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Migrating away from an incumbent platform"

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Should I stay or should I go? Migrating away from an incumbent platform*

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Abstract

We study incumbency advantage in markets with positive consumption externalities. Users of an incumbent platform receive stochastic opportunities to migrate to an entrant and can either accept them or wait for a future opportunity. In some circumstances, users have incentives to delay migration until others have migrated. If they all do so, no migration takes place, even when migration would have been Pareto-superior. We use our framework to identify environments where incumbency advantage is larger. A key result is that having more migration opportunities actually increases incumbency advantage.

Keywords: Platform, Migration, Standardization and Compatibility, Industry Dynamics
JEL Classication Codes: D85, L14, R23, L15, L16

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1 Introduction

The utility of joining a telecommunications or a social media platform, buying a game console, or adopting an industry standard depends on who else has joined the platform, plays the same game, or uses the same standard. Users choose which platform to use, game to purchase, or standards to adopt based on their predictions of the number of users who will make the same choice. Economists and practitioners often believe that this makes entry difficult, i.e., results in *incumbency advantage*. The reasoning is that each user worries that others will not migrate from an incumbent to an entrant platform, even when the latter offers a superior product (see our literature review in section 2). This is easy to understand when switching costs are large but it is harder to explain when incumbency advantage stems exclusively from network externalities, the issue we tackle in this article.[1]

For example, in 2011 Google launched the social network Google+, a direct rival to Facebook. Despite heavy promotion by Google and links to other Google products such as Google Drive and Youtube, Google+ never took off and was ultimately shutdown in 2019. Joshua Gans argued that Google+ was better than Facebook, but not by a wide enough margin to break the barrier of the incumbency advantage. Our contribution can be seen as providing a theoretical underpinning to Professor Gans's analysis.[2]

The topic has policy relevance as incumbency advantage forms the basis of many recent analyses and policy recommendations. For instance, the Explanatory Memorandum of the European Commission's proposed Digital Markets Act argues that, because of network effects and other characteristics of the digital economy, "A few large platforms increasingly act as gateways or gatekeepers between business users and end users and enjoy an entrenched and durable position, . . . ".[3] One can view the recent cases brought by the EU against Facebook, Google, and Amazon as demonstrating these regulatory concerns.

On the other hand, as several authors have emphasized, strong network

[1] In most real world cases, there would be both switching costs and network externalities. As Crémer and Biglaiser (2012) argue, the interaction between the two phenomena is understudied.

[2] See https://hbr.org/2011/07/google-comes-up-short for Professor Gans's analysis and https://en.wikipedia.org/wiki/Google%2B for a longer discussion of Google+.

[3] https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:52020PC0842&from=en, p. 1.

effects do not seem sufficient to generate large incumbency advantage (see, among many others, Evans, Hagiu and Schmalensee (2006), pp. 61–63). An instructive example is the 2018 $7.5 billion acquisition by Microsoft of GitHub, a collaborative coding platform which, at the time of the acquisition, was used by 28 million developers.[4] GitHub is used by closed teams, but it is especially popular in the Open Source community which can access it at zero cost. The European Commission was concerned that Microsoft would exploit the incumbency advantage of GitHub to the detriment of users, for instance by favouring Microsoft's own technologies. Microsoft acknowledged the Commission's concerns, but argued that incumbency advantage is low in this case because GitHub users are sophisticated, well aware of the alternatives, and could easily migrate if the platform were degraded.[5] Ultimately, the acquisition was approved with the following rationale:

> "The market investigation confirmed that Microsoft would not have the market power to undermine the open nature of GitHub to the detriment of competing DevOps tools and cloud services. This is because such behaviour would reduce the value of GitHub for developers, who are willing and able to switch to other platforms." (European Commission (2018), §§ 98–102)

We draw two lessons from this example. First, the European Commission's decision seems reasonable and was the object of little public criticism, but this seems to contradict the view that incumbency advantage is insurmountable. For instance, why is incumbency advantage seemingly low for Github but seemingly high for Google+?

Second, the existing economic literature could not have helped the Commission to evaluate Microsoft's claims. There have been debates around the pervasiveness and size of incumbency advantage in the economy as a whole. However, as we discuss in section 2, there is little theoretical or applied work linking features of the economic environment to the level of incumbency advantage.

[4]Our sources about this acquisition include the following pages, visited on 9 July 2019: http://europa.eu/rapid/press-release_IP-18-6155_en.htm, https://usefyi.com/github-history/, https://www.bloomberg.com/news/articles/2019-06-03/open-source-great-satan-no-more-microsoft-wins-over-skeptics, https://www.theverge.com/2018/6/18/17474284/microsoft-github-acquisition-developer-reaction.

[5]As far as we understand, the EC's analysis assumed that the merger would not affect the size of the incumbency advantage — it was worried that Microsoft would exploit it in ways which were more detrimental than GitHub was able to do on its own.

To explain the relationship between migration across platforms and incumbency advantage, we consider a setting in which a continuum of identical users are initially on an incumbent platform and consider migrating to an entrant platform. We depart from most of the existing literature by allowing individuals to obtain multiple migration opportunities, which arrive stochastically over time. These opportunities can arise, for example, when a user sees an advertisement or hears from a friend about the new platform. Individuals are free to accept a migration opportunity, or to reject it and (possibly) migrate later when another opportunity arises.[6]

Do individuals migrate to an entrant platform when collectively they would be better off doing so? We study the conditions under which there exists a *migration equilibrium* in which individuals migrate to the entrant platform. We say that incumbency advantage is larger if when set of parameters for which a migration equilibrium exists is smaller. Relative to the status quo, all consumers would prefer to migrate immediately and collectively to the entrant. That is, in the game in which each of the users chooses a platform, no-migration is a Pareto inferior equilibrium, but the one which the literature typically focuses on. This contradicts the assumption commonly made in other branches of economic theory, where the Pareto superior equilibrium is often selected. To solve this quandary, we propose a new model of migration between platforms.

We focus on the migration technology and the structure of the economic environment, rather than the beliefs of the agents.[7] For this reason, our emphasis is on allowing the stochastic process that generates migration opportunities to be as general as possible. Moreover, we deliberately study the existence of migration under the beliefs which are most favourable to migration.

In our model, the impediments to collective migration stem from the nature of the migration technology, not from the beliefs of users. In particular, it is often that case that each user would rather wait to migrate until enough other users have migrated, so as not to miss too much of the network value of the incumbent platform. However, if all users act in this way, migration does not take place. This is the cause of incumbency advantage in our model.

[6] Our model shares some features with Farrell and Saloner (1986), Frankel and Pauzner (2000), and Guimaraes and Pereira (2016) where users have multiple opportunities to migrate. In section 2, we discuss our relationship to those articles.

[7] In the literature section, we discuss articles that develop a belief-based approach to the modelling of incumbency advantage.

We make two main contributions. First, we develop a tractable micro-founded model in which incumbency advantage emerges endogenously as a result of the response of users to the migration technology which is available to them. Second, we use this framework to study how the structure of the economic environment and the migration technology affect incumbency advantage.[8]

Many articles analyze how firm strategies can affect user adoption decisions. We take the view that any analysis of competition among platforms must first incorporate user migration decisions and the game played by users, before analyzing how firm pricing, quality, and advertising strategies affect incumbency advantage. Thus, we treat the platforms as passive players in the body of the article.[9]

We present the model and equilibrium in Section 3. Equilibrium has a simple structure: individuals reject all migration opportunities before some threshold in time, and accept all migration opportunities thereafter.

In Section 4, we examine how the structure of the economic environment affects incumbency advantage. Perhaps surprisingly, we find that incumbency advantage is *smaller* when users have only a single migration opportunity rather than multiple ones (Proposition 1): multiple opportunities to migrate reduce the cost of foregoing early opportunities. Furthermore, incumbency advantage is increasing as the speed of the new opportunities increases. We also find that the possibility of multi-homing decreases, but does not eliminate, incumbency advantage. This provides some support to the policy recommendations that competition authorities should pay special attention to practices that hinder multi-homing (see, for instance, Crémer, de Montjoye and Schweitzer, 2019). Finally, we demonstrate that the entrant can *decrease* incumbency advantage by committing to a capacity constraint, so that not all users can join it.

In section 5, we focus on how the process by which users obtain opportunities to migrate affects incumbency advantage. We impose additional structure on utilities, which allows us to further understand how incumbency advantage is influenced by the migration process and informs our discussion of firm behavior.

In Section 6, we consider heterogeneous user preferences (e.g., some users

[8] As we discuss in Section 2, previous articles in the literature were not designed to address how the structure of the economic environment affects incumbency advantage.

[9] In the conclusion we briefly discuss firm strategies.

like the entrant, but others dislike it). In this setting, there can be *staggered* migration equilibria where, initially, only some "eager" users accept their migration opportunities while others wait until enough users have migrated before beginning to migrate themselves. Furthermore, if user preferences are sufficiently polarised, there exists an equilibrium where the different types of users settle on different platforms. In this case, the equilibrium can be inefficient as users do not internalise the network externalities they generate. Conclusions and paths for future research are presented in section 7.

The appendix contains proofs, but also an analysis of a more general migration technology than the one we used in the main text.

2 Literature Review

We know of no econometric evidence of the size of incumbency advantage or of its determinants. On the other hand, there has been a vigorous debate, often based on case studies, on the importance of lock-in which we will not review here (see Farrell and Klemperer (2007) for a survey and discussion). In the remainder of this section, we will focus on the theoretical literature.

Farrell and Saloner (1985) consider a finite number of users who choose sequentially between two standards. They show that they always coordinate on the superior standard. The reasoning is elegant and instructive. The last user who is given the choice to adopt the (Pareto) superior standard does so if the others have joined. The penultimate user, predicting that the last one will join, joins themselves, and so forth. This is known as the "bandwagon" effect. In our model, by contrast, users obtain multiple migration opportunities. This gives users the possibility and incentives to delay migration. In our more realistic setting, incumbency advantage arises endogenously and can preclude migration to a superior platform (see Proposition 1).

Other authors use imperfect information to explain incumbency advantage, in models where users sequentially must make a once-and-for-all decision of which technology to use. Choi (1997) assumes that the quality of a technology becomes known to all users as soon as a single user adopts it. In that article, there is insufficient experimentation with new technologies (relative to the first best), because users fear being stranded by themselves once they adopt. Ochs and Park (2010) analyze an environment where a finite number of players differ in how large a platform must be before they find it worthwhile to join. Each agent knows her own type, but there is aggregate uncertainty about the composition of the pool of players. They show that this uncertainty leads to inefficient adoption decisions. Unlike the above articles, we assume a continuum of users and measurable strategies so that no single user can affect the decisions of the others. As in these articles, migration can also be inefficiently high or low in our setting, but the source of this inefficiency is entirely different from the "bandwagon" effect of early movers on later ones. Instead, incumbency arises endogenously because users prefer to delay migration.

Farrell and Saloner (1986) examine a model of technology standards adoption with network effects. Their focus is on how beliefs can lead to excessive inertia or excessive momentum. They consider two users who receive opportunities to switch according to a Poisson process, similar to the autonomous process we describe in Section 5. As in our model, users have multiple opportunities to switch. Frankel and Pauzner (2000) analyze a technology adoption model where opportunities to switch also arise via a Poisson process. Finding uniqueness of equilibria is the key result of the article. Guimaraes and Pereira (2016) investigate the difference between social welfare and equilibrium in the Frankel and Pauzner framework. Relative to these three articles, our key innovation is to allow for general migration processes. This allows us to study how different migration technologies affect incumbency advantage.

The Farrell and Saloner article is closest to ours, but there are fundamental differences between the two. First, in this article, we consider a continuum of users and, therefore, no user is pivotal.[10] Second, those authors model opportunities to migrate as only arising from a Poisson process; by contrast, we study a general migration technology. As we show in section 5, a Poisson process is a very special case where the incumbency advantage is *independent* of the arrival rate of the migration process. An implication of this is that their model cannot speak to the issues that we discuss in Section 4 such as the rate of migration opportunities changing once a user has a first opportunity, multi-homing, and entrant capacity constraints.

Ostrovsky and Schwarz (2005) analyze a model where there is uncertainty regarding the time at which a firm can adopt a new standard, and a free riding effect can induce the non-adoption of a Pareto dominant standard. By contrast, in our article, there is uncertainty regarding when each agent

[10]The first part of Farrell and Saloner studies a model of successive choice by users to study what inefficiencies can stem from the presence of early adopters of an inferior technology. This is less closely related to this article.

will receive her next opportunity to migrate but the adoption decision of an individual agent does not affect the decisions of other agents.

Some articles explicitly examine the role of platform behavior in consumer adoption dynamics. In Katz and Shapiro (1992), firms compete in price with entry of new consumers over time. Sakovics and Steiner (2012) study a model where a monopoly platform chooses the order in which to attract users and how much to subsidize each of them. Cabral (2019) studies a model of competition between platforms that adjust their prices dynamically. Hałaburda, Jullien and Yehezkel (2020) and Biglaiser and Crémer (2020) allow firms to choose prices to attract consumers, but assume that all consumers make migration decisions after each round of price setting by firms, as do Fudenberg and Tirole (2000). As we discuss in the introduction, we abstract from strategic considerations by firms, and focus on users' decisions. This allows us to study the effect of a general migration technology on incumbency advantage. We see this as a first step towards a fuller understanding of incumbency advantage.

Finally, in a different line of inquiry Gordon, Henry and Murtoz (2018) study the way in which the graph theoretical shape of networks influences the spread of an innovation in a model with local externalities.

3 Model and equilibrium

In this section, we present the model which we use in the applications that follow and the properties of the equilibrium. The proofs are presented in Appendix A. As we discuss at the end of this section (and show in the Appendix), many of the results can be proved for a more general model.

