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"Life Expectancy, Income and Long-Term Care: The Preston Curve Reexamined"

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Life Expectancy, Income and Long-Term Care: The Preston Curve Reexamined*

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Abstract

The Preston Curve - the increasing relation between income per capita and life expectancy - cannot be observed in countries where old-age dependency is widespread (that is, where long-term care (LTC) spending per capita is high). The absence of the Preston Curve in countries with high old-age dependency can be related to two other stylized facts: (1) the inverted-U relation between LTC spending and life expectancy; (2) the inverted-U relation between LTC spending and preventive health investments. This paper develops a two-period OLG model where survival to the old age depends on preventive health spending chosen by individuals while anticipating (fixed) old-age LTC costs. In that model, anticipated LTC costs are shown to have a non-monotonic effect on preventive health investment, thus rationalizing stylized facts (1) and (2). This framework is shown to provide an explanation for the absence of the Preston Curve in countries where old-age dependency is more acute.

Keywords: Preston Curve, life expectancy, OLG models, long-term care.

JEL classification codes: E13, E21, I15, J14.

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1 Introduction

In a seminal paper, Preston (1975) identified a major stylized fact relating demographic outcomes to economic development: the existence of an increasing relation between income per capita and life expectancy at birth. Since the relation identified by Preston is increasing and concave, this stylized fact is known as the Preston Curve. While the Preston Curve was initially identified on the basis of cross-sectional data for the 1900s, the 1930s and the 1960s, this stylized fact remains observed for more recent cross-sectional data. As an illustration, Figure 1 shows the Preston Curve for 185 countries in year 2018.[1]

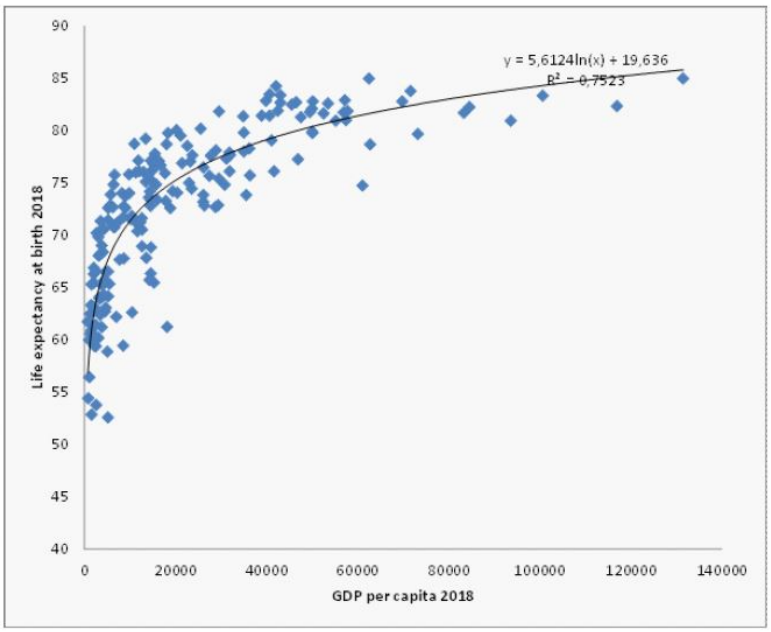


Figure 1: The Preston Curve, 185 countries, 2018.

At the theoretical level, the identification of the Preston Curve gave birth to (long-run) health macroeconomics: life expectancy was introduced in various macroeconomic models, to provide theoretical rationalizations of the increasing relation between income per capita and life expectancy. A first group of models treated survival conditions as parameters *exogenous* to the model, and examined how improvements in survival conditions can be associated to a higher income per capita, through various channels, such as rising labour productiv-

[1]Source of data: GDP per capita (in current PPP international dollars) and life expectancy at birth (men and women, in years) are from The World Bank, *World Development Indicators*, 2023, extracted at.https://databank.worldbank.org/reports. In Appendix A.1, we provide similar Preston Curves for years 1990, 2000 and 2010.

ity associated to a better health, or time horizon effects fostering physical and/or human capital accumulation.[2] A second group of models considered life expectancy as a variable *endogenously* determined within the economy, and studied the joint production of income and life expectancy.[3] Within these models, economic development contributes to improve survival conditions through various channels (e.g. increase in health spending, increase in human capital). In turn, improved survival conditions favor economic development by either favouring savings and/or education through a horizon effect.

These theoretical settings provide various rationalizations of the joint improvement of economic and longevity outcomes, and, hence, of the Preston Curve. However, these models involve an important simplification: they do not account for the possibility of *losses of autonomy at the old age*, that is, for the increasingly widespread occurrence of the long-term care (LTC) phenomenon.[4] Ignoring the LTC phenomenon simplifies the picture: the rise of old-age dependency constitutes a major fact of ageing societies, and is at the origin of substantial costs (see Norton 2000, Cremer et al 2012). The various effects of LTC costs on economic and demographic dynamics need to be examined to understand the dynamics of ageing societies. But taking LTC costs into account matters also from the perspective of understanding the Preston Curve.

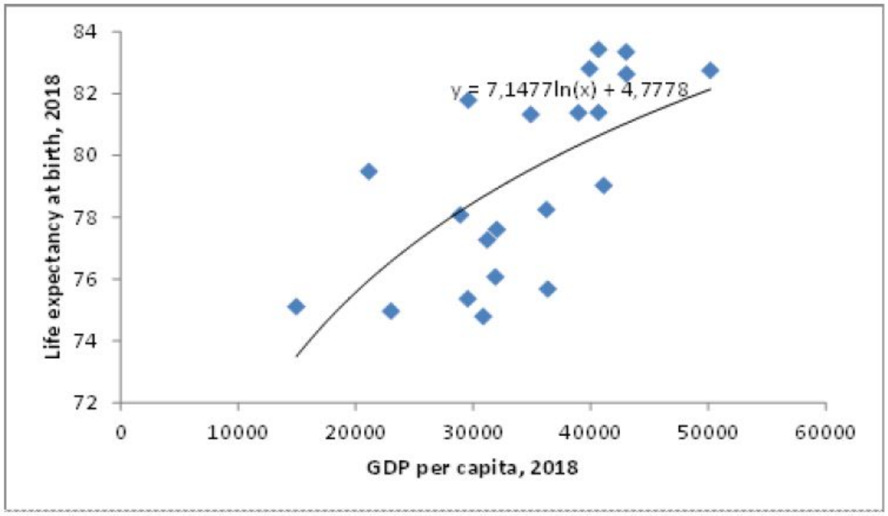
To see this, let us revisit the relationship between income per capita and life expectancy at birth across countries, by separating countries into distinct groups, depending on the prevalence of old-age dependency. For that purpose, we can measure the prevalence of old-age dependency by means of the level of LTC expenditures per capita. Focussing on a subset of the countries presented in Figure 1 - the ones for which we have data on LTC spending in 2018 -, we can divide the subset into two subsamples, based on the level of LTC expenditures per capita.[5] Figure 2a shows the relation between income per capita and life expectancy for

[2]See Ehrlich and Lui (1991), de la Croix and Licandro (1999), Pecchenino and Utendorf (1999), Zhang et al (2001), Boucekkine et al (2002) and Acemoglu and Johnson (2007). A theoretical critique was provided by Hazan and Zoabi (2006) and Hazan (2009).

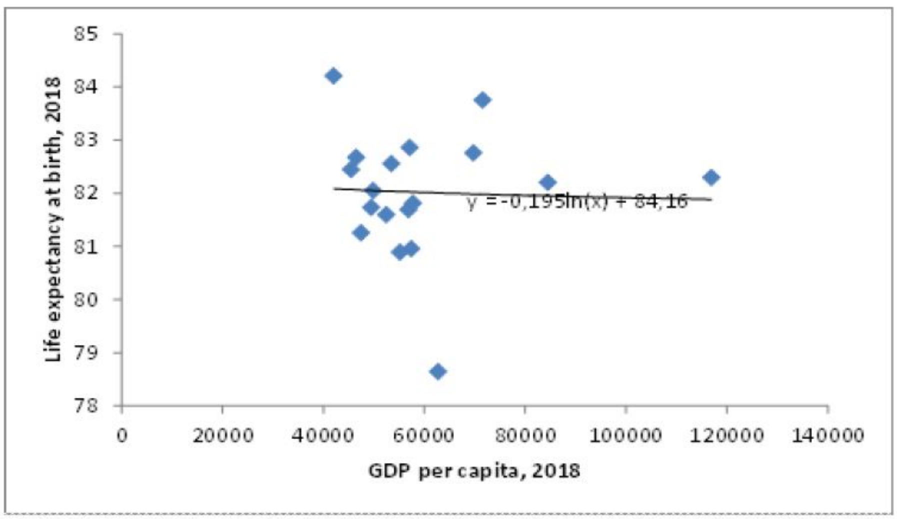
[3]See Blackburn and Cipriani (2002), Chakraborty (2004), Cervelatti and Sunde (2005, 2011), Galor and Moav (2005), Chakraborty and Das (2005), Bhattacharya and Qiao (2007), Pestieau et al (2008), Leung and Wang (2010), Ponthiere (2010), de la Croix and Licandro (2013) and Dalgaard and Strulik (2014).

[4]Exceptions include Canta et al (2016), Pestieau and Ponthiere (2016) and Leroux and Ponthiere (2020).

[5]Source of data: OECD, data on LTC spending per capita, all providers (in current PPPs, 2018). The



(a) The relation between income per capita and life expectancy at birth, for 21 countries with low LTC spending per capita, 2018.



