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Abstract

Advantageous selection occurs when the agents most eager to buy insurance are also the cheapest ones to insure. Hemenway (1990) links it to differences in risk-aversion among agents, implying different prevention efforts, and finally different riskinesses. We argue that it may also appear when agents share the same attitude towards risk, and in the absence of moral hazard. Using a standard asymmetric information setting satisfying a single-crossing property, we show that advantageous selection may occur when several contracts are offered, or when agents also face a non-insurable background risk, or when agents face two mutually exclusive risks that are bundled together. We illustrate this last effect in the context of life care annuities, a product bundling long-term care insurance and annuities, by constructing a numerical example based on Canadian survey data.

Keywords: Propitious selection, positive or negative correlation property, contract bundling, long-term care insurance, annuity

JEL Codes: D82, I13

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1 Introduction

Since the classical contribution of Rothschild and Stiglitz (1976), most of the literature on asymmetric information in insurance markets has highlighted the difficulties associated with adverse selection. As is well-known, these difficulties include the under-provision of insurance, and even sometimes a market breakdown, without trade at all[1]. By contrast, the seminal paper by Hemenway (1990) argues that sometimes selection can be advantageous, meaning that those agents most eager to buy insurance are also the cheapest ones to insure. In such a case, insurers should be more eager to supply insurance, thereby yielding very different market outcomes, as well as different testable predictions.

Hemenway (1990, 1992) and most subsequent papers[2] envision advantageous selection as arising from differences in risk-aversion among agents: more risk-averse agents buy more coverage, and simultaneously exert more self-protection efforts, so that they may end up receiving indemnities less often—i.e., they appear less risky.[3] We instead argue that advantageous selection may occur under various circumstances, including regular settings where agents differ only in riskiness, exhibiting the same attitude towards risk, and from which moral hazard is excluded.

To do so, we rely on the definition of advantageous and adverse selection used in Einav and Finkelstein (2011). This paper mostly focuses on the case where only one insurance contract is offered to agents who are privately informed of their own exogenous riskiness. A change in the contract premium then modifies the agents' decision to insure, and therefore changes the indemnities the insurer expects to pay. The type of selection (adverse or advantageous) is measured by the sign of the slope of the insurer's average cost function with respect to the premium choice: in particular, advantageous selection occurs when "as price is lowered and more individuals opt into the market, the marginal individual opting in has higher expected cost than infra-marginal individuals" (p.124).

[1] See Hendren (2013) for an empirical analysis, and Attar et alii (2021) for a general theory.

[2] See for example de Meza and Webb, 2001; Finkelstein and McGarry, 2006; Cohen and Einav, 2007; Davidoff and Welke, 2007; Fang et al. 2008; Wang et al. 2009; De Donder and Hindriks, 2009.

[3] Hemenway (1990) makes a distinction between favorable and propitious selection, in that the latter is a special case of the former. We use the word advantageous in this paper, noting that the recent literature uses the terms advantageous, favorable and propitious interchangeably.

Hence, the type of selection stems from a composition effect in the pool of subscribers. The effects on equilibrium allocations and welfare are likely to be significant. As an example, consider the case of a single contract.[4] Under adverse selection, this contract may well be unprofitable whatever the premium, because an increase in the premium selects costlier types, leading to a market collapse.[5] By contrast, under advantageous selection, asymmetric information per se cannot yield a market breakdown: if a contract is profitable under complete information, then *a fortiori* it must also be profitable under advantageous selection. This motivates why this search for the sources of advantageous selection may be worth undertaking, and especially so in cases where the demand for insurance is deemed inefficiently low, such as for the market for long-term care insurance, which we shall examine at the end of this paper.

In Section 2, we provide the definition of advantageous and adverse selection of Einav and Finkelstein (2011) for a general insurance economy with hidden types and multiple contracts. Section 3 examines the link between advantageous selection and profits. We first study the canonical situation of perfect Bertrand competition, with discontinuous demand functions in equilibrium, and show that this setting is compatible with advantageous selection. We then move to the more realistic case of continuous demand functions and market power and we show that advantageous selection must be accompanied by a high markup rate, set above the inverse of the demand elasticity for this contract. In particular, this can only occur when maximized profit is positive.

While Section 3 relied on an aggregate demand function, in the next two sections we formulate settings that allow to build demand functions from the individual choices of privately informed consumers. We start in Section 4 with a classical model à la Rothschild-Stiglitz (1976), in which a single-crossing property ensures that riskier agents end up with higher coverage. This Positive Correlation Property (PCP), defined in Chiappori and Salanié (2000), is a global property comparing two distinct contracts: the better-insured agents should appear riskier than less well-insured agents. Still, even in this very regular case we show that advantageous selection may

[4] There is no general consensus on which equilibrium concept to use for insurance markets. It is therefore difficult, and beyond the scope of this paper, to provide a full equilibrium analysis.

[5] See Hendren (2013), and Attar et alii (2021) for a general treatment.

well occur.

Indeed, an increase in the premium of a contract induces some agents to quit, either to subscribe to a contract with less coverage (then these agents must be relatively low-risk agents, according to the single-crossing property), or to a contract with more coverage (for the relatively high-risks agents). The balance between these two opposite effects on the insurer's average cost is thus ambiguous, and so is the nature of selection. Hence, a contract may well face advantageous selection simply because there exists a contract proposing a higher coverage. Also, it is interesting to note that the definition we have chosen does not always agree with the PCP test, which was designed to detect the presence of asymmetric information.[6]

While Section 4 identified a possible source of advantageous selection in a case with only one risk but multiple contracts offering different coverages, in Section 5 we turn to the case of two exclusive risks, and a unique contract. Once more, there is no moral hazard, and agents differ only in their riskiness, which is now a two-dimensional variable. We first assume that the contract covers only the first risk, while the second risk is a non-insurable background risk (Section 5.2). As a consequence, when buying insurance for the first risk, a consumer should take into account the fact that he pays the premium even in the state when the insurable loss did not occur, while the non-insured background loss did occur. This reduces his willingness-to-pay for an insurance contract covering only the first risk, and more so for an agent whose probability to face the background loss is higher. Therefore, participation in an insurance contract depends on private exposure to the background risk. We show by means of an example that agents with a high probability to incur the insurable loss may choose not to insure, because they also display a high probability of the background loss. This proves that advantageous selection may occur even when a single contract is offered. Note that this requires two conditions: that at the level of an agent the correlation of losses is negative (indeed, we assume for simplicity that the two

[6] Making the link with previous empirical investigations, Einav et alii (2011) also notes that "making inferences about marginal individuals is difficult, however. As a result, the early empirical approaches developed strategies that attempt to get around this difficulty by, instead, focusing on comparing averages", and more precisely by "comparing the expected cost of those with insurance to the expected cost of those without (or comparing those with more insurance coverage to those with less coverage)" (p.127). Our contribution here is to emphasize that the PCP and the definition we use do not measure the same phenomenon.

risks are exclusive); and that the correlation of the two riskinesses over the whole population is positive enough.