There is an incumbent platform I, and an entrant platform E, as well as a mass 1 of consumers, which we will also call users. Apart from in Section 6, we assume that all users have the same utility function. If at time $t \in [0, +\infty)$ there is a mass $h_I(t)$ of users on the incumbent platform, and a mass $h_E(t) = 1 - h_I(t)$ users on the entrant platform, the (instantaneous) utility of the users of the incumbent is $u_I(h_I(t))$ and the utility of the users of the entrant is $u_E(h_E(t))$. We assume utility is continuously differentiable and strictly increasing, that is, there are strictly positive network externalities.

At $t = 0$, all users are on the incumbent platform, so $h_I(0) = 1$ and $h_E(0) = 0$. In the main text, we assume that migration only takes place from the incumbent to the entrant, so that h_I is weakly decreasing and h_E

is weakly increasing. In other words, migration to the entrant is irreversible. In Appendix A.7, we show that reversibility of migration does not change our results in any meaningful way.[11]

We assume, as does most of the literature on network externalities, that there are no switching costs. If a user migrates at $t = T$, her lifetime utility is

$$\int_0^T u_I(h_I(t))e^{-rt}\,dt + \int_T^{+\infty} u_E(h_E(t))e^{-rt}\,dt.$$

Notice that this framework is quite flexible. For instance, the entry of a new platform in a market where none existed can be represented by assuming $u_I(h_I) = 0$ for all h_I.[12]

In any infinitesimal interval of time $[t, t+dt]$, each consumer on the incumbent platform is given an opportunity to migrate with probability $\mu(h_I(t)) \times dt$. We call $\mu(h_I(t))$ the *migration process*. The problem is only interesting if we assume $\mu > 0$.[13] as there is a continuum of identical users and the probabilities of receiving migration offers are independent, $\mu(h_I(t))$ is also the proportion of the users of the incumbent platform who receive a migration opportunity during $[t, t+dt]$.

There are two possible (and non-conflicting) interpretations of the migration process μ. It could stem from a psychological process where users remember at random times to re-optimize their choice of platform. It could also stem from the fact that users are made aware of the existence of the entrant platform at random times, for example through advertising or through word of mouth from users who have already migrated. (In Section 5, we examine the interaction between these two phenomena).

We assume that the strategies of the consumers depend only on the mass, but not the identity, of the consumers on the two platforms. Therefore, consumers can "predict" the equilibrium market shares of each platform at each time t and we can rewrite the strategies as a measurable function of

[11]When the migration process μ (defined just below) is bounded, the results do not change at all. When μ is unbounded, there could be equilibria where there is migration from I to E, then a non-zero interval of time in which all the users stay in the Entrant, and then reverse migration. However, this assumes that the benefit of the first mover on the reverse migration path is exactly zero, which is only true of a set of parameters of measure 0.

[12]This is a weakening of our assumption that u_I is strictly increasing, but it is easy to see that our results still hold true in that case.

[13]We have assumed $\mu(h_I)$ depends only on h_I. If, at some t, we had $\mu(h_I(t)) = 0$, then h_I cannot change further, and therefore neither can μ.

time $\phi(t) : \Re^+ \to [0,1]$, interpreted as the probability that the consumer accepts a time t migration opportunity.

Following this strategy, a user migrates during $[t, t+dt]$ if and only if (i) she has the opportunity to do so, which happens with probability $\mu(h(t))\,dt$, and (ii) she accepts this opportunity, which she does with probability $\phi(t)$. Therefore, the probability of migration during the interval $[t, t+dt]$ is $\phi(t) \times \mu(h(t)) \times dt$.

The user is on the incumbent at the end of the interval if she was on the incumbent at the start of the interval and does not migrate. Therefore, taking $h(t)$ as given, the probability $\pi_I(t)$ that, following strategy ϕ, she is on the incumbent at time t satisfies[14]

$$\pi_I(t+dt) = \pi_I(t)\left[1 - \mu(h(t))\phi(t)\,dt\right],$$
$$\Longrightarrow \pi_I(t) = \exp\left[-\int_0^t \mu(h(\tau))\phi(\tau)\,d\tau\right]. \quad (1)$$

The probability that the consumer is on the entrant is $\pi_E(t) = 1 - \pi_I(t)$ and her expected utility is

$$\int_0^\infty \Big[u_I(h_I(t))\pi_I(t) + u_E(h_E(t))\pi_E(t)\Big]e^{-rt}dt$$
$$= \int_0^\infty \Big[u_I(h_I(t)) - u_E(h_E(t))\Big]\pi_I(t)e^{-rt}dt + \int_0^\infty u_E(h_E(t))e^{-rt}dt.$$

The second term of the right-hand side does not depend on ϕ, so users choose a strategy ϕ which maximizes

$$\int_0^\infty [u_I(h_I(t)) - u_E(h_E(t))]\pi_I(t)e^{-rt}dt. \quad (2)$$

subject to (1).

We show in Appendix A that all users use the same strategy in equilibrium. Hence, they have the same probability of being on the incumbent

[14]The second line is derived from the first by using $\pi_I(0) = 1$ and writing

$$\frac{\pi_I(t+dt) - \pi_I(t)}{dt} = \pi_I'(t) = -\pi_I(t)\widetilde{\mu}(t)\phi(t) \Rightarrow \ln(\pi_I(t)) = -\int_0^t \widetilde{\mu}(\tau)\phi(\tau)d\tau.$$

platform at any time t. Because the total mass of consumers is 1, by (1), the mass of users in the incumbent is

$$h_I(t) = \pi_I(t) = \exp\left[-\int_0^t \mu(h_I(\tau))\phi(\tau)\,d\tau\right]. \tag{3}$$

Definition 1 (Equilibrium path)**.** *An equilibrium path is a function $\pi_I(\cdot)$ associated with a strategy $\phi(\cdot)$ that a) maximizes (2) subject to (1) given $h_I(t)$ and b) satisfies (3).*

If $u_E(0) > u_I(1)$, migration is a *dominant strategy*: it is optimal to accept all migration opportunities whatever the migration path h_I. Then, migration is the unique equilibrium. If $u_E(0) \leq u_I(1)$, there exists an equilibrium path with no migration, as $h_I(t) = 1$ and $\phi(t) = 0$ satisfy the definition of an equilibrium path. If $u_E(1) < u_I(0)$, then not migrating is a dominant strategy: users will reject all migration opportunities for any h_I. In the sequel, we are interested in *migration equilibria*, which we now define.

Definition 2 (Migration equilibrium)**.** *A migration equilibrium is an equilibrium path in which a strictly positive mass of consumers migrate. Formally:* $\lim_{t\to+\infty} h_I(t) < 1$.

In Appendix A, we show that, generically (in a sense we make precise there), in any migration equilibrium, all consumers accept all migration opportunities. That is, all consumers migrate as soon as possible and

$$h_I(t) = \exp\left[-\int_0^t \mu\big(h_I(\tau)\big)\,d\tau\right]. \tag{4}$$

In the sequel, h_I will stand for the solution of (4).

Corollary 1, stated just below, is a consequence of Corollary A.3 in Appendix A.5. It characterizes the environments for which there exists a migration equilibrium. The first part assumes that, when all users migrate starting at $t = 0$, the gain of utility from migrating at $t = 0$ (rather than waiting for the next opportunity) is strictly positive. In this case, there exists a unique migration equilibrium where all users accept migration opportunities. As stated above, in this case, there also exists an equilibrium without migration. The second part of the Corollary assumes that, even if she believed that all other users will migrate, a user would choose to wait before migrating at $t = 0$. In this case, there exists no migration equilibrium.

Corollary 1. *If*

$$\int_0^{+\infty} h_I(t)\Big[u_E(h_E(t)) - u_I(h_I(t))\Big]e^{-rt}\,dt > 0, \tag{5}$$

there exists a unique migration equilibrium in which each user accepts all migration opportunities: $\phi(t) = 1$ for a.e. t.

If

$$\int_0^{+\infty} h_I(t)\Big[u_E(h_E(t)) - u_I(h_I(t))\Big]e^{-rt}\,dt < 0, \tag{6}$$

there does not exist a migration equilibrium.

Corollary 1 implies that increasing the attractiveness of the entrant platform and, at the same time, decreasing the attractiveness of the incumbent makes migration more likely. We state this formally in Corollary 2.

Corollary 2. *Suppose that a migration equilibrium exists for a migration process μ and utility functions u_I and u_E. Then, a migration equilibrium also exists for μ and any utility functions $\widehat{u}_I$ and $\widehat{u}_E$ such that $\widehat{u}_I(h_I) \leq u_I(h_I)$ and $\widehat{u}_E(h_E) \geq u_E(h_E)$ for all h_I and h_E.*

In Appendix A, we consider the case where (5) and (6) are equalities, as well as the case where the utility functions and the migration technology depend not only on the number of users on the platform, but also directly on calendar time t. The characterization of equilibria is provided by Proposition A.4 and Corollary A.1: we assume that the entrant would become more attractive over time even in the absence of migration (formally, $u_E(h_E, t) - u_I(h_I, t)$ is increasing in t for all t, h_I and h_E); then, in all migration equilibria there is *threshold* time t_0 such that users reject all migration opportunities for $t < t_0$ and accept all for $t \geq t_0$. In the simpler stationary environments described by Corollary 1, we have $t_0 = 0$. If $u_E(h_E, 0)$ is sufficiently smaller than $u_I(h_I, 0)$), then migration can only begin at $t_0 > 0$.

4 Structural Determinants of Incumbency Advantage

As mentioned in the introduction, we divide our discussion of the determinants of incumbency advantage in two parts. In Section 5, we will examine how the migration process μ influences incumbency advantage. In the current section, we examine how the structural elements of the economic environment influence incumbency advantage. We will discuss the presence of multiple migration opportunities, the ability of users to multi-home, and the role of entrant capacity.

The Importance of Multiple Migration Opportunities

In Farrell and Saloner (1985), users receive a single migration opportunity: migrate now or never. In this section, we show that giving them the opportunity to wait for a subsequent offer *increases* incumbency advantage.

To compare Farrell and Saloner (1985) to the model of Section 3, we consider a version of their model where there exists a continuum of agents. Users have a single opportunity to migrate, which arrives at rate $\mu(h_I(t))$; if they reject it, they must remain on the incumbent platform forever after.

The incentive to migrate is lowest at $t = 0$. At this instant, the discounted utility of accepting migration is $\int_0^{+\infty} u_E(h_E(t))e^{-rt}dt$ and the discounted utility of rejecting it is $\int_0^{+\infty} u_I(h_I(t))e^{-rt}dt$. Therefore, with a single migration opportunity, if

$$\int_0^{+\infty} [u_E(h_E(t) - u_I(h_I(t))]e^{-rt}\, dt > 0. \tag{7}$$

a migration equilibrium exists, and none exist if the left-hand side is strictly smaller to zero. As in the model we presented in Section 3, in a migration equilibrium users accept all migration opportunities. Therefore, the equilibrium migration path in this model is still described by (4) which implies the following proposition.

Proposition 1. [15] *The multiplicity of migration opportunities makes migration less likely:*

- *whenever there exists a migration equilibrium with multiple migration opportunities, there exists one with a single migration opportunity: if Condition (5) holds, so does Condition (7);*

[15] We thank an anonymous referee for insisting that we make this Proposition more informative.

- *whenever there exists no migration equilibrium with a single migration opportunity, there exists no migration equilibrium with multiple migration opportunities: if the left-hand side of Condition (7) is strictly smaller than 0, so is the left-hand side of (5) .*

Proof. We will only prove the first statement; the second follows in the same fashion.

Assume that (5) holds and $u_E(0) < u_I(1)$ (otherwise the result is trivial). There exists $\bar{t}$ such that $u_E(h_E(\bar{t})) = u_I(h_I(\bar{t}))$ with $h_I(\bar{t}) > 0$. Because the function h_I is decreasing, $u_E(h_E(t)) - u_I(h_I(t)) \geq 0$ if and only if $h_I(t) \leq h_I(\bar{t})$ and therefore

$$\int_0^{+\infty} h_I(t)\Big[u_E(h_E(t)) - u_I(h_I(t))\Big]e^{-rt}\,dt \\ \leq h_I(\bar{t})\int_0^{+\infty}[u_E(h_E(t)) - u_I(h_I(t))]e^{-rt}\,dt,$$

and therefore (5) implies (7). □

When users have multiple opportunities to migrate, they have incentives to reject early migration opportunities to avoid being on the entrant platform while it has few adopters. A "take it or leave it" offer favors migration by increasing the cost of refusing a migration opportunity.

To obtain further intuition we define, for any function $g : \Re_+ \to \Re_+$, the expectation under the exponential density re^{-rt}, as

$$\mathbb{E}[g] \stackrel{\text{def}}{=} \int_0^{+\infty} g(t)re^{-rt}\,dt.$$

Similarly, the covariance of any two functions g_1, g_2 is defined as

$$\mathbb{C}\text{ov}[g_1, g_2] = \mathbb{E}\Big[(g_1 - \mathbb{E}[g_1]) \times (g_2 - \mathbb{E}[g_2])\Big].$$

Condition (7) can be rewritten $\mathbb{E}[u_E(h_E(t)) - u_I(h_I(t))] > 0$, and Condition (5) is equivalent to

$$\mathbb{E}\Big[u_E(h_E(t)) - u_I(h_I(t))\Big] \\ \geq -\frac{\mathbb{C}\text{ov}\big[h_I(t), u_E(h_E(t)) - u_I(h_I(t))\big]}{\mathbb{E}[h_I(t)]} > 0, \quad (8)$$

The second inequality is a consequence of the fact that h_I is a decreasing function of t and that $u_E(h_E(t)) - u_I(h_I(t))$ is increasing. Equation (8) has the same left-hand-side as (7). Therefore, the middle term of (8) provides a measure of how much better the entrant has to be for migration to occur when there multiple migration opportunities rather than a single one. Moreover, the covariance term shows that improving the utility on the entrant platform early in the process (while h_I is large), while keeping $\mathbb{E}[h(t)]$ constant makes migration more likely.

