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"Risk-adjusted social discount rates"

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Risk-adjusted social discount rates*

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Abstract

When evaluating public and private investment projects, those that contribute more to the collective risk should be more penalized through an upward adjustment of their discount rate. This paper shows how to estimate the risk-adjusted discount rate for different projects, with applications to the electricity sector. Using the standard framework of consumer theory, we express any investment project's beta in terms of the easier-to-measure price and income elasticities of the goods generated by the project. When considering an investment in production capacity, the beta has a flat term structure, and is positive (negative) for normal (inferior) goods. When considering core infrastructures carrying goods or services, such as energy transmission and distribution assets, the beta has a decreasing term structure with very high values at short horizons for infrastructures facing capacity constraints. We provide a real-case example of a cross-border electricity connection with negative beta for the exporting country.

Keywords: Investment theory, risk-adjusted discount rate, public investment, electricity transmission, capacity investment, cross-border transmission network

JEL codes: H43, D61, L94, G11.

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1 Introduction

The necessity to decarbonize our economies confronts the energy sector to an immense transitional challenge. Huge investment efforts will have to be made over a relatively short period of time. Deep uncertainties affect the outcomes of these investments, from the evolution of energy demand, the ability to capture CO2 in the atmosphere, the capacity to store electricity, the timing of the phasing out of coal and natural gas, the carbon price in the next decades, to the role of biofuels and the architecture of electricity production and of the electricity transmission grid. Because of the long duration of most investment projects in the sector, the discounting rule that is used to value them plays a key role in the decision process, in the allocation of capital in the sector, and eventually in the speed and efficiency at which the energy transition will materialize. Modern asset pricing theory reminds us that it is desirable to adjust project-specific discount rates to the riskiness of the project. According to the Consumption-based Capital Asset Pricing Model (CCAPM), this is measured by its "consumption beta" which is, under proper assumptions recalled in section 3 of this document, obtained from the covariance of the project's return with aggregate consumption growth.

Although the private sector has long been using risk-adjusted discount rates to value investment projects, this practice is much less prevalent in the public sector. This is probably due to difficulties faced by evaluators to measure the social beta of the public investments under scrutiny. Even now, most recovery plans from the COVID-19 crisis contain vast sources of public funding to green our economies, without a proper risk-adjusted assessment.

In this context, our paper makes two main contributions. First, it refines the methodology for estimating project-specific betas, by showing how to take into account the economic characteristics of the supply and demand. Second, it illustrates the importance of the risk-adjustment of discount rates for decision-making by considering specific types of investment (capacity investment, cross-border link) where the betas vary considerably from one project to another. We use a partial equilibrium approach by focusing on each specific project and on its relation to aggregate consumption. We first provide an explicit formula for the beta directly derived from the characteristics of the supply and demand for the flow of goods or services that are generated by the investment. In particular, we clarify the intuitive link that exists between the risk-adjustment coefficient beta and the income-elasticity of the demand for the goods and services generated by the investment under scrutiny. Inferior (superior) goods are associated to assets with a negative (positive) beta. This characterization of betas to the intrinsic economic characteristics of the investment leaves the evaluators with the easier and more classical task of estimating parameters such as the price and income elasticities.

This result is general and is not related to the energy sector specifically. However, this research project was triggered by a social demand originating from various stakeholders active in the French electricity sector. This is due to the fact that public evaluators in France have recently been obliged to adjust discount rates to their project's beta, without any clear guideline. The illustrations presented in this paper come from our interactions with these stakeholders. We first estimated the beta of the electricity sector in France. We obtain a fairly small order of magnitude (< 0.4), in line with the low income-elasticity of the demand for energy. As discussed in section 3, this estimation is not necessarily in line with figures obtained using CAPM (unleveraged) asset beta from companies listed in the sector. The latter approach measures the non-diversifiable market risk of some regulated private projects, and not the systematic socio-economic risk attached to the induced welfare.

The elasticity of supply also affects the beta of the underlying asset. It implies that the CCAPM beta is also affected by the capacity utilization ratio of the asset, which typically vary in a predictable way through the lifetime of the investment. This suggests that risk premia, and their associated betas, have a non-flat term structure. [1] To examine this question, we consider a more specific class of investments in transportation infrastructures (such as a railway infrastructure or an electricity network). We show that their social betas vary greatly, exhibiting a decreasing term structure starting from a surprisingly large value ($\beta > 10$) at short maturities (when capacity is under-utilized) and possibly negative value (when the infrastructure is used to export). As an illustration, we use data provided by the French operator of electricity transmission infrastructure to estimate the beta of an investment in a France-Spain electric cross-border link, which provides, to our knowledge, the first real-case based example of a negative beta. These estimations are based on two polar scenarios designed by the European Network of Transmission Systems Operators for Electricity (ENTSO-E), a relatively conservative one and a second scenario more ambitious with respect to the development of renewable energies.

Those results show how the adjustment of the social beta to risk can radically change the allocation of capital in the public sector. We help decision-makers to identify the investments that provide some form of insurance for our economies (as is the case for projects with negative beta). We know since Aschauer (1989) that investments in "core infrastructure" (i.e. transportation and public utilities) can have a strong and positive impact on private output[2]. Institutions in charge of those infrastructures must constantly anticipate the growth in demand. They must allocate investments accordingly between maintenance, renewal and extension of their networks. The decreasing

[1] Dietz et al. (2017) estimate the term structure of the social beta of green investments aimed at reducing emissions of greenhouse gases. However, their approach does not provide general insights that can apply in other contexts, because it is based on Monte-Carlo simulations of the DICE model, which heavily relies on the assumptions of the model and on the calibration of the distribution for the uncertain parameters.

[2] Although results from early literature appears spurious, recent works confirm that the output elasticity of public capital is significant, and twice higher when considering only core public capital, cf. Bom and Ligthart (2014).

term structure highlighted in our work can play a crucial role in determining the optimal timing of an increase in the size of such infrastructures. The last example considered here is all the more important as it concerns countries belonging to an imperfect monetary union with significant asymmetric shocks that traditional adjustment mechanisms via labor mobility are not enough to mitigate. We know since Frankel and Rose (1998) that the optimal currency area criteria are partly endogeneous. Our work shows that estimating project-specific betas is a valuable means of prioritizing investments that can play an attenuating role in the event of an adverse asymmetric shock.

This paper is part of the attempts to reach academic consensus on these topics and to convince public decision-makers to adopt more efficient discounting rules. As shown by Drupp et al. (2018), there is an emerging consensus among researchers on the risk-free discount rate and, in particular, on the fact that declining discounting rates should be used for risk-free projects.[3] Further work is needed to estimate the risk premia associated with long term projects and to convince public authorities to use risk-adjusted discount rates. Most governments use a much simplified system in which a single discount rate is used independent of the investments' risk profile. This is the case for the UK (Treasury, 2018) for which a constant premium of 1% may be added to the discount rate in order to take account of unpredictable risks both 'catastrophic' and 'systemic' [4]. The United States have also adopted such an approach by selecting a single "risk-adjusted" discount rate of 7%, which is assumed to be the average cost of capital in that country.[5] In the same way as the WACC fallacy observed by Krueger et al. (2015) for the private sector, this is likely to severely distort public investment, leading to under-investment in safe projects and over-investment in riskier ones. [6]

The paper proceeds as follows. Section 2 reviews the literature and section 3 presents a detailed reformulation of the CCAPM discounting system. In section 4, we provide an explicit formula for social betas for projects in which the consumers' willingness to pay and the variable production cost are Cobb-Douglas in aggregate income and quantity. Section 5, 6 and 7 deals with two specific categories of public investment - core infrastructure or cross-border trade infrastructure. Section 8 concludes.

[3]However, as pointed out by Cropper et al. (2014), although this fact is now well established, few countries have adopted such practices - United Kingdom and France did so respectively in 2003 and 2005 while the OMB in the United States still uses a constant discount rate.

[4]According to a recent report made to the Treasury (Freeman et al., 2018), "the view taken in the Green Book is that systematic risk is generally not sufficiently important to necessitate an explicit treatment in CBA". The authors of this report recognize that there are project risks which, when correlated to macroeconomic risks, affect the welfare properties of the project itself.

[5]See OMB (2003). A smaller discount rate of 3% could also be used for a sensitivity analysis. However, some specific public investments are evaluated taking into account some market risk adjustment, for instance government loan guarantees to commercial entreprise using credit ratings (Lucas, 2012).