(b) The relation between income per capita and life expectancy at birth, for 18 countries with high LTC spending per capita, 2018.

Figure 2: The Preston Curve by separating countries w.r.t. the level of LTC expenditures per capita, 2018.

countries with low LTC expenditures per capita, whereas Figure 2b carries out a similar task

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sample of 39 countries was partitioned in two groups, depending on whether LTC spending per capita is below or above the international mean of LTC spending per capita. Group 1 (low LTC spending) includes: Australia, Brazil, Bulgaria, Costa Rica, Croatia, Cyprus, Czech Republic, Estonia, Greece, Hungary, Israel, Italy, Korea, Latvia, Lithuania, Poland, Portugal, Romania, Slovak Republic, Slovenia and Spain. Group 2 (high LTC spending) includes: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Iceland, Ireland, Japan, Luxembourg, Malta, Netherlands, Norway, Sweden, Switzerland, United Kingdom and the for countries with high LTC spending per capita. Whereas the Preston Curve still appears on Figure 2a, the same is not true on Figure 2b: countries that exhibit a sufficiently high level of LTC expenditures per capita are not characterized by an increasing relation between income per capita and life expectancy. *The Preston Curve disappears when one considers countries with a high prevalence of old-age dependency.*

The absence of the Preston Curve for countries with a high LTC prevalence constitutes a stylized fact on its own. This stylized fact is somewhat surprising, and hard to explain: why is it the case that the prevalence of old-age dependency, if sufficiently high, makes the Preston Curve disappear?

At first glance, it is tempting to reply that this question admits a trivial answer: the Preston Curve is *concave*, and countries with a high LTC prevalence exhibit, in general, high income per capita and high life expectancy, so that one can expect that the relation between income per capita and life expectancy is *weaker* for advanced economies. But although intuitive, this explanation is not satisfactory, because of two reasons. First, according to Appendix A.2, if one partitions the entire set of countries of Figure 1 on the basis of income per capita, it is still the case that there is an increasing and concave relation between income per capita and life expectancy in the different groups. Thus our stylized fact about the absence of the Preston Curve under a high LTC prevalence is not reducible to the standard distinction between intermediate and advanced economies. Second, there is a substantial difference between a *weaker* (increasing) relation between income per capita and life expectancy - as suggested by this first explanation - and *a flat or (slightly) decreasing relation* (the stylized fact of Figure 2b). Thus evoking the concavity of the Preston Curve does not do the job of explaining why the Preston Curve vanishes in countries with a high LTC prevalence.

The goal of this paper is to develop a model that can explain why the Preston curve disappears for countries with a high prevalence of old-age dependency. For that purpose, we study a dynamic OLG model where survival to the old age depends on the level of preventive health expenditures chosen by individuals while anticipating LTC costs at the old age. We then use that model to revisit the relationship between income per capita and life expectancy,

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United States.

and to examine its robustness to varying the prevalence of old-age dependency.

Prior to our theoretical explorations, two other - related - stylized facts are worth being highlighted: (1) the inverted-U relation between LTC spending per capita and life expectancy at birth; (2) the inverted-U relation between LTC spending per capita and preventive health expenditures per capita.

Stylized fact (1) is illustrated on Figure 3, which plots 39 countries in the space (LTC spending per capita, life expectancy at birth).[6]

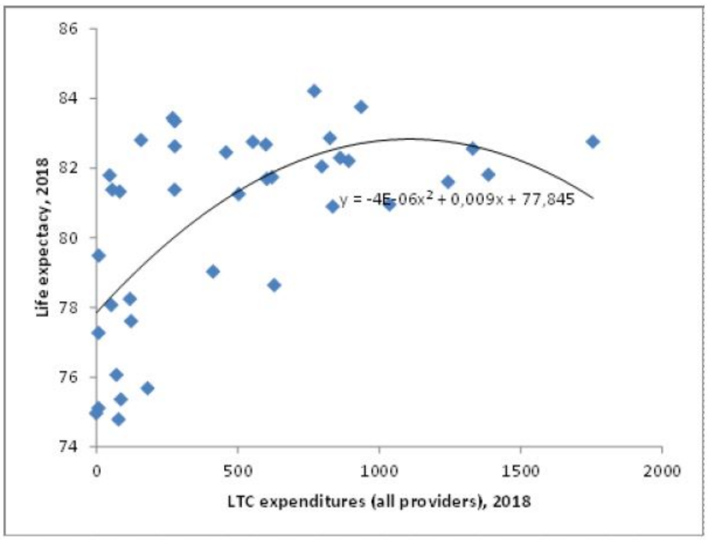


Figure 3: The relation between LTC expenditures per capita (all providers) and life expectancy at birth, 39 countries, 2018.

As Figure 3 reveals, there exists a non-monotonic relation between LTC spending per capita and life expectancy: for economies where LTC costs are low, a rise in LTC costs is associated with a higher life expectancy. However, beyond some of level of LTC spending, more LTC expenditures per capita are associated with a lower life expectancy. Regarding stylized fact (2), the inverted-U relation between LTC spending and preventive health expenditures is illustrated on Figure 4, which plots 39 countries in the space (LTC spending per capita, preventive health spending per capita).[7] In economies where LTC costs are low, a

[6]The sample includes 39 countries plotted on Figures 2a and 2b. Data for LTC expenditures per capita (all providers) in 2018 are from OECD, and expressed in current PPPs. Data for life expectancy at birth, men and women, 2018 are from the World Bank, World Development Indicators.

[7]The sample includes 39 countries plotted on Figures 2a and 2b. Data for LTC expenditures per capita (all providers) in 2018 are from OECD, and expressed in current PPPs. Data for preventive health expenditures

rise in LTC costs is associated with higher preventive health expenditures. On the contrary, once a sufficiently high level of LTC costs is reached, more LTC spending is associated with lower health expenditures. Since preventive health expenditures is usually modeled as an input that raises life expectancy (Chakraborty 2004, Bhattacharjee et al 2017), Figure 4 is consistent with Figure 3.

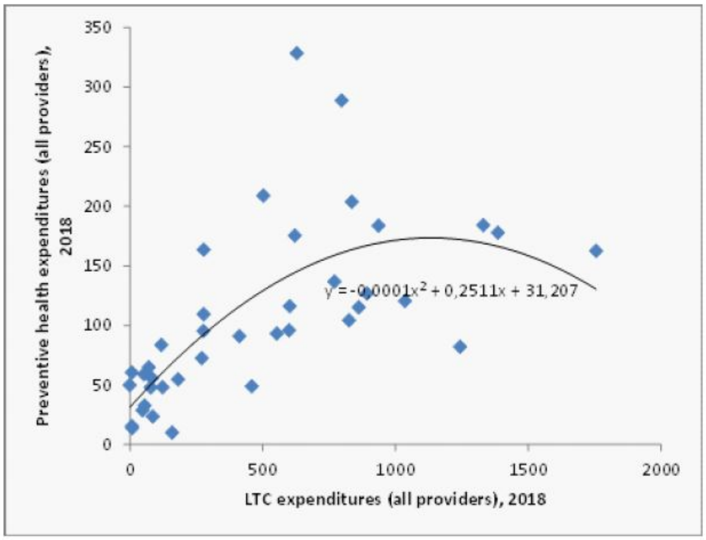


Figure 4: The relation between LTC expenditures per capita (all providers) and preventive health expenditures per capita (all providers), 39 countries, 2018.

Stylized facts (1) and (2) suggest that preventive health expenditures and life expectancy vary non-monotonically with the prevalence of old-age dependency (measured by LTC spending per capita). However, taken on their own, these stylized facts cannot explain the absence of the Preston Curve under high old-age dependency. Reexamining the Preston Curve under old-age dependency requires to connect stylized facts (1) and (2) with the overall *dynamics* of capital accumulation and income growth. This is precisely the task of this paper.

To reexamine the Preston Curve, this paper develops a dynamic OLG model with physical capital accumulation, where preventive health expenditures determining life expectancy are chosen by individuals while anticipating (fixed) old-age LTC costs. In that setting, LTC costs have an ambiguous effect on chosen health expenditures and life expectancy, due to the combination of two effects going in opposite directions. First, old-age LTC costs increase, *ceteris paribus*, the marginal utility of old-age consumption, which encourages savings. As

per capita (all providers) in 2018 are also from OECD, and expressed in current PPPs.

a consequence, by generating higher capital and income, higher LTC costs encourage higher preventive health expenditures, and, hence, yield higher life expectancy. Second, old-age LTC costs reduce, *ceteris paribus*, the marginal utility of investing in preventive health spending, because they make the old age less attractive. Hence, higher LTC costs reduce, *ceteris paribus*, preventive health spending and life expectancy. These two effects going in opposite directions, the relation between LTC costs and life expectancy is ambiguous.

Anticipating our results, we first show that, in our model, savings and preventive health investment are complements along the equilibrium path despite the direct competition for resources. Second, we also show that, *ceteris paribus*, the higher LTC costs are, the higher the savings and the lower the preventive health investment. Third, the dynamics of capital accumulation and those of preventive health investment are shown to be monotonic and to admit a limit which is finite. Interestingly, as old-age dependency favors capital accumulation, a poverty trap in the economy without old-age dependency can vanish once old-age dependency is introduced. Finally, it is shown that anticipated LTC costs have a non-monotonic effect on preventive health investment, thus rationalizing stylized facts (1) and (2). Together, the positive impact of old-age dependency on capital accumulation and the non-monotonic relation between old-age dependency and preventive health spending provide an explanation for the absence of the Preston Curve under a high LTC prevalence.