Two speeds

It seems plausible that a user of the incumbent platform who has refused to migrate will think more often of the possibility of migrating than a user who has not yet been made aware of the existence of the entrant. We show that this *increases* incumbency advantage, modulo the added assumption described in footnote 17.

We modify the model of Section 3 and assume that after a user has had a first opportunity to migrate, she can migrate at any time.[16] We also assume away "delayed migration" equilibria where the users who receive early migration opportunities coordinate on all moving at some later date $t^* > 0$.[17]

Then, a user who is offered her first opportunity to migrate at some time t such that $u_E(h_E(t)) < u_I(h_I(t))$ will not migrate immediately, but "wait" until the instantaneous utilities are equal on the two platforms. This implies that a user who has the opportunity to migrate at $t = 0$ will do so only if $u_E(0) > u_I(1)$ and proves the following proposition. That is, migration only takes place when it is a dominant strategy.

Proposition 2. *If consumers can migrate at any time after their first migration opportunity, no migration equilibrium exists if* $u_E(0) < u_I(1)$.

In Appendix B.1, we consider in a more parameterized model the intermediate region between the two polar cases: users get opportunities to migrate

[16] Equivalently, individuals who have been made aware of the entrant compute the best time to migrate and "set an alarm" to remind themselves to do so at that time.

[17] If $u_E(1) > u_I(1)$ and if all users eventually learn about the existence of the entrant, the users who have learned about the existence of the entrant by some large enough t^* would be better off, collectively and individually, if they migrated simultaneously. We can eliminate these types of equilibria, for instance, by assuming that the entrant platform cannot survive if it has no clients for any interval of time, so that migration must begin at $t = 0$ or not at all.

more frequently, but not continuously, after learning of the existence of the entrant for the first time. The greater the rate at which these opportunities arrive, the *larger* is incumbency advantage (*i.e.,* the smaller the set of utility functions for which a migration equilibrium exists).

The results in this section may help explain why Google's position as the dominant search engine seems entrenched. As many authors (e.g. Athey (2010)) have argued, the possession of data creates a kind of network externalities in search: the more users a search engine has, the more data it can use to optimize its search algorithms, the more relevant the search results and hence the greater the utility of its users. Even if an entrant could convince users that they would collectively be better off coordinating on its platform (so that it can obtain data and provide a better experience), users would individually be better off waiting for enough migration to take place. It is plausible that users feel that once they are aware of the presence of such a competitor they can wait and switch just at the right moment. The model of this section shows that this makes entry *more* difficult. (Of course, the economics of search are much more complicated than our brief description, which is only intended to illustrate our theory.)

Multi-homing

The ability of each user to simultaneously participate in multiple platforms (multi-homing) has been of significant interest to the literature on multi-sided platforms.[18] That literature has stressed that multi-homing increases ex-post competition in the context of competition *in* the market (once users have chosen a platform), but decreases competition *for* for the market (at the point where buyers and sellers choose which platforms to join).

Our one-sided framework emphasizes another benefit of multi-homing which arises in the context of competition *for* the market: it makes migration to a more efficient platform more likely, by decreasing the users' incentives to wait. We conjecture that a similar analysis would hold for multi-sided environments, with important policy implications.

In a report written for the European Commission, Crémer, de Montjoye and Schweitzer (2019) argued that dominant firms should be asked to justify the use of policies that deter multi-homing. This proposal was made on the

[18] For a recent treatment see Belleflamme and Peitz (2019), but discussions of multi-homing appeared in the foundational articles on multi-sided platforms (see Caillaud and Jullien (2003), Rochet and Tirole (2003), and Armstrong (2006)).

basis of an intuition similar to that of this section: multi-homing decreases incumbency advantage, and a dominant firm should be allowed to discourage it only when this has clear pro-competitive consequences (as it sometimes does, for reasons not analysed in this article).

To study this issue, we consider an environment where a user who has received a migration opportunity has three options: a) continue to single-home on the incumbent; b) multi-home on both platforms; or c) single-home on the entrant. A multi-homing user can, at any time but irreversibly, migrate "fully" to the entrant (*i.e.*, abandon the incumbent platform and single-home on the entrant platform).[19]

Let $\overline{h}_{IE}(t)$ be the mass of users who multi-home at time t and $\overline{h}_I(t)$ and $\overline{h}_E(t)$ the mass of individuals who single home on the incumbent and entrant. The $\overline{h}_I(t)$ users who single-home on the incumbent are connected to $h_I(t) = \overline{h}_I(t) + \overline{h}_{IE}(t)$ other users (multi-homers and single-homers on the incumbent). These individuals obtain utility $u_I(h_I(t))$. Similarly, the utility of the $\overline{h}_E(t)$ users who single-home on the entrant platform is $u_E(h_E(t))$, with $h_E(t) = \overline{h}_E(t) + \overline{h}_{IE}(t)$. The utility of the multi-homing users is $\overline{u}_{IE}(\overline{h}_I(t), \overline{h}_E(t), \overline{h}_{IE}(t))$.

Because $\overline{h}_I(t) + \overline{h}_E(t) + \overline{h}_{IE}(t) = 1$, we have $\overline{h}_{IE}(t) = h_I(t) + h_E(t) - 1$, and we simplify notation by writing

$$u_{IE}(h_I(t), h_E(t)) \stackrel{\text{def}}{=} \overline{u}_{IE}(1 - h_E(t), 1 - h_I(t), h_I(t) + h_E(t) - 1).$$

We assume that there is a cost of multi-homing (because individuals split their attention or because they must manage multiple accounts). Therefore, when all other users are on the incumbent, multi-homing yields a lower utility than single-homing on the incumbent (but, of course, a higher utility than single-homing alone on the entrant). Formally:

$$u_E(0) < u_{IE}(1,0) < u_I(1), \qquad u_I(0) < u_{IE}(0,1) < u_E(1).$$

At a time t, a user who has previously accepted a migration opportunity chooses to multi-home if $u_{IE}(h_I(t), h_E(t)) > u_E(h(t))$ and single-homes on the entrant otherwise.[20] Therefore, the decision to accept a migration opportunity is similar to what would happen if multi-homing was not available

[19] As in the single-homing setup, there are no equilibria where *some* consumers move to the entrant and then go back to the incumbent, whether they single or multi-home.

[20] For simplicity, we assume that once a user has migrated, she decides at every point of time whether to multi-home or single-home on the entrant. It could interesting to study what happens if there is a different migration process μ_m which applies to multi-homing users.

and if the utility of joining the entrant were $\max\{u_E(h_E), u_{IE}(1-h_E, h_E)\}$. Corollary 2 then implies that the availability of multi-homing makes migration more likely. Using Corollary 1, we can make this statement more precise.

Proposition 3. *If there does not exist a migration equilibrium with multi-homing, then there also does not exist one without multi-homing. Moreover, if*

$$\int_0^{+\infty} h_E(t) \max\big\{u_E(h_E(t)), u_{IE}(h_I(t), h_E(t))\big\}e^{-rt}dt$$
$$> \int_0^{+\infty} h_I(t)u_I(h_I(t))e^{-rt}\,dt > \int_0^{+\infty} h_E(t)u_E(h_E(t))e^{-rt}\,dt,$$

then, there exists a migration equilibrium with multi-homing and no migration equilibrium without multi-homing.

By adding additional structure to the model, we can say more on the way in which migration takes place in the presence of multi-homing. Assume that the benefits of multi-homing are monotone in h_I and h_E: the difference $u_{IE}(h_I, h_E) - u_E(h_E)$ is strictly decreasing in h_E. This implies the existence of a $\overline{h}_I \in (0, 1)$ such that $u_{IE}(\overline{h}_I, \overline{h}_E) = u_E(\overline{h}_I)$, with $\overline{h}_E = 1 - \overline{h}_I$. Along a migration path, consumers prefer multi-homing to single-homing on the entrant when

$$u_{IE}(h_I(t), h_E(t)) \geq u_E(h_E(t)) \iff h_I(t) \geq \overline{h}.$$

Therefore, there exists a $\bar{t}$ such that multi-homing is preferred to single-homing on the incumbent if and only if $t \leq \bar{t}$. Intuitively, multi-homing is preferred early on, while there is still a significant mass of users only reachable through the incumbent platform. Once a sufficient mass of users multi-homes, the advantage of being connected to the incumbent platform becomes lower than the cost of multi-homing. At that point ($t = \bar{t}$), all multi-homing users simultaneously choose to single-home on the entrant. For $t > \bar{t}$, all users migrate directly from single-homing on the incumbent to single-homing on the entrant platform.

Corollary 3. *If $u_{IE}(1 - h_E, h_E) - u_E(h_E)$ is strictly decreasing in h_E, in any migration equilibrium there exists a $\bar{t}$ such that users who have accepted a migration opportunity choose to multi-home for $t < \bar{t}$, then single-home on the entrant for $t > \bar{t}$.*

Capacity constraints

So far we have assumed that the entrant has the capacity to service all users. We now assume that the entrant has maximum capacity of $1-\kappa<1$. Naively, one might think that reducing the entrant's capacity reduces its network value and therefore makes migration less likely. On the contrary, we show that the fear of being left behind on the incumbent increases the users' incentives to migrate, despite the smaller size of the entrant's network. Thus, it could be in the entrant's best interest to commit to reduce its capacity, as this increases the set of parameters for which the migration equilibrium exists.

Assume that the entrant can only serve $1-\kappa$ agents, with $\kappa\in(0,1)$. Formally, if $h_E(t')=1-\kappa$, then $t>t'\Rightarrow\mu=0$. The utility of a user who does not migrate at $t=0$ is

$$\int_0^T \big[h_I(t)u_I(h_I(t))+h_E(t)u_E(h_E(t))\big]e^{-rt}\,dt \\ +\int_T^\infty [\kappa u_I(\kappa)+(1-\kappa)u_E(1-\kappa)]e^{-rt}dt.$$

If the user accepts to migrate at $t=0$, her utility is

$$\int_0^T u_E(h_E(t))e^{-rt}\,dt+\int_T^\infty u_E(1-\kappa)e^{-rt}dt.$$

Therefore, the condition for the existence of a migration equilibrium is changed from (5) to

$$\int_0^T h_I(t)\Big[u_E(h_E(t))-u_I(h_I(t)\Big]e^{-rt}\,dt+\int_T^\infty \kappa(u_E(1-\kappa)-u_I(\kappa))e^{-rt}dt\geq 0. \tag{9}$$

We assume that the derivative of $h_I[u_E(1-h_I)-u_I(h_I)]$ evaluated at $h_I=0$ is strictly positive. Hence, for κ small enough (and therefore T large enough) and all $t\geq T$,

$$\kappa(u_E(1-\kappa)-u_I(\kappa))>h_I(t)u_E(1-h_I(t))-u_I(h_I(t))$$

and (9) implies (5). This yields the following proposition.

Proposition 4. *There exists a small capacity constraint $\kappa>0$ such that the set of utility functions (u_I,u_E) for which a migration equilibrium exists* *with the capacity constraint $\kappa > 0$ strictly contains the set of utility functions for which a migration equilibrium exists when capacity is unconstrained (i.e., when $\kappa = 0$).*

5 Migration Processes and Incumbency Advantage

In section 4, we have analyzed the influence of the environment on migration between platforms. We now turn to the influence of the migration process itself. Two dimensions need to be considered: its speed (does migration happen quickly?) and its shape (is the rate of migration constant, accelerating or decelerating over time?). We first show, perhaps surprisingly, that the speed is, to a first approximation, irrelevant for the existence of a migration equilibrium. The rest of the section therefore focuses on the shape of the migration process. We begin by introducing linear utility functions, which we use in much of the subsequent analysis. We then develop a measure of coordination of migration and show that coordination decreases incumbency advantage. Finally, we turn to a parameterized example and use it to discuss the effect on migration on the way in which users learn about the entrant.

The speed of migration is irrelevant

One might expect that accelerating the migration process reduces incumbency advantage by decreasing the time that the first migrants spend with few other users on the entrant platform. In this subsection, we show that this intuition is *not* correct, at least when r is small: then, an acceleration or a slowing down of the migration process does not affect the existence of migration equilibria.

It is natural to define an *acceleration* of the migration path h_I as a migration path $\widetilde{h}_I$ such that $\widetilde{h}_I(t) = h_I(\alpha t)$ with $\alpha > 1$. Equivalently,[21] $\widetilde{\mu}(h_I) = \alpha \times \mu(h_I)$. As α becomes larger, migration becomes faster. If $\alpha < 1$, there is a deceleration

We say that there exists a *strict migration equilibrium* if (5) holds strictly: at $t = 0$ a user would strictly prefer to migrate than not migrate.

[21]To see this, note that $\widetilde{h}_I(t) = h_I(\alpha t)$ satisfies $\widetilde{h}'_I(t) = -\widetilde{h}_I(t) \times \widetilde{\mu}(h_I(t))$ as $\widetilde{h}'_I(t) = \alpha h_I(\alpha t)$ and $\widetilde{h}_I(t) \times \widetilde{\mu}(h_I(t)) = h_I(\alpha t) \times \alpha\mu(h_I(\alpha t))$.

