WORKING PAPERS

N° 1429

August 2025

“Regulating investments when both costs and need are private”

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Regulating investments when both costs and need are private.*

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August 1, 2025

Abstract

Large-scale infrastructure investments are often carried out in settings where their eventual usefulness or importance is difficult to predict. This paper studies optimal incentives for investment when the agent undertaking the investment has superior information on two dimensions: the cost of investment and the likelihood it is useful or beneficial to the principal. Usefulness eventually becomes public, but punishments are limited as the regulator aims at ensuring the agent earns non-negative profits irrespective of eventual usefulness. We characterize the optimal screening mechanism and show that the optimal mechanism depends heavily on the degree of asymmetric information about usefulness. When the asymmetry of information about usefulness is severe, the optimal mechanism can feature upward distortions in investment and rent for all agent types.

Key words: monopoly regulation, multidimensional screening.
JEL codes: D81, D82, L51.

*Early work on this paper was supported by a research partnership between the French electricity transmission network RTE and Toulouse School of Economics. We are especially grateful for discussions with Max Papon at RTE. Our work has also been informed by discussion with Dzhordzhio Naldzhiev at the UK energy regulator Ofgem. We are extremely grateful to the Editor and anonymous Referees for their insightful comments and suggestions. We also thank Modibo Camara, David Martimort, Humberto Moreira, Karl Schlag, and the audience at the 2025 SAET Conference in Ischia for their valuable feedback and stimulating discussions. Both authors acknowledge funding from the French National Research Agency (ANR) under the Investments for the Future (Investissements d'Avenir) program (grant ANR-17-EURE-0010). Garrett has been supported by funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement number 714147) and by funding from a UKRI Frontier Research Grant (grant number EP/Z001528/1).

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Introduction.

A critical issue for the regulation of public utilities is incentive provision for large investments. Part of the difficulty for regulation is that the eventual usefulness of these investments can be highly uncertain. Moreover, because the infrastructure provider understands its business better than the regulator, the likelihood that a given investment will be useful can be part of the private information of the regulated firm. Designing incentive schemes for investment in such an environment is potentially challenging because the firm's information is then likely to be *multidimensional*; the firm may have private information on *both* costs and the probable benefit of investment to the principal.

An important example of this kind of scenario is the regulation of investments by an electricity transmission network driven by the anticipated transition to clean energies. For instance, investments in the grid may be needed if the network anticipates the development of large-scale wind farms, but such development takes years and its eventual extent is subject to regulatory uncertainty as well as uncertainty about the eventual energy mix. This may in turn reflect factor prices and societal preferences. Further, the unpredictable evolution of the climate can create uncertainty on both supply and demand sides. These considerations feed into technological uncertainties about the nature and capacity of investments that are likely to be needed, and these uncertainties might in some cases be best understood by the network business.[1]

Setting. To better understand regulation in the above environment, we consider a model in which the agent (the network business, "he") invests on behalf of the principal (the regulator, "she"). The agent aims at maximizing

[1] A described by Broer and Zwart (2013, p. 178), similar concerns arise for local electricity distribution networks, as these "are expected to make significant investments in their grid capacity, or in smarter grids, if electric vehicles become more wide-spread, or as larger penetration of distributed generation - necessary to achieve lower carbon electricity generation - changes local flow patterns."

payments from the principal minus the cost of investment. The principal aims at maximizing the value of investment in case it turns out to be useful minus payments to the agent. At the outset, the agent has private information on two dimensions: the marginal cost of investment and the probability that investment is useful. Uncertainty about usefulness is eventually resolved and usefulness becomes public information. Investment costs, however, remain private.

The presence of multidimensional information leads to a potentially complex screening problem. Similar to Dana (1993), Armstrong (1999), and Armstrong and Rochet (1999), we reduce this complexity by studying the case where there is binary information on each dimension. On one dimension, the agent may have a high or low cost. On the other, the probability that the investment is useful is either high or low. The possibility of investment when the likelihood of usefulness is low relates to a central difficulty faced by regulators in practice. An agent who believes a given investment is unlikely to actually be needed may face incentives to undertake it anyway. This is similar to the problem of "gold plating of assets" familiar to network regulators.

The principal uses money and investment quantities to regulate the agent; in particular, we assume that the agent's actual investment (but not its cost) is perfectly observed by the principal. Furthermore, the usefulness of investment, when it eventually becomes public information, is contractible.

A key restriction on the available mechanisms is that the principal must ensure the agent obtains non-negative profits regardless of whether the investment turns out to be useful.[2] The profit constraints we impose are motivated particularly by a possible regulatory concern for ensuring the agent can ob-

[2] As we discuss below, without a restriction on agent ex-post profits (but with the usual interim participation constraint), the optimal investment policy turns out to coincide with the one when information about the usefulness of investment is symmetric. This policy is easier to characterize and follows from analysis to a large extent already understood in the existing literature.

tain finance. Policies where firms may fail to have costs reimbursed seem likely, at the least, to raise financing costs. We take the extreme view that the regulator therefore aims at ensuring losses do not occur. This focus may be seen as providing a benchmark against which to compare regulatory policies which do not aim at full cost recovery in all instances (e.g., that provide for "cost disallowances" or other penalties). Our restriction on profits is in common with some existing work that has similar motivations; e.g., Krishna et al. (2013) and Krasikov and Lamba (2021). We discuss these contributions further in the Related Literature below.

Findings. We characterize the optimal incentive scheme based on the relative spread of the principal's uncertainty over the two dimensions of the agent's private information. When the principal's uncertainty about probability of usefulness is relatively low (the region below the solid curve in Figure 1, Section 3), the optimal regulation qualitatively resembles the classic regulation scheme which is optimal when the principal is uncertain only about cost: cost-efficient types invest efficiently and earn information rents, while cost-inefficient types earn no rents and may face downward-distorted investment, resolving a familiar rent-efficiency trade-off. By contrast, when the principal's uncertainty about probability of usefulness is relatively high (in the region above the solid curve in Figure 1), the optimal scheme features upward investment distortions for the cost-efficient but low-usefulness type, and, when the spread of uncertainty about the likelihood of usefulness becomes particularly high (in the region above the dashed curve in Figure 1), all types receive information rents.

The possibility of rents for all types relates to our finding of a cycle of binding incentive constraints. First, the cost-inefficient and high-usefulness type makes positive investments given that small investments are assumed highly valued by the principal. The cost-efficient and low-usefulness type then has the opportunity to mimic the cost-inefficient and high-usefulness type at a time when true usefulness is not yet observed. The cost-efficient and low-usefulness type therefore earns a rent. Assuming the cost-efficient and low-usefulness type makes only a small investment, the cost-*in*efficient and low-usefulness type must also be granted a rent to prevent mimicry of the former (noting both types have similar costs of generating only *small* investments). The cost-inefficient and high-usefulness type must also be granted a rent to prevent mimicry of the cost-inefficient and low-usefulness type. This completes the relevant cycle of incentive constraints. The cost-efficient and high-usefulness type earns an additional rent relative to the cost-inefficient and high-usefulness type owing to the lower production cost.

Now consider why we find upward distortions for the cost-efficient and low-usefulness type. The reason is that this can deter mimicry by, and thus help reduce the rents left to, the cost-inefficient and low-usefulness type (and thus reduce rents left to other types as well). An extreme case (considered as an extension to our baseline model in Appendix E) is where low-usefulness types know that investment is useless for sure. In this case, the cost-efficient and low-usefulness type may invest in equilibrium while knowing that investment is certain not to be valued by the principal.

It is worth highlighting that multidimensionality of information and non-negative profit constraints are crucial to the finding of rents for all types and of upward distortions in investment. If cost information is public but the probability of usefulness private, the agent earns no rents and investment is efficient. If usefulness information is public but cost information is private, cost-inefficient types earn no rents but have downward distorted investment while cost-efficient types invest efficiently. If the non-negative profit constraints are omitted and replaced by an interim participation constraint (i.e., the agent, upon learning his type, decides whether to participate), then investments and interim payoffs in the optimal mechanism are the same as in the case where usefulness information is public.

Related Literature. Our work contributes to the literature on multidimensional screening. As mentioned, our analysis is tractable because we follow the approach of Dana (1993), Armstrong (1999), and Armstrong and Rochet (1999) in considering a "2x2" model where there are two dimensions of private information, with a binary signal on each dimension. Similar to our work, this approach allows Armstrong and Armstrong and Rochet to demonstrate the possibility of upward distortions in production (contrary to the usual downward distortions found in one-dimensional models).

Our model is perhaps closest to Armstrong (1999), who studies optimal price regulation of a monopolist with private information about both demand and cost (there, as in our paper, the two dimensions of the agent's type may be thought of as "non-separable" in that they do not relate to separate activities; see Armstrong and Rochet (1999, p. 972)). When demand variation is less significant than cost variation, so that privately desirable prices are ordered in the same way as socially desirable prices, the optimal regulated prices are (weakly) above marginal cost, as in the benchmark case without uncertainty about demand. Otherwise, "the analysis becomes more complex, and sub-marginal pricing may be optimal" (p. 209). The relative magnitude of information asymmetries across the different dimensions of the agent's type therefore plays an important role, as in our paper.

It is perhaps worth pointing out that, contrary to the focus of the wider literature on multidimensional screening (including among others Wilson, 1993, Armstrong, 1996, and Rochet and Choné, 1998), our specification has a "common values" element in that one dimension of the agent's type determines the preferences of the principal and not the agent. Note, however, that Chade et al. (2022) and Gottlieb and Moreira (2023) are recent contributions that treat multidimensional screening in environments with some interdependence in values.

Our paper contributes to a literature on investments under private information on costs where uncertainty about the value of investments is resolved over time. This includes Maeland (1999), Broer and Zwart (2013), Arve (2016), and Willems and Zwart (2018). These papers tend to be motivated by similar applications to our work, but how investments are valued is public information, unlike our main setting of interest. The concern of multidimensional screening therefore does not arise.[3]

Also relevant to our analysis are papers that examine mechanism design under correlated information. For instance, Crémer and McLean (1985, 1988) examine full extraction of agent rents in auctions where bidders have correlated types. Riordan and Sappington (1988) examines an analogous question in a principal-agent environment where, like our paper, there is eventual realization of a public signal that is correlated with the agent's initial private information. While first-best outcomes can often be sustained in these settings, constraints on payments are known to limit the applicability of these results. Papers such as Demski et al. (1988) and Demougin and Garvie (1991) seek to understand the optimal design response to bankruptcy and limited liability constraints when correlated information is available. Our paper also contributes to this agenda, although in a setting where the agent has a multidimensional type, and where the public signal is correlated with only one dimension of the agent's information (that concerning the usefulness of investment to the principal). Hence, in our problem, correlation can be exploited for extracting information only on this dimension.