The present paper is related to the empirical literature studying the Preston Curve (Preston 1975, 1980, 2007), its robustness, its determinants and the underlying (causal) mechanisms at work.[8] With respect to that literature, the contribution of this paper lies in its identification and explanation of a new stylized facts relative to the Preston Curve: the absence of the Preston Curve for countries with a high prevalence of old-age dependency. This paper is also related to the large theoretical literature about the joint dynamics of life expectancy and economic development, which includes, among others, Chakraborty (2004), Cervelatti and Sunde (2005, 2011), Leung and Wang (2010), de la Croix and Licandro (2013) and Dalgaard and Strulik (2014). The contribution of this paper with respect to that literature consists of exploring the effect of introducing old-age dependency on the

[8]Empirical reexaminations of the Preston Curve include Anand and Ravallion (1993), Pritchett and Summers (1996), Bloom and Canning (2007), Mackenbach (2007), Dalgaard and Strulik (2014), Edwards (2016), Jetter et al (2019) and Prados de la Escosura (2023).

joint dynamics of longevity and economic outcomes. Finally, this work is also related to the more recent literature on capital accumulation and LTC, which includes Canta et al (2016), Pestieau and Ponthiere (2016) and Leroux and Ponthiere (2020). Our contribution with respect to these papers lies in our emphasis on interactions between LTC costs and life expectancy outcomes.

The remaining of the paper is organized as follows. Section 2 presents the model. The intertemporal equilibrium is studied in Section 3. Section 4 provides some numerical explorations, and shows how the model can rationalize the absence of the Preston Curve under a high old-age dependency. Concluding remarks are left to Section 5.

2 The economy

We consider a two-period overlapping generations model, where time t goes from 0 to $+\infty$. All agents (individuals and firms) are price-takers, and all markets are competitive.

Without loss of generality, the size of birth cohorts is assumed to be constant.

The life cycle Period 1 of life is young adulthood, during which individuals work, save, consume and invest some resources in their health.[9] A young adult at time t receives a wage w_t, consumes an amount c_t, invests x_t on preventive health expenditures and saves an amount s_t for the old days.

Period 2 of life is old adulthood, during which individuals do not work, and only consume d_{t+1}. Period 2 (the old age) is reached with a probability $\xi(x_t)$. As usual (see, e.g., Bhattacharjee et al, 2017), the longevity function $\xi(\cdot)$ is increasing and concave in the health expenditure input. We assume that $\xi(x_t) \in [0, 1]$ is continuous function such that $\xi'(x_t) > 0$, $\xi''(x_t) < 0$, $\lim_{x_t \to 0} \xi'(x_t) = +\infty$ and $\lim_{x_t \to +\infty} \xi'(x_t) = 0$.

During the old age, the individual faces a positive long-term care (LTC) cost ρ. The parameter ρ can be interpreted as a tractable way of including old-age dependency in our model. The old-age dependency parameter ρ captures, within our model, the cost of LTC, that is, the cost of dealing with the loss of autonomy at the old age.[10] This parameter is

[9] There is an implicit childhood period, which is not studied here.

[10] Note that, if the preference of the individual is a quasi-linear utility function, our model is also equivalent

taken as given by the individual, and is supposed to be constant and perfectly anticipated by the individual.

Production On the production side, production occurs according to a constant returns to scale technology using two inputs, capital K_t and labor L_t:

$$Y_t = F(K_t, L_t) \tag{1}$$

where Y_t is the total output, and $F(\cdot)$ is increasing and concave in its inputs, and homogenous of degree 1.

Thanks to constant returns to scale, output per worker can be written as:

$$y_t = f(k_t) \tag{2}$$

where $k_t = K_t/L_t$ is the capital stock per worker and $f(\cdot)$ is increasing.

Assumption 1. $\forall k > 0,\ f'(k) + kf''(k) > 0.$

This standard assumption is satisfied, for example, by a Cobb-Douglas or a CES production function and assumes that the total return of capital $kf'(k)$ increases as the amount of capital in production increases.

Capital fully depreciates after one period.

Preferences Each individual has preferences taking the Von Neumann Morgenstern form. Normalizing the utility of being dead to 0, each individual maximizes her expected lifetime utility, which takes the form:

$$U(c_t, x_t, d_{t+1}) \equiv u(c_t) + \xi(x_t)v(d_{t+1}) \tag{3}$$

where $u(z)$ and $v(z)$ are continuous functions such that $v(z) > 0$, $u'(z) > 0$, $v'(z) > 0$, $u''(z) < 0$, $v''(z) < 0$, $\lim_{z\to 0} u'(z) = \lim_{z\to 0} v'(z) = +\infty$ and $\lim_{z\to +\infty} u'(z) = \lim_{z\to +\infty} v'(z) = 0$.[11]

to that of an agent having in the second period a probability π of being dependent (and having to finance her LTC costs θ) and a probability $1 - \pi$ of not being dependent, with $\rho = \pi\theta$.

[11] If the utility from consumption in the second period can be negative and the utility in the event of death is normalized to zero, agents then could prefer death to surviving. Assuming $v(z) > 0$ rules out that case, which is not relevant for contemporary economies.

Markets As markets are perfectly competitive, each production factor is paid its marginal product, i.e.:

$$w_t = f(k_t) - k_t f'(k_t) \quad \text{and} \quad R_t = f'(k_t) \tag{4}$$

where w_t is the wage rate, and R_t is one plus the interest rate.

As usual in models with risky lifetime, it is assumed that there exists an annuity market that is perfectly competitive and actuarially fair. Hence the total returns from the savings of individuals who die at the end of their first period will be redistributed to the remaining survivors within the same generation. We thus have:

$$\widetilde{R}_{t+1} = \frac{R_{t+1}}{\xi(x_t)} \tag{5}$$

where $\widetilde{R}_{t+1}$ is the annuity market return on savings.

The optimal individual behavior A young adult at time t chooses consumption c_t, savings s_t and preventive health expenditures x_t so as to maximize her life-cycle utility function $U(c_t, x_t, d_{t+1})$ subject to budget constraints:

$$\max_{s_t, x_t} \quad U(c_t, x_t, d_{t+1}) \equiv u(c_t) + \xi(x_t)v(d_{t+1})$$

$$\text{s.t.} \quad w_t = c_t + x_t + s_t \tag{6}$$

$$d_{t+1} = \widetilde{R}_{t+1} s_t - \rho \tag{7}$$

Assuming that the individual takes into account the effect of her individual choices on the annuity market return $\widetilde{R}$, the maximization's problem of an agent born in t is equivalent to:

$$\max_{s_t, x_t} \phi(s_t, x_t) \equiv u(w_t - x_t - s_t) + \xi(x_t) v\left(\frac{R_{t+1} s_t}{\xi(x_t)} - \rho\right)$$

The optimal values of s_t and x_t are the solutions of the two first order conditions (FOCs):[12]

$$-u'(c_t) + R_{t+1} v'(d_{t+1}) = 0 \tag{8}$$

$$-u'(c_t) + \xi'(x_t)\left[v(d_{t+1}) - \frac{R_{t+1} s_t}{\xi(x_t)} v'(d_{t+1})\right] = 0 \tag{9}$$

[12]The FOCs correspond to $\phi_1'^{\star} = 0$ and $\phi_2'^{\star} = 0$. As $\phi_{11}''^{\star} < 0$, $\phi_{22}''^{\star} < 0$ and $\phi_{11}''^{\star}\phi_{22}''^{\star} - (\phi_{12}''^{\star})^2 > 0$, they are necessary and sufficient to characterize the maximum of $\phi(.)$.

The first FOC is a standard Euler equation. According to equation (8), the marginal rate of substitution between current and future consumption is equal to the expected return on savings. The second FOC, equation (9), captures the trade off between the marginal cost and marginal benefit of preventive health spending. The marginal welfare gain from prevention is increasing in the welfare associated to the old age (first term in brackets), whereas the marginal welfare loss from prevention includes two terms: the standard foregone consumption (first term of the FOC) and the negative effect of preventive investment on the return on savings (second term in brackets).

Merging (8) and (9), we obtain:

$$v(d_{t+1}) - R_{t+1}\left[\frac{1}{\xi'(x_t)} + \frac{s_t}{\xi(x_t)}\right]v'(d_{t+1}) = 0 \tag{10}$$

This condition provides an alternative (implicit) definition of the optimal preventive health investment, which will be used later on in our analyses.

3 Capital accumulation and preventive health investment

Given the full depreciation of capital after one period and the constancy of cohort sizes, the capital to labor ratio at time $t+1$ is equal to the savings per worker at period t, i.e.:

$$k_{t+1} = s_t \tag{11}$$

Our first proposition indicates the relationship between preventive health investment and individual savings at the equilibrium. Intuitively, preventive health investment gives rise to two opposing effects on savings. On the one hand, the resources that are used to invest in health are no longer available for savings. On the other hand, the preventive health investment allows an increase in longevity that encourages more savings.