[6]They show that firms tend to use their weighted-average cost of capital as their unique discount rate and are able to quantify the resulting loss of value for a particular class of investment projects (diversifying acquisition) - about 8% of the deal value on average when the bidder has a lower cost of capital than its target.

2 Related literature

Regarding the choice of the social discount rate, the theory has often been misleading. For example, the celebrated Arrow-Lind theorem (Arrow and Lind (1970)) prevailed over decades among public institutions on both sides of the Atlantic to support the use of a single discount rate to evaluate all public investment projects. The idea was that the mutualization capability of the public sector is so large that the risk of individual projects are washed out by diversification, so that it should evaluate them assuming risk-neutrality. But many economists and most evaluation experts overlooked the fact that Arrow and Lind implicitly assumed that the net benefit of the public projects are statistically independent from each other, so that consumption per capita is certain (Baumstark and Gollier (2014)). In reality, most investment projects, public or private, have benefits that are statistically related to aggregate consumption. This implies that diversification does not fully eliminate risk, and evaluators should be concerned with the impact of public actions on the collective risk eventually borne by the risk-averse citizens.

The right reaction to the fallacious interpretation of the Arrow-Lind theorem was provided by the development of the Consumption-based Capital Asset Pricing Model (CCAPM) by Rubinstein (1976), Lucas (1978) and Breeden (1979). In a Gaussian world, it justifies using a discounting system that combines three ingredients: A ”risk-free” discount rate for projects whose net benefits are independent from aggregate consumption, a systematic risk premium, and a CCAPM cash-flow beta (later on referred to as the ”beta”).[7] The beta is specific to the project, and, potentially, to the maturity of the benefit under scrutiny. It is defined as the elasticity of the net social benefit of the project to a change in aggregate consumption. A positive beta means that the project contributes positively to the macroeconomic risk and should be penalized accordingly by a larger risk-adjusted discount rate. On the contrary, a negative beta signals a project that hedges the macroeconomic risk, and its insurance value should be recognized by a reduced risk-adjusted discount rate. This project-specific risk-adjusted discount rate should in fact be equal to the risk-free rate plus the product of the project's beta by the systematic risk premium. This means that the risk premium determines the intensity of the penalization of a project due to its contribution to the macroeconomic risk.

The literature has mostly been focused on the debate about which risk-free rate and systematic risk premium should be used in this discounting system. This has been done in the context of the risk-free rate puzzle (Weil (1989)) and the equity premium puzzle (Mehra and Prescott (1985)) which state that the CCAPM predicts a risk-free rate that is too large and a systematic risk premium

[7] If there is no serial correlation in the per-period growth rate of aggregate consumption and if the representative agent has constant relative risk aversion, then the risk-free discount rate and the systematic risk premium have a flat term structure, i.e., they should be independent of the maturity under consideration (Gollier (2002, 2016)). Countries like the United Kingdom and France currently use a non-flat term structure for social discount rates, an approach that can be justified by the persistence of shocks to aggregate consumption (Gollier and Weitzman (2010), Gollier (2012)). In fact, the three variables characterizing the efficient discounting system can have a non-flat term structure.

that is too large compared to the asset prices that prevailed during the last century of so. Many resolutions of these puzzles have been explored, with more or less luck, in particular by allowing extreme events (Barro (2006), Julliard and Ghosh (2012)) or by disentangling risk aversion from the aversion to consumption fluctuations combined with adding long run risks in the modeling of the stochastic growth (Bansal and Yaron (2004), Constantinides and Ghosh (2017)).

On the other hand, very little has been made regarding project-specific betas, which is the focus of this paper. As far as we know, Breeden (1980) is the only exception that develops a general theory to obtain the systematic risks of specific projects. More precisely, Breeden derives asset-specific CCAPM betas from a system of demand, and shows how to estimate them from the parameters of this demand system. Also, Breeden et al. (1989) empirically shows that goods with high income elasticities of demand have a high consumption beta, as predicted by the model that we develop in section 4. In contrast, the literature is relatively abundant on how to estimate CAPM equity betas of specific sectors - cf. for in particular Buckland and Fraser (2001) and Paleari and Redondi (2005) for the UK electricity distribution industry. Also, regarding energy generation and transmission planning, several studies develop methods to take into account the decision maker's risk aversion, but they mainly focus on private decision maker and do not try to capture the collective non-diversifiable risk - see Munoz et al. (2017) for a recent review of the literature on this subject. One notable exception is Pierru and Matar (2014) who quantify the CCAPM betas of energy-related public investment project in Saudi Arabia, with a focus on the volatility caused by fluctuations in oil price

3 The CCAPM discounting system

We consider an investment project which creates a market for a new non-durable product or service. It could be a new drug, or an infrastructure in transportation, telecommunication, health, or education for example. We want to measure the socioeconomic value of the flow of net benefits generated by this new market. Once this project has been implemented and at any further date t, it costs $g(x; C_t, \theta_t)$ per capita to produce x units of the product. This cost is a function of the income per capita C_t that prevails at date t, and also of a set of other factors characterized by the vector θ_t, which is assumed to be statistically independent of C_t. We assume that g is increasing and weakly convex in x. The consumer's value (or willingness to pay) per capita associated to consuming x at date t is equal to $f(x; C_t, \nu_t)$. This consumer value is a function of the income per capita and of other factors that are characterized at date t by ν_t, which is assumed to be statistically independent of C_t. We assume that function f is increasing and weakly concave in x. All positive and negative externalities generated in the production and consumption processes are integrated into functions

f and g. [8]

We evaluate the project at date 0, which is the only date at which it can be implemented. This means that there is no option value to delay the investment in this context. Seen from date 0, the future evolution of (C_t, ν_t, θ_t) is uncertain and is characterized by an exogenous joint stochastic process which is arbitrary at this stage of the analysis. We assume a flexible production technology, so that the optimal (or equilibrium) production and consumption x_t at any date conditional to (C_t, ν_t, θ_t) maximizes the net social benefit

$$B_t = B(C_t, \nu_t, \theta_t) = \max_x \quad f(x; C_t, \nu_t) - g(x; C_t, \theta_t). \tag{1}$$

Suppose that the project is scalable, and let s denote the scale of the project. Thus, the net benefit at date t is sB_t. We examine the impact of the implementation of this project on the intertemporal welfare function

$$W(s) = \sum_{t=0}^{+\infty} e^{-\delta t} Eu(C_t + sB_t), \tag{2}$$

where δ is the rate of pure preference for the present, u is the increasing and concave von-Neumann-Morgenstern utility function of the representative agent, and E is the expectation operator conditional to the information available at date 0. At that date, the value $V(s)$ of the flow of the net benefits generated by the project is such that the representative agent is indifferent between implementing the project or receiving $V(s)$ at date 0. This means that $u(C_0 + V(s)) = W(s)$. As is standard in the asset pricing literature and in the cost-benefit methodology, we assume that the project to be evaluated is marginal in the sense that its implementation does not modify the beliefs associated to the dynamics of (C_t, ν_t, θ_t). This is done by assuming that s tends to zero, and by measuring the social value $P = V'(0)$ of the project. This means that

$$P = \sum_{t=0}^{+\infty} e^{-r_t t} EB_t, \tag{3}$$

where the risk-adjusted discount rate of the project for maturity t is defined as follows:

$$r_t = \delta - t^{-1} \log \left(\frac{E\left[B_t u'(C_t)\right]}{u'(C_0) EB_t} \right). \tag{4}$$

We normalize C_0 to unity. In this paper, we use the standard calibration of the consumption-based CAPM for economic growth and risk preferences, as summarized in the following assumption. [9]

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[8]For instance, in the case of transportation and distribution of brown energy, it notably means that the social cost of carbon is included in the function g.

