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“Equilibrium CEO Contract with Belief Heterogeneity”

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Equilibrium CEO Contract with Belief Heterogeneity*

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Abstract

Consider a firm owned by shareholders with heterogeneous beliefs and run by a manager who chooses random production plans. Shareholders do not observe the chosen plan but only its realization. The financial market consists of assets contingent on production realizations. A contract for the manager specifies her compensation as a function of the firm's production and possibly some restrictions to trade in the financial market. Shareholders are unrestricted. We define a concept of equilibrium between the manager and shareholders such that the equilibrium production plan is unanimously preferred by the manager and the shareholders, markets clear and the manager has no incentive to cheat. We first analyze the properties of such equilibria and in particular show that the contract should restrict the manager from trading. We next provide a framework where such equilibria exist. We lastly study the properties of equilibrium compensations when shareholders have beliefs that can be ranked in terms of optimism towards the equilibrium plan. Specific attention is given to their departure from linear compensations.

JEL codes: G32, G34, D24, D51, D53, D70

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1 Introduction

Corporations are often owned by a large number of investors, who may hold different beliefs say on the likelihood of success of a new product, on the payoffs associated to an investment opportunity, or on the future prospects of the market. It is well known that, under some conditions, shareholders with different beliefs can reach an agreement on the preferred plan of action by trading contingent claims in a complete asset market. Even then, however, decisions are often delegated to a manager, and it may be very costly or impossible for shareholders to observe the manager's actions. A classic question for shareholders is how to design a contract that would make the manager act in their interest.

This is the question we address. We consider a firm owned by shareholders with different beliefs and run by a manager. There is only one production and consumption date. The manager chooses a production plan, whose output is uncertain. Shareholders cannot observe the plan chosen by the manager, they only observe the realized production, and they can trade in a financial market that consists in all the assets that are contingent on the realized production. A contract for the manager specifies her compensation as a function of the firm's production and possibly some restrictions to trade in the financial market. We ask whether one can design such a contract so that the manager chooses a plan unanimously preferred by shareholders.

Our analysis builds on Bianchi, Dana and Jouini (2021), who introduce the concept of manager-shareholders equilibrium (in short, m-s equilibrium), defined as follows. Given a compensation scheme and a price, the manager chooses a production plan that maximizes her indirect utility, that is the maximal utility she can obtain by choosing a consumption plan in her budget constraint. Given their expectation about the production plan and future wealth, shareholders choose consumption plans that maximize their utility under their budget constraints. At equilibrium, in addition to the usual market clearing conditions, shareholders should correctly anticipate the plan chosen by the manager and they should unanimously prefer that plan to any other feasible plan. Bianchi et al. (2021) assume that the manager does not face any trading restriction and show that a m-s equilibrium exists only if the manager receives a linear compensation and if she has the same characteristics as the representative shareholder.

This paper is motivated by the fact that the above result can be interpreted as showing the impossibility to reach such an equilibrium. In fact, in practice, it may be impossible to find a manager with the same characteristics as the representative shareholder. As shown in Bianchi et al. (2021), that would require for example finding a manager with a declining or even a stochastic discount rate, even if all shareholders have constant discount rate. This impossibility is tightly linked to the fact that shareholders hold different beliefs, it would not arise if we were to start with a single shareholder. This begs the question of whether commonly observed restrictions to insider trading for the manager can help reaching an equilibrium in case the manager is not like the representative shareholder. If this is the case, the next question is which properties equilibrium compensations have and in particular how they deviate from linearity.[1]

In order to address these questions, we extend the analysis by Bianchi et al. (2021) and allow for restrictions to trade on the manager. In our framework, a m-s equilibrium is a list

[1] As we will show, linear compensations are equilibrium compensations also in our setting when the manager and all the shareholders have the same belief.

of a compensation scheme, a space of allowed transactions, a production plan, consumption plans for shareholders and the manager and a price that fulfill the properties above. The main contribution of this paper is to provide a framework where indeed an equilibrium can be reached if shareholders can restrict the manager's trades in the asset market. In addition, we characterize the associated compensation scheme, and highlight some of its properties that are qualitatively valid irrespective of the exact form of belief heterogeneity.

We start by describing some properties of a m-s equilibrium, showing how it relates to the standard concept of production equilibrium where the initial production set is replaced by a net (of manager's compensation) production set and defining a representative shareholder associated to such a production equilibrium. We then move to our main task of designing a contract for the manager that can lead to a m-s equilibrium. As implied by Bianchi et al. (2021), trading restrictions are necessary when the manager is not the representative shareholder. In fact, in order to characterize an equilibrium compensation, we focus on the case in which the manager is not allowed to trade in the financial market. As we show, the compensation schemes defined in this way are the only candidate equilibrium schemes, even if the manager were to face milder trading restrictions. A key result is that the manager implements an equilibrium plan only if her compensation is designed so that the marginal utility of her compensation is proportional to that of the representative shareholder at equilibrium. In this way, the manager's compensation leads her to mimic the behavior of the representative shareholder at equilibrium.

We can express this condition in terms of a differential equation that the compensation function should satisfy. In general, solving this equation involves a fixed-point problem since both the production equilibrium (if it exists) and the representative shareholder's characteristics depend on the compensation function. Moreover, as detailed below, the fixed-point problem need not be well-defined.

As a useful starting point, we consider the special case in which shareholders have identical beliefs. In this case, our differential equation can be explicitly solved. We show that, if the manager shares the same belief as the shareholders, a linear compensation can be used to implement a m-s equilibrium. If instead the manager is more optimistic than the shareholders (in a sense made precise below), the compensation should moderate the manager's optimism by offering a lower compensation rate when the firm's production turns out to be very large. Similarly, when the manager is more pessimistic than the shareholders, the compensation rate should be larger for larger realizations of production.

Turning to the general case of shareholders with heterogenous beliefs, we address the fixed-point problem by taking a reverse approach. We start by a production set net of the manager's compensation and suppose that there exists a production equilibrium for that set. We obtain the compensation function that solves our differential equation and construct the associated gross production set. We then show that this compensation function is an equilibrium compensation when the production set corresponds to the derived gross production set.

This allows to derive some properties of the associated equilibrium compensation. We show that, as long as the manager holds beliefs that are between the "most optimistic" and the "most pessimistic" (again in a sense made precise below) of the shareholders, an equilibrium compensation has the following property. When output realizations are extreme, the compensation is such that the instantaneous utility of the manager is much larger (in absolute value) than what she would reach with a linear compensation. Notice this does not require having an exact knowledge of the belief of the manager, and it shows that, relative to a linear compensation, an equilibrium compensation induces the manager to attach more importance to how the different production plans behave in terms of extreme realizations.

We conclude by developing an example of a net production set for which we can explicitly compute the production equilibrium and, given the belief of the manager, define an equilibrium compensation in closed form. In this setting, the representative shareholder's belief leads to overestimate the level of risk relative to all shareholders. We show that the associated equilibrium compensation has the properties stated in the previous paragraph, and provide numerical illustrations that highlight the effect of increasing belief heterogeneity.

We think our results have important implications for the study of agency problems. Our model is most closely related to a common agency problem, in which an agent faces multiple principals with possibly conflicting interests (as started by Bernheim and Whinston (1986)). Our key distinctive feature is to embed this problem in a general equilibrium setting, in which the asset market is used to mediate and possibly align shareholders' conflicting interests. This approach allows highlighting the importance of modeling explicitly the equilibrium process leading to the definition of a representative shareholder. As we show, the insights one would get by taking the representative shareholder as given would be different, and possibly misleading, in settings where shareholder heterogeneity is important.

The general equilibrium approach fundamentally changes the incentive problem of the manager, who chooses a production plan and at the same time a consumption plan while possibly trading in the asset market. This opens the possibility for the manager to deviate and implement a different plan, while increasing her utility by trading on her private information and changing her consumption decisions. For this reason, we can abstract from effort costs on the part of the manager; our incentive problem stems from the fact that the manager, unless restricted, can benefit from trading on her information about the chosen production plan.

We show it is necessary to impose trading restrictions to the manager in a setting in which the action she takes cannot be observed by shareholders. Without those restrictions, it would be impossible to find a compensation scheme which induces the manager to choose an equilibrium plan. This provides a rationale for the commonly observed restrictions both to insider trading and to non-exclusive contracts.

We also qualify the view that agency conflicts are minimized when the manager owns a substantial part of the firm's shares, which has motivated the rise in stock compensation. Our analysis instead emphasizes that the compensation rate should vary with the level of production and provide conditions under which this rate should become very large or very small when realizations become extreme. This result stands in contrast to the argument in favor of compensations -such as call options- which encourage risk taking (see e.g. Kadan and Swinkels (2008)).

1.1 Related Literature

Our paper builds on the literature on aggregation of preferences and beliefs in asset markets.[2] Our focus on agency problems between a manager and shareholders is however novel in this literature. Similarly, managerial compensation has typically been studied under the perspective of a representative shareholder (see e.g. Murphy (1999) and Murphy (2012) for reviews). We provide new insights by embedding the choice of the compensation in a stock market equilibrium with heterogeneous shareholders.[3]

Part of this literature has analyzed the characteristics of the optimal contract when the manager and the shareholder hold different beliefs (see e.g. Adrian and Westerfield (2008); Giat, Hackman and Subramanian (2010); Jung and Subramanian (2014)). Our approach is different as we consider a setting with several heterogenous shareholders and, importantly, we introduce the possibility to trade in an asset market. In this way, we can derive endogenously the properties of the representative shareholder and, in addition, study trading restrictions as part of the managerial contract.

In line with the literature on optimal contracting, we emphasize that, in a setting with asymmetric information, it may be beneficial to prevent the manager from trading in the stock market (Fischer (1992)).[4] Our novelty is to show how trading restrictions, and the form of equilibrium compensation, can arise from a different mechanism: the interaction between moral hazard and shareholder heterogeneity. Moreover, as mentioned, moral hazard in our setting is not driven by private effort costs but rather from the fact that the manager's beliefs may differ from those of the representative shareholder and that she can in principle trade on her private knowledge of the production plan.

The way in which information asymmetry is introduced makes our paper in line with the probability approach to general equilibrium developed by Magill and Quinzii (2009). Indeed, we assume that shareholders do not observe states of nature but only the production outcomes, so from their point of view, production plans only differ by the outcomes' probability distribution. Accordingly, we consider contracts that are contingent on the possible realizations firm's production as opposed to being contingent on exogenous states of nature.

Finally, we relate to the literature on firms' objectives when shareholders are heterogeneous. Magill and Quinzii (2002) review fundamental problems posed by market incompleteness, as well as classic contributions addressing these problems. Bisin, Gottardi and Ruta (2016) study competitive equilibria in a production economy with incomplete markets and agency frictions and derive fundamental welfare properties.[5] We instead focus on the design of the compensation scheme and keep shareholders' objective as simple as possible by assuming a form of market

[2] Recent contributions include Detemple and Murthy (1994); Gollier and Zeckhauser (2005); Jouini and Napp (2007); Jouini, Marin and Napp (2010); Cvitanić, Jouini, Malamud and Napp (2012); Xiong and Yan (2010); Bhamra and Uppal (2014); Atmaz and Basak (2018).

[3] Alternative equilibrium models have instead focused on the labor market equilibrium (e.g. Gabaix and Landier (2008)) or on financial market equilibrium with a representative agent (e.g. Diamond and Verrecchia (1982)).

[4] This literature has also pointed out at beneficial aspects of insider trading, such as improving the informational efficiency of market prices (e.g. Leland (1992)). We abstract from this issue as in our settings there are no investors apart from shareholders. We refer to Bhattacharya (2014) for a recent review of these issues.

[5] Other recent contributions include Demichelis and Ritzberger (2011), Magill, Quinzii and Rochet (2015), Crès and Tvede (2021).

completeness (detailed below).

2 Model

We consider a firm owned by a group of shareholders with heterogenous beliefs and run by a manager. There is only one production and consumption date T. The information structure is modeled by a probability space $(\Omega, \mathcal{F}, P)$. The firm produces a consumption good, which we use as numeraire, according to a production plan y. This plan is a random variable and $y(\omega)$ defines the production of the firm at date T in state ω.

We denote by X the space of $\mathcal{F}$ measurable production and consumption random variables x. We denote by X' the space of state-price densities p where, for a given state of the world $\omega \in \Omega$, $p(\omega)$ corresponds to the price of one unit of consumption at date T in state ω. For a given price p, the value of the consumption plan x is $p \cdot x = E\left[px\right]$, where E is the expectation operator under the probability P.

For these expectations to be well defined, we have to further impose that production and consumption plans in X are such that $E|x|^p < \infty$ and that prices in X' are such that $E|p|^q < \infty$, where p and q are such that $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$.[6] We denote by X_+ and X'_+ the set of nonnegative pairs, respectively, in X and X'. For $Y \subset X$, we denote by Y_+ the set $Y \cap X_+$.

In terms of notations, while x and y will be used to denote random consumption and production pairs taking their values in $\mathbb{R}^2$, z will be used to denote vectors in $\mathbb{R}^2$ and, henceforth, generic values taken by x or y. As usual, $x \leq x'$ ($x << x'$) means $x(\omega) \leq x'(\omega)$ ($x(\omega) < x'(\omega)$) almost everywhere, and $x < x'$ means $x \leq x'$ and $x \neq x'$. Finally, we denote by μ the Lebesgue measure on $\mathbb{R}$.

2.1 Production

We let $Y \subset X$ denote the set of production plans. Denote with $N_Y(y)$ the normal cone of Y at y,

$$N_Y(y) = \left\{p \in X' : p \cdot (y' - y) \ \leq 0, \forall\, y' \in Y\right\},$$

which corresponds to the set of linear forms that reach their maximum on Y at y. We will say that $y \in Y$ is positively exposed if there exists $p >> 0$ such that $p \in N_Y(y)$. Note that a positively exposed production plan y is efficient in the sense that it not dominated by other feasible production plans : $\nexists y' \in Y$, $y' > y$. We denote by $\mathrm{Eff}^+(Y)$ the set of positively exposed production plans.

We say that Y is smooth if, for all $\bar{y} \in \mathrm{Eff}^+(Y)$, there exists $p >> 0$ such that $N_Y(\bar{y}) = \{\lambda p : \lambda \geq 0\}$. This condition states that at positively exposed plans, the tangent cone (i.e. the polar of the normal cone) is a half space and it ensures that Y has no outward kink.

We make the following assumptions:

Assumption (P)

P1 $Y = K - X_+$ *where* $K \subset X_+$,

[6] The space X equipped with the norm $\|x\| = \left(\sum_{t=0}^{T} E|x_t|^p\right)^{1/p}$ is then a Banach space whose dual (the space of continuous linear forms on X) is X'.

P2 Y *is closed and smooth,*

P3 *If* $y \in \text{Eff}^+(Y)$, *the random variable* y *has a density* h_y *with* $h_y > 0$, $\mu-$*a.e. on* $(0, \infty)$.

Assumption P1 implies the classical free disposal assumption, $Y - X_+ \subset Y$. Assumption P2 is standard in the general equilibrium literature in finite dimension.[7] Assumption P3 states that for every positively exposed production plan, all positive values are possible.[8] Indeed, this implies that by observing a given realization $(y(\omega))$ of a given production plan in $\text{Eff}^+(Y)$, shareholders are not able to exclude any plan $y \in Y$ from the set of possibly chosen plans. This assumption underlies the information asymmetry between the manager and the shareholders in our model. Bianchi et al. (2021) provide an illustration of Assumptions (P2) and (P3).

2.2 Shareholders

The firm is owned by a group of N shareholders, $i = 1, ..., N$. We denote with ν^i shareholder i's initial endowment of shares, and we assume $\nu^i > 0$ for all i. Shareholders have no other endowments, and they are heterogeneous in their subjective probabilities Q^i. All subjective probabilities are assumed to be equivalent to P and we denote by M^i the density of Q^i with respect to P, $M^i = \frac{dQ^i}{dP}$.

A key ingredient in our analysis is that shareholders do not observe the plan y chosen by the manager nor the state of the world ω. In state ω, their information is given by the realization $(y(\omega))$. As already mentioned, by Assumption P3, the observation of a given series of output realizations does not allow them to infer the chosen plan nor the state of the world. It follows that shareholders can only trade assets whose payoffs are contingent on $y(\omega)$. More formally, let C be the set of contingent contracts $C : X_+ \to X_+$ whose payoffs for a given y are of the form $c(\omega) = C(y(\omega))$, for some measurable functions $C : R_+^* \to R_+$. Given y, shareholders only trade consumption plans in $C(y) = \{C(y), C \in C\}$. We assume that markets are complete, or - more precisely - full spanning, meaning that it is possible to find a portfolio of assets that pays one unit if a given outcome for the firm is realized, and nothing otherwise (as shown in Magill and Quinzii (2009), full spanning is typically much weaker than market completeness).

All shareholders have the same consumption space X_+ and they are assumed to be expected utility maximizers. The expected utility of shareholder i for a contingent consumption plan c is defined as

$$U^i(c) = E\left[M^i u(c)\right], \tag{1}$$

in which u is a CRRA instantaneous utility function (the same for all shareholders).[9] That is

$$u(x) = \frac{1}{\gamma} x^\gamma, \tag{2}$$

for some $\gamma < 1$. We further assume the following:

[7] Note that this assumption is automatically satisfied when the production set is of the form $Y = \{y \in X : E[F(y)] \leq 0\}$ where F is a given function with a bounded derivative. In such a setting, we have $N_Y(y) = \{\lambda F'(y) : \lambda \geq 0\}$ for y such that $E[F(y)] = 0$ and $N_Y(y) = \{0\}$ for y such that $E[F(y)] < 0$.

[8] In this assumption $(0, \infty)$ might be replaced by some (A, B) for $0 < A < B$. In this case, all the considered functions that are defined on $(0, \infty)$ are replaced by functions that are only defined on (A, B).

[9] The assumption that shareholders are only endowed with shares of the firm and that they have the same CRRA utility is crucial for the proof of existence and uniqueness of a production equilibrium.

Assumption (C)

1. *For all* i, M^i *and* $M^i\varsigma^{\gamma-1}$ *belong to* X' for all $\varsigma \in K$,

2. *For all* i, $M^i\varsigma^{\gamma}$ *belong to* $L^1(\Omega, \mathcal{F}, P)$.