As this is shown in Appendix B, there exist two functions $\varphi(\cdot)$ and $\mu(\cdot)$ such that $x_t^\star = \varphi(s_t^\star, \rho)$ and $s_t^\star = \mu(x_t^\star, \rho)$ at the equilibrium. Using the properties of these functions, we can establish the following proposition:

Proposition 1 *Savings and preventive health investment are complements along the equilibrium path despite the direct competition for resources. The higher the old-age dependency* *parameter ρ, the higher the savings and the lower the preventive health investment.*

Proof. See Appendix B. ■

Proposition 1 shows that Proposition 1 of Leung and Wang (2010) remains valid in a framework including old-age dependency: "for the agents' part, there are two motives to save more when health investment increases: on the one hand, a higher health investment raises his life expectancy, so that he has more incentive to save for the old-age income. On the other hand, as health investment becomes higher, its marginal benefit diminishes and saving becomes a more attractive alternative".

Moreover, Proposition 1 allows to capture the impact of anticipated LTC costs on individual decisions. *Ceteris paribus*, the larger the anticipated old-age dependency is (i.e., the higher ρ is), the more the individual will save to bear the cost of this dependency and the less she will invest in health to increase the probability to live the old age.

In order to examine the role of preventive health investment in the capital accumulation more closely, we now focus on cases such that $\phi_{12}^{''\star}$ is non positive.

Assumption 2. At the equilibrium, $\phi_{12}^{''\star} = u''(\cdot) - \frac{sR^2\xi'(\cdot)}{\xi^2(\cdot)}v''(\cdot) \leq 0$.

This standard technical assumption (which will be satisfied in our numerical example where the production function is Cobb-Douglas and utility functions are CES functions) is a sufficient condition for the dynamics in our model to be well defined.[13] Indeed, according to Appendix C, there exist two functions $\psi(\cdot)$ and $\chi(\cdot)$ such that $k_{t+1}^\star = \psi(k_t^\star, \rho)$ and $x_{t+1}^\star = \chi(x_t^\star, \rho)$ at the equilibrium. Using the properties of these functions we can establish the following proposition:

Proposition 2 *The dynamics of capital accumulation and those of preventive health investment are monotonic and admit a limit which is finite. The higher the old-age dependency parameter ρ is, the higher the capital accumulation is.*

Proof. See Appendix C. ■

[13]In the standard Diamond (1965)'s model (framework without LTC cost and with probability one to live to the old age), the relevance of this assumption has been underlined by de la Croix and Michel (2002). They also exhibit conditions under which this assumption is satisfied.

According to initial capital stock k_0, the dynamics of accumulation is monotonic: either increasing or decreasing. It is the same for dynamics of preventive health investment. As dynamics are monotonic, they converge either to $+\infty$, or to zero, or to a finite positive limit which is a positive long-run equilibrium. As usual in OLG model (see, e.g., Jones and Manuelli 1992), we have shown that an infinite limit of the capital accumulation is excluded. In addition, it may exist a poverty trap, i.e., a dynamics of capital accumulation which converges to zero. Thibault (2004) established that the presence of at least one individual who loves its children, whatever the intensity of this love, is sufficient to avoid global contraction of the economy. However, such contraction and poverty traps remain possible in a world without altruism (see, e.g., section 4 of Galor and Ryder 1989). Interestingly, as the introduction of old-age dependency (or the increase of the old-age dependency parameter ρ) allows to increase capital accumulation, a poverty trap in the economy without old-age dependency can vanish once old-age dependency prevails. This important point is graphically illustrated by Figure 5.

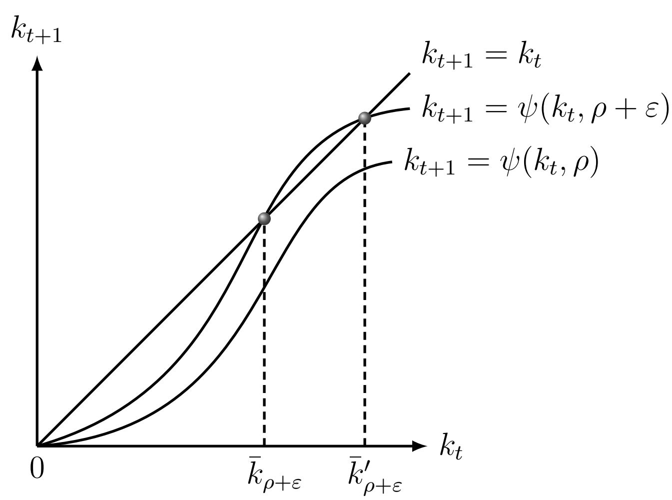


Figure 5: Capital accumulation according to ρ.

On Figure 5, when the old-age dependency parameter is ρ, the dynamics of capital accumulation has no non-trivial stationary equilibrium and, whatever $k_0 > 0$, they converge to $\bar{k} = 0$. An increase of LTC costs from ρ to $\rho + \varepsilon$ implies that the dynamics of capital accumulation has two non-trivial stationary equilibria: one not (locally) stable, denoted $\bar{k}_{\rho+\varepsilon}$, and one locally stable, denoted $\bar{k}'_{\rho+\varepsilon}$. Then, for all $k_0 > \bar{k}_{\rho+\varepsilon}$, the increase of LTC costs ρ allows to avoid a global contraction of the economy.

Following the methodology of Galor and Ryder (1989) or Wendner (2004), we can establish necessary conditions to preclude global contraction[14] or/and sufficient conditions for the existence of a unique (and globally stable non-trivial) stationary equilibrium.[15]

Whereas the impact of old-age dependency on capital accumulation is clear, its effect on preventive health investment is ambiguous. To better understand the economic mechanisms at play, we now focus on the impact of the old-age dependency parameter ρ on a (locally) stable long-run equilibrium. From a non-trivial long-run capital stock $\bar{k}$, we can define $\bar{x} \equiv \varphi(\bar{k}, \rho)$, $\bar{c} \equiv f(\bar{k}) - \bar{k}f'(\bar{k}) - \bar{k} - \bar{x}$ and $\bar{d} \equiv \bar{k}f'(\bar{k})/\xi(\bar{x}) - \rho$. Then, we can establish the following proposition:

Proposition 3 *Let $\bar{x} \equiv \varphi(\bar{k}, \rho)$ where $\bar{k}$ is a (locally) stable long-run capital stock. Then:*

(a) $\partial\bar{k}/\partial\rho$ is positive.

(b) $\partial\bar{x}/\partial\rho$ has the sign of

$$f''(\bar{k})v'(\bar{d}) + \frac{f'(\bar{k})^2 v''(\bar{d})}{\xi(\bar{x})} + (1 + \bar{k}f''(\bar{k}))\Big[1 - \frac{v(\bar{d})v''(\bar{d})}{v'(\bar{d})^2}\Big]u''(\bar{c}) \tag{12}$$

Proof. See Appendix D. ■

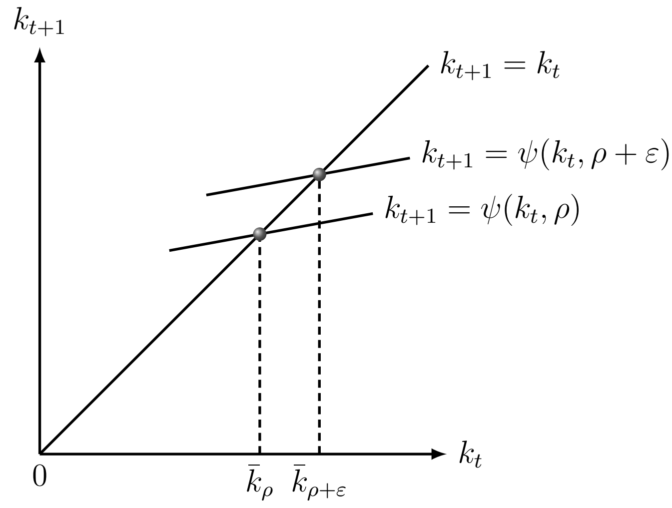
Point (a) is a direct consequence of the fact that the higher the old-age dependency parameter ρ is, the higher is the capital accumulation. As dynamic efficiency plays a critical role in the analysis of fiscal policies, it is useful to understand how the stationary capital stock evolves when dependency costs change.[16] Graphically, it is easy to observe (see Figure

[14]For example, the strengthened Inada condition (i.e., $\lim_{k\to 0} -kf''(k) > 1$) is necessary, ceteris paribus, to the existence of a non-trivial long-run capital stock $\bar{k}$.

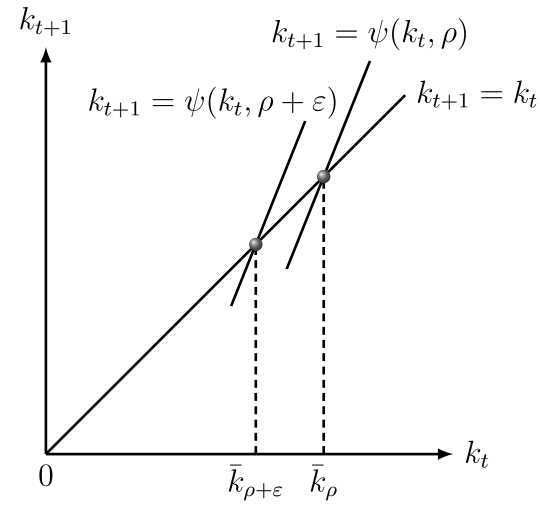
[15]For example, the condition which requires that the labor share is strictly positive close to the origin (i.e., $\lim_{k\to 0} kf'(k)/f(k) < 1$) and the condition which requires that the elasticity of substitution between capital and labor is large enough (i.e., $-f'(k)[f(k) - kf'(k)]/[kf(k)f''(k)] > 1 - k/f(k)$) are, ceteris paribus, sufficient to guarantee the existence of a unique and global non-trivial long-run capital stock $\bar{k}$ if preferences are homothetic.

