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## “Personalized Pricing and Distribution Strategies”

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# Personalized Pricing and Distribution Strategies\*

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## Abstract

The availability of consumer data is inducing a growing number of firms to adopt more personalized pricing policies. This affects both the performance of, and the competition between alternative distribution channels, which in turn has implications for firms' distribution strategies. We develop a formal model to examine a brand manufacturer's choice between mono distribution (selling only through its own direct channel) or dual distribution (selling through an independent retailer as well). We consider different demand patterns, covering both horizontal and vertical differentiation and different pricing regimes, with the manufacturer and retailer each charging personalized prices or a uniform price. We show that dual distribution is optimal for a large number of cases. In particular, this is always the case when the channels are horizontally differentiated, regardless of the pricing regime; moreover, if both firms charge personalized prices, a well-designed wholesale tariff allows them to extract the entire consumer surplus. These insights obtained here for the case of intra-brand competition between vertically related firms are thus in stark contrast to those obtained for inter-brand competition, where personalized pricing dissipates industry profit. With vertical differentiation, dual distribution remains optimal if the manufacturer charges a uniform price. By contrast, under personalized pricing mono distribution can be optimal when the retailer does not expand demand sufficiently. Interestingly, the industry profit may be largest in a hybrid pricing regime, in which the manufacturer forgoes the use of personalized pricing and only the retailer charges personalized prices.

**Keywords:** personalized pricing, distribution strategies, vertical contracting, downstream competition.

**JEL Codes:** L22, L24, L81

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# 1 Introduction

The growing use of the Internet and advances in information technologies enable firms to gather unprecedented volumes of consumer data. This has led to important changes in their pricing policies by allowing them to practice price discrimination at finely-tuned levels. Firms tailor their prices to consumers' purchase history, their physical location, the device they are using, their online search behavior, and so on.<sup>1</sup> For instance, in the apparel and fashion industry, most brand manufacturers, such as Desigual, Guess, and Marc O'Polo, and retailers, such as Zalando and Amazon, use coupons and specific promotions based on their consumer data (e.g., through loyalty programs or information releases about sales) to implement customized prices for consumers.<sup>2</sup>

The trend in data collection has consequences for pricing, but also for many long-term strategic decisions. In this article, we focus on the choice of distribution partners helping manufacturers to reach out to consumers. This is a particularly important issue in the digital age, as technological advances have led to the emergence of new online retail companies, including general retailers such as Amazon Retail, which sells around 12 million products in the US ranging from clothing to grocery items,<sup>3</sup> and specialized ones such as Asos, which distributes fashion and cosmetic products of more than 850 brands. Whether to rely on these independent retailers or only on direct channels is a key question for manufacturers.

Motivated by these trends, the objective of this paper is to identify the implications of data availability for manufacturers' distribution strategies. How does personalized pricing change a manufacturer's incentives to sell through an independent retailer? How is this decision influenced by demand patterns? Does personalized pricing enhance profitability?

To study the strategic interaction between personalized pricing and distribution strategies, we consider a setting with one brand manufacturer, selling directly to final consumers, and one independent retailer. The retailer adds value to the industry but also competes with the manufacturer in the downstream market. That is, there is *intra-brand* competition between a direct distribution channel and an independent channel, the two firms being vertically related through a wholesale contract. Our demand spec-

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<sup>1</sup>For example, Tanner (2014) reports that buyers using a discount site, such as Nextag.com, receive prices as much as 23% lower than direct visitors. Large Internet retailers, such as Amazon and Staples, vary their prices according to customers' geographic location by up to 166%. Companies like Axiom, Datalogix, and ID Analytics, which specialize in developing machine learning algorithms, act as data brokers and help firms to predict a consumer's willingness-to-pay (Bounie *et al.*, 2021).

<sup>2</sup>In practice, firms may not know a consumer's valuation precisely. Although we will consider in this paper the benchmark of perfect information, the insights apply as well to fine-tuned price discrimination.

<sup>3</sup>See Nchannel (2020).

ification allows for positive or negative correlation between consumers' valuations for the two channels; it is therefore sufficiently flexible to encompass classic models of vertical or horizontal differentiation.

For each demand pattern, we consider four different pricing regimes. In the first regime, the manufacturer and the retailer offer uniform prices to final consumers. This represents a market in which consumer tracking is not possible. In the second regime, both firms engage in personalized pricing. This reflects a situation in which the two firms have highly-frequented (e.g., online) stores allowing them to gather very precise consumer data. In the third regime, only the manufacturer can set personalized prices. This represents for example a situation in which, thanks to past purchases, the manufacturer has better consumer data than the retailer. In the fourth regime, only the retailer can set personalized prices. This reflects for instance a situation in which a large retailer offers many products and is thereby able to collect more consumer data than a brand manufacturer. Our analysis therefore captures the fact that, while new technologies allow firms to use personalized pricing as a trend, actual capabilities vary at both the market and the firm level.

We also consider an extended setting in which the pricing regime is endogenous and negotiated by the firms—that is, personalized pricing is available to both firms, and they negotiate whether each of them adopts it or not.<sup>4</sup>

The choice between mono and dual distribution involves a trade-off, as the retailer brings value to the industry but also competes with the manufacturer's direct channel. Yet we show that, when consumers' valuations for the two channels are negatively correlated (as in classic models of horizontal differentiation), dual distribution is always optimal, regardless of the pricing regime. This is because the wholesale price is then particularly effective at limiting intra-brand competition. For instance, a wholesale price equal to the willingness-to-pay of the consumer indifferent between the two channels enables them to segment the market—no firm can then profitably serve the other's core market. Moreover, if both firms can charge personalized prices, adopting this particular wholesale price actually enables them to extract the *entire* consumer surplus (even with a simple two-part tariff contract). The most profitable regime is therefore when both firms charge personalized prices.

These findings stress the importance of distinguishing between *inter-brand* competition and *intra-brand* competition, where the wholesale contract can be used to limit the intensity of competition. In particular, in the case of horizontal differentiation, our insights about the impact of personalized pricing are markedly different from those obtained for inter-brand competition. For example, Thisse and Vives (1988) and Shaf-

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<sup>4</sup>The firms can contract on uniform pricing, for instance, by adopting privacy or fair treatment policies.

fer and Zhang (1995) show that firms are trapped in a prisoner's dilemma in which personalized pricing reduces industry profits. By contrast, in the case of intra-brand competition, personalized pricing, *together* with an appropriate wholesale tariff, maximizes industry profit, and even allows for full consumer surplus extraction.

In the case of positive correlation between consumers' preferences, dual distribution remains optimal if the manufacturer offers a uniform retail price, regardless of the retailer's own pricing policy. Charging an appropriate high wholesale price then suffices again to attenuate the intensity of downstream competition, while still allowing the retailer to expand demand—compared with mono distribution, the manufacturer moreover increases its own (uniform) price and extracts more surplus from high-valuation consumers.

By contrast, if the manufacturer charges personalized prices, then mono distribution (i.e., selling only through the direct channel) may become the optimal strategy—both when the retailer charges a uniform price and when it charges personalized ones. Specifically, relying exclusively on direct distribution is optimal when the retailer does not substantially expand demand, as the effect of increased intra-brand competition then prevails. For example, when both firms can price discriminate, they can price aggressively in each other's strong segment without sacrificing margins in their own core business. As a result, it becomes more difficult to control intra-brand competition without impeding market expansion. Mono distribution is moreover more likely to be optimal when the retailer charges a uniform price, as this reduces its added value. These results show again that wholesale contracting and the possibility of charging personalized prices are crucial when determining the optimal distribution strategy.

Finally, when endogenizing the firms' pricing policies, we obtain the interesting result that it can be profitable for the manufacturer not to use personalized pricing, even if it has the ability to do so. Restricting the manufacturer's pricing policy induces it to focus on its core market, thereby dampening the competitive pressure on the retailer and allowing it to extract more surplus. Hence, a hybrid pricing regime, in which only the retailer charges personalized prices, can achieve the right balance between allowing the retailer to add value and limiting intra-brand competition.

In summary, while personalized pricing triggers a trade-off between rent extraction and increased competition, dual distribution is the optimal strategy in a large number of cases. This is because an appropriate wholesale tariff enables the firms to dampen intra-brand competition. Compared to the literature on personalized pricing between independent firms, our paper therefore shows that charging personalized prices often rises firms' profits—instead of reducing them—if firms are vertically related. Compared to the literature on distribution channels strategies, we show that charging personalized prices can render mono distribution optimal, although only when the man-

ufacturer sets personalized prices and consumer preferences are positively correlated. We discuss in the conclusion the lessons from our analysis, which may provide strategic guidance for contracting with pure retailers.

*Related literature.* The literature on competition with price discrimination has almost exclusively focused on inter-brand competition, that is, competition between completely independent firms. In their seminal paper, Thisse and Vives (1988) analyze the effects of price discrimination for horizontally differentiated firms competing on a Hotelling line. They demonstrate that this leads to a prisoner’s dilemma: firms adopt price discrimination but profits fall due to increased competition.<sup>5</sup> Shaffer and Zhang (1995) highlight a similar prisoner’s dilemma when firms discriminate through coupon targeting and consumers differ in the cost of redeeming coupons. Chen and Iyer (2002) allow firms to choose the proportion of consumers for whom they acquire information. In this case, firms may benefit from consumer addressability and may refrain from acquiring full information.<sup>6</sup>

Choudhary *et al.* (2005) consider instead competition between vertically differentiated firms, and find that pricing strategies can be non-monotonic in consumer valuations. Montes *et al.* (2019) and Chen *et al.* (2020) analyze models in which consumers can prevent firms from exploiting information about their preferences. They show that this possibility can harm consumers and allow firms to benefit from price discrimination.<sup>7</sup>

Our paper studies instead the implications of personalized pricing on intra-brand competition, wholesale contracting, and on the choice of the optimal distribution strategy. To the best of our knowledge, the only two papers analyzing the effects of price discrimination on distribution channels are Liu and Zhang (2006) and Li (2018).

Liu and Zhang (2006) consider a setting in which only the retailer has access to personalized pricing and the manufacturer can open a direct channel charging a uniform price. They show that the adoption of personalized pricing harms the retailer by inducing the manufacturer to charge a higher wholesale price, but can nevertheless be profitable if it deters the manufacturer from entering the downstream market.

Li (2018) determines how behavior-based pricing shapes competition between two manufacturers that sell their products through exclusive retailers.<sup>8</sup> She shows that

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<sup>5</sup>Liu and Serfes (2013) extend the framework of Thisse and Vives (1988) by studying the effects of price discrimination in two-sided markets. Matsumura and Matsushima (2015) show instead that firms may choose not to price discriminate in order to limit rivals’ incentives to engage in cost reduction.

<sup>6</sup>Shaffer and Zhang (2002) show that firms offering higher quality may benefit from personalized pricing, even though competition is fiercer. This is due to a gain in market share, which dominates the effect of lower prices.

<sup>7</sup>For empirical papers on estimates for the profitability of personalized pricing relative to uniform pricing in different set-ups, see e.g. Rossi *et al.* (1996), Shiller (2020), and Dubé and Misra (2021).

<sup>8</sup>Behavior-based price discrimination refers to the practice of charging consumers different prices

channel performance crucially depends on whether only retailers can adopt behavior-based pricing or manufacturers can do so as well.

In contrast to these papers, we focus on an integrated manufacturer's decision to allow a retailer to enter the market, and study the implications of pricing strategies on this decision and on channel performance.

In the strategy literature, the importance of the distribution network and of the supply chain on firm performance has been recognized in several papers—see, e.g., Lassar and Kerr (1996) and Hult *et al.* (2004, 2007). These studies provide empirical contributions, focusing on agency costs or the culture of competitiveness. We show that the pricing instruments, which have radically changed due to increased data availability, may be equally important for the performance of a distribution channel.

Finally, our paper also contributes to the literature on market foreclosure. Several papers show that a vertically integrated firm has the incentive to raise wholesale prices to non-integrated downstream rivals to dampen price competition (see e.g., Salinger, 1988, Ordover *et al.*, 1990, Hart and Tirole, 1990, Chen, 2001, and Bourreau *et al.*, 2011).<sup>9</sup> Our paper contributes to this literature by determining how the possibility of price discrimination affects the integrated firm's choice of whether or not to deny a retailer access to its products.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 considers a simple setting in which consumers' valuations for the two offers are independently distributed. Section 4 analyzes the case where these valuations are negatively correlated, as in the classic Hotelling model of horizontal differentiation. Section 5 considers instead the case of positive correlation, which covers models of vertical differentiation with asymmetric costs. Section 6 introduces the possibility of delegated distribution, that is, mono distribution by the retailer. Finally, Section 7 draws managerial implications and concludes. The proofs of Propositions 1 and 2 are in the Appendix. The Online Appendix contains all other proofs and some additional material.

## 2 The Model

*Supply.* A monopoly manufacturer, firm  $A$ , sells its good to final consumers through a direct distribution channel. In addition, it can also use an independent retailer, firm  $B$ , and choose a dual distribution strategy.<sup>10</sup> In order to highlight the strategic mo-

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dependent on their purchase history, see e.g. Acquisti and Varian (2005), Fudenberg and Tirole (2000), or Choe *et al.* (2018).

<sup>9</sup>Rey and Tirole (2007) provide an overview of this literature.

<sup>10</sup>In Section 6, we also consider the possibility that the manufacturer shuts down its direct channel and distributes only through the independent retailer—i.e., follow a strategy of delegated distribution.

tive for mono or dual distribution, we assume away any fixed costs of opening a new distribution channel. For simplicity, all variable costs are also assumed to be zero.<sup>11</sup>

*Demand.* The two firms offer differentiated distribution services over which final consumers have heterogeneous preferences. Specifically, consumers are indexed by  $x \in [0, X]$  and their utilities from  $A$ 's and  $B$ 's offerings (the “products”, thereafter) are respectively given by:

$$u_A(x) = \max\{1 - x, 0\} \quad \text{and} \quad u_B(x) = \max\{v - sx, 0\},$$

where  $0 < v < 1$  and  $s < 1$ —we allow  $s$  to be negative.<sup>12</sup>

We assume that both products play an effective role. Specifically, letting

$$\hat{x} \equiv \frac{1 - v}{1 - s}$$

denote the consumer type who derives the same value from both products, and

$$\hat{u} \equiv \frac{v - s}{1 - s}$$

denote the corresponding valuation, we maintain the following assumption:

$$\hat{u} > 0 \text{ and } \hat{x} < X. \tag{1}$$

The first condition ensures that both products offer value to some consumers (namely, those close to  $\hat{x}$ ). The second condition then ensures that no product “dominates” the other.

$A$  thus offers a higher utility than  $B$  to consumers  $x < \hat{x}$  whereas  $B$  offers a higher utility to consumers  $x$  with  $\hat{x} < x \leq X$ . For future reference, let

$$\alpha = \int_0^{\hat{x}} [u_A(x) - u_B(x)] dx$$

denote  $A$ 's contribution to the industry monopoly profit,<sup>13</sup> and likewise let

$$\beta = \int_{\hat{x}}^X [u_B(x) - u_A(x)] dx$$

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<sup>11</sup>Introducing constant unit costs would not affect the results as long as no channel dominates the other.

<sup>12</sup>Normalizing to 1 the intercept and the slope of  $u_A(x)$  helps streamlining the exposition. In Online Appendix B, we show that our insights carry over to the case where  $v$  and  $s$  exceed 1; see also the discussion in footnote 14.

<sup>13</sup>That is, the profit that a monopolist controlling both distribution channels could generate under perfect price discrimination.

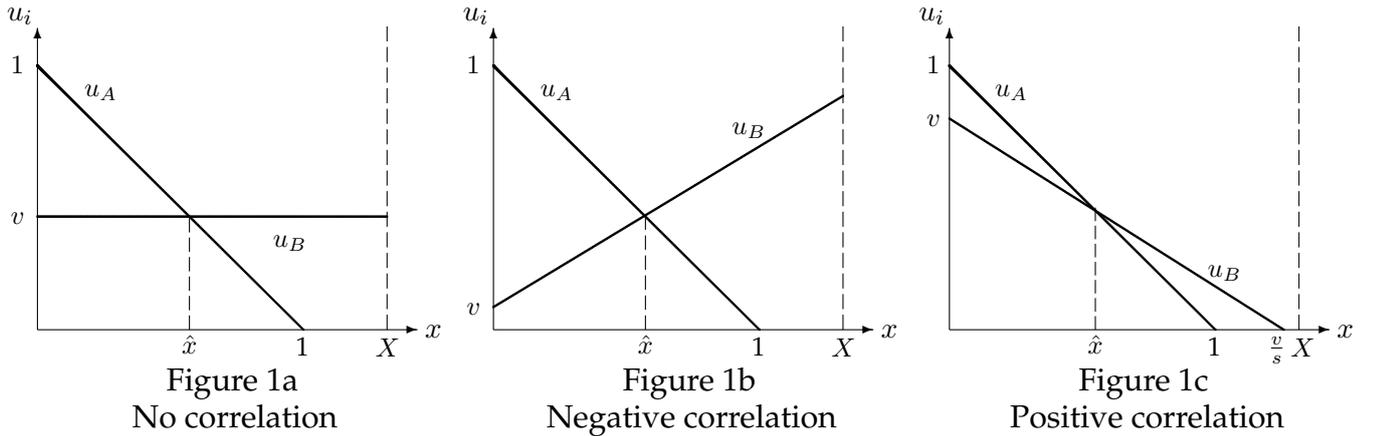
denote  $B$ 's contribution.

Our model is flexible enough to capture different patterns of consumer preferences over the services offered by the manufacturer's direct channel and by the retailer.

If  $s = 0$ , the valuation for  $B$ 's product is the same for all consumers, which implies that there is *no* correlation between the valuations of the two products (see Figure 1a). A practical example could be where consumers have the same valuation for the good but the manufacturer sells it at a physical location, and so consumers face different transportation costs depending on their own location, whereas the distributor sells the good online with the delivery costs being the same for all consumers.

If instead  $s < 0$ , consumers' valuations for the two products are *negatively* correlated (see Figure 1b). An example could be where consumers with the same valuation for the good have heterogeneous preferences for the manufacturer's and the retailer's services. For instance, some customers may particularly value the professional advice or the free testing offered by the manufacturer's channel, whereas others may favor the generous return policies, recommendations, and user feedback offered by the retailer (see, e.g., Acquisti and Varian, 2005).

Finally, if  $s > 0$ , the consumers' valuations are *positively* correlated (see Figure 1c). This may correspond to a situation where consumers who have a higher valuation for the good also have a stronger preference for the manufacturer's channel (e.g., because they regularly visit the manufacturer's store), whereas the others find the retailer's channel more attractive (e.g., because they are active there and discover the manufacturer's good only through browsing).<sup>14</sup>



**Figure 1: Consumers' valuations**

Our setting encompasses the classic models of product differentiation. For exam-

<sup>14</sup>The restriction  $v < 1$  (which, given (1), implies  $s < 1$ ) reflects the assumption that the manufacturer caters to high-valuation consumers; the alternative case (i.e.,  $v > 1$ , implying  $s > 1$ ) is studied in Online Appendix B.

ple, in the Hotelling model of horizontal differentiation, the firms are located at the two ends of a unit-length segment, along which consumers are uniformly distributed, and consumers have a uniform valuation  $V$  for the good but face a constant transportation cost  $t$  per unit of distance. Consumers' utilities are therefore negatively correlated, as consumers who are closer to one product are farther away from the other one.<sup>15</sup>

To take another example, consider a model of vertical differentiation à la Shaked and Sutton (1982), in which firm  $i$  offers quality  $q_i$  at unit cost  $c_i$ , and a consumer of type  $\theta$  derives a utility  $\theta q_i$  from product  $i$ . Consumers' utilities are then positively correlated, as those who value quality more are willing to pay more for both products. Furthermore, if higher quality is more costly, high-valuation consumers are more profitable for the firm with the higher quality, whereas low-valuation consumers are more profitable for the low-quality firm.<sup>16</sup>

*Retail competition.*  $A$  and  $B$  compete in prices for consumers;  $B$  maximizes its retail profit, whereas  $A$  maximizes its total profit, including the wholesale profit.

For each firm, we consider two types of pricing policies: uniform pricing (non-discrimination), in which the firm charges the same price to all consumers, and personalized pricing, in which the firm can perfectly discriminate consumers according to their types. Firm  $i$ 's price is denoted by  $p_i$  under uniform pricing and by  $p_i(x)$  under personalized pricing. Combining firms' pricing policies, there are in total four different pricing regimes: two symmetric regimes (i.e., both firms charge a uniform price, or both firms charge personalized prices), and two hybrid regimes (i.e.,  $A$  charges a uniform price and  $B$  personalized prices, or vice versa).

*Wholesale contracting.* We consider two-part tariffs of the form  $T(q) = F + wq$ , where  $F$  denotes the fixed fee and  $w$  the uniform wholesale price paid by  $B$  to  $A$ , and  $q$  is the quantity bought by  $B$ . We suppose that the outcome of the negotiation over  $w$  and  $F$  is given by the Nash bargaining solution with bargaining power  $a \in [0, 1]$  for  $A$  and  $1 - a$  for  $B$ .

*Timing.* As discussed in the Introduction, the pricing regime is driven by technologies and data considerations: personalized pricing may be available in some industries, and not in others. This leads us to treat the pricing regime as exogenous. However, if personalized pricing is available, firms may have the option of not using it. For this

<sup>15</sup>For instance, the case where  $V = t = 1$  corresponds to our model with  $x$  representing the consumer's location along the segment,  $v = 0$ , and  $s = -1$ .

