WORKING PAPERS

N° 1429

April 2023

"Regulating investments when both costs and need are private"

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Regulating investments when both costs and need are private.*

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April 22, 2023

Abstract

Large-scale infrastructure investments are often carried out in settings where their eventual usefulness or importance is difficult to predict. This paper studies optimal incentives for investment when the agent undertaking the investment has superior information on two dimensions: the cost of investment and the likelihood it is useful or beneficial to the principal. Usefulness eventually becomes public, but punishments are limited as the regulator aims at ensuring the agent earns non-negative profits in each period. We characterize the optimal incentive scheme and show it involves either: (i) investments by the agent even though he knows they are useless and rents to only cost-efficient types, or (ii) rents to all types. The possibility that rent is left to all types contrasts with the usual prediction in static (and also dynamic) mechanism design and arises even though the agent's preferences are stable over time.
Key words: monopoly regulation, real options, multidimensional asymmetric information.
JEL codes: D81, D82, L51.

*This paper has been supported by a research partnership between the French electricity transmission network RTE and Toulouse School of Economics. We are especially grateful for discussions with Max Papon at RTE. Our work has also been informed by discussion with Dzhordzhio Naldzhiev at the UK energy regulator Ofgem. Both authors acknowledge funding from the French National Research Agency (ANR) under the Investments for the Future (Investissements d'Avenir) program (grant ANR-17-EURE-0010). Garrett has been supported by funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement number 714147).

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Introduction.

A critical issue for the regulation of public utilities is incentive provision for large investments. Part of the difficulty for regulation is that the eventual usefulness of these investments can be highly uncertain. Moreover, because the infrastructure provider understands its business better than the regulator, the likelihood that a given investment will be useful can be part of the private information of the regulated firm. Designing incentive schemes for investment in such an environment is potentially challenging because the firm's information is then likely to be *multidimensional*; the firm may have private information on *both* costs and the probable benefit of investment to the principal.

An important example of this kind of scenario is the regulation of investments by an electricity transmission network driven by the anticipated transition to clean energies. For instance, investments in the grid may be needed if the network anticipates the development of large-scale wind farms, but such development takes years and its eventual extent is subject to regulatory uncertainty as well as uncertainty about the eventual energy mix. This may in turn reflect factor prices and societal preferences. Further, the unpredictable evolution of the climate can create uncertainty on both supply and demand sides. These considerations feed into technological uncertainties about the nature and capacity of investments that are likely to be needed, and these uncertainties might in some cases be best understood by the network business.[1]

Setting. To better understand regulation in the above environment, we consider a model in which the agent (the network business, "he") invests on behalf of the principal (the regulator, "she") over two periods. At the outset,

[1]A described by Broer and Zwart (2013, p. 178), similar concerns arise for local electricity distribution networks, as these "are expected to make significant investments in their grid capacity, or in smarter grids, if electric vehicles become more wide-spread, or as larger penetration of distributed generation - necessary to achieve lower carbon electricity generation - changes local flow patterns."

the agent has private information on two dimensions: the marginal cost of investment and the probability that investment is useful. Uncertainty about usefulness is completely resolved in the second period and usefulness then becomes public information. Investment costs, however, remain private and unchanged in the second period.

The presence of multidimensional information leads to a potentially complex screening problem. Similar to Armstrong and Rochet (1999), we reduce this complexity by studying the case where each type comes from a binary distribution. On one dimension, the agent may have a high or low cost. On the other, the agent either believes usefulness is possible, with a given likelihood, or believes that investment is useless for sure. This simplification serves to highlight a central difficulty of economic regulation when investments are uncertain. An agent who believes a given investment is unlikely to actually be needed may face incentives to undertake it anyway (akin to the problem of "gold plating of assets" familiar to network regulators).

The initial investment takes place in period one. At this stage, the usefulness of investment is uncertain and the agent privately knows whether or not the investment is potentially useful. An additional investment may take place in period two after the usefulness of investment becomes public information. The principal uses money and investment quantities to regulate the agent; in particular, we assume that the agent's actual investments (but not its costs) are perfectly observed by the principal.

A key restriction on the available mechanisms is that the principal must ensure the agent obtains non-negative profits in both periods of the interaction.[2] Given that many network businesses recover costs from consumers over long horizons, however, it is important to point out that it is the requirement of cost recovery that is essential here, rather than restrictions on timing. In

[2]As we discuss in Section 2.2, without any restriction on agent profits, the optimal investment policy turns out to coincide with the one when information about the usefulness of investment is symmetric. This policy is easier to characterize and follows from analysis to a large extent already understood in the existing literature.

particular, our profit constraints are shown to be equivalent to the requirement that cost recovery is guaranteed ex-post (i.e., after the realization of all information) in NPV terms over the relevant horizon.

The profit constraints we impose are motivated particularly by a possible regulatory concern for ensuring the agent can obtain finance. Policies where firms may fail to have costs reimbursed seem likely, at the least, to raise financing costs. We take the extreme view that the regulator therefore aims at ensuring losses do not occur. This focus may be seen as providing a benchmark against which regulatory policies which do not aim at full cost recovery in all instances (e.g., that provide for "cost disallowances" or other penalties) can be compared.

Our restriction on per-period profits is in common with some existing work that has similar motivations; e.g., Krishna et al. (2013) and Krasikov and Lamba (2021). We discuss these contributions further in the Related Literature below.

Findings. Our central result is then that optimal screening mechanism in the presence of multidimensional private information involves either: (1) some investments carried out by the agent who *knows* them to be useless, or (2) information rents to all types of agents. The possibility that rents are earned by all agent types contrasts with the usual situation in static mechanism design where cost-inefficient types earn no rents, and with most of literature on dynamic mechanism design.

The basic logic underpinning these findings is the following. Because at least some investment in the first period is assumed extremely valuable when investment is potentially useful, some date-1 investment occurs in this case, and it occurs even if costs are high. Moreover, in this case (cost is high and investment may genuinely be useful), the agent must be compensated for his costs. This creates an opportunity for the low-cost agent who knows investment is useless to claim a high cost and that investment is potentially useful, earning a rent.

Given that the low-cost agent who knows investment is useless earns a rent, the principal has two options. First, if investment is relatively costly or if information rents are not too burdensome, she can aim at inducing the agent who knows investment is useless not to produce even when he is more cost efficient. However, in this case, the high-cost agent who knows investment is useless has the same production cost (from not investing) as the more cost-efficient type, and so earns a rent as well. Since we restrict the principal's scheme not to penalize the agent when claiming production is useless while production turns out to be useful, this means that the agent must also be permitted a rent when he has high costs but knows investment is potentially useful (lest he mimic the high-cost and certainly useless type). Hence, the agent with high costs believing that investment is potentially useful earns a rent, and so then must the more cost-efficient agent who believes investment is potentially useful. Hence, optimal regulatory design can involve positive rents for all agent types.

The principal's other alternative is where the low-cost agent who knows the project is useless is induced to invest. Since the high-cost and certainly useless agent finds such investment more costly, this renders mimicry unattractive. It is then possible to deprive the high-cost agent of rents whether or not he believes investment is likely to be useful.

In qualitative terms, our findings suggest the severity of the agency problem that the principal faces in this environment. We conclude that information asymmetry on the additional dimension (usefulness of investment on top of "classic" cost), combined with period and state-specific nonnegative profit constraints for the agent, leads to a qualitatively high apparent burden for the principal. This burden could potentially be reduced, however, by departing from the objective of guaranteeing cost recovery no matter whether investments turn out to be useful or not.

Related Literature Our work connects the literature on the option value of investment under uncertainty (see Dixit and Pindyck, 1994, and McDonald and Siegel, 1986) with that on the regulation of a monopolist under adverse selection (see Baron and Myerson, 1982). Other work that combines these features includes Maeland (1999), Broer and Zwart (2013), Arve (2016), and Willems and Zwart (2018).[3] In these papers, the benefit of investment to consumers or to the principal evolves stochastically, but unlike our paper it is public information. The concern of multidimensional screening therefore does not arise in these works.[4]

Perhaps the closest paper in terms of the model is Arve (2016). She considers a two-period procurement problem with the possibility of additional investment after uncertainty concerning the benefit to the principal is publicly resolved. The agent undertaking the investment has private information on his marginal cost. A central result is that the optimal contract with perfect commitment resolves a rent-efficiency trade-off by setting the initial investment lower than in the case where no additional investments can be made following the realization of uncertainty about its benefit. We show that the combination of information asymmetries on both costs and benefits can lead to still further investment delays.

Our paper is connected to the literature on dynamic mechanism design such as Baron and Besanko (1984) and Pavan et al. (2014), where agent private information evolves stochastically with time.[5] A key difference is that the realization of uncertainty in the second period of our model is public rather than private to the agent. For this reason, the agent in our model only has an initial information revelation decision that occurs at date 1. As

[3] For further references see Arve and Zwart (2023).

