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Abstract

We study the design of optimal (private and/or social) insurance schemes for formal home care and institutional care. We consider a three period model. Individuals are either in good health, lightly dependent or heavily dependent. Lightly dependent individuals can buy formal home care which reduces the severity of dependency and reduces the probability to become severely dependent in the next period. Severely dependent individuals pay for nursing home care. In both states of dependency individuals can receive a (private or public) insurance benefit (transfers). These benefits can be flat or depend on the formal care consumed (or a combination of the two). These benefits are financed by a premium (or a tax). Individuals may be alive until the end of the last period or die at the beginning of one of the last two periods with a certain probability, which may depend on their state of health.

The *laissez faire* is inefficient because individuals consume a too low level of formal home care and are not insured. The first-best insurance scheme requires a transfer to lightly dependent individuals that, (under some conditions) increases with the amount of formal home care consumed. Severely dependent individuals, on the other hand, must receive a flat transfer (from private or social insurance). The theoretical analysis is illustrated by a calibrated numerical example which show that the expressions have the expected signs under plausible conditions.

Keywords: Long-term care insurance, formal home care, nursing home care.
JEL Codes: I13, I18, H51

1 Introduction

Because of population aging the demand for formal long-term care (LTC) services by the elderly population is likely to grow substantially; see Cremer et al. (2012) and Klimaviciute and Pestieau (2023) for an overview of the relevant evidence. LTC mainly consists in assistance with daily activities, and it is different from health care. Unlike medical care, this assistance does not require highly skilled caregivers, but it is very "labor intensive" and typically not covered by health insurance.

A novel aspect of our study is that it incorporates both home and institutional care, and explicitly models how transitions between levels of dependency (from light to severe) are influenced by the amount of home care received. In this respect, our approach is comparable to therapeutic health care models that incorporate prevention.[1] The key distinction, however, is that home care in our model contributes to both primary and secondary prevention, while institutional care is limited to secondary prevention. Currently, a significant portion—between one-third and one-half—of long-term care is provided informally by family members, primarily daughters (see Barczyk and Kredler, 2018). However, the availability of informal care is expected to decline in the future due to factors such as changing family values, increased female labor force participation, and greater mobility among children. In any case, informal care imposes a substantial burden on caregivers and is not accessible to everyone.[2] As a result, there will be a growing need for formal preventive home care to complement or replace informal care.

[1]See Ellis and Manning (2007) who consider prevention and treatment but concentrate on primary prevention while Barrigozzi (2004) concentrate on treatment and secondary prevention.

[2]See for example Barigozzi et al. (2020), Bonsang and Schoenmaeckers (2015), Cremer et al. (2017), and Cremer et al. (2012).

Individuals with severe dependency require institutional care. In France, for instance, the average stay in a nursing home before death is two years and ten months, with an average monthly net cost of approximately 2,500 euro per person (see DREES, 2014). In the United States, this cost rises to around $7,300 (see Hurd et al., 2017).

For individuals with mild dependency, formal care provided at home can (i) delay the need for institutionalization and (ii) enhance their quality of life. Home care is generally more affordable (see Rizzo, 2016) and preferred by most individuals. This is why the World Health Organization is currently implementing the ICOPE (Integrated Care for Older People) program in various European locations.[3]

The ICOPE program aims to minimize the degree of dependency among older adults. Using a low-cost screening test for individuals aged 65 and over, the program assesses "intrinsic capacity"—an aggregate measure of dependency—based on factors such as cognitive decline, mobility limitations, visual and hearing impairments, undernutrition, and depressive symptoms (see Tavassoli et al., 2022). Based on the resulting score, individuals are categorized as autonomous, lightly dependent, or severely dependent. For those classified as lightly dependent, home care services can (i) help maintain their independence and improve quality of life and (ii) delay progression to severe dependency. Once individuals become severely dependent, however, institutional care becomes necessary. In such cases, care may improve quality of life but does not significantly affect mortality.[4]

Currently, public or private insurers often offer flat-rate reimbursements—

[3]See https://www.who.int/teams/maternal-newborn-child-adolescent-health-and-ageing/ageing-and-health/integrated-care-for-older-people-icope

[4]See García-Andrade et al. (2020 for evidence that only medical expenses are the main predictor of mortality in institutional care.

if any—based on the individual's level of dependency. The objective of this study is to evaluate whether such flat reimbursement schemes are optimal from a welfare perspective. We investigate the optimal design of long-term care (LTC) insurance, considering both formal home care and institutional care.

A novel feature of our model is its inclusion of transitions between dependency states, where the likelihood of progressing from mild to severe dependency is influenced by the level of home care received. In this respect, our framework parallels models of therapeutic healthcare that include prevention strategies.[5]

The key distinction is that home care contributes to both primary and secondary prevention, while institutional care is limited to secondary prevention. Another contribution of this paper is the exploration of nonlinear insurance policies. That is, we determine the optimal policy based on the information available to insurers. In our setting, both home and institutional care are observable, allowing them to be incorporated directly into the insurance scheme.

We consider a model with 3 periods: 0, 1 and 2. In period 0 individuals are either autonomous, lightly dependent or severely dependent. Lightly dependent individuals can buy formal home care which increases their quality of life and reduces the probability to become severely dependent in the next period. Severely dependent individuals pay for nursing home care which also affects their quality of life. In both states of dependency individuals can receive a (private or public) insurance reimbursements. These reimbursements can be flat or depend on the formal care consumed (or a combination of the

[5] See Ellis and Manning (2007) who consider prevention and treatment but concentrate on primary prevention while Barrigozzi (2004) concentrate on treatment and secondary prevention.

two). These benefits are financed by a premium. Individuals may be alive until the end of period 2 or die at the beginning of periods 1 or 2 with a certain probability which can depend on their level of dependency.