[9]As discussed in section 2, more sophisticated versions of the CCAPM solve the classical asset pricing puzzles. They generate more realistic levels for the interest rate and the systematic risk premium without modifying the char-

Assumption 1. *The stochastic process governing aggregate consumption C_t is a geometric Brownian motion with trend μ and volatility σ. Relative risk aversion is a constant γ.*

In this standard CCAPM framework, the pricing of a risk-free benefit and of a claim on aggregate consumption is well-known under this assumption. The interest rate r^f is obtained by applying pricing equation (4) to $B_t = 1$, or more generally to any B_t that is statistically independent of C_t:

$$r^f = \delta - t^{-1} \log \left(E \exp(-\gamma c_t)\right) = \delta + \gamma\mu - 0.5\gamma^2\sigma^2. \tag{5}$$

When the net benefit of the project is a claim on aggregate consumption, i.e., when $B_t = C_t$, the risk-adjusted discount rate net of the interest rate defines the systematic risk premium π. From equations (4) and (5), we have that

$$\pi = \delta - t^{-1} \log \left(\frac{E \exp((1-\gamma)c_t)}{E \exp(c_t)}\right) - r^f = \gamma\sigma^2. \tag{6}$$

Notice that in this standard framework, the interest rate and the systematic risk premium are independent of the maturity, i.e., their term structures are flat.

We are interested in examining the determinant of the risk-adjusted discount rate for other investment projects. Based on the well-established tradition in finance, we define the maturity-specific consumption beta β_t of the investment project in such a way that the risk-adjusted discount rate r_t of the project for maturity t be equal to

$$r_t = r^f + \beta_t \pi. \tag{7}$$

In other words, the project's beta is the scale factor reflecting the systematic risk of the project, and is related to the risk-adjusted discount rate of the project as follows:

$$\beta_t = \frac{r_t - r^f}{\pi}, \tag{8}$$

where r_t, r^f and π are respectively defined by equations (4), (5) and (6). Given the interest rate r^f and the systematic risk premium π, the project's beta fully determines its discount rate. We hereafter characterize the determinants of consumption betas in different economic and technological environments. The following proposition, proved in the Appendix, shows that the beta of a project at any specific date is linked to the OLS regression of $\log(B_t)$ over $\log(C_t)$, and can be considered under some assumption as the income-elasticity of the net benefit of the project.

acterization of the beta discussed in this paper.

Proposition 1. *For a given maturity t, suppose that there exists a pair $(a_t, b_t) \in \mathbb{R}$ such that $\log(B_t) = a_t + b_t \log(C_t) + \varepsilon_t$, with $\exp(\varepsilon_t)$ mean independent of C_t. Then, under Assumption 1, the beta of the investment project for this maturity is $\beta_t = b_t$.*

If it happens that the slope coefficient of the OLS regression is the same at all maturities, the term structure of the beta is constant. It makes sense in that case to refer to "the" consumption beta of the project. Besides, if the net benefit and aggregate consumption are jointly log-normal, the consumption beta of the project is equal to the OLS estimator which is given by the classical formula

$$\beta = \frac{cov(\ln C, \ln B)}{var(\ln C)} = \frac{\rho\sigma_B}{\sigma_C}$$

where ρ denotes the correlation between $\ln B$ and $\ln C$. But in general, the relationship between aggregate consumption and the net benefit of the project is not loglinear : the OLS regression yields a biased estimator of the true consumption beta.

4 Benchmark case: Production capacity

In the section, we consider investments that generate a flow of goods or services. We assume that the consumers' willingness to pay and the variable production cost are Cobb-Douglas in aggregate income and quantity. [10] More precisely, we consider the following consumer value function:

$$f(x; C, \nu) = \nu C^{\rho} \frac{x^{1-\alpha}}{1-\alpha}, \tag{9}$$

where ν is positive, α belongs to $[0, 1]$, and ρ belongs to $\mathbb{R}$. We can interpret $\eta_p^d = -1/\alpha \leq -1$ as the price-elasticity of demand, whereas $\eta_c^d = \rho/\alpha \geq \rho$ is its income-elasticity. We also assume that the variable cost function is as follows:

$$g(x; C, \theta) = \theta C^{\rho'} \frac{x^{1+\alpha'}}{1+\alpha'}, \tag{10}$$

where θ is positive, α' belongs to $\mathbb{R}_+$ and ρ' belongs to $\mathbb{R}$. We can similarly interpret $\eta_p^s = 1/\alpha'$ as the price-elasticity of supply, whereas $\eta_c^s = -\rho'/\alpha'$ is its income-elasticity.

Under those assumptions, the social beta has a flat term structure and can be directly derived from those elasticities as shown in the following.

[10] Consumer value and production cost may take various functional forms. We assume here that they are Cobb-Douglas, which is equivalent to say that they are iso-elastic with respect to aggregate income and quantity. These assumptions seem natural insofar as we consider here the aggregate supply and the aggregate demand for a given product, and do not seek to distinguish at this stage between different types of consumers.

Proposition 2. *Suppose that the consumers' willingness to pay and the variable cost of production of the service generated by the infrastructure are Cobb-Douglas. Under Assumption 1, the beta of the investment in this infrastructure is maturity-independent and equal to*

$$\beta = \frac{\eta_c^d(1+\eta_p^s) - \eta_c^s(1+\eta_p^d)}{\eta_p^s - \eta_p^d}. \tag{11}$$

In the special case in which costs are insensitive to the growth of aggregate consumption, i.e., when ρ' and η_c^s are zero, we obtain the following result:

$$\beta = \eta_c^d \frac{1+\eta_p^s}{\eta_p^s - \eta_p^d}, \tag{12}$$

and, when the price-elasticity of supply takes extreme values, this can be rewritten as:

$$\beta = \begin{cases} -\eta_c^d/\eta_p^d, & \text{if } \eta_p^s = 0, \\ \eta_c^d & \text{if } \eta_p^s \to +\infty. \end{cases} \tag{13}$$

When the price-elasticity of supply is zero, i.e., when the investment project generates a constant flow of goods or services, the beta of the project is the income-elasticity of the consumer's willingness to pay. This is intuitive, since the net social benefit in this case coincides with the consumers' valuation of the good produced. Alternatively, when the price-elasticity of supply goes to infinity, i.e., when the investment project generates a flow of goods or services at constant marginal cost, then the beta of the project is just equal to the income-elasticity of demand.

Observe that, as long as the cost function is (weakly) convex in output, the beta of the investment project has the same sign as the income-elasticity of the demand for the good or service that it generates, i.e. the social beta of public investment projects generating inferior goods is negative. Some welfare and security-related infrastructures (public hospitals, prisons, military spending) satisfy this property. For instance, welfare spending including health safety net is an inferior good across income classes, i.e. alterations in the social situation induce a rise of a risk in income loss and dependence on social assistance (Barth et al., 2015). This is also the case for resources spent for the prison system as well as on rehabilitation of former prisoners since economic conditions are negatively correlated to both delinquency rate and recidivism (Yang, 2017). Other public investments generate a normal good with a low-income elasticity, and consequently have a positive but small social beta. In particular, while health may be considered as a luxury good at the macroeconomic level in some countries (Getzen, 2000), income elasticity of health spending seems to be less than one when one considers exogenous income shocks (Acemoglu et al., 2013). The income elasticity of the value of statistical life also appears to be less than one in high income countries

(Hammitt and Robinson, 2011).

Infrastructure and electricity services are also generally regarded as necessity goods, characterized by an income elasticity of demand lower than one. As an illustration, we give in the following a rough estimation of the beta of a generic investment in the energy sector. More detailed estimates taking into account the specificity of such investments are made in the next sections. To be consistent with the examples discussed thereafter, our calculation is based on the French economy but can easily be applied to other countries. Price-elasticity of residential demand in that country is about 1.5 (Auray et al., 2019), whereas one can consider that the income-elasticity of demand is about 0.4.[11] The relation 13 calibrated using these parameters indicates that the beta is quite low, between 0.27 and 0.4 depending on the price-elasticity of supply. [12] Not surprisingly, this estimation is not always in line with figures obtained by measuring the CAPM market beta from listed companies in that sector. In particular, when the regulation is based on price-cap, private companies carry more risks and have consequently a larger beta. This has been particularly studied in the case of UK electrical distribution companies by Buckland and Fraser (2001) and Paleari and Redondi (2005) who show that the corresponding market beta is then higher than 1. Using an average gearing ratio of 60% (as commonly considered by regulators such as Ofgem), this translates into an unleveraged asset beta greater than 0.4. On the other hand, in period where regulation is based on a rate of return, the market CAPM is about 0.7 according to those two papers, and the corresponding asset beta is then more consistent with our estimation. However, such a comparison remains delicate insofar as the socio-economic beta and market beta do not measure the same thing: the latter measures the non-diversifiable market risk of some regulated private projects, whereas the former reflects the systematic socio-economic risk attached to the induced welfare. It is also important to note that our estimate here is highly dependent on the market and the investment under consideration. In particular, elasticities of demand can vary greatly, depending on the income level and the options of substitution. More importantly, investments in the energy sector generally have a non-constant and very specific marginal cost curve, for example zero marginal cost for renewable energy up to a certain capacity constraint. This can significantly change the risk profile of the investment, and

[11] Income-elasticity of electricity demand is often assumed to be smaller, less than 0.15 (Bakaloglou and Charlier, 2019), but this may be attributed to the correlation between income and home's characteristics. Recent studies taking into account households' heterogeneity show that those with low income (resp. high income) exhibit a low price elasticity of energy demand about 0.2 (resp. a high elasticity about 0.6), cf. Meier et al. (2013) and Hache et al. (2017). The target here is not to suggest a definitive value for the income-elasticity of demand, so we leave that question open at this stage and simply retain here the value of 0.4 used by the French electricity transmission system operator since it appears consistent with those works.

