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Abstract

We study the design of optimal (private and/or social) insurance schemes for formal home care and institutional care. We consider a three period model. Individuals are either in good health, lightly dependent or heavily dependent. Lightly dependent individuals can buy formal home care which reduces the severity of dependency and reduces the probability to become severely dependent in the next period. Severely dependent individuals pay for nursing home care. In both states of dependency individuals can receive a (private or public) insurance benefit (transfers). These benefits can be flat or depend on the formal care consumed (or a combination of the two). These benefits are financed by a premium (or a tax). Individuals may be alive until the end of period 2 or die at the beginning of periods 1 or 2 with a certain probability which may depend on their state of health.

The *laissez faire* is inefficient because individuals consume a too low level of formal home care and are not insured. The first-best insurances scheme requires a transfer to lightly dependent individuals that, (under some conditions) increases with the amount of formal home care consumed. Severely dependent individuals, on the other hand, must receive a flat transfer (from private or social insurance). The theoretical analysis is illustrated by a calibrated numerical example which show that the expressions have the expected signs under plausible conditions.

Keywords: Long-term care insurance, formal home care, nursing home care.
JEL Codes: I13, I18, H51

1 Introduction

Because of population aging the demand for formal long-term care (LTC) services by the elderly population is likely to grow substantially; see Cremer et al. (2012) and Klimaviciute and Pestieau (2023) for an overview of the relevant evidence. LTC mainly consists in assistance with daily activities, and it is different from health care. Unlike medical care, this assistance does not require highly skilled caregivers, but it is very "labor intensive" and typically not covered by health insurance.

Currently a significant part (between one third and one half) of long-term care is provided informally, by family members (mainly daughters); see for instance Barczyk and Kredler (2018). However, the extent of informal care is likely to decrease in the future. This is due for instance to changes in family values, increased labor female labor force participation and mobility of children. In any event informal care imposes a significant cost on caregivers and it is not available to everyone.[1] Consequently, there will be an increasing need for formal care to supplement or replace informal care. While severely dependent individuals require institutional care, formal care provided at home is an alternative for the less severely dependent individuals and it can delay their need for nursery home care. It is cheaper and preferred by most persons. For individuals either of these represents a significant financial risk. As long as individuals are risk averse, standard economic theory suggests that the random and costly nature of LTC makes it precisely the type of risk that calls for private or social insurance protection.[2]

Postal operators can play an important role both as provider of personal

[1] See for instance Barigozzi *et al.* (2020), Bonsang and Schoenmaeckers (2015), Cremer *et al.* (2017) and Cremer *et al.* (2012).

[2] See Cremer and Pestieau (2014) or Cremer and Roeder (2017).

services to the elderly and as insurer (when they also offer financial – banking and insurance – services). These personal services include delivery of medication or meals, remote surveillance, housework, gardening, or simply regular visits. They can be provided directly by mail carriers or via specialized subsidiaries. As to insurance, postal banks offer in any event a wide variety of insurances.

We study the design of optimal (private and/or social) insurance schemes for formal home care and nursery home care. While there is some literature on this subject,[3] our study presents the novel feature that it combines home and institutional care and that the transition between states of dependence (light or severe) is endogenous.[4]

We consider a model with 3 periods: 0, 1 and 2. In period 0 individuals are either in good health, lightly dependent or heavily dependent. Lightly dependent individuals can buy formal home care which reduces the severity of dependency and reduces the probability to become severely dependent in the next period. Severely dependent individuals pay for nursing home care. In both states of dependency individuals can receive a (private or public) insurance benefit (transfers). These benefits can be flat or depend on the formal care consumed (or a combination of the two). These benefits are financed by a premium (or a tax). Individuals may be alive until the end of period 2 or die at the beginning of periods 1 or 2 with a certain probability which may depend on their state of health.

The *laissez-faire* is inefficient because individuals consume a too low level of formal home care and are not insured. We study the first-best (FB) insurance scheme that maximizes expected utility of a representative

[3]See for instance Cremer *et al.* (2016), Drèze *et al.* (2016) or Klimaviciute, J., (2017).

[4]Barigozzi *et al.* (2020) do also allow for both types of formal care but in that setting there is no uncertainy so that the question of insurance design does not arise.

individual and show how it can be implemented by reimbursement rules for home and institutional care.

We show that the decentralization of the FB requires a transfer to lightly dependent individuals that, (under some conditions) increases with the amount of formal home care consumed. The transfer can be entirely public or consist for instance of a flat private insurance payment plus a subsidy; this is a matter of implementation. Severely dependent individuals, on the other hand, must receive a flat transfer (from private or social insurance).

The theoretical analysis is illustrated by a calibrated numerical example which shows that the expressions have the expected signs under plausible conditions and point out some other interesting properties.

These results provide a justification for the role of La Poste Groupe (or other postal operators) as provider of home care (which should be subsidized) and as an insurer providing benefits that depend on the severity of the dependency.

2 The model

Periods and states of nature We consider a model with three periods indexed by $t = 0, 1, 2$. Individuals are endowed with income ω. In each period t, there are 4 states of nature denoted by $\theta_t \in \{G, L, H, D\}$ where G stands for good health, L stands for "lightly" dependent (at home), H stands for severely (heavily) dependent (requiring nursing home care) and D for dead. We assume that death occurs at the beginning of the considered period. The proportion of the initial population who is of type θ in period t is denoted by $\pi_{t,\theta}$. In period 0, we consider only individuals who are alive so that $\pi_{0,D} = 0$. This makes sense because individuals die at the beginning of a period.

Utilities In each period, individuals who are alive pay a premium $\mathcal{P}_r$. Utility of type G individuals in period $t = 0, 1, 2$ is given by

$$U_{t,G} = u\left(\omega - \mathcal{P}_r\right), \tag{1}$$

while utility of type L individuals

$$U_{t,L} = u\left(\omega - \mathcal{P}_r - s_t + R_{t,L}, \theta_L - s_t\right), \tag{2}$$

where s_t denotes home service expenditures and $R_{t,L}$ denotes reimbursement of home services. Utility of individuals of type H is

$$U_{t,H} = u\left(\omega - \mathcal{P}_r - d_t + R_{t,H}, \theta_H - d_t\right), \tag{3}$$

where d_t denotes LTC expenditures (nursing home) and $R_{t,D}$ the reimbursement of these expenditures. In state of nature D we normalize utility to zero

$$U_{t,D} = 0.$$

Expected utility is thus

$$EU = \sum_{t=0,1,2} \sum_{\theta=G,L,H,D} \pi_{t,\theta} U_{t,\theta} \tag{4}$$

where $U_{t,G}, U_{t,L}$ and $U_{t,H}$ are respectively given by (1), (2) and (3).

