THÈSE

En vue de l'obtention du

DOCTORAT DE L'UNIVERSITÉ DE TOULOUSE

Délivré par l'Université Toulouse 1 Capitole

Présentée et soutenue par

Yang YANG

Le 10 septembre 2021

Three Essays on Industrial Organization

Ecole doctorale : **TSE - Toulouse Sciences Economiques**

Spécialité : **Sciences Economiques - Toulouse**

Unité de recherche :
UMR 5314 -TSE-R

Thèse dirigée par
Andrew RHODES et Daniel GARRETT

Jury

M. Ioana CHIOVEANU, Rapporteur
M. Jidong ZHOU, Rapporteur
M. WILFRIED SAND-ZANTMAN, Examinateur
M. Andrew RHODES, Directeur de thèse
Mme Daniel GARRETT, Co-directrice de thèse
M. Bruno JULLIEN, Président

Three Essays on Industrial Organization

Yang Yang

Ph.D. Dissertation
Toulouse School of Economics

Supervised by
Daniel Garrett, Andrew Rhodes

Abstract

My thesis aims to understand the impacts of search frictions generated by the recent development of digital technology, and provide some practical policy implications. The first chapter analyzes the impact of social media on the spread of fake news when the search costs of consumers to find their favorite news are reduced by social media, and then the thesis studies the effects of price transparency in online markets in the second chapter. The third chapter analyzes a monopoly firm's optimal information revelation strategy and return policy when consumers can choose to either buy immediately without knowing their exact match value or search to learn the exact match value.

The first chapter studies a search model to frame the recent debate about social media and fake news. Social media websites have not only helped consumers to find their favorite news but have also enabled fake news producers to spread their stories more easily and widely. This chapter proposes a search model to study the effect of a social media website on the spread of fake news, and its influence on consumer surplus. More specifically, we study the effect of a lower search cost induced by the social media website. We consider a setting where one unit of consumers search sequentially on the social media website to read at most one news. Assume consumers cannot distinguish between true and fake news, and fake news producers are able to produce stories that are good enough to attract consumers. We find that a lower search cost leads to more fake news consumption, but higher consumer surplus.

The second chapter is motivated by the fact that the recent development of price comparison websites has led to increasing price transparency but no quality transparency, and it studies the effect of price transparency (without quality transparency) in a setting with competition among online sellers who compete on price and quality, by comparing situations where consumers learn no information and only price information before searching. We find that price transparency leads to lower prices, and a lower price is always linked to a lower quality. Price transparency also improves consumer surplus. However, price transparency sometimes results in excessive competition on price, if retailers can improve quality with relatively low quality but they did not do that due to the fierce price competition, the efficiency loss can be very large and lead to lower total welfare.

The third chapter analyses a monopoly firm's optimal information revelation strategy and return policy when consumers can choose to either buy immediately without knowing their exact match value or search to learn the exact match value. By paying a return cost which is chosen by the firm, each consumer can return the product and obtain a refund. We find that if the firm is able to give consumers any form of match information, the firm will simply inform each consumer whether or not their match value is above a threshold. This strategy is used as a search deterrence tool, and consumers will just buy directly without knowing the exact match value. The optimal return policy is the one that makes no consumer choose to return the product. The consumer surplus is decreasing in search cost, whereas the total welfare is increasing in search cost. The total welfare reaches the socially efficient level when search cost is large enough, though the consumer surplus is zero in this situation.

Acknowledgement

The completion of my Ph.D. would not be possible without the supports of many people. I am deeply indebted to my two main advisors Daniel Garrett and Andrew Rhodes. Daniel is a person who has all his passion for economic research, he would like to spend one day just to furnish one paragraph of his paper. Andrew is very intelligent and thoughtful, he always has some constructive suggestions, and care about others' feeling. They have been instrumental at each possible stage of my Ph.D. life.

I am also extremely thankful for the guidance from other professors in IO group. Patrick Rey is a terrific economist who can sometimes understand (some part of) a paper better than the author within a very short period of time. Jacques Cremer is well known for his willingness to help young researchers, I especially value his advice on how to write and how to present a theory paper. Alexandre de Cornière, Renato Gomes, Doh-Shin Jeon, Bruno Jullien, and Wilfried Sandmann have read and commented on my chapters and my presentations many times. Each discussion with him helps me have a deeper understanding of my own research, and also spans my knowledge and understanding of IO theory.

I would also like to express my gratitude to my amazing Ph.D. friends. It has always been fun and cool to have their company. I feel extremely lucky to be in such a friendly, diverse, and supportive community. Special thanks to our warmhearted secretaries Ludmila Namolovan-Stephan and Elvire Jalran, who have always been helpful and efficient during the whole period.

Last but not the least, I would like to thank my parents for their unconditional love and limitless support. I would also like to thank my dog Meimei, I feel happy every time when I think about you.

Chapter 1: Social Media and Fake News

Abstract

Social media websites have not only helped consumers to find their favorite news but have also enabled fake news producers to spread their stories more easily and widely. This paper proposes a search model to study the effect of a social media website on the spread of fake news, and its influence on consumer surplus. More specifically, we study the effect of a lower search cost induced by the social media website. We consider a setting where one unit of consumers search sequentially on the social media website to read at most one news. Assume consumers cannot distinguish between true and fake news, and fake news producers are able to produce stories that are good enough to attract consumers. We find that a lower search cost leads to more fake news consumption, but higher consumer surplus.

1 Introduction

People nowadays are more likely to read news on social media, for example, 68 percent of American adults say they at least occasionally get news on social media in a 2018 survey (Matsa and Shearer, 2018). The success of social media, however, comes together with the concerns about fake news. Fierce debates on fake news have arisen since the 2016 US presidential election.[1] In the final three months of the US presidential campaign, the most successful fake news stories on Facebook generated more engagement than the most successful mainstream news stores (Silverman, 2016).

Though social media stimulate debates in many aspects,[2] this paper studies a new and simple search model to frame the following debate: will consumers read more fake news because of social media websites when they make people find both true and fake news more easily at the same time, and what is the welfare effect of that. To answer this question, we make the following assumptions based on recent researches and evidence about fake news. First, we assume people cannot distinguish between true and fake news. According to a 2016 survey (Silverman and Singer-Vine, 2016), fake news headlines fool American adults about 75% of the time. Second, we assume profit is the main motive of the fake news producers.[3] Third, everyone can enter the market and become a fake news producer. The Guardian has a news article that mentioned that people in the town of Veles, Macedonia, started to write fake news stories when they saw their neighbors made money from it.[4] Fake news becomes

[1] The frequency of 'fake news' in Google Trends rose dramatically since September 2016, the same time as the US presidential election. See https://trends.google.com/trends/explore?date=all&q=fake%20news for more details.

[2] For example, people are debating whether social media is responsible for the fact that people have less confidence in the press, and whether social media makes it easier for fake news to fool people. See Gorbach (2018) and Ladd (2012) for more details.

[3] Although politicians also have incentives to produce fake news out of political interests, we will focus on the fake news producers who only care about monetary profit and earn their profits through advertisement. As Gorbach (2018) argues, while one might naturally have assumed that political ideology was the driving force behind the recent spate of what BuzzFeed called 'hyperpartisan' fake news, investigative reporting by the Guardian, Wired, and other outlets revealed that from Macedonia to California, profit was the primary motive.

[4] See the article "How Facebook powers money machines for obscure political 'news' sites" for more details.

much easier to produce recently, the procedure can be as simple as adding a fake story to a picture or a video (Clarke, 2016).

More specifically, we focus on a situation where a social media website reduces the cost of consumers to search for news on the website. The technology improvement helps consumers to find their favorite news more easily, but also helps the spread of fake news at the same time. We find that with a lower search cost, consumers read more fake news in equilibrium but consumer surplus becomes higher.

In more detail, Section 2 introduces the benchmark model. There is a continuum of mainstream news outlets with measure λ_a, and each of them produces a true story. There is a continuum of risk neutral fake news producers with measure $+\infty$ that potentially want to enter the market. Fake news producers incur a fixed cost K to enter the market and can write a fake news story with zero marginal cost after they enter the market. Both authentic news outlets and fake news producers publish their news stories on a social media website, and only news is circulated on the website. Because the focus of this paper is not about authentic news outlets, we intentionally simplify the outlets' problem: the authentic news outlets are ex ante identical, they can only produce true news and the value of these news stories are independent and identically distributed according to an exogenous continuously differentiable distribution. We also simplify the problem of the social media website by assuming its behavior is exogenous.

There is also a unit mass of consumers, who are interested to read at most one news story. Consumers search sequentially on the social media website, they read the title and brief description of a news story, but can only learn its signal because they cannot distinguish between true and fake news. Fake news producers can write a story that looks like an authentic new story with any value. The values of authentic news are vertically differentiated. The value of a story is equal to its signal if it is true, while fake news has zero value. Consumers' optimal strategy is a myopic stopping rule, they stopping search if they find a story with an expected value higher than some level, and then they read the story. The revenue of fake news producers comes from advertisements, if a consumer decides to read a certain news story and click the link of the story, the consumer will be referred to a website that belongs to the fake news producer, the producer of the story then earns a given amount of money. Hence, fake news producers' objective is to attract as many readers as possible. At the beginning of the game, the producers choose whether to enter the market one by one, and every producer can observe the decision of their predecessors.

We find that when search cost is lower, consumers read more fake news, but consumer surplus is higher. In this case, consumers stop search later, and some true news that consumers would like to read before will not be read now. Because fake news can mimic any true news, fake news producers thus still can make sure any of their stories can attract consumers. Therefore, a lower search cost leads to more fake news consumption. However, consumers are still better off with a lower search cost because they benefit directly from the lower search cost, and we find that this direct effect dominates. In equilibrium, fake news producers will not always produce very striking news. They want to make sure their news looks interesting enough to attract consumers, at the same time, they also want their stories to be credible enough.

In Section 3, we consider a more general case in which there are two types of consumers who have different search costs. We find that in equilibrium, the fake news producers only need to make sure the fake stories are good enough to attract low search cost consumers, then all the high search cost consumers are also willing to read these fake stories.

1.1 Related Literature

There is a growing literature on fake news and misinformation. Motivated by the 2016 presidential election, Allcott and Gentzkow (2017) study the incentives of certain outlets to present misleading news. Acemoglu, Ozdaglar, and Siderius (2021) consider a model that can generate a viral spread of misinformation driven by user sharing behavior or endogenous echo chambers. Our work is also related to the large body of economic research on biased news. For example, Besley and Prat (2006), and Gentzkow and Shapiro (2006) study strategic reasons for media bias. None of these papers considers the effect of a lower search cost on the spread of fake news and consumer welfare.

Our model of search builds on the classic works of Anderson and Renault (1999) and Wolinsky (1986). In their models, each firm needs to choose a price to maximize its profit. Whereas in our model, the price is given exogenously, and the prices are not paid by consumers, each producer therefore tries to maximize the demand.

2 A Benchmark Model

There is a continuum of mainstream news outlets with measure $\lambda_a > 0$, and each produces a true story. The behaviors of mainstream news outlets are exogenous. There is a continuum of risk neutral fake news producers with measure $+\infty$ that potentially want to enter the market. A fake news producer (he) incurs a fixed cost K to enter the market and can write a fake news story with zero marginal cost if he enters. Both authentic news outlets and fake news producers publish their news stories on a social media website of which behavior is also exogenous, and only news is circulated on the website. There is also a unit mass of consumers, who are interested to read at most one news story. A consumer (she) can only find news on the social media website.

True news stories are vertically differentiated, their values are realized independently across authentic news outlets, and are randomly drawn from an identical distribution $F(v)$. Let $\underline{v} \geq 0$ and $\overline{v} < +\infty$ be the lower and the upper bound of v. Assume the corresponding density function $f(v)$ exists and is positive and continuous. We assume the fake news producer can write a story that looks like an authentic new story with any value.

The consumer searches sequentially on the social media website, and each search incurs a constant search cost $s > 0$ which is also homogeneous for each consumer. The consumer reads the title and brief description of a news story, but can only learn its signal v since she cannot distinguish between true and fake news. The value of a story is equal to its signal if it is true, while fake news has zero value. We assume reading a news story is costless, and the consumer has a zero outside option.

The revenue of the fake news producer comes from advertisement, and he earns π once a consumer reads his story. At the beginning of the game, the producers choose whether to enter the market one by one, and every producer can observe the decision of their predecessors.

The timing of the game is as follows. At the first stage, fake news producers decide whether or not to enter the market one by one. At the second stage, fake news producers who entered the market choose a signal for each of their fake stories. Then true and fake news stories are published on the social media website. At the third stage, consumers cannot observe the total measure of news, they form (rational) expectations about the measure of fake news producers and the probability that a story of a certain signal is true. They then search sequentially on the social media website and make their reading decisions. After that, fake news producers obtain revenue from advertisements.

2.1 Equilibrium Analysis

We first solve for the consumer's optimal strategy. Focus on symmetric equilibrium, and the equilibrium concept is Perfect Bayesian Equilibrium (PBE). Let λ_f denote the measure of fake news producers that enter the market, and let $G(v)$ denote the fake news producer's signal distribution.

We will focus on the equilibria that the distribution $G(v)$ has a corresponding density function $g(v)$, and the consumer has correct beliefs about $G(v)$ and λ_f.[5] If the consumer finds a story with signal v, she believes it is true with probability $P(v)$ where

$$P(v) = \frac{\lambda_a f(v)}{\lambda_a f(v) + \lambda_f g(v)}.$$

Then let $w(v)$ denote the **expected value** of the story, where

$$w(v) = vP(v).$$

Because the consumer is risk neutral, the expected value of a story is also the consumer's expected utility of reading the story.

Hence, for any given belief about λ_f and $G(v)$, the consumer treats each story as a product of value $w(v)$. The consumer also has a belief about the distribution of $w(v)$ because v follows the distribution $\mu F(v) + (1-\mu)G(v)$, where $\mu = \lambda_a/(\lambda_a + \lambda_f)$ is the probability that a random story is true. From Weitzman (1979), the consumer's optimal search rule in this environment is stationary, she uses a myopic strategy and never goes back to any previously searched story.

[5] In equilibrium, $G(v)$ will never have a mass point as some v. Otherwise, consumers believe the stories of these signals are fake with probability one and never read them.

Let w^* denote the **reservation value** where the consumer stops searching if she finds a news story with expected value higher than w^*, it is uniquely determined by the following stopping rule

$$\int_{\underline{v}}^{\overline{v}} \max\{w(v) - w^*, 0\}\, d\left[\mu F(v) + (1-\mu)\, G(v)\right] = s, \tag{1}$$

since the LHS is strictly decreasing in w^* as long as $w^* < \max_{v\in[\underline{v},\overline{v}]} w(v)$. Suppose that the consumer holds a best story with expected utility w^*, the incremental utility from searching one more story is $\max\{w(v) - w^*, 0\}$ if the new story has a signal v. Hence, the LHS is the expected incremental utility and it must equal to the search cost when w^* is the consumer's optimal stopping time, i.e. the reservation value. Since the expected incremental utility is decreasing in w^*, the consumer should stop searching if the best story she holds is larger than w^* such that the expected incremental utility of one more search is less than the search cost, and vice versa.

Note that the w^* defined in condition (1) depends on the consumer's beliefs about λ_f and $G(v)$. We need to analyze the fake news producer's problem at the first stage to find the equilibrium w^*. The producer's revenue comes from advertisement, he thus wants to make sure the consumer will read his fake news story if she finds it. The following proposition characterizes the equilibrium strategies of consumers and producers.[6]

Proposition 1. *There exists an infinite number of symmetric equilibria, a profile* $\{G(v), w^*, \lambda_f\}$ *is a symmetric equilibrium if and only if*

(i) $v \leq w^* \Rightarrow g(v) = 0$.

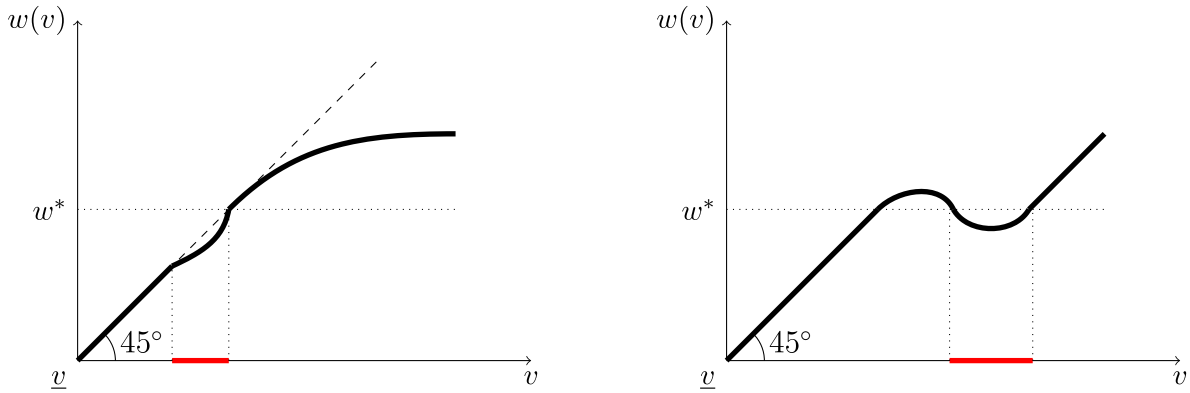
(ii) $v > w^* \Rightarrow w(v) \geq w^*$.

(iii) w^* *is determined by condition* (1).

(iv) The measure of fake news producers in the market is $\lambda_f = \frac{\pi}{K} - \lambda_a\left[1 - F\left(w^*\right)\right]$.

The fake news producer manages to make sure the consumer will read his story once she finds it in any equilibrium. To do that, the producer (i) never writes a boring story that no consumer would like to read even if consumers believe the story is true, and (ii) never

[6]Note that λ_f is not a strategy of fake news producers, we misuse $\{G(v), w^*, \lambda_f\}$ to denote a profile of strategies.



(a) Boring fake news

(b) Too much concentration

Figure 1: Non-equilibrium signal distribution (fake stories in red interval will not be read).

concentrates on certain signals too often because consumers may find the story suspicious otherwise. These are condition (i) and (ii) of Proposition 1.

For example, the fake news with signals lies in the red intervals in figure 1 will not be read. If the reservation value of the consumer is w^*, she never reads any news story with signal $v < w^*$ even if she believes the story is true. On the other hand, if the producer concentrates on certain signals too often, the consumer expects that these stories are too suspicious and will not read them.

From the condition (i) and (ii) of Proposition 1, in any equilibrium we must have

$$max\{w(v) - w^*, 0\} = \begin{cases} w(v) - w^* & v \geq w^* \\ 0 & v < w^* \end{cases},$$

$$\text{and} \int_{w^*}^{\overline{v}} g\left(v\right) dv = 1.$$

Note that $\mu = \lambda_a/(\lambda_a + \lambda_f)$, and substitute the two expressions above, the stopping rule can thus be simplified as

$$\mu \int_{w^*}^{\overline{v}} \left(v - w^*\right) dF\left(v\right) + (1 - \mu)(-w^*) = s. \tag{2}$$

The equation has a straightforward explanation. Suppose the consumer holds a best story of expected value w^*. Then for one more search, with probability μ, she will find a true story, and the expected incremental utility is $\int_{w^*}^{\overline{v}} \left(v - w^*\right) dF(v)$; with probability $1 - \mu$, she will find a fake story and read it, the opportunity cost of reading the fake news is not reading the true news of value w^*.

Note that the reservation value is uniquely pinned down by condition (2) for any given λ_f. There are many ways the producer can make sure $G(v)$ satisfies both condition (i) and (ii) of Proposition 1. For example, the fake news producer can choose a strategy $G(v)$ such that for some positive number $\tilde{v}$,

$$w(v) = \begin{cases} v & \text{if } v < \tilde{v}, \\ \tilde{v} & \text{if } v \geq \tilde{v}. \end{cases}$$

Clearly, the reservation value $w^* < \tilde{v}$.[7]

Note that the reservation value w^* defined in condition (2) depends on the consumer's belief about λ_f, and w^* is decreasing in λ_f. To find out the equilibrium reservation value, we need to analyze the entering decision of fake news producers in the first period.

Note that every consumer will read a story, because there is an infinite number of stories on the website, and they will keep searching until they find a story with an expected value higher than the reservation value, thus, the total demand is one. Any fake news story looks like a story that is interesting and credible enough to attract consumers, while only a proportion $1 - F(w^*)$ of true news is good enough to attract consumers. The demand of the fake news producer is thus $\frac{1}{\lambda_a[1-F(w^*)]+\lambda_f}$. Therefore, the fake news producer's profit is

$$\frac{\pi}{\lambda_a \left[1 - F\left(w^*\right)\right] + \lambda_f} - K.$$

From condition (2), we know that the reservation value w^* is decreasing in λ_f, the producer's profit is thus decreasing λ_f. Note that this expression is true only if $w^* > 0$ because consumers will not search in the first place otherwise.[8] We thus assume $\frac{\pi}{K}$ is not too large (i.e. λ_f is not too large) such that the reservation value w^* is positive. We also assume the profit is positive when $\lambda_f = 0$ (i.e. $\frac{\pi}{K}$ is not so small) such that the equilibrium λ_f is positive.

In the first period, fake news producers keep entering the market until the profit becomes

[7] Actually, this strategy is an equilibrium distribution for any search cost s.

[8] We will show later that w^* is the ex ante consumer surplus.

zero. Therefore, the measure of fake news producers that enter the market is

$$\lambda_f = \frac{\pi}{K} - \lambda_a \left[1 - F\left(w^*\right)\right]. \tag{3}$$

Finally, the equilibrium w^* and λ_f are defined by condition (2) and (3) jointly.

Before we analyze the properties of equilibrium outcome, the following corollary is a useful step to understand the relationship among the different symmetric equilibria.

Corollary 1. *The reservation value w^* is the same in each symmetric equilibrium, and w^* is decreasing in search cost s.*

Proof of Corollary 1. Note that any equilibrium $G(v)$ must satisfy condition (i) and (ii) of Proposition 1. Therefore, the equilibrium w^* and λ_f are defined by condition (2) and (3) jointly. From condition (3), the probability that a story is true is a function of the reservation value w^*: $\mu(w^*) = \frac{\lambda_a}{\frac{\pi}{K}+\lambda_a F(w^*)}$.

Therefore, the equilibrium reservation value is uniquely defined by

$$\mu(w^*) \int_{w^*}^{\overline{v}} \left(v - w^*\right) dF\left(v\right) + \left[1 - \mu(w^*)\right]\left(-w^*\right) = s,$$

because $\mu(w^*)$ is a decreasing function, the LHS of the equation is thus also decreasing in w^*. And w^* is decreasing in s since the RHS is increasing in s. □

To understand why this is the case, recall that $G(v)$ and $g(v)$ disappear in both condition (2) and (3). The consumer reads any fake news story she finds, so the extent that fake news hurts the consumer is the same in any equilibrium. Therefore, reservation value is also the same among different equilibria.

The reservation value w^* cannot exceed the upper bound $\overline{v}$ of the support of $F(v)$ as long as $\lambda_f > 0$. When $s = 0$, the consumer will keep searching until she finds the news story with the highest possible expected value. Therefore, the expected value is equal to the signal when $v < w^*$, and then keeps constant at w^* for $v \geq w^*$. And this is the only equilibrium profile.

The following proposition characterizes the properties of each symmetric equilibrium.

Proposition 2. *A lower search cost leads to:*

(i) more fake news producers entering the market, and more fake news consumption;

(ii) but higher consumer surplus.

As we mentioned before, the demand of each producer is $\frac{1}{\lambda_a[1-F(w^*)]+\lambda_f} = \frac{\pi}{K}$. The amount of fake news consumption is thus

$$\frac{\lambda_f}{\lambda_a\left[1-F(w^*)\right]+\lambda_f} = \frac{\lambda_f K}{\pi}.$$

Because a lower search cost leads to a higher w^* and thus a higher λ_f, the amount of fake news consumption is increasing in s. Like a standard search model, the ex ante consumer surplus is exactly the reservation value when there are infinite options. Therefore, consumer surplus is higher with a lower search cost even if consumers read more fake news.

When search cost is lower, the consumer becomes pickier about the expected quality of news. She thus ignores some true news that she would read when the search cost is higher, but still reads any fake news she finds. Hence, it becomes more profitable for fake news producers, and more producers are willing to enter the market. However, the consumer is better off because the positive effect comes directly from the lower search cost dominates the negative effect of more fake news. To understand why, suppose the reservation value becomes (weakly) smaller when search cost is smaller, then the amount of fake news consumed in equilibrium is (weakly) smaller than before. But the consumer then wants to search more and the reservation value becomes higher because there is less fake news and search cost is smaller, which is a contradiction.

The following lemma analyzes the effect of more authentic news outlets and a lower $\frac{\pi}{K}$.

Lemma 1. *When λ_a is higher, or $\frac{\pi}{K}$ is lower, consumer surplus becomes higher.*

As it is well known, some mainstream news outlets have their social media accounts and publish some selected stories. Consumers are better off if more outlets have their accounts or each of them publishes more stories on social media. This increases the probability that the consumer finds a true news story and therefore decreases the profit of fake news producers.

Another way to fight fake news is to make it harder for fake news producers to enter the market. Technology companies are trying to block their revenue from advertisements. For example, Facebook has put in place a protocol to dry-cut any revenue that has been achieved by spreading false news. And the use of the advertising option will be prohibited. And the use of the advertising option will be prohibited.

3 Heterogeneous Search Costs

We now consider a more general model in which there are two types of consumers h and l with search cost s_h and s_l, respectively. Assume $0 < s_l < s_h$, and the fraction of type i consumers is γ_i for $i \in \{h, l\}$, $\gamma_h + \gamma_l = 1$.

The following lemma is a useful first step in characterizing the symmetric equilibria.

Corollary 2. *If $G(v)$ is an equilibrium signal distribution when $\gamma_l = 1$, it is also an equilibrium signal distribution when $\gamma_h = 1$.*

The model is exactly the same as the benchmark model when either $\gamma_h = 1$ or $\gamma_l = 1$. Hence, the lemma implies that, in the benchmark model, if $G(v)$ is an equilibrium signal distribution when $s = s_l$, it is also an equilibrium signal distribution when $s = s_h$. To understand why, note that in the benchmark model, the equilibrium $G(v)$ needs to satisfies

- $v \leq w^* \Rightarrow g(v) = 0$,
- $v > w^* \Rightarrow w(v) \geq w^*$,

where w^* is the reservation value. Because the reservation value is decreasing in search cost, these two conditions must hold when $s = s_h$ if they hold when $s = s_l$.

