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Long Term Care Insurance with State-Dependent Preferences[1]

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Abstract

We study the demand for Long Term Care (LTC hereafter) insurance in a setting where agents have state-dependent preferences over both a daily life consumption good and LTC expenditures. We assume that dependency creates a demand for LTC expenditures while decreasing the marginal utility of daily life consumption, for any given consumption level. Agents optimize over their consumption of both goods as well as over the amount of LTC insurance. We first show that some agents optimally choose not to insure themselves, while no agent wishes to buy complete insurance, in accordance with the so-called LTC insurance puzzle. At equilibrium, the transfer received from the insurer covers only a fraction of the LTC expenditures. The demand for LTC insurance need not increase with income when preferences are non state-dependent or insurance is actuarially unfair. Also, preferences have to be state-dependent with no insurance bought to rationalize the empirical observation of a higher marginal utility *at equilibrium* when autonomous. Finally, focusing on iso-elastic preferences, we recover the empirical observation that health/LTC expenditures are not very sensitive to income, and we show that LTC insurance as a fraction of income should decrease with income and then become nil above a threshold.

Keywords: Long Term Care Insurance Puzzle, Actuarially Fair Insurance, Risk Aversion, State-dependent Preferences.

JEL Codes: D11, I13.

1 Introduction

Population is aging in most developed countries. According to OECD (2011), the fraction of people aged 80 and above is expected to grow from 4% of the total OECD population in 2010 to 10% in 2050. This demographic trend creates new challenges for policy makers, as aging implies taking care of an ever larger population with very specific health needs, called Long Term Care (hereafter LTC) needs. LTC is defined as "the day-to-day help with activities such as washing and dressing, or help with household activities such as cleaning and cooking" (OECD, 2011). LTC often comes with additional type of support such as medical assistance. Individuals in need of LTC are called dependent.

The costs of LTC are usually large and likely to exhaust most financial resources of the elderly dependent and of his family. For example, Genworth (2018) estimates that the monthly median cost of home care services in the US in 2018 was around US$ 4,000 while that of a semi-private room in a nursing home care was more than US$ 7,000. The risk of needing LTC is also quite large. Brown and Finkelstein (2009) obtain that between 35% and 50% of 65-year-old Americans will be in need of a nursing home at some point. Hurd *et al.* (2013) predict that between 53% and 59% of 50 year old individuals will need LTC services later on in life.

Despite these trends, most people do not insure themselves against the risk of needing LTC. For instance, only 2% of LTC expenditures are financed by private LTC insurance (LTCI hereafter) in OECD countries, with a figure of 7% in the US (OECD, 2011). This lack of insurance is referred to as the LTCI puzzle. A large body of the economics literature, both empirical and theoretical, has tried to explain that puzzle (see Pestieau and Ponthiere (2011) for a survey). Many explanations can be found either on the supply side (adverse selection, rationing effects which increase prices) or on the demand side (crowding out from social programs, substitution with informal family care, risk misperceptions, bequest motives, lack of knowledge of the product and of the LTC costs and institutional support, narrow framing).[1]

This paper provides an explanation for this lack of insurance by focusing on the change in preferences as well as on the change in the composition of the consumption basket when people become dependent. Dependency usually happens at a time in life (at older age, during the retirement period) when individuals enjoy a type of consumption that depends very much

[1] Regarding supply-side explanations, see for example Brown and Finkelstein (2009), Sloan and Norton (1997), Finkelstein and McGarry (2006). Regarding demand-side explanations, see Bonsang (2009), De Donder and Leroux (2014, 2017), Boyer *et al.* (2019, 2020), Gottlieb and Mitchell (2019) among others.

on their health status, such as leisure goods (traveling, attending cultural events, going to restaurants, undertaking physical activities, *etc.*). When dependency strikes, those goods and services become less easily accessible, or may provide less enjoyment.[2] If the marginal utility of those goods decreases with the advent of dependency, individuals may rationally refrain from transferring resources to the dependency state by buying LTCI.

This explanation has received some attention in the empirical literature (as we detail below) but, quite surprisingly, not in the theoretical literature dealing with the demand for LTCI.[3] Our paper proposes a simple theoretical model where we assume state-dependent preferences and where we distinguish daily-life consumption from LTC expenditures (including the health services component). This model enables us to determine the demand for (possibly non actuarially fair) LTCI and how this demand is affected by both the state-dependency of preferences and the variation in the composition of the consumption bundle (between daily-life consumption and health expenditures).

The state-dependency of preferences has long been discussed in the health economics literature, with the seminal contributions of Mossin (1968), Zeckhauser (1970) and Arrow (1974). In that respect, our model is close to the literature on irreplaceable assets (see Zweifel and Eisen, 2012, pp. 81) which shows how the variation in the marginal utility of wealth between good and bad health is likely to affect the demand for insurance.[4] Our model complements that literature since, beyond introducing state-dependent preferences, we also allow for two different types of goods which are, respectively, complement (daily-life consumption) and substitute (LTC including health expenditures) to a good health.

Regarding the specific modeling of private LTCI decisions, to the best of our knowledge, all theory papers (see, among others, Bascans *et al.*, 2017; Courbage and Eeckhoudt, 2012; Cremer and Pestieau, 2014; De Donder and Leroux, 2014; Canta *et al.*, 2016; De Donder and

[2]This argument is especially relevant for LTC, as compared to generic health issues, because (i) for most health ailments, treatments either bring the sufferer back to a good health or at least allow her to function normally, and (ii) dependency is mostly an absorbing state (followed by death) and is precisely defined by the difficulty to perform certain activities, and hence to enjoy certain goods or services.

[3]The survey by Cremer *et al.* (2012) discusses the state-dependence of preferences among the explanations of the LTCI puzzle. They do not develop a thorough model, but their discussion has a brief theoretical modeling perspective.

[4]See also Cook and Graham (1977), Shioshansi (1982), Schlesinger (1984). Rey (2003) and Rey and Rochet (2004) study how the relationship between health, marginal utility of wealth and the cross correlation of risks (including uninsurable ones) influence individual willingness to fully insure against a pecuniary risk. We use interchangeably the terms of marginal utility of consumption, income or wealth to denote their impact on the individual's utility in a specific state of nature (autonomy or dependency). See footnote 16 for the rationale of this equivalence.