Assumption (C) assures that shareholders' marginal utility is well defined in all directions and that their utility is well defined on K.

2.3 Manager

The firm's production plan is chosen by a manager. While we do not model explicitly why shareholders need to delegate this choice to a manager, a standard argument is that they lack the time or the skills needed for implementing the plan, which may require continuous adjustments over time. Manager's characteristics are similar to those of the shareholders. She is an expected utility maximizer with instantaneous utility u, as defined in (2), she has a subjective probability Q^m equivalent to P with density M^m. Her expected utility of a contingent plan c is therefore defined by

$$U_m(c) = E\left[M^m u(c)\right].$$

The manager is given a contract (Φ, W) described by a compensation scheme $\Phi : X_+ \to X_+$ and of a set W of transaction plans she is allowed to make in the contingent claim market, which we describe below. As shareholders can only observe the realized production, the compensation can only depend on the realization $y(\omega)$. Hence $\Phi(y)$ must be of the form

$$\Phi(y)(\omega) = \phi(y(\omega)), \tag{3}$$

for some $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ assumed to be continuous almost everywhere. From now on, we will use the same notation ϕ for $\Phi : X_+ \to X_+$ and for $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ with the convention $\phi(y)(\omega) = \phi(y(\omega))$.

As the compensation cannot exceed the available quantity of consumption good, we necessarily have $\phi(z) \leq z$ for all $z \in \mathbb{R}_+$ and, in particular, $\phi(0) = 0$.

We next describe W, the space of transactions allowed to the manager in the contingent claim market. We assume that W is a closed subspace of X. When $W = \{0\}$, then the manager has no access to the market while when $W = X$, the manager has access to all contingent contracts and she can trade just as shareholders. Given a production plan y, the manager's set of feasible adapted consumptions plans $\mathcal{C}^m(y)$ is

$$\mathcal{C}^m(y) = (\phi(y) + W) \cap \mathcal{C}(y).$$

In particular, when $W = \{0\}$ then $\mathcal{C}^m(y) = \{\phi(y)\}$. When $W = X$, then $\mathcal{C}^m(y) = \mathcal{C}(y)$.

Assumption (F)

1. *For all* $y \in K$ and all $w \in W$, $M_m\phi(y)^{\gamma-1}w$ *belongs to* $L^1(\Omega, \mathcal{F}, P)$,

2. *For all* $y \in K$, $M_m\phi(y)^{\gamma}$ *belong to* $L^1(\Omega, \mathcal{F}, P)$.

Assumption (F) ensures that the manager's marginal utility is well defined in all feasible directions and that her utility is well defined when $y \in K$.

To explain how the manager chooses the production plan, let us introduce the concept of indirect utility of production plans for a given price. Given a production plan y and a price $q \in \mathcal{C}(y)$, let $V_m(y,q)$ be the maximal utility of the consumption plans that the manager can obtain by trading her compensation under her market constraint $c \in \mathcal{C}^m(y)$ and her budget constraint $q \cdot c \leq q \cdot \phi(y)$,

$$V_m(y,q) = \max\{U_m(c),\ c \in \mathcal{C}^m(y),\ q \cdot c \leq q \cdot \phi(y)\}. \tag{4}$$

The manager's maximization problem in Equation (4) differs from what appears in standard agency problems. First, our manager is maximizing her indirect utility, which depends not only on her choice of the production plan y but also on the given price q. Second, the manager's consumption may depend not only on her compensation but also on the possibility to trade in the asset market, which in itself depends on the plan y. This is a key feature that needs to be considered in our equilibrium definition, as we will see.

Similarly, let $V^i(y,q)$ be the maximal utility of the consumption plans that shareholder i can obtain by trading her share of production under her market and budget constraints:

$$V^i(y,q) = \max\{U^i(c),\ c \in \mathcal{C}(y),\ q \cdot c \leq \nu^i(q \cdot (y - \phi(y))\}, \tag{5}$$

where ν^i denotes her initial share and $y - \phi(y)$ is the production left to shareholders after having paid the manager. Equations (4) and (5) show how the manager and the shareholders, respectively, assess the utility associated to the various alternative production plans. They compare their indirect utility under y to the one they would have obtained under any alternative $y' \in Y$, by taking prices q as given.[10]

2.4 Equilibrium

Let us now define our concept of equilibrium between shareholders and the manager. We have in mind a setting with a large number of non-strategic agents. We take a general equilibrium approach in which resource allocation is decentralized through prices and in which prices depend on firm's choices. The only source of risk we consider is -roughly speaking- generated by firm's payoffs and the Arrow-Debreu assets can be seen as options on the firm's payoffs. We adapt the general equilibrium approach so as to account for the information asymmetry between the manager and the shareholders.

Shareholders appoint a manager with a contract (ϕ, W) and delegate to her the choice of the production plan. Given her compensation and a price q, the manager chooses a production plan y that maximizes her indirect utility $V_m(\cdot, q)$ over Y and an optimal consumption plan $C_m(y)$. Shareholders maximize the utility of their consumption plans under their market and budget constraints based on their expectation about the plan chosen by the manager. At

[10] Price taking is important to be able to define a consensus plan (see e.g. Grossman and Stiglitz (1980) for a discussion on price taking behaviors and unanimity). Price taking could also be derived by considering a setting with a large number of identical firms. The analysis would not be affected.

equilibrium, shareholders should correctly anticipate the plan chosen as the manager, they should unanimously prefer the manager's plan to any other plan feasible y and finally, markets should clear.

Definition 1 *A manager-shareholders equilibrium (in short m-s equilibrium) is defined by a contract (ϕ, W), $\phi \neq 0$, a production plan $\hat{y} \in Y$, a list of contingent contracts $(\hat{C}^i)_i$, a contingent contract $\hat{C}_m$, and a price $\hat{q} \in \mathcal{C}(\hat{y})$ such that:*

1. $\hat{c}^i = \hat{C}^i(\hat{y})$ *maximizes* $U^i(c)$ *s.t.* $c \in \mathcal{C}(\hat{y})$, $\hat{q} \cdot c \leq \nu^i(\hat{q} \cdot (\hat{y} - \phi(\hat{y})))$,
2. $\hat{c}_m = \hat{C}_m(\hat{y})$ *maximizes* $U_m(c)$ *s.t.* $c \in \mathcal{C}^m(\hat{y})$, $\hat{q} \cdot c \leq \hat{q} \cdot \phi(\hat{y})$,
3. $\sum_i \hat{c}^i + \hat{c}_m = \hat{y}$,
4. $V_m(\hat{y}, \hat{q}) > V_m(y, \hat{q})$ *for all* $y \in Y$,
5. $U_m(\hat{C}_m(\hat{y})) = \max_{y \in Y} U_m(\hat{C}_m(y))$,
6. $V^i(\hat{y}, \hat{q}) = \max_Y V^i(y, \hat{q})$.

Our definition of m-s equilibrium is adapted from Bianchi et al. (2021) by considering that the manager in our setting receives a contract (ϕ, W) that may specify some trading constraints W, while this was not the case in Bianchi et al. (2021).

From Assumption P3, when y is in $\text{Eff}^+(Y)$, shareholders cannot infer the chosen plan by observing the firm output, they choose their consumption plan based on their expectation about the production plan chosen by the manager. Conditions 4 and 5 imply however that the manager finds it optimal to choose the equilibrium plan $\hat{y}$ and so, at equilibrium, shareholders' expectations are consistent with the plan actually chosen by the manager. An alternative interpretation would be that the manager announces the chosen plan to shareholders (who choose their consumption plans based on this announcement), and Conditions 4 and 5 would imply that the manager has no incentive to misreport her production choice.

Condition 5 is also important to highlight a key distinctive feature of our equilibrium, relative to more standard agency problems in partial equilibrium. In our setting, the manager is choosing a production plan but also a consumption plan $C_m(y)$, possibly by trading in the asset market. As mentioned, this opens the possibility for the manager to implement a different plan and at the same time increase her utility by trading on her private information.

Condition 6 means that, given $\hat{q}$, shareholders unanimously prefer $\hat{y}$ to any other plan y in Y. It is known that when markets are complete and shareholders run a firm according to profit maximization, unanimity holds at equilibrium. We show that unanimity at equilibrium also holds in our framework where decisions are delegated to a manager and shareholders trade contingent claims on production under the condition that there exists at least one shareholder who prefers $\hat{y}$ to any other plan y in Y.[11]

[11] Notice also that we do not explicitly define any participation constraint for the manager. Adding such constraint would be immediate, as discussed in Section 5.

3 Equilibrium Properties

We first show that the manager's equilibrium consumption equals her equilibrium compensation. In other words, in equilibrium the manager does not trade in the financial market. We deduce from the no-trade result that, for a given compensation scheme ϕ, the manager chooses a production plan that strictly maximizes the utility of her compensation on the production set Y. The shareholders solve a production equilibrium problem given the net production set $Y^{\phi} = \{y - \phi(y) : y \in Y\}$.

In order to state our results, let us recall the concept of production equilibrium associated to the production set Y. Note that shareholders are not constrained to trade in $C(y^*)$, where y^* is the equilibrium production plan.

Definition 2 *A production equilibrium associated to the production set $\mathcal{Y}$ is given by a production plan y^*, $y^* \in \mathcal{Y}$, a set of individual consumption plans $\left(c^{*i}\right)_i \in X^N$ and a price $q^* \in X'$ such that*

1. $c^{*i} = \mathrm{argmax}\, U^i(c),\ q^* \cdot c \leq \nu^i(q^* \cdot y^*)$ *for all i,*
2. $y^* = \arg\max_{\mathcal{Y}} q^* \cdot y$,
3. $\sum c^{*i} = y^*$.

We now list some properties of a m-s equilibrium given a contract (ϕ, W).

Theorem 1 *Assume (P) and (C). Let $((\phi, W), (\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ be a m-s equilibrium then*

1. *The functions $z \to \phi(z)$ and $z \to z - \phi(z)$ are both nondecreasing and differentiable,*
2. *Shareholders' income $\hat{q} \cdot (y - \phi(y))$ is maximized over Y,*
3. *The manager does not trade, i.e. $\hat{c}_m = \phi(\hat{y})$,*
4. *The production plan $\hat{y}$ strictly maximizes $U_m(\phi(y))$ over Y, $U_m(\phi(\hat{y})) > U_m(\phi(y))$ for all $y \in Y$,*
5. *The triple $((\hat{c}^i)_i, \hat{q}, \hat{y} - \phi(\hat{y}))$ is a production equilibrium associated to the production set $Y^{\phi} = \{y - \phi(y) : y \in Y\}$ that fulfills $\hat{q} \in \mathcal{C}(\hat{y})$.*
6. *$((\phi, W'), (\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ is a m-s equilibrium for any $W' \subseteq W$. In particular, it is a $(\phi, \{0\})$ m-s equilibrium.*

From Point 6, $W = \{0\}$ plays a special role since any m-s equilibrium $((\phi, W), (\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ remains a m-s equilibrium when the manager is not allowed to trade. We will use this property in the next section, to provide a necessary condition fulfilled by an equilibrium compensation ϕ by setting $W = \{0\}$. The next theorem, together with Theorem 1, provides a characterization of the m-s equilibria associated to a contract $(\phi, \{0\})$.

Theorem 2 *Let* $((\phi,\{0\}),(\hat{c}^i)_i,(\hat{c}_m),\hat{q},\hat{y})$ *be a given list. Assume that C and assertions 3 to 5 of Theorem 1 are fulfilled. Then*
$((\phi,\{0\}),(\hat{c}^i)_i,(\hat{c}_m),\hat{q},\hat{y})$ *is a m-s equilibrium.*

As a consequence, when $(\phi,(\hat{c}^i)_i,(\hat{c}_m),\hat{q},\hat{y})$ are given, there exists W such that $((\phi,W),(\hat{c}^i)_i,(\hat{c}_m),\hat{q},\hat{y})$ is a m-s equilibrium if and only if Assertions 3 to 5 of Theorem 1 are fulfilled.

3.1 Representative Shareholder

We now define the representative shareholder at equilibrium. As standard, she is a fictitious shareholder who - if endowed with the equilibrium production - would have no incentive to trade at equilibrium prices. As we show in the next section, this is a key step for the definition of the equilibrium contract.

As shown in Theorem 1 if $((\phi,W),(\hat{c}^i)_i,(\hat{c}_m),\hat{q},\hat{y})$ is a m-s equilibrium then $((\hat{c}^i)_i,\hat{q},\tilde{y})$ with $\tilde{y}=\hat{y}-\phi(\hat{y})$ is a production equilibrium associated to the net production set $Y^\phi=\{y-\phi(y):y\in Y\}$ that fulfills $\hat{q}\in\mathcal{C}(\hat{y})$. Hence, there exists a representative shareholder associated to this production equilibrium. In order to characterize such shareholder in our setting, let

$$\Lambda=\left\{\lambda\in\mathbb{R}^n_+:\sum_i(\lambda^i)^{\frac{1}{1-\gamma}}=1\right\},\ N(\lambda)=\frac{\left(\sum_i\left(\lambda^iM^i\right)^{\frac{1}{1-\gamma}}\right)^{1-\gamma}}{E\left[\left(\sum_i\left(\lambda^iM^i\right)^{\frac{1}{1-\gamma}}\right)^{1-\gamma}\right]}.$$

As $((\hat{c}^i)_i,\hat{q},\tilde{y})$ is a production equilibrium associated to Y^ϕ that fulfills $\hat{q}\in\mathcal{C}(\hat{y})$ [12], there exists a unique vector of equilibrium utility weights $\left(\tilde{\lambda}^i\right)$ in Λ and $\nu>0$ such that

$$\tilde{\lambda}^iM^i(\hat{c}^i)^{\gamma-1}=\nu\hat{q}\text{ or }\hat{c}^i=(\nu\hat{q})^{\frac{1}{\gamma-1}}\left(\tilde{\lambda}^iM^i\right)^{\frac{1}{1-\gamma}}\text{ for all }i.$$

Summing over i, we obtain:

$$N(\tilde{\lambda})\,(\hat{y}-\phi(\hat{y}))^{\gamma-1}=\nu'\hat{q} \tag{6}$$

for $\nu'=\nu E\left[\left(\sum_i\left(\tilde{\lambda}^iM^i\right)^{\frac{1}{1-\gamma}}\right)^{1-\gamma}\right]^{-1}>0$. As $\hat{q}\in\mathcal{C}(\hat{y}),N(\tilde{\lambda})\in\mathcal{C}(\hat{y})$. Therefore, the representative shareholder has instantaneous utility u and a density $\tilde{M}\in\mathcal{C}(\hat{y})$ determined by

$$\tilde{M}=N(\tilde{\lambda}). \tag{7}$$

Furthermore, as $((\hat{c}^i)_i,\hat{q},\tilde{y})$ is a production equilibrium associated to the net production set $Y^\phi=\{y-\phi(y):y\in Y\}$, $\hat{q}\in N_{Y^\phi}(\tilde{y})$. Hence, from (6), the representative shareholder maximizes her utility on Y^ϕ. Equivalently she maximizes the utility of the net production on Y:

$$\max_Y E\left[N(\tilde{\lambda})u(y-\phi(y))\right]. \tag{8}$$

[12] Note that as $\mathrm{Id}-\phi$ is strictly increasing, it has an inverse and $\mathcal{C}(\tilde{y})=\mathcal{C}(\hat{y})$

4 Equilibrium Contract

We now consider the possibility of designing a contract (ϕ, W) such that there exists $((\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ for which $((\phi, W)(\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ is a m-s equilibrium. When this is the case, we say that (ϕ, W) is an equilibrium contract and that ϕ is an equilibrium compensation. The aim of this section is to define these concepts and to relate them to the fundamentals of our model, the shareholders' and manager's beliefs.

From Theorem 2 and its comment, ϕ is an equilibrium compensation if and only if Y^ϕ has a production equilibrium $((\hat{c}^i)_i, \hat{q}, \tilde{y})$ such that $\hat{q} \in C(\tilde{y})$ and the maximum of $U_m(\phi(y))$ on Y exists and is reached at $\hat{y}$ defined by $\hat{y} - \phi(\hat{y}) = \tilde{y}$.

We start by providing first order conditions for assertions 4 and 5 of Theorem 1. The following corollary establishes a link between the compensation function ϕ, the beliefs of the manager and those of the representative shareholder, as defined in the previous section.

Corollary 3 *Let $((\phi, W)(\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ be a m-s equilibrium. We then have*

$$\phi'(\hat{y}) M^m u'(\phi(\hat{y})) = \mu(1 - \phi'(\hat{y}))\tilde{M} u'(\hat{y} - \phi(\hat{y})), \tag{9}$$

for some $\mu > 0$. Therefore, we must have $M^m \in C(\hat{y})$.

A necessary condition for the existence of m-s equilibria is that the compensation ϕ is such that the gradient of the manager's utility from compensation is proportional to the gradient of the representative shareholder's utility from net production.

When prices are given and the manager faces no trading restrictions, the only feature of a compensation that matters is its market value. However, in our general equilibrium setting, prices are endogenous, and they depend in particular on the production plan chosen by the manager, which in turn depends on her compensation and on her constraints. Hence, the shape of the compensation matters and Equation (9) constrains the shape of the compensation function ϕ.

Without trading restrictions on the manager ($W = X$), as seen in Bianchi et al. (2021), equilibrium requires that the compensation is linear. At the same time, from (9), under linear compensation, the manager shares the same belief as the representative shareholder. When this is not the case, there exists no equilibria and therefore the manager should bear some trading restrictions.

Corollary 4 *Let $((\phi, W)(\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ be a m-s equilibrium. If $W = X$, ϕ is linear. If ϕ is linear then $M^m = \tilde{M}$. If $M^m \neq \tilde{M}$ then $W \subsetneq X$.*

We then consider the other extreme case $W = \{0\}$. From Theorem 1, item 6, the compensation schemes defined when $W = \{0\}$ are the only candidate equilibrium schemes even if the manager were to face milder trading restrictions, $W \neq \{0\}$.