At the third stage, both types of consumers hold some (same) beliefs about λ_f and $G(v)$, then they search according to their own stopping rule. As a result, the fake news producer only needs to make sure the story he writes is interesting and credible enough for type l consumers, because then all the type h consumers will also read his story if they find it. Let w_i^* denote the reservation value of type i consumers for $i \in \{h, l\}$, then w_i^* is defined by the following stopping rule

$$\int_{\underline{v}}^{\overline{v}} max\{w(v) - w_i^*, 0\} d\left[\mu F(v) + (1-\mu)\, G(v)\right] = s_i. \tag{4}$$

Note that $\mu = \frac{\lambda_a}{\lambda_a + \lambda_f}$ is the probability that any random story is true, similar like the benchmark model, the stopping rules can be simplified as

$$\mu \int_{w_h^*}^{\overline{v}} (v - w_h^*)\, dF\,(v) + (1-\mu)(-w_h^*) = s_h,$$
$$\mu \int_{w_l^*}^{\overline{v}} (v - w_l^*)\, dF\,(v) + (1-\mu)(-w_l^*) = s_l.$$

It can be easily checked that $w_l^* > w_h^*$ for any given μ, note that w_i^* is the ex ante consumer surplus of type i consumers, it means that type l consumers always have a higher surplus.

At the first stage, fake news producers know that type i consumers would read true news with signal above w_i^* and any fake story. Following similar arguments, the profit of each fake news producer is

$$\frac{\gamma_h \pi}{\lambda_a \left[1 - F(w_h^*)\right] + \lambda_f} + \frac{\gamma_l \pi}{\lambda_a \left[1 - F(w_l^*)\right] + \lambda_f} - K.$$

Producers thus keep entering the market until they have zero profit. The following proposition summarizes the above analysis.

Proposition 3. *There exists an infinite number of symmetric equilibria. The reservation values w_h^* and w_l^* are the same in each symmetric equilibrium, and $w_h^* < w_l^*$. The profile $\{G(v), w_h^*, w_l^*, \lambda_f\}$ is a symmetric equilibrium if and only if*

(i) $v \le w_l^* \Rightarrow g(v) = 0$.

(ii) $v > w_l^* \Rightarrow w(v) \ge w_l^*$.

(iii) w_h^* and w_l^* are determined by condition (4).

(iv) The measure of fake news producers λ_f is decided by

$$\frac{\gamma_h}{\lambda_a \left[1 - F(w_h^*)\right] + \lambda_f} + \frac{\gamma_l}{\lambda_a \left[1 - F(w_l^*)\right] + \lambda_f} = \frac{K}{\pi}.$$

As we mentioned before, at the third stage, for any given beliefs about λ_f and $G(v)$, each consumer's search behavior is not influenced by others. However, a higher γ_l means more consumers are more likely to read fake news, and hence more fake news producers entering the market at the first stage, which influences the consumers' beliefs about λ_f. Similarly, we can also find that $G(v)$ is an equilibrium signal distribution when $\gamma_l < 1$ only if $G(v)$ is an equilibrium signal distribution when $\gamma_l = 1$, whereas the other way is not necessarily true.

Lemma 2. *Both w_h^* and w_l^* are decreasing in γ_l; w_h^* is increasing in s_l whereas w_l^* is decreasing in s_l.*

According to the analysis of the benchmark model, consumer surplus is higher when γ_l jumps from 0 to 1. Therefore, the average consumer surplus $\gamma_h w_h^* + \gamma_l w_l^*$ is increasing at least at some interval when γ_l is increasing despite the fact that both w_h^* and w_l^* are always decreasing. Because type l consumers are more likely to read fake news, a higher γ_l leads to more fake news producers entering the market, which hurts all the consumers. On the other hand, a higher γ_l means that more consumers enjoy a lower search cost. If we consider γ_l jumps from 0 to 1, the second effect dominates and leads to a higher (average) consumer surplus. Similarly, a lower s_l also leads to more fake news producers entering the market, which reduces w_h. But type l consumers are still better off because they incur less search cost.

4 Conclusion

This paper studies the effect of social media on the spread of fake news and its influence on consumer surplus. We find that consumers read more fake news if a social media website reduces the search cost, but the consumer surplus becomes higher. For future research, we will also discuss a situation where consumers have heterogeneous preferences, and study what is the effect when the social media website can help consumers search for only their preferred type of news.

For future research, we plan to discuss a situation where there are different types of news, and the social media website can help consumers "search in category". It is related to Fershtman, Fishman, and Zhou (2017) who propose a search model with product categories where consumers choose which categories to search and firms respond to such more targeted search by strategically choosing the categories in which to list their products. Another related work is Yang (2013), who finds that when search targetability increases, an additional variety of goods catering to long tail consumers will be provided.

References

Acemoglu, Daron, Asuman Ozdaglar, and James Siderius (2021). *Misinformation: Strategic Sharing, Homophily, and Endogenous Echo Chambers*. Tech. rep. National Bureau of Economic Research.

Allcott, Hunt and Matthew Gentzkow (2017). "Social media and fake news in the 2016 election", *Journal of economic perspectives* 31.2, pp. 211–36.

Anderson, Simon P and Regis Renault (1999). "Pricing, product diversity, and search costs: A Bertrand-Chamberlin-Diamond model", *The RAND Journal of Economics*, pp. 719–735.

Besley, Timothy and Andrea Prat (2006). "Handcuffs for the grabbing hand? Media capture and government accountability", *The American Economic Review* 96.3, pp. 720–736.

Clarke, Oliver (2016). *How To Make Big Money Online With Fake News - Full Guide.*

Fershtman, Chaim, Arthur Fishman, and Jidong Zhou (2017). "Search and Categorization", *International Journal of Industrial Organization.*

Gentzkow, Matthew and Jesse M Shapiro (2006). "Media bias and reputation", *Journal of political Economy* 114.2, pp. 280–316.

Gorbach, Julien (2018). "Not Your Grandpa's Hoax: A Comparative History of Fake News", *American Journalism* 35.2, pp. 236–249.

Ladd, Jonathan M (2012). *Why Americans hate the news media and how it matters.* Princeton University Press.

Matsa, Katerina Eva and Elisa Shearer (2018). "News use across social media platforms 2018", *Pew Research Center* 10.

Silverman, C (2016). *This analysis shows how fake election news stories outperformed real news on Facebook. BuzzFeed, Nov. 16.*

Silverman, Craig and Jeremy Singer-Vine (2016). "Most Americans who see fake news believe it, new survey says", *BuzzFeed News (www. buzzfeed. com). URL: https://www. buzzfeed. com/craigsilverman/fake-newssurvey.*

Weitzman, Martin L (1979). "Optimal search for the best alternative", *Econometrica: Journal of the Econometric Society*, pp. 641–654.

Wolinsky, Asher (1986). "True monopolistic competition as a result of imperfect information", *The Quarterly Journal of Economics* 101.3, pp. 493–511.

Yang, Huanxing (2013). "Targeted search and the long tail effect", *The RAND Journal of Economics* 44.4, pp. 733–756.

Appendix

Proof of Proposition 1. We first show a profile $\{G(v), w^*, \lambda_f\}$ is a symmetric equilibrium if and only if condition (i) to (iv) hold, and then show there exists an infinite number of equilibria.

($\Longrightarrow$) If the profile $\{G(v), w^*, \lambda_f\}$ is an equilibrium, condition (iii) must hold according to Weitzman (1979), and it means that the consumer never reads a story that has a expected value $w(v)$ strictly smaller than w^*. Therefore, the fake news producer has incentive to deviate if he writes a news story with certain signal v such that $w(v) < w^*$, which means that $w(v) \geq w^*$ must be true for any v such that $g(v) > 0$.

For any $v \leq w^*$, $g(v) = 0$ must be true otherwise $w(v) < w^*$. Thus, condition (i) holds. Condition (ii) must also hold, because when $v > w^*$, $w(v) \geq w^*$ is true as long as $g(v)$ is small enough, and consumers will not read the news if $w(v) < w^*$.

Condition (i) and (ii) imply that the consumer only reads news with signal great than w^*, which consists of all fake news and true news with signal greater than w^*. Hence, the profit of the producer is

$$\frac{\pi}{\lambda_a \left[1 - F\left(w^*\right)\right] + \lambda_f} - K.$$

Free entry results in zero profit, which means that $\lambda_f = \frac{\pi}{K} - \lambda_a \left[1 - F\left(w^*\right)\right]$. So condition (iv) holds.

($\Longleftarrow$) If condition (i) and (ii) hold, the producer has no incentive to deviate since the consumer will read the producer's story if she finds it. And condition (iv) guarantees that the producer is indifferent between entering the market or not. The consumer also has no profitable deviation since choosing w^* is the optimal strategy. Therefore the profile $\{G(v), w^*, \lambda_f\}$ is an equilibrium if condition (i) to (iv) hold.

(Existence) To show there exists an infinite number of equilibria, it is equivalent to show there exists an infinite number of profiles $\{G(v), w^*, \lambda_f\}$ that satisfy condition (i) to (iv). Let w^* solve the following equation,

$$\frac{\lambda_a}{\frac{\pi}{K} + \lambda_a F(w^*)} \int_{w^*}^{\overline{v}} \left(v - w^*\right) dF\left(v\right) + \frac{\frac{\pi}{K} + \lambda_a (F(w^*) - 1)}{\frac{\pi}{K} + \lambda_a F(w^*)} (-w^*) = s. \tag{5}$$

The LHS is decreasing in w^*, so w^* is uniquely determined and decreasing in s.

If condition (i) and (ii) hold, consumers will only read a news story with signal greater than w^*, free entry thus leads to zero profit and $\lambda_f = \frac{\pi}{K} - \lambda_a [1 - F(w^*)]$. Substitute $\frac{\pi}{K} - \lambda_a [1 - F(w^*)] = \lambda_f$ and $1 = \int_{w^*}^{\overline{v}} g(v)\, dv$ into equation (5), we then have exactly the stopping rule of the consumer. So condition (iii) and (iv) hold when (i) and (ii) hold.

Therefore, it suffices to show there exists an infinite number of $G(v)$ that satisfy condition (i) and (ii). We first show that there exists a function $g(v)$ satisfies

$$\int_{w^*}^{\overline{v}} g(v)\, dv = 1 \text{ and } w(v) = \begin{cases} \tilde{v} & v \geq \tilde{v} \\ v & v < \tilde{v} \end{cases}$$

It is easy to see $g(v)$ satisfies condition (i) and (ii) when $\tilde{v} > w^*$. Note that $w(v) = \frac{\lambda_a f(v)}{\lambda_a f(v) + \lambda_f g(v)} v$, then the expression of $g(v)$ is

$$g(v) = \begin{cases} \frac{\lambda_a}{\lambda_f \tilde{v}}(v - \tilde{v}) f(v) & v > \tilde{v} \\ 0 & v \leq \tilde{v} \end{cases}$$

Integrate $g(v)$ from w^* to $\overline{v}$, we have $\int_{w^*}^{\overline{v}} g(v)dv = \int_{w^*}^{\overline{v}} \frac{\lambda_a}{\lambda_f \tilde{v}}(v - \tilde{v}) f(v) dv$ which is decreasing in $\tilde{v}$. When $\tilde{v} = w^*$, and substitute the expression of equation (5), we find that

$$\begin{aligned} \int_{w^*}^{\overline{v}} g(v)dv = & \int_{w^*}^{\overline{v}} \frac{\lambda_a}{\lambda_f w^*}(v - w^*) f(v) dv \\ = & \frac{\lambda_a}{\lambda_f w^*} \int_{w^*}^{\overline{v}} (v - w^*) f(v) dv \\ = & 1 + \frac{s(\lambda_f + \lambda_a)}{\lambda_f w^*} > 1. \end{aligned}$$

Therefore, $\tilde{v} > w^*$ and $g(v)$ satisfies condition (i) and (ii). Then for another density function $\tilde{g}(v)$ such that $\tilde{g}(v) = g(v)$ expect for one point $v \in [\tilde{v}, \overline{v}]$, $\tilde{g}(v) = g(v) + \varepsilon$ where ε is very small. It can be easily checked that $\tilde{g}(v)$ also satisfies condition (i) and (ii), and there are infinite number of possible $\tilde{g}(v)$, which completes the proof. □

Proof of Proposition 2. To show statement (i) is true, from the argument mentioned above, w^* is decreasing in s, so λ_f must be decreasing in s as well. In any equilibrium, only fake news and authentic news with value grater than w^* will be read by consumers. Therefore, the probability that the consumer read a fake news is

$$\frac{\lambda_f}{\lambda_a [1 - F(w^*)] + \lambda_f} = \frac{\pi/K - \lambda_a [1 - F(w^*)]}{\pi/K},$$

which is decreasing in s. Because one unit of consumers will each read a story, a lower search cost leads to more fake new consumption.

To show statement (ii) is true, it suffice to show w^* is the ex ante consumer surplus. Note that the value of a fake news story is zero, and consumers only read fake news and true news with signal higher than w^*. The expected value consumers can get from reading read is $\dfrac{\lambda_a \int_{w^*}^{\overline{v}} v dF(v)}{\lambda_a(1-F(w^*))+\lambda_f}$. The only cost is the search cost, with probability $\frac{\lambda_f+(1-F(w^*))\lambda_a}{\lambda_f+\lambda_a}$ consumers will stop searching. Therefore, the expected total search cost is

$$\frac{\lambda_f+(1-F(w^*))\lambda_a}{\lambda_f+\lambda_a}\left[s+\frac{F(w^*)\lambda_a}{\lambda_f+\lambda_a}\times 2s+(\frac{F(w^*)\lambda_a}{\lambda_f+\lambda_a})^2\times 3s+...\right].$$

Substitute the condition (2), calculation reveals that

$$\begin{aligned}
&\frac{\lambda_a \int_{w^*}^{\overline{v}} v dF(v)}{\lambda_a(1-F(w^*))+\lambda_f}-\frac{\lambda_f+(1-F(w^*))\lambda_a}{\lambda_f+\lambda_a}\left[s+\frac{F(w^*)\lambda_a}{\lambda_f+\lambda_a}\times 2s+(\frac{F(w^*)\lambda_a}{\lambda_f+\lambda_a})^2\times 3s+...\right]\\
=&\frac{\lambda_a \int_{w^*}^{\overline{v}} v dF(v)-(\lambda_f+\lambda_a)s}{\lambda_f+(1-F(w^*))\lambda_a}\\
=&w^*,
\end{aligned}$$

which is same as what is found in a standard search model like Anderson and Renault (1999). □

Proof of Lemma 1. Note that ex ante consumer surplus is w^*, so it suffices to show w^* is increasing in λ_a, and decreasing in $\frac{\pi}{K}$.

Multilply both side of equation (5) by $\frac{\pi}{K}+\lambda_a F(w^*)$, and take the total derivative with respect to w^* and λ_a:

$$\underbrace{\left\{\int_{w^*}^{\overline{v}}(v-w^*)\,dF\,(v)+w^*\,[1-F(w^*)]-sF(w^*)\right\}}_{>0}d\lambda_a-\underbrace{\left[\frac{\pi}{K}+\lambda_a f(w^*)(w^*+s)\right]}_{>0}dw^*=0$$

The first part is positive because $\int_{w^*}^{\overline{v}}(v-w^*)\,dF\,(v)>s>sF(w^*)$. Therefore, $\frac{dw^*}{d\lambda_a}>0$.

Similarly, we have

$$\underbrace{(w^*+s)}_{>0}d\frac{\pi}{K}+\underbrace{\left[\frac{\pi}{K}+\lambda_a f(w^*)(w^*+s)\right]}_{>0}dw^*=0.$$

Therefore, $\frac{dw^*}{d\frac{\pi}{K}}<0$. □

Proof of Corollary 2. Note that the model would be exactly the same as the benchmark model when either $\gamma_h = 1$ or $\gamma_l = 1$. It suffice to show that $G(v)$ satisfies the condition (i) and (ii) of Proposition 1 when $\gamma_h = 1$ if it is an equilibrium signal distribution when $\gamma_l = 1$. And this argument is true since the reservation value is decreasing in search cost. □

Proof of Proposition 3. We first show a profile $\{G(v), w^*, \lambda_f\}$ is a symmetric equilibrium if and only if condition (i) to (iv) hold, and then show there exists an infinite number of equilibria.

($\Longrightarrow$) If the profile $\{G(v), w_h^*, w_l^*, \lambda_a\}$ is an equilibrium, condition (iii) must hold. And this means that the type i consumers never read a story with expected value $w(v)$ strictly smaller than w_i^* for $i \in \{h, l\}$. At the same time, the reservation value of type l consumers is higher $w_l > w_h$ since w_i^* is decreasing in s_i and $s_l < s_h$. Hence, the fake news producer has incentive to deviate if the producer writes a news story with certain signal v such that $w(v) < w_l^*$, which means that $w(v) \geq w_l^*$ must be true for any v such that $g(v) > 0$.

For similar arguments that we provided in the proof of Proposition 1, condition (i) and (ii) hold. It implies that the type i consumers only read news with signal great than w^*, which consists of all fake news and true news with signal greater than w_i^*. Hence, the profit of the producer is

$$\frac{\gamma_h \pi}{\lambda_a \left[1 - F(w_h^*)\right] + \lambda_f} + \frac{\gamma_l \pi}{\lambda_a \left[1 - F(w_l^*)\right] + \lambda_f} - K.$$

Free entry results in zero profit, which means that condition (iv) holds.

($\Longleftarrow$) If condition (i) and (ii) hold, the producer has no incentive to deviate since the consumer will read the producer's story if she finds it. And condition (iv) guarantees that the producer is indifferent between entering the market or not. The type i consumers also have no profitable deviation since choosing w^* is the optimal strategy for $i \in \{h, l\}$. Therefore the profile $\{G(v), w_h^*, w_l^*, \lambda_a\}$ is an equilibrium if condition (i) to (iv) hold.

(Existence) To show there exists an infinite number of equilibria, it is equivalent to show there exists an infinite number of profiles $\{G(v), w_h^*, w_l^*, \lambda_a\}$ that satisfy condition (i) to (iv).

Let w_h^* and w_l^* solve the following system of equations

$$\mu \int_{w_h^*}^{\overline{v}} (v - w_h^*)\, dF(v) + (1-\mu)(-w_h^*) = s_h, \tag{6}$$

$$\mu \int_{w_l^*}^{\overline{v}} (v - w_l^*)\, dF(v) + (1-\mu)(-w_l^*) = s_l, \tag{7}$$

$$\frac{\gamma_h}{\lambda_a [1 - F(w_h^*)] + \lambda_f} + \frac{\gamma_l}{\lambda_a [1 - F(w_l^*)] + \lambda_f} = \frac{K}{\pi}, \tag{8}$$

where $\mu = \lambda_a / (\lambda_a + \lambda_f)$.

When λ_f is larger, μ is smaller, then w_h^* and w_l^* must be smaller to make sure condition (6) and (7) satisfy. Therefore, the LHS of condition (8) is decreasing in λ_f, w_h^* and w_l^* are also uniquely determined.

Following the same arguments as before (the proof of Proposition 1), condition (iii) and (iv) hold if condition (i) and (ii) are correct. Then we can show that condition (i) and (ii) hold if the density function $g(v)$ satisfies

$$\int_{w_l^*}^{\overline{v}} g(v)\, dv = 1 \text{ and } w(v) = \begin{cases} \tilde{v} & v \geq \tilde{v} \\ v & v < \tilde{v} \end{cases}$$

The rest arguments are also the same as before, and we complete the proof. □

Proof of Lemma 2. To show statement (i) is true, we take the total derivative of equation (6), (7), and (8) with respect to λ_f, w_h^*, w_l^*, γ_h and γ_l. Note that $\gamma_h = 1 - \gamma_l$ implies that $d\gamma_h = -d\gamma_l$, let $Y_i = \lambda_a [1 - F(w_i^*)] + \lambda_f$ for $i \in \{h, l\}$, we have

$$\left(\gamma_h Y_l^2 + \gamma_l Y_h^2\right) d\lambda_f = \gamma_h \lambda_a f(w_h^*) Y_l^2 dw_h^* + \gamma_l \lambda_a f(w_l^*) Y_h^2 dw_l^* + Y_h Y_l \lambda_a [F(w_l^*) - F(w_h^*)]\, d\gamma_l$$

$$Y_h dw_h^* + (w_h^* + s_h) d\lambda_f = 0,$$

$$Y_l dw_l^* + (w_l^* + s_l) d\lambda_f = 0.$$

Eliminating λ_f gives

$$\frac{dw_h^*}{d\gamma_l} = -\frac{Y_h Y_l \lambda_a [F(w_l^*) - F(w_h^*)]}{\frac{\left(\gamma_h Y_h^2 + \gamma_l Y_l^2\right) Y_h}{w_h^* + s_h} + \gamma_h \lambda_a f(w_h^*) Y_l^2 + \frac{\gamma_l \lambda_a Y_h^3 (w_l^* + s_l) f(w_l^*)}{Y_l (w_h^* + s_h)}}$$

$$\frac{dw_l^*}{d\gamma_l} = -\frac{Y_h Y_l \lambda_a [F(w_l^*) - F(w_h^*)]}{\frac{\left(\gamma_h Y_l^2 + \gamma_l Y_h^2\right) Y_l}{w_l^* + s_l} + \gamma_l \lambda_a f(w_l^*) Y_h^2 + \frac{\gamma_h \lambda_a Y_l^3 (w_h^* + s_h) f(w_h^*)}{Y_h (w_l^* + s_l)}}$$

Therefore, both w_h^* and w_l^* are decreasing in γ_l because $F(w_h^*) < F(w_l^*)$.

To show statement (ii) is true, we find that the total derivative of equation (6), (7), and (8) with respect to λ_f, w_h^*, w_l^*, and s_l are

$$\left(\gamma_h Y_l^2 + \gamma_l Y_h^2\right) d\lambda_f = \gamma_h \lambda_a f(w_h^*) Y_l^2 dw_h^* + \gamma_l \lambda_a f(w_l^*) Y_h^2 dw_l^*$$
$$Y_h dw_h^* + (w_h^* + s_h) d\lambda_f = 0,$$
$$Y_l dw_l^* + (w_l^* + s_l) d\lambda_f + (\lambda_a + \lambda_f) ds_l = 0.$$

Rearranging the equations gives

$$\frac{dw_h^*}{ds_l} = \frac{\lambda_a + \lambda_f}{\frac{Y_h Y_l (\gamma_h Y_l^2 + \gamma_l Y_h^2) + Y_l^3 \gamma_h \lambda_a f(w_h^*)(w_h^* + s_h)}{(w_h^* + s_h)\gamma_l \lambda_a f(w_l^*) Y_h^2} + \frac{w_l^* + s_l}{w_h^* + s_h} Y_h} > 0,$$
$$\frac{dw_l^*}{ds_l} = -\frac{\lambda_a + \lambda_f}{\frac{Y_h (\gamma_h Y_l^2 + \gamma_l Y_h^2) + Y_l^2 \gamma_h \lambda_a f(w_h^*)(w_h^* + s_h)}{(w_h^* + s_h)\gamma_l \lambda_a f(w_l^*) Y_h} \frac{w_l^* + s_l}{w_h^* + s_h} + 1} < 0.$$

□

Chapter 2: Price transparency in online markets

Abstract

The recent development of price comparison websites has led to increasing price transparency but no quality transparency. This paper studies the effect of price transparency (without quality transparency) in a setting with competition among online sellers who compete on price and quality, by comparing situations where consumers learn no information and only price information before searching. We find that price transparency leads to lower prices, and a lower price is always linked to a lower quality. Price transparency also improves consumer surplus. However, price transparency sometimes results in excessive competition on price, if retailers can improve quality with relatively low quality but they do not do that due to the fierce price competition, the efficiency loss can be very large and lead to lower total welfare.

1 Introduction

Efforts by online sellers to improve their product and service quality (free accessories, extended warranties, fast delivery, and so on), as well as the prices, are usually considered as key dimensions affecting sales. Yet *information* about these choices also plays an important role in determining demand. The recent development of price comparison websites has led to increasing price transparency, but quality transparency remains unchanged. Indeed in the European Commission's e-commerce sector inquiry, the respondent manufacturers argued that price comparison websites "typically focus mainly on price and do not [...] allow retailers to differentiate themselves sufficiently in terms of [...] quality [...]".

People normally believe that price transparency leads to more intensive price competition thus lower prices, and a lower price is often linked to a lower quality. For example, the e-commerce sector inquiry argued that decreased prices and margins could "reduce incentives of specialized retailers to invest in quality [...]". Then a straightforward question arises: what is the welfare effect of price transparency? Moreover, the argument "price transparency leads to lower prices" implicitly implies that consumers prefer a cheaper product even when they have taken quality into consideration. However, consumers sometimes prefer more expensive products in reality when they believe "you get what you pay for". So how can we unify these conflicting observations?

We argue that the key to answering these questions is that whether retailers can improve quality with relatively low effort cost. If retailers cannot do it, they will not make much effort to improve quality anyway. The problem is closed to a one dimensional (price) competition game where price transparency leads to higher consumer surplus and total welfare, and consumers always prefer cheaper products. If retailers can easily improve quality, they have incentives to make high efforts when they choose high prices because they have concerns about missed sales, whereby some consumers may forgo purchasing the product when faced with a very low quality. However, as the e-commerce sector inquiry worried, reduced prices and margins induced by fierce price competition erodes retailers' effort incentives to invest in quality, the efficiency loss can be very large such that price transparency leads to lower total welfare.

Section 3 introduces the framework. We modify a canonical clearinghouse model by adding quality. As is conventional, there are more than two retailers selling homogeneous products. Retailers can exert efforts to increase the quality of their service and of their product, and consumers benefit from a retailer's effort only if they purchase from this retailer.[1] Examples include fast delivery, assembly, warranties, gift wrapping, packaging, maintenance, and so forth. Consumers are divided into two groups: "loyal" consumers only buy from a designated 'local' retailer, while "non-loyal" consumers use all the information available to make the best decision.

