delta_2$. Finally, by construction of γ, i.e., Step 2, for all $\delta > 0$ there exists a low type allocation $(Q^P + q_L, Q^P c + q_L c_x)$ in the image of γ so that $|Q^P - Q_L^{JHG}| < \delta$ and $0 < q_L < \delta$. In particular, we can choose such (Q^P, q_L) for $\delta = \min\{\delta_1, \delta_2\}$. Then pick $q_H \in \arg\max_q U_H(Q^P + q, Q^P c + qc_H)$.

Thus constructed aggregate trades $(Q^P, Q^P c), (q_L, q_L c_x)$ and $(q_H, q_H c_H)$ are incentive compatible, competitively priced and conditionally efficient. For δ sufficiently small they moreover satisfy large pooling, because the JHG allocation satisfies large pooling. □

.7 Proof of Theorem 2

Proof of Lemma 4

Proof. First, it follows from Condition 4 and $t > q^\ell c$ that

$$U_L(Q_L, T_L) = \max_q U_L(Q_L + q, T_L + q\,c) \geq \max_q U_L(q_L + q, t_L + q\,c).$$

This is weakly greater than the utility when purchasing at least one latent contract: Condition 2 asserts that buyers can at most once purchase contract (q_L, t_L) (possibly part of a latent contract $(q_L + q^{n\ell}, t_L + t^{n\ell})$; and due to Condition 2 and $t > q^\ell c$ any other contract offered has unit price weakly greater than c.

Second, observe that by construction and Lemma 3

$$\begin{aligned} &U_H(Q_H, T_H) = U_H(Q_L + q^\ell, T_L + t^\ell) \\ &\geq \max_q U_H(Q_L + q^\ell + q, T_L + t^\ell + q\,c) = \max_q U_H(q_L + q, t_L + q\,c). \end{aligned}$$

Thereby derived utility is weakly greater than purchasing any latent contract for identical reasons. □

Proof of Lemma 5

Proof. First, consider a large cream-skimming deviation (q', t') in either group $\mathcal{K}_1$ or $\mathcal{K}_2$ so that $U_L(q', t') > U_L(Q_L, T_L)$. Then the flatter-curvature assumption implies that $U_H(q'+q^\ell, t'+t^\ell) > U_H(Q_L + q^\ell, T_L + t^\ell) = U_H(Q_H, T_H)$. So, the latent contract (q^ℓ, t^ℓ) blocks this deviation.

Analogously, consider a small cream-skimming deviation (q', t') in either group $\mathcal{K}_1$ or $\mathcal{K}_2$ that pivots on the latent contract $(q^{(n-1)\ell}, t^{(n-1)\ell})$ where $n > 1$. Or, $U_L(q'+q^{(n-1)\ell}, t'+t^{(n-1)\ell}) > U_L(Q_L, T_L)$. Then the flatter-curvature assumption implies that $U_H(q'+q^{(n-1)\ell}+q^\ell, t'+t^{(n-1)\ell}+t^\ell) > U_H(Q_L+q^\ell, T_L+t^\ell) = U_H(Q_H, T_H)$. So, the latent contract $(q^{n\ell}, t^{n\ell})$ blocks this deviation.

Finally, we show that there does not exist a small cream-skimming deviation (q', t') in either group $\mathcal{K}_1$ or $\mathcal{K}_2$ that pivots on the latent contract $(q^{N\ell}, t^{N\ell})$ and thereby exclusively attracts low type buyers. If it did, due to incentive compatibility, $U_L(q' + q^{N\ell}, t' + t^{N\ell}) > U_L(Q_L, T_L) \geq U_L(Q_H, T_H)$. Yet since $q^{N\ell} > Q_H$, also $U_H(q' + q^{N\ell}, t' + t^{N\ell}) > U_H(Q_H, T_H)$ due to single-crossing. Analogously, one cannot exclusively attract low type buyers by pivoting on the latent contracts $(q_L + q^{N\ell}, t_L + t^{N\ell})$, $(q_H + q^{N\ell}, t_H + t^{N\ell})$ and $(Q^P + q^{N\ell}, t^P + t^{N\ell})$.

Second, consider a cream-skimming deviation (q', t') in group $\mathcal{K}_1$ that pivots on a contract $(q_L+q^{(n-1)\ell}, t_L+t^{(n-1\ell)})$ in group $\mathcal{K}_2$ where $1 \leq n \leq N$. Or, $U_L(q'+q_L+q^{(n-1)\ell}, t'+t_L+t^{(n-1)\ell}) > U_L(Q_L, T_L)$. Then the flatter-curvature assumption implies that $U_H(q' + q_L + q^{(n-1)\ell} + q^\ell, t' + t_L + t^{(n-1)\ell} + t^\ell) > U_H(Q_L + q^\ell, T_L + t^\ell) = U_H(Q_H, T_H)$. So, the contract $(q_L + q^{n\ell}, t_L + t^{n\ell})$ in group $\mathcal{K}_2$ blocks this deviation. (The same argument applies for cream-skimming deviations that pivot on a contract $(q_H + q^{(n-1)\ell}, t_H + t^{(n-1\ell)})$ in the group $\mathcal{K}_2$ where $1 \leq n \leq N$.).

Third, consider a cream-skimming deviation (q', t') in group $\mathcal{K}_2$ that pivots on a contract $(Q^P + q^{(n-1)\ell}, T^P + t^{(n-1\ell)})$ in group $\mathcal{K}_1$ where $1 \leq n \leq N$. Or, $U_L(q' + Q^P + q^{(n-1)\ell}, t' + T^P + t^{(n-1)\ell}) > U_L(Q_L, T_L)$. Then the flatter-curvature assumption implies that $U_H(q' + Q^P + q^{(n-1)\ell} + q^\ell, t' + T^P + t^{(n-1)\ell} + t^\ell) > U_H(Q_L + q^\ell, T_L + t^\ell) = U_H(Q_H, T_H)$. So, the contract $(Q^P + q^{n\ell}, T^P + t^{n\ell})$ in group $\mathcal{K}_1$ blocks this deviation.

□

Proof of Lemma 6

Proof. First observe that a deviation (q', t') exclusively targeting high type buyers must satisfy $q' c_H < t'$ in order to be profitable. Then recall that $q^\ell c < t^\ell$. It follows that if a deviating contract (q', t') pivots on a latent contract that is different from $(q_L + q^{n\ell}, t_L + t^{n\ell})$, then the total unit cost of the quantity traded following the deviation must exceed c (due to competitive pricing, Condition 2). But then (q', t') traded in conjunction with the latent contract can impossibly be advantageous to high type buyers, for on-path utility satisfies $U_H(Q_H, T_H) = \max_q U_H(q_L + q^\ell + q, t_L + t^\ell + q\, c) \geq \max_q U_H(q^\ell + q, t^\ell + q\, c)$ (due to Lemma 3, moreover $t_L \leq q_L\, c$ due to competitive pricing in Condition 2).

Thus consider a deviation (q', t') in group $\mathcal{K}_1$ that pivots on a contract of the form $(q_L + q^{n\ell}, t_L + t^{n\ell})$. If $n = 0$ this deviation is not profitable and at the same time advantageous to high type buyers due to Condition 3. Thus suppose that $1 \leq n$. If high type buyers were better off by trading (q', t'), it follows (due to Lemma 3) that

$$U_H(q' + q_L + q^{n\ell}, t' + t_L + t^{n\ell}) > U_H(Q_H, T_H) = \max_q U_H(q_L + q^\ell + q, t_L + t^\ell + q\,c)$$
$$\geq U_H(q' + q^{n\ell} + q_L, t_L + t^\ell + (q' + q^{n\ell} + q_L - q_L - q^\ell)c).$$

It must therefore hold that

$$t' + t^{n\ell} + t_L < t_L + t^\ell + (q' + q^{n\ell} - q^\ell)c \qquad \Leftrightarrow \qquad t' < q^{(n-1)\ell}c - t^{(n-1)\ell} + q'c.$$

Since $q^\ell c < t^\ell$, this implies that $t' < q'c$ and so (q', t') must be lossmaking. □

.8 Proof of Proposition 3

Proof of Proposition 3, claim 1. Denote (Q^P, T^P) the equilibrium pooling allocation. If there exists a non-zero quantity $Q_H - Q^P = \arg\max_{q \geq 0} U_H(Q^P + q, T^P + qc_H)$, an aftermarket seller can propose the contract $(Q_H - Q^P, (Q_H - Q^P)c_H + \epsilon)$. For $\epsilon > 0$ sufficiently low, this contract attracts type H and is profitable. Whence (Q^P, T^P) is not a serendipitous-aftermarket-proof.

If to the contrary pooling is conditionally efficient, no profitable pooling nor a separating contract targeting high type buyers will generate the desired demand. Cream-skimming deviations uniquely targeting low type buyers, meanwhile, will be infeasible due to single-crossing. □

Proof of Proposition 3, claim 2. To prove the claim it suffices to consider aftermarket cream-skimming deviations that uniquely target the low type. For conditional efficiency ensures that $U_H(Q_H, T_H) = \max_{q \geq 0} U_H(Q_H + q, T_H + qc_H)$ and $U_L(Q_L, T_L) = \max_{q \geq 0} U_L(Q_L + q, T_L + qc)$, so that neither a pooling deviation nor a deviation uniquely targeting high type buyers can be both incentive compatible and profitable. The proof that there do not exist profitable cream-skimming deviations relies on the following arguments:

Step 1: For any three contracts $(Q_1, T_1), (Q_2, T_2)$ and (q', t') so that $Q_2 > Q_1$ satisfying $U_H(Q_1, T_1) = U_H(Q_2, T_2)$ and $U_H(Q_2, T_2) < U_H(Q_2 + q', T_2 + t')$: it must hold that $\frac{T_2 - T_1}{Q_2 - Q_1} > \frac{T_2 - T_1 + t'}{Q_2 - Q_1 + q'}$.

This is an immediate consequence of strict quasi-concavity. Otherwise, there exists a contract (Q_3, T_3) : $Q_3 > Q_2$ with $\frac{T_3 - T_2}{Q_3 - Q_2} = \frac{T_2 - T_1}{Q_2 - Q_1}$ and $U_H(Q_3, T_3) \geq U_H(Q_2, T_2)$. And, due to strict quasi-concavity, it must hold that $U_H(\lambda Q_1 + (1 - \lambda)Q_3, \lambda T_1 + (1 - \lambda)T_3) > \min\{U_H(Q_1, T_1), U_H(Q_3, T_3)\} = U_H(Q_1, T_1) = U_H(Q_2, T_2)$. Yet when setting $\hat{\lambda} = \frac{Q_3 - Q_2}{Q_3 - Q_1}$ (which is equal to $\frac{T_3 - T_2}{T_3 - T_1}$ by construction), it thereby follows that $U_H(Q_2, T_2) = U_H(\hat{\lambda} Q_1 + (1 - \hat{\lambda})Q_3, \hat{\lambda} T_1 + (1 - \hat{\lambda})T_3) > U_H(Q_2, T_2)$, which establishes the desired contradiction.

Step 2: For any three contracts $(Q_1, T_1), (Q_2, T_2)$ and (q', t') so that $Q_2 > Q_1$ satisfying $U_H(Q_1, T_1) = U_H(Q_2, T_2)$ and $U_H(Q_2, T_2) < U_H(Q_2 + q', T_2 + t')$: it must hold that $U_H(Q_1, T_1) <$

$U_H(Q_1+q', T_1+t')$.

This is also a consequence of strict quasi-concavity: $U_H(\lambda Q_1 + (1-\lambda)(Q_2+q'), \lambda T_1 + (1-\lambda)(T_2+t')) > \min\{U_H(Q_1,T_1), U_H(Q_2+q', T_2+t')\} = U_H(Q_1,T_1)$ for all $\lambda \in (0,1)$. Then set $\hat{\lambda} = \frac{Q_2-Q_1}{Q_2-Q_1+q'}$ so that $\hat{\lambda} Q_1 + (1-\hat{\lambda})(Q_2+q') = Q_1+q'$. In effect, $\hat{\lambda} T_1 + (1-\hat{\lambda})(T_2+t') = T_1+t'+(T_2-T_1)+\hat{\lambda}(t'-T_2+T_1) = T_1+t'+(T_2-T_1)-(Q_2-Q_1)\frac{T_2-T_1+t'}{Q_2-Q_1+q'} > T_1+t'+(T_2-T_1)-(Q_2-Q_1)\frac{T_2-T_1}{Q_2-Q_1} = T_1+t'$ where the inequality is due to the step 1. Since utility is decreasing in transfers, this proves that $U_H(Q_1+q', T_1+t') > U_H(\hat{\lambda}(Q_1,T_1)+(1-\hat{\lambda})(Q_2+q', T_2+t')) > U_H(Q_1,T_1)$.

Step 3: For any "Pooling + Separating" allocation $(Q_L,T_L),(Q_H,T_H)$ satisfying conditions 1-4 **??**, the unique principal latent contract (q^ℓ, t^ℓ) (see Assumption **??** and Lemma 3) satisfies $Q_L + q^\ell > Q_H$.

To see this, recall that $U_H(Q_L+q^\ell, T_L+t^\ell) = U_H(Q_H,T_H)$. Then first, due to strict quasi-concavity for $\lambda = 1/2$, it holds that $U_H(Q_H + 1/2(Q_L+q^\ell-Q_H), T_H + 1/2(T_L+t^\ell - T_H)) > U_H(Q_H,T_H) = \max_q U_H(Q_H+q, T_H + q\ c_H) \geq U_H(Q_H+1/2(Q_L+q^\ell-Q_H), T_H + 1/2(Q_L+q^\ell-Q_H)c_H)$ where the equality is due to conditional efficiency. It follows that $T_L+t^\ell-T_H < c_H(Q_L+q^\ell-Q_H)$. Second, due to strict quasi-concavity for $\lambda = 1/2$ it similarly holds that $U_H(Q_L+q^\ell+1/2(Q_H-Q_L-q^\ell), T_L+t^\ell+1/2(T_H-T_L-t^\ell)) > U_H(Q_L+q^\ell, T_L+t^\ell) = \max_q U_H(Q_L+q^\ell+q, T_L+t^\ell+q\,c) \geq U_H(Q_L+q^\ell+1/2(Q_H-Q_L-q^\ell), T_L+t^\ell+1/2(Q_H-Q_L-q^\ell)c)$ where the equality is due to Lemma 3. It follows that $T_H - T_L - t^\ell < c(Q_H-Q_L-q^\ell)$. To conclude, combining the conclusions of the first and the second argument establishes that $(Q_L+q^\ell-Q_H)c_H > (Q_L+q^\ell-Q_H)c$. This can impossibly hold if $Q_L+q^\ell \leq Q_H$.

Step 4: We now prove claim (ii): Consider an aftermarket cream-skimming deviation (q',t') so that $U_L(Q_L+q', T_L+t') > U_L(Q_L,T_L)$. Then Assumption **??** ensures that $U_H(Q_L+q^\ell+q', T_L+t^\ell+t', U_H(Q_L+q^\ell, T_L+t^\ell) = U_H(Q_H,T_H)$. Then set $(Q_1,T_1) = (Q_H,T_H)$ and $(Q_2,T_2) = (Q_L+q^\ell, T_L+t^\ell)$. Step 3 implies that $Q_2 > Q_1$. Then step 2 implies that $U_H(Q_1,T_1) < U_H(Q_1+q', T_1+t')$, as claimed. □

Proof of Proposition 3 claim 3 . To begin with, we show that there do not exist cream-skimming aftermarket deviations uniquely attracting low type buyers. Indeed, due to Assumption 6 there exists a (unique due to Lemma 3) contract (q^ℓ,t^ℓ) satisfying $U_H(Q_L+q^\ell, T_L+t^\ell) = U_H(Q_H,T_H)$ so that $U_L(Q_L+q, T_L+t) \geq U_L(Q_L,T_L)$ implies $U_H(Q_L+q^\ell+q, T_L+t^\ell+t) > U_H(Q_L+q^\ell, T_L+t^\ell)$ for all (q,t) (although strictly speaking we only require this for positive (q,t)). And by Steps 2 and 3 from the proof of Claim 2, $Q_L+q^\ell > Q_H$, and $U_H(Q_L+q^\ell+q, T_L+t^\ell+t) > U_H(Q_L+q^\ell, T_L+t^\ell)$ implies that $U_H(Q_H+q, T_H+t) > U_H(Q_H,T_H)$. It follows that no cream-skimming deviation that uniquely attracts low type buyers exists.

We then prove the "if" claim. If $0 = \arg\max_{q\geq 0} U_L(Q_L+q, T_L+qc)$, no profitable pooling contract can attract low type buyers. And due to the conditional efficiency of (Q_H^{RS}, T_H^{RS}), no separating contract targeting high type buyers can generate positive profit.

We then prove the "only if" claim: If to the contrary $q^P \in \arg\max_{q\geq 0} U_L(Q_L+q, T_L+qc)$, there exists $\epsilon > 0$ so that $U_L(Q_L + q^P, T_L + q^P c + \epsilon) > U_L(Q_L, T_L)$. In effect, this contract surely attracts type L buyers and is already profitable and conditional on attracting both buyer types. $\square$

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Chapter 2: Adverse Selection and Equilibrium Existence in Almost Nonexclusive Markets

Abstract

Considering fully nonexclusive competitive markets, where privately informed buyers can trade with arbitrarily many sellers at the same time. Attar, Mariotti, and Salanié (2014) show the unique equilibrium candidate is the JHG allocation: low and high-risk buyers pool for a basic layer of insurance, and high-risk buyers purchase an additional layer of insurance. However, they also show that this allocation equilibrium never exists. Here I show that an equilibrium exists if we impose an upper bound on the number of sellers that buyers can jointly trade with. Equilibrium existence requires a strong assumption frequently made in the literature in the context of quadratic or CARA utility functions: translated indifference curves must have an identical shape across types. My findings shed light on the disconnection between continuous type models (associated with existence results) and discrete type models (associated with the absence thereof). It suggests that the choice of parametrization of utility functions is critical.

1 Introduction

There are two main strands of studies in the screening of menu games based on the competitive market plagued by adverse selection. Under exclusive competition, where each buyer can only trade with at most one seller, the seminal paper Rothschild and Stiglitz (1976), characterized the equilibrium allocation which is a fully separated allocation consisting of full insurance to high-risk and partial insurance to low-risk types. Under nonexclusive competition, where buyers can trade with any sellers, Attar et al. (2014) showed that the Rothschild-Stiglitz (RS) allocation is not an equilibrium under nonexclusive competition. They also showed that the existence of equilibrium relies on a special assumption on the preference of low-risk types—market breakdown for low types, thus, the equilibrium is Akerlof's allocation, where only high-risk types can have positive trade. However, for a more rational preference such as Inada's condition for low-risk types, there is no equilibrium under nonexclusive competition markets. This result was disquieting, as there are many markets with the feature of nonexclusivity in real life, and the equilibrium seems to exist and some of them include the pooling policy for all types.

There are many insurance markets that have the feature of nonexclusively trade—or multiple contracting. An example is the annuity market in the United Kingdom in 2013, five million people owns six million annuities. Similarly, in the "Mutuelle" in France, each buyer can purchase one social security and an additional purchase of one "Mutuelle" from different companies. Similar cases as in Germany, the Netherlands, and Switzerland, where more than half of the population holds more than one policy of health insurance. We have some observations among some of these markets: First, the real markets include some pooling contracts, like social security, or basic health insurance—this is different from the prediction in the nonexclusive model which contains no pooling policy in the equilibrium; Second, even some insurance markets have the feature of nonexclusive-trade—they can trade with more than one sellers, however, there are some restrictions on nonexclusive or an upper bound number of sellers they can trade.[1] Based on these observations, we model with the same model as Attar et al. (2014), but add some upper bound restriction on the number of sellers, show that we can restore the equilibrium under a restrictive nonexclusive market. This equilibrium allocation contains some pooling policy for both types of buyers which is consistent with our observations.[2]

This paper studies a simple framework that is similar to the model in Attar et al. (2014). There are two types (low and high) of the continuum of buyers who owns private information at their own risk (or unit serving cost). The buyers' preferences are concave and satisfy the single-crossing property. There are a finite number of sellers competing by posting menus of contracts, where a contract specifies the quantity-transfer(or coverage-premium) pair. After observing the menus in the market, buyers can trade with sellers according to the rule set by

[1] Such as "Mutuelle", each buyer can purchase at most one social security and at most "Mutuelle".

[2] This equilibrium allocation is Janyes-Hellwig-Glosten allocation, concluded by Jaynes (1978), Hellwig (1988), and Glosten (1994).

the planner, they can choose sellers and contracts from sellers to trade.

In this context, to study the screening game among different environments, or market structures— that is, a trading rule that specifies the subset of sellers with whom buyers can jointly trade— I analyze all market structures into two main parts: Partially exclusive market structures, and Never exclusive market structures. In a partially exclusive market structure with competition, in which there are some sellers have the ability to offer exclusively trade contracts. In the setting of simultaneous games and menu screening games, I show that the unique equilibrium candidate is still the Rothschild-Stiglitz(RS) allocation in Lemma 1: it is a fully separated allocation, in which high type (bad type) buyers can achieve the full insurance while low type buyers get partial insurance. The uniqueness of RS allocation will not change even if some sellers can offer menus to trade nonexclusively. That is to say, the existence of exclusively trade sellers prevents any equilibrium allocation with pooling trade. [3]

Under the menu screening game, if buyers' preferences satisfy Inada's condition, this paper shows that in any Never exclusive market structures— those where no seller can prohibit further side trading, the unique equilibrium candidate is the JHG allocation as in Theorem 1. This allocation consists of a partial pooling layer for both types—this means that cross-subsidy happens among these parts of contracts, which never happens in the partially exclusive market structure, then an additional layer in which high type buyers trade fairly with sellers.

There are two main observations with the result of uniqueness among the never exclusive market structures: First, the equilibrium allocation candidate suggests no separating contracts with unit price equal to the cost serving low-risk type buyers. Low-risk types can only trade pooling contracts due to the screening of the menu with a double deviation, that is to say, one seller can propose a menu with a contract of lower price contracts deviation to attract the low type, while a proposing another contract attract high type buyers and take advantage of separating contracts for low types to get positive profit.[4] Second, compared to the results in nonexclusive competition, this paper extends the uniqueness of equilibrium candidates to any market structures containing any kind of feature of nonexclusivity. Theorem 1 shows the uniqueness of the equilibrium candidate even doesn't require a competitive feature of market structure, it suggests that the side-trading has a very strong impact on the equilibrium allocation candidate. In an economy with side trading, the sellers' ability to propose the menu contracts prevents any other possible allocations as equilibrium candidates.

Then, this paper replicates the nonexistence of equilibrium in Proposition 2 to show that JHG allocation is not an equilibrium in the nonexcluisve environment under Inada's condition.[5]

[3]In this paper, I will not focus more on the exclusive feature. My main research contributes to the feature of nonexclusively trade. But the question such as the results of monopoly exclusively trade, and the existence of equilibrium under a partially exclusive market structure can be pursued.

[4]The double deviation is first proposed by Attar et al. (2014), they showed the JHG allocation can be destroyed by a double deviation, which trades a little loss-making contract to attract high type while trading a profit-making contract with low type, and the total profits are positive.

[5]In Attar et al. (2014), they argue with special preference such that market breakdown for low-risk types, there is an equilibrium similar to Akerlof's allocation. But it is not what we focused on.

The main conflict to sustain JHG allocation as equilibrium is to keep competitive and block double deviation at the same time: competitive requires JHG allocation that can be alternatively provided by different sellers, a double deviation which consists of a lower unit price contract compared to pooling to profit-making trade with lower types only, while a lower unit price contract top up on pooling layer with very small loss-making on high types.

Different from Attar et al. (2014), this paper next shows that JHG allocation can be an equilibrium under some middle-ground market structures which even has features of nonexclusivity. Start with the simplest structure, the "1+1" market structure, in which sellers are separated into two subgroups so that buyers can trade with at most one seller from each group. The restriction of the largest number of trades ensures a competitive market among sellers within small numbers of sellers, and it is impossible for a seller to take advantage of other sellers' allocations due to the restriction on the number of trades.[6]. In the "1+1" market structure, we sustain the JHG allocation as an equilibrium in Theorem 2. Similar to the work of Attar et al. (2022), equilibrium existence hinges on so-called latent contracts, i.e., contracts that are not traded actively in equilibrium but that play a role to deter cream-skimming deviations uniquely targeting low-risk buyers. Then if one cream-skimming deviation happens, it also attracts high-risk types with the combining of latent contracts. [7] To implement the JHG allocation with latent contracts under the menu game, I need a strong assumption frequently made in the literature in the context of quadratic[8] or CARA utility functions with identical risk preference: translated indifference curves must have an identical shape across types. Under this assumption, the specific latent contract prevents the cream-skimming deviation and also the double deviation based on latent contracts.

Theorem 3 proved that a JHG allocation can also be an equilibrium in some other never exclusive market structures such as "$\lambda+1$" and "$\lambda_1+\lambda_2$" market structure. [9] Compared these middle ground market structures with nonexclusive market structure, there are two features in my model are the keys to sustaining the JHG allocation as the equilibrium in an environment with side trading. The first key is the segmentation of the market, which means dividing sellers into two or more groups.[10] This segmentation can help sellers in the same groups compete and offer the same contracts. The second key is the upper bound trade number of sellers, this restriction helps the market sustain competition within very small numbers of sellers and prevents the double deviation taking advantage of the same groups' active contracts.[11] Actually,

[6]This restriction prevent the double deviations proposed by Attar et al. (2014)

[7]The cream-skimming deviation is profitable due to JHG consisting of pooling contacts for low-risk types, which has a unit price higher than the serving cost of low-risk types, thus, a lower unit price can still be higher than the unit cost for serving low-risk type only.

[8]In the financial market with nonexclusive, Biais et al. (2000) use the quadratic utility function, to solve the equilibrium allocation among continuous types model.

[9]"$\lambda+1$" market structure in which sellers are separated into two subgroups so that buyers can trade with at most λ sellers from group 1 and at most one seller from group 2, and "$\lambda_1+\lambda_2$" market structure in which sellers are separated into two subgroups so that buyers can trade with at most λ_1 sellers from group 1 and at most λ_2 sellers from group 2.

[10]Only requires the number of groups is larger than buyers types.

[11]In real life, this restriction is very common, buyers trade with very limited numbers of sellers due to

along with similar steps on "$\lambda_1 + \lambda_2$" market structure, any never exclusive market structure has the features of segmentation and upper bound on at least one of the groups will sustain the JHG allocation as the equilibrium under side trading menu game. [12]

Related Literature

First, this paper is related to the literature that the discussion of using different game settings or alternative equilibrium concepts to deal with the nonexistence of equilibrium problems among the competitive markets under adverse selection. There are two main branches among the literature on the nonexistence problems, which are exclusive and nonexclusive competition.

Under exclusive competition, Rothschild and Stiglitz (1976) studied the insurance market model, they showed that when the proportion of good types is big, the nonexistence of equilibrium will be a problem of this model. To solve his problem, Wilson (1977) and Riley (1979) tried to solve the nonexistence problem either by proposing the new withdrawal equilibrium concept with a new equilibrium of fully pooling (Wilson (1977)) or by proposing a new additional offer with restoring the RS equilibrium (Riley (1979)). Then by using the menu setting and equilibrium concept in Wilson (1977), Miyazaki (1977) and Spence (1978) showed a new equilibrium allocation called Miyazaki-Wilson-Spence (MWS) which is separating, maximizes the good types' utility conditional on zero profit and incentive-compatible condition. In Netzer and Scheuer (2014), they assume sellers can choose to be inactive with little cost after proposing an offer, and show an MWS allocation is the unique equilibrium in an exclusive market. Azevedo and Gottlieb (2017) revisit the Rothschild and Stiglitz (1976) model and define a new equilibrium concept and show RS allocation is a unique equilibrium among all proportions. Mimra and Wambach (2014) reviewed the literature and showed differences in the literature of the setting on insurance game.

Under nonexclusive competition, Attar et al. (2014) is the closed work to this paper. In a simultaneous nonexclusive game and menu is allowed, Attar et al. (2014) shows that Jaynes-Hellwig-Glosten (JHG) allocation is the unique equilibrium candidate if Inada's condition is satisfied. However, there also proved that JHG allocation is not an equilibrium due to double deviation can be profitable. Thus, the equilibrium in their economy only happens when the exclusion of good types happens—so it is reduced as Akerlof (1970) allocation. To restore the JHG allocation, Stiglitz et al. (2020) propose the model on the complex information disclosure setting, which is similar to the information sharing as in Jaynes (1978) and Hellwig (1988), to restore the JHG allocation as equilibrium in nonexclusively trade model. The regulation on cross-subsidy from one contract to another contract of one seller is another method to deal with the nonexistence of JHG allocation, as shown in Attar et al. (2022). Different from focusing

limitations on searching cost, information availability, restriction on location, and buyers' patience.

[12] In Asriyan and Vanasco (2022), they show that with a design on senior security which exclusively pays based on return, they can sustain JHG allocation as the equilibrium. It actually one of the special market structures of the middle grounds which satisfy the segmentation (senior security and others) and exclusively trade of senior security.

on the simultaneous game, Attar et al. (2021) use an ascending auction to implement the JHG allocation in a nonexclusive environment. Asriyan and Vanasco (2022) employ the withdrawal contracts similar to Netzer and Scheuer (2014), and add a design of senior security, showing that JHG is an equilibrium in the financial market. Compared with the literature mentioned above, this paper sticks to the simultaneous game, and without introducing the regulation, withdrawal, or information disclosure, shows that only weak restricting on trade numbers with sellers can also sustain the JHG allocation, which provides a new policy suggestion to the planner.

Then, the latent contract plays a very important role to sustain the equilibrium, because the latent contract can block the cream-skimming deviations which aim to attract good type only. Latent contracts were first introduced in the context of competing mechanisms in the seminal paper by Peters (2001). Since latent contracts have mainly appeared under multiple contracting in the literature on moral hazard. Hellwig (1983) argues that latent contracts can deter entry into the insurance market when agents' effort decisions are not contractible. This can result in positive equilibrium profits. Based on the adverse selectionAttar et al. (2011), extending the environment of Akerlof (1970) to non-exclusive contracting and sustaining the equilibrium with latent contact. Attar et al. (2022) use regulation and latent contract to sustain the JHG allocation. Huang and Sandmann (2022) shared the same setting with Attar et al. (2022), and use market structure, showing that new equilibria have forms of "pooling + separating".

My paper contributes to the implementation of Glosten (1994) allocation, it sheds light on the disconnection between continuous type models (associated with existence results) and discrete type models (associated with the absence thereof). Biais et al. (2000) studied the financial markets, in which uninformed market-makers compete in a nonexclusive way by supplying liquidity to an informed insider. They constructed an equilibrium in which market-makers post convex price schedules and shared positive profit due to the oligopoly. They also showed that when the number of sellers is big enough(competitive), the equilibrium becomes the Glosten (1994) equilibrium which the ask prices are equal to upper tail expectations. However, the literature shows different results in the model of the discrete type. Attar et al. (2014) show there is no JHG allocation in simultaneous menu game and Akerlof (1970) can be an equilibrium only if the preference is special, and Ales et al. (2014) study nonexclusive competition under three types of model among the framework of Attar et al. (2014), suggest the Akerlof (1970) results with some latent contracts. Different from Akerlof's results, under the assumption of perfect translation, which is the same as the preference of Biais et al. (2000), my paper uses latent contract in a restrictive trade number, I sustain the JHG allocation, this finding is consistent with Biais et al. (2000) competitive situation.