[12] Price-elasticity of supply depends on the time frame considered, but also on whether the elasticity of capacity or the elasticity of output with a given capacity is considered. Besides, empirical research in energy elasticities has mainly focused on demand elasticity rather than supply elasticity. Available results provide fairly broad ranges of value for non-renewable energy (Wiser and Bolinger, 2007), whereas empirical evidence for renewable energy are still scarce and difficult to generalize (Johnson, 2014).

therefore its beta, as shown in the next two sections.

5 Core infrastructure investment

As mentioned in the introduction, public investments can have a strong and positive impact on private production, particularly with regards to ”core infrastructure” : transportation (motorways, railway, mass transit, telecom infrastructure) and public utilities (water distribution systems and sewers, electrical and gas facilities). These investments allow the transportation of goods or information, and are characterized by capacity constraints : decision-makers must see whether it is necessary to widen a road, increase the size of an electricity transmission line or expand the broadband capacity of a fiber optic network. In the next two sections, we make the following assumption regarding uncertainty:

Assumption 2. *Both consumer value and marginal cost evolve stochastically through time according to multiplicative parameters ν_t and θ_t that have no drift, i.e. a constant expected value, and are independent of C_t.*

Let's first assume that the investment project is about creating an infrastructure with fixed maximum capacity K and a constant marginal cost below that capacity. This case corresponds to the following total cost function:

$$g(x; C, \theta) = \begin{cases} \theta x, & \text{if } x \leq K; \\ +\infty, & \text{if } x > K. \end{cases} \tag{14}$$

We maintain the assumption of a multiplicative consumer surplus denoted f as in (9). Contrary to the previous case, the net benefit $B = f - g$ is not a power function of consumption, so that the CCAPM beta is more complex to estimate. The following proposition shows that the term structure of the beta of the investment project is decreasing.

Proposition 3. *Suppose that the consumers' willingness to pay for the service generated by the infrastructure is Cobb-Douglas, and that the marginal cost of this service is constant under a threshold, and infinite above, as defined by relation 14. Then, under Assumption 1 and 2 with positive growth $\mu > 0$, the beta of the investment is decreasing, going from η_c^d to $\rho \leq \eta_c^d$.*

This result follows quite intuitively from the previous section : since the demand is expected to grow over time, the capacity constraint is almost surely not binding in the short run and almost surely binding at very long maturities, so that these two polar cases correspond to those discussed previously after proposition 2, that is to say with respectively a price-elasticity of supply going to zero or to infinity.

The decreasing term structure of the beta is even more pronounced when we consider an investment that marginally increases the capacity K of the infrastructure. The result set forth in the following proposition reflects the fact that the probability of benefiting from the marginal increase of capacity tends to zero in the short term (resp. to 1 in long run), and becomes very sensitive to economic growth (resp. insensitive).

Proposition 4. *Under assumptions of proposition 3, and with the additional hypothesis that the service is a normal good ($\rho \geq 0$), the beta of a marginal increment in the capacity infrastructure has the following properties:*

- *Assuming a positive trend of growth ($\mu > 0$), its term structure tends to ρ asymptotically.*
- *It tends to infinity with K.*

The two previous propositions are illustrated by numerical examples presented in Figure 1. This decreasing term structure significantly impacts the expected value of the project. For instance, taking into account the sloping term structure corrects the expected value by about 20% downward when the additional marginal infrastructure induces the highest discounted benefit (i.e. in this particular example with an horizon of 50 years). These considerations can play a crucial role in determining the optimal timing of an increase in the size of an infrastructure. This is all the more important as infrastructure managers must constantly anticipate the growth of demand and invest at the right time to renew or expand their networks.

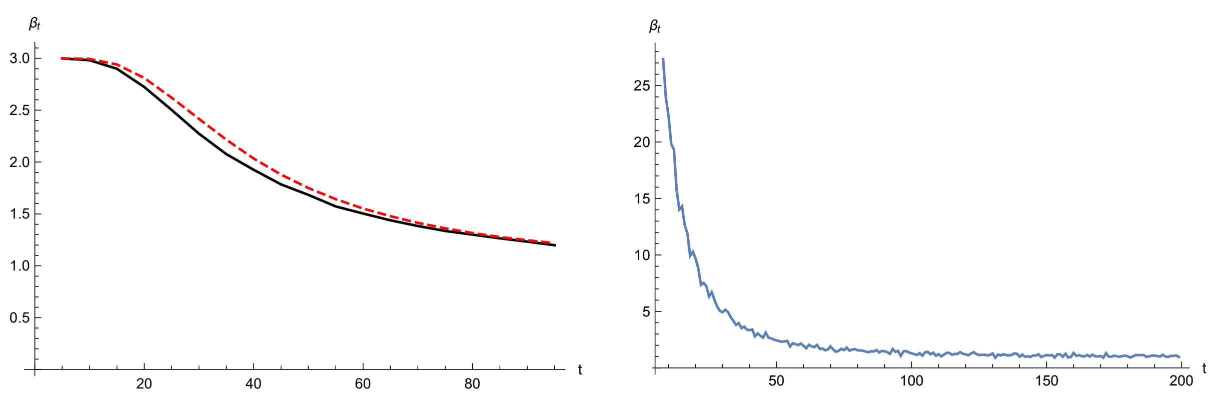

Fig. 1. The term structure of the CCAPM cash-flow beta in the case of a capacity investment $K = 4$ (left) and in the case of a marginal capacity investment from $K = 4$ to 4.1 (right). We assume $\alpha = 1/3$, $\rho = 1$, $\delta = 0$, $\gamma = 2$, $\log(\theta_t) \sim N(0, 0.001^2 t)$ and $\log(C_t) \sim N(0.02t, 0.04^2 t)$.

A methodological remark must be made at this stage. As observed in Proposition 1, the beta of a project is equal to the regressor of $\log(B_t)$ over $\log(C_t)$ when these two random variables are jointly Normal. It is tempting to measure the parameter of a project in this way, but the underlying assumptions are often not verified. This is illustrated by the previous example. In that case, for intermediary maturities, the support of the distribution of the demand x^d contains K so that the logarithm of the net benefit B_t is not linear in the logarithm of aggregate income C_t. In Figure 1, we have depicted on the left the term structure of the cash-flow beta of the capacity investment when using this econometric approach of the beta (dashed curve) and when using the correct approach (plain curve) based on equation (8). The OLS beta tends to slightly overestimate the correctly estimated beta at intermediary maturities. It is only at very short or long maturities that the econometric approach is unbiased. This is due to the fact that the probability of (not) being capacity constrained at those maturities is almost zero, so that the pair $(\log(C_t), \log(B_t))$ is almost jointly Normal.

