"Radicalization of Islam or Peddling Radicalism?
Lessons from the French Experience"

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Radicalization of Islam or Peddling Radicalism?

Lessons from the French Experience

by

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Abstract: A simple game-theoretic model is used to end the sterile intellectual trench war between those who analyze each instance of a community's radicalization process as a self-contained phenomenon and those who prefer to embed such episodes within a more encompassing social framework. In the model, two groups labeled "Islamic" and "Nativists" are competing using radicalization as a tool to enlarge their share of the limelight in the media. Exogenous shocks are then shown to entail both idiosyncratic responses and interactions between the two groups. The French "radicalized decade" 2011-2020, which witnessed both the Bataclan highly lethal Jihadist attack in 2015 and the populist *gilets jaunes* massive uprising from 2018 to the COVID-related lockdown in 2020, among other radicalization events, is used to put some of the model's insight to work. A simple extension of the model is used to shed some light on the emerging Islamo-Leftist and Lefto-Populist tacit collusions, which suggest that the radical left is somehow breaking apart, thus probably boosting in fact the collective radicalization process.

Introduction

Like in many other parts of the world, more and more vocal politicians are nowadays pointing at political Islam as a social and cultural threat on the French nation. At least two of the main potential candidates in the current 2022 presidential election in France are daily voicing thinly veiled references to various conspiracy theories involving the Muslims in their campaign speeches. The incumbent Macron Administration has even denounced a form of "separatism" on the part of the most politicized of the latter and legislated explicitly against that just a few months before the electoral campaign started. At the same time, most of the main candidates have drifted to the extremes of the political spectrum, making on the right-wing side hard-liner stands on the issue of immigration control and on the left-hand side some pleas for radical redistribution, as well as some thinly veiled anti-Jews allusions. Most voters clearly understand "Islamic threat" when politicians talk about "migrants", as very few people are afraid of immigration coming from inside the EU.

At a time when this radicalization of the political debates was just affecting some marginal politicians, former Prime Minister Edouard Balladur (center-right) published a strong warning against policies that might be perceived as a mark of religious intolerance against Muslims (Balladur, 2005). In particular, he pointed out that the ban on the headscarf (*hijab*) for girls at school should be lifted to avoid provoking a strong rigidification of the ethno-religious identity issue. His vibrant call fell on deaf ears, and ten years later, the identity crisis had to be taken on board as a fact, if only because of its extremely violent expressions. Political scientists and sociologists then felt compelled to search for a theory explaining its causes, with the implicit hope that this could show the way out of the apparent deadlock. In the best Parisian Academic tradition, a spectacular conflict started to oppose two of the most famous specialists of Islamism and Jihadism of the place. In a book published in 2015, Gilles Kepel defended the view that the root cause of Islamic radicalization and the rise of Jihadism in France were to be found in the mosques (Kepel, 2015). His analysis is based on a wealth of interview-based field work that brings out the proselytizing that takes place at or around the mosques, often supported by external influences. About a year later, Olivier Roy analyzed instead Islamism as an ideological cover hiding a deeper radicalism, reflecting a more general generational phenomenon (Roy, 2016). This debate among specialists largely spilt over beyond Academic circles and Cécile Daumas made it accessible to a larger public in a deeply researched article published in 2016 in the daily newspaper *Libération*. She clearly spelt out

that these are two one-dimensional visions of causality that are confronted head on, using quite a strong language. This verbal duel amply fed the conversations of a large part of the Francophone Intelligentsia at the time. Nevertheless, she somehow suggested that each contending author had a valuable piece of the puzzle. The present authors subscribe to this point of view, and the present paper tries to find a way to come out of the turmoil within a more inclusive framework.

A similar debate appeared in the Anglophone literature, in a more muffled style. Petter Nesser (2016) shows how Islamic terrorism arose in Europe, by scrutinizing in detail the channels of influence of the various Middle Eastern conflicts that percolated into Europe. He emphasizes like Gilles Kepel how much this external Jihadist influence was decisive in the radicalization of European Islamism and the subsequent rise in violent Jihadism. By contrast, various voices arose to recast the rise in Radical Islamism in a broader perspective, somehow the way Olivier Roy does. Gambetta and Hertog (2016) bring out some striking similarities between radical Islam and white supremacist or nativist extremism. They emphasize the commonalities between the profiles of the most lethal terrorists on the two sides, showing statistically, using reasonably sized samples of terrorists' biographies, that highly educated activists are often implicated on both sides. They find a shockingly dominant number of engineers among them, especially among the most lethal ones. Azam and Ferrero (2017) show the equally striking similarities between the behaviors of Islamist murderers like Mohamed Merah and Mehdi Nemmouche, who perpetrated spectacular attacks in Toulouse, Montauban and Brussels, respectively in 2012 and 2014, of some killers from the European extreme right as well, like Anders Breivik, who killed 77 teenagers from a socialist family background, and lastly of those that kill children in schools following the pattern established in Columbine, USA. They draw on the analysis of the Herostratos syndrome presented by Borowitz (2005), which they embed in a participation-game framework where the players seek self-glorification and social recognition by perpetrating odious deeds. They suggest that the media play a key part in amplifying this appetite for infamy for those who already have such a tendency, by creating a kind of emulation between the killers.

The present paper tries to shed some light on this debate, based on a fundamental methodological diagnosis. It ascribes the violence of the polemics between Kepel and Roy, and the resulting relative sterility of their verbal trench war, to the use of a too narrow concept of causality, one that is too close to the simplest mechanical physicist's one. On one side,

Jihadism results from the radicalization of Islam, partly due to external influences as documented by Kepel, and on the other side, it is the "Islamization of radicalism" that is at work, as advocated by Roy. The next section shows graphically that you can't have it both ways without falling either into absurd predictions or concluding trivially that either one or the other must be wrong. However, both diagnoses seem to hold a piece of a puzzle that the reader would like to be put together when the excitement of the authors' Academic duel is dissipated. There is too much convincing evidence and brilliant insights in these duelists' work to accept to throw much of it away.

The subsequent sections offer a change in methodological perspective that opens the way to the desirable synthesis, within an "analytic narrative" framework (Bates, et al., 1998). It shows that the social sciences in general can hardly be content with a too narrow causality concept and that a broader vision opens more fruitful perspectives. We must first embed Islamist radicalization in a broader framework where other brands of radicalization may also exist. Here, we understand the word radicalization as the process that pushes many persons from a given social category or grouping to cross the line for inflicting significant costs to the society at large and we leave the psychological overtones that this word often implies at bay. This process could involve murderous attacks, or other violent actions that entail the very expensive loss of income or property for many citizens. For the sake of simplicity, we label "populist" the alternative type of radicalization that develops beside the Islamic one in our setting. What makes that step decisive is the assumption that there is an interaction between the levels of radicalization developed by each group. Various avenues could be invoked to justify such an assumption. In the light of the French experience with the rise of Jihadism, on the one hand, and the emergence of the *gilets jaunes* (yellow waistcoats) movement, on the other hand, we suggest that the strengthening of a group consciousness and the defense of a collective self image lie at the heart of this interaction between the two groups' radicalization processes. A very simple game-theoretic model enables us to produce a graphical analysis of the equilibrium of the game that can take place between these two opposite brands of radicalization, the "Islamic" and the "populist" ones, which interact in fact in an amplifying fashion. A mathematical appendix is provided at the end of the paper for the sake of the readers who care for more calculation.