[4] Grenadier and Wang (2005) is another study where there is investment by an agent and the benefit to the principal evolves stochastically, but here the agent has private information about the benefit. Again, screening is one dimensional in this paper.

[5] Perhaps most related to our problem, work on dynamic screening includes applications to non-time separable regulatory investment problems with stochastically evolving types. An instance is Zwart (2021), where the agent's private types are investment costs.

mentioned above, our requirement of non-negative payoffs in all periods is essentially the same assumption explored in the dynamic mechanism design papers of Krishna et al. (2013) and Krasikov and Lamba (2021), where agent types are private and evolve stochastically. Note that these papers generally predict positive expected rents for all types of agent both because the agent is protected from negative payoffs and because, even if his current cost is high, it will fall with positive probability at future dates. In contrast, we obtain the possibility of positive rents for all types *even though* agent preferences (in particular, costs) are stable over time.[6]

Also relevant to our analysis are papers that examine mechanism design under correlated information. For instance, Crémer and McLean (1985, 1988) examine full extraction of agent rents in auctions where bidders have correlated types. Riordan and Sappington (1988) examines an analogous question in a principal-agent environment where, like our paper, there is eventual realization of a public signal that is correlated with the agent's initial private information. While first-best outcomes can often be sustained in these settings, constraints on payments are known to limit the applicability of these results. Papers such as Demski et al. (1988) and Demougin and Garvie (1991) seek to understand the optimal design response to bankruptcy and limited liability constraints when correlated information is available. Our paper also contributes to this agenda, although in a setting where the agent has a multidimensional type, and where the public signal is correlated with only one dimension of the agent's information (that concerning the usefulness of investment to the principal). Hence, in our problem, correlation can be exploited for extracting information only on this dimension.

Finally, our paper relates to work on multidimensional screening by a mo-

[6]Some other work in dynamic mechanism design has also identified the possibility of positive rents to all types due to the fact that an agent's future preferences are uncertain. To give an example, Garrett (2017) studies a dynamic mechanism design problem where the agent arrives over time and can choose when to participate in the mechanism. Again related to the fact that types will evolve stochastically in the future, even the "worst" type of agent can expect a positive rent.

nopolist. Different to classic contributions such as Wilson (1993), Armstrong (1996), Rochet and Choné (1998), and most of the literature that follows, our specification has a "common values" element in that one dimension of the agent's type determines the preferences of the principal and not the agent. Note, however, that Chade et al. (2022) and Gottlieb (2023) are recent contributions that treat multidimensional screening in environments with some interdependence in values. As mentioned, our analysis is tractable because we follow the approach of Armstrong and Rochet (1999) in considering a "2x2" model where there are two dimensions of private information, with binary distributions for both dimensions.

Roadmap. Section 1 describes the model. Section 2 establishes two useful benchmarks: the optimal mechanism when information is symmetric, and the optimal mechanism when the agent has superior information solely on the cost of investment while information on the usefulness of investment is symmetric. Section 3 characterizes the optimal mechanism when agent private information is multidimensional. Section 4 concludes.

1 Basic model.

The agent ("he") operates a network business on behalf of the principal ("she"). The agent's role is to make possible investments in network enhancement over the course of two periods. The benefits of investment are potentially experienced over this time, and the parties have a common discount factor $\delta \in (0, 1)$. The principal designs a regulatory contract that aims at maximizing the expected discounted benefits of investment net of payments to the agent. Both principal and agent are assumed to be able to fully commit to this contract.

The investment technology is described as follows.

Investment technology. Investment in network enhancement is possible in two successive time periods indexed with $t = 1, 2$. Investment in period t is denoted by q_t. The agent incurs only variable investment costs, with a constant marginal cost c that is the same in both periods.

The initial (irreversible) investment at the beginning of the first period delivers gross return $\alpha S(q_1)$ at the end of the period, where $S(\cdot)$ is strictly increasing concave function such that $S(0) = 0$, S is bounded above and hence $\lim_{q\to\infty} S'(q) = 0$, and $\lim_{q\downarrow 0} S'(q) = \infty$ (Inada conditions). Parameter α captures the usefulness or need for the investment and is uncertain when the initial investment takes place (we provide details below). In particular, it can either be that the investment is not at all beneficial, $\alpha = 0$, or it has a positive benefit to the principal, $A > 0$. The possibility that investment is not at all beneficial (rather than merely providing small benefits) is a convenient simplification.

While the value of α is uncertain in the initial period, its value is realized by the second period, at which point it is commonly observed by the parties and directly contractible. In the second period, the agent can make an additional investment q_2 (on top of the initial investment q_1 which does not depreciate). The gross benefit to the principal realized at the end of period 2 is equal to $\alpha S(q_1 + q_2)$.

A couple of remarks are worth making regarding these assumptions. First, note that the Inada condition as investment approaches zero implies that the principal has a strong incentive to induce at least some investment, including in the first period, in case $\alpha = A$ with positive probability. This means that shutting down when investment is expected to be useful with positive probability will not be part of the principal's optimal policy. This assumption is therefore convenient as it reduces the number of cases.

A second remark is that we assume first-period investment creates a benefit to the principal at date 1, even before the usefulness of investment becomes common knowledge. Our interpretation is that it is important for the principal to have network investments installed at date 1 to avoid delays if investment turns out to be useful. The actual benefits to the principal might properly be understood as occurring over some time between date 1 and date 2 (when the need for the investment might be known, but before the players interact for a second time at date 2).

Information. It is commonly known that the marginal cost of investment c may take either relatively low value $\underline{c} > 0$ or relatively high value $\overline{c} > \underline{c}$:

$$c = \begin{cases} \underline{c} \text{ with probability } \lambda, \\ \overline{c} \text{ with probability } 1-\lambda. \end{cases}$$

The agent privately knows the realization of c. Parameter $\Delta c = \overline{c} - \underline{c}$ is a measure of the principal's uncertainty about the marginal cost.

Note that our view that the agent has private information on costs is in line with that taken by Baron and Myerson (1982). We believe that our model remains relevant even when the regulator is able to monitor accounting costs ex post. This is because some components of the relevant economic costs, such as the costs of managerial effort to undertake an investment, or opportunity costs of other foregone investments in the network, may never be fully understood by the regulator.

The agent also has superior information regarding the distribution of parameter α. Indeed, the agent knows at date 1 that α will turn out to equal A with a probability p. We therefore have:

$$\alpha = \begin{cases} A \text{ with probability } p, \\ 0 \text{ with probability } 1-p. \end{cases}$$

We allow that p can take two possible values: either a strictly positive value h or a value zero (in which case the investment is certainly useless). It is commonly known at the outset that p takes the strictly positive value with a probability r, and also that p is distributed independently of the marginal cost c. Thus, we have

$$p = \begin{cases} h \text{ with probability } r, \\ 0 \text{ with probability } 1 - r. \end{cases}$$

We assume that the probability h is not too high, and in particular,

$$h(1+\delta) < 1. \tag{1}$$

Inequality (1) is equivalent to assuming that the option value of delaying investment is positive, so we avoid corner solutions where the only investment takes place at date 1.

We denote the agent's two dimensional type with $\theta = (c, p)$ and we name its four possible realizations as follows: "good" $g = (\underline{c}, h)$; "expensive" $e = (\overline{c}, h)$; "useless but cheap" $u = (\underline{c}, 0)$; and "bad" $b = (\overline{c}, 0)$. The set of possible types is denoted with $\Theta = \{g, u, e, b\}$. Good and expensive types are termed "potentially useful" while the other two types are referred to as "certainly useless". Good and useless but cheap types are termed "cost efficient", while expensive and bad types are termed "cost inefficient".

We use notation $\mu(\theta)$ for the probability of the agent's type being θ. By the above distributional assumptions,

$$\mu(g) = \lambda r,\ \mu(e) = (1-\lambda) r,\ \mu(u) = \lambda(1-r),\ \mu(b) = (1-\lambda)(1-r).$$

Regulation subject to financing constraints. Investments and the second-period realization of α are publicly observable and verifiable. Without loss of generality (Dasgupta, Hammond and Maskin, 1979; Myerson, 1981), the principal commits to a direct mechanism:

$$(q_1(m), T_1(m), q_2(m, A), T_2(m, A), q_2(m, 0), T_2(m, 0)),$$

where $m = (\widetilde{c}, \widetilde{p})$ is the agent's report of type θ, q_t is investment required at date t, and T_t is the agent's compensation received at date t.

We assume that a regulatory objective is to ensure the agent receives non-negative profits in both periods for both realizations of α. Formally, we require that, for all $\theta \in \Theta$, and all $\alpha \in \{0, A\}$,

$$T_1(\theta) - cq_1(\theta) \geqslant 0 \text{ and } T_2(\theta, \alpha) - cq_2(\theta, \alpha) \geqslant 0. \tag{2}$$

Note that constraints (2) are required to be met only when the agent truthfully reports costs. The idea is that the regulatory objectives need to be met only in equilibrium, and are thus not required to hold when the agent deviates by misrepresenting costs. Implicit in the above, however, we assume that part of the regulatory objective is to ensure the satisfaction of financing constraints even if the agent turns out to be "wrong" about the usefulness of regulation, and hence impose the constraints also when the agent claims the investment to be certainly useless while it turn out at date 2 to be useful. This avoids a discontinuity in the constraints of the regulatory problem if we instead consider an environment where the agent, in place of being certain that investment is useless, merely believes it is useless with very high probability. We address these nearby environments in Section 3.1 and show that our characterization of optimal mechanisms (in Proposition 3 below) continues to hold approximately.