Pestieau, 2017; Klimaviciute and Pestieau, 2018; Courbage and Montoliu-Montes, 2018) assume that individuals consume a composite good (including LTC services in case of dependency) and that preferences over this good are not affected by the loss of autonomy. More precisely, this literature models dependency as a fixed monetary loss (i.e., a "ransom"), so that marginal utility of consumption under dependency is higher than under autonomy, inducing all individuals to fully insure when LTCI is available at actuarially fair terms.[5]

On the empirical side, the state-dependency of utilities is mentioned in surveys such as Brown and Finkelstein (2009, 2011) and Davidoff (2013). This empirical literature on state-dependent preferences in the context of the loss of autonomy has failed so far to generate a consensus. On the one hand, Lillard and Weiss (1997), and Ameriks *et al.* (2020), find that marginal utility is higher when dependent than when autonomous. On the other hand, Finkelstein *et al.* (2013), Hong *et al.* (2015) and Koijen *et al.* (2016) obtain the opposite result.[6] These papers differ in their modeling of income, consumption, and in the scenarios considered. One crucial assumption is whether agents consume the same composite good in sickness and in health (as in Lillard and Weiss (1997) and Ameriks *et al.* (2020)) or whether agents' consumption is unbundled between LTC/health expenditures and other goods (as in Davidoff (2013), Hong *et al.* (2015), Finkelstein *et al.* (2013)). Finkelstein *et al.* (2013) note that it is a priori ambiguous whether the marginal utility of consumption rises or falls with deteriorating health, given that some goods (e.g., travel) are complements to good health whereas other goods (e.g., prepared meals or assistance with self-care) are substitutes for good health. However, they obtain, using subjective well-being measures from the Health and Retirement Survey, that a one standard deviation increase in an individual's number of chronic diseases is associated with a 10%-25% decline in marginal utility of consumption. Moreover, Blundell *et al.* (2020) find that most of the effect of temporary drops

[5] There are three recent exceptions, De Nardi *et al.* (2016), Achou (2020) and Leroux *et al.* (2021), that differ from this paper in their objectives and contributions. Leroux *et al.* (2021) assume a composite good and lower marginal utility of income under dependency together with extra LTC spending so as to ensure that individuals partially insure themselves against dependency. Their objective is to study a normative problem in which an ex-post egalitarian social planner wishes to compensate old-age dependent agents as well as short-lived agents for their unluckiness. De Nardi *et al.* (2016) and Achou (2020) distinguish between pure health expenditures and the consumption of non-medical goods, when modelling individuals' preferences. Again their objectives are very different from ours as Achou (2020) studies the welfare consequences of homestead exemption in the Medicaid program while De Nardi *et al.* (2016) study the impact of Medicaid on redistribution.

[6] Lillard and Weiss (1997) study individuals' saving and consumption decisions at the end of life using a sample of individuals aged 65 and more, so that the bad health status could be interpreted as becoming dependent. Koijen *et al.* (2016) estimate a health-state dependent utility function to analyze its effect on the observed demand for insurance products. Given their sample selection, their "sick" state may also be interpreted as being in need of LTC. Finkelstein *et al.* (2013) consider the number of chronic diseases, which are closely linked to the advent of dependency.

in health on consumption stems from the reduction in the marginal utility of consumption that they generate.

We proceed as in Finkelstein *et al.* (2013), Bajari *et al.* (2014), Hong *et al.* (2015), De Nardi *et al.* (2016) and Achou (2020) by unbundling consumption and assuming that agents derive specific utility from LTC (including health) services only when dependent. In that case, the marginal utility of any given amount of non-LTC (or "daily life") consumption is lower than when autonomous, by assumption. All agents face the same probability of becoming dependent, and differ in income. Before the advent of dependency, they choose how much LTCI to buy. We allow for loading costs/actuarially unfair insurance.

We obtain that agents always buy less than full LTCI, with some agents preferring not to buy any insurance at all, even when LTCI is actuarially fair. Our generalization of the irreplaceable assets theory can then explain why there is little LTCI in practice. Moreover, we obtain that the transfer received from the insurer at equilibrium covers only a fraction of the LTC expenses. This can be related to LTCI contracts observed worldwide. In certain countries, such as Canada for instance, firms sell a simple indemnity contract, with a transfer conditional only on the advent of dependency, with no constraint on the use of the indemnity by the recipient. In other countries, such as the US, the insured individual receives a (partial) reimbursement of incurred LTCI expenses. In our model, the US-style constraints that all the insurance transfer be used for LTC expenditures are not binding at equilibrium.

We study how the demand for insurance varies with income. We obtain that, while the amount of LTCI should always increase with income when preferences are non state-dependent and insurance is actuarially fair, this need not be the case when either assumption is not satisfied. Note that lower marginal utility when dependent (than when autonomous) *for any given* daily life consumption level need not imply lower marginal utility of income *at equilibrium*. The reason is that, at the same time, dependency creates LTC needs, so that the amount of daily life consumption is lower than income when dependent, unlike when autonomous, which in turn increases marginal utility. We show that, *at equilibrium*, a higher marginal utility when autonomous (as found in the empirical literature surveyed above) can only occur if preferences are state-dependent and if agents do not buy LTCI at equilibrium (the empirical observation that has given rise to the "LTCI puzzle" literature). Moreover, actuarial unfairness is neither a necessary nor a sufficient condition for higher marginal utility of income *at equilibrium* when autonomous. This result then rationalizes empirical evidence, showing that state-dependency is superior conditional on our model assumptions.

We then follow the empirical literature and assume iso-elastic preferences. This literature has obtained that the relative risk aversion for LTC (including health) expenditures is higher than for (non LTC) consumption. We then obtain, in accordance with this empirical literature, that the elasticity of LTC/health expenditures to income is low. We also establish in this case that the share of income that an individual devotes to LTCI at equilibrium decreases with income, and becomes nil above a threshold income level.

The paper is organized as follows. Section 2 presents the model, including the individual choices, how they vary with respect to income, and the comparison of marginal utility of income when dependent and when autonomous. Section 3 introduces iso-elastic utility functions. Section 4 concludes. Most proofs are relegated to an Appendix.