[16]Diamond (1965) showed that an OLG economy can reach a long-run equilibrium with capital over-accumulation relative to the Golden Rule introduced by Phelps (1961). Such an economy is said to be dynamically inefficient since a Pareto-improvement can be achieved by allowing the current generation to devour a portion of the capital stock and leaving the consumption of all future generations intact. This result, further improved by Galor and Ryder (1991), has motivated a large body of research (see, e.g., Thibault 2008, 2016). Here, it is crucial to note that endogenizing health investment and longevity does not leave

6) that the capital stock $\bar{k}$ decreases with respect to ρ when it is not (locally) stable (i.e., when $\psi_1'(\bar{k}, \rho) > 1$) while it increases when it is (locally) stable (i.e., $\psi_1'(\bar{k}, \rho) < 1$).



a – Case where $\bar{k}$ is (locally) stable.



b – Case where $\bar{k}$ is not (locally) stable.

Figure 6: Capital accumulation at the neighborhood of $\bar{k}$ according to ρ.

To understand point (b), we can distinguish the direct effect of an increase of the old-age dependency ρ from its indirect effects. The direct effect is clear and is mentioned Proposition 1: the higher ρ is, the lower an agent will invest in preventive health spending to increase the probability $\xi(x)$ to live a dependency period. The indirect effects are related to the fact that an increase of ρ modifies the prices w and R by increasing the capital stock k. According to Proposition 1, savings and health investment are complements. Then, using (11), an increase of k can lead to an increase of x. This first indirect effect is due to the change of the disposable income Rs in the second period.[17] A second indirect effect exists and is due to the change of the income $w - s$ available in the first period to consume c and invest x. The sign of this indirect effect is then crucially related to the one of $\partial(w - s)/\partial k$,

the optimal capital accumulation unchanged. Importantly, de la Croix and Ponthiere (2010) show that the capital per worker maximizing long-run consumption per head is inferior to the standard Golden Rule when the only role of health is to enhance longevity.

[17]Indeed, Proposition 1 is based on Equation (10), which only depends on R, s, d and x. According to Assumption 1, $\partial Rs/\partial k$ is positive. Then, given x, disposable income Rs increases when k increases. Given k, according to (9), $\partial \xi(x)v(d(x))/\partial x$ is positive. Consequently, using (9) and (10), an increase in k lead to an increase in x.

i.e. to the one of $-[1+\bar{k}f''(\bar{k})]$.

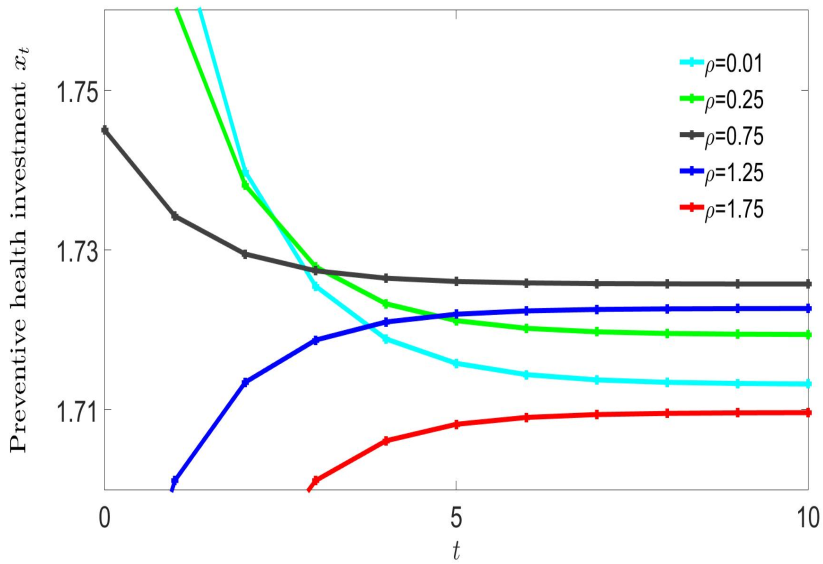
Thus, we are in the presence of a direct effect that tends to increase the long-run health investment, an indirect effect that tends to lower it and another effect whose impact depends on the sign of $1+\bar{k}f''(\bar{k})$. Importantly, when $1+\bar{k}f''(\bar{k})$ is positive, $\partial(c+x)/\partial k$ is negative and the latter indirect effect is the most powerful: as the three terms of (12) are negative, we have that an increase in ρ implies a decrease in x. Consequently, according to Assumption 1, $\partial\bar{x}/\partial\rho$ is also negative when $f'(k) \leq 1$ i.e. when $\bar{k}$ is high enough. Moreover, assuming (as in the Cobb-Douglas case) that $f''(k) + kf'''(k)$ is positive, there exists a threshold of k above which $1 + kf''(k)$ is positive. Combined with point (a), we have that the higher ρ is, the more likely it is that $\partial\bar{x}/\partial\rho$ is negative. On the other hand, it is possible for ρ low enough that $\partial\bar{x}/\partial\rho$ is positive. According to (12), a necessary condition to have such a result is that $1+\bar{k}f''(\bar{k})$ is negative. In this case, the two indirect effects lead to an increase of x, while the direct effect leads to a decrease of x. Using a numerical example, the next section will illustrate the possibility that an increase of LTC costs ρ implies an increase of long-run preventive health investment $\bar{x}$.

4 A numerical illustration

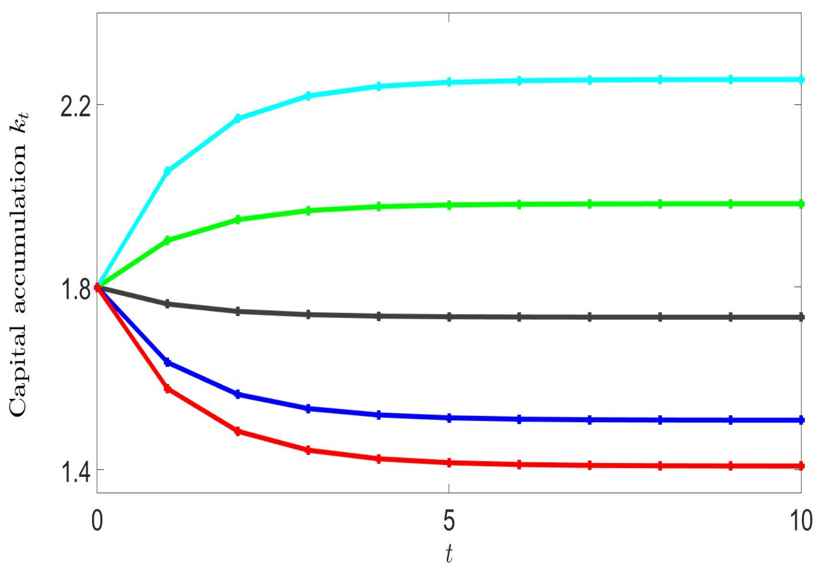
In addition to taking into account the impact of LTC costs, another contribution of our study – contrary to the OLG literature which study links between health investment, life expectancy and economic development – is to have established results on the dynamics of preventive health expenditures and capital accumulation by considering general (and not specific) functional forms for $u(\cdot)$, $v(\cdot)$, $f(\cdot)$ and $\xi(\cdot)$. To illustrate our findings we now assume the following functions:

$$\begin{cases} u(z) = v(z) = z^{1-\gamma}/(1-\gamma) \text{ with } \gamma \in (0,1). \\ f(k) = Ak^{\alpha} \text{ with } A > 0 \text{ and } \alpha \in (0,1). \\ \xi(x) = p_0 + \bar{p}\sqrt{\frac{x}{1+x}} \text{ with } p_0 \in (0,1),\, \bar{p} \in (0,1) \text{ and } p_0 + \bar{p} \leq 1. \end{cases}$$

The forms of the utility[18] and production functions are quite standard. The function $\xi(\cdot)$, used by Chen (2007) and Leung and Wang (2010), consists of two parts, which are related to two kinds of health capital respectively: the inherent health capital and the supplementary health investment. To calibrate our model, we choose the following parameters: $\gamma = 0.9$, $A = 7$, $\alpha = 0.4$, $p_0 = 0.2$, $\bar{p} = 0.7$.



(a) The dynamics of x_t.



(b) The dynamics of k_t.

Figure 7: The dynamics of capital accumulation and of preventive health investment.

[18] We assume the restriction $0 < \gamma < 1$ to avoid the agents preferring death to surviving.

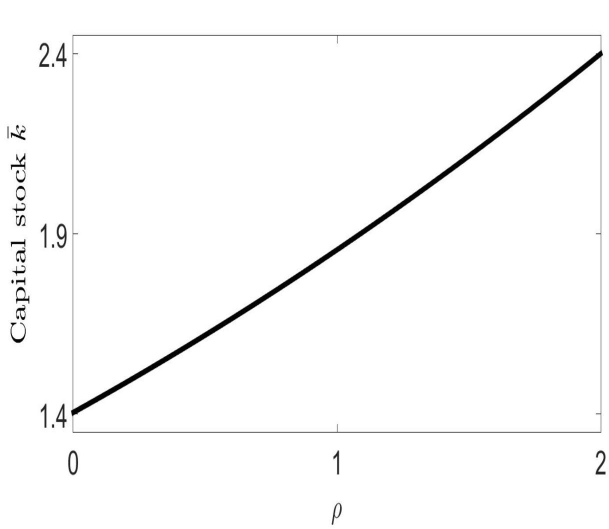
We begin to focus on the dynamics of capital accumulation and of preventive health investment according to the old-age dependency parameter ρ. For five different values of ρ (0.01, 0.25, 0.75, 1.25 and 1.75), these dynamics are computed on Figure 7 over a horizon of 10 generations from $k_0 = 1.8$. This initial value allows us to obtain both increasing and decreasing dynamics: for the three lowest values of ρ, the dynamics of capital accumulation and preventive health investment are decreasing, while they are increasing for the two largest values of ρ. In accordance with Proposition 2, the dynamics are monotonic and we have at each period that the larger the old-age dependency parameter ρ is, the larger the capital stock k_t is.