This paper is organized as follows. Section 2 introduces the model setting with a menu, the equilibrium notion, and the definition of market structure and competitive structure. Section 3 discusses the uniqueness of equilibrium candidates among all competitive partially exclusive market structures. Section 4 shows strong uniqueness results with the market structure containing no seller can propose exclusively trade contracts. Section 5 replicates the result of the nonexistence of equilibrium in nonexclusive competition, and shows the equilibrium can be restored with some middle never exclusive structures. Section 6 concludes the paper.

2 Set-up

We study a competitive market plagued by adverse selection. Our model features an insurance economy in which buyers with an exogenously given high and low-risk profile purchase coverage in exchange for a premium. Our description of the economy (preferences, cost, and the space of admissible contracts) is identical to that in Attar et al. (2014) and encompasses the classical set-up in Rothschild and Stiglitz (1976). Different from the two models in the literature, We innovate in that we introduce the concept of a *market structure*, i.e., a trading rule that specifies the subsets of sellers that the buyers can jointly. The definition admits as special cases the market structures where each buyer can trade with at most one (exclusive competition) and with arbitrarily many (nonexclusive competition) sellers. By using the concepts of market structure, we can study the middle ground situations which include both features of nonexclusive and exclusive environments.

2.1 An Insurance Economy

We study competitive pricing under adverse selection in the insurance economy, buyers purchase coverage in exchange for an insurance premium or we also call it quantity and transfer pair. Buyers in the economy hold private information while sellers propose a different menu of contracts[13] to attract different buyers.

Buyers and Sellers There are two buyers, indexed by their type H, L. Types are buyers' private information, and the commonly known proportion is m_L to type L and $m_H \equiv 1 - m_L$ to type H. [14]

We consider a finite set of sellers $\mathcal{K} = \{1, ..., K\}, K \geq 2$ and a continuum of buyers that are characterized by their type $\theta \in \{L, H\}$. Sellers compete by proposing contracts $(q^k, t^k) \in \mathbb{R}_+^2$ specifying a quantity and a tariff. Types are non-contractible so that all buyers can select identical trades if they so wish. A contract (q, t) between a seller and a buyer covers a fixed fraction $q \geq 0$ of the loss for a premium t, with a unit price of $\frac{t}{q}$. Sellers are perfectly substitutable in that the cost of providing aggregate quantity q to some buyer type θ does not depend on the identity nor quantity provided by individual sellers. If a seller trade a contract (q, t) with a buyer of type θ, then the seller earns an expected profit $t - c_\theta q$, where c_θ is the marginal cost of serving buyer type $\theta \in \{L, H\}$.

[13] In my companion paper Huang and Sandmann (2022), we discuss the situation if sellers can only propose a single contract or regulation on cross-subsidy of menus, it showed different results from this paper.

[14] The model can be interpreted as if there was a continuum of buyers, but for descriptive purposes, it is best to think of a single consumer of unknown types.

A buyer type θ holds preferences over aggregate trades; if she trades with sellers $M \subseteq \{1, ..., K\}$ her aggregate trades are henceforth given by $Q = \sum_{k \in M} q^k$ and $T = \sum_{k \in M} t^k$ the corresponding aggregate quantity and tariff. Preferences over aggregate trades are represented by a utility $u_\theta(Q, T)$ which is quasi-concave, increasing in its first, and decreasing in its second argument, twice continuously differentiable, and satisfying the Inada's condition. All buyers have as a common outside option the null trade so that $u_\theta(0, 0) = 0$. Finally, buyer types are ordered in the sense that higher types have a greater propensity to consume.

Assumptions First, we impose regularity conditions so that the buyer's demand is well-behaved.[15]

Assumption 1 (quasi-concavity). *$u_\theta(q, t)$ is strictly quasi-concave, i.e. $\forall \alpha \in (0, 1)$, and $(q_1, t_1) \neq (q_2, t_2)$ it holds that $u_\theta(\alpha(q_1, t_1) + (1 - \alpha)(q_2, t_2)) > \min\{u_\theta(q_1, t_1), u_\theta(q_2, t_2)\}$.*

Second, we assume that types are ordered so that high buyer types' demand exceeds low buyer types' demand:[16]

Assumption 2 (single-crossing). *For all (q, t) and (q', t') so that $q' > q$ it holds that $u_L(q', t') \geq u_L(q, t) \quad \Rightarrow \quad u_H(q', t') > u_H(q, t)$.*

Cost. As what we mentioned before, trading a contract (q, t) with a buyer type θ earns the seller an expected profit $t - c_\theta q$. Here c_θ denotes the marginal cost of serving type θ. In line with a model of adverse selection, we assume that those buyer types most eager to trade, i.e., high types H, are also the most costly to serve.

Assumption 3 (Adverse Selection). *$c_H > c_L$.*

In here, we denote m_H as the proportion of type H and m_L as the proportion of type L buyers so that the average marginal cost is $c = c_H m_H + c_L m_L$. Thus, a contract (q, t) serving for both types of buyers will provide $t - cq$ expected profit for the seller.

Assumption 4 (Twice Differentiable). *For each θ, u_θ is continuously differentiable.*

Assumption 5 (Smoothing Demand). *$\arg\max_{Q \geq 0} u_\theta(Q, Qc_x)$ is finite $\forall\, c_x > 0$ and $\theta \in \{L, H\}$. And $\tau_\theta(0, 0)$ satisfies Inada's condition.*

Notice that strict quasi-concavity implies that $\arg\max_{Q \geq 0} u_\theta(Q, Qc_x)$ is a singleton and is a positive demand. Hence, Assumption 5 suggests that for any unit price tariff, any type of buyers would have positive and finite quantity unit price.

[15] In the section of existence, we introduce further regularity conditions: a perfect translation assumption that ensures the existence of an equilibrium.

[16] Provided that utility is differentiable, this is equivalent to assuming that the slope of the indifference curve $\tau_H(Q, T) = -\frac{\partial_1 u_\theta(Q,T)}{\partial_2 u_\theta(Q,T)}$ is greater for higher types, i.e., $\tau_H(Q, T) > \tau_L(Q, T)$.

2.2 Market Structure

The key innovation of our framework is the concept of a market structure. A market structure specifies which sellers a buyer can jointly trade with. So it is a subset of the power set of all sellers.[17] In this paper, I mainly compare the market structure of nonexclusive competition which put no restriction on joint trade, and the middle ground market structure with the mainly nonexclusive feature but add a few exclusive features. I will show that any nonexclusive feature will lead to a unique equilibrium candidate, but restrictive nonexclusive competition helps with the existence problem,

Definition 1. *A market structure $\mathcal{M}$ is a (non-empty) collection of subsets of sellers with whom a buyer can jointly trade: $\mathcal{M} \subseteq \mathcal{P}(\{1, ..., K\}) \equiv \mathcal{P}(\{\text{all sellers}\})$.*

The two polar cases considered in the literature are defined as follows:

Example 1. (i) *Exclusive competition $\mathcal{M} = \big\{\emptyset, \{1\}, \{2\}, ..., \{K\}\big\}$.*

(ii) *Nonexclusive competition : $\mathcal{M} = \mathcal{P}(\{1, ..., K\})$*

To remind what is the description of market structures, the $\mathcal{P}(\{1, ..., K\})$ denotes the power set, i.e., the set of all subsets of $\{1, ..., K\}$. As shown in the example 1 (i), when $\mathcal{M}$ is the sets includes a singleton of each seller, it means that a buyer can only trade with at most one sellers which is actually the exclusive market structure as proposed in Rothschild and Stiglitz (1976). In the example 1 (ii), when the available sets for the buyer are power sets of all sellers, it is equivalent to say a buyer can trade with any sellers, thus, it is a nonexclusive market structure as proposed in Attar et al. (2014).

In line with the two polar cases of exclusive and nonexclusive market structures, our focus is on competitive market structures. We, therefore, require that the sellers' offers can be declined by the buyers and that each seller is replaceable.

Definition 2. *A market structure $\mathcal{M}$ is competitive if*

- *buyers can trade with any subset of a feasible set of trading partners, i.e., for all $M \in \mathcal{M}$ and $j \in \mathcal{K}$, if $j \in M$, then also $M \setminus \{j\} \in \mathcal{M}$;*
- *each seller is replaceable,[18] i.e. for all $M \in \mathcal{M}$ and $j \in \mathcal{K}$, if $j \in M$, then there exist distinct $k_1 \in \mathcal{K} \setminus M$ so that $M \cup \{k_1\} \setminus \{j\} \in \mathcal{M}$.*

The ability to decline offers allows the buyers to play some sellers against others by threatening to accept only a subset of the offers they receive. Bernheim and Whinston Bernheim and Whinston (1986) call this arrangement delegated common agency. Then, we want to focus the market structures on competition, that is to say, any sellers should not have monopoly power,

[17] $\mathcal{P}\{\textit{all sellers}\}$ denotes the power sets of all sellers, which includes any subsets of any sellers.

[18] we don't need the twice replaceable as in Huang and Sandmann (2022) due to the setting of menus can provide the competition to the market.

so we require that each seller can be replaced by at least one of the other sellers. This arrangement ensures that an active seller is always competing with an inactive seller and preserves undercutting incentives.

2.3 Equilibrium

One can imagine a benevolent planner that decides on jointly feasible trades before the market opens. For now, we shall take the (competitive) market structure as given and focus our analysis on the ensuing equilibria.

The Simultaneous Game with Menu. We consider a competitive screening game in which firms compete in menus of contracts for quantity-tariff pairs. Given a fixed market structure $\mathcal{M} \subseteq \mathcal{P}(\{1, ..., K\})$, the game unfolds as follows:

- Stage 1: Each seller k proposes a **menu of contracts**, i.e., a set $\mathcal{C}^k \subset \mathbb{R}^2$ which includes at least the no trade contract $(0, 0)$.
- Stage 2: Each buyer learns her type, selects some $M \in \mathcal{M}$ and derives utility $u_\theta\big(\sum_{k\in M} q^k, \sum_{k\in M} t^k\big)$.

Strategy for Buyers: A pure strategy for type θ is first to choose one available seller set $M \in \mathcal{M}$ and then choose a function that maps each available menu profile (from the chosen market structure M) $\left(C^1, \ldots, C^k\right)$ into a vector of contracts $\left(\left(q^1, t^1\right), \ldots, \left(q^k, t^k\right)\right) \in C^1 \times \cdots \times C^k$ with each $k \in M$, and $M \in \mathcal{M}$. To ensure that type θ 's utility-maximization problem

$$\max \left\{ u_\theta \left(\sum_k q^k, \sum_k t^k \right) : \left(q^k, t^k\right) \in C^k \text{ for each } k \in M \right\}$$

always has a solution, we require the buyers' menus sets are compact. Then our equilibrium concept is perfect Bayesian equilibrium in pure strategies. That is to say, buyers purchase the contracts to maximize the utility according to the market structure rule. And sellers will propose the menu to maximize the expected profit. Throughout the paper, we focus on **pure-strategy** equilibria.

3 Partially Exclusive: Rothschild-Stiglitz Allocation

To study all possible equilibrium in various market structures, we can divide all market structures into Partially exclusive and Never exclusive market structures. The first contains at least one seller who can propose an exclusive menu to the market, while in the second no seller has this ability. In this section, we will focus on the Partially exclusive market, the show the unique equilibrium is Rothschild-Stiglitz allocation, and as we all know, it exists in an exclusive market when the proportion of high type is large[19].

[19] Also, if the proportion of low type is big, then there is no equilibrium

Definition 3. *A market structure $\mathcal{M}$ is partially exclusive if there exists a seller that has the right to exclusively trade with buyers, i.e.,* $\max_{M \in \mathcal{M}: k \in M} |M| = 1$ *for some* $k \in \mathcal{K}$.

As in Rothschild and Stiglitz (1976), they define a separating equilibrium allocation in the exclusive competition, which is defined as follows:

Definition 4 (Rothschild-Stiglitz (RS))**.** *The RS allocation is the separating allocation* (Q_L^{RS}, T_L^{RS}) *and* (Q_H^{RS}, T_H^{RS}) *where*

$$
\begin{aligned}
Q_H^{RS} &= \arg\max_{Q_H \geq 0} u_H(Q_H, c_H Q_H), \quad T_H^{RS} = c_H Q_H^{RS} \\
Q_L^{RS} &= \arg\max_{Q_L \geq 0} u_L(Q_L, c_L Q_L), \quad T_L^{RS} = c_L Q_H^{RS} \\
&\textit{subject to } \; u_H(Q_H^{RS}, T_H^{RS}) \geq u_H(Q_L^{RS}, T_L^{RS}).
\end{aligned}
$$

This separating allocation (Also see Figure 1) provides us a way to solve the problem of asymmetric information, which includes full insurance to high type while only giving partial insurance to low types. Then the high-risk types would have no interest to mimic the low-risk types due to the rationing of the quantity. Actually, as in Lemma 1, we show this kind of allocation is also the unique equilibrium candidate for any market structure that includes at least competitive sellers who have exclusive ability[20].

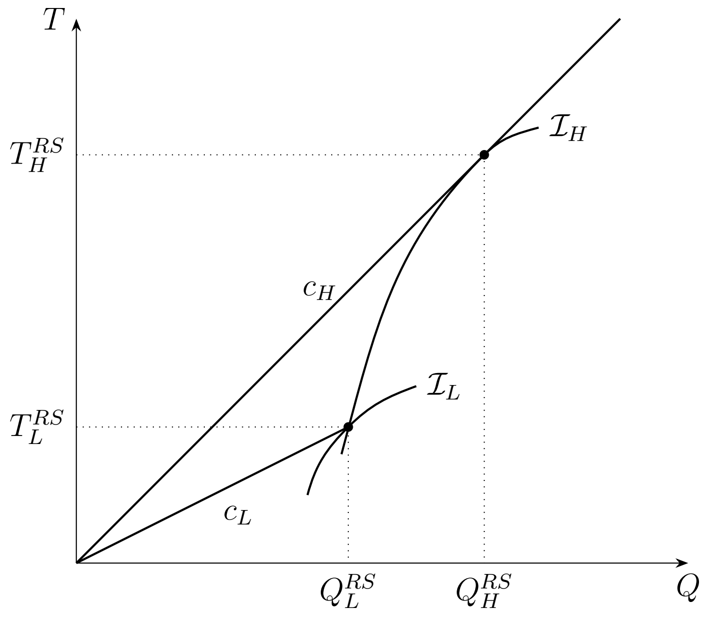

Figure 1: Rothschild-Stiglitz Allocation

Lemma 1. *Posit Assumptions 1, 2 and 3 . The RS separating allocation is the unique equilibrium candidate allocation under a partially exclusive and competitive market structure.*

[20]As the same logic with Rothschild and Stiglitz (1976), this kind of allocation is an equilibrium in some situations

Proof. Fix a partially exclusive and competitive market structure and consider an equilibrium allocation (Q_L, T_L) and (Q_H, T_H).

Step 1. we show that $T_L = Q_L\, c_L$. First, $T_L \geq Q_L\, c_L$. Otherwise, there exists at least one seller who makes a negative profit. Then this seller can choose to be inactive instead. Second, $T_L \leq Q_L\, c_L$. Otherwise, suppose that $T_L > Q_L c_L + \epsilon$. Since the market structure is competitive and partially exclusive, there exist at least three exclusive sellers, i.e., sellers k for whom $\max_{M\in\mathcal{M}:k\in M} |M| = 1$. Then at least one of these three is inactive and makes zero profit on-path. A profitable deviation for an exclusive seller consists in offering a cream-skimming contract (q', t') so that $u_L(q', t') > u_L(Q_L, T_L)$ and $u_H(q', t') < u_H(Q_L, T_L)$. Following standard arguments (using single-crossing and continuity of the utility function), such a contract (q', t') always exists and can be chosen to be arbitrarily close to (Q_L, T_L). In effect, one can choose a (q', t') that is profitable conditional on trading with low type buyers only: $q' < Q_L + \frac{\epsilon}{2}$ and $t' > T_L - \frac{\epsilon}{2} > Q_L c_L + \frac{\epsilon}{2} > q' c_L + \frac{\epsilon}{2}$. And since $u_H(Q_L, T_L) \leq u_H(Q_H, T_H)$ due to incentive compatibility, the exclusive contract (q', t') only attracts low type buyers.

Step 2. we show that $T_H = Q_H\, c_H$. First, step 1 implies that in equilibrium no pooling contract can be actively traded. Since sellers serving high types cannot make a negative profit, it follows that $T_H \geq Q_H\, c_H$. And due to Bertrand's competition, the unit price for serving high type buyers must be smaller or equal to c_H. As a result, we have that $T_H = Q_H\, c_H$.

Step 3. we observe that the allocation must be efficient. This means that $Q_H = \arg\max_{Q\geq 0} u_H(Q, c_H Q)$, and $Q_L = \arg\max_{Q\geq 0} u_L(Q, c_L Q)$ subject to high type incentive compatibility, i.e., $u_H(Q_H, Q_H c_H) \geq u_H(Q_L, Q_L c_L)$. But this is the Rothschild-Stiglitz allocation so efficiency follows from their familiar arguments.

□

4 Never Exclusive: Uniqueness of Equilibrium

In this section, our focus will be the possible equilibrium in all the possible never exclusive market structures, which are the market structure in which no seller has exclusive power. We denote the equilibrium allocation by (Q_θ, T_θ) which is the aggregate trades of type θ, we also specify the individual trades of type θ trade with buyer k as (q^k_θ, t^k_θ). Then by using the single-crossing assumption, we have $Q_H \geq Q_L$. Next, we will first study some necessary conditions and second show the existence of equilibrium. We also define the u_θ as the equilibrium utility of each type θ.

Definition 5. *A market structure $\mathcal{M}$ is never-exclusive if no seller has the right to exclusively trade with buyers, i.e.,* $\max_{M\in\mathcal{M}:k\in M} |M| \neq 1$ *for all* $k \in \mathcal{K}$.

In the never-exclusive environment, if a buyer type θ trade with some the seller k with a quantity-transfer pair (q,t), we define the indirect utility as follow:

$$z_\theta^{-k}(q,t)=\max\left\{u_\theta\left(q+\sum_{l\neq k}q^l,t+\sum_{l\neq k}t^l\right):\left(q^l,t^l\right)\in C^l \text{ for each } l\in M, l\neq k\right\}$$

This means that the buyer's utility may not rely on one single seller, thus any deviation that aims to attract buyers should consider the interaction of other sellers' menus of contracts.

Denote the $s_\theta^k \equiv t_\theta^k - c_\theta q_\theta^k$ the individual profits when seller k trade with type θ buyer, then then the expected individual profits is $s^k \equiv m_L s_L^k + m_H s_H^k$. Let $B_H \equiv T_H - T_L - c_H(Q_H - Q_L)$, which is the top-up layer aggregate profit. Then denote that $b_H^k \equiv t_H^k - t_L^k - c_H(q_H^k - q_L^k)$ the difference profit on different types of seller k's trade.

Lemma 2. *In any equilibrium, we have $B_H \leq 0$ in any equilibrium.*

Proof. Suppose, to the contrary, $B_H > 0$, due to $B_H = \sum_k s_H^k$,then there is a seller k who trade in the equilibrium with menu $\{(q_L^k,t_L^k),(q_H^k,t_H^k)\}$, such that $B_H > b_H^k$, that is to say,

$$\begin{aligned}&T_H - T_L - c_H(Q_H - Q_L) > t_H^k - t_L^k - c_H(q_H^k - q_L^k)\\ \Rightarrow &t_L^k + T_H - T_L - \epsilon - c_H(q_L^k + Q_H - Q_L) > t_H^k - c_H q_H^k = s_H^k \quad (\epsilon > 0 \text{ small enough})\end{aligned}$$

Then seller k can propose a deviation $\{(q',t'),(q_L^k,t_L^k)\}$, with $(q',t') = (q_L^k + Q_H - Q_L, t_L^k + T_H - T_L - \epsilon)$ where ϵ small enough, then buyer H is better off if trade $(Q_L - q_L^k, T_L - t_L^k)$ from other sellers and then trade (q',t') with seller k. This is because $z_H^{-k}(q',t') \geq u_H(Q_H, T_H - \epsilon) > u_H = z_H^{-k}(q_H^k,t_H^k)$. Then if (q',t') not attractive for low types, the aggregate profit of deviation is

$$m_H[t_L^k + T_H - T_L - \epsilon - c_H(q_L^k + Q_H - Q_L)] + m_L s_L^k > m_H s_H^k + m_L s_L^k$$

And if (q',t') also be better than (q_L^k,t_L^k), then the profit of trade with low type is $[t_L^k + T_H - T_L - \epsilon - c_L(q_L^k + Q_H - Q_L)] = s_L^k + B_H + (c_H - c_L)(Q_H - Q_L) - \epsilon > s_L^k$, due to $B_H > 0$. So in any situation , $\{(q',t'),(q_L^k,t_L^k)\}$ is a profitable deviation for seller k. Contradiction. As a result, we should have that $B_H \leq 0$.

□

Lemma 3. *In any equilibrium, we have $T_L \leq cQ_L$.*

Proof. To the contrary, suppose if $T_L > cQ_L$, denote the aggregate profit on type L is $S_L \equiv \sum_k s_L^k$, then $S_L = T_L - c_L Q_L > T_L - cQ_L > 0$.

If there exists one k such that $S_L > s_L^k$, then we claim that $s^k \geq S = T_L - cQ_L + m_H B_H$, otherwise, k can propose a deviation of $(q',t') = (Q_L, T_L - \epsilon)$, this is attractive for low type, due to $z_L^{-k}(q',t') \geq u_L(Q_L, T_L - \epsilon) > u_L$, even if it attracts high type also, it derives the total profit of $T_L - cQ_L - \epsilon = T_L - cQ_L - \epsilon + m_H B_H - m_H B_H = S - m_H B_H - \epsilon$, when ϵ is small enough we have that the deviation profit is equal to the S. This also means that other sellers earn zero profit, due to $S = s^k \equiv \sum_k s^k$. However, in this situation, any seller j can deviate to propose $(q', t') = (Q_L, T_L - \epsilon)$ to get strict positive profit, so contradiction, as a result, we have that $T_L \leq cQ_L$.

□

Lemma 4. *In any equilibrium, $B_H \geq 0$.*

Proof. Suppose to the contrary,if the equilibrium we have that $B_H < 0$. Then the aggregate profit of the whole economy is $S \equiv \sum_k s^k = m_L(T_L - c_L Q_L) + m_H(T_H - c_H Q_H) = T_L - cQ_L + m_H B_H < 0$, due to $T_L \leq cQ_L$ according to Lemma 3 and $B_H < 0$. It means there always exists one seller k with $s^k < 0$, in that situation, a profitable deviation for k is chosen to provide a null contract. Thus, as a result, we have that $B_H \geq 0$. □

Proposition 1 (Zero-Profit)**.** *In any equilibrium, $S = 0$ and so each seller earns zero profit, moreover, the equilibrium allocation $(Q_L, T_L), (Q_H, T_H)$ should satisfy that:*

$$T_L = Q_L\, c$$
$$T_H - T_L = (Q_H - Q_L)\, c_H$$

Proof. First, by combining Lemma 2 and 4 we have $B_H = 0$. And the total profit of the economy $S \geq 0$, but in the same time $S \equiv \sum_k s^k = m_L(T_L - c_L Q_L) + m_H(T_H - c_H Q_H) = T_L - cQ_L + m_H B_H = T_L - Q_L\, c \leq 0$, thus, we have that $S = 0$ and $T_L - Q_L\, c = 0$, which proved the proposition. □

Lemma 5. *In any equilibrium, $s^k_H \leq 0$.*

The intuition for Lemma 5 is as follows. If $s^k_H > 0$ for some seller k with a contract of (q^k_H, t^k_H), then this seller can deviate to only propose this contract in the market. Compared to zero profit of menu, this deviation can at least have a profit of s^k_H, if it attracts low types, it is more profitable. Thus, we must have that $s^k_H \leq 0$.

Lemma 6. *In any equilibrium, for each active seller k, $t^k_L = q^k_L\, c$, $b^k_H = 0$,*

Proof. First, we show that $b^k_H \geq 0$. (1) when $s^k_H = 0$, according to that $s^k = 0$, we have that $s^k_L = 0$, so that $q_L = 0, b^k_H = 0$ (2)when $s^k_H < 0$, then $s^k_L > 0$ due to $s^k - 0$. Then assume to the contrary if $b^k_H < 0$, that is to say, $t_H - c_H q^k_H < t^k_L - c_H q^k_L$, seller k can deviate to only provide (q^k_L, t^k_L) instead of the menu, if attracts both types, it is better than the old one because of $b^k_H < 0$, if only attracts low types, $s^k_L > 0$, it is still a profitable deviation. Thus, $b^k_H \geq 0$ for any active seller k.

Second, we show that $b^k_H = 0$. According to Proposition 1, $B_H \equiv \sum_k b^k_H = 0$ and we have $b^k_H \geq 0$,then we have $b^k_H = 0$.

Last, we show that $t^k_L = c\, q^k_L$. From Proposition 1, we have $s^k \equiv t^k_L - cq^k_L + m_H b^k_H = 0$, and we know $b^k_H = 0$, then it means $t^k_L = c\, q^k_L$.

then for seller k, it is a profitable deviation that only propose (q^k_L, t^k_L): due to $s^k_H \leq 0$ □

Lemma 7. *If $\tau_L(0,0) > c$, then in the equilibrium allocation, the marginal utility of type θ buyers should equal the tail expectation serving cost from type θ, $\tau_L(Q_L, T_L) = c$, and $\tau_H(Q_H, T_H) = c_H$ if $\tau_H(Q_L, T_L) > c_H$.*

Proof. We first show that $\tau_L(Q_L, T_L) = c$, if it is not satisfied, we assume that a seller k trades with low type in the equilibrium with (q_L^k, t_L^k) we have that $t_L^k = cq_L^k$ according to Lemma 6, thus this seller k can propose deviation as $(q^{k'}, t^{k'}) = (q_L^k + \delta, t_L^k + \epsilon)$ with δ and ϵ chosen so that:

$$\tau_L(Q_L, T_L)\delta > \epsilon > c\delta$$

When δ and ϵ are small enough, "$\tau_L(Q_L, T_L)\delta > \epsilon$" implies that the new contract is attractive for low type buyers. (If δ is positive, it is the case of undersupply; If δ is negative, it is the case of oversupply). Then even if $(q^{k'}, t^{k'})$ attracts high type buyers, then the seller could still achieve a positive profit with $\epsilon - c\delta > 0$ with second inequality, thus a positive profitable deviation, thus,$\tau_L(Q_L, T_L) = c$. Using a similar logic, we can prove that $\tau_H(Q_H, T_H) = c_H$. □

Theorem 1 (Uniqueness)**.** *Posit Assumption 5, under any never-exclusive market structure, the unique equilibrium candidate is JHG allocation: (Q_L, T_L) and (Q_H, T_H) defined as below:*

$$\begin{aligned} Q_L &= argmax\{u_L(Q, cQ) : Q \geq 0\} \\ T_L &= cQ_L \\ Q_H - Q_L &= argmax\{u_H(Q_L + Q, T_L + c_H Q) : Q \geq 0\} \\ T_H - T_L &= c_H(Q_H - Q_L) \end{aligned}$$

The Theorem 1 is the result of all the Lemma mentioned in this section. The candidate equilibrium allocation consists of two layers of quantity and transfer: the first layer is priced with average cost serving for both types, while the second layer is priced with unit cost serving for high types. Two layers are break-even.(As shown in Figure 2).

Theorem 1 states that in any never exclusive market, the unique equilibrium is JHG allocation. It doesn't require a competitive feature for the market structure, or totally nonexclusive—thus, any structure with at least two sellers trading nonexclusively will achieve this result, no matter fully nonexclusive or not competitive. JHG allocation is the unique equilibrium candidate.

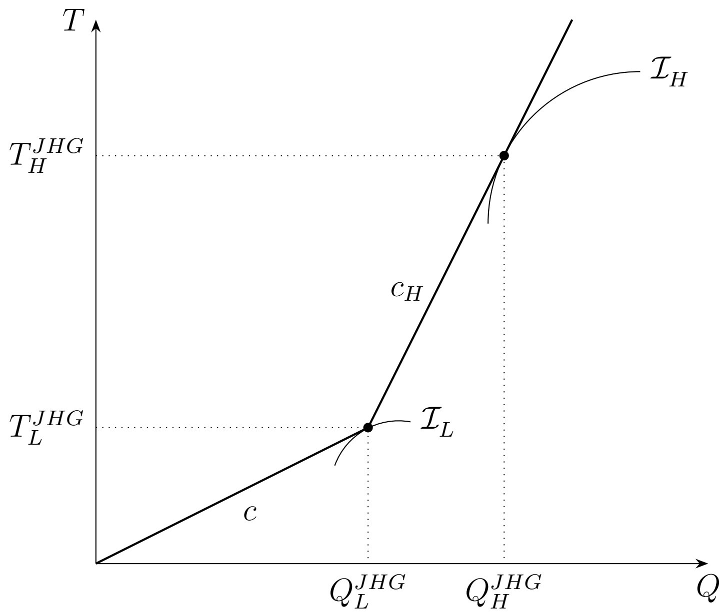

Figure 2: Nonexclusive Competition:JHG Allocation

5 Nonexistence in Nonexclusive and Existence in Restrictive Nonexclusive

5.1 Nonexistence of Equilibrium in Nonexclusive Market

From Theorem 1, we know that the only possible equilibrium candidate with never exclusive market structure is JHG allocation with (Q_L, T_L) and (Q_H, T_H). However, as in Attar et al. (2014), this allocation is not an equilibrium in a nonexclusive market structure.

Lemma 8. *In the equilibrium of nonexclusive market structure, for any active seller k, the seller can trade (Q_L, T_L) with buyers other than k.*

The intuition of Lemma 8 is that if one seller can replace the seller k to get the aggregate allocation (Q_L, T_L), then seller k can deviate and achieve positive profit. Actually, according to Lemma 6, we have $t_L^k = cq_L^k$, then if k has no competitor, she can deviate to propose $(q_L, t_L + \epsilon)$ to get positive profit [21].

Proposition 2. *JHG allocation is not an equilibrium in the nonexclusive market structure.*

Proof. Suppose JHG allocation is an equilibrium in the nonexclusive market structure. Then we will show that any active seller k can propose a double deviation menu to have a positive profit.