2 The model

2.1 State-dependent utilities

Individual derive utility both from the consumption c of a “daily-life”, non-LTC, good and from their health status h. In old age, an agent can be either autonomous, denoted by a or dependent, denoted by d. Her health status depends on both whether she is dependent or autonomous, and on the amount of LTC (including health) expenditures z she consumes. The utility of an agent in state $i = \{a, d\}$ is denoted by

$$U_i(c, z) = u_i(c) - h_i(z),$$

so that we make the simplifying assumption that utility is separable in consumption and in health status, in both states of the world.[7]

[7] This separability assumption is made by most empirical papers: see Finkelstein et al. (2013), De Nardi *et al.* (2016), Achou (2020) and Bajari *et al.* (2014). These papers further assume iso-elastic utility functions, as we do in Section 3. Note that most of our results (Propositions 1 and 4 as well as most of Proposition 2) carry through to the case of complementarity between the health status and daily-life consumption (proof available upon request). The variation of c_a^* with w in Proposition 2 as well as Proposition 3 would require additional assumptions on the third-order derivatives of the utility function in case of dependence.

The utility of "daily-life" (or non-LTC) consumption is state-dependent with

$$
\begin{aligned}
u_i'(.) &> 0, u_i''(.) < 0\,,\ \forall i \in \{a,d\}, \\
\lim_{c \to 0} u_i'(c) &= \infty, \\
u_a(x) &> u_d(x) \text{ and } u_a'(x) > u_d'(x).
\end{aligned}
$$

The first line is standard, with increasing and concave utility from consumption, independently of the dependency status, while the second line is the usual Inada condition. The third line states that when autonomous, both the marginal and the absolute utility of consuming any given amount x are higher than when dependent. This reflects the observation that "daily life" consumption (such as restaurants, travel, clothing, active leisure, etc.) is more enjoyable when in good health than when dependent. In that sense, daily-life consumption and a good health are assumed to be complements.

We assume for simplicity that autonomous agents need no LTC expenditures ($h_a(z) = 0, \forall z$) and always choose $z = 0$.[8] As for dependent agents, we assume that $h_d(z) = h(z) \geq 0$, $h'(z) \leq 0$, $h''(z) \geq 0$ and $h'(0) \to -\infty$. In words, LTC expenditures generate infinite utility at the margin when $z = 0$, with decreasing marginal utility as z increases (recall that we subtract $h(z) > 0$ to obtain the individual's utility). LTC expenditures and a good health (i.e. autonomy) are substitutes.

Therefore, the agent's utility when autonomous is

$$U_a(c, z) = u_a(c)$$

while it is

$$U_d(c, z) = u_d(c) - h(z)$$

when dependent.

As mentioned in the introduction, most theoretical papers about LTCI (Canta *et al.*, 2016; Cremer and Pestieau, 2014; De Donder and Leroux, 2014; De Donder and Pestieau, 2017; Klimaviciute and Pestieau, 2018) differ from ours in at least two respects. First, they do not model state-dependent preferences and the advent of dependency is introduced as a ransom deducted from income. Second, they do not make the distinction between the utility obtained

[8] The crucial assumption we need is that marginal utility of LTC expenditures is higher under dependency than under autonomy: -$h_d'(z) > -h_a'(z)$. Assuming that $h_a'(z) = 0$ is then without further loss of generality.

from consuming daily-life goods and from consuming LTC goods. In our framework, this would translate into $h(x) = 0, \forall x$, and $u_a(c) = u_d(c) = u(c)$ where c is a unique composite good, with a fixed monetary loss z when dependent. This would imply that the utility under autonomy is higher than under dependency ($u(c) > u(c-z)$) with the opposite relationship for marginal utility ($u'(c-z) > u'(c)$). Distinguishing between daily-life consumption and health expenditures also allows us to model endogenous LTC health expenditures.

2.2 Individual choices

At the time of taking the decision to get insured against the LTC risk, all agents face the same probability $p \in [0,1]$ of becoming dependent. Private insurance contracts are described by the premium $t \geq 0$ paid in return for a LTC benefit $\kappa t/p$ in case of dependency. The degree of actuarial fairness of the LTC is accounted for through the parameter $0 \leq \kappa \leq 1$. If the insurance market is perfectly competitive (profits of insurance firms are driven to zero) with no loading costs, agents face an actuarially fair insurance market and $\kappa = 1$.[9] With loading costs, we have $\kappa < 1$ and the insurance offers actuarially unfair returns.[10] Note that the empirical literature has established that existing LTCI contracts are actuarially unfair, although their degree of unfairness is not especially large compared with other types of contracts and by itself does not explain the low demand for LTCI.[11] Our modelling allows us to disentangle the impact of actuarial (un)fairness and of state-dependent preferences in explaining this low demand.

We denote an individual by the income w she is endowed with. Agents choose simultaneously the amount of insurance premium $t \geq 0$ and the amount of LTC expenditures $z \geq 0$ in case of dependency to maximize their expected utility function:[12]

$$EU(t,z) = (1-p)u_a(c_a) + p[u_d(c_d) - h(z)],$$

where $c_a = w - t$ is consumption if autonomous while $c_d = w - t + \kappa\frac{t}{p} - z$ is consumption if dependent.

[9] This modeling also corresponds to the case of a non redistributive public LTC insurance.

[10] Our approach is equivalent to assuming that agents pay a premium of t/κ to receive an indemnity of t/p. In that formulation, the loading cost (i.e., percentage of the premium paid above the actuarially fair one) is $1/\kappa$.

[11] Brown and Finkelstein (2007) estimate that $\kappa = 0.82$ while Achou (2021) finds that $\kappa = 0.94$ for women.

[12] Assuming a two-period model in which the individual is in good health with certainty and pays a premium in the first period, but may become dependent and obtain a LTC benefit in the second, would allow us to introduce saving, which would in turn reduce the willingness to pay for insurance. Appendix B shows that we would obtain similar results, so we stick with the simpler model throughout the paper. Also, whether z is chosen at the same time as t, or later on when dependency arises, is of no consequence here since we assume away time inconsistency or any other behavioral problem.

First-order conditions with respect to LTC expenditures, z and the premium paid, t are:[13]

$$\begin{aligned} \frac{\partial EU}{\partial z} &= -u_d'(c_d) - h'(z) = 0, &(1)\\ \frac{\partial EU}{\partial t} &= (\kappa - p)u_d'(c_d) - (1-p)u_a'(c_a) \leq 0. &(2) \end{aligned}$$

We denote by (z^*, t^*) the solution to this system of two equations, with the corresponding consumption levels c_a^* and c_d^*. The assumption that $h'(0) \to -\infty$ implies that equation (1) always holds with equality, so that $z^* > 0$. It will prove useful to denote by z^0 the optimal level of LTC expenditures (satisfying equation (1)) when $t = 0$. In that case, we denote the consumption bundle as $(z^0,\, c_d^0,\, c_a^0)$ with $z^0 > 0$, $c_a^0 = w > c_d^0 = w - z^0$.