From the initial condition k_0, note that the value of x_0 obtained is decreasing with ρ. Things are therefore more complicated for the dynamics of preventive health investment, because the starting point of those which are decreasing is greater than of those that are increasing. From a certain period, the order of magnitudes of preventive health investment does not change, but this order is not monotonous with ρ.[19] Indeed, from $t \geq 5$ we have $x_t|_{\rho=1.75} < x_t|_{\rho=0.01} < x_t|_{\rho=0.25} < x_t|_{\rho=1.25} < x_t|_{\rho=0.75}$. To give an explanation to this result, Figure 8 presents the values of the long-run capital stock $\bar{k}$, the long-run preventive health investment $\bar{x}$ and the long-run life expectancy $1 + \xi(\bar{x})$ according to ρ.[20]

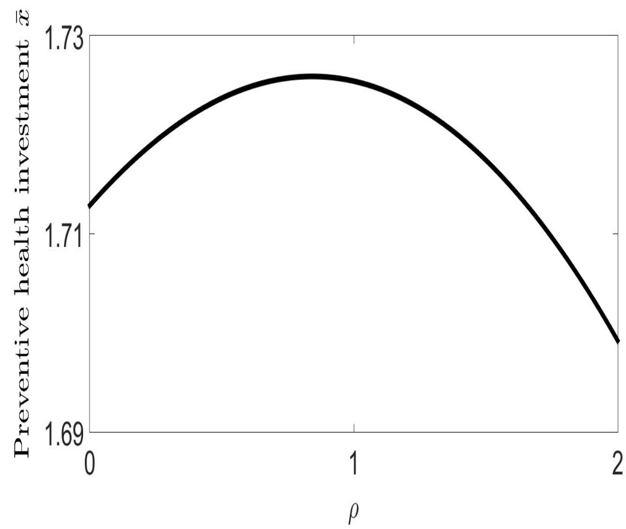
In accordance to the first part of Proposition 3, the long-run capital stock $\bar{k}$ is increasing in the old-age dependency parameter ρ (see Figure 8a). Interestingly, Figure 8b shows that the long-run preventive health investment $\bar{x}$ exhibits an inverted-U shape: it increases (resp. decreases) in the old-age dependency parameter if ρ is low enough (resp. high enough). This inverted-U relationship between LTC costs and preventive health expenditures coincides with the stylized fact (2) presented in Section 1. Our model is thus able to rationalize the existence of an inverted-U relation between LTC costs and preventive health spending as shown on Figure 4 (Section 1). Moreover, given that x is here the only input in the production of survival conditions, our model can also rationalize the stylized fact (1) presented in Section

[19] Note that, with the CRRA utility function $v(.)$, we have $1 - v(z)v''(z)/v'^2 = 1/(1-\gamma)$. Then the third term of (12) tends to ∞ when γ tend to 1. With the Cobb-Douglas function $f(.)$, $1 + kf''(k)$ increases from $-\infty$ to 1 when k increases from 0 to $+\infty$.

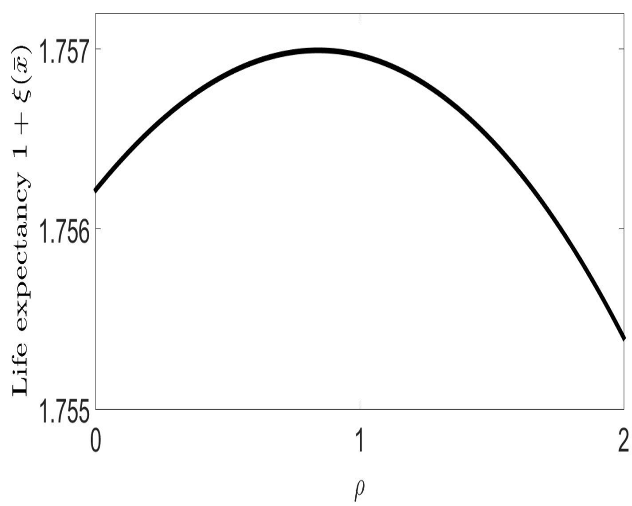
[20] In Appendix E, we draw the value of $\phi_{12}^{''\star} \equiv \phi_{12}^{''}(\bar{k}, \bar{x})$ according to ρ to show that Assumption 2 is satisfied in our example for all $\rho \in [0, 2]$.



(a) $\bar{k}$ according to ρ.



(b) $\bar{x}$ according to ρ.



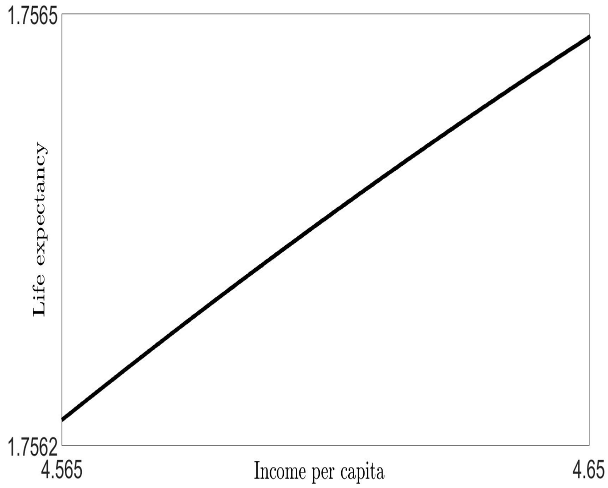
(c) $1 + \xi(\bar{x})$ according to ρ.

Figure 8: Long-run capital stock, preventive health investment and life expectancy according to ρ.

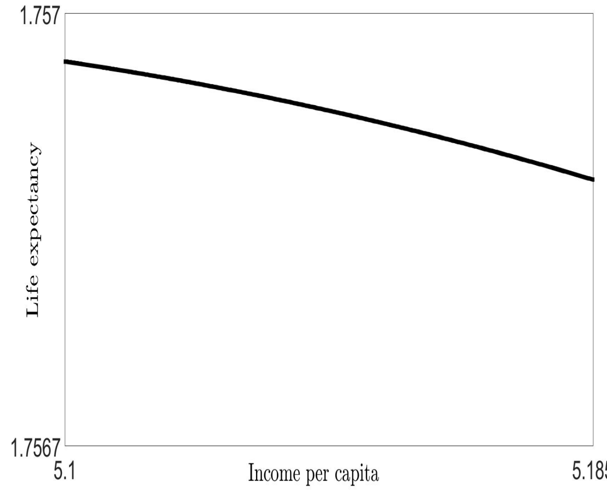
1, that is, the existence of an inverted-U relation between LTC costs and life expectancy (Figure 3).

Taken together, Figures 8a and 8c illustrate that our theoretical framework can also provide an explanation for the main stylized fact identified at the beginning of this paper: the prevalence of the Preston Curve for countries with low old-age dependency, as well as the disappearance of the Preston Curve for countries where old-age dependency is high.

To see this, note first that, based on Figure 8a and Figure 8c, we can conclude that, for countries with a low old-age dependency parameter ρ, there is an increasing relation between output per worker and life expectancy, whereas, for countries with high LTC costs, there is a decreasing relation between output per worker and life expectancy. These relations, which concern output *per worker* and not output *per capita*, do not provide the final word on the prevalence of the Preston Curve. However, since life expectancy varies along a short interval, those relations have unambiguous implications regarding the Preston Curve, as shown on Figure 9.



(a) When ρ vary from 0 to 0.165.



(b) When ρ vary from 0.995 to 1.16.

Figure 9: The simulated Preston Curve.

Countries with low old-age dependency are characterized by an increasing relation between output per capita and life expectancy, that is, by the Preston Curve (see Figure 9a). On the contrary, countries with a high prevalence of old-age dependency exhibit a decreasing relation between output per capita and life expectancy, that is, the absence of the Preston Curve (see Figure 9b). Thus this model can replicate the disappearance of the Preston Curve when old-age dependency is high, in line with the stylized fact highlighted in Section 1 (Figure 2b).

5 Concluding remarks

Identified more than 40 years ago, the Preston Curve constitutes a major stylized fact for long-run macroeconomics: this establishes nothing less than an increasing relation between economic prosperity (measured by income per capita) and longevity outcomes (measured by life expectancy at birth).

While the Preston Curve was largely reexamined during the last decades, both at the theoretical and the empirical levels, this paper focused on a particular aspect: its robustness once one distinguishes countries on the basis of the prevalence of old-age dependency. Adopting that particular perspective was motivated by the fact that old-age dependency becomes an increasingly acute problem for ageing societies, at the origin of large LTC costs. Partitioning the sample of countries based on the prevalence of old-age dependency (measured by LTC spending per capita) yielded a somewhat negative result: the Preston Curve does not hold for countries with a high old-age dependency.