To identify the influence of information, we mainly focus on the following two situations. First, as a benchmark, consumers learn no information before searching and they incur a search cost to visit a retailer to learn its price and quality. Second, non-loyal consumers can use a price comparison website to learn all prices before searching (price transparency), their search actions are therefore directed by the observed prices. We call them the benchmark scenario and price directed search scenario.

Section 3 also studies the benchmark scenario, retailers always choose an efficient effort that maximizes consumers' gain from quality minus retailers' effort cost. Conditional on equilibrium effort, the equilibrium is qualitatively similar to that of Diamond (1971), retailers choose the monopoly price and quality in equilibrium.[2] Since consumers cannot observe anything before searching, the number of consumers visiting a (monopoly) retailer is fixed, and exerting the efficient effort allows the retailer to obtain the highest margin while keeping the per-consumer demand unchanged.

Section 4 considers the price directed search scenario which is more complicated. We first need to pin down non-loyal consumers' beliefs about efforts according to their observed prices. For example, if a consumer sees two retailers charging 100 euros and 110 euros respectively for the same sunglasses on the price comparison website, which one should he or she expect to be better? We employ a belief concept summarized by In and Wright (2018). The idea is that non-loyal consumers believe that a retailer must choose the effort that is optimal for it given its price and given all the players' equilibrium strategies. We find that the belief about a retailer's effort is uniquely pinned down by its price. Different from the benchmark scenario, retailers choose lower efforts because they cannot commit to the efficient effort. When the price is lower, they always have incentives to slightly reduce their efforts due to the search cost.

[1]Concerns about effort free-riding have also recently led to new attention because of price transparency. Examples of recent studies are Wang and Wright (2020) and Janssen and Ke (2020); see the European Commission E-commerce Sector Inquiry for more information.

[2]The corresponding monopoly effort is the efficient effort.

Non-loyal consumers therefore will visit the retailer with the highest expected surplus according to observed prices and the corresponding beliefs about efforts. If retailers cannot easily improve quality, efforts do not have a large influence on consumers' net surplus. Non-loyal consumers will simply visit the retailer with the lowest price, and the equilibrium is similar to that of Varian (1980). However, if retailers can easily improve quality, we provide conditions such that consumers associate a higher price with a higher surplus. As we mentioned before, the reduced margin caused by a lower price can reduce retailers' effort incentives, which will have a large influence on the expected surplus in this case. Therefore, retailers cannot attract more consumers by choosing lower prices. This property can result in a mixed strategy equilibrium with a mass point at the bottom or middle of the equilibrium support and can even lead to a pure strategy equilibrium, which is quite different from the results of Varian (1980).

Section 5 studies the welfare analysis. Compared with the benchmark scenario, price transparency always increases consumer surplus, because retailers are willing to provide better offers to attract non-loyal consumers. However, the effect of price transparency on total welfare is ambiguous; the benchmark scenario involves more efficient efforts, while the price directed search scenario involves lower prices. We find that total welfare in the price directed search scenario can be either higher or lower than that of the benchmark scenario. The key determinants are the fraction of non-loyal consumers and whether retailers can easily improve quality.

The price directed search scenario is exactly the same as the benchmark scenario when there are no non-loyal consumers. A higher fraction of non-loyal consumers increases the total welfare of the price directed search scenario when the fraction is small enough. A higher fraction of non-loyal consumers has a relatively smaller effect on industry profit than on consumer surplus, because retailers choose prices and efforts that are close to the monopoly price and effort in this situation.

When the fraction of non-loyal consumers is large, the effect of a higher fraction on total welfare in the price directed search scenario depends on whether retailers cannot easily improve quality or not. If retailers cannot easily improve quality, the total welfare of the price directed search scenario is always increasing in the fraction of non-loyal consumers. The effect of lower prices dominates the effect of efficient efforts because a higher effort does not have much effect on quality. If retailers can easily improve quality, the total welfare of the price directed search scenario is sometimes decreasing in the fraction of non-loyal consumers and can be lower than the total welfare of the benchmark scenario. A relatively large fraction of non-loyal consumers leads to excessive competition, which reduces the retailers' effort incentives, and the effect of efficient efforts therefore dominants the effect of lower prices.

Section 6 analyzes the policy implication. We first explore the implications of these insights for resale price maintenance regulations. We do so by introducing a minimum price in the price directed search scenario. We find that having a minimum price slightly above the lower bound of the equilibrium pricing support strictly improves the total welfare in the price directed search scenario. There exists an optimal minimum price since total welfare is bounded. With the optimal minimum price, the corresponding consumer surplus and total welfare are strictly higher than those of the benchmark scenario. Having a minimum price prevents retailers from choosing very low prices and inefficient efforts, and it results in less risky price distribution since retailers choose the minimum price with a positive probability. Banning resale price maintenance therefore might reduce consumer surplus and total welfare.

We then argue the importance of quality transparency by examining the situation when non-loyal consumers know all the prices and qualities before searching (full information scenario). We find that retailers always choose the efficient effort and the competition for non-loyal consumers leads to price dispersion like in Varian (1980). Consumers know the net surplus of purchasing from each retailer before searching. Therefore, both the number of consumers visiting a retailer and per-consumer demand depends on the observed net surplus, and exerting the efficient effort allows the retailer to obtain the highest margin while keeping the net surplus unchanged. Compared with the price directed search scenario, retailers can compete in a more efficient way since they can commit to the efficient effort. Like before, having a minimum price slightly above the lower bound of the equilibrium pricing support strictly improves the consumer surplus and total welfare in the full information scenario, and the consumer surplus and total welfare with the optimal minimum price are strictly higher in the full information scenario than in the price directed search scenario.

To complete the analysis, in section 7, we finally examine the case where retailers can obfuscate consumers about their price before the consumers visit the retailer. The

Chinese platform Taobao offers one such example. When consumers search for a given product, say, the iPhone 8 128 G, they can see a list of offers with prices. However, retailers are able to obfuscate by listing the price of the iPhone 8 64 G, and consumers can never know the true price until they click the link. Consumers are able to observe the rating scores of the retailers, which is a proxy for retailers' quality. We find that even if consumers can observe perfectly accurate quality information before searching, retailers will choose the monopoly price and effort without price transparency. Like the price directed search scenario, the belief about price is uniquely pinned down by observed quality. Different from the price directed search scenario, consumers believe that the best offer they can get is when they see a retailer chooses the monopoly quality, and they expect that the retailer also chooses the monopoly price. The equilibrium outcome is therefore exactly the same as that of the benchmark scenario, even though the logic is very different.

1.1 Related Literature

This paper is related to the clearinghouse literature which has been used to rationalize the price dispersion of homogeneous products observed in both offline and online markets. Prominent early examples are Varian (1980), Stahl (1989), and Baye, Kovenock, and De Vries (1992). In these models, some consumers have access to price information by consulting an "information clearinghouse" (e.g. price comparison websites). Consumers are then split into "loyal" consumers who are only willing to buy from a designated firm, and "non-loyal" consumers who only buy from the firm charging the lowest price. Baye and Morgan (2001) generalize the framework by endogenizing the decisions of firms and consumers and the fees charged by the clearinghouse. Shelegia and Wilson (2020) further generalize the model by allowing for multiple dimensions of firm heterogeneity. The key difference between this literature and our work is that, in our model, consumers care about two attributes: price and quality. "Non-loyal" consumers may not choose the lowest price, because their search behavior depends on their beliefs about quality.

This paper also relates to the literature on price as a signal of quality. Two most relevant papers are Wolinsky (2005) and Dubovik and Janssen (2012), they also study an environment where quality is endogenous and observable (before purchasing), and their equilibrium beliefs on qualities according to observed prices also satisfy the idea of In and Wright (2018).[3] Wolinsky (2005) finds similar results that price transparency leads to higher consumer surplus and lower total welfare,[4] he studies a heterogeneous products market and focuses on a symmetric equilibrium, while this paper studies a homogeneous product market which is more relevant to the debate about the price comparison websites and allows us to study how consumers react to prices when they take quality into consideration. Dubovik and Janssen (2012) study a similar model as this paper. In their paper, retailers have incentives to improve quality because they want to attract "shoppers" who know all information before searching, while in our paper, retailers have incentives to improve quality because they want to increase the probability that consumers purchase the product. This difference also allows us to provide more insights into the problem.

This paper also relates to the large and growing literature on prominence and ordered search models.[5] Early examples of match search models include Armstrong, Vickers, and Zhou (2009), Rhodes (2011), Armstrong and Zhou (2011), Arbatskaya (2007) and Zhou (2011).[6] These papers consider models where the match value contains only one attribute. In three recent papers (Choi, Dai, and Kim, 2018, Haan, Moraga-González, and Petrikaitė, 2018 and Anderson, Engers, and Savelle, 2020), the match value is composed of two attributes, and the order is decided by partially observed product information and advertised prices. Similar to our paper, Wilson (2010) and Ding and Zhang (2018) study the ordered search problem in a clearinghouse model, but neither of them considers the effect of quality on consumers' utility. Chioveanu (2019) also use this framework to study the ordered search problem when each firm can obfuscate consumers about its price. In this paper, the product is homogeneous, and consumers' valuation and the prices are known before searching. The unobserved attribute is endogenous in this paper, and the equilibrium depends on the beliefs of consumers based on the observed attribute.

Finally, this paper relates to the literature on consumer search and firms' service

[3] Other examples include Wilson (1980), Klein and Leffler (1981), Wolinsky (1983), Ellingsen (1997), and Bester (1998). These papers either assume quality is exogenous, or consumers cannot fully observe quality before purchasing.

[4] In this paper, price transparency can lead to either higher or lower total welfare.

[5] See Armstrong (2017) for a recent survey.

[6] The search order is exogenous in Armstrong, Vickers, and Zhou (2009) and Rhodes (2011), firms can influence the order by paying money in Armstrong and Zhou (2011), the order is determined by the search cost in Arbatskaya (2007), and in Zhou (2011), the order is decided by the consumers' beliefs about the equilibrium prices.

and quality. The potential for free-riding is discussed by Janssen and Ke (2020), who consider a situation where retailers can provide services that enhance the utility of buying the product at any retailer. They find that firms that provide service and those that do not can coexist in equilibrium. Wolinsky (2005), Fishman and Levy (2015) and Moraga-González and Sun (2020) consider a situation where firms can make efforts to increase product quality. Chen and Zhang (2019) study a similar model but with experience goods. In our model, consumers form a belief about quality according to observed prices, which means that the search is ordered.

2 An Illustrative Example

To introduce the main ideas in the simplest possible way, we consider the following example. There are two online retailers and a continuum of consumers interested in buying a book. Consumers' valuation for the book v is uniformly distribution from \$0 to \$10. If retailers make no effort, the delivery time will be long, and the book will not be in a good condition. We assume consumers suffer \$5 in this situation, which means that the corresponding (service and product) quality is $q_0 = -5$. Retailers can also incur an effort to improve the quality at a cost \$2, the corresponding quality is q_+. We assume $q_0 < q_+ < 0$.

Consumers' net surplus of purchasing a product with price p and quality q is:

$$v + q - p.$$

Consumers need to visit a retailer to buy the product, the cost of visiting the first retailer is zero, and the cost of visiting the second retailer is \$10, which implies that consumers never search twice.[7]

We compare two scenarios to study the effect of price transparency, 1) there is no price comparison website and consumers need to visit a retailer to learn its price and quality; 2) there exists a price comparison website, consumers learn both prices before searching, but they still need to visit a retailer to learn its quality and purchase the product.

If there is no price comparison website, we can show that both retailers will choose the monopoly price and quality. Like Diamond (1971), consumers do not search more than

[7]We use this assumption to simplify the analysis, but actually we only need the second search cost to any strictly positive number.

once, retailers cannot attract more consumers by choosing a lower price, they therefore just behave like a monopoly. Each retailer has incentives to deviate if some consumers visit the retailer and the retailer does not choose the monopoly price and quality. If there exists a price comparison website, consumers can use it to learn both prices before searching. Price is a signal of quality, and consumers need to compare expected $p - q$.

Let us first consider the situation where retailers cannot easily improve quality such that $q_+ = -4.999$. Retailers will always make zero effort because it is too difficult to improve quality. If retailers make effect \$2, the quality is improved by just \$0.001, the margin is decreasing a lot, but the demand is increasing only a little bit. The problem is like a one dimensional price competition model.

Without the price comparison website, retailers choose the monopoly price (and zero effort). The corresponding price is \$2.5, the consumer surplus is \$0.3125 and total welfare is \$0.9375. With the price comparison website, price transparency leads to Bertrand competition equilibrium, because consumers believe retailers will always choose zero effort, and they can compare prices directly before searching. The corresponding price is \$0, the consumer surplus and total welfare are both \$1.25. As is common in price competition models, price transparency improves the consumer surplus and total welfare.

We then consider the situation where retailers can easily improve quality such that $q_+ = -2$. In this case, retailers have incentives to choose strictly positive effort when the price is large enough. For the same reason, without the price comparison website, retailers behave like a monopoly. The corresponding monopoly price is \$5 and monopoly effort is \$2, and the consumer surplus is \$0.45 and total welfare is \$1.35. If retailers choose zero price and zero effort when there exists a price comparison website, price transparency improves the consumer surplus (\$1.25 vs \$0.45) but reduces the total welfare (\$1.25 vs \$1.35). We will argue why this is the equilibrium outcome later.

To answer the question that whether consumers prefer cheaper or more expensive products when there exists a price comparison website, we study consumers' beliefs about the qualities according to observed prices. Following a belief concept summarized by In and Wright (2018), we argue that consumers believe each retailer must choose an effort that can maximize its per-consumer profit, $(p - e)\,[1 - F(p - q)]$, given its price. Each retailer understands that consumers never search twice, and will choose an effort that maximizes the per-consumer profit according to its price.

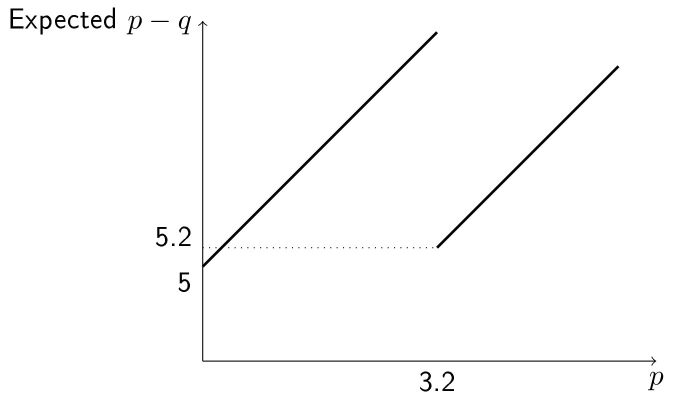


Figure 1: Expected $p - q$ as a function of price

When $q_+ = -4.999$, it is clear that consumers should always visit the one with the lower price since retailers will always make zero effort. When $q_+ = -2$, we find that the per-consumer profit is higher when $e = 2$, $(p-2)[1-F(p-2)] \geq p[1-F(p-5)]$, for any $p \geq 3.2$; and the per-consumer profit is higher when $e = 0$, $(p-2)[1-F(p-2)] < p[1-F(p-5)]$, for any $p < 3.2$. For example, if a retailer chooses price \$3.3, it is not reasonable to believe the retailer chooses zero effort. Because for the given price \$3.3, the per-consumer profit with strictly positive effort (\$0.611) is higher than that with zero effort (\$0.561). Figure 1 shows the relationship between the expected $p - q$ and price, whether consumers prefer cheaper or more expensive product depends on their expected $p - q$. To attract more consumers, both retailers choose $p = e = 0$ in equilibrium because the expected $p - q$ reaches its minimum when $p = 0$.

3 The Model

There are $n \geq 2$ retailers selling a homogeneous product. There is also a mass one of risk-neutral consumers who have unit demand and a zero outside option. Retailers compete by choosing a price p and an effort $e \geq 0$, the cost of effort e is equal to e.

A consumer with valuation v obtains the following surplus when purchasing from a retailer:

$$v + q(e) - p,$$

where p is the retailer's price, and $q(e)$ is the quality provided by the retailer when it exerts effort e. Consumers' valuations of the product v are independently distributed according to a twice-differentiable c.d.f. $F(v)$ with support $[0, v_{max}]$, and the corresponding density function $f(v)$ is strictly positive on $[0, v_{max}]$. Assume that $1-F(v)$ is strictly log-concave. [8]

We assume that the quality function, $q(e)$, has the following properties:

$$q(e) < 0,\ q'(e) > 0,\ q''(e) < 0 \text{ for any } e,$$
$$q'(0) > 1,\ v_{max} + q(0) > 0.$$

Quality being negative is merely a convenient normalization, which captures the idea that consumers do not enjoy the process of shopping. It has a natural interpretation in our setting where quality could represent service attributes such as delivery speed. With effort, a retailer can reduce the buyer's disutility of waiting for delivery.[9] The concavity of the quality function $q(e)$ means that there are diminishing returns to effort. Assumption $q'(0) > 1$ excludes the situation where retailers would never make positive efforts. It is assumed that $v_{max} + q(0) > 0$, so there exist some consumers willing to buy the product when there is zero effort as long as the price is sufficiently low, which also implies that the profit of a monopoly retailer is strictly positive.

A fraction $\lambda \in (0, 1)$ of consumers are non-loyal, who can choose any retailer, while the rest of them $(1-\lambda)$ are loyal consumers who are only willing to buy from a designated 'local' retailer. Retailers share the loyal consumers equally. Both the valuation v and loyalty are consumers' private information and are independent of each other. Consumers need to visit a retailer to buy the product, which incurs a search cost; they learn its price and quality once upon arrival at the retailer (if they do not know it before). Loyal consumers search only once and the (first) search cost is zero; for non-loyal consumers, the search cost to visit the first retailer is also zero, then it equals $s > 0$ for each additional retailer that they visit. Consumers can return to any previously searched retailer without cost. If non-loyal consumers are indifferent between some retailers, they randomly (with equal probability) visit one of them.

The timing is as follows: At stage one, retailers simultaneously choose price and effort. At stage two, loyal consumers visit their local retailers and learn its price and quality. Non-loyal consumers search retailers and learn their prices and qualities sequentially.

We will mainly focus on two different situations where consumers have different in-

[8]Strictly log-concavity means there is a strictly decreasing reversed hazard rate.

[9]The analysis is the same if we assume $q(e)$ can be positive when effort is large as long as $e > q(e)$ is satisfied.

formation before searching. In the benchmark scenario, consumers know nothing before they search a retailer. In the price directed search scenario, non-loyal consumers know all the prices before searching.

3.1 Preliminary Analysis

We first characterize the per-consumer profit function. Note that consumer preferences can be defined over the *adjusted price* ψ:

$$\psi = p - q(e).$$

A consumer chooses to purchase the product only if his or her valuation is greater than the adjusted price (positive surplus), and this probability is $1 - F(p - q(e))$. Define per-consumer profit $\pi(p, e)$ to be a function of price p and effort e:

$$\pi(p, e) = [1 - F(p - q(e))] (p - e).$$

For a reason that will be become clear soon, we now consider the behavior of a monopoly retailer and the properties of per-consumer profit functions. Let (p^m, e^m) maximize per-consumer profit:

$$(p^m, e^m) \in \underset{(p,e)}{\operatorname{argmax}}\, \pi(p, e).$$

Let $\pi^m = \pi(p^m, e^m)$ and $\psi^m = p^m - q(e^m)$ denote the monopoly per-consumer profit and monopoly adjusted price. And let e^* denote the *efficient effort* that maximizes the utility of quality minus the effort cost, $q(e) - e$. Note that e^* being positive is guaranteed by assumption $q'(0) > 1$. The next lemma characterizes the properties of per-consumer profit functions and monopoly behaviors.

Lemma 1. *The pair (p^m, e^m) is unique, the monopoly effort equals to the efficient effort, $e^m = e^*$, and the monopoly price is $p^m = e^m + \frac{1-F(p^m-q(e^m))}{f(p^m-q(e^m))}$.*

To understand that the monopoly effort equals the efficient effort, consider the per-consumer profit as a function of adjusted price and effort:

$$[1 - F(\psi)] (\psi + q(e) - e). \tag{1}$$

For any given adjusted price that leads to a strictly positive per-consumer demand ($1 - F(\psi) > 0$),[10] the per-consumer profit function is strictly concave in e and reaches its

[10]We focus on this situation since a monopoly retailer can always obtain a strictly positive profit because of assumption $v_{max} + q(0) > 0$.

maximum when $e = e^*$, which means that choosing the efficient effort makes retailers able to obtain the highest margin while simultaneously keeping the per-consumer demand unchanged. Then, we can treat $e = e^*$ as a fixed marginal cost, the strict log-concavity of $1 - F(v)$ guarantees that the per-consumer profit function is strictly quasi-concave in p, and p^m can be obtained by solving the first order condition.

3.2 Equilibrium of the benchmark scenario

We begin by analyzing the benchmark scenario where non-loyal consumers know nothing before searching. We look for rational expectations equilibria, in which consumers' beliefs about prices and service qualities are correct. As is conventional, the equilibrium concept is PBE, and we assume consumers hold passive beliefs about the strategies of other retailers if they find an unexpected adjusted price.

Proposition 1 (Diamond Paradox)**.** *There exists a unique equilibrium where all the retailers choose the monopoly price and quality, and they share non-loyal consumers equally.*

The intuition is the same as that of Diamond (1971), according to the tie-breaking rule, non-loyal consumers randomly choose a retailer to visit because they believe that all retailers choose the same adjusted price. Given this belief, retailers cannot attract more non-loyal consumers by choosing a lower adjusted price due to search cost, and are therefore willing to choose the monopoly price and quality. In equilibrium, consumers' beliefs about prices and service qualities are correct, as is required for rational expectation equilibria, so they will only search once.

If there exists some retailers (or one retailer) do not choose the monopoly price and quality, they have incentives to deviate a little bit because consumers will not leave and search again due to the search cost, and the per-consumer profit can be strictly higher.[11]

[11] As we mentioned before, choosing an effort closer to e^* can strictly increase the per-consumer profit. If these retailers choose the monopoly quality but not the monopoly price, choosing a price closer to the monopoly can strictly increase the per-consumer profit because $1 - F(v)$ is strictly log-concave, and $\pi(p, e^m)$ is strictly quasi-concave in p.

4 Price Directed Search

To examine the effect of price transparency, we discuss the price directed search scenario where non-loyal consumers observe all the prices before searching. It is a signaling game, and the equilibrium concept is PBE. In the first subsection, we introduce a reasonable way to describe the consumers' belief about qualities (and adjusted price) according to observed prices, and then we analyze the properties of this belief. In the second subsection, we discuss the equilibrium outcomes.

4.1 The Belief

In this scenario, one of the retailers' actions (quality) is not observed by consumers but can be signaled to consumers through another action (price). The difficulty is how to uniquely pin down the consumer's belief about unobserved qualities, otherwise, the model could suffer from a plethora of equilibria.[12]

To solve this problem, we follow the idea of a belief concept summarized by In and Wright (2018), who propose to use a hypothetical reordered game in which retailers choose prices before qualities, then subgame perfection would pin down consumers' belief about each retailer's choice of quality, which should be optimally chosen given the observed prices and all the other players' equilibrium strategies.[13] However, if prices and qualities are chosen simultaneously, we cannot use subgame perfection to pin down the consumers' belief about the unobserved qualities for off-equilibrium levels of the observed prices.

This reordering should not matter. No matter retailers make the choices of price and quality simultaneously or sequentially, they have the same payoff relevant information. Even if the choices of quality are set at the same time as the choices of price, retailers should set the qualities according to their prices. Hence, rational retailers would make the same choices for price and quality irrespective of they are set sequentially or at the same time. Rational consumers, reasoning in this way, believe that each retailer's choice of quality should be chosen optimally given the observed price and all the other

[12] See In and Wright (2018) for more details.

[13] The idea of this belief concept has been adopted by many papers, for example, McAfee and Schwartz (1994) and Rey and Vergé (2004) use wary belief to pin down each retailer's belief about their rival's offer. In and Wright (2018) analyze a broader range of games, and show wary belief is a special case of their more general reordering invariance belief.

players' equilibrium strategies. That is, to form the beliefs about the unobserved qualities, consumers can treat the observed prices as if they were set first.

> *Consumers believe a retailer must choose a quality that is optimal for it given its price and given all the other players' equilibrium strategy.*

We therefore argue that the belief (both on-path and off-path) of consumers about a retailer's effort is a function of its (observed) price, let $e_w(p)$ denote the belief, it satisfies:

$$e_w(p) \in \operatorname*{argmax}_e \pi(p, e). \tag{2}$$

To better understand this idea, imagine retailers choose prices and then choose efforts. The game has not changed because retailers If all the non-loyal consumers hold the belief $e_w(p)$, they will visit the retailer with the lowest $p - q(e_w(p))$, and the retailer has no incentive to deviate because the quality level maximizes its profit conditional on some consumers visit it.

All the retailer's equilibrium strategies and other retailer's realized prices do not matter for the determination of $e_w(p)$. As we mentioned before, consumers believe a retailer must choose a quality that maximizes its profit, although each retailer's profit is not independent of other retailers? strategies, conditional on the retailer's price and the retailer being visited by a consumer, its profit from that consumer is independent of other retailer's strategies.

For any given belief about unobserved qualities, consumers make their visit decision only according to observed prices. Due to the search cost, consumers will not search again if they find the quality of a retailer is only slightly lower than what they have expected. Therefore, a retailer has an incentive to deviate if it does not choose an effort level that maximizes the per-consumer profit for the given price. Note that this relies on the assumption that quality investment is flexible to adjust.

The following corollary is a useful step before we study the belief $e_w(p)$. Let Ω denote the set of pairs (p, e) that result in strictly positive per-consumer profits,

$$\Omega = \{(p, e) : p - q(e) < v_{max} \text{ and } p > e\}.$$

Corollary 1. *If $1 - F(v)$ is strictly log-concave for $v \in [0, v_{max}]$, the per-consumer profit function $\pi(p, e)$ is strictly quasi-concave in effort e for any given price p when $(p, e) \in \Omega$.*

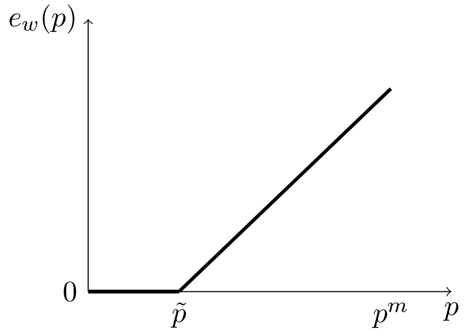


Figure 2: The belief $e_w(p)$

If non-loyal consumers believe that a retailer with price p must choose effort $e_w(p)$, retailers do not have an incentive to deviate. A lower effort strictly reduces the per-consumer profit, and it may also decrease the number of consumers staying at the retailer; a higher effort cannot attract more consumers to visit the retailer, and it strictly reduces the per-consumer profit.