Our first proposition below provides a simple explanation as to why agents may not insure themselves against dependency even though they are risk averse and may incur extra expenses when dependent. This proposition generalizes the result of under-insurance obtained in the irreplaceable assets literature mentioned in the introduction to the case where (i) agents consume different goods when the damage occurs and (ii) they optimize over the quantities of the two goods purchased.

Proposition 1 *(i) An agent chooses to (resp., not to) buy LTCI if* $(\kappa - p)u_d'(w - z^0) > (1 - p)u_a'(w)$ *(resp.,*$\leq$*).*
(ii) If the agent decides to insure herself, the level of LTCI coverage is incomplete, that is $c_d^* < c_a^*$ *and* $t^* < pz^*/\kappa$.
(iii) (a) $z^0 \leq z^* \leq z^0 + t^*(\kappa - p)/p$ *with strict inequalities iff* $t^* > 0$. *(b)* $c_a^0 \geq c_a^* > c_d^* \geq c_d^0$ *with strict inequalities iff* $t^* > 0$.

Proof. (i) The agent decides to buy insurance if and only if her marginal gain from buying insurance is positive when $t = 0$, namely

$$(\kappa - p)u_d'(w - z^0) - (1-p)u_a'(w) > 0. \tag{3}$$

(ii) We now assume that $(\kappa - p)u_d'(w - z^0) > (1 - p)u_a'(w)$ so that the agent buys LTCI at equilibrium. In that case, the FOC with respect to t holds with equality and t^* is defined by

$$(\kappa - p)u_d'(w - t^* - z^* + \kappa\frac{t^*}{p}) = (1-p)u_a'(w - t^*) \tag{4}$$

[13] We assume that second-order conditions are satisfied.

where z^* is defined by $-u'_d(w-t^*-z^*+\kappa t^*/p) = h'(z^*)$. Equation (4) together with $u'_d(x) < u'_a(x)$ and with the concavity of both u_d and u_a imply that $c^*_d < c^*_a$ and thus that $t^* < pz^*/\kappa$.
(iii) See Appendix A. ■

First note that a necessary (although not sufficient) condition for t to be positive is that the degree of actuarial fairness be greater than the probability to become dependent– i.e. $\kappa > p$ – since this is a necessary condition for the payment received when dependent to be larger than the premium paid.

The advent of dependency has two impacts of opposite signs on the demand for insurance. On the one hand, dependent agents bear additional expenses z^*, inducing them to insure themselves so as to smooth consumption. On the other hand, dependency reduces the marginal utility of daily life consumption, decreasing the incentive to insure and transfer resources to the bad state of the world. Depending on which effect dominates, the agent chooses to insure herself or not. Part (i) above shows that a necessary condition for the agents' demand for LTCI to be positive is that their marginal utility of consumption be larger when dependent than when autonomous, when the former is measured at the consumption level obtained in the absence of insurance (i.e., when $t = 0$ so that $z = z^0$).[14] This condition is also sufficient if the insurance is actuarially fair ($\kappa = 1$).

The second result in Proposition 1 that $t^* < pz^*/\kappa$ implies that agents optimally buy more LTC expenditures than the insurance transfer received, whatever the degree of fairness of the LTCI contracts, κ. This has an interesting implication when looking at the LTCI contracts used in practice. We model a simple indemnity contract, with a transfer conditional only on the advent of dependency. In such contracts (observed in Canada, for instance), the transfer can be used by the recipient at her discretion, and need not fund exclusively LTC expenditures. In other countries, such as the US, the insured receives a (partial) reimbursement of incurred LTC expenditures, so that the transfer received is lower than these expenses. In our model, it is a property of the equilibrium allocation that the transfer is lower than the LTC expenses incurred (as measured by z). In other words, the US-style constraints that all the transfer be used for LTC expenditures are not binding at equilibrium.

The third result of Proposition 1 compares the allocation with and without insurance and

[14] Actually, if $(\kappa - p)u'_d(w - z^0) < (1 - p)u'_a(w)$, agents would want to transfer resources from dependency towards autonomy (i.e., they would prefer $t^* < 0$). We are unaware of the existence of such financial instruments, which is why we restrict t to be non-negative.

shows that part of the insurance transfer, net of the premium paid (i.e., $t^*\kappa/p - t^*$), is used to finance increased LTC expenditures while the remainder is used to increase the non-LTC consumption level, so as to partially compensate for the loss in daily-life consumption utility due to dependency.

We now look at how the optimal insurance behavior varies with individual income, w.

2.3 Comparative statics with respect to income

In this section, we explore how the agent's insurance behavior varies with her income, w. Our results are summarized in the following proposition:

Proposition 2 *We obtain that:*

1. *LTC expenditures z^* are increasing with w.*
2. *The amount, t^*, of LTCI bought is increasing in w if $R_a^A(c_a^*) \geq R_d^A(c_d^*)$ (where $R_i^A(c_i) = -u_i''(c_i)/u_i'(c_i)$ for $i = \{a, d\}$) .*
3. *Consumption levels c_d^*, c_a^* increase with w.*
4. *We have*
$$0 < \frac{dc_d^0}{dw}, \frac{dz^0}{dw} < 1.$$

Proposition 2 shows that LTC expenditures are a normal good (part 1), as well as non-LTC expenditures, whether the insurance amount is chosen optimally to be positive (part 3) or is nil (part 4).

Part 2 of Proposition 2 is obtained by differentiating the optimality condition for an interior t^*, equation (2), with respect to w,

$$\begin{aligned}\frac{\partial^2 EU}{\partial t \partial w} &= (\kappa - p)u_d''(c_d^*)(1 - \frac{dz^*}{dw}) - u_a''(c_a^*)(1-p) \\ &= \left[(\kappa - p)u_d''(c_d^*) - (1-p)u_a''(c_a^*)\right] - (\kappa - p)u_d''(c_d^*)\frac{dz^*}{dw} \qquad (5) \\ &= (\kappa - p)\{u_d'(c_d^*)[R_a^A(c_a^*) - R_d^A(c_d^*)] - u_d''(c_d^*)\frac{dz^*}{dw}\} \qquad (6)\end{aligned}$$

where we made use of (2) to obtain the last line. Using the implicit function theorem, this expression has the same sign as dt^*/dw.[15]

[15] Alternatively, using Cramer's rule would allow us to obtain the same results.