Let $((\phi, W)(\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ be a m-s equilibrium and let $\tilde{y} = \hat{y} - \phi(\hat{y}) = (\mathrm{Id} - \phi)(\hat{y})$ be the equilibrium net production plan. As $N(\tilde{\lambda}) \in \mathcal{C}(\tilde{y})$ and $M^m \in \mathcal{C}(\tilde{y})$, there exists a measurable function $h : \mathbb{R}_+ \to \mathbb{R}_+$ such that

$$\frac{N(\tilde{\lambda})}{M^m} = h(\tilde{y}).$$

From (9), we obtain that

$$\phi'(\hat{y})u'(\phi(\hat{y})) = \mu(1 - \phi'(\hat{y}))h(\tilde{y})u'(\hat{y} - \phi(\hat{y})), \tag{10}$$

or

$$\phi'(z)\phi(z)^{\gamma-1} = \mu(1 - \phi'(z))(z - \phi(z))^{\gamma-1}h(z - \phi(z)), \text{ for all } z. \tag{11}$$

Equations (10) and (11) provide first-order conditions at $\hat{y}$ such that the manager and representative shareholder are aligned in the choice of the production plan. Note that while $\hat{y}$ does not appear explicitly in Equation (11), it enters in the definition of h as that involves the Lagrange multipliers associated to the equilibrium net production plan $\tilde{y} = \hat{y} - \phi(\hat{y})$.

Although (11) seems to be a standard differential equation, it raises a fixed-point problem. Indeed, the solutions of the above differential equation depend on h, which depends on the production equilibrium associated to Y^ϕ. An equilibrium compensation ϕ must be such that the differential equation associated to the production equilibrium of Y^ϕ has ϕ as solution. However, for a given ϕ the existence of a production equilibrium associated to Y^ϕ is not guaranteed.[13] When h and ϕ are independent, Equation (11) may be solved to determine ϕ. This is the case when the shareholders have the same beliefs, as we consider in Section 5. It is also the case when the net production set (and the net production equilibrium) are taken as primitive as h is then exogenously given, as detailed in Section 6.

5 Homogeneous Shareholders

In this section, we consider the case in which all shareholders and the manager have the same belief, and the case in which all shareholders have the same belief, but that is different from the manager's belief. As mentioned, in these cases, Equation (11) can be solved explicitly and this allows to build useful intuitions for the more general cases considered in Section 6.

5.1 The Common Belief Case

Suppose that shareholders and the manager have identical beliefs. We show that in this case a m-s equilibrium can be implemented with a linear compensation, where we say that ϕ is linear if there exists $0 < \alpha < 1$ such that $\phi^\alpha(y) = \alpha y$. In fact, when $M^i = M^m$ for all i, $h(z) = 1$ and from Equation (11), we obtain the following result.

Corollary 5 *Assume $M^i = M^m$ for all i. If there exists a production equilibrium $((\hat{c}^i)_i, \hat{q}, \hat{y})$ associated to Y such that $\hat{q} \in \mathcal{C}(\hat{y})$, then any linear compensation ϕ^α with $0 < \alpha < 1$ is an*

[13] Along the same line, to show that $((\phi, W), (\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ is a m-s equilibrium, given ϕ, we must prove existence of a production equilibrium $((\hat{c}^i)_i, \hat{q}, \tilde{y})$ associated to Y^ϕ. We then have to find a ϕ such that the maximum of $U_m(\phi(y))$ on Y exists and is reached at $\hat{y}$ defined by $\hat{y} - \phi(\hat{y}) = \tilde{y}$. We are thus led to find a zero of a correspondence $\phi \rightrightarrows (\mathrm{Id} - \phi)^{-1}(\tilde{y}) - \arg\max_Y U_m(\phi(y))$. However, this correspondence may not be well defined as for a given ϕ the existence of a production equilibrium associated to Y^ϕ is not guaranteed nor that of a solution to the problem $\max_Y U_m(\phi(y))$.

equilibrium compensation.

It might seem puzzling that the compensation is defined up to a multiplicative constant α. In order to define such α, one may assume that the manager has a reservation utility level U^* so that we must have $U^m(\phi(\hat{y})) \geq U^*$ (condition RU) and that shareholders minimize the compensation cost, that is, $\hat{q} \cdot \phi(\hat{y}) \leq \hat{q} \cdot \varphi(\hat{y})$ for any other equilibrium compensation φ (condition CC). Under these conditions, it is easy to show that α is uniquely pinned down as

$$\alpha = \left(\frac{U^*}{U^m(\hat{y})} \right)^{1/\gamma}.$$

One can also show that, among the compensation schemes that implement the equilibrium production plan $\hat{y}$ and satisfy condition RU, the linear compensation scheme is the only one satisfying condition CC (see Appendix B).

5.2 A Unique Shareholder Example

Let us now consider a situation in which there is a unique shareholder whose belief is M (or, equivalently, all shareholders have the common belief M) while the manager has a belief M^m. We express these beliefs as a function of the equilibrium production $\hat{y}$, assumed to follow a log-normal distribution. More precisely, the shareholder believes that $\ln \hat{y} \sim \mathcal{N}(a, \sigma^2)$ while the manager believes that $\ln \hat{y} \sim N(b, \sigma^2)$. We focus on the case in which disagreement is large, and we say that the manager is very pessimistic relative to the shareholder when $a - b \geq (1-\gamma)\sigma^2$, and that she is very optimistic relative to the shareholder when $a-b \leq -(1-\gamma)\sigma^2$. The following corollary defines some properties of the equilibrium compensation in this setting.

Corollary 6 *Let $((\phi, W), (\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ be a m-s equilibrium and let us assume that the shareholders believe that $\ln \hat{y} \sim \mathcal{N}(a, \sigma^2)$ while the manager believes that $\ln \hat{y} \sim \mathcal{N}(b, \sigma^2)$. If the manager is very pessimistic relative to the shareholder, the share of the production left to the shareholders should be bounded above. If she is very optimistic relative to the shareholder, her compensation should be bounded above.*

Hence, when the manager is optimistic, the equilibrium compensation should moderate the manager's optimism by lowering her compensation rate when the realized production is very large. Similarly, when the manager is pessimistic, the compensation rate should instead be large when production is large. These assumptions are clearly special, and they are expressed directly in terms of $\hat{y}$, but they allow to convey our intuition in the simplest form. A more general treatment of disagreement is developed in the next section.

6 Heterogeneous Shareholders

As mentioned in the previous section, in the case of heterogenous beliefs, Equation (11) involves a fixed-point problem as h depends on ϕ. In this section, we overcome this dependence by assuming that the net production set is given as primitive and that there exists a production equilibrium on that set. As h is then a data of the model, h and ϕ are independent and Equation (11) may be explicitly solved to determine ϕ. We first characterize the solutions of Equation (11) that fulfill $\phi(0) = 0$ and provide conditions insuring that such a solution is an equilibrium compensation. Hence, this compensation be embedded into a m-s equilibrium associated to a well-chosen gross production set. We next analyze the effect of belief heterogeneity on the shape of the compensation. Finally, a fully worked-out example illustrates our methodology.

6.1 From Gross to Net Production

The setting is as follows: Let $\mathcal{Y}$ be a set for which there exists a production equilibrium $((\hat{c}^i)_i, \hat{q}, \tilde{y})$ such that $\tilde{y}$ has a positive density on $(0, a)$ for $a \in R_+ \cup \{\infty\}$ and $\hat{q} \in \mathcal{C}(\tilde{y})$. The set $\mathcal{Y}$ may be considered as the net production set. Let $\tilde{\lambda}$ be the production equilibrium Lagrange multipliers. From equation (6), as $\hat{q} \in \mathcal{C}(\tilde{y})$, $N(\tilde{\lambda}) \in \mathcal{C}(\tilde{y})$. Assume that $M^m \in \mathcal{C}(\tilde{y})$. Then $\frac{N(\tilde{\lambda})}{M^m} \in \mathcal{C}(\tilde{y})$. Let $h : (0, a) \to R_+$ be the measurable function such that

$$\frac{N(\tilde{\lambda})}{M^m} = h(\tilde{y}). \tag{12}$$

Let ϕ_μ with $\phi_\mu(0) = 0$ verify the following differential equation:

$$\phi_\mu'(z)\phi_\mu(z)^{\gamma-1} = \mu(1 - \phi_\mu'(z))(z - \phi_\mu(z))^{\gamma-1}h(z - \phi_\mu(z)), z \in R_+. \tag{13}$$

It may easily be seen that a solution ϕ_μ of (13) has the property that $\phi_\mu(z) \neq 0$ if $z \neq 0$ and is such that ϕ_μ and $(\mathrm{Id} - \phi_\mu)$ are strictly increasing .

Our next goal is to provide conditions on h that insure the existence of solutions to (13) defined on R_+ that fulfill $\phi_\mu(0) = 0$ and to characterize them. This will allow us to state technical conditions insuring that the list $((\phi_\mu, \{0\}), (\hat{c}^i)_i, \phi_\mu(\hat{y}), \hat{q}, \hat{y})$ with $\hat{y}$ defined by $\hat{y} - \phi_\mu(\hat{y}) = \tilde{y}$, is a m-s equilibrium associated to the gross production set $Y_\mu = (\mathrm{Id} - \phi_\mu)^{-1}(\mathcal{Y})$.

To this end, let us consider the compensation as a function of net production, in other words, the function ψ_μ such that

$$\phi_\mu(y) = \psi_\mu(y - \phi_\mu(y)), \text{ for all } y. \tag{14}$$

As the function $\mathrm{Id} - \phi_\mu$ is strictly increasing, it is invertible and

$$\psi_\mu(z) = \phi_\mu((\mathrm{Id} - \phi_\mu)^{-1}(z)), \text{ for all } z \in (0, a).$$

Conversely as $\mathrm{Id} + \psi_\mu = (\mathrm{Id} - \phi_\mu + \phi_\mu) \circ (\mathrm{Id} - \phi_\mu)^{-1} = (\mathrm{Id} - \phi_\mu)^{-1}$, from (14) we obtain:

$$\phi_\mu(z) = \psi_\mu((\mathrm{Id} + \psi_\mu)^{-1}(z)), \text{ for all } z \in (0, a). \tag{15}$$

Let us show that ψ_μ verifies a differential equation which solutions may be given in closed form. Differentiating (14), we get

$$\phi'(z) = (1 - \phi'(z))\psi'(z - \phi(z)).$$

From Equation (13), we obtain that:

$$\psi'_\mu(z-\phi_\mu(z))\psi_\mu(z-\phi_\mu(z))^{\gamma-1} = \mu(z-\phi_\mu(z))^{\gamma-1}h(z-\phi_\mu(z)), \text{ for all } z,$$

or equivalently

$$\psi'_\mu(z)\psi_\mu(z)^{\gamma-1} = \mu z^{\gamma-1}h(z), \text{ for all } z \in (0,a). \tag{16}$$

Let us assume that h satisfies the following condition.[14]

Assumption (H) For $\gamma > 0$, the integral of $h(u)u^{\gamma-1}$ converges at 0. For $\gamma < 0$, the integral of $h(u)u^{\gamma-1}$ is divergent at 0 and convergent at a when $a = \infty$.

Let ψ_μ be the solution of (16) that verifies $\psi_\mu(0) = 0$, We then have

$$\begin{cases} \psi_\mu(z)^\gamma = \gamma\mu \int_0^z h(u)u^{\gamma-1}du & \text{when } \gamma > 0, \\ \psi_\mu(z)^\gamma = C - \gamma\mu \int_z^a h(u)u^{\gamma-1}du & \text{when } \gamma < 0. \end{cases} \tag{17}$$

Under Assumption (H), ψ_μ is well defined, continuous and increasing. Recalling (15), we then have

$$\phi_\mu(z) = \psi_\mu((\mathrm{Id} + \psi_\mu)^{-1}(z)). \tag{18}$$

Then ϕ_μ is well defined, satisfies $\phi_\mu(0) = 0$. As usual $\phi_\mu : X_+ \to X_+$ and $\psi_\mu : X_+ \to X_+$ are defined by $\phi_\mu(y)(\omega) = \phi_\mu(y(\omega))$ and $\psi_\mu(y)(\omega) = \psi_\mu(y(\omega))$ a.e.

Finally, let the gross production set Y_μ be defined by $Y_\mu = \mathcal{Y} + \psi_\mu(\mathcal{Y})$. Note that $Y_\mu = (\mathrm{Id} - \phi_\mu)^{-1}\mathcal{Y}$. Hence, it is the natural gross production set when the net production set is given by $\mathcal{Y}$ and the compensation in terms of net (resp. gross) production is given by ψ_μ (resp. ϕ_μ).

We can now state an existence result for an m-s equilibrium associated to Y_μ and for the existence of an equilibrium compensation ϕ_μ.

Theorem 7 *Let $\mathcal{Y}$ be smooth and let $((\hat{c}^i)_i, \hat{q}, \tilde{y})$ be a production equilibrium associated to $\mathcal{Y}$ such that $\hat{q} \in \mathcal{C}(\tilde{y})$ and $M^m \in \mathcal{C}(\tilde{y})$. Let $\mathcal{Y}$ be considered as the net production set and ψ_μ defined by (17) be the compensation as a function of the net production. Let the compensation as a function of the gross production ϕ_μ be given by (15) and the gross production set be given by $Y_\mu = \mathcal{Y} + \psi_\mu(\mathcal{Y})$. Assume further (C) and (H) and $u \circ \psi_\mu$ strictly concave, then $((\phi_\mu, \{0\}), (\hat{c}^i)_i, \phi_\mu(\hat{y}), \hat{q}, \hat{y})$ with $\hat{y} = \tilde{y} + \psi_\mu(\tilde{y})$ is a m-s equilibrium associated to the production set $Y_\mu = \mathcal{Y} + \psi_\mu(\mathcal{Y})$.*

In Appendix C, we provide conditions under which there exists a production equilibrium $((\hat{c}^i)_i, \hat{q}, \tilde{y})$ associated to $\mathcal{Y}$ and conditions under which $((\phi_\mu, \{0\}), (\hat{c}^i)_i, \phi_\mu(\hat{y}), \hat{q}, \tilde{y} + \psi_\mu(\tilde{y}))$ is a m-s equilibrium associated to the production set $Y_\mu = \mathcal{Y} + \psi_\mu(\mathcal{Y})$.

6.2 Properties of the Equilibrium Compensation

In this subsection, we consider the same setting as in 6.1 and we wish to highlight some properties of the compensation schemes defined by Equations (17) and (14). In particular, our aim is to highlight the effect of heterogenous beliefs on the shape of the compensation. We introduce an assumption that implies some boundary properties on h. We say that shareholder i is more

[14] The Assumptions at 0 are, for instance, satisfied if there exists some $\varepsilon > 0$ such that $\lim_{z\to 0} h(z)z^{\gamma-\varepsilon} = 0$ when $\gamma > 0$, and $h(z)z^\gamma$ is bounded away from 0 at 0 when $\gamma < 0$.

optimistic than shareholder j (or that shareholder j is more pessimistic than shareholder i) with respect to the net production y if[15]

$$\lim_{y\to\infty}\frac{M^i}{M^j}=\infty \text{ and } \lim_{y\to 0}\frac{M^i}{M^j}=0. \tag{19}$$

Note that this definition generalizes the definition of optimism/pessimism introduced in Section 5.2. Indeed, if i believes that $\ln y \sim N(a_i,\sigma^2)$, we have that $\frac{M^i}{M^j}$ is proportional to $y^{\frac{(a_i-a_j)}{\sigma^2}}$.

We assume that there exists some shareholder i more optimistic than the manager and some shareholder j more pessimistic than the manager with respect to $\tilde{y}=\hat{y}-\phi_\mu(\hat{y})$. In particular, there exists a shareholder i and a shareholder j such that

$$\lim_{\tilde{y}\to\infty}\frac{M^i}{M^m}=\infty \text{ and } \lim_{\tilde{y}\to 0}\frac{M^m}{M^j}=0. \tag{20}$$

It is then easy to check that,

$$\lim_{\tilde{y}\to\infty}\frac{N(\tilde{\lambda})}{M^m}=\lim_{\tilde{y}\to 0}\frac{N(\tilde{\lambda})}{M^m}=\infty. \tag{21}$$

The reason is that

$$\frac{N(\tilde{\lambda})}{M^m}>\frac{\tilde{\lambda}^i M^i}{M^m} \text{ for all } i,$$

and that holds in particular for any shareholder who is more optimistic than the manager. That gives the result in (21) for $\tilde{y}\to\infty$. Similarly, we have

$$\frac{M^m}{N(\tilde{\lambda})}<\frac{M^m}{\tilde{\lambda}^j M^j},$$

where j is the most pessimistic shareholder. That gives the result in (21) for $\tilde{y}\to 0$. Therefore, under Assumption(20), from Equation (12), $h(\tilde{y})=\frac{N(\tilde{\lambda})}{M^m}\to\infty$ as $\tilde{y}\to\infty$ or $\tilde{y}\to 0$ or equivalently

$$\lim_{z\to 0}h(z)=\lim_{z\to\infty}h(z)=\infty. \tag{22}$$

This can be expressed by considering the compensation rate, defined by the function $\chi:(0,\infty)\to\mathbb{R}_+$ such that

$$\chi_\mu(z)=\frac{\psi_\mu(z)}{z},\ z\in(0,\infty).$$

We can show the following result.

Corollary 8 *Assume that $M^i\neq M^j$ for some i and j. Assume that (H) and (20) hold. If there exists a m-s equilibrium, then the compensation rate in terms of net production verifies*

$$\begin{aligned}\lim_{z\to 0}\chi_\mu(z)=\lim_{z\to\infty}\chi_\mu(z)=0,\ \textit{for } \gamma<0,\\ \lim_{z\to 0}\chi_\mu(z)=\lim_{z\to\infty}\chi_\mu(z)=\infty,\ \textit{for } \gamma>0.\end{aligned} \tag{23}$$

Corollary 8 shows that the equilibrium compensation is such that the instantaneous utility

[15] We assume that the support of the net production plan is $(0,\infty)$. All the results can be easily adapted to the $(0,a)$ case.

of the manager for extreme realizations is much larger (in absolute value) than what she would reach with a linear compensation. Relative to a linear compensation, this induces her to attach more importance to how the different production plans behave in terms of extreme realizations. As the ratio between the representative shareholder's and the manager's belief is such that $\tilde{M}/M^m \to \infty$ as $z \to \infty$ or $z \to 0$, from Equation (16), the compensation counterbalances this effect by having that the ratio between the manager's and the representative shareholder's marginal utilities goes to infinity as $z \to \infty$ or $z \to 0$. As $\lim_{z\to 0} h(z) = \lim_{z\to\infty} h(z) = \infty$, it follows from equation (16) that the ratio

$$\frac{\psi'_\mu(z)\psi_\mu(z)^{\gamma-1}}{z^{\gamma-1}} \to \infty,$$

as $z \to 0$ or $z \to \infty$. As shown in the proof of Corollary 8, this implies that $\frac{(\psi_\mu(z))^\gamma}{z^\gamma} = \chi_\mu(z)^\gamma$ tends to infinity as $z \to 0$ or $z \to \infty$, which gives the properties of $\chi_\mu(z)$ depending on the sign of γ as stated in (23).