Transition probabilities and proportions of types The probability to be in state θ at period t when the state in period $t-1$ was θ_{t-1} is denoted $p_{t,\theta}\left(\theta_{t-1}\right)$. Consequently proportions of the different types are given by

$$\pi_{t,\theta} = \sum_{\theta_{t-1} \in \{G,L,H,D\}} p_{t,\theta}\left(\theta_{t-1}\right) \pi_{t-1,\theta_{t-1}}. \tag{5}$$

Note that by definition proportions of types in any period must add up to 1 so that we have

$$\sum_{\theta \in \{G,L,H,D\}} \pi_{t,\theta} = 1 \text{ for every } t$$

Let P_t denote matrix of transition probabilities defined by

$$P_t = \begin{pmatrix} p_{t,G}(G) & p_{t,L}(G) & p_{t,H}(G) & p_{t,D}(G) \\ p_{t,G}(L) & p_{t,L}(L) & p_{t,H}(L) & p_{t,D}(L) \\ p_{t,G}(H) & p_{t,L}(H) & p_{t,H}(H) & p_{t,D}(H) \\ p_{t,G}(D) & p_{t,L}(D) & p_{t,H}(D) & p_{2,D}(D) \end{pmatrix} \quad (6)$$

where the sum of the elements of each line of P_t is equal to 1.

We make the following assumptions:

$$\pi_{0,D} = 0 \quad (7)$$

$$p_{t,G}(L) = p_{t,G}(H) = p_{t,G}(D) = 0 \quad (8)$$

$$p_{t,L}(H) = p_{t,L}(D) = 0 \quad (9)$$

$$p_{t,H}(G) = p_{tH}(D) = 0 \quad (10)$$

$$p_{t,D}(H) \geq p_{t,D}(L) \quad (11)$$

Equation (7) is a normalization i.e. $\pi_{0,G} + \pi_{0,L} + \pi_{0,H} = 1$; we consider only individuals who are alive in period 0 which is formally equivalent to assuming $\pi_{0,D} = 0$. The other equations imply that dependency is a "one way road"; there is no way to return to a less dependent state and the deterioration of the state is gradual. Furthermore death is of course an "absorbing state"; there is no resurrection. In particular, equation (8) says that the probability to be in state G at period t is zero if the individual was in state L or H or D in the previous period. Similarly, equation (9) says that the probability of people in state L in period t is zero if the individual was in state H or D in the previous period. Equation (10) says that the probability of people in state H in period t is zero if the individual was in state G or D in the previous period. Finally, expression (11) implies that mortality is larger among the severely dependent than among the lightly dependent. This assumption is not essential for our analysis but it simplifies the interpretation of some expressions and reduces the number of cases to be considered.

Furthermore, we assume that for individuals who are in state L in period $t-1$ the probability of remaining in state L or moving to state H at date t is endogenous and depends upon the level of home care services. Formally, we write $p_{t,L}\left(L, s_{t-1}\right)$ and $p_{t,H}\left(L, s_{t-1}\right)$ with $\partial p_{t,L}\left(L, s_{t-1}\right)/\partial s_{t-1} > 0$ and $\partial p_{t,H}\left(L, s_{t-1}\right)/\partial s_{t-1} < 0$. And we must have $\partial p_{t,L}\left(L, s_{t-1}\right)/\partial s_{t-1} + \partial p_{t,H}\left(L, s_{t-1}\right)/\partial s_{t-1} = 0$. In words, lightly dependent individuals are less likely to become heavily dependent is they benefit from more formal home care services.

With these assumptions, one can rewrite the transition matrix as follow

$$P_t = \begin{pmatrix} p_{t,G}\left(G\right) & p_{t,L}\left(G\right) & 0 & p_{t,D}\left(G\right) \\ 0 & p_{t,L}\left(L, s_{t-1}\right) & p_{t,H}\left(L, s_{t-1}\right) & p_{t,D}\left(L\right) \\ 0 & 0 & p_{t,H}\left(H\right) & p_{t,D}\left(H\right) \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{12}$$

In period 0 the types' proportions are given. In the subsequent periods, they are then obtained from the initial proportion together with the transition probabilities. In period 1, we have

$$\pi_{1,G} = p_{1,G}\left(G\right)\pi_{0,G}, \tag{13}$$

$$\pi_{1,L}\left(s_0\right) = p_{1,L}\left(G\right)\pi_{0,G} + p_{1,L}\left(L, s_0\right)\pi_{0,L}, \tag{14}$$

$$\pi_{1,H}(s_0) = p_{1,H}\left(L, s_0\right)\pi_{0,L} + p_{1,H}\left(H\right)\pi_{0,H}, \tag{15}$$

$$\pi_{1,D} = p_{1,D}\left(G\right)\pi_{0,G} + p_{1,D}\left(L\right)\pi_{0,L} + p_{1,D}\left(H\right)\pi_{0,H}. \tag{16}$$

Observe that the proportions of types L and H depend on home care services, received by the lightly dependent in period 0, s_0. This is because the transition probability between these two states depends on s_0.

Similarly in period 2 we have

$$\pi_{2,G} = p_{2,G}\,(G)\,\pi_{1,G}, \tag{17}$$

$$\pi_{2,L}(s_0, s_1) = p_{2,L}\,(G)\,\pi_{1,G} + p_{2,L}\,(L, s_1)\,\pi_{1,L}(s_0), \tag{18}$$

$$\pi_{2,H}(s_0, s_1) = p_{2,H}\,(L, s_1)\,\pi_{1,L}(s_0) + p_{2,H}\,(H)\,\pi_{1,H}(s_0), \tag{19}$$

$$\pi_{2,D}(s_0) = p_{2,D}\,(G)\,\pi_{1,G} + p_{2,D}\,(L)\,\pi_{1,L}(s_0) + p_{2,D}\,(H)\,\pi_{1,H}(s_0). \tag{20}$$

Now the proportions depend on both s_0 and s_1. The proportions in period 1 depend on s_0, while the transition probabilities between L and H from period 1 to 2 depend on s_1. Transitions and states of nature in each period are illustrated in Figure 1.

Differentiating (14), (15), (20), (18) and (19) with respect to s_0 and s_1 shows how the shares of types are affected by home care. The expressions are provided in Appendix A and they are used when determining the optimal policy. Here we restrict ourselves to stating their signs.

$$\frac{\partial \pi_{1,L}(s_0)}{\partial s_0} > 0 \tag{21}$$

$$\frac{\partial \pi_{1,H}(s_0)}{\partial s_0} < 0 \tag{22}$$

$$\frac{\partial \pi_{2,L}(s_0, s_1)}{\partial s_0} > 0 \tag{23}$$

$$\frac{\partial \pi_{2,H}(s_0, s_1)}{\partial s_0} = \pi_{0,L}\frac{\partial p_{1,L}\,(L, s_0)}{\partial s_0}[p_{2,H}\,(L, s_1) - p_{2,H}\,(H)] \lesseqgtr 0 \tag{24}$$

$$\frac{\partial \pi_{2,D}(s_0, s_1)}{\partial s_0} = \pi_{0,L}\frac{\partial p_{1,L}\,(L, s_0)}{\partial s_0}[p_{2,D}\,(L) - p_{2,D}\,(H)] < 0 \tag{25}$$

Equations (21)–(23) and (25) follow directly from our assumptions on the impact of s on the transition probabilities. Roughly speaking, an increase in s_0 increases the proportion of lightly dependent individuals at the expense of the severely dependent in subsequent periods. This in turn results in a