Finally, it is worth highlighting that our finding of rents for all types is unusual in the mechanism design literature broadly. For instance, it does not occur in the aforementioned literature on multidimensional screening in static settings. The reason is that if all types *were* to strictly prefer participation in the mechanism as compared to their outside option, the mechanism could be perturbed by uniformly lowering rents without disturbing incentive compatibility.

[3] Grenadier and Wang (2005) is another study where there is investment by an agent and the benefit to the principal evolves stochastically, but in that work the agent has private information about the benefit. Again, screening is one dimensional in that paper.

However, positive rents to all types are found in some instances of the literature on dynamic mechanism design, where agents learn private information about their types over time.[4] For example, Krishna et al. (2013) and Krasikov and Lamba (2021), impose per-period non-negative profit constraints and find that all types may expect a positive dynamic rent owing to the possibility that they have favorable types (costs) affecting their own payoffs in the future. Similarly, in settings with dynamic participation decisions such as Garrett (2017) and Bergemann and Strack (2022), agents may earn positive rents owing to the possibility of delaying participation until types affecting their payoffs (valuations) become favorable. Our finding of positive rents for all types is different, however – the information which is learned after time in our model does not directly enter the agent's payoff.

Roadmap. Section 1 describes the model. Section 2 establishes two useful benchmarks: the optimal mechanism when information is symmetric, and the optimal mechanism when the agent has superior information solely on either the cost of investment or the usefulness but not both. Section 3 characterizes the optimal mechanism when agent private information is multidimensional. Section 4 concludes.

1 Basic model.

This section introduces our central model. We delay some comments on the model until the end of the section.

The agent ("he") operates a network business on behalf of the principal ("she"). The agent's role is to make possible investments in network enhancement. The principal designs a regulatory contract that aims at maximizing

[4] Classic references in the dynamic mechanism design literature include Baron and Besanko (1984) and Pavan et al. (2014). Our paper is loosely related to this literature as uncertainty is also resolved after time in our model, although the information is revealed publicly.

the expected benefits of investment net of payments to the agent. Both principal and agent are assumed to be able to fully commit to this contract. The investment technology is described as follows.

Investment technology. Investment in network enhancement is denoted by q. The agent incurs only variable investment costs, with a constant marginal cost c. The investment delivers the principal gross return $\alpha S(q)$, where $S(\cdot)$ is a strictly increasing, strictly concave, and continuously differentiable function satisfying $S(0) = 0$. We assume, moreover, that S is bounded above and hence $\lim_{q \to \infty} S'(q) = 0$, and also that $\lim_{q \downarrow 0} S'(q) = \infty$ (these last two conditions are the "Inada conditions"). Parameter α captures the usefulness or need for the investment and is uncertain when the investment takes place. In particular, it can be that the investment is not at all beneficial, $\alpha = 0$, or it has a positive benefit to the principal, in which case $\alpha = 1$.[5] While the value of α is uncertain when the investment takes place (see the summary of the timing of events at the end of this section), its realization is commonly observed by the parties and directly contractible.

Information. It is commonly known that the marginal cost of investment c may take either a relatively low value $\underline{c} > 0$ or a relatively high value $\overline{c} > \underline{c}$. The set of possible costs is denoted with $\mathbb{C} = \{\underline{c}, \overline{c}\}$. It is commonly known at the outset that c takes the low value with a probability λ in $(0, 1)$:

$$c = \begin{cases} \underline{c} \text{ with probability } \lambda, \\ \overline{c} \text{ with probability } 1 - \lambda. \end{cases}$$

We use standard notation $\Delta c = \overline{c} - \underline{c}$ for the range of costs. The agent privately knows the realization of c.

The agent also has superior information regarding the distribution of parameter α. Indeed, the agent knows that α will turn out to equal 1 with a

[5]The possibility that investment is not at all beneficial (rather than merely providing small benefits) is a convenient simplification.

probability p in $(0,1)$. We therefore have:

$$\alpha = \begin{cases} 1 \text{ with probability } p, \\ 0 \text{ with probability } 1-p. \end{cases}$$

We allow that the probability of usefulness p can take two possible values: either a relatively high value h or a lower value l, where $0 < l < h < 1$. (Our results extend to cases where the probabilities l and h take the extreme values (that is, $l = 0$ and/or $h = 1$), as specified in Appendix E.) The set of possible realizations of p is denoted with $\mathbb{P} = \{l, h\}$. It is commonly known at the outset that p takes the value h with a probability r, and also that p is distributed independently of the marginal cost c. Thus, we have

$$p = \begin{cases} h \text{ with probability } r, \\ l \text{ with probability } 1-r. \end{cases}$$

We let $\Delta p = h - l$.

We denote the agent's two dimensional type by (c, p), and we use notation $\mu(c, p)$ for the probability of the agent's type being (c, p). By the above distributional assumptions,

$$\mu(\underline{c}, h) = \lambda r,\ \mu(\overline{c}, h) = (1-\lambda) r,\ \mu(\underline{c}, l) = \lambda(1-r),\ \mu(\overline{c}, l) = (1-\lambda)(1-r).$$

Regulation subject to financing constraints. Investments and realization of α are publicly observable and verifiable. Without loss of generality (Dasgupta et al., 1979; Myerson, 1981), the principal commits to a direct mechanism:

$$(q(\widehat{c}, \widehat{p}), T(\widehat{c}, \widehat{p}, 1), T(\widehat{c}, \widehat{p}, 0)), \tag{1}$$

where $(\widehat{c}, \widehat{p})$ is the agent's report of type (c, p), q is the required investment, and T is the agent's compensation.

We assume that a regulatory objective is to ensure the agent receives non-negative profits in equilibrium. Given our focus on direct mechanisms, this means when the agent reports his information truthfully. Formally, we require that, for all types $(c, p) \in \mathbb{C} \times \mathbb{P}$, and all $\alpha \in \{0, 1\}$,

$$T(c, p, \alpha) - cq(c, p) \geqslant 0. \tag{2}$$

Payoffs. With the description of the mechanism in place, it may now be helpful to summarize the payoffs of the players. The principal's payoff in the direct mechanism, given agent report $(\widehat{c}, \widehat{p})$ and the realization of usefulness α, is

$$\alpha S(q(\widehat{c}, \widehat{p})) - T(\widehat{c}, \widehat{p}, \alpha).$$

The agent's payoff, given true type (c, p), report $(\widehat{c}, \widehat{p})$, and given the realization of usefulness α, is

$$T(\widehat{c}, \widehat{p}, \alpha) - cq(\widehat{c}, \widehat{p}).$$

Note that the agent cares about the investment cost but not directly about its usefulness. Conversely, the principal values the usefulness of the investment but is not directly concerned with its cost.

Timing of events. The timing of events can be summarised as follows.

1. The agent privately learns his type (c, p).
2. The principal commits to a direct mechanism (1).
3. The agent sends message $(\widehat{c}, \widehat{p})$ to the mechanism.
4. The agent invests quantity $q(\widehat{c}, \widehat{p})$.
5. The uncertainty about usefulness α is publicly resolved and the agent receives transfer $T(\widehat{c}, \widehat{p}, \alpha)$.

1.1 Discussion of the model.

We now provide some comments on the model.

Private information on costs. It is worth highlighting that, while usefulness eventually becomes commonly observed, the agent's costs remain his private information. This is the usual case in regulatory models with private costs such as Baron and Myerson (1982). It is therefore important to interpret costs as those that a regulator would not eventually readily observe. Examples may include managerial costs of effort to undertake investment, or opportunity costs of other foregone investments in the network.

Financing constraints. As mentioned in the Introduction, a possible rationale for a regulatory objective that adheres to financial constraints is that subjecting firms to the risk of losses may increase financing costs (not only would firms face increased risk, but they would need to ensure sufficient reserves to weather any losses imposed by the regulatory scheme). While we expect there is a trade-off in practice, we simplify in this paper by taking the extreme view that the regulatory scheme should be designed to avoid any losses ex post. Our results might then be seen as a benchmark against which improvements in regulatory performance due to introducing some penalties can be measured. (Such penalties might include, for instance, what regulators call "cost disallowances" when investments turn out not to be useful.)

A further reason for our choice of focus can be understood by considering what happens if we remove the non-negative profit constraints and replace them with an interim participation constraint (as is more commonly studied). That is, when the principal commits to the mechanism, the agent (knowing his type) decides whether to accept it and participate or to reject it for a type-independent outside option (typically normalized to zero). Then, the fact that the agent is initially privately informed about usefulness would not affect regulatory performance: investments under the optimal mechanism would be the same as if the probability of usefulness were public, and the analysis would be fairly easily anticipated in light of the existing literature. As we discuss further in Section 2, this is because the agent could effectively be asked to "bet" at date 1 on the observed future state and then subjected to rewards or (unrestricted) penalties to incentivize revelation of the relevant information on usefulness. Our analysis is therefore pertinent to understanding optimal regulation when such "bets", and in particular the corresponding penalties, are restricted.

2 Helpful benchmarks.

In this section, we consider three benchmarks that will be useful for understanding the solution to the problem of interest.

