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Equilibrium (non-)Existence in Games with Competing Principals

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Abstract

We study competing-mechanism games, in which multiple principals contract with multiple agents. We reconsider the issue of non-existence of an equilibrium as first raised by Myerson (1982). In the context of his example, we establish the existence of a perfect Bayesian equilibrium. We clarify that Myerson (1982)'s non-existence result is an implication of the additional requirement he imposes, that each principal selects his preferred continuation equilibrium in the agents' game.

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1 Introduction

Competition in several market settings is modeled as an extensive-form game in which principals post mechanisms to deal with multiple agents. The competing auctions (McAfee (1993); Peters and Severinov (1997)), and competitive search (Wright et al. (2021)) models offer prominent examples of this approach.

Despite the increased economic relevance of competing-mechanism approaches, we still lack a comprehensive characterization of the corresponding market equilibria. While, following Epstein and Peters (1999), the literature has extended the revelation principle to these contexts, the general issue of equilibrium existence remains largely unexplored. Indeed, existence of an equilibrium in games with multiple principals has only been established for the particular case of a single agent (Carmona and Fajardo (2009)). With several agents, the celebrated example of Myerson (1982) provides an instance of equilibrium non-existence. In Myerson's approach, the presence of multiple principals is at the root of the non-existence, since it generates a fundamental discontinuity in the optimal choice of a principal's mechanism.[1] His analysis is framed in terms of competition between principal-agent (manufacturer-retailer) hierarchies, which makes the non-existence result potentially problematic for applications.

We propose a reinterpretation of this result. We argue that, in the example, the non-existence is implied by the requirement that, at equilibrium, each principal chooses his optimal incentive-compatible mechanism. This guarantees that he does not have a profitable deviation *regardless* of the continuation equilibrium selected by agents. Such requirement need not be satisfied by any perfect bayesian equilibria (PBE) of the competing-mechanism game. We eventually show that existence of a PBE can be established in the example.

We next formalize the competitive game between principal-agent hierarchies (Section 2) and illustrate our result (Section 3).

2 Competing Hierarchies

We consider multiple principals (indexed by $j \in \mathcal{J} = \{1, \ldots, J\}$) contracting with multiple agents (indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$). Each agent i has a private type ω^i in the finite set Ω^i, with $\Omega = \times_{i \in \mathcal{I}} \Omega^i$.

We denote $y_j \in Y_j$ a decision for principal j, with Y_j finite and $Y = \times_{j \in \mathcal{J}} Y_j$. We let $v_j : Y \times \Omega \to \mathbb{R}$ and $u^i : Y \times \Omega \to \mathbb{R}$ be the payoffs of principal j and of agent i, respectively.

[1] "[...] the set of feasible (incentive-compatible) mechanisms for principal j varies upper-semicontinuously in the other (mechanism), rather than continuously as is required by the existence theorem of Debreu (1952)" (Myerson, 1982, p. 78).

An allocation $\phi : \Omega \to \Delta Y$ is a mapping associating to each state $\omega \in \Omega$ a probability distribution over Y.

Communication occurs via the public mechanisms posted by principals, and via the messages that agents privately send them. Each agent contracts with one principal only: let $i_j \in \mathcal{I}_j$ be the typical agent dealing with principal j. Clearly, $\underset{j\in\mathcal{J}}{\cup}\mathcal{I}_j = \mathcal{I}$, and $\mathcal{I}_j \cap \mathcal{I}_{j'} = \emptyset$ for each (j, j'). We call *hierarchy* j the collection of principal j and his agents. Each $i_j \in \mathcal{I}_j$ sends a message m^i_j in the finite set $M^i_j \supseteq \Omega^i$ to principal j.[2]

Formally, a mechanism for principal j is a mapping $\gamma_j : M_j \to \Delta(Y_j)$. We let Γ_j be the set of mechanisms available to principal j, with $\Gamma = \underset{j\in\mathcal{J}}{\times}\Gamma_j$. The corresponding game, denoted G^Γ, unfolds as follows. First, principals simultaneously commit to mechanisms. Given the observed mechanisms $(\gamma_1, \gamma_2, \ldots, \gamma_J)$, and their private types $(\omega^1, \ldots, \omega^I)$, agents simultaneously send a message to the principal of their hierarchy. Finally, decisions are implemented, lotteries realize, and payoffs accrue. A (pure) strategy for principal j is a mechanism $\gamma_j \in \Gamma_j$. A strategy λ^{i_j} for agent i_j associates to every profile of mechanisms a probability distribution over M^i_j, for each realized type. This an instance of Myerson (1982) model, in which agents take no physical actions.