6 Transfrontier trade infrastructure

We consider now an investment that creates an infrastructure to trade goods between two countries. It could be a transport infrastructure such as a railway infrastructure, or a transfrontier electricity connection. Such infrastructure is particularly useful to transfer a tradable good from the country where the equilibrium price of this good is low to the other country where it is larger. We use the same notation as before by adding a subscript $i = 1, 2$ to represent the two countries. In autarky, the welfare of country i is given by

$$V_i = \max_{x_i} f_i(x_i, C_i) - g_i(x_i, \theta_i). \tag{15}$$

Let us consider a trade infrastructure of size K whose flow of net benefits is equally shared by the two countries. The social surplus generated by the infrastructure is then defined as follows:

$$\begin{aligned} S(K) \quad = \quad & \max_{x_1,x_2,y_1,y_2} \sum_{i\in\{1,2\}} \left(f_i(x_i, C_i) - g_i(y_i, \theta_i)\right) \quad - \quad \sum_{i\in\{1,2\}} V_i \\ \text{s.t.} \quad & y_1 + y_2 = x_1 + x_2 \\ & |y_1 - x_1| \leq K, \end{aligned}$$

where x_i and y_i denote respectively the local consumption and local production that may be exchanged. For simplicity, we assume here that the sharing of the social surplus between the two countries is such that country i receives share k_i, with $k_1 + k_2 = 1$. We are interested in measuring the social benefit of marginally increasing the capacity of the transfrontier infrastructure and its correlation with aggregate consumption.

For the sake of illustration, let us consider a concrete example, where two countries exchange electricity through a high-voltage electricity line. In that case, when a country has an instantaneous surplus of zero-marginal-cost renewable energy, it may use the transfrontier electricity connection to export this energy to the other country. The occurrence of such an event depends upon weather conditions, as well as upon local demand. In particular, an exporting country that slips into recession will consume less electricity and might then be able to exploit more the transfrontier connection to its own benefit. The correlation between the economic growth of this country and the expected benefit of the cross-border link is therefore negative, i.e. the beta is negative for an exporting country and similar reasoning indicates that the beta should be positive for an importing country. This result is stated more rigorously in the following proposition:

Proposition 5. *If a transfrontier trade infrastructure is entirely used to export a normal good from one specific country to the other, then the beta of the social value of the infrastructure is negative for the exporting country, and positive for the importing one.*

In most cases, the trade infrastructure can be used to transfer goods in both directions. For instance, in the case of a high-voltage electricity line, depending on weather events and time of the year, a country may export energy some days, and import energy the rest of the time through the cross-border infrastructure. The value at date t of the investment is equal to

$$B_t = \Sigma \varrho_{t,\theta} B_{t,\theta} = \Sigma \varrho_{t,\theta} \frac{\partial S_{t,\theta}}{\partial K},$$

where $S_{t,\theta}$ is the social surplus of country 1 at date t when the weather state θ occurs. We denote $\varrho_{t,\theta}$ for the probability of that state. We are interested in measuring the income-elasticity of this benefit B_t. To give the intuition, if the estimation with an OLS-regressor of the logarithm was correct, the beta would be exactly the weighted average of the expected value of the beta associated to each climatic event, that is $\beta = \Sigma \varrho' \beta_{t,\theta}$ where $\beta_{t,\theta} = dLn(B_{t,\theta})/dLn(w)$, for the modified weight $\varrho'_t = \varrho_{t,\theta} B_{t,\theta} / \Sigma \varrho_{i,\theta'} B_{i,\theta'}$.

This suggests that Proposition 5 can be generalized when the direction of the flow of trades is uncertain. In practice, this method is biased because the distribution of the log of social benefits is not normal, and because its relation with log consumption is not linear, as in the example discussed at the end of the previous section. A direct evaluation based on a Monte-Carlo simulation is necessary to obtain an accurate estimate of the beta, which is done in the following example. We assume that the two countries have the same constant price elasticities of demand and supply, and that the marginal cost is independent of income, so that the consumer surplus and the cost function are given by (with $\rho_i > 0$).

$$f_i(x_i, C_i) = C_i^{\rho_i} \frac{x_i^{1-\alpha}}{1-\alpha} \qquad g_i(y_i, \theta_i) = \theta_i \frac{y_i^{1+\alpha'}}{1+\alpha'}, \tag{16}$$

We see in Figure 2 that the same ideas continue to hold: the country which is expected to predominantly use the line as an exporter should use a negative beta to evaluate that trading infras-

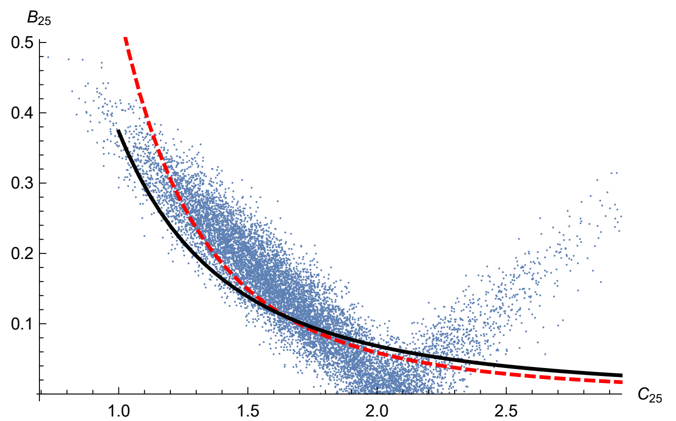

Fig. 2. In the case of the small transfrontier trade infrastructure, and from the point of view of the initially exporting country 1, Monte-Carlo simulation of (C_{1t}, B_{1t}) for a $t = 25$ years maturity. As before, we choose $\gamma = 2$ and $\delta = 0$, and also assume that $\rho_1 = \rho_2 = \alpha' = 1$, $\alpha = 1/3$. We suppose further that $(C_{1t}, C_{2t}, \theta_{1t}, \theta_{2t})$ follows independent geometric brownian processes with $\mu_{C_1} = \mu_{C_2} = 0.02$, $\sigma_{C_1} = 0.04$, $\sigma_{C_2} = 0.01$, $\mu_{\theta_1} = \mu_{\theta_2} = 0$ and $\sigma_{\theta_1} = \sigma_{\theta_2} = 0.001$. Initial values are $C_{10} = C_{20} = 1$, $\theta_{10} = 1$ and $\theta_{20} = 2$. Surplus is equally shared, that is $k_1 = k_2 = 0.5$. The dashed curve illustrates as in the previous figure the bias estimation with an OLS regressor.

tructure project. One key element is that although the initial aggregate consumption levels are the same, country 1 has a smaller initial variable cost which implies that, initially, country 1 is indeed exporting to country 2. The statistical relationship between the aggregate consumption in country 1 and the net benefit of the infrastructure for that country is well fit by a negative exponential on the left of this figure, where economic growth in that country is low enough to preserve its exporting advantage. However, for larger growth rates of C_1, the demand and the local price in that country will grow faster, transforming the country into an importer. This explains why the relationship between aggregate consumption in country 1 and its net benefit of the infrastructure becomes positive on the right part of the figure. The first effect dominates, so the beta is negative for this country ($\beta = -2.47$ at the horizon $t = 25$ year selected in the figure). For longer maturities, the initial comparative advantage of country 1 is more likely to disappear because of the uncertain growth in local demands and costs. This tends to bring the beta of the infrastructure of country 1 closer to zero.

7 Application to the France-Spain cross-border electricity connection

We apply the methodology developed in the previous section to a concrete example detailed in the online Appendix, the investment in an additional cross-border electricity connection between

France and Spain. This was done based on data provided by the French electricity transmission system operator (RTE). Uncertainty on demand and supply is captured by 12 climatic scenarios, giving for each hour of year 2030 consumers' demand and available renewable energy. This takes also into account the available production capacity of different types of non-renewable energy (gas, hard coal, light oil, lignite and nuclear) in both countries. We consider two polar scenarios within the Ten-Year Network Development Plan designed by the community of European Network of Transmission System Operators for Electricity (ENTSO-E) : the fist one is a conservative one with about 40% renewable energy whereas the second one reflects an ambitious path with 60% of load supplied by renewable energy in 2030. [13] As illustrated by the following graph, the second scenario foresees more renewable energy production capacities, and a significant decline in nuclear generation in France