As mentioned in the Introduction, it is worth pointing out that the essential requirement expressed in the equation set (2) is not the timing of payments but that the agent obtains a guarantee of eventual cost reimbursement. In particular, our two-period model is equivalent to one where we replace the profit constraints of equation set (2) with the following requirement. For all $\theta \in \Theta$, and all $\alpha \in \{0, A\}$,

$$T_1(\theta) - cq_1(\theta) + \delta(T_2(\theta, \alpha) - cq_2(\theta, \alpha)) \geqslant 0. \tag{3}$$

The inequality (3) clearly follows if the inequalities in equation set (2) hold. Conversely, if the inequality (3) holds, keeping investments unchanged, we can consider adjusted transfers $\check{T}_1$ and $\check{T}_2$ that satisfy the inequalities in (2) as follows. For all θ, $\check{T}_1(\theta) = cq_1(\theta)$ (so that non-negative profits are guaranteed at date 1). Also, for all θ and α,

$$\check{T}_2(\theta, \alpha) = T_2(\theta, \alpha) + \tfrac{1}{\delta}(T_1(\theta) - cq_1(\theta)).$$

Date-2 profits are then, for all θ and α,

$$\check{T}_2(\theta, \alpha) - cq_2(\theta, \alpha) = T_2(\theta, \alpha) - cq_2(\theta, \alpha) + \tfrac{1}{\delta}(T_1(\theta) - cq_1(\theta))$$

which is non-negative by the inequality (3). The NPV of payments to the agent are unchanged across all types θ and both realizations of α, and hence agent incentives are unaffected. The constraint in the inequality (3) can be understood as a requirement that the NPV of agent profits are non-negative *ex post*.

A possible rationale for a regulatory objective that adheres to financial constraints is that subjecting firms to the risk of losses in any period may increase financing costs (not only would firms face increased risk, but they would need to ensure sufficient reserves to weather any losses imposed by the regulatory scheme). While we expect there is a trade-off in practice, we simplify in this paper by taking the extreme view that the regulatory scheme should be designed to avoid any losses ex post. Our results might then be seen as a benchmark against which improvements in regulatory performance due to introducing some penalties can be measured. (Such penalties might include, for instance, “cost disallowances” when investments turn out not to be useful.)

One reason for our choice of focus is also that, without any such profit constraints, the fact that the agent is initially privately informed about usefulness would not affect regulatory performance (private information about usefulness would not play a role in determining the optimal investments, and the analysis would be fairly easily anticipated in light of the existing literature). As we discuss further in Section 2.2, this is because the agent could effectively be asked to “bet” at date 1 on the observed future state and then subjected to rewards or (unrestricted) penalties to incentivize revelation of the relevant information on usefulness. Our analysis is therefore pertinent to understanding optimal regulation when such "bets", and in particular the corresponding penalties, are restricted.

Payoffs. With the description of the mechanism in place, it may now be useful to summarize the intertemporal payoffs of the players. The principal's payoff in the direct mechanism under truthful reporting, given θ and α, is

$$\alpha\left(S(q_1(\theta)) + \delta S(q_1(\theta) + q_2(\theta, \alpha))\right) - \left(T_1(\theta) + \delta T_2(\theta, \alpha)\right).$$

The agent's payoff, given θ and α, is

$$T_1(\theta) - cq_1(\theta) + \delta\left(T_2(\theta, \alpha) - cq_2(\theta, \alpha)\right).$$

2 Helpful benchmarks.

In this section, we consider two benchmarks that will be useful for understanding the solution to the problem of interest.

2.1 Complete information benchmark.

The first benchmark is the situation in which the principal knows θ. We refer to this solution as "first best" (and denote the first-best policy by superscript "FB"). In this case, the principal finds it optimal to choose investments efficiently, compensating their cost to the agent. Given θ, the optimal investment pattern $(q_1^{FB}(\theta),\, q_2^{FB}(\theta, A), q_2^{FB}(\theta, 0))$ maximizes the expected net return

$$\begin{aligned} W(\theta) = pAS(q_1(\theta)) - cq_1(\theta) + \delta pAS(q_1(\theta) + q_2(\theta, A)) \\ -\delta c\left(pq_2(\theta, A) + (1-p)\, q_2(\theta, 0)\right). \end{aligned} \tag{4}$$

Payments are set according to:

$$T_1^{FB}(\theta) = cq_1^{FB}(\theta),\ T_2^{FB}(\theta, \alpha) = cq_2^{FB}(\theta, \alpha). \tag{5}$$

We then have the following result.

Proposition 1. *If the investment is potentially useful, that is $p = h$, then the first-best investments are:*

$$q_1^{FB}(\theta) = F\left(\frac{c}{A}\frac{1-\delta h}{h}\right),\ q_2^{FB}(\theta, A) = F\left(\frac{c}{A}\right) - F\left(\frac{c}{A}\frac{1-\delta h}{h}\right),\ q_2^{FB}(\theta, 0) = 0, \tag{6}$$

where $F = (S')^{-1}$. Otherwise, i.e. if $p = 0$,

$$q_1^{FB}(\theta) = q_2^{FB}(\theta, 0) = 0 \tag{7}$$

and the choice of $q_2(\theta, A)$ is irrelevant.

The formal proof of Proposition 1 is in Appendix A. Note that the efficient initial investment by useful types solves a trade-off between a higher initial investment, hence a higher return in period 1 in case of positive realization of α, and an option of waiting in order to learn the realization of α. The higher probability h, the higher the expected return from the initial investment and the less valuable is the option "wait-and-see". The inequality (1) is a necessary and sufficient condition for this option to be valuable (see Appendix A).

2.2 Benchmark: no asymmetry on usefulness.

Suppose now that the anticipated likelihood of usefulness p is commonly observed and is contractible, while the marginal cost of investment c is the agent's private information. The principal's information is thus worse than in the "first-best" case discussed in Section 2.1, but it is better than in the model of interest. This situation is familiar from the literature cited in the Introduction, and the analysis is similar to many other settings with information asymmetry on a single dimension (for a treatment of such models, see Laffont and Martimort, 2002).

We now find it useful to introduce notation that we use throughout the rest of the paper (including in Section 3 where the value of p is the agent's private information). For any true type θ and message m,

$$U_1(\theta, m) = T_1(m) - cq_1(m), \; U_2(\theta, m, A) = T_2(\theta, m, A) - cq_2(\theta, m, A),$$

$$\text{and } U_2(\theta, m, 0) = T_2(\theta, m, 0) - cq_2(\theta, m, 0).$$

These correspond to the agent's rents in each period. We will find it useful to view these rents as control variables in solving the principal's problem, rather than maximizing with respect to transfers.

As further notation, we write

$$U(\theta, m) = U_1(\theta, m) + \delta\left[pU_2(\theta, m, A) + (1-p)U_2(\theta, m, 0)\right]$$

for the expected rents by the agent of type θ sending message m.

In addition, in a more or less standard abuse, we introduce notation for period and state specific rents earned in equilibrium. We write:

$$U(\theta) = U(\theta, \theta) \text{ and } U_1(\theta) = U_1(\theta, \theta)$$

for the expected lifetime and period 1 rents earned by the agent revealing his type truthfully. Then,

$$P(\theta) = U_2(\theta, \theta, A)$$

denotes the agent's "delayed" rents when he truthfully reveals his type and the investment turns out to be useful. On the other hand,

$$B(\theta) = U_2(\theta, \theta, 0)$$

denotes the agent's "delayed" rent when the agent truthfully reveals his type and the investment turns out *not* to be useful.

By the revelation principle, we search for an optimal screening mechanism among direct mechanisms incentivizing the agent to truthfully report his marginal cost of investment c. Formally, given the commonly known likelihood that investment is useful p, the principal's problem is

$$\max_{\{q_1(\theta), q_2(\theta,\alpha), U(\theta), P(\theta), B(\theta)\}_{\theta\in\{(\underline{c},p),(\overline{c},p)\},\alpha\in\{0,A\}}} \sum_{\theta\in\{(\underline{c},p),(\overline{c},p)\}} \mu(\theta)\left[W(\theta) - U(\theta)\right]$$

subject to two incentive compatibility constraints

$$U(\theta) \geqslant U(\theta, m) \ \text{ for } \theta, m \text{ in the set } \{(\underline{c}, p), (\overline{c}, p)\},$$

and six nonnegative profit constraints. These profit constraints require that, for each θ in the set $\{(\underline{c}, p), (\overline{c}, p)\}$,

$$U(\theta) - \delta[pP(\theta) + (1-p)B(\theta)] \geqslant 0,$$

$$P(\theta) \geqslant 0, \text{ and } B(\theta) \geqslant 0.$$

We will mark the solution to the above problem with "hat" $\widehat{\cdot}$. It is trivial if the investment is definitely useless, that is, $p = 0$. Indeed, the efficient no-investment pattern (7) creates no incentive issues, and is therefore optimal.