However, the value added produced by this simple modelling exercise and its graphical analysis goes beyond a mere methodological clarification; the main benefit derived

from this framework is to clearly generate the comparative-static predictions that describe the radicalization mechanisms brought out by the two antagonistic authors mentioned above within a unified framework. The model allows us to study the predicted impacts of external shocks, including those analyzed by Kepel (2015) and Nesser (2016), but also the kind of policy handles that the government could use to mitigate the radicalization process. Another kind of external shocks that are liable to increase the level of radicalization may involve some side effects from other government policies. This is brought out econometrically by Azam and Bhatia (2017) in the case of India, where local government or militia violence against some minority groups that respond predictably by undertaking terrorist attacks seems to be engineered purposefully by local governments with a view to clear the ground for mining firms. A similar type of provocation is found econometrically by Azam and Thelen (2018), showing that the rise of transnational terrorist attacks produced by citizens from overseas countries where US soldiers are deployed, especially in or near oil-exporting ones, seems to be knowingly accepted as a side effect of that military presence. This indirect incidence is relevant for France, if only because she is included in the recommended targets lists of some terrorist organizations, as an important ally of the US. Moreover, section four suggests that the French government's attempts at raising the taxes on fossil fuel consumption are liable to play a similar part in triggering populist uprisings. A key benefit generated by this kind of comparative-static analysis is to give a precise meaning to the concept of "peddling radicalism" that we use in the title of the paper. This expression refers to the process whereby an initial radicalization response to a group-specific exogenous shock propagates by triggering similar responses by other groups. This sheds some useful light on the process of "Islamization of radicalism" put forward by Roy (2016), as well as on the reverse interaction whereby a populist shock could trigger an Islamic response.

The fruitfulness of this framework is then put to the test by discussing how the *gilets jaunes* (yellow waistcoats) movement embodies a real "populist" radicalization, inflicting a massive disruption on the French economy, whose cost is evaluated at several billion euros. The movement was triggered by a government plan to raise taxes on diesel fuel for cars and trucks, for ecological reasons, without any preliminary concertation. It was based on weekly monstrous marches in the main cities involving hundreds of thousands of demonstrators that affected negatively the central inner-city shopping areas every Saturday, the usual peak day for commercial business. It also involved the blocking occupation of hundreds of important roundabouts and crossroads, especially on the motorway feeder roads as well as of toll booths.

The tourism sector's turnover fell drastically below the normal pattern, inflicting several billion-euro worth of lost activity. Moreover, as is often the case, these massive marches harbored some classic "black blocs", i.e., gangs of masked marchers breaking shop windows and vandalizing urban public goods, whose cost also amounts to several billion euros. These weekly demonstrations and nearly permanent roadblocks lasted for about two years. The *gilets jaunes* movement typically side-stepped the standard institutional framework of political demonstrations, like political parties and trade unions, and used a strikingly "anti-system" rhetoric (Le Bart, 2020). It was coordinated using the social media by independent working-class people from the lower tail of the wage distribution. Some surveys have shown that these people had mainly voted either for extreme right-wing or extreme left-wing parties at the previous presidential election. The convergence of these two extremes of the political spectrum, such that national tricolor flags and communist red flags were often flying side by side in many demonstrations and makeshift road-side encampments, is a telling phenomenon that surprised many. The description of this anti-system movement seems to provide some strong support for the "peddling radicalism" hypothesis alongside radical Islamism. A simple extension of the model suggests that the tacit collusion between the far left and the far right alluded to above is *per se* a cause of enhanced radicalization. The paper concludes by suggesting that the radicalization of various categories of people in France mainly reflects the abyssal gap that exists between them and the centralized government and public sector. This is probably true in many other countries where radicalized minorities have emerged in the recent past.

1. Jihadism and Radicalism: The Mono-Causal Dead End.

For the sake of simplicity, one can summarize the two antagonistic positions defended by Kepel and Roy as resulting from the inversion of the direction of causality between the impacts of Islamism and radicalism between the two approaches. For Kepel, an increase in Islamism entails an increase in radicalism, while the impact works the other around for Roy. Figure 1 captures these two alternative causal links by positively sloped straight lines in the {Islamism, Radicalism} space. It assumes that the Kepel effect, whereby Islamism causes radicalization, has a steeper slope than the Roy effect in this space, which maintains that radicalism determines radical Islamism. The opposite assumption is examined in figure 2 below. This relative-slope assumption means for example that if an exogenous increase in radicalism occurs, it triggers an increase in Islamism as a reflection of the Roy mechanism,

which entails in turn a second-round impact on radicalism by the Kepel mechanism that is weaker than the initial shock, i.e., a decreasing incremental response. The opposite case is analyzed below. The intersection point R may be called the "reconciliation" equilibrium where the two mechanisms function simultaneously in a consistent fashion, i.e., where the level of Islamism I_R produced by the Roy mechanism for a given level of radicalism E_R creates in turn by the Kepel mechanism the same level of radicalism. This consistency property does not exist at any other point of this $\{I, E\}$ space. Elsewhere in this space, the Kepel mechanism would in fact produce a different level of radicalism than the initial one, entailing the chain reaction of I and E chasing each other in turn in the convergence process.
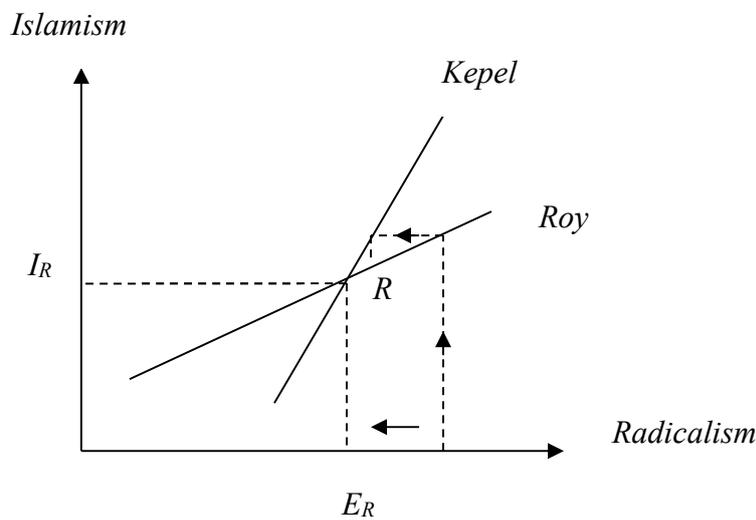

Figure 1: Kepel and Roy Combined

This is illustrated in figure 1 by the arrows born by dashed lines, starting to the right of E_R and moving to the left after hitting the Roy-labeled line, and then moving downwards after hitting the Kepel line, etc., all the way until point R is reached. Then, the iteration between the two effects stops at the equilibrium point, which is therefore regarded as stable, as any initial shock away from R tends to be corrected by the subsequent iterative dynamics triggered in turn by the two causal effects. A similar convergence process could be initiated by starting with an initial point located to the left of E_R. This stability property is required if we want to use the diagram to meaningfully simulate the potential impacts of any parameter change that might shift exogenously either line. This concept of stability was first used in economics by Cournot (1838, p.132) in his mathematical analysis of the duopoly. We obviously need any

useful equilibrium model to have this convergence property if we want to use it for predicting the responses of the two endogenous variables to any exogenous shock that is liable to shift one of the lines or the other by looking at the changed position of the equilibrium point. This requirement is known as "Samuelson's Correspondence Principle" (Samuelson, 1947).
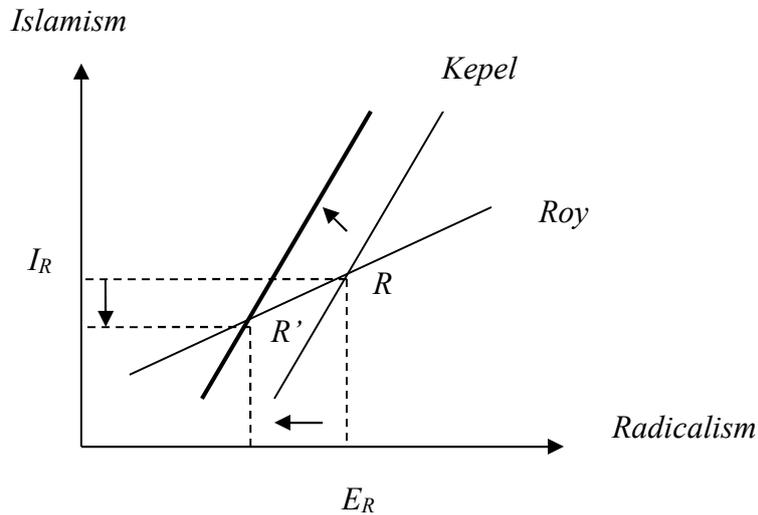