If consumers have a belief other than $e_w(p)$, then the retailers have an incentive to deviate a little bit by choosing an effort closer to $e_w(p)$. It can increase the per-consumer profit because $\pi(p, e)$ is strictly quasi-concave in effort e for any given price p when $(p, e) \in \Omega$, and consumers will not visit other retailers when they see unexpected effort due to the search cost.[14]

Before characterizing the properties of the belief $e_w(p)$ and consumers' search behavior, we first introduce two parameters. Let e_{max} solve $e - q(e) = v_{max}$,[15] note that v_{max} is the upper bound of the distribution of the valuation v. Note that $(p, e) \in \Omega$ implies

$$e - q(e) < p - q(e) < v_{max},$$
$$p - q(p) < p - q(e) < v_{max}.$$

We can conclude that $(p, e) \in \Omega \implies e < p < e_{max}$, which means that a retailer's profit is at most zero if it chooses a price or effort above e_{max}.[16] Because the monopoly price is strictly positive, we must have $p^m < e_{max}$. Let $\tilde{p}$ solve the first order condition $\frac{\partial \pi(p,e)}{\partial e} = 0$

[14]We can focus on $(p, e) \in \Omega$ since retailers can at least focus on its loyal consumers and make strictly positive profits.

[15]The parameter e_{max} is unique because $e - q(e)$ is convex, and $e - q(e) < v_{max}$ when $e = 0$.

[16]If a retailer chooses a very high price, it cannot make consumers buy the product by providing high quality because it is very costly ($q''(e) < 0$); on the other hand, if the retailer chooses a very high quality, it needs to charge a high price to cover the cost, which makes no one willing to purchase the product.

at $e = 0$, which implies that $e_w(\tilde{p}) = 0$ because $\pi(p, e)$ is strictly quasi-concave in e. The following lemma characterizes the properties of the belief $e_w(p)$.

Lemma 2. *The belief $e_w(p)$ has the following properties.*

I. When $p < e_{max}$, the belief $e_w(p)$ is unique, weakly increasing and satisfies:

(i) when $p \leq \tilde{p}$, $e_w(p) = 0$.

(ii) when $\tilde{p} < p < e_{max}$, $e_w(p)$ solves the equation:

$$\frac{\partial \pi(p, e)}{\partial e} = f(p - q(e)) \left[-\frac{1 - F(p - q(e))}{f(p - q(e))} + (p - e)q'(e) \right] = 0 \tag{3}$$

(iii) when $p \leq p^m$, $e'_w(p) < 1$.

II. When $p \geq e_{max}$, the belief on effort $e_w(p)$ is not unique, but consumers always believe that the surplus of buying from the retailer must be negative, $p - q(e_w(p)) \geq v_{max}$.

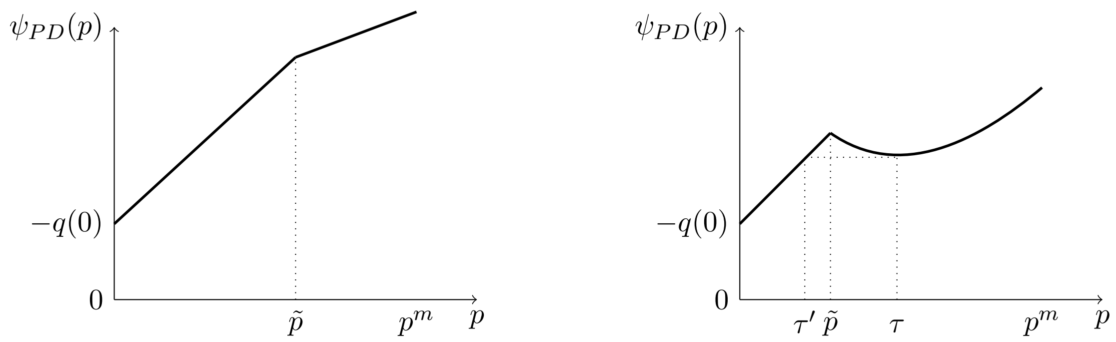
An example of $e_w(p)$ when $p < e_{max}$ is provided in Figure 2, we find that a lower price is always linked to a lower quality. When price is very small such that $p < \tilde{p}$, the corresponding margin is so small such that it is not profitable for the retailer to make a positive effort to improve per-consumer demand. When price is larger than $\tilde{p}$ but smaller than e_{max}, a higher price means a higher margin, which makes the retailer have incentives to exert more effort to increase the per-consumer demand. When price is so large such that $p \geq e_{max}$ (which is not depicted in the figure), the profit of the retailer is at most zero, so consumers believe that the retailer must choose an effort that induces zero demand, which means that the net surplus is negative for every consumer ($p - q(e_w(p)) \geq v_{max}$).

To answer the question that whether consumers associate a higher price with a higher surplus or lower surplus, now we discuss the belief of adjusted price (or expected adjusted price) which is denoted by $\psi_{PD}(p) = p - e_w(p)$. Let $\tilde{q} = \frac{1+\sqrt{1-4q''(0)\tilde{p}}}{2}$,[17] the following lemma characterizes the properties of the expected adjusted price.

Lemma 3. *If $q'(e)$ is log-concave,*[18] *the expected adjusted price $\psi_{PD}(p)$ has the following properties:*

(i) when retailers cannot easily improve quality such that $q'(0) \leq \tilde{q}$, the expected adjusted price is increasing in p for $p \in [0, p^m]$;

(ii) when retailers can easily improve quality such that $q'(0) > \tilde{q}$, the expected adjusted price is first increasing, then decreasing and finally increasing again in p for $p \in [0, p^m]$.



(a) Situation (i): $q'(0) \leq \tilde{q}$

(b) Situation (ii): $q'(0) > \tilde{q}$

Figure 3: Expected adjusted price

Examples of expected adjusted price are provided in Figure 3. When $p < \tilde{p}$, the expected adjusted price $\psi(p)$ is linear in p because retailers make zero effort. When the price is slightly above $\tilde{p}$, price has both direct and indirect (through $e_w(p)$) effects on the adjusted price. If retailers cannot easily improve quality, quality does not have a big effect on players' behaviors, the direct effect dominates, and consumers believe that a lower price means a lower adjusted price. If retailers can easily improve quality, quality has a big impact on players' behaviors, the indirect effect dominates, and consumers associate a higher price with a higher surplus. When price becomes larger, the direct effect dominates again and the expected adjusted price is increasing since the quality function is concave.

4.2 Equilibrium Analysis

In this section, we solve for a symmetric equilibrium where all the non-loyal consumers hold the belief $e_w(p)$ and all the retailers choose the same mixed strategy $G_{PD}(p, e)$. Since retailers also find it optimal to choose effort equal to $e_w(p)$ for a given p, we can simply focus on the pricing strategy $G_{PD}(p)$ to describe the equilibrium.

We will characterize the equilibrium when retailers cannot easily improve quality ($q'(0) \leq \tilde{q}$) and when they can easily improve quality ($q'(0) > \tilde{q}$) separately.

[17] $\tilde{q}$ is well defined because $q''(0) < 0$.

[18] The result still holds when $q'(e)$ is not log-concave but $q''(e)/q'(e)$ is increasing not too fast.

4.2.1 Retailers cannot easily improve quality

Lemma 4. *When retailers cannot easily improve quality such that $q'(0) \leq \tilde{q}$, there exists a unique symmetric mixed strategy equilibrium. The equilibrium strategy $G_{PD}(p)$ and lower bound of the equilibrium support $\left[\underline{p}, p^m\right]$ are determined by the indifference conditions:*

$$\left[\frac{1-\lambda}{n} + (1 - G_{PD}(p_i))^{n-1}\lambda\right] \pi(p_i, e_w(e_i)) = \frac{1-\lambda}{n}\pi^m \tag{4}$$

$$\frac{1+(n-1)\lambda}{n}\pi(\underline{p}, e_w(\underline{p})) = \frac{1-\lambda}{n}\pi^m \tag{5}$$

The equilibrium outcome is similar to that of Varian (1980) when retailers cannot easily improve quality, because quality does not have a big effect on players' behaviors. Since the expected adjust price is monotone in price, non-loyal consumers will always first visit the retailer with the lowest price. In equilibrium, retailers should be indifferent among any price in the equilibrium support. Compared with the benchmark scenario, retailers are willing to choose lower prices to attract non-loyal consumers, and they choose lower prices more often when there are more non-loyal consumers.

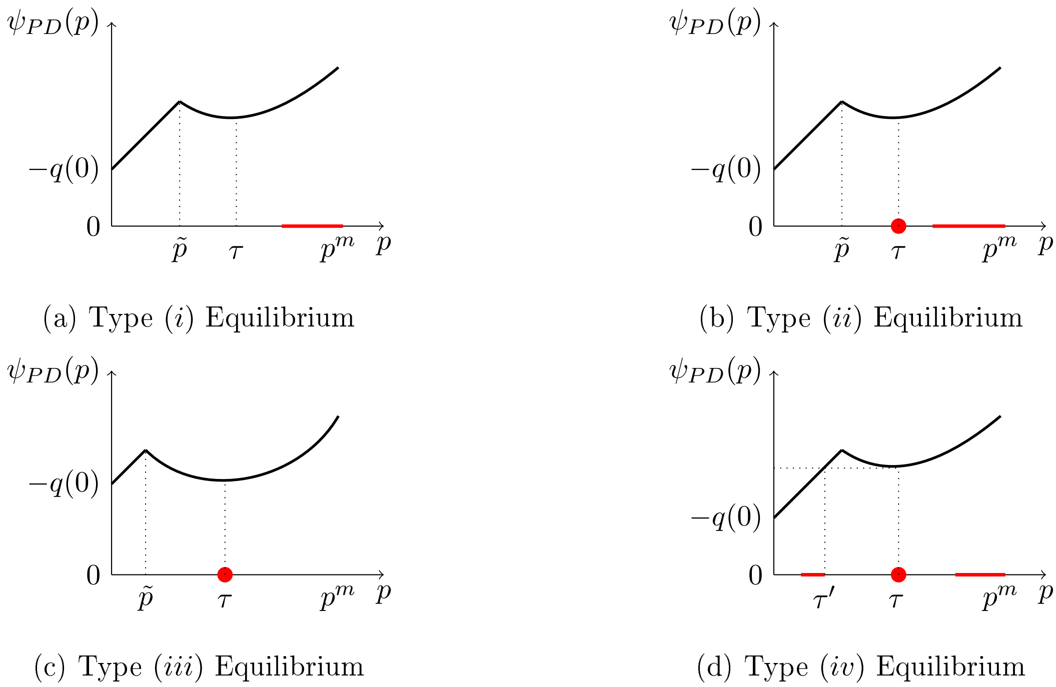
4.2.2 Retailers can easily improve quality

Let τ be the price that minimizes the adjusted price for $p \in [\tilde{p}, p^m]$, let $\tau' < \tau$ solve $\tau - q(e_w(\tau)) = \tau' - q(e_w(\tau'))$, see figure 3b as an example. And let λ_τ, $\lambda_{\tau'}$, $\lambda_{\tau p}$, and $\lambda_{\tau\tau'}$ solve the following equations:[19]

$$\begin{aligned}
\frac{1-\lambda}{n}\pi^m &= \frac{1+(n-1)\lambda}{n}\pi(\tau, e_w(\tau)), \\
\frac{1-\lambda}{n}\pi^m &= \frac{1+(n-1)\lambda}{n}\pi(\tau', 0), \\
\frac{1}{n}\pi(\tau, e_w(\tau)) &= \frac{1-\lambda}{n}\pi^m, \\
\frac{1}{n}\pi(\tau, e_w(\tau)) &= \frac{1+(n-1)\lambda}{n}\pi(\tau', 0).
\end{aligned}$$

As what we will show later, there are only two relationships between the four parameters. One is $\lambda_\tau < \lambda_{\tau p} < \lambda_{\tau'} \leq \lambda_{\tau\tau'}$, and $\lambda_{\tau'}$ and $\lambda_{\tau\tau'}$ could be equal to 1; the other possibility is $\lambda_\tau < \lambda_{\tau\tau'} < \lambda_{\tau'} < \lambda_{\tau p} < 1$. The following proposition characterizes the equilibrium.

[19] We assume τ' is a negative number when $\tau - q(e_w(\tau)) \leq -q(0)$, $\lambda_{\tau'}$ and $\lambda_{\tau\tau'}$ are assumed to be 1 at this time.



(a) Type (i) Equilibrium

(b) Type (ii) Equilibrium

(c) Type (iii) Equilibrium

(d) Type (iv) Equilibrium

Figure 4: Equilibrium price support (red line/dot) when $q'(0) > \tilde{q}$

Proposition 2. *If retailers can easily improve quality such that $q'(0) > \tilde{q}$, the game has a unique symmetric equilibrium and the equilibrium outcome depends on the fraction of non-loyal consumers λ and the curvature of the quality function $q(e)$.*

(i) When $\lambda \leq \lambda_\tau$, there exists a unique symmetric mixed strategy equilibrium such as the one described in lemma 4.

(ii) When $\lambda_\tau < \lambda \leq \min\{\lambda_{\tau'}, \lambda_{\tau p}\}$, there exists a symmetric mixed strategy equilibrium with noncontinuous support $\{\tau\} \bigcup [\underline{p}', p^m]$. The equilibrium strategy $G_{PD}(p)$ involves a mass point at τ.

(iii) When $\lambda_{\tau p} < \lambda \leq \lambda_{\tau\tau'}$, there exists a unique pure strategy equilibrium where retailers choose $p = \tau$.

(iv) When $\lambda > \max\{\lambda_{\tau'}, \lambda_{\tau\tau'}\}$, there exists a symmetric mixed strategy equilibrium with noncontinuous support $[\underline{p}, \tau') \bigcup \{\tau\} \bigcup [\underline{p}'', p^m]$ or $[\underline{p}, \tau') \bigcup \{\tau\}$. The equilibrium strategy $G_{PD}(p)$ involves a mass point at τ.

The equilibrium price supports for different types of equilibrium are provided in figure 4, as we will explain later, they are determined by the fraction of non-loyal consumers and the curvature of quality function $q(e)$.

When the fraction of non-loyal consumers is small, retailers want to focus on their loyal consumers and choose prices close to the monopoly price. According to Lemma 3, the expected adjusted price is monotone for $p \geq \tau$. Hence, the equilibrium outcome is the same as that in lemma 4 when the fraction of non-loyal consumers is small enough such that retailers have no incentive to choose a price below τ. And this is guaranteed by $\lambda \leq \lambda_\tau$, because λ_τ solves the condition that a retailer is indifferent between choosing the monopoly price to just have its loyal consumers and choosing τ to attract all non-loyal consumers. Therefore, we have type (i) equilibrium when the fraction of non-loyal consumers is small enough such that $\lambda \leq \lambda_\tau$.

When the fraction of non-loyal consumers is moderate ($\lambda > \lambda_\tau$, but not too large), retailers have incentives to choose prices lower than τ to attract all the non-loyal consumers according to the definition of λ_τ, but they cannot because non-loyal consumers associate a higher price with a higher surplus when $p \in (\tilde{p}, \tau)$. Choosing any price such that $p \in [\tau', \tau)$ is strictly dominated by choosing τ, because choosing τ weakly increases the demand and strictly increases per-consumer profit.[20] To attract non-loyal consumers, retailers thus choose τ with strictly positive probability. In standard clearinghouse models, retailers do not choose a price $p < p^m$ with strictly positive probability because their rivals can choose a price slightly lower than p, but this does not happen in our model because choosing τ dominates any slightly lower price.

Note that $\lambda_{\tau'}$ solves the condition that retailers are indifferent between choosing the monopoly price and choosing a very lower price τ' to attract all non-loyal consumers, and $\lambda_{\tau p}$ solves the condition that a retailer is indifferent between choosing τ (when all the other retailers choose τ and they share non-loyal consumers equally) and choosing the monopoly price. The condition $\lambda \leq \min\{\lambda_{\tau'}, \lambda_{\tau p}\}$ guarantees that the fraction of non-loyal consumers is not so large such that retailers want to choose a price lower than τ' or choose τ with probability one, hence, we have type (ii) equilibrium in this case.

When the fraction of non-loyal consumers is large, it could be either type (iii) or type (iv) equilibrium. According to the definition of $\lambda_{\tau p}$ and $\lambda_{\tau\tau'}$, the condition $\lambda_{\tau p} < \lambda < \lambda_{\tau\tau'}$ guarantees that retailers have no incentive to choose either p^m or τ' which is necessary for type (iii) equilibrium. Similarly, type (iv) equilibrium involves very low prices (smaller than τ'), which requires that $\lambda > \max\{\lambda_{\tau'}, \lambda_{\tau\tau'}\}$.

Whether the equilibrium is type (iii) or type (iv) depends on the curvature of the

[20] According to the envelope theorem, p only has a direct effect on $\pi(p, e_w(p))$, per-consumer profit $\pi(p, e_w(p))$ is therefore increasing in price.

quality function $q(e)$. First, when $q(e)$ is increasing moderately fast, we have $\lambda_{\tau\tau'} < \lambda_{\tau p}$ which means that there exists no type (iii) equilibrium. Because it is more profitable for retailers to choose a price smaller than τ' to attract all non-loyal consumers than choosing τ with probability one, the quality only has a moderate effect on consumers' utility. Second, when $q(e)$ is increasing very fast, we have $\lambda_{\tau p} < \lambda_{\tau\tau'}$, and there exist type (iii) equilibrium for sure. Note that $\lambda_{\tau'}$ and $\lambda_{\tau\tau'}$ could be equal to 1, there exists type (iv) equilibrium in this case only if $\lambda_{\tau\tau'} < 1$. The relationship between parameters and equilibrium types is provided in figure 5.

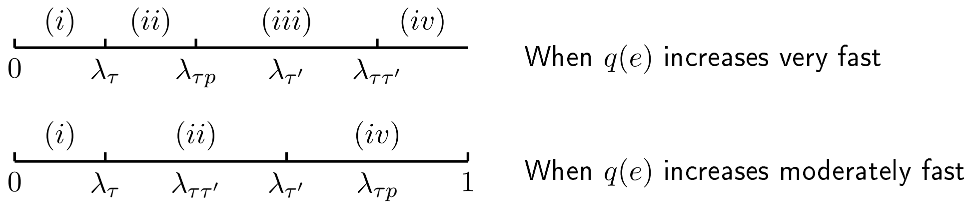


Figure 5: Relationship among λ_τ, $\lambda_{\tau'}$, $\lambda_{\tau p}$, and $\lambda_{\tau\tau'}$

Note that the existence of the pure strategy equilibrium is quite different from other clearinghouse literature. As we mentioned before, the existence of a mass point comes from the fact that retailers cannot choose prices slightly lower than τ to attract non-loyal consumers. There are two reasons that the mass point can be equal to 1. On the one hand, retailers do not want to choose very high prices because they want to attract some non-loyal consumers. On the other hand, it is very costly to attract all non-loyal consumers by choosing very low prices, they could get high profits from loyal consumers because they can easily improve quality.

5 Welfare Analysis

Proposition 3. *Compared with the benchmark scenario, the price directed search scenario (i) always involves higher consumer surplus, (ii) always involves higher total welfare when the fraction of non-loyal consumers λ is sufficiently small, and (iii) sometimes leads to lower total welfare when λ is relatively large.*

Price transparency improves consumer surplus because retailers are willing to choose better offers to attract non-loyal consumers. We therefore focus on the effect of price transparency on total welfare. Let TW_m and TW_{PD} denote the total welfare of the benchmark and price directed search scenarios,

$$TW_m = W(p^m, e^m),$$
$$TW_{PD} = \lambda \int_{p \in supp_{PD}} W(p, e_w(p))\, \mathrm{d}\left[1 - (1 - G_{PD}(p))^n\right] + (1 - \lambda) \int_{p \in supp_{PD}} W(p, e_w(p))\, \mathrm{d}G_{PD}(p),$$

where $supp_{PD}$ is equilibrium support of the price directed search scenario, and *per-transaction welfare* $W(p, e)$ is defined as:

$$W(p, e) = \int_{p-q(e)}^{v_{max}} [v + q(e) - p]\, dF(v) + \pi(p, e).$$

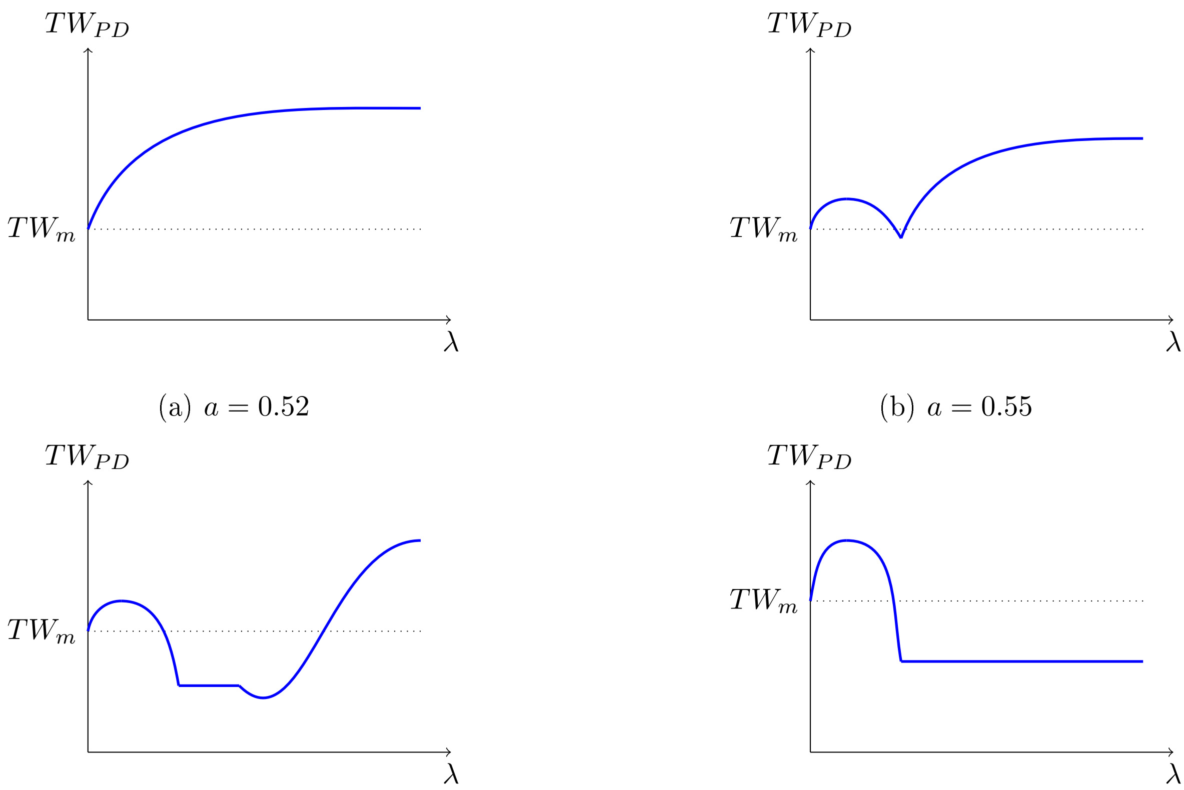
Per-transaction welfare is maximized when all the retailers choose $p = e = e^*$. Choosing the efficient effort results in the highest profit while keeping the consumer surplus unchanged.[21] Then we can treat $e = e^*$ as a fixed marginal cost, as is well known, $W(p, e)$ is maximized when price equals the marginal cost.

Total welfare is therefore maximized when all retailers choose $p = e = e^*$. Intuitively, total welfare is higher if retailers choose (p, e) close to (e^*, e^*) with higher probability. Compared with the benchmark scenario, the price directed search scenario involves lower prices but also lower efforts, the resulting welfare effect is thus unclear.

Price transparency leads to strictly higher total welfare when the fraction of non-loyal consumers λ is sufficiently small. Note that the two scenarios have exactly the same equilibrium outcome when there are no non-loyal consumers, i.e. $\lambda = 0$. Total welfare in the benchmark scenario is not influenced by the value of λ. While in the price directed search scenario, a slightly positive λ can induce retailers to slightly lower their adjusted prices to attract these consumers, it has a larger effect on consumer surplus than on industry profit because the prices and qualities are close to their monopoly level.

The total welfare in the price directed search scenario will keep increasing in the fraction of non-loyal consumers if $W(p, e_w(p))$ is monotone in p. However, it is impossible to analytically analyze the change in total welfare when $W(p, e_w(p))$ is not monotone. Compared with the monopoly level per-transaction welfare $W(p^m, e^m)$, a lower price sometimes results in lower per-transaction welfare, and the mixed strategy equilibrium makes it difficult to say how often this occurs or determine the extent of the difference.

[21] To understand this result, we treat the per-transaction welfare as a function is adjusted price ψ and effort e. The first part of per-transaction welfare only depends on ψ, while the second part is maximized when $e = e^*$ for any ψ as we have shown before.



(a) $a = 0.52$ (b) $a = 0.55$ (c) $a = 0.6$ (d) $a = 0.8$

Figure 6: Total welfare

We therefore use the following example to simulate the model: valuation v is uniformly distributed on $[0, 1]$, $q(e) = -a(e-1)^2$ and $n = 2$. It is easier for retailers to improve quality if a is larger.[22] It is easy to check that the uniform distribution satisfies strictly log-concavity; to make $q(e)$ compatible with the assumptions we made before, we assume $a \in (\frac{1}{2}, 1)$ because

$$q'(e) > 0,\ q''(e) < 0,\ q'(0) > 1,\ v_{max} + q(0) > 0 \iff a \in (\frac{1}{2}, 1).$$

We present the results of simulations of total welfare in Figure 6. Note that the total welfare in the benchmark scenario is constant for any value of λ. Figure 6a shows the change in total welfare when per-transaction welfare $W(p, e_w(p))$ is monotone, the total welfare of the price directed search scenario is increasing because retailers cannot easily improve quality. In Figure 6b, if it is relatively easy for retailers to improve quality, total welfare is decreasing when λ is relatively large for the reason mentioned above. The

[22] We can define the quality function to be $q(e) = a - 1 - a(e-1)^2$ if we want to fix $q(0)$, the analysis would be exactly the same.