Suppose that the investment is potentially useful, that is, $p = h$. The analysis of the principal's optimal mechanism is by now relatively standard in the literature. In order to provide the agent of good type g with incentives for revealing cost efficiency, the principal must pay him rents at least

$$U(g) = \ \Delta c\left(\widehat{q}_1(e) + \delta h \widehat{q}_2(e, A)\right). \tag{8}$$

Because these rents are increasing in the expected investment by the agent of expensive type e, the principal distorts this investment downwards as compared to real-option value benchmark in Proposition 1. This reflects the classic rents-efficiency trade-off. Specifically, the investment pattern by the agent of type e is the following:

$$\widehat{q}_1(e) = F\left(\frac{\widehat{c}}{A}\frac{1-\delta h}{h}\right) \ \text{ where } \widehat{c} = \overline{c} + \Delta c \frac{\lambda}{1-\lambda}, \tag{9}$$

$$\widehat{q}_2(e, A) = F\left(\frac{\widehat{c}}{A}\right) - F\left(\frac{\widehat{c}}{A}\frac{1-\delta h}{h}\right), \text{ and } \widehat{q}_2(e, 0) = 0. \tag{10}$$

Note that inequality (1) is necessary and sufficient for the additional investment to be positive, as in the first best case (see Appendix B). Also, note that the good type g invests efficiently, which is the classic "no distortions at the top" result. The main insights are summarized as follows.

Proposition 2. *Suppose that the principal knows profitability of investment p, while its marginal cost c is the agent's private information. Then, the optimal screening mechanism is the following.*

(i) The agent of either type for which investment is certainly useless does not invest and is not paid.

(ii) Investments by the agent of the good type are efficient. He receives expected information rents given in Equation (8).

(iii) Investments by the agent of expensive type are specified by Equation (9) and by the set of equations (10). The initial investment, and the total investment in case investment turns out to be useful at date 2, are downward distorted relative to first-best levels. These investments are reimbursed without any excess in either period (no rents).

Hence, when the agent has superior information solely on cost, the principal has to provide rents only to the "good" (cost-efficient and potentially useful) type g. In order to decrease these rents, she optimally distorts the investments of the "expensive" (cost-inefficient and potentially useful) type e downwards as compared to the real-option value benchmark in Proposition 1, as described in Proposition 2. This finding aligns with earlier literature on regulation of investment under uncertainty in the presence of asymmetric information on cost.

We conclude this section with two remarks regarding the role of participation constraints. First, we note that the principal's payoff and the optimal investment plan would be the same as in Proposition 2 if, instead of non-negative profits at either date and in either state (as in our setting), the agent needed to be guaranteed non-negative profits only in expected terms from the perspective of date 1. That is, our results would not change if we imposed only a participation constraint at date 1, which has been the most common approach in the literature on dynamic mechanism design.[7]

Second, and related to our discussion of financing constraints in Section

[7]The proof (available upon request) is similar to that of Proposition 2.

1, we note that if non-negative profit constraints were replaced by the more common date-1 participation constraint, then information asymmetries relating to the likelihood investment is useful would not constrain the principal's performance in the regulatory contract. In particular, the principal's equilibrium payoff, and the equilibrium investments, would be the same as for the regulatory contract characterized in Proposition 2. The reason is that we could augment the mechanism in Proposition 2 with "bets" on the state such that the agent breaks even on the bet if taking the option coinciding to a truthful report of p and makes sufficiently large expected losses otherwise (this is a standard argument and similar, for instance, to Riordan and Sappington, 1988).

3 The optimal screening mechanism.

Now, consider our model in which both parameters c and p are the agent's private information (and where we impose the financing constraints introduced in Section 1). The optimal screening mechanism solves

$$\max_{\{q_1(\theta),q_2(\theta,\alpha),U(\theta),P(\theta),B(\theta)\}_{\theta\in\Theta,\alpha\in\{0,A\}}} \sum_{\theta\in\Theta}\mu(\theta)\left[W(\theta)-U(\theta)\right] \tag{11}$$

subject to twelve incentive compatibility constraints and twelve non-negative profit constraints. The incentive compatibility constraints require that any type $\theta\in\Theta$ has no incentive to lie:

$$U(\theta)\geqslant U(\theta,m)\ \text{ for any } \theta\in\Theta \text{ and any } m\in\Theta\backslash\{\theta\}. \tag{12}$$

The non-negative profit constraints require that any type θ receives non-negative profit in period 1 and also in period 2 for either realization of α when he truthfully reports his type. That is, for any $\theta\in\Theta$,

$$P(\theta), B(\theta)\geqslant 0, \text{ and}$$

$$U(\theta)-\delta\left[pP(\theta)+(1-p)B(\theta)\right]\geqslant 0.$$

We call the mechanism solving the above problem the "second best" and mark it with an upper index SB.

Because the jointly observed realization of α is an informative signal on p, the principal maximally relaxes the incentive constraints (12) by allocating rents conditionally on this signal (see Appendix C). Rents to the agent of either potentially useful type (if any) are paid in the form of profits in period 2 conditional on α taking positive value A. This means that

$$U_1(\theta) = B(\theta) = 0 \text{ and } U(\theta) = \delta h P(\theta) \text{ for either } \theta \in \{g, e\}. \quad (13)$$

On the other hand, rents to the agent of either certainly useless type (if any) are paid in the form of profits at date 2 conditional on α taking value zero:

$$U_1(\theta) = P(\theta) = 0 \text{ and } U(\theta) = \delta B(\theta) \text{ for either } \theta \in \{u, b\}. \quad (14)$$

There is no loss in considering agent rents that are paid with the timing described above, and so we focus on this timing throughout the characterization of the second-best mechanism. We determine this second-best mechanism in Appendix D. This turns out to be generically unique up to the timing of investment in case the agent is certain that investment is useless.

To anticipate our characterization of the second-best mechanism, note that if the principal uses the mechanism in Proposition 2, the agent of useless but cheap type u gains rents $\Delta c \widehat{q}_1(e)$ by pretending that his type is expensive. This observation suggests that, in the second-best problem, ignoring other incentive constraints, the useless but cheap type could be incentivized to reveal his type and not to produce if paid a rent

$$B^{SB}(u) = \tfrac{\Delta c}{\delta} q_1^{SB}(e) \quad (15)$$

at date 2 when $\alpha = 0$. This bonus, however, would attract the agent of the bad type. If the useless but cheap type u does not produce, then the bad type can access precisely the same rents effectively because this type is not at a cost disadvantage when not producing anything.

This discussion suggests, and we will show, that there are two kinds of optimal responses from the principal. One is to accept that the bad type will indeed earn a rent while not investing. In this case, note that because the expensive type e can mimic the bad type, also type e will earn a rent. We will then observe that all types of the agent expect a rent from the perspective of date 1. In many environments (especially static, but also most dynamic mechanism design environments) the possibility of rents for all types does not survive a simple uniform reduction in transfers to the agent for all types, which would increase the principal's payoff. Here, however, at the optimal mechanism to be characterized below, it is not possible to reduce payments to the agent uniformly (i.e., independently of type) at date 1 because of the non-negative profit constraints for this period. Similarly, a uniform reduction in payments at date 2 (independent of both type and realization of α) is not possible due to non-negative profit constraints for this period.

The principal's second possible response is to induce the useless but cheap type u to invest a positive amount. This works to reduce the rents that must be granted to the bad type b, as this type faces higher costs from the same level of investment. When such a policy is optimal, we are able to show that the principal pursues this logic to the point where the bad type earns no rents. Similarly, then, the expensive type e will earn no rents.

Now, let us describe the principal's optimal policy in more detail, starting with the case where the cost-inefficient types are deprived of any rent. Here, we will find that the useless but cheap type's total discounted investment is the same as the expensive type's investment at date 1 (this is sufficient to deprive the bad type b of any rent). That is, we have

$$q^{SB}(u) = q_1^{SB}(e), \text{ where} \tag{16}$$

$$q^{SB}(u) = q_1^{SB}(u) + \delta q_2^{SB}(u, 0).$$

Only cost-efficient types receive rents, and these are described by the set of equations (13) and (14), as well as Equation (15) together with

$$P^{SB}(g) = \frac{\Delta c}{\delta h}\left(q_1^{SB}(e) + \delta h q_2^{SB}(e, A)\right). \tag{17}$$

The corresponding downward distortion of investments is described by the following equations:

$$q_1^{SB}(e) = F\left(\frac{\widehat{c}}{A}\frac{1-\delta h}{h} + \frac{\bar{c}}{hA}\frac{\lambda(1-r)}{(1-\lambda)r}\right),\ q_2^{SB}(e, A) = F\left(\frac{\widehat{c}}{A}\right) - q_1^{SB}(e). \tag{18}$$

Note that this implies total investment by type e in case investment proves useful, $q_1^{SB}(e)+q_2^{SB}(e, A)$, is the same as for the optimal mechanism of Section 2.2 where there was no asymmetric information about usefulness. However, the initial investment $q_1^{SB}(e)$ is *more* distorted, which implies that investment by type e is more delayed.