Complete information benchmark. The first benchmark is the situation in which the principal knows (c, p). We refer to this solution as "first best" (and denote the first-best policy by superscript "FB"). In this case, the principal finds it optimal to choose investment $q^{FB}(c, p)$ efficiently, that is, so as to maximize the expected net return

$$W(c, p) = pS(q(c, p)) - cq(c, p). \tag{3}$$

The mechanism compensates the investment cost to the agent: for both realisations of α,

$$T^{FB}(c, p, \alpha) = cq^{FB}(c, p). \tag{4}$$

Proposition 1. *First-best investments are given, for all (c, p), by*

$$pS'\left(q^{FB}(c, p)\right) = c. \tag{5}$$

Naturally, for any given probability of usefulness p, the first-best investment by the cost-efficient type $(\underline{c}, p)$ is higher than that of the cost-inefficient type $(\overline{c}, p)$:

$$q^{FB}(\underline{c}, p) > q^{FB}(\overline{c}, p). \tag{6}$$

Similarly, for any given cost level c, the first-best investment by the high-probability of usefulness type (c, h) is higher than that of the low-probability of usefulness type (c, l):

$$q^{FB}(c, h) > q^{FB}(c, l). \tag{7}$$

At the same time, the first-best investment by the cost-inefficient but high-probability of usefulness type $(\overline{c}, h)$ may be either higher or lower than that of the cost-efficient but low-probability of usefulness type $(\underline{c}, l)$, depending on the relative dispersion of probability of usefulness and cost types. Specifically,

$$q^{FB}(\overline{c}, h) > q^{FB}(\underline{c}, l) \text{ if and only if } \tfrac{\Delta c}{\underline{c}} < \tfrac{\Delta p}{l}. \tag{8}$$

Benchmark: no asymmetry on cost. Our second benchmark is the situation in which the probability of usefulness p is the agent's private information while the marginal cost of investment c is commonly observed and contractible.[6] Then, the principal can achieve the "first-best" outcome by compensating the cost of efficient investment without any excess. Under such compensation scheme the agent has no incentives to lie about the probability of usefulness.

Benchmark: no asymmetry on probability of usefulness. The last benchmark we consider is the situation in which the probability of usefulness p is commonly observed and is contractible, while the marginal cost of investment c is the agent's private information. The analysis is similar to many other settings with information asymmetry on a single dimension (for a treatment of such models, see Laffont and Martimort, 2002).

We find it convenient to introduce here notation that will be used in the rest of the paper (where both cost and the probability of usefulness will be the agent's private information). For any true type (c, p) and message $(\widehat{c}, \widehat{p})$,

$$U(c, p, \widehat{c}, \widehat{p}, \alpha) = T(\widehat{c}, \widehat{p}, \alpha) - cq(\widehat{c}, \widehat{p}).$$

This corresponds to the agent's rents, as they depend on the realization of α, when true type is (c, p) and the message sent is $(\widehat{c}, \widehat{p})$. We will find it useful

[6] We are grateful to an anonymous referee for suggesting that we consider this benchmark.

to view these rents as control variables in solving the principal's problem, rather than maximizing with respect to transfers.

As further notation, we write

$$U(c, p, \widehat{c}, \widehat{p}) = pU(c, p, \widehat{c}, \widehat{p}, 1) + (1 - p) U(c, p, \widehat{c}, \widehat{p}, 0)$$

for the expected rents by the agent of type (c, p) sending message $(\widehat{c}, \widehat{p})$. In addition, in a more or less standard abuse, we introduce the following notation for rents earned in equilibrium. We write:

$$U(c, p) = U(c, p, c, p)$$

for the expected rents earned by the agent revealing his type truthfully. Then,

$$P(c, p) = U(c, p, c, p, 1)$$

denotes the agent's rents when he truthfully reveals his type and the investment turns out to be useful. On the other hand,

$$B(c, p) = U(c, p, c, p, 0)$$

denotes the agent's rents when the agent truthfully reveals his type and the investment turns out to be useless.

The present task is to determine the optimal mechanism when the probability of usefulness p is public information, so as a notational device we allow p to enter the mechanism directly (instead of the agent reporting it). We thus search for an optimal screening mechanism among direct mechanisms incentivising the agent to truthfully report his marginal cost of investment c. Therefore, given the commonly known probability of usefulness p, the principal's problem is to maximize by choice of $\{q(c, p), P(c, p), B(c, p) : c \in \mathbb{C}\}$

$$\begin{aligned} &\lambda [W(\underline{c}, p) - pP(\underline{c}, p) - (1 - p)B(\underline{c}, p)] \\ &+(1 - \lambda) [W(\overline{c}, p) - pP(\overline{c}, p) - (1 - p)B(\overline{c}, p)] \end{aligned} \tag{9}$$

subject to the incentive compatibility constraints

$$U(c,p) \geqslant U(c,p,\widehat{c},p) \text{ for any } c \in \mathbb{C} \text{ and any } \widehat{c} \in \mathbb{C} \setminus \{c\}, \tag{10}$$

together with the non-negative profit constraints. These profit constraints require that, for each $c \in \mathbb{C}$,

$$P(c,p) \geqslant 0 \text{ and } B(c,p) \geqslant 0. \tag{11}$$

We will mark the solution to the above problem with a "tilde" $\widetilde{\cdot}$. The analysis of the principal's optimal mechanism is by now relatively standard in the literature (see Appendix A for details). In order to provide the agent of either cost-efficient type with incentives for revealing cost efficiency, the principal must pay him rents:

$$U(\underline{c},h) = \Delta c \widetilde{q}(\overline{c},h) \text{ and } U(\underline{c},l) = \Delta c \widetilde{q}(\overline{c},l). \tag{12}$$

The principal distorts investments by cost-inefficient types downwards as compared to the benchmark in Proposition 1. This reflects the classic rents-efficiency trade-off. Specifically,

$$hS'(\widetilde{q}(\overline{c},h)) = \overline{c} + \Delta c \tfrac{\lambda}{1-\lambda},\ lS'(\widetilde{q}(\overline{c},l)) = \overline{c} + \Delta c \tfrac{\lambda}{1-\lambda}. \tag{13}$$

The cost efficient types invest efficiently, which is the classic "no distortions at the top" result. The main insights are summarized as follows.

Proposition 2. *Suppose that the principal knows probability of usefulness p, while cost c is the agent's private information. Then, the optimal screening mechanism is the following. Investments by the agent of either cost-efficient type are efficient, as described by set of equations (5). These types receive expected information rents given in set of equations (12). Investments by the agent of cost-inefficient types are distorted downwards, as specified by set of equations (13). These investments are reimbursed without any excess (no rents).*

We conclude this section with a remark regarding the role of the non-negative profit constraints related to our discussion of financing constraints in Section 1. Suppose that the state-specific non-negative profit constraints were replaced by the more commonly imposed interim participation constraint. That is, the agent, after learning his type, has the option to participate in the mechanism or to reject it for an outside option, normalised to zero. Then information asymmetries relating to the likelihood investment is useful would not constrain the principal's performance in the regulatory contract. In particular, the principal's expected equilibrium payoff, and the agent's interim expected payoffs and equilibrium investments, would be the same as for the regulatory contract characterized in Proposition 2. The reason is that we could augment the mechanism in Proposition 2 with "bets" on the state such that the agent breaks even on the bet if taking the option coinciding to a truthful report of p and makes sufficiently large expected losses otherwise (this is a standard argument and similar, for instance, to Riordan and Sappington, 1988).

3 The optimal screening mechanism.

Now, consider our model in which both parameters c and p are the agent's private information (and where we impose the financing constraints introduced in Section 1). The optimal screening mechanism solves

$$\max_{\{q(c,p),P(c,p),B(c,p)\}_{(c,p)\in\mathbb{C}\times\mathbb{P}}} \sum_{(c,p)\in\mathbb{C}\times\mathbb{P}} \mu(c,p)\left[W(c,p) - pP(c,p) - (1-p)B(c,p)\right] \tag{14}$$

subject to twelve incentive compatibility constraints and twelve non-negative profit constraints. The incentive compatibility constraints require that any type (c,p) has no incentive to lie:

$$U(c,p) \geqslant U(c,p,\widehat{c},\widehat{p}) \text{ for any } (c,p)\in\mathbb{C}\times\mathbb{P} \text{ and any } (\widehat{c},\widehat{p})\in\mathbb{C}\times\mathbb{P}\setminus\{(c,p)\}\,. \tag{15}$$

The non-negative profit constraints require that any type (c, p) receives non-negative profit for either realization of α when he truthfully reports his type. That is, for any $(c, p) \in \mathbb{C} \times \mathbb{P}$,

$$P(c, p) \geqslant 0, \text{ and } B(c, p) \geqslant 0. \tag{16}$$

We call the mechanism solving the above problem the "second best" and mark the mechanism and associated variables (e.g., rents) with an upper index SB.

Because the jointly observed realization of α is an informative signal on p, the principal maximally relaxes the incentive constraints guaranteeing that the agent reports probability p truthfully (without affecting the remaining constraints) by allocating rents conditionally on this signal. Rents to the agent of either type with a high probability of usefulness are paid in the form of profits conditional on α taking value 1. This means that

$$B(c, h) = 0 \text{ and } U(c, h) = hP(c, h) \text{ for either } c \in \mathbb{C}. \tag{17}$$

On the other hand, rents to the agent of either type with a low probability of usefulness are paid in the form of profits conditional on α taking value zero:

$$P(c, l) = 0 \text{ and } U(c, l) = (1 - l)B(c, l) \text{ for either } c \in \mathbb{C}. \tag{18}$$

There is no loss in considering agent rents that are paid as described above, and so we focus on this allocation of rents throughout the characterization of the second-best mechanism. The incentive constraints (15) for such allocation of rents are specified in Appendix B.

3.1 Low difference in probability of usefulness.

Note that if the principal uses the mechanism described in Proposition 2, the agent of type $(\underline{c}, l)$ has an incentive to misreport his type as $(\overline{c}, h)$. In particular, the expected rents assigned in the mechanism of Proposition 2 are

$\Delta c\widetilde{q}(\overline{c},l)$. The rents available by mimicking type $(\overline{c},h)$ are $\Delta c\widetilde{q}(\overline{c},h)$. The latter is larger because investment is higher for type $(\overline{c},h)$ than $(\overline{c},l)$.

This consideration suggests that both cost-efficient types should be paid rents sufficient to deter them from mimicking the cost-inefficient type investing the highest quantity. A natural candidate for the cost-inefficient type investing the highest quantity is $(\overline{c},h)$. Following this logic, our initial focus is on parameters such that the second-best mechanism grants expected rents

$$hP^{SB}(\underline{c},h) = (1-l)\,B^{SB}(\underline{c},l) = \Delta c q^{SB}(\overline{c},h) \tag{19}$$

to the cost-efficient types, offers no rents to cost-inefficient types, and distorts the investment of type $(\overline{c},h)$ downwards to resolve the rents-efficiency trade-off:

$$hS'\left(q^{SB}(\overline{c},h)\right) = \overline{c} + \tfrac{\lambda}{(1-\lambda)r}\Delta c. \tag{20}$$

Investments for all other agent types are at the first-best level. This will turn out to represent the second-best mechanism if and only if two incentive problems do not arise.