Following the standard approach to competing mechanisms (Epstein and Peters (1999)), we focus on the PBE of G^Γ. The strategies $\gamma = (\gamma_j, \gamma_{-j})$ and $\lambda = (\lambda^{i_j}, \lambda^{-i_j})_{j\in\mathcal{J}}$, constitute a PBE if:

1. λ is a continuation equilibrium. That is, for every $\gamma \in \Gamma$, the strategies $(\lambda^{i_j}, \lambda^{-i_j})_{j\in\mathcal{J}}$ form a Bayes-Nash equilibrium of the subgame γ;

2. Given λ, the strategies (γ_j, γ_{-j}) form a Nash equilibrium of the principals' game.

A mechanism is *direct* if agents can only communicate their types to the principal. We denote $\gamma^D_j : \underset{i_j\in\mathcal{I}_j}{\times}\Omega^{i_j} \to \Delta(Y_j)$ a direct mechanism for principal j, with $\gamma^D_j \in \Gamma^D_j \subseteq \Gamma_j$, and let G^D be the corresponding game.

For a given array of direct mechanisms γ^D_{-j}, a mechanism γ^D_j is *incentive compatible* if it induces a continuation equilibrium in which the agents of each hierarchy j are truthful to principal j, under the belief that the same occurs in each hierarchy $-j$. Thus, γ^D_j can be incentive compatible given γ^D_{-j}, but not relative to $\tilde{\gamma}^D_{-j} \neq \gamma^D_{-j}$.[3]

Myerson (1982) analyses the competition between principals as a *generalized* single-principal problem. That is, he restricts principals to direct mechanisms, and focuses on "principals'

[2] The analysis extends to situations in which all relevant sets are infinite (Attar et al. (2021)).

[3] This possibility is already acknowledged by Myerson (1982, p.77). See also McAfee (1993, p. 1288).

equilibria", in which each principal chooses an optimal incentive-compatible mechanism. Together with the revelation-principle result of his Proposition 2, this amounts to let each principal select his preferred continuation equilibrium. This choice, we shall argue, is key to the non-existence problem he documents.

3 Restoring Existence

We revisit the example in Section 4 of Myerson (1982). Consider two principals, each of them dealing with only one agent. The type set of each agent $i = 1, 2$ is $\Omega^i = \{\alpha, \beta\}$; types are independent, with $prob(\alpha) = prob(\beta) = 1/2$.

Each principal $j = 1, 2$ takes decisions in $Y_j = \{A, B, C\}$. In the matrix below, the first number is principal j's payoff, and the second one is that of agent i_j of type ω^{i_j}.

	$\omega^{i_j} = \alpha$	$\omega^{i_j} = \beta$
A	$6, 1$	$0, z^{i_j}$
B	$0, z^{i_j}$	$6, 1$
C	$5, 0$	$5, 0$

z^{i_j} is determined by principal $-j$'s decision. Specifically,

$$z^1 = \begin{cases} 2 & \text{if } y_2 \in \{A, B\} \\ 1 & \text{if } y_2 = C \end{cases}$$

and

$$z^2 = \begin{cases} 1 & \text{if } y_1 \in \{A, B\} \\ 2 & \text{if } y_1 = C \end{cases}$$

The non-existence result. Proposition 3 in Myerson (1982) shows that, in this example, there is no *principals' equilibrium.* Consider P1: if P2's equilibrium mechanism implements C with probability one, P1's optimal incentive-compatible mechanism selects A if A1 reports α, and B if she reports β, which gives him a payoff of 6. In all other cases, P1 cannot achieve a payoff above 5, and his optimal mechanism is to select C regardless of A1's message. P2's optimal mechanism is exactly reversed, that is, it makes available A and B when P1 plays C, and viceversa. This implies the non-existence of a principals' equilibrium.

Existence can however be restored if one focuses on PBE, in line with the competing-mechanisms literature. We establish the result in the simple game in which principals post direct mechanisms.

Lemma 1 *The allocation $\phi(\omega) = (C, C)$ for each $\omega \in \{\alpha, \beta\}^2$ can be supported in a PBE of G^D.*

Proof Denote γ^* the mechanism yielding C for every agent's report, and let both principals post γ^*, which guarantees that the desired allocation is obtained on the equilibrium path. We specify the agents' equilibrium strategies as follows. In each subgame (γ_1, γ_2),

(i) Type α of agent i_j, whenever indifferent between several reports in γ_j, picks any message $m^i_j(\alpha) \in \{\alpha, \beta\}$ maximizing the probability that γ_j implements B. We denote $\gamma_j(B|m^i_j(\alpha)) \geq 0$ the corresponding probability.