The alternative policy, termed hereafter "rents to all", involves no production by certainly useless types. It then involves paying the same delayed bonus to the agent with a bad type as to the agent with a useless but cheap type:

$$B^{SB}(b) = \frac{\Delta c}{\delta} q_1^{SB}(e). \tag{19}$$

This calls for upgrading delayed profits by potentially useful types correspondingly:

$$P^{SB}(e) = \frac{1-h}{h} B^{SB}(b) = (1-h)\frac{\Delta c}{\delta h} q_1^{SB}(e), \text{ and} \tag{20}$$

$$P^{SB}(g) = P^{SB}(e) + \frac{\Delta c}{\delta h}\left(q_1^{SB}(e) + \delta h q_2^{SB}(e, A)\right). \tag{21}$$

In order to decrease agent profits, the principal again distorts investments by the expensive type e, as this type's investments satisfy

$$q_1^{SB}(e) = F\left(\frac{\widehat{c}}{A}\frac{1-\delta h}{h} + \frac{\Delta c}{hA}\frac{1-hr}{(1-\lambda)r}\right) \text{ and } q_2^{SB}(e, A) = F\left(\frac{\widehat{c}}{A}\right) - q_1^{SB}(e). \tag{22}$$

Again, note that the sum of type e's investments in case investment proves useful, $q_1^{SB}(e) + q_2^{SB}(e, A)$, turns out to be the same as in the benchmark where there is no asymmetric information on the usefulness of investments.

Relative to that benchmark, the initial investment $q_1^{SB}(e)$ is more distorted, i.e. investments are again *more delayed* as a result of the additional information asymmetry.

Of these two approaches, the former (production by the useless but cheap type to avoid rents to cost-inefficient types) is optimal if and only if the expected cost of useless investments by the agent of the useless but cheap type is weakly below the expected cost of inflated information rents from non-investment (which is increasing in the spread of the principal's uncertainty on the cost of investment Δc). In particular, the condition is

$$\lambda (1 - r) \underline{c} \leqslant \Delta c (r (1 - h) + (1 - r)(1 - \lambda)). \tag{23}$$

We can summarize the key observations as follows.

Proposition 3 (second best). *A second-best mechanism can be described as follows. The investments by the agent of both good and bad types are efficient: for* $\theta \in \{g, b\}$ *and* $\alpha \in \{0, A\}$,

$$q_1^{SB}(\theta) = q_1^{FB}(\theta) \text{ and } q_2^{SB}(\theta, \alpha) = q_2^{FB}(\theta, \alpha).$$

(i) If inequality (23) holds, the investments by the agent of the useless but cheap type are distorted upwards in any way allowed by Equation (16), while the investments by the agent of the expensive type are distorted downwards and delayed as described by set of equations (18). Only cost-efficient types receive information rents, and these rents are described by sets of equations (13) and (14) and equations (15) and (17).

(ii) If the inequality in Equation (23) is reversed, the agent of useless but cheap type does not invest (which is efficient), while the investments by the agent of expensive type are distorted downwards and delayed as described by set of equations (22). All types receive information rents, specified by sets of equations (13) and (14), by Equation (15), and by the equations (19) to (21).

As discussed above, the result provides a qualitative sense in which optimal regulation has undesirable features, suggesting the high cost to the principal of information asymmetries. Either there is production by an agent who knows investment is useless to the principal (Case (i) in the proposition) or all agents can expect a positive rent at the time they agree to the contract (date 1).

It is worth reiterating the reason why the agent can (from the perspective of date 1) expect positive rents for all types in Case (ii) of the Proposition. We have shown (see the discussion above as well as Appendix C) that it is optimal to maximally separate the potentially useful from the certainly useless types by backloading the payment of all rents to date 2, and then paying rents to the potentially useful types when investment turns out to be useful and to the certainly useless types when it turns out to indeed be useless. This implies that zero rents are paid at date 1 and also at date 2 for *some* realization of α. It is therefore not possible to uniformly reduce the agent's rents either at date 1 or at date 2 without violating the non-negative profit constraints. This offers a partial reason why positive expected rents for all types can survive in our environment, while it is not a prediction of most other mechanism design settings.

3.1 Continuity: almost certainly useless types.

As mentioned, our focus on the case where the agent may know at the outset that investment is certainly useless is a simplification, but anticipates some continuity of the optimal mechanism if we instead consider agents who view the probability of usefulness as being positive but small. We now suppose that p can take two values: either h as originally considered (the highest probability with which investment is useful), or l, which is a low probability that will be taken to zero. All other aspects of the design problem are unchanged and held fixed as we vary l. Now, types u and b anticipate at date 1 that the probability that $\alpha = A$ is l rather than zero.

We make an additional assumption that per-period investments are bounded above by some $\bar{q}$ which is large enough to accommodate any of the investments made under the second-best policy of Proposition 3. This bound will ensure that the second-best investment does not explode for type u when $\alpha = A$ is realized, which occurs with a vanishingly small probability as $l \to 0$.

The same argument as appears in Appendix C can be applied to the new incentive contraints when $0 < l < h$ to obtain the same conclusion as presented in equation sets (13) and (14). That is, it is optimal for all rents to be delivered in the second period as follows. Types who believe investment is useful with probability h only receive rents when the investment is useful ($\alpha = A$): $P(g)$ for the good type and $P(e)$ for the expensive type. Types who believe investment is useful with probability l only receive rents when investment is useless ($\alpha = 0$): $B(u)$ for the useless but cheap type and $B(b)$ for the bad type. This timing for the payment of rents relaxes all incentive constraints and we restrict attention to such policies.

We now view a policy $M(l)$ as a collection of investment decisions $\{q_1(\theta), q_2(\theta, 0), q_2(\theta, A)\}_{\theta\in\Theta}$ together with the rents $B(u)$, $B(b)$, $P(g)$, and $P(e)$. Applying the Theorem of the Maximum, we find the following result (proved in Appendix E).

Proposition 4 (second best with almost certainly useless types). *Consider the principal's mechanism design problem parameterized by the probability l attached to $\alpha = A$ by types u and b. The set of solutions $M^{SB}(l)$ is locally non-empty and upper hemi-continuous at $l = 0$.*

Upper hemi-continuity has the implication that, considering any solutions to the second-best problem with the probability of usefulness $l > 0$, investments and rents become arbitrarily close to those described in Proposition 3 as we take l to zero. In other words, our characterization also holds approximately if types u and b are merely "close to certain" that they are useless.

4 Conclusions.

We have described the optimal regulation of investments under uncertainty when the agent holds superior information on both the cost of investment and the likelihood it is useful to the principal. We focused on the case where, out of concern for the firm's ability to finance the investments, the regulation aims at ensuring non-negative profits in any period.

Our main result was that, contrary to the case where information on usefulness is symmetric, optimal regulation involves either investment by the agent when he is certain it is useless, or positive expected rents to the agent for all types. In the second case we mean that, from the perspective of the time the agent agrees to the regulatory contract at date 1, the agent always expects to earn positive rents from the relationship. While such a possibility has been obtained elsewhere in the dynamic mechanism design literature (see the Related Literature), we establish this result in an environment where the agent's preferences are stable over time (as investment costs are constant across time).

Our findings are arguably pessimistic about the nature of optimal regulation, as they are suggestive of a high burden of asymmetric information on the principal. We nonetheless see our results as a possible benchmark against which regulatory policy might aim to improve. For instance, they suggest the advantage of allowing that network businesses recuperate expenditures only gradually over time, and determining that some costs are then "disallowed" and hence not reimbursed in case investment turns out not to be useful. Alternatively, the regulator may seek to reduce the extent of information asymmetries concerning the necessity of investments by requiring the submission of detailed investment plans and understanding the parameters of the network themselves (many regulators such as CRE in France and Ofgem in the UK require network businesses to provide detailed plans). A third possibility is that the regulator could provide payments only sufficient to cover low-cost investments, thus depriving firms of rent but possibly foregoing some beneficial investments that have higher costs. Such a policy was never optimal for the regulator in our model due to the Inada conditions (i.e., because we assumed that small amounts of investment were highly beneficial). The role of these kinds of regulatory policies, as well as their practical implementation, remain possible areas for future research.

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A. Proof of Proposition 1.

Step 1. It is easy to see that no investment is optimal when $p = 0$ in either period, and that no investment is optimal at date 2 in case $\alpha = 0$. When

$p = h$ and yet $\alpha = A$, the amount of investment is irrelevant (as this event has probability zero).