Definition 3. *There exists a strict migration equilibrium for small r if there exists $\bar{r}$ such that a strict migration equilibrium exists for all $r < \bar{r}$.*

It is then easy to prove that for small r, the set of utility functions for which there exists a strict migration equilibrium is not affected by an acceleration of the migration process, as stated formally in the following proposition.

Proposition 5. *If a strict migration equilibrium exists for small r, then it also exists if the migration process μ is accelerated or decelerated.*

In other words: if there exists $\bar{r}$ such that for all $r < \bar{r}$ there is a migration equilibrium, then for all accelerations or decelerations of the process, there exists a $\bar{r}'$ (usually different from $\bar{r}$) such that a migration equilibrium exists for all $r < \bar{r}'$.

Proof. To show the result for an acceleration, assume that (5) holds strictly for h_I for all $r < \bar{r}$. Let $\alpha > 1$ and $\widetilde{h}_I(t) = h_I(\alpha t)$. Then, by the change of variable $u = t/\alpha$, (5) holds with h_I replaced by $\widetilde{h}_I$ for all $r < \alpha\bar{r}$. The proof is the same for a deceleration. □

The intuition for this result is actually quite simple: an acceleration of the migration process makes migration more attractive as the number of other users on the entrant platform will increase faster. It also makes waiting for a subsequent migration opportunity, which will come sooner, more attractive. At the limit, when r is very small, these two effects exactly compensate each other.

Linear Utilities

Up to this point, our results hold for general utility functions and migration opportunities processes. In much of the rest of the article, we will consider linear utilities of the form

$$\begin{cases} u_I(h_I) = b_I h_I, \\ u_E(h_E) = b_E h_E + k_E. \end{cases}$$

This linear specification allows platforms to differ in the strength of network effects (b_E, b_I) and/or in their "stand-alone" quality k_E (without loss of generality, the stand-alone value of the incumbent is normalised to zero). In this setting, migration is a dominant strategy if $k_E > b_I$, and not migrating is a dominant strategy if $k_E < -b_E$.

Corollary 1 implies that there exists a migration equilibrium if and only if

$$\frac{b_E + k_E}{b_E + b_I} \geq \frac{\int_{t_0}^{+\infty} h_I^2(t) e^{-rt} dt}{\int_{t_0}^{+\infty} h_I(t) e^{-rt} dt}, \tag{10}$$

with $h_I(t)$ defined by (4).

The left-hand-side of (10) depends only on the preferences of the users and is a measure of the quality advantage required of the entrant for a migration equilibrium to exist, given the migration path $h_I(t)$. The right-hand side takes values in to $(0, 1)$ as $h_I^2(t) \leq h_I(t)$ for all t, and depends only on the migration process. Therefore, the right-hand side is a measure of the incumbency advantage associated with the migration path $h_I(t)$.

If the left-hand-side of (10) is greater than 1, then there exists a migration equilibrium for any $h_I(t)$: this occurs when $k_E > b_I$ (migration is a dominant strategy). If the left-hand-side is negative, individuals will not migrate for any $h_I(t)$: this occurs when $k_E + b_E < 0$ (not migrating is a dominant strategy).

Increases in k_E or b_E make migration more likely.[22] A proportional increase in the network effect parameters b_E and b_I always decreases the left hand side of (10) and thus shrinks the set of migration processes for which a migration equilibrium exists. Intuitively, an overall increase in the strength of network effects increases the cost of early migration and therefore makes users less eager to start the migration process.

Synchronicity

In this section, we show how the *distribution over time* of migration opportunities affects incumbency advantage. Perhaps surprisingly, migration technologies that allow many individuals to migrate within a short period of time results in *larger* incumbency advantage.[23]

[22] An increase in b_E decreases the left-hand side of (10) if $b_I < k_E$ but, in this case, the left-hand side is greater than 1 so migration is a dominant strategy.

[23] An exception is if most individuals can migrate immediately. Furthermore, to be clear, we are not considering communication between individuals that allows them to coordinate their migration timing. We are concerned only with properties of the migration technology μ and the corresponding migration path h_I.

To pursue this inquiry formally, recall our definition of $\mathbb{E}[g]$ at the beginning of Section 4. Then, the variance of g is defined by

$$\mathbb{V}[g] = \mathbb{E}[(g - \mathbb{E}[g])^2].$$

Using these definitions, and assuming linear utilities, we re-write Condition (10) as follows

$$\frac{b_E + k_E}{b_E + b_I} \geq \frac{\int_0^{+\infty} h_I^2(t)e^{-rt}dt}{\int_0^{+\infty} h_I(t)e^{-rt}dt} = \mathbb{E}[h_I] + \frac{\mathbb{V}[h_I]}{\mathbb{E}[h_I]}.$$

This yields the following lemma.

Lemma 1. *Assume utilities are linear. Consider two migration processes h_I and $\widetilde{h}_I$ with $\mathbb{E}[h_I] = \mathbb{E}[\widetilde{h}_I]$. Suppose that $\mathbb{V}[\widetilde{h}_I] \geq \mathbb{V}[h_I]$. If there is no migration equilibrium with path h_I, then there is no migration equilibrium with path $\widetilde{h}_I$.*

This re-writing of (10) shows that incumbency advantage depends on two moments of the equilibrium migration path $h_I(t)$. First, $\mathbb{E}[h_I]$ captures how fast and early migration opportunities are available to individuals. Intuitively, if many individuals migrate early, then $h_I(t)$ decreases quickly, so the integral that defines $\mathbb{E}[h_I]$ (Equation (4)) is small.

Second, $\mathbb{V}[h_I]/\mathbb{E}[h_I]$ captures the *synchronicity* of the migration process. Holding fixed $\mathbb{E}[h_I]$, the variance $\mathbb{V}[h_I]$ is large when most individuals are able to migrate within a short period of time. Conversely, $\mathbb{V}[h_I]$ is small when some individuals migrate early in the process, but others are only able to migrate after a significant delay.

To build intuition for $\mathbb{V}[h_I]$ and its consequences for incumbency advantage, consider the three migration paths illustrated in Figure 1. Migration occurs during $t \in [0, 1]$. For different values of $d \in [0, 1/2]$, the paths h_I are piecewise linear in three intervals, defined by the points $(0, 1)$, $(d, \frac{1}{2} + d)$, $(1 - d, \frac{1}{2} - d)$, and $(1, 0)$. For simplicity we consider the limit as $r \to 0$ so $\mathbb{E}[h_I] = 1/2$ for each path, but $\mathbb{V}[h_I]$ is increasing in d.

The migration path for $d = 0.05$ is shown with the solid line. It has low synchronicity (low $\mathbb{V}[h_I] \approx 0.01$): two waves of rapid migration are separated by a long period of stalled migration.[24] To this low synchronicity

[24] Notice that synchronicity is measured over the entire migration process and holding fixed $\mathbb{E}[h_I]$, so this process is described as having low synchronicity even though half of individuals migrate within a short period of time.

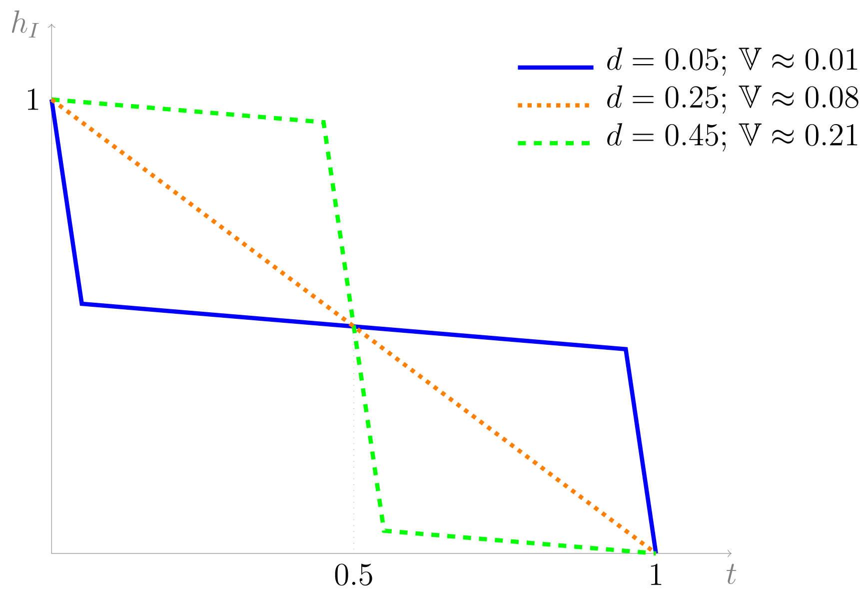

Figure 1: Three functions with the same value of $\mathbb{E}[h_I] = 1/2$ but different values of $\mathbb{V}[h_I]$.

corresponds a small incumbency advantage. Individuals will accept early migration opportunities for two reasons. First, plentiful early opportunities mean that the network value of the entrant will increase rapidly. Second, as further migration opportunities may only occur late in the migration process, rejecting an early opportunity means a high probability of finding oneself in the incumbent for a long time.

Second, the migration path for $d = 0.25$ is shown with a dotted line. It exhibits a constant rate of migration and $\mathbb{V}[h_I] = 1/12 \approx 0.08$. This migration path is more synchronous than the first, and yields a higher incumbency advantage as migration occurs uniformly throughout the migration period.

Third, the migration path for $d = 0.45$ is shown with a dashed line. Almost all individuals migrate at t close to 0.5, and the value of $\mathbb{V}[h_I] \approx 0.21$ is large. The path exhibits large synchronicity to which corresponds a large incumbency advantage. Users reject early migration opportunities because there is a high probability of being in the large mass of individuals who will migrate nearly simultaneously without being in relative isolation on the entrant for a long time period.

In Figure 1, paths with a large value of d (large incumbency advantage) are "above" paths with a low value of d for $t < 1/2$, and are "below" those paths for $t > 1/2$. The next result generalizes this intuition for more general paths.

Proposition 6. *Assume utilities are linear and consider two migration processes h_I and $\widetilde{h}_I$ with $\mathbb{E}[h_I] = \mathbb{E}[\widetilde{h}_I]$ and such that, for some $\bar{t}$, $\widetilde{h}_I(t) \geq h_I(t)$ if and only if $t \leq \bar{t}$. If there is no migration under the process h_I, then there is no migration under $\widetilde{h}_I$.*

Proof. We show $\mathbb{V}[\widetilde{h}_I] > \mathbb{V}[h_I]$. Let $\gamma(t) = \widetilde{h}_I(t) - h_I(t)$, which implies $\mathbb{E}[\gamma] = 0$. Recall h_I is decreasing. On $t \in [0, \bar{t}]$, $\gamma \geq 0$ and $h_I \geq h(\bar{t})$. On $t \in [\bar{t}, \infty)$, $\gamma \leq 0$ and $h_I \leq h(\bar{t})$. Therefore $\mathbb{E}[\gamma h_I] \geq h(\bar{t})\mathbb{E}[\gamma] = 0$. Then,

$$\begin{aligned}
\mathbb{V}[\widetilde{h}_I] &= \mathbb{E}[h_I^2] + \mathbb{E}[\gamma^2] + 2\mathbb{E}[\gamma h_I] - \mathbb{E}[\widetilde{h}_I]^2 \\
&= \left(\mathbb{E}[h_I^2] - \mathbb{E}[\widetilde{h}_I]^2\right) + \mathbb{E}[\gamma^2] + 2\mathbb{E}[\gamma h_I] \\
&\geq \left(\mathbb{E}[h_I^2] - \mathbb{E}[h_I]^2\right) + \mathbb{E}[\gamma^2] > \mathbb{V}[h_I].
\end{aligned}$$

□

Proposition 6 compares two migration paths where $\mathbb{E}[h_I] = \mathbb{E}[\widetilde{h}_I]$. Under $\widetilde{h}_I$, opportunities are concentrated in the middle of the migration process, but under h_I opportunities are more frequent at the start and end of the migration process. Therefore, h_I is less synchronous and results in lower incumbency advantage.

Advertising & Word of Mouth

In order to provide further insights about incumbency advantage, we now specialise the model by assuming that migration stems from the mixture of two easily interpretable processes — see Figure 2 for a graphic illustration. The first process is a constant rate of opportunities to migrate, which we call an *autonomous* process. A leading example would be advertising that occurs at a constant rate.[25] Formally, during any "small" interval of time of length dt, every user on the incumbent platform has a probability $s \times$

[25] Other forms of "one to many" communication would yield similar migration patterns. The frequency with which users see advertisements or other information is constant over time, and each advertisement reminds the user of the possibility to migrate to the entrant platform and thus generates a migration opportunity.

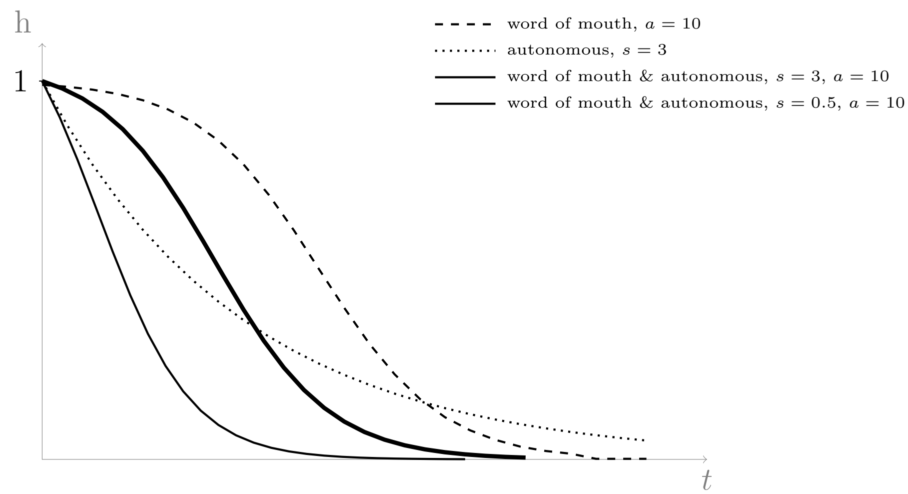

Figure 2: Migration paths as a function of the autonomous parameter s and the word of mouth parameter a.

dt of obtaining a migration opportunity. The migration process $\mu = s$ is independent of h_I, h_E, and t.[26]

The second process consists of *word of mouth*: users learn about the new platform via pairwise meetings with other users. Formally, in an interval of time of length dt, a user meets another user with probability $a \times dt$. Assuming random matching, each user on the incumbent platform has a probability $a \times h_E(t) \times dt$ of meeting a user of the entrant platform. In this case, the user who has migrated informs the one who has not migrated, and a migration opportunity is generated.