For an active seller k, consider a menu that consists of the non-trade contract and the contract $(q_1, t_1) = (Q_H - Q_L, T_H - T_L - \epsilon_1)$ and the contract $(q_2, t_2) = (q_L^k - \delta_2, t_L^k - \epsilon_2)$, for

[21] If there exists some other seller can have $u_L(Q^{-k}, T^{-K}) = u_L$, then $Q^{-k} \neq Q_L$ suggests that $T^{-k} < cQ^{-k}$, so k can pivoting on (Q^{-k}, T^{-k}) to have positive profit.

some $\epsilon_1, \epsilon_2, \delta_2$ small enough. and with specific relations of three small variables to ensure that $z_L^{-k}(q_1, t_1) \geq u_L(Q_L - \delta_2, T_L - \epsilon_2) > u_L$ and due to Lemma 8 we can have $z_H^{-k}(q_2, t_2) > u_H > z_H^{-k}(q_1, t_1)$. Then the total profit of this deviation is:

$$m_L(t_L^k - \epsilon_2 - c_L(q_L^k - \delta_2)) - m_H(T_H - T_L - \epsilon_1 - c_H(Q_H - Q_L))$$

When $\epsilon_1, \epsilon_2, \delta_1$ is small enough, this is equal to the $m_L(t_L^k - c_L q_L^k) > m_L(t_L^k - c q_L^k) = 0$. Thus, these two contracts ensure that seller k can get a positive profit, which is a desirable deviation. □

Thus, Theorem 1 and Proposition 2 suggests that if $\tau_L(0,0) > c_L$, there is no equilibrium in the nonexclusive environment. As shown above, the two contracts pair can always destroy the unique equilibrium candidate under a fully nonexclusive environment, we call that **Double Deviation**. However, we also observe that in real life, there are many markets that have nonexclusive features, how can we understand the equilibrium in real life? In the next subsection, we show the existence of some restrictive nonexclusive market structures, that is to say, if there exists some restriction on the nonexclusive feature, JHG allocation is an equilibrium allocation.

5.2 Existence of Equilibrium in "1+1" Market Structure

In this part, we would like to show the possible implementation of the equilibrium with JHG allocation in "1+1" market structure. First , the definition of "1+1" market structure is as follow:

Definition 6 ("1+1" market structure)**.** *$\mathcal{M}$ is a "1+1" market structure if the sellers $\mathcal{K} = \{1, ..., K\}$ can be partitioned into two disjoint subgroups $\mathcal{K}_1$ and $\mathcal{K}_2$ so that buyers can never trade with two sellers from the same subgroup at the same time:*

$$\mathcal{M} = \big\{\{j, k\} : j \in \mathcal{K}_1 \cup \{\emptyset\},\, k \in \mathcal{K}_2 \cup \{\emptyset\}\big\}.$$

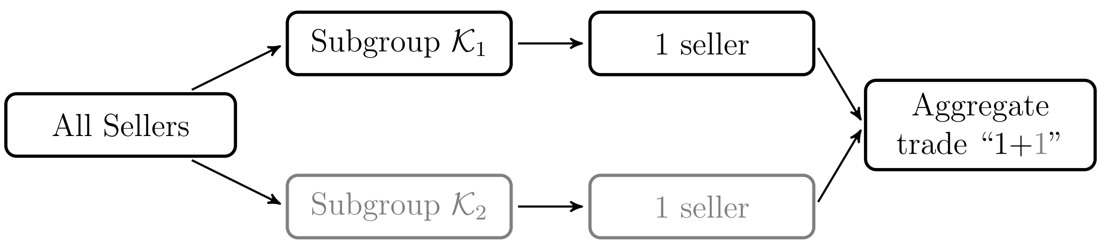

Figure 3: The "1+1" market structure. Buyers select at most one seller from each subgroup.

We next show the construction of the equilibrium allocation with JHG allocation in "1+1" market structure. The main difference of "1+1" market structure and nonexclusive market structure is the double deviation in nonexclusive market structure is not available in "1+1" market structure because of the exclusivity in each subgroup.

5.2.1 Equilibrium: On-path Menus

The menus in subgroup $\mathcal{K}_1$ support the JHG allocation.

We wanted that the sellers in subgroup $\mathcal{K}_1$ could propose at least the pooling contract (Q_L, T_L), due to the sellers can propose a menu of contracts or a tariff specifies on different quantities, we hope that the sellers who propose this kind of pooling could also help the market block the cream-skimming deviation in $\mathcal{K}_2$, then we can construct at least two sellers in $\mathcal{K}_1$ provides the tariff with

$$T(Q) = 1_{\{Q \leq Q_L\}} cQ + 1_{\{Q > Q_L\}} [cQ_L + c_H(Q - Q_L)] \tag{1}$$

Actually, this kind of tariff is entry-proof to the sellers of $\mathcal{K}_2$, and due to buyers can at most purchase from one seller in $\mathcal{K}_1$, so the sellers who propose this kind of tariff in $\mathcal{K}_1$ will not affect the possible profit of others. Let $\mathcal{C}_1^*$ be the menu of this tariff in subgroup $\mathcal{K}_1$.

The menus in $\mathcal{K}_2$ with additional separating contracts. The sellers in $\mathcal{K}_2$ were designed to provide the additional contracts $(Q_H - Q_L, T_H - T_L)$, to block some potential deviation, we construct at least two sellers in $\mathcal{K}_2$ propose the tariff with

$$T(Q) = c_H Q \tag{2}$$

Let $\mathcal{C}_2^*$ be the menu of this tariff in subgroup $\mathcal{K}_1$.

5.2.2 Equilibrium: Off-path Latent contract

The latent contract in $\mathcal{K}_2$ to block the large cream-skimming deviation in $\mathcal{K}_1$

Followed with Attar et al. (2022), we will start dealing with the large cream-skimming deviations. It is a deviation designed for attracting type L only, even if she doesn't trade with other contracts. We call them **large cream-skimming deviations** which proposes a large quantity-tariff (Q', T') pair and gives a better utility than $u_L(Q_L, T_L)$ and attracts low type . To deal with this kind of deviation, we would like to find a latent contract to block the possible large cream-skimming deviations:

Definition 7. *A contract (q^ℓ, t^ℓ) blocks large cream-skimming deviations if:*

$$\textit{for each } (q,t), u_L(q,t) \geq u_L(Q_L, T_H) \textit{ implies } u_H(q + q^\ell, t + t^\ell) \geq u_H(Q_H, T_H) \tag{3}$$

We would like to find a single latent that could block ll large cream-skimming deviations. Then, we can have that the only possible candidate (q^ℓ, t^ℓ) which can block all large cream-skimming deviations is the contract defined as below:

$$\begin{aligned} &u_H\left(Q_L + q^\ell, T_L + t^\ell\right) = u_H\left(Q_H, T_H\right) \\ &\tau_H\left(Q_L + q^\ell, T_L + t^\ell\right) = c \end{aligned} \tag{4}$$

To avoid the large cream-skimming in $\mathcal{K}_1$, we need that there is at least one seller in subgroup $\mathcal{K}_2$ propose (q^ℓ, t^ℓ) defined by equation 4. With the assumption of Perfect translation, this kind of contract in $\mathcal{K}_2$ can block the cream-skimming deviation in $\mathcal{K}_1$.

The latent contract in $\mathcal{K}_2$ to block the small cream-skimming deviation in $\mathcal{K}_1$
There are two possible small deviations in $\mathcal{K}_1$: (1) the first is to propose some (q, t) in $\mathcal{K}_1$ and attracts low type if she can combine (q, t) with latent contact (q^ℓ, t^ℓ) to get better utility: in this case, Let n_ℓ be the smallest integral number satisfies $n_\ell q^\ell > Q_H$, we just need to construct the seller who proposes menu with n_ℓ latent contract also has the contracts as :

$$(q^{k\ell}, t^{k\ell}) \equiv k \times (q_\ell, t_\ell) \tag{5}$$

. Let $\mathcal{C}_l^1$ be the menu of this tariff in subgroup $\mathcal{K}_1$. (2) The second case is (q, t) in $\mathcal{K}_1$ and attracts low type if she can combine (q, t) with additional separating contracts tariff, in this case : we just need to construct at least one seller in $\mathcal{K}_2$ propose the tariff with quantity great than q^ℓ as :

$$T(Q + q^\ell) = t^\ell + c_H Q \tag{6}$$

Let $\mathcal{C}_l^2$ be the menu of this tariff in subgroup $\mathcal{K}_1$. Then all small skimming deviations are blocked by these $\mathcal{C}_l^1$ and $\mathcal{C}_l^2$ of latent contracts.

Assumption 6. ***Flatter Curvatures (ii) (Perfect Translation):***[22]

For each θ, Gaussian Curvature $\kappa_\theta > 0$. Moreover, the following statements holds:

For all Q_L, Q_H, T_L, T_H, if $\tau_L(Q_L, T_L) = \tau_H(Q_H, T_H)$, then $\kappa_L(Q_L, T_L) = \kappa_H(Q_H, T_H)$.

Theorem 2. *Under Assumption 6, JHG allocation is an equilibrium in which there are at least two sellers propose $\mathcal{C}_1^*$ in subgroup $\mathcal{K}_1$, and there are at least two sellers propose $\mathcal{C}_2^*$, $\mathcal{C}_l^1$ and $\mathcal{C}_l^2$ in subgroup $\mathcal{K}_2$.*

To ensure that JHG allocation is an allocation, we need to deal with two main deviations: the first one is the cream-skimming deviation, we could deal with that by using the latent contract which is similar which what we did in Attar et al. (2022). The second deviation is a double deviation, in which a seller in subgroup $\mathcal{K}_1$ tries to screen low type and high type by proposing a menu of contracts. Actually, in a totally nonexclusive environment, no equilibrium exists if the low type trade with positive contracts(Attar, Mariotti and Salanié (2014)). This

[22] This is the Assumption C(ii) in Attar et al. (2022), and CARA utility function and quadratic function are some examples that satisfy this assumption. The Gaussian Curvature is defined as

$$\kappa_\theta(Q_0, T_0) \equiv \frac{1}{\|\nabla U_\theta\|^3} \begin{vmatrix} -\nabla^2 u_\theta & \nabla u_\theta \\ -\nabla u_\theta^\top & 0 \end{vmatrix} (Q_0, T_0) = -\frac{\frac{\partial^2 \mathcal{I}_i}{\partial Q^2}(Q_0, u_\theta(Q_0, T_0))}{\left\{1 + \left[\frac{\partial \mathcal{I}_i}{\partial Q}(Q_0, u_\theta(Q_0, T_0))\right]^2\right\}^{\frac{3}{2}}}$$

is because that one seller can have profitable deviation by posting two contracts, one contract a little bit cheaper than (Q_L, T_L), the other pivoting on the (Q_L, T_L). In our "1+1" market structure, this kind of double deviation is impossible because of the exclusivity of the subgroup $\mathcal{K}_1$, the pivoting contract is exclusive with the (Q_L, T_L). Thus, the double deviation which destroys equilibrium in nonexclusive does not work in "1+1" market structure. But another possible double deviation is using one pivoting contract on a latent contract and then attracting high type buyers, and the other contract attracts low type buyers. Under assumption Flatter curvature (ii), any pivoting contract based on a latent contract to attract high type should have a unit price lower than c, and by the perfect translate indifference curve, it would also attract low type, which means that deviation is not profitable.

5.2.3 Other Restrictive Market Structures: "$\lambda + 1$"and "$\lambda_1 + \lambda_2$"

After the motivating example of "$1 + 1$" market structure case, we can along the same trick of proving the uniqueness, get that JHG allocation is the unique possible equilibrium in menu game without regulation in the "λ+1" [23] and "λ_1+λ_2"[24] structure. Then we will construct that JHG allocation can be an equilibrium if some assumption can be sustained in the preference.

Theorem 3. *If the preference of buyers satisfies the perfect translation property (Flatter Curvature (ii)), then in "$\lambda + 1$" and "$\lambda_1 + \lambda_2$" market structures, JHG allocation is also the equilibrium.*

The trick of existence is similar to "$1 + 1$" case: Firstly, we need the sellers in subgroup $\mathcal{K}_1$ propose the entry-proof tariff, which ensures that there is no deviation in subgroup $\mathcal{K}_2$. Then we need the sellers in $\mathcal{K}_2$ to propose the complementary contract with unit price c_H, and then the latent contract which can block the cream-deviation in subgroup $\mathcal{K}_1$. Take the "$\lambda + 1$" structure as the example, there exist at least $\lambda + 1$ sellers propose the menu with $T(Q) = 1_{\{Q \le \frac{Q_L}{\lambda}\}} cQ + 1_{\{Q > \frac{Q_L}{\lambda}\}} [c\frac{Q_L}{\lambda} + c_H(Q - \frac{Q_L}{\lambda})]$. It is easy to get that combination of λ sellers in $\mathcal{K}_1$ forms an entry-proof tariff, thus, there is no profitable deviation in $\mathcal{K}_2$. And due to the restriction of trade at most "λ" seller in $\mathcal{K}_1$, it is not possible for high type trade pooling contract larger than Q_L. Thus, the possible profitable deviation only exists in $\mathcal{K}_1$ group sellers.

For the cream-skimming deviation in $\mathcal{K}_1$, the latent contract in $\mathcal{K}_2$ can block it. Then the only possible profitable deviation in $\mathcal{K}_1$ is double deviation which propose a menu with different contracts c_L^k, c_H^k to attract different types, where $c_L^k = (q_1^k, t_1^k), c_H^k = (q_2^k, t_2^k)$.$c_L^k$ attracts low type only if the unit price is lower than c: (1)If c_H^k combine complementary on latent contract in $\mathcal{K}_2$, then we will have that $t_2^k < cq_2^k$, and by perfect translation of utility function if c_H^k is preferred by high type than c_L^k with a combination of latent contract, then

[23] Divide sellers into two subgroups, trade at most λ sellers in group 1 and at most one seller in group 2
[24] Divide sellers into two subgroup, trade at most λ_1 sellers in group 1 and at most λ_2 seller in group 2

low type also prefer c_H^k , but the average cost for serving both types is c, negative profit. (2) If there is no high type combined with the latent contract in this menu, firstly, if a buyer chooses one of the contracts of the deviation, the tariff given by other sellers in this $\mathcal{K}_1$ is $T'(Q) = 1_{\{Q \le \frac{\lambda-1}{\lambda} Q_L\}} cQ + 1_{\{Q > \frac{\lambda-1}{\lambda} Q_L\}} [c \frac{\lambda-1}{\lambda} Q_L + c_H(Q - \frac{\lambda-1}{\lambda} Q_L)]$.

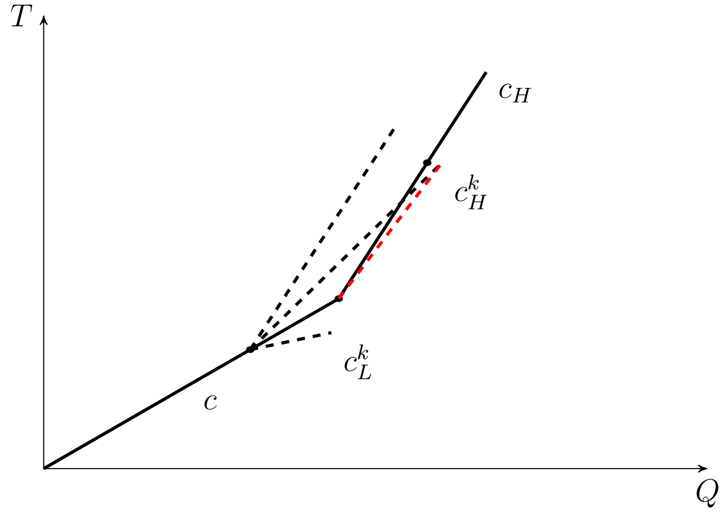

Figure 4: The small double deviation

Then we can divide the possible c_L^k in two cases, the first one is that $q_1^k < \frac{1}{\lambda} Q_L$, in this case, c_H^k attracts high type only if the combination of c_H^k can give better utility than (Q_H, T_H) , but this indicates that the total profit of c_L^k and c_H^k will be negative. As in Figure 4, c_H^k can be split as the $\frac{1}{\lambda} Q_L + (q_2', t_2')$ with $t_2' < c_H q_2'$, so it is straightforward to get the total profit is negative.

As in Figure 5, the second possible deviation is that $q_1^k > Q_L$, there are two cases for the high type's contracts: c_H^k can have a unit price lower than c_L^k or greater than c_L^k. When c_H^k has a lower unit price and attracts high type , the convexity of the available tariff suggests that c_H^k also more attractive for the low type, which means c_H^k attracts both types with a unit price lower c, which is not profitable. Then c_H^k has a larger unit price of c_L^k, by the same trick in small double deviation, the total profit is negative. To conclude, JHG allocation can be an equilibrium in this menu game.

6 Conclusion

From the viewpoint of different market structures, this paper analyzed the market outcomes among all the competitive market structures where the market is plagued by adverse selection. We found that the equilibrium allocation candidates are different with two main market structures: Partially exclusive market structures and Never exclusive market structures, which depend on whether include at least one seller has the ability to exclusively trade with buyers.

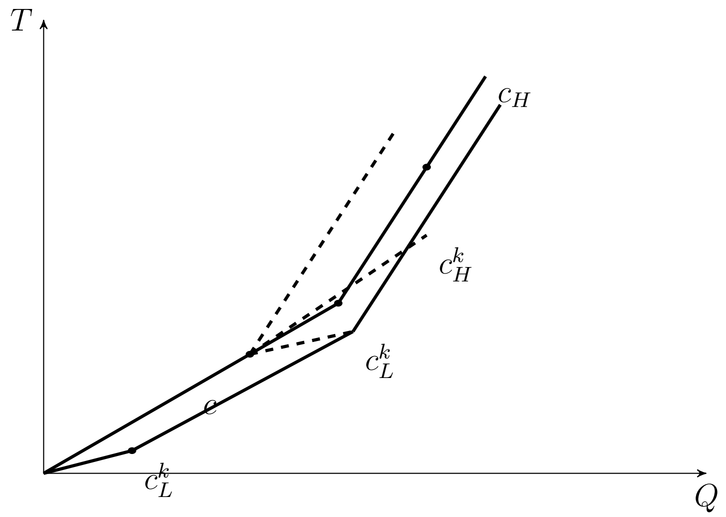

Figure 5: The large double deviation

The first results show that, under any partially exclusive competitive markets, the equilibrium allocation is the same as those obtained in an exclusive environment where buyers compete through any menu offers. Thus, in any competitive partially exclusive structure, the unique equilibrium candidate is one type of buyer gets full insurance (or the efficient allocation) and the other type of buyer achieves partial insurance (or rationed with quantity).

However, under Inada's condition, in any never exclusive market structure, I found that the unique equilibrium candidate is the JHG allocation as in Theorem 1: both types purchase a pooling layer contracts with the unit price equal to the average serving cost, and then high-risk types purchase additional layers of contracts with unit price equal to serving cost of high type. The Theorem of uniqueness even doesn't require a competitive feature of market structure, it suggests that the not exclusive trade has a very strong impact on the equilibrium allocation. In the not exclusive trade, each seller has less ability to control the possible buyers' trade with other sellers, which provides each seller more deviation space.

In the nonexclusive market structure, I replicate the result in Attar et al. (2014) to show that the JHG allocation is not an equilibrium. The nonexistence of equilibrium is because, in nonexclusive trade, there are very large spaces of deviations that one seller can take advantage of **any** other sellers' contracts, which makes screening more difficult and it is difficult to satisfy the competitive and no profitable deviations requirements. Based on competitive contracts in nonexclusive competition, one seller can propose a double deviation that attracts different types of buyers to get positive profit.

Then I showed that, under the assumption of perfect translation, in the "1+1","$\lambda + 1$", "$\lambda_1 + \lambda_2$" market structure, JHG allocation is the equilibrium allocation as in Theorem 2 and 3. In these market structures, we can still find the feature of nonexclusive competition, any seller's menu of contracts depends on and affects other sellers' contracts. But there exist some restrictions on the number of sellers that one buyer can maximize trade with, this new feature helps the sellers prevent double deviations based on the competitive contracts. The restriction on the number of sellers also decreases the sets of deviations that one seller can propose. In real life the restriction on the number of sellers could happen not only due to the restriction from the planner but also comes from the restriction on buyers' ability to trade with all sellers or the searching cost for different places sellers. The introduction of restrictions on the number of sellers' trade will not lose the feature of nonexclusive, however, it restores the existence of the equilibrium of the economy with the nonexclusive competition.

In the end, it is important to stress that the existence of JHG allocation as equilibrium relies on the assumption of perfect translation utility, which means high and low type buyers have the same shape of indifference curve. The CARA and the quadratic utility function are two examples that satisfy this assumption. Biais et al. (2000) also uses the quadratic utility function to show the existence results in the setting of a continuous types model, in their paper, they show that when the number of sellers becomes large enough(no seller is indispensable or competitive), the equilibrium converges to the Glosten allocation. In my paper, I use the restriction on the number of trades and more sellers than the restriction of sellers to ensure a competitive feature of the market and then show the existence of Glosten allocation as an equilibrium in a discrete type model. This findings shed light on the disconnection between continuous type models (associated with existence results) and discrete type models (associated with the absence thereof).

Appendix

.1 Proof of Theorem 2

Step 1, we construct the equilibrium menus of each group:

For the subgroup $\mathcal{K}_1$:

1. At least two sellers propose the entry-proof tariff with $T(Q) = 1_{\{Q \leq Q_L\}} cQ + 1_{\{Q > Q_L\}}[cQ_L + c_H(Q - Q_L)]$.
2. Other sellers could propose the same menu as 1 or null contracts

For the subgroup $\mathcal{K}_2$:

1. At least two sellers propose the separating tariff with: $T(Q) = c_H Q$
2. At least one sellers propose the menu tariff with: $T(Q + q^\ell) = t^\ell + c_H Q \quad (Q \geq 0)$
3. At least one sellers propose the menu with contracts : $(q_k^\ell, t_k^\ell) \equiv k \times (q_\ell, t_\ell)$, where $k \leq n_\ell$.
4. Other sellers could propose one of 1,2,3 or null menus.

Then to get the JHG allocation, type L and type H purchase (Q_L, T_L) in $\mathcal{K}_1$ and then purchase $(Q_H - Q_L, T_H - T_L)$ in subgroup $\mathcal{K}_2$.

Step 2, We show that it is best response to both types:

(1) Consider first for type L : we know that all the contracts in two subgroup have the premium rate at least c , so the maximal utility that type L can get $u_L^* \leq max\{u_L(Q, cQ) : Q \geq 0\} = u_L(Q_L, T_L)$, so it is best response for type L.

(2) Then for type H buyers: (1) If combining with any latent contract , given any other menu has a unit price at least c, the combination of latent contract and others will locate at least the line with slope c that supports her upper contour set of (Q_H, T_H), which gives type H worse utility. (2) If not combining with latent contract, $\tau_H(Q_H, T_H) = c_H$ means no additional contracts with unit price of c_H is attractive for type H, and the maximal pooling quantity of type H can have is Q_L, thus the max utility of type H $u_H^* \leq max\{u_H(Q_L + Q, T_L + c_H^*) : Q \geq 0\} = u_H(Q_H, T_H)$.Thus, it is also the best response to type H buyers.

Step 3, We then prove that no seller has a profitable deviation that only attracts type H buyers. According to the construction in step 1, type H can always get the utility $u_H(Q_H, T_H)$ even if any one of the sellers deviates to other menus. Thus, given the utility $u_H(Q_H, T_H)$, a contract (q, t) attracts type H should satisfy that $t < c_H q$, but if only attracts type 2 buyers, it is not profitable deviation.

Step 4, We next prove that no seller has a profitable deviation that only attracts type L buyers.

Firstly, if one seller deviates to other menus, type L can still achieve the utility $u_L(Q_L, T_L)$ with other sellers with the construction in step 1. Next, if a seller in $\mathcal{K}_2$ tries to propose a contract (q, t) to attract type L buyers, according to Attar, Mariotti and Salanié (2021a), due to the tariff in $\mathcal{K}_1$ is entry-proof to the market, any seller in $\mathcal{K}_2$ can at most achieve 0 profit given the menu in $\mathcal{K}_1$.

Then, there is a seller proposing (q, t) as cream-skimming deviation in $\mathcal{K}_1$, then to attract type L consumers, it requires that $t \leq cq$. And this contract is profitable only if $t > c_L q$. So any cream-skimming deviation must belong to the cone :

$$X \equiv \{(q, t) : cq \geq t \geq c_L q\} \tag{7}$$

We can discuss the cream-skimming deviation in two cases: Large cream-skimming deviation in $\mathcal{K}_1$, Small cream-skimming deviation in $\mathcal{K}_1$.

case (1), Large cream-skimming deviation in $\mathcal{K}_1$.that is a contract $(q, t) \in X$ proposed by some seller in $\mathcal{K}_1$ such that :

$$u_L(q, t) > u_L(Q_L, T_L) \tag{8}$$

By using the lemma 4 and with the latent contract (q^ℓ, t^ℓ) in $\mathcal{K}_2$, we have that :

$$u_H(q + q^\ell, t + t^\ell) > u_H(Q_H, T_H) \tag{9}$$

So (q, t) in $\mathcal{K}_1$ also attracts type H with the latent contract (q^ℓ, t^ℓ)in subgroup $\mathcal{K}_2$. However, if (q, t) trades with both types, the marginal serving cost is c, and $t \leq cq$ means this contract can get at most 0 profit.

Case (2), Small cream-skimming deviation in $\mathcal{K}_1$.

1. If a small cream-skimming deviation (q, t), type L can combine (q, t) and a contract $(Q, c_H Q)$ for some Q in subgroup $\mathcal{K}_2$. which makes that

$$u_L(q + Q, t + c_H Q) > u_L(Q_L, T_L) \tag{10}$$

Then by using the lemma 2 , (17) also means that

$$u_H(q + Q + q^\ell, t + c_H Q + t^\ell) > u_H(Q_H, T_H) \tag{11}$$

while the contract of $(Q + q^\ell, c_H Q + t^\ell)$ is one of latent contract as construction in step 1. so this small deviation was blocked by the latent contract in subgroup $\mathcal{K}_2$.

2. If a small cream-skimming deviation is (q, t), and type L can combine (q, t) and latent contract (q^ℓ_k, t^ℓ_k).$(1 \leq k \leq n_\ell - 1)$ in subgroup $\mathcal{K}_2$ to get better utility, then this deviation will be blocked by $(q^\ell_{(k+1)}, t^\ell_{(k+1)})$. and then if type L combines (q, t) with $(q^\ell_{n_\ell}, t^\ell_{n_\ell})$, the aggregate quantity is larger than (Q_H, T_H), so still attract type H also. Thus, the (q, t) satisfies this case also attracts type H .

Step 5, In the end, we show that no seller k has profitable deviation that attracts both types with menu $\{c^k_L, c^k_H\}$.

If $c^k_L = c^k_H$, then it is a pooling contract to market and trying to attract both types with contract (q^k, t^k). We know that $u_L(Q_L, T_L) = max\{u_L(Q, cQ) : Q \geq 0\}$, then (q^k, t^k) attracts both types only if $t^k < cq^k$, but the serving cost to both type is c, which means the deviation with $t^k < cq^k$ is not profitable.

If $c^k_L \neq c^k_L$, then we can discuss the case as follow:

1. If the seller is in the subgroup $\mathcal{K}_2$, then given the tariff in subgroup $\mathcal{K}_1$ is entry-proof to the market with nonexclusivity , due to $\mathcal{K}_1$ and $\mathcal{K}_2$ are non-exclusive trade, then the maximal profit of seller k can get is 0, so it is not profitable deviation.

2. If the seller is in subgroup $\mathcal{K}_1$, and $c^k_L = (q^k_1, t^k_1), c^k_H = (q^k_2, t^k_2)$ with $q^k_1 \leq Q_L$. c^k_L attracts type L only if $t^k_1 < cq^k_1$, then if c^k_H is attractive for type H without combination with other contracts, which means c^k_H gives better utility than (Q_H, T_H), it is easy to conclude that the total profit of c^k_L, c^k_H is negative.If c^k_H attract type H with combination of other contract in $\mathcal{K}_2$, then we can discuss the situation in two cases:
   (1) type H can combine c^k_H and the contract with unit price c_H in $\mathcal{K}_2$ to have better utility, then we will have that $q^k_2 > Q_L \geq q^k_1$, there exist one (q', t') such that $c^k_H = (Q_1, T_L) + (q', t')$ with $t' < c_H q'$, then the aggregate profit of c^k_L, c^k_H will be negative.

   (2) if type H combine c^k_H with a latent contract in $\mathcal{K}_2$, then we have that $t^k_2 < cq^k_2$ and type H prefers c^k_H than c^k_L, then due to the assumption flatter curvature (ii), by perfect translate with the latent contract, we will also have that type L also prefers c^k_H than c^k_L, but the serving cost of both types is c which makes the deviation of seller get negative profit again. Thus, there is no profitable deviation in this case.

.2 Proof of Theorem 3

The uniqueness is the same as the trick in the "$1+1$" partial exclusive structure, we then show the existence of equilibrium in the cases mentioned.

Step 1, For the structure of "$\lambda + 1$", we construct the equilibrium menus of each group:

For the subgroup $\mathcal{K}_1$:

1. At least $\lambda + 1$ sellers propose the tariff with $T(Q) = 1_{\{Q \leq \frac{Q_L}{\lambda}\}} cQ + 1_{\{Q > \frac{Q_L}{\lambda}\}} [c\frac{Q_L}{\lambda} + c_H(Q - \frac{Q_L}{\lambda})]$.
2. Other sellers could propose the same menu as 1 or null contracts

For the subgroup $\mathcal{K}_2$:

1. At least two sellers propose the separating tariff with: $T(Q) = c_H Q$
2. At least one seller propose the menu tariff with $T(Q + q^\ell) = t^\ell + c_H Q \quad (Q \geq 0)$
3. At least one seller proposes the menu with contracts: $(q_k^\ell, t_k^\ell) \equiv k \times (q_\ell, t_\ell)$, where $k \leq n_\ell$.
4. Other sellers could propose one of 1,2,3 or null menus.

Then to get the JHG allocation, type 1 and type 2 purchase (Q_L, T_L) in $\mathcal{K}_1$ and then purchase $(Q_H - Q_L, T_H - T_L)$ in subgroup $\mathcal{K}_2$.

Step 2, We show that it is best response to both types:

(1) Consider first for type 1 : we know that all the contracts in two subgroups have the premium rate at least c , so the maximal utility that type 1 can get $u_L^* \leq max\{u_L(Q, cQ) : Q \geq 0\} = u_L(Q_L, T_L)$, so it is the best response for type 1.

(2) Then for type 2 buyers: (1) If combining with any latent contract, given any other menu has a unit price at least c, the combination of latent contract and others will locate at least the line with slope c that supports her upper contour set of (Q_H, T_H), which gives type 2 worse utility. (2) If not combined with the latent contract, $\tau_H(Q_H, T_H) = c_H$ means no additional contracts with a unit price of c_H is attractive for type 2, and the maximal pooling quantity of type 2 can have is Q_L, thus the max utility of type 2 $u_H^* \leq max\{u_H(Q_L + Q, T_L + c_H^*) : Q \geq 0\} = u_H(Q_H, T_H)$.Thus, it is also the best response to type 2 buyers.