This paper proposed to develop a simple dynamic model that can account for this stylized fact. That model is extremely parsimonious: it involves only two ages of life (active and inactive), one input in the survival process (preventive health spending) and one old-age dependency parameter (capturing LTC costs). The model is also general because it assumes general functional forms for utility functions and production functions. But despite its parsimony and its generality, our model can provide a rationalization of the absence of the Preston Curve for countries with high old-age dependency. The intuition goes as follows. In our model, while a higher old-age dependency necessarily favours capital accumulation and output, this has a non-monotonic effect on preventive health investment and life expectancy. As a consequence, our analysis can explain that, beyond some degree of old-age dependency, the Preston Curve does no longer prevail, in line with the data.

To conclude, it should be stressed here that this paper provides only one possible explanation for this stylized fact, whereas many other explanations are possible. In particular, one may want to develop alternative, possibly less parsimonious, models, including, for instance, endogenous LTC spending (through living arrangement choices, nursing home choices, etc.). Of course this alternative path could also be most relevant for the issue at stake, and could deliver other interesting analyses, but it is important to emphasize that, from the mere perspective of explaining why the Preston Curve disappears in high old-age dependency countries, a simpler model with fixed LTC costs can do the job.

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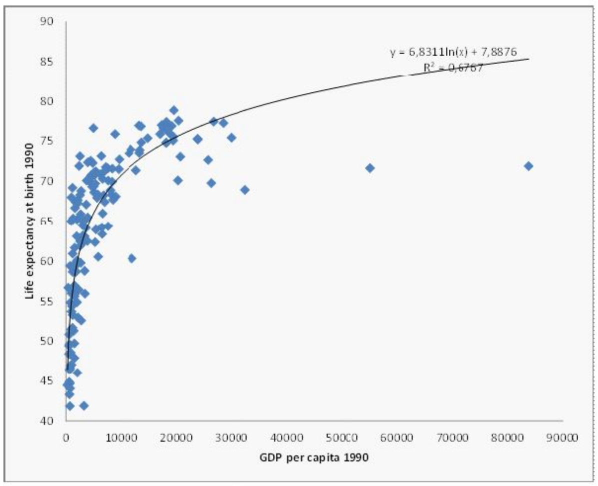
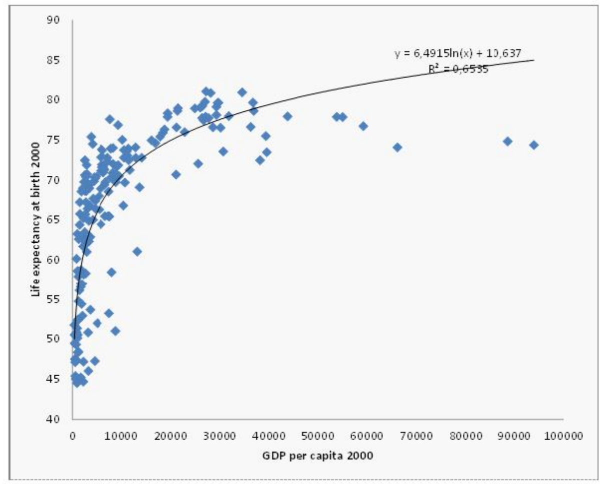
Appendix

Appendix A – The Preston Curve.

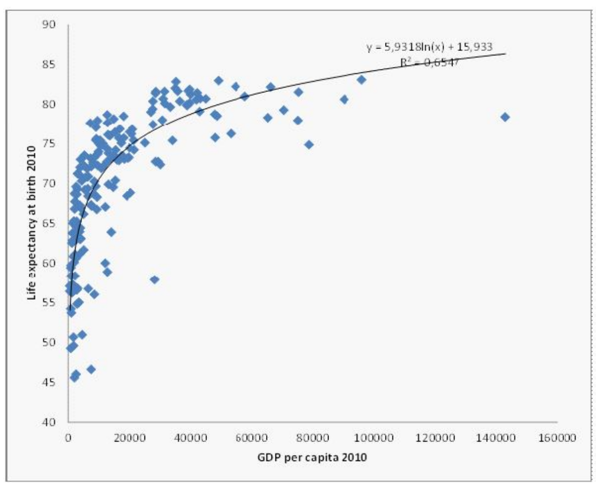
Appendix A.1 – The Preston Curve since 1990.



(a) The Preston Curve, 157 countries, 1990



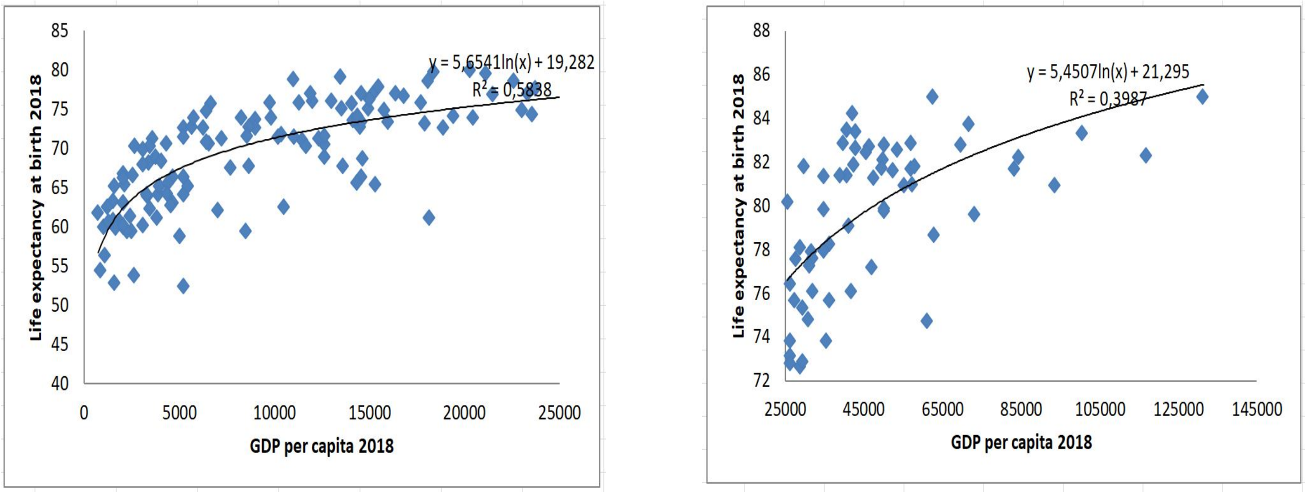
(b) The Preston Curve, 184 countries, 2000



(c) The Preston Curve, 187 countries, 2010

Figure A.1: The Preston Curve since 1990.

Appendix A.2 – The Preston Curve based on whether GDP per capita is below or above 25000\$, 2018.



(a) The Preston Curve when GDP per capita is below 25000\$, 2018.

(b) The Preston Curve when GDP per capita is above 25000\$, 2018.

Figure A.2: The Preston Curve based on whether GDP per capita is below or above 25000\$, 2018.

Appendix B – Proof of Proposition 1.

Using (4) and (11), (7) implies $d_{t+1} = \Delta(s_t, x_t, \rho) = s_t f'(s_t)/\xi(x_t) - \rho$. Then, we have $\Delta'_2(s_t, x_t, \rho) = -s_t f'(s_t)\xi'(x_t)/[\xi(x_t)]^2 < 0$, $\Delta'_3(s_t, x_t, \rho) = -1$ and, according to Assumption 1, $\Delta'_1(s_t, x_t, \rho) = [s_t f''(s_t) + f'(s_t)]/\xi(x_t) > 0$. According to (10), (4) and (11) we have $H(s_t, x_t, \rho) = v[\Delta(s_t, x_t, \rho)] - f'(s_t)[1/\xi'(x_t) + s_t/\xi(x_t)]v'[\Delta(s_t, x_t, \rho)] = 0$. After computations, we have $H'_1(s_t, x_t, \rho) = -f''(s_t)v'[\Delta(s_t, x_t, \rho)]/\xi'(x_t) - f'(s_t)\Delta'_1(s_t, x_t, \rho)[1/\xi'(x_t) + s_t/\xi(x_t)]v''[\Delta(s_t, x_t, \rho)]$, $H'_2(s_t, x_t, \rho) = f'(s_t)\xi''(x_t)v'[\Delta(s_t, x_t, \rho)]/\xi'(x_t)^2 - f'(s_t)\Delta'_2(s_t, x_t, \rho)$ $[1/\xi'(x_t)+s_t/\xi(x_t)]v''[\Delta(s_t, x_t, \rho)]$ and $H'_3(s_t, x_t, \rho) = f'(s_t)[1/\xi'(x_t)+s_t/\xi(x_t)]v''[\Delta(s_t, x_t, \rho)] - v'[\Delta(s_t, x_t, \rho)]$. Then, we have $H'_1(s_t, x_t, \rho) > 0$, $H'_2(s_t, x_t, \rho) < 0$ and $H'_3(s_t, x_t, \rho) < 0$. Consequently, there exists two functions $\varphi(.)$ and $\mu(.)$ such that $x^\star_t = \varphi(s^\star_t, \rho)$ and $s^\star_t = \mu(x^\star_t, \rho)$ with $\varphi'_1(s^\star_t, \rho) = -H'_1(s^\star_t, x^\star_t, \rho)/H'_2(s^\star_t, x^\star_t, \rho) > 0$, $\varphi'_2(s^\star_t, \rho) = -H'_3(s^\star_t, x^\star_t, \rho)/H'_2(s^\star_t, x^\star_t, \rho) < 0$, $\mu'_1(x^\star_t, \rho) = -H'_2(s^\star_t, x^\star_t, \rho)/H'_1(s^\star_t, x^\star_t, \rho) > 0$ and $\mu'_2(x^\star_t, \rho) = -H'_3(s^\star_t, x^\star_t, \rho)/H'_1(s^\star_t, x^\star_t, \rho) < 0$. □

Appendix C – Proof of Proposition 2.