The first problem is that the information rents (19) earned by the agent of type $(\underline{c},l)$ may attract the agent of type $(\overline{c},l)$. This happens when the putative second best investment $q^{SB}(\overline{c},h)$ (satisfying equation (20)) exceeds the first-best investment by type $(\underline{c},l)$. Intuitively, for this problem not to arise, the difference Δp between the probabilities h and l must not be too large. Indeed, the problem is avoided if and only if the dispersion in probability of usefulness is sufficiently low relative to the dispersion in cost, namely

$$\tfrac{\Delta p}{l} \leqslant \tfrac{\Delta c}{\underline{c}}\left(1 + \tfrac{\lambda}{(1-\lambda)r}\right). \tag{21}$$

Condition (21) is satisfied in the area below the solid line in Figure 1 (we describe Figure 1 in more detail in Section 3.3 below). We assume condition (21) is satisfied in the present subsection, delaying the case where it fails to Subsection 3.2.

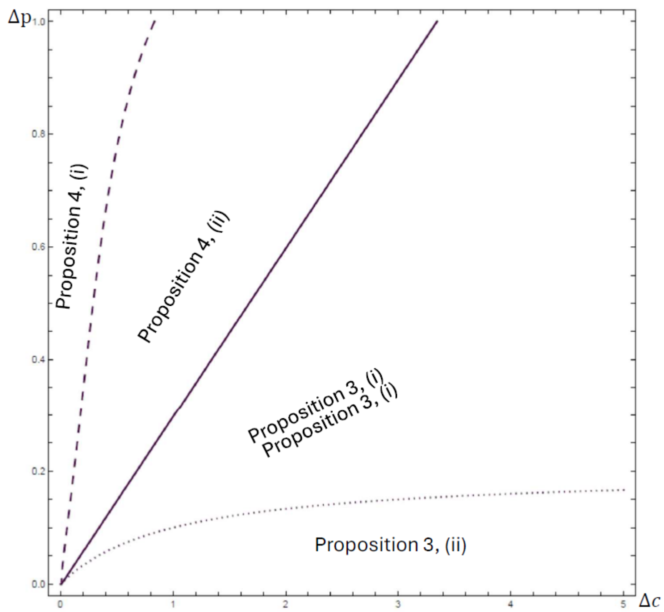

Figure 1: the optimal incentive scheme based on the relative spread of the principal's uncertainty over the two dimensions of the agent's private information ($\lambda = r = \frac{1}{2}$, $l = 0.1$, $\underline{c} = 1$).

The second problem is that rents (19) earned by the cost-efficient types may not suffice to prevent them from mimicking type $(\overline{c}, l)$. This occurs where the putative second-best investment by type $(\overline{c}, h)$ (again, see equation (20)) is smaller than the first-best investment by type $(\overline{c}, l)$. This is ruled out if and only if

$$\frac{\Delta c}{\overline{c}} \frac{\lambda}{(1-\lambda)r} \leqslant \frac{\Delta p}{l}. \tag{22}$$

This corresponds to the area above the dotted curve in Figure 1.

Otherwise, if the inequality in (22) fails, the principal needs to find a way of preventing the agent of either cost-efficient type from misreporting his type as $(\overline{c}, l)$. In this case the optimal mechanism involves pooling, with cost-inefficient types choosing the same investment

$$q_{L} = q^{SB}(\bar{c}, h) = q^{SB}(\bar{c}, l), \tag{23}$$

and cost-efficient types earning rents proportional to that pooled investment level:

$$hP^{SB}(\underline{c}, h) = (1-l)\, B^{SB}(\underline{c}, l) = \Delta c q_{L}. \tag{24}$$

To address the rents-efficiency trade-off, the investment q_L by the cost-inefficient types is distorted downwards:

$$(rh + (1-r)l)\, S'(q_{L}) = \bar{c} + \Delta c \tfrac{\lambda}{1-\lambda}. \tag{25}$$

Proposition 3. *Suppose that inequality (21) holds. Then, the optimal screening mechanism is as follows. Investments by the agent of either cost-efficient type remain at the first-best level. Information rents are attributed solely to cost-efficient agent types.*

(i) If inequality (22) holds, these rents are proportional to the investment by the agent of type $(\bar{c}, h)$, as specified in the set of equations (19). This investment is distorted downwards according to equation (20), while the investment by the agent of type $(\bar{c}, l)$ remains efficient.

(ii) If inequality (22) is violated, both cost-inefficient types invest the same (inefficiently low) amount specified in equation (25) and the cost-efficient types receive rents (24) proportional to this pooled investment level.

Comparison of Propositions 2 and 3 suggests that, when the dispersion of the probability of usefulness is relatively low (in the area below the solid line in Figure 1), the burden of asymmetric information on this additional dimension implies what may seem more minor adjustments to the optimal mechanism. The classic "efficiency at the top" result is preserved, with rents continuing to be attributed solely to the cost-efficient agent types. These rents are proportional to the investment by the agent of type $(\bar{c}, h)$, which is distorted downwards. This distortion, in turn, leads to pooling of investments by the cost-inefficient types when the principal's uncertainty about probability of usefulness is sufficiently low.

Concerning uniqueness of the optimal mechanism, note that the optimal mechanism characterized in Proposition 3 is uniquely determined among those that allocate rents as set out in equations (17) and (18). Conditioning rents on the realisation of α in this way is, however, not necessary for optimality. For instance, there is an optimal mechanism where cost-efficient types are paid their expected rents independently of the realization of α (i.e., $P(\underline{c}, p) = B(\underline{c}, p) = U(\underline{c}, p)$ for both values of p). However, for any optimal mechanism that does not allocate rents according to (17) and (18), there is another one that does so and where the agent makes the same investments and earns the same interim rents (by "interim" it is meant conditional on type (c, p) but not on realization of α). We can conclude that investments and *interim* rents are uniquely determined by the requirement of optimality.

3.2 High difference in probability of usefulness.

Suppose now that inequality (21) is violated, corresponding to the area above the solid line in Figure 1. If the principal applies the mechanism from Part (i) of Proposition 3, she encounters the incentive problem outlined in the previous section: the first-best investment by type $(\underline{c}, l)$ is smaller than the second-best investment by type $(\bar{c}, h)$. As a result, the rents paid to type $(\underline{c}, l)$ attract the agent of type $(\bar{c}, l)$.[7]

This seems to suggest, and indeed we will show, that there are then two kinds of optimal policies for the principal. One is to accept that type $(\bar{c}, l)$ will indeed earn a rent. In this case, note that because type $(\bar{c}, h)$ can mimic the type $(\bar{c}, l)$, also type $(\bar{c}, h)$ will earn a rent. We will then observe that all types of the agent expect a rent. In many environments the possibility of rents for all types does not survive a simple uniform reduction in transfers to the agent for all types, which would increase the principal's payoff while

[7]Note that type $(\bar{c}, l)$ has a stronger incentive to mimic type $(\underline{c}, l)$ than type $(\bar{c}, h)$ does. The reason is that information rents are allocated conditional on the realization of α being aligned with the agent's message.

preserving incentive constraints. Here, however, at the optimal mechanism to be characterized below, it is not possible to reduce payments to the agent uniformly (i.e., independently of type and of the realization of usefulness) because of non-negative profit constraints.

The second kind of optimal policy for the principal involves distorting the investment by type $(\underline{c}, l)$ upwards. This works to reduce the rents that must be granted to type $(\overline{c}, l)$, as this type faces higher costs from the same level of investment.

Now, let us describe the principal's optimal policy in more detail, starting with the case where the principal pursues the logic of distorting the investment by type $(\underline{c}, l)$ upwards to the point where the cost-inefficient types are deprived of any rent. Here, we will find that:

$$q^{SB}(\underline{c}, l) = q^{SB}(\overline{c}, h). \tag{26}$$

Only cost-efficient types receive rents, and these are described by the set of equations (19). The corresponding downward distortion of investment by type $(\overline{c}, h)$, together with the upward distortion for type $(\underline{c}, l)$, is described by the equation

$$((1-\lambda) rh + \lambda (1-r) l) S'\left(q^{SB}(\overline{c}, h)\right) = r(1-\lambda)\overline{c} + \lambda(1-r)\underline{c} + \lambda\Delta c \tag{27}$$

together with equation (26). Equation (27) balances the marginal return from increasing the pooled level of investment for $q^{SB}(\overline{c}, h)$ and $q^{SB}(\underline{c}, l)$ with the direct marginal cost of this investment $r(1-\lambda)\overline{c} + \lambda(1-r)\underline{c}$ plus the marginal cost of higher information rents to the cost efficient types $\lambda\Delta c$.

The alternative policy, termed hereafter "*rents to all*", involves paying a positive rent to the agent of type $(\overline{c}, l)$ that depends on the difference between investment by $(\overline{c}, h)$ and investment by $(\underline{c}, l)$. We denote this difference by $\Delta q \equiv q(\overline{c}, h) - q(\underline{c}, l)$. In particular, the second-best mechanism involves:

$$(1-l) B^{SB}(\overline{c}, l) = \tfrac{h(1-l)\Delta c \Delta q^{SB}}{\Delta p}, \text{ where} \tag{28}$$

$$\Delta q^{SB} = q^{SB}\left(\overline{c}, h\right) - q^{SB}\left(\underline{c}, l\right). \tag{29}$$

The higher the value of Δq the smaller the upward distortion in investment for type $(\underline{c}, l)$. Leaving rents to the agent of type $(\overline{c}, l)$ calls for upgrading rents to other types correspondingly:

$$hP^{SB}\left(\overline{c}, h\right) = \tfrac{h(1-h)\Delta c \Delta q^{SB}}{\Delta p}, \tag{30}$$

$$hP^{SB}\left(\underline{c}, h\right) = \tfrac{h(1-h)\Delta c \Delta q^{SB}}{\Delta p} + \Delta c q^{SB}\left(\overline{c}, h\right), \tag{31}$$

$$\left(1-l\right) B^{SB}\left(\underline{c}, l\right) = \tfrac{l(1-h)\Delta c \Delta q^{SB}}{\Delta p} + \Delta c q^{SB}\left(\overline{c}, h\right). \tag{32}$$

The marginal increase in expected rents from increasing Δq (and hence reducing the upward distortion in investment by type $(\underline{c}, l)$) is equal to:[8]

$$\Delta R = \tfrac{\Delta c}{\Delta p}\left(rh(1-h) + (1-r)\left(1-\lambda\right) h(1-l) + \lambda\left(1-r\right) l(1-h)\right).$$

In the second-best mechanism, the principal again distorts investment by type $(\overline{c}, h)$ downwards:

$$r\left(1-\lambda\right) hS'\left(q^{SB}\left(\overline{c}, h\right)\right) = r\left(1-\lambda\right)\overline{c} + \lambda \Delta c + \Delta R. \tag{33}$$

The distortion for type $(\underline{c}, l)$ is upwards:

$$\lambda\left(1-r\right) lS'\left(q^{SB}\left(\underline{c}, l\right)\right) = \underline{c}\lambda\left(1-r\right) - \Delta R. \tag{34}$$

By the concavity of function $S(\cdot)$, and equations (29), (33) and (34), the difference in investments Δq^{SB} is positive, and so all types earn strictly positive rents if, and only if,

$$\tfrac{r(1-\lambda)\overline{c} + \lambda \Delta c + \Delta R}{hr(1-\lambda)} < \tfrac{\underline{c}\lambda(1-r) - \Delta R}{l\lambda(1-r)}. \tag{35}$$

Note that inequality (35) implies:

$$\underline{c}\lambda\left(1-r\right) > \Delta R\ . \tag{36}$$

[8] Formally, ΔR is equal to the partial derivative of the expectation of information rents, as given by (28) through (32), with respect to Δq^{SB}.