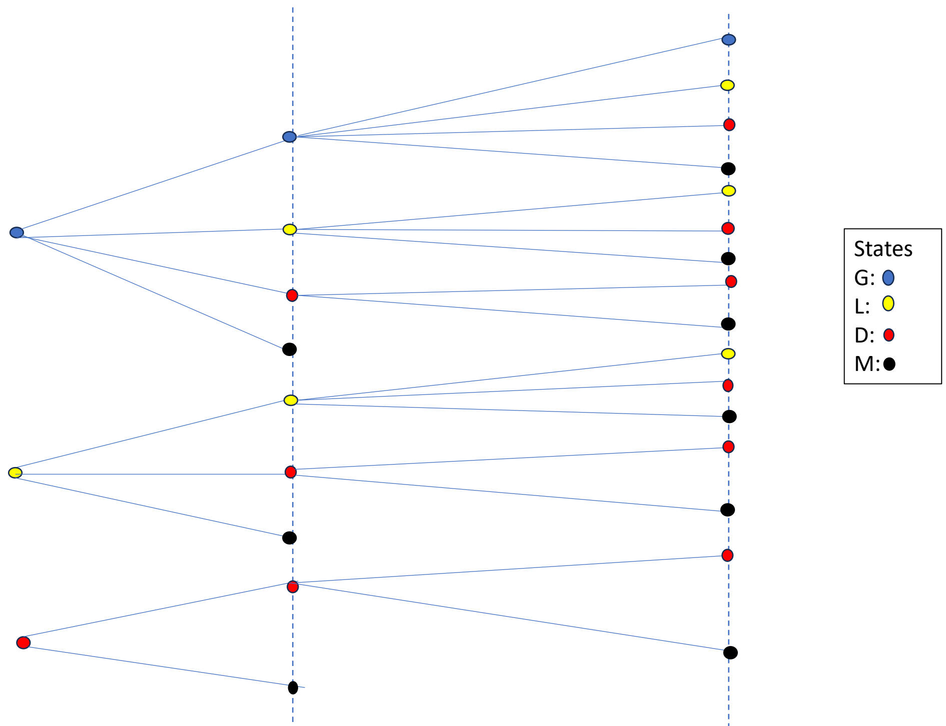

Figure 1: Timing, transition probabilities and proportions. The columns refer to periods 0, 1 and 2 respectively.

decrease of the proportion of individuals who are no longer alive in period 2 as stated in (25). This impact on mortality also explains why the sign of (24) is ambiguous; while s_0 increases the proportion of lightly dependent in period 2 it also reduces mortality in period 1 which explains that some individuals who would have otherwise died earlier end up severely dependent in period 2. This indirect effect via mortality affects some of our results below.

Because individuals are risk averse, it is clear that insurance coverage for care expenses would be desirable. The question to which we now turn is how the insurance contract should be designed. To address this question we first characterize the optimal allocation and then determine how it can be decentralized via an insurance contract covering home and nursing home care.

3 First best

The first-best solution is obtained by maximizing the expected utility of a representative individual subject to the resource constraint. Formally, the problem is

$$\begin{aligned}
&\max_{\mathcal{P}_r, R_{t,L}, R_{t,D}, s_t, d_t} EU \\
\text{s.t.}\quad & \left[3-\pi_{1,D}-\pi_{2,D}\left(s_0\right)\right]\mathcal{P}_r \\
& -\pi_{0,L}R_{0,L}-\pi_{0,H}R_{0,H} \\
& -\pi_{1,L}(s_0)R_{1,L}-\pi_{1,H}\left(s_0\right)R_{1,H} \\
& -\pi_{2,L}(s_0,s_1)R_{2,L}-\pi_{2,D}(s_0,s_1)R_{2,H}\geq 0
\end{aligned}$$

where the expected utility is given by

$$\begin{aligned}
EU = {} & \pi_{0,G} u \left(\omega - \mathcal{P}_r\right) \\
& + \pi_{0,L} u \left(\omega - \mathcal{P}_r - s_0 + R_{0,L}, \theta_L - s_0\right) \\
& + \pi_{0,H} u \left(\omega - \mathcal{P}_r - d_0 + R_{0,H}, \theta_H - d_0\right) \\
& + \pi_{1,G} u \left(\omega - \mathcal{P}_r\right) \\
& + \pi_{1,L} \left(s_0\right) u \left(\omega - \mathcal{P}_r - s_1 + R_{1,L}, \theta_L - s_1\right) \\
& + \pi_{1,H} \left(s_0\right) u \left(\omega - \mathcal{P}_r - d_1 + R_{1,H}, \theta_H - d_1\right) \\
& + \pi_{2,G} u \left(\omega - \mathcal{P}_r\right) \\
& + \pi_{2,L} \left(s_0, s_1\right) u \left(\omega - \mathcal{P}_r - s_2 + R_{2,L}, \theta_L - s_2\right) \\
& + \pi_{2,H} \left(s_0, s_1\right) u \left(\omega - \mathcal{P}_r - d_2 + R_{2,H}, \theta_H - d_2\right)
\end{aligned}$$

Denoting μ the Lagrange multiplier associated with resource constraint, and u_c the partial derivative of u with respect to its first argument the FOC w.r.t $\mathcal{P}_r$ is:

$$\begin{aligned}
& - \left(\pi_{0,G} + \pi_{1,G} + \pi_{2,G}\right) u' \left(\omega - \mathcal{P}_r\right) \\
& - \pi_{0,L} u_c \left(\omega - \mathcal{P}_r - s_0 + R_{0,L}, \theta_L - s_0\right) \\
& - \pi_{0,H} u_c \left(\omega - \mathcal{P}_r - d_0 + R_{0,H}, \theta_H - d_0\right) \\
& - \pi_{1,L} \left(s_0\right) u_c \left(\omega - \mathcal{P}_r - s_1 + R_{1,L}, \theta_L - s_1\right) \\
& - \pi_{1,H} \left(s_0\right) u_c \left(\omega - \mathcal{P}_r - d_1 + R_{1,H}, \theta_H - d_1\right) \\
& - \pi_{2,L} \left(s_0, s_1\right) u_c \left(\omega - \mathcal{P}_r - s_2 + R_{2,L}, \theta_L - s_2\right) \\
& - \pi_{2,H} \left(s_0, s_1\right) u_c \left(\omega - \mathcal{P}_r - d_2 + R_{2,H}, \theta_H - d_2\right) \\
& + \mu \left(3 - \pi_{2,D} \left(s_0\right)\right) = 0 \qquad (26)
\end{aligned}$$