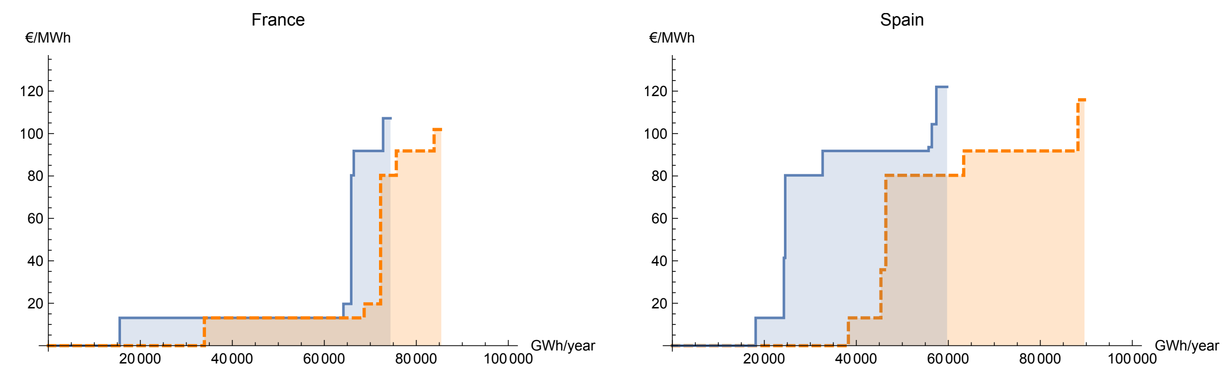

Fig. 3. Marginal cost structure of energy production in France and Spain for the first (thick line) and the second scenario (dahed line)

We estimate the beta of the benefit characterized by equation (21) for a project that increases marginally the existing cross-border connection between France and Spain. Results are presented in Table 1. The left part of the table gives results without taking account of the correlation between the economic cycles in France and Spain. As expected, the beta of the project is negative for the exporting country (France), and positive for the importing one (Spain). In the first scenario, about 97% of the traded energy is French nuclear electricity. This explains why the beta is strongly negative for France. In the second scenario, the usage is more balanced between the two countries (about one fourth of traded energy is exported from Spain). The high beta for Spain in this case can be explained by a wider price differential between the two countries (i.e. zero marginal cost renewable energy or cheaper nuclear energy vs. more costly fossile energy), and also because the additional link is less used. Indeed, in this second scenario the high investments in renewable

[13] These scenarios corresponds more precisely to the so-called vision 1 ("slow progress" scenario) and vision 4 ("green revolution" scenario) of the Ten-Year Network Development Plan released in 2014 by ENTSO-E.

energies on both sides of the border have made the two countries less dependent on each other. [14]

	Without correlation		With correlation		Union of
	France	Spain	France	Spain	two countries
First Scenario	-0.98	0.47	-0.58	0.21	0.06
Second Scenario	-0.49	1.6	1.1	1.4	1.96

Table 1: Beta of a marginal investment to increase the size of the cross-border link. With the exception of the left side of this table, these estimates are made with a correlation coefficient between the economic cycles $\rho = 0.52$. The first and the second scenarios take into account investment and demand forecasts of the respectively conservative and "green revolution" scenarios designed by the European Network of Transmission System Operators for Electricity.

The right part of the table describes estimation taking into account the correlation between macroeconomic cycles, which is significantly positive within the European Union, and then the beta for the two countries taken as a whole. This is done using standard deviations and correlations measured by Christodoulakis et al. (1995). According to their estimations, the correlation between the macroeconomic cycles in France and Spain is equal to 0.52. There is no obvious relationship between betas with and without correlations: a symmetric or an asymmetric shocks on GDP of both countries may have various effect on the benefit induced by the cross-border connection. It depends on the initial level of electricity production in the two countries at a given time of the year, and whether the shock leads to a change in the marginal cost of energy. We see that in the first scenario, taking into account correlation pushes the betas closer to zero. The beta remains negative for France whereas the two-countries beta is close to zero. The result in the second scenario is very different, for the reasons presented in the previous paragraph. Getting a very different consumption beta depending on scenarios and investments' type is not surprising and shows how important it is to carry out such assessments.

All this matters a lot as far as an incomplete monetary union is concerned. Indeed, the above example concerns the euro area, which remains quite distant from an optimal monetary zone in the sense of Mundell. We know since Eichengreen (1992) that it is composed of countries experiencing significant asymmetric shocks, which traditional adjustment mechanisms via labor mobility are not sufficient to mitigate.[15] In the presence of asymmetric shocks, a negative shock in the exporting country reduces the local price whereas price would rise on the other side of the border if a positive shock were affecting at the same time the importing country. This boosts the net benefit of the

[14] The value of the investment in the second scenario in less than half the value obtained in the first scenario. As observed in section 5, when we consider marginal investment in capacities, the gain induced by this investment is all the more sensitive to economic growth as it is less used.

[15] Whether the level of synchronization of business cycle has significantly increased or not since then is still an open debate, cf. De Haan et al. (2008) for a survey of that empirical literature.

infrastructure even more than when shocks are idiosyncratic. Thus, some investment in a cross-border infrastructure can play an attenuating role in the event of an asymmetric shock. Other similar investments may have instead a strong positive beta and reinforce economic divergence in the event of an asymmetric shock. The tools presented here are a valuable way to distinguish and prioritize investments with this factor in mind.

Overall, these analyses are of course not enough to assess the value of this cross-border investment, and must be seen as illustrating an important factor for estimating the net present value of such projects. The difference in the beta obtained for both scenarios show how essential it is to adjust for non-diversifiable risk. Beyond that, a full estimate must take into account the entire time horizon (i.e. carry such estimation for different maturities) and the ability to change the timing of the investment (using real option analysis). As seen in the previous section, CCAPM betas have a very specific term structure when considering capacity investments, which has a significant impact when estimating real option values. On the other hand, the increasing amount of renewable generation significantly modifies the risk profile of these investments. How it will change the value of a cross-border link is not obvious a priori, and shall depend on the impact of the associated volatility on the non-diversifiable risk (in the example considered here, higher betas in scenario 2 mean that the substantial green investments planned in this scenario makes the two countries less dependent on each other in the event of a macroeconomic shock). We leave those questions for further study.

8 Conclusion

Discounting remains a crucial, controversial and complex ingredient of modern investment theory. It traditionally serves the bi-dimensional purpose of penalizing cash flow for being either delayed and/or risky. These two dimensions are taken care of by two independent parameters, the risk-free rate and the systematic risk premium, for which a wide literature exists that elaborates on their level and term structure. A full discounting system also requires a methodology for determining the beta accrued to the risk component of the project-specific discount rate. In the simplest case in which the net benefit of the investment project under scrutiny and aggregate consumption are jointly log-normal, this ”cash-flow” beta is just the income-elasticity of the net benefit. We show in this paper that this situation prevails when the investment project is to build an infrastructure that produces a good or service for which the consumers' willingness to pay and the variable production costs are Cobb-Douglas in income and quantity. Specifically, we obtain an explicit formula for the cash-flow beta of such an investment in terms of price and income elasticities of the supply and demand functions, and develop the idea that the beta is increasing in the income-elasticity of demand. When the variable cost function is insensitive to aggregate consumption, the beta is positive for a normal good, and negative for an inferior good, as is intuitive. But this is a very special case. When considering the broad class of investments in ”core infrastructure” (transportation and public utilities), we show that the term structure of the beta is decreasing when the infrastructure has limited capacity, and takes possibly negative value when it is used to export.

The models used in the first sections of this paper are schematic in order to provide stylized results, in particular the relation between market elasticities and betas, as well as the impact of capacity constraints on the term structure of the beta. The approach developed in those sections can be implemented in a similar way using more realistic and more complex models, like what is done in the last section of this document based on data provided by the French electricity transmission system operator. We assumed in this paper take-it-or-leave-it-forever investment opportunities, thereby leaving aside the problem of real option values. Because the future benefits of an investment in a capacity infrastructure generally depend upon whether this capacity will be expanded or not when the capacity constraint will become binding, the problem of estimating the beta cannot be separated from the problem of estimating real option values. We leave this important question for further research.