The *laissez-faire* is inefficient because individuals consume a too low level of formal home care and are not insured. We study the first-best (FB) insurance scheme that maximizes expected utility of a representative individual and show how it can be implemented by reimbursement rules for home and institutional care.

We show that the decentralization of the FB requires a transfer to lightly dependent individuals that, (under some conditions) increases with the amount of formal home care consumed. The transfer can be entirely public or consist for instance of a flat private insurance payment plus a subsidy; this is a matter of implementation. Severely dependent individuals, on the other hand, must receive a flat transfer (from private or social insurance).

The theoretical analysis is illustrated by a calibrated numerical example which shows that the expressions have the expected signs under plausible conditions and point out some other interesting properties. These results provide a justification for prevention operators, as provider of home care (which should be subsidized) and as an insurer providing benefits that depend on the severity of the dependency.

2 The model

2.1 Periods and states of nature

We consider a model with three periods indexed by $t = 0, 1, 2$. Individuals are endowed with a given level of wealth ω for every $t = 0, 1, 2$. In each period t, there are 4 states of nature denoted by $i = \{G, L, H, D\}$ where

G stands for autonomous, L stands for "lightly" dependent (at home), H stands for severely dependent (requiring institutional care) and D for dead. Finally, we assume that death occurs at the beginning of the considered period. The proportion of the initial population who is of type i in period t is denoted by $\pi_{t,i}$. In period 0, we consider only individuals who are alive so that $\pi_{0,D} = 0$. This makes sense because individuals die at the beginning of a period.

2.2 Utilities

The VNM utility of a type G individual in period t is given by

$$U_{t,G} = v(c_{t,G})$$

where $c_{t,i}$ denotes the consumption level at date t of an individual with type $i = G, L, H$ while the utility of an individual with type $i = L, H$ at date t is given by:

$$U_{t,i} = u(c_{t,i}, q_{t,i}) \text{ for } t = 0, 1, 2 \text{ and } i = L, H. \tag{1}$$

where $q_{t,i}$ is a measure of quality of life and is given by $q_{t,L} = s_t - \theta_{t,L}$ and $q_{t,H} = d_t - \theta_{t,H}$. The exogenous variables $\theta_{t,L}$ and $\theta_{t,H}$ represent the loss of quality of life when in state L and H. The variables $s_{L,t}$ and $d_{H,t}$ respectively denote home care and institutional care expenses which increase quality of life in the associated state. We assume that $\partial u/\partial c_{t,i} = u^c_{t,i} > 0$, $\partial^2 u/\partial c^2_{t,i} < 0$ and $\partial u_{t,i}/\partial q_{t,i} = u^q_{t,i} > 0$ so that the utility is increasing with the level of consumption and the quality of life and is strictly concave with the level of consumption. We further assume $\partial^2 u/\partial c_{t,i}\partial q_{t,i} \geq 0$ so that the marginal utility of consumption is increasing with quality of life. We normalize utility to zero in state D so that $U_{t,D} = 0$ for every $t = 1, 2$. When alive, individuals pay a premium $\mathcal{P}_r$ and are reimbursed by an amount equal to $R_{t,i}$ so that

$$c_{t,G} = \omega - \mathcal{P}_r, \;\; c_{t,L} = \omega - \mathcal{P}_r + R_{t,L} - s_t, \tag{2}$$

$$c_{t,H} = \omega - \mathcal{P}_r + R_{t,H} - d_t. \tag{3}$$

The expected utility at time 0 is thus given by:

$$EU = \sum_{t=0,1,2} \sum_{i=G,L,H} \pi_{t,i} U_{t,i}. \tag{4}$$

2.3 Transition probabilities and proportions of types

Proportions of the different types at time t are given by

$$\pi_{t,i} = \sum_{j \in \{G,L,H,D\}} p_{t,j} \pi_{t-\,1,i} \tag{5}$$

and the transition matrix is given by a right stochastic matrix P_t.given by In order to follow the views of WHO about the ICOPE program (see the introduction), the matrix is gradual except for death and there is no way back when dependent

$$P_t = \begin{pmatrix} p_{t,G}\,(G) & p_{t,L}(G) & p_{t,H}(G) & p_{t,D}(G) \\ 0 & p_{t,L}(s_{t-1}, L) & p_{t,H}(s_{t-1}, L) & p_{t,D}(L) \\ 0 & 0 & p_{t,H}\,(H) & p_{t,D}(H) \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{6}$$

for every $t = 1, 2$. For individuals who are in state L in period $t-1$, the probability of remaining in state L or moving to state H at date t is endogenous and depends upon the level of home care services s_{t-1}. Again, the role of home care services is to maintain as much as possible individuals in state L in order not to jump to state H. However, the occurrence of death can exogenously happen in any state of the previous period.[6] Formally, we can

[6]Again the paper does not deal with life increasing health expenses but LTC expenses so that death occurrence does not depend on LTC expenditures.

write $p_{t,L}(s_{t-1}, L)$ and $p_{t,H}(s_{t-1}, L)$ with $p_{t,L}^{s_{t-1}} = \partial p_{t,L}(s_{t-1}, L)/\partial s_{t-1} > 0$ and $p_{t,H}^{s_{t-1}} = \partial p_{t,H}(s_{t_{-1}}, H)/\partial s_{t-1} < 0$. And we must have $p_{t,L}^{s_{t-1}} + p_{t,H}^{s_{t-1}} = 0$. In words, lightly dependent individuals are less likely to become severely dependent if they benefit from more home care services. We further assume that mortality is larger among the severely dependent than among the lightly dependent i.e. $p_{t,D}(L) < p_{t,D}(H)$ for every $t = 0, 1, 2$. This assumption is not essential for our analysis but it simplifies the interpretation of some expressions and reduces the number of cases to be considered.