Let us see the consequences of (23) on ϕ_μ, that is the compensation defined in terms of gross production, and let us define

$$\varkappa_\mu(z) = \frac{\phi_\mu(z)}{z}, \ z \in (0,\infty).$$

Corollary 9 *Assume $M^i \neq M^j$ for some i and j. Suppose Assumption (H) and (20) hold. If there exists a m-s equilibrium, then the compensation rate verifies*

$$\lim_{z\to 0} \varkappa_\mu(z) = \lim_{z\to\infty} \varkappa_\mu(z) = 0, \ \text{for } \gamma < 0,$$
$$\lim_{z\to 0} \varkappa_\mu(z) = \lim_{z\to\infty} \varkappa_\mu(z) = 1, \ \text{for } \gamma > 0.$$

Until now, in this section, we assumed that there is a shareholder who is more pessimistic than the manager and another one who is more optimistic. When the manager is more optimistic (pessimistic) than all the shareholders, we have the following result.

Corollary 10 *Assume (H) and that the manager is more optimistic than all the shareholders. If there exists a m-s equilibrium, then the compensation rates verify*

$$\chi_\mu(z) \to_0 \infty, \ \chi_\mu(z) \to_\infty 0, \ \varkappa_\mu(z) \to_0 1 \ \text{and} \ \varkappa_\mu(z) \to_\infty 0.$$

If the manager is more pessimistic than all the shareholders, then

$$\chi_\mu(z) \to_0 0, \ \chi_\mu(z) \to_\infty \infty, \ \varkappa_\mu(z) \to_0 0 \ \text{and} \ \varkappa_\mu(z) \to_\infty 1.$$

6.3 Example

Let the net production set Y^ϕ be given by

$$\mathcal{Y}_{a,b,\theta_0} = \left\{ \exp\left(m(\theta)t + \theta\sqrt{t}\widetilde{x} \right) : \theta \geq 0 \right\},$$

where $\widetilde{x} \sim \mathcal{N}(0,1)$, $m(\theta) = a - b(\theta - \theta_0)^2$ and a, b and θ_0 are given positive constants. The value of θ is set by the manager and a given choice of θ generates the production plan

$y_\theta = \exp\left(m(\theta)t + \theta\sqrt{t}\widetilde{x}\right)$.

We assume there are two shareholders with CRRA utility as in (2) with $\gamma = -1$. Shareholders only consume at time t and they have heterogeneous beliefs indexed by β. A shareholder of type β believes that $\widetilde{x} \sim \mathcal{N}(\beta, 1)$ and we assume $\beta \in \{\delta, -\delta\}$ with $\delta > 0$. We have

$$M^\beta = \frac{1}{\sqrt{2\pi}} \exp -\frac{(\widetilde{x} - \beta)^2}{2}, \;\; \beta \in \{\delta, -\delta\}.$$

All densities are expressed with respect to Lebesgue measure. We denote with ν_β the proportion of stocks of shareholder of type β. In order to ensure the existence of a production equilibrium, we assume that

$$\frac{1}{\delta^2}(1 - 2b) + \frac{1}{1 + \exp -\frac{\delta^2}{2}} < 0. \tag{24}$$

Proposition 11 *Under (24), there exists a unique net production equilibrium* $(\hat{c}^\delta, \hat{c}^{-\delta}, \hat{q}, \tilde{y})$ *such that* $\hat{q} \in C(\tilde{y})$ *with* $\tilde{y} = \exp\left(m(\tilde{\theta})t + \tilde{\theta}\sqrt{t}\widetilde{x}\right)$ *and*

$$\tilde{\theta} = \frac{2b}{2b+1}\theta_0 + \frac{1}{\sqrt{t}}\frac{\delta}{2b+1}\left(2\nu_\delta - 1\right), \tag{25}$$

$$\begin{aligned}
\hat{q} &= \left(\lambda_{\tilde{\theta}} \exp -\frac{(\widetilde{x} - \delta)^2}{4} + \exp -\frac{(\widetilde{x} + \delta)^2}{4}\right)^2 (\tilde{y})^{-2}, \\
\hat{c}^\delta &= \frac{\lambda_{\tilde{\theta}} \exp -\frac{(\widetilde{x} - \delta)^2}{4}}{\lambda_{\tilde{\theta}} \exp -\frac{(\widetilde{x} - \delta)^2}{4} + \exp -\frac{(\widetilde{x} + \delta)^2}{4}} \tilde{y}, \\
\hat{c}^{-\delta} &= \frac{\exp -\frac{(\widetilde{x} + \delta)^2}{4}}{\lambda_{\tilde{\theta}} \exp -\frac{(\widetilde{x} - \delta)^2}{4} + \exp -\frac{(\widetilde{x} + \delta)^2}{4}} \tilde{y}
\end{aligned}$$

where $\lambda_{\tilde{\theta}}$ *the square root of shareholder's* δ *equilibrium utility weight when that of shareholder* $-\delta$ *is normalized to one is the nonnegative solution of*

$$\lambda^2 \exp(-\tilde{\theta}\delta\sqrt{t})\nu_{-\delta} + \lambda \exp(-\frac{\delta^2}{2})(2\nu_{-\delta} - 1) + \exp(\tilde{\theta}\delta\sqrt{t})(\nu_{-\delta} - 1) = 0. \tag{26}$$

We notice from (25) that the equilibrium production plan induces a larger θ and so a larger exposure to the random variable $\widetilde{x}$ when shareholder of type δ has at least half of the shares ($2\nu_\delta > 1$) and is very optimistic (δ is large). If shareholders have the same endowment ($2\nu_\delta = 1$), $\hat{\theta}$ is always lower than θ_0 since any $\theta > \theta_0$ would increase the exposure to risk and at the same time decrease $m(\theta)$.

Let the representative shareholder belief be defined by

$$\tilde{M} = \frac{\left(\lambda_{\tilde{\theta}} \exp -\frac{(\widetilde{x} - \delta)^2}{4} + \exp -\frac{(\widetilde{x} + \delta)^2}{4}\right)^2}{E\left[\left(\lambda_{\tilde{\theta}} \exp -\frac{(\widetilde{x} - \delta)^2}{4} + \exp -\frac{(\widetilde{x} + \delta)^2}{4}\right)^2\right]}.$$

Note that as $C(\tilde{y}) = C(\widetilde{x})$, $\hat{q} \in C(\tilde{y})$ and $\tilde{M} \in C(\tilde{y})$ and this example fulfills the hypotheses of the previous section.

From Corollary 4, if $((\phi, W), (\hat{c}^{\delta}, \hat{c}^{-\delta}), \hat{c}_m, \hat{q}, \hat{y})$ with $\hat{c}_m = \phi(\hat{y})$ and $\tilde{y} = \hat{y} - \phi(\hat{y}))$ is a m-s equilibrium without trading restrictions on the manager, then ϕ must be linear. Furthermore, if ϕ is linear, then the manager implements the equilibrium plan $\hat{y}$ only if she has the representative shareholder belief ($M^m = \tilde{M}$). This means that unless the manager belief is given by $\tilde{M}$, we have $W \neq X$ and the existence of an m-s equilibrium requires the manager to be constrained.

Remark 12 *Suppose that the manager receives a linear compensation. Let $E^m[\widetilde{x}]$ and $\mathrm{VAR}^m[\widetilde{x}]$ denote respectively the mean and the variance of $\widetilde{x}$ as perceived by the manager. We then have*

$$E^m[\widetilde{x}] = \frac{(\lambda_{\tilde{\theta}}^2 - 1)\delta}{\lambda_{\tilde{\theta}}^2 + 1 + 2\lambda_{\tilde{\theta}} \exp -\frac{\delta^2}{2}}.$$

$$\mathrm{VAR}^m[\widetilde{x}] = \frac{\left(\lambda_{\tilde{\theta}}^2 + 1\right)(1+\delta^2) + 2\lambda_{\tilde{\theta}} \exp\left(-\frac{\delta^2}{2}\right)}{\lambda_{\tilde{\theta}}^2 + 1 + 2\lambda_{\tilde{\theta}} \exp -\frac{\delta^2}{2}} - \left(\frac{(\lambda_{\tilde{\theta}}^2 - 1)\delta}{\lambda_{\tilde{\theta}}^2 + 1 + 2\lambda_{\tilde{\theta}} \exp -\frac{\delta^2}{2}}\right)^2.$$

We remark (see the appendix) that $E^m[\widetilde{x}]$ is a weighted average between the mean as perceived by the optimistic shareholder (δ) and that perceived by the pessimistic shareholder ($-\delta$), with weight depending on ν_δ. When $\nu_\delta = 0$, $\lambda_{\tilde{\theta}} = 0$ and $E^m[\widetilde{x}] = -\delta$ while if $\nu_\delta = 1$, $\lambda_{\tilde{\theta}} = \infty$ and $E^m[\widetilde{x}] = \delta$. Moreover, $E^m[\widetilde{x}] > 0$ if and only if $\lambda_{\tilde{\theta}} > 1$, which is true if ν_δ is sufficiently large. On the other hand, while shareholders agree that $\mathrm{VAR}[\widetilde{x}] = 1$, we have (see the appendix) $\mathrm{VAR}^m[\widetilde{x}] > 1$. Hence, the manager should overestimate the variance of $\widetilde{x}$. In other words, a manager with a linear compensation evaluating the risk as shareholders would underestimate the level of risk relative to what would lead her to choose $\hat{y}$. Hence, without constraints ($W = X$), the m-s equilibrium exists only if the manager overestimates the risk with respect to the shareholders.

Assuming next that the manager cannot trade, let us search for a compensation ψ in terms of net production such that $((\phi, \{0\}), (\hat{c}^{\delta}, \hat{c}^{-\delta}), \hat{c}_m, \hat{q}, \hat{y})$ with ϕ defined by (14), $\hat{y} = \tilde{y} + \psi(\tilde{y})$ and $\hat{c}_m = \phi(\hat{y})$ is a m-s equilibrium. Let us assume that the manager has a belief which coincides with the objective one ($\widetilde{x} \sim \mathcal{N}(0,1)$) and that shareholders have the same endowment,

$$\nu_\delta = \nu_{-\delta} = \frac{1}{2}. \tag{27}$$

Condition (27) implies

$$\lambda_{\tilde{\theta}} = \exp\left(\delta\tilde{\theta}\sqrt{t}\right).$$

It is also convenient to define the variable

$$k = \frac{\delta}{2\tilde{\theta}\sqrt{t}}.$$

In order to ensure that the resulting compensation is nonnegative for all $y \in (0, \infty)$, we need to impose $2k \leq 1$. The compensation is given by Equation (17) with

$$h(y) = \exp\left(4k\tilde{\theta}^2 t\right) \exp(-2k\mu(\tilde{\theta})t) y^{2k} + \exp(2km(\tilde{\theta})t) y^{-2k} + \exp\left(2k\tilde{\theta}^2 t\right).$$

From (17), we obtain

$$\psi_{(\mu,C)}(y) = \frac{\mu y}{C\mu y + 1 + \frac{y^{-2k}}{2k+1}\exp(2kt(m(\tilde{\theta}) - \tilde{\theta}^2)) - \frac{y^{2k}}{2k-1}\exp(2kt(\tilde{\theta}^2 - m(\tilde{\theta})))}, \tag{28}$$

where μ and C are arbitrary positive scalars. Note that $u \circ \psi_{(\mu,C)} = -\frac{1}{\psi_{(\mu,C)}}$ is a negative combination of a constant and of three negative power functions. Hence $u \circ \psi_{(\mu,C)}$ is concave. We can then state the following:

Proposition 13 *Suppose that $\nu_\delta = 1/2$, $2k \leq 1$ and that the manager correctly believes that $\widetilde{x} \sim \mathcal{N}(0,1)$. There exists a two parameters family of compensation functions of net production which implement a m-s equilibrium associated to the net production set $\mathcal{Y}$ given by Equation (28).*

The corresponding compensation in terms of gross production follows from (14):

$$\phi_{(\mu,C)}(z) = \psi_{(\mu,C)}((\mathrm{Id} + \psi_{(\mu,C)})^{-1}(z))$$

It is easy to verify that $\lim_{z\to 0} h(z) = \lim_{z\to\infty} h(z) = \infty$. Hence, the compensation in (28) has the properties highlighted in Corollary 8: the compensation rate $\chi(y)$ goes to zero as $y \to 0$ or $y \to \infty$. Further properties are illustrated in a simple numerical example.

Suppose $C = 0, a = \frac{1}{2}(\theta_0 - 1)$ and $b = \frac{1}{2(\theta_0 - 1)}$. That gives $\tilde{\theta} = 1$ and $m(\tilde{\theta}) = 0$. Suppose also $\mu = 1$, $t = 1$, $\theta_0 = 3/2$ and $\delta = 1/2$ (shareholders' beliefs deviate from the objective belief by $\frac{1}{2}$ standard deviation). In Figure 1, we plot the resulting compensation $\psi(y)$ (left panel) and compensation rate $\chi(y)$ (right panel) as a function of the net production. We observe that the compensation rate $\chi(y)$ is inverted U-shaped while the compensation $\psi(y)$ is convex as $y \to 0$ and concave as $y \to \infty$ (the convexity close to zero is not easily showed in the graph as it would require a change of scale close to zero). In Figure 2, we plot the corresponding compensations as function of the gross production, and we observe that those functions have the same shape as the ones in Figure 1.

In order to see the effect of shareholder heterogeneity, notice first that if shareholders agreed on the true distribution of $\widetilde{x}$ (that is, $\delta = -\delta = 0$) we would have $k = 0$ and expression (28) would give $\psi(y) = \tau y$ and (the linear compensation). In Figure 1, we also plot an equilibrium compensation rate $\chi(y)$ and compensation $\psi(y)$ for $\delta = 1/4$ (dashed lines) and $\delta = 3/4$ (dotted lines). In Figure 2, we plot the corresponding compensations as function of the gross production. As intuitive, the larger shareholders heterogeneity is (the larger δ), the larger are the required deviations from linearity.

7 Conclusion

We have shown that, when shareholders have different beliefs, reaching a manager-shareholders equilibrium requires imposing some trading restrictions on the manager. As already mentioned, when prices are given and the manager faces no trading restrictions, the only feature of a compensation that matters is its market value. However, in our general equilibrium setting, prices are endogenous, an equilibrium compensation cannot just be summarized by its market value, its shape as a function of the production realization matters. This is in particular analyzed for well chosen families of beliefs.

While we have defined equilibrium compensations when shareholders can trade any asset contingent on the firm's output and the manager is prevented from trading, an interesting next step would be to consider intermediate cases in which shareholders have access to an incomplete market or in which the manager may face milder trading restrictions.[16] We have shown that the set of compensation schemes identified when the manager has no access to the market are the only equilibrium candidates even under milder trading restrictions. More generally, one could investigate which minimal trading restrictions would allow to achieve a manager-shareholders equilibrium.

Another natural extension would be to introduce multiple firms. This would allow to consider a richer contracting space, both in terms of trading restrictions and in terms of compensation, and possibly to investigate issues of competition among firms for managerial talent. One could also consider more general forms of endowment for shareholders. These are in our view very interesting avenues for future research.

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8 Figures

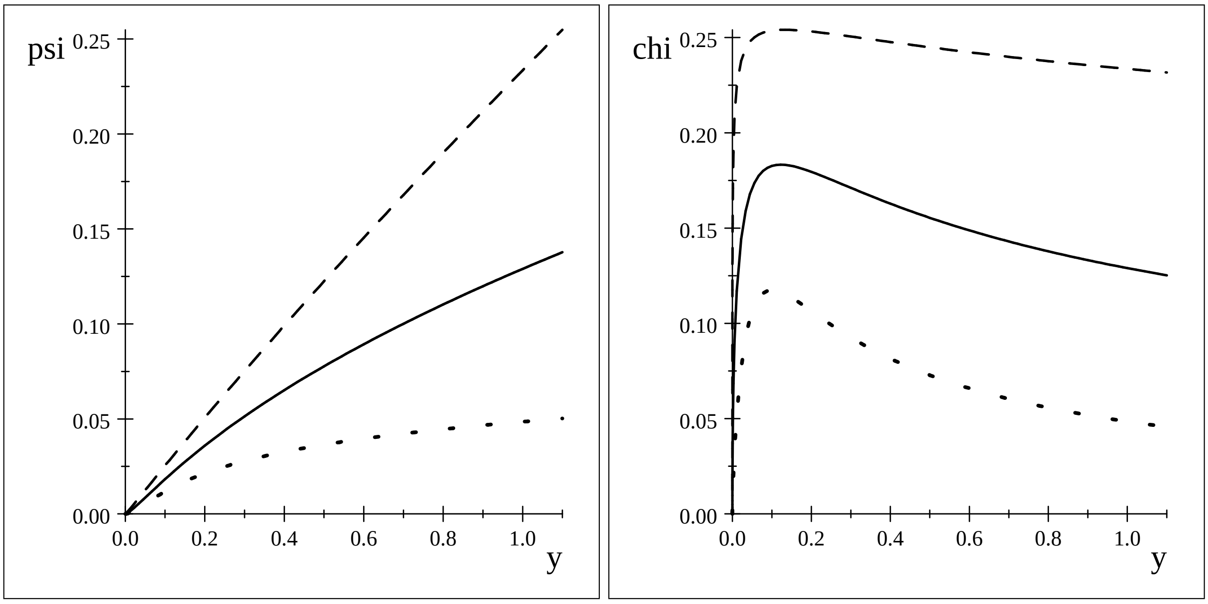

Figure 1. Equilibrium compensation $\psi(y)$ (left panel) and compensation rate $\chi(y)$ (right panel) as a function of the net production for $\delta = 1/4$ (dashed), $\delta = 1/2$ (solid) and $\delta = 3/4$ (dotted).