The first-order conditions (FOCs) with respect to $R_{t,i}$ are:

$$\pi_{0,L}u_c\left(\omega - \mathcal{P}_r - s_0 + R_{0,L}, \theta_L - s_0\right) = \mu\pi_{0,L}, \tag{27}$$

$$\pi_{0,H}u_c\left(\omega - \mathcal{P}_r - d_0 + R_{0,H}, \theta_H - d_0\right) = \mu\pi_{0,H}, \tag{28}$$

$$\pi_{1,L}\left(s_0\right)u_c\left(\omega - \mathcal{P}_r - s_1 + R_{1,L}, \theta_L - s_1\right) = \mu\pi_{1,L}\left(s_0\right), \tag{29}$$

$$\pi_{1,H}\left(s_0\right)u_c\left(\omega - \mathcal{P}_r - d_1 + R_{1,H}, \theta_L - d_1\right) = \mu\pi_{1,H}\left(s_0\right), \tag{30}$$

$$\pi_{2,L}\left(s_0, s_1\right)u_c\left(\omega - \mathcal{P}_r - s_2 + R_{2,L}, \theta_L - s_2\right) = \mu\pi_{2,L}\left(s_0, s_1\right), \tag{31}$$

$$\pi_{2,H}\left(s_0, s_1\right)u_c\left(\omega - \mathcal{P}_r - d_2 + R_{2,L}, \theta_L - d_2\right) = \mu\pi_{2,H}\left(s_0, s_1\right). \tag{32}$$

These conditions require that the marginal utility of income is the same in all states of nature (when alive). In other words, there is full insurance. The FOC w.r.t d_t for $t = 0, 1, 2$ are

$$u_c\left(\omega - \mathcal{P}_r - d_t + R_{0,t}, \theta_H - d_t\right) = -u_\theta\left(\omega - \mathcal{P}_r - d_t + R_{t,H}, \theta_H - d_t\right) \tag{33}$$

Marginal costs of d equal marginal benefits in all periods. Both are expressed in terms of utility.

Finally, the FOCs with respect to s_t are

$$\begin{aligned}
&-\pi_{0,L}u_c\left(\omega - \mathcal{P}_r - s_0 + R_{0,L}, \theta_L - s_0\right) - \pi_{0,L}u_\theta\left(\omega - \mathcal{P}_r - s_0 + R_{0,L}, \theta_L - s_0\right)\\
&+\frac{\partial\pi_{1,L}\left(s_0\right)}{\partial s_0}U_{1,L} + \frac{\partial\pi_{1,H}\left(s_0\right)}{\partial s_0}U_{1,H} + \frac{\partial\pi_{2,L}\left(s_0,s_1\right)}{\partial s_0}U_{2,L} + \frac{\partial\pi_{2,H}\left(s_0,s_1\right)}{\partial s_0}U_{2,H}\\
&+\mu[-\frac{\partial\pi_{2,D}\left(s_0\right)}{\partial s_0}\mathcal{P}_r - \frac{\partial\pi_{1,L}\left(s_0\right)}{\partial s_0}R_{1,L} - \frac{\partial\pi_{1,H}\left(s_0\right)}{\partial s_0}R_{1,H}\\
&-\frac{\partial\pi_{2,L}\left(s_0,s_1\right)}{\partial s_0}R_{2,L} - \frac{\partial\pi_{2,H}\left(s_0,s_1\right)}{\partial s_0}R_{2,H}] = 0,
\end{aligned} \tag{34}$$

$$\begin{aligned}
&-\pi_{1,L}u_c\left(\omega - \mathcal{P}_r - s_1 + R_{1,L}, \theta_L - s_1\right) -\\
&\pi_{1,L}u_\theta\left(\omega - \mathcal{P}_r - s_1 + R_{0,L}, \theta_L - s_1\right)\\
&+\frac{\partial\pi_{2,L}\left(s_0,s_1\right)}{\partial s_1}U_{2,L} + \frac{\partial\pi_{2,H}\left(s_0,s_1\right)}{\partial s_1}U_{2,H}\\
&+\mu\left[-\frac{\partial\pi_{2,L}\left(s_0,s_1\right)}{\partial s_1}R_{2,L} - \frac{\partial\pi_{2,H}\left(s_0,s_1\right)}{\partial s_1}R_{2,H}\right] = 0,
\end{aligned} \tag{35}$$

$$-\pi_{2,L}u_c\left(\omega-\mathcal{P}_r-s_2+R_{2,L},\theta_L-s_2\right)-\pi_{2,L}u_\theta\left(\omega-\mathcal{P}_r-s_2+R_{2,L},\theta_L-s_2\right)=0. \quad (36)$$

These conditions also require equalization of marginal costs and benefits for formal home care s. The expressions are more complex than for d because in addition to the direct effect in the considered period, there are indirect effects via the impact of s on the probability of dependency in subsequent periods. These indirect effects do not arise in period 2 which is the last period; see (36).

4 Decentralization

In the previous section we have assumed that the insurer directly controls all the relevant variables including expenditures on care, s and d. We now show how this solution can be decentralized via an appropriate insurance contract $(\mathcal{P}_r, R_{t,L}(s_t), R_{t,H}(d_t))$ which specifies the premium and the reimbursement rules for care expenses. In other words we let individuals choose their levels of s_t and d_t taking the insurance contract $(\mathcal{P}_r, R_{t,L}(s_t), R_{t,H}(d_t))$, as given. The main question is then how the reimbursement rules should be designed. Specifically should they involve a flat payment in each state of nature or entail a full or partial reimbursement of care expenses?

Individuals then solve

$$
\begin{aligned}
\max_{s_t,d_t} EU =& \pi_{0,G} u\left(\omega - \mathcal{P}_r\right) \\
&+ \pi_{0,L} u\left(\omega - \mathcal{P}_r - s_0 + R_{0,L}\left(s_0\right), \theta_L - s_0\right) \\
&+ \pi_{0,H} u\left(\omega - \mathcal{P}_r - d_0 + R_{0,H}\left(t_0\right), \theta_H - d_0\right) \\
&+ \pi_{1,G} u\left(\omega - \mathcal{P}_r\right) \\
&+ \pi_{1,L}\left(s_0\right) u\left(\omega - \mathcal{P}_r - s_1 + R_{1,L}\left(s_1\right), \theta_L - s_1\right) \\
&+ \pi_{1,H}\left(s_0\right) u\left(\omega - \mathcal{P}_r - d_1 + R_{1,H}\left(d_1\right), \theta_H - d_1\right) \\
&+ \pi_{2,G} u\left(\omega - \mathcal{P}_r\right) \\
&+ \pi_{2,L}\left(s_0, s_1\right) u\left(\omega - \mathcal{P}_r - s_2 + R_{2,L}\left(s_2\right), \theta_L - s_2\right) \\
&+ \pi_{2,H}\left(s_0, s_1\right) u\left(\omega - \mathcal{P}_r - d_2 + R_{2,H}\left(d_2\right), \theta_H - d_2\right).
\end{aligned}
$$