Figure 2: Impact of an exogenous Islamization shock

Such a "comparative statics" analysis is performed at figure 2 where the Kepel line is shifted upwards relative to the case of figure 1 by assuming an exogenous increase in Islamism. This means that the level of Islamism is increased for any given initial level of radicalism, e.g., because of a foreign intervention in support of rigorous religious practice or Jihad. The thick line represents the resulting new position of the Kepel line after this external shock. This might be interpreted as depicting the impact of an external stimulation of Islamic religious practice of the kind described by Kepel's field work. Unfortunately, this exercise leads to an absurd prediction in this case. Because of the relative slopes of the two lines imposed by the stability requirement, we know that the Kepel line must be steeper than the Roy line. This entails that the equilibrium point shifts from R to R', thus depicting a simultaneous decrease of radicalism and Islamism. This is an absurd prediction.

What happens if we inverse the assumption regarding the relative slopes of the two lines? Figure 3 allows us to derive the consequences of such a change in specification. It simply uses figure 1 and swaps the labels of the two lines. Unfortunately, this move is unable to reconcile the two authors' viewpoints in a relevant framework. This is because the

equilibrium point R is not stable anymore in this case and does not comply with Samuelson's Correspondence Principle mentioned above. This can be checked by repeating the thought experiment performed with figure 1, i.e., looking at the consequence of starting from an initial point located to the right of E_R. The Roy line is now positioned above the Kepel line in that part of the space, entailing a higher level of Islamization, which triggers in turn an increase in radicalism, etc. As shown by the directions of the little arrows borne by the dashed lines, a conceptually explosive chain reaction gets started which would push both the levels of Islamism and radicalism to ever increasing levels up to infinity, i.e., to an Apocalypse exceeding all the predictions of any of the various ancient scriptures.
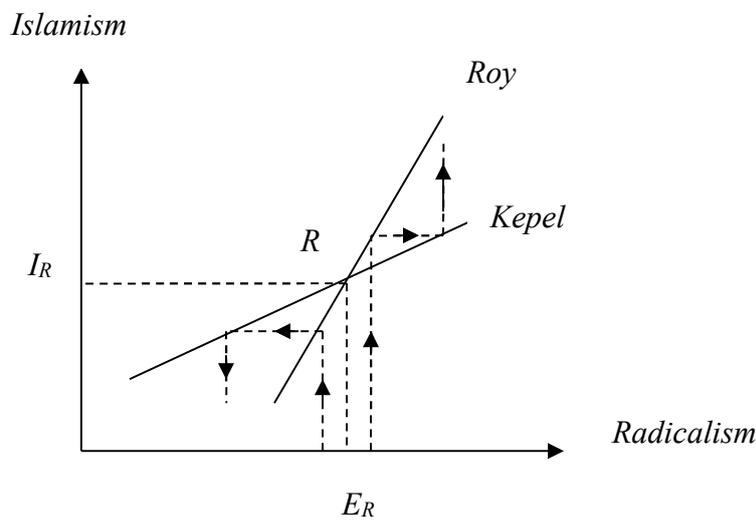
Figure 3 : The Explosive Case

The symmetric thought experiment can be performed along similar lines to determine the consequence of an initial level of radicalism located to the left-hand side of E_R, i.e., "too low" relative to the equilibrium level corresponding to point R. In this case, the chain reaction would start leftwards, leading necessarily the model to a world freed from Islamism and radicalism. Hence, even if we started exactly at the equilibrium point R, then even a tiny exogenous shock resulting from a small autonomous change in the external influences mentioned above would send our world either to the zero or the infinite of Islamism and radicalism. In other words, any hope of reconciling Gilles Kepel's and Olivier Roy's viewpoints within a mono-causal framework must be given up, unless one is prepared to assume that the two lines are exactly identical, a perfectly silly assumption. Therefore, beside

the three absurd or unrealistic predictions brought out above, this mono-causal framework only yields a disappointing single conclusion of a bitter dullness: either one or the other must be wrong. This zero-sum game choice might help to explain why the two authors engaged in such a fierce theoretical dueling.

Fortunately, there exists at least one synthetic approach that allows us to take on board the scientific contributions made by both Gilles Kepel and Olivier Roy. It only requires a small intellectual investment in game theory, which is offered in the next section.

2. A Game-Theoretic Reconciliation

Let us now set up a simple analytical framework in which R_I represents the level of Islamic radicalization and R_P the level of populist radicalization. A specialist's study of the radicalization phenomena would clearly require much finer distinctions, but this level of simplification yields a good combination of simplicity and analytical insight. Similarly, we assume for the sake of simplicity that there are only two groups having the potential to develop the type of radicalization analyzed here, which might end up producing dangerous or even murderous attacks, while the real world obviously involves a larger set of relevant groups. We call them "Muslim" and "Nativist", with a view to motivate a bit the analysis with a broad brush, keeping in mind the drastic simplification thus involved. We keep "Islamist", "Jihadist" and "populist" to describe the outcomes of their potential radicalization processes.

Assume that each community maximizes an objective function that represents what Montesquieu (1748) called its "general spirit", which captures somehow the dominant state of mind prevailing in the group. This aggregate might result from idiosyncratic processes generated by the group's informal institutions, charismatic leaders, or religious or professional associations, or in some cases from a spontaneous grassroots phenomenon. Some examples are described in what follows. This kind of collective Welfare functions immediately raises a red flag for economists: which axiom of Arrow's 'possibility' theorem has been dropped to make it possible? This theorem gives a set of axioms that make it impossible to determine such a function. To start with, it assumes that the individual preferences to be aggregated are given and immutable, among other things. The most useful amendment to that list of axioms is to consider the fact that a lot of social processes are precisely about influencing people's preferences. Responding to a comment by Amartya Sen, Ken Arrow had this beautiful

answer: "Amartya is correct that I did not address the question of conversation and dialog in the formation of preferences, the meaning of changing your opinion when talking to someone else" (Arrow, 2014, p.58). By contrast, this issue is the focus of George Akerlof's "Loyalty Filters" theory, well summarized by the following quote: "When people go through experiences, frequently their loyalties, or their values, change. I call these value-changing experiences "loyalty filters"" (Akerlof, 1983, p.54). Azam (2012) uses this idea, focusing on how education may change people's worldview and turn some of them into suicide terrorists. This kind of experiences are undoubtedly what is at stake in the formation of ethno-linguistic, religious, or other forms of collective identity. In our model, we use a very simple specification to capture a potentially crucial form of endogenous collective preference changes to explain the radicalization process. In an intriguing case study, Khosrokhavar (2022) documents how Olivier Corel, whose real name is Abdel Ilah Al-Dandachi, provided in his community in Artigat, near Toulouse, a father figure that was decisive in the radicalization of several young Islamists. What is striking is that he was not using any indoctrination strategy, using instead an inspiring silent attitude to let the aspiring activists speak out their radical tendencies in a cumulative fashion. Domingo (2022) provides a more detailed analysis of this case. El Janati (2022) provides a deep analysis of another charismatic figure of the Jihadist movement in Toulouse, using a significantly different approach. Mamadou Daffé, a researcher in biochemistry at Paul Sabatier University in Toulouse, was also the Imam of the Le Mirail mosque, on the Western outskirts of Toulouse. His was a strategy of creating a Muslim-friendly haven, where young people could strengthen their self image and dream of the neo-Umma, a concept related to the international Caliphate. Many of them eventually left to join ISIS in Syria.