Step 3, We then prove that no seller has a profitable deviation that only attracts type 2 buyers. According to the construction in step 1, type 2 can always get the utility $u_H(Q_H, T_H)$ even if any one of the sellers deviates to other menus. Thus, given the utility $u_H(Q_H, T_H)$, a contract (q, t) attracts type 2 should satisfy that $t < c_H q$, but if only attracts type 2 buyers, it is not profitable deviation.

Step 4, Cream-skimming deviation: We next prove that no seller has a profitable deviation that only attracts type 1 buyers.

Firstly, if one seller deviates to other menus, type 1 can still achieve the utility $u_L(Q_L, T_L)$ with other sellers with the construction in step 1. Next, if a seller in $\mathcal{K}_2$ tries to propose a contract (q, t) to attract type 1 buyers, according to Attar, Mariotti, and Salanié (2021a), due to the tariff in $\mathcal{K}_1$ is entry-proof to the market, any seller in $\mathcal{K}_2$ can at most achieve 0 profit given the menu in $\mathcal{K}_1$.

Then, there is a seller proposes (q, t) as cream-skimming deviation in $\mathcal{K}_1$, then to attract type 1 consumers, it requires that $t \leq cq$. And this contract is profitable only if $t > c_L q$. So any cream-skimming deviation must belong to the cone :

$$X \equiv \{(q, t) : cq \geq t \geq c_L q\} \tag{12}$$

We can discuss the cream-skimming deviation in two cases: Large cream-skimming deviation in $\mathcal{K}_1$,Small cream-skimming deviation in $\mathcal{K}_1$.

case (1), Large cream-skimming deviation in $\mathcal{K}_1$.that is a contract $(q, t) \in X$ proposed by some seller in $\mathcal{K}_1$ such that :

$$u_L(q, t) > u_L(Q_L, T_L) \tag{13}$$

By using the lemma 4 and with the latent contract (q^ℓ, t^ℓ) in $\mathcal{K}_2$, we have that :

$$u_H(q + q^\ell, t + t^\ell) > u_H(Q_H, T_H) \tag{14}$$

So (q, t) in $\mathcal{K}_1$ also attracts type 2 with the latent contract (q^ℓ, t^ℓ)in subgroup $\mathcal{K}_2$. However, if (q, t) trades with both types, the marginal serving cost is c, and $t \leq cq$ means this contract can get at most 0 profit.

Case (2), Small cream-skimming deviation in $\mathcal{K}_1$.

1. If a small cream-skimming deviation (q, t), type 1 can combine (q, t) and a contract $(Q, c_H Q)$ for some Q in subgroup $\mathcal{K}_2$. which makes that

$$u_L(q + Q, t + c_H Q) > u_L(Q_L, T_L) \tag{15}$$

   Then by using the lemma 2 , (17) also means that

$$u_H(q + Q + q^\ell, t + c_H Q + t^\ell) > u_H(Q_H, T_H) \tag{16}$$

while the contract of $(Q+q^\ell, c_H Q + t^\ell)$ is one of latent contract as construction in step 1. so this small deviation was blocked by the latent contract in subgroup $\mathcal{K}_2$.

2. If a small cream-skimming deviation is (q,t), and type 1 can combine (q,t) and latent contract (q_k^ℓ, t_k^ℓ).$(1 \le k \le n_\ell - 1)$ in subgroup $\mathcal{K}_2$ to get better utility, then this deviation will be blocked by $(q^\ell_{(k+1)}, t^\ell_{(k+1)})$. and then if type 1 combines (q,t) with $(q^\ell_{n_\ell}, t^\ell_{n_\ell})$, the aggregate quantity is larger than (Q_H, T_H), so still attract type 2 also. Thus, the (q,t) satisfies this case also attracts type 2 .

Step 5, In the end, we show that no seller k has profitable deviation that attracts both types with menu $\{c_L^k, c_H^k\}$.

If $c_L^k = c_H^k$, then it is a pooling contract to market and trying to attract both types with contract (q^k, t^k). We know that $u_L(Q_L, T_L) = max\{u_L(Q, cQ) : Q \ge 0\}$, then (q^k, t^k) attracts both types only if $t^k < cq^k$, but the serving cost to both type is c, which means the deviation with $t^k < cq^k$ is not profitable.

If $c_L^k \ne c_L^k$, then we can discuss the case as follow:

1. If the seller is in the subgroup $\mathcal{K}_2$, then given the tariff in subgroup $\mathcal{K}_1$ is entry-proof to the market with nonexclusivity , due to $\mathcal{K}_1$ and $\mathcal{K}_2$ are non-exclusive trade, then the maximal profit of seller k can get is 0, so it is not profitable deviation.

   Then we consider the profitable deviation that attracts different types with different contracts within one menu. In most menu games with nonexclusive structures, double-deviation is a usual problem for sustaining the equilibrium. However, in the "$\lambda + 1$" structure, we will show that the partial exclusive feature can help us prevent the classical double deviation which as proposed in Attar, Mariotti and Salanie (2014): when the double deviation happens in subgroup $\mathcal{K}_1$, then, there will be two contracts c_L^k, c_H^k which aim type 1 and type 2, where $c_L^k = (q_1^k, t_1^k), c_H^k = (q_2^k, t_2^k)$.

   We discuss the situation in two different cases: Case (1), given the deviation c_L^k, c_H^k in $\mathcal{K}_1$, type 2 would not combine with the latent contract in subgroup $\mathcal{K}_2$; Case(2) , given the deviation c_L^k, c_H^k in $\mathcal{K}_1$, type 2 would combine with the latent contract in subgroup $\mathcal{K}_2$.

   In all the situations, we need to be aware of that, if buyer choose the menu from a seller who propose some deviation, then with the restriction of trade at most λ sellers in $\mathcal{K}_1$, the available tariff in this structure is $T^*(Q) = 1_{\{Q \le \frac{\lambda-1}{\lambda} Q_L\}} cQ + 1_{\{Q > \frac{\lambda-1}{\lambda} Q_L\}} [c\frac{\lambda-1}{\lambda} Q_L + c_H(Q -$

$\frac{\lambda-1}{\lambda}Q_L)]$.

Case (1.1) small contract c_L^k: When $q_1^k \leq \frac{1}{\lambda}Q_L$, then type 1 chooses contracts from other $\lambda-1$ sellers in $\mathcal{K}_1$, and combined with c_L^k. In this case, c_L^k attracts type 1 only if $t_1^k < cq_1^k$, then due to (Q_H, T_H) is available for type 2, then c_H^k attracts type 2 only if $\frac{\lambda-1}{\lambda}(Q_L, T_L) + c_H^k$ give better utility than (Q_H, T_H), however, this kind of c_H^k and c_L^k will result negative profit in total.

Case (1.2) Large contract c_L^k: When $q_1^k > \frac{1}{\lambda}Q_L$, then c_L^k attracts type 1 only if $t_1^k < cq_1^k$. Considering the contract c_H^k:

(1) if $q_2^k < q_1^k$, then c_H^k attracts type 2 only if c_H^k has a lower unit price and the convex contour tariff combined with c_H^k contains the convex contour tariff combined with c_L^k, but in this case, c_H^k is more attractive than c_L^k for type 1 also, the serving cost of both types is c while we have that the unit price of c_H^k is lower than c, unprofitable.

(2) if $q_2^k \geq q_1^k$, we know that $q_2^k \geq q_1^k > \frac{1}{\lambda}Q_L$. Then we can always find a (q_2', t_2') such that $c_H^k = (q_2^{k\prime}, t_2^{k\prime}) + c_L^k$, due to c_H^k is more attractive than c_L^k to type 1, then we know that $t_2' < c_H q_2^{k\prime}$, then the profit of this double deviation is $m_2(t_2^{k\prime} - c_H q_2^{k\prime}) + t_1^k - cq_1^k < 0$, not profitable.

Case (2) Double deviation based on latent contract in $\mathcal{K}_2$. if type 2 combines c_H^k with a latent contract in $\mathcal{K}_2$, due to it is better than JHG, then we have that $t_2^k < cq_2^k$ and means that type 2 prefers c_H^k than c_L^k, then due to the assumption flatter curvature (ii), by perfect translation with the latent contract, we will also have that type 1 also prefers c_H^k than c_L^k, but the serving cost of both types is c which makes the deviation get negative profit again. Thus, there is no profitable deviation in this case. Q.E.D

Then, to prove the existence in $\mathcal{K}_1 + \mathcal{K}_2$ structure, we just need to change a little bit in step 1 and construct the equilibrium menus of each group as follows:

For the subgroup $\mathcal{K}_1$:

(a) At least $\mathcal{K}_1 + 1$ sellers propose the tariff with $T(Q) = 1_{\{Q \leq \frac{Q_L}{\mathcal{K}_1}\}} cQ + 1_{\{Q > \frac{Q_L}{\mathcal{K}_1}\}}[c\frac{Q_L}{\mathcal{K}_1} + c_H(Q - \frac{Q_L}{\mathcal{K}_1})]$.

(b) Other sellers could propose the same menu as 1 or null contracts

For the subgroup $\mathcal{K}_2$:

(a) At least two sellers propose the separating tariff with: $T(Q) = c_H Q$

(b) At least one seller propose the menu tariff with $T(Q + q^\ell) = t^\ell + c_H Q \quad (Q \geq 0)$

(c) At least one seller proposes the menu with contracts: $(q_k^\ell, t_k^\ell) \equiv k \times (q_\ell, t_\ell)$, where $k \leq n_\ell$.

(d) Other sellers could propose one of 1,2,3 or null menus.

Then to get the JHG allocation, type 1 and type 2 purchase (Q_L, T_L) in $\mathcal{K}_1$ and then purchase $(Q_H - Q_L, T_H - T_L)$ in subgroup $\mathcal{K}_2$. Q.E.D

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Chapter 3: Multiple Contracting in Annuity Markets

Abstract

The annuity market represents a special segment of the insurance industry, where buyers acquire contracts during an initial period and receive payments as long as they are alive in subsequent periods. This market is characterized by nonexclusive trade and multiple periods, with considerable evidence pointing to the presence of adverse selection. In their research, Attar, Mariotti, and Salanié investigated nonexclusive trade in the insurance market within a static context, identifying the JHG tariff as entry-proof and budget balanced. Our study reveals that a JHG-similar tariff remains entry-proof when the annuity contract indemnity is consistent across the market. However, without indemnity restrictions, the demand for annuity contracts in later periods may crow out the high-risk demand from earlier periods, causing low-risk individuals to consume more annuities in the initial period and rendering the JHG-similar tariff non-entry-proof. To address this issue, we introduce an optional cross-period policy for planners to consider. By integrating the JHG-similar tariff with this policy, we demonstrate that the tariff becomes entry-proof once again, and the resulting allocation is budget balanced. These findings prove to be robust in any multi-period environment.

1 Introduction

Nonexclusive competition or multiple contracting is a widespread phenomenon in the insurance markets. Especially in annuities, for example, in the UK in 2013, five million people own six million annuities. Most of the markets of health insurance also have the feature of non-exclusively trade, for example, health insurance in France consists of a basic coverage proposed by the government, and can additionally purchase contracts from private firms. In the literature, most of the paper captures the feature of *nonexclusive* and *adverse selection* in these kinds of markets, such as Jaynes (1978), Hellwig (1988),Glosten (1994), Attar et al. (2011, 2014, 2019, 2020, 2021, 2022), all of them studied the non-exclusively trade under adverse selection, and show an allocation named *JHG*[1] played an important role in this kind of economy. None of their paper considers this nonexclusive trade into multiple periods—However, it is very natural to see multiple period payments in annuities markets—buyers purchase the quantity in the initial period and get payment conditional on alive. This paper studies an insurance economy with the features of *adverse selection, nonexclusive competition, and multiple periods.*

Adverse selection is a commonly occurs in various markets, such as insurance markets, financial markets, labor markets, used goods markets and so on.[2] It arises due to the presence of asymmetric information or private information, which can have harmful effects on the market. The classic example of inefficiency is the "lemons market" problem described by Akerlof (1970). In this analysis, sellers possess private information about the quality of their products, which allows low-quality sellers to masquerade as high-quality sellers. As a result, asymmetric information can lead to overtrading or market breakdown, compared to situations where perfect information is available.

To address the problems of existence and efficiency under adverse selection: Rothschild and Stiglitz (1976) (RS) proposed the use of menu contracts that offer different quantities and prices to separate high-risk and low-risk agents and incentivize them to reveal their type mon. the effectiveness of using incentive compatibility to analyze adverse selection problems. Other researchers, such as Wilson (1977) and Miyazaki (1977), have proposed the "Wilson foresight" approach to defining a new equilibrium concept in adverse Add selection, Crocker and Snow (1985) introduced the concept of using incentive compatibility for agents and a break even condition for principals to achieve a second-best efficiency allocation. These papers provide a range of models that can be used to study all ms adverse selection. The papers mentioned above assume that the trades between buyers and sellers are exclusive, meaning that a buyer can only trade with at most one seller. But in many real-world scenarios, buyers may have the option to trade with multiple sellers. This nonexclusivity can further complicate the issue of adverse selection and its potential solutions.

[1] Jaynes (1978), Hellwig (1988),Glosten (1994): we use JHG allocation later to describe one allocation which mainly concludes from their research on nonexclusive environment.

[2] This could happen in any markets with asymmetric information, in that situation, we call adverse selection happens if sellers and buyers rank the trade in different direction.

In markets where sellers cannot monitor all the trades between buyers and other sellers, non-exclusive trading among buyers is possible. An example of such a market is the annuity market. However, under the assumption of non-exclusivity and asymmetric information, the previously mentioned allocation may not be incentive-compatible, and alternative solutions may need to be considered to address the issue of adverse selection.To address this issue, Attar et al. (2011, 2014) analyzed a two-risk type agent model in an environment with non-exclusive trading. In their paper, they defined a robust incentive-compatible condition that states that a contract is robust incentive-compatible when no other entry can make a profitable contract in the market. They found that if an entrant can propose arbitrary contracts to the market, the only allocation that satisfies the entry-proof condition is the allocation implemented by the JHG tariff. The JHG allocation is characterized by different prices of coverage in each layer, where the marginal price of coverage in each layer is equal to the average cost for all consumers who purchase it. This allocation was first described by Jaynes (1978), Hellwig (1988), and Glosten (1994).

In Attar et al. (2021), a more general model is presented to show that for multi-type agents, the JHG tariff is again the only entry-proof tariff. The JHG tariff is characterized by the seller providing a unit price equal to the average cost of all agents who will purchase it. Since different types of agents have different demands for the contract, this kind of allocation has many different layers of price. Higher-risk types will purchase more contracts than lower-risk types, and the seller who provides this kind of tariff is also budget-balanced. In previous work by Attar, Mariotti, and Salani'e, they provided a non-exclusive environment to study the possible equilibriums in a static model. The R-S allocation is no longer an equilibrium in the non-exclusive environment, and they also showed that the JHG allocation could be entry-proof in both two-agent and multiple-agent cases in the static model. However, all their results are based on the static environment, which is suitable for most insurance markets. In dynamic markets such as annuity markets, the entry-proof tariff could be different. Therefore, the main focus of this paper is to find the entry-proof tariff in annuity markets, which can provide a possible budget-balanced and robustly incentive-compatible allocation.

Compared to the static non-exclusive model, annuity markets that involve multiple period payments exhibit a new feature. In traditional insurance, risk-neutral sellers provide contracts consisting of coverage and premium, while risk-averse buyers pay the premium and receive coverage only in the event of an unfavorable occurrence. This kind of insurance hedges the risk of buyers not losing too much. In annuity markets, risk-neutral sellers provide contracts consisting of annuity fees and annuity payments. Risk-averse buyers pay the annuity fees at the beginning and receive payments in later periods, only if they are still alive. Agents purchase annuity contracts to hedge the risk of living too long without sufficient financial resources. Essentially, annuity contracts are also insurance contracts, with the annuity fee as the premium, and the unfavorable event being alive, and the annuity payment as coverage. The dynamic setting of annuity markets provides us with more variables to study.

There are several reasons why we can study annuity markets based on the research of Attar, Mariotti, and Salani'e. Firstly, annuity markets are special insurance markets, and there is evidence that this market exhibits adverse selection between buyers and sellers. As previously introduced, the risk for agents in annuity markets is living too long, so they need to purchase annuities to hedge their longevity risk. For the same price layer, higher longevity agents clearly want to buy more annuities, while sellers have less incentive to sell to higher longevity agents. Thus, adverse selection can occur if buyers privately know their own type. As shown by Finkelstein and Poterba (2002, 2004), using empirical data from the UK annuity market, differential mortality experiences for annuitants who purchase different types of contracts are consistent with standard models of adverse selection in which individuals have private information about their risk. This evidence supports the assumption that buyers have private information about their mortality type, which can be incorporated into our model.

In addition, annuity market contracts also satisfy the condition of non-exclusivity. In most annuity markets, agents with private information can choose to combine multiple contracts. Therefore, annuity markets also satisfy the non-exclusivity environment of the basic model in Attar et al. (2014). As a result, annuity markets not only exhibit adverse selection but also satisfy the non-exclusive trading rule. This kind of market satisfies all the conditions described in Attar, Mariotti, and Salani'e (2011, 2014, 2016). Based on these similar features, we can begin our analysis with the same setting used in the nonexclusive markets papers.

In this paper, we aim to study dynamic contracts in annuity markets. To accomplish this, we first introduce a three-period model with two different types of buyers. Higher-type buyers also have a higher probability of living longer. We get one simple result if we take the setting of Hosseini (2015) , which the annuity payment in the market is the same, and the seller provides the price of the payment. Similar to the results obtained by Attar, Mariotti, and Salani'e in their static case, we find that the JHG allocation is the only entry-proof allocation under the assumption of the same annuity payment in each period in the market. Thus, the results in static model extends to multiple periods model if idemnity is the same in each period.

If we assume that the annuity payment can vary across different periods, we find that the JHG-similar allocation may not be an entry-proof allocation in some cases. This is because the low-risk buyer, who knows they will live shorter, will consume a lot of payment in the early period but very little in the later period. On the other hand, the high-risk buyer who knows they may live longer will purchase and consume more in total but may consume less than the low-risk buyer in the early period due to consumption smoothing. We refer to this phenomenon as the "**squeeze-out effect**," where later period consumption demand crowds out early period consumption. We have proven that if the squeeze-out effect occurs, the JHG-similar allocation will not be entry-proof.

We provide a solution for the seller's planner to restore the JHG-similar tariff entry-proof. We call it an optional cross-period policy, which allows buyers to exchange their payments from one period to a future period conditional on being alive, and the exchange rate exactly equals the alive probability at that period. By combining the cross-period policy, even if the squeeze-out effect happens, high-risk buyers can still purchase the JHG allocation and then exchange it to the later period. As a result, if a planner combines the JHG-similar tariff with this policy, the tariff remains entry-proof even if the squeeze-out effect occurs. What's more, we also show that this cross-period policy is entry-proof and budget-balanced in any cross-periods. Thus, this new policy provide the planner a very simple way to restore the entry-proof tariff and it will not bring any new deviations.

We also extend these results to multiple period cases, as the squeeze-out effect can also occur in these scenarios. However, we have proven that this effect only happens because later period consumption crowds out early period consumption. A planner can provide an optional cross-period policy combined with the JHG-similar tariff, making the tariff entry-proof again.

Related Literature

There is a growing body of literature on studying adverse selection and nonexclusive markets. The most related literature to our paper is based on the foundation proposed by Attar, Mariotti, and Salanié. They revisit the Akerlof model in Attar et al. (2011), which shows that the nonexclusive setting can also achieve equilibrium with some latent contract. Then, in Attar et al. (2014), they show that the Akerlof allocation is the unique equilibrium in a nonexclusive competition economy. However, the JHG allocation, which is concluded from Jaynes (1978), Hellwig (1983), and Glosten (1994), is not an equilibrium in this simultaneous game economy due to the possible double deviations. Based on this, Attar et al. (2021) show that the analysis of entry-proof allocation is essential to a nonexclusive economy. They demonstrate that the JHG allocation is the unique entry-proof allocation and can be an equilibrium in an ascending auction. Attar et al. (2022) also show that if the planner prohibits cross-subsidy contracts, the JHG allocation is the unique equilibrium. Other literature, such as Stiglitz et al. (2020), shows that introducing bilateral endogenous information disclosure results in the JHG allocation being the unique equilibrium. Meanwhile, Asriyan and Vanasco (2022) uses the withdraw game and security design to show that if the principal can propose one exclusive contract with collateral, the JHG allocation can be sustained in equilibrium. However, different from our paper, all of these papers above mainly focus on a one-period model without the influence from other periods.

In terms of multiple period models, previous literature has mainly focused on the impact of social security on the annuity market, such as Hosseini (2015), which uses a dynamic model to show the effect on social welfare and annuity market prices. However, the assumption of same payment in the dynamic model makes it similar to the static model results. Rothschild (2015) uses a three-period model in the annuity market to find efficient allocation for benevolent planners. They propose a compulsory environment with nonexclusivity and linear pricing in the annuity market and find some efficient allocations for the planner. However, they assume all buyers should use all their money to purchase the annuity, which is different from our model, and they also impose linear restrictions on the dynamic model. In this paper, we focus on the feature of dynamic contracts in annuity markets and aim to find entry-proof allocations while comparing them with Attar, Mariotti, and Salani'e's static case results.

Section 2 describes some basic definitions of entry-proof allocation and shows that JHG allocation is entry-proof in the static case. Section 3 shows the three-period dynamic model in the annuity market and discusses the entry-proof allocation in the same indemnity situation and then shows a counter-example in the situation of different indemnity. Section 4 studies the results with a multiple-period model in the annuity market. Section 5 two discussions based on our dynamics model. Section 6 is the conclusion of this paper.

2 Model Setting

In this section, I first present a simple *static model* in nonexclusive insurance markets proposed by Attar et al. (2019), Attar et al. (2021), and Attar et al. (2022). which could give us an introduction that the trade rule and some feature in a nonexclusive environment. Then I will define the *entry-proof* tariff, and also show that the *Jaynes-Hellig-Glosten (JHG)* tariff is the entry-proof and the allocation implemented by it is *Budget-balanced*. The basic setting and replication of the static results can help us easily realize the framework of the nonexclusive environment, and enlighten us on the approach to finding the entry-proof tariff in a dynamic model.

2.1 Insurance Markets Under Nonexclusively Competition

In an insurance market, a risk-averse buyer who has private information could purchase the contracts from several risk-neutral sellers. The asymmetric information between buyers and sellers will result in adverse selection. The sellers in this economy seek nonnegative profit while the buyers try to maximize their preference utility. The detail of this economy is as below:

The buyers:

There are two kinds of buyers (high- and low-risk types) who privately know their type θ, $where\ \theta = H, L$ with the proportion of m_H, m_L with $m_L + m_H = 1$. A type θ buyer's preference is based on the *aggregate* coverage-premium (Q, T) contracts she has purchased in the market, and it is represented by utility function $U_\theta(Q, T)$. It is natural to assume that $U_\theta(Q, T)$ is twice continuously differentiable, increases with the coverage, and decreases with the premium T. We define the type θ's marginal rate of substitution of coverage for premium as $\tau_\theta \equiv -\frac{\frac{\partial U_\theta}{\partial Q}}{\frac{\partial U_\theta}{\partial T}}$, we assume that in this economy that single-crossing condition is always satisfied. which is defined as:

Assumption 1 (Single-crossing)**.** *For any* $(Q, T) \in R_+ \times R$*,* $\tau_H(Q, T) > \tau_L(Q, T)$*.*

It means that given any pair of aggregate contracts in the markets, high-risk type buyers are more eager to purchase more coverage than low-risk type buyers.

The sellers:

There are enough risk-neutral sellers in the insurance markets. A seller could propose any coverage-premium (or quantity-transfer) contract $(q, t) \in R^+ \times R$ to the market. If a seller sells the contract (q, t) to type θ buyers, then the profit for this seller is $t - v_\theta q$. where v_θ is the unit cost for serving the contract with coverage q to type θ buyers. Budget balance for a seller means that the contracts traded in the market should let the seller earn a nonnegative profit, otherwise the seller can choose to deviate to become inactive. We also assume the common-value condition assumption holds, which is :

Assumption 2 (Common Value)**.** *high-risk type buyers have a higher cost than low-risk type buyers :* $v_H > v_L$*.*

Thus for the same premium, sellers are less eager to sell coverage to high-risk type buyers, however, combining with Assumption 1, high-risk types are more eager to trade, thus, Adverse selection happens in this economics. We also define $v = m_H v_H + m_L v_L$ as the average cost for the seller if he sells coverage to both types of buyers.

Trade rules:

As we mentioned in the introduction, the sellers can only monitor their own trade with the buyers but can**not** monitor the trade between buyers and other sellers. Thus the buyers may combine many coverage-premium contracts from the market, if a buyer does not trade with some sellers, we can say the trade of them is $(0, 0)$. As a result. If K is the set of sellers in the market who trade with a buyer, then her aggregate trade is $(Q, T) \equiv (\sum_{k \in K} q^k, \sum_{k \in K} t^k)$. Similar to the literature, we define the incentive compatibility and budget-balanced with aggregate allocations quantity-transfer pairs as:

Definition 1 (Incentive Compatibility)**.** *An allocation* (Q_L, T_L) *and* (Q_H, T_H) *is incentive compatible if* $U_L(Q_L, T_L) \geq U_L(Q_H, T_H)$ *and* $U_H(Q_H, T_H) \geq U_H(Q_L, T_L)$*.*

Definition 2 (Budget-Balanced)**.** *An allocation* (Q_L, T_L) *and* (Q_H, T_H) *is Budget-Balanced for the market if* $m_L(T_L - v_L Q_L) + m_H(T_H - v_H Q_H) \geq 0$*.* [3]

In this nonexclusive environment, it is easy to find that the allocation in Rothschild and Stiglitz (1976) (RS allocation) is no longer incentive compatible. Because no seller can monitor

[3] A seller is budget-balanced if she earns a nonnegative aggregate profit.

all the trades in the markets and a new entrant can propose a contract that attracts both types based on RS allocation and make it profitable. Thus to find the possible incentive-compatible condition, it needs to be robust to entrants into the markets, which is an entry-proof tariff. In this paper, we use many definitions in Attar et al. (2021). which is as below:

2.2 Entry-Proof Tariff

Firstly, for all the coverage-premium pairs, we can without loss of generality use the tariff specifying a transfer to be $T(Q)$ to represent the premium as a function of coverage demand. Then we consider a situation that a planner could propose a tariff that can be robust to the new entrants.

Definition 3 (Entry-proof). *The tariff T^P is entry-proof if, for any tariff T^E offered by an entrant, there exists for each buyer θ a solution (q_θ^P, q_θ^E) to type θ's problem:*

$$max\{U_\theta(q^P + q^E, T^P(q^P) + T^E(q^E) : q^P \geq 0 \text{ and } q^E \geq 0\} \tag{1}$$

such that the expected profit of the entrant is at most zero,

$$m_H[T^E(q_H^E) - v_H q_H^E] + m_L[T^E(q_L^E) - v_L q_L^E] \leq 0 \tag{2}$$

The intuition behind the entry-proof tariff is that if a planner proposes this tariff to the market, no seller can profitably offer additional contracts that attract some buyers who trade with a planner. If we could find a tariff that is entry-proof, and the allocation implemented by this tariff is also budget balanced for the planner. Then this kind of allocation proposed by planners is actually a benevolent monopolistic equilibrium to the markets. And also, as in Attar et al. (2021), the entry-proof allocation can be the equilibrium under the competitive ascending auction. Thus, it is important to find entry-proof tariffs in the nonexclusive environment.

In this paper, we will focus on finding the entry-proof and budget-balanced allocation in a three-period and multiple-period environment. In the next subsection, we show that the JHG tariff with JHG allocation is actually entry-proof and budget balanced in the static environment. After that, we will study the entry-proof tariff in dynamic cases.

2.3 JHG Tariff: Entry-Proof and Budget-Balanced

JHG allocation was first studied by Jaynes (1978), Hellwig (1988) Glosten (1994), and then Attar et al. (2014, 2019, 2020, 2021) generalize it in the model to analyze the feature of JHG model. In JHG allocation, the planner proposes two layers of price to the markets. In the first layer, which is the basic coverage layer, the unit price of coverage is equal to the average cost v of the contract, both the high-risk type and low-risk type will buy it. In the second layer, which is the additional coverage layer, the unit price for buying coverage is equal to the cost of serving for high-risk type, and only high-risk type buyers purchase it. Actually, each layer of coverage is fairly priced given the types who purchase it, the size of each layer is also optimally chosen by the buyers. Let (Q_L, T_L) is the optimal pair for low-risk type buyers, (Q_H, T_H) is the optimal pair for high-risk type buyers, the detail of JHG allocation is as below:

Definition 4 (Jaynes-Hellwig-Glosten (JHG))**.** *The JHG allocation is the partially pooling allocation* (Q_L^P, T_L^P) *and* (Q_H^P, T_H^P) *where*

$$\begin{aligned} Q_L^P &= \arg\max_{Q\geq 0} U_L(Q, cQ), \\ T_L^P &= cQ_L^P \\ Q_H^P - Q_L^P &= \arg\max_{Q\geq 0} U_H(Q_L^P + Q, T_L^P + v_H Q) \\ T_H^P - T_H^P &= v_H(Q_H^P - Q_L^P). \end{aligned}$$

Because each layer is actually fairly priced, it is obviously budget balanced in this kind of allocation. According to this kind of allocation, it is easy to induce the JHG tariff which could implement the JHG allocation for the planner as below: (the JHG allocation is as in Figure 1)

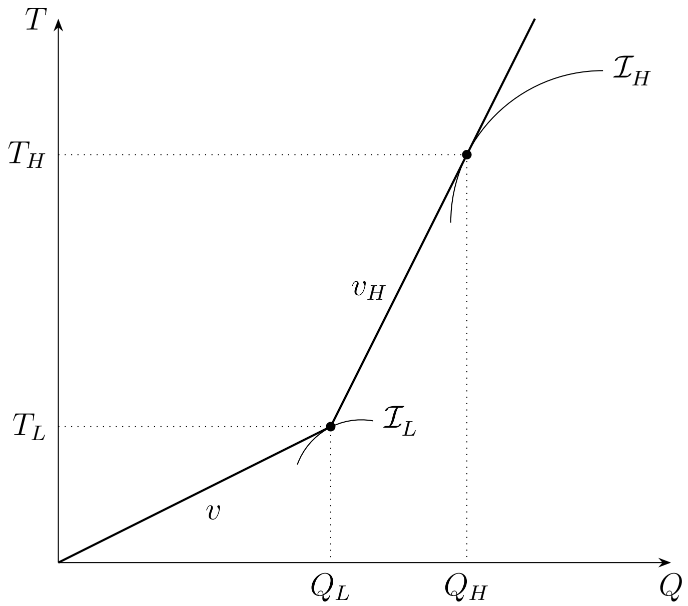

Figure 1: JHG Allocation

Definition 5 (JHG Tariff)**.** *JHG tariff is a two-layer unit price tariff, which gives break-even to the market given the different buyers' adequate purchases according to this tariff. Thus, it is defined by:*

$$T^P = 1_{\{Q\leq Q_L\}} cQ + 1_{\{Q>Q_L\}}[cQ_L + v_H(Q - Q_L)] \tag{3}$$

In Attar et al. (2019, 2021), they give very detailed proof that JHG allocation is the unique entry-proof and budget-balanced allocation. In this subsection, we just show some short intuition about the proof. First, any tariff that has a higher unit price than the JHG tariff is not entry-proof. Otherwise, a new entrant can always propose a tariff just a little above the JHG tariff, it is attractive for buyers and profitable for the entrant. Second, any tariff that has a lower unit price than JHG couldn't satisfy the budget-balanced condition. Now for the JHG tariff, if an entrant is trying to attract only high-risk type buyers, she needs to propose a contract with a unit price less than v_H, but it is not profitable for her. If an entrant is trying to attract low-risk type buyers, she needs to propose a contract with a unit price less than v, but by using the single-crossing condition, if one contract attracts low-risk type, it is also attractable for high-risk type buyers. As a result, both types of buyers buy the contract with a unit price of less than v, but the cost for serving this contract is v, thus it is also not profitable for entry. Thus, the JHG tariff is entry-proof in this market. This is the result of the static case, the single-crossing condition secures that high-risk type buyers are more eager to purchase more coverage than low-risk type buyers, thus we need to use the average cost rather than only cost for low-risk type buyers in the first layer.