<sup>16</sup>Consider for instance the case where  $A$  offers  $q_A = 1$  at cost  $c > 0$ ,  $B$  offers  $q_B = q < 1$  at zero cost, and  $\theta$  is uniformly distributed between 0 and  $1 + c$ . This corresponds to our model with  $x = 1 + c - \theta$ ,  $s = q$ ,  $v = q(1 + c)$ , interpreting  $p_A$  as firm  $A$ 's net margin (i.e., net of its cost  $c$ ) and  $u_A(x)$  as the net value of its product. Indeed, we then have  $u_A(x) = \theta q_A - c = (1 + c - x) \times 1 - c = 1 - x$  and  $u_B(x) = \theta q_B = (1 + c - x)q = v - sx$ . Papers on inter-brand competition mostly focus on the case in which costs are the same for the two firms, implying that product  $A$  dominates product  $B$ ; however, for intra-brand competition, mono distribution is always optimal in that case.

reason, we also consider an extension where firms can contract as well on their retail pricing policies.

The timing of the game is as follows:

- Stage 1:  $A$  and  $B$  negotiate the wholesale contract.
- Stage 2: Active firms set their retail prices. Consumers then observe all retail prices and decide whether or not to buy, and from which firm to buy. If active,  $B$  then orders the quantity from  $A$  to satisfy its demand.

In the first stage, firms can share their joint profit through the fixed fee; hence, they seek to maximize the industry profit. Dual distribution thus constitutes the optimal strategy if it generates a higher industry profit than mono distribution.<sup>17</sup>

In the second stage, for symmetric pricing regimes, firms simultaneously set their prices. For asymmetric pricing regimes, we follow Thisse and Vives (1988), Liu and Zhang (2006), and Choe *et al.* (2018) in assuming that the firm charging a uniform price, say firm  $i$ , acts as a price leader: it sets  $p_i$  before the competitor sets its personalized prices  $p_{-i}(x)$ . This assumption ensures the existence of a pure-strategy Nash equilibrium. As pointed out by Thisse and Vives (1988), it is natural for asymmetric regimes, as firm  $i$  can announce and advertise its uniform price in advance, whereas this may be too complex or costly for the competitor. In addition, as noted by Choe *et al.* (2018) and Chen *et al.* (2020), the adjustment of a uniform price is a higher-level managerial decision, that is slower in practice than the adjustment of personalized prices.

*Solution concept.* Our solution concept is subgame perfection.<sup>18</sup> In the case of price discrimination, asymmetric Bertrand competition for each consumer  $x$  is known to generate multiple equilibria. Following the literature, we focus on the equilibrium in which the firm offering the lower value prices at cost.<sup>19</sup>

*Remark: wholesale personalized pricing.* We focus on the case in which personalized pricing may be possible at the retail but not at the wholesale level. That is, the wholesale tariff cannot be conditioned on consumers' types. While this would allow the firms to maximize the industry profit, it is usually infeasible. First, manufacturers are often unable to monitor which consumers their retailers are selling to; and even if they

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<sup>17</sup>For the sake of exposition, we assume that dual distribution arises only if it generates strictly more profit. Introducing an arbitrary negotiation cost would support this tie-breaking rule.

<sup>18</sup>Formally, subgame perfection applies from stage 2 onwards. In stage 1, Nash bargaining could also be achieved as the equilibrium of a non-cooperative random-proposer game in which each firm gets to make a take-it-or-leave-it offer with a probability reflecting its bargaining power. To obtain a deterministic outcome, it suffices to introduce a preliminary stage in which one firm (either one) makes an offer, with the random-proposer game acting as default option.

<sup>19</sup>This is the unique Coalition-Proof Nash equilibrium (in particular, it is the Pareto-dominant equilibrium from the firms' standpoint) and is also the unique trembling-hand perfect equilibrium.

could obtain that information, it would be difficult to verify it in a court of law. Second, through direct interaction with its customers, the retailer may have access to data that is not available to the manufacturer.

### 3 Independent valuations

We start with the situation in which there is no correlation between consumers' valuations for the two products:  $s = 0$ . As is evident from Figure 1a, dual distribution enhances demand. At the same time,  $B$  may cannibalize  $A$ 's sales and the resulting intra-brand competition may dissipate profits. However, this concern can be mitigated via the wholesale contract—namely, by adjusting appropriately the wholesale price paid by  $B$ . The following proposition shows that this instrument is indeed effective, to the point that it enables the firms to obtain the industry monopoly profit; as a result, dual distribution is always optimal.

**Proposition 1** *If  $s = 0$ , then in all pricing regimes the firms can achieve the integrated monopoly outcome; hence, dual distribution is the optimal strategy.*

As consumers' valuations for product  $B$  are constant, firms can avoid competition by setting the wholesale price at that level:  $w = v$ . This induces  $B$  to charge a retail price of  $v$  to all consumers, regardless of whether it can personalize its prices or not: any higher price would discourage consumers, and any lower price would generate a loss.  $B$  therefore offers zero net value to consumers, which eliminates the risk of cannibalization. If  $A$  can charge personalized prices, it can then extract all the surplus from consumers  $x < \hat{x}$ , and let  $B$  extract all the surplus from the other consumers. If instead  $A$  must charge a uniform price, it will set the optimal price that a monopolist controlling both channels would choose, as it can secure through the wholesale price the full surplus (namely,  $v$ ) of the consumers served by  $B$ . In both cases,  $A$  obtains (gross of the fixed fee) the industry monopoly profit given the pricing regime; dual distribution is therefore optimal.

As the industry profit is maximal when  $A$  charges personalized prices, firms will choose this regime if they can contract on it.

### 4 Negative correlation of valuations

We next turn to the situation of negative correlation between consumers' valuations:  $s < 0$ . The next proposition shows that the wholesale price provides again an effective tool for controlling intra-brand competition:

**Proposition 2** *If  $s < 0$ , then in all pricing regimes the firms can achieve the integrated monopoly outcome; hence, dual distribution is the optimal strategy.*

When consumers' valuations are negatively correlated, firms can again control intra-brand competition and obtain the industry monopoly profit, regardless of the pricing regime.<sup>20</sup> For instance, when both firms can charge personalized prices, agreeing on a wholesale price  $w = \hat{u}$  (i.e., equal to the valuation of the indifferent consumer  $x = \hat{x}$ ) eliminates any risk of cannibalization while allowing each firm to extract all the surplus from its core market:  $B$  cannot profitably offer any positive value to consumers  $x < \hat{x}$  (for whom  $u_B(x) < \hat{u} = w$ ), and so  $A$  can charge them the full value (i.e.  $p_A(x) = u_A(x)$ ); conversely,  $A$  has no incentive to serve consumers  $x > \hat{x}$  (for whom  $u_A(x) < w$ ), as it earns more through the wholesale price, and so  $B$  can extract all their surplus (by charging them  $p_B(x) = u_B(x)$ ).

The same logic carries over to the other pricing regimes: while double marginalization may call for a wholesale price lower than  $\hat{u}$ , maintaining it at a high enough level still ensures that the consumer indifferent between the two firms obtains zero utility. This in turn avoids any risk of cannibalization, while allowing each firm to generate as much profit from its own market segment as an integrated monopolist would.

As the industry profit is maximal when both channels charge personalized prices, the firms will choose this pricing regime if they can contract on it. The associated profits generated are illustrated in Figure 2. The hatched area to the left of  $\hat{x}$  represents the retail profits of  $A$ . On the right of  $\hat{x}$ , the bottom rectangular area is the wholesale profit of  $A$ , whereas the upper triangular area is the retail profit of  $B$ . Together, these areas represent total consumer surplus, which is fully extracted.

That personalized pricing allows firms to maximize the industry profit is in sharp contrast to the findings of classic papers on personalized pricing in the context of *inter-brand* competition (see, e.g., Thisse and Vives, 1988, and Shaffer and Zhang, 1995). In that case—which, in our setting, amounts to imposing a cost-based tariff (namely,  $w = F = 0$ )—the possibility of personalized pricing leads to a prisoner's dilemma in which firms end-up choosing personalized pricing even though the industry profit is lower than with uniform pricing. By contrast, in the case of *intra-brand* competition between distribution channels, a well-designed wholesale tariff enables the firms to perform better under personalized pricing—and even to extract the entire consumer surplus. In other words, the common wisdom for inter-brand competition does not apply to intra-brand competition.

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<sup>20</sup>As the proof of Proposition 2 makes clear, this insight does not depend on the linearity of the demand specification.

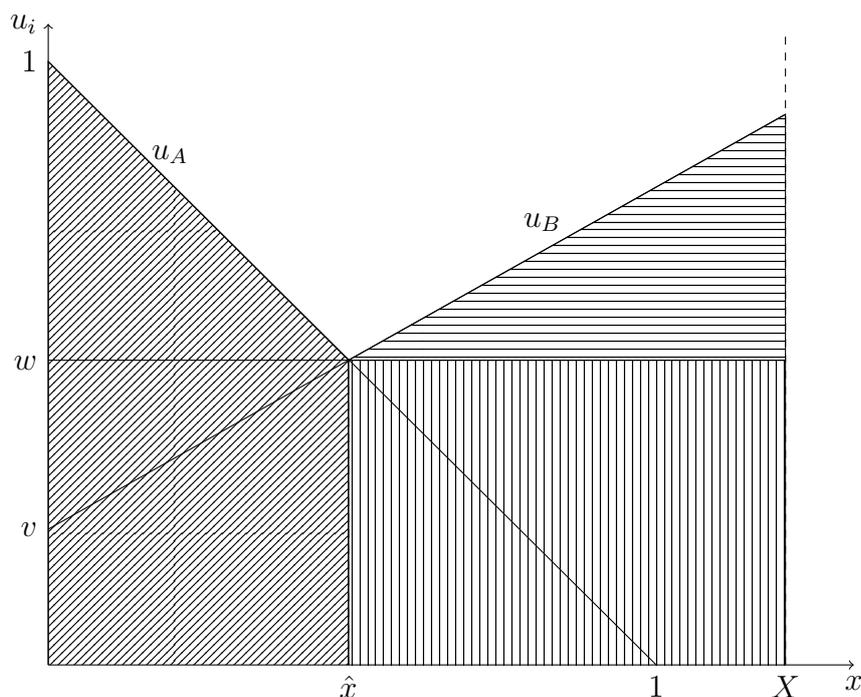


Figure 2: Industry profit with personalized pricing by both firms

## 5 Positive correlation of valuations

In this section, we consider the situation in which consumers' valuations are positively correlated:  $s > 0$ . The conditions  $\hat{x} > 0$  and  $\hat{u} > 0$  then imply  $v > s$ . To simplify the exposition, we restrict attention to the case in which  $X \geq v/s$ , that is,  $X$  is large enough that it does not limit the demand for  $B$ .<sup>21</sup> None of our qualitative results hinges on this assumption, but it helps to convey our insights in a concise way.

With positive correlation, the *same* consumers (i.e., those with a low  $x$ ) have the highest valuations for *both* firms' products. Therefore, it is no longer possible to find a wholesale price that entirely eliminates intra-brand competition and yet allows  $B$  to enhance total demand. We show below that intra-brand competition remains manageable when  $A$  charges uniform prices (Section 5.1), but becomes a more serious issue when  $A$  can charge personalized prices (Section 5.2).

### 5.1 Uniform Pricing by $A$

The next proposition shows that dual distribution remains optimal under uniform pricing by  $A$ :

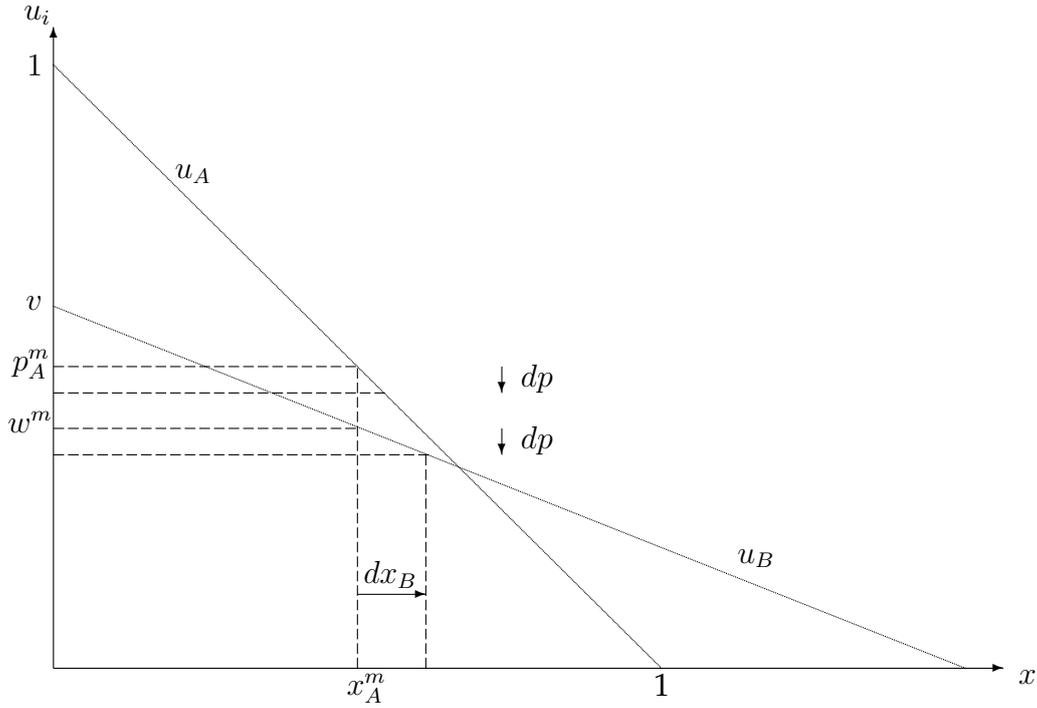
**Proposition 3** *If  $s > 0$  and  $A$  charges a uniform price, dual distribution is the optimal strategy, regardless of  $B$ 's pricing policy.*

<sup>21</sup>I.e., any  $x$  with  $v - sx \geq 0$  is such that  $x \leq X$ .

Proposition 3 establishes that, as long as  $A$  charges uniform prices, there always exists a wholesale price that appropriately limits the risk of cannibalization while allowing  $B$  to expand consumer participation. We sketch below the underlying arguments.

If only  $A$  is active, it faces the monopoly demand  $1 - p_A$ ; it thus charges the monopoly price  $p_A^m = 1/2$ , serves consumers  $x \leq x_A^m = 1/2$ , and obtains a profit of (the subscript  $U$  stands for *Uniform pricing*)  $\Pi_U^m = 1/4$ .

Consider now the situation, illustrated by Figure 3, in which  $A$  and  $B$  are both active and charge a uniform price. To see that dual distribution leads to a higher industry profit than mono distribution, note first that the firms can replicate the outcome of mono distribution by setting  $w = w^m \equiv u_B(x_A^m)$ . This enables  $A$  to charge the monopoly price  $p_A^m$  by preventing  $B$ , which must charge at least  $w^m$ , from profitably attracting any consumer. Indeed, consumers with  $x > x_A^m$  are not willing to pay  $w^m$ , and those with  $x < x_A^m$  prefer  $A$ 's product at price  $p_A^m = u_A(x_A^m)$  to  $B$ 's product at any price  $p_B \geq w^m = u_B(x_A^m)$ .



**Figure 3:** Uniform Pricing

Consider now a small reduction in the wholesale price,  $w < w^m$ . This generates a retail equilibrium in which  $B$  serves some consumers at price  $p_B = w^m - dp$ . In this retail equilibrium,  $B$  cannot obtain a negative profit and  $A$  cannot obtain less than what it would earn by charging  $\hat{p}_A = p_A^m - dp = u_A(x_A^m) - dp$ , so as to maintain its market share,  $x_A = x_A^m$ . Doing so would lead  $B$  to sell a quantity  $dx_B$  implicitly given by  $dp \equiv -u'_B(x_A^m) dx_B$ . Hence, the industry profit cannot be lower than  $\pi_A + \pi_B \geq [(p_A^m - dp) x_A^m + w dx_B] + 0 \simeq \Pi_U^m + u'_B(x_A^m) x_A^m dx_B + u_B(x_A^m) dx_B =$

$\Pi_U^m + \frac{d}{dx} [u_B(x)x] \Big|_{x=x_A^m} dx_B$ , which exceeds  $A$ 's monopoly profit,  $\Pi_U^m$ : as  $B$  caters to low-valuation consumers, it faces a more elastic monopolistic demand (that is,  $|u'_B(x)|/u_B(x) < |u'_A(x)|/u_A(x)$ );<sup>22</sup> hence, its monopolistic output exceeds  $x_A^m$  (that is,  $\frac{d}{dx} [u_B(x)x] \Big|_{x=x_A^m} > 0$ ).

This shows that the industry profit is always larger if  $B$  is marginally active. This insight does not hinge on the demand being linear; it holds more generally as long as  $B$ 's monopolistic output exceeds that of  $A$ .<sup>23</sup> Note, however, that the *equilibrium* wholesale price may be substantially lower than  $w^m$ , and so  $B$ 's market share may be substantial.

Consider now the regime in which  $A$  still charges a uniform price but  $B$  charges personalized prices. The intuition why dual distribution is optimal in this case is illustrated by Figure 4, which depicts the equilibrium prices under uniform pricing,  $p_A^*$  and  $p_B^*$ , and the retail prices of  $B$  that would emerge if the two firms opted for dual distribution and set  $w = p_B^*$  and  $p_A = p_A^*$ .<sup>24</sup>

$A$  then serves consumers  $x < x_A^*$  (for whom  $u_A(x_A^*) - u_B(x_A^*) = w$ ), whereas  $B$  serves consumers between  $x_A^*$  and  $x_B^*$  (for whom  $u_B(x_B^*) = w$ ), and charges them prices equal to  $\min \{p_A^* + u_B(x) - u_A(x), u_B(x)\}$ . The resulting industry profit is larger than under uniform pricing: in the segment served by  $A$ , the profit is the same because  $p_A = p_A^*$ ; by contrast, in the segment served by  $B$ ,  $B$  charges a strictly higher price than  $p_B^*$ . Moreover, given the wholesale tariff  $w = p_B^*$ ,  $A$  would find it optimal to set a higher retail price than  $p_A^*$ ,<sup>25</sup> implying an even higher profit for both  $A$  (by revealed preferences) and  $B$  (due to weaker competitive pressure). Because opting for dual distribution was already optimal with uniform pricing, and yields even more profits in the regime with personalized pricing by  $B$ , it also dominates mono distribution in the latter regime.

## 5.2 Personalized Pricing by $A$

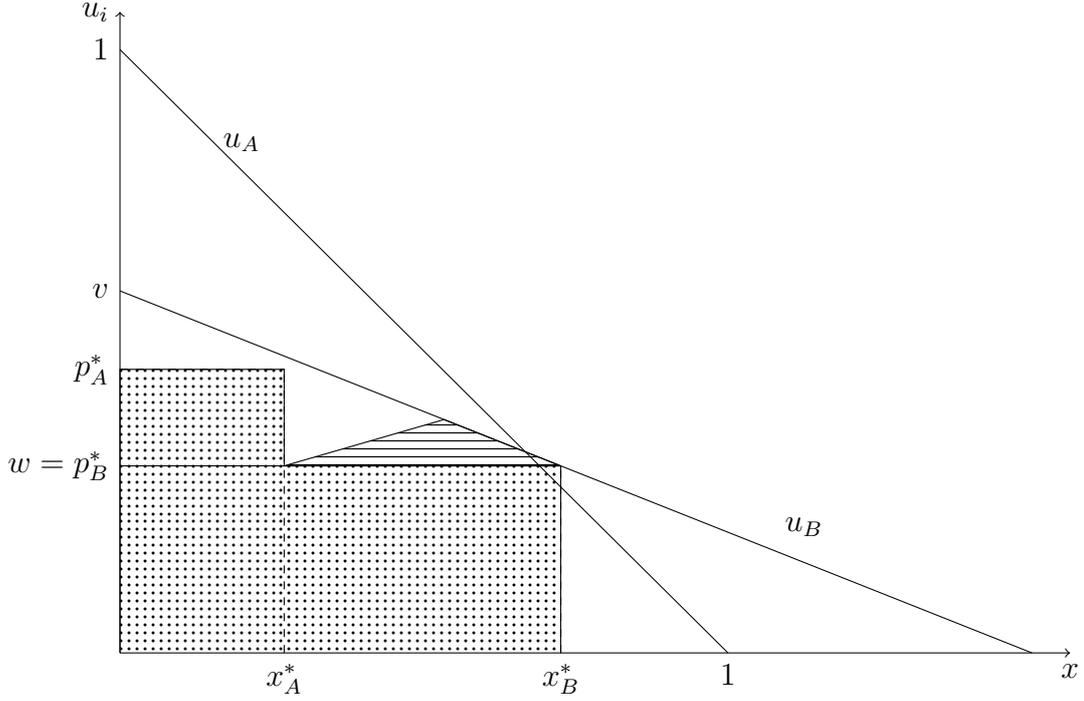
Intra-brand competition becomes a more serious issue when  $A$  can charge personalized prices; indeed, the next proposition shows that dual distribution is then optimal only when  $B$ 's contribution to the industry profit is large enough:

<sup>22</sup>This is due to the fact that  $v > s$ .

<sup>23</sup>See Online Appendix C.

<sup>24</sup>Remember that  $A$  acts as a price leader in this regime:  $w$  and  $p_A$  are chosen before  $B$  sets its retail prices.

<sup>25</sup>Increasing  $p_A$  by  $dp_A$  reduces the demand by  $dx_A$  (which, given the sequential timing adopted for pricing decisions and  $w = p_B^*$ , is the same as when both firms charge uniform prices); it thus increases profit by  $x_A^* dp_A + (p_A^* - w) dx_A > 0$ , where the inequality stems from the fact that (i)  $p_A^*$  constitutes  $A$ 's best-response under uniform pricing, and (ii) the wholesale price is here higher than the equilibrium wholesale price  $w_{UU}^*$  that sustains  $p_A^*$  and  $p_B^*$  under uniform pricing (i.e.,  $w = p_B^* > w_{UU}^*$ ).