Like economists often do, we will call this collective welfare function a utility function, taking "utility" in the broader sense of the Latin *utilitas*, which includes all kinds of advantages that accrue to the maximizing agent. For each community, radicalization entails both benefits and costs, and "utility" is the measure of the difference between these two terms. Many different avenues could be explored to determine what constitutes a benefit for the group, as assessed by its "general spirit". One possible component of group identity may derive from genealogy. Typically, this is the case in clanic societies. Azam (2014) discusses how such clanic ties have been harnessed to develop a homegrown type of democracy in the case of Somaliland. However, in most European countries, several other mechanisms can result in the crystallization of some group identity. This may involve religion, political or

trade union affiliation, supporting of soccer or rugby teams ("*Allez le Stade*"), etc. Ferret et al. (2022) develop the concept of "imaginary family", or "neo-Umma", to explain one mechanism by which a person can link up emotionally to a wider group. Domingo (2022) uses the expression "the Merah clan" to describe how a cluster of Jihadist tendencies developed in the Merah broken family, in a very informative monographic study. This is a way to restore some pride for people whose self-image has been tarnished by their life experience, involving may be family or professional disappointments, and the resulting loneliness or lack of social connections. He shows how Mohamed Merah's killing by the police in Toulouse, putting an end to his series of seven murders, made him a collective identity-defining hero for many members of his extended family. This is very close to Ferrero's "Cult of Martyrs" theory (Ferrero, 2013). In a related but slightly different vein, Debray (2005) uses the concept of "human communions". Moreover, Ferret (2022) illustrates how Mohamed Merah odious deed created a model for other groups to emulate. He shows that the nine Jihadists of the Ripoll cell, in Spanish Catalonia, who killed eventually 16 people in Barcelona and Cambrils in 2017, claimed to have carefully planned their murderous acts "*à la Merah*".

For a group to deliver this kind of services to its members, it must reach a critical mass and have some mental cohesion. One path to get to that point is for the group to capture a fair share of the limelight in the media. Islamic radicalization has been a very effective approach to amplify the Muslim group's presence in the news. Similarly, many observers have pointed out that the *gilets jaunes* movement has mobilized mostly underdog or "invisible" people (Le Bart, 2020) who feel unconnected to the wider society because of their low income or lack of professional qualification to reach the same objective (Algan et al., 2019, Le Bart, 2020). They then succeeded in getting a lot of attention in the country and abroad. Hence, in that context, the benefits of radicalization in a group may result from its enhanced importance in the media, and this entails a competition for the limelight with other groups. In other words, the benefit derived by the group from the radicalization of some of its members may be the mitigation of the threat entailed by the level of the other community's radicalization, which is liable to affect possibly both the self-image and the security of the group's members. Each community has an incentive to face up to such implicit threats, to avoid any increased feeling of inferiority for its members. An important dimension of this competition between groups is provided by the social media, which manage to reach a very large number of viewers outside the control of the conventional mass media. Kepel (2015), for the French Jihadism case,

Abdel Bari Atwan (2015) and Stern and Berger (2015) (among others) for ISIS, and Boulouque (2019), for the *gilets jaunes*, all provide fine descriptions of the use of the social media in this context. This technology opens new avenues for competition between the different groups, each one trying hard to match the innovations adopted by the others, because of their potential impact on the value they attach to their radicalization levels. The benefits may also derive from external influences, like the Salafist increasing aura of holiness for the Muslims, or the increased political visibility of populist movements elsewhere in Europe and the US for the Nativists. These influences have the potential to enhance the allure of radical Islamist or Populist postures by endowing them with brighter images. The costs include individual human costs of activism, like personal risk-taking that could even result in imprisonment or self-sacrifice in perpetrating suicide attacks, as well as the increased social pressure on non-activist members of the community, increased police surveillance, profiling, and discrimination, etc.

To capture this interaction between the groups, let's note $f(R_P, \mu) \geq 0$ the value attached by the Muslims per unit of radicalization within their own community R_I. It is an increasing function of its two arguments, R_P, the radicalization level of the other group, and μ that represents the possible external influences. Hence, the benefit for this community produced by its radicalization level reads as $f(R_P, \mu) R_I$. Similarly, for the Nativists, we note $g(R_I, \pi) \geq 0$ the value that they attach per unit of their own radicalization R_P. It is also an increasing function of its two arguments and π also represents the exogenous influences that can increase this collective value. Let us assume also that these two functions are concave with respect to the other group's radicalization level, to capture a decreasing marginal impact. In other words, this assumption means that the positive impact of an additional unit of the other group's radicalization on this value decreases as the level already achieved by them is higher. The benefit of the nativist community's radicalization may thus be written as $g(R_I, \pi) R_P$.

The drawback of their radicalization is captured for each community by assuming that the unit of measurement applied to the phenomena ("utils") is such that the costs borne by each community, measured in the same units as the benefits, may be written as the square of their respective levels of radicalization R_I^2 and R_P^2. This specification captures the idea that

radicalization inflict more drawbacks, the higher its level already is. Ultimately, it could lead to urban guerilla if it was pushed too far. Given this notation, we naturally assume that each community seeks to maximize the difference between the benefits and costs of its radicalization, given the radicalization level of the other community. Their respective utility functions U_M or U_N are thus written as follows:

$$U_M = f\left(R_P, \mu\right) R_I - R_I^2 \quad \text{et} \quad U_N = g\left(R_I, \pi\right) R_P - R_P^2. \tag{1}$$

The chosen specification for these utility functions allows us to perform very simply these maximization exercises. Their first-order conditions are fulfilled when the per-unit benefits of their radicalizations, $f\left(R_p, \mu\right)$ and $g\left(R_I, \pi\right)$, respectively, are equal to their marginal costs, $2R_I$ and $2R_P$. These conditions allow us to write the radicalization levels chosen by the two groups as:

$$R_I^* = f\left(R_P, \mu\right)\big/2 \quad \text{and} \quad R_P^* = g\left(R_I, \pi\right)\big/2. \tag{2}$$

Hence, our model boils down to the pair of equations (2) linking the two endogenous variables R_I and R_P. Notice that there is some symmetry between these two equations, whose structures are very similar. This approximate symmetry can be easily seen by noticing that one can be approximately deduced from the other just by swapping R_I and R_P between these two equations. The precise specifications of $f(.)$ and $g(.)$ will of course remain different, as well as the parameters μ and π. This approximate symmetry is used to construct the diagram presented below.