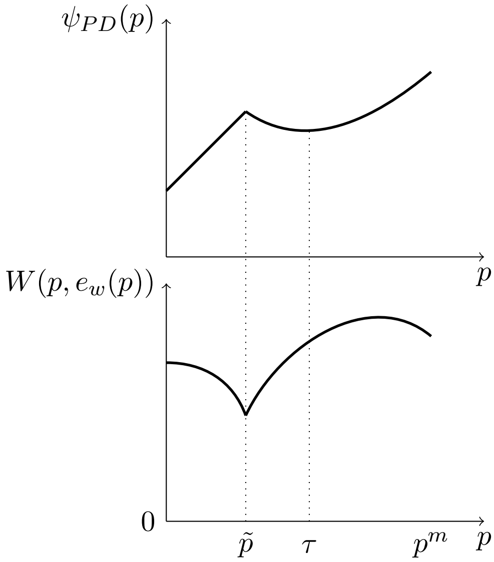


Figure 7: Per-transaction welfare and expected adjusted price

total welfare is increasing again because retailers begin to choose very low prices, which dominates the effect of inefficient effort providing. Part of the graph is flat in Figure 6c because it is a pure strategy equilibrium where all the retailers choose $p = \tau$ in that interval. As λ keeps increasing, retailers are willing to choose very low prices to attract non-loyal consumers, and there is mixed strategy equilibrium. However, if it is very easy for retailers to improve quality, the expected adjusted price is higher when $p = \tau$ than $p = 0$. As we can see from Figure 6d, there is a pure strategy equilibrium when λ is above some level.

The key reason that TW_{PD} can be decreasing in λ is the following: when it is relatively easy for retailers to improve quality, a relatively large fraction of non-loyal consumers leads to too much competition, it reduces retailers' effort incentives and makes retailers' profits decrease very fast. As we can see from Figure 7, retailers will keep lower their prices until $p = \tau$ ($\psi_{PD}(p)$ reaches the local minimum at $p = \tau$ for $p \geq \tilde{p}$) to attract non-loyal consumers. However, a price around τ is too low for total welfare because a slightly higher price can only hurt consumer surplus in the sense of second-order effect, but it has a first-order effect on profit.

As noted in the introduction, price comparison websites create downward pressure on prices and therefore reduce margins, which can further reduce the incentive of retailers to provide high quality. Although price transparency always results in lower adjusted prices which is good for consumers, it may have a big influence on retailers' profit due to inefficient effort providing, which can finally lead to lower total welfare than the situation without price transparency.

6 Policy Implications

This section sheds some light on the regulation of resale price maintenance. We also examine the effect of quality transparency, to further argue why it is important for the price comparison website to show more accurate information on quality.

6.1 Resale price maintenance

In this subsection, we examine the effect of resale price maintenance in the price directed search scenario. We modify the model by assuming retailers are only allowed to choose prices above a minimum price p_{min}. Let $TW_{PD}(p_{min})$ denote the total welfare in the price directed search scenario with the minimum price p_{min}. Since the total welfare is bounded, there exists an optimal minimum price p^*_{min} such that

$$p^*_{min} \in \underset{p_{min}}{\operatorname{argmax}} TW_{PD}(p_{min}).$$

Note that $supp_{PD}$ is the equilibrium price support of the price directed search scenario, we have the following proposition.

Proposition 4. *In the price directed search scenario, if $q'(e)$ is log-concave and the valuation v follows the uniform distribution, a minimum price p_{min} slightly above the lower bound of $supp_{PD}$ strictly increases the total welfare. The consumer surplus and total welfare with the optimal minimum price p^*_{min} are strictly higher than those in the benchmark scenario.*

Resale price maintenance can increase total welfare in two aspects. First, it prevents retailers from choosing low and inefficient qualities. Instead of choosing prices lower than the minimum price, retailers choose the minimum price with strictly positive probability. Note that a higher price is always linked to a higher quality, retailers also choose more efficient quality more often. Second, the equilibrium adjusted price is less dispersed since the minimum price is chosen with strictly positive probability. The industry profit is not changed because retailers can always choose the monopoly price and quality and focus on their loyal consumers, while consumer surplus becomes higher because equilibrium adjusted price is less 'risky'.

As in Armstrong and Vickers (2001), we view retailers as choosing the per-consumer profit π rather than price and effort. Since retailers never choose prices such that $p \in [\tau', \tau)$, for $p \in supp_{PD}$, there is a one to one correspondence between π and p and π is increasing in p. Let $v_{PD}(p)$ denote consumers' average net surplus in the price directed search scenario

$$v_{PD}(p) = \int_{p-q(e_w(p))}^{v_{max}} v - (p - q(e_w(p)))dF(v),$$

and let $V_{PD}(\pi(p, e_w(p))) \equiv v_{PD}(p)$ denote the average net surplus as a function of per-consumer profit. Let $G_{PD\pi}(\pi)$ and $supp_{PD\pi}$ denote the corresponding equilibrium strategy and equilibrium support, the consumer surplus therefore can be described as

$$\underbrace{\lambda \int_{\pi \in supp_{PD\pi}} V_{PD}(\pi) \;\, \mathrm{d}\left[1 - (1 - G_{PD\pi}(\pi))^n\right]}_{\text{non-loyal consumer surplus}} + \underbrace{(1-\lambda) \int_{\pi \in supp_{PD\pi}} V_{PD}(\pi) \;\, \mathrm{d}G_{PD\pi}(\pi)}_{\text{loyal consumer surplus}}$$

$$= \int_{\pi \in supp_{PD\pi}} \underbrace{V_{PD}(\pi)}_{\text{net surplus}} \;\, \mathrm{d}\underbrace{\left[\lambda(1 - (1 - G_{PD\pi}(\pi))^n) + (1-\lambda)G_{PD\pi}(\pi)\right]}_{\text{distribution of } \pi}.$$

Since the industry profit, or expected per-consumer profit, is not influenced by the minimum price when p_{min} is not too large,

$$\int_{\pi \in supp_{PD}} \pi \;\, \mathrm{d}\left[\lambda(1 - (1 - G_{PD\pi}(\pi))^n) + (1-\lambda)G_{PD\pi}(\pi)\right] = (1-\lambda)\pi^m.$$

As we can see from Figure 8, the distribution of the per-consumer profit with the minimum price is therefore a mean-preserving contraction, because the equilibrium with the minimum price involves a mass point at $\pi(p_{min}, e_w(p_{min}))$. Consumer surplus is thus higher with the minimum price when $V_{PD}(\pi)$ is concave.

To see $V_{PD}(\pi)$ is concave, differentiating $V_{PD}(\pi(p, e_w(p))) \equiv v_{PD}(p)$ shows that

$$V'_{PD}(\pi(p, e_w(p))) = -\frac{\psi'_{PD}(p)}{1 - \frac{f(p-q(e_w(p)))}{1-F(p-q(e_w(p)))}(p - e_w(p))}$$

Note that $\pi(p, e_w(p))$ is increasing in price for $p \in supp_{PD}$, and $\frac{f(p-q(e_w(p)))}{1-F(p-q(e_w(p)))}(p - e_w(p))$ is increasing in p due to log-concavity of $1 - F(v)$. $V_{PD}(\pi)$ is concave if $\psi'_{PD}(p)$ is increasing or decreasing not so fast, which can be guaranteed by the assumptions that $q'(e)$ is log-concave and v follows the uniform distribution.

Retailers choose p_{min} with higher probability when p_{min} is increasing until it becomes a pure strategy equilibrium, let p'_{min} denote the corresponding minimum price. Consumer

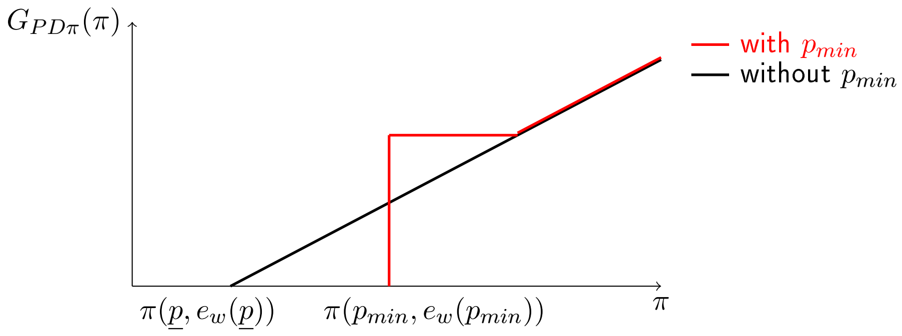


Figure 8: Equilibrium strategy and minimum price

surplus keeps increasing in this process, while industry profit keeps constant. However, it is not necessarily true that $p'_{min} = p^*_{min}$. Note that total welfare is $w(p'_{min}, e_w(p'_{min}))$ with the minimum price p'_{min} since it is a pure strategy equilibrium. As we can see from Figure 7, a higher minimum price leads to higher total welfare when $W(p_{min}, e_w(p_{min}))$ is increasing at p'_{min}. By choosing a higher minimum price, the industry profit becomes higher but consumer surplus becomes lower. Total welfare becomes larger because the efficiency loss from low quality is more important than the gain from lower price.

6.2 Quality transparency

As we discussed before, there are many concerns about the low quality problems. And the Competition and Market Authority of the UK recommend Digital comparison tools "work with sector regulators and suppliers to improve the effectiveness of quality metrics in order to mitigate against the risk of hollowing-out (a reduction of product quality as a result of undue focus on price)". To further argue the importance of quality transparency, we discuss the *full information scenario* where non-loyal consumers observe both price and quality information before searching.

Let $supp_{FI}$ denote the equilibrium price support, let $TW_{FI}(p_{min})$ denote the total welfare when there exists a minimum price p_{min}, and let p^{**}_{min} denote the optimal minimum price

$$p^{**}_{min} \in \underset{p_{min}}{\operatorname{argmax}} \, TW_{FI}(p_{min}).$$

The following the proposition characterizes the effect of quality transparency.

Proposition 5. *In the full information scenario, there exists a unique symmetric mixed strategy equilibrium, and retailers always choose the efficient effort e^*. A minimum price*

p_{min} slightly above the lower bound of $supp_{FI}$ strictly increases the consumer surplus and total welfare. And the consumer surplus and total welfare with the optimal minimum price p^{**}_{min} are strictly higher than those with p^{*}_{min} in the price directed search scenario.

The inefficiency quality problem is not happening again in the full information scenario since retailers always choose the efficient effort e^*. And with a minimum price, consumer surplus and total welfare become larger because of the less dispersed prices.

The full information scenario is similar to Varian (1980). Since non-loyal consumers can observe both prices and service qualities before searching, the retailer with the lowest adjusted price will attract all the non-loyal consumers. If there is no tie, the measure of consumers visiting retailer i is

$$\begin{cases} \frac{1+(n-1)\lambda}{n} & \text{if } \forall\ j \neq i,\ \psi_i < \psi_j, \\ \frac{1-\lambda}{n} & \text{if } \exists\ j \neq i,\ \psi_i > \psi_j. \end{cases}$$

And retailers that choose the lowest adjusted price share non-loyal consumers equally if there is a tie. Since the expected measure of consumers visiting a retailer only depends on its adjusted price, the retailer will choose the price and quality accordingly to maximize its per-consumer profit. For any adjusted price ψ, there exists a unique price and effort that maximizes a retailer's per-consumer profit

$$e = e^*$$

$$p = \psi + q(e^*)$$

The same as the benchmark scenario, retailers always choose the efficient effort in this scenario because it allows the retailers to obtain the highest margin while keeping the demand unchanged. Then the game can be understood as a Varian's model where retailers always choose the efficient effort. When $\lambda = 1$, the Bertrand competition involves that all the retailers choose $e = p = e^*$.

Since retailers are able to compete in a more efficient way in this scenario. Like before, we find that consumers do not like riskier per-consumer profit distribution. Construct a decreasing function $V_{FI}(\pi)$ for the full information scenario, we find

$$V'_{FI}(\pi(p, e^m)) = -\frac{1}{1 - \frac{f(p-q(e^m))}{1-F(p-q(e^m))}(p - e^m)},$$

$V_{FI}(\pi)$ is concave because both $\pi(p, e^m)$ and $\frac{f(p-q(e^m))}{1-F(p-q(e^m))}(p - e^m)$ are increasing in price for $p \in supp_{FI}$.

A minimum price p_{min} slightly above the lower bound of $supp_{FI}$ results in a mean preserving contract of the distribution of π. The industry profit is unchanged but consumer surplus is strictly higher. The optimal minimum price solves $\pi(p^{**}_{min}, e^m) = (1-\lambda)\pi(p^m, e^m)$, where retailers choose pure strategy in equilibrium. Any $p_{min} > p^{**}_{min}$ results in higher industry profit but lower total welfare, while any $p_{min} < p^{**}_{min}$ results in the same industry profit but a mean preserving spread of the distribution of π.

7 Extension: Quality Directed Search

Finally, we consider a scenario in which only quality can be observed before searching. Similar to the analysis above, consumers' equilibrium strategy depends on their belief on unobserved prices. Following the same belief concept, we argue that the (on-path and off-path) belief about price $p_w(e)$ is a function of the retailer's effort such that:

$$p_w(e) \in \operatorname*{argmax}_p \pi(p, e). \tag{6}$$

Perhaps surprisingly, retailers cannot attract more consumers by choosing high efforts. To describe the consumers' behavior, let $\psi_{QD} = p_w(e) - q(e)$ denote the expected adjusted price.

Proposition 6. *If $1 - F(v)$ is log-concave, the expected adjusted price reaches its minimum when $e = e^*$. There exists a unique equilibrium where all retailers choose the monopoly price and quality.*

Similar to the belief $e_w(p)$ in the price directed search scenario, the belief $p_w(e)$ has the following properties: (1) when $e < e_{max}$, there exists a unique $p_w(e)$ that solves the equation

$$p - e = \frac{1 - F(p - q(e))}{f(p - q(e))}, \tag{7}$$

and (2) when $e \geq e_{max}$, $p_w(e)$ is not unique, and the expected adjusted price is very large such that $v_{max} \leq p_w(e) - q(e)$.

Focus on $e < e_{max}$, since $p_w(e)$ solves equation (7) and $\psi_{PD} = p_w(e) - q(e)$, we know ψ_{PD} solves the following equation:

$$q(e) - e = \frac{1 - F(\psi_{PD})}{f(\psi_{PD})} - \psi_{PD}.$$

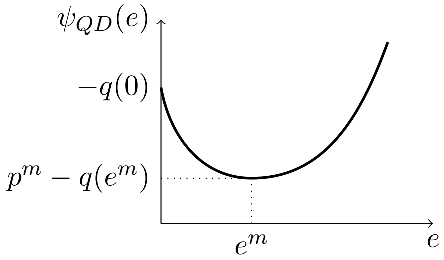


Figure 9: Expected adjusted price ψ_{QD}

Note that $\frac{1-F(\psi)}{f(\psi)} - \psi$ is decreasing (log-concavity of $1-F$), it is clear that the expected adjusted price $\psi_{QD}(e)$ is quasi-convex and reaches its minimum when $e = e^*$, as we can see from Figure 9. Higher effort increases price through two channels: higher cost and higher demand due to higher quality. At $e = e^*$, a slightly higher effort increases both price and quality by one unit; for $e < e^*$, a slightly higher effort increases quality more than price due to concavity of the quality function, and the opposite is true for $e > e*$.

There exists a unique equilibrium where all the retailers choose the monopoly price and effort, because choosing e^* (and correspondingly $p = p_w(e^*) = p^m$) is a strictly dominant strategy. Choosing a different effort weakly decreases the measure of consumers visiting the retailer, and strictly decreases the per-consumer profit. Non-loyal consumers believe all the retailers choose the same adjusted price, thus randomly visit a retailer. Therefore, even if the price comparison website can provide perfectly accurate information on quality, retailers will still behave like a monopoly as long as they can obfuscate.

8 Conclusion

This paper studies the role of information in a setting with competition among online sellers. By comparing the benchmark and price directed search scenarios, we find that although price transparency always leads to higher consumer surplus, the price directed search scenario may either results in higher or lower total welfare than the benchmark scenario. The result depends on the fraction of non-loyal consumers and whether retailers can easily improve quality.

Directions for further research. For the sake of tractability, we assume that consumers value the quality in the same way, which means that quality is vertically differentiated. An alternative to relax this assumption is to assume that consumers have different valuations for the quality. If some consumers are inattentive to quality, it is not clear whether price transparency improves consumer surplus or not.

We also assume that quality is one dimensional which is a big simplification of reality. Fast delivery can be quite important for some consumers, but means nothing to others. Then price can only poorly signal the quality, because a retailer may provide some service that a consumer does not care about.

Finally, we assume that retailers equally share the loyal consumers, and consumers randomly visit a retailer when indifferent. In practice, retailers like Amazon have more loyal consumers than other retailers, and consumers are more likely to visit Amazon if indifferent. In theoretical work, Shelegia and Wilson (2020) study a generalized 'clearinghouse' framework that allows for multiple dimensions of firm heterogeneity.

References

Anderson, Simon P, Maxim Engers, and Daniel Savelle (2020). "Ordered Search: Equilibrium and Optimum", *working paper*.

Arbatskaya, Maria (2007). "Ordered search", *The RAND Journal of Economics* 38.1, pp. 119–126.

Armstrong, Mark (2017). "Ordered consumer search", *Journal of the European Economic Association* 15.5, pp. 989–1024.

Armstrong, Mark and John Vickers (2001). "Competitive price discrimination", *The Rand Journal of economics*, pp. 579–605.

Armstrong, Mark, John Vickers, and Jidong Zhou (2009). "Prominence and consumer search", *The RAND Journal of Economics* 40.2, pp. 209–233.

Armstrong, Mark and Jidong Zhou (2011). "Paying for prominence", *The Economic Journal* 121.556, F368–F395.

Baye, Michael R, Dan Kovenock, and Casper G De Vries (1992). "It takes two to tango: equilibria in a model of sales", *Games and Economic Behavior* 4.4, pp. 493–510.

Baye, Michael R and John Morgan (2001). "Information gatekeepers on the internet and the competitiveness of homogeneous product markets", *American Economic Review* 91.3, pp. 454–474.

Bester, Helmut (1998). “Quality uncertainty mitigates product differentiation”, *The RAND Journal of Economics*, pp. 828–844.

Chen, Yongmin and Tianle Zhang (2019). “Experience Goods and Consumer Search”, *working paper.*

Chioveanu, Ioana (2019). “Prominence, complexity, and pricing”, *International Journal of Industrial Organization* 63, pp. 551–582.

Choi, Michael, Anovia Yifan Dai, and Kyungmin Kim (2018). “Consumer search and price competition”, *Econometrica* 86.4, pp. 1257–1281.

Diamond, Peter A (1971). “A model of price adjustment”, *Journal of economic theory* 3.2, pp. 156–168.

Ding, Yucheng and Tianle Zhang (2018). “Price-directed consumer search”, *International Journal of Industrial Organization* 58, pp. 106–135.

Dubovik, Andrei and Maarten CW Janssen (2012). “Oligopolistic competition in price and quality”, *Games and Economic Behavior* 75.1, pp. 120–138.

Ellingsen, Tore (1997). “Price signals quality: The case of perfectly inelastic demand”, *International Journal of Industrial Organization* 16.1, pp. 43–61.

Fishman, Arthur and Nadav Levy (2015). “Search costs and investment in quality”, *The Journal of Industrial Economics* 63.4, pp. 625–641.

Haan, Marco A, José L Moraga-González, and Vaiva Petrikaitė (2018). “A model of directed consumer search”, *International Journal of Industrial Organization* 61, pp. 223–255.

In, Younghwan and Julian Wright (2018). “Signaling private choices”, *The Review of Economic Studies* 85.1, pp. 558–580.

Janssen, Maarten CW and T Tony Ke (2020). “Searching for service”, *American Economic Journal: Microeconomics* 12.1, pp. 188–219.

Klein, Benjamin and Keith B Leffler (1981). “The role of market forces in assuring contractual performance”, *Journal of political Economy* 89.4, pp. 615–641.

McAfee, R Preston and Marius Schwartz (1994). “Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity”, *The American Economic Review*, pp. 210–230.

Moraga-González, José L and Yajie Sun (2020). “Product Quality and Consumer Search”, *working paper.*

Rey, Patrick and Thibaud Vergé (2004). “Bilateral control with vertical contracts”, *RAND Journal of Economics*, pp. 728–746.

Rhodes, Andrew (2011). “Can prominence matter even in an almost frictionless market?”, *The Economic Journal* 121.556, F297–F308.

Shelegia, Sandro and Chris M Wilson (2020). “A Generalized Model of Advertised Sales”, *American Economic Journal: Microeconomics. forthcoming.*

Stahl, Dale O (1989). “Oligopolistic pricing with sequential consumer search”, *The American Economic Review*, pp. 700–712.

Varian, Hal R (1980). “A model of sales”, *The American Economic Review* 70.4, pp. 651–659.

Wang, Chengsi and Julian Wright (2020). “Search platforms: Showrooming and price parity clauses”, *the RAND Journal of Economics* 51.1, pp. 32–58.

Wilson, Charles (1980). “The nature of equilibrium in markets with adverse selection”, *The Bell Journal of Economics*, pp. 108–130.

Wilson, Chris M (2010). “Ordered search and equilibrium obfuscation”, *International Journal of Industrial Organization* 28.5, pp. 496–506.

Wolinsky, Asher (1983). “Prices as signals of product quality”, *The review of economic studies* 50.4, pp. 647–658.

— (2005). “Procurement via sequential search”, *Journal of Political Economy* 113.4, pp. 785–810.

Zhou, Jidong (2011). “Ordered search in differentiated markets”, *International Journal of Industrial Organization* 29.2, pp. 253–262.

Appendices

Proof of Lemma 1. Rewrite the per-consumer profit as a function of adjusted price and effort:

$$[1 - F(\psi)]\,(\psi + q(e) - e).$$

Per-consumer profit is maximized when $e = e^*$ for any ψ satisfied $1 - F(\psi) > 0$, so $e^m = e^*$ must be true.

Now we show there is a unique ψ^m that maximizes the per-consumer profit given

$e = e^*$, take the first order derivative with respect to ψ and substitute $e = e^*$:

$$f(\psi)\left[\frac{1-F(\psi)}{f(\psi)} - \psi + e^* - q(e^*)\right]$$

Because $1 - F(v)$ is strictly log-concave, $\frac{1-F(\psi)}{f(\psi)} - \psi + e^* - q(e^*)$ is strictly decreasing in ψ. When $\psi = e^* - q(e^*)$, $\frac{1-F(\psi)}{f(\psi)} - \psi + e^* - q(e^*) = \frac{1-F(e^*-q(e^*))}{f(e^*-q(e^*))}$ which is strictly positive because $e^* - q(e^*) < 0 - q(0) < v_{max}$; when $\psi = v_{max}$, $\frac{1-F(\psi)}{f(\psi)} - \psi + e^* - q(e^*) = e^* - q(e^*) - v_{max}$ which is strictly negative. So there exists a unique ψ^m such that $e^* - q(e^*) < \psi^m < v_{max}$ that maximizes the per-consumer profit function, and

$$\psi^m = \frac{1-F(\psi^m)}{f(\psi^m)} + e^* - q(e^*)$$

Substitute $\psi^m = p^m - q(e^m)$, we obtain the expression for p^m. □

Proof of Proposition 1. We first show it is not an equilibrium if there exists at least one retailer who chooses $(p, e) \neq (p^m, e^m)$. Suppose not, and there exists an equilibrium where some retailers choose some $(p, e) \neq (p^m, e^m)$. Let $(p', e') \neq (p^m, e^m)$ denote the pair such that $p' - q(e')$ is the lowest among them. Note that in equilibrium, non-loyal consumers will first visit the retailer with the lowest expected adjusted price, and the beliefs should be correct.

If $p' - q(e') > \psi^m$, the retailer (retailers) who chooses (p', e') has incentive to deviate. The retailer can choose (p^m, e^m) which strictly improves per-consumer profit but will not influence the measure of the consumers that visit the retailer. If $p' - q(e') < \psi^m$, the retailer (retailers) also has profitable deviations. The retailers can increase the adjusted price a little bit which strictly improves the per-consumer profit but will not influence the measure of the consumers that visit the retailer because of the search cost.

Second, we show it is an equilibrium where all the retailers choose the monopoly price and quality. If non-loyal consumers believe all the retailers will choose the same adjusted price ψ^m, they will just randomly choose a retailer to visit because of the assumption on the tie-breaking rule. And they will not search again for the same reason. Retailers also have no incentive to deviate because they cannot attract more consumers to visit them. Non-loyal consumers' beliefs are therefore correct, and retailers share non-loyal consumers equally because of the tie breaking rule mentioned before, so it is an equilibrium. □

Proof of Corollary 1. Take the first order derivative of $\pi(p, e)$ with respect to e:

$$\frac{\partial \pi(p,e)}{\partial e} = f(p - q(e))\left[-\frac{1-F(p-q(e))}{f(p-q(e))} + (p-e)q'(e)\right]$$

To show $\pi(p,e)$ is strictly quasi-concave in e for given p such that $(p,e) \in \Omega$, we only need to show $-\frac{1-F(p-q(e))}{f(p-q(e))} + (p-e)q'(e)$ is strictly decreasing in e for $(p,e) \in \Omega$. Note that $(p,e) \in \Omega$ means that the per-consumer demand is always strictly positive, i.e. $1-F(p-q(e)) > 0$, the first part $-\frac{1-F(p-q(e))}{f(p-q(e))}$ is therefore strictly decreasing in e because $1-F(v)$ is strictly log-concave for $v < v_{max}$ and $q'(e) > 0$. And the second part $(p-e)q'(e)$ is also strictly decreasing in e because $q''(e) < 0$. Therefore, $-\frac{1-F(p-q(e))}{f(p-q(e))} + (p-e)q'(e)$ is strictly decreasing in e, and $\pi(p,e)$ is strictly quasi-concave in e for given p such that $(p,e) \in \Omega$.