Let $C(k_{t+1}, k_t, \rho) = f(k_t) - k_t f'(k_t) - k_{t+1} - \varphi(k_{t+1}, \rho)$ and $D(k_{t+1}, \rho) = k_{t+1} f'(k_{t+1})/\xi[\varphi($

$k_{t+1}, \rho)] - \rho$. According to (8) and Appendix B, we have $\Gamma(k_{t+1}, k_t, \rho) = -u'[C(k_{t+1}, k_t, \rho)] + f'(k_{t+1})v'[D(k_{t+1}, \rho)] = 0$. After computations, we obtain $\Gamma'_1(k_{t+1}, k_t, \rho) = u''[C(k_{t+1}, k_t, \rho)] + f''(k_{t+1})v'[D(k_{t+1}, \rho)] + f'(k_{t+1})[k_{t+1}f''(k_{t+1}) + f'(k_{t+1})]v''[D(k_{t+1}, \rho)]/\xi[\varphi(k_{t+1}, \rho)] + \varphi'_1(k_{t+1}, \rho)\phi''^{\star}_{12}$, $\Gamma'_2(k_{t+1}, k_t, \rho) = k_t f''(k_t)u''[C(k_{t+1}, k_t, \rho)]$ and $\Gamma'_3(k_{t+1}, k_t, \rho) = -f'(k_{t+1})v''[D(k_{t+1}, \rho)] + \varphi'_2(k_{t+1}, \rho)\phi''^{\star}_{12}$. Then, according to Appendix B, $\Gamma'_1(k_{t+1}, k_t, \rho) < 0$, $\Gamma'_2(k_{t+1}, k_t, \rho) > 0$ and $\Gamma'_3(k_{t+1}, k_t, \rho) > 0$. Consequently, there exists a function $\psi(.)$ such that $k^\star_{t+1} = \psi(k^\star_t, \rho)$ with $\psi'_1(k^\star_t, \rho) = -\Gamma'_2(k^\star_{t+1}, k^\star_t, \rho)/\Gamma'_1(k^\star_{t+1}, k^\star_t, \rho) > 0$ and $\psi'_2(k^\star_t, \rho) = -\Gamma'_3(k^\star_{t+1}, k^\star_t, \rho)/\Gamma'_1(k^\star_{t+1}, k^\star_t, \rho) > 0$. As $\psi'_1(k^\star_t, \rho) > 0$, the dynamics of capital accumulation are monotonic. They admit (at least) a finite limit $\bar{k}$ because $\lim_{k_t \to +\infty} \psi(k_t, \rho)/k_t = 0$. Indeed, according to (6), $0 \leq s_t \leq w_t$, i.e. $0 \leq \psi(k_t, \rho)/k_t \leq f(k_t)/k_t - f'(k_t)$. As $\lim_{k_t \to +\infty} f(k_t)/k_t - f'(k_t) = 0$ (see the technical appendix A.1.4 of de la Croix and Michel, 2002) we have $\lim_{k_t \to +\infty} \psi(k_t, \rho)/k_t = 0$.

Using functions $\psi(.)$ and $\mu(.)$ we have $\mu(x^\star_{t+1}, \rho) = \psi[\mu(x^\star_t, \rho), \rho]$. Then $\Omega(x_{t+1}, x_t, \rho) = \mu(x_{t+1}, \rho) - \psi[\mu(x_t, \rho), \rho] = 0$. After computations we have $\Omega'_1(x_{t+1}, x_t, \rho) = \mu'_1(x_{t+1}, \rho) > 0$ and $\Omega'_2(x_{t+1}, x_t, \rho) = -\mu'_1(x_t, \rho)\psi'_1[\mu(x_t, \rho), \rho] < 0$. Consequently, there exists a function $\chi(.)$ such that $x^\star_{t+1} = \chi(x^\star_t, \rho)$ with $\chi'_1(x^\star_t, \rho) = -\Omega'_2(x^\star_{t+1}, x^\star_t, \rho)/\Omega'_1(x^\star_{t+1}, x^\star_t, \rho) > 0$. As $\chi'_1(x^\star_t, \rho) > 0$, the dynamics of health investment are monotonic. As the dynamics of capital accumulation admit (at least) a finite limit $\bar{k}$, the dynamics of health investment admit (at least) a finite limit $\bar{x}$ such that $\bar{x} = \varphi(\bar{k}, \rho)$. □

Appendix D – Proof of Proposition 3.

A capital stock $\bar{k}$ is a long-run equilibrium if and only if $\nabla(\bar{k}, \rho) = 0$ with $\nabla(k, \rho) = k - \psi(k, \rho)$. Moreover, $\bar{k}$ is (locally) stable if and only if $\psi'_1(\bar{k}, \rho) < 1$, i.e. if and only if $\nabla'_1(\bar{k}, \rho) > 0$, i.e. (according to Appendix C) if and only if $\Gamma'_1(\bar{k}, \bar{k}, \rho) + \Gamma'_2(\bar{k}, \bar{k}, \rho) < 0$.

(*a*) As $\nabla(\bar{k}, \rho) = 0$, $\nabla'_1(\bar{k}, \rho) > 0$ and $\nabla'_2(\bar{k}, \rho) = -\psi'_2(\bar{k}, \rho) < 0$ we have $\partial\bar{k}/\partial\rho = -\nabla'_2(\bar{k}, \rho)/\nabla'_1(\bar{k}, \rho) > 0$.

(*b*) According to Appendix B and (11), $\bar{x} = \varphi(\bar{k}, \rho)$. Then $\partial\bar{x}/\partial\rho = \partial\bar{k}/\partial\rho \times \varphi'_1(\bar{k}, \rho) + \varphi'_2(\bar{k}, \rho)$. Then according to Appendix C, $\partial\bar{x}/\partial\rho$ has the sign of $\bar{F} = \bar{\Gamma}'_3(\bar{k}, \bar{k}, \rho)\bar{\varphi}'_1(\bar{k}, \rho) - (\bar{\Gamma}'_1(\bar{k}, \bar{k}, \rho) + \bar{\Gamma}'_2(\bar{k}, \bar{k}, \rho))\bar{\varphi}'_2(\bar{k}, \rho)$. After computations, $\bar{F}$ has the sign of $-f'(\bar{k}\bar{H}'_1(\bar{k}, \bar{x}, \rho))v''(\bar{d}) - [(1 + \bar{k}f''(\bar{k}))u''(\bar{c}) + f''(\bar{k})v'(\bar{d}) + f'(\bar{k})(f'(\bar{k}) + \bar{k}f''(\bar{k}))v''(\bar{d})/\xi(\bar{x})]\bar{H}'_3(\bar{k}, \bar{x}, \rho)$ with $\bar{c} = f(\bar{k}) - \bar{k}f'(\bar{k}) - \bar{k} - \bar{x}$ and $\bar{d} = \bar{k}f'(\bar{k})/\xi(\bar{x}) - \rho$. Consequently, $\partial\bar{x}/\partial\rho$ has the sign of $f''(\bar{k})v'(\bar{d})^2 +$

$f'(\bar{k})^2 v'(\bar{d}) v''(\bar{d})/\xi(\bar{x}) + (1 + \bar{k} f''(\bar{k}))[v'(\bar{d}) - f'(\bar{k})(1/\xi'(\bar{x}) + \bar{k}/\xi(\bar{x})) v''(\bar{d})] u''(\bar{c})$, i.e., using (10), the sign of $f''(\bar{k}) v'(\bar{d}) + f'(\bar{k})^2 v''(\bar{d})/\xi(\bar{x}) + (1 + \bar{k} f''(\bar{k}))[1 - v(\bar{d}) v''(\bar{d})/v'(\bar{d})^2] u''(\bar{c})$. □

Appendix E – Assumption 2 and Calibration.

The following figure ensures that Assumption 2 is satisfied in our example of Section 4.

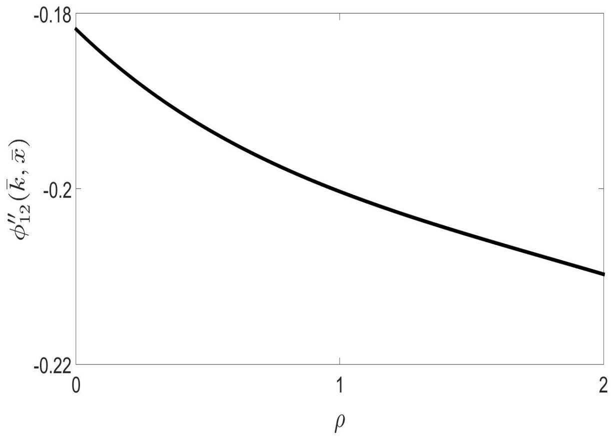


Figure A.3: The value of $\phi''^{\star}_{12} = \phi''_{12}(\bar{k}, \bar{x})$ according to ρ.

Indeed, $\phi''^{\star}_{12} < 0$ for all $\rho \in [0, 2]$. □