The Nash equilibrium of this model is defined by the consistency condition that the two equations in (2) are simultaneously holding for $R_I = R_I^*$ and $R_P = R_P^*$. This equilibrium concept has first been used by Cournot (1838), whose analysis was however a bit less precise from a mathematical point of view than the one developed by Nash more than a century later. Figure 4 represents the determination of this equilibrium by the point $\left\{R_I^N, R_P^N\right\}$ in the $\left\{R_I, R_P\right\}$ space, at the intersection N of the two curves representing the two equations in (2). The concavity of one is reflected in the convexity of the other, because of the

approximate symmetry described above. One immediately checks that this equilibrium is stable as defined above by performing the kind of thought experiment of the type of the chain reactions described by the little arrows borne by the dashed lines, as in figures 1 and 3.
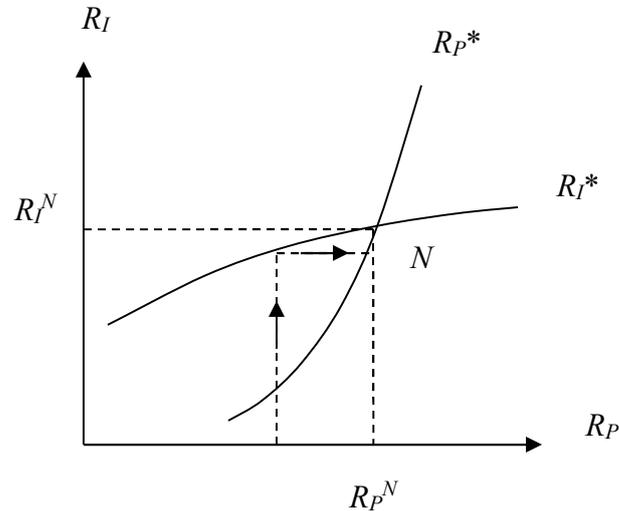

Figure 4 : Equilibrium Radicalization Levels

3. The Peddling Radicalism Process

We can now gauge the fruitfulness of this small Nash equilibrium model by looking at the kind of predictions that it yields regarding the consequences of some changes in the exogenous parameters μ and π. A Kepel-type shock is here captured by an increase in the μ parameter, which entails an upward shift of the R_I^* line. Figure 5 shows that the Nash equilibrium point responds to this shock by shifting to the North-East of the $\{R_I, R_P\}$ space. This shift of point N may be decomposed into a first direct vertical shift, for the given initial value of R_P^N, followed by a chain reaction of the two groups' levels of radicalization that increase in turn one after the other. This prediction does not strain common sense, and the model's theoretical foundations provide a natural way of separating the impact of the direct partial effect, which is the focus of Gilles Kepel's book, from the resulting interactive increases of the two types of radicalizations. The latter effect is close to Roy's analysis, as it captures a general climate of radicalization well.
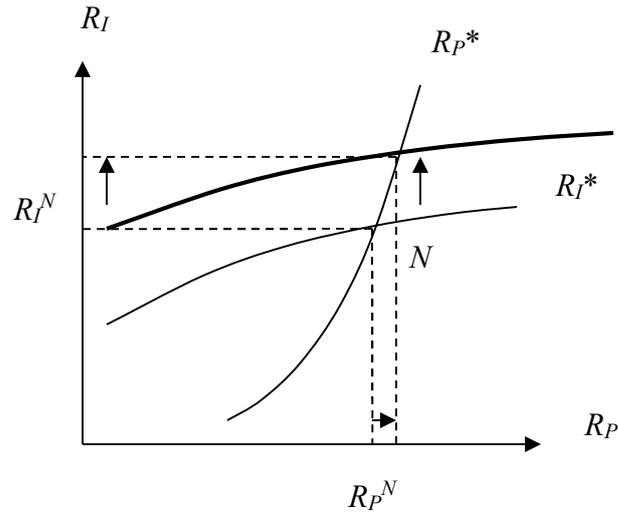
Figure 5: Impact of a Kepel-type Islamic Shock

The impact of an autonomous increase in the other type of radicalization analyzed here, that we labeled populist, can be analyzed symmetrically in the model by looking at the impact of an increase in π. This is depicted at figure 6, where the exogenous increase in populist radicalization is captured by a shift to the right of the R_P* curve. We find again in a symmetric fashion a direct partial effect of the populist radicalization, followed by a chain reaction where the two types of radicalizations interact by increasing in turn one after the other. The latter chain reaction undoubtedly captures the "Islamization of radicalism" that Olivier Roy refers to. It involves an increase in Islamic radicalization that does not respond to any idiosyncratic causal shock on the Muslim side. This is the reason why Roy's contentious distinction between the radicalization of Islam and the Islamization of radicalism is an important theoretical *caveat*: the cause of a puzzling buildup of Islamic radicalization does not necessarily have to be sought in the Muslim world but might be a response to a shock happening somewhere else in the radicalization field. Because of the alternate concavity and convexity of the two curves, one could notice that if the two shocks have an equal amplitude, in terms of shifts of R_I and R_P, each measured in the relevant dimension, the direct impact of the shocks always seems to be stronger than the induced response in the neighborhood of the equilibrium point analyzed here. In fact, this remark has no general validity, and other cases could be generated, depending on the steepness of the curves in that neighborhood.
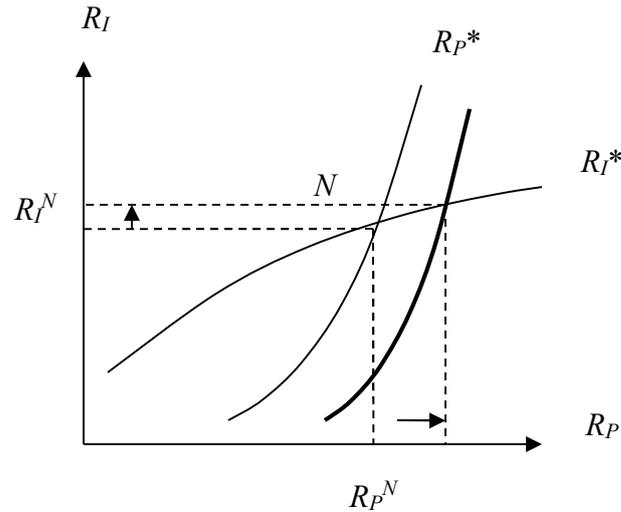
Figure 6: Impact of a Populist Shock

To sum up, the foregoing section has brought out two main propositions: (i) idiosyncratic shocks like the one analyzed by Kepel (2015) and Nesser (2015) are crucial determinants of the changes in the radicalization level of the groups. However, it also brought out that (ii) the different groups' radicalization processes can also be triggered directly in a chain reaction responding to the other group's radicalization process, as emphasized by Roy (2016). Without making any claim at exhaustivity, the model presented above focuses on the competition for a share of the limelight in the media as the mainstay of these interactions between the two group's radicalization processes, either directly via pure emulation, or indirectly via the groups' innovations in the field of communication strategies and information technologies. The next section offers a glance at the French experience during the 2011-2020 decade, in which many events testify that that the groups producing them were undergoing a radicalization process. The narrative presented sheds some light on the model's predictions and suggest some marginal adjustments to take more facts on board.

4. Lessons from the 2011-2020 Radicalized Decade in France

The second decade of the 21st century witnessed many lethal Jihadist attacks in various French cities and populist marches involving strong rioting events, including the infamous tagging and sacking of the *Arc de triomphe*, and a lot of broken shop windows and looting of luxury shops on the *Champs Elysées*. Table 1 gives a partial list of these events. The civil war

in Syria was a major focal point on the Islamist side, attracting more than 1000 young French Muslims and recent converts who went to Syria to fight either for Al Qaeda (*al Nusra*) or ISIS. The converts were often drawn from right-wing populist backgrounds and Taguieff uses the expression "Islamo-Nazi" to refer to this surprising connection (Taguieff, 2021). However, he provides some illuminating historical background showing the age-old links existing between political Islam and Nazism, obviously bonded by a common anti-Jews hatred. The Grand Mufti of Jerusalem Haj Amin al-Husseini started to stir up anti-Jews riots in Palestine as early as the 1920s. Because of their implications for his relationships with the British authorities at the time, these frictions got him an impressive reputation as an anti-imperialist activist that survives even to this day. As early as 1933, al-Husseini established close links with the Third Reich and was invited to Berlin where he met Hitler and the Nazi top brass. In 1943, Hitler ordered by decree the creation of the 13th division of the Waffen-SS, called *Handschar* (dagger), mainly involving Muslims from Bosnia-Herzegovina. It was confirmed after the war that al-Husseini had played a key part in getting that unit created. There are photographs showing him doing the Nazi salute and strutting down the line of men. This division inflicted an extreme level of violence on the Serbian civilian population and the Jews. Taguieff carefully traces a chain of people who kept this Islamo-Nazi legacy alive to this day, via the Muslim Brotherhood in particular. Some former Nazi officials found refuge in the Middle East and North-Africa, and even converted to Islam in some cases, like the former SS Johan von Lees, alias Omar Amin, who died in Cairo in 1965, where he had worked as an adviser to President Nasser. About a third of the French Jihadists in Syria came back to France, being tightly filtered at the sealed border, and most of them ended up in jail (Thomson, 2016). However, this was not true during the early years of the war, so that some of the lethal Jihadist activists of the 2010s had in fact spent some time in Syria and returned freely.