□

Proof of Lemma 2. We first show $\tilde{p}$ is unique and $\tilde{p} < p^m$ before we prove the main results. Note that $\tilde{p}$ solves $p = \frac{1-F(p-q(0))}{f(p-q(0))q'(0)}$, the right hand side is decreasing in p because $1-F(v)$ is log-concave, hence, there exists a unique $\tilde{p}$. As we will show later, $\frac{\partial^2 ln\pi(p,e)}{\partial p \partial e} > 0$ is true. Therefore, we must have $p^m > \tilde{p}$ because $e^m > 0$.

(i) We show $\text{argmax}_e\, \pi(p,e)$ is increasing in p, which is equivalent to show $\pi(p,e)$ obeys single crossing difference. Since $f'(v)(1-F(v)) + f^2(v) > 0$ (log-concavity) for any $v \leq v_{max}$. Take the cross partial derivative of $ln\pi(p,e)$, we have:

$$\frac{\partial^2 ln\pi(p,e)}{\partial p \partial e} = \frac{1}{(p-e)^2} + q'(e)\frac{f'(p-q(e))(1-F(p-q(e))) + f^2(p-q(e))}{1-F(p-q(e))} > 0$$

According to Milgrom-Shannon theorem, $\text{argmax}_e\, \pi(p,e)$ is increasing in p.

Note that $\tilde{p}$ satisfies $\frac{\partial \pi(\tilde{p},0)}{\partial e} = 0$, it means that the optimal corresponding effort is zero, in other words $e_w(\tilde{p}) = 0$. Because $\text{argmax}_e\, \pi(p,e)$ is increasing in p, $e_w(p) = 0$ for any $p \leq \tilde{p}$.

(ii) As we have shown before, $\pi(p,e)$ is strictly quasi-concave in e as long as $(p,e) \in \Omega$, which is equivalent to $e \in (\max\{q^{-1}(p-v_{max}), 0\}, p)$. According to condition (3), it suffices to show that the effort e that solves $-\frac{1-F(p-q(e))}{f(p-q(e))} + (p-e)q'(e) = 0$ is positive when $p > \tilde{p}$. As we argued before, $-\frac{1-F(p-q(e))}{f(p-q(e))} + (p-e)q'(e)$ is decreasing in e. When $e = 0$, $-\frac{1-F(p-q(e))}{f(p-q(e))} + (p-e)q'(e) = -\frac{1-F(p-q(0))}{f(p-q(0))} + pq'(0)$ which is strictly positive if $p > \tilde{p}$; when $e = q^{-1}(p-v_{max})$, $-\frac{1-F(p-q(e))}{f(p-q(e))} + (p-e)q'(e) = (p-q^{-1}(p-v_{max}))q'(q^{-1}(p-v_{max}))$ which is also strictly positive; when $e = p$, $-\frac{1-F(p-q(e))}{f(p-q(e))} + (p-e)q'(e) = -\frac{1-F(e-q(e))}{f(e-q(e))}$ which is strictly negative. Hence, there exists a unique e that satisfies condition (3) and gives the highest per-consumer profit.

(iii) now we show $0 \leq e'_w(p) < 1$. Totally differentiate equation (3), and substitute the expression of condition (3), i.e. $p-e=\frac{1-F(p-q(e))}{f(p-q(e))q'(e)}$:

$$\begin{aligned}\frac{\mathrm{d}e}{\mathrm{d}p} &= \frac{q'(e)f'(p-q(e))(p-e)+q'(e)f(p-q(e))+f(p-q(e))}{2q'(e)f(p-q(e))+(q'(e))^2f'(p-q(e))(p-e)-q''(e)f(p-q(e))(p-e)} \\ &= \frac{f'(p-q(e))\frac{1-F(p-q(e))}{f(p-q(e))}+q'(e)f(p-q(e))+f(p-q(e))}{2q'(e)f(p-q(e))+q'(e)f'(p-q(e))\frac{1-F(p-q(e))}{f(p-q(e))}-\frac{q''(e)}{q'(e)}(1-F(p-q(e)))}\end{aligned}$$

The numerator is positive because $f'(p-q(e))\frac{1-F(p-q(e))}{f(p-q(e))}+f(p-q(e))>0$ (log-concavity). We now show the denominator is larger than the numerator, subtracting the numerator from the denominator gives

$$(q'(e)-1)\left[f(p-q(e))+f'(p-q(e))\frac{1-F(p-q(e))}{f(p-q(e))}\right]-\frac{q''(e)}{q'(e)}(1-F(p-q(e)))$$

The effort is smaller than e^m when $p<p^m$, so $q'(e)-1>0$. Since $q''(e)<0$, the whole expression is strictly positive, and we complete our proof.

□

Proof of Lemma 3. It is easy to check that $\psi_{PD}(p)=p-q(0)$ is linearly increasing in p when $p\leq\tilde{p}$.

The sign of $\mathrm{d}\psi_{PD}(p)/\,\mathrm{d}p$ depends on the shape of $q(e)$ when $p>\tilde{p}$. Substitute the expression of $\mathrm{d}e_w(p)/\,\mathrm{d}p$, we have:

$$\frac{\mathrm{d}\psi_{PD}(p)}{\mathrm{d}p}=\frac{q'(e)(q'(e)-1)+q''(e)(p-e)}{-2q'(e)-(q'(e))^2\frac{f'(p-q(e))}{f(p-q(e))}(p-e)+q''(e)(p-e)}$$

As we have shown before, the denominator is negative. While the sign of the numerator depends on the shape of $q(e)$.

(i) we check the right derivative at $\tilde{p}$. The corresponding numerator is $q'(0)(q'(0)-1)+q''(0)\tilde{p}$. It is easy to see $q'(0)(q'(0)-1)$ is increasing in $q'(0)$ because $q'(e)>1$. Note that $\tilde{p}=\frac{1-F(\tilde{p}-q(0))}{f(\tilde{p}-q(0))q'(0)}$, hence, $q''(0)\tilde{p}$ is increasing in $q'(0)$ because $\tilde{p}$ is decreasing in $q'(0)$ and $q''(e)<0$. The numerator is thus increasing in $q'(0)$ at $p=\tilde{p}$. It equals to $+\infty$ when $q'(0)=+\infty$; it equals to $q''(0)f(\tilde{p}-q(0))\tilde{p}<0$ when $q'(0)\to 1$. So there exists a unique $\tilde{q}\in(1,+\infty)$ such that the numerator is positive (respectively negaitve) when $q'(0)<\tilde{q}$ (respectively $q'(0)>\tilde{q}$).

(ii) we check the derivative at p^m. Since $e_w(p^m)=e^m$ and $q'(e^m)=1$, the numerator is negative at p^m.

(iii) we show the numerator is decreasing in p. From lemma 2, $0\leq e'_w(p)<1$, so $p-e_w(p)$ is increasing in p. Note $q'(e)(q'(e)-1)$ is increasing in $q'(e)$, which means it is decreasing in $e_w(p)$ and p. So the numerator is decreasing in p if $q''(e)$ is weakly decreasing or increasing not so fast. And we complete the proof. □

Proof of Lemma 4. When $q'(0) \leq \tilde{q}$, we show there exists a unique symmetric equilibrium where retailers choose the strategy $G_{PD}(p)$ and non-loyal consumers form beliefs $e_w(p)$.

To show it is an equilibrium. First, as we argued before, retailers' best response is to choose $e = e_w(p)$ for any given price. Second, according to the construction, each retailer's expected profit is the same for $p \in [\underline{p}, p^m]$ given all the other retailers do not deviate. Choosing a price smaller than $\underline{p}$ or higher than p^m has no influence on the measure of consumers visiting the retailer but can strictly lower the per-consumer profit. Finally, non-loyal consumers therefore have correct beliefs about the quality, and will just visit the retailer with the lowest expected adjusted price.

To show the uniqueness. First, there is no pure strategy equilibrium because of the existence of non-loyal consumers. Second, the upper bound of the equilibrium support must be p^m. If the upper bound $\overline{p}$ is strictly higher than p^m, choosing $p = p^m$ instead of $\overline{p}$ can strictly increase per-consumer profit and weakly increase the measure of consumers visiting the retailer. If the upper bound $\overline{p}$ is strictly lower than p^m, choosing $p = p^m$ instead of $\overline{p}$ has no effect on the measure of visiting consumers but can strictly increase the per-consumer profit. Third, there must involve no mass point. Otherwise, choosing a price slightly below the mass point is a profitable deviation. And choosing mass point at $\underline{p}$ gives strictly lower expected profit than choosing p^m. Thus, we finish the proof. □

Proposition (Details of Proposition 2)**.** *If retailers can easily improve quality such that $q'(0) > \tilde{q}$, the game has a unique symmetric equilibrium and the equilibrium outcome depends on the fraction of non-loyal consumers λ.*

(i) When $\lambda \leq \lambda_\tau$, there exists a unique symmetric mixed strategy equilibrium like the one in lemma 4.

(ii) When $\lambda_\tau < \lambda < \min\{\lambda_{\tau'}, \lambda_{\tau p}\}$, there exists a unique symmetric mixed strategy equilibrium with noncontinuous support $\{\tau\} \bigcup [\underline{p}', p^m]$. The equilibrium strategy $G_{PD}(p)$ involves a mass point α_1 at τ, $G_{PD}(p)$, α_1 and $\underline{p}'$ are determined by equation (4) and

$$\left[\frac{1-\lambda}{n} + \sum_{i=0}^{n-1} \binom{n-1}{i} (1-\alpha_1)^i \alpha_1^{n-i-1} \frac{\lambda}{n-i}\right] \pi(\tau, e_w(\tau)) = \frac{1-\lambda}{n}\pi^m,$$

$$\alpha_1 = G_{PD}(\underline{p}').$$

(iii) When $\lambda_{\tau p} < \lambda < \lambda_{\tau\tau'}$, there exists a unique pure strategy equilibrium where retailers choose $p = \tau$.

(iv) When $\lambda > \max\{\lambda_{\tau'}, \lambda_{\tau\tau'}\}$, there exists a unique symmetric mixed strategy equilibrium with noncontinuous support $\left[\underline{p}, \tau'\right) \bigcup \{\tau\} \bigcup \left[\underline{p}'', p^m\right]$ if the following system of equations of α_2 and $\underline{p}''$ have a solution.

$$\left[\frac{1-\lambda}{n} + \sum_{i=0}^{n-1} \binom{n-1}{i} (1 - G_{PD}(\tau') - \alpha_2)^i \alpha_2^{n-i-1} \frac{\lambda}{n-i}\right] \pi(\tau, e_w(\tau)) = \frac{1-\lambda}{n}\pi^m,$$

$$\alpha_2 = G_{PD}(\underline{p}'') - G_{PD}(\tau').$$

Where $G_{PD}(p)$ is the equilibrium strategy that involves a mass point α_2 at τ, and $G_{PD}(p)$ is determined by equation (4).

Otherwise, there exists a unique symmetric mixed strategy equilibrium with noncontinuous support $\left[\underline{p}, \tau'\right) \bigcup \{\tau\}$. The equilibrium strategy $G_{PD}(p)$ involves a mass point α_2 at τ, $G_{PD}(p)$ and α_2 are determined by:

$$\left[\frac{1-\lambda}{n} + (1 - G_{PD}(p))^{n-1}\lambda\right] \pi(p, 0) = \left[\frac{1-\lambda}{n} + \frac{\alpha_2^{n-1}\lambda}{n}\right] \pi(\tau, e_w(\tau)),$$

$$\alpha_2 = 1 - G_{PD}(\tau').$$

Proof of Proposition 2. We first show there are only two possible relationships among λ_τ, $\lambda_{\tau'}$, $\lambda_{\tau p}$, and $\lambda_{\tau\tau'}$. Note that the four parameters solve the following conditions respectively

$$\frac{1-\lambda}{n}\pi^m = \frac{1+(n-1)\lambda}{n}\pi(\tau, e_w(\tau)), \tag{8}$$

$$\frac{1-\lambda}{n}\pi^m = \frac{1+(n-1)\lambda}{n}\pi(\tau', 0), \tag{9}$$

$$\frac{1}{n}\pi(\tau, e_w(\tau)) = \frac{1-\lambda}{n}\pi^m, \tag{10}$$

$$\frac{1}{n}\pi(\tau, e_w(\tau)) = \frac{1+(n-1)\lambda}{n}\pi(\tau', 0). \tag{11}$$

It can easily check that λ_τ is lowest among the four, we only need to compare $\lambda_{\tau'}$, $\lambda_{\tau p}$, and $\lambda_{\tau\tau'}$.

When $\tau' \leq 0$, we have $\lambda_{\tau'} = 1$ and $\lambda_{\tau\tau'} = 1$ according to our assumption. Therefore, we have $\lambda_{\tau p} < \lambda_{\tau'} = \lambda_{\tau\tau'}$ because $0 < \lambda_{\tau p} < 1$ is true according to condition (10). When $\tau' > 0$, there are two possible situations. If $\lambda_{\tau'} < \lambda_{\tau p}$, then according to condition (9), (10), and (11), we have

$$\frac{1+(n-1)\lambda_{\tau'}}{n}\pi(\tau', 0) > \frac{1}{n}\pi(\tau, e_w(\tau)) = \frac{1+(n-1)\lambda_{\tau\tau'}}{n}\pi(\tau', 0),$$

which means that $\lambda_{\tau\tau'} < \lambda_{\tau'}$. If $\lambda_{\tau'} > \lambda_{\tau p}$, following similar arguments, we must have $\lambda_{\tau\tau'} > \lambda_{\tau'} > \lambda_{\tau p}$.

In conclusion, there are only two possible relationships among λ_{τ}, $\lambda_{\tau'}$, $\lambda_{\tau p}$, and $\lambda_{\tau\tau'}$:

$$\lambda_{\tau\tau'} \geq \lambda_{\tau'} > \lambda_{\tau p} > \lambda_{\tau},$$

$$\lambda_{\tau p} > \lambda_{\tau'} > \lambda_{\tau\tau'} > \lambda_{\tau}.$$

We then show there exists a unique symmetric equilibrium where retailers' pricing strategy is $G_{PD}(p)$ they choose their effort level according to $e = e_w(p)$, and non-loyal consumers have correct beliefs.

(i) When $\lambda \leq \lambda_{\tau}$, it is equivalent to $\tau \leq \underline{p}$, the proof is the same as the that of Lemma 4 because adjusted price is monotone in price for $p \in [\underline{p}, p^m]$.

(ii) When $\lambda_{\tau} < \lambda < \min\{\lambda_{\tau'}, \lambda_{\tau p}\}$, if all the other retailers choose price distribution $G_{PD}(p)$, any retailer's expected profit is the same for $p \in \{\tau\} \bigcup [\underline{p}', p^m]$ according to the construction. As we mentioned before, retailers never choose a price in $[\tau', \tau)$. They will never choose a price smaller than τ' to attract all the non-loyal consumers because $\lambda < \lambda_{\tau'}$, they will not choose τ with probability one because $\lambda < \lambda_{\tau p}$. Choosing a price in $(\tau, \underline{p}')$ is also not profitable, if the retailer chooses $\underline{p}'$ instead, it strictly increases the per-consumer profit, and will not lose any consumer.

(iii) When $\lambda_{\tau p} < \lambda < \lambda_{\tau\tau'}$, the unique symmetric equilibrium is a pure strategy equilibrium where every retailer chooses $p = \tau$, and the profit of each retailer is $\frac{1}{n}\pi(\tau, e_w(\tau))$. The retailers do not have incentive to choose a price higher than τ, because the highest profit of this deviation is $\frac{1-\lambda}{n}\pi^m$, which is small than $\frac{1}{n}\pi(\tau, e_w(\tau))$ because $\lambda \geq \lambda_{\tau p}$. The retailers also have no incentive to choose a price slight lower than τ' because $\lambda < \lambda_{\tau\tau'}$.

(iv) When $\lambda > \max\{\lambda_{\tau'}, \lambda_{\tau\tau'}\}$, the situation is similar to type (ii) equilibrium. Retailers are indifferent to any price in the equilibrium price support according to the construction. If retailers do not have incentives to choose prices around p^m, we have the second kind of equilibrium, otherwise, we have the first kind of equilibrium. □

Proof of Proposition 3. We first show $\frac{\mathrm{d}W(p, e_w(p))}{\mathrm{d}p}$ is negative when $p = p^m$. We use q and q' to denote $q(e_w(p))$ and $q'(e_w(p))$ for convenience, take the derivative of $W(p, e_w(p))$

with respect to p, and substitute the equation (3)

$$\begin{aligned}\frac{\mathrm{d}W(p,e_w(p))}{\mathrm{d}p} =& [1-F(p-q)]\,(q'-1)e_w'(p) - f(p-q)(p-e_w(p))(1-q'e_w'(p)) \\ =& f(p-q)(p-e)(q'^2e_w'(p)-1)\end{aligned}$$

At $p=p^m$, because $e_w(p^m)=e^*$ and $q'(e^*)=1$, and $e_w'(p)<1$ as we have shown before, simple calculation reveals that $\frac{\mathrm{d}W(p,e_w(p))}{\mathrm{d}p}|_{p=p^m}<0$.

Now we prove the main part of the proposition. Because total welfare of both benchmark and price directed search situations are the same when $\lambda=0$. To prove $TW_{PD}>TW_m$ when λ is sufficiently small, we only need to show TW_{PD} is increasing in λ when λ is close to zero. Then we use an example to show TW_{PD} can be lower than TW_m when λ is relatively large.

I. Note that according to proposition 2, when λ is close to zero, the equilibrium support is continuous, and the equilibrium strategy $G_{PD}(p)$ and lower bound of equilibrium support are also determined by λ:

$$\begin{aligned}(1-G_{PD}(p_i))^{n-1} &= \frac{1-\lambda}{n\lambda}\left(\frac{\pi^m}{\pi(p_i,e_w(e_i))}-1\right)\\ \frac{1+(n-1)\lambda}{n}\pi(\underline{p},e_w(\underline{p})) &= \frac{1-\lambda}{n}\pi^m\end{aligned}$$

We use $G_{PD}(p,\lambda)$ and $\underline{p}(\lambda)$ to denote them for convenience. Take the derivative of TW_{PD} with respect to λ at $\lambda=0$:

$$\begin{aligned}\frac{\mathrm{d}TW_{PD}}{\mathrm{d}\lambda}|_{\lambda=0} =& \frac{\mathrm{d}\int_{\underline{p}(\lambda)}^{p^m} W(p,e_w(p))\,\mathrm{d}G_{PD}(p,\lambda)}{\mathrm{d}\lambda}|_{\lambda=0}\\ =& \lim_{\Delta\to 0}\frac{\int_{\underline{p}(\Delta)}^{p^m} W(p,e_w(p))\,\mathrm{d}G_{PD}(p,\Delta)-W(p^m,e^m)}{\Delta}.\end{aligned}$$

It is strictly positive since $W(p,e_w(p))>W(p^m,e^m)$ for p very close to p^m.

II. Now we use an example to show TW_{PD} can be lower than TW_m when λ is relatively large. Let the valuation v uniform distributed on $[0,1]$, $q(e)=-0.85(e-1)^2$ and $n=2$. Then we have $\tau\approx 0.432$, and $\psi_{PD}(\tau)\approx 0.846$ which is lower than $\psi_{PD}(0)=0.85$, which means that there is a pure strategy equilibrium when λ is large enough. We find $TW_{PD}\approx 0.0319$ which is lower than $TW_m\approx 0.0324$. □

Proof of Proposition 4. We view retailers as choosing the per-consumer profit π rather than the price p and effort e.

(i) If the equilibrium without the minimum price is type (i) or type (ii). Let p_{min} solves $\pi(p_{min}, e_w(p_{min})) = (1-\lambda)\pi^m$, there exists a unique equilibrium where all the retailers choose $\pi(p_{min}, e_w(p_{min}))$ and share non-loyal consumers equally. Choosing a higher per-consumer profit is not profitable due to definition of p_{min}, while choosing a lower per-consumer profit is not allowed. There exists no other pure strategy equilibrium, otherwise at least one retailer has incentive to deviate to $\pi(p_{min}, e_w(p_{min}))$. There exists no other mixed strategy equilibrium for the same reason, choosing $\pi(p_{min}, e_w(p_{min}))$ results in a strictly higher expected profit if retailers sometimes choose higher per-consumer profit.

The new equilibrium results in the same industry profit and a strictly higher consumer surplus. The distribution of profit without the minimum price is a mean-preserving spread of that with the minimum price. We argue that $p^*_{min} = \text{argmax}_p W(p, e_w(p))$ for $p \in [p_{min}, p^m]$. The total welfare is higher than that in the benchmark scenario since $W(p_{min}, e_w(p_{min})) > W(p^m, e^m)$. The industry profit is lower than that in the benchmark scenario, so consumer surplus is also higher.

(ii) If the equilibrium without the minimum price is type (iii). We argue that $p^*_{min} = \text{argmax}_p W(p, e_w(p))$ for $p \in [\tau, p^m]$. Since $\frac{\mathrm{d}W(p,e_w(p))}{\mathrm{d}p}|_{p=\tau} > 0$, we know the minimum price strictly improves the total welfare. It is clear that consumer surplus is reduced by the minimum price.

(iii) If the equilibrium without the minimum price is type (iv), the argument is same as type (i). □

Proof of proposition 5. We will show that retailers always choose the efficient effort, and the equilibrium pricing strategy $G_{FI}(p)$ and support $\left[\underline{p}_{FI}, p^m\right]$ are decided by indifference conditions:

$$\left[\frac{1-\lambda}{n} + (1-G_{FI}(p))^{n-1}\lambda\right]\pi(p, e^m) = \frac{1-\lambda}{n}\pi^m \tag{12}$$

$$\frac{1+(n-1)\lambda}{n}\pi(\underline{p}_{FI}, e^m) = \frac{1-\lambda}{n}\pi^m \tag{13}$$

We first show for any given adjusted price ψ_i, there exists unique price and effort that maximize the per-consumer profit. The maximization problem is:

$$\max_{p,e} \pi(p, e)$$

$$s.t.\ p - q \leq \psi_i$$

Let μ denote the Lagrangian multiplier, the Lagrangian is:

$$\mathcal{L} = [1 - F(p - q(e))]\,(p - e) + \mu(\psi_i - p - c(e))$$

The solution is given by first order conditions:

$$\begin{aligned} 1 - F - f \cdot (p - e) - \mu &= 0 \\ -fc' \cdot (p - e) - (1 - F) - \mu c' &= 0 \\ \psi_i - p - c(e) &= 0 \end{aligned} \implies \begin{aligned} e &= e^* \\ p &= \psi_i - c(e^*) \end{aligned}$$

Therefore, retailers always choose the efficient effort e^*, and non-loyal consumers will just visit the one with the lowest price. The rest of the proof is the same as the proof of lemma 4.

Because retailers always choose $e = e^*$ in equilibrium, consumer surplus is concave in per-consumer profit π according to Armstrong and Vickers (2001). Therefore, following the same argument mentioned before, a p_{min} slightly higher than the lower bound of the equilibrium price support strictly improves consumer surplus and total welfare.

Let p^{**}_{min} solve $\pi(p^{**}_{min}, e^m) = (1 - \lambda)\pi^m$, p^{**}_{min} is uniquely defined because $\pi(p, e^m)$ is increasing for $p \in [0, p^m]$. When $p_{min} = p^{**}_{min}$, all the retailers choose $p = p_{min}$ in equilibrium, and the industry profit remains to be $(1 - \lambda)\pi^m$. For any $p_{min} < p^{**}_{min}$, the equilibrium industry profit remains to be $(1 - \lambda)\pi^m$, but consumer surplus is strictly lower because of the mean-preserving spread argument. For any $p_{min} > p^{**}_{min}$, retailers choose pure strategy p_{min} and e^m in equilibrium, the total welfare is lower as we can see from the graph of $W(p, e)$. Hence, there exists a unique p^{**}_{min} such that $p^{**}_{min} \in \text{argmax}_{p_{min}} TW_{FI}(p_{min})$.

It is easy to check that $TW_{FI}(p^{**}_{min}) > TW_{PD}(p^{*}_{min})$. Because the industry profit is weakly lower in the full information scenario, the corresponding consumer surplus is strictly higher. □

Proof of Proposition 6. We first show the properties of the belief $p_w(e)$, then analyze the properties of the expected adjusted price ψ_{QD}, finally we prove there is a unique equilibrium.

I. We first show that (1) when $e < e_{max}$, there exists a unique $p_w(e)$ that solves the equation $p - e = \frac{1-F(p-q(e))}{f(p-q(e))}$, and (2) when $e \geq e_{max}$, $p_w(e)$ is not unique, and the expected surplus $v - p_w(e) + q(e)$ is negative.

When $e < e_{max}$, as we showed previously, $\pi(p, e)$ is log-concave in p for any given e, so there exists a unique price p maximizes the per-consumer profit. Take the first order derivative of per-consumer profit with respect to price:

$$(1 - F)\left[1 - \frac{f}{1 - F}(p - e)\right]$$

Since $1 - F(v)$ is log-concave, $1 - \frac{f}{1-F}(p - e)$ is decreasing for $p \leq v_{max} - c(e)$, it is strictly positive when $p = e$ because $p - q < v_{max}$ and $p - e = 0$; it is negative when $p = v_{max} - c(e)$. Thus, there exists a unique price that maximizes the per-consumer profit, and $p_w(e_i)$ solves the first order condition:

$$p - e = \frac{1 - F(p - q(e))}{f(p - q(e))}$$

When $e \geq e_{max}$, we find that the expected net surplus is negative. If a retailer chooses a given effort $e \geq e_{max}$, he will never choose a price strictly smaller than e because per-consumer profit is strictly negative. If the retailer chooses a price $p \geq e$, then no one will purchase the product because $p - q(e) > v_{max}$. Therefore, the retailer must choose a price $p \geq e$, and consumers have a negative net surplus if they purchase the product.

II. Focus on $e < e_{max}$, since $p_w(e)$ solves equation (2) and $\psi_{PD} = p_w(e) - q(e)$, ψ_{PD} solves the following equation

$$q(e) - e = \frac{1 - F(\psi)}{f(\psi)} - \psi.$$

Note that $\frac{1-F(\psi)}{f(\psi)} - \psi$ is decreasing in ψ (log-concavity of $1 - F(v)$), it is easy to see expected adjust price is quasi-concave and reaches its minimum when $e = e^*$.