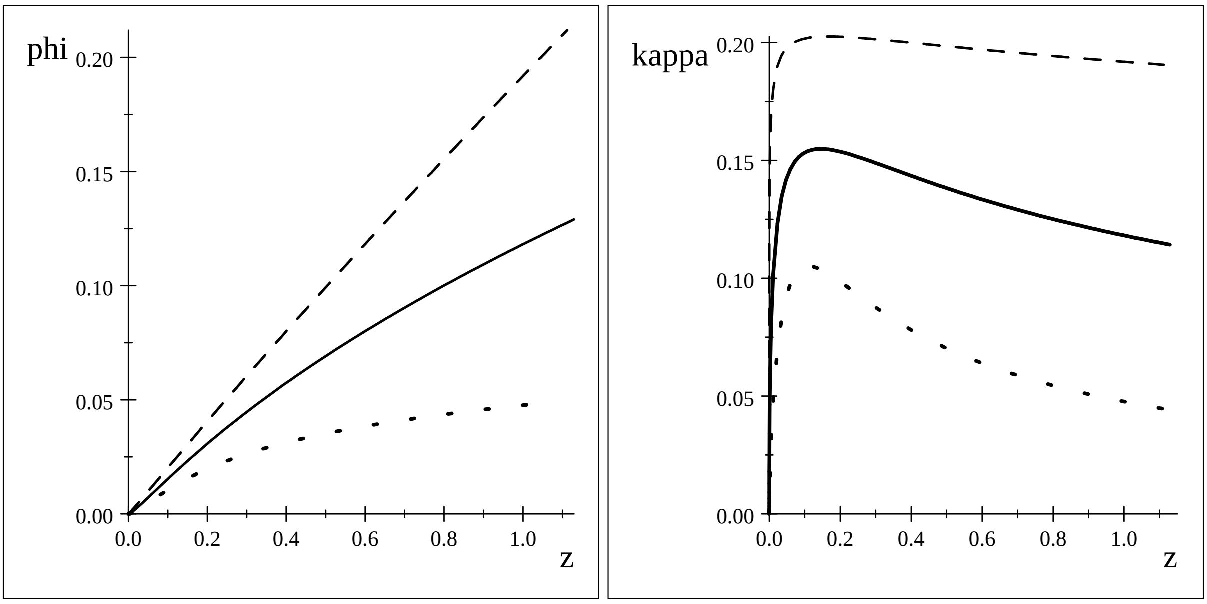

Figure 2. Equilibrium compensation $\phi(z)$(left panel) and compensation rate $\varkappa(z)$(right panel) as a function of the gross production for $\delta = 1/4$(dashed), $\delta = 1/2$(solid) and $\delta = 3/4$(dotted).

9 Appendix A

Proof of Theorem 1. The proof that $z \to \phi(z)$ and $z \to z - \phi(z)$ are monotone and differentiable is in Appendix B, where we also show that when some consumption plan c does not satisfy the market constraints $c \in \mathcal{C}(y)$ for some $y \in Y$, the market constraints are satisfied for some slight perturbation of y.

PROOF OF THE SHAREHOLDERS' INCOME MAXIMIZATION PROPERTY

We first show that at equilibrium, the price $\hat{q}$ should be strictly positive. We then show that $\hat{y}$ should maximize on Y_+ any shareholder's income at price $\hat{q}$,i.e. $\hat{q}\cdot(\hat{y}-\phi(\hat{y})) = \max_Y \hat{q}\cdot(y-\phi(y))$ and $\hat{y} \in \mathrm{Eff}^+(Y)$. Let us first show that $\hat{q} \gg 0$. Indeed if for some t, $\hat{q}_t \leq 0$ on a set A of positive measure, the shareholders' and manager's demand could be arbitrarily large on A violating assertion 3 of the definition of equilibrium.

Let (ϕ, W) be given and $((\hat{c}^i)_i, \hat{c}_m, \hat{q}, \hat{y})$ be a m-s equilibrium. From items 2 and 4 in Definition 1, the manager solves the following problem:

$$\max_{y \in Y} \max_{c} U_m(c) \text{ s.t. } c \in \mathcal{C}^m(y) \text{ and } \hat{q} \cdot c \leq \hat{q} \cdot \phi(y). \tag{29}$$

As $\hat{c}_m$ solves Assertion 2 and U_m is increasing, $\hat{c}_m = \phi(\hat{y}) + \hat{w}$ for some $\hat{w} \in W \cap \mathcal{C}(\hat{y})$ and $\hat{q} \cdot \hat{c}_m = \hat{q} \cdot \phi(\hat{y})$. Let c be a consumption plan of the form, $c = \phi(y) + \hat{w}$ for some $y \in K$ such that $U_m(c)$ is well defined with $U_m(c) > U_m(\hat{c}_m)$. Since ϕ is nondecreasing, we have $\min\left[u\left(\phi(y) + \hat{w}\right), u\left(\phi(y)\right)\right] \leq u\left(\phi(y) + (1-\varepsilon)\,\hat{w}\right) \leq \max\left[u\left(\phi(y) + \hat{w}\right), u\left(\phi(y)\right)\right]$ and $U_m(\phi(y) + (1-\varepsilon)\,\hat{w})$ is well defined. We have $u((1-\varepsilon)\left(\phi(y) + \hat{w}\right)) = (1-\varepsilon)^{\gamma}\, u(\phi(y) + \hat{w})$ and $U_m((1-\varepsilon)\left(\phi(y) + \hat{w}\right))$ is also well-defined and $U_m((1-\varepsilon)\left(\phi(y) + \hat{w}\right)) > U_m(\hat{c}_m)$ for ε sufficiently small. Let $x = \hat{y}$ and $x' = \phi(y)$, by Lemma 14 there exists $x'' \in X_+$ such that

$0 \leq \phi(y) - x'' \leq \varepsilon\phi(y)$ and $\mathcal{C}(\hat{y}) \subset \mathcal{C}(x'')$. Since ϕ is continuous nondecreasing with $\phi(0) = 0$ and since we have $x'' \leq \phi(y)$, we may define y'' pointwise by $y''(\omega) = \inf\{z : \phi(z) = x''(\omega)\}$. We have $\phi(y'') = x''$, $\mathcal{C}(\hat{y}) \subset \mathcal{C}(x'') \subset \mathcal{C}(y'')$ and $y'' \leq y$ which gives $y'' \in Y$ If we take $c'' = x'' + (1-\varepsilon)\,\hat{w} = \phi(y'') + (1-\varepsilon)\,\hat{w}$, we have

$$(1-\varepsilon)\,(\phi(y) + \hat{w}) \leq c'' \leq \phi(y) + (1-\varepsilon)\,\hat{w}$$

hence $U_m(c'')$ is well defined with $U_m(c'') > U_m(\hat{c}_m)$, $\hat{w} \in \mathcal{C}(y'')$ and $\hat{q} \cdot (\phi(y'') + (1-\varepsilon)\,\hat{w}) = \hat{q} \cdot \phi(y'')$ which contradicts (29). Therefore, we have

$$U_m(\phi(\hat{y}) + \hat{w}) \geq \max_K U_m(\phi(y) + \hat{w}).$$

Assume that there exists $y' \in K$ such that $\hat{q}\cdot(y' - \phi(y')) > \hat{q}\cdot(\hat{y} - \phi(\hat{y}))$. As ϕ is $1-$Lipschitz and by Lemma 14, there exists y'' close to y' such that $\hat{q}\cdot(y'' - \phi(y'')) > \hat{q}\cdot(\hat{y} - \phi(\hat{y}))$, $\mathcal{C}(\hat{y}) \subset \mathcal{C}(y'')$ and $y'' \in Y$. From the definition of the indirect utility, we have $V^i(y'', \hat{q}) > V^i(\hat{y}, \hat{q})$ which violates assertion 4 of the definition of equilibrium. Therefore, $\hat{q} \cdot (\hat{y} - \phi(\hat{y})) \geq \max_K \hat{q} \cdot (y - \phi(y))$.

Let us assume that there exists $y' \in Y$ such that $y' > \hat{y}$. Without loss of generality, we may assume that $y' \in K$. We have $U_m(\phi(\hat{y}) + \hat{w}) \geq U_m(\phi(y') + \hat{w})$ and since U_m is increasing we have $\phi(\hat{y}) = \phi(y')$ on $A = \{y' > \hat{y}\}$. This gives $\hat{q} \cdot (y' - \phi(y'))1_A > \hat{q} \cdot (\hat{y} - \phi(\hat{y}))1_A$ and $\hat{q} \cdot (y' - \phi(y'))1_{\Omega\backslash A} = \hat{q} \cdot (\hat{y} - \phi(\hat{y}))1_{\Omega\backslash A}$ which contradicts the fact that $\hat{q} \cdot (\hat{y} - \phi(\hat{y})) \geq \max_K \hat{q} \cdot (y - \phi(y))$. Hence $\hat{y} \in K$ and from there

$$\hat{q} \cdot (\hat{y} - \phi(\hat{y})) = \max_K \hat{q} \cdot (y - \phi(y)) = \max_Y \hat{q} \cdot (y - \phi(y)), \tag{30}$$

$$U_m(\phi(\hat{y}) + \hat{w}) = \max_K U_m(\phi(y) + \hat{w}) = \max_Y U_m(\phi(y) + \hat{w}). \tag{31}$$

Let $\partial\phi(z) = \overline{co}\left\{\lim_{h\to 0} \frac{\phi(z+h)-\phi(z)}{h}\right\}$. As ϕ is nondecreasing and $1-$Lipschitz, we have $\partial\phi(z) = \left[\phi'_-(z), \phi'_+(z)\right] \subset [0,1]$ for all z with $\phi'_-(z) = \phi'_+(z) = \phi'(z)$ when $\phi'(z)$ exists (which is the case almost everywhere on $\mathbb{R}_+$). Let $y \in Y$, $v = y - \hat{y}$ and $0 < \varepsilon < 1$. We have $\hat{y} + \varepsilon v \in Y$ and $\hat{q} \cdot (\hat{y} - \phi(\hat{y})) - \hat{q} \cdot (\hat{y} + \varepsilon v - \phi(\hat{y} + \varepsilon v)) = -\hat{q} \cdot \phi(\hat{y})) - \hat{q} \cdot (\varepsilon v - \phi(\hat{y} + \varepsilon v)) \geq 0$ or

$$-\hat{q} \cdot v + \hat{q} \cdot \frac{\phi(\hat{y} + \varepsilon v) - \phi(\hat{y})}{\varepsilon} \geq 0.$$

We have $0 \leq \hat{q}\left(1 - \frac{\phi(\hat{y}+\varepsilon v)-\phi(\hat{y})}{\varepsilon v}\right) v 1_{v\geq 0}$ hence, by Fatou's Lemma, $E\left[\hat{q}v1_{v\geq 0}\right] - E\left[\hat{q}\phi'_+(\hat{y})v1_{v\geq 0}\right] \leq E\left[\hat{q}v1_{v\geq 0}\right] - \limsup_{\varepsilon\to 0} E\left[\hat{q}\frac{\phi(\hat{y}+\varepsilon v)-\phi(\hat{y})}{\varepsilon}1_{v\geq 0}\right]$ or $E\left[\hat{q}\phi'_+(\hat{y})v1_{v\geq 0}\right] \geq \limsup_{\varepsilon\to 0} E\left[\hat{q}\frac{\phi(\hat{y}+\varepsilon v)-\phi(\hat{y})}{\varepsilon}1_{v\geq 0}\right]$. Similarly, from $0 \leq -\hat{q}\frac{\phi(\hat{y}+\varepsilon v)-\phi(\hat{y})}{\varepsilon v} v 1_{v\leq 0}$ we derive $-E\left[\hat{q}\phi'_-(\hat{y})v1_{v\leq 0}\right] \leq -\liminf_{\varepsilon\to 0} E\left[\hat{q}\frac{\phi(\hat{y}+\varepsilon v)-\phi(\hat{y})}{\varepsilon}1_{v\leq 0}\right]$

or $E\left[\hat{q}\phi'_{-}(\hat{y})v1_{v\leq 0}\right] \geq \liminf_{\varepsilon\to 0} E\left[\hat{q}\frac{\phi(\hat{y}+\varepsilon v)-\phi(\hat{y})}{\varepsilon}1_{v\leq 0}\right]$. Therefore,

$$
\begin{aligned}
&E\left[\hat{q}\phi'_{+}(\hat{y})v1_{v\geq 0}\right] + E\left[\hat{q}\phi'_{-}(\hat{y})v1_{v\leq 0}\right] \\
&\geq \limsup_{\varepsilon\to 0} E\left[\hat{q}\frac{\phi(\hat{y}+\varepsilon v)-\phi(\hat{y})}{\varepsilon}1_{v\geq 0}\right] + \liminf_{\varepsilon\to 0} E\left[\hat{q}\frac{\phi(\hat{y}+\varepsilon v)-\phi(\hat{y})}{\varepsilon}1_{v\leq 0}\right] \\
&\geq \liminf_{\varepsilon\to 0} E\left[\hat{q}\frac{\phi(\hat{y}+\varepsilon v)-\phi(\hat{y})}{\varepsilon}\right] \qquad (32)\\
&\geq \hat{q}v.
\end{aligned}
$$

To prove (32), it suffices to consider a sequence (ε_n) such that $E\left[\hat{q}\frac{\phi(\hat{y}+\varepsilon_n v)-\phi(\hat{y})}{\varepsilon}1_{v\leq 0}\right]$ converges to $\liminf_{\varepsilon\to 0} E\left[\hat{q}\frac{\phi(\hat{y}+\varepsilon v)-\phi(\hat{y})}{\varepsilon}1_{v\leq 0}\right]$. The sequence $E\left[\hat{q}\frac{\phi(\hat{y}+\varepsilon_n v)-\phi(\hat{y})}{\varepsilon}1_{v\geq 0}\right]$ is bounded above by $E\left[\hat{q}v1_{v\geq 0}\right]$ and (ε_n) admits a subsequence that converges to some $\ell \leq \limsup_{\varepsilon\to 0} E\left[\hat{q}\frac{\phi(\hat{y}+\varepsilon v)-\phi(\hat{y})}{\varepsilon}1_{v\geq 0}\right]$. Along this subsequence, $E\left[\hat{q}\frac{\phi(\hat{y}+\varepsilon_n v)-\phi(\hat{y})}{\varepsilon}\right]$ converges to $\ell+ \liminf_{\varepsilon\to 0} E\left[\hat{q}\frac{\phi(\hat{y}+\varepsilon v)-\phi(\hat{y})}{\varepsilon}1_{v\leq 0}\right]$ which gives $\ell+ \liminf_{\varepsilon\to 0} E\left[\hat{q}\frac{\phi(\hat{y}+\varepsilon v)-\phi(\hat{y})}{\varepsilon}1_{v\leq 0}\right] \geq \liminf_{\varepsilon\to 0} E\left[\hat{q}\frac{\phi(\hat{y}+\varepsilon v)-\phi(\hat{y})}{\varepsilon}\right]$ then (32).

To summarize, we have $\hat{q}(1-\phi'_{+}(\hat{y})1_{v\geq 0}-\phi'_{-}(\hat{y})1_{v\leq 0})\cdot v \leq 0$ for all v. Therefore, we have $p_1 = \hat{q}(1-\phi'_{+}(\hat{y})1_{v\geq 0}-\phi'_{-}(\hat{y})1_{v\leq 0}) \in N_Y(\hat{y})$.

We also have $U_m(\phi(\hat{y})+\hat{w}) - U_m(\phi(y+\varepsilon v)+\hat{w}) \geq 0$ and $\frac{U_m(\phi(\hat{y})+\hat{w})-U_m(\phi(\hat{y}+\varepsilon v)+\hat{w})}{\varepsilon v}v \geq 0$ and, by Fatou's Lemma, $E\left[\left(\phi'_{+}(\hat{y})1_{v\geq 0}+\phi'_{-}(\hat{y})1_{v\leq 0}\right)M_m u'(\phi(\hat{y})+\hat{w})v\right] \leq 0$ for all v which leads to $p_2 = \left(\phi'_{+}(\hat{y})1_{v\geq 0}+\phi'_{-}(\hat{y})1_{v\leq 0}\right)M_m u'(\phi(\hat{y})+\hat{w}) \in N_Y(\hat{y})$. Note that $p_2 = 0$ if and only if $\left(\phi'_{+}(\hat{y})1_{v\geq 0}+\phi'_{-}(\hat{y})1_{v\leq 0}\right) = 0$ which implies $p_2 \neq 0$. We have then $0 \ll \hat{p} = p_1 + p_2 \in N_Y(\hat{y})$ and $\hat{y} \in \mathrm{Eff}^+(Y)$. Therefore, $\hat{y}$ has a density $h_{\hat{y}}$ with $h_{\hat{y}} > 0$, $\mu-$a.e. on $\mathbb{R}_+^*$ and $\phi'_{+}(\hat{y}) = \phi'_{-}(\hat{y}) = \phi'(\hat{y})$ a.e. Since $\hat{y} \in \mathrm{Eff}^+(Y)$, $N_Y(\hat{y})$ is generated by a single vector, $(1-\phi'(\hat{y}))\hat{q}$ and $\phi'(\hat{y})U'_m(\phi(\hat{y})+\hat{w})$ are proportional.