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9 Proofs

Proof of proposition 1: By assumption, the net benefit B_t equals $C_t^{b_t}\exp(a_t+\varepsilon_t)$ and, since a_t is certain and $E[\exp(\varepsilon_t)|C_t]$ is independent of C_t, equation (4) implies that

$$r_t=\delta-t^{-1}\log\left(\frac{EC_t^{b_t-\gamma}}{EC_t^{b_t}}\right)=\delta-t^{-1}\log\left(\frac{E\exp((b_t-\gamma)c_t)}{E\exp(b_tc_t)}\right).$$

Under assumption 1 , consumption c_t is normally distributed with mean μt and variance $\sigma^2 t$, so that

$$r_t=\delta-t^{-1}\log\left(\frac{\exp\left((b_t-\gamma)\mu t+0.5(b_t-\gamma)^2\sigma^2 t\right)}{\exp\left(b_t\mu t+0.5b_t^2\sigma^2 t\right)}\right).$$

This simplifies to $r_t=r^f+b_t\pi$, where r^f and π are defined respectively by equation (5) and (6). Plugging this into equation (8) yields $\beta_t=b_t$. ■

Proof of proposition 2: The consumer value function yields the demand function:

$$x^d(p;C,\nu)=\nu^{\frac{1}{\alpha}}C^{\frac{\rho}{\alpha}}p^{-\frac{1}{\alpha}} \tag{17}$$

whereas the variable cost function yields the supply function:

$$x^s(p;C,\theta)=\theta^{-\frac{1}{\alpha'}}C^{-\frac{\rho'}{\alpha'}}p^{\frac{1}{\alpha'}}.$$

Solving program (1) for the optimal production and consumption of the non-durable product yields

$$B(C_t,\nu_t,\theta_t)=\frac{(\alpha+\alpha')\nu_t^{\frac{1+\alpha'}{\alpha+\alpha'}}\theta_t^{\frac{\alpha-1}{\alpha+\alpha'}}}{(1-\alpha)(1+\alpha')}C_t^{\beta},$$

with

$$\beta=\frac{\rho(1+\alpha')+\rho'(\alpha-1)}{\alpha+\alpha'}. \tag{18}$$

Proposition 1 can then be applied to this benchmark case, equation (11) being a simple rewriting of equation (18). Besides, one can easily see that this holds also when the parameters θ and ν are random variables independent of C_t.

Proof of proposition 3: It yields the following net benefit:

$$B(C_t,\nu_t,\theta_t)=\begin{cases}\frac{\alpha}{1-\alpha}\nu_t^{\frac{1}{\alpha}}\theta_t^{\frac{\alpha-1}{\alpha}}C_t^{\frac{\rho}{\alpha}}, & \text{if } x^d(\theta_t;C_t,\nu_t)\le K;\\ \nu_tC_t^{\rho}\frac{K^{1-\alpha}}{1-\alpha}-\theta_tK, & \text{if } x^d(\theta_t;C_t,\nu_t)>K,\end{cases} \tag{19}$$

where x^d is the demand function defined by equation (17). We assumed that the capacity of the infrastructure to be built is large enough to guarantee almost surely that the capacity constraint will not be binding in the short run. In that case, equation (19) implies that the logarithm of the net benefit is a linear function of log income plus a white noise, with a slope coefficient $\rho/\alpha = \eta_c^d$. This means that the beta of the project is equal to that income-elasticity of demand in the short run. For very long maturities for which the demand is assumed to be so large that the capacity constraint is almost surely be binding, equation (19) tells us that the net benefit combines a gross benefit whose beta is ρ and a gross cost whose beta is zero. Since this latter is bounded, the beta of the project tends asymptotically to $\rho \leq \eta_c^d$.

Proof of proposition 4 : Using equation (19), we see that the net benefit in this case has the shape of a call option:

$$B(C_t, \nu_t, \theta_t) = \begin{cases} 0, & \text{if } \nu_t C_t^\rho K^{-\alpha} \leq \theta_t; \\ \nu_t C_t^\rho K^{-\alpha} - \theta_t, & \text{if } \nu_t C_t^\rho K^{-\alpha} > \theta_t, \end{cases} \tag{20}$$

Under assumption 1, the log of consumption at time t is a normal distribution with mean μt and variance $\sigma^2 t$. If we additionnally suppose that $\nu_t = \theta_t = 1$, the expected benefit $E[B(C)]$ can be rewritten as

$$K^{-\alpha} e^{\rho^2\sigma^2 t/2 + \rho\mu t} \phi(\frac{\alpha \log K - \rho\mu t - \rho^2\sigma^2 t}{\rho\sigma\sqrt{t}}) - \phi(\frac{\alpha \log K - \rho\mu t}{\rho\sigma\sqrt{t}})$$

using the relation

$$\frac{1}{\sqrt{2\pi}\sigma} \int_z^{+\infty} e^{ac} e^{-(c-\mu)^2/2\sigma^2} dc = e^{a^2\sigma^2/2 + \mu a} \phi(\frac{z - \mu - a\sigma^2}{\sigma})$$

where $\phi(z) = (2\pi)^{-1/2} \int_z^{+\infty} e^{-u^2/2} du$. When $t \to \infty$, we see that $E[B(C_t)] \sim K^{-\alpha} e^{(\rho^2\sigma^2/2 + \mu\rho)t}$ and, similarly, $E[B(C_t)C_t^{-\gamma}] \sim K^{-\alpha} e^{((\rho-\gamma)^2\sigma^2/2 + \mu(\rho-\gamma))t}$, so that $r_t \sim r_f + \rho\gamma\sigma^2$. This shows that the beta of this incremental project tends asymptotically to ρ when the demand for the infrastructure is expected to grow over time. Reversely, when K tends to ∞, since $E[B(C_t)C_t^{-\gamma}] < E[B(C_t)]K^{-\gamma\alpha/\rho}$, the risk-adjusted discount rate for a given maturity t tends to ∞. If we assume instead that ν_t and θ_t are random variables independent of C_t, the same proof works by taking the expected value after replacing K by $(\theta_t/\nu_t)^{1/\alpha} K$.

Proof of proposition 5: Let's assume that the infrastructure is entirely used to transfer goods from country 1 to country 2. This means that country 1 has a comparative advantage in terms of production costs or that country 2's demand is higher. Consequently $p_2 > p_1$ where p_i is the equilibrium price in country i. We have $x_i = y_i + (-1)^i K$ and, by the envelop theorem, the social surplus of increasing marginally K is equal to $p_2 - p_1$. It's the productivity gain generated by the investment, which marginally transfers the production of the good from the high marginal cost country to the low marginal cost one. The general formula that gives the social benefit of the project accruing to country i is then

$$B_i = k_i \left| p_2 - p_1 \right| . \tag{21}$$

Equation (21) states that the benefit of the extension of the infrastructure accruing to country i just corresponds to its share of its global value creation. Let's now examine the beta of the marginal investment project from the point of view of the exporting country 1. We assumed that shocks to the two countries are idiosyncratic and that the good is normal. Consequently, a positive shock on country 1's growth increases demand in that country, and thus the price p_1, whereas it has no impact on p_2. Therefore, the net benefit for the exporting country 1 is the difference between two flows, one with a zero beta, and the other with a positive beta. This implies a negative beta for this exporting country. Symmetrically, the beta of the infrastructure is positive for the importing country.

10 Technical appendix (not for publication) : Cross-border electricity connection between France and Spain

As explained in the core text, we approximate the beta as the income-elasticity of the net expected benefit induced by the investment project. This benefit is equal to the cost reductions allowed by sending cheaper energy from one country to the other - for instance, more french nuclear energy to Spanish in winter, or more spanish solar energy to France during the summer, thus reducing the need to use more expensive fossile fuels. A refinement of this approach would be to take into account the impact of a marginal investment in a cross-border link on the terms of trade. It can be quite significant here because electricity is an homogeneous good that is sold in bulk at a price equal to its marginal production cost. Consequently, a slight change in the cross-border transport infrastructure can trigger a jump from a price equal to the zero marginal cost of renewable energy to the non-zero marginal cost of fossil energy. It will induce a gain of trade for the exporting country, and an equivalent loss for the importing country. We do not consider that in our evaluation of electricity betas because it does not affect global welfare, since it is a zero-sum transfer between countries and also because this effect is very transient and very sensitive to assumptions with respect to profit sharing. [16]

In order to value the benefit induced by the investment project of the French electricity transmission system operator (RTE), we calculate the optimal allocation of electricity in both countries for a given size of the cross-border link. To do this, we use RTE forecasts of energy consumption and production every hour of the year in France and Spain. For each of these time segments, we determine the optimal production and distribution of energy between the two countries that minimize marginal costs. Any increase in the size of the cross-border link make it possible to better optimize costs. The net benefit of the project is equivalent to these additional cost reductions, and is shared equally between the two countries. We estimate the net expected benefit using twelve equiprobable climatic random draws provided by RTE. This requires therefore to do $12 \times 365 \times 24 = 105120$ such calculations.