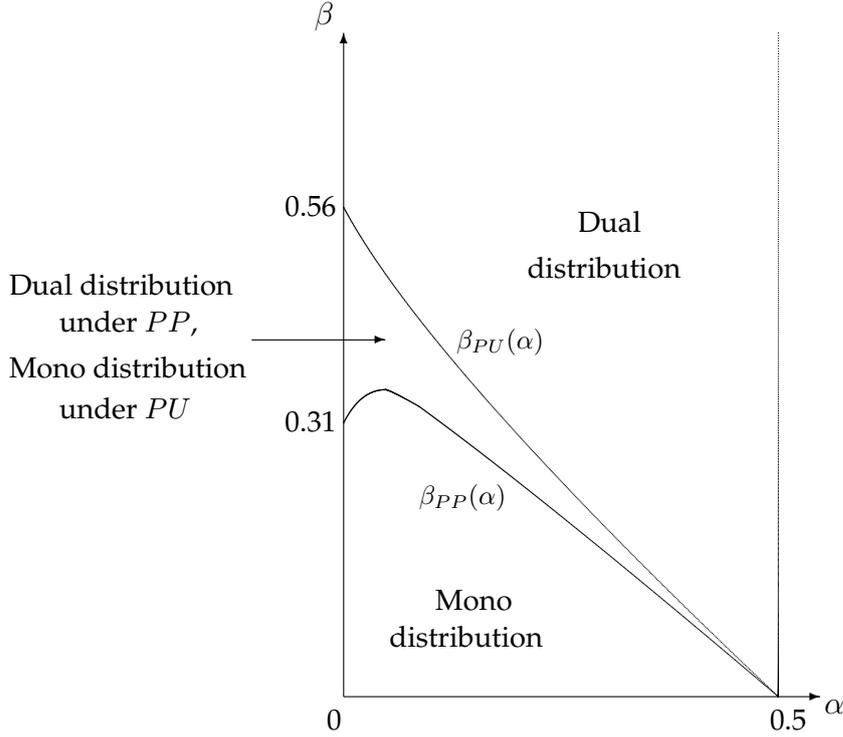
**Figure 4:** Profit with personalized pricing by  $B$  only, if  $w = p_B^*$  and  $p_A = p_A^*$

**Proposition 4** *If  $s > 0$  and  $A$  charges personalized prices, then:*

- (i) *dual distribution is optimal only when  $B$ 's contribution to the industry profit is sufficiently large: for any  $B$ 's pricing policy  $Z \in \{P, U\}$ , there exists a threshold  $\beta_{PZ}(\alpha)$  such that dual distribution is the optimal strategy if and only if  $\beta > \beta_{PZ}(\alpha)$ .*
- (ii) *dual distribution is more likely to be optimal when  $B$  offers personalized prices:  $\beta_{PU}(\alpha) > \beta_{PP}(\alpha)$ .*

The equilibrium configurations under personalized pricing by  $A$  are depicted in Figure 5.<sup>26</sup> Dual distribution remains an optimal strategy only when  $B$  brings a sufficiently large added-value (i.e.,  $\beta$  is large enough). That dual distribution may no longer be optimal is particularly true when  $B$  charges a uniform price (i.e.,  $\beta_{PU}(\alpha) > \beta_{PP}(\alpha)$ ). Interestingly, the threshold values are for the most part decreasing in  $\alpha$  (the only exception concerns  $\beta_{PP}(\alpha)$  for very low values of  $\alpha$ ); hence, keeping  $\beta$  constant, dual distribution is more likely to be optimal when  $A$ 's contribution to the industry profit is large enough. This is because a large  $\alpha$  means that  $B$  offers a low utility to those consumers who can already be served by  $A$ , and contributes to the industry profit mostly by expanding demand from low-value consumers; hence, the risk of cannibalization becomes more limited.

<sup>26</sup>Since  $u_A(x) = 1 - x$ ,  $\alpha < 0.5$ .



**Figure 5:** Equilibrium configurations in the pricing regimes  $PP$  and  $PU$

We sketch below the analysis underlying Proposition 4. Under mono distribution,  $A$  charges each consumer  $x$  a price  $u_A(x)$  and thus obtains a profit of (the subscript  $P$  standing for *Personalized pricing*)  $\Pi_P^m = \int_0^{\bar{x}^A} (1-x) dx = 1/2$ . We now compare this profit to that obtained under dual distribution, starting with the symmetric regime in which both firms charge personalized prices.

*Retail competition.* As firms now compete for each consumer  $x$ , three cases can arise.

If  $u_B(x) < w$ , then  $B$  cannot offer a positive value to consumers without incurring a loss;  $A$  then charges the monopoly price  $p_A = u_A(x)$ .

If instead  $u_A(x) < w \leq u_B(x)$ ,  $A$  would have to price below  $w$  to win the consumer, and is therefore better off letting  $B$  serve this consumer; hence,  $B$  wins the competition by charging the monopoly price  $u_B(x)$  (and  $A$  charges a price equal to its opportunity cost  $w$ ).

Finally, when  $w \leq u_A(x), u_B(x)$ ,  $A$ 's profit from such a consumer type is either  $p_A(x)$ , if  $A$  serves the consumer itself, or  $w$ , if instead  $B$  serves the consumer. Thus,  $w$  constitutes  $A$ 's opportunity cost of serving the consumer as well as  $B$ 's real cost. A standard Bertrand argument then applies: for consumers  $x$  with  $u_i(x) > u_j(x)$ , for  $i \neq j \in \{A, B\}$ , firm  $i$  wins the competition and sells to the consumer at price  $p_i(x) = w + u_i(x) - u_j(x)$ , whereas the other firm sets  $p_j(x) = w$ .

*Wholesale negotiation.* We now turn to the determination of the wholesale contract. We

first note that for any wholesale price  $w$  above  $\hat{u}$ ,  $B$  is inactive in equilibrium: it is dominated by  $A$  in the consumer segment  $x < \hat{x}$ , and cannot offer a positive value at a profitable price in the segment  $x > \hat{x}$ . The profit thus cannot exceed  $\Pi_P^m$ .

If the firms negotiate a wholesale price  $w \leq \hat{u}$ , they are both active in the continuation equilibrium. Let:

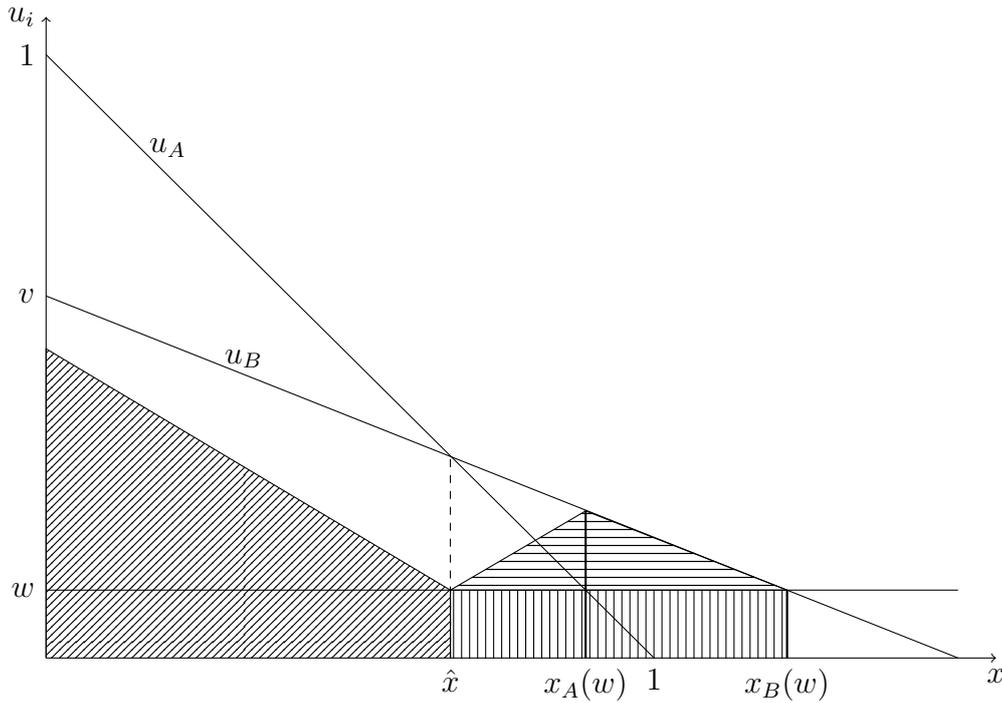
$$x_A(w) \equiv 1 - w \quad \text{and} \quad x_B(w) \equiv \frac{v - w}{s}, \quad (2)$$

denote the marginal consumers willing to buy product  $A$  and  $B$ , respectively, at price  $w$ . The profits of the two firms at the retail stage can be expressed as  $\Pi_A$  and  $\Pi_B$ , where:<sup>27</sup>

$$\Pi_A = \int_0^{\hat{x}} [w + u_A(x) - u_B(x)] dx + w [x_B(w) - \hat{x}],$$

and:

$$\Pi_B = \int_{\hat{x}}^{x_A(w)} [u_B(x) - u_A(x)] dx + \int_{x_A(w)}^{x_B(w)} [u_B(x) - w] dx.$$



**Figure 6:** Industry profit in the pricing regime  $PP$

These profit functions are illustrated by Figure 6, where the hatched area represents the industry profit. The first term in  $A$ 's profit comes from consumers  $x < \hat{x}$  (first region in Figure 6):  $A$  offers them a higher net value, and serves them at price  $w +$

<sup>27</sup>As we show in the proof of Proposition 4, conditional on reaching an agreement (i.e.,  $w < \hat{u}$ ), the firms negotiate a wholesale price so that  $B$  expands potential demand; that is,  $B$  sells to consumers who would not be interested in buying from  $A$  at any positive price (i.e.,  $w$  is sufficiently low that  $x_B(w) > 1$ ).

$u_A(x) - u_B(x)$ . The second term in  $A$ 's profit reflects the wholesale revenue generated by consumers served by  $B$  (the rectangular area between  $\hat{x}$  and  $x_B(w)$  in the figure).  $B$ 's profit comes from consumers for whom it offers a higher net value, and can also be split in two parts. The first term corresponds to consumers  $\hat{x} < x < x_A(w)$  (second triangle), for whom both firms compete, and so  $B$  only earns a margin  $u_B(x) - u_A(x)$ . The second term corresponds to consumers  $x_A(w) < x < x_B(w)$  (third triangle), to whom  $A$  offers a lower net value than  $w$ , and so  $B$  can extract the full value and earn a margin  $u_B(x) - w$ .

At the negotiation stage, the firms maximize the industry profit:

$$\Pi(w) = \int_0^{x_A(w)} [w + |u_B(x) - u_A(x)|] dx + \int_{x_A(w)}^{x_B(w)} u_B(x) dx.$$

Taking the derivative with respect to  $w$  (and using  $u_i(x_i(w)) = w$  for  $i = A, B$ ) yields:

$$\Pi'(w) = x_A(w) + wx'_B(w).$$

When choosing the wholesale price  $w$ , firms face a trade-off. By increasing  $w$ , the firms obtain a higher benefit from the infra-marginal consumers in the range  $x < x_A(w)$ : as the two firms compete for these consumers, an increase in  $w$  increases the final consumer price by the same amount. However, increasing  $w$  has also a negative effect on the marginal consumer,  $x_B(w)$ , for whom  $B$  can charge the full value,  $v_B(x_B(w)) = w$ . By contrast, the revenue from consumers between  $x_A(w)$  and  $x_B(w)$  is unchanged, as these consumers continue buying from  $B$  and pay their reservation price.

Using (2), the first-order condition  $x_A(w) + wx'_B(w) = 0$  yields<sup>28</sup>

$$w_{PP}^* = \frac{s}{1+s}.$$

The associated industry profit is:

$$\Pi_{PP}^* = \frac{s(1+3s) - 4s(1+s)v + (1+s)^2 v^2}{2s(1-s^2)}, \quad (3)$$

which is larger than the monopoly profit  $\Pi_P^m = 1/2$  if and only if:

$$v > \frac{s}{1+s} (2 + \sqrt{1-s}), \quad (4)$$

where the right-hand side increases with  $s$ . It follows that dual distribution is optimal when  $v$  is large and/or  $s$  is small, that is, when  $B$ 's contribution to the industry profit,  $\beta$ , is large; the formal analysis shows that this condition indeed holds when  $\beta$  is above

<sup>28</sup>The profit function is concave as  $\Pi''(w) = -(1 + 1/s) < 0$ .

a certain threshold, which depends only on  $A$ 's own contribution to the industry profit,  $\alpha$ .

Proposition 4 shows that, contrary to the case of uniform pricing by  $A$ , mono distribution may prevail when both firms charge personalized prices. This may appear surprising, as personalized pricing enables the firms to share the market efficiently: consumers  $x < \hat{x}$  (resp.,  $x > \hat{x}$ ) buy from  $A$  (resp.,  $B$ )—when instead  $A$  charges a uniform price,  $B$  attracts some consumers from  $A$ 's core market. However, personalized pricing also allows the firms to lower their prices for marginal consumers (even down to cost) without sacrificing profit on infra-marginal ones. The resulting competition in  $A$ 's core market—namely, high-valuation consumers—can dissipate profits below that of mono distribution. By contrast, under uniform pricing the firms tend to focus on their respective core market, and as a result dual distribution is always optimal.

We next turn to the situation in which only  $A$  charges personalized prices. Proposition 4 shows that mono distribution is again optimal if  $\beta$  is sufficiently small. As in the case of personalized pricing by both firms, dual distribution triggers competition for high-valuation consumers that can reduce the industry profit below that of mono distribution. Interestingly, mono distribution is even optimal for a larger range in the hybrid regime (as illustrated in Figure 6). Indeed, dual distribution occurs in the hybrid regime if and only if

$$v > \frac{s}{1+s} \left[ 2 + \sqrt{\frac{(1-s)(2+5s+s^2)}{1+2s}} \right], \quad (5)$$

where the right-hand side is strictly larger than that of (4). This is because, while uniform pricing by  $B$  dampens competition for high-valuation consumers, it also limits  $B$ 's ability to expand the market and extract surplus from low-valuation consumers. As a result, the industry profit generated by dual distribution is lower in the hybrid regime.<sup>29</sup>

It is worth noting that the common wisdom, according to which personalized pricing dissipates profits in the case of inter-brand competition, does not apply to the vertical differentiation context considered here: for low values of  $s$ , the industry profit under inter-brand competition is actually larger when both firms charge personalized rather than uniform prices.<sup>30</sup> Moreover, whether personalized pricing increases or reduces profits under inter-brand competition provides little guidance for the optimal distribution strategy. In particular, there are situations in which personalized pricing

<sup>29</sup>Interestingly, small-scale distribution by the retailer is never optimal when  $A$  charges personalized prices—dual distribution is then optimal only when  $B$  serves more than half of the market in pricing regime  $PP$  and more than 40% of the market in pricing regime  $PU$ .

<sup>30</sup>See Online Appendix D.

would increase *both* firms' profits in the case of inter-brand competition and yet, mono distribution is optimal under personalized pricing in the case of intra-brand competition.

### 5.3 Endogenous Pricing Regime

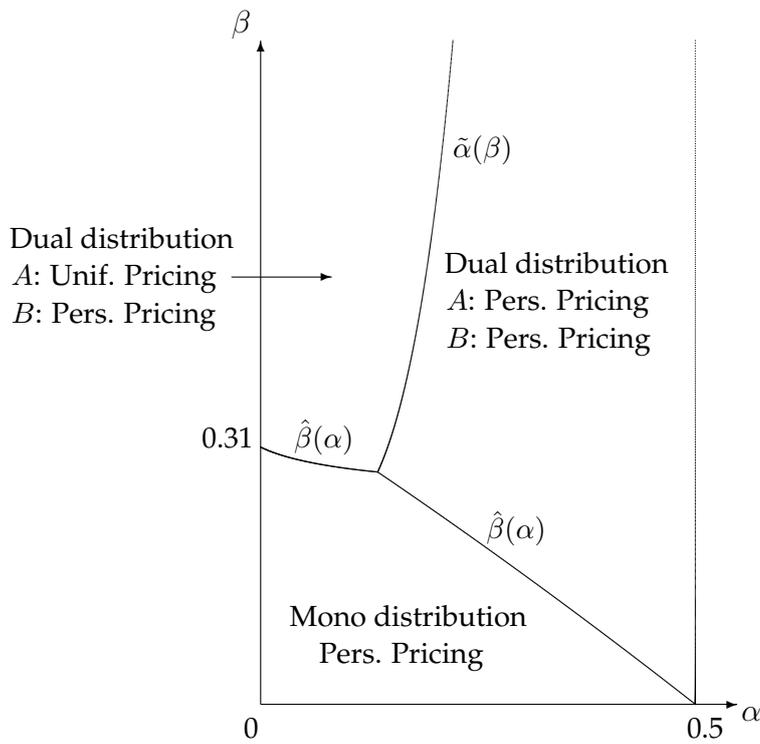
As we have seen, in the absence of correlation and in the case of negative correlation, the pricing regime in which both firms charge personalized prices enables them to extract the entire consumer surplus; hence, it is optimal for them to opt for this pricing regime if they can. By contrast, in the case of positive correlation firms can no longer extract the entire consumer surplus. As a consequence, firms may favor a different pricing regime. To study this issue, in this section we suppose that both pricing policies (i.e., personalized or uniform prices) are available to both firms, and let firms contract on the pricing regime. Their negotiation thus covers now four elements: the per-unit price  $w$ , the fixed fee  $F$ , and the two firms' pricing policies. As before, the firms seek to maximize the industry profit, conditional on retail prices being chosen independently later on.

The next proposition shows that different pricing regimes may be optimal:

**Proposition 5** *If  $s > 0$  and firms can contract on pricing policies, then personalized pricing by  $B$  is always optimal (whenever it is active) and there exists thresholds  $\hat{\beta}(\alpha)$  and  $\tilde{\alpha}(\beta)$  such that:*

- (i) *dual distribution is optimal if and only if  $B$ 's contribution is large enough, namely,  $\beta \geq \hat{\beta}(\alpha)$ ;*
- (ii) *conditional on dual distribution being optimal, personalized pricing by  $A$  is optimal if and only if its contribution is large enough, namely,  $\alpha \geq \tilde{\alpha}(\beta)$ .*

Figure 7 illustrates these insights. If  $B$  does not expand demand significantly (i.e.,  $\beta$  is small), then mono distribution maximizes the industry profit, as it avoids downstream competition and enhances  $A$ 's ability to price discriminate. When instead  $B$  brings enough value, the firms opt for dual distribution and it is always optimal to allow  $B$  to charge personalized prices, which enables it to extract more surplus from low-valuation consumers. By contrast, while restricting  $A$  to a uniform price limits its ability to extract consumer surplus from high-valuation consumers, it also induces it to focus on its core market, which relaxes the competitive pressure on  $B$  and enables it to extract more surplus from medium-range consumers. Proposition 5 shows that doing so is indeed optimal when  $A$  does not generate too much value (i.e.,  $\alpha$  is relatively small). As a consequence, it may be optimal for  $A$  to refrain from charging personalized prices even if it can do so.



**Figure 7:** Endogenous pricing regime

The above discussion assumes that firms negotiate about the pricing regime. Another plausible scenario has each firm independently choosing its pricing policy. The timing of the game could then be as follows. In the first stage, firms agree on whether to use dual distribution or not. In the second stage, the firms then simultaneously and independently select a pricing policy (i.e., personalized or uniform pricing). Thereafter, in case of dual distribution, firms negotiate the wholesale contract consisting of a per-unit price  $w$  and a fixed fee  $F$ . Finally, active firms set their retail prices. This timing reflects the idea that prices are easier to adjust than channel infrastructures and general pricing policies.

In such a scenario, the profit of a firm depends on its outside option and on its share of the joint additional profit.<sup>31</sup> The latter is determined by its bargaining power. As  $B$ 's outside option (namely, zero profit) is independent of its pricing policy, it still has an incentive to maximize the profit to be shared; hence, it again opts for personalized pricing, regardless of its bargaining power. If  $A$  has sufficient bargaining power, it will also seek to maximize the joint profit, in which case the equilibrium outcome remains as characterized by Proposition 5. By contrast, if the manufacturer has little bargaining power, it will instead seek to maximize its outside option by choosing personalized pricing even if this sacrifices some industry profit.

<sup>31</sup>See Online Appendix E for a detailed analysis.

## 6 Delegated Distribution

We assumed so far that  $A$  always uses its direct channel. In this section, we extend our analysis by allowing firms to opt for mono distribution by  $B$ —i.e., delegated distribution. By the same logic as before, doing so is optimal for the firms when it maximizes the industry profit.

The next proposition shows that shutting down  $A$ 's direct channel can indeed be optimal when consumer valuations are positively correlated. This occurs if the direct channel does not contribute much to the industry profit (i.e., when  $\alpha$  is small). To state the result in a concise way, in what follows the subscript  $YZ \in \{UU, UP, PU, PP\}$  refers to the pricing regime in which  $A$ 's pricing policy is  $Y$  and  $B$ 's pricing policy is  $Z$ :

**Proposition 6** *When delegated distribution is feasible:*

- (i) *if  $s \leq 0$ , then delegated distribution is never optimal.*
- (ii) *if  $s > 0$ , then in every pricing regime  $YZ$  there exists a threshold  $\alpha_{YZ}^+(\beta)$  such that delegated distribution is optimal if and only if  $\alpha < \alpha_{YZ}^+(\beta)$ ; the thresholds moreover satisfy  $\alpha_{UP}^+(\beta) > \alpha_{PP}^+(\beta) > \alpha_{UU}^+(\beta) > \alpha_{PU}^+(\beta)$ .*

When consumer valuations are independent (i.e.,  $s = 0$ ) or negatively correlated (i.e.,  $s < 0$ ), we have shown in Sections 3 and 4 that dual distribution enables the firms to obtain the profit of an integrated monopolist. As  $A$ 's channel adds value to the industry, dual distribution always dominates delegated distribution.