Recall that t is chosen in order to equalize marginal utilities of non-LTC consumptions c in both states of the world, given the choice of LTC expenditures z. Increasing w then has both a *direct* and an *indirect* impact on the marginal expected utility of t. The direct impact (first term in (5)) is that a higher income affects the balance of marginal utilities across both states of the world, for a given z, while the indirect impact (second term in (5)) operates through the variation of z generated by an increase in w. As we know that z increases with w, this indirect impact is always positive, as the individual wishes to transfer more resources to the dependency state. The sign of the direct impact is a priori ambiguous, and depends on the comparison of the coefficients of absolute risk aversion, as shown in (6).

When insurance is actuarially fair and preferences are not state-dependent, the agent equalizes consumption in both states so that the direct effect disappears, and higher income agents always prefer a larger value of t, thanks to the indirect impact stated above. The direct effect is generically non nil when insurance is not actuarially fair or when preferences are state-dependent. If the first term in (5) is positive, marginal utility decreases more slowly with income when dependent than when autonomous, which induces the agent to transfer more income to the dependency state–i.e., to increase t. In that case, both (direct and indirect) effects concur to increase t^*.

The first term in (6) shows that the comparison of the degree of concavity of the state-dependent utility functions can be expressed in terms of risk aversion, with a higher risk aversion coefficient when autonomous (compared to the dependency state) translating into a more concave utility, and thus a larger decrease in marginal utility when income increases. Note that this remains true whether insurance is actuarially fair or not. Finally, when the first term in (5) or (6) is negative, the sign of the total derivative of t^* with respect to w is ambiguous, so that t^* may be non monotone in income.

The following proposition shows that focusing instead on the insurance *rate*, denoted $\tau^* = t^*/w$, allows us to sign the impact of income in a non-ambiguous way.

Proposition 3 *When strictly positive, the insurance rate τ^* is increasing (resp. decreasing) in w when $R_z^R(z^*)\varepsilon_{z^*,w} < R_a^R(c_a^*)$ (resp. $>$), with $R_z^R(z) = -h''(z)z/h'(z)$, $R_a^R(c) = -u_a''(c)c/u_a'(c)$ and $\varepsilon_{z^*,w}$ is the elasticity of LTC expenditures to income, measured at the preferred choice of the individual.*

The role of risk aversion in Proposition 3 runs as follows. A large risk aversion for daily-life consumption when autonomous means that the function $u_a(c)$ is very concave, so that an increase in income induces the agent to transfer a larger fraction of her income to the dependency state. Analogously, a large risk aversion for LTC expenditures means that the function $h(z)$ is very convex, so that the agent does not wish to increase her LTC expenditures by much and actually decreases the share of her income devoted to LTCI.

As for the role of the elasticity of LTC expenditures to income, note that, for given values of $R_z^R(z)$ and $R_a^R(c_a)$, a low value of $\varepsilon_{z,w}$ means that the individual has to transfer proportionally more income to the dependency state if she wants to increase her LTC expenditures–i.e., she increases τ^*.

Finally, note that τ^* increasing with w implies that t^* increases with w, while τ^* decreasing with w is compatible with t^* either increasing or decreasing with w.

2.4 Comparison of equilibrium marginal utilities across states

In this section, we compare equilibrium marginal utilities across the two states of the world (autonomy and dependency). It will prove handy to introduce a specific notation for the equilibrium utility in each state as, respectively,

$$V_a = u_a(c_a^*) = u_a(w_a). \tag{7}$$

$$V_d = u_d(c_d^*) - h(z^*) = u_d(w_d - z^*) - h(z^*), \tag{8}$$

where the income in each state of nature, after buying the insurance, is defined by

$$\begin{aligned} w_a &= w - t^*, \\ w_d &= w - t^*(1 - \frac{\kappa}{p}). \end{aligned}$$

Totally differentiating these utilities and using (1), we obtain that[16]

$$\begin{aligned} dV_a &= u_a'(c_a^*)dc_a = u_a'(c_a^*)dw_a, \\ dV_d &= u_d'(c_d^*)dc_d - h'(z^*)dz = u_d'(c_d^*)[dc_d + dz] = u_d'(c_d^*)dw_d \end{aligned}$$

Using (2), we obtain that

$$\frac{1-p}{\kappa - p}\frac{dV_a}{dw_a} = \frac{1-p}{\kappa - p}u_a'(c_a^*) \geq u_d'(c_d^*) = \frac{dV_d}{dw_d}, \tag{9}$$

[16] This explains the equivalence between marginal utility of income and of consumption mentioned in the introduction to the paper.

with a strict inequality only if $t^* = 0$.

It is worth looking more closely at equation (9), to conclude that

Proposition 4 *The only case where $dV_a/dw_a > dV_d/dw_d$ at equilibrium is when preferences are state-dependent with $t^* = 0$.*

Proof.

1. $\kappa = 1$ and $t^* > 0$ imply that $dV_a/dw_a = dV_d/dw_d$.

2. $\kappa < 1$ and $t^* > 0$ imply that (a) $(1-p)/(\kappa - p) > 1$ and, (b) that (9) holds with equality, implying that $dV_a/dw_a < dV_d/dw_d$.

3. $\kappa = 1$ and $t^* = 0$ imply that $u'_a(w) \geq u'_d(w - z^0)$ (by the FOC (2)) so that $dV_a/dw_a \geq dV_d/dw_d$. Note that this case cannot arise with NSD preferences, since it would mean $u'(w) \geq u'(w - z^0)$, which is impossible given that $z^0 > 0$ and that $u''(x) < 0$.

4. $\kappa < 1$ and $t^* = 0$ imply that $\frac{1-p}{\kappa-p} u'_a(w) \geq u'_d(w - z^0)$ with $(1-p)/(\kappa-p) > 1$. Note that (a) in the case of non-state dependent preferences, we always have $u'(w) < u'(w - z^0)$ so that $dV_a/dw_a < dV_d/dw_d$, while (b) with SD preferences, we may have that $u'_a(w) > u'_d(w - z^0)$ so that we may obtain that $dV_a/dw_a > dV_d/dw_d$.