Step 2. The only case that needs attention is then where $p = h$. Here we are maximizing the expression in Equation (4) by choice of $q_1(\theta)$ and $q_2(\theta, A)$. Note that, by the Inada conditions, we may assume that these investments are bounded above and that $q_1(\theta)$ is bounded away from zero. The objective is continuous and strictly concave, and so a unique solution to the maximization problem exists.

Necessary conditions for optimality are then:

$$\begin{cases} hAS'(q_1(\theta)) + \delta hAS'(q_1(\theta) + q_2(\theta, A)) = c, \text{and} \\ AS'(q_1(\theta) + q_2(\theta, A)) \leq c, \end{cases} \tag{24}$$

where the inequality is an equality in case $q_2(\theta, A) > 0$.

By Equation (1), in case $q_2(\theta, A) = 0$, we have that if the first statement in Equation (24) holds, then the second statement (i.e., the weak inequality) is violated. Therefore, we must have $q_2(\theta, A) > 0$ and both the equations in (24) hold as equalities.

Plugging the expression for $AS'(q_1(\theta) + q_2(\theta, A))$ from the latter equation of system (24) into its former equation, we find

$$hAS'(q_1(\theta)) = c(1 - \delta h),$$

which is equivalent to the first equation in set (6). Combining the first equation in set (6) with the second equation in system (24), we find the second equation in set (6).

B. Proof of Proposition 2.

Step 1. The case where $p = 0$ is immediate and omitted.

Step 2. Suppose that $p = h$. Consider the principal's problem in more detail:

$$\max_{\{q_1(\theta), q_2(\theta,\alpha), U(\theta), P(\theta), B(\theta)\}_{\theta\in\{e,g\},\alpha\in\{0,A\}}} \sum_{\theta\in\{e,g\}} \mu(\theta)\left[W(\theta) - U(\theta)\right]$$

subject to two incentive compatibility constraints

$$U(g) \geqslant U(e) + \Delta c\left(q_1(e) + \delta h q_2(e, A) + (1-h)\, q_2(e, 0)\right), \tag{25}$$

$$U(e) \geqslant U(g) - \Delta c\left(q_1(g) + \delta h q_2(g, A) + (1-h)\, q_2(g, 0)\right), \tag{26}$$

and six nonnegative profit constraints

$$U(g) - \delta\left[hP(g) + (1-h)\, B(g)\right] \geqslant 0, \tag{27}$$

$$U(e) - \delta\left[hP(e) + (1-h)\, B(e)\right] \geqslant 0, \tag{28}$$

$$P(g) \geqslant 0,\ B(g) \geqslant 0, \tag{29}$$

$$P(e) \geqslant 0,\ B(e) \geqslant 0. \tag{30}$$

Step 3. Let us solve the above problem under constraints (25), (28) and (30), ignoring the other constraints ("the relaxed problem").
Step 3.1. Given that rents are costly for the principal, the constraints of the relaxed problem are binding:

$$P(e) = B(e) = U(e) = 0,$$

$$U(g) = U(e) + \Delta c\left(q_1(e) + \delta h q_2(e, A) + (1-h)\, q_2(e, 0)\right). \tag{31}$$

Step 3.2. Notice that setting $q_2(e, 0) = 0$ is efficient and it relaxes the incentive compatibility constraint (31). Furthermore, in the relaxed problem $q_2(g, 0) = 0$ because it is efficient (we ignore for the moment constraint (26)).
Step 3.3. By Steps 3.1 and 3.2, the relaxed problem is equivalent to

$$\max_{\{q_1(\theta), q_2(\theta, A)\}_{\theta\in\{e,g\}}} \lambda\left[W(g) - \Delta c\left(q_1(e) + \delta h q_2(e, A)\right)\right] + (1-\lambda)\, W(e).$$

The analysis of this relaxed optimization program is analogous to the first-best problem considered in Proposition 1, but with a modified marginal cost for the "expensive" type (the "virtual marginal cost" in place of the true marginal cost). The same reasoning as for Proposition 1 therefore allows us to conclude that $q_1(g), q_1(e), q_2(g,A), q_2(e,A) > 0$, and investments are as specified in Proposition 2. We refer to these investments now as $\widehat{q}_1(g), \widehat{q}_1(e), \widehat{q}_2(g,A)$, and $\widehat{q}_2(e,A)$.

Step 4. By set of nonnegative profit constraints (28) and (30), the agent of type e is reimbursed for investment in each period:

$$\widehat{T}_1(e) = \overline{c}\widehat{q}_1(e), \widehat{T}_2(e,A) = \overline{c}\widehat{q}_2(e,A), \widehat{T}_2(e,0) = 0.$$

Step 5. Now recall the original problem. There are many ways to ensure that the non-negative profit constraints (27) and (29) are met through a judicious choice of payments to the good type g. For instance, it will be enough that $P(g) = B(g) = 0$ so that the good type earns no rents at date 2. This means specifying

$$\widehat{T}_2(g,A) = \underline{c}\widehat{q}_2(g,A), \text{ and } \widehat{T}_2(g,0) = 0.$$

We can then ensure that type g receives the necessary rents at date 1 by specifying

$$\widehat{T}_1(g) = \underline{c}\widehat{q}_1(g) + \Delta c\left(\widehat{q}_1(e) + \delta h \widehat{q}_2(e,A)\right).$$

Step 6. It remains to verify that the solution of the relaxed problem satisfies the ignored incentive compatibility constraint (26).

This constraint is satisfied provided that the payoff of type e when mimicking type g, as given by

$$\Delta c(\widehat{q}_1(e) + \delta h \widehat{q}_2(e,A)) - \Delta c(\widehat{q}_1(g) + \delta h \widehat{q}_2(g,A), \tag{32}$$

is non-positive. This holds, however, because $\widehat{q}_1(g) > \widehat{q}_1(e)$ and $\widehat{q}_1(g) + \widehat{q}_2(g,A) > \widehat{q}_1(e) + \widehat{q}_2(e,A)$, so that

$$(1-\delta h)\widehat{q}_1(g) + \delta h(\widehat{q}_1(g) + \widehat{q}_2(g,A)) > (1-\delta h)\widehat{q}_1(e) + \delta h(\widehat{q}_1(e) + \widehat{q}_2(e,A))$$

which establishes that the expression in Equation (32) is negative. This completes the proof.

C. An optimal timing for the payment of rents.

This appendix concerns an optimal specification for the timing of rents to the agent in the second-best mechanism of Section 3. Consider the set of incentive compatibility constraints (12) under the direct mechanism:

$$U(g) \geqslant U(e) + \Delta c \left(q_1(e) + \delta h q_2(e, A) + \delta(1-h) q_2(e, 0)\right),$$

$$U(g) \geqslant U(b) + \Delta c \left(q_1(b) + \delta h q_2(b, A) + \delta(1-h) q_2(b, 0)\right) + \delta h \left(P(b) - B(b)\right), \tag{33}$$

$$U(g) \geqslant U(u) + \delta h \left(P(u) - B(u)\right), \tag{34}$$

$$U(u) \geqslant U(b) + \Delta c \left(q_1(b) + \delta q_2(b, 0)\right), \tag{35}$$

$$U(u) \geqslant U(e) + \Delta c \left(q_1(e) + \delta q_2(e, 0)\right) + \delta h \left(B(e) - P(e)\right), \tag{36}$$

$$U(u) \geqslant U(g) + \delta h \left(B(g) - P(g)\right), \tag{37}$$

$$U(e) \geqslant U(u) - \Delta c \left(q_1(u) + \delta h q_2(u, A) + \delta(1-h) q_2(u, 0)\right) + \delta h \left(P(u) - B(u)\right), \tag{38}$$

$$U(e) \geqslant U(g) - \Delta c \left(q_1(g) + \delta h q_2(g, A) + \delta(1-h) q_2(g, 0)\right), \tag{39}$$

$$U(e) \geqslant U(b) + \delta h \left(P(b) - B(b)\right), \tag{40}$$

$$U(b) \geqslant U(e) + \delta h \left(B(e) - P(e)\right), \tag{41}$$

$$U(b) \geqslant U(u) - \Delta c \left(q_1(u) + \delta q_2(u, 0)\right), \tag{42}$$

$$U(b) \geqslant U(g) - \Delta c \left(q_1(g) + \delta q_2(g, 0)\right) + \delta h \left(B(g) - P(g)\right). \tag{43}$$

When the principal distributes the rents as explained in Equations (13) and (14), the incentive constraints (33), (34), (36) to (38), (40), (41), and (43), are maximally relaxed, while the remaining constraints are not affected.