When instead consumer valuations are positively correlated (i.e.,  $s > 0$ ), shutting down  $A$ 's channel is optimal whenever its contribution  $\alpha$  is too small. As we have seen, the wholesale price constitutes a less effective tool for controlling intra-brand competition; shutting down  $A$ 's channel instead allows  $B$  to profit most from its ability to serve more consumers than  $A$ . In addition, the range in which delegated distribution is optimal is larger if  $B$  charges personalized prices rather than a uniform price, and, given  $B$ 's pricing policy, is also larger if  $A$  can only set a uniform price. This is obvious when comparing mono distribution by  $A$  with delegated distribution, as the profit achieved with a single channel is maximal when it can extract the entire surplus through personalized pricing. The proposition shows that the insight carries over when considering dual distribution as well.<sup>32</sup>

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<sup>32</sup>When the thresholds are positive—implying that delegated distribution can be optimal—they are strictly increasing in  $\beta$ , reflecting the fact that delegated distribution is more attractive when  $B$ 's contribution is sufficiently large.

## 7 Conclusion

This paper analyzes the effects of personalized pricing on the incentives of a brand manufacturer to opt for dual distribution. Adding an independent distribution channel enables the manufacturer to reach out to different consumer groups but triggers intra-brand competition with its own distribution channel. We show that, while the benefit of dual distribution depends on the interplay between personalized pricing and the demand pattern, dual distribution is nevertheless optimal in a large number of cases.

If consumers' preferences for the two channels are negatively correlated, then dual distribution is optimal regardless of whether any of them can or cannot charge personalized prices. This is because the wholesale tariff provides in this case a sufficiently effective tool to control intra-brand competition. Moreover, personalized pricing by both firms enables them to extract the entire consumer surplus. This result is in sharp contrast to the case of inter-brand competition, where the possibility of charging personalized prices instead reduces both firms' profit. That the wholesale tariff can limit the intensity of intra-brand competition is therefore key for assessing the effects of personalized pricing.

If instead consumer's preferences for the two channels are positively correlated, dual distribution remains optimal when the manufacturer charges a uniform price to consumers. By contrast, when the manufacturer charges personalized prices, the wholesale tariff is less effective at controlling intra-brand competition and the manufacturer opts for dual distribution only when the independent retailer expands demand significantly. Interestingly, a hybrid regime—in which only the retailer charges personalized prices—may yield the highest industry profit. The manufacturer then extracts less surplus from high-valuation consumers, but benefits from reduced intra-brand competition. The manufacturer may thus optimally forgo charging personalized prices even if it has the possibility of doing so.

An important managerial implication of our analysis is that the extent to which price discrimination is feasible not only affects the pricing strategy but, depending on channels' positioning, may also affect the optimal distribution network. If the pure retailer appeals foremost to a different consumer segment, expanding the distribution network is profitable and the possibility of price discrimination at a finely-grained level helps firms to reap larger profits. Instead, if the high-valuation consumers are the same for both firms, a more cautious use of new distribution channels is appropriate when the possibility of price discrimination exists. The brand manufacturer then risks fiercer competition, which may be detrimental to profits. A more profitable strategy could then be to rely on mono distribution by the manufacturer or by the retailer, in order to

avoid intra-brand competition.

Another implication is that adopting a non-discriminatory pricing policy can be profitable for manufacturers in case both firms cater mainly to high-valuation consumers. This is particularly true for companies facing the opportunity of distributing their products through a data-intensive retailer, which can perform price discrimination and expand demand substantially. In that case, not using consumer data for its own distribution channel can achieve the right balance between rent extraction (by the retailer) and the avoidance of fierce intra-brand competition.

We conclude by briefly discussing two interesting avenues for future research emerging from our model. First, we considered a situation in which a firm may set personalized prices for all consumers. Alternatively, firms may only have data on their previous customers. A dynamic extension of our model in which firms set a uniform price in early periods but can offer personalized prices to its customers in subsequent periods can shed light on how learning about consumer preferences shapes distribution as well as pricing decisions. Second, our model assumes that the brand manufacturer does not face competition from rival manufacturers. This is a reasonable assumption in markets in which brands are strongly differentiated and helps singling out the effects at work in a clear way. Analyzing whether new effects emerge with competition between manufacturers, and the resulting implications for wholesale contracts in such an extended framework, constitutes a fruitful direction for future research.

## Appendix

### Proof of Proposition 1

Suppose that the two firms negotiate a wholesale tariff with  $w = v$ . Then, in the continuation equilibrium at stage 2,  $B$  charges a retail price equal to  $v$ , regardless of whether it can set personalized prices or only a uniform one. As explained in the text, a higher wholesale price would lead to no demand for  $B$  and a lower wholesale price would generate a loss.

If  $A$  can charge personalized prices, it will then set a price of  $u_A(x)$  to all consumers  $x$ , with  $x \in [0, \hat{x}]$ , and extract the entire surplus from these consumers. Moreover, as  $A$  obtains a profit of  $v$  if  $B$  serves a consumer, and  $u_A(x) < v$  for all consumers  $x$ , with  $x \in (\hat{x}, X]$ , it is optimal for  $A$  to charge a price larger than  $u_A(x)$  to these consumers;  $B$  then sells to these consumers at their valuation. It follows that the industry profit is equal to the consumer surplus under dual distribution; hence, it is strictly larger than the profit with mono distribution.

If  $A$  must charge a uniform price, its maximization problem is equivalent to the maximization problem of a two-channel monopolist. This is due to the fact that a two-channel monopolist would set a price of  $v$  to consumers who buy from  $B$ 's channel and, given this price, maximizes its profit with respect to the price it charges to consumers buying from  $A$ 's channel. As  $B$  sets a price of  $v$  under dual distribution, and  $A$  obtains this as the wholesale price, the two maximization problems are equivalent. As  $B$  adds value to the industry, the profit is strictly higher than the profit under mono distribution.

## Proof of Proposition 2

We show that, under dual distribution, an appropriate choice of the wholesale price enables the firms, in all pricing regimes, to achieve the same profit as an integrated monopoly. As the industry monopoly profit is maximal when both channels are active, it follows that dual distribution is the optimal strategy.

### Uniform pricing by both firms

We first analyze the case where both firms charge uniform prices.<sup>33</sup> Two cases can arise. If an integrated firm controlling both channels would find it optimal to set prices leading to local monopolies, then the same outcome can be replicated with dual distribution and a wholesale price equal to 0. The market shares are then respectively:

$$x_A^m \equiv \arg \max_x u_A(x) x = \frac{1}{2}$$

and  $X - x_B^m$ , where

$$x_B^m \equiv \arg \max_x u_B(x) (X - x) = \frac{X}{2} - \frac{v}{-2s}.$$

This case arises when  $x_B^m \geq x_A^m$ , which amounts to  $v + s(X - 1) \leq 0$ .

If instead  $x_B^m < x_A^m$ , an integrated firm would fully cover the market and give zero utility to the marginal consumer  $x$  (i.e., the consumer indifferent between the two firms) by charging  $p_A = u_A(x)$  and  $p_B = u_B(x)$ ; the associated profit is:

$$u_A(x) x + u_B(x) (X - x),$$

which is concave in  $x$  and maximal for  $x = \tilde{x}$ , which satisfies  $x_B^m < \tilde{x} < x_A^m$  and is the

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<sup>33</sup>Without loss of generality, we focus on the case in which  $A$  cannot serve all consumers, that is,  $X > 1$ .

unique solution to:

$$u_A(\tilde{x}) + u'_A(\tilde{x})\tilde{x} = u_B(\tilde{x}) - u'_B(\tilde{x})(X - \tilde{x}). \quad (6)$$

We now show that an appropriate choice of the wholesale price enables the firms to replicate this outcome with dual distribution. When facing a wholesale price  $w$ , if  $B$  enjoys a local monopoly its profit can be expressed as a function of the marginal consumer  $x$  (uniquely characterized by  $p_B = u_B(x)$ ):

$$\hat{\pi}_B^m(x; w) \equiv [u_B(x) - w](X - x),$$

which is concave and maximal for  $x = x_B^m(w)$ , the unique solution to:

$$u_B(x_B^m) - w = u'_B(x_B^m)(X - x_B^m).$$

Suppose now that the firms agree on the wholesale price:

$$w = \tilde{w} \equiv (x_B^m)^{-1}(\tilde{x}) = u_B(\tilde{x}) - u'_B(\tilde{x})(X - \tilde{x}). \quad (7)$$

We now show that this wholesale price induces the firms to charge  $\check{p}_A \equiv u_A(\tilde{x})$  and  $\check{p}_B \equiv u_B(\tilde{x})$ , thus replicating the integrated monopoly outcome.<sup>34</sup> By construction,  $B$  has no incentive to increase its price, as this would increase  $x$  and generate a profit  $\hat{\pi}_B^m(x; \tilde{w})$ , which is precisely maximal for  $x = \tilde{x}$ . If instead  $B$  seeks to expand its market share to  $X - x > X - \tilde{x}$ , it must set a price  $p_B = u_B(x) - [u_A(x) - \check{p}_A]$ , and would thus obtain a profit equal to:

$$\hat{\pi}_B^c(x; \tilde{w}) \equiv [u_B(x) - u_A(x) + \check{p}_A - \tilde{w}](X - x),$$

which is concave in  $x$  and satisfies:

$$\begin{aligned} \frac{\partial \hat{\pi}_B^c}{\partial x}(\tilde{x}; \tilde{w}) &= [u'_B(\tilde{x}) - u'_A(\tilde{x})](X - \tilde{x}) - [u_B(\tilde{x}) - u_A(\tilde{x}) + \check{p}_A - \tilde{w}] \\ &= -u'_A(\tilde{x})(X - \tilde{x}) \\ &> 0, \end{aligned}$$

where the second equality stems from  $\check{p}_A \equiv u_A(\tilde{x})$  and (7), and the inequality stems from  $u'_A(\tilde{x}) < 0 < X - \tilde{x}$ ; hence,  $B$  has no incentive to reduce  $x$  and, thus, to expand its market share.

Likewise, if  $A$  seeks to increase its price and thus reduce its market share  $x$ , it would

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<sup>34</sup>Other retail equilibria—with a lower market share for  $B$ —exist for this wholesale price. However, the firms can agree on the appropriate market sharing, and have no incentive to deviate from it.

obtain a profit equal to

$$\hat{\pi}_A^m(x; w) \equiv u_A(x)x + w(X - \tilde{x}),$$

which is concave and maximal for  $x_A^m > \tilde{x}$ ; hence,  $A$  has no incentive to reduce its market share below  $\tilde{x}$ . Finally, if  $A$  seeks instead to expand its market share, it must set a price  $p_A = u_A(x) - [u_B(x) - \check{p}_B]$ , and would thus obtain a profit equal to:

$$\hat{\pi}_A^c(x; \check{w}) \equiv [u_A(x) - u_B(x) + \check{p}_B]x + \check{w}(X - x),$$

which is concave in  $x$  and satisfies:

$$\begin{aligned} \frac{\partial \hat{\pi}_A^c}{\partial x}(\tilde{x}; \check{w}) &= [u'_A(\tilde{x}) - u'_B(\tilde{x})]\tilde{x} + u_A(\tilde{x}) - u_B(\tilde{x}) + \check{p}_B - \check{w} \\ &= -u'_B(\tilde{x})\tilde{x} \\ &< 0, \end{aligned}$$

where the second equality stems from (i)  $\check{p}_B \equiv u_B(\tilde{x})$  and (ii) (6) and (7) (which together imply  $\check{w} \equiv u_A(\tilde{x}) + u'_A(\tilde{x})\tilde{x}$ ), whereas the inequality stems from  $u'_B(\tilde{x}) > 0$  and  $\tilde{x} > 0$ . Hence,  $A$  has no incentive to expand its market share beyond  $\tilde{x}$ .

### Personalized pricing by $A$ only

We now turn to the pricing regime in which  $A$  charges personalized prices and  $B$  a uniform one. Again, two cases can arise. If an integrated firm would find it optimal to set prices leading to local monopolies, then the same outcome can be achieved under dual distribution with a wholesale price equal to 0.  $A$  then charges  $p_A(x) = u_A(x)$  to all consumers  $x \leq 1$ , whereas, as before,  $B$  charges its uniform monopoly price to all consumers  $x \geq x_B^m = (v + sX)/2s$ ; this case thus arises when  $x_B^m \geq 1$ , which amounts to  $v + s(X - 2) \leq 0$ .

If instead  $x_B^m < 1$ , an integrated firm would again fully cover the market. It would therefore extract all the surplus from consumers up to some marginal consumer  $x$  and charge  $p_B = u_B(x)$  to the others, generating a profit given by:

$$\int_0^x u_A(y) dy + u_B(x)(X - x),$$

which is again concave in  $x$ ; let  $x = \tilde{x}$  denote the optimal value of the marginal consumer, which satisfies  $x_B^m < \tilde{x} < 1$  and is the unique solution to:

$$u_A(\tilde{x}) = u_B(\tilde{x}) - u'_B(\tilde{x})(X - \tilde{x}). \quad (8)$$

We now show that the wholesale price characterized by:

$$w = \tilde{w} \equiv (x_B^m)^{-1}(\tilde{x}) = u_B(\tilde{x}) - u'_B(\tilde{x})(X - \tilde{x}) \quad (9)$$

induces the firms to replicate the integrated monopoly outcome, by having  $A$  charge  $\tilde{p}_A(x) = u_A(x)$  and serve all consumers  $x < \tilde{x}$ , and  $B$  charge  $\tilde{p}_B = u_B(\tilde{x})$  and serve all consumers  $x > \tilde{x}$ . Given that  $A$  extracts all the surplus from the consumers it serves,  $B$  can obtain any market share  $X - x$  by setting  $p_B = u_B(x)$ ;  $B$ 's profit is thus given by  $\hat{\pi}_B^m(x; \tilde{w})$ , which by construction is maximal for  $x = x_B^m(\tilde{w}) = \tilde{x}$ . Furthermore, it follows from (8) and (9) that  $\tilde{w} = u_A(\tilde{x})$ . Hence,  $A$  has no incentive to serve consumers  $x > \tilde{x}$ , as it could not obtain from them more than  $u_A(x) < u_A(\tilde{x}) = w$ , which it can secure by letting  $B$  serve these consumers; conversely,  $A$  prefers to serve and extract all the surplus from consumers  $x < \tilde{x}$  rather than letting  $B$  supply them.

### Personalized pricing by $B$ only

We now consider the pricing regime in which  $B$  charges personalized prices and  $A$  a uniform one. The assumption  $v > 0$  rules out the possibility that an integrated firm would find it optimal to set prices leading to local monopolies.<sup>35</sup> An integrated firm would thus fully cover the market and give zero utility to the marginal consumer  $x$ , that is, it would charge  $p_A = u_A(x)$  to consumers up to  $x$  and extract all the surplus from the others; the associated profit is:

$$u_A(x)x + \int_x^X u_B(y) dy,$$

which is concave in  $x$ ; let  $x = \check{x}$  denote the optimal value of the marginal consumer, which satisfies  $v/(-s) < \check{x} < x_A^m$ , and is the unique solution to:

$$u_A(\check{x}) + u'_A(\check{x})(\check{x}) = u_B(\check{x}). \quad (10)$$

We now show that the wholesale price characterized by:

$$w = \check{w} \equiv u_B(\check{x}) \quad (11)$$

induces the firms to replicate the integrated monopoly outcome, by having  $B$  charge  $\check{p}_B(x) = u_B(x)$  and serve all consumers  $x > \check{x}$ , and  $A$  charge  $\check{p}_A = u_A(\check{x})$  and serve all consumers  $x < \check{x}$ . Given this wholesale price,  $B$  has no incentive to serve consumers  $x < \check{x}$ , to which it cannot charge more than  $u_B(x) < u_B(\check{x}) = \check{w}$ . Furthermore, as  $A$

<sup>35</sup>In a situation of local monopolies,  $B$  should serve all consumers with positive valuations for its product (and extract all of their surplus); the assumption  $v > 0$  would then leave no market share to  $A$ , contradicting the fact that  $A$  is better positioned to serve consumers close to  $x = 0$ .

offers no value to consumers  $x > \check{x}$  (for whom  $u_A(x) < u_A(\check{x}) = \check{p}_A$ ),  $B$  can charge them  $\check{p}_B(x) = u_B(x) > \check{w}$  and thus has an incentive to serve them.

Conversely, as  $B$  extracts all the surplus from the consumers it serves,  $A$  can obtain any market share  $x$  by setting  $p_A = u_A(x)$ . If  $A$  seeks to reduce its market share, its profit at the retail stage is given by:

$$\check{\pi}_A^m(x; \check{w}) \equiv u_A(x)x + \check{w}(X - \check{x}),$$

which is concave and maximal for  $x_A^m > \check{x}$ ; hence,  $A$  has no incentive to reduce its market share. If instead  $A$  seeks to expand its market share, its profit at the retail stage becomes:

$$\check{\pi}_A^c(x; \check{w}) \equiv u_A(x)x + \check{w}(X - x),$$

which is concave and satisfies

$$\frac{\partial \check{\pi}_A^c}{\partial x}(\check{x}; \check{w}) = u_A(\check{x}) - \check{w} + u'_A(\check{x})\check{x} = 0,$$

where the last equality stems from (10) and (11). Hence,  $A$  has no incentive to expand its market share beyond  $\check{x}$ .

### **Personalized pricing by both firms**

Finally, if both firms charge personalized prices then, as noted in the main text, the wholesale price  $w = \hat{u}$  enables the firms to replicate the integrated monopoly outcome, by inducing  $B$  to extract all the surplus from consumers  $x \in (\hat{x}, X]$  and  $A$  to extract all the surplus from consumers  $x \in [0, \hat{x}]$ .

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# Personalized Pricing and Distribution Strategies

## Online Appendix (not for publication)

This Online Appendix consists of five sections. In Section A, we provide the proofs of Propositions 3 - 6. In Section B, we analyze the demand pattern where  $v$  and  $s$  exceed 1 and show that our main insights carry over to this case. In Section C, we extend Proposition 3 to more general demands. In Section D, we show that comparing the profits from personalized pricing and uniform pricing in case of inter-brand competition provides little guidance for the optimal distribution strategy in the case of intra-brand competition. Finally, in Section E, we consider the equilibrium pricing regime in the game in which each firm chooses its pricing policy individually.

### A Proofs of Propositions 3, 4, 5, and 6

#### Proof of Proposition 3

##### Uniform pricing by $B$

We first analyze the situation of uniform pricing by both firms. To solve for the subgame-perfect equilibrium, we proceed by backward induction and first determine the reaction functions in the downstream stage. To simplify the exposition, we proceed under the assumption that both demands are positive in equilibrium and verify later that this is in fact true. If both firms are active, they charge retail prices  $p_A$  and  $p_B$  such that some consumers favor  $A$  whereas others favor  $B$ . Let  $x_{AB} > 0$  denote the consumer indifferent between buying from  $A$  or  $B$ , and  $x_B > 0$  denote the consumer indifferent between buying from  $B$  and not buying:

$$x_{AB}(p_A, p_B) = \frac{1 - p_A - v + p_B}{1 - s} \quad \text{and} \quad x_B(p_B) = \frac{v - p_B}{s}.$$

The demands for  $A$  and  $B$  are, respectively,  $x_{AB}$  and  $x_B - x_{AB}$ . Their profit functions (gross of the fixed fee) are then  $\Pi_A = x_{AB}(p_A, p_B)p_A + [x_B(p_B) - x_{AB}(p_A, p_B)]w$  and  $\Pi_B = [x_B(p_B) - x_{AB}(p_A, p_B)](p_B - w)$ . The linearity of the demand functions ensures that firms' profit functions are strictly concave in their prices; hence, firms' reaction functions are characterized by the first-order conditions, which yield:

$$p_A(p_B; w) = \frac{1 - v + p_B + w}{2} \quad \text{and} \quad p_B(p_A; w) = \frac{v + w}{2} - \frac{s(1 - p_A)}{2}.$$

Combining these reaction functions yields the equilibrium retail prices, as a function of the wholesale price  $w$ :

$$p_A(w) = \frac{2 - s + 3w - v}{4 - s},$$

$$p_B(w) = \frac{v(2 - s) + w(2 + s) - s}{4 - s}.$$

The associated demands are  $D_A(w) = x_{AB}(p_A(w), p_B(w))$  and  $D_B(w) = x_B(p_B(w)) - x_{AB}(p_A(w), p_B(w))$ .