In the same model, but allowing now both risks to be insured (Section 5.3), we show how a contract bundling together coverage against the two risks may face advantageous selection, even though every contract that would cover only one of these risks would face adverse selection. This leads to the observation that issuing bundled insurance contracts may help insurers avoid the difficulties associated with adverse selection. Section 6 constructs a numerical example of this idea by bundling annuity contracts and Long-Term Care (LTC hereafter) insurance. Annuity contracts ensure a longevity risk, for which empirical studies tend to support the existence of adverse selection.[7] LTC corresponds to the risk of becoming dependent at old age and of needing "day-to-day help with activities such as washing and dressing, or help with household activities such as cleaning and cooking" (OECD, 2011). While this risk realizes with high probability and implies significant expenses, most people choose not to insure against it. OECD (2011) has estimated that only 2% of LTC expenditures are financed by private LTC insurance (LTCI hereafter) in OECD countries, while the figure is 7% in the US. This is often referred to as the LTCI puzzle. Adverse selection is often mentioned as one reason why this market is so little developed.[8] Notice that in both cases, moral hazard can be assumed away.[9]

Creating an insurance contract that would insure against both the longevity risk and the LTC risk could alleviate simultaneously both adverse selection problems. Such a contract was proposed as a 'life care annuity' contract in Murtaugh et al. (2001) and Brown and Warshawsky (2013). Ameriks et al. (2011) provides survey evidence of a significant potential demand for such a product. Intuitively, agents with poor health face high prices for LTC insurance, while they are less costly on the annuity side because of their reduced life expectancy. Conversely, there exists a group of lucky agents with a high longevity in good health, meaning that they are high

[7] Some classical references are Finkelstein and Poterba (2002, 2004), and Rothschild (2009).

[8] On the LTCI puzzle and the importance of adverse selection, see among others, Pauly (1990), Sloan and Norton (1997), Brown and Finkelstein (2007, 2009), Pestieau and PonthiÃšre (2012), Lockwood (2018) and Boyer et al. (2019, 2020).

[9] In as much as the need for help in day-to-day activities can be measured objectively, the indemnity served only depends on the advent of dependency, and is not conditional on the type or amount of help received.

risks for annuity sellers and low risks for LTCI sellers. Bundling the two insurance products would thus likely reduce the extent of adverse selection. In this article, we even argue that bundling could generate advantageous selection. Using Canadian survey data, we are indeed able to recover the two-dimensional riskiness probabilities for a dataset of 2.000 individuals, and under reasonable assumptions on preferences we construct a numerical example of a bundled contract displaying advantageous selection.

2 Framework and definitions

In this section, we describe a general insurance economy with hidden types. As discussed above, we depart from most of the existing literature by providing rationales for advantageous selection which do not hinge on moral hazard.

Consider a fixed population of observationally identical consumers. Each consumer faces a risk, which is represented by a stochastic, individual-specific state of the world, whose realization is public. An insurance contract is a premium $P \geq 0$ to be paid upfront, and an indemnity function $I(.)$ specifying the amount to be paid as a function of the realization of the state.

Each consumer is endowed with a private information α, that may bear both on his preferences and riskiness, and whose distribution in the population is known.[10] An individual with type α who buys the contract $(P, I(.))$ obtains a payoff $V(P, I(.), \alpha)$, and for the moment we only assume that this payoff is decreasing with the premium P. The insurer's revenue is P, minus the cost associated to the payment of indemnities that we represent as a function $c(I(.), \alpha)$. Notice that a consequence of excluding moral hazard is that this cost does not depend on P.[11] Notice also that this definition of costs implies constant returns to scale: total profits are simply the sum of the profits made on each consumer.

As an illustration, in the Rothschild-Stiglitz (1976) model, agents face a binomial risk on their wealth and differ through their riskiness α distributed on $[0, 1]$. Then, an insurance contract

[10] From Section 4 on, we assume that agents differ only in riskiness.

[11] Moral hazard would re-introduce P as an argument of c, either because the agent's choice of effort may be impacted by the value of the premium through wealth effects, or – more perversely– because the agent may be indifferent between several effort levels and may change his selection as a function of the premium.

is simply a pair (P, I), and payoffs and costs are:

$$V(P, I, \alpha) = \alpha u(w - L + I - P) + (1 - \alpha)u(w - P) \qquad c(I, \alpha) = \alpha I,$$

where w is the initial wealth level, L is the loss, and $u(.)$ is an increasing and concave function.

The market situation we consider throughout the paper is as follows. The focus is on a particular contract $(P, I(.))$, while other available contracts $(P_j, I_j(.))_{j \in J}$ are fixed. A consumer α selects the contract under study if and only if the following participation constraint holds:

$$PC(\alpha): \qquad V(P, I(.), \alpha) \geq \max_{j \in J} V(P_j, I_j(.), \alpha).$$

This inequality defines a set of subscribers, together with the subscribers' distribution of types. If this set is non-empty, and ignoring ties, we can define the insurer's average cost for this contract as a function

$$AC(P, I(.), (P_j, I_j(.))_{j \in J}) = E[c(I(.), \alpha) | PC(\alpha) \text{ holds}],$$

where the expectation is taken over the set of types. Because the emphasis will be put on changes in the premium P, we shall often omit arguments and use the lighter notation $AC(P)$. As in Einav et al. (2010a) and Einav and Finkelstein (2011), we adopt the following definition:

Definition 1 *A contract $(P, I(.))$ is said to face adverse selection when $AC(.)$ is increasing with the premium at P, while it faces advantageous selection when $AC(.)$ is decreasing with the premium at P.*

These definitions formalize the idea that a change in premium affects the composition of the pool of subscribers, in a manner that may harm or benefit the insurer. In fact, in the absence of moral hazard this composition effect is the unique channel through which a change in premium has some impact on the expected cost of the contract.[12]

[12]Interestingly, the survey in Einav et al. (2010b) allows for moral hazard, but uses a different definition: There is adverse selection if $AC(P)$ is greater than the average cost of insuring the *whole* population with the contract $(P, I(.))$, assuming that each agent would select the optimal effort under that contract. This definition has the drawback of requiring a precise knowledge of the whole population, while the definition we use allows to only consider the effects of an increase in the premium of a contract on the population of subscribers to this contract.

3 Advantageous selection and profits

To study this composition effect, one has to evaluate what would happen if the premium were to be modified. At this stage, it is worth noting that many theoretical analyses of insurance markets assume a discrete set of types, together with perfect competition in prices. In such cases, equilibrium incentive constraints bind for a nonzero fraction of consumers, leading to discontinuous demand functions.[13] We first show that advantageous selection may occur in such a framework, before turning to the probably more realistic scenario where demand is continuous in price.