(ii) Type β of agent i_j, whenever indifferent between several reports in γ_j, picks any message $m^i_j(\beta) \in \{\alpha, \beta\}$ maximizing the probability that γ_j implements A. We denote $\gamma_j(A|m^i_j(\beta)) \geq 0$ the corresponding probability.

We now show that no principal has a profitable deviation from γ^*. Consider P1 deviating to some $\gamma_1 \neq \gamma^*$, and the behavior of A1 in the subgame (γ_1, γ^*). Since P2 sticks to γ^*, one has $z^1 = 1$, which implies that both types of A1 have the same incentives.

Any such type is indifferent between reporting α or β to P1 if and only if:

$$\gamma_1(A|\alpha) + \gamma_1(B|\alpha) = \gamma_1(A|\beta) + \gamma_1(B|\beta). \tag{1}$$

Suppose that γ_1 satisfies (1). If

$$\gamma_1(A|\alpha) \leq \gamma_1(A|\beta) \quad \Longleftrightarrow \quad \gamma_1(B|\alpha) \geq \gamma_1(B|\beta), \tag{2}$$

then, given $(i) - (ii)$, both types of A1 are truthful to P1. The payoff to P1 is

$$\frac{1}{2}\big(6\gamma_1(A|\alpha) + 5\gamma_1(C|\alpha)\big) + \frac{1}{2}\big(6\gamma_1(B|\beta) + 5\gamma_1(C|\beta)\big) =$$
$$= 5 + \frac{1}{2}\big(\gamma_1(A|\alpha) - 5\gamma_1(A|\beta)\big) + \frac{1}{2}\big(\gamma_1(B|\beta) - 5\gamma_1(B|\alpha)\big) \leq 5,$$

since $\gamma_1(A|m) + \gamma_1(B|m) + \gamma_1(C|m) = 1$, for each $m \in \{\alpha, \beta\}$. The deviation is therefore unprofitable. If, instead, (1) is satisfied while (2) is not, we get

$$\gamma_1(A|\alpha) > \gamma_1(A|\beta) \quad \Longleftrightarrow \quad \gamma_1(B|\alpha) < \gamma_1(B|\beta), \tag{3}$$

in which case no type of A1 is truthful to P1, and his payoff is

$$\frac{1}{2}\big(6\gamma_1(A|\beta) + 5\gamma_1(C|\beta)\big) + \frac{1}{2}\big(6\gamma_1(B|\alpha) + 5\gamma_1(C|\alpha)\big) =$$
$$= 5 + \frac{1}{2}\big(\gamma_1(A|\beta) - 5\gamma_1(A|\alpha)\big) + \frac{1}{2}\big(\gamma_1(B|\beta) - 5\gamma_1(B|\alpha)\big) < 5,$$

which, again, makes the deviation unprofitable.

Finally, if (1) does not hold, neither type of A1 is indifferent, and they will send the same message $m \in \{\alpha, \beta\}$ to P1. His payoff is

$$\frac{1}{2}\big(6\gamma_1(A|m) + 5\gamma_1(C|m)\big) + \frac{1}{2}\big(6\gamma_1(B|m) + 5\gamma_1(C|m)\big) = \frac{1}{2}\big(6 + 4\gamma_1(C|m)\big),$$

which is not greater than 5 for each $\gamma_1(C|m) \in [0,1]$.

A similar reasoning applies to P2. ■

The proof exploits the fact that principals do not have full control over the agents' coordination in the continuation game. Specifically, the PBE notion does not prevent agents from punishing principals via untruthful behaviors.

In the language of competing mechanisms, the notion of principals' equilibrium is recovered by that of *strongly-robust* equilibria.[4] Formally, a strongly-robust equilibrium (SRE) of G^{Γ} is a PBE (γ, λ), in which, for every j:

$$V_j\left(\gamma, \lambda\right) \geq V_j\left(\gamma_j', \gamma_{-j}, \lambda'\right) \qquad \forall \gamma_j' \in \Gamma_j, \forall \lambda',$$

with λ' being a profile of agent's equilibrium strategies in the subgame $\left(\gamma_j', \gamma_{-j}\right)$.

In a SRE, each principal believes that, in every subgame, agents will coordinate on his preferred continuation equilibrium. It is easy to check that in the above example a SRE does not exist.

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[4]See Epstein and Peters (1999), and Han (2007). The application of SRE concept extends beyond the competing-hierarchy model.

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