Let $c_{t,\theta}$ denote consumption in period t of an individual in state θ. The FOCs with respect to s_0, s_1, s_2 are respectively given by

$$
\begin{aligned}
&-\pi_{0,L} u_c\left(c_{0,L}, \theta_L - s_0\right) - \pi_{0,L} u_\theta\left(c_{0,L}, \theta_L - s_0\right) + \\
&+\frac{\partial \pi_{1,L}\left(s_0\right)}{\partial s_0} u_{1,L} + \frac{\partial \pi_{1,H}\left(s_0\right)}{\partial s_0} u_{1,H} \\
&+\frac{\partial \pi_{2,L}\left(s_0\right)}{\partial s_0} u_{2,L} + \frac{\partial \pi_{2,H}\left(s_0\right)}{\partial s_0} u_{2,H} \\
&+\pi_{0,L} \frac{\partial R_{0,L}\left(s_0\right)}{\partial s_0} u_c\left(c_{0,L}, \theta_L - s_0\right) = 0 \qquad (37) \\
&-\pi_{1,L} u_c\left(c_{1,L}, \theta_L - s_1\right) - \pi_{1L} u_\theta\left(c_{1,L}, \theta_L - s_1\right) \\
&+\frac{\partial \pi_{2,L}}{\partial s_1} u_{2,L} + \frac{\partial \pi_{2,H}}{\partial s_1} u_{2,H} \\
&+\pi_{1,L} \frac{\partial R_{1,L}\left(s_1\right)}{\partial s_1} u_c\left(c_{1,L}, \theta_L - s_1\right) = 0 \qquad (38) \\
&\quad - \pi_{2,L} \frac{\partial R_{2,L}\left(s_2\right)}{s_2} u_c\left(c_{2,L}, \theta_L - s_2\right) \\
&-\pi_{2,L} u_c\left(c_{2,L}, \theta_L - s_2\right) - \pi_{2,L} u_\theta\left(c_{2,L}, \theta_L - s_2\right) = 0 \qquad (39)
\end{aligned}
$$

While the FOCs with respect to d_t for $t = 0, 1, 2$ are given by

$$\pi_{t,H} \frac{\partial R_{t,H}}{d_t} u_c \left(c_{t,H}, \theta_H - d_t\right) + u_c \left(c_{t,H}, \theta_H - d_t\right) = -u_\theta \left(c_{t,H}, \theta_H - d_t\right). \quad (40)$$

To study the properties of the implementing reimbursement rule, we have to combine the individual's FOCs (37)–(40) with the conditions characterizing the first best, namely, (26)–(36). We will successively consider care in case of severe dependence and then for light dependence.

4.1 Severe dependence

Combining (33) and (40) yields $\partial R_{t,L}/\partial d_t = 0$ for all $t = 0, 1, 2$. Consequently, insurance coverage of care in case of severe dependence involves a flat payment. There is no marginal subsidy, so that the level of care is not distorted. In practice this means that it can depend on the severity of dependence but not on actual expenditures.

4.2 Light dependence

4.2.1 Period 0

In case of light dependence in period 0 individuals consume care services of s_0 which affects the shares of dependent individuals in the subsequent periods. This affects their own probability of dependence and thus their expected utility. This is spontaneously taken into account when individuals choose their s_0. However, it also affects the insurers budget constraint and this effect is ignored by individuals who take the insurance contract as given. This explains that the reimbursement scheme must be designed to correct for this bias in the individual's choice. Consequently, a simple flat payment will no longer be sufficient.

To study how s_0 should be reimbursed we start by combining (34) with

(37) which yields

$$\frac{\pi_{0,L}\frac{\partial R_{0,L}(s_0)}{\partial s_0}u_c\left(c_{0,L},\theta_L - s_0\right)}{\mu}$$
$$= -\frac{\partial \pi_{2,D}\left(s_0\right)}{\partial s_0}\mathcal{P}_r - \frac{\partial \pi_{1,L}\left(s_0\right)}{\partial s_0}R_{1,L}$$
$$- \frac{\partial \pi_{1,H}\left(s_0\right)}{\partial s_0}R_{1,H} - \frac{\partial \pi_{2,L}\left(s_0,s_1\right)}{\partial s_0}R_{2,L} - \frac{\partial \pi_{2,H}\left(s_0,s_1\right)}{\partial s_0}R_{2,H}. \quad (41)$$

Equation (41) shows that $\partial R_{0,L}\left(s_0\right)/\partial s_0$ has the same sign as the (right-hand-side) RHS of this equation. We show in Appendix B that this RHS can be rearranged to obtain

$$- \frac{\partial \pi_{2,D}\left(s_0\right)}{\partial s_0} - \frac{\partial \pi_{1,L}\left(s_0\right)}{\partial s_0}R_{1,L} - \frac{\partial \pi_{1,H}\left(s_0\right)}{\partial s_0}R_{1,H}$$
$$- \frac{\partial \pi_{2,L}\left(s_0,s_1\right)}{\partial s_0}R_{2,L} - \frac{\partial \pi_{2,H}\left(s_0,s_1\right)}{\partial s_0}R_{2,H} =$$
$$\frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}\pi_{0,L}\left(R_{1,H} - R_{1,L}\right) + \frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}\pi_{0,L}\left(p_{2,D}\left(H\right) - p_{2,D}\left(L\right)\right)\mathcal{P}_r$$
$$+ \frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}\pi_{0,L}\left(p_{2,H}\left(H\right) - p_{2,H}\left(L,s_1\right)\right)R_{2,H}$$
$$+ \frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}\pi_{0,L}\left(p_{2,D}\left(L\right) - p_{2,D}\left(H\right)\right)R_{2,H}. \quad (42)$$

As long as $R_{t,H} > R_{t,L}$ all terms of this expression are positive except for the last one. Indeed one can expect that $p_{2,D}\left(H\right) \geq p_{2,D}\left(L\right)$ which means that the mortality rate of severely dependent persons is at least as large as that of lightly dependent.[5] Consequently we have $\partial R_{0,L}\left(s_0\right)/\partial s_0 > 0$ so that home care in period 0 must be subsidized as long as $\left(p_{2,D}\left(H\right) - p_{2,D}\left(L\right)\right)$ is not too large.

Intuitively the different effects can be explained as follows. Recall that while individuals anticipate the direct impact of s_0 on their expected utility, they do not take into account the impact on the insurers budget constraint.

[5] See expression (11).

When $(p_{2,D}(H) - p_{2,D}(L))$ is not too large this effect is positive and to achieve the appropriate level of s_0 a subsidy is required.

Let us have a closer look at the different terms of (42) which represent the relevant effects. A first effect $\pi_{0,L}(R_{1,H} - R_{1,L})\,\partial p_{1,L}(L, s_0)/\partial s_0$ takes place in period 1. Increasing s_0 increases the share of individuals of type L and decreases the share of type H in period 1 so that as long as $R_{1,H} > R_{1,L}$, this effect is positive. A second effect is a premium effect taking place in period 2: increasing s_0 increases the share of individuals of type L and decreases the one of type H in the second period so that it implies a positive effect on the resource constraint as long as $p_{2,D}(H) > p_{2,D}(L)$ that is if the mortality rate is higher among the highly dependent individual than the one among the lightly dependent. A third effect takes place in period 2: a change in expenditures $p_{2,L}(L, s_1)(R_{2,H} - R_{2,L}) > 0$. Its extent depends on the proportion of type L individuals in period one which in turn depends on s_0. These three effects go in the same direction.

A fourth effect is that because the proportion of type L in period 1 increases and that of type H decreases, mortality decreases; and with more individual alive in period 2, expenditures on the severely dependent $R_{2,H}$ increase.