Proof of the no-trade property

From Assertion 5 of Definition 1

$$\max_{y\in Y} E[M^m u(\hat{C}_m(y)] = U_m(\hat{C}_m(\hat{y}))].$$

Let us show that this implies that $\hat{C}_m(z)$ is nondecreasing in z. Let us define

$A = \left\{(z,z') \in \left(\mathbb{R}_+^*\right)^2 : (z-z')(\hat{C}_m(z)-\hat{C}_m(z')) < 0\right\}$ and let us assume that $\mu\otimes\mu\,(A) > 0$. By Fubini, there exists z^* such that $\mu\,(B) > 0$ with $B = \left\{z > z^* : \hat{C}_m(z^*) > \hat{C}_m(z))\right\}$. Let us consider $\hat{y}'$ such that $\hat{y}' = z^*$ on $\{\hat{y} \in B\}$ and $\hat{y}' = \hat{y}$ elsewhere. We have $\hat{y}' \leq \hat{y}$ and then $\hat{y}' \in Y$ and $\hat{C}_m(\hat{y}') > \hat{C}_m(\hat{y})$ which contradicts the fact that $\hat{y}$ maximizes $U_m(\hat{C}_m(\hat{y}))$ on Y. Hence, $\hat{C}_m(z)$ is nondecreasing and $\hat{C}_m(z)$ admits a derivative with respect to z almost everywhere. Let us denote by $\partial\hat{C}_m(z) = \overline{co}\left\{\lim_{h\to 0}\frac{\hat{C}_m(z+h)-\hat{C}_m(z)}{h}\right\}$. As $\hat{C}_m$ is nondecreasing, we have $\partial\hat{C}_m(z) = \left[\hat{C}'_{m,-}(z), \hat{C}'_{m,+}(z)\right] \subset [0,\infty]$ for all z with $\hat{C}'_{m,-}(z) = \hat{C}'_{m,+}(z) = \hat{C}'_m(z)$ almost everywhere on $\mathbb{R}_+$. Let $y \in Y$, $v = y - \hat{y}$ and $0 < \varepsilon < 1$. We have $\hat{y}+\varepsilon v \in Y$ and $U_m(\hat{C}_m(\hat{y})) \geq U_m(\hat{C}_m(\hat{y})+\varepsilon v)$ and $\frac{U_m(\hat{C}_m(\hat{y}))-U_m(\hat{C}_m(\hat{y})+\varepsilon v)}{\varepsilon v}v \geq 0$ which, by Fatou's Lemma gives $E\left[\left(\hat{C}'_{m,+}(\hat{y})1_{v\geq 0}+\hat{C}'_{m,-}(\hat{y})1_{v\leq 0}\right)M_m u'(\hat{C}_m(\hat{y})(\hat{y}))v\right] \leq 0$ for all v and then $\hat{C}'_{m,+}(\hat{y})1_{v\geq 0} + \hat{C}'_{m,-}(\hat{y})1_{v\leq 0} \in N_Y(\hat{y})$. As $\hat{y} \in \mathrm{Eff}^+(Y)$, it has a density $h_{\hat{y}}$ with $h_{\hat{y}} > 0$, $\mu-$a.e. on $\mathbb{R}_+^*$and

$\hat{C}'_{m,-}(\hat{y}) = \hat{C}'_{m,+}(\hat{y}) = \hat{C}'_m(\hat{y})$ almost everywhere on Ω which gives $\hat{C}'_m(\hat{y}) \in N_Y(\hat{y})$ or

$$\hat{C}'_m(\hat{y}) M^m u'(\hat{c}_m) = \nu \hat{p}, \tag{33}$$

for some $\nu > 0$. From Equations (31) and (33), we thus obtain that :

$$\left(\hat{C}'_m - \nu\phi'\right)(\hat{y}) M^m u'(\hat{c}_m) = 0 \tag{34}$$

As $\hat{y} \in \text{Eff}^+(Y)$, it takes all possible values in $(0, \infty)$ and we have

$$\hat{C}'_m(z) = \nu\phi'(z), \ a.e.$$

Integrating with respect to z, we obtain

$$\hat{C}_m(z) = \nu\phi(z) + k \tag{35}$$

for some constant k. From item 3 of Definition 1, we have $\hat{C}_m(z) \leq z$ hence $\hat{C}_m(0) = 0$ which implies $k = 0$ and

$$\hat{C}_m(z) = \nu\phi(z), \ \forall z \in \mathbb{R}_+. \tag{36}$$

As $\hat{q} \cdot \hat{C}_m = \hat{q} \cdot \phi(\hat{y})$, $\nu = 1$ and $\hat{C}_m = \phi$.

Proof of $U_m(\phi(\hat{y})) > U_m(\phi(y))$ for all $y \in Y - \{\hat{y}\}$

To prove Assertion 4, from item 2 of Definition 1 and from the last result, we have

$$V_m(\hat{y}, \hat{q}) = U_m(\hat{C}_m(\hat{y})) = U_m(\phi(\hat{y})).$$

As $\phi(y)$ is a feasible consumption plan for the manager when she is given y, $V_m(y, \hat{q}) \geq U_m(\phi(y))$, therefore

$$V_m(\hat{y}, \hat{q}) = U_m(\phi(\hat{y})) > V_m(y, \hat{q}) \geq U_m(\phi(y)), \forall y \in Y - \{\hat{y}\},$$

proving Assertion 4.

Proof of the production equilibrium property

From the first item in the definition of the m-s equilibrium, $\hat{c}^i$ maximizes $U^i(c)$ s.t. $c \in \mathcal{C}(\hat{y})$, $\hat{q} \cdot c \leq \nu^i(\hat{q} \cdot (\hat{y} - \phi(\hat{y})))$. Let $c \notin \mathcal{C}(\hat{y})$ such that $\hat{q} \cdot c \leq \nu^i(\hat{q} \cdot (\hat{y} - \phi(\hat{y})))$. Taking the expectation conditional to $\hat{y}$, we have $\hat{q} \cdot E\left[c|\,\hat{y}\right] \leq \nu^i(\hat{q} \cdot (\hat{y} - \phi(\hat{y})))$. Since $E\left[c|\,\hat{y}\right] \in \mathcal{C}(\hat{y})$, $U^i(E\left[c|\,\hat{y}\right]) \leq U^i(\hat{c}^i)$ and, by concavity of U^i, $U^i(c) \leq U^i(\hat{c}^i)$. We therefore have that $\hat{c}^i$ maximizes $U^i(c)$ s.t. $\hat{q} \cdot c \leq \nu^i(\hat{q} \cdot (\hat{y} - \phi(\hat{y})))$, for all i. We conclude the proof of Assertion 5 thanks to Assertion 2 and the market clearing condition.

Proof of the equilibrium property for $W' \subset W$

In order to prove assertion 6, it suffices to remark that if the manager does no trade when allowed to make transactions in W, she will not trade when only allowed to make transactions in $W' \subset W$ and since all the other conditions do not involve W, we have the result. ■

Proof of Theorem 2. Let $\hat{q} \in \mathcal{C}(\hat{y})$ and $((\hat{c}^i)_i, \hat{q}, \hat{y} - \phi(\hat{y}))$ be a production equilibrium. Then $\hat{c}^i$ maximizes $U^i(c)$ s.t. $\hat{q} \cdot c \leq \nu^i(\hat{q} \cdot (\hat{y} - \phi(\hat{y})))$. As $\hat{q} \cdot \hat{c}^i = \nu^i(\hat{q} \cdot (\hat{y} - \phi(\hat{y})))$, taking the conditional expectation with respect to $\hat{y}$, we obtain that $\hat{q} \cdot E\left[\hat{c}^i \middle|\, \hat{y}\right] = \nu^i(\hat{q} \cdot (\hat{y} - \phi(\hat{y})))$. Hence

$E\left[\hat{c}^i \middle| \hat{y}\right]$ is budget feasible and from Jensen's conditional inequality, as U^i is strictly concave, $U^i(\hat{c}^i) \leq U^i(E\left[\hat{c}^i \middle| \hat{y}\right])$ with a strict inequality unless $\hat{c}^i = E\left[\hat{c}^i \middle| \hat{y}\right]$. As the strict inequality would contradict the definition of $\hat{c}^i$, we have $\hat{c}^i = E\left[\hat{c}^i \middle| \hat{y}\right]$ and therefore $\hat{c}^i \in \mathcal{C}(\hat{y})$ for every i. It also follows that $V^i(\hat{y}, \hat{q}) \geq V^i(y, \hat{q})$ for any y and i. The remainder is immediate. ■

Proof of Corollary 3. From Assertion 2 of Theorem 1, we have $\hat{q} \cdot (\hat{y} - \phi(\hat{y})) = \max_Y \hat{q} \cdot (y - \phi(y))$. From the first order condition, we have $(1 - \phi'(\hat{y}))\hat{q} \in N_Y(\hat{y})$. We also have $U_m(\phi(\hat{y})) = \max_Y U_m(\phi(y))$ which gives $\phi'(\hat{y}) M_m u(\phi(\hat{y})) \in N_Y(\hat{y})$. Finally, by definition of the representative shareholder, we have $N(\lambda^*) u'(\hat{y} - \phi(\hat{y})) = \nu \hat{q}$. Since all elements in $N_Y(\hat{y})$ are positively proportional, we have the result. ■

Proof of Corollary 4. The 2 first assertions are from Bianchi et al. (2021). The third one is immediate. ■

Proof of the strict monotonicity of ϕ and $\mathrm{Id} - \phi$. Since ϕ is nondecreasing, let I an interval on which it is constant. We have $\phi' = 0$ on I and then $\mu(1 - \phi'(z))(z - \phi(z))^{\gamma - 1} g(z) = 0$ on I. Since all the beliefs are equivalent and since $\hat{y}$ is such that $h_{\hat{y}} > 0$, we have $g > 0$ a.e. on I. Furthermore, $\phi'(z) = 0 \neq 1$ on I and ϕ cannot be equal to z a.e. on I. We have then $\mu = 0$ and $\phi' = 0$ everywhere and ϕ is constant. Since $\phi(0) = 0$ we have $\phi = 0$ which is excluded by condition 4 in the definition of a m-s equilibrium. We have then that ϕ is increasing. The result on $z \to z - \phi(z)$ is derived similarly. ■

Proof of Corollary 5. When $M^i = M^m =: M$ for all i, $h = 1$ and (11) becomes: for some $\mu > 0$

$$\phi'(z)\phi(z)^{\gamma-1} = \mu(1 - \phi'(z))(z - \phi(z))^{\gamma-1}, \text{ for all } z \geq 0.$$

It may easily be verified that for any $\gamma < 1$ and any $0 < \alpha < 1$, $\phi^\alpha(z) = \alpha z$ is a solution of the differential equation above. If there exists a production equilibrium $((\hat{c}^i)_i, \hat{q}, \hat{y})$ associated to Y such that $\hat{q} \in C(\hat{y})$, $((1-\alpha)\hat{c}^i, \hat{q}, (1-\alpha)\hat{y})$ is a production equilibrium associated to $(1-\alpha)Y$ such that $\hat{q} \in C(\hat{y})$ and $\hat{y}$ solves $\max_Y E(Mu(y))$. By homogeneity of u, it also solves $\max_Y E(Mu(\phi^\alpha(y))$. The list $((\phi^\alpha, \{0\}), ((1-\alpha)\hat{c}^i)_i, \alpha\hat{y}, \hat{q}, \hat{y})$ thus fulfills the conditions of theorem 2. ■

Proof of Corollary 6. It is easy to see that $h(\tilde{y}) = \exp\left(\frac{b-a}{\sigma^2} \ln \hat{y}\right) \exp(\frac{a^2-b^2}{\sigma^2}) = \beta \hat{y}^\alpha$ with $\beta = \exp(\frac{a^2-b^2}{\sigma^2})$ and $\alpha = \frac{b-a}{\sigma^2}$. Equation (11) becomes $\phi'(z)\phi^{\gamma-1}(z) = \beta\mu(1 - \phi'(z))(z - \phi(z))^{\gamma-1}\phi(z)^\alpha$ or $\phi'(z) = \frac{\beta\mu z^\alpha \phi(z)^{1-\gamma}}{(z-\phi(z))^{1-\gamma} + \beta\mu z^\alpha \phi(z)^{1-\gamma}} = A(z, \phi(z))$ with $A(z,s) = \frac{\beta\mu z^\alpha \mu s^{1-\gamma}}{(z-s)^{1-\gamma} + \beta\mu z^\alpha \mu s^{1-\gamma}}$ and $\frac{\partial A}{\partial xz} = z^{\alpha-1} s^{-\gamma+1} \frac{\beta\mu}{(z-s)^\gamma} \frac{-z+z\alpha+z\gamma-s\alpha}{\left((z-s)^{-\gamma+1} + z^\alpha s^{-\gamma+1}\beta\mu\right)^2}$ which has the sign of $-z + z\alpha + z\gamma - s\alpha = 0$ that cancels only for $z = s\frac{\alpha}{\alpha+\gamma-1}$. Hence, for $\frac{\alpha}{\alpha+\gamma-1} > 1$ or $\alpha > 1 - \gamma$, we have $A(s,s) = 1$, $\lim A_{z\to\infty}(z,s) = 1$ and $A(s\frac{\alpha}{\alpha+\gamma-1}, s) = \frac{\left(s\frac{\alpha}{\alpha+\gamma-1}\right)^\alpha \beta\mu}{\left(\frac{1-\gamma}{\alpha+\gamma-1}\right)^{1-\gamma} + \left(s\frac{\alpha}{\alpha+\gamma-1}\right)^\alpha \beta\mu} < 1$ is then a minimum for $A(z,s)$ on $[s, \infty)$. We have then $\phi'(z) \geq \frac{1}{1+B\phi(z)^{-\alpha}}$ with $B = \frac{1}{\beta\mu}\left(\frac{\alpha}{\alpha+\gamma-1}\right)^{-\alpha}\left(\frac{1-\gamma}{\alpha+\gamma-1}\right)^{1-\gamma}$ which easily leads to $\lim_\infty \phi(z) = \infty$ and $\lim_\infty \phi'(z) = 1$. We have then $\phi'(z)\left(1 + B\phi(z)^{-\alpha}\right) \geq 1$ and integrating between 1 and z gives $\phi(z) - \phi(1) + \frac{1}{(1-\alpha)} B\left(\phi^{1-\alpha}(z) - \phi^{1-\alpha}(1)\right) \geq z - 1$. We have seen that $\lim_\infty \phi(z) = \infty$ hence, for $\sup(1, 1-\gamma) < \alpha$, we have $\lim_\infty \phi^{1-\alpha}(z) = 0$ which gives $\lim_\infty \frac{\phi(z)}{z} = 1$ and $0 \geq \phi(z) - z \geq -\phi(1) - \frac{1}{(1-\alpha)} B\left(\phi^{1-\alpha}(z) - \phi^{1-\alpha}(1)\right) \geq -\phi(1) + \frac{1}{(1-\alpha)} B\phi^{1-\alpha}(1)$ and the share $\hat{y} - \phi(\hat{y})$ of the production left to the shareholder is bounded above.

Remark that replacing α by $-\alpha$, and $\phi(z)$ by $z - \phi(z)$ does not modify our differential equation above which means that the behavior of manager's compensation $\phi(\hat{y})$ for $\alpha > 0$ is the same as the behavior of the share of the production $\hat{y} - \phi(\hat{y})$ left to the shareholder for $\alpha < 0$ and conversely. The case $-\alpha > \sup(1, 1-\gamma)$ is then immediate. ■

Proof of Corollary 8. For $\gamma > 0$, $\psi_\mu(z)^\gamma = \gamma\mu \int_0^z h(u) u^{\gamma-1} du$ and $\frac{\psi_\mu(z)^\gamma}{z} \sim_0 \gamma\mu h(z) z^{\gamma-1}$. Hence $\left(\frac{\psi_\mu(z)}{z}\right)^\gamma = z^{1-\gamma} \frac{\psi_\mu(z)^\gamma}{z} \sim_0 \gamma\mu h(z)$ and $\frac{\psi_\mu(z)}{z} \to_0 \infty$. Let u be given, for u sufficiently large, we have $h(u) > M$ and $\frac{\psi_\mu(z)^\gamma}{z} = \gamma\mu \frac{1}{z} \int_0^z h(u) u^{\gamma-1} du > \mu M z^{\gamma-1}$ for z sufficiently large. Hence, for z sufficiently large, we have $\left(\frac{\psi_\mu(z)}{z}\right)^\gamma > \gamma\mu M$ and $\frac{\psi_\mu(z)}{z} \to_\infty \infty$. The case $\gamma < 0$ is treated similarly. ■

Proof of Corollary 9. We have $\phi(z) = \psi_\mu((\mathrm{Id} + \psi_\mu)^{-1}(z))$ and $\varkappa_\mu(z) = \frac{\psi_\mu((\mathrm{Id}+\psi_\mu)^{-1}(z))}{(\mathrm{Id}+\psi_\mu)^{-1}(z)} \frac{(\mathrm{Id}+\psi_\mu)^{-1}(z)}{z}$. If $x = (\mathrm{Id} + \psi_\mu)^{-1}(z)$, we have $\psi_\mu(x) + x = z$ then $x \le z$ and, in particular, $\frac{(\mathrm{Id}+\psi_\mu)^{-1}(z)}{z} \le 1$ and $\lim_{z\to 0} (\mathrm{Id} + \psi_\mu)^{-1}(z) = 0$. Since ψ_μ is nondecreasing and defined on $\mathbb{R}_+$, it is also easy to check that $\lim_{z\to\infty} (\mathrm{Id} + \psi_\mu)^{-1}(z) = \infty$ If $\gamma < 0$, $\lim_{z\to 0,\infty} \frac{\psi_\mu(z)}{z} = 0$ and then $\lim_{z\to 0,\infty} \frac{\psi_\mu((\mathrm{Id}+\psi_\mu)^{-1}(z))}{(\mathrm{Id}+\psi_\mu)^{-1}(z)} = 0$ which gives $\lim_{z\to 0,\infty} \varkappa_\mu(z) = 0$. If $\gamma > 0$, we have $\frac{x\chi_\mu(x)}{z} + \frac{x}{z} = 1$ and the second term is negligible with respect to the first one for x (or equivalently for z) near to 0 or to ∞. Since we have $\frac{x\chi_\mu(x)}{z} = \frac{\psi_\mu(x)}{z} = \varkappa_\mu(z)$, it comes $\lim_{z\to 0,\infty} \varkappa_\mu(z) = 1$.