In the first stage,  $A$  and  $B$  negotiate over  $w$  and  $F$ , taking into account that, in the second stage, each firm sets its retail price so as to maximize its own profit. Hence, they set  $w$  to maximize the industry profit and use  $F$  to share it according to their bargaining powers and outside options.<sup>1</sup> The industry profit is given by:

$$x_{AB}(p_A(w), p_B(w))p_A(w) + [x_B(p_B(w)) - x_{AB}(p_A(w), p_B(w))]p_B(w)$$

This profit is again a strictly concave function of  $w$ , as its second-order derivative is given by:

$$\Pi''(w) = -\frac{2(4 + 5s)}{s(4 - s)^2} < 0.$$

Hence, the equilibrium wholesale price is characterized by the first-order condition, leading to (the subscript  $UU$  stands for *Uniform pricing by A and B*):

$$w_{UU}^* = \frac{s(4(1 + v) + s)}{2(4 + 5s)}.$$

Inserting the equilibrium prices into the demand functions  $D_A$  and  $D_B$  yields:

$$D_A^* = \frac{2(2 - v) + s(3 - 4v - s)}{2(4 + 5s)(1 - s)} \quad \text{and} \quad D_B^* = \frac{(2 + s)(v - s)}{s(4 + 5s)(1 - s)}.$$

The assumption that the two demand functions intersect at a positive valuation (i.e.,  $v > s$ ) ensures that both equilibrium demands are positive. Indeed,  $D_A^*$  is strictly falling in  $v$  and is equal to  $(2 + s)/(2(4 + 5s)) > 0$  at the highest possible value of  $v$ , which is 1. Direct inspection of  $D_B$  shows that it is positive for  $v > s$ . As  $w^*$  constitutes a global maximum in the relevant range, and achieving  $D_B = 0$  is feasible with a high enough  $w$ , it follows that in equilibrium it is optimal for the firms to generate positive

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<sup>1</sup>Specifically,  $A$ 's outside option is its profit from mono distribution whereas  $B$ 's outside option is zero.

sales for  $B$ . Indeed, the resulting profit, equal to:

$$\Pi_{UU}^* = \frac{s(5s + 4 - s^2) + 4v(1 + s)(v - 2s)}{4s(1 - s)(4 + 5s)}, \quad (12)$$

exceeds the monopoly profit that  $A$  can obtain with mono distribution,  $\Pi_U^m$ :

$$\Pi_{UU}^* - \Pi_U^m = \frac{(1 + s)(v - s)^2}{s(4 + 5s)(1 - s)} > 0.$$

### Personalized pricing by $B$

We next analyze the case in which  $B$  charges personalized prices (and  $A$  still a uniform one). Given  $w$  and  $p_A$ ,  $B$ 's price response is such that consumers  $x$  with  $u_A(x) - p_A > u_B(x) - w$ , or:

$$x < \tilde{x}(w, p_A) = \frac{1 - p_A - v + w}{1 - s},$$

end-up buying from  $A$ . Instead, consumers  $\tilde{x}(w, p_A) < x < \tilde{x}_B(w)$ , with  $\tilde{x}_B(w) = (v - w)/s$ , end-up buying from  $B$  at price  $p_B(x) = u_B(x) - \max\{u_A(x) - p_A, 0\}$ .  $A$ 's variable profit (gross of the fee  $F$ ) is therefore given by:

$$p_A \tilde{x}(w, p_A) + w [\tilde{x}_B(w) - \tilde{x}(w, p_A)].$$

Optimizing this with respect to  $p_A$  yields:

$$p_A(w) = w + \frac{1 - v}{2}.$$

We now turn to the wholesale stage. The two firms seek to maximize the industry profit given by:

$$\Pi = p_A(w) \tilde{x}(w) + \int_{\tilde{x}(w)}^{\hat{x}(w)} [p_A(w) + u_B(x) - u_A(x)] dx + \int_{\hat{x}(w)}^{\tilde{x}_B(w)} u_B(x) dx, \quad (13)$$

where  $p_A(w) = w + (1 - v)/2$ ,  $\tilde{x}(w) = (1 - p_A - v + w)/(1 - s)$ ,  $\hat{x}(w) = (1 + v)/2 - w$ , and  $\tilde{x}_B(w) = (v - w)/s$ . Maximizing this profit with respect to  $w$  yields (the subscript  $UP$  stands for the pricing regime in which  $A$  sets a *Uniform price* and  $B$  *Personalized prices*):<sup>2</sup>

$$w_{UP}^* = \frac{s(1 + v)}{2(1 + s)}.$$

Inserting  $w = w_{UP}^*$  into (13), we obtain that the industry profit is given by:

$$\Pi_{UP}^* = \frac{s + 2s^2 - 4vs - 2vs^2 + 2v^2 + v^2s}{4s(1 - s^2)}. \quad (14)$$

<sup>2</sup>It is straightforward to check that the industry profit is a concave function of  $w$ .

As  $\Pi_{UU}^* > \Pi_{UU}^m$ , to show that  $\Pi_{UP}^* > \Pi_{UP}^m$  it suffices to show that  $\Pi_{UP}^* > \Pi_{UU}^*$ . Indeed, we obtain

$$\Pi_{UP}^* - \Pi_{UU}^* = \frac{(s-v)^2}{4s} \frac{4+6s+s^2}{(4+5s)(1-s^2)} > 0. \quad (15)$$

## Proof of Proposition 4

### Personalized pricing by $B$

We start with the situation of personalized pricing by both firms. As noted in the main text, if the wholesale price  $w$  is such that  $w \geq \hat{u} = (v-s)/(1-s)$ ,  $B$  will be inactive;<sup>3</sup> hence, the industry profit cannot be larger than  $\Pi_P^m$ .

We now focus on  $w \leq \hat{u}$ . We need to distinguish whether or not  $B$  finds it profitable to supply (some) consumers uninterested in  $A$ 's product. From Figure 6, such consumers exist if and only if  $x_B(w) > 1$ . The latter inequality can only hold if  $w$  is sufficiently low, that is  $w < \underline{w} \equiv v-s$ . Note that  $\underline{w} = (1-s)\hat{u} < \hat{u}$ .

### Region $w \leq \underline{w}$

In this region, in which  $x_B(w) \geq 1$ , as shown in the text, the industry profit is given by:

$$\Pi(w) = \int_0^{x_A(w)} [w + |u_B(x) - u_A(x)|] dx + \int_{x_A(w)}^{x_B(w)} u_B(x) dx.$$

It is strictly concave in  $w$ : using  $u_A(x_A(w)) = u_B(x_B(w)) = w$ , we have:

$$\Pi'(w) = x_A(w) + w \frac{dx_B(w)}{dw} = 1 - w - \frac{w}{s} = 1 - \frac{w(1+s)}{s},$$

and thus:

$$\Pi''(w) = -\frac{1+s}{s} < 0.$$

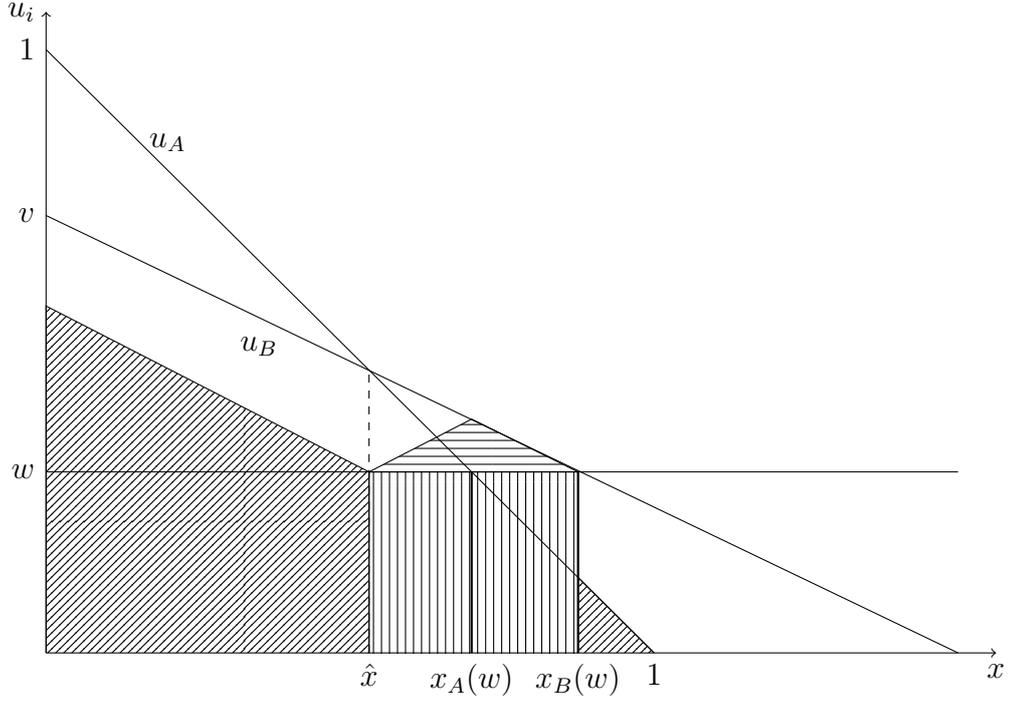
### Region $\underline{w} < w \leq \hat{u}$

If instead  $w > \underline{w}$ , the industry profit includes an additional term, as illustrated by Figure 8. This term corresponds to consumers in the region  $x_B(w) < x \leq 1$ :  $B$  does not find it profitable to supply these consumers (as  $u_B(x) < w$ ), but they are still willing to buy from  $A$ , which can extract their full surplus. The industry profit can then be written as:

$$\Pi(w) = \int_0^{x_A(w)} [w + |u_B(x) - u_A(x)|] dx + \int_{x_A(w)}^{x_B(w)} u_B(x) dx + \int_{x_B(w)}^1 u_A(x) dx.$$

---

<sup>3</sup>Recall that  $\hat{u} = u_A(\hat{x}) = u_B(\hat{x})$ .



**Figure 8:** Profits if  $w > \underline{w}$

The first-order derivative is equal to:

$$\begin{aligned}
 \Pi'(w) &= x_A(w) + [w - u_A(x_B(w))] \frac{dx_B(w)}{dw} \\
 &= (1-w) \left(1 + \frac{1}{s}\right) - (v-w) \frac{1}{s^2} \\
 &= \frac{s^2 + s - v + (1-s-s^2)w}{s^2}.
 \end{aligned}$$

Hence:

$$\begin{aligned}
 \Pi'_-(\hat{u}) &= \frac{1-v}{1-s}, \\
 \Pi'_+(w) &= \frac{1+s}{s} \left( \frac{s(2+s)}{1+s} - v \right), \\
 \Pi''(w) &= \frac{1-s-s^2}{s^2}.
 \end{aligned}$$

It follows that  $\Pi(w)$  is strictly concave in  $w$  if:

$$s > \hat{s} = \frac{\sqrt{5}-1}{2} \simeq 0.618,$$

and is instead weakly convex if  $s \leq \hat{s}$ ; in addition,  $\Pi'(\hat{u}) > 0$  whereas  $\Pi'(w) \geq 0$  if and only if:

$$v \leq \hat{v}(s) \equiv \frac{s(2+s)}{1+s},$$

where  $\hat{v}(s)$  increases with  $s$  and exceeds 1 for  $s \geq \hat{s}$ . Furthermore, not only is the profit function  $\Pi(w)$  continuous at  $w = \underline{w}$ , its derivative  $\Pi'(w)$  is also continuous:

$$\Pi'_-(\underline{w}) = \left(1 - \frac{1+s}{s}w\right)\Big|_{w=v-s} = \frac{1+s}{s} \left(\frac{s(2+s)}{1+s} - v\right) = \Pi'_+(\underline{w}).$$

### Optimal distribution policy

As long as  $w \geq \hat{u}$ ,  $B$  cannot attract any consumer at any profitable price: hence, doing so cannot be more profitable than mono distribution. Furthermore, if  $v \leq \hat{v}(s)$ , then  $\Pi'(\underline{w}) \geq 0$ , implying that dual distribution cannot be more profitable than mono distribution:

- in the range  $\underline{w} \leq w \leq \hat{u}$ , the profit function  $\Pi(w)$  is increasing, as it is quadratic and its derivative is non-negative at both ends of the range (namely,  $\Pi'(\underline{w}) \geq 0$  and  $\Pi'(\hat{u}) > 0$ );
- in the range  $w \leq \underline{w}$ , the profit function  $\Pi(w)$  is again increasing, as it is concave and its derivative is non-negative at the upper end of the range (namely,  $\Pi'(\underline{w}) \geq 0$ );
- it follows that the profit achieved under dual distribution cannot exceed  $\Pi(\hat{u})$ , which is less profitable than mono distribution.

As already noted,  $\hat{v}(s)$  is increasing in  $s$  in the range  $s \in [0, 1]$ , and satisfies  $\hat{v}(s) \geq 1$  for  $s \geq \hat{s}$ . It follows that, if  $s \geq \hat{s}$ , then dual distribution cannot be more profitable than mono distribution, as we then have  $\hat{v}(s) \geq 1 (> v)$ .

If instead  $s < \hat{s}$  and  $v > \hat{v}(s)$ , then  $\Pi'(\underline{w}) < 0$ . From the analysis for the region  $w \leq \underline{w}$  above, the first-order condition  $\Pi'(w) = 0$  then determines the candidate optimal wholesale price, which is given by:

$$w = w_{PP}^* = \frac{s}{1+s} \in (0, \underline{w}).$$

The corresponding profit is:

$$\Pi_{PP}^* = \frac{1}{2} \frac{s(1+3s) - 4s(1+s)v + (1+s)^2 v^2}{s(1-s^2)}.$$

Compared with the profit from mono distribution,  $\Pi_P^m$ , dual distribution introduces a change in profit equal to:

$$\frac{1}{2} \frac{s^2(s+3) - 4s(1+s)v + (1+s)^2 v^2}{s(1-s^2)}.$$

The numerator of this expression is a convex quadratic polynomial of  $v$  and its roots are:

$$\frac{s(2 - \sqrt{1-s})}{1+s} \text{ and } \frac{s(2 + \sqrt{1-s})}{1+s}.$$

Furthermore,  $\hat{v}(s)$  lies between these two roots in the relevant range  $s < \hat{s}$ :

$$\begin{aligned} \frac{\frac{s(2+\sqrt{1-s})}{1+s}}{\hat{v}(s)} &= \frac{\frac{s(2+\sqrt{1-s})}{1+s}}{\frac{s(2+s)}{1+s}} = \frac{2 - \sqrt{1-s}}{2+s} < 1, \\ \frac{\frac{s(2+\sqrt{1-s})}{1+s}}{\hat{v}(s)} &= \frac{\frac{s(2+\sqrt{1-s})}{1+s}}{\frac{s(2+s)}{1+s}} = \frac{2 + \sqrt{1-s}}{2+s} > 1, \end{aligned}$$

where the last inequality stems from  $\sqrt{1-s} > s$  in the relevant range  $s < \hat{s}$ . It follows that dual distribution is more profitable than mono distribution if and only if  $s < \hat{s}$  and  $v$  exceeds the larger root, that is, if:

$$v > \tilde{v}(s) \equiv \frac{s(2 + \sqrt{1-s})}{1+s}$$

Note that  $\tilde{v}(s)$  is increasing in  $s$  in the range  $s \leq \hat{s}$ , and exceeds 1 in the range  $s \geq \hat{s}$ . Hence, as  $v < 1$ , the condition  $v > \tilde{v}(s)$  implies  $s < \hat{s}$ .

We next show that the threshold  $\tilde{v}(s)$  translates into a threshold  $\beta(\alpha)$ , such that dual distribution is optimal if and only if  $\beta > \beta(\alpha)$ . Let:

$$\Omega \equiv \{(v, s) \text{ satisfying } 0 < s < v < 1\}$$

denote the set of admissible values. For any  $(v, s) \in \Omega$ , the channels' contributions are respectively given by:

$$\alpha = \hat{\alpha}(v, s) \equiv \int_0^{\frac{1-v}{1-s}} [1 - x - (v - sx)] dx = \frac{(1-v)^2}{2(1-s)}$$

and:

$$\beta = \hat{\beta}(v, s) \equiv \int_{\frac{1-v}{1-s}}^1 [v - sx - (1-x)] dx + \int_1^{\frac{v}{s}} [v - sx] dx = \frac{(v-s)^2}{2s(1-s)},$$

where:

$$\frac{\partial \hat{\alpha}}{\partial v} = -\frac{1-v}{1-s} < 0 \quad \text{and} \quad \frac{\partial \hat{\alpha}}{\partial s} = \frac{(1-v)^2}{2(1-s)^2} > 0, \quad (16)$$

and:

$$\frac{\partial \hat{\beta}}{\partial v} = \frac{v-s}{s(1-s)} > 0 \quad \text{and} \quad \frac{\partial \hat{\beta}}{\partial s} = -\frac{(v-s)(v+s-2vs)}{2(1-s)^2 s^2} < 0. \quad (17)$$

Let:

$$\hat{\Omega} \equiv \left\{ (\alpha, \beta) \mid \exists (v, s) \in \Omega \text{ such that } (\hat{\alpha}(v, s), \hat{\beta}(v, s)) = (\alpha, \beta) \right\}$$

denote the set of admissible values for  $\alpha$  and  $\beta$ . We now show that there exists a bijection between  $\Omega$  and  $\hat{\Omega}$ . To see this, fix  $(\alpha, \beta) \in \hat{\Omega}$  and consider the slopes of the iso- $\alpha$  curve:

$$\hat{\alpha}(v, s) = \alpha$$

and of the iso- $\beta$  curve:

$$\hat{\beta}(v, s) = \beta$$

in the space  $(v, s)$ . The slopes of these curves satisfy:

$$\left. \frac{dv}{ds} \right|_{\hat{\beta}(v,s)=\beta} = \frac{-\frac{\partial \hat{\beta}}{\partial s}(v, s)}{\frac{\partial \hat{\beta}}{\partial v}(v, s)} \Big|_{\hat{\beta}(v,s)=\beta} = \frac{v + s - 2vs}{2s(1-s)} \Big|_{\hat{\beta}(v,s)=\beta} > \frac{1}{2},$$

where the last inequality is due to the fact that  $v + s - 2vs > s(1-s)$  for  $v \in (0, 1)$ ,<sup>4</sup> and:

$$\left. \frac{dv}{ds} \right|_{\hat{\alpha}(v,s)=\alpha} < \frac{-\frac{\partial \hat{\alpha}}{\partial s}(v, s)}{\frac{\partial \hat{\alpha}}{\partial v}(v, s)} \Big|_{\hat{\alpha}(v,s)=\alpha} = \frac{1-v}{2(1-s)} \Big|_{\hat{\alpha}(v,s)=\alpha} < \frac{1}{2}.$$

It follows that

$$\left. \frac{dv}{ds} \right|_{\hat{\beta}(v,s)=\beta} > \left. \frac{dv}{ds} \right|_{\hat{\alpha}(v,s)=\alpha},$$

implying that the iso- $\alpha$  and iso- $\beta$  curves intersect at most once. Hence, for any  $(\alpha, \beta) \in \hat{\Omega}$ , there exists a unique  $(v, s) \in \Omega$  such that  $(\hat{\alpha}(v, s), \hat{\beta}(v, s)) = (\alpha, \beta)$ . Let  $\hat{v}(\alpha, \beta)$  and  $\hat{s}(\alpha, \beta)$  denote these values.

By construction, we have:

$$\hat{\alpha}(\hat{v}(\alpha, \beta), \hat{s}(\alpha, \beta)) = \alpha \quad \text{and} \quad \hat{\beta}(\hat{v}(\alpha, \beta), \hat{s}(\alpha, \beta)) = \beta. \quad (18)$$

Differentiating these equalities with respect to  $\alpha$  yields (dropping the arguments for ease of exposition):

$$\begin{aligned} \frac{\partial \hat{\alpha}}{\partial v} \frac{\partial \hat{v}}{\partial \alpha} + \frac{\partial \hat{\alpha}}{\partial s} \frac{\partial \hat{s}}{\partial \alpha} &= 1, \\ \frac{\partial \hat{\beta}}{\partial v} \frac{\partial \hat{v}}{\partial \alpha} + \frac{\partial \hat{\beta}}{\partial s} \frac{\partial \hat{s}}{\partial \alpha} &= 0. \end{aligned}$$

<sup>4</sup>To see this, note that  $v + s - 2vs - s(1-s) = v(1-v) + (v-s)^2$ .

Solving for  $\frac{\partial \hat{v}}{\partial \alpha}$  and  $\frac{\partial \hat{s}}{\partial \alpha}$  then yields:

$$\frac{\partial \hat{v}}{\partial \alpha}(\alpha, \beta) = \left. \frac{\frac{\partial \hat{\beta}}{\partial s}(v, s)}{D(v, s)} \right|_{(v,s)=(\hat{v}(\alpha,\beta), \hat{s}(\alpha,\beta))}, \quad (19)$$

$$\frac{\partial \hat{s}}{\partial \alpha}(\alpha, \beta) = \left. \frac{-\frac{\partial \hat{\beta}}{\partial v}(v, s)}{D(v, s)} \right|_{(v,s)=(\hat{v}(\alpha,\beta), \hat{s}(\alpha,\beta))}, \quad (20)$$

where:

$$\begin{aligned} D(v, s) &\equiv \frac{\partial \hat{\alpha}}{\partial v}(v, s) \frac{\partial \hat{\beta}}{\partial s}(v, s) - \frac{\partial \hat{\alpha}}{\partial s}(v, s) \frac{\partial \hat{\beta}}{\partial v}(v, s) \\ &= -\frac{\partial \hat{\alpha}}{\partial v}(v, s) \frac{\partial \hat{\beta}}{\partial v}(v, s) \left[ \frac{-\frac{\partial \hat{\beta}}{\partial s}(v, s)}{\frac{\partial \hat{\beta}}{\partial v}(v, s)} - \frac{-\frac{\partial \hat{\alpha}}{\partial s}(v, s)}{\frac{\partial \hat{\alpha}}{\partial v}(v, s)} \right] > 0, \end{aligned}$$

where the inequality stems from  $\frac{\partial \hat{\alpha}}{\partial v}(v, s) < 0 < \frac{\partial \hat{\beta}}{\partial v}(v, s)$  and, as already noted:

$$\frac{-\frac{\partial \hat{\beta}}{\partial s}(v, s)}{\frac{\partial \hat{\beta}}{\partial v}(v, s)} > \frac{1}{2} > \frac{-\frac{\partial \hat{\alpha}}{\partial s}(v, s)}{\frac{\partial \hat{\alpha}}{\partial v}(v, s)}.$$

It then follows from  $\frac{\partial \hat{\beta}}{\partial s}(v, s) < 0 < \frac{\partial \hat{\beta}}{\partial v}(v, s)$  that:

$$\frac{\partial \hat{v}}{\partial \alpha}(\alpha, \beta) < 0 \quad \text{and} \quad \frac{\partial \hat{s}}{\partial \alpha}(\alpha, \beta) < 0.$$

Likewise, differentiating the equalities in (18) with respect to  $\beta$  yields:

$$\begin{aligned} \frac{\partial \hat{\alpha}}{\partial v} \frac{\partial \hat{v}}{\partial \beta} + \frac{\partial \hat{\alpha}}{\partial s} \frac{\partial \hat{s}}{\partial \beta} &= 0, \\ \frac{\partial \hat{\beta}}{\partial v} \frac{\partial \hat{v}}{\partial \beta} + \frac{\partial \hat{\beta}}{\partial s} \frac{\partial \hat{s}}{\partial \beta} &= 1. \end{aligned}$$

Solving for  $\frac{\partial \hat{v}}{\partial \beta}$  and  $\frac{\partial \hat{s}}{\partial \beta}$  then yields:

$$\frac{\partial \hat{v}}{\partial \beta}(\alpha, \beta) = \left. \frac{-\frac{\partial \hat{\alpha}}{\partial s}(v, s)}{D(v, s)} \right|_{(v,s)=(\hat{v}(\alpha,\beta), \hat{s}(\alpha,\beta))} < 0, \quad (21)$$

$$\frac{\partial \hat{s}}{\partial \beta}(\alpha, \beta) = \left. \frac{\frac{\partial \hat{\alpha}}{\partial v}(v, s)}{D(v, s)} \right|_{(v,s)=(\hat{v}(\alpha,\beta), \hat{s}(\alpha,\beta))} < 0, \quad (22)$$

where the inequalities stem from  $D(v, s) > 0$  and  $\frac{\partial \hat{\alpha}}{\partial v}(v, s) < 0 < \frac{\partial \hat{\alpha}}{\partial s}(v, s)$ .