To sum up and roughly speaking s_0 has mostly positive effects on the insurer's budget because it reduces the proportion of severe dependency amongst the individual who are still alive in period 2. However, it also increases the number of individuals who survive until period 2 and some of them will be severely dependent.[6]

[6]Increasing longevity increases utility but this is already accounted for in individuals' decisions. But it does have a negative impact on the insurer's budget.

4.2.2 Period 1

We now turn to s_1 which has also indirect effects but only in period 2. Consequently expressions are simpler. Combining (35) with (38) yields

$$\frac{\pi_{1,L}\frac{\partial R_{1,L}(s_1)}{\partial s_1}u_c\left(c_{1,L},\theta_L - s_1\right)}{\mu} = -\frac{\partial \pi_{2,L}\left(s_0,s_1\right)}{\partial s_1}R_{2,L} - \frac{\partial \pi_{2,H}\left(s_0,s_1\right)}{\partial s_1}R_{2,H} \tag{43}$$

Equation (43) shows that $\partial R_{1,L}\left(s_1\right)/\partial s_1$ has the same sign as the RHS of (43). Rearranging this expression we obtain

$$\begin{aligned}
&-\frac{\partial \pi_{2,L}\left(s_0,s_1\right)}{\partial s_1}R_{2,L} - \frac{\partial \pi_{2,H}\left(s_0,s_1\right)}{\partial s_1}R_{2,H} \\
&= \pi_{0,L}p_{1,L}\left(L,s_0\right)\frac{\partial p_{2,L}\left(L,s_1\right)}{\partial s_1}R_{2,H} - \pi_{0,L}p_{1,L}\left(L,s_0\right)\frac{\partial p_{2,L}\left(L,s_1\right)}{\partial s_1}R_{2,L} \\
&= \pi_{0,L}p_{1,L}\left(L,s_0\right)\frac{\partial p_{2,L}\left(L,s_1\right)}{\partial s_1}\left(R_{2,H} - R_{2,L}\right) > 0,
\end{aligned}$$

so that $\partial R_{1,L}\left(s_1\right)/\partial s_1 > 0$. In other words we obtain an unambiguous result and a subsidy on s_1 is always desirable. Intuitively, a larger s_1 increases the share of individuals of type L and decreases the share of type H in period 2. Consequently, as long as $R_{2,H} > R_{2,L}$, s_1 has a positive effect on total care reimbursements (an effect which is not spontaneously taken into account by individuals) and should be subsidized.

5 Numerical results

To illustrate our findings and to show that expression (42) has the expected sign when transition probabilities are calibrated to plausible levels.

Assume that utilities are given by

$$U_{t,G} = \log\left(1 + c_{t,G}\right)$$

$$U_{t,L} = \log\left(1 + c_{t,L}\right) + \left(s_t - \theta_L\right)^2$$

and

$$U_{t,H} = \log(1 + c_{t,H}) + (d_t - \theta_H)^2$$

for $t = 0, 1, 2$.

For the sake of calibration we assume that $t = 0$ corresponds to age 70, $t = 1$ to age 80 and $t = 2$ to age 90. Initial proportions are: $\pi_{0,G} = 0.6$, $\pi_{0,L} = 0.3$ and $\pi_{0,H} = 0.1$; see DRESS, 2010.

The transition matrices based on currently observed probabilities are taken from Fuino and Wagner (2018). We assume that these transition matrices correspond to $s_t = 0$ for $t = 0, 1, 2$.[7]

$$P_1 = \begin{pmatrix} 0.7 & 0.14 & 0.03 & 0.13 \\ 0 & 0.56 & 0.12 & 0.32 \\ 0 & 0 & 0.1 & 0.9 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{44}$$

$$P_2 = \begin{pmatrix} 0.25 & 0.25 & 0.2 & 0.3 \\ 0 & 0.5 & 0.18 & 0.32 \\ 0 & 0 & 0.68 & 0.32 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{45}$$

The probability functions are then specified as

$$p_{1,L}(L, s) = 1/(1 + \exp[\beta_{1,L} - \alpha s]) \tag{46}$$

$$p_{2,L}(L, s) = 1/(1 + \exp[\beta_{2,L} - \alpha s]), \tag{47}$$

where $\alpha > 0$ measures how formal home, s, care affects the probability p_L. The larger is α, the larger is the impact of s on the probability to stay in state L.

To calibrate $\beta_{1,L}$ and $\beta_{2,L}$ we set s to zero in (46) and (47) and solve to obtain the probability given in (44) and (45). This yields the following two

[7] This is a matter of normalization. In reality, subsidies to formal home care already exist so that the levels we have have to be understood as coming in addition to the current levels.

equations

$$p_{1,L}(L,0) = 1/(1+\exp[\beta_{1,L}]) = 0.56$$
$$p_{2,L}(L,0) = 1/(1+\exp[\beta_{2,L}]) = 0.5$$

and we obtain $\beta_{1,L} = -0.24$ and $\beta_{2,L} \approx 0$.

Substituting these values into equations (46) and (47) we obtain the expressions for $p_{1,H}(L,s)$ and $p_{2,H}(L,s)$ by using the property that in any given period the probabilities of the different states of nature must add up to one which yields

$$p_{1,H}(L,s) = 1 - p_{1,D}(L) - 1/(1+\exp[\beta_{1,L} - \alpha s]),$$
$$p_{2,H}(L,s) = 1 - p_{2,D}(L) - 1/(1+\exp[\beta_{2,L} - \alpha s]),$$

where $p_{1,D}(L)$ and $p_{2,D}(L)$ are given by (44) and (45).

Table 1 and Table 2 present allocations and marginal reimbursement rates $\partial R_{0,L}(s_0)/\partial s_0$ and $\partial R_{1,L}(s_1)/\partial s_1$ for two values of α and θ_H and with $w = 100$ and $\theta_L = 1$.

First, and foremost, these results confirm that the marginal reimbursement rate of s_1, $\partial R_{1,L}(s_1)/\partial s_1$, is positive for the considered calibration of transition probabilities. In addition, the illustration brings about a number of interesting properties. In particular, transfers to type H individuals, that is $R_{0,H}$, $R_{1,H}$ and $R_{2,H}$, as well as expenses d_0, d_1 and d_2 do not differ with respect to age. This is line with equations $(28), (30)$, (32) and (33) which state that marginal utility of consumption should be equal for everybody and that the marginal utility of consumption is equal to the marginal effect of d on utility given here by $2(\theta - d)$. Given that marginal utilities of consumption are equal to $1/(1+c)$, the marginal utility of consumption is low so that d is set very closed to θ_H. This property does not depend on

$\theta_H = 200$	$\alpha = 1.5$	$\alpha = 0.9$
P_r	15.35	22.31
$R_{0,L}$	21.38	19.74
$R_{1,L}$	21.27	19.63
$R_{2,L}$	21.09	19.45
$R_{0,H}$	220.09	218.45
$R_{1,H}$	220.09	218.45
$R_{2,H}$	220.09	218.45
d_0	199.99	199.99
d_1	199.99	199.99
d_2	199.99	199.99
s_0	1.28	1.28
s_1	1.17	1.17
s_2	0.99	0.99
$\partial R_{0,L}(s_0)/\partial s_0$	0.13	0.13
$\partial R_{1,L}(s_1)/\partial s_1$	0.07	0.07

Table 1: Allocation and marginal reimbursment rates when $\theta_H = 200$.