Consider the canonical situation of perfect Bertrand competition, so that in equilibrium all firms post the same price P, share the market equally, and get zero profits: hence, the price P equals the average cost of the contract sold by all firms, $AC(P)$. No firm has an incentive to increase its price unilaterally, since it would lose all demand. A firm decreasing unilaterally its price slightly below P would attract customers that were previously uninsured and under advantageous selection, this results in a loss for the deviating firm. Hence, advantageous selection seems compatible with a Bertrand equilibrium with zero profit. Actually, the only case which is precluded in the Bertrand equilibrium is the one where $AC'(P) > 1$ (*i.e.*, very large adverse selection effects), since this would result in a profitable downward price deviation.

We now move to the polar case where the insurer offering the contract under study enjoys some market power, so that he faces a continuous demand function $D(P)$, equal to the measure of agents for whom the participation constraint $PC(\alpha)$ holds. Then, the insurer's profits are

$$\Pi(P) = (P - AC(P))D(P).$$

Assuming differentiability, the first-order condition for profit maximization is

$$\Pi'(P) = (1 - AC'(P))D(P) + (P - AC(P))D'(P) = 0.$$

Making use of the demand elasticity, defined as usual as

$$\varepsilon(P) = -\frac{PD'(P)}{D(P)} > 0,$$

[13]This property indeed holds for equilibria in Rothschild and Stiglitz (1976) or de Meza and Webb (2001).

with $D'(P) < 0$, we obtain a modified rule for monopoly pricing:

$$\frac{P - AC(P)}{P} = (1 - AC'(P))\frac{1}{\varepsilon(P)}. \tag{1}$$

Intuitively, under advantageous selection, the insurer is tempted to raise the premium in order to keep only the less costly customers: indeed, this favorable composition effect reduces the average cost and raises the profit margin. To counterbalance this effect, it has to be that consumers who would stop buying the contract after the rise in premium are both numerous and highly profitable. Therefore, profit-maximizing pricing is consistent with advantageous selection only when both the profit margin and the demand elasticity are high enough, as expressed in the above equality. We thus have shown the following result:

Result 1 *When the insurer enjoys market power on a given insurance contract and the demand function is differentiable in its price, the presence of advantageous selection must be accompanied by a high markup rate, set above the inverse of the elasticity of demand for this contract. Therefore, equilibrium profits must be strictly positive.*

This result may be seen as providing a test for advantageous selection. It also points to the fact that whatever the market structure, a study of selection requires to know who the marginal consumers are, and how they react to a change in the premium. This is what we intend to do in the next sections. We make no specific assumption regarding competition in the insurance market and rather look at the conditions under which advantageous selection (as given by Definition 1) occurs for a given set of insurance contracts.

4 Advantageous selection under single-crossing

As recalled in the Introduction, Hemenway (1990, 1992) and many subsequent works single out a rationale for advantageous selection which relies on agents having different attitudes towards risk, and choosing both their coverage and their prevention effort. The driving force is that more risk-averse agents choose both a higher coverage and higher prevention efforts, and it may be that this second effect is strong enough to reduce the riskiness of insured agents below that of uninsured agents.

We focus here instead on a class of settings that have been widely used in the literature in order to study the effects of asymmetric information. In this class derived from the Rothschild and Stiglitz (1976) model, agents differ only in riskiness, and those with a higher probability of loss are more costly to insure. For the same reason, they are also more eager to buy more coverage. This single-crossing property creates a rich structure. For a given set of insurance contracts, the subscribers of each contract form an interval of risk types, and these intervals are ordered by coverage.

We first show that this structure also arises in equilibrium for a much more general class of models. Formally, consider the case where α is unidimensional, with a differentiable distribution F on the interval $[0, 1]$. Suppose that consumers face a monetary risk L continuously distributed over some interval, and that higher types face a higher risk, in the sense of the Monotone Likelihood Ratio Property (MLRP hereafter).[14] Each agent is an expected utility maximizer, and gets the payoff

$$V(P, I(.), \alpha) \equiv E[u(w - P + I(L) - L)|\alpha]$$

when he subscribes to the insurance contract $(P, I(.))$. Note that all agents share the same utility function, and thus the same risk attitude.

As in Chiappori et al. (2006), we assume that available contracts specify indemnity functions $I(.)$ that are increasing with the loss, and in addition that the amount which is not covered increases with the loss, i.e., $L - I(L)$ is increasing with L (for example, because one does not want the agent to hide the true extent of their losses). Finally, we assume that available contracts can be ranked by coverage, in the following sense: a contract $(P_2, I_2(.))$ covers more than a contract $(P_1, I_1(.))$ if and only if the difference $I_2(L) - I_1(L)$ is increasing in L. Many classes of real-life contracts satisfy these requirements, for example contracts with deductibles ($I(L) = \max\{L - D, 0\}$), or co-insurance contracts ($I(L) = \delta L$, $\delta < 1$).

[14] This property expresses that the ratio of conditional densities $g(L|\alpha')/g(L|\alpha)$ is increasing in L when $\alpha' > \alpha$, thus making higher losses more likely for higher types.

Our last assumption is that the individual cost function $c(I(.),\alpha)$ increases with the expected payment $E[I(L)|\alpha]$. Then MLRP yields two properties. First, higher types are more costly to insure. Second, if a type prefers a contract that covers more to a contract that covers less, then so do all higher types.[15] This single-crossing property implies that the population of types is segmented into intervals, with higher (and thus costlier) types choosing contracts with higher coverage. This generalized model thus yields the same market structure as the Rothschild-Stiglitz model.

It is remarkable that advantageous selection may still be observed in such a standard setting. Indeed, the set of subscribers to a contract $(P, I(.))$ forms an interval of types $[\underline{\alpha}(P), \overline{\alpha}(P)]$. Notice that the lower bound, $\underline{\alpha}(P)$ is increasing with P, while the upper bound, $\overline{\alpha}(P)$ is decreasing with P. Indeed, when P increases, low-risk agents prefer switching to a contract with less coverage, and high-risk agents to one with more coverage.

Consider first the particular case where only one contract is offered on the market. This contract attracts all types above the threshold $\underline{\alpha}(P)$, so that $\overline{\alpha}(P) = 1$, and we obtain

$$AC(P) = \frac{\int_{\underline{\alpha}(P)}^{1} c(I(.),\alpha)dF(\alpha)}{1 - F(\underline{\alpha}(P))}.$$

In this case, adverse selection must prevail: the marginal consumers who leave when the premium is increased are those consumers with the lowest types $\underline{\alpha}(P)$, and these types display the lowest cost among all subscribers. Indeed, one easily shows that the derivative $AC'(P)$ has the sign of

$$f(\underline{\alpha}(P))\underline{\alpha}'(P)[AC(P) - c(I(.),\underline{\alpha}(P))],$$

which is nonnegative.