The condition:

$$v > \tilde{v}(s) = \frac{s(2 + \sqrt{1-s})}{1+s}$$

amounts to:

$$\Phi(\alpha, \beta) \equiv \hat{v}(\alpha, \beta) - \tilde{v}(\hat{s}(\alpha, \beta)) > 0,$$

where:

$$\frac{\partial \Phi}{\partial \beta}(\alpha, \beta) = \frac{\partial \hat{v}}{\partial \beta}(\alpha, \beta) - \tilde{v}'(s) \frac{\partial \hat{s}}{\partial \beta}(\alpha, \beta). \quad (23)$$

Using (21), (22) and (23), we have:

$$\frac{\partial \Phi}{\partial \beta}(\alpha, \beta) = \left[ \tilde{v}'(s) - \frac{(1-v)}{2(1-s)} \right] \frac{1-v}{(1-s)D(v, s)} \Big|_{(v,s)=(\hat{v}(\alpha,\beta), \hat{s}(\alpha,\beta))}.$$

To show that  $\frac{\partial \Phi}{\partial \beta}(\alpha, \beta) > 0$ , it thus suffices to show that:

$$\tilde{v}'(s) - \frac{1-v}{2(1-s)} > 0$$

in the relevant range  $v > \tilde{v}(s)$ . A sufficient condition is therefore  $\tilde{\psi}(s) > 0$ , where:

$$\begin{aligned} \tilde{\psi}(s) &\equiv \tilde{v}'(s) - \frac{1-\tilde{v}(s)}{2(1-s)} = \\ &= \frac{d}{ds} \left( \frac{s(2+\sqrt{1-s})}{1+s} \right) - \frac{1-\frac{s(2+\sqrt{1-s})}{1+s}}{2(1-s)} = \frac{2(1-s) + (3-s)\sqrt{1-s}}{2(1+s)^2(1-s)}, \end{aligned}$$

which is indeed positive for  $s \in (0, 1)$ . It follows that there exists a unique value of  $\beta$ , which we denote  $\beta_{PP}(\alpha)$ , such that dual distribution is strictly optimal if and only if  $\beta > \beta_{PP}(\alpha)$ .

### Uniform pricing by $B$

We now move to the hybrid regime in which  $A$  charges personalized prices and  $B$  a uniform one. Again, we solve the game by backward induction. Consider first  $A$ 's price response to a given  $w$  and  $p_B$ . Two market share configurations can occur, depending on the value of  $p_B$ . When  $p_B$  is relatively small, the marginal consumer indifferent between buying from  $B$  and not buying,  $x_B = (v - p_B)/s$ , exceeds 1.  $A$ 's best response is to serve consumers  $x$  with  $v_A(x) - w > u_B(x) - p_B$ , or:

$$x < x_{AB}(w) \equiv \frac{1-w-v+p_B}{1-s},$$

whereas consumers  $x$  with  $x_{AB}(w) < x < x_B$  end-up buying from  $B$ .

Instead, when  $p_B$  is high enough so that  $x_B < 1$ , a third demand region exists between  $x_B$  and 1 in which consumers end-up buying from  $A$ . The thresholds for the first two demand regions are the same as in the market share configuration above.

We start with the first case. If  $B$  serves consumers  $x$  with  $x_{AB}(w) < x < x_B$ , its profit is:

$$(p_B - w) \left( \frac{v - p_B}{s} - \frac{1 - w - v + p_B}{1 - s} \right).$$

Maximizing with respect to  $p_B$  yields:

$$p_B(w) = \frac{v + w - s(1 - w)}{2}. \quad (24)$$

We now turn to the negotiation at the wholesale stage. Because  $A$  charges to each consumer  $x$  a personalized price of  $1 - x - v + sx + p_B(w)$ , the industry profit is:

$$\int_0^{\frac{1-w-v+p_B(w)}{1-s}} [1 - x - v + sx + p_B(w)] dx + p_B(w) \left( \frac{v - p_B(w)}{s} - \frac{1 - w - v + p_B(w)}{1 - s} \right), \quad (25)$$

with  $p_B(w)$  given by (24). Maximizing (25) with respect to  $w$  yields (the subscript  $PU$  stands for the pricing regime in which  $A$  sets *Personalized prices* and  $B$  a *Uniform price*)<sup>5</sup>

$$w_{PU}^* = \frac{s[2 + s + v]}{2 + 5s + s^2}.$$

This market configuration is only valid if  $x_B \geq 1$ . Comparing the two thresholds at the equilibrium values, we obtain that the inequality is fulfilled if and only if:

$$v \geq \frac{s(2 + 4s + s^2)}{1 + 2s}. \quad (26)$$

As  $v$  is bounded above by 1, this inequality can only be satisfied if  $s^2(4 + s) - 1 \leq 0$ , which, in the relevant range  $s \in (0, 1)$ , amounts to

$$s \leq \check{s} \simeq 0.473.$$

Inserting  $w = w_{PU}^*$  in (25), the resulting industry profit is:

$$\Pi_{PU}^* = \frac{v^2 + s(2 - 4v + 3v^2) + s^2(5 - 8v + 2v^2) - s^3}{2s(1 - s)(2 + 5s + s^2)}. \quad (27)$$

Compared with the profit from mono distribution,  $\Pi_P^m$ , the profit from dual distribution is larger if and only if:<sup>6</sup>

$$v \geq \check{v}(s) \equiv s \frac{2 + \sqrt{\frac{(1-s)(2+5s+s^2)}{1+2s}}}{1 + s}. \quad (28)$$

<sup>5</sup>The maximization problem is strictly concave.

<sup>6</sup>Because  $\Pi_{PU}^*$  is a convex quadratic polynomial in  $v$ , the equation  $\Pi_{PU}^* - \Pi_P^m = 0$  has two roots. The lower one is below zero, and the larger one is  $\check{v}(s)$ .

It can be checked that  $\check{v}(s)$  is increasing in  $s$  in the range  $s \in (0, \check{s})$  and exceeds 1 in the range  $s \in (\check{s}, 1)$ . As  $v < 1$ , the condition  $v \geq \check{v}(s)$  implies  $s < \check{s}$ . Moreover,  $\check{v}(s)$  is indeed larger than the right-hand side of (26) for  $s < \check{s}$ . Hence, if the first market configuration is valid, both firms are active if and only if (28) holds.

We now turn to the second market configuration. In this case, the industry profit is:

$$\int_0^{\frac{1-w-v+p_B(w)}{1-s}} [1-x-v+sx+p_B(w)] dx \quad (29)$$

$$+p_B(w) \left( \frac{v-p_B}{s} - \frac{1-w-v+p_B(w)}{1-s} \right) + \int_{\frac{v-p_B}{s}}^1 [1-x] dx,$$

with  $p_B(w)$  again given by (24). Maximizing with respect to  $w$ , we obtain that the second-order condition for an interior solution is fulfilled if and only if  $s^2(4+s)-1 > 0$ , resulting in a wholesale price of:

$$\frac{s^3 + s^2(3+v) + s(1-v) - v}{s^3 + 4s^2 - 1}.$$

However, at this wholesale price,  $B$ 's demand is negative for  $s^2(4+s)-1 > 0$ . As a consequence, in case the maximization problem is concave, mono distribution is optimal. Instead, if  $s^2(4+s) - 1 \leq 0$ , the maximization problem (29) is convex. It follows that  $w$  is optimally set either so high that  $B$  is not active, which results in mono distribution, or so low that  $x_B \geq 1$ .

In the latter case, the first market configuration is valid if (26) holds. Instead, if (26) is not fulfilled, the optimal  $w$  is set such that  $x_B$  exactly equals 1, or  $(v - p_B(w))/s = 1$ , with  $p_B(w)$  given by (24). Solving the last equation for  $w$  yields  $w = (v - s)/(1 + s)$ . The resulting industry profit is:

$$\frac{1 + v^2 - 2sv(1-v) - s^2(1+4v) + 2s^3 + s^4}{2(1-s)(1+s)^2}.$$

Compared with  $\Pi_P^m$ , the profit with dual distribution is larger if and only if:

$$v \geq s + (1+s) \sqrt{\frac{s(1-s)}{1+2s}}. \quad (30)$$

However, the right-hand side of (30) is larger than the right-hand side of (26) for all values of  $s$ , with  $s^2(4+s) - 1 \leq 0$ . Since the profit function with  $x_B = 1$  is only valid if (26) does not hold, (30) cannot be fulfilled for any admissible value of  $s$ . Hence, mono distribution is optimal in this case.

As a consequence, dual distribution is more profitable than mono distribution if and only if  $v \geq \check{v}(s)$ , which can only hold if  $s < \check{s}$ . In the same way as in the first part of

the proof (i.e., personalized pricing by  $B$ ), we can show that the existence of a unique threshold value  $\check{v}(s)$  implies that there is also a unique threshold value for  $\beta$ , denoted by  $\beta_{PU}(\alpha)$ , such that dual distribution is optimal if and only if  $\beta > \beta_{PU}(\alpha)$ . Following the same steps as before, a sufficient condition is:

$$\check{\psi}(s) \equiv \check{v}'(s) - \frac{1 - \check{v}(s)}{2(1-s)} > 0,$$

which indeed holds:

$$\begin{aligned} \check{v}'(s) - \frac{1 - \check{v}(s)}{2(1-s)} &= \frac{d}{ds} \left( s \frac{2 + \sqrt{\frac{(1-s)(2+5s+s^2)}{1+2s}}}{1+s} \right) - \frac{1-s}{2(1-s)} \\ &= \frac{3-s}{2(1+s)^2} + \frac{4 + 19s + 25s^2 + 8s^3 + 2s^4}{2(1+s)^2(1+2s)^2 \sqrt{\frac{(2+5s+s^2)}{(1-s)(1+2s)}}}, \end{aligned}$$

which is strictly positive for all  $s \in (0, 1)$ .

Finally, we compare the two thresholds  $\tilde{v}(s)$  and  $\check{v}(s)$  with each other. Note that  $\check{v}(s)$  can be written as  $s(2 + \delta(s)\sqrt{1-s})/(1+s)$  with  $\delta(s) = \sqrt{(2+5s+s^2)/(1+2s)}$ , whereas  $\tilde{v}(s)$  is  $s(2 + \sqrt{1-s})/(1+s)$ . Since:

$$\sqrt{\frac{2+5s+s^2}{1+2s}} > 1,$$

mono distribution is optimal for a larger range in the hybrid regime—i.e., the regime in which only  $B$  charges a uniform price—compared to the symmetric regime in which both firms charge personalized prices. Due to the one-to-one mapping between the thresholds in the  $v$ - $s$ -plane and the thresholds in the  $\alpha$ - $\beta$ -plane, it follows that  $\beta_{PU}(\alpha) > \beta_{PP}(\alpha)$ .

## Proof of Proposition 5

Conditional on opting for mono distribution, firm  $A$  obviously favors personalized pricing, which yields the profit  $\Pi_P^m = 1/2$ . Conditional on opting for dual distribution, firms negotiate over the pricing regime that maximizes the industry profit.

The profits generated by the pricing regimes in which  $A$  charges uniform prices are  $\Pi_{UU}^*$  and  $\Pi_{UP}^*$ , respectively given by (12) and (14), which satisfy  $\Pi_{UP}^* - \Pi_{UU}^* > 0$  from (15). Therefore, firms never choose the uniform pricing regime.

The profits generated by the two pricing regimes in which  $A$  charges personalized

prices are  $\Pi_{PP}$  and  $\Pi_{PU}$ , respectively given by (3) and (27), which satisfy:

$$\Pi_{PP}^* - \Pi_{PU}^* = \frac{(1 + 3s + s^2)(2s - v - sv)^2}{2s(1 - s^2)(2 + 5s + s^2)} > 0.$$

Therefore, firms never choose the hybrid regime.

The relevant options are therefore mono distribution with personalized pricing, yielding  $\Pi_P^m$ , and dual distribution with personalized pricing by  $B$  and either personalized pricing or uniform pricing by  $A$ , yielding respectively  $\Pi_{PP}^*$  and  $\Pi_{UP}^*$ .

Comparing the relevant profits from dual distribution yields:

$$\Pi_{PP}^* - \Pi_{UP}^* = \frac{(1 - v)(1 + 4s - v(3 + 2s))}{4(1 - s^2)},$$

which is positive if:

$$v < \lambda(s) \equiv \frac{1 + 4s}{3 + 2s}.$$

This condition amounts to:

$$\Lambda(\alpha, \beta) \equiv \hat{v}(\alpha, \beta) - \lambda(\hat{s}(\alpha, \beta)) < 0.$$

Differentiating  $\Lambda(\alpha, \beta)$  with respect to  $\alpha$  yields:

$$\frac{\partial \Lambda}{\partial \alpha}(\alpha, \beta) = \frac{\partial \hat{v}}{\partial \alpha}(\alpha, \beta) - \lambda'(s) \frac{\partial \hat{s}}{\partial \alpha}(\alpha, \beta).$$

Using (16), (17), (19), and (20), this amounts to:

$$\frac{\partial \Lambda}{\partial \alpha}(\alpha, \beta) = \left[ \lambda'(s) - \frac{v + s - 2vs}{2s(1 - s)} \right] \frac{v - s}{(1 - s)sD(v, s)} \Big|_{(v, s) = (\hat{v}(\alpha, \beta), \hat{s}(\alpha, \beta))}.$$

To show that  $\frac{\partial \Lambda}{\partial \alpha}(\alpha, \beta) < 0$  in the relevant range  $v < \lambda(s)$ , it again suffices to show that:

$$\lambda'(s) - \frac{\lambda(s) + s - 2s\lambda(s)}{2s(1 - s)} < 0.$$

Simplifying this condition yields  $-3(1 + 4s^2) / [2s(3 + 2s)^2] < 0$ , which is indeed holds because  $s \in (0, 1)$ . It follows that there exists a unique solution for  $\alpha$ , which we denote  $\tilde{\alpha}(\beta)$ , such that  $\Pi_{PP}^* > \Pi_{UP}^*$  if and only if  $\alpha > \tilde{\alpha}(\beta)$ .

From Proposition 4,  $\Pi_{PP}^* > \Pi_P^m$  if and only if  $\beta > \beta_{PP}(\alpha)$ . It follows that dual distribution together with personalized pricing by both firms is optimal if and only if  $\alpha > \tilde{\alpha}(\beta)$  and  $\beta > \beta_{PP}(\alpha)$ .

Finally, the comparison of  $\Pi_{UP}^*$  with  $\Pi_P^m$  yields:

$$\Pi_{UP}^* > \Pi_P^m \iff \frac{s + 2s^2 - 4vs - 2vs^2 + 2v^2 + v^2s}{4s(1-s^2)} > \frac{1}{2}.$$

This holds if

$$v > s + \sqrt{\frac{s(1-s^2)}{2+s}} \quad \text{or} \quad v < s - \sqrt{\frac{s(1-s^2)}{2+s}}.$$

Because  $v > s$ , the only relevant case is:

$$v > \hat{\lambda}(s) \equiv s + \sqrt{\frac{s(1-s^2)}{2+s}}.$$

Following the same steps as above, the existence of a unique threshold value for  $v$  as a function of  $s$  implies that there exists a unique solution for  $\beta$ , which we denote  $\beta_{UP}(\alpha)$ , such that  $\Pi_{UP}^* > \Pi_P^m$  if and only if  $\beta > \beta_{UP}(\alpha)$ . In particular, the condition

$$\frac{\partial \left( s + \sqrt{\frac{s(1-s^2)}{2+s}} \right)}{\partial s} - \frac{1-v}{2(1-s)} > 0$$

is fulfilled in the relevant range where  $v > s + \sqrt{\frac{s(1-s^2)}{2+s}}$  because the left-hand side is equal to:

$$\frac{1}{2} + \frac{(1-s)(2+4s+s^2)}{(2+s)^2 \sqrt{\frac{s(1-s^2)}{2+s}}},$$

which is strictly positive for all  $s \in (0, 1)$ .

It can be checked that, in the relevant range  $\alpha \in (0, 1/2)$ ,  $\beta_{UP}(\alpha) \leq \beta_{PP}(\alpha)$  if and only if  $\alpha \leq \hat{\alpha} \simeq 0.12$ . It follows that mono distribution is optimal if and only if:

$$\beta < \hat{\beta}(\alpha) \equiv \min \{ \beta_{UP}(\alpha), \beta_{PP}(\alpha) \} = \begin{cases} \beta_{UP}(\alpha) & \text{for } \alpha \leq \hat{\alpha}, \\ \beta_{PP}(\alpha) & \text{for } \hat{\alpha} < \alpha < 1/2. \end{cases}$$

## Proof of Proposition 6

### Independent valuations and negative correlation of valuations

As discussed in Sections 3 and 4, if  $s \leq 0$  then, in all pricing regimes, dual distribution enables the firms to obtain the profit of an integrated monopolist controlling both channels. As  $A$  adds value to the industry, delegated distribution is never optimal.

### Positive correlation of valuations

We start with the analysis of the regime in which both firms set uniform prices. If  $A$ 's channel is shut down,  $B$  sets the monopoly price in the retail market, equal to  $v/2$ , and the industry profit is  $v^2/(4s)$ . Comparing this profit with the industry profit under dual distribution, which is given  $\Pi_{UU}^*$ , yields that delegated distribution gives a higher industry profit if and only if:

$$v \geq v_{UU}^+(s) \equiv \frac{4(1+s) - (1-s)\sqrt{4+5s}}{3+5s}.$$

It is straightforward to check that this inequality always holds at  $v = 1$  but is never fulfilled at  $v = s$ . In addition,  $v_{UU}^+(s) \in (0, 1)$  for all  $s \in (0, 1)$ , that is  $v_{UU}^+(s)$  is in the interior of the admissible range.

Second, we analyze the pricing regime  $UP$ . Without the presence of  $A$ 's channel,  $B$  extracts the entire surplus in the retail market, which implies that the industry profit is  $v^2/(2s)$ . Instead, with dual distribution, the industry profit is  $\Pi_{UP}^*$ . Comparing the two profits yields:

$$\frac{v^2}{2s} \geq \Pi_{UP}^* \quad \Leftrightarrow \quad v \geq v_{UP}^+ \equiv \frac{2+s - \sqrt{3(1+s)(1-s)}}{1+2s}.$$

This inequality always holds at the upper bound  $v = 1$ . At the lower bound  $v = s$ , the inequality is also fulfilled if  $s \geq 1/2$ . Hence, delegated distribution is optimal in this regime for all admissible values of  $v$  if  $s \geq 1/2$ , and, for  $v \geq v_{UP}^+(s)$  if  $s < 1/2$ .

Third, we turn to the regime  $PU$ . Without  $A$ 's channel, the industry profit is  $v^2/(4s)$  because  $B$  charges only a uniform price. Instead, if  $A$ 's channel is open, dual distribution is optimal if and only if:

$$v \geq s \frac{2 + \sqrt{\frac{(1-s)(2+5s+s^2)}{1+2s}}}{1+s} \quad (31)$$

and leads to a profit of  $\Pi_{PU}^*$ ; otherwise, mono distribution by  $A$  is optimal with a profit of  $\Pi_P^m = 1/2$ . Comparing  $\Pi_P^m$  with the profit from delegated distribution (i.e.,  $v^2/(4s)$ ) yields that the latter is larger if and only if  $v \geq \sqrt{2s}$ . As the upper bound of  $v$  equals 1, this inequality can only be fulfilled if  $s \leq 0.5$ . Because this comparison is only relevant if (31) does not hold, we need to check if  $\sqrt{2s}$  is smaller than the right-hand side of (31). Thus is true if and only if  $0.357 \lesssim s$ . It follows that for  $0.357 \lesssim s \leq 0.5$ , delegated distribution is optimal if  $v \geq \sqrt{2s}$ , whereas for  $s > 0.5$ , delegated distribution can never be optimal.

Instead, if dual distribution is optimal in case  $A$ 's channel is open, we obtain:

$$\frac{v^2}{4s} \geq \Pi_{PU}^* \Leftrightarrow v \geq \frac{4(1+2s) - (1-s)\sqrt{2(2+5s+s^2)}}{3+8s+s^2}. \quad (32)$$

This threshold is larger than the one of the right-hand side of (32) if and only if  $0 \leq s < 0.357$ , approximately. Hence, delegated distribution is optimal if  $v \geq v_{PU}$ , with:

$$v_{PU}^+(s) \equiv \begin{cases} \frac{4(1+2s) - (1-s)\sqrt{2(2+5s+s^2)}}{3+8s+s^2} & \text{if } 0 < s \lesssim 0.357; \\ \sqrt{2s} & \text{if } 0.357 \gtrsim s < 1; \end{cases}$$

Finally, proceeding in the same way for the regime  $PP$ , we obtain that delegated distribution is optimal if:

$$v_{PP}^+(s) \equiv \begin{cases} 1 - \sqrt{\frac{1-s}{2(1+s)}} & \text{if } 0 < s \lesssim 0.157; \\ \sqrt{s} & \text{if } 0.157 \gtrsim s < 1; \end{cases}$$

We now compare the ranges for which delegated distribution is optimal in the four pricing regimes. We start with a comparison of the regimes  $PU$  with  $UU$ . In the latter, delegated distribution is optimal if  $v \geq v_{UU}^+(s)$  holds, whereas in the former delegated distribution is optimal if  $v \geq v_{PU}^+(s)$ . We start with the case  $0 \leq s < 0.357$ . The difference:

$$v_{PU}^+(s) - v_{UU}^+(s) \Leftrightarrow \frac{4(1+2s) - (1-s)\sqrt{2(2+5s+s^2)}}{3+8s+s^2} - \frac{4(1+s) - (1-s)\sqrt{4+5s}}{3+5s}$$

equals 0 at the lower bound  $s = 0$ . Instead, at the upper bound it is approximately equal to 0.034. The difference is also increasing in  $s$ , which implies that it is positive for all  $s$  between 0 and 0.357. In the range,  $0.357 < s \leq 1$ , the relevant comparison is:

$$\sqrt{2s} - \frac{4(1+s) - (1-s)\sqrt{4+5s}}{3+5s}.$$

It is easy to check that this difference is again equal to 0.034 at the lower bound. At the upper bound, it is equal to  $\sqrt{2} - 1 > 0$ . It is also increasing for all values of  $s$  in the range between 0.357 and 1. It follows that  $v_{PU}^+(s) > v_{UU}^+(s)$ , which implies that the range in which delegated distribution is optimal in the regime  $PU$  is a subset of the one in the regime  $UU$ .