D. Proof of Proposition 3.

Step 1 specifies the set of incentive constraints (12) under our assumption on the timing of rents paid to the agent:

$$P(g) \geqslant P(e) + \tfrac{\Delta c}{\delta h}\left(q_1(e) + \delta h q_2(e, A) + \delta(1-h)\, q_2(e, 0)\right), \tag{44}$$

$$P(g) \geqslant \tfrac{1-h}{h} B(b) + \tfrac{\Delta c}{\delta h}\left(q_1(b) + \delta h q_2(b, A) + \delta(1-h)\, q_2(b, 0)\right), \tag{45}$$

$$P(g) \geqslant \tfrac{1-h}{h} B(u), \tag{46}$$

$$B(u) \geqslant B(b) + \tfrac{\Delta c}{\delta}\left(q_1(b) + \delta q_2(b, 0)\right), \tag{47}$$

$$B(u) \geqslant \tfrac{\Delta c}{\delta}\left(q_1(e) + \delta q_2(e, 0)\right), \tag{48}$$

$$B(u) \geqslant 0, \tag{49}$$

$$P(e) \geqslant \tfrac{1-h}{h} B(u) - \tfrac{\Delta c}{\delta h}\left(q_1(u) + \delta h q_2(u, A) + \delta(1-h)\, q_2(u, 0)\right), \tag{50}$$

$$P(e) \geqslant P(g) - \tfrac{\Delta c}{\delta h}\left(q_1(g) + \delta h q_2(g, A) + \delta(1-h)\, q_2(g, 0)\right), \tag{51}$$

$$P(e) \geqslant \tfrac{1-h}{h} B(b), \tag{52}$$

$$B(b) \geqslant 0, \tag{53}$$

$$B(b) \geqslant B(u) - \tfrac{\Delta c}{\delta}\left(q_1(u) + \delta q_2(u, 0)\right), \tag{54}$$

$$B(b) \geqslant -\tfrac{\Delta c}{\delta}\left(q_1(g) + \delta q_2(g, 0)\right). \tag{55}$$

Step 2 asks which incentive constraints are potentially relevant.

Step 2.1. Inequalities (49), (53) and (55) follow from nonnegative profit constraints. Hence, the set of relevant incentive constrains belongs to (44) to (48), (50) to (52), and (54).

Step 2.2. By Step 2.1, inequality (54) is the unique relevant incentive constraint that is a lower bound on $B(b)$. Because the principal's objective is decreasing in $B(b)$, we may assume that

$$B(b) = \max\left\{B(u) - \tfrac{\Delta c}{\delta}\left(q_1(u) + \delta q_2(u, 0)\right), 0\right\}. \tag{56}$$

We may then view $B(b)$ as determined by the choice of $B(u)$ according to this equation.

Step 2.3. Equation (56) together with Equation (52) imply that Equation (50) is satisfied. Hence, we can ignore also constraint (50).

Step 2.4. Inequalities (45) and (47) are relaxed and the remaining incentive constraints are unaffected when investments by the agent of bad type are efficient (zero). Therefore,

$$q_1^{SB}(b) = q_2^{SB}(b, \alpha) = 0 \text{ for all } \alpha \text{ in set } \{0, A\}. \tag{57}$$

Step 2.5. By Equation (56) and set of equations (57), inequality (47) follows from nonnegative profit constraints. Therefore, inequality (48) is the unique relevant lower constraint on $B(u)$. Because the principal's objective is decreasing in $B(u)$, we may then assume that

$$B(u) = \tfrac{\Delta c}{\delta}\left(q_1(e) + \delta q_2(e, 0)\right). \tag{58}$$

Note that setting $q_2(e, 0) = 0$ is efficient, it permits a reduction of $B(u)$ through the equality (58), and it relaxes incentive constraint (44) without any effect on the remaining incentive constraints. Therefore,

$$q_2^{SB}(e, 0) = 0. \tag{59}$$

By equations (58) and (59),

$$B(u) = \tfrac{\Delta c}{\delta} q_1(e). \tag{60}$$

Step 2.6. By Equation (56) and set of equations (57), inequality (45) follows from inequality (46). Hence, there are two constraints limiting the choice of $P(g)$ from below, namely

$$P(g) \geqslant P(e) + \tfrac{\Delta c}{\delta h}\left(q_1(e) + \delta h q_2(e, A)\right) \text{ and} \tag{61}$$

$$P(g) \geqslant (1 - h)\tfrac{\Delta c}{\delta h} q_1(e). \tag{62}$$

By the non-negative profit constraint $P(e) \geqslant 0$, inequality (62) follows from inequality (61). Therefore, inequality (61) is the unique constraint limiting $P(g)$ from below. Because the principal's objective is decreasing in $P(g)$, we may assume

$$P(g) = P(e) + \tfrac{\Delta c}{\delta h}\left(q_1(e) + \delta h q_2(e, A)\right). \tag{63}$$

<u>Step 3</u>. Conjecture now that we can ignore constraint (51). Then the inequality (52) is the only relevant lower constraint on $P(e)$ and so can be assumed to hold with equality. If this conjecture is correct, then information rents are uniquely determined, in any optimal mechanism satisfying our assumptions on the timing of rents, by the investment decisions through the inequality (52) holding with equality, as well as the equalities (56), (60) and (63).

We therefore obtain

$$B(u) = \tfrac{\Delta c}{\delta} q_1(e), \tag{64}$$

together with

$$B(b) = \max\left\{\tfrac{\Delta c}{\delta} q_1(e) - \tfrac{\Delta c}{\delta}\left(q_1(u) + \delta q_2(u, 0)\right), 0\right\}, \tag{65}$$

also

$$P(e) = \tfrac{1-h}{h} \max\left\{\tfrac{\Delta c}{\delta} q_1(e) - \tfrac{\Delta c}{\delta}\left(q_1(u) + \delta q_2(u, 0)\right), 0\right\}, \tag{66}$$

and

$$\begin{aligned} P(g) = \tfrac{1-h}{h} \max\left\{\tfrac{\Delta c}{\delta} q_1(e) - \tfrac{\Delta c}{\delta}\left(q_1(u) + \delta q_2(u, 0)\right), 0\right\} \\ + \tfrac{\Delta c}{\delta h}\left(q_1(e) + \delta h q_2(e, A)\right). \end{aligned} \tag{67}$$

<u>Step 4</u> now solves problem (11) given the satisfaction of (64) to (67).

<u>Step 4.1</u> specifies problem (11) incorporating these values for rents. This yields the problem of maximizing by choice of $q_1(e)$, $q_2(e, A)$, $q_1(g)$, $q_2(g, A)$, $q_2(g, 0)$, $q_1(u)$, and $q_2(u, 0)$ the expression

$$\begin{array}{l}\lambda r W(g)+(1-\lambda)\, r W(e) \\ -\lambda r \Delta c\left(q_1(e)+\delta h q_2(e, A)\right) \\ -\Delta c \max \left\{q_1(e)-q(u), 0\right\}(r(1-h)+(1-\lambda)(1-r)) \\ -\lambda(1-r)\left[\underline{c} q(u)+\Delta c q_1(e)\right]\end{array} \tag{68}$$

where $q(u)=q_1(u)+\delta q_2(u,0)$.

Step 4.2 shows that investments by the agent of the good type are efficient:

$$q_1^{SB}(g)=q_1^{FB}(g),\ q_2^{SB}(g,\alpha)=q_2^{FB}(g,\alpha) \text{ for either } \alpha \text{ in set } \{0,A\}. \tag{69}$$

This follows from optimization with respect to $q_1(g)$ and $q_2(g,\alpha)$.

Step 4.3 shows that $q^{SB}(u) \in \left\{0, q_1^{SB}(e)\right\}$. Indeed, any positive investment by the agent of useless type is wasteful. It allows saving rents as long as it lies no higher than $q_1^{SB}(e)$. Therefore,

$$q^{SB}(u) \leqslant q_1^{SB}(e) \tag{70}$$

(where $q^{SB}(u)=q_1^{SB}(u)+\delta q_2^{SB}(u,0)$). By set of equations (69), the principal's problem (68) is equivalent to maximizing by choice of $q_1(e)$, $q_2(e,A)$, $q_1(u)$, and $q_2(u,0)$

$$\begin{array}{c} r(1-\lambda)W(e)-\lambda r \Delta c\left(q_1(e)+\delta h q_2(e, A)\right) \\ -\Delta c\left(q_1(e)-q(u)\right)(r(1-h)+(1-\lambda)(1-r)) \\ -\lambda(1-r)\left[\underline{c} q(u)+\Delta c q_1(e)\right]\end{array} \tag{71}$$

subject to inequality $q(u) \leqslant q_1(e)$. Problem (71) is linear in $q(u)$ with coefficient which is equal to the difference between the right- and the left-hand sides of inequality (23). If this coefficient is positive, that is, inequality (23) holds, constraint $q(u) \leqslant q_1(e)$ is binding:

$$q^{SB}(u)=q_1^{SB}(e). \tag{72}$$

Otherwise, $q^{SB}(u)=0$.