Table 1: Select Chronology of Radical Events (France 2011-2020)

Year	Jihadist Event	External Event	Populist Event	External Event
2011	- *Charlie Hebdo* satirical magazine's premises set on fire by a *Molotoff* cocktail thrown by two Islamists	- Ossama Ben Laden killed in Abbottabad by US special forces - Publication of a drawing of Mahomet in a special issue titled *Charia Hebdo* - Arab Spring in Arab countries.		
2012	- Mohammed Merah kills three French Muslim soldiers in Toulouse and Montauban and four Jewish school children and teacher in Toulouse	- The Islamic State in Iraq becomes ISIS following its expansion into Syria.	- *Manif pour tous* (march for all), between 300 000 and 1 million marchers estimates. Extreme-right stewards in some cities. Homophobic slogans.	- Taubira Law to authorize same-sex marriage.
2013			- "Red Woolen Hats" uprising in Brittany and other places - *Manif pour tous,* continued, massive demonstration in May	- Carbon tax on fuel on motorways project. - Taubira Law, continued.
2014	- Mehdi Nemmouche kills Jews in Brussels	- Daesh proclaims the new Caliphate		
2015	- The Kouachi brothers attack *Charlie Hebdo*, killing 12 people, on behalf of Al Qaeda - Series of lethal attacks in the Paris area, culminating in the Bataclan attack killing 90+40 people on behalf of the Islamic State.	- Caliphate thrives		
2016	- Catholic priest Jacques Hamel assassinated by two Islamists in his church - Truck driven in the crowd in Nice killing 89	- Caliphate thrives	- 'Labor Law' project riots in main cities	- "El Khomri" Labor Law discussed in Parliament
2017	- Car driven in the crowd in Barcelona's *Ramblas*	- Caliphate thrives		
2018	- Revolver attack (ISIS claim) in a Christmas market in Strasbourg, killing five.	- Caliphate thrives	- *Gilets jaunes* uprising in main cities and roundabouts	- Announced tax increase on diesel fuel for cars and trucks
2019		- Collapse of the New Caliphate	- Continued	Pres. Macron opens a series of *Grand débat* face-to- face meetings.
2020	- Knife attacks against people praying in the Cathedral in Nice - History high-school teacher Samuel Paty decapitated by a Chechen Islamist for showing the *Charlie Hebdo* drawing of Mahomet in class.		- COVID lockdown discourages *gilets jaunes* marchers.	

A surprising set of partners popping up in this competition for the limelight between the Islamists and the Populists comes from the far left, including hard-liner socialists, communists, and Trotskyites. Caroline Fourest (2005) traces the origins of this connection to the rise of the Anti-globalization movement around the turn of the century, which converged over time with the Islamist intelligentsia inspired by the Muslim brotherhood. The resulting unholy alliance has been dubbed "Islamo-leftist" (*islamo-gauchiste*) initially by Pierre-André Taguieff, an expression that became viral in the recent political debate. It was crowned with some form of officialdom when the Minister of Education Jean-Michel Blanquer voiced it on the Europe 1 radio channel in commenting on the beheading of high-school history teacher Samuel Paty on October 22, 2020. The Minister of Higher Education Frédérique Vidal has used it again recently in February 2022 to announce that she was about to launch a special inquiry about the kind of affirmative action and anti-White discrimination that is alleged to be implemented in various Universities, inspired by this ideological framework. Taguieff (2021) provides an up-to-date account of its main tenets. However, this evolution seems so far to remain limited to the intelligentsia, without much impact on the grassroots. May be, this should be credited to the Islamo-leftists, as many of their actions seem clearly targeted at opening new channels of inclusion, a point that is not perceived by their critics. Both Fourest (2005) and Taguieff (2021) take a strong deontological approach rigidly defending fundamental republican first principles, without much attention paid to more consequentialist strategic considerations. However, some of the opinion survey data cited by Taguieff suggest that the Islamo-leftist strategy has gone too far, as some of the most worrying Jihadist claims seem to have percolated deeply into the mental setting of average middle school children. In terms of our model, this might suggest that the R_I^* curve has shifted upwards as depicted in figure 5, may be just a little.

By contrast, numerous communist red flags were highly visible in the *gilets jaunes* marches and roadblocks and remained so up to the end of the movement. This suggests that many far-leftist marchers joined autonomously the populist movement, what can be interpreted as an external shock that shifted the R_P^* curve to the right, as depicted in figure 6. This apparently spontaneous collusion between the two extremes seems to testify of a yawning gap opening inside the French far left, revealing a divorce between its intelligentsia and its grassroots. This is reflected in the polls that estimate that the socialist party candidate

Anne Hidalgo hovers around 2% in the predicted votes for the coming 2022 presidential election. Algan et al. (2019) present some intriguing statistical analysis of the differences and similarities between the radical leftist and right-wing populist supporters of this movement. All of them are characterized by low incomes and a basic distrust of both formal institutions and other people, while they differ significantly in terms of educational achievement. The "Lefto-populists", as we may call them by analogy with the Islamo-leftist expression used above, have a significantly higher level of education, creating thus a resentment for being unfairly treated by the labor market. The latter feeling seems absent among the right-wing populist supporters. Boulouque (2019) brings out another intriguing difference, involving a kind of mediatic specialization. As the left-wing populist activists were often seasoned former trade unionists or party members, they handled much better their show in the urban marches, managing to get a larger share of the limelight with the conventional media like the main TV and radio channels. By contrast, the right-wing populist supporters of the *gilets jaunes* seem to be more attracted by the social networks and seem to have largely outcompeted the others in this field. In both cases, the mainly spontaneous large mobilization was undoubtedly targeted by the marchers and roadblock occupiers at increasing their share of the limelight.

Our quick review of the French experience thus brought out the collusive behavior of the radical left and right wings, which came as a shock to observers used to the standard left-right axis for categorizing political positions. Analyzing this issue in depth would require an extension to a three-group model, which we leave for further research. The next section shows that our model opens a larger opportunity set to the players, provided they resort to more sophisticated modes of interaction than those allowed within the Nash-Equilibrium framework.

5. Some Room for Collusion

Our model captures endogenously how the two players' radicalization processes affect the value that they attach to their own radicalization levels. The previous sections have shown that this very simple specification of the players' endogenous preference formation yields some important clues about some classic debates regarding radicalization. However, the brief review of the French experience of the previous decade performed above has brought out an additional issue, namely that the limits of the different groups are not in fact watertight, as shown by various examples of informal collusion between different radical groups. We have

seen how the Far Left split somehow, with its intelligentsia providing support for the Islamists and its grassroots joining the Nativists. It is likely that these leftist sub-groups were trying to capture some of the limelight already focused on their hosts. In so doing, they most probably enhanced sizably the levels of radicalization of both the Islamists and the Nativists. This can be interpreted as a form of amorphous collusion, where the Leftists grafted on both sides created some additional communication channels, beside the enhanced numbers of participants. Analyzing this new kind of collusive behavior is outside the scope of this paper and calls for further research. Here, we will be content with showing that there is room for this type of maneuvering in our model, as there are gains to be reaped by going beyond the simple Nash equilibrium.