We combine these two processes into the overall migration process

$$\mu(h(t)) = s + ah_E(t) = a(\sigma - h_I(t)).$$

where $\sigma = (s + a)/a \in (1, +\infty)$ captures the relative importance of s, the autonomous component of the migration process.[27] With σ close to 1, word of mouth is dominant.[28] With σ very large, the autonomous process dominates.

[26] This is the process assumed, for instance, by Farrell and Saloner (1986).

[27] This equation defines the Bass diffusion process (Bass, 1969, first equation on p. 217), although our interpretation is slightly different.

[28] We must have $\sigma > 1$ for migration to occur. If $\sigma = 1$, the migration process $\mu = ah_E$

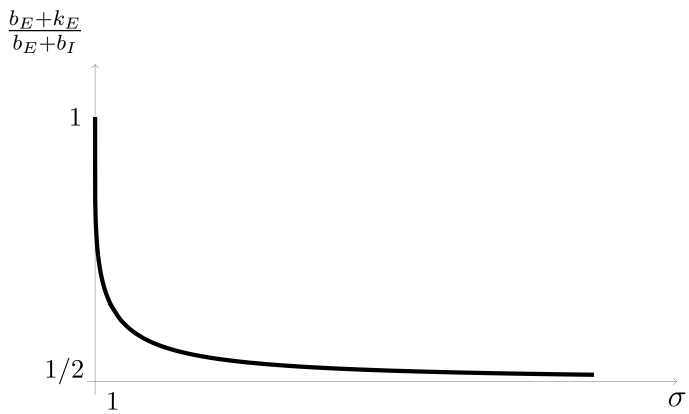

Figure 3: The cutoff $(b_E + k_E)/(b_E + b_I)$ as a function of σ, as described in (12). Notice that the function converges to 1 as $\sigma \to 1$.

Then, (4) implies that the migration path has the closed form solution[29]

$$h_I(t) = \frac{\sigma}{1 + (\sigma - 1)e^{\sigma a t}}. \tag{11}$$

In Appendix B.2 we show that in the limit as $r \to 0$, Equation (10) becomes

$$\frac{b_E + k_E}{b_E + b_I} \geq \frac{\int_0^{+\infty} h_I^2(t)dt}{\int_0^{+\infty} h_I(t)dt} = \sigma - \frac{1}{\ln \sigma - \ln(\sigma - 1)}. \tag{12}$$

The right-hand side, plotted in Figure 3, is decreasing in σ — this is proved in Appendix B.2. This proves the following proposition.

Proposition 7. *In the limit as $r \to 0$, incumbency advantage increases when the word of mouth component of the migration process becomes more prominent compared to the autonomous component: the set of parameters for which a migration equilibrium exists shrinks as σ decreases.*

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is purely "word of mouth." In this case, the initial condition $h(0) = 1$ implies $h'(t) = 0$ for all t.

[29](11) implies $h_I(0) = 1$ and

$$h_I'(t) = -\frac{\sigma \times (\sigma - 1) \times \sigma a e^{\sigma a t}}{(1 + (\sigma - 1)e^{\sigma a t})^2} = -\frac{(\sigma - 1) \times \sigma a e^{\sigma a t}}{1 + (\sigma - 1)e^{\sigma a t}} h_I(t) = -a(\sigma - h_I(t))h_I(t).$$

At the limit as $\sigma \to 1$, the word of mouth component dominates and a migration equilibrium exists if and only if $k_E \geq b_I$, *i.e.*, when migration is a dominant strategy. Initial migration is very slow, early migrants enjoy low network externalities for a long period and only migrate if they are better off "alone" on the entrant platform than with all the other users on the incumbent. Given that migration is efficient whenever $k_E + b_E > b_I$, there exist regions of the parameter space where migration is socially desirable but no migration equilibrium exists: as discussed in Appendix A.6, there is excessive inertia.

At the other extreme, as $\sigma \to \infty$, the migration process becomes purely autonomous and a migration equilibrium exists if and only if $k_E \geq (b_I - b_E)/2$. Migration is socially desirable if $k_E \geq b_I - b_E$, so there can be insufficient or excessive migration.

An important benchmark is the case where the migration process is purely autonomous and network externalities are equally strong on both platforms ($b_E = b_I$). In this case, migration takes place if and only if it is efficient ($k_E \geq 0$).

The results in this section may shed light on the acquisition of GitHub by Microsoft, which we discussed in the introduction. Programmers presumably learn about alternative platforms to GitHub from online news sources or bulletin boards, *i.e.*, from "one to many" modes of communication. That is, the bulletin board is very much like an advertisement that many users see simultaneously. Under this interpretation the migration process is closer to autonomous than to word of mouth — σ is close to 1. Then, there would be migration when Microsoft degraded GitHub's quality and a better platform appeared, as the European Commission agreed.

6 Heterogeneous users

So far, we have assumed that all users share the same preferences. We now present a parameterized example that enables us to discuss some of the ways in which heterogeneity of preferences influences incumbency advantage. Our main results are presented in Proposition 9: inefficient equilibria can obtain because of excessive segregation of users across different platforms.

We assume linear utility. All the users obtain $u_I = bh_I$ on the incumbent platform. Utility on the entrant platform is

$$\begin{cases} bh_E + k_H & \text{with } k_H > 0 \text{ for a mass } p_H \text{ of } \textit{eager} \text{ users,} \\ bh_E + k_L & \text{with } k_L < 0 \text{ for a mass } p_L = 1 - p_H \text{ of } \textit{reluctant} \text{ users.} \end{cases}$$

We call $\overline{k} = p_H k_H + p_L k_L$ the (average) quality advantage of the entrant platform. Migration opportunities arise solely based on the autonomous process, so $\mu(h_I) = s > 0$, for all t. There is no discounting (we consider the limit as $r \to 0$).

Migration equilibria with heterogeneous users

We focus on the "maximal-migration equilibria", that is, the equilibria in which the greatest number of users migrate and do so as early as possible. In these equilibria, eager users (if they migrate) accept migration opportunities for all $t \geq 0$, and reluctant users (if they migrate) accept all migration opportunities for all $t \geq T_L$ for some some $T_L \geq 0$. The equilibrium migration path is

$$h_I(t) = \begin{cases} p_H e^{-st} + (1 - p_H) & t \in [0, T_L], \\ p_H e^{-st} + (1 - p_H) e^{-s(t - T_L)} & t \geq T_L. \end{cases} \tag{13}$$

We obtain the following proposition, which is illustrated by Figure 4.

Proposition 8. *The maximal-migration equilibria satisfy:*

- *If and only if $\overline{k} > 0$ and $bp_H > -k_L$, the maximal migration equilibrium is a "staggered" equilibrium where eager users accept all migration opportunities and reluctant users accept all migration opportunities for $t \geq T_L$, with T_L defined by*

$$p_H(1 - e^{-sT_L}) = -k_L/b. \tag{14}$$

- *If and only if $k_H > (1 - p_H)b$ and $bp_H \leq -k_L$, the maximal migration equilibrium is a "segregated" equilibrium where eager users accept all migration opportunities and reluctant users never migrate.*

- *In all other cases, there exists no migration in any equilibrium.*

Proof. See Appendix B.3. □

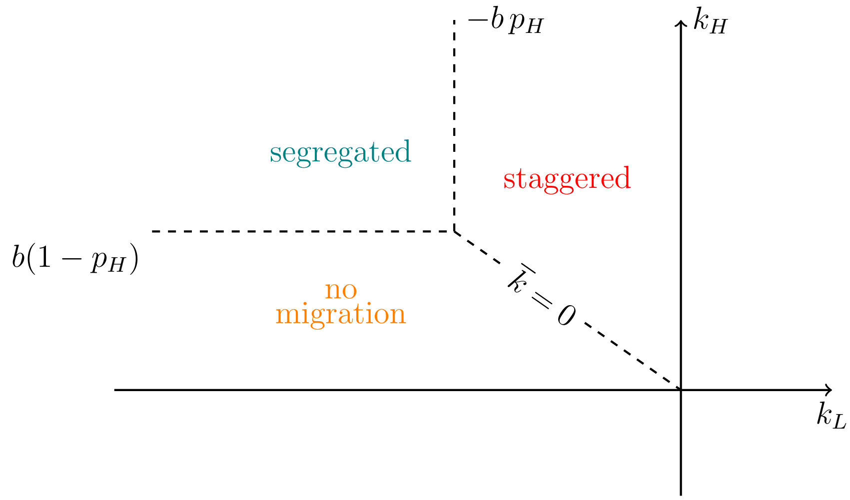

Figure 4: Types of equilibria with heterogeneous users.

We make three remarks about Proposition 8. First, for reasons similar to those discussed after Corollary A.4, in any maximal-migration equilibrium, eager types accept all migration opportunities. In staggered migration equilibria, reluctant types start accepting migration opportunities at time T_L, which is the instant at which they derive the same utility by migrating immediately or by waiting for the next opportunity.

Second, notice that the migration path is now directly affected by the preferences of users. As $k_L \to 0$, both types migrate at $t = 0$. For $k_L < 0$, reluctant users postpone migration so $T_L > 0$.

Third, as one would expect from the analysis of the autonomous migration process with homogeneous users in Section 5, in a segregated equilibrium, eager users migrate if and only if it is efficient for them to do so knowing that reluctant users will not migrate. That is, if the quality benefit of the entrant platform is greater than the loss of the externality benefits stemming from the absence of the reluctant users.

The results of this section shed light on the successful entry of Facebook against what was arguably the premier social media platform of the time, Myspace. Facebook effectively appealed to a significantly different audience by initially targeting Ivy League undergraduates. Eventually, this resulted in a segregated equilibrium where, currently, Myspace still exists but has taken

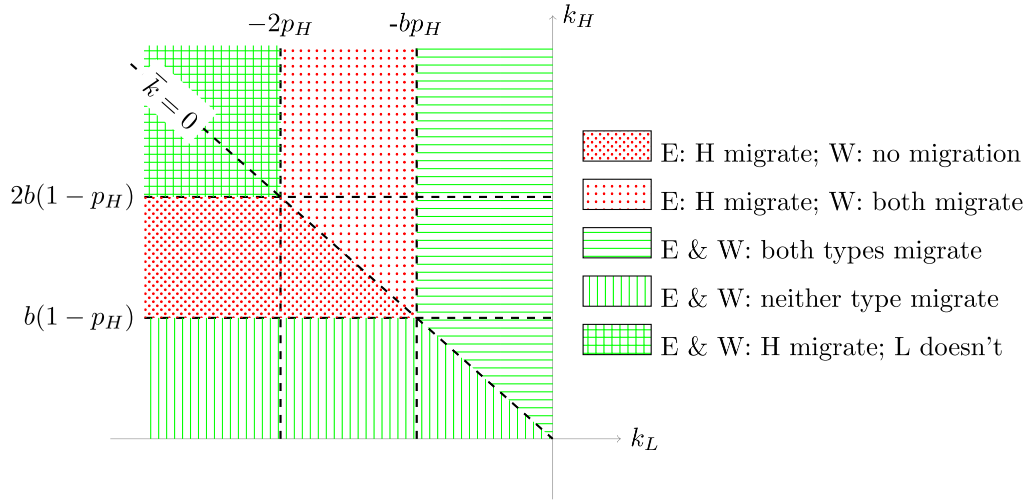

Figure 5: **Equilibria and welfare with 2 types of users.** The legend should be read as follows. E and W indicate respectively the Equilibrium and Social welfare maximising configurations. For instance, the first line shows that, for the relevant configuration of parameters, the eager consumers migrate even though it would be socially optimal not to have any migration at all. The bottom line indicates that, in equilibrium, eager consumers migrate whereas reluctant consumers do not, and this is socially optimal.

a niche position as a platform focused primarily on music and art. A similar reasoning may also help contribute to explain the unsuccessful entry attempt of Google+ against Facebook: the incumbency advantage of Facebook was enhanced by the low heterogeneity of the of the users.

Welfare with Heterogenous Users

We now discuss the relationship between equilibrium and efficiency in the model with preference heterogeneity. Because $r = 0$, the welfare lost during migration can be ignored. Instead, we focus on long run welfare, that is

$$\begin{cases} b & \text{without migration,} \\ b + p_H k_H + (1-p_H)\,k_L & \text{if all users migrate,} \\ b\,(1-p_H)^2 + p_H\,(bp_H + k_H) & \text{if only eager users migrate.} \end{cases}$$

Figure 5 illustrates the socially optimal behaviour and contrasts it with the equilibrium behaviour described in Proposition 8. The main conclusions are summarized in the following proposition.