Therefore, the investment $q^{SB}(\underline{c}, l)$ given by equation (34) is well defined.

Inequality (35) is a necessary and sufficient condition for the latter approach (rents to all types) to dominate the former (upward distortion of investments by type $(\underline{c}, l)$ to the point of matching the investment by type $(\overline{c}, h)$ so as to avoid rents to cost-inefficient types).

Proposition 4. *Suppose that inequality (21) is violated. Then, the second-best mechanism is as follows. The investments by the agent of types $(\underline{c}, h)$ and $(\overline{c}, l)$ are efficient.*
(i) If inequality (35) holds, all types receive information rents, specified by equations (28) through (32). The investment by agent of type $(\overline{c}, h)$ is distorted downwards as described by equation (33), while the investment by the agent of type $(\underline{c}, l)$ is distorted upwards but remains below that of type $(\overline{c}, h)$, as described by equation (34).
(ii) Otherwise, the investment by the agent of type $(\underline{c}, l)$ is distorted upwards to match the investment by the agent of type $(\overline{c}, h)$, which is distorted downwards, as described by equation (27). Only the cost-efficient types receive information rents, and these rents are described by set of equations (19).

Proposition 4 captures two notable features of optimal mechanisms when the difference in probabilities of usefulness is relatively high. Specifically, we find inefficiently high investment levels by the agent of cost-efficient type $(\underline{c}, l)$. Furthermore, when the difference in probabilities of usefulness is sufficiently large, all agent types can expect to receive a positive information rent at the time of contracting. Note that, as for the case of Proposition 3, the optimal mechanism is unique in terms of the implied investments and interim expected rents.

3.3 Comparing the different cases.

Comparison of Propositions 3 and 4 shows that the optimal regulation depends critically on the relative magnitude of information asymmetry along two dimensions, as illustrated in Figure 1. In the region below the solid line, the asymmetry regarding probability of usefulness is relatively small. As discussed above, the optimal regulatory mechanism more closely resembles the one in which the principal is uncertain only about the cost of investment: it features "efficiency at the top" (as investments by the cost-efficient types are undistorted) and information rents accrue only to the cost-efficient types, as described in Proposition 3. This region is further divided into two parts by the dotted curve: above that curve investment by type $(\bar{c}, h)$ is above the first-best investment by type $(\bar{c}, l)$; investment by type $(\bar{c}, l)$ is therefore not distorted. Below the dotted curve, a sufficient distortion in investment by the agent of type $(\bar{c}, h)$ calls for pooling of investment levels by the cost-inefficient types.

The optimal mechanism in both cases of Proposition 3 is solved for in the appendix (Appendix C) by formulating a "relaxed optimisation program" which ignores many of the incentive constraints. The remaining constraints are then subsequently verified. The incentive constraints imposed in the relaxed program offer insights into the form of the optimal mechanism. The left panel of Figure 2 shows the relevant incentive constraints in Part (i) of Proposition 3, while the right panel of Figure 2 shows the relevant incentive constraints in Part (ii). In the first case, the relevant incentive constraints are those ensuring cost-efficient types do not mimic the cost-inefficient high-usefulness type $(\bar{c}, h)$ and the latter type's investment is downward distorted. In the second case, we may think of incentive constraints pointing to both cost-inefficient types (i.e., to both $(\bar{c}, l)$ and $(\bar{c}, h)$) as being active and both types' investments are downward distorted. In both cases, potential mimicry by the cost-inefficient types can be ignored, and so these types do not earn a rent.

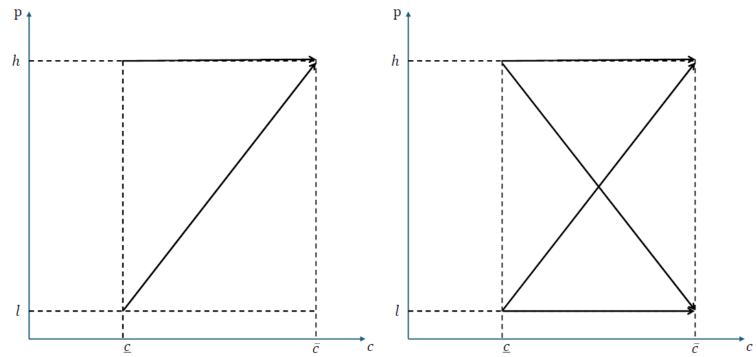

Figure 2: binding incentive constraints in Proposition 3.

In the region above the solid line of Figure 1, the presence of information asymmetry along the usefulness dimension has what may seem more marked effects on the optimal mechanism, as described by Proposition 4. On the one hand, there is inefficiently high investment for type $(\underline{c}, l)$. On the other, all types of the agent may earn strictly positive information rents and indeed this occurs above the dashed curve in what corresponds to Case (i) of Proposition 4.

The optimal mechanism in Proposition 4 is solved by considering a relaxed program where we impose the incentive constraints depicted in Figure 3. As discussed above, the rents-to-all-types result in Case (i) of the proposition can be understood from the cycle of incentive constraints, starting with the observation that type $(\underline{c}, l)$ can earn a positive rent by mimicking $(\bar{c}, h)$. One subsequently observes that, if investment by $(\underline{c}, l)$ is not too high, then $(\bar{c}, l)$ must also earn a rent to prevent mimicry of the former type. Then also $(\bar{c}, h)$ must earn a rent due to the possibility of mimicking $(\bar{c}, l)$. Considering, then, Case (ii) of Proposition 4, we note that same cycle of incentive constraints is relevant, but investment by type $(\underline{c}, l)$ is high enough that rents do not flow to type $(\bar{c}, l)$ and therefore also not to type $(\bar{c}, h)$.

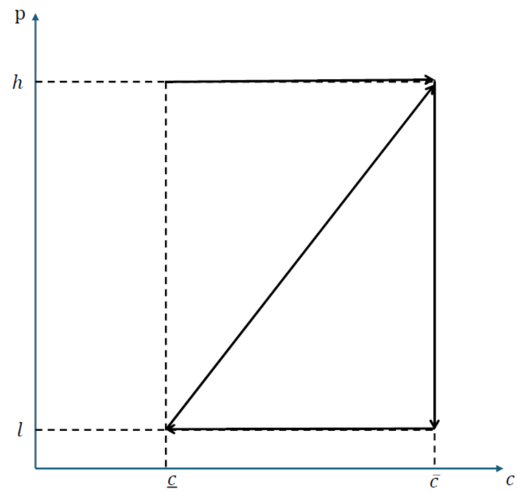

Figure 3: binding incentive constraints in Proposition 4.

Small value of l. It is worth remarking on what happens as $l \to 0$ holding other parameters fixed (Appendix E considers the case where $l = 0$ as well as the possibility that $h = 1$). Note that Condition (21) fails for small enough l, and hence the optimal mechanism must be described by Proposition 4. If the inequality

$$\underline{c}\lambda\left(1-r\right) > \Delta c\left(r\left(1-h\right)+\left(1-r\right)\left(1-\lambda\right)\right) \tag{37}$$

holds, then for small enough l, Case (i) of Proposition 4 holds and all types earn positive expected rents. Consistent with the inequality (37), the intuition is that this should occur when investment by type $(\underline{c}, l)$ is costly relative to the size of information rents, which are proportional to Δc.

If the reverse of inequality (37) holds then, as $l \to 0$, Case (ii) of Proposition 4 applies and (using equation (27)) investment by type $(\underline{c}, l)$ approaches the positive value $q^*(\underline{c}, l)$ satisfying

$$rh\left(1-\lambda\right)S'\left(q^*\left(\underline{c}, l\right)\right) = r\left(1-\lambda\right)\overline{c} + \lambda\left(1-r\right)\underline{c} + \lambda\Delta c. \tag{38}$$

This occurs even though the first-best investment $q^{FB}(\underline{c}, l)$ given by (5) approaches zero as $l \rightarrow 0$. This establishes that the second-best mechanism may call for a positive level of investment bounded away from zero even when investment is essentially useless to the principal and the first-best investment vanishes.

4 Conclusions.

We have described the optimal regulation of investments under uncertainty when the agent holds superior information on both the cost of investment and the likelihood it is useful to the principal. We focused on the case where, out of concern for the firm's ability to finance the investments, the regulation aims at ensuring non-negative profits in any period.

Our main result was that, as long as the variation in probability of usefulness is relatively low, the optimal regulation qualitatively resembles that in a setting with uncertainty only about cost. Otherwise, when the agent's private information on the probability of usefulness is more pronounced, there are upward distortions in investment (by type $(\underline{c}, l)$) and all types may earn strictly positive information rents.

This finding suggests that the burden of two-dimensional screening depends crucially on how easy it is to align interests between the principal and the agent. In particular, when the information asymmetry on the usefulness of investment is severe, alignment of interests is more difficult, and the principal responds by inducing upward distortions in investment and granting rents for all types. The features of the optimal mechanism therefore appear further from the one-dimensional screening problem with private information only on costs. This suggests an apparent parallel with Armstrong (1999), where the additional dimension of private information is on demand faced by the regulated monopolist and the features of the screening problem are transformed when uncertainty on this dimension is sufficiently pronounced.

Whether a deeper connection exists could be worthy of further exploration.