From a technical point of view, this is not surprising for a trained economist. The way we model the interaction between the groups' levels of radicalization involves a clear instance of externalities. Decisions made by one group affect the environment in which the other group makes its own decisions, and vice versa. Moreover, it affects their preferences positively, insofar as one group's enhanced radicalization increases the value that the other group attaches to its own radicalization level. An economist's instinct should then hint that this externality is bound to result in a below-optimal choice of radicalization levels because neither group internalizes the benefit that this decision produces for the other group. A standard economic argument suggests that our Nash-equilibrium framework might unrealistically preclude the search for more rewarding solutions by the players. This is due to the extreme simplicity of the exchange of information between the groups that this equilibrium concept allows. The almost universal access to social networks probably creates a tension about this assumption. Without venturing into the jungle of cooperative game theory, we can still take a few steps to clarify this point. We can prove easily that there is some scope for improvement for the two players' strategic choices as there are, so to speak, large-denomination euro notes lying on the pavement. This is done by showing that our Nash-equilibrium point N lies below the so-called Pareto frontier, i.e., that it leaves some room for increasing the utility level reached by either player without reducing the other player's utility.

Let us first define the indifference map of the Muslim group in the $\{R_I, R_P\}$ space by performing the following thought barter experiment: by how much would the Muslim group accept to change its own radicalization level in return for a small increase $d R_P$ of the Nativist

group's radicalization level, while remaining as well off as before at a utility level $\bar{U}_M$? To find the answer, let us take the total differential of U_M from (1) to write:

$$dU_M = 0 = \left(f(.) - 2R\right)dR_I + \left(\partial f(.)/\partial R_P\right)dR_P = $$
$$2\left(R_I^* - R_I\right)dR_I + \left(\partial f(.)/\partial R_P\right)dR_P \qquad . \tag{3}$$

Rearranging the terms, we can write the implicit barter price in the question above as:

$$\left.\frac{dR_I}{dR_P}\right|_{\bar{U}_M} = \frac{-\left(\partial f(.)/\partial R_P\right)}{2\left(R_I^* - R_I\right)}. \tag{4}$$

Hence, an indifference curve of the Muslim group in this space would be vertical where $R_I = R_I^*$, negatively sloped where $R_I < R_I^*$, and positively sloped where $R_I > R_I^*$. Therefore, such a member of the Islamist group's indifference map would intersect the R_I^* curve vertically, and slope upwards above it, and downwards below. The one that gives the utility level reached by this group in the Nash equilibrium is drawn in figure 7 with a thicker line that intersects the R_I^* curve vertically at the Nash equilibrium point N and is labeled U_M^N.

Using an approximately symmetrical argument we can easily find that an indifference curve of the Nativist group in this space has a slope equal to:

$$\left.\frac{dR_I}{dR_P}\right|_{\bar{U}_N} = \frac{-2\left(R_P^* - R_P\right)}{\left(\partial g(.)/\partial R_I\right)}. \tag{5}$$

Hence, an indifference curve of the Nativist group would be horizontal where $R_P = R_P^*$, negatively sloped where $R_P < R_P^*$, and positively sloped where $R_P > R_P^*$. This is represented in figure 7 by a U-shaped curve that intersects the R_P^* curve horizontally, slopes downwards to its left, and upwards to its right. The utility level reached by this group in the Nash equilibrium is shown in figure 7 by the thicker U-shaped curve through point N and is labeled U_N^N.

A glance at the diagram shows that there is a non-empty set between U_M^N and U_N^N to the North-East of point N that contains all the combinations of R_I and R_P yielding at least as much utility to both players as at point N. Only the points located on either U_M^N or U_N^N to the North-East of N would only benefit strictly one of them. This non-empty set captures graphically what the title of this section calls "room for collusion", as it shows that there are many $\{R_I, R_P\}$ pairs that would be strictly preferred by the two sides to the outcome produced by the initial Nash equilibrium. Let's call it the "bargaining set". All of them clearly demand from the players to use more sophisticated strategies than the Nash equilibrium analyzed above. In the latter, each player seeks to optimize its own choice while taking the other one's choice as given. Relaxing a bit that rigid rule opens the way to allow the players to reap a better deal. The standard approach to analyze how people manage to improve on Nash equilibrium outcomes is provided by bargaining theory, which has produced a host of models; that's why it is dubbed "jungle" at the beginning of this section. Most of these models analyze a two-player setting where the agreed outcome is described as a contract, assuming implicitly that there exists a third player, e.g., a judiciary institution, which can force the contractors to keep their words. Oliver Hart (1995) suggests that this can work even with very simple "incomplete contracts" in the case of two parties investing complementary assets in a firm. However, it would strain credulity to assume that the Islamists and the Populists could easily exchange information and directly strike a bargain. These are strongly decentralized movements, distrustful of most kinds of formal institutions (Algan, 2019, LeBart, 2020), in which hatred of the other group is often stirred high by some activists or charismatic leaders. Another very imaginative application of contract theory was presented by Azariadis (1975), using the concept of implicit contracts, i.e., contracts that reflect social customs rather than judicially enforced binding promises. This might be more relevant for our particular context. In their analysis of the Herostratos syndrome, Azam and Ferrero (2019) use a related idea whereby a terrorist may sacrifice his own life in return for a high probability of being remembered for a long time after his death, or simply to get an instant international Warholian recognition thanks to the media. Ferrero (2013) had shown earlier how the perspective of a "cult" developing to keep the memory of the deed alive can fruitfully be analyzed similarly. Even the Marxian concept of "class consciousness" plays this kind of role in the theory of proletarian revolution, by providing the incentives for selfless behavior (Wright Mills, 1962). In many religions, it is an implicit contract with God that promises other-worldly rewards in return for a righteous conduct. It would be foolish to try and identify precisely what kind of

implicit or explicit contracts are liable in fact to convince the members of the two groups analyzed here to go beyond the Nash equilibrium, which might entail as seen above that they will somehow internalize the externality that their behavior generates. It might simply be some clever strategic thinking by some leaders or other charismatic agents, as illustrated by Domingo (2022) and El Janati (2022) in the case of Jihadists in the Toulouse area. Moreover, there is no reason to believe that all the group members respond to the same incentives. However, in the French case in the 2010s, a more likely mechanism seems to explain quite simply (if not exclusively) how the two sides were led unwittingly to seize the opportunity to increase their utility levels beyond the initial Nash equilibrium. As emphasized above, the split of the Far Left, some of its members joining each of the two sides, plausibly provided the incentives for the two R_I and R_P curves to shift, upwards and rightwards, respectively, pushing the final Nash equilibrium point North-Easterly inside the initial bargaining set.