■

First, if agents insure themselves ($t^* > 0$) when insurance is actuarially fair ($\kappa = 1$), we obtain the well known result of equalization of marginal utilities, whether preferences are state dependent (SD) or not (NSD). Actually, the only difference between the two formulations is that an actuarially fair insurance is a sufficient condition to have $t^* > 0$ only with NSD preferences. With actuarially unfair but positive insurance ($\kappa < 1$, $t^* > 0$), we obtain that marginal utility is higher when dependent than when autonomous both with SD and NSD preferences. We obtain the opposite ranking of marginal utilities when $t^* = 0$ and $\kappa = 1$ (a case which cannot occur with NSD preferences). Finally, when $\kappa < 1$ and $t^* = 0$, marginal utility is always (weakly) higher when dependent with NSD, but may be lower with SD preferences.[17]

The intuition for these results goes as follows. Looking at expression (9), dependency has two impacts on the marginal utility of income. First, with state-dependent preferences, dependency

[17] It is actually easy to find examples where $u'_a(c_a) > u'(c_d)$ when $t^* = 0$ within the class of iso-elastic utility functions studied in Section 3, by setting γ low and κ close to 1, so that the state dependent dimension overrides the loading costs dimension in the comparison of marginal utilities.

decreases marginal utility for any given consumption level, by assumption. Second, dependency creates needs for LTC expenditures, decreasing the amount of non-LTC consumption bought at equilibrium from any given income level. This in turn increases marginal utility of non-LTC consumption, due to the concavity of the utility function. If the second effect dominates the first one, then agents do buy insurance at equilibrium ($t^* > 0$). They equalize marginal utilities if insurance is fair ($\kappa = 1$), but fall short of that if LTCI is unfair ($\kappa < 1$), so that the marginal utility of income is larger when dependent than when autonomous at equilibrium. If the first effect dominates the second, marginal utility of income is larger when autonomous than when dependent even in the absence of insurance, and agents have no incentive to buy LTCI.

Proposition 4 is consistent with empirical evidence, showing that conditional on the model assumptions, the assumption of state-dependency is superior. Empirical papers such as Finkelstein *et al.* (2013) obtain that marginal utility is lower when dependent than when autonomous at equilibrium. Proposition 4 then implies both that people do not buy LTCI (the very subject of the LTCI puzzle) and that preferences are state-dependent. Note especially that actuarially unfair insurance ($\kappa < 1$) is neither necessary nor sufficient to obtain this result.

3 Iso-elastic utility functions

The introduction of the widely used iso-elastic functional forms for the utility of daily-life consumption and health expenditures (as in Finkelstein *et al.*, 2013; Bajari *et al.*, 2014; Hong *et al.*, 2015; De Nardi *et al.,* 2016; Ameriks *et al.,* 2020; Achou, 2020) allows us to shed more light on the LTC insurance and consumption behavior of agents.

3.1 State-dependent preferences for non-LTC consumption

We assume the following form for state-dependent preferences for non-LTC consumption.

Assumption 1

$$u_d(x) = \gamma u_a(x)$$

where $\gamma \in]0,1[$ *is the same for all agents, with*

$$u_a(x) = \frac{x^{1-\beta}}{1-\beta},$$

and $\beta < 1$.

This formulation implies that the coefficient of relative risk aversion (β) is smaller than one, which is consistent with empirical evidence. For instance, Karagyozova and Siegelman (2012) review the empirical literature regarding the estimation of the relative risk aversion coefficient and find that assuming that it is smaller than 1 is reasonable.[18] We now show that how the elasticity of LTC expenditures to income compares with unity determines both whether low or high income individuals choose not to buy LTCI, and whether LTC expenditures as a share of income increase or decrease with income when agents do not buy LTCI (either because their most-preferred level is nil, or because such insurance is not available).

Proposition 5 *When Assumption 1 holds, $\varepsilon_{z^0,w} = (dz^0/dw)(w/z^0) < 1$ (resp., >1) implies that (i) agents with an income lower than a threshold $\tilde{w}$ (resp., higher) defined in the Appendix insure themselves at equilibrium, and that (ii) z^0/w decreases (resp., increases) with w.*

The intuition for this result runs as follows. Recall that agents buy insurance if their marginal utility of (non-LTC) consumption when dependent is larger than when autonomous, in the case where LTC expenditures are financed from their own resources (*i.e.*, we have $(\kappa - p)u'_d(w - z^0) > (1 - p)u'_a(w)$). Recall also that the advent of dependency has two impacts of opposite signs on the demand for LTCI. First, it reduces the marginal utility of non-LTC consumption (depressing the demand for LTCI). Second, it increases the need for LTC expenditures, and thus decreases the income available for the non-LTC consumption good (increasing the demand for LTCI). If the elasticity of LTC expenditures to income, $\varepsilon_{z^0,w}$, is smaller than one, then high-income agents do not increase much their demand for LTC expenditures, z^0, muting the second effect above and resulting in high-income agents preferring not to buy LTCI. This explains part (i) of the above proposition. At the same time, the share of LTC expenditures in income w decreases (part (ii) of the above proposition). The opposite occurs when $\varepsilon_{z^0,w} > 1$.

Observe that Proposition 5 does not depend on any functional form assumption for the utility obtained from LTC expenditures when dependent.

3.2 Preferences for LTC expenditures

We now introduce a functional form for the benefit obtained from LTC expenditures when dependent.

[18] Similarly, Holt and Laury (2002) find that 64% of respondents have a coefficient between 0.15 and 0.97. Chetty (2006) finds a mean value for that coefficient equal to 0.71.

Assumption 2

$$h(z) = A - \frac{z^{1-\alpha}}{1-\alpha},$$

where A is a large positive constant and $\alpha < 1$.

Note that the constant A is introduced only to make sure that agents are better-off autonomous than dependent, for any value of $c_a = c_d$ and any value of $z \geq 0$, but will play no role in the rest of the analysis.

The following lemma (proved in the Appendix) will be useful when looking for the conditions underlying the variation of the insurance rate τ^* with respect to w.