On the applied side, our findings are arguably pessimistic about the nature of optimal regulation of investment projects with high uncertainty about their eventual usefulness or importance, as they are suggestive of a high burden of asymmetric information on the principal in such settings. We nonetheless see our results as a possible benchmark against which regulatory policy might aim to improve. For instance, they highlight the value of "disallowing" and hence not reimbursing some costs in case investment turns out not to be useful. Alternatively, the regulator may seek to reduce the extent of information asymmetries concerning the necessity of investments by requiring the submission of detailed investment plans and understanding the parameters of the network themselves (many regulators such as CRE in France and Ofgem in the UK require network businesses to provide detailed plans). A third possibility is that the regulator could provide payments only sufficient to cover low-cost investments, thus depriving firms of rent but possibly foregoing some beneficial investments that have higher costs. Such a policy was never optimal for the regulator in our model due to the Inada conditions (i.e., because we assumed that small amounts of investment were highly beneficial). The role of these kinds of regulatory policies, as well as their practical implementation, also remain possible areas for future research.

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A. Proof of Propositions 1 and 2.

Proof of Proposition 1. We are maximizing the expression in equation (3) by choice of $q(c,p)$. By the Inada conditions and strict concavity of the objective, there is a unique optimizer, and this is strictly positive. By differentiability of the objective, it is characterized by a first-order condition, in particular by equation (5).[9]

Proof of Proposition 2. Consider the principal's problem of maximizing the expression in (9) subject to incentive compatibility constraints:

$$U\left(\underline{c},p\right) \geqslant U\left(\bar{c},p\right) + \Delta c q(\bar{c},p), \tag{39}$$

$$U\left(\bar{c},p\right) \geqslant U\left(\underline{c},p\right) - \Delta c q(\underline{c},p), \tag{40}$$

and nonnegative profit constraints:

$$P\left(\bar{c},p\right) \geqslant 0,\ B\left(\bar{c},p\right) \geqslant 0, \tag{41}$$

$$P\left(\underline{c},p\right) \geqslant 0,\ B\left(\underline{c},p\right) \geqslant 0. \tag{42}$$

Let us solve the above problem under constraint (39) and set of constraints (41), ignoring the other constraints ("the relaxed problem"). Given that rents are costly for the principal, the constraints of the relaxed problem are binding:

$$P\left(\bar{c},p\right) = B\left(\bar{c},p\right) = U\left(\bar{c},p\right) = 0,$$

[9] We continue to apply this reasoning in the proofs of Propositions 2 through 4, without restating it explicitly.

$$U\left(\underline{c},p\right)=\Delta cq(\overline{c},p). \tag{43}$$

Therefore, the relaxed problem is equivalent to

$$\max_{q(\overline{c},p),q(\underline{c},p)}\lambda\left[W(\underline{c},p)-\Delta cq(\overline{c},p)\right]+(1-\lambda)\,W(\overline{c},p).$$

The analysis of this optimization program is analogous to the first-best problem considered in Proposition 1, but with a modified marginal cost for the cost-inefficient type $(\overline{c},p)$: the "virtual marginal cost" in place of the true marginal cost. The investments $\widetilde{q}(c,p)$ solving this problem are given by set of equations (13).

It remains to verify that the solution of the relaxed problem satisfies the ignored constraints of the original problem. There are infinitely many ways to allocate rents (43) so that

$$pP\left(\underline{c},p\right)+(1-p)\,B\left(\underline{c},p\right)=\Delta cq(\overline{c},p)$$

and the ignored non-negative profit constraints (42) hold. The ignored incentive constraint (40) holds because, using the set of equations (13), $\widetilde{q}(\underline{c},p)>\widetilde{q}(\overline{c},p)$.

B. The incentive constraints with allocation of rents conditional on the realized usefulness.

Suppose that the rents are allocated conditional on realization of α as specified by set of equations (17) and (18). Then, the set of incentive constraints (15) is as follows.

$$hP\left(\underline{c},h\right)\geqslant hP\left(\overline{c},h\right)+\Delta cq\left(\overline{c},h\right), \qquad ((\underline{c},h)\to(\overline{c},h))$$

$$hP\left(\underline{c},h\right)\geqslant(1-h)\,B\left(\overline{c},l\right)+\Delta cq\left(\overline{c},l\right), \qquad ((\underline{c},h)\to(\overline{c},l))$$

$$hP\left(\underline{c},h\right)\geqslant(1-h)\,B\left(\underline{c},l\right), \qquad ((\underline{c},h)\to(\underline{c},l))$$

$$(1-l)\,B\left(\underline{c},l\right) \geqslant (1-l)\,B\left(\overline{c},l\right) + \Delta cq\left(\overline{c},l\right), \qquad \left(\left(\underline{c},l\right) \to \left(\overline{c},l\right)\right)$$

$$(1-l)\,B\left(\underline{c},l\right) \geqslant lP\left(\underline{c},h\right), \qquad \left(\left(\underline{c},l\right) \to \left(\underline{c},h\right)\right)$$

$$(1-l)\,B\left(\underline{c},l\right) \geqslant lP\left(\overline{c},h\right) + \Delta cq\left(\overline{c},h\right), \qquad \left(\left(\underline{c},l\right) \to \left(\overline{c},h\right)\right)$$

$$hP\left(\overline{c},h\right) \geqslant (1-h)\,B\left(\underline{c},l\right) - \Delta cq\left(\underline{c},l\right), \qquad \left(\left(\overline{c},h\right) \to \left(\underline{c},l\right)\right)$$

$$hP\left(\overline{c},h\right) \geqslant hP\left(\underline{c},h\right) - \Delta cq\left(\underline{c},h\right), \qquad \left(\left(\overline{c},h\right) \to \left(\underline{c},h\right)\right)$$

$$hP\left(\overline{c},h\right) \geqslant (1-h)\,B\left(\overline{c},l\right), \qquad \left(\left(\overline{c},h\right) \to \left(\overline{c},l\right)\right)$$

$$(1-l)\,B\left(\overline{c},l\right) \geqslant lP\left(\overline{c},h\right), \qquad \left(\left(\overline{c},l\right) \to \left(\overline{c},h\right)\right)$$

$$(1-l)\,B\left(\overline{c},l\right) \geqslant (1-l)\,B\left(\underline{c},l\right) - \Delta cq\left(\underline{c},l\right), \qquad \left(\left(\overline{c},l\right) \to \left(\underline{c},l\right)\right)$$

$$(1-l)\,B\left(\overline{c},l\right) \geqslant lP\left(\underline{c},h\right) - \Delta cq\left(\underline{c},h\right). \qquad \left(\left(\overline{c},l\right) \to \left(\underline{c},h\right)\right)$$

C. Proof of Proposition 3.

Case (i).

Suppose first that both inequalities (21) and (22) hold.

Solution to relaxed problem. Consider the relaxed problem maximizing the objective (14) subject to the set of non-negative profit constraints (16) and two incentive constraints: $\left(\left(\underline{c},h\right) \to \left(\overline{c},h\right)\right)$ and $\left(\left(\underline{c},l\right) \to \left(\overline{c},h\right)\right)$. Because the objective (14) decreases in rents, both incentive constraints of the relaxed problem are binding:

$$hP\left(\underline{c},h\right) = hP\left(\overline{c},h\right) + \Delta cq\left(\overline{c},h\right), \tag{44}$$

$$(1-l)\,B\left(\underline{c},l\right) = lP\left(\overline{c},h\right) + \Delta cq\left(\overline{c},h\right). \tag{45}$$

Given that we are ignoring for the moment the other incentive constraints, the non-negative profit constraints by the cost inefficient types are binding:

$$P\left(\overline{c},h\right) = 0,\ B\left(\overline{c},l\right) = 0. \tag{46}$$

By set of equations (44), (45) and (46), the expected rents to the cost efficient types are given by set of equations (19). Using the first-order approach to solving the optimization problem (14) subject to sets of constraints (46) and (19), we find that the investment by type $(\overline{c}, h)$ is distorted downwards to optimally solve a rent-efficiency trade-off, as described by equation (20), while the investments by all other types are efficient:

$$q^{SB}(c,p) = q^{FB}(c,p) \text{ for any } (c,p) \text{ in set } C \setminus \{(\overline{c},h)\}. \tag{47}$$

Verification of omitted constraints. We now ask under which conditions the above solution to the relaxed problem satisfies the omitted incentive constraints of the original problem?

The incentive constraints $((\underline{c},h) \to (\underline{c},l))$ and $((\underline{c},l) \to (\underline{c},h))$ hold (strictly) by set of equations (19) and true inequality $l < h$.

The incentive constraints $((\overline{c},h) \to (\overline{c},l))$ and $((\overline{c},l) \to (\overline{c},h))$ hold by set of equations (46).

The incentive constraints $((\underline{c},h) \to (\overline{c},l))$ and $((\underline{c},l) \to (\overline{c},l))$ hold if and only if

$$q^{FB}(\overline{c},l) \leqslant q^{SB}(\overline{c},h). \tag{48}$$

By set of equations (5) and (20), inequality (48) is equivalent to inequality (22).

By sets of equations (46) and (19), the incentive constraints $((\overline{c},h) \to (\underline{c},l))$ and $((\overline{c},l) \to (\underline{c},l))$ are equivalent to inequality

$$q^{SB}(\overline{c},h) \leqslant q^{FB}(\underline{c},l). \tag{49}$$

By equation (20) and set of equations (47), inequality (49) is equivalent to inequality (21).

By sets of equations (46) and (19), inequalities $((\overline{c},h) \to (\underline{c},h))$ and $((\overline{c},l) \to (\underline{c},h))$ follow from inequality

$$q^{SB}(\overline{c},h) \leqslant q^{FB}(\underline{c},h), \tag{50}$$

which follows from set of inequalities

$$q^{SB}(\bar{c},h) < q^{FB}(\bar{c},h) < q^{FB}(\underline{c},h)$$

implied by equation (20) and set of equations (5).

Case (ii).