Next, we compare  $v_{UU}^+(s)$  with  $v_{PP}^+(s)$ . In the latter regime, we need to distinguish whether  $s$  is below or above approximately 0.157. We start again with the former case.

The difference:

$$v_{UU}^+(s) - v_{PP}^+(s) \Leftrightarrow \frac{4(1+s) - (1-s)\sqrt{4+5s}}{3+5s} - 1 - \sqrt{\frac{1-s}{2(1+s)}}$$

equals  $1/\sqrt{2} - 1/3 > 0$  at  $s = 0$ , it is approximately equal to 0.339 at  $s = 0.157$ , and increasing for all values of  $s$  between 0 and 0.157. Turning to the range  $0.157 < s \leq 1$ , the relevant comparison is:

$$\frac{4(1+s) - (1-s)\sqrt{4+5s}}{3+5s} - \sqrt{s},$$

which is approximately equal to 0.339 at  $s = 0.157$ , decreasing in  $s$  for  $s \in (0.157, 1]$ , and equal to 0 at  $s = 1$ . It follows that  $v_{UU}^+(s) > v_{PP}^+(s)$ .

Finally, we compare  $v_{PP}^+(s)$  with  $v_{UP}^+(s)$ , noting that in the regime  $UP$ , delegated distribution maximizes the industry profit for all  $s \geq 1/2$ . We start with the region  $0 \leq s < 0.157$ . At the lower bound  $s = 0$ , the difference  $v_{PP}^+(s) - v_{UP}^+(s)$  equals  $\sqrt{3} - 1 - 1/\sqrt{2} = 0.025 > 0$ , and the upper bound  $s = 0.157$ , this difference is approximately 0.056. The difference is also increasing in  $s \in [0, 0.157)$ , which implies that it is positive in the entire range. Turning to the range  $0.157 < s \leq 0.5$ , the difference is approximately equal to 0.056 at  $s = 0.157$ , is decreasing in  $s$  for  $s < 1$ , and equals 0 at  $s = 1$ ; hence the difference is positive for  $s \in [0, 0.157)$ . It follows that  $v_{PP}^+(s) > v_{UP}^+(s)$ .

As a consequence, the ordering of the thresholds is:  $v_{UP}^+(s) < v_{PP}^+(s) < v_{UU}^+(s) < v_{PU}^+(s)$ . In the same way as in the proof of the previous propositions, we can show that for each threshold, there is a corresponding threshold in the  $\alpha$ - $\beta$ -plane, such that delegated distribution is optimal if and only if  $\alpha$  is below a threshold in the respective pricing regime. These thresholds, which are all increasing in  $\beta$ , can be ordered accordingly, that is,  $\alpha_{UP}^+(\beta) > \alpha_{PP}^+(\beta) > \alpha_{UU}^+(\beta) > \alpha_{PU}^+(\beta)$ .

## B Positive Correlation of Valuations with $v$ and $s$ being larger than 1

In this appendix, we consider a demand pattern of positive correlation of valuations with  $s > 1$ . The assumptions  $\hat{x} > 0$  and  $\hat{u} > 0$  then imply  $v \in (1, s)$ . This demand pattern corresponds to a situation of vertical differentiation between firms in which  $B$  offers the high-end product.

We first provide a proposition that characterizes the optimal distribution strategy and pricing regime (part (i)), as well as the optimality of delegated distribution (part (ii)). We then explain the intuition behind the results. Finally, we provide the proof of

the proposition.

**Proposition** *In the case of positive correlation of valuations with  $B$  offering the high-end product:*

(i) *In the absence of delegated distribution: (a) dual distribution is the optimal strategy in all pricing regimes; (b) if firms can contract on their pricing policies, personalized pricing by both firms is optimal.*

(ii) *If delegated distribution is feasible, in each pricing regime  $YZ \in \{UU, UP, PP\}$  there exists a threshold  $\alpha_{YZ}^{++}(\beta)$  such that delegated distribution is optimal if  $\alpha < \alpha_{YZ}^{++}(\beta)$ ; by contrast, it is never optimal in the regime  $PU$ . The thresholds can be ordered as follows:*

$$\alpha_{UP}^{++}(\beta) > \alpha_{PP}^{++}(\beta) > \alpha_{UU}^{++}(\beta) \quad \text{if } \beta > \check{\beta},$$

$$\alpha_{UP}^{++}(\beta) > \alpha_{UU}^{++}(\beta) > \alpha_{PP}^{++}(\beta) \quad \text{if } \beta < \check{\beta},$$

where  $\check{\beta} \approx 0.82$ .

If valuations are positively correlated and  $B$  is the one catering to high-end consumers, dual distribution is optimal, regardless of the pricing regime. To see why, consider first the situation where  $A$  charges a uniform price, and suppose that the firms negotiate a wholesale price equal to  $A$ 's monopoly price under mono distribution,  $p_U^m$ . In the continuation equilibrium,  $A$  can then secure the monopoly profit  $\Pi_U^m$  by charging  $p_U^m$ : this can only increase sales—regardless of the price charged by  $B$ —and  $A$  obtains the same margin on every sale, regardless of which firm make it. Furthermore, as  $B$  can charge a higher price to high-valuation consumers, the industry profit increases compared to mono distribution. The argument carries over when  $A$  charges personalized prices, setting the wholesale price to the highest of  $A$ 's monopoly prices, that is,  $w = 1$ —in this case,  $A$  is strictly better off if  $B$ 's makes a sale.

In addition, the industry profit is highest in the regime in which both firms charge personalized prices. Indeed, the optimal wholesale price then exceeds  $\hat{u}$ —i.e., the crossing point of the consumers' utility functions. This prevents  $B$  from competing for consumers in  $A$ 's core segment, as consumers who have a higher utility from  $A$ 's product have a utility from  $B$ 's product that is lower than  $\hat{u}$ .  $A$  thus acts as a monopolist towards these consumers, and can extract more surplus with personalized pricing. A wholesale price above  $\hat{u}$  also gives  $A$  a large revenue when  $B$  serves a consumer. This dampens competition in  $B$ 's core segment, and there as well, charging personalized prices enables  $B$  to increase its profit.

The proposition therefore confirms a general theme of the paper: With intra-brand competition, dual distribution is optimal for a large number of cases. This is due to the fact that an appropriately chosen wholesale tariff enables the firms to dampen compe-

tition.

Part (ii) of the proposition shows that delegated distribution can be optimal in three of the four pricing regimes. The only pricing regime in which this does not occur is *PU*. In this regime, *A* can extract the entire consumer surplus from low-valuation consumers whereas *B* cannot extract the entire surplus from any consumer (due to competition). This implies that using *A*'s direct channel is always profitable. Conversely, *A* contributes least value to the industry monopoly profit in the regime *UP*; hence, delegated distribution occurs for the largest parameter range there. Comparing the two symmetric pricing regimes, delegated distribution is optimal for a larger range in the regime *PP* than in *UU* if  $\beta$  is large. In that case, *A* is not a strong competitor to *B* for high-valuation consumers; shutting down the direct channel is then more profitable if *B* can extract the surplus from high-valuation consumers through personalized pricing.

### Proof of the Proposition

We start with part (i), statement (a)—i.e., we show that dual distribution is the optimal strategy in all pricing regimes (in the absence of delegated distribution). We first analyze the situation in which *A* sets a uniform price and then the one in which *A* sets personalized prices.

#### Uniform pricing by *A*

Suppose that the firms agreed on the wholesale tariff  $F = 0$  and  $w = p_A^m$ .

Assume first that *B* also charges a uniform price. In the continuation equilibrium, *B* then charges a price  $p_B > w = p_A^m$  on all consumers served and obtains a positive share the market,<sup>7</sup> whereas *A* charges some price  $p_A^+$ .<sup>8</sup> Suppose now that *A* deviates and charges the mono-distribution price  $p_A^m$ . Following this deviation, total demand (weakly) exceeds that of the mono-distribution outcome (as consumers have more choice) and *A* obtains a margin  $p_A^m$  (either directly through  $p_A$  or indirectly through  $w$ ). It follows that *B*'s profit is strictly positive after the deviation, and *A*'s profit is weakly larger. Therefore, in the continuation equilibrium in which *A* best responds to  $p_B$ , the same result must hold. As a consequence, the tariff  $F = 0$  and  $w = p_A^m$  yields a continuation equilibrium that strictly increases *B*'s profit and weakly increases *A*'s profit.

<sup>7</sup>To see this, suppose instead that *B* does not attract any consumer. In that case, *B* prices at cost (i.e.,  $p_B = w = p_A^m$ ) and *A* prices so as to attract consumers with type  $x = 0$  (i.e.,  $p_A \leq p_A^m - (v - 1) < p_A^m$ ), and obtains a profit lower than  $\Pi_A^m$  (as it charges  $p_A \neq p_A^m$ , and *B* attracts no additional consumer). But then, *A* would profitably deviate by charging  $p_A' = p_A^m$ : compared with the mono distribution outcome, this would (weakly) expand demand (as consumers can now buy from both firms) and *A* would obtain the same margin on each customer (either directly through  $p_A'$ , or indirectly through  $w$ ); hence, the deviation brings a profit of at least  $\Pi_A^m$ .

<sup>8</sup>Compared with mono distribution, *B* now charges a "lower" price (the mono distribution outcome can be interpreted as *B* charging  $p_B \geq v$ ), and in response *A* also lowers its own price.

The same reasoning applies to the case in which  $B$  charges personalized prices. In the continuation equilibrium, after firms agreed on a tariff  $F = 0$  and  $w = p_A^m$ ,  $B$ 's demand is positive, and  $A$  charges some price  $p_A^{++}$ . A deviation to  $p_A = p_A^m$  then gives  $A$  a profit that is weakly larger than  $\Pi_A^m$  due to the fact that it obtains the same price  $p_A^m$  on all consumers served and demand weakly exceeds that with mono distribution.

### Personalized pricing by $A$

Under mono distribution,  $A$  charges each consumer a price  $u_A(x)$ . Suppose now that the firms agree on the two-part tariff  $F = 0$  and  $w = 1$ . In the continuation equilibrium,  $B$  obtains a positive demand from consumers close enough to  $x = 0$ : these consumers have a net willingness-to-pay for  $B$ 's product equal to  $u_B(x)$ , which is larger than  $B$ 's wholesale price  $w = 1$ , and  $A$  earns a higher margin by letting  $B$  serve these consumers, as  $w = 1 > u_A(x)$ . Therefore,  $A$  charges prices larger than or equal to  $w$  to the consumers served by  $B$ , and obtains a higher margin compared to mono distribution. This holds regardless of whether  $B$  charges personalized prices or a uniform price. In addition,  $B$  does not offer a positive net utility to consumers it does not serve: if it did,  $B$  would serve these consumers because  $A$  is better off by letting  $B$  serve these consumers and get a margin  $w > u_A(x)$  instead of serving the consumers itself. It follows that  $A$  can still charge a price of  $p_A(x) = u_A(x)$  to these consumers. Hence,  $A$  obtains the same profit as under mono distribution from consumers that it serves but a strictly larger profit from consumers served by  $B$ . Therefore, the two-part tariff  $F = 0$  and  $w = 1$  increases the profits of both firms.

### Endogenous Pricing Policy

We next prove statement (b) of part (i), that is, we show that if firms can contract on their pricing policies, they choose the regime of personalized pricing by both firms. To simplify the exposition, we focus on the case  $X > 1$  (i.e.,  $X$  is large enough that it does not limit  $A$ 's demand). All our results also hold if this inequality was not fulfilled.

We start with the regime in which both firms set a uniform price. Following the analysis in Section 5 and the proof of Proposition 3, we denote the consumer indifferent between buying from  $B$  or  $A$  by  $x_{BA} > 0$ , whereby:

$$x_{BA}(p_A, p_B) = \frac{v - p_B - 1 + p_A}{s - 1},$$

and the consumer indifferent between buying from  $A$  and not buying by  $x_A > 0$ , whereby:

$$x_B(p_A) = v - p_A.$$

The profit functions (gross of the fixed fee) are then  $\Pi_A = x_{BA}(p_A, p_B)w + [x_A(p_A) - x_{BA}(p_A, p_B)]p_A$  and  $\Pi_B = x_{BA}(p_A, p_B)(p_B - w)$ . Solving for the equilibrium retail prices,

as a function of the wholesale price  $w$ , yields:

$$\begin{aligned} p_A(w) &= \frac{2 - 1 + 3w - v}{4s - 1}, \\ p_B(w) &= \frac{v(2s - 1) + w(2s + 1) - s}{4s - 1}. \end{aligned}$$

The associated demands are  $D_A(w) = x_A(p_A(w)) - x_{BA}(p_A(w), p_B(w))$  and  $D_B(w) = x_{BA}(p_A(w), p_B(w))$ . In the first stage, firms choose  $w$  so as to maximize the industry profit,  $\Pi(w) = p_A(w) D_A(w) + p_B(w) D_B(w)$ . Inserting the respective demands and prices, we obtain that equilibrium wholesale price is:

$$w_{UU}^* = \frac{s(1 + 4v + 4s)}{2(5 + 4s)}.$$

Inserting the equilibrium prices into the industry profit yields:

$$\Pi_{UU}^* = \frac{5s + 4s^2 - 1 + 4v(1 + s)(2 - v)}{4(s - 1)(5 + 4s)}$$

We next turn to the case in which only  $B$  charges personalized prices. Given  $w$  and  $p_A$ ,  $B$ 's price response is such that consumers  $x$  with  $v_B(x) - w > v_A(x) - p_A$ , or:

$$x < \frac{v - w - 1 + p_A}{s - 1},$$

buy from  $B$ .  $A$ 's maximization problem with respect to  $p_A$  is therefore given by:

$$w \left( \frac{v - w - 1 + p_A}{s - 1} \right) + p_A \left( 1 - p_A - \frac{v - w - 1 + p_A}{s - 1} \right),$$

leading to an optimal  $p_A$  of:

$$p_A(w) = \frac{w s - v}{s - 2s}.$$

We now turn to the wholesale stage. The two firms seek to maximize the industry profit given by:

$$\Pi = p_A(w) \left( 1 - p_A(w) - \frac{v - w - 1 + p_A(w)}{s - 1} \right) + \int_0^{\frac{v - w - 1 + p_A(w)}{s - 1}} [p_A(w) + v_B(x) - v_A(x)] dx.$$

Maximizing this profit with respect to  $w$  yields  $w_{UP}^* = (s + v) / (2(1 + s))$ . Inserting  $w = w_{UP}^*$  into the industry profit yields:

$$\Pi_{UP}^* = \frac{s(2 + s) - v(1 + 2s)(2 - v)}{4(1 + s)(s - 1)}.$$

Comparing  $\Pi_{UP}^*$  and  $\Pi_{UU}^*$ , we obtain:

$$\Pi_{UP}^* - \Pi_{UU}^* = \frac{(v-1)^2(1+4s^2+6s)}{4(5+4s)(1+s)(s-1)} > 0.$$

Third, we determine the industry profit in case both firms charge personalized prices. Following the analysis in Section 5 and the proof of Proposition 4, taking into account that the optimal wholesale price will be between  $\hat{u}$  and 1, the industry profit in this case can be written as:

$$\Pi = \int_0^{1-w} [w + v_B(x) - v_A(x)] dx + \int_{1-w}^{\frac{v-w}{s}} [v_B(x)(x)] dx + \int_{\frac{v-w}{s}}^1 [v_A(x)(x)] dx.$$

The first term is the profit from consumers close  $x = 0$ , who have a higher valuation than  $w$  for the product of each firm, which implies that the two firms compete for these consumers. The second term is the profit from consumers in the middle, whose valuation for  $B$ 's product is above  $w$  but their valuation for  $A$ 's product is below  $w$ . As  $w$  is  $A$ 's opportunity cost,  $A$  is better off when  $B$  serves these consumers, which implies that  $B$  extracts their entire consumer surplus. Finally, the third term is the profit from consumer whose valuation for  $B$ 's product is below  $w$ , which implies that  $A$  can extract their consumer surplus. Maximizing with respect to  $w$ , we obtain that the optimal wholesale price is:

$$w_{PP}^* = \frac{1+s-v}{s^2-1+s},$$

leading to an industry profit of:

$$\Pi_{PP}^* = \frac{(1+s)^2 - v(2+s)(2-v)}{2(s^2-1+s)}.$$

We next turn to the regime in which only  $A$  sets personalized prices. Following the same steps as in the proof of Proposition 2,  $A$ 's best response to  $p_B$  is to serve all consumers  $x$  with  $v_A(x) - w > v_B(x) - p_B$ . Since, as shown above,  $B$  is always active in equilibrium, this implies that  $A$  serves consumers  $x$ , such that  $(v - p_B)/s < x \leq 1$ . Therefore,  $B$ 's profit at the retail stage is:

$$(p_B - w) \left( \frac{v - p_B}{s} \right),$$

leading to an optimal retail price of  $p_B(w) = (v + w)/w$ . Turning to the wholesale

stage, firms maximize the industry profit, given by:

$$p_B(w) \left( \frac{v - p_B(w)}{s} \right) + \int_{\frac{v - p_B(w)}{s}}^1 v_A(x) dx,$$

with respect to  $w$ . This yields  $w_{PU}^* = (2s - v) / (2s - 1)$ . Inserting this into the industry profit yields:

$$\Pi_{PU}^* = \frac{2s + v(v - 2)}{2(2s - 1)}.$$

Comparing  $\Pi_{PP}^*$  and  $\Pi_{PU}^*$ , we obtain:

$$\Pi_{PP}^* - \Pi_{PU}^* = \frac{(v - 1)^2 (s^2 + 2s - 1)}{2(s^2 - 1 + s)(2s - 1)} > 0.$$

Finally, to determine the optimal pricing regime in the negotiation, we need to compare  $\Pi_{PP}^*$  with  $\Pi_{UP}^*$ . The sign of the difference between the two profits is given by:

$$\text{sign} \{ \Pi_{PP}^* - \Pi_{UP}^* \} = \text{sign} \{ (s^2 - 2)(1 + s^2 + s) + v(2 - v)(3 - s^2 + s) \}. \quad (33)$$

At the lower bound of  $v$ —i.e.,  $v = 1$ —the right-hand side of (33) equals  $(s - 1)(s + 1)(s^2 + s - 1)$ , which is strictly positive due to the fact that  $s > 1$ . At the upper bound of  $v$ —i.e.,  $v = s$ —the right-hand side of (33) equals  $2(s - 1)(s^3 - s + 1)$ , which is again strictly positive as  $s > 1$ . Finally,  $\Pi_{PP}^* - \Pi_{UP}^*$  is a concave second-order polynomial in  $v$ , which implies that the difference must be positive in the admissible range, given that it is positive at the limits of this range; hence,  $\Pi_{PP}^* > \Pi_{UP}^*$  for all  $v$  in the admissible range. Therefore, firms choose the regime  $PP$  in case they can contract on their pricing policies.

### Delegated Distribution

We last prove the statement in part (ii) of the proposition. To do so, we proceed in the same way as in the proof of Proposition 6 by comparing the respective profits from dual distribution (i.e.,  $\Pi_{UU}^*$ ,  $\Pi_{UP}^*$ ,  $\Pi_{PP}^*$ , and  $\Pi_{PU}^*$ ) with the profits from delegated distribution. The latter are  $v^2/(4s)$ , in case  $B$  sets only a uniform price, and  $v^2/(2s)$ , in case  $B$  sets personalized prices.

Starting with the regime in which both firms set a uniform price, the comparison between  $\Pi_{UU}^*$  and  $v^2/(4s)$  yields that  $\Pi_{UU}^* - v^2/(4s) = (s - 1)/(4s) > 0$  at the lower bound of  $v$  (i.e.,  $v = 1$ ) and  $\Pi_{UU}^* - v^2/(4s) = -((s - 1)/(4(5 + 4s))) < 0$  at the upper bound of  $v$  (i.e.,  $v = s$ ). In addition, the difference is strictly decreasing in  $v$ , and the

threshold value of  $v$  at which both profits are equal to each other is:

$$v_{UU}^{++}(s) \equiv \frac{4s(1+s) - (s-1)\sqrt{s(5+4s)}}{5+3s}.$$

It is easy to check that the threshold is below  $s$  for all  $s > 1$ . Therefore, delegated distribution is optimal if and only if  $v > v_{UU}^{++}(s)$ .

Turning to the regime in which only  $B$  sets personalized prices, we obtain that  $\Pi_{UP}^* - v^2/(2s)$  is positive at the lower bound of  $v$  if and only if  $s > 2$  but strictly negative at the upper bound of  $v$ . The difference is again strictly decreasing in  $v$ . If a solution to  $\Pi_{UP}^* = v^2/(4s)$  in the admissible range exists, which is the case if  $s > 2$ , the threshold value at which both profits are equal to each other is:

$$v_{UP}^{++}(s) \equiv \frac{s \left( 2s + 1 - \sqrt{3(s+1)(s-1)} \right)}{2+s}.$$

This threshold is again below  $s$  for  $s > 1$ , which implies that delegated distribution is optimal if and only if  $v > v_{UP}^{++}(s)$ .