$\theta_H = 400$	$\alpha = 1.5$	$\alpha = 0.9$
P_r	17.25	30.12
$R_{0,L}$	21.12	18.19
$R_{1,L}$	20.96	17.98
$R_{2,L}$	20.64	17.62
$R_{0,H}$	419.64	416.62
$R_{1,H}$	419.64	416.62
$R_{2,H}$	419.64	416.62
d_0	399	399
d_1	399	399
d_2	399	399
s_0	1.47	1.56
s_1	1.30	1.35
s_2	0.99	0.99
$\partial R_{0,L}(s_0)/\partial s_0$	0.21	0.26
$\partial R_{1,L}(s_1)/\partial s_1$	0.14	0.16

Table 2: Allocation and marginal reimbursment rates when $\theta_H = 400$.

the specification of probabilities but is due to the separability of the utility functions.

Furthermore the results show that formal home care s decreases with age. This is because s_0 has a higher impact than s_1 on total welfare: s_0 has not only an impact on the probability $p_{1,L}$ but also on $p_{2,L}$. As a consequence, the transfers R to type L are decreasing with age so that marginal utility of consumption in state L is the same in all periods. Since s_2 has no impact, it is set such that the marginal utility of consumption is the same in all states which again implies that s_2 is very close to θ_L.

Finally, the parameter α does not appear to have a very significant impact on the marginal subsidies and on s_0 and s_1. We present the results only for two values but simulations with different values have produced similar results.

6 Concluding comments

We have studied the design of optimal (private and/or social) insurance schemes for formal home care and nursery home care. Our results point out a potential new role for postal operators as providers of formal home care and insurers. We have illustrated the role of formal home care which has both direct effect (on utility) and indirect effects (on insurers budget constraint via transition probabilities). More precisely, formal home care provided to lightly dependent individuals reduces their risk of becoming severely dependent. We have shown that the *laissez-faire* is inefficient because individuals consume a too low level of formal home care and are not insured. The optimal insurance scheme implies a transfer to lightly dependent individuals that (under some conditions) increases with the amount of formal home care consumed. Severely dependent individuals, on the other hand, must receive a flat transfer. The theoretical analysis is illustrated by a calibrated numerical example which has shown that the expressions have the expected signs under plausible conditions.

Our model is simple and can be extended in several ways. First, one could consider more periods and possibly an infinite horizon setting where the number of periods extends until individual's death. This would complicate the expressions but not change the results. Second, we concentrate on full information setting. In particular, severity of dependency is observable; this is a quite standard assumption. A more restrictive assumption is that we have not considered *ex post* moral hazard. This can only reinforce results for institutional care (see Cremer *et al.* 2016) but may mitigate subsidy on formal home care. Last but not least, we have also ignored *ex ante* heterogeneity which would introduce income redistribution. This would significantly complicate the analysis and one would have to consider the interaction with income taxation. However, overall, this is likely to reinforce the case for insurance coverage as described.

References

[1] Barczyk D. and M. Kredler, (2018), "Evaluating long-term-care policy options, taking the family seriously", *Review of Economic Studies*, 85, 766–809. 93(6), 1045–1076.

[2] Barigozzi, F., H. Cremer and K. Roeder (2020) "Caregivers in the Family: Daughters, Sons and Social Norms", *European Economic Review*, **130,** Article 103589.

[3] Bonsang, E. and J. Schoenmaeckers, (2015), "Long-term care insurance and the family: does the availability of potential caregivers substitute

for long-term care insurance?", In: Börsch-Supan, A., T. Kneip and H. Litwin (Eds.), *Ageing in Europe - Supporting Policies for an Inclusive Society*, De Gruyter, Chapter 33.

[4] Cremer, H. and P. Pestieau, (2014), "Social long-term care insurance and redistribution", International tax and Public Finance, 21, 955–974.

[5] Cremer, H., P. Pestieau and G. Ponthières, (2012), "The economics of Long-Term-Care: A survey", *Nordic Economic Policy Review*, **2**, 107–148.

[6] Cremer, H., Gahvari, F. and P. Pestieau, (2017), "Uncertain altruism and the provision of long term care", *Journal of Public Economics*, **151**, 12–24.

[7] Cremer, H., Lozachmeur, J.-M. and P. Pestieau, (2016). "The design of long-term care insurance contracts", *Journal of Health Economics*, **50**, 330–339.

[8] Cremer, H. and K. Roeder, (2017), "Social insurance with competitive insurance markets and risk misperception", *Journal of Public Economics*, **146**, 138–147.

[9] DRESS (2010), "Une approche de l'autonomie chez les adultes et les personnes âgées, *Etudes et Résultats, **718**.*

[10] Drèze, J., P. Pestieau, and E. Schokkaert, (2016), "Arrow's theorem of the deductible and long-term care insurance", *Economics Letters*, **148**, 103–105.

[11] Klimaviciute, J., (2017), “Long-term care insurance and intra-family moral hazard: Fixed vs proportional insurance benefits”, *The Geneva Risk and Insurance Review*, **42**, 87–116.

[12] Klimaviciute, J. and P. Pestieau, (2023), “The economics of long-term care: An overview”, *Journal of Economic Surveys*, **37**, 1192–1213.

[13] Fuino, M. and J Wagner, (2018), Long-term care models and dependence probability tables by acuity level: New empirical evidence from Switzerland, *Insurance: Mathematics and Economics*, **81**, 51-70.

Appendix

A Impact of s_0 and s_1 on the shares of the different types

Differentiating (14), (15), (20), (18) and (19) with respect to s_0 and s_1 and using

$$\partial p_{1,L}(L, s_0)/\partial s_0 + \partial p_{1,H}(L, s_0)/\partial s_0 = 0,$$
$$\partial p_{2,L}(L, s_1)/\partial s_1 + \partial p_{2,H}(L, s_1)/\partial s_1 = 0,$$

yields

$$\frac{\partial \pi_{1,L}(s_0)}{\partial s_0} = \pi_{0,L} \frac{\partial p_{1,L}(L, s_0)}{\partial s_0} > 0 \tag{A1}$$

$$\frac{\partial \pi_{1,H}(s_0)}{\partial s_0} = \pi_{0,L} \frac{\partial p_{1,H}(L, s_0)}{\partial s_0} < 0 \tag{A2}$$

$$\begin{aligned}\frac{\partial \pi_{2,L}(s_0, s_1)}{\partial s_0} &= \frac{\partial \pi_{1,L}(s_0)}{\partial s_0} p_{2,L}(L, s_1) \\ &= \pi_{0,L} p_{2,L}(L, s_1) \frac{\partial p_{1,L}(L, s_0)}{\partial s_0} > 0 \end{aligned} \tag{A3}$$