Proposition 9. *With heterogenous users, any inefficiency stems from excess segregation rather than coordination on the wrong platform: the eager types migrate and the reluctant types do not when it would be optimal either for both types to migrate or for no user to migrate. These inefficiencies arise for intermediate values of k_H or k_L.*

The three green shaded areas in Figure 5 are the regions of the parameter space where a migration equilibrium exists and is also the welfare maximising outcome, whereas in the red regions the equilibrium is not welfare maximising.

First, if $k_H/|k_L|$ is large, there exists a migration equilibrium where both types migrate. This is socially desirable as the mild aversion of reluctant users is not enough to justify the loss in network externalities that would result from segregation.[30] Second, if k_L is very negative and k_H not too large, so that $\overline{k} < 0$, a migration equilibrium does not exist. In this case, migration is also socially undesirable because preferences are, overall, in favor of the incumbent and the mild preferences of eager types for the entrant are not enough to justify segregation. Third, if preferences are sufficiently polarised (both $|k_L|$ and k_H large), there exists a migration equilibrium where only eager users migrate. This is socially optimal because each type has an extreme preference for a different platform.

In the red regions of Figure 5, the equilibrium outcome is socially undesirable. The inefficiency is always due to excessive segregation: types k_H migrate and types k_L do not, when it would be optimal for all users to be in the same platform to maximise network externalities. If $|k_L|$ is greater than k_H, the socially optimal outcome is for all types to remain on the incumbent platform. If k_H is larger than $|k_L|$, it is optimal for all users to migrate.

[30] Migration is staggered, but because we are considering the limit as $r \to 0$, this delay does not affect long run welfare.

7 Conclusions and paths for future research

As we have discussed in Section 2, the literature contains very few discussions of the migration process between platforms, and our analysis could be expanded in many directions. For instance, we could add a more sophisticated description of the determinants of the migration decisions of the users. In particular, if the agents belonged to a more structured network, the decisions of their "neighbors" would be the most important determinant of their decision to migrate. Kempe, Kleinberg and Tardos (2015) have studied this problem,[31] where the agents are represented as nodes in a graph. However, they assume exogenous rules for migration, such as in their "linear threshold model" where an agent adopts the innovation if a sufficient number of her neighbors do.[32] It would be interesting to study a similar model with a more solid game theoretical basis. However, Kempe et al. show that the problem is computationally difficult even without this complication. Thinking of the proper representation of the bounded rationality of agents for such decisions would be of great interest.

Another line of investigation which should be pursued is the link between the migration process and the strategy of firms, which can influence u_I and u_E through their choice of price and quality. It would be of great interest to understand how the migration process affects these choices. When discussing the Microsoft-GitHub merger in the introduction and in Section 5, we have argued that the fact that the migration process was close to autonomous would reduce the incentives of the incumbent to decrease the quality of the platform. It would be important to have a more formal analysis of this point.

Platforms can also influence the migration process itself. In Section 4, we examined a special case of this when we showed how the entrant can decrease incumbency advantage by restricting the number of users who can migrate. Other strategies are available; for instance different forms of advertising would lead to different migration processes. Similarly, our discussion of multi-homing is preliminary and stark. In reality, platforms are rarely perfect substitutes and agents communicate through multiple means: one might

[31] In fact, they study the diffusion of an innovation in a network, but because agents care about the adoption of other users, the problem is similar to that of a choice of platform.

[32] This is a very simplified description of that model. Actually, each agent exerts a (exogenous) weight on the decision by his neighbors to imitate this adoption of the innovation. An agent adopts the innovation if the sum of these weights for his neighbors who have accepted the innovation is large enough.

send, to the same contact, e-mails, text messages and WhatsApp messages and also know that he or she will see one's Facebook or Twitter posts. In that case, the decision that users face is a decision about the subset of platforms to join. We believe that some of the same considerations that apply in the framework of this article would apply, but more formal applied and theoretical analysis would be useful.

Finally, we have focussed our attention on one-sided models. It has been claimed by many that Craigslist, whose main line of business is as a classified advertisement site, is an example of an inefficient incumbent platform; the design of the website has not changed substantially for years. Despite this lack of innovation, it has been able to maintain its market position even when facing entry from well financed and well known firms such as Facebook. We think that many of our insights would also be applicable in two-sided models: the free rider problem may be even harder to overcome in that context.

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Appendix

A Model and equilibrium

We now present both proofs and a more formal version of the theory of Section 3, under more general hypotheses. In particular, we allow the migration process μ and the utility functions to depend on both h_I and directly on calendar time t. We also distinguish more clearly than in the main text the results that stem from the individual behavior of the users and those that are strictly properties of the equilibrium.

A.1 Single-user setup

We begin by considering the problem of a single user of the incumbent platform I who must decide whether to accept an opportunity to migrate to an entrant platform E.

At time $t \geq 0$, the instantaneous utility of the user is $\widetilde{u}_I(t)$ on the incumbent platform and $\widetilde{u}_E(t)$ on the entrant platform (we endogenize these utilities in section A.4 where we allow them to depend explicitly on the mass of users in the platforms). Because other users are migrating, or due to other factors such as pre-determined changes in the design of the platforms or in prices, we assume that the difference in utilities is continuous and weakly increasing in t:[33]

$$\frac{d}{dt}[\widetilde{u}_E(t) - \widetilde{u}_I(t)] \geq 0.$$

In any infinitesimal interval of time $[t, t+dt]$, a consumer on the incumbent platform has a probability $\widetilde{\mu}(t) \times dt$ to receive an opportunity to migrate from I to E.

A strategy for the consumer is a measurable function $\phi(t) : \Re^+ \to [0, 1]$ which is interpreted as the probability that the agent accepts a migration opportunity that arises at time t. The probability of migration during $[t, t + dt]$ is therefore $\phi(t) \times \widetilde{\mu}(t) \times dt$.

[33] If there is migration, one would expect $\widetilde{u}_E(0) - \widetilde{u}_I(0) < 0$ and $\lim_{t\to+\infty} \widetilde{u}_E(t) - \widetilde{u}_I(t) > 0$, but these conditions are not necessary for the results of this section.

As in the main text, the probability $\pi_I(t)$ that the consumer is on the incumbent at time t satisfies

$$\pi_I(t + dt) = \pi_I(t) \left[1 - \widetilde{\mu}(t)\phi(t)dt\right].$$

This implies

$$\pi_I(t) = \exp\left[-\int_0^t \widetilde{\mu}(\tau)\phi(\tau)d\tau\right].$$

The utility of the user is

$$\int_0^\infty \left[\widetilde{u}_I(t)\pi_I(t) + \widetilde{u}_E(t)\pi_E(t)\right]e^{-rt}dt$$
$$= \int_0^\infty \left[\widetilde{u}_I(t) - \widetilde{u}_E(t)\right]\pi_I(t)e^{-rt}dt + \int_0^\infty \widetilde{u}_E(t)e^{-rt}dt.$$

and therefore her problem is to choose a strategy ϕ which maximizes

$$\int_0^\infty [\widetilde{u}_I(t) - \widetilde{u}_E(t)]\pi_I(t)e^{-rt}dt \tag{A.1}$$

subject to (1).

A.2 A generalized single agent problem

As a first step in the characterization of the solutions of the single agent's problem, we describe the solutions of a more general problem. This more general problem makes the structure of the proof clearer, and we hope that the more general result could be useful in future research. A more general proof also helps avoid wrong interpretations of some of our results, as we discuss below.

Setting up the generalized problem

Let $v : \Re^+ \to \Re$ be a continuous, differentiable, and weakly decreasing function with $v(0) > 0$ and $\lim_{t\to+\infty} v(t)$ finite. Let $g : \Re^+ \to (0, 1]$ be continuous and strictly decreasing with $g(0) = 1$.[34] Consider also $\mu : \Re^+ \to \Re^{++}$ with $\mu(t) > 0$ for all t. User strategies are $\phi(t) : \Re^+ \to [0, 1]$.

[34] Note that $g(t) > 0, \forall t$ implies migration will "last forever"". However, the same results hold when migration ends a finite time.

In the setting above, $v(t) = \widetilde{u}_I(t) - \widetilde{u}_E(t)$ and $g(x) = \exp(-x)$. The functions μ, ϕ have the same interpretation as above.

We define

$$\pi(t;\phi) = g\left(\int_0^t \mu(\tau)\phi(\tau)\,d\tau\right).$$

We then consider the following maximisation problem:

$$\max_{\phi} \int_0^{+\infty} v(t)\pi(t;\phi)e^{-rt}\,dt,$$

whose solution we will call ϕ^*. Notice that this takes the same form as the individuals' problem described in (A.1).

Users choose threshold strategies

Towards a characterisation of individual optimal strategies (Proposition A.3), we first show that the users' optimal strategies are threshold strategies (Proposition A.1), which we begin by defining.

Definition A.1 (Threshold strategy)**.** *A strategy $\phi(t)$ is a threshold strategy if there exists a $t_0 \geq 0$ such that $\phi^*(t) = 0$ on $[0, t_0)$ and $\phi^*(t) = 1$ on $(t_0, +\infty)$. We will say that t_0 is a threshold.*

To prove Proposition A.1, we require several preliminary results (Lemmas A.1 to A.3). Lemma A.1 shows that, if utility is always greater in the incumbent, then no migration opportunities are accepted.

Lemma A.1. *If $v(t) > 0$ for all t, then $\phi^*(t) = 0$ for almost all t.*[35]

Proof. If ϕ is strictly greater than 0 on a measurable interval, then there exists a t' such that $\pi(t;\phi) < 1$ for all t greater than t'. Therefore

$$\int_0^{+\infty} v(t)\pi(t;\phi)e^{-rt}\,dt < \int_0^{+\infty} v(t)e^{-rt}\,dt,$$

which is attainable with $\phi(t) = 0$ for all $t \geq 0$. □

[35] If $\pi(t;\phi^*) = 0$ for t large enough, then the value of $\phi^*(t)$ does not affect the objective function and it could be less than 1. To simplify the exposition, we do not explicitly mention this point.

Lemma A.2 below shows that, once utility in the entrant platform is greater than or equal to the utility on the incumbent platform, all migration opportunities are accepted.

Lemma A.2. *For almost all* $t > \underline{T}^0 \stackrel{def}{=} \min\{t \mid v(t) \leq 0\}$*, we have* $\phi^*(t) = 1$*.*

Proof. Because v is decreasing and continuous, $v(t) \leq 0$ for all $t \geq \underline{T}^0$.

Let $\widetilde{\phi}(t) = \phi^*(t)$ for $t \leq \underline{T}^0$ and to 1 for $t > \underline{T}^0$. Clearly, $\pi(t; \widetilde{\pi}) = \pi(t; \pi^*)$ for $t \leq \underline{T}^0$. For $t > \underline{T}^0$, we have

$$\begin{aligned}
\int_0^t \mu(\tau)\widetilde{\phi}(\tau)\, d\tau &= \int_0^{\underline{T}^0} \mu(\tau)\widetilde{\phi}(\tau)\, d\tau + \int_{\underline{T}^0}^t \mu(\tau)\widetilde{\phi}(\tau)\, d\tau \\
&= \int_0^{\underline{T}^0} \mu(\tau)\phi^*(\tau)\, d\tau + \int_{\underline{T}^0}^t \mu(\tau)\widetilde{\phi}(\tau)\, d\tau \\
&\geq \int_0^{\underline{T}^0} \mu(\tau)\phi^*(\tau)\, d\tau + \int_{\underline{T}^0}^t \mu(\tau)\phi^*(\tau)\, d\tau,
\end{aligned}$$

which implies, because g is decreasing, $\pi(t; \widetilde{\phi}) \leq \pi(t; \pi^*)$ with a strict inequality if $\phi^*(t)$ is not almost always equal to 1 for $\tau \in (\underline{T}^0, t)$.

Therefore

$$\begin{aligned}
\int_0^{+\infty} v(t)\pi(t; \widetilde{\phi})e^{-rt}\, dt &= \int_0^{\underline{T}^0} v(t)\pi(t; \widetilde{\phi})e^{-rt}\, dt + \int_{\underline{T}^0}^{+\infty} v(t)\pi(t; \widetilde{\phi})e^{-rt}\, dt \\
&\geq \int_0^{\underline{T}^0} v(t)\pi(t; \pi^*)e^{-rt}\, dt + \int_{\underline{T}^0}^{+\infty} v(t)\pi(t; \pi^*)e^{-rt}\, dt \\
&= \int_0^{+\infty} v(t)\pi(t; \pi^*)e^{-rt}\, dt
\end{aligned}$$

with a strict inequality if $\phi^*(t)$ is not almost always equal to 1, which proves the result. □

Lemma A.3 below shows there exist a threshold t_0 such that individuals reject all migration opportunities before the threshold, and accept all opportunities after it. To prove Lemma A.3 we assume a putative optimal strategy ϕ for which this is not the case, and then build another threshold strategy $\widetilde{\phi}(t)$ that delivers greater utility.

Lemma A.3. *There exist a* $t_0 \in [0, \underline{T}^0]$ *such that* $\phi^*(t) = 0$ *for almost all* $t \in [0, t_0]$ *and* $\phi^*(t) = 1$ *for almost all* $t \in [t_0, \underline{T}^0]$.

Proof. For $T \leq \underline{T}^0$ let $h(T) \overset{\text{def}}{=} \int_T^{\underline{T}^0} \mu(\tau)\, d\tau$. The function h is continuous and decreasing on $[0, \underline{T}^0]$ and satisfies

$$h(0) = \int_0^{\underline{T}^0} \mu(\tau) d\tau \geq \int_0^{\underline{T}^0} \mu(\tau)\, \pi(\tau; \phi^*) d\tau \geq 0 = h(\underline{T}^0).$$

Therefore there exists t_0 such that $h(t_0) = \int_0^{\underline{T}^0} \mu(\tau)\pi(\tau; \phi^*)\, d\tau$.