In this static environment, under the nonexclusivity environment, we conclude that the JHG tariff is entry-proof and budget balanced, this tariff provides a two-layer unit price to the market, and the contract in each layer is actually break-even given who purchased it. This kind of tariff enlightens us on the idea to study the entry-proof tariff in all nonexclusivity environments. In the next section, we use this kind of idea to find the entry-proof tariff in a dynamic environment, there is some similar trick in the dynamic case, like the form of an entry-proof tariff. however, there is also something new in a dynamic model, JHG-similar tariff may not be an entry-proof tariff.

3 Three Period Model

According to the results in the static case, we can know that it is very important to find the entry-proof and budget-balanced allocation for the planner. In the later parts of this paper, **we focus on finding the entry-proof tariff in dynamic case**. The intuition from the static case shows us that the JHG tariff which gives different layer prices to the market seems a good candidate for an entry-proof tariff. In this section, we will introduce JHG-similar tariff in a dynamic environment, and show what will happen if the planner proposes JHG-similar tariff in dynamic cases. And we will find that JHG's similar tariff can be an entry-proof tariff in many cases. However, in some cases, the JHG-similar tariff is not enough to be an entry-proof tariff. (All the characters used in this section have no relation with the last section.)

3.1 Model Setting

Basic environment: It is a three-period economy. We can treat that the planner wants to design some annuity contracts policy for buyers which should be entry-proof to the market. In period 0, all the buyers own w_0 wealth, they could purchase the annuity contracts which will give them a cash flow in period 1 and period 2. The buyers will choose the annuity contracts to max their total utility in three periods. Assume there is no discount in this economy.

The buyers: There are two types of buyers who are privately informed of their type. high-risk type buyers have a higher probability of alive longer in each period. Let p_1^H, p_2^H represent the alive probability of high-risk type buyers in period 1 and period 2. Then p_1^L, p_2^L is the alive probability of low-risk type buyers in period 1 and period 2. let m_H, m_L as the proportions of high-risk type buyers and low-risk type buyers. Let $\overline{p}_1 = m_H p_1^H + m_L p_1^L$ and $\overline{p}_2 = m_H p_2^H + m_L p_2^L$ represent the average alive probability of period 1 and 2. If a buyer chooses the annuity contract $C = (T, c_1, c_2)$ from the market. then the preference is given as: (u is increasing and concave)

$$U^\theta(C) = u(w_0 - T) + p_1^\theta u(c_1) + p_2^\theta u(c_2), \theta = H, L$$

In this paper, we use the common assumption in annuities such as Rothschild (2015),

Assumption 3 (Higher Longevity). *The high-risk type buyers have higher longevity, in any period, they have a higher probability to survive in the next period. i.e.* p_t^θ *satisfy that* $p_1^H \geq p_1^L$ *and* $\frac{p_2^H}{p_1^H} \geq \frac{p_2^L}{p_1^L}$

The planner and sellers: The planner is trying to design the annuity contracts for the market which can be entry-proof and budget balanced. The other sellers are trying to attract buyers and maximize their profit. The contract C is consist of total fees T based on period 1's payment c_1 and period 2's payment c_2. Then the profits for the planner selling the contract to i-type are:

$$\pi^\theta(C) = T - p_1^\theta c_1 - p_2^\theta c_2, i = H, L$$

3.2 Same Indemnity: Same as Static Model

In Hosseini (2015), he uses a model in which the annuity payment is the same in each period. And the price of the annuity is depending on this payment. This kind of setting is used in many annuity contracts, the same indemnity contracts are very usual in the real world. In this subsection, we use this simple setting to study the entry-proof allocation for planners. Thus, we transfer this condition into the assumption: the sellers can only provide the annuity contracts that have the same payment in each period. which means that $c_1 = c_2$ is **compulsory** for every seller. Let the annuity fees as the function of annuity payments, then the contract tariff proposed by sellers consists of the form $(T(c), c, c)$.

According to the trick in the static model, we describe a JHG-similar allocation, which also consists of two layers of contracts, in the first layer, both two types will buy the annuity contract. And after that, the high-risk types will purchase additional annuity contracts in the second layer. The first layer c^{L*} is optimal for low-risk type buyers at average price $v = \overline{p}_1 + \overline{p}_2$:

$$\begin{aligned} c^{L*} &\equiv argmax\{U^L(v_L); c \geq 0\} \\ where\ v_L &= (vc, c, c) \\ T^{L*} &= v * c^{L*} \end{aligned}$$

Then high-risk type will buy the additional annuity contracts with unit price $p^H = p_1^H + p_2^H$, with optimal quantity c^{H*} that:

$$\begin{aligned} c^{H*} - c^{L*} &\equiv argmax\{U^H(v_L^* + C_2); c \geq 0\} \\ where\ v_L^* &= (vc^{L*}, c^{L*}, c^{L*}), v_H = (p^H c, c, c). \\ T^{H*} &= p^H(c^{H*} - c^{L*}) \end{aligned}$$

Then the JHG-similar tariff in the same indemnity case is that:

$$T^P(c) \equiv 1_{\{c \leq c^{L*}\}} vc + 1_{\{c > c^{L*}\}}[vc^{L*} + p^H(c - c^{L*})] \tag{4}$$

Actually, in the setting of the same indemnity case, it is easy to find that the JHG-similar tariff is entry-proof and the allocation implemented by it is budget balanced. This is due to the high-risk type buyers always having higher demand in this same indemnity setting, thus single crossing condition is satisfied in every period. So we can use the same trick with a static case to prove the JHG-similar tariff is entry-proof in this dynamic model. The policy that serves the market with a basic layer and an additional layer seems can be also used in a dynamic model. In this special dynamic model, we can find that the JHG-similar tariff plays an important role in the entry-proof tariff. Followed with this idea, we move to a more general three-period model which relaxes the restriction on indemnity.

3.3 Model: No Restrictions on Indemnity

In this section, we discuss a model in that buyers can choose different payments of annuity contracts in different periods, this means that the contracts of annuity may have different indemnities in the market, this setting is consistent with the setting of Hosseini (2015). This kind of assumption is more related to the various sellers in the annuity market, every seller in the market could design contracts to attract buyers. Thus, there is no incentive to this market that restrict the payment on the same indemnity.

In the model of free choice of indemnity, the annuity contracts' tariffs have the form of $C = (T(c_1, c_2), c_1, c_2)$. In this case, one period's demand for an annuity may affect the other period's demand. Thus, the possible entry-proof allocation could be more complicated.

First, following the trick before, we also introduce a JHG-similar allocation in this general model, which has the same intuition as what we did, but due to different periods could have different payments, we let the JHG-similar allocation priced on each period: In the first layer, both type purchase the contracts at the average price, low-risk type buyers choose an optimal quantity $c_t^{L^*}$ at the average price $\overline{p}_t$ in each period, then in the second layer, high-risk types could buy an additional annuity in each period at a unit price of p_t^H, and buying $c_t^{H*} - c_t^{L*}$. which is that:

$$
\begin{aligned}
(c_1^{L*}, c_2^{L*}) \equiv argmax\{U^L(v_L); c_1 \geq 0, c_2 \geq 0\} \\
where\ v_L = (\overline{p}_1 c_1 + \overline{p}_2 c_2, c_1, c_2) \\
T^{L*} = \overline{p}_1 c_1^{L*} + \overline{p}_2 c_2^{L*}
\end{aligned}
$$

Then high-risk type agents could choose additional annuity contracts with the:

$$
\begin{aligned}
(c_1^{H*} - c_1^{L*}, c_2^{H*} - c_2^{L*}) \equiv argmax\{U^H(v_L^* + v_H); c_1 \geq 0, c_2 \geq 0\} \\
where\ v_L^* = (T^{L*}, c_1^{L*}, c_2^{L*}), v_H = (p_1^H c_1 + p_2^H c_2, c_1, c_2) \\
T^{H*} - T^{L^*} = p_1^H (c_1^{H*} - c_1^{L*}) + p_2^H (c_2^{H*} - c_2^{L*})
\end{aligned}
$$

We can define a **JHG-similar tariff** with the form with:

$$
\begin{aligned}
T^P(c_1, c_2) \equiv T_1(c_1) + T_2(c_2) \\
where\ T_t(c_t) \equiv 1_{\{c_t \leq c_t^{L*}\}} \overline{p}_t c_t + 1_{\{c_t > c_t^{L*}\}} [\overline{p}_t c_t^{L*} + p_t^H (c_t - c_t^{L*})]
\end{aligned}
$$

The JHG-similar allocation and JHG-similar tariff are defined above. According to the results in the static model and in the dynamic same indemnity model, it seems this kind of tariff is a good candidate for entry-proof and budget-balanced tariff. This idea makes some sense because that **if** high-risk type buyers have higher demand in every period, then in period t the lowest unit price which achieves break-even in the first layer for the seller is at least $\overline{p}_t$. And the lowest unit price which achieves break-even in the second layer for the seller is at least p_t^H.The break-even unit price here is consistent with the JHG-similar tariff.

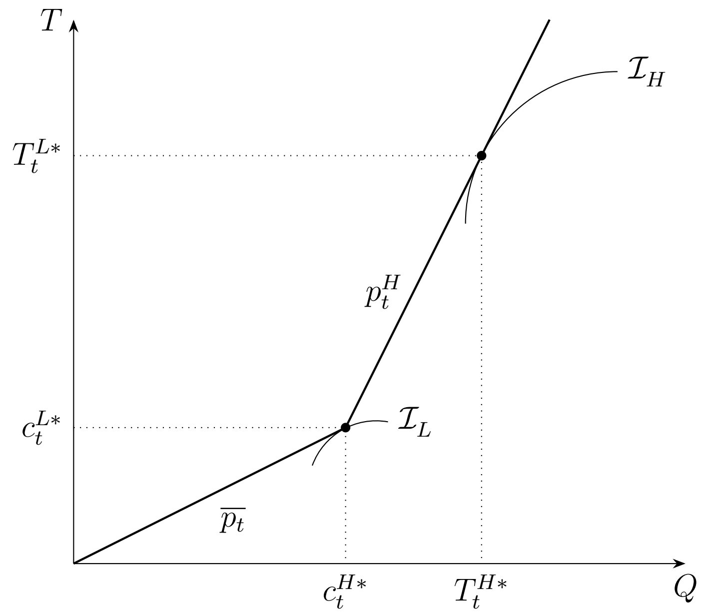

Figure 2: JHG-similar Tariff

It is easy to prove that any entry-proof tariff T^{EP} should have a unit price less than JHG-similar tariff T^P in every period. Otherwise, if there is a period that has higher pricing than T^P, then an entry can provide annuity contracts which only provide a little higher price than T_t in every period, it is attractable for buyers and it is profitable for this entrant. Thus, any entry-proof contract should have a lower tariff than JHG-similar tariff, that is to say, $T^{EP} \leq T^P$. **In the static cases**, because the high-risk type will always buy more quantity, thus by using budget balanced, we can get that any budget balanced tariff should have higher pricing than T^P, as a result, $T^{EP} = T^P$. **However**, in this dynamic setting, one period's demand for an annuity could reduce the other period's demand. It is **not always** that the high-risk type has higher demand in each period. Thus, the budget balance constraint may not work in some cases. We can see an extreme example that the JHG-similar tariff is not an entry-proof tariff.

3.4 Counter Example: JHG-similar Tariff is Not Entry-Proof

In this part, we will show an example in the dynamic model: even though the high-risk type has a higher alive probability, there still exists some period that the high-risk type buyers have lower consumption demand than low-risk type buyers. And if this kind of case happens, then the JHG-similar tariff is not entry-proof.

Assuming the initial wealth is $w_0 = 1$, the proportion of two type buyers is $m_H = 0.99, m_L = 0,01$. So the market mainly consists of high-risk type buyers. The high-risk type agents can be alive for certain in period 1 and almost alive for certain in period 2, that is to say, $p_1^H = 1, p_2^H = 0.99$. But for low-risk type agents, the alive probability is 0.99 in period 1 but with 0 probability alive in period 2, thus, $p_1^L = 0.99, p_2^L = 0$. No low-risk types can survive in

Period 2, but almost all of them can be alive in Period 1. From the viewpoint of consumption of smoothing, High types would prefer to almost averagely consume in every period, but low types prefer to almost averagely consume in periods 0 and 1.

Then by calculation, the break-even price of the first layer is $\overline{p}_1 = 0.9999, \overline{p}_2 = 0.9801$. If we propose a JHG-similar tariff to this market, the optimal quantity for the low-risk type is $c_1^{L*} \approx \frac{1}{2}, c_2^{L*} = 0$, while the optimal quantity for high-risk type is $c_1^{H*} \approx c_2^{H*} \approx \frac{1}{3}$. The high-risk type buyers will purchase **less** quantity than low-risk type buyers in period 1. Demand in period 2 **crowds out** the demand in period 1. Then we can find that an entrant could propose a contract aimed at period 1 with a unit price $0.9999 - \epsilon$, this contract can attract both types of buyers at the same time, and it is profitable. The budget-balanced constraint doesn't work when the low-risk type has higher demand in some period. (In here, the single crossing condition is not satisfying in period 1).

Intuition in this example: It is natural that high-risk type buyers with longevity will buy more annuity contracts in total, but with consumption smoothing, it is also possible that high-risk type buyers purchase less quantity than low-risk type buyers who focus more on early period. If there is only one basic contract in the market, there is no doubt that if the low-risk type buys it, the high-risk type will also buy at least the same contract as low-risk type buyers. But with the additional contract, the later period's demand for contracts may reduce the demand of the early period, which may result that the high-risk type having lower demand than the low-risk type in the early period. This is a big difference between the dynamic model and the static model: In the dynamic model, the different period consumption choices can affect each other. If this interaction is not severe, JHG-similar tariff is still entry-proof, and if this interaction of every period is severe, JHG-similar tariff is not entry-proof. We will define the cut-off below, and discuss the entry-proof tariff according to the cut-off condition.

Definition 6 (Squeeze Out Effect)**.** *Given the JHG-similar tariff, if high-risk type buyers have lower consumption than low-risk type buyers in some periods, we say that a squeeze-out effect exists in this period. i.e. If in a period t, we have $c_t^L > c_t^H$ for a given JHG-similar tariff, then we call Squeeze out effect happens.*

As what we saw in the special example, the squeeze-out effect happens in period 1, as a result, the JHG-similar tariff becomes not an entry-proof tariff. In the next, we discuss the case depending on whether the squeeze-out effect happens.

3.5 No Squeeze Out Effect in Economy

In this part, we focus on the condition that the interaction of consumption is not severe, and there is no squeeze-out effect happening in every period. That is to say, given the JHG tariff, the high-risk type buyers always have higher consumption demand in every period. Intuitively, this condition is similar to the "single crossing" condition, thus, following the idea of the static model, this condition makes the JHG-similar tariff entry-proof. Then we can conclude it in the proposition as below:

Proposition 1 (JHG Tariff is Entry-Proof Case)**.** *Given the JHG-similar tariff to the market, if there is no squeeze-out effect exist, then JHG-similar tariff is entry-proof, and the allocation implemented by it is budget balanced.*

The main idea of proposition 1 is as below :

First, all the results are based on the condition that **no** squeeze-out effect given JHG-similar tariff. It means that if a seller proposes JHG-similar tariff to the buyers, the high-risk type buyers always choose higher consumption in period 1 and period 2. According to this assumption, we change this kind of condition into a mathematical expression, let $(c_1^{L*}, T_1^{L*}), (c_2^{L^*}, T_2^{L*})$ be the contract choose by the low-risk type buyers. where the total contracts is $C^{L*} = (T_1^{L*} + T_2^{L*}, c_1^{L*}, c_2^{L^*})$. Similar, we let $(c_1^{H*}, T_1^{H*}), (c_2^{H^*}, T_2^{H*})$ is the contract choose by high-risk type buyers. Then we can get that direct relationship in this step is that $c_t^{L*} \leq c_t^{H*}$ and $T_t^{L*} \leq T_t^{H*}$.(no squeeze-out).

Then, based on the assumption, we want to prove that JHG's similar tariff is entry-proof. That is to say, no entry can attract some buyers in this market and at the same time get profit. If an entry proposed some higher tariff into this market, it is not attractive for buyers, so there is no effect. Then if an entrant wants to attract some buyers, it must provide a unit price lower than the JHG tariff. Then if the entrant aims to high-risk type buyers, they must propose some unit price lower than p_t^H, but the cost for serving high-risk type is p_t^H, thus also unprofitable. As a result, if an entrant wants to get a profit, the only way is that attract low-risk type buyers in some period. But later, we will show that any contract that attracts low-risk type will also attract high-risk type buyers, but the cost for serving both types is $\overline{p}_t$ which equals JHG tariff low price.

Thus, we only need to prove that any lower-price contract attracts low-risk type buyers and also attracts high-risk type buyers. We divide this problem into two lemmas:

Lemma 1. *No lower price contract can attract only low-risk type buyers in period 1.*

Lemma 2. *No lower price contract can attract only low-risk type buyers in period 2.*

The proof of Lemma 1 and 2 are attached in the appendix. These two lemmas tell us that under the assumption that no squeeze-out effect happens, any contract attracts low-risk type buyers in any period and also attracts high-risk type buyers. This means the lowest price for the first layer is at least equal to the break-even level $\overline{p}_t$, and the second layer price is equal to the cost for serving high-risk type buyers. This setting is consistent with the JHG-similar tariff. As a result, if there is no squeeze-out effect in the economy, then JHG-similar tariff is entry-proof and the allocation implemented by it is budget balanced.

3.6 Squeeze Out Effect Exists in Economy

In the special example, we have seen a situation where there exists a squeeze-out effect in some periods. Then a new entry can propose a contract to attract both types and get positive profit. This deviation always exists if high-risk type buyers have lower demand than low-risk type buyers. Thus, we can conclude that if the squeeze-out effect exists in the economy, then the JHG-similar tariff is not entry-proof. In the proof of lemma 2, we show that the squeeze-out effect only happens in period 1 in this three-period model(Intuitively, for any basic price in period 2, high-risk type buyers always have higher consumption demand in period 2, because they live longer.). Later, we try to propose a policy to make the JHG-similar to become entry-proof even squeeze-out effect happens.

Definition 7 (Optional Cross-period Policy)**.** *A cross-period policy is a conditional insurance for different periods. It can exchange the consumption from period 1 to 2 with a unit price of* $\frac{p_2^H}{p_1^H}$ *and payment conditional on the state of alive.*

Under Optional cross-period policy: Assume now that the planner could propose an optional contract that can change the payments from period 1 to period 2, the payment of period 2 is dependent on the condition alive of buyers. And let the unit price of c_2 is $c_2 * \frac{p_2^H}{p_1^H}$ in payment at period 1. Then we can easily prove that $\overline{p}_1 * \frac{p_2^H}{p_1^H} = (m_H p_1^H + m_L p_1^L) * \frac{p_2^H}{p_1^H} \geq (m_H + m_L \frac{p_2^L}{P_2^H}) p_2^H = \overline{p}_2$. and $\overline{p}_1 * \frac{p_2^H}{p_1^H} \leq p_1^H * \frac{p_2^H}{p_1^H} = p_2^H$. Thus, we will have the inequality that

$$p_2^H \geq \overline{p}_1 * \frac{p_2^H}{p_1^H} \geq \overline{p}_2 \tag{5}$$

If we combine this optional cross-period policy with JHG-similar tariff, we could find that it is always better for high-risk type buyers to get the quantity at least as same as low-risk type buyers' quantity. Firstly, the squeeze-out happens when period 2's demand is relatively high. In that case, the marginal price for buying c_2 for high-risk type buyers is p_2^H, but if she purchases annuity payments from period 1 and then transfers it to c_2, the marginal cost is $\overline{p}_1 * \frac{p_2^H}{p_1^H}$ which has a lower cost for buying this contract. Thus, even if the squeeze-out effect can exist in the early period, high-risk buyers could also have no less demand in the early period if they could use this cross-period policy. (or have a higher preference to buy this annuity contract in period 1), if an entrant proposes a contract in period 1 with a unit price less than $\overline{p}_1$, due to (5), it is more attractive for high-risk type buyers. Thus, if we combine the JHG tariff with a cross-period policy, high-risk type buyers still have higher demand in this period, the single-crossing condition could hold again.

Will this optional cross-period policy affect other demands? The answer is **no**. Firstly, it is not attractive to low-risk type buyers. According to the right-hand side of (5), it is not better for low-risk type buyers to buy an annuity in the first period and transfer to payments in the second period. Secondly, it is not attractive for high-risk type buyers if there is no squeeze-out effect. Due to $p_1^H * \frac{p_2^H}{p_1^H} = p_2^H$, it is no different for high-risk type buy an annuity in period 2 or buy it in period 1 and transfer. As a result, this kind of policy is only effective when the squeeze-out effect exists.

Is this cross-period policy budget balanced for the planner? Yes, we can find that if high-risk type buyers buy some c_2, she needs to pay $c_2 * \frac{p_2^H}{p_1^H}$ unit of c_1 to the planner, and after that, only $\frac{p_2^H}{p_1^H}$ proportion of high-risk type buyers could alive and receive c_2 in period 2. As a result, the cost for a planner is also $c_2 * \frac{p_2^H}{p_1^H}$. Thus, the policy is budget balanced.

Is this kind of policy (only for cross-period contracts) entry-proof for the market? Yes, if an entrant wants to propose another cross-period policy only to attract high-risk type buyers, the lowest unit price is $\frac{p_2^H}{p_1^H}$. But if she wants to attract low-risk type buyers, she needs to provide the policy with a unit transfer price lower than $\frac{\overline{p}_2}{\overline{p}_1}$, however, this will lead high-risk type trade more cross-period policy. which will also result in negative profit.

Will the squeeze-out effect happen in period 2 of the JHG tariff? Intuitively, high-risk type of buyers would always want to purchase more contracts than low-risk type buyers for the same price policy in the later period. Now, we will prove that in any case, the JHG-similar tariff will lead the high-risk type buys more annuity than low-risk type buyers in period 2. Let c_1^L, c_2^L be the annuity contract chosen by low-risk type buyers and c_1^H, c_2^H be the annuity contract chosen by high-risk type buyers. Next, we will prove that $c_2^H \geq c_2^L$. For the corner solutions, it is obvious that the market satisfies this condition. For non-corner solutions, we assume that if $c_2^H < c_2^L$, then we must have that $c_1^H > c_1^L$. This means that the marginal utility for buying an annuity in period 2 with price $\overline{p}_2$ is lower than the marginal utility for buying an annuity in period 1 with price p_1^H at point (c_1^L, c_2^L). which is equivalent with that $p_2^H u(c_2^L - \epsilon) + p_1^H u(c_1^L + \frac{\overline{p}_2 \epsilon}{p_1^H})$ should great than 0, if ϵ is small. But take derivation of ϵ and let it close to 0, we could find that value is $\overline{p}_2 u'(c_1^L) - p_2^H u'(c_2^L) = u'(w_0 - \overline{p}_1 c_1^L - \overline{p}_2 c_2^L)(\frac{\overline{p}_2 \overline{p}_1}{p_1^L} - \frac{\overline{p}_2 p_2^H}{p_2^L}) < 0$, contradiction. Thus, the high-risk type buyers will always purchase more annuities in period 2 than the low-risk type in JHG-similar tariff. Thus, there is no squeeze-out phenomenon that happens in period 2 for high-risk type buyers. (Actually, using the same trick, we can show that for any tariff provided in the first layer in each period, the high-risk type buyers will always have a higher preference for buying more annuity in period 2.) Thus single-crossing condition is always satisfied in period 2. Thus, if an entrant proposes some contracts in period 2 could attract low-risk type and also attracts high-risk type buyers.

From the analysis above, we can conclude that the cross-period policy combined with the JHG-similar tariff is entry-proof, and this policy is only effective when there exists a squeeze-out effect. As a result, we have the proposition below:

Proposition 2. *When there exists a squeeze-out effect in the economy, the cross-period policy and the JHG-similar tariff together are entry-proof, and the allocation implemented by them is budget balanced.*

As a result of Proposition 1 and 2, we have the conclusion in the three-period model: **(1)** If there is no squeeze-out effect happen in period 1, then high-risk type buyers have higher demand of annuity payments in every period, the single crossing condition hold in every period, JHG-similar tariff is entry-proof and allocation is budget balanced in this three-period model. In this case, low-risk type buyers purchase basic layer annuities in every period, and high-risk type buyers purchase basic and additional layer annuities in every period. **(2)** If the squeeze-out effect exists in period 1, then JHG-similar tariff combined with cross-period policy is entry-proof. In this case, low-risk type buyers purchase and consume the basic layer annuity in every period, and high-risk type buyers purchase the basic layer in the same quantity as the low-risk type in period 1 but will transfer some quantity to period 2. Then the annuity contract for high-risk type buyers consists of **three layers** in period 2: basic layer with a unit price of $\overline{p}_2$, transfer price layer with unit price $\overline{p}_1 * \frac{p_2^H}{p_1^H}$, and the additional price layer with unit price p_2^H. Also, this kind of allocation is entry-proof and budget balanced even if the squeeze-out effect exists.

4 Multiple Period Model

In three period case, we presented that if the planner only proposes JHG-similar tariff, the possible squeeze-out effect results in the single crossing condition failure in period 1. And the optional cross-period policy combined with JHG-similar tariff could be entry-proof again. In this section, we put our dynamic contracts in a multiple-period model, firstly showing that the squeeze-out effect happens only because that later period demand crowds out the consumption in the early period. Then propose a set of cross-period policies which could transfer early-period annuity payments to later period annuity payments. As a result, the JHG-similar tariff combined with the optional cross-period policy is entry-proof again.

4.1 Model Setting

Basic environment: There are $T_0 + 1$ period in this general model. In period 0, buyers could choose any kind of annuity contract from the market, and also could purchase different contracts from different sellers. And after that, in the later T_0 period, the annuity contract could give the buyers a cash flow for every period on the condition alive of buyers. The buyers will purchase the annuity to maximize their total utility in all periods. Assume there is no discount in this economy.

The buyers: there are two types of buyers who are privately informed of their type. high-risk type buyers have a higher probability live longer in each period. Let p_t^H, p_t^L represent the alive probability of high-risk type and low-risk type buyers in period t, m_H, m_L is the proportion of high-risk type and low-risk type buyers in period 0. then let the $\overline{p}_t = m_H p_t^H + m_L p_t^L$ as the average alive probability of period t. If a buyer chooses the annuity contract $C = (T, c_1, ..., c_{T_0})$ from the market (C could be a combining of many contracts). Then the preference is given as (u is increasing and concave.)

$$U^\theta(C) = u(w_0 - T) + p_1^\theta u(c_1) + ... + p_t^\theta u(c_t) + ... + p_{T_0}^\theta u(c_{T_0}), \theta = H, L$$

In this multiple-period case, we assume that the alive probabilities p_t^θ satisfy that $p_1^H \geq p_1^L$ and $\frac{p_{t+1}^H}{p_t^H} \geq \frac{p_{t+1}^L}{p_t^L}$. This means that high-risk type buyers have higher longevity. And in each period, they have a higher probability to survive in the next period.

The planner and sellers: The planner is trying to design the annuity contracts for the market which can be entry-proof and budget balanced. The other sellers are trying to attract buyers and be profitable. The contracts' tariff C is consist of total fees $T(c_1, ..., c_{T_0})$, period 1's annuity payment c_1, period t's annuity payment c_t. Then the profits for the planner selling the contract to i-type are :

$$\pi^\theta(C) = T - p_1^\theta c_1 - ... - p_t^\theta c_t - .... - p_{T_0}^\theta c_{T_0}, \theta = H, L$$

4.2 JHG-similar Tariff in Multiple Periods

In this subsection, the annuity contracts could have different payments in different periods, so the contract tariff of annuity is like the form of $C = (T(c_1, ..., c_{T_0}), c_1, ...c_{T_0})$. In this case, one period's demand for an annuity could reduce the other period's demand. We define the JHG-similar tariff that is actually proposed JHG tariff in every period.

Firstly, we also introduce a JHG-similar allocation in this general multiple period model, which has the same intuition as what we did: In the first layer, both types buy the contracts at the average price, low-risk type buyers choose an optimal quantity $c_t^{L^*}$ at the average price $\overline{p}_t$ in each period, then in the second layer, high-risk type could buy an additional annuity in each period at a unit price of p_t^H, and buying $c_t^{H*} - c_t^{L*}$. which is that:

$$(c_1^{L*}, ..., c_{T_0}^{L*}) \equiv argmax\{U^L(v_L); c_1 \geq 0, ..., c_{T_0} \geq 0\}$$
$$where\ v_L = (\overline{p}_1 c_1 + ..., + \overline{p}_{T_0} c_{T_0}, c_1, ..., c_{T_0})$$
$$T^{L*} = \overline{p}_1 c_1^{L*} + \overline{p}_2 c_2^{L*} + ... + \overline{p}_{T_0} c_{T_0}^{L*}$$

Then high-risk type agents could choose additional annuity contracts with :

$$(c_1^{H*} - c_1^{L*}, ..., c_{T_0}^{H*} - c_{T_0}^{L*}) \equiv argmax\{U^H(v_L^* + v_H); c_1 \geq 0, ..., c_{T_0} \geq 0\}$$
$$where\ v_L^* = (T^{L*}, c_1^{L*}, ..., c_{T_0}^{L*}), v_H = (p_1^H c_1 + ... + p_{T_0}^H c_{T_0}, c_1, .., c_{T_0})$$
$$T^{H*} - T^{L^*} = p_1^H(c_1^{H*} - c_1^{L*}) + p_2^H(c_2^{H*} - c_2^{L*}) + ... + p_{T_0}^H(c_{T_0}^{H*} - c_{T_0}^{L*})$$

We can call a JHG-similar tariff in $T_0 + 1$ period with the form with:

$$T^P(c_1, c_2, ..., c_{T_0}) \equiv T_1(c_1) + T_2(c_2) + ... + T_T(c_{T_0})$$
$$where\ T_t(c_t) \equiv 1_{\{c_t \leq c_t^{L*}\}} \overline{p}_t c_t + 1_{\{c_t > c_t^{L*}\}}[\overline{p}_t c_t^{L*} + p_t^H(c_t - c_t^{L*})]$$

Again, any entry-proof tariff T^{EP} should be no great than T^P in every period pricing. Otherwise, if there is a period that has higher pricing than T^P, then an entry can provide an annuity contract which only provides payment at this period and just a little higher price than T_t, it is attractable for buyers and it is profitable for this entrant. Thus, any entry-proof contract should have a lower tariff than JHG-similar tariff, that is to say, $T^{EP} \leq T^P$. If there is no squeeze-out effect happens in this dynamic model, due to high-risk type buyers having higher demand in every period, which means that the single crossing condition hold in every period. Thus, by using budget-balanced and single crossing conditions, we can get that any budget-balanced tariff should have higher pricing than T^P, as a result, $T^{EP} = T^P$.