Figure 7: The Pareto Frontier

Let us now characterize a bit more precisely the benefits that could accrue to the groups if they managed to move beyond the initial Nash Equilibrium, may be under some external influence. Most contract or bargaining models present mechanism whereby the chosen outcome will be Pareto-efficient. This means that such contracts will be such that no player could be made better off without making the other one worse off. This excludes all the points where the two groups' indifference curves are not tangent with each other, like the thick ones at figure 7 as shown above, and a two-dimensional *continuum* of others. Figure 7

shows an example where the two indifference curves labeled $\tilde{U}_M$ and $\tilde{U}_N$ are tangent. The tangency point $\{\tilde{R}_I, \tilde{R}_P\}$ is visibly such that any variation from it would necessarily yield a new point below or to the left, respectively, of at least one of the two initial tangent indifference curves or both in some cases. The Pareto frontier is defined as the curve linking all the Pareto-efficient points having this property within the bargaining set. The dashed thick curve so labeled represents a plausible instance of that Pareto frontier. Figure 7 clearly shows that all the points belonging to this frontier yield a higher utility level to the two groups than what they would get in the Nash equilibrium, except for the two ends, where only one of the players strictly benefits. It also shows that all these points (i) entail higher levels of radicalization for both groups than the Nash equilibrium point, and (ii) make at least one of the two players better off, and most of the time both. In other words, this curve provides incentives for the two players to find ways to collude to produce more radicalization by the two groups than the Nash equilibrium. This might explain why the original marchers or activists did not reject the turncoats. However, it also yields a *caveat*: there is no point in pushing radicalization beyond that frontier because, starting hypothetically from this frontier, any gains for the two players obtained by increasing their radicalization further, could equally well be achieved by cutting them instead.

One might then conjecture that a series of unplanned individual decisions, like the increased numbers of marchers supplied by the radical left to join the *gilets jaunes* demonstrations, the emergence of the Islamo-leftists interacting with the Jihadists, or even the non negligible number of recent Muslim converts from the populist right that joined the Jihad, all played a significant part in boosting the radicalization levels on both sides depicted in this model. All these radicalized people seem to share a strong distrust of formal institutions, rather than any well-defined Left/Right affiliation. This shows the way to further research.

Conclusion

The analysis developed in the present paper helped us to take a few methodological steps forward by showing that our understanding of the real world can be significantly enriched by replacing a mono-causal framework by a bi-causal or even multi-causal one. Here the two contending analyses presented by Gilles Kepel and Olivier Roy are combined within a Nash equilibrium of a two-player game, representing the interaction between two potentially

conflicting communities. We have labeled these two groups Muslim and Nativist, without making any claim at exhaustiveness. This is of course a daring simplification relative to the real world. It allows us in return to deepen our intuition about the interactions between the external influences and the general climate of radicalization that can tentatively explain the concomitant rise in Jihadism and populism that can be observed in many countries. A glance at the French experience of the 2010s has provided a finer understanding of the relevant "external" influences, by pointing out among other things the split that occurred in the traditional French Far Left between supporting the Islamists and the Populists, which probably boosted the radicalization processes on both sides. In terms of political strategy, the attempt at synthesis presented here clearly suggests that the right path to follow to avoid the escalation of violence might profitably involve a simultaneous intervention on the two (or more) radicalization fronts. The recently imposed regulations about hate speech online by the EU, and somehow implemented by the main social media companies (Bradford, 2020), probably reduce the outreach of the various radicalized groups in the world. This might in turn dampen the peddling radicalism process that this paper brought out.

Mathematical Appendix

The Model

The two groups respectively seek to maximize the following two utility functions, taking the other group's radicalization level as given:

$$U_I = f\left(R_P, \mu\right) R_I - R_I^2 \text{ and } U_P = g\left(R_I, \pi\right) R_P - R_P^2. \tag{A.1}$$

The first-order conditions for optimality are:

$$f\left(R_P, \mu\right) = 2\, R_I \text{ and } g\left(R_I, \pi\right) = 2 R_P, \tag{A.2}$$

which can be arranged as the following best-response functions:

$$R_I^* = f\left(R_P, \mu\right)\big/2 \text{ and } R_P^* = g\left(R_I, \pi\right)\big/2. \tag{A.3}$$

The Nash Equilibrium

The Nash equilibrium holds when the following consistency conditions simultaneously hold:

$$R_I^* = f\left(R_P^*, \mu\right)\big/2 \text{ and } R_P^* = g\left(R_P^*, \pi\right)\big/2.$$ (A.4)

This allows us to derive the reduced-form equations:

$$R_I^* = R_I^*\left(\mu, \pi\right) \text{ and } R_P^* = R_P^*\left(\mu, \pi\right),$$ (A.5)

such that:

$$\frac{\partial R_I^*}{\partial \mu} = \frac{\left(\dfrac{\partial f(.)}{2\partial \mu}\right)}{1 - \left(\dfrac{\partial f(.)}{4\partial R_P^*}\right)\left(\dfrac{\partial g(.)}{\partial R_I^*}\right)} > 0 \text{ and } \frac{\partial R_I^*}{\partial \pi} = \frac{\left(\dfrac{\partial f(.)}{4\partial R_P^*}\right)\left(\dfrac{\partial g(.)}{\partial \pi}\right)}{1 - \left(\dfrac{\partial f(.)}{4\partial R_P^*}\right)\left(\dfrac{\partial g(.)}{\partial R_I^*}\right)} > 0,$$ (A.6)

and

$$\frac{\partial R_P^*}{\partial \mu} = \frac{\left(\dfrac{\partial g(.)}{4\partial R_I^*}\right)\left(\dfrac{\partial f(.)}{\partial \mu}\right)}{1 - \left(\dfrac{\partial g(.)}{4\partial R_I^*}\right)\left(\dfrac{\partial f(.)}{\partial R_P^*}\right)} > 0 \text{ and } \frac{\partial R_P^*}{\partial \pi} = \frac{\left(\dfrac{\partial g(.)}{2\partial \pi}\right)}{1 - \left(\dfrac{\partial g(.)}{4\partial R_I^*}\right)\left(\dfrac{\partial f(.)}{\partial R_P^*}\right)} > 0.$$ (A.7)

We know from the diagrams that these four derivatives are positive, as written above. Analytically, this is ensured by the fact the slope of the R_P^* curve is always larger than the slope of the R_I^* curve in the neighborhood of the equilibrium point, given the alternate concave-convex shapes of the curves, i.e., that $1\big/\left(\partial g(.)\big/\partial R_I\right) > \partial f(.)\big/\partial R_P$, which ensures that the denominator is positive.

Proof

The proof of the claims made at (A.6) and (A.7) runs as follows: solving the two-equation system (A.4) using total differentials yields:

$$d R_I^* = \left(\frac{1}{2}\right)\left(\frac{\partial f(.)}{\partial R_P^*}d R_P^* + \frac{\partial f(.)}{\partial \mu}d \mu\right) \text{ and } d R_P^* = \left(\frac{1}{2}\right)\left(\frac{\partial g(.)}{\partial R_I^*}d R_I^* + \frac{\partial g(.)}{\partial \pi}d \pi\right)$$ (A.8)

By substitution, we get the two following equations:

$$d R_I^* = \left(\frac{1}{2}\right)\left(\frac{\partial f(.)}{2\partial R_P^*}\left(\frac{\partial g(.)}{\partial R_I^*}d R_I^* + \frac{\partial g(.)}{\partial \pi}d\pi\right) + \frac{\partial f(.)}{\partial \mu}d\mu\right) \qquad (A.9)$$

and

$$d R_P^* = \left(\frac{1}{2}\right)\left(\frac{\partial g(.)}{2\partial R_I^*}\left(\frac{\partial f(.)}{\partial R_P^*}d R_P^* + \frac{\partial f(.)}{\partial \mu}d\mu\right) + \frac{\partial g(.)}{\partial \pi}d\pi\right) \qquad (A.10)$$

By re-arranging the terms, we then get easily (A.6) and (A.7).

The Pareto-Frontier

We can define the Pareto frontier as the set of points that solve equivalently either of the following two problems: $\max_{R_I, R_P} U_M$ s.t. $U_N \geq \bar{U}_N$ or $\max_{R_I, R_P} U_N$ s.t. $U_M \geq \bar{U}_M$, for every given values of $\bar{U}_I$ and $\bar{U}_P$ in the relevant range, i.e., such that $\bar{U}_I \geq U_I^N$ and $\bar{U}_P \geq U_P^N$.

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