Suppose now that inequality (22) is violated, that is,

$$\tfrac{\Delta p}{l} < \tfrac{\Delta c}{\bar{c}}\tfrac{\lambda}{(1-\lambda)r}. \tag{51}$$

The relaxed problem. Consider the relaxed problem with objective (14) subject to the set of non-negative profit constraints (16) and the incentive constraints $((\underline{c},h) \to (\bar{c},h))$, $((\underline{c},h) \to (\bar{c},l))$, $((\underline{c},l) \to (\bar{c},h))$ and $((\underline{c},l) \to (\bar{c},l))$. Since we do not take into account any incentive constraints for the cost-inefficient types, they receive no rents, as described by the set of equations (46). As a result, the incentive constraints $((\underline{c},h) \to (\bar{c},h))$, $((\underline{c},h) \to (\bar{c},l))$, $((\underline{c},l) \to (\bar{c},h))$, $((\underline{c},l) \to (\bar{c},l))$ can be rewritten as follows:

$$hP(\underline{c},h) \geqslant \Delta c \max\{q(\bar{c},h), q(\bar{c},l)\}, \tag{52}$$

$$(1-l)B(\underline{c},l) \geqslant \Delta c \max\{q(\bar{c},h), q(\bar{c},l)\}. \tag{53}$$

Proof of equation (23). We prove equation (23) by contradiction. Suppose first that in the solution to the relaxed problem

$$q(\bar{c},h) > q(\bar{c},l). \tag{54}$$

Then, the incentive constraints $((\underline{c},h) \to (\bar{c},l))$ and $((\underline{c},l) \to (\bar{c},l))$ are slack while the incentive constraints $((\underline{c},h) \to (\bar{c},h))$ and $((\underline{c},l) \to (\bar{c},h))$ are binding. By the same argument as in Part (i) of Proposition 3, $q(\bar{c},l) = q^{FB}(\bar{c},l)$ and $q(\bar{c},h)$ is given by equation (20). However, by inequality (51),

$q(\overline{c},h)$ given by equation (20) is smaller than $q^{FB}(\overline{c},l)$, which contradicts our supposition (54).

Suppose now that in the solution to the relaxed problem

$$q(\overline{c},h) < q(\overline{c},l). \tag{55}$$

Then, the incentive constraints $((\underline{c},h) \rightarrow (\overline{c},h))$ and $((\underline{c},l) \rightarrow (\overline{c},h))$ are slack, while the incentive constraints $((\underline{c},h) \rightarrow (\overline{c},l))$ and $((\underline{c},l) \rightarrow (\overline{c},l))$ are binding. Hence, the set of incentive constraints (52) and (53) is equivalent to the following set of equations:

$$hP(\underline{c},h) = (1-l)B(\underline{c},l) = \Delta c q(\overline{c},l). \tag{56}$$

By plugging the set of equations (56) into the objective function (14) and considering necessary conditions for optimality of $q(\overline{c},l)$ and $q(\overline{c},h)$ we find:

$$lS'(q(\overline{c},l)) = \overline{c} + \tfrac{\lambda}{(1-\lambda)(1-r)}\Delta c, \tag{57}$$

$$q(\overline{c},h) = q^{FB}(\overline{c},h). \tag{58}$$

By equation (57) and set of equations (5),

$$q(\overline{c},l) < q^{FB}(\overline{c},l). \tag{59}$$

By inequality (59) and inequality (7),

$$q(\overline{c},l) < q^{FB}(\overline{c},h), \tag{60}$$

which contradicts our supposition (55).

The only remaining possibility is that types $(\overline{c},h)$ and $(\overline{c},l)$ invest the same quantity, as specified in the set of equations (23). We denote this quantity by q_L.

Solution to the relaxed problem. By the set of equations (23), the sets of incentive constraints (52) and (53) can be equivalently written as follows:

$$hP\left(\underline{c},h\right) \geqslant \Delta c q_{L} \text{ and } (1-l)B\left(\underline{c},l\right) \geqslant \Delta c q_{L}. \tag{61}$$

Because the principal's objective (14) decreases in rents, and we are, for the moment, ignoring all incentive constraints except those in set (61), these constraints must be binding. Therefore, the cost-efficient types receive rents proportional to the investment q_{L}, as described by the set of equations (24). Plugging the set of equations (24) into the objective (14) and maximizing with respect to investment q_{L}, we find that it is given by equation (25).

Verification of omitted incentive constraints. We will now verify that under the mechanism described by equations (46) and (23) to (25), the omitted incentive constraints hold. The incentive constraints $((\underline{c},h) \to (\underline{c},l))$ and $((\underline{c},l) \to (\underline{c},h))$ follow from the set of equations (24) and true inequality $l < h$. The incentive constraints $((\overline{c},h) \to (\overline{c},l))$ and $((\overline{c},l) \to (\overline{c},h))$ follow from set of equations (46). It remains to verify the incentive constraints $((\overline{c},h) \to (\underline{c},l))$, $((\overline{c},h) \to (\underline{c},h))$, $((\overline{c},l) \to (\underline{c},l))$ and $((\overline{c},l) \to (\underline{c},h))$ which require that the cost-inefficient types do not mimic the cost-efficient types. These constraints hold if and only if

$$q_{L} \leqslant \min\left\{q^{FB}\left(\underline{c},h\right), q^{FB}\left(\underline{c},l\right)\right\} = q^{FB}\left(\underline{c},l\right). \tag{62}$$

By concavity of $S\left(\cdot\right)$, and equations (5) and (25), this is equivalent to inequality

$$\tfrac{\Delta p}{l} \leqslant \tfrac{\Delta c}{r\underline{c}}\left(1+\tfrac{\lambda}{1-\lambda}\right),$$

which follows from inequality (51).

D. Proof of Proposition 4.

Suppose that inequality (21) does not hold, that is,

$$\tfrac{\Delta p}{l} > \tfrac{\Delta c}{\underline{c}}\left(1 + \tfrac{\lambda}{(1-\lambda) r}\right). \tag{63}$$

The relaxed problem. Consider the relaxed optimization problem with objective (14) subject to the set of non-negative profit constraints (16) and four incentive constraints: $((\underline{c}, h) \to (\bar{c}, h))$, $((\underline{c}, l) \to (\bar{c}, h))$, $((\bar{c}, h) \to (\bar{c}, l))$ and $((\bar{c}, l) \to (\underline{c}, l))$. Since each of these constraints pertains to a different agent type and the principal's objective is decreasing in rents, we can conclude that in any solution to the relaxed program:

$$hP(\underline{c}, h) = hP(\bar{c}, h) + \Delta c q(\bar{c}, h). \tag{64}$$

$$(1-l) B(\underline{c}, l) = lP(\bar{c}, h) + \Delta c q(\bar{c}, h). \tag{65}$$

$$hP(\bar{c}, h) = (1-h) B(\bar{c}, l). \tag{66}$$

$$(1-l) B(\bar{c}, l) = \max\{(1-l) B(\underline{c}, l) - \Delta c q(\underline{c}, l), 0\}. \tag{67}$$

By equations (65) and (67),

$$(1-l) B(\bar{c}, l) = \max\{lP(\bar{c}, h) + \Delta c \Delta q, 0\}, \tag{68}$$

where $\Delta q \equiv q(\bar{c}, h) - q(\underline{c}, l)$ as defined in the main text. Let us prove that

$$lP(\bar{c}, h) + \Delta c \Delta q \geqslant 0, \text{ and therefore} \tag{69}$$

$$(1-l) B(\bar{c}, l) = lP(\bar{c}, h) + \Delta c \Delta q. \tag{70}$$

Suppose, by contradiction, that in a solution to the relaxed program

$$lP(\bar{c}, h) + \Delta c \Delta q < 0. \tag{71}$$

Then, by equations (64) to (67),

$$B(\bar{c}, l) = P(\bar{c}, h) = 0, \tag{72}$$

$$hP\left(\underline{c},h\right)=\left(1-l\right)B\left(\underline{c},l\right)=\Delta cq\left(\bar{c},h\right). \tag{73}$$

Given (71), together with the satisfaction of equations (64) to (67), local perturbations of $q\left(\underline{c},l\right)$ do not affect rents, while local perturbations of $q\left(\bar{c},h\right)$ affect rents through equations (73). Therefore, it is necessary for an optimum in the relaxed program that investments are given by Proposition 3, Part (i). But this would imply that investments by types $\left(\bar{c},h\right)$ and $\left(\underline{c},l\right)$ are such that, by inequality (63), $\Delta q>0$. This contradicts equation (71); so equation (70) is true.

Solving the system of equations (64) through (66) and (70) for the rents earned by different types, we find that these rents are given by

$$\left(1-l\right)B\left(\bar{c},l\right)=\tfrac{h(1-l)\Delta c\Delta q}{\Delta p}, \tag{74}$$

$$hP\left(\bar{c},h\right)=\tfrac{h(1-h)\Delta c\Delta q}{\Delta p}, \tag{75}$$

$$hP\left(\underline{c},h\right)=\tfrac{h(1-h)\Delta c\Delta q}{\Delta p}+\Delta cq\left(\bar{c},h\right), \tag{76}$$

$$\left(1-l\right)B\left(\underline{c},l\right)=\tfrac{l(1-h)\Delta c\Delta q}{\Delta p}+\Delta cq\left(\bar{c},h\right). \tag{77}$$

Note that, because we have established the inequality in (69), at an optimum in our relaxed program we must have

$$\Delta q\geqslant 0. \tag{78}$$

The expressions for rents are valid when this holds, and we will impose this restriction in what follows.

We plug equations (74) through (77) into the objective function (14):

$$\begin{aligned}
&\sum_{c\in\mathbb{C}}\mu\left(c,p\right)\left[W\left(c,p\right)-pP\left(c,p\right)-(1-p)B(c,p)\right]\\
=\;&\lambda r\left(hS\left(q\left(\underline{c},h\right)\right)-\underline{c}q\left(\underline{c},h\right)-\tfrac{\Delta c\Delta q}{\Delta p}h\left(1-h\right)-\Delta cq\left(\bar{c},h\right)\right)\\
&+\lambda\left(1-r\right)\left(lS\left(q\left(\bar{c},h\right)-\Delta q\right)-\underline{c}\left(q\left(\bar{c},h\right)-\Delta q\right)-\tfrac{\Delta c\Delta q}{\Delta p}l\left(1-h\right)-\Delta cq\left(\bar{c},h\right)\right)\\
&+\left(1-\lambda\right)r\left(hS\left(q\left(\bar{c},h\right)\right)-\bar{c}q\left(\bar{c},h\right)-\tfrac{\Delta c\Delta q}{\Delta p}h\left(1-h\right)\right)\\
&+\left(1-\lambda\right)\left(1-r\right)\left(lS\left(q\left(\bar{c},l\right)\right)-\bar{c}q\left(\bar{c},l\right)-\tfrac{\Delta c\Delta q}{\Delta p}h\left(1-l\right)\right).
\end{aligned} \tag{79}$$

We maximize the resulting expression (79) with respect to the investment quantities for different types, using Δq as a control variable in place of $q(\underline{c}, l)$, subject to constraint (78). Since investments by types $(\underline{c}, h)$ and $(\overline{c}, l)$ have no effect on rents, they are chosen efficiently. The following two sections specify the optimal choice of $q(\overline{c}, h)$ and $\Delta q \geqslant 0$.