Let $\widetilde{\phi}$ be defined by

$$\widetilde{\phi}(t) = \begin{cases} 0 & \text{for } t \leq t_0, \\ 1 & \text{for } t \in (t_0, \underline{T}^0], \\ \phi^*(t) & \text{for } t \geq \underline{T}^0. \end{cases}$$

This implies

$$\int_0^t \mu(\tau)\widetilde{\phi}(\tau)\, d\tau \leq \int_0^t \mu(\tau)\phi^*(\tau)\, d\tau \text{ for } t \in [0, t_0],$$

$$\int_0^t \mu(\tau)\widetilde{\phi}(\tau)\, d\tau = \underbrace{\int_0^{\underline{T}^0} \mu(\tau)\widetilde{\phi}(\tau)\, d\tau}_{=\int_0^{\underline{T}^0} \mu(\tau)\phi^*(\tau)\, d\tau} - \underbrace{\int_t^{\underline{T}^0} \mu(\tau)\widetilde{\phi}(\tau)\, d\tau}_{\geq \int_t^{\underline{T}^0} \mu(\tau)\phi^*(\tau)\, d\tau} \leq \int_0^t \mu(\tau)\phi^*(\tau)\, d\tau$$

$$\text{for } t \in [t_0, \underline{T}^0],$$

and

$$\int_0^t \mu(\tau)\widetilde{\phi}(\tau)\, d\tau = \int_0^{\underline{T}^0} \mu(\tau)\widetilde{\phi}(\tau)\, d\tau + \int_{\underline{T}^0}^t \mu(\tau)\widetilde{\phi}(\tau)\, d\tau = \int_0^t \mu(\tau)\phi^*(\tau)\, d\tau$$

$$\text{for } t \geq \underline{T}^0.$$

Because g is decreasing, this implies

$$\widetilde{\pi}(t) = \pi^*(t) \text{ for } t \geq \underline{T}^0$$

when $v(t)$ is negative, and

$$\widetilde{\pi}(t) \geq \pi^*(t) \text{ for } t \leq \underline{T}^0$$

when $v(t)$ is positive, with a strict inequality if $\phi^*(t) \neq \widetilde{\phi}(t)$ on a subset of $[0, \underline{T}^0]$ of measure greater than 0 and proves the lemma. □

We are therefore able to establish the following result.

Proposition A.1. *Either $\phi^*(t) = 0$ for all t, or $\phi^*(t)$ is a threshold strategy.*

Proof. A direct consequence of Lemmas A.1 to A.3. □

Characterization of single user's optimal strategy

In this section we characterise the optimal threshold strategies. First, Proposition A.2 describes the first and second order conditions for the user's choice of threshold. Then, we formalize the characterization of the individual's optimal strategy in Proposition A.3. Recall that, in the main body of the article, $g(x) = \exp(-x)$ which is concave. However, below we prove a more general result.

Proposition A.2. *If the function $g(\cdot)$ is twice differentiable and concave, then a threshold strategy with threshold t_0 is optimal if and only if*

$$t_0 \times \int_{t_0}^{+\infty} v(t) g' \left(\int_0^t \mu(\tau)\phi(\tau)\, d\tau \right) e^{-rt}\, dt = 0. \tag{A.2}$$

Proof. By Proposition A.1 the optimal t_0 maximizes

$$\int_0^{t_0} v(t) e^{-rt}\, dt + \int_{t_0}^{+\infty} v(t) g \left(\int_{t_0}^t \mu(\tau)\, d\tau \right) e^{-rt}\, dt.$$

After elimination of two terms which cancel out, the derivative is equal to

$$\int_{t_0}^{+\infty} \left[v(t) g' \left(\int_{t_0}^t \mu(\tau)\, d\tau \right) \times \left(-\widetilde{\mu}(t_0) \right) \right] e^{-rt}\, dt$$
$$= -\widetilde{\mu}(t_0) \int_{t_0}^{+\infty} \left[v(t) g' \left(\int_{t_0}^t \mu(\tau) \right] d\tau \right) e^{-rt}\, dt. \tag{A.3}$$

By assumption $\widetilde{\mu}$ is strictly positive, we have therefore proved that condition (A.2) is a necessary condition. To see that it is sufficient, note that

$$\frac{d}{dt_0} \left[\int_{t_0}^{+\infty} v(t)\, g' \left(\int_{t_0}^t \mu(\tau)\, d\tau \right) e^{-rt}\, dt \right]$$
$$= -v(t_0) g'(0) e^{-rt_0} - \widetilde{\mu}(t_0) \int_{t_0}^{+\infty} g'' \left(\int_{t_0}^t \mu(\tau)\, d\tau \right) e^{-rt}\, dt.$$

The first term is positive because $v(t_0) > 0$ on the relevant range and g is decreasing. So is the second term when g is concave. Hence, the derivative of the second term of the right-hand side of (A.3) is negative, which implies that the derivative is negative everywhere if it is for $t_0 = 0$ and cannot be equal to 0 more than once. □

A.3 Characterization of the user's optimal strategy in the model of section A.1.

We now revert to the more specialized migration model of Section A.1, to prepare the characterization of the of the equilibrium in the "migration game" which the users play.

Proposition A.3 below is a direct consequence of Proposition A.1. It states that, once a user has started to accept migration opportunities with positive probability, she will accept all future opportunities with probability one; therefore optimal strategies are threshold strategies. Proposition A.3 also characterises the optimal threshold.

Proposition A.3. *If* $\widetilde{u}_E(0) - \widetilde{u}_I(0) \geq 0$*, the user accepts all migration opportunities:* $\phi^*(t) = 1$ *for almost all* t*.*

If $\lim_{t\to+\infty} \widetilde{u}_E(t) - \widetilde{u}_I(t) \leq 0$*, the user accepts no migration opportunities:* $\phi^*(t) = 0$ *for almost all* t*.*

Otherwise (if $\widetilde{u}_E(0) - \widetilde{u}_I(0) < 0$ *and* $\lim_{t\to+\infty} \widetilde{u}_E(t) - \widetilde{u}_I(t) > 0$*), there exists a unique*[36] $t_0 < \inf\{t : \widetilde{u}_E(t) - \widetilde{u}_I(t) \geq 0\}$ *such that the user does not migrate before* t_0 *and accepts all migration opportunities afterwards:*

$$\phi^*(t) = \begin{cases} 0 & \text{for almost all } t < t_0, \\ 1 & \text{for almost all } t > t_0. \end{cases}$$

Moreover, t_0 *satisfies either*

$$t_0 = 0 \quad \text{and} \quad \int_{t_0}^{+\infty} [\widetilde{u}_E(t) - \widetilde{u}_I(t)]\, \pi_I(t)\, e^{-rt} dt \geq 0, \tag{A.4a}$$

$$\text{or} \quad t_0 > 0 \quad \text{and} \quad \int_{t_0}^{+\infty} [\widetilde{u}_E(t) - \widetilde{u}_I(t)]\, \pi_I(t)\, e^{-rt} dt = 0, \tag{A.4b}$$

[36] In the more general setup of proposition A.1, the inequality that t_0 satisfies is weak. In the context of the model in the main text, the user will want to migrate if she has an opportunity to do so "just before" $\widetilde{u}_E(t) - \widetilde{u}_I(t) = 0$ in order not to be stranded on the incumbent platform when it yields a lower utility. Hence, the strict inequality.

with $\pi_I(t)$ *defined by* (1).

Once $\widetilde{u}_E(t) - \widetilde{u}_I(t) > 0$, the user accepts all migration opportunities ($\phi^*(t) = 1$). She will start accepting migration opportunities sometime before the time at which $\widetilde{u}_E(t)$ is equal to $\widetilde{u}_I(t)$; if she waited until $\widetilde{u}_E(t) - \widetilde{u}_I(t) \geq 0$, then she would find herself on the incumbent platform with probability 1 at a time where the incumbent platform has lower value than the entrant platform. She prefers migrating when the entrant platform still yields slightly less utility than the incumbent platform.[37]

Note that the fact that the strategies are threshold strategies is a property stemming from the utility maximization of the individuals, not a property of the equilibrium *per se*.

A.4 Equilibrium

We now embed the individual optimisation problem into the equilibrium model.

At time t the (instantaneous) utility of the users of the incumbent is $u_I(h_I, t)$ and the utility of users on the entrant is $u_E(h_E, t)$. Notice that, contrary to what we assume in the main text, here we allow the utility of the consumers to depend on calendar time t directly. We assume that utility functions are continuously differentiable in both arguments, strictly increasing in their first arguments, and $u_E(h_E, t) - u_I(h_I, t)$ is weakly increasing in t for all h_I and h_E.

For instance, in the absence of switching costs, the lifetime utility of a user who migrates at time T is

$$\int_0^T u_I(h_I(t), t)e^{-rt}\, dt + \int_T^{+\infty} u_E(h_E(t), t)e^{-rt}\, dt.$$

In any infinitesimal interval of time $[t, t + dt]$, each consumer on the incumbent platform is given an irreversible opportunity to migrate with prob-

[37]It is tempting to interpret the integrals in (A.4a) and (A.4b) as the future discounted utility of the user. For instance, (A.4b) would state that if $t_0 > 0$, then the discounted utility of the user from t_0 on is equal to 0. However, this is an artefact of the exponential function. As the proof in Appendix A.2 makes clear, these integrals represent the *marginal* utility.

ability $\mu(h_I(t),t) \times dt$.[38] Here, the $\widetilde{\mu}(t)$ of Section A.1 is to be interpreted as $\mu(h_I(t),t)$. That is, we allow the migration process to depend directly on t. As there is a continuum of users, a share $\mu(h_I(t),t)$ of the users on the incumbent platform receives a migration opportunity during the infinitesimal interval $[t, t+dt]$.

The definition of the problem of the individuals and of equilibrium are the same as in Section 3. Proposition A.3 shows that there is only one optimal strategy. That is, the solution to the individual's utility maximisation problem is a singleton. Therefore consumers, who are ex-ante identical, all follow the same strategy, a property which we had assumed in the main text. This leads us to the following proposition.

Proposition A.4. *In any migration equilibrium, all consumers use the same threshold strategy. There exists a t_0 such that $h_I(t) = 1$ for $t \leq t_0$ and $h_I'(t) = -\mu\big(h_I(t),t\big) \times h_I(t)$ for all $t > t_0$.*

The following corollary is a direct consequence of Proposition A.4 and plays an important role in the sequel.

Corollary A.1. *There exists a migration equilibrium if and only for some t_0*

$$\int_{t_0}^{+\infty} h_I(t)\Big[u_E\big(h_E(t),t\big) - u_I\big(h_I(t),t\big)\Big]e^{-r(t-t_0)}\,dt \geq 0 \tag{A.5}$$

with

$$h_I(t) = \begin{cases} 1 & \text{if } t < t_0, \\ 1 - \displaystyle\int_{t_0}^{t} \mu(h_I(\tau),\tau)h_I(\tau)\,d\tau & \text{if } t \geq t_0, \end{cases}$$
$$h_E(t) = 1 - h_I(t). \tag{A.6}$$

Moreover, (A.5) must be satisfied as an equality whenever $t_0 > 0$.

Proof. If there exists a migration equilibrium, there exists a date $t = t_0$ after which users accept their first opportunity to migrate. The discounted utility of a user who migrates at $t = t_0$ is

$$\int_{t_0}^{+\infty} u_E(h_E(t),t)e^{-r(t-t_0)}\,dt, \tag{A.7}$$

[38] In full generality, μ would be a function of h_E as well as h_I. However, we assume that users all belong to one and only one platform, so $h_E(t) = 1 - h_I(t)$ for all t. Therefore, the argument h_E can be omitted without loss of generality.

where $h_E(t)$, defined by (A.6). If she chooses to wait for the next opportunity, by (3) she will be on the incumbent platform with probability $h_I(t)$ and therefore on the entrant platform with probability $h_E(t)$.

This yields an expected utility of

$$\int_{t_0}^{+\infty} \Big[h_I(t)u_I(h(t),t) + (1-h_I(t))u_E(h_E(t),t)\Big] e^{-r(t-t_0)}\,dt \geq 0. \tag{A.8}$$

Condition (A.5) states that (A.7) is greater than (A.8), and therefore that at time t_0 users prefer migrating than waiting for the next opportunity.

If (A.5) were a strict inequality with $t_0 > 0$, then a user who receives an opportunity to migrate just before t_0 would have strict incentives to accept it. □

Corollary A.1 implies that the parallel of Corollary 2 also holds without stationarity. We state this formally in the following corollary.

Corollary A.2. *Suppose a migration equilibrium exists for a migration process μ and utility functions u_I and u_E. Then, a migration equilibrium also exists for μ and any utility functions $\widehat{u}_I$ and $\widehat{u}_E$ such that $\widehat{u}_I(h_I,t) \leq u_I(h_I,t)$ and $\widehat{u}_E(h_E,t) \geq u_E(h_E,t)$ for all t, h_I and h_E.*

A.5 Stationarity

In the main text we have assumed that the environment is stationary (i.e., that the utilities u_I and u_E and the migration process μ do not depend directly on t). Corollary 1 is a direct consequence of the following Corollary, which itself is a direct consequence of the analysis of Section A.4.

Corollary A.3. *In stationary environments, there exists a migration equilibrium if and only if*

$$\int_0^{+\infty} h_I(t)\Big[u_E\big(h_E(t)\big) - u_I\big(h_I(t)\big)\Big] e^{-rt}\,dt \geq 0,$$

with

$$h_I(t) = 1 - \int_0^t \mu\big(h_I(\tau)\big) \times h_I(\tau)\,d\tau. \tag{A.9}$$

If inequality (5) holds strictly, there exists a unique migration equilibrium in which migration starts at $t = 0$: $h_I(t) < 1$ for all $t > 0$.