However, in the more general case where several contracts are offered, there might exist a contract offering a higher coverage, and in that case the upper bound $\overline{\alpha}(P)$ is not trivial anymore. Then we have

$$AC(P) = \frac{\int_{\underline{\alpha}(P)}^{\overline{\alpha}(P)} c(I(.),\alpha)dF(\alpha)}{F(\overline{\alpha}(P)) - F(\underline{\alpha}(P))},$$

[15] A proof of this result is provided in Appendix.

and we get that the derivative $AC'(P)$ has the same sign as

$$f(\underline{\alpha}(P))\underline{\alpha}'(P)[AC(P) - c(I(.), \underline{\alpha}(P))] + f(\overline{\alpha}(P))\overline{\alpha}'(P)[c(I(.), \overline{\alpha}(P)) - AC(P)]. \tag{2}$$

An increase in P now has two effects. Marginal subscribers with the lowest types decide to migrate to a contract with a lower coverage (the first term above), while marginal subscribers with the highest type instead migrate to a contract with higher coverage (the second term). The sum of these effects is ambiguous, as $\overline{\alpha}'(P) \leq 0 \leq \underline{\alpha}'(P)$.[16]

To summarize, we have shown that *adverse and advantageous selection are mirror images of each other*. They appear simultaneously, for each contract, as soon as the set of contracts is sufficiently rich. The assumptions we made in this section are often said to formalize the idea of adverse selection – but in fact they generate advantageous selection as well. At the extremes, the contract with the lowest coverage (maybe the null contract) must exhibit advantageous selection, while the contract with the highest coverage must exhibit adverse selection; in between, the nature of selection depends not only on costs, preferences, and the distribution of types, but also on the presence and characteristics of other contracts.

Result 2 *A contract may face advantageous selection when there exists an alternative insurance contract offering higher coverage.*

Notice that our assumptions are strong enough to imply that whatever the contracts offered, riskier types end up with more coverage, thus yielding a positive correlation between riskiness and coverage. This positive correlation property (PCP, see Chiappori and Salanié (2000)) is often a consequence of well-structured theoretical models à la Rothschild-Stiglitz (1976), where riskier agents are more eager to buy more coverage.[17] It is interesting to relate this property to our definition of selection. From an empirical viewpoint, our definition focuses on a single

[16] Even when postulating uniformly distributed types and costs $c(I(.), \alpha)$ linear in α, expression (2) only tells us to check the sign of $\underline{\alpha}'(P) + \overline{\alpha}'(P)$. Despite these strong assumptions, this sign still depends on detailed properties of neighboring contracts and remains ambiguous.

[17] Chiappori et al. (2006) use a revealed preference argument to show that a similar property also obtains in very general models, provided profits (defined as the difference between premiums and expected indemnities) are non-increasing with coverage. As underlined in de Meza and Webb (2001), a negative correlation property may also appear in a competitive equilibrium when fixed costs per insured agents are significant.

contract, but requires to estimate how demand changes as a function of the premium; while the correlation test requires to collect data for at least two contracts (or to compare the population of insured to the population of uninsured agents). Einav and Finkelstein (2011) argue that their definition is equivalent to this correlation property in the specific one-contract setting they study, where $AC(.)$ is in addition assumed globally monotonic in P. We have shown in this section that this equivalence does not hold anymore when several contracts are offered, because the composition effect plays opposing roles downward (toward contracts with less coverage) and upward (toward contracts with more coverage).

5 Advantageous selection, background risk and bundling

While Section 4 identified a possible source of advantageous selection in a case with only one risk but multiple contracts offering different coverages, in this Section we turn to the case of two exclusive risks, and a unique contract. This will allow us to identify two other sources of advantageous selection, namely the existence of a background risk, and the bundling of two risks into a unique insurance contract. Note that assuming that only one contract is proposed ensures that finding advantageous selection cannot be attributed to the existence of alternative contracts providing higher coverage.

5.1 The setting

For each agent, there are three mutually exclusive individual states of the world, labelled 0, 1 and 2. An agent ending up in state i incurs a monetary loss equal to L_i, with $L_0 = 0$ so that state 0 is the state without loss. An agent's type $\alpha = (\alpha_1, \alpha_2)$ specifies the probabilities of ending up in state $i = \{1, 2\}$, with $\alpha_1 + \alpha_2 < 1$. Types are distributed in the population of agents according to a c.d.f. F. An insurer proposes to these agents a single contract (P, I_1, I_2). If an agent of type (α_1, α_2) buys it, then the insurer's cost is simply the expectation of indemnities paid:

$$c(I_1, I_2, \alpha) = \alpha_1 I_1 + \alpha_2 I_2,$$

and the expected utility of an agent with initial wealth w is[18]

$$V(P, I_1, I_2, \alpha) = \alpha_1 u(w - L_1 + I_1 - P) + \alpha_2 u(w - L_2 + I_2 - P) + (1 - \alpha_1 - \alpha_2) u(w - P).$$

where u is strictly increasing and strictly concave in income, and is the same for all agents.

5.2 Background risk

One intriguing question is whether advantageous selection can occur for contracts that cover a single risk, say the risk of being in state 1, while individuals face also a second uninsurable risk (equivalently, a background risk), say risk 2. Thus, consider the contract $(P, I_1, 0)$. A type α buys this contract if and only if $V(P, I_1, 0, \alpha) \geq V(0, 0, 0, \alpha)$, or equivalently

$$\begin{aligned} &\alpha_1 u(w - P - L_1 + I_1) + \alpha_2 u(w - L_2 - P) + (1 - \alpha_1 - \alpha_2) u(w - P) \\ &\qquad \geq \alpha_1 u(w - L_1) + \alpha_2 u(w - L_2) + (1 - \alpha_1 - \alpha_2) u(w). \end{aligned}$$

This participation constraint can be rewritten as

$$\begin{aligned} \alpha_1 u(w - P - L_1 + I_1) + (1 - \alpha_1) u(w - P) \quad &\geq \quad \alpha_1 u(w - L_1) + (1 - \alpha_1) u(w) \\ &+ \quad \alpha_2 [u(w - L_2) - u(w - L_2 - P) - (u(w) - u(w - P))]. \end{aligned}$$

We recognize on the first line the usual trade-off associated to the risk α_1 of a single loss L_1, but on the second-line an additional term appears. The bracketed expression turns out to be positive under risk-aversion, and is increasing in P. This means that a higher α_2 makes the agent more reluctant to participate, and more sensitive to an increase in the premium. Intuitively, because risks are exclusive, the only impact of the background risk is to increase the utility cost of paying the contract premium. This simple observation means that even in the case of a contract that covers only one risk, the participation frontier is not trivial.