Lemma 1 *When Assumptions 1 and 2 hold, the elasticity of LTC expenditures with respect to income is such that, for $z \in \{z^0, z^*\}$, (i) if $\beta = \alpha$, $\varepsilon_{z,w} = 1$, (ii) if $\beta > \alpha$, $\varepsilon_{z,w} > 1$, (iii) if $\beta < \alpha$, $\varepsilon_{z,w} < 1$.*

Under Assumptions 1 and 2, we obtain that the coefficients of relative risk aversion are $R_r^R(z) = \alpha$, and $R_a^R(c) = R_d^R(c) = \beta$. If $\alpha > \beta$, agents are more averse to variations in LTC expenditures z than in consumption c. The choice of z is then less sensitive to variations in income w than the choice of consumption c, resulting in an elasticity of LTC expenditures to income lower than the elasticity of daily-consumption to income.[19]

Note that the empirical evidence strongly suggests that $\alpha > \beta$ (see Bajari *et al.*, 2014; De Nardi *et al.*, 2016 and Achou, 2020)– i.e., that the relative risk aversion coefficient regarding health/LTC expenditures is higher than the one regarding non-LTC consumption. Moreover, De Nardi *et al.* (2021, figure 5) find little variations of z with permanent income which is consistent with Lemma 1, under the assumption that $\alpha > \beta$. This is also confirmed by Blundell *et al.* (2020) who obtain that income shocks affect non durable consumption but not medical expenses.

[19] To see this, we fully differentiate eq.(1) with respect to w,

$$-u_d''(c_d^*)\frac{dc_d^*}{dw} - h''(z^*)\frac{dz^*}{dw} = 0,$$

and we obtain after some rearrangements that

$$\varepsilon_{c_d^*,w} = \frac{R_z^R(z^*)}{R_d^R(c_d^*)}\varepsilon_{z^*,w}.$$

Since $R_d^R(c_d^*) = \beta$ and $R_z^R(z^*) = \alpha$ under Assumptions 1 and 2, we have that $\varepsilon_{c_d^*,w} > \varepsilon_{z^*,w}$ if $\alpha > \beta$.

We now look at how the absolute amount of LTCI bought at equilibrium (i.e., t^*) varies with income when preferences are iso-elastic and $\alpha > \beta$. We obtain that $R_a^A(c_a^*) = \beta/c_a^* < R_a^A(c_d^*) = \beta/c_d^*$ so that the direct effect of w on the variation of t^* observed in (6) is always negative, while the indirect effect is positive. The overall impact of w on t^* is then ambiguous, but we nevertheless obtain that the variation of t^* with respect to w is smaller than if preferences were not state-dependent (in which case there is no direct effect).

We then look at how the the income share of LTCI, τ^*, varies with income in the next proposition:

Proposition 6 *When Assumptions 1 and 2 hold, with $\alpha > \beta$, τ^* decreases with w up to the threshold $\tilde{w}$ above which $\tau^* = 0$.*

We first study the reasons why agents decide to insure or not, depending on the comparison between α and β. Lemma 1 has shown that $\varepsilon_{z^0,w} < 1$ if $\alpha > \beta$, while Proposition 5 has shown that, in this case, only poor people (*i.e.*, with income smaller than $\tilde{w}$) insure themselves.

We now move to the comparative statics of τ^* with respect to w when $\tau^* > 0$. Note first that Proposition 3 together with Lemma 1 are inconclusive, since Lemma 1 implies that $\varepsilon_{z^*,w} < 1$ when $\alpha = R_z^R(z) > \beta = R_a^R(c)$. The intuition for Proposition 6 then runs as follows. Because of the larger curvature of the utility for LTC expenditures in comparison to the curvature of daily-life consumption utility, a higher income w translates into a small increase in z. This small increase in z can then be financed by a smaller share of LTCI premium in income, so that τ^* decreases with w. Observe that this complements Proposition 5 which has shown that low income agents insure themselves at equilibrium when $\varepsilon_{z^0,w} < 1$ (which, by Lemma 1, corresponds to $\alpha > \beta$).

Proposition 6 then constitutes a testable implication of our model. To the best of our knowledge, empirical papers looking at the relationship between LTCI and income or wealth report aggregate results, such as the proportion of individuals *holding* LTCI in various wealth quantiles.[20] Testing Proposition 6 would require to collect information on the premium paid by individuals for LTC insurance. This information is available in the Health and Retirement Survey (HRS) but, to the best of our knowledge, has not yet been used for that purpose. One challenge

[20] See for instance Lockwood (2018) and Mommaerts (2020) who report that the proportion of individuals holding LTCI increases with wealth.

would consist in controlling adequately for pre-existing public insurance (i.e. Medicaid), which is likely to insure more the poor people than the rich ones. This may require combining both HRS and Medicare Current Beneficiary Survey (MCBS) data as only the latter contains information on Medicaid payments.

4 Conclusions

We have followed a positive approach where individuals choose how much LTCI to buy, in a setting where autonomous agents only care about daily-life, non-LTC, consumption while dependent agents also care about LTC expenditures. We assume from the outset that the marginal utility of non-LTC consumption is lower when dependent than when autonomous, for any given consumption level, as suggested by the empirical literature, and in stark contrast with most of the theoretical literature on LTC. We then study the consequences of this assumption for the demand for LTCI and for consumption behavior.

We first obtain that some individuals optimally choose not to buy any LTCI, while no one buys full insurance even with actuarially fair insurance. Also, the transfer received from the insurer at equilibrium covers only a fraction of the LTC expenses. While non state-dependent preferences together with actuarial fairness imply that the amount of LTCI increases with income, this need not be the case when either assumption is not satisfied. Preferences have to be state-dependent with no insurance bought to rationalize the empirical observation of a higher marginal utility when autonomous than when dependent, *at equilibrium*, whether LTCI is actuarially fair or not. Finally, focusing on iso-elastic preferences, we recover the empirical observation that health/LTC expenditures are not very sensitive to income when risk aversion is higher for health/LTC expenditures than for non-LTC expenditures, and we show that this ensures that LTCI as a fraction of income should decrease with income and then become nil above a threshold.

The model we propose, while very simple, delivers results which are in line with the empirical literature, and which cannot be generated only with non state-dependent preferences and actuarially unfair LTCI. The LTCI puzzle vanishes when state-dependency is introduced, since low insurance take-up becomes a rational behavior. Yet, our model also raises new empirical questions, in particular as to how the amount of LTCI and its share in income vary with individual income. Extending the analysis of Blundell *et al.* (2020) to permanent health shocks, such an empirical estimation could try and disentangle the different effects at play (state-dependency of preferences vs straightforward income effects). These empirical questions are certainly worth exploring both for governments and for private insurers.