4.3 Squeeze-Out Effect and Solution

In the three periods, we show an approach that if there is no squeeze-out effect, then the JHG-similar tariff is entry-proof, this proposition is also robust in the multiple-period model and the proof is the same as before. In this part, we focus on the case that the squeeze-out effect exists in multiple period models, propose the general cross-period policy and prove that the JHG-similar tariff combined with this policy is entry-proof again. In the multiple-period case, there are more periods, and the squeeze-out effect could also happen. We will prove that if there exists a squeeze-out effect, the reason only comes from the later period's demand crowding out the early period's demand, with no reverse case. To study what kind of form it could be, we first introduce some lemma in multiple cases:

Lemma 3. *In the JHG-similar tariff, If the squeeze-out effect happened in some period t, then it also happened in all period τ with $\tau \leq t$.*

Firstly, the squeeze-out effect happened in the period only if the demand for low-risk type in this period is great than 0. Then if we assume that $c_1^L, c_2^L, ..., c_{T_0}^L$ represents the amount chosen by low-risk type in JHG-similar tariff. And if there is no corner solution (the corner solution is not a squeeze-out effect), then we will have that:

$$-\overline{p}_t u'(w_0 - T^P(c_1^L, ..., c_{T_0}^L)) + p_t^L u'(c_t^L) = 0 \tag{6}$$

$$which\ means\ u'(c_t^L) = \frac{\overline{p}_t}{p_t^L} u'(w_0 - T^P(c_1^L, ..., c_{T_0}^L)) \tag{7}$$

Then, let us think about the reason why the squeeze-out effect happens. Actually, if there is only one layer of price provided by that planner, the high-risk type buyers always buy at least quantity v_L^t in each period because $p_t^H \geq p_t^L$. Thus, a squeeze-out effect happens when there exists that the marginal utility comes from some period with first-layer pricing is lower than the marginal utility comes from some other period with second-layer pricing. That is to say, if the squeeze-out effect happened in some period t, then we can induce that for ϵ small enough, we will have that there exist some period k such that for high-risk type buyers $p_t^H u(c_t^L - \epsilon) + p_k^H u(c_k^H + \frac{\overline{p}_t \epsilon}{p_k}) > 0$. If ϵ is small enough, this means that :

$$\overline{p}_t u'(c_k^H) - p_t^H u'(c_t^L) > 0 \tag{8}$$

This is equivalent that $u'(c_k^H) > \frac{p_t^H}{\overline{p}_t} u'(c_t^L)$, then according to (11) and the assumption that $\frac{p_t^H}{p_\tau^H} \geq \frac{p_t^L}{p_\tau^L}$ for $\tau < t$, we will have that $u'(c_k^H) > \frac{p_t^H}{\overline{p}_t} u'(c_t^L) \geq \frac{p_\tau^H}{\overline{p}_\tau} u'(c_\tau^L)$. This is equivalent with that $\overline{p}_\tau u'(c_k^H) - p_\tau^H u'(c_\tau^L) > 0$, that is also to say for the high-risk type buyers, the marginal utility for buying c_τ^L at price $\overline{p}_\tau$at period τ is less than the marginal utility of buying c_k^H at unit price p_k^H in period k. This means that the squeeze-out effect also happens for $\tau < t$. Thus, we can get the result in lemma 3.

Lemma 4. *In JHG-similar tariff, the squeeze-out effect will not happen in the last period.*

Intuitively, high-risk type buyers will have higher demand in a later period than low-risk type buyers because they know that they have a higher probability to live longer. Thus lemma 4 is to say that in the last period, high-risk type buyers will always buy more annuity payments than low-risk type buyers. There are two methods to prove this lemma. The first method is as showed in three-period cases, that no period could have a higher marginal utility with unit price p_k^H than the last period's marginal utility with unit price $\overline{p}_T$.

The second method to prove lemma 4 is using the result of lemma 3. If we assume that in the last period, the squeeze-out effect happens, then by using the lemma 3 , we will have that squeeze-out effect happens in every period. This means that in every period, high-risk type buyers purchase less annuity payment, which contradicts that $p_t^H \geq p_t^L$. As the result, the high-risk type buyers had at least bought more annuities in the last period than low-risk type buyers.

With the introduction of the Lemma 3 and 4, we can see that the structure of the squeeze-out effect in JHG is a similar tariff. That is, it may happen in some early period, and after some period, the high-risk type will have higher demand than low-risk types in every later period,this means that the squeeze-out effect happens only because that later period demand squeeze-out early period's demand, no other cases. And when the squeeze-out effect happens, the single crossing condition does not hold if only under JHG-similar tariff. Thus, we need to find a policy that can change this situation. An optional cross-period could solve this kind of problem. Now assume that the planner provides a cross-period policy that can transfer any period annuity to the period later, by using a numeric way, the planner could provide a transfer contract to two periods t and $t+k$, which can transfer the annuity payment from period t to $t+k$, and with a unit price of $\frac{p_{t+k}^H}{p_t^H}$.In the next lemma, we can know how the optional cross-period policy affects the JHG-similar tariff.

Lemma 5. *For any cross-period between t and $t+k$ with unit price $\frac{p_{t+k}^H}{p_t}$, we always have that* $p_{t+k}^H \geq \overline{p}_t * \frac{p_{t+k}^H}{p_t^H} \geq \overline{p}_{t+k}$.

Firstly, we prove the lemma 5, for the left-hand side of lemma 3, due to that $p_t^H \geq \overline{p}_t$, we can get that $p_t^H * \frac{p_{t+k}^H}{p_t^H} \geq \overline{p}_t * \frac{p_{t+k}^H}{p_t^H}$, which is equivalent to LHS of lemma. For the RHS of lemma 5, we have that $\overline{p}_t = m_H p_t^H + m_L p_t^L$, $\overline{p}_t * \frac{p_{t+k}^H}{p_t} = (m_H p_t^H + m_L p_t^L) * \frac{p_{t+k}^H}{p_t^H} = m_H p_{t+k}^H + m_L p_t^L * \frac{p_{t+k}^H}{p_t^H} \geq m_H p_{t+k}^H + m_L p_t^L * \frac{p_{t+k}^L}{p_t^L}$ (according to the assumption that $\frac{p_{t+k}^H}{p_t^H} \geq \frac{p_{t+k}^L}{p_t^L}$), which is equivalent to the RHS of lemma 5. So we finished the proof.

What lemma 5 could tell us? First, if squeeze-out happens in some period t because of some period $t+k$, the cross-period policy is attractive to high-risk type buyers, because he can buy the annuity contract with unit price $\overline{p}_t$ at period t and then change it to the annuity payment at period $t+k$. By using inequality $p_{t+k}^H \geq \overline{p}_t * \frac{p_{t+k}^H}{p_t^H}$, it is better for him to trade with this kind of cross-period policy. But due to that $\overline{p}_t * \frac{p_{t+k}^H}{p_t^H} \geq \overline{p}_{t+k}$, this kind of contract will not attract low-risk type buyers in period t.

Second, if there is no squeeze-out effect happened in period t and $t+k$, then trade cross-period policy means that high-risk type buyers buy some unit of annuity with unit price p_t^H at period t and then change it to payment in period $t+k$. The cost for high-risk type buyers directly buy an annuity or buy it in period t and change it to $t+k$'s payment is the same, because $p_t^H * \frac{p_{t+k}^H}{p_t^H} = p_{t+k}^H$. From the analysis above, the optional cross-period policy only affects high-risk type buyers when the squeeze-out effect happened. In this kind of case, the cost for the planner to provide this kind of policy with c_{t+k} unit is $c_{t+k} * \frac{p^H_{t+k}}{p^H_t}$ in the period, which is same as the price of it, thus, this cross-period policy is also budget balanced.

Then, by using this optional cross-period policy and combining it with the JHG-similar tariff, we can have the results as below:

Lemma 6. *Combining the JHG-similar tariff with optional cross-period policy, the high-risk type buyers always have a higher demand for annuity contracts in each period*

If the squeeze-out effect happens in some period t, according to lemma 3 and 4, this is because of some later period $t + k$'s demand with unit price p^H_{t+k} , then the high-risk type buyers can purchase the contract in period t and then transfer to period $t + k$ with unit price $\overline{p}_t * \frac{p_{t+k}}{p_t}$ which is less than p^H_{t+k} by using lemma 5. Thus, if squeeze-out happens in some period, the JHG-similar tariff adds cross-period policy could make the high-risk type buyers always have a higher demand for the annuity contract in every period. The single crossing condition hold in every period. As a result, the JHG-similar tariff combined with cross-period policy is entry-proof again.

Results: In the multiple period case:(1) If there is no squeeze-out happens in any period, then JHG-similar tariff is entry-proof in this economy. In JHG-similar allocation, low-risk type buyers purchase basic annuity contracts in each period, high-risk type buyers purchase basic and additional annuity contracts in every period, this is also entry-proof. (2) If the squeeze-out effect happens in some period, then it must happen in the period early than this period. In this case, the JHG-similar tariff combined with an optional cross-period policy could be entry-proof. In this kind of allocation, the low-risk type buys the annuity contract at the basic layer price. high-risk type buyers purchase the same quantity as low-risk type buyers in the squeeze-out effect happened period, and purchase basic and additional annuity contract in another period, the contract in no squeeze-out period consist of a basic price layer, cross-period transfer price layers, and additional price layer. This new allocation is entry-proof and budget balanced to planners even if a squeeze-out effect happens.

5 Extension

5.1 The elements affect squeeze-out condition

In this paper, we proposed the squeeze-out condition as a cut-off to distinguish the cases in a dynamic model. If the squeeze-out effect exists, it means the interaction of consumption is severe for high-risk type buyers, then the JHG-similar tariff is not enough to be entry-proof. In this part, we want to study this condition itself, try to answer how the variable affects this condition, and get some intuition, all the discussion is based on the three-period model.

We recall that squeeze-out effect: In some periods, the high-risk type consumers consume less than low-risk type consumers. In the three-period model, it means that low-risk type buyers have higher consumption in period 1. To find the condition when will the squeeze-out effect happen, we just need to find the **limit case**: At some point, the high-risk type buyers and low-risk type buyers have the same consumption in period 1.

First, I define some terms as below: Given JHG tariff to the market, the low-risk type chooses the contract with $(T_L^*, c_1^{L*}, c_2^{L*})$, the high-risk type chooses the contract with $(T_H^*, c_1^{H*}, c_2^{H*})$. Then the limit case is that in the equilibrium, $c_1^{L*} = c_1^{H*}$ and there is no incentive for high-risk type buyers to change the consumption from period 1 to period 2. Considering the utility function of high-risk type buyers, we may propose two cases to discuss the condition, which is as below:

$$u(w_0 - T_H^*) + p_1^H * u(c_1^{L*} - \epsilon) + p_2^H * u(c_2^{H*} + \frac{\overline{p}_1 * \epsilon}{p_2^H}) \tag{9}$$

$$u(w_0 - T_H^*) + p_1^H * u(c_1^{L*} + \frac{p_2^H * \epsilon}{\overline{p}_1}) + p_2^H * u(c_2^{H*} - \epsilon) \tag{10}$$

It seems that both (9) and (10) can study the squeeze-out effect, but actually (10) is not in the limit case, what we want to know is to find the condition that squeeze-out effect happens, and $c_1^{L*} = c_1^{H*}$ is not squeeze-out. so we rule out the discussion of (10).
From (9), we let ϵ close to 0, and take the derivation of ϵ, if the derivation is positive, this means that change consumption from period 1 to period 2 is a better choice of high-risk type buyers, that is to say, the optimal consumption of high-risk type buyers in period 1 is lower than c_1^{L*}, so the squeeze-out effect will happen. This condition can be written as

$$\overline{p} * u'(c_2^{H*}) - p_1^H * u'(c_1^{L*}) > 0 \tag{11}$$

And due to the high-risk type and low-risk type choose the optimal choice in the market, then we can get three FOC conditions in this market;(for high-risk type, FOC in period 1 is as same as low type)

$$-u'(w_0 - T_L^*) * \overline{p}_1 + p_1^L * u'(c_1^{L*}) = 0 \tag{12}$$

$$-u'(w_0 - T_L^*) * \overline{p}_2 + p_2^L * u'(c_2^{L*}) = 0 \tag{13}$$

$$-u'(w_0 - T_H^*) * p_2^H + p_2^H * u'(c_2^{H*}) = 0 \tag{14}$$

If we use (12) and (14) into (11), then by calculation, the squeeze-out effect happen is equivalent to that:

$$\frac{u'(w_0 - T_H^*)}{u'(w_0 - T_L^*)} > \frac{p_1^H}{p_1^L} \tag{15}$$

If we use (12) and (13) into (11), by calculation, the squeeze-out effect happen is equivalent to that:

$$\frac{u'(c_2^{H*})}{u'(c_2^{L*})} > \frac{p_1^H}{p_1^L} * \frac{p_2^L}{\bar{p}_2} \tag{16}$$

Then we can get some simple intuitions from (15) and (16):

(1) The alive probability in period 1 can affect the squeeze-out condition, the difference in period 1 is bigger, and the less probability squeeze-out effect happens in this period.

(2) The alive probability in period 2 can also affect the squeeze-out condition but is different from period 1, the difference in period 2 is bigger, and the more probability squeeze-out effect happens.(Because it can increase the total fees for high-risk type buyers)

5.2 Compulsory purchasing at initial period

In Rothschild (2015), he proposed a three-period model with features including compulsory and nonexclusivity in the annuity market. He argues that the government will let the agent deposit a fixed sum of money in their compulsory-retirement account. And they are required to spend all the accounts to choose the annuity plan. This kind of setting simplifies the model form, which means that all the sellers will receive the same amount w_0, and they compete with different annuity payment plan $c = (c_1, c_2)$. The buyers' utility function also becomes $U^i(c) = p_1^i u(c_1) + p_2^i u(c_2)$. This kind of setting gives us a good figure to study the problem. In this kind of setting, C.Rothschild solves the efficient allocation for planners, which both types' contracts are away from first best.

However, when we try to use this model to find the entry-proof tariff for planners, we find that there is no entry-proof tariff in this model. Even the JHG-similar tariff combined with cross-period policy in this kind of model, we can still find the profitable deviation from the new entrants. Compare this compulsory model with our model, the key difference is that in the compulsory model, all the buyers will use all the money to purchase the contract. In our model, the buyers can choose the amount to purchase the contract. While we can use the quantity and price together to screen different types in our model, C.Rothschild's model can only screen the types by different quantities. And the low-risk type can not reduce purchasing in this model, as a result, there is no entry-proof tariff in this model.

6 Conclusions

This paper mainly focuses on the annuity market, study the adverse selection in the nonexclusive and dynamic environment, based on the results of Attar et al. (2019, 2021) , we extend it to the annuity market to find the entry-proof and budget-balanced allocation to planners in the dynamic model. The JHG-similar tariff plays a very important role in a static environment, actually, if we study the case that the payment of annuity market is the same in every period, the JHG-similar tariff is also entry-proof to the market, this is because that demand of annuity of high-risk type higher low-risk type in every period, the single crossing condition hold in this same indemnity case. However, if we assume that the payment of annuity could be different in everything period, we showed that later period consumption to high-risk type buyers could crowd out early period consumption which may lead to the demand for high-risk type buyers lower than demand for low-risk type buyers in the early period. We call this phenomenon the "squeeze-out effect", if the squeeze-out effect happens in a dynamic model, we show that JHG-similar tariffs are no longer entry-proof. To solve this problem, we propose an optional policy for a planner, this policy allows the buyers to transfer the payment from the early period to the later period. If the squeeze-out effect happens, then high-risk type buyers can use this cross-period policy to buy the annuity contract in the early period and transfer it to a later period and have a lower cost in the later period, which give high-risk type buyers more demand for annuity contract in this period. Thus, combining JHG and cross-period policy, the high-risk type buyers have higher demand than low-risk type buyers in every period, single crossing condition holds again. As a result, if there is no squeeze-out effect happens in the dynamic model, JHG-similar tariff is entry-proof and implements a budget-balanced allocation. If the squeeze-out effect happens in the dynamic model, the JHG-similar tariff plus cross-period policy is also entry-proof and it is also implementing a new budget balance allocation. This result can be extended to multiple period cases, the only thing we need to clarify is that the squeeze-out effect only happens because later period demand crowds out early period demand as showed in lemma 1 and lemma 2.

There are more possible researches that could be done along with this paper: Firstly, we only assume two types of buyers in this dynamic model, and we also tried to find some results for multiple agent cases in a dynamic model. However, it is not as straightforward as what we did in the annuity market. It is difficult to define the new cross-period policy for planners in multiple agent cases, but it is valuable to study this problem. Secondly, Rothschild(2015) proposed a dynamic model to study the annuity markets with compulsory purchasing, in which is that he assumes the buyers need to consume all their annuity account's wealth to buy the contracts(That means he needs to use out w_0 in our dynamic model).In his setting, he finds that the efficient allocation for a monopoly social planner is different from what we found before.

But if we use the same setting to find the entry-proof allocation in competitive markets, it is strange to find there is no entry-proof tariff in his economy. The compulsory contracts in the annuity market are also valuable to study.

Appendix

A Proof of lemma 1

Assume that an entrant proposes a contact (c_1^L, T_1^L) to attract low-risk type in period 1, which means this contract has a lower unit price $p_1 < \overline{p}_1$. If $T_1^L \leq T_1^H$, then obviously this contract can also attract high-risk type buyers because the fees have no effect on period 2's consumption, high-risk type buyers just need to substitute the contract JHG in period 1 to this new contract, it has a lower cost for high type. Thus in this kind of case, it is also attractive for high-risk type buyers. If $T_1^L > T_1^H$, then we assume that given this new contract, the low-risk type chooses a new optimal contract in every period is that $(c_1^L, T_1^L), (c_2^L, T_2^L)$, then it is easy to get that $c_2^L \leq c_2^{L*}$. Now by using FOC in the model, we can have some equations as below:

1:For only JHG-similar tariff case, we use the FOC to low-risk type buyers and have that:

$$-u'(w_0 - T_L^*) * \overline{p}_1 + p_1^L * u'(c_1^{L*}) = 0 \tag{17}$$

$$-u'(w_0 - T_L^*) * \overline{p}_2 + p_2^L * u'(c_2^{L*}) = 0 \tag{18}$$

2: For JHG-similar tariff and new entrant case, we also use the FOC to low-risk type buyers and get the condition as below:

$$-u'(w_0 - T_L) * \overline{p}_1 + p_1^L * u'(c_1^L) = 0 \tag{19}$$

$$-u'(w_0 - T_L) * \overline{p}_2 + p_2^L * u'(c_2^L) = 0 \tag{20}$$

Now we use our assumption that there is no squeeze-out effect in period 1, this means that for high-risk type buyers, at the point (c_1^{L*}, T_1^{L*}) they still have the incentive to purchase more annuity contract in period 1. If we use ϵ to change purchases from periods 1 to 2, it is unprofitable. which is that $p_1^H * u(c_1^{L*} - \epsilon) + p_2^H * u(c_2^{L*} + \frac{\overline{p}_1 * \epsilon}{p_2^H})$ is not a better choice. Take ϵ close to 0, this is equivalent to that :

$$\frac{p_1^H}{\overline{p}_1} * u'(c_1^{L*}) - u'(c_2^{L*}) > 0 \tag{21}$$

What we want to prove: we want that the new contract is also attractive to high-risk type buyers. this means that at the point of (c_1^L, T_1^L), the high-risk types also have an incentive to purchase an annuity contract in period 1. Use the same trick, what we want to get is that :

$$\frac{p_1^H}{p_1} * u'(c_1^L) - u'(c_2^{L*}) > 0 \tag{22}$$

By using equation (19) and (20), we have that :

$$\frac{u'(c_1^L)}{p_1} = \frac{u'(w_0 - T^L)}{p_1^L} = \frac{p_2^L * u'(c_2^L)}{\overline{p}_2} \tag{23}$$

By using equation (17) and (18), we have that

$$\frac{u'(c_1^{L*})}{\overline{p}_1} = \frac{u'(w_0 - T^{L*})}{p_1^L} = \frac{p_2^L * u'(c_2^{L*})}{\overline{p}_2} \tag{24}$$

And due to $c_2^L \leq c_2^{L*}$, we always have that $u'(c_2^L) \geq u'(c_2^{L*})$. thus if we have condition (21), we can always have that condition (22) is satisfied. This means that high-risk type buyers are also attractable to this cheap contract. So No lower price contract can attract only low-risk type buyers in period 1.

B Proof of lemma 2

(the characters in this part has no relationship with the last part)

We want to prove that, if given a lower unit price in period 2 to attract low-risk type buyers, it is also attractable to high type. Actually, in this period, we can prove a stronger proposition. which is that: for any two given basic unit prices in periods 1 and 2, we let it as p_1, p_2, if low-risk type buyers choose the optimal contract are $(c_1, T_1) and (c_2, T_2)$, high-risk type buyers always have higher demand than low-risk type in period 2 . Now assume given basic price p_1, p_2, then the optimal quantity chosen by low-risk type is given by FOC:

$$-u'(w_0 - T^L) * p_1 + p_1^L * u'(c_1^L) = 0 \tag{25}$$

$$-u'(w_0 - T^L) * p_2 + p_2^L * u'(c_2^L) = 0 \tag{26}$$

Then for high-risk type buyers at the point of (c_1^L, c_2^L), if they can still have the same unit price p_2, the high-risk type buyer can choose to purchase a little more annuity in period 2 and purchase less in period 1 and get positive utility change. The deviation is as $p_1^H * u(c_1^l - \epsilon) + p_2^H * u(c_2^L + \frac{p_1 * \epsilon}{p_2})$, take ϵ close to 0,and take first order. we get that this is equivalent to:

$$p_2^H * \frac{p_1}{p_2} * u'(c_2^L) - p_1^H * u'(c_1^L) \tag{27}$$

By using the equation of (25) and (26), we can have that (27) is always great than 0. which means that for any basic price, high-risk type always has higher demand in period 2. As a result, if an entrant tries to propose a lower price in period 2 to attract low-risk type buyers, this contract always attracts high-risk type buyers.

Using the result of lemma 1 and lemma 2 , any entrant trying to attract low-risk type buyers always attracts high-risk type buyers, but the cost for serving both types is at least $\overline{p}_t$, which is the basic price in JHG-similar tariff. Thus, if there is no squeeze-out effect in the dynamic model, then the JHG-similar tariff is entry-proof and budget balanced.

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Chapter 4: Social Efficiency with Linear Side Trading

Abstract

This paper examines entry-proof allocations under linear contract entrants. While the JHG allocation is the unique entry-proof allocation for arbitrary tariff entrants, this paper shows that there are multiple feasible entry-proof allocations, such as the Pauly line allocation and two-part zero-profit allocations. The study provides conditions for the existence of non-Entry-Proof-Pareto-Dominated (EPPD) allocations from a planner's perspective and demonstrates that the absence of a planner in a linear pricing market is less efficient than in a market with the planner's intervention. However, the paper finds that linear side trading can often destroy efficiency in the convex tariff setting, even with non-convex tariffs and restrictive linear side trading. Furthermore, the Non-EPPD allocation can coincide with the second-best allocation under non-convex tariffs in some cases. The paper also shows that any entry-proof efficient allocation should provide an average unit price of c to L type buyers if there is no second-best allocation that coincides with the Non-EPPD allocation.

1 Introduction

This paper serves as a complementary study to the research presented in Attar et al. (2020). In traditional research on resource allocation problems under private information, efficient allocations must satisfy incentive compatibility constraints and lie on the Pareto efficient frontier, also known as second-best allocations (Myerson (1979), Harris and Townsend (1981)). However, Attar et al. (2020) demonstrates that if the planner cannot prevent potential side trading by entrants,[1] the only feasible incentive-compatible and entry-proof allocations are reduced to the unique allocation proposed by Jaynes (1978), Hellwig (1983), Glosten (1994), the JHG allocation. They argue that side trading undermines the planner's ability to redistribute resources among different types of buyers. In most cases, the JHG allocation is not the second-best allocation, suggesting that side trading with arbitrary tariff entrants impairs the efficiency of resource distribution.

However, the assumption of arbitrary tariffs appears to grant excessive power to entrants, constraining the planner's ability to distribute resources effectively. In real-world scenarios, sellers or entrants possess limited capabilities to monitor trades between themselves and buyers. Additionally, buyers can often trade anonymously with all entrants in markets such as financial and annuity markets, which operate with linear pricing. Empirical studies by Finkelstein and Poterba (2002, 2004), Cawley and Philipson (1999), and Chiappori and Salanie (2000) reveal that linear contracting is prevalent in insurance markets. Theoretical models on optimal allocation under private information with hidden trades typically limit entrants to proposing only linear contracts, as seen in Cole and Kocherlakota (2001) and Farhi et al. (2009). In this paper, we investigate the planner's feasible allocations that are robust to linear contract entrants, aligning with the literature while granting less power to entrants. Our aim is to determine whether there are more feasible entry-proof allocations for the planner and to assess whether the presence of linear contracting entrants compromises efficiency.

We address the issues in a setting similar to the one presented in Attar et al. (2020), using the insurance market as a specific example. In this economy, there are three kinds of actors: the planner and entrants (sellers), who sell a divisible good to buyers; and the buyers themselves. The planner can offer any type of tariff, while the entrants are limited to proposing linear tariffs. Buyers are privately informed about their types and can be classified as either high-type or low-type.

High-type buyers are more costly for sellers to serve, as their common value cost is higher than that of low-type buyers. Adverse selection exists in this economy, indicating that high-type buyers are more inclined to trade larger quantities than low-type buyers, in line with the single-crossing assumption. The utility of buyers is dependent on the aggregate quantity-transfer pair they traded with sellers, increasing with quantity but decreasing with the transfer. The planner is unable to monitor trades between entrants and buyers, allowing buyers to

[1]Entrants in side trading can propose any tariff of contracts.

trade non-exclusively with both entrants and the planner. Due to private information and the possibility of side trading, the planner must take into account incentive-compatible and entry-proof allocation strategies to implement resource distribution effectively.

Since entrants can only offer linear tariffs to the market, we first identify the break-even line if only one linear tariff serves the market. This break-even line is essentially the Pauly line, which has been studied by Pauly (1974). Under the Pauly line allocation, high-type buyers purchase a larger quantity than low-type buyers, but their average unit prices are equal. This average price, c^*, is greater than the unit cost c for serving both types at the same quantity. If a planner proposes a linear tariff with the Pauly line unit price, then this tariff is both entry-proof and incentive feasible. This is because any linear tariff offered by entrants with a price greater than c^* would not be attractive to buyers, and any linear tariff with a price lower than c^* would result in negative profit for the entrants. By utilizing the results in Attar et al. (2020), the JHG allocation is also an entry-proof allocation and robust against any linear side trading in tariffs. [2]

Under the presence of linear pricing entrants, the basic tariff of an entry-proof tariff is no longer limited to the serving cost for both types, c. Any basic tariff with a unit price $c_1 \in [c, c^*]$ can be part of entry-proof tariffs. As discussed in subsections 3.4 and 3.5, we demonstrate that there are numerous entry-proof tariffs with linear side trading. We then define an allocation as Entry-Proof-Pareto-Dominated (EPPD) if there is any other feasible entry-proof allocation that Pareto dominates it. We subsequently discuss the planner's tariff in two situations: convex tariffs and arbitrary tariffs.[3]

Under the convex tariff setting, we provide a necessary condition to a non-EPPD allocation in Theorem 1, which is that marginal substitution of high types should equal to serving cost of low types, and marginal substitution of low type should equal to average serving unit price. As a result of Theorem 1, Pauly line allocation is an EPPD allocation, which means if all the sellers in the market can only propose linear contracts, then the intervention of the planner always improves the efficiency, also the planner should never propose the Pauly line allocation to the market. Under the convex tariff, Theorem 2 shows that JHG allocation is the unique non-EPPD allocation if the proportion of high type buyers is big enough, this provides the suggestion for the planner to consider the JHG allocation is high type buyers account for main buyers. However, Proposition 3 shows that the second-best will not coincide with the non-EPPD allocation unde convex tariff setting.

According to Theorem 3, under a non-convex tariff setting, certain second-best allocations can coincide with a non-efficient pooled (NEP) allocation. This situation arises when the marginal substitution of low type is equal to the marginal low type cost c_L, a scenario that

[2]JHG allocation: The marginal price of any quantity is the upper-tail conditional expectation of the cost of serving the types who buy at least that amount. Both types purchase basic contracts at the unit price for serving both types, and high types buy additional contracts at the unit price for serving high types.

[3]It is worth noting that, in terms of nonconvex tariffs, there is significant debate on whether the planner should provide a convex tariff due to the potential of excessive cross-subsidization from low-type to high-type buyers.

cannot occur under a convex tariff. However, if there exists no point within the low type feasible sets where $MRS = c_L$, then there is no second-best allocation that can withstand linear side trading. Consequently, in such cases, all non-EPPD allocations should offer L type buyers an average unit price of c.

To sum up, compared to arbitrary contracts side trading, linear contracting side trading permits the planner to create a wider range of feasible entry-proof allocations. Among these, the Pauly line allocation and JHG allocation are two notable options that also satisfy entry-proof. If the planner is limited to proposing a convex tariff, then the JHG allocation may be the most efficient choice. However, the Pauly line allocation is always Pareto dominated by some other entry-proof allocations. Additionally, even with linear side trading allowing more feasible entry-proof allocations for the planner, it is still possible that potential side trading undermines the overall efficiency of the economy.

Related Literature

The most related literature to this paper is Attar et al. (2020). This paper examines nonexclusive trade and linear pricing of entrants, which can both be seen as the seller's ability to monitor trade. Specifically, a seller's ability to monitor trade can allow for menu contracts to screen different demand buyers, leading to higher prices for higher quantities, while the absence of monitoring can result in linear contracts. Additionally, a seller's ability to monitor the trade between other sellers and buyers can enable the use of exclusive conditions in contracts to prevent side trading. This paper is related to adverse selection, exclusive trade and nonexclusive trade, and linear pricing.

Under the condition of trade limited to one unit of goods, if all sellers can monitor the trade of others but not their own, which means the buyers can only propose linear contracts and can only trade with one seller, then the market outcome is the Akerlof allocation introduced by Akerlof (1970). If all sellers can monitor their own trade but not that of others, which means the buyers can propose the menu contracts but can non-exclusively trade with other sellers, the market outcome is also the Akerlof allocation according to Attar et al. (2011). The market outcome with Akerlof is only low types can trade, so inefficient of the market.