Let us consider the case where the manager is more optimistic than all the shareholders. we have then $\lim_{\tilde{y}\to\infty} \frac{N(\tilde{\lambda})}{M^m} = 0$ and $\lim_{\tilde{y}\to 0} \frac{N(\tilde{\lambda})}{M^m} = \infty$, $\lim_\infty h(z) = 0$ and $\lim_0 h(z) = \infty$. ■

Proof of Corollary 10. If the manager is more optimistic than all the shareholders and $\gamma > 0$, the function $g : v \to h(v^{\frac{1}{\gamma}})$ is such that $\lim_{s\to\infty} g(s) = 0$ and then $\lim_{s\to\infty} \frac{1}{s}\int_0^s g(v) dv = 0$ and $\lim_{z\to\infty} \frac{\psi_\mu(z)^\gamma}{z^\gamma} = \lim_{z\to\infty} \gamma\mu \frac{1}{z^\gamma} \int_0^{z^\gamma} h(v^{\frac{1}{\gamma}}) dv = 0$. The limit in 0, the case $\gamma < 0$ and the case where the manager is more pessimistic than all the shareholders are treated similarly. The properties on $\varkappa$ are derived as in the proof of Corollary 9. ■

Proof of Proposition 11. Let $y_\theta = \exp\left(m(\theta)t + \theta\sqrt{t}\widetilde{x}\right)$ where θ is given. Let us show that the y_θ-exchange equilibrium $\left(c^{\theta,\delta}, c^{\theta,-\delta}, q_\theta\right)$ is given by

$$q_\theta = \left(\frac{y_\theta}{\lambda_\theta \exp -\frac{(\widetilde{x}-\delta)^2}{4} + \exp -\frac{(\widetilde{x}+\delta)^2}{4}}\right)^{-2}$$

$$c^{\theta,\delta} = \frac{\lambda_\theta \exp -\frac{(\widetilde{x}-\beta)^2}{4}}{\lambda_\theta \exp -\frac{(\widetilde{x}-\delta)^2}{4} + \exp -\frac{(\widetilde{x}+\delta)^2}{4}} y_\theta$$

$$c^{\theta,-\delta} = \frac{\exp -\frac{(\widetilde{x}+\delta)^2}{4}}{\lambda_\theta \exp -\frac{(\widetilde{x}-\delta)^2}{4} + \exp -\frac{(\widetilde{x}+\delta)^2}{4}} y_\theta,$$

where λ_θ the square root of shareholder δ equilibrium utility weight (when that of the other shareholder is normalized to one) is the nonnegative solution of

$$\lambda^2 \exp(-\theta\delta\sqrt{t})\nu_{-\delta} + \lambda \exp(-\frac{\delta^2}{2})(2\nu_{-\delta} - 1) + \exp(\theta\delta\sqrt{t})(\nu_{-\delta} - 1) = 0. \quad (37)$$

We have

$$q_\theta = \lambda_\theta^2 M_\delta u'\left(c^{\theta,\delta}\right) = M_{-\delta} u'(c^{\theta,-\delta}) \text{ and } y_\theta = c^{\theta,\delta} + c^{\theta,-\delta}$$

and $\left(c^{\theta,\delta}, c^{\theta,-\delta}\right)$ satisfies the first-order conditions for utility maximization as well as the market clearing condition. We need to check that the budget constraint is also satisfied, i.e.

$E\left[q_\theta c^{\theta,-\delta}\right] = \nu_{-\delta} E\left[q_\theta y_\theta\right]$. After simple calculations, this constraint appears to be equivalent to (37). It is also straightforward to show that (37) admits only one positive solution. Let us finally find $\hat{\theta}$ such that $(\hat{c}^\delta, \hat{c}^{-\delta}, \hat{q}, \tilde{y})$ is a production equilibrium. Since $\left(c^{\tilde{\theta},\delta}, c^{\tilde{\theta},-\delta}, q_{\tilde{\theta}}\right)$ is already a $y_{\tilde{\theta}}$-exchange equilibrium, we need only take care of the profit maximization constraint. For this purpose, let us define $g(\theta, \sigma) = E\left[q_\theta y_\sigma\right]$ and let $\tilde{\theta}$ be such that $g_\sigma(\tilde{\theta}, \tilde{\theta}) = 0$ which can be rewritten as

$$\tilde{\theta} = \frac{2b}{2b+1} s_0 + \frac{1}{\sqrt{t}} \frac{\delta}{2b+1} \left(1 - 2\nu_{-\delta}\right).$$

For such a $\tilde{\theta}$, $\tilde{y} = y_{\tilde{\theta}}$ satisfies the first order condition for profit maximization. Under condition (24), we can show that $g_\sigma(\tilde{\theta}, \sigma)$ is positive for $\sigma < \tilde{\theta}$ and negative for $\sigma > \tilde{\theta}$ which means that $g(\tilde{\theta}, \sigma)$ reaches its maximum for $\sigma = \tilde{\theta}$ and $\left(c^{\tilde{\theta},\delta}, c^{\tilde{\theta},-\delta}, q_{\tilde{\theta}}\right)$ is a production equilibrium,

■

Proof of Remark 12. The analytic expressions of $E^m\left[\widetilde{x}\right]$ and $\mathrm{Var}^m\left[\widetilde{x}\right]$ are obtained through straightforward computations. From there, we easily derive,

$$-\delta \leq \frac{\lambda_{\tilde{\theta}} - 1}{\lambda_{\tilde{\theta}} + 1}\delta \leq E^m\left[\widetilde{x}\right] \leq \frac{\lambda_{\tilde{\theta}}^2 - 1}{\lambda_{\tilde{\theta}}^2 + 1}\delta \leq \delta,$$

hence if $\lambda_{\tilde{\theta}} < 1$ and $\lambda_{\tilde{\theta}} \neq 0$ (or $\nu_\delta \neq 0$), we have

$$\mathrm{Var}^m\left[\widetilde{x}\right] > 1 + \delta^2 \frac{(\lambda_{\tilde{\theta}}^2 + 1)}{(\lambda_{\tilde{\theta}} + 1)^2} - \delta^2 \frac{(\lambda_{\tilde{\theta}} - 1)^2}{(\lambda_{\theta^*} + 1)^2} > 1.$$

While for $\lambda_{\tilde{\theta}} \geq 1$ and if $\lambda_{\tilde{\theta}} \neq \infty$ (or $\nu_\delta \neq 1$), we have

$$\mathrm{Var}^m\left[\widetilde{x}\right] > 1 + \delta^2 \frac{(\lambda_{\tilde{\theta}}^2 + 1)}{(\lambda_{\tilde{\theta}} + 1)^2} - \delta^2 \left(\frac{\lambda_{\tilde{\theta}}^2 - 1}{\lambda_{\tilde{\theta}}^2 + 1}\right)^2 > 1.$$

■

10 Online Appendix

10.1 Appendix B

Proof of Theorem 1, Point 1.. Proof of the monotonicity of $z \to \phi(z)$ and $z \to z - \phi(z)$

Let us show first that $z \to \phi(z)$ and $z \to z - \phi(z)$ are both nondecreasing a.e. in the sense $\mu \otimes \mu \left(\left\{(z,z') \in \mathbb{R}^2 : (z-z')(\varphi(z) - \varphi(z')) < 0\right\}\right) = 0$.

We have $\hat{c}_m \in \mathcal{C}(\hat{y})$ and there exists a nonnegative measurable function C such that $\hat{c}_m = C(\hat{y})$. By Lebesgue Theorem, the derivative of $z \to \int_0^z C(s)ds$ exists and is equal to $C(z)$ almost everywhere. Let $F(z) = E\left[M_m | \, \hat{y} = z\right]$ and let us assume that $\mu(G) = 0$ where $G = \{z : F(z) = 0\}$. We have $0 = \int_{z \in G} F(z) h_y(z) dz = E\left[M_m 1_{\hat{y} \in G}\right]$ and $M_m > 0$ a.e. which gives $M_m = 0$ a.e. on G and since $M_m > 0$ a.e., we have $\mu(G) = 0$.

Let D the set of points z such that $\left(\int_0^z C(s)ds\right)' = C(z)$, ϕ is continuous at z, $F(z) > 0$ and $h_y(z) > 0$. We have $\mu(\mathbb{R}_+ \backslash D) = 0$.

Let us assume that ϕ is not nondecreasing and let

$$A = \left\{(z,z') \in \left(\mathbb{R}_+^*\right)^2 : (z-z')(\phi(z) - \phi_T(z')) < 0\right\}.$$

We have, $\mu^2(A) > 0$. Without loss of generality, we may replace A by $A \cap C^2$.

Let $(a,b) \in A$ with $a < b$ and then

$$\phi(a) > \phi(b) \; .$$

For $\eta > 0$, there exists $\varepsilon > 0$ such that $|\phi(z) - \phi(a)| < \eta\phi(a)$ for $z \in I = [a-\varepsilon, a+\varepsilon)$ and $|\phi(z) - \phi(b)| < \eta\phi(b)$ for $z \in J = [b-\varepsilon, b+\varepsilon)$. If η and ε are chosen such that $\frac{1-\eta}{1+\eta}\frac{\phi(a)}{\phi(b)} > 1$ and $\varepsilon < \frac{b-a}{2}$, we have $I < J$ and $\phi(I) > \phi(J)$. It is immediate that $\phi(a) < \frac{1}{1-\eta}\phi(z)$ for $z \in I$ and $\phi(b) < \frac{1}{1-\eta}\phi(z)$ for $z \in J$.

Let $f : \mathbb{R} \to \mathbb{R} \setminus J$ be defined by $f(z) = \frac{1}{2}(a-\varepsilon) + \frac{1}{2}z$ on I, $f(z) = a + \frac{1}{2}(z - b + \varepsilon)$ on J and $f(z) = z$ elsewhere. We have $f(z) \leq z$ for all z and f admits an inverse denoted by g.

If $z \in I$ then $f(z) \in I$, and $|\phi(z) - \phi(f(z))| < 2\eta\phi(a) < \frac{2\eta}{1-\eta}\phi(z)$. We further impose $\frac{2\eta}{1-\eta} < 1$ and we have $\phi(f(z)) > (1 - \frac{2\eta}{1-\eta})\phi(z)$.

If $z \in J$ then $f(z) \in I$ and

$$\phi(z) - \phi(f(z)) < (1+\eta)\,\phi(b) - (1-\eta)\,\phi(a) < 0. \tag{38}$$

We also have $\phi(f(z)) < (1+\eta)\,\phi(a) < k\phi(z)$ with $k = \frac{1+\eta}{1-\eta}\frac{\phi(a)}{\phi(b)} > 1$.

Elsewhere, we have $\phi(z) = \phi(f(z))$.

Let us define the random variable $\tilde{y}$ by $\tilde{y} = f(\hat{y})$. We have $\tilde{y} \leq \hat{y}$ then $\tilde{y} \in Y$. By definition of g, we have $\hat{y} = g(\tilde{y})$. They generate then the same information structure and we have $\mathcal{C}(\tilde{y}) = \mathcal{C}(\hat{y})$.

By definition of the indirect utility function, $V_m(\hat{y}, \hat{q}) = U_m(\hat{c}_m)$. Let $\hat{w}$ be defined by $\hat{w} = \hat{c}_m - \phi(\hat{y})$, we have $\hat{w} \in \mathcal{C}(\hat{y}) \cap W$ and $\hat{q} \cdot \hat{w} \leq 0$. If we define $\tilde{c}_m$ by $\tilde{c}_m = \phi(\tilde{y}) + (1 - \varepsilon\frac{2\eta}{1-\eta})\hat{w}$, $\tilde{c}_m$ is a function of $\tilde{y}$. We further impose $\varepsilon < 1$.

On $\{\hat{y} \in I\}$, $\tilde{c}_m = \phi(f(\hat{y})) + (1-\varepsilon\frac{2\eta}{1-\eta})\hat{w} > (1-\frac{2\eta}{1-\eta})(\phi(\hat{y})+\hat{w}) \geq (1-\frac{2\eta}{1-\eta})\hat{c}_m \geq 0$. From there, we derive $u(\hat{c}_m) - u(\tilde{c}_m) \leq \left(1-(1-\frac{2\eta}{1-\eta})^\gamma\right) u(\hat{c}_m)$. By the market clearing condition, we have $\hat{c}_m \leq \hat{y}$ and $u(\hat{c}_m) - u(\tilde{c}_m) \leq \left(1-(1-\frac{2\eta}{1-\eta})^\gamma\right) u(\hat{y})$.

On $\{\hat{y} \in J\}$, $\tilde{c}_m = \phi(f(\hat{y})) + (1-\varepsilon\frac{2\eta}{1-\eta})\hat{w} > \phi(\hat{y}) + (1-\frac{2\eta}{1-\eta})\hat{w} > (1-\frac{2\eta}{1-\eta})(\phi(\hat{y})+\hat{w}) \geq 0$. Furthermore, since u is increasing and concave, we have

$$\begin{aligned}
& u(\hat{c}_m) - u(\tilde{c}_m) \\
& \leq u\left(\phi(\hat{y})+\hat{w}\right) - u\left(\phi(f(\hat{y})) + (1-\varepsilon\frac{2\eta}{1-\eta})\hat{w}\right) \\
& \leq \left(\phi(\hat{y}) - \phi(f(\hat{y})) + \varepsilon\frac{2\eta}{1-\eta}\hat{w}\right) u'(c).
\end{aligned}$$

where $c = \hat{c}_m$ if $\phi(\hat{y}) - \phi(f(\hat{y})) + \varepsilon\frac{2\eta}{1-\eta}\hat{w} \geq 0$ and $c = \phi(f(\hat{y})) + (1-\varepsilon\frac{2\eta}{1-\eta})\hat{w} \leq k\left(\phi(\hat{y})+\hat{w}\right)$ elsewhere.

From there and using (38), we have

$$\begin{aligned}
& u(\hat{c}_m) - u(\tilde{c}_m) \\
& \leq \sup_{h=1,k} \left((1+\eta)\,\phi(b) - (1-\eta)\,\phi(a) + \varepsilon\frac{2\eta}{1-\eta}\hat{w}\right) u'\left(h\hat{c}_m\right)
\end{aligned}$$

and since $(1+\eta)\,\phi(b) - (1-\eta)\,\phi(a) < 0$, we have

$$\begin{aligned}
& u(\hat{c}_m) - u(\tilde{c}_m) \\
& \leq k^{\gamma-1}\left((1+\eta)\,\phi(b) - (1-\eta)\,\phi(a)\right) u'\left(\hat{c}_m\right) + \varepsilon\frac{2\eta}{1-\eta}\left(k^{\gamma-1}\hat{w}1_{\hat{w}\geq 0} + \hat{w}1_{\hat{w}\geq 0}\right) u'\left(\hat{c}_m\right) \\
& \leq k^{\gamma-1}\left((1+\eta)\,\phi(b) - (1-\eta)\,\phi(a)\right) u'\left(\hat{y}\right) + \varepsilon\frac{2\eta}{1-\eta}\left(k^{\gamma-1}\hat{w}1_{\hat{w}\geq 0} + \hat{w}1_{\hat{w}\geq 0}\right) u'\left(\hat{c}_m\right)
\end{aligned}$$

Finally, on $\{\hat{y} \notin I \cup J\}$, $\tilde{c}_m = \phi(\hat{y}) + (1-\varepsilon\frac{2\eta}{1-\eta})\hat{w} \geq (1-\varepsilon\frac{2\eta}{1-\eta})(\phi(\hat{y})+\hat{w}) = (1-\varepsilon\frac{2\eta}{1-\eta})\hat{c}_m \geq 0$. Furthermore, since u is increasing $u(\hat{c}_m) - u(\tilde{c}_m) \leq u(\hat{c}_m) - u\left((1-\varepsilon\frac{2\eta}{1-\eta})\hat{c}_m\right) \leq u(\hat{c}_m)\left(1-(1-\varepsilon\frac{2\eta}{1-\eta})^\gamma\right) \leq u(\hat{y})\left(1-(1-\varepsilon\frac{2\eta}{1-\eta})^\gamma\right)$.

We have then $\tilde{c}_m \geq 0$ a.e. and then $\tilde{c}_m \in \mathcal{C}^m(\tilde{y})$. Furthermore, we have $\hat{q}\cdot\tilde{c}_m = \hat{q}\cdot\phi(\tilde{y}) + \hat{q}\cdot\hat{w} \leq \hat{q}\cdot\phi(\tilde{y})$ which gives $V_m(\tilde{y},\hat{q}) \geq U_m(\tilde{c}_m)$. From Condition 4 in the equilibrium definition, we have

$$U_m(\hat{c}_m) = V_m(\hat{y},\hat{q}) > V_m(\tilde{y},\hat{q}) \geq U_m(\tilde{c}_m). \tag{39}$$

However, from the above inequalities on $u(\hat{c}_m(\omega)) - u(\tilde{c}_m(\omega))$, we have

$$\begin{aligned}
&\frac{1}{2\varepsilon}\left[U_m(\hat{c}_m) - U_m(\tilde{c}_m)\right] \\
&\quad \le \frac{1}{2\varepsilon}(1-(1-\frac{2\eta}{1-\eta})^{\gamma})E\left[M_m u(\hat{y})1_{\hat{y}\in I}\right] \\
&\quad + \frac{1}{2\varepsilon}k^{\gamma-1}\left((1+\eta)\,\phi(b) - (1-\eta)\,\phi(a)\right)E\left[M_m u'(\hat{y})\,1_{\hat{y}\in J}\right] \\
&\quad + \frac{\eta}{1-\eta}E\left[M_m\left(k^{\gamma-1}\hat{w}1_{\hat{w}\ge 0} + \hat{w}1_{\hat{w}\ge 0}\right)u'(\hat{c}_m)1_{\hat{y}\in J}\right] \\
&\quad + \frac{1}{2\varepsilon}(1-(1-\varepsilon\frac{2\eta}{1-\eta})^{\gamma})E\left[M_m u(\hat{c}_m)1_{\hat{y}\notin I\cup J}\right].
\end{aligned} \tag{40}$$