Differentiating $\pi_{t,i}$ as given by (5) with respect to s_0 and s_1 shows how the shares of types are affected by home care. The full expressions are provided in Appendix A and they are used when determining the optimal policy. Here we restrict ourselves to stating their signs.

$$\pi_{1,L}^{s_0} > 0, \pi_{1,H}^{s_0} < 0, \ \pi_{2,L}^{s_0} > 0, \tag{7}$$

$$\pi_{2,H}^{s_0} = \pi_{0,L} p_{1,L}^{s_0} [p_{2,H}(s_1, L) - p_{2,H}(H)] \lesseqgtr 0, \tag{8}$$

$$\pi_{2,D}^{s_0} = \pi_{0,L} p_{1,L}^{s_0} [p_{2,D}(L) - p_{2,D}(H)] < 0, \tag{9}$$

$$\pi_{2,L}^{s_1} > 0, \pi_{2,H}^{s_1} < 0. \tag{10}$$

Equations (7)–(10) follow directly from our assumptions on the impact of s on the transition probabilities. Roughly speaking, an increase in s_0 increases the proportion of lightly dependent individuals at the expense of the severely dependent in subsequent periods. This in turn results in a decrease of the proportion of individuals who are no longer alive in period 2 as stated in (9). This impact on mortality also explains why the sign of (8) is ambiguous; while s_0 increases the proportion of lightly dependent in period 2 it also reduces mortality in period 1 which explains that some individuals who would have otherwise died earlier end up being severely dependent in period 2. This indirect effect via mortality affects some of our results below.

Finally, because individuals are risk-averse, it is clear that insurance coverage $(R_{t,i}, s_{t,i}, d_{t,i})$ for both lightly and severely dependent care expenses would be desirable in all states L and H. The question to which we now turn is how the insurance contract should be designed. To address this question we first characterize the optimal allocation and then determine how it can be decentralized via an insurance contract covering home care in state L and nursing home care in state H as a function of home and institutional care s_t and d_t.

3 First best

The social planner chooses the premium $\mathcal{P}_r$, the reimbursements $R_{t,L}$ and $R_{t,H}$, home care s_t services and institutional care d_t such that it solves:

$$\max_{\mathcal{P}_r, R_{t,L}, R_{tH}, s_t, d_t} EU = \sum_{t=0,1,2} \sum_{i=G,L,H,D} \pi_{t,i} U_{t,i}$$
$$\text{s.to: } \mathcal{P}_r \sum_{t=0,1,2} \sum_{i=G,L,H} \pi_{t,i} - \sum_{i=L,H,} \sum_{t=0,1,2} (\pi_{t,i} R_{t,i} + s_t + d_t) \geq 0. \tag{11}$$

Denoting by μ the Lagrange multiplier associated to the resource constraint, we show in appendix B that the marginal utility of consumption is the same in all states of nature (when alive):

$$v'(c_{t,G}) = u^c_{t,i} = u^c_{t,j} \text{ for every } t = 0, 1, 2 \text{ and } i, j = L, H. \tag{12}$$

In other words, there is full insurance. We also show that:

$$u^c_{t,H} = u^q_{t,H} \text{ for every } t = 0, 2, 3 \tag{13}$$

so that the marginal utility of consumption in state H equals the marginal benefit of quality of life in state H for $t = 0, 1, 2$. Finally, we show that

$$
\begin{aligned}
&-\pi_{0,L}u^c_{0,L} - \pi_{0,L}u^q_{0,L} + \pi^{s_0}_{1,L}U_{1,L} + \pi^{s_0}_{1,H}U_{1,H} + \pi^{s_0}_{2,L}U_{2,L} + \pi^{s_0}_{2,H}U_{2,H} \\
&-\mu[\pi^{s_0}_{2,D}\mathcal{P}_r + \pi^{s_0}_{1,L}R_{1,L} + \pi^{s_0}_{1,H}R_{1,H} + \pi^{s_0}_{2,L}R_{2,L} + \pi^{s_0}_{2,H}R_{2,H}] = 0, \qquad (14) \\
&-\pi_{1,L}u^c_{1,L} - \pi_{1,L}u^q_{1,L} + \pi^{s_1}_{2,L}U_{2,L} + \pi^{s_1}_{2,H}U_{2,H} \\
&-\mu[\pi^{s_1}_{2,L}R_{2,L} - \pi^{s_1}_{2,H}R_{2,H}] = 0 \qquad (15)
\end{aligned}
$$

and

$$u^c_{2,L} = u^q_{2,L}. \qquad (16)$$

These conditions also require equalization of marginal costs and benefits for formal home care s. The expressions are more complex than for d because in addition to the direct effect in the considered period, there are indirect effects via the impact of s on the probability of dependency in subsequent periods. These indirect effects do not arise in period 2 which is the last period; see (16).

4 Decentralization

In the previous section we have assumed that the insurer directly controls all the relevant variables including expenditures on care, s and d. We now show how this solution can be decentralized via an appropriate insurance contract $(\mathcal{P}_r, R_{t,L}(s_t), R_{t,H}(d_t))$ which specifies the premium and the reimbursement rules for care expenses in states L and H. In other words we let individuals choose their levels of s_t and d_t taking the insurance contract $(\mathcal{P}_r, R_{t,L}(s_t), R_{t,H}(d_t))$, as given. The main question is then how the reimbursement rules should be designed. Specifically should they involve a flat payment in each state of nature or entail a full or partial reimbursement of care? Faced with the contract $(\mathcal{P}_r, R_{t,L}(s_t), R_{t,H}(d_t))$, the problem of the individual is given by:

$$\max_{s_t,d_t} EU = \sum_t \pi_{t,G} v(\omega - \mathcal{P}_r) + \sum_t \pi_{t,L} u(\omega - \mathcal{P}_r - s_t + R_{t,L}(s_t), s_t - \theta_L)$$
$$+ \sum_t \pi_{t,H} u(\omega - \mathcal{P}_r - d_t + R_{t,H}(d_t), d_t - \theta_H)$$