We now show that this contract may face advantageous selection, even though it is the only contract on the market. Consider the simple case of full coverage: $I_1 = L_1$. Then the

[18] In this section, we abstract from the possibility of state-dependent preferences as our rationale for the emergence of advantageous selection is orthogonal to this issue. In Section 6, we consider state-dependent preferences because they seem indispensable when modeling the risk of death.

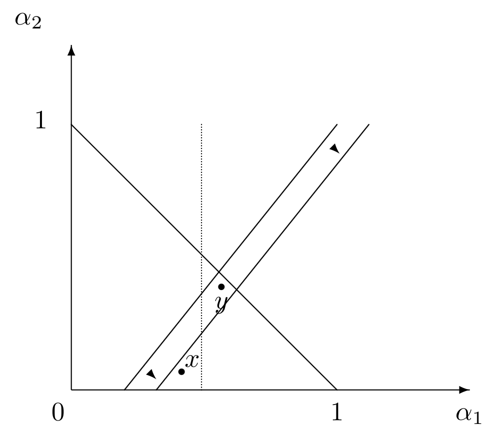
participation constraint can be rewritten as

$$b\alpha_1 - a \geq \alpha_2,$$

where

$$a = \frac{u(w) - u(w-P)}{u(w-L_2) - u(w-L_2-P) - (u(w) - u(w-P))} > 0$$
$$b = \frac{u(w) - u(w-L_1)}{u(w-L_2) - u(w-L_2-P) - (u(w) - u(w-P))} > 0.$$

where a may increase or decrease with P (because of wealth effects) while b decreases with P. This participation frontier is represented in the space of types in Figure 1 by an ascending solid line. Agents with types (α_1, α_2) above the frontier prefer not to buy the contract while those with types below do. The descending solid line is the line $\alpha_2 + \alpha_1 = 1$, that constrains feasible types. The bottom triangle is thus the set of subscribers to the contract under study.



Note: Types in the bottom triangle buy the contract; the dotted line is an iso-cost curve.

Figure 1: Advantageous selection with a background risk

To illustrate our argument, consider the special case where types take only two values x and y, as shown in the figure. The iso-cost curve corresponding to the average cost is the vertical dotted line, with an equation given by

$$AC(P) = E[\alpha_1 I_1 | b\alpha_1 - a \geq \alpha_2].$$

Being an average cost, it must thus lie in between x and y, as shown on Figure 1. An increase in P then shifts the ascending solid line in the direction of the arrows, into a new ascending line (it is parallel to the initial line under constant absolute risk-aversion). One sees on Figure 1 that this move indeed induces type y agents to quit buying the contract because their high probability of a background risk reduces their willingness-to-pay for insurance, while type x agents choose to stay. In this case, an increase in P leads to a reduction in $AC(P)$, and advantageous selection occurs.[19] We have shown the following result:

Result 3 *The presence of a background risk makes advantageous selection possible, even in the simple case of an insurance contract designed to cover only one risk.*

Note that the introduction of the background risk L_2 is the only difference with the Rothschild-Stiglitz model. To check for the role of correlations in the above result, one has to carefully distinguish two notions. We assumed that the two risks were exclusive, and thus a strong form of *negative correlation between losses.* Alternatively, one could assume that the two losses are independent, thus creating four different states of nature instead of three. It can be shown that under constant absolute risk-aversion, the new participation constraint is independent from the value of α_2, and therefore that we are back to a standard model where adverse selection must hold; taking wealth effects into account considerably complicates the picture, and we will not follow this avenue further. Going back to our case where risks are exclusive, one sees from the above reasoning that a form of *positive correlation between riskinesses* has to hold for advantageous selection to prevail: when the premium is increased, agents with high values for α_1 are the first to leave because they also display high values for α_2, and thus are more sensitive to this increase in the premium.

5.3 Contract bundling

Let us now study whether advantageous selection may occur when both risks are insured through a single contract. We proceed step-by-step.

[19] The negative correlation property holds for sure in the final situation, as riskier types are not covered anymore. The correlation property thus agrees with our definition in this very specific case.

Participation and expected cost under bundling A bundling contract (P, I_1, I_2) provides insurance against both risks. We therefore assume $I_1, I_2 > P > 0$. An agent of type (α_1, α_2) buys the contract if and only if $V(P, I_1, I_2; \alpha) \geq V(0, 0, 0; \alpha)$, or equivalently

$$PC(\alpha): \qquad \alpha_1 A_1 + \alpha_2 A_2 \geq H,$$

where

$$H = u(w) - u(w - P) > 0$$

can be interpreted as the utility cost of the premium in the absence of loss, while

$$A_i = u(w - L_i + I_i - P) - u(w - L_i) + H > 0 \qquad i = 1, 2 \tag{3}$$

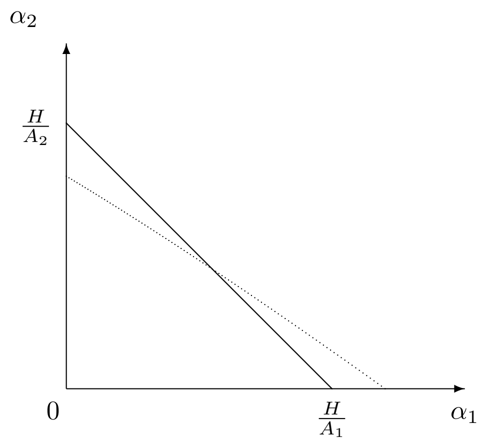
adds to H the utility gain from being insured when state i realizes. Note that A_1, A_2 and H are independent of the agent's type. Figure 2 represents this set of types, whose frontier is the solid line $\alpha_1 A_1 + \alpha_2 A_2 = H$. Agents whose type lies above the frontier buy the insurance contract (P, I_1, I_2), while types below the frontier do not. Because agents dislike increases in the premium, such an increase must shift the frontier upwards, meaning that both H/A_1 and H/A_2 must increase with P; this is readily verified using the above formulas.

Moving to the insurer's side, an iso-cost curve is $\alpha_1 I_1 + \alpha_2 I_2 = c$ for some constant cost c. One of these iso-cost curves is represented by the dotted line in Figure 2. Note that there is no a priori reason why iso-cost and participation lines should have the same slope, a point on which we shall come back soon. Finally, the expected cost of the contract is

$$AC(P) = E[\alpha_1 I_1 + \alpha_2 I_2 | PC(\alpha) \text{ holds}]. \tag{4}$$

Necessary conditions for the existence of advantageous selection Advantageous selection necessitates that an increase in the premium lead some high-cost types to unsubscribe. These marginal subscribers' types must thus belong to the participation frontier, with individual costs $c(I_1, I_2, \alpha)$ above the average cost $AC(P)$. At this point, essentially two cases may occur.