Let us show that $M_m u'(\hat{c}_m)\,\hat{w} \in L^1(\Omega, \mathcal{F}, P)$ and for this purpose let us consider separately its positive part and its negative part. Since u is increasing, the sign of $M_m u'(\hat{c}_m)\,\hat{w}$ only depends on the sign of $\hat{w}$. Let us define c_ε by $c_\varepsilon = \phi(\hat{y}) + (1-\varepsilon)\,\hat{w}$, we have $c_\varepsilon \in \mathcal{C}(\hat{y}) \cap W$ and $\hat{q}\cdot c_\varepsilon \le \hat{q}\cdot\phi(\hat{y})$ which gives $U(c_\varepsilon) \le U(\hat{c}_m)$. For $\hat{w} \ge 0$, we have $0 \le \frac{u(\hat{c}_m)-u(c_\varepsilon)}{\varepsilon} \le u'(\phi(\hat{y}))\,\hat{w}$ and since $\frac{u(\hat{c}_m)-u(c_\varepsilon)}{\varepsilon} \to u'(\hat{c}_m)\,\hat{w}$, Assumption (F) and the dominated convergence Theorem give $\lim_{\varepsilon\to 0} E\left[M_m\frac{u(\hat{c}_m)-u(c_\varepsilon)}{\varepsilon}1_{\hat{w}\ge 0}\right] = E\left[M_m u'(\phi(\hat{y}))\,\hat{w}1_{\hat{w}\ge 0}\right] < \infty$. Let us assume that $-E\left[M_m u'(\hat{c}_m)\,\hat{w}1_{\hat{w}\le 0}\right] = \infty$. Since u is increasing, we have $\left(u(c_\varepsilon) - u(\hat{c}_m)\right)1_{\hat{w}\le 0} \ge 0$ and by Fatou's Lemma, we have $\liminf_{\varepsilon\to 0} E\left[M_m\frac{u(c_\varepsilon)-u(\hat{c}_m)}{\varepsilon}1_{\hat{w}\le 0}\right] \ge -E\left[u'(\hat{c}_m)\,\hat{w}1_{\hat{w}\le 0}\right] = \infty$. From there $\lim_{\varepsilon\to 0} E\left[M_m\frac{u(c_\varepsilon)-u(\hat{c}_m)}{\varepsilon}\right] = \infty$ which gives $U(c_\varepsilon) > U(\hat{c}_m)$ for ε sufficiently small. We have then a contradiction and then $-E\left[M_m u'(\phi(\hat{y}))\,\hat{w}1_{\hat{w}\le 0}\right] < \infty$ which gives $L^1(\Omega, \mathcal{F}, P)$. As a consequence, we have

$$\lim_{\varepsilon\to 0} E\left[M_m\left(k^{\gamma-1}\hat{w}1_{\hat{w}\ge 0} + \hat{w}1_{\hat{w}\ge 0}\right)u'(\hat{c}_m)1_{\hat{y}\in J}\right] = 0.$$

Taking the limit when ε goes to 0, in (40), we obtain

$$\begin{aligned}
&\lim_{\varepsilon\to 0}\frac{1}{2\varepsilon}\left[U_m(\hat{c}_m) - U_m(\tilde{c}_m)\right] \\
&= (1-(1-\frac{2\eta}{1-\eta})^{\gamma})F(a)u(a) \\
&+ k^{\gamma-1}\left((1+\eta)\,\phi(b) - (1-\eta)\,\phi(a)\right)F(a)u'(a) \\
&+ \gamma\frac{\eta}{1-\eta}E\left[M_m u(\hat{c}_m)\right].
\end{aligned}$$

Taking the limit when η goes to 0, we obtain $k^{\gamma-1}\left(\phi(b) - \phi(a)\right)F(a)u'(a) < 0$.

Hence, for η and ε sufficiently small, $U_m(\phi(\hat{y}) + \hat{w}) < U_m(\phi(\tilde{y}) + \hat{w})$ which contradicts 39. Therefore, the compensation $z \to \phi(z)$ is increasing.

Similarly, replacing the manager by one of the shareholders, we obtain that the net production $z \to z - \phi(z)$ is increasing.

Proof of the differentiability of $z \to \phi(z)$ and $z \to z - \phi(z)$

Since $z \to \phi(z)$ and $z \to z - \phi(z)$ are nondecreasing then $z \to \phi(z)$ is 1-Lipshitz and then differentiable a.e.

Lemma 14 *For $(x, x') \in X_+^2$ and $\varepsilon > 0$, there exists $x'' \in X_+$ such that $0 \le x' - x'' \le \varepsilon x'$ and $\mathcal{C}(x) \subset \mathcal{C}(x'')$. In particular, if $x' \in Y$ then $x'' \in Y$.*

■

Proof of Lemma 14. Let $s_n = (1+\varepsilon)^{\frac{n}{2}}$ for $n \in \mathbb{Z}$. The family $S = (s_n)_{n\in\mathbb{Z}}$ is an ordered family with $\lim_{\infty} s_n = \infty$, $\lim_{-\infty} s_n = 0$. Let $z \geq 0$ and $z' > 0$. There exits $n \in \mathbb{Z}$ such that $s_n \leq z' < s_{n+1}$ and we define h by $h(z,z') = s_n - (s_n - s_{n-1})\frac{z}{1+z}$. We have $0 < (1-\varepsilon)z' < (1-\varepsilon)s_{n+1} \leq s_{n-1} < h(z,z') \leq s_n \leq z' < s_{n+1}$. Let us suppose now that we know $h(z,z')$ without knowing z nor z'. The inequalities $s_{n-1} < h(z,z') \leq s_n$ uniquely define a pair (s_n, s_{n-1}). z is uniquely determined by the equation $h(z,z') = s_n - (s_n - s_{n-1})\frac{z}{1+z}$. Let $(x,x') \in X_+^2$, and let $x'' = h(x,x')$. We have $(1-\varepsilon)x' < x'' \leq x'$ or $0 \leq x' - x'' \leq \varepsilon x'$. Furthermore, knowing x'' permits to determine x and we have $\mathcal{C}(x) \subset \mathcal{C}(x'')$. Finally, if $x' \in Y$ then $x'' \leq x'$ leads to $x'' \in Y$. ■

Corollary 15 *We assume that $M^i = M^m$ for all i, and that there exists a unique production equilibrium $((\hat{c}^i)_i, \hat{q}, \hat{y})$ that satisfies $U^m(\hat{y}) \geq U^*$, then*

1. *$((\hat{\phi}, \hat{W}), ((1-\alpha)\hat{c}^i)_i, \alpha\hat{y}, \hat{q}, \hat{y})$ with $\hat{\phi}(z) = \alpha z$ and $\alpha = \left(\frac{U^*}{U^m(\hat{y})}\right)^{1/\gamma}$ is a m-s equilibrium that satisfies (RU),*
2. *if $((\check{\phi}, \check{W}), (\check{c}^i)_i, \check{\phi}(\hat{y}), \check{q}, \hat{y})$ is a m-s equilibrium that satisfies (RU) then $\hat{q}\cdot\hat{\phi}(\hat{y}) \leq \hat{q}\cdot\check{\phi}(\hat{y})$.*

Proof of Corollary 15. The case $\gamma > 0$ is immediate. The first assertion is also immediate and we denote by $((\hat{\phi}, \hat{W}), ((1-\alpha)\hat{c}^i)_i, \alpha\hat{y}, \hat{q}, \hat{y})$ with $\hat{\phi}(z) = \alpha z$ and $\alpha = \left(\frac{U^*}{U^m(\hat{y})}\right)^{1/\gamma}$ the linear compensation m-s equilibrium that satisfies (RU). Without any loss of generality, we may assume that $M^i = 1$ for all i and we have $\hat{q} = \hat{y}^{\gamma-1}$. Let us assume that $\gamma < 0$ and let us consider $((\check{\phi}, \check{W}), (\check{c}^i)_i, \check{\phi}(\hat{y}), \check{q}, \hat{y})$ a m-s equilibrium that satisfies (RU) and that minimizes $\hat{q}\cdot\check{\phi}(\hat{y})$ among all the other compensation schemes that are compatible with $\hat{y}$. By Corollary 5, we may restrict our attention to the compensation schemes $\check{\phi} = \phi_{\mu,C}$ with $\phi_{\mu,C}^{\gamma}(z) = C + \mu(z - \phi_{\mu,C}(z))^{\gamma}$ for some (μ, C). For a given z, we have

$$\frac{d\phi_{\mu,C}(z)}{dC} = \frac{1}{\gamma\left(\mu\left(-\phi_{\mu,C}(z)+z\right)^{\gamma-1} + \phi_{\mu,C}(z)^{\gamma-1}\right)} < 0,$$
$$\frac{d\phi_{\mu,C}(z)}{d\mu} = \frac{\left(-\phi_{\mu,C}(z)+z\right)^{\gamma}}{\gamma\left(\mu\left(-\phi_{\mu,C}(z)+z\right)^{\gamma-1} + \phi_{\mu,C}(z)^{\gamma-1}\right)} < 0,$$

and $\varphi_{\mu,C}(y)$ is decreasing in μ and C. Both quantities $\hat{q}\cdot\varphi_{\mu,C}(\hat{y})$ and $U_m(\hat{c}_m) = U_m(\varphi_{\mu,C}(\hat{y}))$ are then decreasing in μ and C. The compensation cost minimization leads then to $U_m(\hat{\phi}(\hat{y})) = U^*$. We may then restrict our attention to the pairs (μ, C) that satisfy $\frac{1}{\gamma}E\left[\varphi_{\mu,C}^{\gamma}(\hat{y})\right] = U^*$. By the implicit functions Theorem, we have

$$\frac{d\mu}{dC} = -\frac{E\left[\phi_{\mu,C}^{\gamma-1}(\hat{y})\frac{1}{\left(\mu\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma-1}+\phi_{\mu,C}(\hat{y})^{\gamma-1}\right)}\right]}{E\left[\phi_{\mu,C}^{\gamma-1}(\hat{y})\frac{\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma}}{\left(\mu\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma-1}+\phi_{\mu,C}(\hat{y})^{\gamma-1}\right)}\right]}$$

and since the shareholders minimize $\hat{q} \cdot \phi_{\mu,C}(\hat{y})$, this leads to the following first order condition

$$\frac{d}{dC}\hat{q} \cdot \phi_{\mu,C}(\hat{y}) + \frac{d\mu}{dC}\frac{d}{d\mu}\hat{q} \cdot \phi_{\mu,C}(\hat{y}) = 0.$$

As $\hat{q} = \hat{y}^{\gamma-1}$, we have

$$E\left[\hat{y}^{\gamma-1}\frac{1}{\gamma\left(\mu\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma-1}+\phi_{\mu,C}(\hat{y})^{\gamma-1}\right)}\right]$$
$$-\frac{E\left[\phi^{\gamma-1}_{\mu,C}(\hat{y})\frac{1}{\left(\mu\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma-1}+\phi_{\mu,C}(\hat{y})^{\gamma-1}\right)}\right]}{E\left[\phi^{\gamma-1}_{\mu,C}(\hat{y})\frac{\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma}}{\left(\mu\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma-1}+\phi_{\mu,C}(\hat{y})^{\gamma-1}\right)}\right]}E\left[\hat{y}^{\gamma-1}\frac{\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma}}{\gamma\left(\mu\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma-1}+\phi_{\mu,C}(\hat{y})^{\gamma-1}\right)}\right]$$
$$=0$$

or

$$E\left[M\frac{\hat{y}^{\gamma-1}}{\phi^{\gamma-1}_{\mu,C}(\hat{y})}\right]E\left[M\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma}\right]-E\left[M\right]E\left[M\frac{\hat{y}^{\gamma-1}}{\phi^{\gamma-1}_{\mu,C}(\hat{y})}\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma}\right]=0$$

with $M = \phi^{\gamma-1}_{\mu,C}(\hat{y})\frac{1}{\left(\mu\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma-1}+\phi_{\mu,C}(\hat{y})^{\gamma-1}\right)}$ or, equivalently,

$$E^{*}\left[\frac{\hat{y}^{\gamma-1}}{\phi^{\gamma-1}_{\mu,C}(\hat{y})}\right]E^{*}\left[\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma}\right]-E^{*}\left[\frac{\hat{y}^{\gamma-1}}{\phi^{\gamma-1}_{\mu,C}(\hat{y})}\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma}\right]=0$$

where E^{*} is the expectation under the probability whose density with respect to P is given by $\frac{M}{E[M]}$. This condition can be rewritten as

$$\text{COV}^{*}\left(\left(\frac{\hat{y}}{\phi_{\mu,C}(\hat{y})}\right)^{\gamma-1},\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma}\right)=0.$$

On one hand, we know that$-\phi_{\mu,C}(z)+z$ increases with z and $\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma}$ is then decreasing with $\hat{y}$. On the other hand, from $\phi^{\gamma}_{\mu,C}(z) = C + \mu(z-\phi_{\mu,C}(z))^{\gamma}$ we derive $\frac{z}{\phi_{\mu,C}(z)} = \frac{1}{\phi_{\mu,C}(z)}\left(\left(\frac{\phi^{\gamma}_{\mu,C}(z)-C}{\mu}\right)^{\frac{1}{\gamma}}+\phi_{\mu,C}(z)\right) = G(\phi_{\mu,C}(z))$ with $G(z) = \frac{1}{z}\left(\left(\frac{z^{\gamma}-C}{\mu}\right)^{\frac{1}{\gamma}}+z\right)$. We have $G'(x) = C\frac{\left(-\frac{1}{\mu}(C-x^{\gamma})\right)^{\frac{1}{\gamma}}}{x^{2}(-C+x^{\gamma})}$ and G is increasing for $C>0$ and $\left(\frac{\hat{y}}{\phi_{\mu,C}(\hat{y})}\right)^{\gamma-1}$ is then decreasing with $\hat{y}$. The random variables $\left(\frac{\hat{y}}{\phi_{\mu,C}(\hat{y})}\right)^{\gamma-1}$ and $\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma}$ are then comonotonic and we necessarily have $\text{COV}^{*}\left(\left(\frac{\hat{y}}{\phi_{\mu,C}(\hat{y})}\right)^{\gamma-1},\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma}\right)>0$. Similarly, for $C<0$ we have $\text{COV}^{*}\left(\left(\frac{\hat{y}}{\phi_{\mu,C}(\hat{y})}\right)^{\gamma-1},\left(-\phi_{\mu,C}(\hat{y})+\hat{y}\right)^{\gamma}\right)<0$. The first order condition leads then to $C=0$ and $\phi_{\mu,C}$ is linear with $\phi_{\mu,C}(z)=az$ for some a. With the reservation utility condition we conclude that $\check{\phi}=\hat{\phi}$. ■

10.2 Appendix C

PROOF OF THEOREM 7

Proof. Let us show that $((\phi_\mu, \{0\}), (\hat{c}^i)_i, \phi_\mu(\hat{y}), \hat{q}, \hat{y})$ with $\hat{y} = \tilde{y} + \psi_\mu(\tilde{y})$ is a m-s equilibrium associated to the production set $Y_\mu = \mathcal{Y} + \psi_\mu(\mathcal{Y})$. From Theorem 2, as $Y^{\phi_\mu} = \mathcal{Y}$ has a production equilibrium $((\hat{c}^i)_i, \hat{q}, \tilde{y})$, it remains to show that $\hat{y} = \tilde{y} + \psi_\mu(\tilde{y})$ strictly maximizes $u(\phi_\mu(y))$ on Y_μ or equivalently as $Y_\mu = \mathcal{Y} + \psi_\mu(\mathcal{Y})$ that $\tilde{y}$ strictly maximizes $u(\psi_\mu(y))$ on $\mathcal{Y}$. As $u \circ \psi_\mu$ is strictly concave, it is equivalent to show that $M^m \psi'_\mu(\tilde{y}) u'(\psi_\mu(\tilde{y})) \in N_{\mathcal{Y}}(\tilde{y})$. From (12) and (16), we have

$$M^m \psi'_\mu(\tilde{y}) \psi_\mu(\tilde{y})^{\gamma-1} = \mu N(\tilde{\lambda}) \tilde{y}^{\gamma-1} = \mu N(\tilde{\lambda}) u'(\tilde{y}).$$

As the representative shareholder of the production equilibrium $((\hat{c}^i)_i, \hat{q}, \tilde{y})$ maximizes at $\tilde{y}$ the utility of the net production on $\mathcal{Y}$ which is smooth, we have $N(\tilde{\lambda}) u'(\tilde{y}) \in N_{\mathcal{Y}}(\tilde{y})$ proving the desired result. ■

As the statement of Theorem 7 requires the existence of a production equilibrium associated to $\mathcal{Y}$, let us provide conditions on $\mathcal{Y}$ under which a production equilibrium exists. To this end, let us first introduce a definition and an assumption. We will say that $\mathcal{Y}$ is strictly convex from above if for $(y_1, y_2) \in \mathcal{Y}^2$ and $t \in (0,1)$, there exists $y \in \mathcal{Y}$ such that $ty_1 + (1-t)y_2 < y$.[17]

Assumption (P')

P'1 $\mathcal{Y} = K - X_+$ where $K \subset X_+$ and such that $0 < \varsigma \leq K \leq \Xi$,

P'2 $\mathcal{Y}$ is closed, strictly convex from above and smooth,

P'3 If $y \in \mathrm{Eff}^+(\mathcal{Y})$, the random variable y has a positive density on $(0, a_y)$ for some $a_y \in R \cup \{\infty\}$.

P'4 For all i, $M^i \varsigma^{\gamma-1}$ belongs to X' and $M^i \varsigma^\gamma$ and $M^i \Xi^\gamma$ belong to $L^1(\Omega, \mathcal{F}, P)$.

Assumption (P') provides conditions on $\mathcal{Y}$ under which a production equilibrium exists. The following corollary can then be easily derived.

Corollary 16 *Assume (P'), then there exists a unique production equilibrium $((\hat{c}^i)_i, \hat{q}, \tilde{y})$ associated to $\mathcal{Y}$. Let ψ_μ be defined by (17) and ϕ_μ by (18). Assume (H), $\hat{q} \in \mathcal{C}(\tilde{y})$, $M^m \in \mathcal{C}(\tilde{y})$ and $u \circ \psi_\mu$ strictly concave. Then $((\phi_\mu, \{0\}), (\hat{c}^i)_i, \phi_\mu(\hat{y}), \hat{q}, \hat{y})$ with $\hat{y} = \tilde{y} + \psi_\mu(\tilde{y})$ is a m-s equilibrium associated to the production set $Y_\mu = \mathcal{Y} + \psi_\mu(\mathcal{Y})$.*

Proof. The existence and uniqueness result is adapted from Bianchi et al. (2021). The rest is immediate. ■

[17] Let us recall that $\mathcal{Y}$ is strictly convex when for all $(y_1, y_2) \in Y^2$ and $t \in (0,1)$, there exists $ty_1 + (1-t)y_2 \in \mathrm{int}(\mathcal{Y})$. It is immediate that strict convexity implies strict convexity from above. However strict convexity is a much stronger condition and requires, in particular, a nonempty interior.