where $\pi_{t,i} = \sum_{j \in \{G,L,H,D\}} p_{t,j}(.)\, \pi_{t-1,i}$. The FOCs with respect to s_0, s_1, s_2 are respectively given by

$$- \left(1 - \frac{\partial R_{0,L}(s_0)}{\partial s_0}\right) \pi_{0,L} u^c_{0,L} + \pi_{0,L} u^q_{0,L} +$$
$$+ \pi^{s_0}_{1,L} U_{1,L} + \pi^{s_0}_{1,H} U_{1,H} + \pi^{s_0}_{2,L} U_{2,L} + \pi^{s_0}_{2,H} U_{2,H} = 0, \tag{17}$$
$$- \left(1 - \frac{\partial R_{1,L}(s_1)}{\partial s_1}\right) \pi_{1,L} u^c_{1,L} + \pi_{1L} u^q_{1,L}$$
$$+ \pi^{s_1}_{2,L} U_{2,L} + \pi^{s_1}_{2,H} U_{2,H} = 0, \tag{18}$$
$$- \pi_{2,L} \left(1 - \frac{\partial R_{2,L}(s_2)}{\partial s_2}\right) u^c_{2,L} + \pi_{2,L} u^q_{2,L} = 0. \tag{19}$$

While the FOCs with respect to d_t for $t = 0, 1, 2$ are given by:

$$\left(1 - \frac{\partial R_{t,H}}{\partial d_t}\right) u^c_{t,H} = u^q_{tH}. \tag{20}$$

To study the properties of the implementing reimbursement rule, we have to combine the individual's FOCs (17)–(20) with the conditions characterizing the first best, namely, (12)–(16). We will successively consider care in case of severe dependence and then for light dependence.

4.1 Severe dependence

Combining (13) and (20) yields $\partial R_{t,H}/\partial d_t = 0$ for all $t = 0, 1, 2$ so that $u^c_{t,H} = u^q_{t,H}$ for every $t = 0, 1, 2$. Consequently, insurance coverage of institutional care (that is in the case of severe dependence) involves a flat payment. There is no marginal subsidy, so that the level of care is not distorted. In practice this means that it can depend on the severity of dependence but not on actual expenditures.

4.2 Light dependence

4.2.1 Period 0

In case of light dependence in period 0 individuals consume care services of s_0 which affects the shares of dependent individuals in the subsequent periods. This affects their own probability of dependence and thus their expected utility. This is spontaneously taken into account when individuals choose their s_0. However, it also affects the insurers budget constraint and this effect is ignored by individuals who take the insurance contract as given. This explains that the reimbursement scheme must be designed to correct for this bias in the individual's choice. Consequently, a simple flat payment will no longer be sufficient.

To study how s_0 should be reimbursed we start by combining (14) with (17) which yields

$$\begin{aligned}&\frac{\partial R_{0,L}\left(s_0\right)}{\partial s_0}\frac{\pi_{0,L}u^c_{0,L}}{\mu}\\&= -\pi^{s_0}_{2,D}\mathcal{P}_r - \pi^{s_0}_{1,L}R_{1,L} - \pi^{s_0}_{1,H}R_{1,H}\\&- \pi^{s_0}_{2,L}R_{2,L} - \pi^{s_0}_{2,H}R_{2,H}.\end{aligned}\tag{21}$$

Equation (21) shows that $\partial R_{0,L}\left(s_0\right)/\partial s_0$ has the same sign as the (right-hand-side) RHS of this equation. We show in Appendix C that this RHS can be rearranged to obtain

$$\begin{aligned}\frac{\partial R_{0,L}\left(s_0\right)}{\partial s_0}\frac{\pi_{0,L}u^c_{0,L}}{\mu} &= \pi_{0,L}\mathcal{P}_r p^{s_0}_{1,L}[p_{2,D}\left(H\right)-p_{2,D}\left(L\right)] \\ &+ \pi_{0,L}p^{s_0}_{1,L}[R_{1,H}-R_{1,L}] \\ &+ \pi_{0,L}p^{s_0}_{1,L}p_{2L}\left(L,s_1\right)\left(R_{2H}-R_{2L}\right) \\ &+ \pi_{0,L}p^{s_0}_{1,L}\left[p_{2,D}\left(L\right)-p_{2,D}\left(H\right)\right]R_{2H}\right]. \qquad (22)\end{aligned}$$

As long as $R_{t,H} > R_{t,L}$ all terms of this expression are positive except for the last one. Indeed one can expect that $p_{2,D}\left(H\right) \geq p_{2,D}\left(L\right)$ which means that the mortality rate of severely dependent persons is at least as large as that of lightly dependent. Consequently we have $\partial R_{0,L}\left(s_0\right)/\partial s_0 > 0$ so that home care in period 0 must be subsidized as long as $(p_{2,D}\left(H\right) - p_{2,D}\left(L\right))$ is not too large.

Intuitively the different effects can be explained as follows. Recall that while individuals anticipate the direct impact of s_0 on their expected utility, they do not take into account the impact on the insurers budget constraint. Let us have a closer look at the different terms of (22) which represent the relevant effects. A first effect is a premium effect taking place in period 2: increasing s_0 increases the share of individuals of type L and decreases the one of type H in the second period so that it implies a positive effect on the resource constraint as long as $p_{2,D}\left(H\right) > p_{2,D}\left(L\right)$ that is if the mortality rate is higher among the highly dependent individual than the one among the lightly dependent. A second effect $\pi_{0,L}p^{s_0}_{1,L}\left(R_{1,H}-R_{1,L}\right)$ takes place in period 1. Increasing s_0 increases the share of individuals of type L and decreases the share of type H in period 1 so that as long as $R_{1,H} > R_{1,L}$, this effect is positive. A third effect takes place in period 2: a change in expenditures $p_{2,L}\left(L,s_1\right)\left(R_{2,H}-R_{2,L}\right) > 0$. Its extent depends on the proportion of type L individuals in period one which in turn depends on

s_0. These three effects go in the same direction. A fourth effect is that because the proportion of type L in period 1 increases and that of type H decreases, mortality decreases; and with more individual alive in period 2, expenditures on the severely dependent $R_{2,H}$ increase.