It may be that all types on the participation frontier have the same cost. In that situation, the slope of the participation frontier equals the slope of an iso-cost curve, i.e., $I_1/I_2 = A_1/A_2$.



Note: Types above the solid line buy the bundling contract; the dotted line is an iso-cost curve.

Figure 2: Advantageous selection under contract bundling

In this case, all types above the frontier display a higher cost than those at the frontier, and the iso-cost corresponding to the average cost $AC(P)$ must also lie above the participation frontier. Then advantageous selection cannot occur: an increase in P moves the participation frontier upward and all consumers who stop buying the contract are the lowest cost ones.

Alternatively and generically, types on the participation frontier may exhibit different costs. To fix ideas, suppose that the participation frontier is steeper than iso-cost curves, i.e., $I_1/I_2 < A_1/A_2$, which is the case illustrated on Figure 2 (the opposite inequality would define a symmetrical case and would be dealt with similarly). Then, individual costs decrease when one goes down the frontier, from the northwest to the southeast. A necessary condition to obtain advantageous selection is that the highest cost on the frontier (which equals $\alpha_2 I_2$ at $(\alpha_1, \alpha_2) = (0, H/A_2)$) is larger than $AC(P)$. Since $AC(P)$ is an average cost, the corresponding iso-cost line must lie above the one representing the lowest cost in the population of subscribers, which is also the lowest cost on the frontier (which equals $\alpha_1 I_1$ at $(\alpha_1, \alpha_2) = (H/A_1, 0)$). Therefore, a necessary condition for advantageous selection to occur is

$$\frac{I_1}{A_1} < \frac{AC(P)}{H} < \frac{I_2}{A_2}. \tag{5}$$

Moreover, if the premium increases, some agents decide not to buy the contract anymore, and the participation constraint moves upward. For advantageous selection to arise, an additional necessary condition is that the measure of agents exiting the market to the left of the intersection of the iso-cost and the frontier (that is, with costs above the average) be sufficiently large compared to the measure of agents exiting the market to the right of the intersection (with costs below the average).

Existence of advantageous selection under full insurance We now provide a simple example to show that advantageous selection may indeed occur. To do so, we assume full insurance for both risks ($I_1 = L_1$ and $I_2 = L_2$), so that we now have

$$A_i = u(w) - u(w - L_i),\ i = 1, 2 \qquad H = u(w) - u(w - P).$$

In particular, the slope of the participation frontier A_1/A_2 becomes independent of P. This is also the case for the slope of the iso-cost curve which now equals L_1/L_2. It is interesting to note that, due to risk-aversion, the ratio

$$\frac{A_i}{L_i} = \frac{u(w) - u(w - L_i)}{L_i} \tag{6}$$

is increasing in L_i. Therefore, when $L_1 = L_2$ the two slopes are equal, and advantageous selection cannot occur, as already observed. In fact, in such a situation, we are back to a Rotschild-Stiglitz model with one contract covering against a single loss that realizes with probability $\alpha_1 + \alpha_2$.

Assume now $L_1 > L_2$. Since A_i/L_i increases with L_i, the slope of the participation frontier is larger (in absolute value) than the slope of the iso-costs, as in Figure 2, and therefore the highest-cost types on the participation frontier are those with a type close to $(0, H/A_2)$. We obtain the paradoxical result that, although the indemnity is larger for the first loss than for the second one (since $I_1 = L_1 > I_2 = L_2$), the minimum cost for the insurer on the participation frontier is attained for a type with a high probability of the largest loss (i.e. $\alpha_1 = H/A_1$). The intuition for this result comes from the behavior of agents. Since $L_1 > L_2$, insurance against state 1 is more valuable to agents than insurance against state 2. This translates into a smaller participation threshold (i.e., minimum risk α_i to buy the insurance contract) for contract 1 (i.e.

$H/A_1 < H/A_2$). Due to risk-aversion, this effect on the loss probability for the insurer more than compensates for the higher loss L_1.

We now propose a distribution of types under which adverse selection holds for every unbundled contract of the form $(P, I_1, 0)$ or $(P, 0, I_2)$, while advantageous selection occurs for at least one bundled contract. Consider the case where a proportion 1/2 of agents face only risk 1, with a loss probability α_1 uniformly distributed on $[0, 1]$; and the other half face only risk 2, with α_2 also uniformly distributed. Then only agents in the first group may be attracted by a contract $(P, I_1, 0)$, and this simple structure Ã la Rothschild-Stiglitz immediately implies that adverse selection must prevail for such a contract; and similarly for contracts of the form $(P, 0, I_2)$.

There remains to show that advantageous selection may happen for some full-insurance contract (P, L_1, L_2). We need participation from both groups, so let us set P so that $P < L_2 < L_1$, or equivalently $H < A_2 < A_1$. Then the average cost is

$$AC(P) = \frac{\int_{\frac{H}{A_1} \leq \alpha_1 \leq 1} \alpha_1 L_1 d\alpha_1 + \int_{\frac{H}{A_2} \leq \alpha_2 \leq 1} \alpha_2 L_2 d\alpha_2}{\int_{\frac{H}{A_1} \leq \alpha_1 \leq 1} d\alpha_1 + \int_{\frac{H}{A_2} \leq \alpha_2 \leq 1} d\alpha_2}.$$

Result 4 *One can find a Constant Relative Risk Aversion utility function and parameters* (w, L_1, L_2) *such that* $AC(P)$ *is decreasing on a non-empty interval of values of* P*, for the full insurance contract* $(P, I_1 = L_1, I_2 = L_2)$.

An example with a relative risk aversion equal to 2 is provided in the Appendix.

6 The life care annuity market

We now switch to the study of a specific case where bundling different risks into a single contract may create advantageous selection. These two risks are the longevity risk, and the Long Term Care risk. The first risk may be covered by annuities, while the second one is usually dealt with by medical insurances. In both cases, adverse selection plays a role, as consumers who expect a long life are more likely to purchase annuities (notice that they are also less likely to need LTC); while those with poor health are more eager to buy a LTC insurance (but have a lower life expectancy). Consequently, bundling these two risks into a single insurance contract may well reduce the extent of adverse selection, as underlined in Murtaugh et al. (2001) and Brown and Warshawsky (2013). Here, we aim at constructing a reasonable numerical example based on Canadian data to explore whether this bundled contract could face advantageous selection, even though each risk dimension taken separately gives rise to adverse selection.[20]

To inform our numerical example, we use data from a survey conducted in 2016 on 2,000 respondents aged 50 to 70, from Ontario and QuÃ©bec, representative of the older population in these two provinces.[21] The survey includes many socio-demographic and health information about each participant. The answers to these questions can be fed directly into a health microsimulation model (called COMPAS), which provides estimates of personalized lifetime exposure to death and disability from socio-economic and health individual characteristics. More precisely, in the COMPAS model, each individual has multiple characteristics: socio-demographic (such as age, sex, immigration status, education level), health related (diseases such as diabetes, high blood pressure, heart diseases, stroke, cancer, lung diseases, or dementia), risk factors (smoking, obesity), etc. Based on these characteristics, the core of the model consists of a Markovian transition model of the health state variables listed above. This microsimulation model then estimates the individuals' probability to be alive at 85 years old as well as the probability of ever entering a nursing home; note that this event typically takes place around or after this age. We refer the reader to Boisclair et al. (2016) for more detailed information on the COMPAS model.