Proceeding in the same way in the regime in which both firms set personalized prices yields that  $\Pi_{PP}^* - v^2/(2s)$  is positive at the lower bound of  $v$ , negative at the upper bound of  $v$ , and strictly decreasing in  $v$ . The threshold value at which the two profits are equal to each other is:<sup>9</sup>

$$v_{PP}^{++}(s) \equiv \frac{s(2+s) - \sqrt{s(s^2+s-1)}}{1+s};$$

hence, delegated distribution is optimal if and only if  $v > v_{PP}^{++}(s)$ .

Finally, in the regime in which only  $A$  sets personalized prices, the difference between  $\Pi_{PU}^* - v^2/(4s)$  is:

$$\frac{(2s-v)^2}{4s(2s-1)},$$

which is strictly positive for all parameters in the admissible range.

Therefore, we have shown that, in case of positive correlation of valuations with  $v \in (1, s)$  and  $s > 1$ , delegated distribution is optimal if  $v > v_{YZ}^{++}(s)$  in the regimes  $\{Y, Z\}$  in  $\{UU, UP, PP\}$ , but it is never optimal in the regime  $PU$ .

In the same way as in previous proofs, we can show that, for the pricing regimes  $UU$ ,  $UP$ , and  $PP$ , there is a unique threshold in the  $\alpha$ - $\beta$ -plane. We denote the respective thresholds by  $\alpha_{UU}^{++}(\beta)$ ,  $\alpha_{UP}^{++}(\beta)$ , and  $\alpha_{PP}^{++}(\beta)$ .

We next compare the threshold values in the regimes  $UU$ ,  $UP$ , and  $PP$ . We start with a comparison between  $v_{UU}$  and  $v_{UP}$ . Taking the difference between  $v_{UU}^{++}(s)$  and

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<sup>9</sup>This threshold is also below  $s$  for  $s > 1$ .

$v_{UP}^{++}(s)$  yields:

$$\frac{s(5+3s)\sqrt{3(s+1)(s-1)} - (s-1)\left[s(3+2s) - (2+s)\sqrt{s(5+4s)}\right]}{(2+s)(5+3s)},$$

which is equal to 0 at the lower bound of  $s$  (i.e.,  $s = 1$ )<sup>10</sup> but strictly increasing in  $s$  for all  $s > 1$ . It follows that  $v_{UU}^{++}(s) > v_{UP}^{++}(s)$ , which implies  $\alpha_{UU}^{++}(\beta) < \alpha_{UP}^{++}(\beta)$ .

Next, we compare  $v_{PP}^{++}(s)$  and  $v_{UP}^{++}(s)$ . Taking the difference between  $v_{PP}^{++}(s)$  and  $v_{UP}^{++}(s)$  yields:

$$\frac{s(1+s)\sqrt{3(s+1)(s-1)} - s(s^2 - s - 3) - (2+s)\sqrt{s(s^2 + s - 1)}}{(2+s)(1+s)},$$

which is again 0 at  $s = 1$  but strictly increasing in  $s$  for all  $s > 1$ ; hence,  $v_{PP}^{++}(s) > v_{UP}^{++}(s)$ , or  $\alpha_{PP}^{++}(\beta) < \alpha_{UP}^{++}(\beta)$ .

Lastly, the difference between  $v_{UU}^{++}(s)$  and  $v_{PP}^{++}(s)$  is given by:

$$\frac{s(s^2 - 3s - 6) + s(5+3s)\sqrt{s(s^2 + s - 1)} - (1+s)(1-s)\sqrt{s(5+4s)}}{(1+s)(5+3s)}.$$

This difference is again 0 at  $s = 1$ . It is increasing in  $s$  for  $s \searrow 1$ , which implies that  $v_{UU}^{++}(s) > v_{PP}^{++}(s)$  for values of  $s$  close to 1. By contrast, the difference is negative as  $s \rightarrow \infty$ , which implies that  $v_{UU}^{++}(s) < v_{PP}^{++}(s)$  for very high values of  $s$ . Translating this into the respective thresholds values of  $\alpha$  implies that  $\alpha_{UU}^{++}(\beta) < \alpha_{PP}^{++}(\beta)$  for values of  $\beta$  close to 0 and  $\alpha_{UU}^{++}(\beta) > \alpha_{PP}^{++}(\beta)$  for sufficiently large values of  $\beta$ . Solving  $\alpha_{UU}^{++}(\beta) = \alpha_{PP}^{++}(\beta)$  yields that there is a unique threshold for  $\beta$  in the admissible range, which is approximately equal to 0.82. This threshold is denoted  $\check{\beta}$  in the proposition.

## C Generalization of Proposition 3

In this appendix, we generalize part Proposition 3 to an extended setting in which consumers with unit demand obtain values, net of distribution costs, of  $u_A(x)$  and  $u_B(x)$  for the products of the two firms, where  $u_A(\cdot)$  and  $u_B(\cdot)$  are both twice continuously differentiable,  $x$  is distributed according to a twice continuously differentiable c.d.f.  $G(x)$  over  $\mathbb{R}_+$  and:

- $\forall x \in \mathbb{R}_+, u'_A(x) < u'_B(x) < 0$ ;
- $u_i(\bar{x}_i) = 0$  for some  $\bar{x}_i > 0$ ; and
- $u_A(\hat{x}) = u_B(\hat{x}) > 0$  for some  $\hat{x} > 0$ .

<sup>10</sup>Recall that  $s \geq 1$  if  $B$  offers the high-quality product.

This implies that, as in our baseline model, the curves  $u_A(\hat{x})$  and  $u_B(\hat{x})$  intersect exactly once, and this intersection occurs in the positive quadrant.

Let:

$$D_i^m(p_i) \equiv G(u_i^{-1}(p_i)),$$

denote the monopolistic demand for firm  $i$ 's product:

$$p_i^m \equiv \arg \max_{p_i} p_i D_i^m(p_i),$$

denote firm  $i$ 's monopoly price:

$$x_i^m \equiv u_i^{-1}(p_i^m),$$

denote the location of the associated marginal consumer, and:

$$q_i^m \equiv D_i^m(p_i^m) = G(x_i^m) \quad \text{and} \quad \pi_i^m \equiv p_i^m q_i^m,$$

denote the monopoly output and profit, respectively. Our working assumption is that  $B$  would seek to serve more consumers than  $A$  in these monopoly situations:

**Assumption A:**  $B$ 's monopoly profit function is strictly quasi-concave and  $q_B^m > q_A^m$ .

Let  $w^m \equiv u_B(x_A^m)$ . For  $w \geq w^m$ , there exists a continuation equilibrium in which  $A$  charges its monopoly price,  $p_A^m$ , and  $B$  does not serve any consumer (e.g., by charging  $p_B = w^m$ ). If instead  $w < w^m$ , both firms can obtain a positive market share:  $A$  then faces a demand:

$$D_A(p_A, p_B) \equiv G(\Delta^{-1}(p_A - p_B)),$$

where:

$$\Delta(x) \equiv u_A(x) - u_B(x),$$

whereas  $B$  faces a demand given by:

$$D_B(p_A, p_B) \equiv D_B^m(p_B) - D_A(p_A, p_B).$$

For the sake of exposition, we will assume that there then exists an equilibrium where both firms obtain a positive market share, which is moreover "well-behaved":

**Assumptions B:** For any  $w \leq w^m$ , there exists a unique downstream equilibrium,  $(p_A^e(w), p_B^e(w))$ , where  $p_A^e(w)$  and  $p_B^e(w)$  are continuous and increasing in  $w$ , and such that  $p_A^e(w^m) = p_A^m$  and  $p_B^e(w^m) = w^m$ .

We have:

**Proposition:** Under Assumptions A and B, dual distribution is the optimal strategy if valuations are positively correlated and A charges a uniform price.

**Proof:** We first consider the regime in which both firms charge a uniform price. Starting from a situation in which the firms negotiate  $w = w^m$ , and thus A obtains  $\Pi_A^m$ , consider a small reduction in the wholesale price from  $w^m$  to  $w < w^m$ , together with a fixed fee,  $F(w)$ , designed to appropriate B's profit (or almost all of it, to ensure acceptance). A then obtains (almost all of) the industry profit, which can be expressed as:

$$\Pi(w) = \Pi_A(w) + \Pi_B(w),$$

where:

$$\begin{aligned}\Pi_A(w) &= p_A^e(w) D_A(p_A^e(w), p_B^e(w)) + w D_B(p_A^e(w), p_B^e(w)) + F(w), \\ \Pi_B(w) &= [p_B^e(w) - w] D_B(p_A^e(w), p_B^e(w)) - F(w).\end{aligned}$$

By deviating from the downstream equilibrium and charging:

$$\hat{p}_A(w) = p_B^e(w) - u_B(x_A^m) + u_A(x_A^m) = p_A^m + p_B^e(w) - w^m,$$

A would maintain its output of  $q_A^m$ , and generate an output  $\hat{q}_B = D_B^m(p_B^e(w)) - q_A^m$  for B. Therefore:

$$\begin{aligned}\Pi_A(w) &\geq \hat{p}_A(w) D_A(\hat{p}_A(w), p_B^e(w)) + w D_B(\hat{p}_A(w), p_B^e(w)) + F(w) \\ &= [p_A^m + p_B^e(w) - w^m] q_A^m + w [D_B^m(p_B^e(w)) - q_A^m] + F(w) \\ &= \pi_A^m + [p_B^e(w) - w - w^m] q_A^m + w D_B^m(p_B^e(w)) + F(w).\end{aligned}$$

Likewise, noting that B could always choose to deviate from the downstream equilibrium and charge  $p_B = w$ , we have:

$$\Pi_B(w) \geq -F(w).$$

Adding these two inequalities yields (recalling that  $\Pi(w) = \Pi_A(w) + \Pi_B(w)$ ):

$$\Pi(w) - \pi_A^m \geq \phi(w) \equiv [p_B^e(w) - w - w^m] q_A^m + w D_B^m(p_B^e(w)).$$

Note that  $\phi(w^m) = 0$  because  $p_B^e(w^m) = w^m$  and  $D_B^m(w^m) = G(x_A^m) = q_A^m$ . Taking the derivative of  $\phi(w)$  and evaluating it at  $w = w^m$ , we obtain (again using  $p_B^e(w^m) = w^m$

and  $q_A^m = D_B^m(w^m)$ ):

$$\begin{aligned}\phi'(w^m) &= \left[ \frac{dp_B^e}{dw}(w) - 1 \right] q_A^m + D_B^m(w^m) + w \frac{dD_B^m}{dp_B^e}(p_B^e(w)) \frac{dp_B^e}{dw}(w) \\ &= \frac{dp_B^e}{dw}(w) \left[ D_B^m(w^m) + w^m \frac{dD_B^m}{dp_B^e}(w^m) \right],\end{aligned}$$

where the expression within bracket is negative from Assumption A.<sup>11</sup> It follows that a reduction of  $w$  below  $w^m$  is strictly profitable, implying that dual distribution is the unique optimal mode of distribution.

Turning to the hybrid regime in which  $B$  charges personalized prices, the same logic as in the main text can be applied. In particular, setting  $p_A = p_A^*$  and  $w = p_B^*$ , where  $p_A^*$  and  $p_B^*$  are the equilibrium retail prices under uniform pricing, delivers a higher industry profit than dual distribution with uniform pricing, and therefore also a higher profit than  $\Pi_U^m$ .

## D Inter-brand versus intra-brand competition

In this appendix, we analyze a situation of inter-brand competition between two completely independent firms in the scenario of positive correlation of valuations. Specifically, we compare the profits of the two symmetric regimes (i.e., uniform pricing by both firms and personalized pricing by both firms) with each other. We show that the insights from such an analysis are misleading if applied to a situation of intra-brand competition between two channels, which is partly governed by the negotiated wholesale contract.

The case of inter-brand competition is equivalent to a situation in which no wholesale contract exists, that is, the wholesale price  $w$  equals zero. We can then apply the same techniques as in the proof of Propositions 3 and 4 to solve for the aggregate profits of the firms in the two pricing regimes via inserting  $w = 0$  in the respective equations. This yields:

$$\Pi_{UU}^A(w = 0) = \frac{(2 - v - s)^2}{(4 - s)^2(1 - s)} \quad \text{and} \quad \Pi_{UU}^B(w = 0) = \frac{(2v - s(1 + v))^2}{s(4 - s)^2(1 - s)}$$

for the uniform pricing regime, and:

$$\Pi_{PP}^A(w = 0) = \frac{(1 - v)^2}{2(1 - s)} \quad \text{and} \quad \Pi_{PP}^B(w = 0) = \frac{(v - s)^2}{2s(1 - s)}$$

<sup>11</sup>Indeed, Assumption A implies that firm  $B$ 's optimal monopoly demand is strictly larger than  $q_A^m = D_B^m(w^m)$ ; hence, firm  $B$ 's monopoly price is below  $w^m$ , which implies that the first-order condition evaluated at  $w^m$  is negative.

for the personalized pricing regime. Comparing  $\Pi_{PP}^i(w = 0)$  with  $\Pi_{UU}^i(w = 0)$ ,  $i = A, B$ , we obtain that  $A$  benefits from personalized pricing if and only if:

$$v \leq v_A(s) \equiv \frac{(\sqrt{2} - 1)(2\sqrt{2} - s)}{4 - \sqrt{2} - s},$$

and  $B$  benefits from personalized pricing if and only if:

$$v \geq v_B(s) \equiv \frac{s(\sqrt{2} + 1)(4 - \sqrt{2} - s)}{2\sqrt{2} + s}.$$

Turning to the joint profits, the difference between the firms' aggregate profits with personalized pricing and their profits with uniform pricing (i.e.,  $\Pi_{PP}^A(w = 0) + \Pi_{PP}^B(w = 0) - \Pi_{UU}^A(w = 0) - \Pi_{UU}^B(w = 0)$ ) is:

$$\frac{1}{2} \frac{(8 + 14s - 9s^2 + s^3)(v^2 + s) - 4vs(12 - 6s + s^2)}{(1 - s)s(4 - s)^2}. \quad (34)$$

The numerator is a convex quadratic polynomial of  $v$  with the two roots:

$$\underline{v}(s) \equiv \frac{2s(12 - 6s + s^2) - (1 - s)(4 - s)\sqrt{s(12s - 4 - s^2)}}{8 + 14s - 9s^2 + s^3}$$

and

$$\bar{v}(s) \equiv \frac{2s(12 - 6s + s^2) + (1 - s)(4 - s)\sqrt{s(12s - 4 - s^2)}}{8 + 14s - 9s^2 + s^3}.$$

It is straightforward to check that for  $s < 2(3 - 2\sqrt{2}) \approx 0.343$ , the numerator of (34) is always positive (i.e., no root exists in this case), which implies that firms' aggregate profits are larger with personalized prices. In addition, for these values of  $s$ ,  $v_A(s) > v_B(s)$ , which implies that both firms benefit from personalized pricing if and only if  $v_B(s) \leq v \leq v_A(s)$ . Instead, for  $s \geq 2(3 - 2\sqrt{2})$ , the joint profit with personalized pricing is larger if  $v \leq \underline{v}(s)$  or  $v \geq \bar{v}(s)$ . The threshold  $\bar{v}(s)$  is increasing in  $s$  and reaches the upper bound for  $v$ , which is 1, at  $s = 5 - \sqrt{17} \approx 0.877$ . Similarly,  $\underline{v}(s)$  is decreasing in  $s$  and reaches the lower bound for  $v$ , which is  $s$ , also at  $s = 5 - \sqrt{17} \approx 0.877$ . It follows that for  $s > 5 - \sqrt{17}$ , the joint profit from uniform pricing is higher in the admissible range for  $v$ .

The result shows that inter-brand competition is less fierce in a situation in which valuations are positively correlated than in one with negative correlation of valuations. As mentioned in the main text, Thisse and Vives (1988) and Shaffer and Zhang (1995) show that in a Hotelling model—which implies a negative correlation of valuations—aggregate profits with personalized pricing are unambiguously lower than those with uniform pricing. Instead, our analysis shows that with positive correlation of valua-

tion, this is not necessarily true and there is indeed a sizable range in which the opposite result occurs.

The comparison between the profits from personalized pricing and from uniform pricing in case of inter-brand competition provides little guidance for the question whether or not dual distribution is optimal in case of intra-brand competition. In particular, as shown in the proof of Proposition 4, mono distribution is optimal under intra-brand competition if  $v$  is close to  $s$ —i.e., if  $v < s(2 + \sqrt{1-s})/(1+s)$ . Instead, as shown above, with inter-brand competition, for all values of  $s \leq 0.877$  personalized pricing leads to higher aggregate profits than uniform pricing if  $v$  is close to  $s$ . Moreover, mono distribution can be optimal even if personalized pricing increases the profits of *both* firms in the case of inter-brand competition. The latter occurs if  $v_B(s) \leq v \leq v_A(s)$ . From the proof of Proposition 4, mono distribution under intra-brand competition is optimal if and only if  $v \geq \tilde{v}(s)$ . It is straightforward to check that  $\tilde{v}(s) > v_B(s)$  for  $s \in (0, 1)$  and  $\tilde{v}(s) < v_A(s)$  for  $s \lesssim 0.230$ ; hence, the situation occurs for  $s \in (0, 0.230)$  if  $v_B(s) \leq v \leq \tilde{v}(s)$ .

## E Independent Pricing Policy Choices

In this appendix, we analyze the game laid out in the second part of Section 5.3—i.e.,  $A$  and  $B$  independently choose their individual pricing policy (i.e., uniform pricing or personalized pricing).

In Sections 5.1 and 5.2, we determined for each of the four pricing regimes the result of the wholesale contract negotiation between  $A$  and  $B$ . We now move one stage backwards and analyze stage 2 of the game. If the distribution strategy chosen in the first stage is mono distribution,  $A$  chooses personalized pricing, which leads to a profit of  $1/2$  for  $A$ .

Now suppose that, in the first stage, the firms agree to use dual distribution. We start with the pricing policy of  $B$ . First, we note that comparing  $B$ 's profit from personalized pricing with its profit from uniform pricing yields that the difference does not depend on  $B$ 's bargaining power  $1-a$ . The reason is that  $B$ 's outside option equals zero. This implies that in any combination of pricing choices of  $A$  and  $B$ ,  $B$  always receives the industry profit from dual distribution minus  $A$ 's outside option, multiplied by  $1-a$ ; hence,  $1-a$  cancels out in any comparison. Therefore,  $B$  chooses the pricing policy that maximizes the industry profit, given  $A$ 's pricing policy.

If  $A$  chooses uniform pricing, we know from the proof of Proposition 3 that:

$$\Pi_{UP}^* - \Pi_{UU}^* = \frac{(s-v)^2}{4s} \frac{4+6s+s^2}{(4+5s)(1-s^2)} > 0,$$

which implies that  $B$ 's best response is to choose personalized pricing. Similarly, if  $A$  chooses personalized pricing, we know from the proof of Proposition 5 that:

$$\Pi_{PP}^* - \Pi_{PU}^* = \frac{(1 + 3s + s^2)(v(1 + s) - 2s)^2}{2s(1 - s^2)(2 + 5s + s^2)} > 0;$$

hence, choosing personalized pricing is again the best response of  $B$ . Therefore, regardless of  $A$ 's choice, it is optimal for  $B$  individually to choose personalized pricing.

We next consider  $A$ 's optimal individual choice. As personalized pricing is a dominant strategy for  $B$ ,  $A$  needs to compare the profit it receives in the symmetric regime  $PP$  with the profit it receives in the hybrid regime  $UP$ . In either case, due to the fact that  $B$ 's outside option is zero,  $A$  receives the resulting industry profit, multiplied by  $a$ , plus its outside option, multiplied by  $1 - a$ . Therefore,  $A$ 's profit when choosing uniform pricing is:

$$a \left( \frac{s + 2s^2 - 4vs - 2vs^2 + 2v^2 + v^2s}{4s(1 - s^2)} \right) + (1 - a)\frac{1}{4}, \quad (35)$$

and its profit when personalized pricing is:

$$a \left( \frac{s(1 + 3s) - 4vs(1 + s) + v^2(1 + s)^2}{2s(1 - s^2)} \right) + (1 - a)\frac{1}{2}. \quad (36)$$

Comparing (35) with (36) yields that  $A$ 's profit from personalized pricing is higher if and only if:

$$a \leq \frac{(1 + s)(1 - s)}{2v(2 + 3s) - v^2(3 + 2s) - s(4 + s)}.$$

It is straightforward to check that the right-hand side of this inequality is strictly positive for all  $v \in (0, 1)$  and  $s \in (0, v)$ . Moreover, it is smaller than 1 as long as  $v \geq (1 + 4s)/(3 + 2s)$ . If the reverse holds true, the inequality is always fulfilled as  $a \in (0, 1)$ , which implies that  $A$  always chooses personalized pricing. Instead, if the inequality holds,  $A$  chooses uniform pricing if  $a$  is sufficiently large.

Finally, we analyze the first stage of the game in which firms agree on whether or not to use dual distribution.  $A$  is only willing to do so if its resulting profit is larger than the one it obtains from mono distribution. As stated above, the latter profit is  $1/2$  because  $A$  will always choose personalized pricing in that case. Therefore, the equilibrium distribution regime can be determined by comparing the profit in (35) with  $1/2$ , in case  $a > (1 + s)(1 - s)/[2v(2 + 3s) - v^2(3 + 2s) - s(4 + s)]$  and comparing the profit in (36) with  $1/2$ , in case  $a \leq (1 + s)(1 - s)/[2v(2 + 3s) - v^2(3 + 2s) - s(4 + s)]$ . Doing so yields that, for  $a$  sufficiently high, the equilibrium outcome is qualitatively similar to that of the game in which firms negotiate the pricing regime. In particular, if  $a \rightarrow 1$ ,

it is evident that the profits in (35) and (36) go to the industry profits in the respective pricing regimes. Therefore,  $A$ 's comparison of whether to use dual distribution or not is the same as in the game in which the pricing regime is negotiated, which leads to the same thresholds as in Proposition 5. Instead, if  $a$  is low,  $A$  receives only a small part of the profit from dual distribution, which implies that mono distribution occurs for a larger range in equilibrium.