To sum up and roughly speaking s_0 has mostly positive effects on the insurer's budget because it reduces the proportion of severe dependency amongst the individual who are still alive in period 2. However, it also increases the number of individuals who survive until period 2 and some of them will be severely dependent.[7]

4.2.2 Period 1

We now turn to s_1 which has also indirect effects but only in period 2. Consequently expressions are simpler. Combining (15) with (18) yields

$$\frac{\partial R_{1,L}\left(s_1\right)}{\partial s_1}\frac{\pi_{1,L}u^c_{1,L}}{\mu} = -\pi^{s_1}_{2,L}R_{2,L} - \pi^{s_1}_{2,H}R_{2,H}. \tag{23}$$

Rearranging this expression we obtain

$$\frac{\partial R_{1,L}\left(s_1\right)}{\partial s_1}\frac{\pi_{1,L}u^c_{1,L}}{\mu} = \pi_{0,L}p_{1,L}\left(L,s_0\right)\frac{\partial p_{2,L}\left(L,s_1\right)}{\partial s_1}\left(R_{2,H} - R_{2,L}\right)$$

so that $\partial R_{1,L}\left(s_1\right)/\partial s_1 > 0$. In other words we obtain an unambiguous result and a subsidy on s_1 is always desirable. Intuitively, a larger s_1 increases the share of individuals of type L and decreases the share of type H in period 2. Consequently, as long as $R_{2,H} > R_{2,L}$, s_1 has a positive effect on total care reimbursements (an effect which is not spontaneously taken into account by individuals) and should be subsidized.

[7]Increasing longevity increases utility but this is already accounted for in individuals' decisions. However, it does have a negative impact on the insurer's budget.

5 Numerical results

To illustrate our findings, and to show that expression (22) has the expected sign when transition probabilities are calibrated to plausible levels, we now present some numerical illustrations. We assume that VNM utilities are given by

$$U_{t,G} = \log(1 + c_{t,G}),$$

$$U_{t,L} = \log(1 + c_{t,L}) + (s_t - \theta_L)^2,$$

and

$$U_{t,H} = \log(1 + c_{t,H}) + (d_t - \theta_H)^2$$

for $t = 0, 1, 2$.

For the sake of calibration we assume that $t = 0$ corresponds to age 70, $t = 1$ to age 80 and $t = 2$ to age 90. Initial proportions are: $\pi_{0,G} = 0.6$, $\pi_{0,L} = 0.3$ and $\pi_{0,H} = 0.1$; see DRESS, 2010. The transition matrices based on currently observed probabilities are taken from Fuino and Wagner (2018). We assume that these transition matrices correspond to $s_t = 0$ for $t = 0, 1, 2$.[8]

$$P_1 = \begin{pmatrix} 0.7 & 0.14 & 0.03 & 0.13 \\ 0 & 0.56 & 0.12 & 0.32 \\ 0 & 0 & 0.1 & 0.9 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{24}$$

and

$$P_2 = \begin{pmatrix} 0.25 & 0.25 & 0.2 & 0.3 \\ 0 & 0.5 & 0.18 & 0.32 \\ 0 & 0 & 0.68 & 0.32 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{25}$$

[8]This is a matter of normalization. In reality, subsidies to formal home care already exist so that the levels have to be understood as coming in addition to the current levels.

The probability functions are then specified as

$$p_{1,L}(L,s) = 1/(1+\exp[\beta_{1,L} - \alpha s]), \tag{26}$$

$$p_{2,L}(L,s) = 1/(1+\exp[\beta_{2,L} - \alpha s]) \tag{27}$$

where $\alpha > 0$ measures how formal home, s, care affects the probability p_L. The larger is α, the larger is the impact of s on the probability to stay in state L.

To calibrate $\beta_{1,L}$ and $\beta_{2,L}$ we set s to zero in (26) and (27) and solve to obtain the probability given in (24) and (25). This yields the following two equations

$$p_{1,L}(L,0) = 1/(1+\exp[\beta_{1,L}]) = 0.56,$$
$$p_{2,L}(L,0) = 1/(1+\exp[\beta_{2,L}]) = 0.5$$

and we obtain $\beta_{1,L} = -0.24$ and $\beta_{2,L} \approx 0$.

Substituting these values into equations (26) and (27) we obtain the expressions for $p_{1,H}(L,s)$ and $p_{2,H}(L,s)$ by using the property that in any given period the probabilities of the different states of nature must add up to one which yields

$$p_{1,H}(L,s) = 1 - p_{1,D}(L) - 1/(1+\exp[\beta_{1,L} - \alpha s]),$$
$$p_{2,H}(L,s) = 1 - p_{2,D}(L) - 1/(1+\exp[\beta_{2,L} - \alpha s])$$

where $p_{1,D}(L)$ and $p_{2,D}(L)$ are given by (24) and (25). Table 1 and Table 2 present allocations and marginal reimbursement rates $\partial R_{0,L}(s_0)/\partial s_0$ and $\partial R_{1,L}(s_1)/\partial s_1$ for two values of α and θ_H and with $w = 100$ and $\theta_L = 1$.