$$\begin{aligned}\frac{\partial \pi_{2,H}(s_0, s_1)}{\partial s_0} &= \frac{\partial \pi_{1,L}(s_0)}{\partial s_0} p_{2,H}(L, s_1) + \frac{\partial \pi_{1,H}(s_0)}{\partial s_0} p_{2,H}(H) \\ &= \pi_{0,L} p_{2,H}(L, s_1) \frac{\partial p_{1,L}(L, s_0)}{\partial s_0} + \pi_{0,L} p_{2,H}(H) \frac{\partial p_{1,H}(L, s_0)}{\partial s_0} \\ &= \pi_{0,L} \frac{\partial p_{1,L}(L, s_0)}{\partial s_0} [p_{2,H}(L, s_1) - p_{2,H}(H)] < 0 \end{aligned} \tag{A4}$$

$$\begin{aligned}\frac{\partial \pi_{2,D}(s_0, s_1)}{\partial s_0} &= \frac{\partial \pi_{1,L}(s_0)}{\partial s_0} p_{2,D}(L) + \frac{\partial \pi_{1,H}(s_0)}{\partial s_0} p_{2,D}(H) \\ &= \pi_{0,L} p_{2,D}(L) \frac{\partial p_{1,L}(L, s_0)}{\partial s_0} + \pi_{0,L} p_{2,D}(H) \frac{\partial p_{1,H}(L, s_0)}{\partial s_0} \\ &= \pi_{0,L} \frac{\partial p_{1,L}(L, s_0)}{\partial s_0} [p_{2,D}(L) - p_{2,D}(H)] < 0 \end{aligned} \tag{A5}$$

$$\frac{\partial \pi_{2,L}(s_0, s_1)}{\partial s_1} = \pi_{1,L}(s_0) \frac{\partial p_{2,L}(L, s_1)}{\partial s_1} = \pi_{0,L} p_{1,L}(L, s_0) \frac{\partial p_{2,L}(L, s_1)}{\partial s_1} > 0 \tag{A6}$$

$$\frac{\partial \pi_{2,H}(s_0, s_1)}{\partial s_1} = \pi_{1,L}(s_0) \frac{\partial p_{2,H}(L, s_1)}{\partial s_1} = \pi_{0,L} p_{1,L}(L, s_0) \frac{\partial p_{2,H}(L, s_1)}{\partial s_1} < 0 \tag{A7}$$

B Proof of expression (42)

Using expressions (A1)–(A5) we successively obtain

$$
\begin{aligned}
&-\frac{\partial \pi_{2,D}\left(s_0\right)}{\partial s_0}\mathcal{P}_r-\frac{\partial \pi_{1,L}\left(s_0\right)}{\partial s_0}R_{1,L}-\frac{\partial \pi_{1,H}\left(s_0\right)}{\partial s_0}R_{1,H}\\
&-\frac{\partial \pi_{2,L}\left(s_0,s_1\right)}{\partial s_0}R_{2,L}-\frac{\partial \pi_{2,H}\left(s_0,s_1\right)}{\partial s_0}R_{2,H}=\\
&-\pi_{0,L}p_{2,D}\left(L\right)\frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}\mathcal{P}_r-\pi_{0,L}p_{2,D}\left(H\right)\frac{\partial p_{1,H}\left(L,s_0\right)}{\partial s_0}\mathcal{P}_r\\
&-\pi_{0,L}\frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}R_{1,L}-\pi_{0,L}\frac{\partial p_{1,H}\left(L,s_0\right)}{\partial s_0}R_{1,H}\\
&-\pi_{0,L}p_{2,L}\left(L,s_1\right)\frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}R_{2,L}-\pi_{0,L}p_{2,H}\left(L,s_1\right)\frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}R_{2,H}\\
&-\pi_{0,L}p_{2,H}\left(H\right)\frac{\partial p_{1,H}\left(L,s_0\right)}{\partial s_0}R_{2H}=\\
&\frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}\pi_{0,L}\left(R_{1,H}-R_{1,L}\right)+\frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}\pi_{0,L}\left(p_{2,D}\left(H\right)-p_{2,D}\left(L\right)\right)\mathcal{P}_r\\
&+\frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}\pi_{0,L}\left[\left(p_{2,H}\left(H\right)-p_{2,H}\left(L,s_1\right)\right)\right]R_{2H}-\frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}\pi_{0,L}p_{2,L}\left(L,s_1\right)R_{2L}
\end{aligned}
\tag{A8}
$$

The last line of this expression can be rearranged as follows

$$
\begin{aligned}
&\frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}\pi_{0,L}\left[\left(p_{2,H}\left(H\right)-p_{2,H}\left(L,s_1\right)\right)\right]R_{2H}\\
&-\frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}\pi_{0,L}p_{2,L}\left(L,s_1\right)R_{2L}\\
&=\frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}\pi_{0,L}\left[\left(1-p_{2,D}\left(H\right)-1+p_{2,D}\left(L\right)+p_{2L}\left(L,s_1\right)\right)\right]R_{2H}\\
&-\frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}\pi_{0,L}p_{2,L}\left(L,s_1\right)R_{2L}\\
&=\frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}\pi_{0,L}\left(\left[\left(-p_{2,D}\left(H\right)+p_{2,D}\left(L\right)+p_{2L}\left(L,s_1\right)\right)R_{2H}\right]-p_{2,L}\left(L,s_1\right)R_{2L}\right)\\
&=\frac{\partial p_{1,L}\left(L,s_0\right)}{\partial s_0}\pi_{0,L}\left[\left(p_{2,D}\left(L\right)-p_{2,D}\left(H\right)\right)R_{2H}+p_{2L}\left(L,s_1\right)\left(R_{2H}-R_{2L}\right)\right]
\end{aligned}
$$

Consequently, expression (A8), which corresponds to the RHS of (41) and determines the sign of $\partial R_{0,L}(s_0)/\partial s_0$ can be rewritten as

$$\begin{aligned}
&-\frac{\partial \pi_{2,D}(s_0)}{\partial s_0}-\frac{\partial \pi_{1,L}(s_0)}{\partial s_0}R_{1,L}-\frac{\partial \pi_{1,H}(s_0)}{\partial s_0}R_{1,H}\\
&-\frac{\partial \pi_{2,L}(s_0,s_1)}{\partial s_0}R_{2,L}-\frac{\partial \pi_{2,H}(s_0,s_1)}{\partial s_0}R_{2,H}=\\
&\frac{\partial p_{1,L}(L,s_0)}{\partial s_0}\pi_{0,L}(R_{1,H}-R_{1,L})+\frac{\partial p_{1,L}(L,s_0)}{\partial s_0}\pi_{0,L}(p_{2,D}(H)-p_{2,D}(L))\mathcal{P}_r\\
&+\frac{\partial p_{1,L}(L,s_0)}{\partial s_0}\pi_{0,L}\left[(p_{2,H}(H)-p_{2,H}(L,s_1))\right]R_{2,H}\\
&+\frac{\partial p_{1,L}(L,s_0)}{\partial s_0}\pi_{0,L}(p_{2,D}(L)-p_{2,D}(H))R_{2,H}.
\end{aligned}$$