From now on, we shall distinguish three states of nature. State 0 corresponds to death before 85; state 1 is associated to a long life with bad health, during which costly LTC is needed; finally, state 2 is associated to a long and healthy life, with intermediate needs for higher revenues. Each individual in the survey is thus characterized by the probabilities α_1 and α_2 of being in state 1 or 2. Our data enable us to recover the type (α_1, α_2) of each individual since the probability

[20] For simplicity, the simulations below rely on strong assumptions, and accordingly we refrain from any statement concerning, for example, welfare gains from bundling.

[21] Data are available at `https://dataverse.scholarsportal.info/dataset.xhtml?persistentId=doi:10.5683/SP2/PP5U7Y`. To correct for the under- or overrepresentation of some socio-demographic groups in the sample, we reweighed the data using the Labor Force Survey of 2014. More details about this survey are given in Boyer et al. (2019, 2020).

to be alive at 85 corresponds to $\alpha_1 + \alpha_2$ while the probability of entering a nursing equals α_1. Figure 3 represents this cloud of 2,000 points in the types' space. We also present descriptive statistics in Table 1.

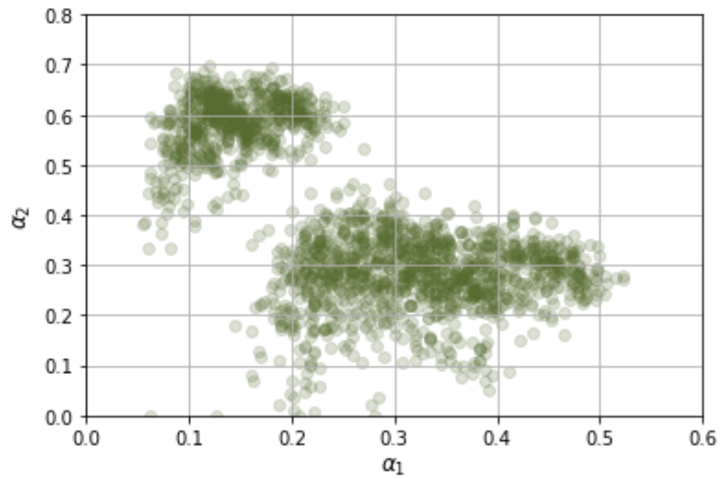


Figure 3: Probability distribution of (α_1, α_2).

	α_1	α_2	$1 - \alpha_1 - \alpha_2$
mean	0.262	0.386	0.352
std	0.112	0.16	0.119
min	0.055	0.000	0.125
max	0.523	0.698	0.938

Table 1: Distribution of types for the 2,000 individuals.

Interestingly, the figure allows to distinguish two groups. The lucky group in the Northwest displays a high probability α_2 of good health; while the other (unlucky) group displays a high risk of needing LTC (a high α_1). Note that the correlation coefficient between α_1 and α_2 is equal to -0.67. For each risk insured separately, we thus expect adverse selection to prevail, as it is a one-dimensional risk without moral hazard, and the correlation of riskinesses is negative (see our comments at the end of subsection 5.2). But this negative correlation also means that those agents who need LTC do not have a high demand for annuities contingent to good health, and vice-versa.

Formally, the losses in each state can be defined as follows. The loss L_2 in state 2 can be construed as the income needed when alive (and, hence, justifying the buying of annuities), while $L_1 - L_2 > 0$ can be seen as the cost of LTC services. A life care annuity contract (P, I_1, I_2) consists in providing an annuity I_2 together with LTC insurance coverage $(I_1 - I_2 > 0)$. For an agent with income w and type $\alpha = (\alpha_1, \alpha_2)$, the expected utility from buying this contract is

$$V(P, I_1, I_2, \alpha) = \alpha_1 u(w - L_1 + I_1 - P) + \alpha_2 u(w - L_2 + I_2 - P) + (1 - \alpha_1 - \alpha_2)v(w - P),$$

where we assume the same utility function $u(x)$ when alive (whenever dependent or not), but a different utility function $v(x)$ when dead.[22]

Using this model, we now show that it is possible to construct a reasonable numerical example of life care annuity contracts exhibiting advantageous selection by bundling together the risk of a long life under good health, and the risk of becoming dependent. In this numerical exercise, we assume the following functional forms and parameter values:

$$u(x) = \frac{x^{1-\varepsilon}}{1-\varepsilon}, \qquad v(x) = \beta u(x)$$

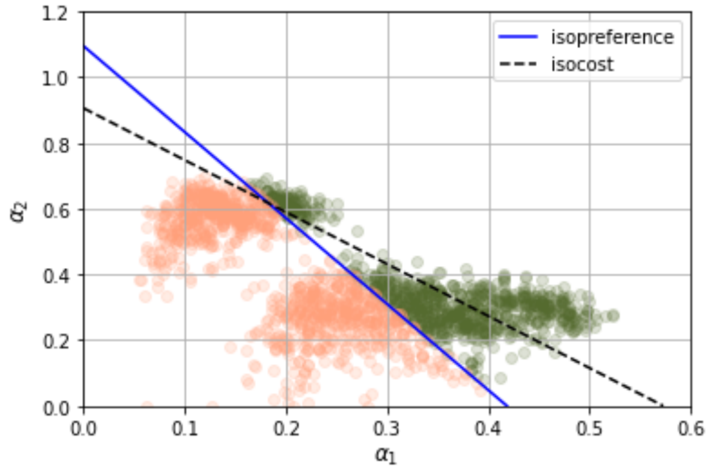
where we have chosen $\beta = 0.8$ and $\varepsilon = 0.8$ for their plausibility.[23] The utility of leaving a bequest is thus smaller than the utility of consuming the same amount when alive. In our sample, the average income is $107,000$ CAD with median $72,000$ CAD, so we have set $w = 100,000$ CAD, $L_1 = 87,000$ CAD, $L_2 = 55,0000$ CAD.

Figure 4 is the simulated equivalent of Figure 2 in Section 5.3, for a premium $P = 57,850$CAD. In the figure, the blue line represents the participation frontier (see equation (3)) and the dotted line represents the iso-cost line corresponding to the average cost $AC(P)$ (see equation (4)). In this example, the necessary condition for advantageous selection (5) is satisfied since the participation frontier has a steeper slope than the iso-average-cost line, and since both intersect.