In a market with no quantity restrictions on trade, if all sellers cannot monitor their own or others' trades, the market is analogous to Pauly (1974) allocation. If all sellers can monitor their own and others' trades, it is an exclusive economy as studied by Rothschild and Stiglitz (1976), Wilson (1977), Miyazaki (1977) and so on. If all sellers can only monitor their own trades, but not others, it is a nonexclusive economy previously studied by Attar et al. (2014, 2021, 2022), which also related to the work of Jaynes (1978), Hellwig (1988), Glosten (1994), Stiglitz et al. (2020). If the sellers not only can monitor the trade but also can make the contract one other sellers' contract, it is the economy of Yamashita (2010) and Szentes (2015).

This paper specifically studies the scenario where only one seller can monitor their own trade. From a planner's perspective, intervention can improve social efficiency in this case.

This paper also provides insight into seller competition: in a Pauly economy, a seller who can monitor their own trade may become a monopoly in the market without lower costs or product breakthroughs. This implies that a monopoly on information can lead to a monopoly in the market.

The paper is structured as follows. Section 2 outlines the model's setting, while Section 3 provides examples of entry-proof allocations and defines EPPD. Section 4 examines the efficiency of the convex tariff planner, demonstrating that the planner's intervention improves efficiency. Section 5 explores the efficiency of arbitrary tariff planners, highlighting instances where the second-best allocation can only coincide with non-convex tariffs and demonstrating that even linear side trading can sometimes undermine efficiency. Finally, Section 6 presents the conclusions.

2 The Model Setting

In the economy, there are three types of actors: buyers, sellers (referred to as "planners" in this context), and possible entrants (other non-planner sellers). Buyers have access to private information about their preferences and can purchase goods from multiple sellers. Planners can monitor their own trades but cannot observe trades between buyers and other sellers, which makes side trading possible. This means that planners only have knowledge of their own contracts with buyers and cannot prevent buyers from purchasing contracts from other sellers. Possible entrants can also provide contracts to the market, but unlike planners, they cannot monitor trades with buyers and therefore cannot distinguish between buyers who have already purchased contracts. This results in the contracts provided by entrants being linear in nature. The detailed model setting is identical to that of Attar et al. (2020), except that in this scenario, only linear side trading contracts are permitted for entrants.

Buyers. Our model consists of two types of buyers: type L for low risk and type H for high risk. Each buyer's type is their private information, and their preferences over the quantity-transfer pair $(q, t) \in \mathbb{R}^+ \times \mathbb{R}$ differ depending on their type.

Let m_L and m_H define the proportion of these two types in the market, where $m_L + m_H = 1$. Buyers can purchase goods in exchange for monetary transfers. The preference of type θ is denoted as u_θ and is determined by the aggregate contracts (q, t) she purchased. u_θ is strictly quasiconcave and differentiable. $\partial_q u_\theta \geq 0$ and $\partial_t u_\theta < 0$, which means the utility nondecreasing with quantity q and decreasing with tariff t. The marginal rate of substitution is defined as:

$$\tau_\theta \equiv -\frac{\partial_q u_\theta}{\partial_t u_\theta}$$

In general, we take the marginal rate of substitution (MRS) to satisfy Inada's condition, which means that $\tau_\theta(q, t)$ approaches 0 as q grows large along the curve. This ensures that type θ's demand at any unit price is finite. Additionally, as in other literature, we assume the single-crossing assumption (SC):

Assumption 1 (Single-Crossing(SC))**.** *For any pairs* $(q,t) \in \mathbb{R}^+ \times \mathbb{R}$, $\tau_H(q,t) > \tau_L(q,t)$

This assumption says that type H buyers have a higher incentive to purchase more quantity of goods than type L buyers. The insurance market is a special case of this setting, where q represents the coverage and t represents the premium in the insurance contract. In this context, the SC assumption implies that type H buyers are more likely to purchase more coverage and pay higher premiums than type L buyers. This is because type H buyers have a higher aversion to risk and value the protection that insurance provides more than type L buyers.

The SC assumption is widely used in the literature on adverse selection and insurance markets. It is a critical assumption that ensures that the equilibrium allocation in the market is well-defined and that buyers of different types purchase different quantities of goods or services. Furthermore, the SC assumption is supported by empirical evidence from insurance markets, where risk-averse individuals tend to purchase more insurance coverage than risk-tolerant individuals.

Sellers: The Planner and Entrants. In our model, there is a planner who can observe her own trade with buyers, but cannot observe other trades in the market. This means that the planner can propose any type of contract to the market, but buyers are not restricted to only purchasing contracts from the planner. Additionally, the private information about buyer types is not available to sellers, which means that sellers cannot distinguish between different types of buyers based on observable characteristics.

The unit cost of goods depends on the type of buyer who purchased it and is denoted as c_θ for serving type θ. If the planner serves a type θ buyer with a contract (q,t), then the profit for this trade is $t - c_\theta q$. In a general setting, serving high-risk types typically incurs a higher unit cost than serving low-risk types. Therefore, we assume the common-value (CV) assumption:

Assumption 2 (Common Value (CV))**.** *H type buyers are more costly to serve:* $c_H \geq c_L$

In addition to the planner and buyers, there are also potential entrants in this economy who can privately sign contracts with buyers. We consider the situation they cannot monitor the total quantity of goods purchased by a buyer, and thus, they can only propose linear contracts to buyers based on unit prices. Because the planner cannot monitor the trades between buyers and entrants, the potential trades may disrupt the planner's plan in the economy. Furthermore, since buyers can purchase multiple contracts, this economy operates in a non-exclusive environment. As a result, the planner needs to consider the effect of potential entrants before proposing contracts to the market.

For example, the allocation proposed by Rothschild and Stiglitz (1976) is not sustainable in this economy due to the presence of potential entrants. The Rothschild-Stiglitz (RS) allocation involves a fully separated allocation consisting of full insurance to high-risk types and partial insurance to low-risk types under exclusive competition, where each buyer can only trade with at most one seller. However, in a non-exclusive environment with potential entrants, this allocation is not an equilibrium because buyers can purchase additional contracts from entrants, and entrants have the incentive to offer contracts that pivots the planner's contracts. Therefore, to sustain the allocation, the planner must design contracts that not only induce truthful revelation of buyer types but also are robust to the potential competition from entrants, which we call entry-proof allocation later.

Under the SC and CV assumptions, high-risk types are weakly more eager to purchase more goods than low-risk types. However, the sellers are less eager to sell goods to high-risk types, and they cannot distinguish between high-risk and low-risk types. As a result, adverse selection occurs in this economy. We define $c \equiv m_L c_L + m_H c_H$ as the average cost of serving both types of buyers. The SC and CV assumptions are common in the literature on adverse selection and insurance markets. They ensure that high-risk types have a higher willingness to pay for goods or services than low-risk types and that the sellers and buyers have the same information about the distribution of costs.

Trading Rule and Incentive Feasible. The market operates through a three-stage process. Firstly, the planner proposes a contract set or tariff to the market, which can take various forms, including different layers of unit prices. Then, the entrants and buyers observe the planner's tariff, and entrants can provide a unit price to buyers, effectively proposing linear contracts to the market. Finally, buyers can choose contracts from the market, and due to the assumption of side trading, they can combine contracts from multiple sellers. We refer to this type of trade as nonexclusively trade.

To model the market, we can use a simple set of parameters to illustrate what happens. A **tariff** $T(q)$ is a price plan for different quantities of goods q. $T(0) = 0$ means that buyers can choose not to purchase, while $T(q) = \infty$ means that the seller does not want to sell q quantity. Initially, the planner proposes the tariff $T^P(q)$ to the market. Then, the entrants can propose linear tariffs to the market, taking the form of $T^E(q) = p^E q$. Finally, buyers can choose any contract in the market and try to achieve maximum utility.

Suppose a buyer chooses (q^P, t^P) from the planner and selects contracts from K different entrants. Then, her aggregate trade is $(Q, T) \equiv (q^P + \sum_{k \in K} q^k, t^P + \sum_{k \in K} p^k q^k)$. This three-stage process allows us to analyze the impact of different types of contracts and pricing strategies on market outcomes, including the effects of adverse selection and potential competition from entrants.

We will now define some terms used in the model, based on Attar et al. (2020).

An allocation $(q_\theta, t_\theta)_{\theta=L,H}$ is **budget-feasible** if $m_L(t_L - c_L q_L) + m_H(t_H - c_H q_H) \geq 0$. This means that the planner will not have a negative profit when achieving the allocation $(q_\theta, t_\theta)_{\theta=L,H}$.

Additionally, an allocation $(q_\theta, t_\theta)\theta = L, H$ is **incentive-compatible** if it satisfies $IC_{L \to H}$: $u_L(q_L, t_L) \geq u_L(q_H, t_H)$ and $IC_{H \to L} : u_H(q_H, t_H) \geq u_H(q_L, t_L)$.

An allocation is **incentive-feasible** if it is both budget-feasible and incentive-compatible.

Finally, a second-best allocation is Pareto-efficient in the set of all incentive-feasible allocations that do not rely on transfers from other buyers or entrants. These definitions will be used in the later model to analyze the properties of different allocation schemes and pricing strategies, and to identify the conditions under which Pareto-efficient outcomes can be achieved despite the challenges posed by non-exclusivity and adverse selection.

3 Incentive-Feasible Allocations: Examples

In this section, we will introduce the concept of an entry-proof allocation and the corresponding tariff that can implement this allocation. We will then use the assumption of linear side trading to identify allocations that are robust to such trading. Finally, we will discuss whether certain second-best allocations are robust to linear side trading, and analyze the implications of this for achieving efficient outcomes in the presence of non-exclusivity and adverse selection.

3.1 Definition of Entry-Proof Allocation

Implementation The tariff T implements the allocation $((q_L, t_L), (q_H, t_H)$ if for each θ:

$$q_\theta \in argmax\{u_\theta(q, T(q)) : q \geq 0\}$$
$$t_\theta = T(q_\theta)$$

In the context of our model, an entry-proof allocation refers to the optimal choice of contracts for buyers when they can select from the tariff proposed by the planner and the linear contracts proposed by the entrants. Thus, the allocation that satisfies this definition is also incentive-compatible. A tariff refers to the price plan that a seller provides for different quantities of goods q. Specifically, for a seller, the tariff T specifies the price to be paid for each quantity of goods purchased. As we introduced before, $T(0) = 0$ means the buyer can choose not to purchase from this seller, while $T(q) = \infty$ indicates that the seller does not allow the purchase of q quantity of goods. $t = T(q)$ represents that buyer can purchase quantity q with the transfer t. On the other hand, the entrants, being unable to monitor their trades with buyers, can only propose a unit price of p for the coverage they offer. Thus, their tariff is simply pq. Then we define a tariff T^P as entry-proof to linear entrants when **no** linear entrant can achieve positive profit for a given T^P, which is as below:

Entry-proof Tariff with Linear Entrants: The tariff T^P is entry-proof with linear entrants if , for any unit price p^E offered by an entrant, there exists for each buyer θ a solution (q^P_θ, q^E_θ) to type i's problem:

$$max\{u_\theta(q^P + q^E, T^P(q^P) + p^E q^E) : q^P \geq 0 \; and \; q^E \geq 0\} \quad (1)$$

such that the expected profit of the entrant is at most zero,

$$p^E[m_L q_L^E + m_H q_H^E] \leq m_L v_L q_L^E + m_H v_H q_H^E \tag{2}$$

An allocation is said to be robust to linear side trading if it can be implemented by an entry-proof tariff. In the following subsection, we will first introduce some entry-proof tariffs and allocations implemented by them. We will then examine possible second-best allocations and discuss whether they are robust to linear side trading.

3.2 The Break Even Line

Initially, we aim to determine the cutoff linear unit price that would enable a single entrant to make a non-negative profit in the absence of any other sellers. To identify the break-even line, we can consider the scenario where the entrant captures all the buyers in the market. Due to the single-crossing (SC) assumption, we know that any linear contract that attracts low-risk buyers will also attract high-risk buyers. Consequently, high-risk buyers will always purchase higher quantities of goods than low-risk buyers.

We begin by finding the break-even linear unit price for an entrant who attracts all buyers in the market. Let $c = m_L c_L + m_H c_H$ as the average cost of serving both types of buyers. Intuitively, if a seller provides a unit price equal to c, the high-risk buyers will choose a higher quantity than low-risk buyers due to the SC assumption. As a result, the seller will have a negative profit. However, if the seller provides a unit price equal to c_H, she will receive zero profit from high-risk buyers and positive profit from low-risk buyers. Thus, we can find a line between c and c_H that gives zero profit for the entrant. Let c^* be the lowest unit price that satisfies this condition, and let $(q_\theta^*, t_\theta^*)_{\theta=L,H}$ be the allocation implemented by this price, then we can state the following condition:

$$\begin{aligned}
(m_L q_L^* + m_H q_H^*)v^* &= m_L q_L^* c_L + m_H q_H^* c_H \\
-\frac{\partial u_L}{\partial q_L^*} / \frac{\partial u_L}{\partial t_L^*} &= c^* \\
-\frac{\partial u_H}{\partial q_H^*} / \frac{\partial u_H}{\partial t_H^*} &= c^* \\
t_\theta &= c^* q_\theta^*
\end{aligned}$$

We then define an allocation as entry-proof if it is the optimal choice for buyers when selecting from a tariff, and if any entrant providing a linear tariff below c^* will have a negative profit. This condition is satisfied when the profit of the entrant is zero, which we can express as $m_L(t_L^* - c_L q_L^*) + m_H(t_H^* - c_H q_H^*) = 0$. Here, $(q_\theta^*, t_\theta^*)_{\theta=L,H}$ is the optimal allocation purchased by type θ buyers given this linear tariff, and $q_H^* > q_L^*$ due to the SC assumption. Geometrically, the optimal points come from the tangent between the zero-profit line and the indifference curves, so we can express the condition as MRS equals the marginal unit price.

Once we have found the break-even line for entrants, we can compare different allocations and determine if they are entry-proof for linear pricing entrants. In the next subsection, we will present some allocations that are entry-proof under this condition.

3.3 Pauly Line Allocation

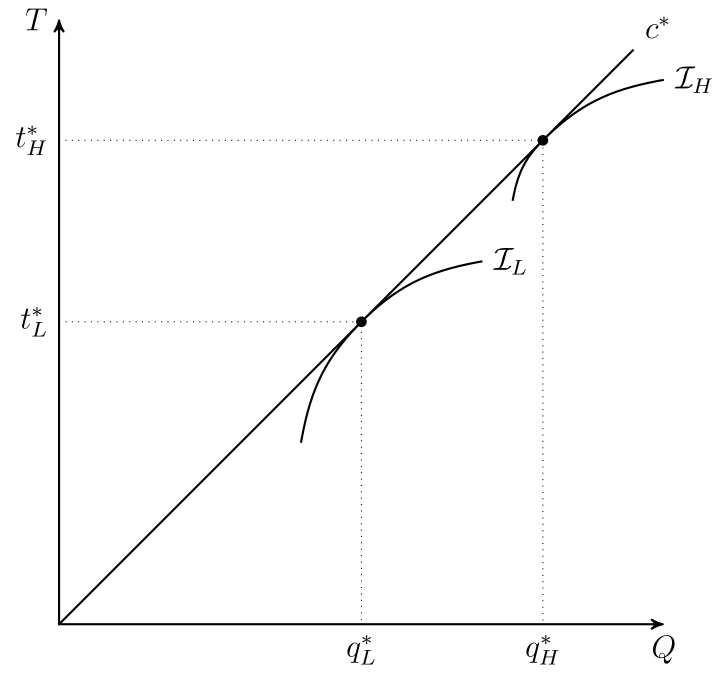

Figure 1: Pauly Line Allocation

According to the definition of the linear pricing break-even line, a seller providing a linear tariff to the market can attract the whole market and make a profit only if the slope of the contract is above the line with slope c^*, which we call the Pauly line.[4]

Intuitively, if the planner proposes only the Pauly line tariff to the market, it is an entry-proof with linear contract entrants. The planner can set the tariff as $T^P(q) = c^*q$, which is entry-proof if entrants can only propose linear contracts $T^E(q) = pq^E$, where p is the unit price proposed by the entrant. The allocation (q_θ^*, t_θ^*) is the budget-feasible allocation robust to linear side trading.

If the planner offers contracts with the Pauly line tariff, any entrant whose unit price is greater than c^* will not be attractive to both type L and type H buyers. This means that if the entrant's unit price p is above c^*, we will have:

$$argmax\{u_\theta(q^E + q^P, T^P(q^P) + T^E(q^E)\} = argmax\{u_\theta(q^P, T^P(q^P))\}$$

[4]Pauly (1974) show the allocation implemented by this tariff is the equilibrium in the linear nonexclusive competition.

Recall the definition of the linear pricing break-even line or Pauly line, which is the lowest unit price for an entrant to earn zero profit when attracting the whole market. If a seller offers a linear contract with a slope (unit price) above the Pauly line, it will be profitable; otherwise, it will not be profitable.

If the planner offers the contracts with a Pauly line tariff $T^P(q) = c^*q$ to the market, it is an entry-proof allocation with linear contract entrants. Any entrant whose unit price is above c^* will have no deal with buyers. On the other hand, if an entrant offers a unit price below c^*, she will attract all types of buyers. In this situation, $argmax\{U_\theta(q^E + q^P, T^P(q^P) + T^E(q^E)\} = argmax\{u_\theta(q^E, T^E(q^E))\}$, thus, her profit will be negative. Therefore, whatever unit price entrants set, they will not have a positive profit.

In summary, the Pauly line allocation is an entry-proof allocation if entrants can only offer linear contracts. The allocation (q^*_θ, t^*_θ) implemented by the Pauly line tariff is the budget-feasible allocation robust to linear side trading, in this tariff, the planner proposes the linear tariff with a unit price of c^*.

Proposition 1 (Pauly Line Tariff and Allocation)**.** *The Pauly line tariff $T^{Pauly}(q) = c^*q$ is an entry-proof tariff, the allocation implemented by the Pauly line tariff is incentive-feasible.*

3.4 JHG Allocation

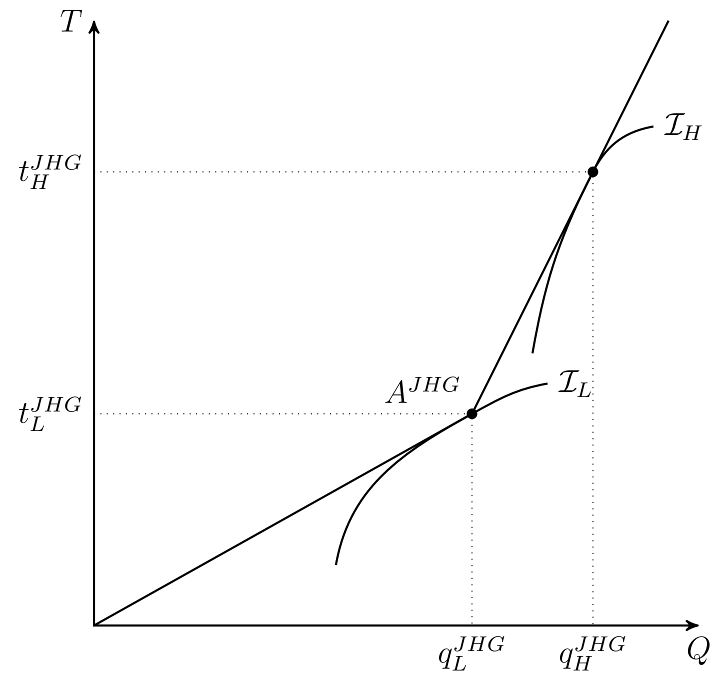

Figure 2: JHG Allloncation

The JHG allocation was first introduced by Jaynes (1978), Hellwig (1988), and Glosten (1994), and was later formalized and analyzed by Attar et al. (2020). They showed, under the assumption of non-exclusivity, that the JHG allocation is the only entry-proof and budget-balanced allocation with arbitrary entrants. In this subsection, we will provide a brief overview of the JHG allocation and present proof that the JHG tariff is entry-proof with linear pricing entrants.

In the JHG allocation, the planner first offers basic contracts to the market with a unit price of c, where $c = m_L c_L + m_H c_H$, representing the average cost of serving both types of buyers. The basic contract is a limit order with a maximum quantity that buyers can choose equal to the optimal quantity for type L buyers under the unit price of c, which we denote as q_L^{JHG}. Then, an additional contract is provided with a unit price of c_H, which type H buyers can combine with the basic contract to obtain their optimal quantity of contracts, denoted as q_H^{JHG}. The JHG allocation $(q_\theta^{JHG}, t_\theta^{JHG})_{\theta=L,H}$ is then defined as follows:

$$\begin{aligned} q_L^{JHG} &= argmax\{u_L(q, cq) : q \geq 0\} \\ t_L^{JHG} &= cq_L^{JHG} \\ q_H^{JHG} - q_L^{JHG} &= argmax\{u_H(q_L^{JHG} + q, t_L^{JHG} + c_H q) : q \geq 0\} \\ t_H^{JHG} - t_L^{JHG} &= c_H(q_H^{JHG} - q_L^{JHG}) \end{aligned}$$

In the JHG allocation, both type L and type H buyers purchase the basic contract with quantity-transfer pairs (q_L^{JHG}, t_L^{JHG}) and (q_L^{JHG}, t_L^{JHG}) respectively, where q_L^{JHG} is the optimal quantity for type L given the unit price of c_L. After that, high-risk type buyers purchase an additional quantity with a higher unit price c_H. The profit for the planner is 0 in the JHG allocation since the serving cost of both types in the basic contract is $m_L c_L q_L^{JHG} + m_H c_H q_L^{JHG}$, and the transfer is $m_L t_L^{JHG} + m_H t_L^{JHG} = cq_L^{JHG} = m_L c_L q_L^{JHG} + m_H c_H q_L^{JHG}$. For the additional contract, the serving cost is equal to the unit price of the contract. Thus, the JHG allocation is budget-feasible.

The JHG allocation $(q_\theta^{JHG}, t_\theta^{JHG})_{\theta=L,H}$ can be implemented by the JHG tariff, which is as follows:

$$T^{JHG}(q) = 1q \leq q_L^{JHG} cq + 1_{q > q_L^{JHG}}[cq_L^{JHG} + c_H(q - q_L^{JHG})]$$

For any linear pricing entrant, if they want to attract low-risk type L buyers, they can only offer a contract with a unit price p^E below c. However, if they set the unit price lower than c, they cannot separate high-risk type H buyers, and thus they will attract the whole market. But, as we proved before, any slope below c^* is not profitable. It is not possible to attract type L buyers profitably. If the new entrant wants to only attract type H buyers, then they must set the unit price lower than c_H to have any market. But obviously, it is not profitable if the entrant sets the unit price below c_H and only makes a contract with type H buyers. As a result, the new entrants cannot find a profitable way to enter this market. Thus, JHG allocation is a budget-feasible allocation and robust to linear side trading."

Proposition 2 (JHG Tariff and Allocation). *The JHG tariff $T^{JHG}(q)$ is an entry-proof tariff, the allocation implemented by the JHG tariff is incentive-feasible.*

3.5 Two-Part Tariff Allocations

We have analyzed two allocations previously. The Pauly line allocation is a linear allocation that sets the unit price at c^*. On the other hand, the JHG allocation is a two-part tariff where the basic tariff part has a unit price equal to c, while the second part of the tariff has a unit price equal to c_H. Given these two allocations, we are now curious to investigate what would happen if the unit price of the basic contract is set between c and c^*.

Firstly, we can choose any c_1 as the unit price of the basic layer serving both types, where c_1 is between c and c^*. Let (q_L^1, t_L^1) be the optimal quantity-transfer pair chosen by type L buyers given the unit price of c_1. We can now consider a tariff (T^{P_1}) as follows:

$$T^{P_1}(q) = 1_{\{q \leq q_L^1\}} c_1 q + 1_{\{q > q_L^1\}}[c_1 q_L^1 + c_H(q - q_L^1)]$$

Under the tariff $T^{P_1}(q)$, using a similar analysis of JHG allocation, the planner can implement the entry proof allocation $(q_L^1, t_L^1), (q_H^1, t_H^1)$ for type L and H buyers. In this allocation, for the basic contract, the cost for the planner is $(m_L c_L + m_H c_H) q_L^1$, and the transfer is $c_1 q_L^1 > c q_L^1 = (m_L c_L + m_H c_H) q_L^1$. The additional contract has zero profit because the serving cost is equal to the unit price of the tariff. Thus, this kind of contract actually provides the planner with a positive profit, making it budget feasible. Moreover, for entry-proofness, this allocation is very similar to the JHG allocation. If the entrant sets the unit price below c_1, the entrant gets the whole market but incurs a negative profit. If the entrant sets the unit price above c_H, they cannot attract any buyers. If the entrant offers the contract with a unit price between c_1 and c_H, buyer 2 will make a contract with them, but it still would be unprofitable for the entrant. As a result, this allocation is a profitable and entry-proof allocation.

Based on the profitable allocation we found above, considering the similar structure of tariff we can keep the same tariff for the basic layer and then cross-subsidize to the H type more to obtain a zero-profit allocation. Thus, we can also set any c_1 between c and c^* as the unit price of the basic layer tariff and use the profit from the basic layer to subsidize the H type buyer. Then we can find a unit price $c_2 < c_H$ that results in a zero-profit allocation. The details of the allocation $(q_L^1, t_L^1), (q_H^2, t_L^2)$ are as follows:

$$\begin{aligned}
q_L^1 &= argmax\{u_L(q, c_1 q) : q \geq 0\} \\
t_L^1 &= c_1 q_L^1 \\
q_H^2 - q_L^1 &= argmax\{u_H(q_L^1 + q, t_L^1 + c_2 q) : q \geq 0\} \\
t_H^2 - t_L^1 &= c_2(q_H^2 - q_L^1)
\end{aligned}$$

c_2 is solved by the zero-profit condition. For any different c_1 between c and c^*, we can always solve for one c_2 that satisfies the zero-profit condition:

$$
\begin{aligned}
q_L^1 c_1 + m_H c_2 (q_H^2 - q_L^1) &= m_L q_L^1 c_L + m_H q_H^2 c_H \\
-\frac{\partial u_L}{\partial q_L^1} / \frac{\partial u_L}{\partial t_L^1} &= c_1 \\
-\frac{\partial u_H}{\partial q_H^2} / \frac{\partial u_H}{\partial t_H^2} &= c_2 \\
t_L^1 &= c_1 q_L^1 \\
t_H^2 - t_L^1 &= c_2 (q_H^2 - q_L^1)
\end{aligned}
$$

This two-part zero profit allocation can be implemented by tariff $T^{P_2}(q)$ as below:

$$
T^{P_2}(q) = 1_{\{q \leq q_L^1\}} c_1 q + 1_{\{q > q_L^1\}} [c_1 q_L^1 + c_2 (q - q_L^1)]
$$

One of the zero-profit conditions is based on the first equation, where the left-hand side represents the profit of the two-part allocation and the right-hand side represents the cost of serving both types. The second and third equations indicate that (q_L^1, t_L^1) and (q_H^2, t_H^2) are the optimal choices given the tariff $T^{P_2}(q)$. Under this allocation scheme, the new entrant cannot set a price lower than c_1 as doing so would result in a negative profit. If the entrant sets a price above c_1, they can only attract type buyers, and this will only be profitable if the unit price is above $c_H > c_2$. However, in this scenario, the entrant's contract will be worse than the planner's allocation, resulting in zero profit. Therefore, this allocation is both budget-feasible and robust to linear side trading.

Comments:

The Pauly line allocation and JHG allocation can be considered as special cases of the two-part zero-profit allocation, representing two limit cases. Given the tariff $T^{P_2}(q) = 1_{q \leq q_L^1} c_1 q + 1_{q > q_L^1} [c_1 q_L^1 + c_2 (q - q_L^1)]$, there are two possible choices. First, if we choose $c_1 = c$, the second part unit price becomes $c_2 = c_H$, making it equivalent to the JHG allocation. Second, if we choose $c_1 = c^*$, by applying the zero-profit condition, we find that $c_2 = c^*$. Therefore, the JHG allocation and Pauly line allocation are two special cases of the two-part zero-profit allocation. It is also simple to deduce that the unit price between c and c^* can serve as the basic tariff of a single entry-proof tariff.

There are numerous entry-proof allocations available if entrants can only offer linear contracts. From a planner's perspective, it is desirable to eliminate all inefficient allocations. To this end, we can define an inefficient allocation as follows:

Definition 1 (Entry Proof Pareto Dominated (EPPD))**.** *An allocation $(q'_\theta, t'_\theta)_{\theta=L,H}$ is said to be Entry Proof Pareto Dominated if there exists another allocation $(q_\theta, t_\theta)_{\theta=L,H}$ such that $u_\theta(q_\theta, t_\theta) \geq u_\theta(q'_\theta, t'_\theta)$ with at least one inequality, and the allocation $(q_\theta, t_\theta)_{\theta=L,H}$ is entry-proof to linear entrants.*

Using this definition, it can be easily demonstrated that all profitable allocations are in fact

Pareto dominated by some zero-profit allocations. As a planner, if an allocation is EPPD, it would not be chosen for the market, because a better allocation can always be found.

4 Incenvtive-Feasible Allocations with Convex Tariff

In this section, our focus is on the convex tariff for the planner. The consideration of a convex tariff allows us to easily determine the range of entry-proof basic contracts, which have a unit price that lies between the average cost, c, and the zero-profit unit price on the Pauly line, c^*. Furthermore, in real-life applications, non-convex tariffs have been associated with debates on fairness due to the excessive cross-subsidy from L types to H types, making it burdensome for L type buyers. Thus, we concentrate on the convex tariff setting in this section.

4.1 Two Part Convex Tariff with Zero Profit Frontier

In the previous section, we began our analysis with the entry-proof tariff to identify zero-profit allocations. We discussed a particular type of zero-profit allocation in which the planner offers a basic contract to both buyer types. Then based on this basic quantity, the planner then provides an additional layer of linear contracts to high-type buyers, allowing both types to choose their optimal quantities according to the two-part tariff. In this section, we expand our analysis to encompass all zero-profit allocations, which enables us to identify non-EPPD allocations with implementable tariffs.