First, and foremost, these results confirm that the marginal reimbursement rate of s_1, $\partial R_{1,L}(s_1)/\partial s_1$, is positive for the considered calibration of transition probabilities. In addition, the illustration brings about a number

$\theta_H = 200$	$\alpha = 1.5$	$\alpha = 0.9$
P_r	15.35	22.31
$R_{0,L}$	21.38	19.74
$R_{1,L}$	21.27	19.63
$R_{2,L}$	21.09	19.45
$R_{0,H}$	220.09	218.45
$R_{1,H}$	220.09	218.45
$R_{2,H}$	220.09	218.45
d_0	199.99	199.99
d_1	199.99	199.99
d_2	199.99	199.99
s_0	1.28	1.28
s_1	1.17	1.17
s_2	0.99	0.99
$\partial R_{0,L}(s_0)/\partial s_0$	0.13	0.13
$\partial R_{1,L}(s_1)/\partial s_1$	0.07	0.07

Table 1: Allocation and marginal reimbursment rates when $\theta_H = 200$.

$\theta_H = 400$	$\alpha = 1.5$	$\alpha = 0.9$
P_r	17.25	30.12
$R_{0,L}$	21.12	18.19
$R_{1,L}$	20.96	17.98
$R_{2,L}$	20.64	17.62
$R_{0,H}$	419.64	416.62
$R_{1,H}$	419.64	416.62
$R_{2,H}$	419.64	416.62
d_0	399	399
d_1	399	399
d_2	399	399
s_0	1.47	1.56
s_1	1.30	1.35
s_2	0.99	0.99
$\partial R_{0,L}(s_0)/\partial s_0$	0.21	0.26
$\partial R_{1,L}(s_1)/\partial s_1$	0.14	0.16

Table 2: Allocation and marginal reimbursment rates when $\theta_H = 400$.

of interesting properties. In particular, transfers to type H individuals, that is $R_{0,H}$, $R_{1,H}$ and $R_{2,H}$, as well as expenses d_0, d_1 and d_2 do not differ with respect to age. This is line with equations (12) and (13) which state that marginal utility of consumption should be equal for everybody and that the marginal utility of consumption is equal to the marginal effect of d on utility given here by $2\,(d-\theta)$. Given that marginal utilities of consumption are equal to $1/\,(1+c)$, the marginal utility of consumption is low so that d is set very closed to θ_H. This property does not depend on the specification of probabilities but is due to the separability of the utility functions.

Furthermore the results show that formal home care s decreases with age. This is because s_0 has a higher impact than s_1 on total welfare: s_0 has not only an impact on the probability $p_{1,L}$ but also on $p_{2,L}$. As a consequence, the transfers R to type L are decreasing with age so that marginal utility of consumption in state L is the same in all periods. Since s_2 has no impact, it is set such that the marginal utility of consumption is the same in all states which again implies that s_2 is very close to θ_L.

Finally, the parameter α does not appear to have a very significant impact on the marginal subsidies and on s_0 and s_1. We present the results only for two values but simulations with different values have produced similar results.

6 Concluding comments

We have studied the design of optimal (private and/or social) insurance schemes for formal home care and nursery home care. We have illustrated the role of formal home care which has both direct effect (on utility) and indirect effects (on insurers budget constraint via transition probabilities). More precisely, formal home care provided to lightly dependent individuals reduces their risk of becoming severely dependent. We have shown that the *laissez-faire* is inefficient because individuals consume a too low level of formal home care and are not insured. The optimal insurance scheme implies a transfer to lightly dependent individuals that (under some conditions) increases with the amount of formal home care consumed. Severely dependent individuals, on the other hand, must receive a flat transfer. The theoretical analysis is illustrated by a calibrated numerical example which has shown that the expressions have the expected signs under plausible conditions.

Our model is simple and can be extended in several ways. First, one could consider more periods and possibly an infinite horizon setting where the number of periods extends until individual's death. This would complicate the expressions but not change the results. Second, we concentrate on full information setting. In particular, severity of dependency is observable; this is a quite standard assumption. A more restrictive assumption is that we have not considered *ex post* moral hazard. This can only reinforce results for institutional care (see Cremer *et al.* 2016) but may mitigate subsidy on formal home care. Last but not least, we have also ignored *ex ante* heterogeneity which would introduce income redistribution. This would significantly complicate the analysis and one would have to consider the interaction with income taxation. However, overall, this is likely to reinforce the case for insurance coverage as described.

References

[1] Barczyk D. and M. Kredler, (2018), "Evaluating long-term-care policy options, taking the family seriously", *Review of Economic Studies*, **85**, 766–809. 93(6), 1045–1076.

[2] Barigozzi, F., (2004), Reimbursing preventive care, *The Geneva Papers on Risk and Insurance Theory*, **29**, 165–186.

[3] Barigozzi, F., H. Cremer and K. Roeder (2020) "Caregivers in the Family: Daughters, Sons and Social Norms", *European Economic Review*, **130,** Article 103589.

[4] Bonsang, E. and J. Schoenmaeckers, (2015), "Long-term care insurance and the family: does the availability of potential caregivers substitute for long-term care insurance?", In: Börsch-Supan, A., T. Kneip and H. Litwin (Eds.), *Ageing in Europe - Supporting Policies for an Inclusive Society*, De Gruyter, Chapter 33.

[5] Cremer, H. and P. Pestieau, (2014), "Social long-term care insurance and redistribution", International tax and Public Finance, **21**, 955–974.

[6] Cremer, H., P. Pestieau and G. Ponthières, (2012), "The economics of Long-Term-Care: A survey", *Nordic Economic Policy Review*, **2**, 107–148.

[7] Cremer, H., Gahvari, F. and P. Pestieau, (2017), "Uncertain altruism and the provision of long term care", *Journal of Public Economics*, **151**, 12–24.

[8] Cremer, H., Lozachmeur, J.-M. and P. Pestieau, (2016). "The design of long-term care insurance contracts", *Journal of Health Economics*, **50**, 330–339.

[9] Cremer, H. and K. Roeder, (2017), "Social insurance with competitive insurance markets and risk misperception", *Journal of Public Economics*, **146**, 138–147.

[10] DRESS (2010), “Une approche de l’autonomie chez les adultes et les personnes âgées”, *Etudes et Résultats, **718**.*

[11] Drèze, J., P. Pestieau, and E. Schokkaert, (2016), “Arrow’s theorem of the deductible and long-term care insurance”, *Economics Letters*, **148**, 103–105.