[22]This function $v(x)$ can be seen as a "joy of giving" utility–i.e., the individual derives utility from leaving some bequests in case of death. The modeling is similar, for instance, to Glomm and Ravikumar (1992), Kopczuk and Lupton (2007), Piketty and Saez (2013) and Fleurbaey et al. (2022).

[23]Assuming a coefficient of relative risk aversion (i.e. ε) smaller than one is supported by empirical evidence (see for instance, Karagyozova and Siegelman, 2012; Holt and Laury, 2002; Chetty, 2006). Assuming different values for β and ε does not qualitatively change our results.

Orange dots account for agents choosing not to buy the contract, while green dots represent individuals buying the contract.



Note: the intersection arises at $(0.183, 0.618)$.

Figure 4: Advantageous selection

For this contract, we find that a local increase in P lowers the average cost of the contract, generating advantageous selection. When P increases, some individuals located on or immediately above the participation frontier stop buying the contract. Note that the isocost curves are flatter than the participation constraint in the (α_1,α_2) space. As P increases, two groups of agents stop buying the contract. The first group (located to the left of the intersection between the two lines) has a large probability of remaining healthy but a probability to need formal LTC low enough to induce them to stop buying the bundled contract. The second group (located to the right of the intersection) exhibits a large LTC risk, but a probability of a long healthy life small enough that they quit buying the more expensive contract. The former group (made of agents with a larger-than-average cost among contract buyers) is large enough compared to the latter (with lower-than-average cost) that the average cost of the contract decreases when its price is increased. Advantageous selection may then occur when bundling the longevity and LTC risks within life care annuities. Note that this argument necessitates a negative correlation between the longevity risk and the LTC risk, so that, as we just described the agents who are more (resp. less) costly on the annuity side of the contract are also less (resp. more) costly on the LTC side of the contract.

7 Conclusion

In this paper, we examine the effects and the possible sources of advantageous selection, in the absence of moral hazard. If the demand for a specific insurance contract is continuous in its price, we show that the presence of advantageous selection leads to large equilibrium markup rates, above the inverse of the elasticity of demand. We then study a large class of environments where the single-crossing property implies the positive correlation property, and show how adverse and advantageous selections are mirror images of each other. More precisely, when several contracts are offered, the increase in the price of a single contract induces both low-cost and high-cost agents to stop buying this contract, so that the type of selection in equilibrium is essentially an empirical question. Finally, we study an environment with two mutually exclusive risks, and show that advantageous selection may arise either when one risk is a background, uninsurable one, or when the two risks are bundled into the same contract. The necessary condition for advantageous selection to arise has a simple geometrical interpretation, stating that the iso-preference curve (delineating the set of buyers from non buyers) has to intersect the iso-average-cost line in the probabilities space. We then provide two numerical examples showing that advantageous selection can indeed occur, the first based on the simplest setting (uniform probability distributions of perfectly negatively correlated risks) and the second obtained from a Canadian survey covering the survival and long-term care risks.

These results show that advantageous selection may appear in various settings without moral hazard, when agents differ only in their riskiness, and not in their attitude toward risk. This is due to a composition effect: the set of clients to a contract changes as the premium changes, in ways that are specific to each situation. This effect is also impacted by the presence of market power, and by the existence of contracts offering higher coverage. Therefore, the nature of selection is likely to remain an empirical question, in particular when it comes to bundling different risks. Our study of this complicated geometry opens up the possibility of building new contracts for which issues associated with adverse selection are alleviated or even annihilated, and that would fare better on markets plagued by asymmetric information.

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Appendix

A study of the single-crossing property in Section 4

Let $\alpha < \alpha'$, and consider two contracts $(P_1, I_1(.))$ and $(P_2, I_2(.))$. Define $w_i(L) = w - P_i - L + I_i(L)$. If the second contract covers more, then there exists L_0 such that $u(w_2(L)) - u(w_1(L))$ is at most zero when $L < L_0$, and is at least zero when $L > L_0$. Then the difference in payoffs for type α'

$$\int_L [u(w_2(L)) - u(w_1(L))]dG(L|\alpha')$$

can be split in two integrals

$$\int_{L<L_0} [u(w_2(L)) - u(w_1(L))]\frac{g(L|\alpha')}{g(L|\alpha)}dG(L|\alpha) + \int_{L>L_0} [u(w_2(L)) - u(w_1(L))]\frac{g(L|\alpha')}{g(L|\alpha)}dG(L|\alpha).$$

Recall that MLRP states that the ratio of densities is increasing with L. Therefore, because $u(w_2) - u(w_1)$ changes sign at L_0, this expression is at least $g(L_0|\alpha')/g(L_0|\alpha)$, times

$$\int_{L<L_0} [u(w_2(L)) - u(w_1(L))]dG(L|\alpha) + \int_{L>L_0} [u(w_2(L)) - u(w_1(L))]dG(L|\alpha),$$

which is the difference in payoffs for type α. This proves that if α prefers the second contract to the first, then so does α', as announced.

Proof of Result 4: We have

$$AC(P) = \frac{1}{2}\frac{L_1(1 - (\frac{H}{A_1})^2) + L_2(1 - (\frac{H}{A_2})^2)}{1 - \frac{H}{A_1} + 1 - \frac{H}{A_2}} = \frac{1}{4}(L_1 + L_2)\frac{1 - bH^2}{1 - aH},$$

where

$$a = \frac{1}{2}(\frac{1}{A_1} + \frac{1}{A_2}) \in [\frac{1}{A_1}, \frac{1}{A_2}] \qquad b = \frac{1}{L_1 + L_2}(\frac{L_1}{A_1^2} + \frac{L_2}{A_2^2}).$$

Since $H = u(w) - u(w - P)$ is an increasing function of P, and $A_i = u(w) - u(w - L_i)$ does not depend on P, we directly differentiate with respect to H to get that $AC'(P)$ has the same sign as

$$-2bH(1 - aH) + (1 - bH^2)a = -2bH + abH^2 + a. \tag{7}$$

This expression is decreasing in H for $H < 1/a$, which holds since $H < A_2 < A_1$. Therefore, if the derivative of the average cost is negative at $H = A_2$, then it is negative on a nonempty interval $[H_0, A_2]$, where H_0 is the smallest solution to equation (7). There remains to provide a numerical example. Take for example a CRRA function with relative risk-aversion equal to 2, and the following parameters:

$$u(x) = -1/x, w = 10, L_1 = 9, L_2 = 7.$$

Then we obtain

$$A_1 = 0.9, A_2 = 0.233, a = 2.698, b = 8.73$$

and the derivative $(-2bH + abH^2 + a)$ is negative for $H \in [0.22, 0.2333]$, or equivalently for $P \in [6.871, 7]$.