If (5) is an equality, the strategy ϕ is an equilibrium if and only if there exists a $t_0 \geq 0$ such that $\phi(t) = 0$ for $t < t_0$ and $\phi(t) = 1$ for $t > t_0$.

A.6 Welfare

The focus of this article is on the positive analysis of migration, but at times we discuss welfare. Assuming stationarity, if migration starts at $t = 0$, social welfare is

$$\int_0^{+\infty} \Big[h_I(t)u_I(h_I(t)) + h_E(t)u_E(h_E(t))\Big]e^{-rt}\,dt.$$

Migration is efficient if this expression is greater than the aggregate welfare when all users remain on the incumbent platform, $\int_0^{+\infty} u_I(1)e^{-rt}\,dt$. Assuming $\lim_{t\to+\infty} h_I(t) = 0$, migration is efficient, in the limit as $r \to 0$, if and only if

$$u_E(1) > u_I(1),$$

as in a model with instantaneous migration. In section 5, we provide examples that show there can be excessive inertia as well as excessive migration.

A.7 Reversibility of migration

Up to this point, we have assumed that migration only takes place from the incumbent to the entrant platform. That is, we assumed that migration is irreversible. We now argue that this assumption is not restrictive.

When migration is reversible (with migration between the entrant and the incumbent also satisfying the general properties assumed in this article), we have the following cases:

– if μ is bounded above (for instance, as it is in the case of a Poisson process), then the equilibria described in the main text are the only equilibria;

– otherwise, the only additional equilibria compared to those we have analyzed are scenarios in which a first stage all the users migrate from I to E; in a second stage of sufficiently long duration all the users stay on E; and then all the users migrate back to I (maybe, with several iterations of this process). With stationarity, these scenarios are only possible if the net benefit of the migration from E to I is equal to zero for a user who obtains a migration opportunity at $t = 0$.

Henceforth, we assume that migration from E to I is possible, and that migration opportunities arise at the same rate for individuals in either platform. The reasoning of Section A.4 carries over and, for nearly all t, users will accept migration opportunities with probability 0 or 1. Notice that, because individuals are identical, for nearly any time t agents cannot be migrating in opposite directions.

In the interest of brevity, instead of presenting a complete analysis, we show that it is not possible to have an equilibrium where there is *partial* migration to E, followed by a period of no migration, followed by a return to I. More formally, it is not possible to have an equilibrium in which the following three conditions hold: a) between $t \in [0, T^-]$ users accept with probability 1 migration from I to E, and for $t \geq T^+ \geq T^-$, they accept with probability 1 migration from E to I; b) they refuse all migration opportunities for $t \in [T^-, T^+]$; and c) both $h_I(t)$ and $h_E(t)$ are strictly positive (and constant, of course) on $t \in [T^-, T^+]$.

We first introduce some notation. Let $W_E(t)$ be the value of being on the entrant at a time t and, similarly, let $W_I(t)$ be the value of being on the incumbent. For instance, a consumer who has the opportunity to migrate from I to E at time t does so whenever $W_E(t) > W_I(t)$.

We must have $W_I(t) \leq W_E(t)$ for $t < T^+$ and

$$W_I(t) \geq W_E(t) \text{ for } t > T^+. \tag{A.10}$$

Therefore,

$$W_I(T^+) = W_E(T^-). \tag{A.11}$$

Now, consider a $t > T^+$, but close enough to T^+ that the probability that a user has an opportunity to migrate during $[T^+, t]$ can be neglected. If there is reverse migration (from E to I), we must have $W_I(t) \geq W_E(t)$. We have

$$W_I(t) - W_E(t) = W_I(T^+) - W_E(T^+) + \int_{T^+}^{t} [u_I(h_I(\tau)) - u_E(h_E(\tau))] \, d\tau.$$

The right-hand side of this equality is strictly negative a) because (A.11) holds and b) because there would be no migration if $u_I(h_I(T^-)) = u_I(h_I(T^+))$ were not strictly smaller than $u_E(h_E(T^-)) = u_E(h_E(T^+))$ and therefore $u_I(h_I(\tau)) - u_E(h_E(\tau)) < 0$ for τ close enough to T^+. This contradicts (A.10), and concludes the proof.

The reasoning also holds if $T^- = T^+$ (migration from E to I starts immediately after migration from I to E has finished) and if $h_I(T^+) = 0$, *i.e.*, all the users first migrate to E.

As a conclusion, the only possible back and forth migration is complete migration from I to E, followed by a period where all the users are on platform E, followed by a complete migration back to I, with the process continuing in the same pattern. Then, reverse migration resembles the migration described by equation (A.4b) in Proposition A.3 with $t_0 > 0$: the first user who migrates has zero gain from it. If μ is bounded above, then h_I is always strictly positive and therefore reverse migration is impossible.

B Extensions and proofs

B.1 Two speeds

We extend the analysis of the "two speeds" model of Section 4: after a first opportunity to migrate, subsequent opportunities arrive faster than the first, but not infinitely fast. For simplicity, the basic migration process is "autonomous": $\mu_I(h) = s$. After refusing a first opportunity, users of the incumbent platform receive additional opportunities to migrate according to an accelerated process $\mu(h_I) = \alpha s$. We are mostly interested in the case of $\alpha > 1$, but the derivations are valid for any α. With linear utilities, a user who migrates at $t = 0$ and expects others to follow obtains a benefit equal to

$$\int_0^{\infty} [b_E\, h_E(t) + k_E] e^{-rt} dt.$$

The density function of the time of the next opportunity is e^{-ast}/as. Therefore, a user who waits for her next opportunity[39] to migrate will at time t be on the incumbent platform with probability $e^{-\alpha st}$ and on the entrant platform with probability $1 - e^{-\alpha st}$. Her discounted utility is

$$\begin{aligned}
&\int_0^{+\infty} \Big[e^{-\alpha st}(b_I\, h_I(t)) + (1 - e^{-\alpha st})(b_E\, h_E(t) + k_E) \Big] e^{-rt}\, dt \\
&\qquad = \int_0^{\infty} \Big[b_E\, h_E(t) + k_E \Big] e^{-rt} dt \\
&\qquad\quad + \int_0^{+\infty} e^{-\alpha st} \Big[-(b_E + k_E) + (b_I + b_E) h_I(t) \Big] e^{-rt} dt.
\end{aligned}$$

By (4), $h_I(t) = e^{-st}$ and there exists a migration equilibrium if the second term of the right-hand side of the equation is positive, *i.e.,* if

$$0 \le -\frac{b_E + k_E}{\alpha s + r} + \frac{b_I + b_E}{\alpha s + s + r}.$$

[39] It is straightforward to prove that it is not optimal to wait for a later opportunity.

Proposition B.5. *With linear utilities and first opportunities arising according to the autonomous migration process of parameter s and future opportunities according to the autonomous process of parameter $\alpha \times s$, a migration equilibrium exists if and only if*

$$\frac{b_I - k_E}{b_E + k_E} \leq \frac{s}{\alpha s + r}. \tag{B.1}$$

A more frequent arrival rate of subsequent opportunities (higher α) makes migration less likely.

Proof. The last statement is simply a consequence of the fact that the right-hand side of (B.1) is decreasing in s. □

B.2 Proofs for the autonomous model of Section 5

Proof of Equation (12)

For some migration processes, the numerator and denominator of (10) are not well defined when $r = 0$, but, as we show below, they are when $h_I(t)$ satisfies (11).

We first derive an expression for $\int_0^{+\infty} h_I(t)\, dt$. Because

$$\frac{d}{dt}\left[\sigma t - \frac{\ln[1 + (\sigma - 1)e^{\sigma a t}]}{a}\right] = \sigma - \frac{(\sigma - 1)\sigma a e^{\sigma a t}}{a(1 + (\sigma - 1)e^{\sigma a t})} = h_I(t),$$

we have

$$\int_0^{+\infty} h_I(t)\, dt = \left[\sigma t - \frac{\ln[1 + (\sigma - 1)e^{\sigma a t}]}{a}\right]_0^{+\infty}. \tag{B.2}$$

Also

$$\begin{aligned}
&\lim_{t \to +\infty}\left[\sigma t - \frac{\ln(1 + (\sigma - 1)e^{\sigma a t})}{a}\right] \\
&\quad = \lim_{t \to +\infty}\left[\sigma t - \frac{\ln[(\sigma - 1)e^{\sigma a t}]}{a} - \ln\left(1 + \frac{1}{(\sigma - 1)e^{\sigma a t}}\right)\right] \\
&\quad = \lim_{t \to +\infty}\left[\sigma t - \frac{\ln(\sigma - 1)}{a} - \sigma t - \frac{1}{(\sigma - 1)e^{\sigma a t}}\right] = -\frac{\ln(\sigma - 1)}{a}
\end{aligned}$$

and

$$\sigma t - \frac{\ln[1 + (\sigma - 1)e^{\sigma a t}]}{a}\bigg|_{t=0} = -\frac{\ln \sigma}{a}.$$

Therefore, from (B.2)

$$\int_0^{+\infty} h_I(t)\,dt = \frac{\ln\sigma - \ln(\sigma-1)}{a}.$$

We now compute $\int_0^{+\infty} h_I^2(t)\,dt$. Note that $h_I'(t) = -\mu(h_I(t)) \times h_I(t)$ implies $h_I^2(t) = h_I'(t)/a + \sigma h(t)$, we have

$$\int_0^{+\infty} h_I^2(t)\,dt = \frac{[h_I(t)]_0^{+\infty}}{a} + \sigma \int_0^{+\infty} h_I(t)\,dt$$
$$= \frac{-1}{a} + \sigma\frac{\ln\sigma - \ln(\sigma-1)}{a} = \frac{\sigma(\ln\sigma - \ln(\sigma-1)) - 1}{a},$$

which yields the equality in (12).

The right-hand-side of (12) is decreasing in σ

The derivative of the right-hand side of (12) with respect to σ is

$$1 + \frac{\frac{1}{\sigma} - \frac{1}{\sigma-1}}{(\ln\sigma - \ln(\sigma-1))^2} = 1 - \frac{1}{\sigma(\sigma-1)(\ln\sigma - \ln(\sigma-1))^2} > 0,$$

where the inequality is a consequence of the fact that, by strict concavity of the function ln, we have

$$\ln\sigma - \ln(\sigma-1) < \left.\frac{\partial \ln}{\partial\sigma}\right|_{\sigma=\sigma-1} \times (\sigma - (\sigma-1)) = \frac{1}{\sigma-1}.$$

B.3 Proofs for Section 6

The two lemmas in this appendix assume the hypotheses of Section 6.

Lemma B.1. *If eager users begin migrating at time 0 and reluctant users begin migrating at time $t \geq T_L > 0$,where T_L satisfies (14).*

Proof. Under the hypotheses of the lemma, for $t \geq T_L$, a reluctant user is on the incumbent platform with probability $e^{-s(t-T_L)}$. Migrating at time T_L

yields the same utility than waiting for the next opportunity; therefore

$$\int_{T_L}^{+\infty} [bh_E(t) + k_L] e^{-rt}\, dt$$
$$= \int_{T_L}^{+\infty} \Big[e^{-s(t-T)} bh_I(t) + (1 - e^{-s(t-T)}) [bh_E(t) + k_L] \Big] e^{-rt}\, dt$$
$$= \int_{T_L}^{+\infty} [bh_E(t) + k_L] e^{-rt}\, dt$$
$$+ \int_{T_L}^{+\infty} e^{-s(t-T)} [2bh_I(t) - b - k_L] e^{-rt}\, dt.$$

This implies $\int_{T_L}^{\infty} e^{-s(t-T)} [2bh_I(t) - b - k_L] e^{-rt}\, dt = 0$ and therefore, by (13) and taking the limit as $r \to 0$,

$$\frac{k_L + b}{s} = \frac{b}{s} \left[(1 - p_H) + e^{-sT} p_H \right],$$

which implies (14). □

Lemma B.2. *Eager users migrate at time $t = 0$ if*

$$\begin{cases} k_H \geq -(1 - p_H) k_L / p_H & \text{if reluctant users begin migrating at } T_L < +\infty, \\ k_H \geq b\,(1 - p_H) & \text{otherwise.} \end{cases}$$

Proof. An eager user migrates at time 0 rather than wait for the next opportunity if

$$\int_0^{+\infty} [bh_W(t) + k_H] e^{-rt}\, dt \geq$$
$$\int_0^{+\infty} \Big[e^{-st} bh_I(t) + (1 - e^{-st}) [bh_E(t) + k_H] \Big] e^{-rt}\, dt$$
$$\iff \int_0^{+\infty} [b + k_H] e^{-(r+s)t}\, dt \geq \int_0^{+\infty} 2b e^{-(r+s)t} h_I(t)\, dt$$

Using (13), this is equivalent to

$$\frac{k_H + b}{2b(s + r)} \geq \int_0^{T_L} \Big[e^{-(r+2s)t} p_H + e^{-(r+s)t} (1 - p_H) \Big]\, dt$$
$$+ \int_{T_L}^{+\infty} \Big[e^{-(r+2s)t} p_H + e^{-(r+s)(t - T_L)} (1 - p_H) \Big]\, dt.$$

As $r \to 0$ and using (14), this condition is equivalent to $k_H/b \geq (1 - p_H)(1 - e^{-sT_L}) = -(1 - p_H)k_L/(bp_H)$. This completes the proof for the case $T_L < +\infty$.

The result for $T_L = +\infty$ follows trivially. It is equivalent to the fact that for purely autonomous migration process and $r \to 0$, migration takes place if and only if it is efficient. □