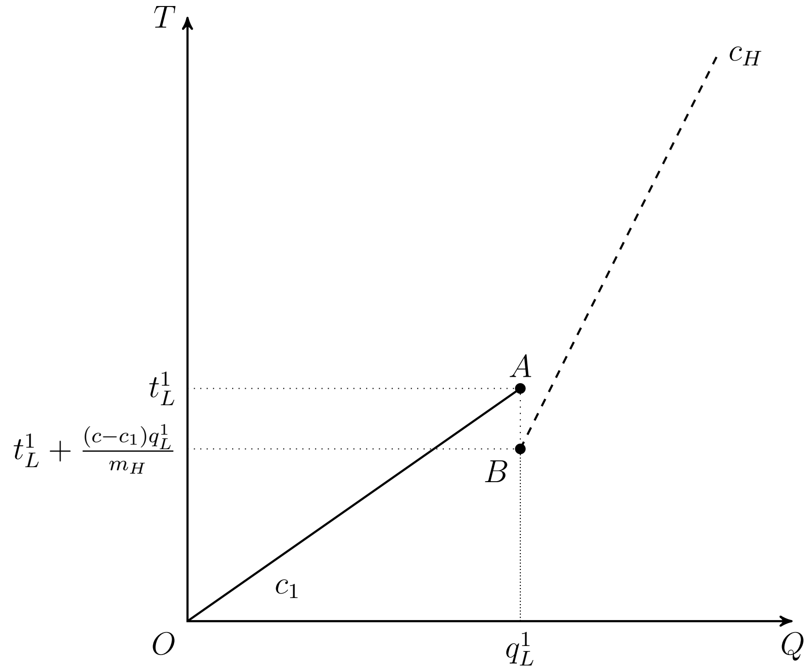

Figure 3: Zero Profit Frontier

Starting from zero profit, we can consider that the planner uses the profit from the first layer to cross-subsidize buyers in the second layer. Let (q_L^1, t_L^1) represent the quantity-transfer pair for both L and H types in the basic layer, and (q_H^{ZF}, t_H^{ZF}) denote the allocation for H type buyers in the second layer. By accounting for the zero profit of all allocations, for a given (q_L^1, t_L^1), we can determine the zero-profit frontier for (q_H^{ZF}, t_H^{ZF}),[5] which must satisfy specific conditions.

$$m_H(t_H^{ZF} - t_L^1 - c_H(q_H^{ZF} - q_L^1)) = (c - c_1)q_L^1 \tag{3}$$

In equation (3), the absolute value of $q_L^1(c_1 - c)$ on the right-hand side (RHS) represents the profit for serving both types in the first layer, which corresponds to the amount of cross-subsidy from the basic layer. On the left-hand side (LHS), the planner serves only high-type buyers, which corresponds to the m_H proportion. The expression $t_H^{ZF} - t_L^1 - c_H(q_H^{ZF} - q_L^1)$ represents the profit of the additional quantity-transfer pair purchased by H types. Equation (3) illustrates the function of all zero-profit allocations for H types, given that the L type chooses the allocation (q_L^1, t_L^1).

Interestingly, based on the zero-profit frontier equation, we can deduce that the frontier is actually a line. This line has a slope of c_H and passes through the point $(q_L^1, t_L^1 + \frac{(c-c_1)q_L^1}{m_H})$. The starting point of this zero-profit frontier lies just below the L type buyers' allocation due to the profit from the first layer, and the frontier has a slope of c_H higher than the first layer, given that the marginal serving cost for the H type is c_H. As shown in Figure 3, the dashed line starting at point B represents the zero-profit frontier.

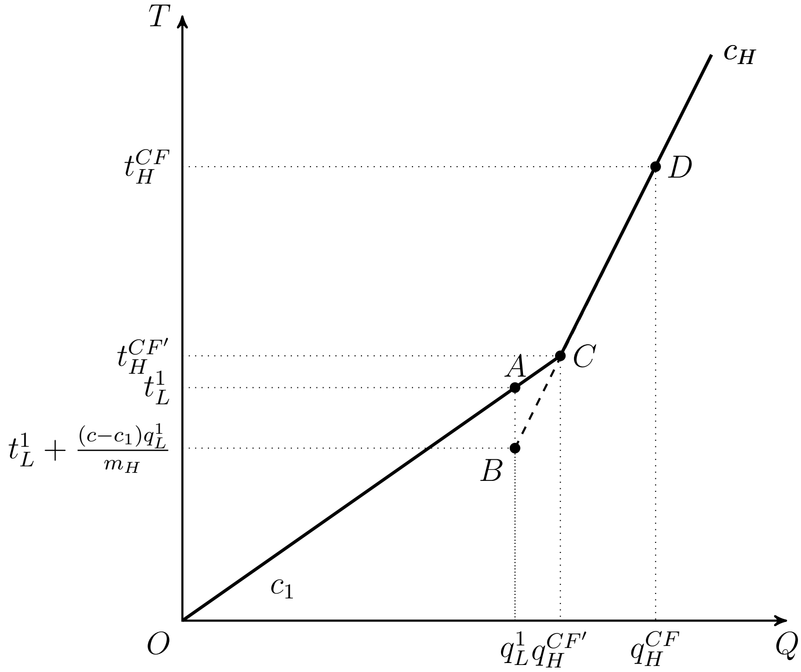

Figure 4: Convex Tariff

We aim to find the most efficient allocation for H type buyers under the convex tariff, given the zero-profit frontier. As illustrated in Figure 4, the planner can propose a tariff along the line $OACD$, where OAC represents the basic layer with a unit price of c_1, and CD is situated

[5]ZF: zero-profit frontier

on the zero-profit frontier.

In the first scenario, if the indifference curve of the H type has a tangent point above point C (assume at $D(q_H^{CF}, t_H^{CF})$),[6] then the most efficient allocation for H types is (q_H^{CF}, t_H^{CF}), given the zero-profit frontier. In this situation, L type buyers trade (q_L^1, t_L^1), while H type buyers trade (q_H^{CF}, t_H^{CF}). Additionally, we have $MRS_H(D) = c_H$ and $MRS_L(A) = c_1$.

In the second scenario, if $MRS_H(C) \leq c_H$, it implies that the indifference curve of the H type has a tangent point with the zero-profit frontier below C. In this situation, the most efficient allocation for H type buyers is $C(q_H^{CF'}, t_H^{CF'})$[7] under the convex tariff. Consequently, both scenarios and their respective allocations can be implemented by the line $OACD$, which represents the convex tariff $T_{c_1}^{CF}(q)$:

$$T_{c_1}^{CF}(q) = 1_{\{q \leq q_H^{CF'}\}} c_1 q + 1_{\{q > q_H^{CF'}\}} [c_1 q_H^{CF'} + c_H(q - q_H^{CF'})]$$

The zero profit frontier gives us all the possible feasible allocations for H type buyers given L types allocation is (q_L^1, t_L^1). Under the convex tariff condition, the tariff $T_{c_1}^{CF}(q)$ can implement the most efficient allocation for H type buyers given (q_L^1, t_L^1).

4.2 Efficiency of Pauly Line and JHG allocation

In this subsection, we revisit the two special allocations found in the literature to determine if they are efficient under a convex tariff. The Pauly line allocation represents a market equilibrium in the absence of a planner, with all sellers offering only linear price tariffs. The JHG allocation is the sole entry-proof allocation when entrants can provide any arbitrary contract. The JHG allocation consistently results in higher utility for type L buyers in all cases, but it does not always yield higher utility for type H buyers. In this subsection, we compare the previously mentioned allocations to demonstrate that the planner's involvement improves efficiency when markets can only provide linear contracts.

Theorem 1 (Entry-Proof-Pareto-Dominated)**.** *Under the convex tariff, given incentive-feasible entry-proof allocation is* $(q_\theta, t_\theta)_{\theta=L,H}$. *Any non-EPPD allocation should satisfy* $MRS_L(q_L, c_1 q_L) = c_1$, *and* $MRS_H(q_H, t_H) = c_H$ *if* $q_L \neq q_H$.

Firstly, under a convex tariff, suppose for a given $c_1 \in [c, c^*]$, the allocation for L type buyers $(q_L, c_1 q_L)$ doesn't satisfy $MRS_L(q_L, c_1 q_L) = c_1$. On the one hand If $MRS_L(q_L, c_1 q_L) < c_1$, then under convex tariff, it never attracts L type buyers with quantity q_L. On the other hand, if $MRS_L(q_L, c_1 q_L) > c_1$, then the tariff $T_{c_1}^{CF}(q)$ gives a better allocation for L type buyers with optimal quantity given c_1, due to $MRS_L(q_L, c_1 q_L) = c_1$, and it also gives a better zero-profit frontier for H type buyers due to it has higher cross-subsidy for H type buyers, so it is Pareto improvement for the markets.

[6]CF: Convex-tariff with Frontier.

[7]By using the intersection of two lines, we can determine that $q_H^{CF'} = q_L^1(1 + \frac{c_1 - c}{m_H(c_H - c_1)}), t_H^{CF'} = c_1 q_H^{CF'}$

Secondly, in terms of H type buyers allocation. Suppose $MRS_H(q_H, t_H) > c_H$, the allocation is not an entry-proof allocation. Due to this situation, the entrants can propose the linear tariff with a unit cost $c_H + \epsilon$ and it is attractive for H type buyers and profitable.Thus, we have $MRS_H(q_H, t_H) \leq c_H$. Then suppose $MRS_H(q_H, t_H) < c_H$, the only non-EPPD situation is that $t_L = c_1 q_L$ and $t_H = c_1 q_H$, and (q_H, t_H) located in the zero profit frontier. Then the planner can slightly change the basic unit price from c_1 to $c_1 - \epsilon$ to have a new convex entry-proof tariff $T^{CF}_{c_1-\epsilon}(q)$. If ϵ is small enough, the new tariff provides better allocation for both L and H type buyers. In terms of L type buyers' allocation, it gives a lower unit price. in terms of H type buyers' allocation, it can provide a better utility for H type buyers when ϵ is small enough. So the new tariff $T^{CF}_{c_1-\epsilon}(q)$ gives a better entry-proof allocation.

Lemma 1 (Pauly Line is EPPD)**.** *The Pauly line allocation is Pareto Dominated by other Entry-Proof allocations.*

In Pauly line allocation, $MRS_L(q_L^*, t_L^*) = c^*$, however $MRS_H(q_H^*, t_H^*) = c^* \neq c_H$, so according to Theorem 1, Pauly line allocation is EPPD allocation. The intuition is as shown in figure 5, given Pauly line allocation where A, B are the points for L and H type buyers. Then by choosing $c_1 = c^* - \epsilon$ with ϵ small enough and then proposing the zero-profit frontier tariff $T^{CF}_{c_1}(q)$ (the red line in figure 5) to the market. It can implement more efficient allocation with A', B' for L, H type buyers and more efficient.

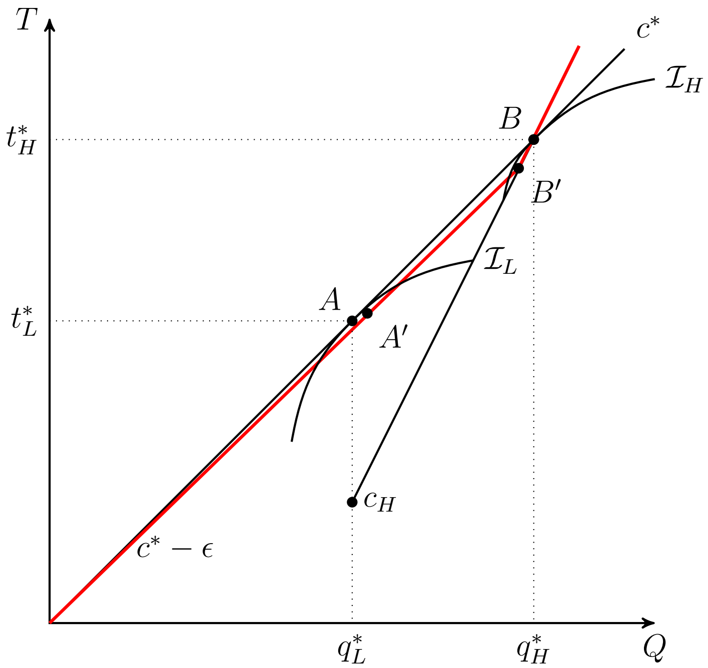

Figure 5: Pauly Line EPPD

Comments: Lemma 1 demonstrates that a planner's involvement can increase the utility of buyers. In the absence of a planner, the only equilibrium is the Pauly line allocation, which is Pareto dominated by other possible allocations that can arise when the planner participates in the market. Therefore, the planner should not choose the Pauly line allocation as her preferred plan for the market, as it is inferior to other potential allocations that could be achieved with her participation. So compared with only linear contracts markets, the intervention of the planner can improve social efficiency.

Theorem 2 (JHG Efficiency)**.** *Under a convex tariff, if the proportion of type H buyers is large enough, JHG allocation becomes the most efficient allocation for both types of buyers, and the unique non-EPPD allocation is JHG allocation.*

Firstly, it is evident that among all convex basic contract tariffs, type L buyers always derive higher utility in JHG allocation since it offers the lowest unit price for them.

To demonstrate the efficiency of JHG allocation compared to other convex tariff allocations, we aim to establish that any zero-profit frontier corresponding to $c_1 \in [c, c^*]$ yields a suboptimal allocation in comparison to JHG allocation. Since any zero-profit frontier has the same slope c_H, we only need to determine if JHG Tariff is located to the right of the zero-profit frontier. To satisfy this condition, we compare point $A^{JHG}(q_L^1, t_L^1 + \frac{(c-c_1)q_L^1}{m_H})$ in Figure 4 and point $B(q_L^{JHG}, t_L^{JHG})$ in Figure 2. JHG allocation is more efficient for H-type buyers than any other allocation on the zero-profit frontier only if the slope of $A^{JHG}B$ is smaller than c_H, which can be expressed as:

$$\frac{t_L^{JHG} - (t_L^1 + \frac{(c-c_1)q_L^1}{m_H})}{q_L^{JHG} - q_L^1} \leq c_H \tag{4}$$

According to the equation in JHG allocation and the zero profit frontier, we have $t_L^{JHG} = cq_L^{JHG}$, and $t_L^1 = c_1 q_L^1$. We have the relation that $m_H + m_L = 1, m_H c_H + m_L c_L = c, c_H - c = m_L(c_H - c_L)$, so we can simplify the inequality (4) as:

$$m_H \geq \frac{q_L^1}{q_L^{JHG}} \times \frac{c_1 - c_L}{c_H - c_L} \tag{5}$$

As $c_1 > c$, it follows that $q_L^1 \leq q_L$. Moreover, since $c_1 < c^* < c_H$, the RHS of (5) is smaller than $\frac{c_1 - c_L}{c_H - c_L} < \frac{c^* - c_L}{c_H - c_L} < 1$. Consequently, if m_H is sufficiently large, we have $m_H > \frac{q_L^1}{q_L^{JHG}} \times \frac{c_1 - c_L}{c_H - c_L}$, which implies that JHG allocation results in better utility for H-type buyers. Therefore, we conclude that JHG allocation is the most efficient allocation among all convex tariff allocations if the proportion of H type buyers is large enough.

Comments: Unlike Pauly line allocation, JHG allocation is not an EPPD (Envy-Free, Proportional-Dislike) entry-proof allocation. However, if the proportion of H-type buyers is sufficiently large, all other convex tariffs are Pareto-dominated by JHG allocation. Hence, the planner can focus on the JHG tariff as the unique tariff in the market when m_H is large enough.

4.3 Second-Best and Robust to Side Trading

In Attar et al. (2020), it is demonstrated that if a second-best allocation is robust to arbitrary tariff side trading, then the second-best allocation coincides with JHG allocation, and

$\tau_H(q_L^{JHG}, t_L^{JHG}) < c_H$. Otherwise, there is no second-best allocation that can be robust to entrants.[8]

In this paper, we investigate whether relaxing the restrictions on entrants, under the linear contracts and convex tariff setting, leads to additional possible second-best allocations that are robust to linear side trading. To facilitate our analysis, we begin by introducing a lemma quoted from Attar et al. (2020).

Lemma 2 (Second-Best Criteria)**.** *In any second-best allocation* $(q_\theta, t_\theta)_{i=L,H}$*:*

- *If* $IC_{H\to L}$*is slack, then* $\tau_L(q_L, t_L) \leq c_L$*, with equality if* $q_L > 0$*.*[9]
- *If* $IC_{L\to H}$ *is slack, then* $\tau_H(q_H, t_H) = c_H$

Then we can discuss the problem in the situation as follow: (1)Both $IC_{H\to L}$ and $IC_{L\to H}$ are binding; (2)$IC_{H\to L}$ is binding and $IC_{L\to H}$ is slack; (3)$IC_{H\to L}$ is slack and $IC_{L\to H}$ is binding; (4) Both $IC_{H\to L}$ and $IC_{L\to H}$ are slack.

(1): If both $IC_{H\to L}$ and $IC_{L\to H}$ are binding. By using the assumption 1 and both binding, we can easily get that $q_L = q_H$, in that case, the second-best allocation is actually JHG allocation, with that $\tau_H(q_L, t_L) < c_H$, which is same as in Attar et al. (2020).

(2): If $IC_{H\to L}$ is binding and $IC_{L\to H}$ is slack, then according to Lemma 2, we have $\tau_H(q_H, t_H) = c_H$, and the indifference curve of $u_H(q_H, t_H)$ is concave. Thus, the point (q_L, t_L) must lie to the right of the tangent line with (q_H, t_H), which implies that the average unit price for an additional contract is above c_H. However, in this scenario, new entrants can offer contracts slightly above c_H to attract type H buyers, which can generate positive profit. Therefore, there is **no** entry-proof tariff that satisfies this case.

(3): If $IC_{H\to L}$ is slack and $IC_{L\to H}$ is binding, then according to Lemma 2, we have $\tau_L(q_L, t_L) = c_L$. Due to the concavity of the indifference curve, the point (q_H, t_H) lies to the right of the tangent line with (q_L, t_L). Thus, the average price for an additional contract is below c_L, which implies that the average unit price for (q_L, t_L) should be above c. However, if we consider the fact that $\tau_L(q_L, t_L) = c_L$, there is no convex basic contract entry-proof tariff with an average price above $c > c_L$ that satisfies all the conditions. Therefore, there is **no** convex basic contract entry-proof tariff that satisfies this case.

(4): If both $IC_{H\to L}$ and $IC_{L\to H}$ are slack.This means that $q_H > q_L$ under assumption 1. By using Lemma 2, we have that $\tau_L(q_L, t_L) = c_L$ due to Inada's condition and $\tau_H(q_H, t_H) = c_H$. But under convex tariff, according to Theorem 1 we have that any non-EPPD allocation should satisfy that $MRS_L(q_L, c_1 q_L) = c_1$ with $c_1 \geq c > c_L$, and $MRS_H(q_H, t_H) = c_H$. Thus, there is no second-best allocation in this situation that coincides with convex tariff entry-proof allocation. In conclusion with the four cases discussed above

[8]In the special case where JHG allocation coincides with the pooling allocation, we assume Inada's condition, so we do not have the situation $\tau_L(0, 0) < c_L$.

[9]$IC_{H\to L}$ is the incentive compatible condition: $u_H(q_H, t_H) \geq u_H(q_L, t_L)$.

Proposition 3 (Inefficiency under Convex Tariff). *If the planner proposes only a convex tariff to the market and $\tau_H(q_L^{JHG}, t_L^{JHG}) > c_H$, then even if entrants can only propose linear pricing, the planner cannot implement the second-best allocation which robust to side trading.*

5 Incenvtive-Feasible Allocations in Any Tariff

In the last section, we discuss the case that the planner can only provide a convex basic contract to the market, which means that for a given average unit price of a basic contract, type L buyers can choose the most suitable quantity for them. However, if the planner can also provide the non-convex contract as a basic contract, type L may choose more quantity than they want at an average price.

In this section, we discuss the case that the planner can let type L choose more quantity through a non-convex basic tariff. Then we discuss under nonconvex tariff, what is the zero profit frontier for type H buyers. After that, we will discuss the possible non-EPPD allocation and the relationship with the second-best allocation.

5.1 Basic Contract with Non-convex Tariff

Firstly, if the planner can only provide a convex basic tariff with a given average unit price c_1, the highest quantity she sells to type L buyers is actually q_L^1, which is the point with $\tau_L(q_L^1, t_L^1) = c_1$. With the convex basic tariff, the planner can not let type L buyers to purchase a quantity of more than q_L^1.

However, if we allow the planner to provide a non-convex basic tariff to the market, she can use a non-convex tariff to let type L buyers purchase quantity $q_L^{1'}$ more than q_L^1 and assume $k_0 = \tau_L(q_L^{1'}, t_L^{1'}) < c_1$, then a tariff $T(q) = t_L^{1'} - k_0 q_L^{1'} + k_0 q$ can let L type buyers choose the quantity $q_L^{1'}$ even the average unit price is $c_1 > k_0$, where k_0 is the $MRS_L(q_L^{1'}, t_L^{1'})$. Here the non-convex tariff is similar to a fixed fee before purchasing the contracts. By using this kind of non-convex tariff, the planner can make the L type buyers cross subsidy more to H type buyers.

If the planner can propose a nonconvex tariff to implement H type buyers' preferred allocation. We would like to ask what are the entry-proof non-convex basic tariff allocations for type L buyers. It is easy to conclude that the planner can implement any $(q_L^{1'}, t_L^{1'})$ such that:

$$u_L(q_L^{1'}, t_L^{1'}) \geq u_L(q_L^*, t_L^*) \tag{6}$$

$$t_L^{1'} \geq c q_L^{1'} \tag{7}$$

The inequality (6) means that $(q_L^{1'}, t_L^{1'})$ gives L type buyers higher utility compared with only linear contracts Pauly allocation (q_L^*, t_L^*). If this inequality is not satisfied, then an entrant can provide a contract with unit price $c^* + \epsilon$ are better off for type L buyers, even this new contract also attracts type H buyers, it is profitable because $c^* + \epsilon > c^*$. The inequality (7) means that the planner should receive nonnegative profit. To see the reason, suppose $t_L^{1'} < cq_L^{1'}$ and denote the allocation for H type buyers is $(q_H^{1'}, t_H^{1'})$, if $\frac{t_H^{1'}-t_L^{1'}}{q_H^{1'}-q_L^{1'}} > c_H$, then an entrant can propose the linear contracts with $T(q) = (c_H + \epsilon)q$ to attract H type buyers and make the positive profit. If $\frac{t_H^{1'}-t_L^{1'}}{q_H^{1'}-q_L^{1'}} \leq c_H$, the aggregate profit for the planner is $t_L^{1'} - cq_L^{1'} + m_H(t_H^{1'} - c_H q_H^{1'}) < 0$, which achieve negative profit. As a result, the feasible possible entry-proof allocation for L types should satisfy the inequality (6) and (7).

5.2 Efficient Zero Profit Frontier Rules

From the feasible allocation for type L buyers, there are many different points for the planner to choose. In this subsection, we want to study how can a planner choose the allocation for buyers to make the allocation more efficient, and avoid the EPPD allocation. We give some lemmas to show how L types allocation affects zero profit frontier.

Lemma 3 (Quantity). *Assume the L type contracts (q^1, t^1) located the line with $t = c_1 q$, then higher quantity q_L^1 gives a better zero profit frontier for H type buyers.*

This lemma tells us that for the same basic average unit price tariff, the higher quantity purchased by type L buyers, the better the zero profit frontier type H buyers can get. This is actually the low-type buyers subside more to high-type buyers.

Assume (q^1, t^1) and $(q^{1'}, t^{1'})$ are two points located in the same average cost line with slope c_1 and $q^{1'} > q^1$. Then the corresponding starting point of zero profit frontier is $(q^1, t^1 - \frac{t^1 - q^1 c}{m_H})$ and $(q^{1'}, t^{1'} - \frac{t^{1'} - q^{1'} c}{m_H})$. The slope between this two starting points is:

$$\begin{aligned}\frac{c_1(q^{1'} - q^1) - (q^{1'} - q^1)(c_1 - c)/m_H}{(q^{1'} - q^1)} &= c_H + \frac{m_L}{m_H}(c_L - c_1) \\ &< c_H\end{aligned}$$

This inequality tells us that the starting point $(q^1, t^1 - \frac{t^1 - q^1 c}{m_H})$ is strictly on the left side of zero profit frontier correspond with $(q^{1'}, t^{1'} - \frac{t^{1'} - q^{1'} c}{m_H})$. Thus, given the same average unit price of a basic contract, the higher quantity purchased by type L buyers, the better the zero profit frontier type H buyers can get.

Assume that for a given average unit price c_1, the optimal choice for type L buyers is (q_L^1, t_L^1), then we can have proposition below:

Proposition 4 (Inefficiency). *For any average unit basic price c_1, any allocation for type L buyers with quantity $q^1 < q_L^1$ is EPPD.*

Compare with (q_L^1, t_L^1) and (q^1, t^1) we have $u_L(q_L^1, t_L^1) \succ u_L(q^1, t^1)$, and according to Lemma 3 the (q_L^1, t_L^1) also gives better zero profit frontier to high type buyers, thus the allocation with (q^1, t^1) for L type buyers is EPPD.

Lemma 4 (Marginal Rate of Substitution)**.** *Assume the quantity-pairs* (q_l, t_l) *gives the same utility for L type buyers,that is* $u_L(q_l, t_l) = u_0$*. Then the highest zero profit frontier is given by the point with* $\tau_L(q_l, t_l) = c_L$*. If there is no point satisfying* $\tau_L(q_l, t_l) = c_L$ *in feasible,* (q_l, t_l) *supports the highest zero profit frontier only if* $t_l = cq_l$*.*

The intuition is that for a point (q, t) locate in an indifference curve for type L buyers. Then the starting point of zero profit frontier correspond to (q, t) is $(q_s, t_s) = (q, t - \frac{t-qc}{m_H})$. The slope(changing with the indifference utility curve) of this point is:

$$\begin{aligned}\frac{\partial t_s}{\partial q_s} &= \frac{\partial t}{\partial q} - \frac{1}{m_H}\frac{\partial t}{\partial q} + \frac{c}{m_H} \\ &= -\frac{m_L}{m_H}\frac{\partial t}{\partial q} + \frac{c}{m_H} \\ &= \frac{m_L}{m_H}(c_L - \tau_L) + c_H\end{aligned}$$

According to the equation above:

If $\tau_L > c_L$, then the slope of starting point is below c_H, then we can find some point in the right of (q, t) which gives a better zero point frontier to type H buyers.

If $\tau_L < c_L$, then the slope of starting point is great than c_H, then we can find some point in the left of (q, t) which gives a better zero point frontier to type H buyers.

If $\tau_L = c_L$, the curve of starting point has a tangent with zero profit frontier, thus, there is no other point have a better frontier than this point.

If there is no point satisfy $\tau_L(q, t) = c_L$ in the type L feasible allocation, by using that $\tau_L(q, t)$ is decreasing with q, the highest zero profit frontier is given by the corner allocation which locates in line with slope c.

Lemma 5 (Pareto Efficient)**.** *Assume in the L type buyer's entry-proof feasible allocations, we have* $\tau_L(q_l^1, t_l^1) = \tau_L(q_l^2, t_l^2) = c_L$ *and* $u_L(q_l^1, t_l^1) < u_L(q_l^2, t_l^2)$*, then the L type allocation* (q_l^1, t_l^1) *gives a better zero profit frontier for H type buyers than* (q_l^2, t_l^2)*.*

The lemma 5 means that given $\tau_L = c_1$, increase of L type buyers' utility will decrease H type buyers' utility in the Non-EPPD allocation. This provides us with possible allocations for second-best allocations. To prove lemma 5, by using similar logic of proving Lemma 3, we just need to prove that the slope between the start point of zero profit frontier $(q_l^1, t_l^1 - \frac{t_l^1 - q_l^1 c}{m_H}))$ and $(q_l^2, t_l^2 - \frac{t_l^2 - q_l^2 c}{m_H}))$ is great than c_H. So it is equivalent to $c_L > \frac{t_l^2 - t_l^2}{q_l^2 - q_l^1}$, due to $\tau_L = c_L$, so it holds for sure.

5.3 Second-best and Robust to Side Trading

Under the convex tariff, we show that the linear side trading can prevent the planner implement the second-best tariff, and if the proportion of H type buyers is large enough, JHG allocation is the Pareto Dominates other entry-proof allocations. In this subsection, we show the relationship between second-best allocation and Non-EPPD allocation under any tariff.

Theorem 3 (Second-Best and Non-EPPD). *Assume (q_L^E, t_L^E) satisfy that $u_L(q_L^E, t_L^E) = u_L(q_L^*, t_L^*)$, $t_l^E = cq_l^E$ and $q_L^E > q_L^{JHG}$, then:*

- *If $\tau_L(q_L^E, t_L^E) \leq c_L$, then there exists second-best allocation coincides with the Non-EPPD allocation with linear entrants.*
- *If $\tau_L(q_L^E, t_L^E) > c_L$, then the second-best allocation is robust to linear side trading only if it is a pooling allocation.*
- *If $\tau_L(q_L^E, t_L^E) > c_L$, in any non-EPPD allocation, the L type buyers' quantity-transfer pair should be located in the line of $t = cq$.*

The first condition of Theorem 3 states that if there exists a feasible entry-proof allocation with $\tau_L = c_L$, then this allocation, which we call the Non-EPPD allocation, will coincide with the second-best allocation. This is because of Lemmas 4 and 5. Specifically, given the same level of utility for type L, the quantity-transfer pair with $\tau_L = c_H$ gives the most efficient zero-profit frontier. Moreover, any increase in the utility level of type L will decrease the utility level of type H, as Lemma 5 implies. Therefore, there is no other allocation that can Pareto dominate the Non-EPPD allocation, making it the second-best allocation.

The second condition of Theorem 3 is the same as the result in Attar et al. (2020), and it only holds when the pooling allocation has $\tau_H < c_H$. The third condition, on the other hand, is derived from the analysis of Lemma 4. Specifically, given any fixed level of utility for type L buyers, when $\tau_L > c_L$, a lower value of τ_L results in a better zero-profit frontier. Consequently, given the same utility level for type L buyers, the lowest value of τ_L satisfies the budget balance constraint $t = cq$.

Comments: In contrast to the convex tariff case, under linear side trading, it is possible for the second-best allocation to coincide with the non-EPPD allocation when the planner is free to propose any tariff. The tariff that implements the second-best allocation can be viewed as a fixed transfer plus a quantity-related transfer. This provides evidence that non-convex tariffs can improve social efficiency. However, there may be situations where even if the planner has the flexibility to propose any tariff, the second-best allocation cannot be made robust to linear side trading. In such cases, we know that the non-EPPD allocation will always result in an average unit price of c for type L buyers.

6 Conclusion

This paper investigates all entry-proof allocations under linear contract entrants. In contrast to the results of uniqueness under arbitrary tariff entrants, where the JHG allocation is the unique entry-proof allocation, we show that there are many other entry-proof allocations that are also efficient, such as the Pauly line allocation and two-part zero-profit allocations. From the planner's perspective, we provide conditions for the existence of EPPD allocations. Based on these conditions, we show that the Pauly line allocation is an EPPD allocation, implying that the absence of a planner in a linear pricing market is inefficient compared to a market with the planner's intervention.

However, in the convex tariff setting, there is usually no second-best allocation that coincides with the Non-EPPD allocation. This means that linear side trading can destroy efficiency if the planner can only provide a convex tariff. Moreover, if the proportion of H type buyers is sufficiently large, the JHG allocation will be the most efficient allocation.

If the planner can propose a non-convex tariff, then some second-best allocations can coincide with a non-EPPD allocation. However, in other situations, there is no second-best allocation that is robust to linear side trading. This means that even if the planner can propose flexible tariffs and under very restrictive side trading, the efficiency of the market can still be affected by side trading. If there is no second-best allocation that coincides with the Non-EPPD allocation, then any entry-proof efficient allocation should give an average unit price of c to L type buyers.

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