Case (i).

Suppose first that inequality (35) holds. Let us maximise the objective function (79) ignoring for the moment constraint (78) (we will check later that the resulting solution satisfies that constraint). By concavity of the objective function (79), the unique solution is given by the first-order conditions

$$\begin{gathered}(1-\lambda) rhS'(q(\overline{c}, h)) + \lambda(1-r) lS'(q(\overline{c}, h) - \Delta q) = \\ \underline{c}\lambda(1-r) + \overline{c}r(1-\lambda) + \lambda\Delta c,\end{gathered} \tag{80}$$

$$\lambda(1-r) lS'(q(\overline{c}, h) - \Delta q) = \lambda\underline{c}(1-r) - \tfrac{\Delta c}{\Delta p}(h(1-h) + \Delta p(1-r)(h-\lambda)). \tag{81}$$

Substituting this expression for $\lambda(1-r) lS'(q(\overline{c}, h) - \Delta q)$ given by equation (81) into equation (80), we obtain equations (33) and (34). Notice that (35) implies that the right-hand side of equation (34) is positive, hence, the investment quantity

$$q(\underline{c}, l) = q(\overline{c}, h) - \Delta q$$

is well-defined. Furthermore, by equations (33) and (34), and concavity of function $S(\cdot)$ inequality (35) is equivalent to inequality

$$q(\underline{c}, l) < q(\overline{c}, h), \tag{82}$$

hence, Δq is, indeed, positive, and so the constraint (78) which we have ignored is verified.

Verification of omitted incentive constraints. Now we have our candidate for the second-best mechanism, we substitute the expressions for rents from equations (28) through (32) into the omitted incentive constraints to verify that these constraints are satisfied. We find that the incentive constraint $((\underline{c}, h) \to (\overline{c}, l))$ is equivalent to inequality

$$q^{FB}(\overline{c}, l) \leqslant q^{SB}(\overline{c}, h). \tag{83}$$

Note that by set of inequalities (6),

$$q^{FB}(\overline{c}, l) < q^{FB}(\underline{c}, l) \tag{84}$$

Furthermore, by equation (34), set of equations (5) and concavity of function $S(\cdot)$, investment $q(\underline{c}, l)$ is distorted upwards:

$$q^{FB}(\underline{c}, l) < q^{SB}(\underline{c}, l). \tag{85}$$

Inequality (83) follows from inequalities (84), (85), and (82).

The incentive constraint $((\underline{c}, h) \to (\underline{c}, l))$ is equivalent to true inequality

$$\Delta q^{SB}(1-h) + \Delta p q^{SB}(\overline{c}, h) \geqslant 0.$$

We find that the incentive constraint $((\underline{c}, l) \to (\overline{c}, l))$ is equivalent to inequality

$$q^{SB}(\underline{c}, l) \geqslant q^{FB}(\overline{c}, l),$$

which follows from set of inequalities

$$q^{SB}(\underline{c}, l) > q^{FB}(\underline{c}, l) > q^{FB}(\overline{c}, l).$$

The incentive constraint $((\underline{c}, l) \to (\underline{c}, h))$ is equivalent to true inequality

$$q^{SB}(\overline{c}, l) \geqslant \tfrac{l}{h} q^{SB}(\overline{c}, l).$$

The incentive constraint $((\overline{c}, h) \to (\underline{c}, l))$ reduces to the true inequality

$$q^{SB}(\underline{c}, l) \geq 0.$$

The incentive constraint $((\overline{c},h)\to(\underline{c},h))$ is equivalent to inequality

$$q^{FB}\left(\underline{c},h\right)\geqslant q^{SB}\left(\overline{c},h\right),$$

which follows from set of inequalities

$$q^{FB}\left(\underline{c},h\right)>q^{FB}\left(\overline{c},h\right)>q^{SB}\left(\overline{c},h\right). \tag{86}$$

The incentive constraint $((\overline{c},l)\to(\overline{c},h))$ is equivalent to true inequality

$$\Delta c\Delta q^{SB}\geqslant 0.$$

The incentive constraint $((\overline{c},l)\to(\underline{c},h))$ is equivalent to inequality

$$\Delta q^{SB}\geqslant \tfrac{l}{h}q^{SB}\left(\overline{c},h\right)-q^{FB}\left(\underline{c},h\right),$$

which follows from inequality $\Delta q^{SB}>0$ and set of inequalities (86).

Case (ii).

Suppose that inequality (35) does not hold, that is,

$$\tfrac{r(1-\lambda)\overline{c}+\lambda\Delta c+\Delta R}{hr(1-\lambda)}\geqslant\tfrac{\underline{c}\lambda(1-r)-\Delta R}{l\lambda(1-r)}. \tag{87}$$

Let us prove that, in our solution to the relaxed program, $\Delta q=0$. Suppose, by contradiction, that $\Delta q>0$. Then, the investments by types $(\overline{c},h)$ and $(\underline{c},l)$ are given by equations (33) and (34). By inequality (87) and concavity of function $S(\cdot)$, the investment by types $(\overline{c},h)$ is weakly below that by type $(\underline{c},l)$, that is, $\Delta q\leqslant 0$, which contradicts our supposition $\Delta q>0$. Hence, this supposition is false and so $\Delta q\leqslant 0$. Combining the latter inequality with constraint (78), we conclude that $\Delta q=0$ and the equation (26) holds. Substituting $q\left(\underline{c},l\right)=q\left(\overline{c},h\right)$ into the expression (79) and maximizing with respect to $q\left(\overline{c},h\right)$, we find that $q\left(\overline{c},h\right)$ is given by equation (27).

Remark A.1. *Note that $q\left(\overline{c},h\right)$ given by equation (27) lies below $q^{FB}\left(\overline{c},h\right)$, while $q\left(\underline{c},l\right)$ given by equations (26) and (27) lies above $q^{FB}\left(\underline{c},l\right)$.*

These statements follow from equations (5) and (27) and concavity of function $S(\cdot)$. Indeed, the first statement is equivalent to the true inequality

$$\frac{r(1-\lambda)\overline{c}+\lambda(1-r)\underline{c}+\lambda\Delta c}{hr(1-\lambda)+l\lambda(1-r)} > \frac{\overline{c}}{h}.$$

The second statement is equivalent to inequality

$$\frac{r(1-\lambda)\overline{c}+\lambda(1-r)\underline{c}+\lambda\Delta c}{hr(1-\lambda)+l\lambda(1-r)} < \frac{\underline{c}}{l},$$

which we find is equivalent to the negation of inequality (21).

Verification of omitted incentive constraints. By equations (28) through (32),

$$B^{SB}(\overline{c},l) = P^{SB}(\overline{c},h) = 0, \tag{88}$$

$$hP^{SB}(\underline{c},h) = (1-l)\,B^{SB}(\underline{c},l) = \Delta c q(\overline{c},h). \tag{89}$$

We plug the sets of equations (88) and (89) into the omitted incentive constraints in order to verify that they hold.

The incentive constraint $((\underline{c},h) \to (\overline{c},l))$ is equivalent to inequality

$$q^{SB}(\overline{c},h) \geqslant q^{FB}(\overline{c},l), \tag{90}$$

which follows from inequalities (6), equation (26) and Remark A.1:

$$q^{SB}(\overline{c},h) = q^{SB}(\underline{c},l) > q^{FB}(\underline{c},l) > q^{FB}(\overline{c},l).$$

The incentive constraint $((\underline{c},h) \to (\underline{c},l))$ holds due to the true inequality

$$1 \geqslant \frac{1-h}{1-l}. \tag{91}$$

The incentive constraint $((\underline{c},l) \to (\overline{c},l))$ is equivalent to the above inequality (90).

The incentive constraint $((\underline{c},l) \to (\underline{c},h))$ holds due to true inequality

$$1 \geqslant \frac{l}{h}. \tag{92}$$

By equation (26), the incentive constraint $((\bar{c},h) \to (\underline{c},l))$ holds due to the true inequality

$$1 \geqslant \tfrac{1-h}{1-l}.$$

The incentive constraint $((\bar{c},h) \to (\underline{c},h))$ follows from the same set of inequalities as (86).

The incentive constraint $((\bar{c},l) \to (\bar{c},h))$ holds (as an equality): $0=0$.

The incentive constraint $((\bar{c},l) \to (\underline{c},h))$ is equivalent to inequality

$$\tfrac{l}{h} q^{SB}(\bar{c},h) \leqslant q^{FB}(\underline{c},h)$$

which follows from the same set of inequalities as (86) and true inequality (92).

E. Extreme values of probabilities l and h.

In the main text we have considered $0 < l < h < 1$. This appendix extends Propositions 3 and 4 to situations in which $l = 0$ and/or $h = 1$. Note that we continue to assume that the constraints (16) hold, meaning that profits must be non-negative for both realisations of α. This is notwithstanding that $\alpha = 0$ or $\alpha = 1$ may now be inconsistent with the agent's report, indicating that the agent has not been truthful.

If $h = 1$ and $l \in (0,1)$, the proofs of Propositions 3 and 4 go through. It is worth mentioning that the rents (28) earned by the agent of type $(\bar{c},l)$ in Proposition 4, Part (ii) do not inflate rents earned by the other types: terms multiplied by $(1-h)$ in equations (30) to (32) are equal to 0.

Suppose that $l = 0$ and $0 < h \leqslant 1$. If inequality (36) (equivalently, inequality (37)) holds, the optimal screening mechanism is described by Part (i) of Proposition 4. If inequality (36) is reversed, it is described by Part (ii) of Proposition 4. The proof of this statement follows closely the proof of Proposition 4. The only difference is that the objective (79) is linear in Δq with a coefficient equal to the difference between the left and right sides of inequality (36). Therefore, the solution is then "bang-bang". One possibility is that we have $q(\underline{c}, l) = q^{FB}(\underline{c}, l) = 0$ and, if $h < 1$, there are rents for all types. The other is that there is investment by type $(\underline{c}, l)$ to reduce information rents and $\Delta q = 0$. In the latter case, note that investment occurs even though it is useless to the principal.