III. Note that choosing e^* (correspondingly $p = p_w(e^*) = p^m$) is a strictly dominate strategy. Choosing a different effort weakly decreases the measure of consumers visiting the retailer, and strictly decreases the per-consumer profit. Non-loyal consumers believe all the retailers choose the same adjusted price, thus randomly visit a retailer. Therefore, the equilibrium outcome is the same as in the benchmark case. □

Chapter 3: Search or Buy Directly

Abstract

This article analyses a monopoly firm's optimal information revelation strategy and return policy when consumers can choose to either buy immediately without knowing their exact match value or search to learn the exact match value. By paying a return cost which is chosen by the firm, each consumer can return the product and obtain a refund. We find that if the firm is able to give consumers any form of match information, the firm will simply inform each consumer whether or not their match value is above a threshold. This strategy is used as a search deterrence tool, and consumers will just buy directly without knowing the exact match value. The optimal return policy is the one that makes no consumer choose to return the product. The consumer surplus is decreasing in search cost, whereas the total welfare is increasing in search cost. The total welfare reaches the socially efficient level when search cost is large enough, though the consumer surplus is zero in this situation.

1 Introduction

Consumers now have more options when they want to purchase a product but do not know their match value. Besides visiting a brick-and-mortar store to learn the price and match value, each consumer can also choose to shop online. They can first visit the website to learn the price, return policy, and some basic information about the product which enables them to update their beliefs about the match value. Then they can choose to buy directly online or visit the brick-and-mortar store to learn their exact match value. If consumers choose to buy directly, by paying return costs, they may return the products and obtain refunds once they find out that they do not like the product.

Several authors have studied firms' optimal product information disclosure strategy (e.g., Anderson and Renault, 2006, Wang, 2017). To the best of our knowledge, however, no previous work has discussed that in a situation when consumers can return the product if they do not like it. This article fills the gap by studying the optimal information revelation strategy and return policy of a monopoly firm. We find that the profit maximization strategy always involves no consumer searching, and the firm frequently wants to restrict the information it provides to consumers, which leads to the fact that some consumers purchase the product when their match value is lower than the price. Search cost plays an important role even no consumer chooses to search in equilibrium, intriguingly, the total welfare is increasing in consumer search cost.

We consider a monopoly firm that sells a product to one unit of consumers, their match value is ex ante unknown to both parties. The firm first chooses the price p, return cost r, and how much information to reveal on its website. Consumers visit the website to learn p, r, and some basic information that enables them to update their beliefs about their match value. Then, they choose to buy immediately or incur a search cost to learn the exact match value (visit the brick-and-mortar store). We assume the firm can give consumers any form of match information.[1]

We find that the firm's optimal information revelation strategy is just to inform each

[1]See Anderson and Renault (2006) for more details about how the firm may convey any information by means of announcing characteristics of the product.

consumer whether or not the match value exceeds some critical threshold level, the price and return policy are chosen accordingly to make sure consumers will not search, quit the game, or return the product. For any given price p, when consumers choose to search, they will buy the product if and only if their match value is higher than the price. However, the firm can choose to inform consumers whether or not their match value is above $p - \epsilon$, and choose a very large return cost. When ϵ is sufficiently small, consumers will choose to buy directly to avoid the search cost, even if with some probability the match value can be lower than the price. Because the firm only needs to inform consumers whether or not their match value is above some level, the threshold strategy is good enough to convey all the necessary information.

Although consumers do not search in equilibrium, the size of the search cost plays an important role. The firm's profit and total welfare are increasing in search cost, whereas consumer surplus is decreasing in that. When search cost is very large, consumers will never choose to search. Therefore, the firm can inform consumers whether or not their match value is above the marginal cost of production, choose a very large return cost, and charge a price that is equal to the expected match value (conditional on the value is above the marginal cost). By doing this, the allocation is efficient because any consumer who has a match value above the marginal cost will purchase the product, the firm can obtain all the surplus, but consumer surplus is zero.

When search cost is very small, consumers are always willing to search if either the price or return cost is too high. To make sure consumers do not search, the firm needs to choose a threshold that is closer to the price when search cost is decreasing, and thus fewer people purchase the product. Hence, the firm's profit and total welfare are also decreasing, but consumer surplus is increasing because less likely consumers purchase the product when their match value is below the price. When search cost is not too large or too small, the optimal strategy of the firm makes consumers indifferent among buying directly, searching, and quitting the game, which means that consumer surplus is zero. The profit and total welfare are increasing in search cost because the threshold is higher and more consumers will purchase the product.

1.1 Literature Review

The most closely related work is the article by Anderson and Renault (2006). They discuss the optimal product information revelation strategy of a monopoly firm in a search model which builds on Wolinsky (1986) and Anderson and Renault (1999). In their model, consumers need to incur a search cost to visit the firm in order to make purchases. If consumers are perfectly informed about the match value, the search cost causes a hold-up problem as in Diamond (1971). Partial information disclosure solves this problem because consumers do not know the exact match value and will visit as long as the expected match value is high enough.

Our model differs from Anderson and Renault (2006) by allowing consumers to buy immediately without knowing the exact match value, which is very common in online shopping. The hold-up problem disappears even when consumers are fully informed about the match value because consumers can observe the price in our model. However, even if they cannot observe the price, the fact that consumers can buy the product without incurring the search cost leads to no hold-up problem. The firm always finds it optimal to partially inform consumers,[2] different from Anderson and Renault (2006), this is because the firm wants to invite more consumers to buy directly such that some of them purchase the product even if their match value is lower than the price.

Another related work is the article by Wang (2017), he considers the product information revelation problem in a search model where consumers can buy directly without searching, but consumers cannot return the product in his model. Similarly, he finds that the firm will choose partial information disclosure to deter search. However, under his setting, partial disclosure is used only when search costs are relatively small. Bar-Isaac, Caruana, and Cuñat (2010) discuss a similar problem by assuming the firm can change search costs, and consumers can choose to buy without delay or search. They find that it can be optimal to choose an intermediate level of opaqueness (i.e., medium-level search cost), which induces high-valuation consumers to buy without delay and low-valuation consumers to search.

Our work also relates to the articles about return policy. Petrikaitė (2018) studies a

[2] This is true unless the marginal cost of production is zero where the firm chooses to reveal no information.

duopoly model where consumers buy horizontally differentiated products directly without knowing exact the match value, consumers can then return any item and obtain refunds by paying return costs, but consumers cannot search in her model. She finds that price competition is more intense compared with a standard search model, and the symmetric equilibrium price is lower when returns are higher. Hinnosaar and Kawai (2020) analyze a bilateral trade model, and there is uncertainty about the prior information that a potential buyer might have about the product. They find that the firm always benefits from offering a refund policy, and it is optimal for the firm to choose a simple mechanism that combines a generous refund policy with random non-refundable discounts. Lubensky and Schmidbauer (2020) studies a model where a pre-purchase trial allows a consumer to learn both a product's quality and how well it matches her idiosyncratic taste. They find that a free return policy benefits consumers but decreases welfare when there are sufficient gains from trade.

2 Model Setup

A monopoly firm sells a single good, which it produces at a marginal cost $c \geq 0$. There is a unit mass of risk-neutral consumers, each of them (she) buys at most one unit of the good. Each consumer's match value v is an i.i.d. draw from the distribution function $F(v)$ on the support $[0, 1]$. The associated density function $f(v)$ is strictly positive, twice continuously differentiable, and log-concave. The match value v is initially unknown to the firm and to the consumer.

The consumer first visits the website of the firm to learn the price and some information of the good without any cost. Then she updates her beliefs about the match value, and can (i) search to learn her exact match value which incurs a search cost $s > 0$; or (ii) buy directly without knowing the exact match value; or (iii) choose the zero outside option. If the consumer chooses to buy directly, she learns the match value by using it, and then can either keep or return the good.

If the consumer learns the exact match value by searching, she will buy the good if and only if $v \geq p$ where p is the price chosen by the firm. If she buys directly, she will keep the good if and only if $v - p \geq -r$ where r is the return cost she needs to pay the firm. The return cost r is also chosen by the firm.[3] If the consumer is indifferent among searching, buying directly, and the outside option, we assume she buys directly; if she is indifferent between returning and keeping the good, we assume she keeps it.

Except for the marginal cost c, the management cost of each return for the firm is $c_r < 1$, and we assume all the other costs are zero. The firm chooses a price p, a return cost r, and an information revelation strategy. To make the analysis as general as possible, following Anderson and Renault (2006), we assume the firm reveals the information of the good by choosing an information transmission mechanism (ITM). An ITM induces a probability measure over the joint space of match values and signals which are sent by the firm.[4] The consumer can update her belief about the match value in a Bayesian manner. Hence, an ITM is a probability space $([0,1] \times I, \Sigma([0,1]) \times H, P)$ where $\Sigma([0,1])$ denotes the σ-field of Borel sets in $[0,1]$, I is a set of signals, H is a σ-field of subsets of I, and P is a probability measure over $[0,1] \times I$ that satisfies $P(v \leq x) = F(x)$ for all $x \in [0,1]$.

The timing is as follows: At stage one, the firm chooses the price p, return cost r, and information revelation strategy (ITM). At stage two, each consumer observes p, r, and a signal, then decides to buy directly, search, or choose the outside option. At stage three, the consumer chooses whether to buy the good or not if she searched at stage two; the consumer either keeps or returns the good if she bought directly at stage two.

2.1 Preliminary Analysis

We first briefly discuss the situation the consumers can only search (or quit). Note that if the consumer learns her match value v by searching, she purchases the good if and only if $v \geq p$. Therefore, if the strategy of the firm makes no consumer buy directly, the firm's profit is at most the **monopoly profit**

$$\pi^m = (p^m - c)\left[1 - F(p^m)\right]$$

[3]Here we assume the physical cost of the consumer to return the good is zero.

[4]It does not matter whether the firm knows the consumer's actual match value or not, see Anderson and Renault (2006) for more details.

where $p^m = \text{argmax}_p (p - c)\left[1 - F(p)\right]$.[5] We know p^m and π^m are well defined because $f(v)$ is log-concave.

We find that the firm can obtain a strictly higher profit than π^m if the firm simply chooses a threshold information revelation strategy and a very small return cost r. The threshold strategy involves a signal set $I = \{\tilde{v}^+, \tilde{v}^-\}$, and the consumer receives the signal $\tilde{v}^+$ if and only if her match value is above the threshold $\tilde{v}$.

At the second period, when the consumer receives the signal $\tilde{v}^+$, she chooses to buy directly if and only if

$$\begin{aligned} &E\left[\max\{v-p,-r\}\,|\tilde{v}^+\right] \geq E\left[\max\{v-p,0\}\,|\tilde{v}^+\right] - s, \\ &E\left[\max\{v-p,-r\}\,|\tilde{v}^+\right] \geq 0. \end{aligned}$$

Because $E\left[\max\{v-p,-r\}\,|\tilde{v}^+\right]$ is weakly decreasing in r, and it is positive when $r = 0$ for any $\tilde{v}$ and $p < 1$, these two constraints hold when r is very small. Then let $\tilde{v} = p - r$, the consumer will buy directly when she receives the signal $\tilde{v}^+$ for r sufficiently close to 0, and she will not return the product because she returns the product only if $v < p - r$.

Hence, suppose the firm chooses a given very small return cost r and the threshold strategy $\tilde{v} = p - r$. If the firm chooses $p = p^m$, it can obtain a profit that is strictly higher than π^m because the demand $1 - F(p^m - r)$ is higher than before $1 - F(p^m)$. The firm can choose a slightly higher price and obtain even more profit.[6] Intuitively, consumers are willing to (1) pay more and (2) sometimes buy the product when $v < p$, in order to avoid the search cost.

We finally argue that the firm can obtain the highest possible profit $\int_c^1 (v-c)dF(v)$ when the search cost is large enough. Note that the maximum total welfare is $\int_c^1 (v-c)dF(v)$ which is achieved when any consumer who has a match value higher than c purchases the good, which means that it is also the highest possible profit conditional on positive consumer surplus.

Consumer will never search because the search cost is very large, the firm only needs to make sure ex ante consumer surplus is positive if they choose to buy directly. The firm

[5] Consumers would prefer the outside option if the search cost is large enough.

[6] The optimal price is increasing in r because it solves $p - c = \frac{1-F(p-r)}{f(p-r)}$ and $f(v)$ is log-concave.

thus can choose $\tilde{v} = c$, $p = E[v|v \geq c]$, and any $r \geq p - c$. Then any consumer who has a match value above c buys directly because they pay their expected match value, and ex ante consumer surplus is zero. The correspond profit is thus

$$\begin{aligned} & [1 - F(c)] \{E[v|v \geq c] - c\} \\ = & [1 - F(c)] \left\{ \frac{\int_c^1 v dF(v)}{1 - F(c)} - c \right\} \\ = & \int_c^1 (v - c) dF(v), \end{aligned}$$

which is exactly the highest total welfare. This outcome is socially efficient like the one of Bertrand competition, but the firm obtains all the surplus.

3 Equilibrium Analysis

We first argue that for any ITM, the firm's optimal strategy leads to no consumer searching, which means that it is never optimal for the firm to make some consumers search and others buy directly.

Lemma 1. *The firm's profit is maximized when no consumer chooses to search.*

The arguments are similar. Suppose for any price p, return cost r, and ITM chosen by the firm such that there exist some consumers who choose to search at stage three. The firm can do better by informing these consumers whether or not their match value is above $p - \epsilon$, where ϵ is a very small positive number. To avoid incurring the search costs, these consumers will buy directly instead, and the profit becomes higher.

The following lemma shows that the firm can simply focus on a threshold information revelation strategy that conveys a message that informs the consumer whether or not her match value exceeds the critical threshold.

Lemma 2. *For any price $p \in (c, 1)$ and any positive return cost r, the firm cannot do better than informing the consumer whether or not her match value is above some critical threshold $\tilde{v} \in [p - r, p]$.*

Given some price p and return cost r, the threshold $\tilde{v}$ is good enough to get consumers with high match value to buy directly. This result is the same as the findings of Anderson and Renault (2006). The firm never chooses a $\tilde{v}$ greater than p because a slightly lower $\tilde{v}$ weakly increases the profit, it also does not choose a $\tilde{v}$ lower than $p-r$ because the threshold leads to some returns, and a slightly higher threshold can increase consumers' expectation about consumer surplus.

Hence, the firm's profit maximization problem can be written as

$$\max_{p,\tilde{v}}(p-c)\left[1-F(\tilde{v})\right]$$

$$s.t. \int_{\tilde{v}}^{p}(p-v)dF(v) \leq \int_{p}^{1}(v-p)dF(v) \qquad \text{(IR)}$$

$$\int_{\tilde{v}}^{p}(p-v)dF(v) \leq s\left[1-F(\tilde{v})\right] \qquad \text{(IC)}$$

The firm chooses a price p and threshold $\tilde{v}$ to maximize the profit, subject to two constraints that those consumers who have the match value above $\tilde{v}$ will not search (IC) or quit the game (IR), and the return cost can be any number higher than $p-\tilde{v}$.

To understand the two constraints, if a consumer chooses to buy directly, with some probability her match value is lower than the price, and the corresponding negative utility (in absolute value) is $\int_{\tilde{v}}^{p}\frac{p-v}{1-F(\tilde{v})}dF(v)$. To make sure the consumer does not want to search or quit, this negative utility must be smaller than the search cost, and it must also be smaller than the corresponding positive utility $\int_{p}^{1}\frac{v-p}{1-F(\tilde{v})}dF(v)$ when her match value is higher than the price.

Let $s_2=\int_{c}^{E[v|v\geq c]}\frac{E[v|v\geq c]-v}{1-F(c)}dF(v)$, and we assume[7]

$$\left(E[v|v\geq c]-c\right)\left[1-F\left(E[v|v\geq c]\right)\right] \geq \int_{c}^{E[v|v\geq c]}\left[E[v|v\geq c]-v\right]dF(v). \qquad \text{(A1)}$$

The following proposition summarizes the properties of the firm's optimal strategy.

Proposition 1. *If $f(v)$ is log-concave, and assumption A1 is true, there exists a positive number $s_1 < s_2$ such that*

(i) when $s < s_1$, IC binds, the price and profit are increasing in search cost while $\tilde{v}$ is decreasing in search cost;

[7] Assumption A1 holds for the uniform distribution.

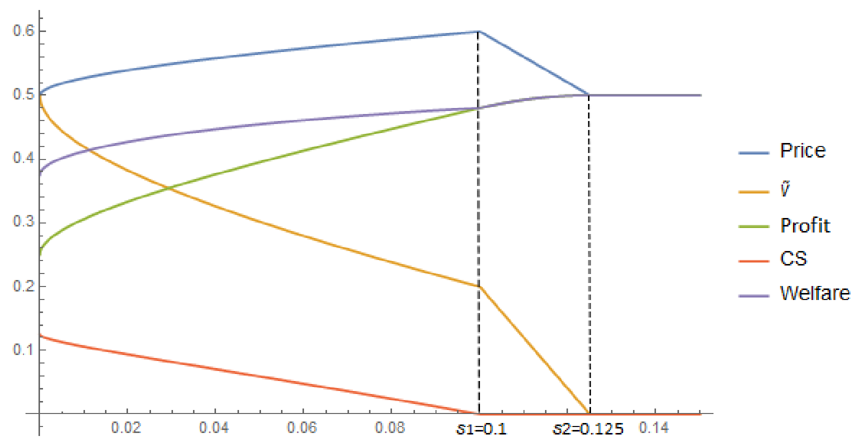


Figure 1: Example: $c = 0$ and $F(v) = v$.

(ii) when $s_1 \leq s \leq s_2$, both IC and IR bind, the profit is increasing in search cost while the price and $\tilde{v}$ are decreasing in search cost;

(iii) when $s > s_2$, IR binds, the outcome is constant and socially efficient, but the seller obtains all the surplus.

Figure 1 depicts an example when $c = 0$ and v follows the uniform distribution. For small search cost s, the consumer tends to search to learn the exact match value. The firm thus needs to reveal accurate information ($\tilde{v}$ close to p) to make sure (IC) holds, which can guarantee that (IR) holds because s is small. Hence, only IC constraint binds. When s is increasing, (IC) is more relaxed, the firm can obtain more profit by choosing a lower $\tilde{v}$ to let more consumers purchase, and choosing a higher price to let consumers pay more. As a result, profit is increasing and consumer surplus is decreasing. Note that total welfare is higher when more consumers purchase the product and their match value is greater than c, total welfare is therefore decreasing in $\tilde{v}$ (and increasing in s).

This situation continues until $s = s_1$, where the consumer surplus becomes zero, which means that (IR) constraint starts to bind from now. Both (IC) and (IR) constraints bind when $s_1 < s < s_2$. As s is increasing, the firm reduces $\tilde{v}$ to let more consumers buy the product, but the price is decreasing to make sure consumer surplus is positive (zero).

When the search cost is large enough such that $s \geq s_2$, consumers do not search because it is too costly, the firm only needs to make sure that (IR) constraint holds. Let (IR) constraint bind, the firm's problem can thus be stated as

$$\max_{\tilde{v}} \int_{\tilde{v}}^{1} (v-c)dF(v)$$

It is clear that the profit reaches the maximum when the threshold $\tilde{v} = c$, and the corresponding price is $p = E[v|v \geq c]$.

4 Conclusion

We have analyzed the optimal information revelation strategy and return policy of a monopoly firm. Generally, the firm has no incentive to provide precise information on product characteristics, because partial information leads to more demand, some consumers with the match value lower than the price will also buy the product. The firm just needs to provide threshold match information, which merely informs the consumer if her utility is above some value. Consumer surplus is decreasing in search cost, but total welfare is increasing in that. The allocation is efficient when the search cost is very large, but the firm obtains all the surplus in this situation.

References

Anderson, Simon P and Regis Renault (1999). "Pricing, product diversity, and search costs: A Bertrand-Chamberlin-Diamond model", *The RAND Journal of Economics*, pp. 719–735.

Anderson, Simon P and Régis Renault (2006). "Advertising content", *American Economic Review* 96.1, pp. 93–113.

Bar-Isaac, Heski, Guillermo Caruana, and Vicente Cuñat (2010). "Information gathering and marketing", *Journal of Economics & Management Strategy* 19.2, pp. 375–401.

Hinnosaar, Toomas and Keiichi Kawai (2020). "Robust pricing with refunds", *The RAND Journal of Economics.*

Lubensky, Dmitry and Eric Schmidbauer (2020). "Free Product Trials: Disclosing Quality and Match Value", *Economic Inquiry* 58.4, pp. 1565–1576.

Petrikaitė, Vaiva (2018). “A search model of costly product returns”, *International Journal of Industrial Organization* 58, pp. 236–251.

Wang, Chengsi (2017). “Advertising as a search deterrent”, *The RAND Journal of Economics* 48.4, pp. 949–971.

Wolinsky, Asher (1986). “True monopolistic competition as a result of imperfect information”, *The Quarterly Journal of Economics* 101.3, pp. 493–511.

Appendices

Proof of Lemma 1. If a firm's strategy (an ITM, price p, and return cost r) makes some consumers want to search, we will show that the firm can obtain strictly higher profit by choosing a new ITM. Let $([0,1]\times I, \Sigma([0,1])\times H, P)$ and $([0,1]\times I', \Sigma([0,1])\times H', P')$ denote the old and new ITM, and let $D_s \subseteq I$ denote the set of signals such that consumers choose to search if they receive a signal $i \in D_s$, the old ITM leads to $P(i \in D_s) > 0$.

Let $I' = (I \setminus D_s) \cup \{i^*, i'\}$ where $i^*, i' \notin I$. And P' is the same as P for $i \in I \setminus D_s$; for $i \in D_s$, consumers will receive signal i^* if their match value is greater than $p - \epsilon$, they receive i' otherwise.

For any $i \in D_s$, consumers choose to search instead of quitting, which means that

$$E\left[\max\left\{v-p,0\right\}|i\in D_s\right]-s\geq 0.$$

The following condition must be true when $\epsilon = 0$

$$E\left[\max\left\{v-p,-r\right\}|i^*\right] > E\left[\max\left\{v-p,0\right\}|i\in D_s\right]-s\geq 0,$$

because consumers know their match value is greater than the price and the search cost is strictly positive. Hence, when ϵ is sufficiently small and $\epsilon \leq r$, consumers will choose to buy directly and will not return the product if they receive signal i^*. These consumers believe that their match value is very likely to be greater than the price, and the difference is very small even if their match value is lower than the price. The new ITM increases the demand because consumers with the match value $p - \epsilon < v < p$ will now buy the product, therefore, the firm's profit becomes higher with the new ITM. □

Proof of lemma 2. As we showed in Lemma 1, the optimal strategy of the firm must lead to no consumer searching. In part (I) we show that for any strategy that leads to no consumer searching, the firm can obtain at least the same profit by just focusing on informing the consumer whether or not her match value is above some critical threshold $\tilde{v}$, and in part (II) we show that the necessary condition for profit maximization is that the critical threshold must be $\tilde{v} \in [p-r, p]$.

(I) Let ITM $([0,1] \times I, \Sigma([0,1]) \times H, P)$, price p, and return cost r denote a strategy that leads to no consumer searching, let us define the good news set to be the largest set $D_g \in H$, such that for all $i \in D_g$, the consumer prefers buying directly to choosing the outside option and searching,

$$\mathbb{E}\left[\max\{v-p,-r\}|i \in D_g, P\right] \geq 0, \tag{1}$$

$$\mathbb{E}\left[\max\{v-p,-r\}|i \in D_g, P\right] \geq \mathbb{E}\left[\max\{v-p,0\}|i \in D_g, P\right] - s, \tag{2}$$

where the argument P indicates that the expectation is taken using probability P. Then it suffices to show that there exists another ITM with signal set $I' = \{i^*, i'\}$ and probability P' such that for $i^* \in I'$ and some $\tilde{v}$,

(*i*) $P'(i = i^*|v \geq \tilde{v}) = 1$ and $P'(i = i^*|v < \tilde{v}) = 0$;

(*ii*) The probability of buying directly is at least as large as in the initial ITM.

If $P(i \in D_g) = 0$, the claim trivially holds, we therefore focus on the situation where $P(i \in D_g) > 0$. Let $\tilde{v}$ be the unique solution to $1 - F(\tilde{v}) = P(i \in D_g)$, and let $P'(v \leq x) = F(x)$ for $x \in [0,1]$ (P' is consistent with the distribution of the match value v), $P'(i = i^*|v) = 1$ if $v \geq \tilde{v}$ and $P'(i = i'|v) = 1$ if $v < \tilde{v}$.

By construction, $([0,1] \times I, \Sigma([0,1]) \times H', P')$ is an ITM that satisfies (*i*), where H' consists of all subsets of I'. Let us define the conditional cumulative distribution functions $G'(\cdot|i^*)$ and $G(\cdot|D_g)$ on $[0,1]$ by $G'(x|i^*) = P'(v \leq x|i^*)$ and $G(x|D_g) = P(v \leq x|D_g)$ for all $x \in [0,1]$.

We first show that the consumer prefers buying directly than choosing the outside option when they receive signal i^*. Using integration by parts we have

$$\begin{aligned}
&\mathbb{E}\left[\max\{v-p,-r\}|i^*,P'\right]\\
=&\int_{p-r}^{1}(v-p)dG'(v|i^*)-rG'(p-r|i^*)\\
=&\int_{p-r}^{1}\left[1-G'(v|i^*)\right]dv-r.
\end{aligned}$$

According to condition (1), if $G(v|D_g)\geq G'(v|i^*)$ is true, then we know the consumer will not choose the outside option because

$$\begin{aligned}
&\mathbb{E}\left[\max\{v-p,-r\}|i^*,P'\right]-\mathbb{E}\left[\max\{v-p,-r\}|D_g,P\right]\\
=&\int_{p-r}^{1}\left[G(v|D_g)-G'(v|i^*)\right]dv\geq 0.
\end{aligned}$$

To show $G(v|D_g)\geq G'(v|i^*)$, it suffices to prove $P'(v>x|i^*)\geq P(v>x|s\in D_g)$ for any $x\in[0,1]$. Because $P(i\in D_g)=1-F(\tilde{v})=P'(i=i^*)$, we have $P'(i=i^*|v>x)=1\geq P(i\in D_g|v>x)$ when $x\geq\tilde{v}$; otherwise, we have

$$\begin{aligned}
P'(i=i^*|v>x)=&\frac{1-F(\tilde{v})}{1-F(x)}\\
=&\frac{P(i\in D_g)}{1-F(x)}\\
\geq& P(i\in D_g|v>x).
\end{aligned}$$

Which means that $P'(i=i^*|v>x)\geq P(i\in D_g|v>x)$ for any $x\in[0,1]$. We therefore have

$$\begin{aligned}
P'(v>x|i^*)=&\frac{P'(i=i^*|v>x)\left[1-F(x)\right]}{P'(i=i^*)}\\
\geq&\frac{P(i\in D_g|v>x)\left[1-F(x)\right]}{P(i\in D_g)}\\
=&P(v>x|s\in D_g),
\end{aligned}$$

which complete the proof of this part.