[12] Ellis, R., and W. Manning, (2007), Optimal health insurance for prevention and treatment, *Journal of Health Economics*, **26**, 1128–1150.

[13] Klimaviciute, J., (2017), “Long-term care insurance and intra-family moral hazard: Fixed vs proportional insurance benefits”, *The Geneva Risk and Insurance Review*, **42**, 87–116.

[14] Klimaviciute, J. and P. Pestieau, (2023), “The economics of long-term care: An overview”, *Journal of Economic Surveys*, **37**, 1192–1213.

[15] Fuino, M. and J Wagner, (2018), “Long-term care models and dependence probability tables by acuity level: New empirical evidence from Switzerland”, *Insurance: Mathematics and Economics*, **81**, 51-70.

[16] García-Andrade, M.M., Santaeugenia-González, S.J., Solá Hermida, J.C., Baixauli-Alacreu, S., Santabalbina, F.J.T. , (2020), “Risk Factors for Mortality in Nursing Home Residents: An Observational Study”, *Geriatrics*, **5(71)**, 1-16..

[17] Hurd, M.D, P. Michaud, and S. Rohwedder, (2017), “Distribution of lifetime nursing home use and of out-of-pocket spending”, *Proc. Natl. Acad. Sci. U.S.A* 114, **37**, 9838-9842.

[18] Rizzo, Z.O, Baker D., G. Mccavay, and M. Tinetty, (1996), "The Cost-Effectiveness of a Multifactorial Targeted Prevention Program for Falls Among Community Elderly Persons", *Medical Care*, **34(9)**, 954-969.

[19] Tavassoli, N., De Souto Barreto, P., Berbon, C., Mathieu C., de Kerimel, J., Lafont, C., Takeda, C., Carrie, I., Piau, A., Jouffrey, T., Andrieu, S., Nourhashemi, F., Beard R., Soto Martin, M.E., Vellas, B.(2022), "Implementation of the WHO integrated care for older people (ICOPE) programme in clinical practice: a prospective study", *The Lancet Healthy Longevity* **3(6),** 394-404.

Appendix

A Impact of s_0 and s_1 on the shares of the different types

Differentiating (5) with respect to s_0 and s_1 and using $p_{t,L}^{s_{t-1}} + p_{t,H}^{s_{t-1}} = 0$ for $t =, 1, 2$ yields

$$\pi_{1,L}^{s_0} = \pi_{0,L} p_{1,L}^{s_0} > 0, \tag{A1}$$

$$\pi_{1,H}^{s_0} = \pi_{0,L} p_{1,H}^{s_0} < 0, \tag{A2}$$

$$\pi_{2,L}^{s_0} = \pi_{1,L}^{s_0} p_{2,L}(L, s_1) = \pi_{0,L} p_{1,L}^{s_0} p_{2,L}(L, s_1) > 0 \tag{A3}$$

$$\begin{aligned}\pi_{2,H}^{s_0} &= \pi_{1,L}^{s_0} p_{2,H}(L, s_1) + \pi_{1,H}^{s_0} p_{2,H}(H) \\ &= \pi_{0,L} p_{2,H}(L, s_1) p_{1,L}^{s_0} + \pi_{0,L} p_{2,H}(H) p_{1,H}^{s_0} \\ &= \pi_{0,L} p_{1,L}^{s_0} [p_{2,H}(L, s_1) - p_{2,H}(H)] < 0,\end{aligned} \tag{A4}$$

$$\begin{aligned}\pi_{2,D}^{s_0} &= \pi_{1,L}^{s_0} p_{2,D}(L) + \pi_{1,H}^{s_0} p_{2,D}(H) \\ &= \pi_{0,L} p_{2,D}(L) p_{1,L}^{s_0} + \pi_{0,L} p_{2,D}(H) p_{1,H}^{s_0} \\ &= \pi_{0,L} p_{1,L}^{s_0} [p_{2,D}(L) - p_{2,D}(H)] < 0,\end{aligned} \tag{A5}$$

$$\pi_{2,L}^{s_1} = \pi_{1,L} p_{2,L}^{s_1} = \pi_{0,L} p_{1,L}(L, s_0) p_{2,L}^{s_1} > 0, \tag{A6}$$

$$\pi_{2,H}^{s_1} = \pi_{1,L} p_{2,H}^{s_1} = \pi_{0,L} p_{1,L}(L, s_0) p_{2,H}^{s_1} < 0. \tag{A7}$$

B The fist order conditions of the First Best

Using the assumptions made on P_t, the First Best problem amounts to solve:

$$\begin{aligned}
\max_{\mathcal{P}_r, R_{t,\theta L}, s_t} EU = {} & \pi_{0,G} u\left(\omega - \mathcal{P}_r\right) \\
& + \pi_{0,L} u\left(\omega - \mathcal{P}_r - s_0 + R_{0,L}, \theta_L - s_0\right) \\
& + \pi_{0,H} u\left(\omega - \mathcal{P}_r - d_0 + R_{0,H}, \theta_H - d_0\right) \\
& + \pi_{1,G} u\left(\omega - \mathcal{P}_r\right) \\
& + \pi_{1,L}\left(s_0\right) u\left(\omega - \mathcal{P}_r - s_1 + R_{1,L}, \theta_L - s_1\right) \\
& + \pi_{1,H}\left(s_0\right) u\left(\omega - \mathcal{P}_r - d_1 + R_{1,H}, \theta_H - d_1\right) \\
& + \pi_{2,G} u\left(\omega - \mathcal{P}_r\right) \\
& + \pi_{2,L}\left(s_0, s_1\right) u\left(\omega - \mathcal{P}_r - s_2 + R_{2,L}, \theta_L - s_2\right) \\
& + \pi_{2,H}\left(s_0, s_1\right) u\left(\omega - \mathcal{P}_r - d_2 + R_{2,H}, \theta_H - d_2\right), \\
\text{s.to: } & \left[3 - \pi_{1,D} - \pi_{2,D}\left(s_0\right)\right] \mathcal{P}_r - \pi_{0,L} R_{0,L} - \pi_{0,H} R_{0,H} \\
& - \pi_{1,L}(s_0) R_{1,L} - \pi_{1,H}\left(s_0\right) R_{1,H} \\
& - \pi_{2,L}(s_0, s_1) R_{2,L} - \pi_{2,D}(s_0, s_1) R_{2,H} \geq 0. \qquad \text{(A8)}
\end{aligned}$$