Step 4.4 considers the situation in which inequality (23) holds. By equation (72), $q_1^{SB}(e)$ and $q_2^{SB}(e,A)$ solve

$$\max_{q_1(e),q_2(e,A)} r(1-\lambda)W(e) - \lambda r\Delta c\left(q_1\left(e\right) + \delta h q_2\left(e,A\right)\right) - \lambda\left(1-r\right)\overline{c}q_1\left(e\right).$$

Using the Inada conditions and concavity of the objective, there exists a unique solution to this optimization. Using the first-order conditions, we find that $q_1^{SB}\left(e\right)$ and $q_2^{SB}\left(e,A\right)$ are given by set of equations (18).

Step 4.5 considers the situation in which inequality (23) is violated. Then, we find that investment by the agent of useless type is zero and hence efficient. By an analogous argument to the previous case, using the first-order conditions, we find that the investments $q_1^{SB}\left(e\right)$ and $q_2^{SB}\left(e,A\right)$ by the agent of expensive type are given by set of equations (22).

Step 5 verifies that constraint (51), ignored in Step 4, is satisfied in the above solution.

Step 5.1 supposes that inequality (23) holds. Then, by sets of equations (69) and (18),

$$\begin{aligned} & q_1^{SB}\left(e\right) + \delta h q_2^{SB}\left(e,A\right) < q_1^{FB}\left(e\right) + \delta h q_2^{FB}\left(e,A\right) \\ & < q_1^{FB}\left(g\right) + \delta h q_2^{FB}\left(e,g\right) = q_1^{SB}\left(g\right) + \delta h q_2^{SB}\left(e,g\right). \end{aligned} \tag{73}$$

therefore, the incentive constraint (51) is verified.

Step 5.2 supposes that inequality (23) is reversed. By sets of equations (69) and (22), set of inequalities (73) holds. Once again, the incentive constraint (51) ignored in Step 4 is verified.

Step 6 concludes with noticing that $q_1^{SB}\left(e\right)$ is positive by the Inada conditions, while $q_2^{SB}\left(e,A\right)$ is positive by comparison to the case with no information asymmetry on usefulness (see Section 2.2 and the discussion in the main text). This is true no matter whether $q_1^{SB}\left(e\right)$ and $q_2^{SB}\left(e,A\right)$ are given by set of equations (22) or (18).

Step 7 notes that, except when the condition (23) holds as in equality, the characterization obtained here is unique (recalling that only the discounted investment of the useless but cheap type $q(u)$ is uniquely pinned down by this characterization, and recalling that the allocation of rents over time was shown in Appendix C to be optimal, though not necessarily uniquely so). Uniqueness follows because the investments determined by our optimizations above yield unique investment choices (up to the timing of investments by type u), and because rents are then uniquely determined by Equations (64)-(67). When (23) holds as an equality, both characterizations in Part (i) and Part (ii) of the proposition are valid.

E. Proof of Proposition 4.

Step 1. Let $l \in [0, h)$. We first list the set of incentive constraints, analogous to Equations (44) to (55) in the previous proof:

$$P(g) \geqslant P(e) + \frac{\Delta c}{\delta h}\left(q_1(e) + \delta h q_2(e, A) + \delta(1-h) q_2(e, 0)\right), \tag{74}$$

$$P(g) \geqslant \frac{1-h}{h} B(b) + \frac{\Delta c}{\delta h}\left(q_1(b) + \delta h q_2(b, A) + \delta(1-h) q_2(b, 0)\right), \tag{75}$$

$$P(g) \geqslant \frac{1-h}{h} B(u), \tag{76}$$

$$B(u) \geqslant B(b) + \frac{\Delta c}{\delta(1-l)}\left(q_1(b) + \delta l q_2(b, A)\right), \tag{77}$$

$$B(u) \geqslant \frac{l}{1-l} P(e) + \frac{\Delta c}{\delta(1-l)}\left(q_1(e) + \delta l q_2(e, A) + \delta(1-l) q_2(e, 0)\right), \tag{78}$$

$$B(u) \geqslant \frac{l}{1-l} P(g), \tag{79}$$

$$P(e) \geqslant \frac{1-h}{h} B(u) - \frac{\Delta c}{\delta h}\left(q_1(u) + \delta h q_2(u, A) + \delta(1-h) q_2(u, 0)\right), \tag{80}$$

$$P(e) \geqslant P(g) - \frac{\Delta c}{\delta h}\left(q_1(g) + \delta h q_2(g, A) + \delta(1-h) q_2(g, 0)\right), \tag{81}$$

$$P(e) \geqslant \frac{1-h}{h} B(b), \tag{82}$$

$$B(b) \geqslant \frac{l}{1-l} P(e), \tag{83}$$

$$B(b) \geqslant B(u) - \frac{\Delta c}{\delta(1-l)}\left(q_1(u) + \delta l q_2(u, A) + \delta(1-l) q_2(u, 0)\right), \tag{84}$$

$$B(b) \geqslant \frac{l}{1-l} P(g) - \frac{\Delta c}{\delta(1-l)}\left(q_1(g) + \delta l q_2(u, A) + \delta(1-l) q_2(u, 0)\right). \tag{85}$$

The principal faces a choice of policy $M(l)$ as described in the main text, which is subject to the incentive constraints above, to the requirement that $0 \leq q_1(\theta), q_2(\theta, \alpha) \leq \bar{q}$ for all $\theta \in \Theta$ and $\alpha \in \{0, A\}$, and to the requirement that the second-period rents $P(g)$, $P(e)$, $B(u)$, and $B(b)$ are non-negative. In addition, note that, because S is bounded above, and yet an available policy is to induce no investment and pay the agent zero, we may assume these rents are also bounded above (say by some $\bar{R} > 0$) and we impose this bound from now on.

Step 2. For each $l < h$, let $\Gamma(l)$ denote the set of policies $M(l)$ satisfying the above constraints, and note that $\Gamma(l)$ is compact. We will show in addition that there is $\bar{l} \in (0, h)$ such that $\Gamma(l)$ is a continuous correspondence over $[0, \bar{l}]$. Upper hemi-continuity of Γ follows because the constraint set is always non-empty (it includes the null policy where the agent does not invest and is paid nothing) and because the right-hand side of Equations (74) to (85) are continuous in l.

To obtain lower hemi-continuity over this set for small enough $\bar{l}$, observe that there is a choice of M, call it M^*, such that the following is true. There exists $\bar{l}, \kappa > 0$ such that all incentive constraints in Equations (74) to (85) hold by at least κ for all $l \in [0, \bar{l}]$ (i.e., the difference between the left and right-hand sides is at least κ). To see this, it is enough to consider a policy M^* where the only positive investments are $q_1(g) = q_1(u) = \bar{q}$, and where $B(b) = \varepsilon$, $B(u) = 2\varepsilon$, $P(g) = 4\varepsilon\frac{1-h}{h}$, and $P(e) = 2\varepsilon\frac{1-h}{h}$ for some ε sufficiently small. (For ε small enough, all incentive constraints are strict for $l = 0$, and hence by continuity also for l in a sufficiently small neighborhood of zero.) Note that this also means that M^* can be chosen to satisfy the upper and lower bounds on investments and rents.

Taking the values of $\bar{l}$ and κ selected above, consider any $l' \in [0, \bar{l}]$ and any corresponding policy $M' \in \Gamma(l')$. Consider any sequence (l_n) in $[0, \bar{l}]$ with $l_n \to l'$. Note that we can find a sequence (M_n) with $M_n \to M'$ and $M_n \in \Gamma(l_n)$ for all n as follows. For each n, put $M_n = (1 - \alpha_n)M' + \alpha_n M^*$, where $\alpha_n \in [0,1)$ is chosen to ensure that $M_n \in \Gamma(l_n)$. That we can choose (α_n) so that $\alpha_n \to 0$ follows because policy M', to the extent it violates the constraints of the program for l_n, only violates the incentive constraints (74) to (85) and then only by a vanishing amount as $n \to \infty$ (this is true by continuity of the right-hand side of these constraints in l). Moreover, for policy $(1-\alpha_n)M' + \alpha_n M^*$, the differences between the left and right sides of constraints (74) to (85) are linear in α_n.

Step 3. Now consider the principal's objective for $l \in [0, \bar{l}]$. This is to maximize by choice of a policy $M \in \Gamma(l)$ the expression

$$\begin{aligned}&\mu(g)(W(g) - \delta h P(g)) + \mu(e)(W(e) - \delta h P(e)) \\ &+\mu(u)(W(u) - \delta(1-l)B(u)) + \mu(b)(W(b) - \delta(1-l)B(b)),\end{aligned}$$

where recall that $W(\theta)$ represents the discounted expected surplus generated by type θ under the investments $q_1(\theta), q_2(\theta, \alpha)$ undertaken by type θ, which note depends also on the probability l. Fixing the other parameters, this expression is continuous in both M and l. Moreover, as argued above, the constraint set $\Gamma(l)$ is non-empty, compact-valued and continuous on $[0, \bar{l}]$. We conclude by the Theorem of the Maximum that the set of solutions $M^{SB}(l)$ is an upper hemi-continuous correspondence on $[0, \bar{l}]$, and hence is upper hemi-continuous in particular at $l = 0$. This concludes the proof.