We then show that the consumer prefers buying directly then searching when they receive signal i^*. For similar reasons, we have

$$\begin{aligned}
&\mathbb{E}\left[\max\{v-p,0\}|i^*,P'\right]-\mathbb{E}\left[\max\{v-p,0\}|i\in D_g,P\right]\\
=&\int_p^1\left[G(v|D_g)-G'(v|i^*)\right]dv\\
\leq&\int_{p-r}^1\left[G(v|D_g)-G'(v|i^*)\right]dv\\
=&\mathbb{E}\left[\max\{v-p,-r\}|i^*,P'\right]-\mathbb{E}\left[\max\{v-p,-r\}|D_g,P\right].
\end{aligned}$$

According to condition (2), we must have

$$\mathbb{E}\left[\max\{v-p,-r\}|i^*,P'\right]\geq\mathbb{E}\left[\max\{v-p,0\}|i^*,P'\right]-s,$$

Therefore, the consumer will not search or choose the outside option if she observes signal i^*, and the probability of buying directly is at least as large as that in the original ITM, which completes the proof of this part.

(II) We first show that it is profitable for the firm to choose a lower critical threshold $\tilde{v}$ when $\tilde{v}>p$. There are two possible situations: (i) only those with match value above $\tilde{v}$ buy directly (others choose the outside option), or (ii) everyone buys directly.

In situation (i), no one returns the good because their match value is strictly higher than p. By choosing a slightly lower critical threshold (for example $(\tilde{v}+p)/2$), more consumers will buy directly, and the profit becomes higher. They do not choose the outside option because they believe their match value is higher than the price, similarly, they do not search to avoid the search costs.

In situation (ii), the consumer will return the good if $v<p-r$ when $p-r>0$. Therefore, the firm can choose $\tilde{v}'=p-r$ to avoid some management costs. The profit becomes $(p-c)\left[1-F(\tilde{v}')\right]$ which is strictly higher than before $(p-c)\left[1-F(\tilde{v}')\right]-c_rF(\tilde{v}')$. When $p-r\leq 0$, no one returns the product because the return cost is too high. Choosing $\tilde{v}'=c$ can increase ex ante consumer surplus, thus make the firm being able to charge a higher price.

We then show that it is profitable for the firm to choose a slight higher critical threshold $\tilde{v}$ when $\tilde{v}<p-r$. We focus on the situation when $p-r>0$, otherwise, the firm is better off by choosing $\tilde{v}'=c$ as we mentioned before. Note that we have already showed that the firm can just focus on a strategy with a threshold $\tilde{v}$ that makes no consumer searching. The consumer will buy directly if her match value is greater than $\tilde{v}$, and she will return the good if $v < p - r$. If the firm chooses $\tilde{v}' = p - r$, the consumer will not choose the outside option because the surplus of buying directly is higher. To see the consumer will not search, let ex ante surplus of buying directly minus that of searching,

$$\begin{aligned}&\frac{\int_{p-r}^{1}(v-p)dF(v) - r\left[F(p-r) - F(\tilde{v})\right]}{1-F(\tilde{v})} - \{\frac{\int_{p}^{1}(v-p)dF(v)}{1-F(\tilde{v})} - s\}\\ =&\frac{\int_{p-r}^{p}(v-p)dF(v) - r\left[F(p-r) - F(\tilde{v})\right]}{1-F(\tilde{v})} + s\end{aligned}$$

The consumer will not search if the expression above is increasing in $\tilde{v}$. Take the derivative with respect to $\tilde{v}$, and using integration by parts we have

$$\begin{aligned}&\frac{rf(\tilde{v})\left[1-F(\tilde{v})\right] + f(\tilde{v})\left[\int_{p-r}^{p}(v-p)dF(v) - r\left[F(p-r) - F(\tilde{v})\right]\right]}{\left[1-F(\tilde{v})\right]^2}\\ =&\frac{rf(\tilde{v})\left[1-F(p-r)\right] + f(\tilde{v})\int_{p-r}^{p}(v-p)dF(v)}{\left[1-F(\tilde{v})\right]^2}\\ =&\frac{f(\tilde{v})\int_{p-r}^{p}\left[1-F(v)\right]dv}{\left[1-F(\tilde{v})\right]^2} \geq 0.\end{aligned}$$

Therefore, profit becomes $(p-c)\left[1-F(\tilde{v}')\right]$ which is strictly higher than before $(p-c)\left[1-F(\tilde{v}')\right] - c_r\left[F(\tilde{v}') - F(\tilde{v})\right]$, and we finish the proof. □

Proof of Proposition 1. Take a putative optimum and suppose no constraint binds. Since no constraint binds, the firm can locally increase the price, and each constraint is still slack, but then profit has gone up, which contradicts the initial supposition of an optimum. Therefore, at least one constraint must be binding.

(1) When s is very large, consumers never choose to search. We therefore solve the maximization problem with only the (IR) constraint binding, and then show the (IC) constraint is satisfied when $s \geq s_2$. The binding (IR) constraint gives $p = \frac{\int_{\tilde{v}}^{1} v dF(v)}{1-F(\tilde{v})}$. Substitute this expression into the profit function, we have

$$\begin{aligned}&\left[\frac{\int_{\tilde{v}}^{1} v dF(v)}{1-F(\tilde{v})} - c\right]\left[1-F(\tilde{v})\right]\\ =&\int_{\tilde{v}}^{1}(v-c)dF(v)\end{aligned}$$

The unique maximum is reached when $\tilde{v} = c$, and the corresponding price is $p = E[v|v \geq c]$. It is easy to check that the (IC) condition is indeed satisfied.

(2) When s is very close to zero, the binding (IC) constraint implies that (IR) constraint holds. We therefore solve the maximization problem with only the (IC) constraint binding, and then show the (IR) constraint holds when s is smaller than some positive number s_1 and $s_1 < s_2$.

(2.1) Let λ denote the Lagrangian multiplier, the Lagrangian is:

$$\mathcal{L} = (p - c)\left[1 - F(\tilde{v})\right] + \lambda\{s\left[1 - F(\tilde{v})\right] - \int_{\tilde{v}}^{p} (p - v) dF(v)\}.$$

First order conditions are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial p} = 0 : &\quad 1 - F(\tilde{v}) - \lambda\left[F(p) - F(\tilde{v})\right] = 0, \\
\frac{\partial \mathcal{L}}{\partial \tilde{v}} = 0 : &\quad p - c + \lambda s - \lambda(p - \tilde{v}) = 0, \\
\frac{\partial \mathcal{L}}{\partial \lambda} = 0 : &\quad s\left[1 - F(\tilde{v})\right] - \int_{\tilde{v}}^{p} (p - v) dF(v) = 0.
\end{aligned}$$

Eliminating λ gives

$$(p - c)\left[F(p) - F(\tilde{v})\right] = (p - \tilde{v} - s)\left[1 - F(\tilde{v})\right]. \tag{3}$$

(2.2) To show the optimal price p and the threshold $\tilde{v}$ are uniquely determined by equation (3) and the binding (IC) condition. We treat these two conditions as functions $p = K(\tilde{v})$ and $p = K_{IC}(\tilde{v})$, because we showed before it is never optimal to choose $p < \tilde{v}$, then it suffices to show the functions have a unique intersection when $p \geq \tilde{v}$.

Take total derivative of the binding (IC) condition with respect to p and $\tilde{v}$, we find that

$$\begin{aligned}
\frac{dK_{IC}(\tilde{v})}{d\tilde{v}} &= \frac{f(\tilde{v})}{F(p) - F(\tilde{v})}(p - \tilde{v} - s) \\
&= \frac{f(\tilde{v})}{F(p) - F(\tilde{v})}\left(p - \tilde{v} - \frac{\int_{\tilde{v}}^{p}(p - v)dF(v)}{1 - F(\tilde{v})}\right) \\
&= \frac{f(\tilde{v}) \int_{\tilde{v}}^{p} 1 - F(v) dv}{\left[F(p) - F(\tilde{v})\right]\left[1 - F(\tilde{v})\right]} \geq 0
\end{aligned}$$

To know the sign of $\frac{dK(\tilde{v})}{d\tilde{v}}$, rearranging equation (3) gives

$$(p - c)\left[1 - F(p)\right] = (\tilde{v} + s - c)\left[1 - F(\tilde{v})\right].$$

Then, $\tilde{v} \leq \tilde{v}_h$ must be true where $\tilde{v}_h$ solves $(\tilde{v} + s - c)[1 - F(\tilde{v})] = p^m$, otherwise, there is no solution of p. Apparently $\tilde{v}_h < p^m$ as long as $s > 0$. There are two solutions of p for any $\tilde{v} < \tilde{v}_h$ (one is greater than p^m, the other is smaller than p^m), because $(p - c)[1 - F(p)]$ is quasi-concave. As what we will show later, $p > p^m$, we therefore choose the solution that is greater than p^m, and $\frac{dK(\tilde{v})}{d\tilde{v}} < 0$.

We first argue there is an intersection point when $s = 0$. From the binding (IC) condition, we know $p \to \tilde{v}$ when $s \to 0$. Substitute the binding (IC) condition into equation (3), and integrate by part, we find

$$p - c = \frac{(p - \tilde{v})[1 - F(\tilde{v})] - \int_{\tilde{v}}^{p}(p - v)dF(v)}{F(p) - F(\tilde{v})} = \frac{\int_{\tilde{v}}^{p}[1 - F(v)]\,dv}{F(p) - F(\tilde{v})}. \tag{4}$$

using L'Hôpital's rule, we can see

$$\lim_{p \to \tilde{v}} \frac{\int_{\tilde{v}}^{p}[1 - F(v)]\,dv}{F(p) - F(\tilde{v})} = \frac{1 - F(p)}{f(p)},$$

which means that $s \to 0$ leads to

$$p - c = \frac{1 - F(p)}{f(p)}.$$

Therefore, both the price and threshold are equal to the monopoly price p^m when $s \to 0$, which means that there is an intersection point at $\tilde{v} = p^m$.

When s is increasing, we argue that the function $K(\tilde{v})$ is moving downwards, and $K_{IC}(\tilde{v})$ is moving upwards. Take total derivatives of condition 3 and the binding (IC) condition with respect to p and s,

$$K(\tilde{v}): \qquad \frac{dp}{ds} = \frac{1 - F(\tilde{v})}{1 - F(p) - f(p)(p - c)} \leq 0,$$
$$K_{IC}(\tilde{v}): \qquad \frac{dp}{ds} = \frac{1 - F(\tilde{v})}{F(p) - F(\tilde{v})} \geq 0.$$

Hence, there must be an intersection point as long as s is not too large, we then show the intersection point exists as long as $s \leq s_2$.

It suffice to show $K(c) \geq K_{IC}(c)$ when $s = s_2$. Note that $s_2 = \int_{c}^{E[v|v\geq c]} \frac{E[v|v\geq c] - v}{1 - F(c)} dF(v)$, we have $K_{IC}(c) = E[v|v \geq c]$ because $p = K_{IC}(c)$ solves

$$\int_{c}^{p}(p - v)dF(v) = s_2[1 - F(c)] = \int_{c}^{E[v|v\geq c]}(E[v|v \geq c] - v)dF(v).$$

Similarly, $p = K(c)$ solves condition (3) when $\tilde{v} = c$

$$(p-c)\left[1-F(p)\right] = s_2\left[1-F(c)\right].$$

According to the assumption

$$(E[v|v \geq c] - c)\left[1 - F\left(E[v|v \geq c]\right)\right] \geq \int_c^{E[v|v\geq c]} \left[E[v|v \geq c] - v\right] dF(v). \tag{A1}$$

We have $K(c) \geq E[v|v \geq c] = K_{IC}(c)$ when $s = s_2$. We thus can conclude that there must be an intersection point whenever $s \leq s_2$.

(2.3) To analyze how the optimal price and threshold evolve with the search cost, take the total derivative with respect to p, $\tilde{v}$, and s,

$$\begin{cases} -\left[1-F(p)-(p-c)f(p)\right]dp + \left[f(\tilde{v})(c-\tilde{v}-s) + 1 - F(\tilde{v})\right]d\tilde{v} + \left[1-F(\tilde{v})\right]ds = 0, \\ \left[F(p)-F(\tilde{v})\right]dp - f(\tilde{v})(p-\tilde{v}-s)d\tilde{v} - \left[1-F(\tilde{v})\right]ds = 0. \end{cases}$$

Note that rearranging equation (3) gives

$$\begin{aligned} p - \tilde{v} - s &= (p-c)\frac{F(p)-F(\tilde{v})}{1-F(\tilde{v})} \\ c - \tilde{v} - s &= -(p-c)\frac{1-F(p)}{1-F(\tilde{v})} \end{aligned}$$

Substitute these expressions, we have

$$\begin{aligned} \frac{dp}{ds} &= -\frac{\left[1-F(\tilde{v})\right]\left[1-F(\tilde{v})-(p-c)f(\tilde{v})\right]}{\left[F(p)-F(\tilde{v})\right]\left\{f(\tilde{v})(p-c)\left[2-2F(p)-f(p)(p-c)\right]-\left[1-F(\tilde{v})\right]^2\right\}}, \\ \frac{d\tilde{v}}{ds} &= -\frac{\left[1-F(\tilde{v})\right]\left[1-F(p)-(p-c)f(p))-F(p)+F(\tilde{v})\right]}{\left[F(p)-F(\tilde{v})\right]\left\{f(\tilde{v})(p-c)\left[2-2F(p)-f(p)(p-c)\right]-\left[1-F(\tilde{v})\right]^2\right\}}. \end{aligned}$$

As we mentioned before, the firm chooses $p = \tilde{v} = p^m$ when $s \to 0$. Applying L'Hôpital's rule gives

$$\begin{aligned} \lim_{s\to 0}\frac{dp}{ds} &= +\infty, \\ \lim_{s\to 0}\frac{d\tilde{v}}{ds} &= -\infty. \end{aligned}$$

Therefore, we find that $p > p^m > \tilde{v}$ when s is close to zero, which means that $F(p) - F(\tilde{v}) > 0$ and $1 - F(p) - (p-c)f(p) < 0$.

We then check the signs of $\frac{dp}{ds}$ and $\frac{d\tilde{v}}{ds}$ when s is not close to zero. To do that, we first show

$$[1-F(\tilde{v})]^2 \geq f(\tilde{v})(p-c)\left[2-2F(p)-f(p)(p-c)\right]$$

Note that $(1-F(v))/f(v)$ is decreasing because $f(v)$ is log-concave, we have $[1-F(\tilde{v})]^2 \geq [1-F(p)]^2 \frac{f(\tilde{v})}{f(p)}$. So it suffice to show

$$\begin{aligned}
&[1-F(p)]^2 \geq f(p)(p-c)\left[2-2F(p)-f(p)(p-c)\right] \\
\Leftrightarrow &[1-F(p)-f(p)(p-c)]^2 \geq 0
\end{aligned}$$

which trivially holds.

We then show

$$1-F(\tilde{v})-(p-c)f(\tilde{v}) \geq 0,$$

substitute condition (4), it becomes

$$[1-F(\tilde{v})]\left[F(p)-F(\tilde{v})\right] \geq f(\tilde{v}) \int_{\tilde{v}}^{p} [1-F(v)]\, dv.$$

This condition trivially holds when $p=\tilde{v}$, so we only need to show the left hand side is increasing faster in p. Take the derivatives of both sides with respect to p, we find that it is true

$$f(p)\left[1-F(\tilde{v})\right] \geq f(\tilde{v})\left[1-F(p)\right],$$

since $f(v)$ is log-concave.

Therefore, we can conclude that $\frac{dp}{ds} \geq 0$ and $\frac{d\tilde{v}}{ds} \leq 0$ as long as only (IC) constraint is binding.

(2.4) Let s_1 be the highest search cost such that only (IC) constraint binds, we now show $s_1 < s_2$. Note that ex ante consumer surplus (if only (IC) binds) $\int_{\tilde{v}}^{1}(v-p)dF(v)$ is strictly positive when s is small, and it is decreasing in s because

$$\frac{d\int_{\tilde{v}}^{1}(v-p)dF(v)}{ds} = (p-\tilde{v})f(\tilde{v})\frac{d\tilde{v}}{ds} - [1-F(\tilde{v})]\,\frac{dp}{ds} \leq 0.$$

To show that $s_1 < s_2$, it suffices to show $\int_{\tilde{v}}^{1}(v-p)dF(v) < 0$ (if only (IC) binds) when $s = s_2$.

Suppose it is not true, $\int_{\tilde{v}}^{1}(v-p)dF(v) \geq 0$ (if only (IC) binds) when $s = s_2$, and the optimal price p and the threshold $\tilde{v}$ are determined by equation (3) and the binding (IC) condition. We treat the binding (IR) condition $\int_{\tilde{v}}^{1}(v-p)dF(v)=0$ as function $p=K_{IR}(\tilde{v})$, then to prove there is a contradiction, it suffices to show the intersection point of $K(\tilde{v})$ and $K_{IC}(\tilde{v})$ is above $K_{IR}(\tilde{v})$.

From the binding (IR) constraint, $p=\frac{\int_{\tilde{v}}^{1} vdF(v)}{1-F(\tilde{v})}$, then integrate by part,

$$p-\tilde{v}=\frac{\int_{\tilde{v}}^{1}\left[1-F(v)\right]dv}{1-F(\tilde{v})}$$

Then following similar steps, we find that

$$\begin{aligned}\frac{dK_{IR}(\tilde{v})}{d\tilde{v}} &= \frac{f(\tilde{v})(p-\tilde{v})}{1-F(\tilde{v})}\\ &= \frac{f(\tilde{v})}{1-F(\tilde{v})}\frac{\int_{\tilde{v}}^{1}\left[1-F(v)\right]dv}{1-F(\tilde{v})}\\ &< \frac{f(\tilde{v})}{1-F(\tilde{v})}\frac{\int_{\tilde{v}}^{p}1-F(v)dv}{F(p)-F(\tilde{v})}=\frac{dK_{IC}(\tilde{v})}{d\tilde{v}}.\end{aligned}$$

We can go from the second line to the third if $\frac{\int_{\tilde{v}}^{p}1-F(v)dv}{F(p)-F(\tilde{v})}$ is decreasing in p, and to prove it is true, take the derivatives with respect to p

$$\begin{aligned}\frac{d}{dp}\frac{\int_{\tilde{v}}^{p}1-F(v)dv}{F(p)-F(\tilde{v})} &= \frac{\left[1-F(p)\right]\left[F(p)-F(\tilde{v})\right]-f(p)\int_{\tilde{v}}^{p}\left[1-F(v)\right]dv}{\left[F(p)-F(\tilde{v})\right]^{2}}\\ &= \frac{\int_{\tilde{v}}^{p}\left\{f(v)\left[1-F(p)\right]-f(p)\left[1-F(v)\right]\right\}dv}{\left[F(p)-F(\tilde{v})\right]^{2}}<0,\end{aligned}$$

we know $f(v)\left[1-F(p)\right]-f(p)\left[1-F(v)\right]$ is negative because $\frac{1-F(x)}{f(x)}$ is decreasing.

Then, as we can see from figure 2, $K_{IR}(\tilde{v})$ is increasing slower than $K_{IC}(\tilde{v})$, the intersection point is therefore above the $K_{IR}(\tilde{v})$, which means that (IR) constraint does not hold. Hence, we can conclude that $s_1<s_2$.

(2.5) To complete the proof, we finally check the second order conditions. The bordered Hessian matrix is

$$H^{b}=\begin{bmatrix}0 & -\lambda\left[F(p)-F(\tilde{v})\right] & \lambda f(\tilde{v})(p-\tilde{v}-s)\\ -\lambda\left[F(p)-F(\tilde{v})\right] & -\lambda f(p) & f(\tilde{v})(\lambda-1)\\ \lambda f(\tilde{v})(p-\tilde{v}-s) & f(\tilde{v})(\lambda-1) & -\lambda f(\tilde{v})\end{bmatrix}$$

The determinant of the principal minor is $-\left\{\lambda\left[F(p)-F(\tilde{v})\right]\right\}^{2}$ which is clearly negative.

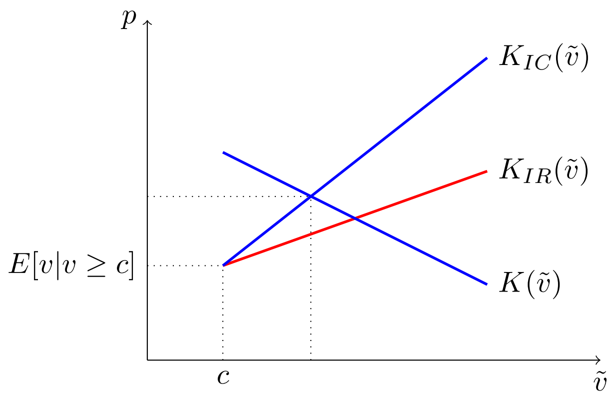


Figure 2: When $s = s_2$ and only (IC) binds

Eliminating λ, the determinant of H^b is

$$\begin{aligned}
&\lambda\left[F(p)-F(\tilde{v})\right]\left\{\lambda^2\left[F(p)-F(\tilde{v})\right]f(\tilde{v})-\lambda(\lambda-1)f(\tilde{v})(p-\tilde{v}-s)\right\}\\
&+\lambda f(\tilde{v})(p-\tilde{v}-s)\left\{-\lambda(\lambda-1)f(\tilde{v})\left[F(p)-F(\tilde{v})\right]+\lambda^2 f(\tilde{v})f(p)(p-\tilde{v}-s)\right\}\\
=&\left[1-F(\tilde{v})\right]^2-2\left[1-F(p)\right]f(\tilde{v})(p-c)+f(\tilde{v})f(p)(p-c)^2\\
\geq&\frac{f(\tilde{v})}{f(p)}\left\{\left[1-F(p)\right]^2-2\left[1-F(p)\right]f(p)(p-c)+\left[f(p)(p-c)\right]^2\right\}\\
=&\frac{f(\tilde{v})}{f(p)}\left[1-F(p)-f(p)(p-c)\right]^2>0,
\end{aligned}$$

which complete the proof.

(3) When $s_1 < s < s_2$, the only possible result is that both constraints bind. Hence, the price p and threshold $\tilde{v}$ are determined by the binding (IC) and (IR) conditions

$$\int_{\tilde{v}}^{p}(p-v)dF(v)=\int_{p}^{1}(v-p)dF(v),$$
$$\int_{\tilde{v}}^{p}(p-v)dF(v)=s\left[1-F(\tilde{v})\right].$$

As we have showed before, $K_{IR}(\tilde{v})$ is increasing slower than $K_{IC}(\tilde{v})$. When s is increasing, $K_{IR}(\tilde{v})$ is not influenced, whereas $K_{IC}(\tilde{v})$ is moving upwards. The two functions have a unique intersection both when $s = s_2$ and $s = s_1$, therefore, there must be a unique intersection for any $s_1 < s < s_2$.

Take total derivatives with respect to p, $\tilde{v}$, and s, and rearrange the equations, we have

$$\frac{dp}{ds} = \frac{(p-\tilde{v})[1-F(\tilde{v})]}{(p-\tilde{v})[F(p)-1]+s[1-F(\tilde{v})]},$$
$$\frac{d\tilde{v}}{ds} = \frac{[1-F(\tilde{v})]^2/f(\tilde{v})}{(p-\tilde{v})[F(p)-1]+s[1-F(\tilde{v})]}.$$

The signs of $\frac{dp}{ds}$ and $\frac{d\tilde{v}}{ds}$ are the same, and depend on the sign of their common denominator $(p-\tilde{v})[F(p)-1]+s[1-F(\tilde{v})]$.

As we mentioned before, the binding (IR) constraint gives

$$p-\tilde{v} = \frac{\int_{\tilde{v}}^1 [1-F(v)]\,dv}{1-F(\tilde{v})}.$$

Using both binding (IC) and (IR) conditions gives

$$s[1-F(\tilde{v})] = \int_p^1 (v-p)dF(v) = \int_p^1 (1-F(v))dv.$$

Substitute these expression into the denominator, we find that

$$\begin{aligned}
&(p-\tilde{v})[F(p)-1]+s[1-F(\tilde{v})] \\
=&[1-F(p)]\left\{\frac{\int_p^1 [1-F(v)]\,dv}{1-F(p)} - \frac{\int_{\tilde{v}}^1 [1-F(v)]\,dv}{1-F(\tilde{v})}\right\}
\end{aligned}$$

Therefore, $\frac{dp}{ds} \leq 0$ and $\frac{d\tilde{v}}{ds} \leq 0$ as long as $\frac{\int_x^1 [1-F(v)]dv}{1-F(x)}$ is decreasing in x. To show this is true, take derivative with respect to x, we have

$$\begin{aligned}
\frac{d}{dx}\frac{\int_x^1 [1-F(v)]\,dv}{1-F(x)} =& \frac{-[1-F(x)]^2 + \int_x^1 f(x)[1-F(v)]\,dv}{[1-F(x)]^2} \\
\leq& \frac{-[1-F(x)]^2 + \int_x^1 f(v)[1-F(x)]\,dv}{[1-F(x)]^2} = 0.
\end{aligned}$$

We have $f(x)[1-F(v)] \leq f(v)[1-F(x)]$ as long as $v \geq x$ because $(1-F(v))/f(v)$ is decreasing, therefore, we can go from the first line to the second. And we complete the proof. □