The first order conditions with respect to $\mathcal{P}_r$ yield:

$$\begin{aligned}
& - \left(\pi_{0,G} + \pi_{1,G} + \pi_{2,G}\right) v\left(\omega - \mathcal{P}_r\right) \\
& - \pi_{0,L} u^c_{0,L} - \pi_{0,H} u^c_{0,H} - \pi_{1,L} u^c_{1,L} - \pi_{1,H} u^c_{1,H} \\
& - \pi_{2,L} u^c_{2,L} - \pi_{2,H} u^c_{2,H} + \mu\left(3 - \pi_{2,D}\left(s_0\right)\right) = 0. \qquad \text{(A9)}
\end{aligned}$$

The first order conditions with respect to $R_{t,i}$ are successively given by:

$$u^c_{0,L}\left(\omega - \mathcal{P}_r - s_0 + R_{0,L}, q_{0,L}\right) = \mu, \tag{A10}$$

$$u^c_{0,H}\left(\omega - \mathcal{P}_r - d_0 + R_{0,H}, q_{0,H}\right) = \mu, \tag{A11}$$

$$u^c_{1,L}\left(\omega - \mathcal{P}_r - s_1 + R_{1,L}, q_{1,L}\right) = \mu,$$

$$u^c_{1,H}\left(\omega - \mathcal{P}_r - d_1 + R_{1,H}, q_{1,H1}\right) = \mu, \tag{A12}$$

$$u^c_{2,L}\left(\omega - \mathcal{P}_r - s_2 + R_{2,L}, q_{2,L}\right) = \mu, \tag{A13}$$

$$u^c_{2,H}\left(\omega - \mathcal{P}_r - d_2 + R_{2,H}, q_{2,H}\right) = \mu. \tag{A14}$$

which combined with (A9) yield $v^c = u^c_{t,i} = u^c_{t,j}$ for every t, i and j.

C Proof of expression (22)

Using expressions (A1)–(A5) and using $p^{s_{t-1}}_{t,L} + p^{s_{t-1}}_{t,H} = 0$, we successively obtain

$$\begin{aligned}
&\pi^{s_0}_{2,D}\mathcal{P}_r + \pi^{s_0}_{1,L}R_{1,L} + \pi^{s_0}_{1,H}R_{1,H} \\
&+ \pi^{s_0}_{2,L}R_{2,L} + \pi^{s_0}_{2,H}R_{2,H} \\
&= \pi_{0,L}p_{2,D}\left(L\right)p^{s_0}_{1,L}\mathcal{P}_r + \pi_{0,L}p_{2,D}\left(H\right)p^{s_0}_{1,H}\mathcal{P}_r \\
&+ \pi_{0,L}p^{s_0}_{1,L}R_{1,L} + \pi_{0,L}p^{s_0}_{1,H}R_{1,H} \\
&+ \pi_{0,L}p_{2,L}\left(L, s_1\right)p^{s_0}_{1,L}R_{2,L} - \pi_{0,L}p_{2,H}\left(L, s_1\right)p^{s_0}_{1,L}R_{2,H} \\
&+ \pi_{0,L}p_{2,H}\left(H\right)p^{s_0}_{1,H}R_{2H} \\
&= \pi_{0,L}\mathcal{P}_r p^{s_0}_{1,L}[p_{2,D}\left(L\right) - p_{2,D}\left(H\right)] \\
&+ \pi_{0,L}p^{s_0}_{1,L}[R_{1,L} - R_{1,H}] \\
&+ \pi_{0,L}p^{s_0}_{1,L}p_{2,L}\left(L, s_1\right)[R_{2,L} - R_{2,H}] \\
&+ \pi_{0,L}p^{s_0}_{1,H}p_{2,H}\left(H\right)R_{2H}.
\end{aligned} \tag{A15}$$

The two last lines of equation ($A15$) can be rewritten as:

$$
\begin{aligned}
&\pi_{0,L}p_{1,L}^{s_0}p_{2,L}\left(L,s_1\right)\left[R_{2,L}-R_{2,H}\right]+\pi_{0,L}p_{1,H}^{s_0}p_{2,H}\left(H\right)R_{2H}\\
&=\pi_{0,L}p_{1,L}^{s_0}R_{2,H}[p_{2,L}\left(L,s_1\right)-p_{2,H}\left(H\right)]+\pi_{0,L}p_{1,L}^{s_0}p_{2,L}\left(L,s_1\right)R_{2,L}\\
&=\pi_{0,L}p_{1,L}^{s_0}R_{2,H}[1-p_{2,D}\left(L\right)-p_{2L}\left(L,s_1\right)-1+p_{2,D}\left(H\right)]+\pi_{0,L}p_{1,L}^{s_0}p_{2,L}\left(L,s_1\right)R_{2,L}\\
&=\pi_{0,L}p_{1,L}^{s_0}\Big[\left(p_{2,D}\left(H\right)-p_{2,D}\left(L\right)\right)R_{2H}-\pi_{0,L}p_{1,L}^{s_0}p_{2L}\left(L,s_1\right)\left(R_{2H}-R_{2L}\right)\Big]
\end{aligned}
$$

which yields (22).