This is done according to two polar scenarios within the Ten-Year Network Development Plan designed in 2014 by the community of European Network of Transmission System Operators for Electricity (ENTSO-E) : the fist one (vision 1 or ”slow progress” scenario) is a conservative one with about 40% renewable energy whereas the second one (vision 4 or ”green revolution” scenario) reflects an ambitious path with 60% of load supplied by renewable energy in 2030. This is illustrated by the right part of figure 4 which shows the aggregate average monthly production of renewable

[16] An empirical evaluation using RTE’s data shows that the impact can be positive or negative at very specific time horizons, and close to zero elsewhere depending on the expected volume of energy produced in each countries. More precisely, the marginal cost curve of electricity can be described by a step function, so that the impact on trade will be zero when the expected volume of electricity is located on the flat part of this curve.

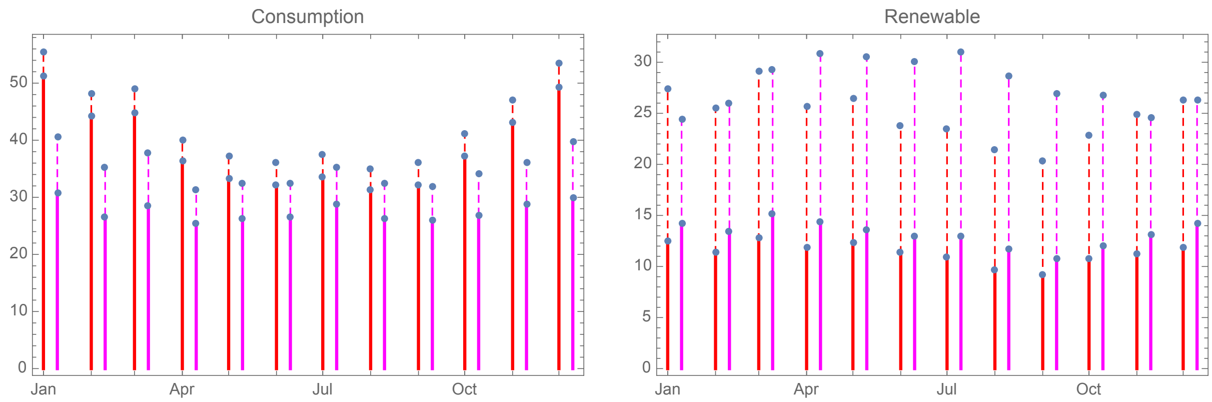


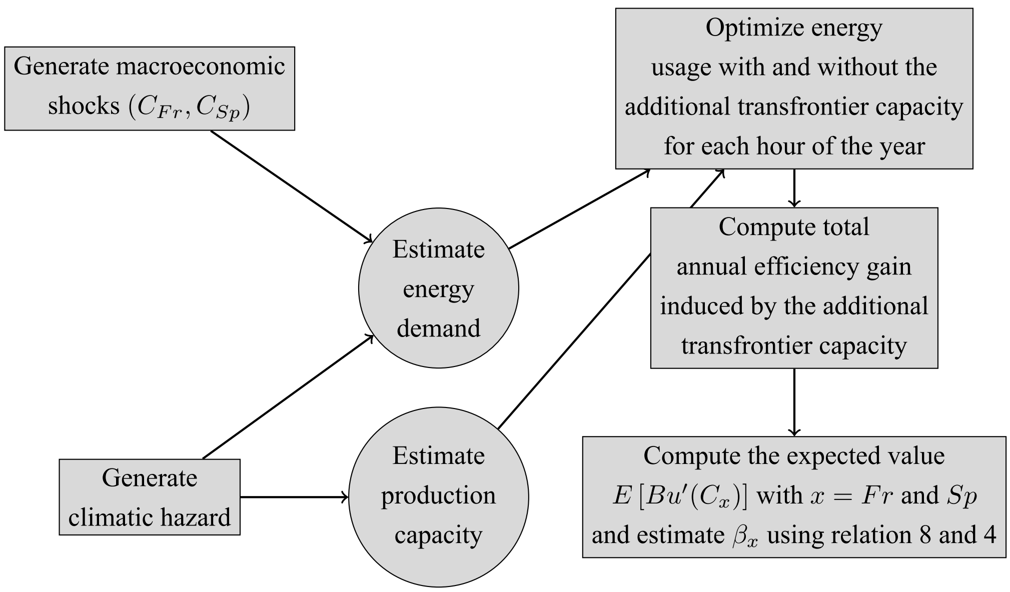
Fig. 4. Electricity consumption and renewable energy production in France and Spain for the first (thick line) and the second scenario (dahed line)

energies (including solar, wind and hydro power). The carbon price is higher in the second scenario, which reduces the consumption of certain fossil fuels (gas in particular), and leads to an increase in the total electricity consumption as indicated in the left part of figure 4. The table below shows that this second scenario also predicts a significant drop in nuclear power generation in France, as well as fossil energy in both countries (both in terms of lower capacity and higher variable cost).

	First scenario			Second scenario		
	variable cost (€/MWh)	Maximum capacity (GWh/year)		variable cost (€/MWh)	Maximum capacity (GWh/year)	
		France	Spain		France	Spain
Nuclear	13,1	56000	7070	13,1	40000	7070
Hard coal	69,1		2525	115,9		1420
Hard coal	60,8	1740		101,9	1740	
Hard coal	53,3		760	89		
Hard coal	42,3		310	35,8		1068
Lignite	40		1060	104,4		
Gas	76,4	6885	24840	83,6	8900	24840
Gas	104,9	578	8400	114,8	3580	16930
Light oil	264,6	1753		247,8	3750	

Fig. 5. Cost structure and maximum production capacity per country.

We measure the sensitivity of the net benefit, calculated by the method described above, to slight variations in the GDP of each of the two countries. This allows us to get the beta for both countries. The following picture describes the process followed to carry out the estimation. We first simulate two random events : macroeconomic shocks on France and Spain (taking into account when mentioned the correlation between macroeconomic cycles) and climatic hazard using 12 climate scenarios of equal weight provided by RTE. These climate scenarios give the impact of weather conditions on energy demand and renewable energy production capacity. The income-elasticity of demand presented in section 4 is then used to get the energy demand in each country. All these data are given for each hour of the year. In particular, we have a detailed marginal cost curve for each country, similar to those presented in figure 3 but for each hour of the year. We then compute the optimal allocation of energy to meet demand while minimizing cost. The cross-border electricity connection allows better optimization. From this net benefit B induced by the cross-border investment (corresponding to a reduction in energy costs and obtained for each random events), we estimate the β using the relations 8 and 4 presented at the beginning of this document. This takes a lot of computing time, but simple OLS estimations provide also good approximations in that case.



The following table presents the results of these estimates for both scenarios, both for the marginal investment project and the total investment project in the cross-border link. As expected, and in all four cases considered, the beta of the project is negative for the exporting country (France), and positive for the importing one (Spain). As explained in the core text, taking into account correlation has a strong (and non linear) impact on the beta. The estimation for both countries taken as a whole is obtained by the same methodology presented in the previous graph when considering the aggregated impact $C_{tot} = C_{Fr} + C_{Sp}$.

	Total investment		Marginal Investment		Marginal Investment with correlation		Union of both countries
	France	Spain	France	Spain	France	Spain	Both
First Scenario	-0.60	0.33	-0.98	0.47	-0.48	0.22	0.06
Second Scenario	-0.05	1.08	-0.49	1.6	1.12	1.41	1.94

Table 2: Beta of a marginal investment to increase the size of the cross-border link.

The following table illustrates the explanations provided in the main text. In the first scenario, about 97% of the traded energy is french nuclear electricity, which explain why the beta is strongly negative for France. In the second scenario, the usage is more balanced between the two countries : about one fourth of traded energy is exported from Spain, mainly renewable and gas. The fact that the beta is higher in absolute value when we consider the marginal project is quite intuitive. Indeed, the use of this additional cross-border capacity is more rare (only during certain peaks of demand or production deficits), which makes it naturally more sensitive to economic activity.

	Scenario 1		**Scenario 2**	
	France	**Spain**	**France**	**Spain**
Renewable	0%	0,08%	1,10%	13%
Nuclear	97%	0,04%	73%	1,60%
Hard coal	0,00%	0,181%	0,0029%	1,7%
Lignite	0%	0,0046%	0%	0%
Gas	1,33%	1%	1,85%	7,30%
Light oil	0,00028%	0%	0,0027%	0%
Total Share of export per country	98%	1%	76%	24%

Fig. 6. Detailed use of the transfrontier link