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Appendix A: proofs

Proof of Proposition 1

(iii) (a) If $t^* = 0$, we have by definition that $z^0 = z^* = z^0 + t^*(\kappa - p)/p$. Assume then that $t^* > 0$. Assume by contradiction that $z^0 > z^*$, so that $-h'(z^0) < -h'(z^*)$. Comparing the equation (1) with z^* and z^0, this implies that

$$
\begin{aligned}
& u_d'(w - z^0) < u_d'(w - z^* + t^*(\kappa - p)/p) \\
\Leftrightarrow \quad & w - z^0 > w - z^* + t^*(\kappa - p)/p \\
\Leftrightarrow \quad & z^* > z^0 + t^*(\kappa - p)/p
\end{aligned}
$$

which contradicts that $z^0 > z^*$. Proceeding similarly then shows that $z^0 < z^*$ implies that $z^* < z^0 + t^*(\kappa - p)/p$ when $\kappa > p$.

(b) If $t^* = 0$, we have by definition that $c_a^0 = c_a^* = w > c_d^* = w - z^* = c_d^0 = w - z^0$. Assume then that $t^* > 0$ so that $\kappa > p$. We have that $c_a^0 = w > c_a^* = w - t^* > c_d^* = w + t^*(\kappa - p)/p - z^* = w - t^* + (t^*\kappa/p - z^*)$ since Proposition 1 (ii) has shown that $z^* > t^*\kappa/p$. Finally, $c_d^* > c_d^0$ since $z^* < z^0 + t^*(\kappa - p)/p$.

Proof of Proposition 2

1. In order to find the variation of health expenditures with income, we apply Cramer's rule on equations (1) and (2). We obtain that $0 < dz^*/dw < 1/p$.

2. See text below Proposition 2.

3. Since z^* increases with w, the FOC for z (equation (1)) implies that c_d^* increases with w as well. Using the FOC for t (equation (2)), we obtain that $c_a^* = w - t^*$ increases with w, so that $dt^*/dw < 1$.

4. Fully differentiating the FOC for z (equation (1)) when $t = 0$, with respect to w, we obtain, after some rearrangements, that

$$\frac{dz^0}{dw} = \frac{u_d''(c_d^0)}{u_d''(c_d^0) - h''(z^0)} \in [0, 1].$$

This in turn implies that $dc_d^0/dw \in [0, 1]$.

Proof of Proposition 3

Changing notations for $\tau = t/w$ and replacing (1) into (2), we obtain:

$$-(\kappa - p)h'(z^*) - u_a'(w(1 - \tau^*)) = 0 \tag{10}$$

Fully differentiating this expression with respect to w, we get

$$\frac{d\tau^*}{dw} = \frac{1}{wu_a''(c_a^*)}\left[(\kappa - p)h''(z^*)\frac{dz^*}{dw} + u_a''(c_a^*)(1 - \tau^*)\right].$$

From Proposition 2, we have that $dz^*/dw > 0$. Using (10) and rearranging terms, we obtain that

$$\frac{d\tau^*}{dw} = \frac{u_a'(c_a^*)}{w^2 u_a''(c_a^*)}\left[R_z^R(z^*)\varepsilon_{z^*,w} - R_a^R(c_a^*)\right]$$

where $R_a^R(c) = -u_a''(c)c/u_a'(c)$ and $R_z^R(z) = -h''(z)z/h'(z)$.

Proof of Proposition 5

(i) Using equation (3), we obtain that $t^* > 0$ if and only if

$$\gamma > \frac{1 - p}{\kappa - p}\left(\frac{w - z^0}{w}\right)^{\beta}. \tag{11}$$

Denote by $\tilde{w}$ the value of w equalizing both sides of (11), namely

$$\gamma(\kappa - p)(\tilde{w} - z^0)^{-\beta} - (1 - p)\tilde{w}^{-\beta} = 0.$$

Observe that the right hand side of (11) is increasing (resp., decreasing) in w if and only if the elasticity of health expenditure to income, $\varepsilon_{z^0,w} = (dz^0/dw)(w/z^0)$, is smaller (resp., larger) than one, so that the value of $\tilde{w}$ is unique when it exists. When $\varepsilon_{z^0,w} < 1$, agents with $w < \tilde{w}$ insure themselves, while agents with $w > \tilde{w}$ insure themselves if $\varepsilon_{z^0,w} > 1$. Note that we set $\tilde{w} = +\infty$ if (i) (11) is satisfied for all agents (i.e., including the highest income) when $\varepsilon_{z^0,w} < 1$ (so that everyone insures at equilibrium) or if (11) is not satisfied, even for the highest income when $\varepsilon_{z^0,w} > 1$ (so that no one insures at equilibrium). Alternatively, we set $\tilde{w} = 0$ if (i) (11) is satisfied for no agent (i.e., including the lowest income) when $\varepsilon_{z^0,w} < 1$ (so that no one insures at equilibrium) or if (11) is satisfied for all agents (including the lowest income) when $\varepsilon_{z^0,w} > 1$ (so that everyone insures at equilibrium).

(ii) Denoting the derivative of z^0 with respect to w by $z^{0\prime}$, we have

$$\frac{dz^0/w}{dw} = \frac{z^{0\prime}w - z^0}{w^2}$$

so that

$$\begin{aligned} \frac{dz^0/w}{dw} &< 0 \Leftrightarrow z^{0\prime}w < z^0 \\ &\Leftrightarrow \varepsilon_{z^0,w} < 1. \end{aligned}$$

Proof of Lemma 1

Assume first that $t^* > 0$. Let us make a change of variables in (1) and (2) with $\tau = t^*/w$ and $\bar{z} = z^*/w$:

$$\begin{aligned} -\gamma(1-\tau+\frac{\kappa}{p}\tau-\bar{z})^{-\beta} + \bar{z}^{-\alpha}w^{\beta-\alpha} &= 0, \qquad (12)\\ (\kappa-p)\gamma(1-\tau+\frac{\kappa}{p}\tau-\bar{z})^{-\beta} - (1-p)(1-\tau)^{-\beta} &= 0, \qquad (13) \end{aligned}$$

when t^* is interior. Applying Cramer's rule on the above two equations, $d\bar{z}/dw < 0$ if $\beta > \alpha$. The reverse is true for $\beta < \alpha$. Recognizing that $d\bar{z}/dw$ can be rewritten as

$$\frac{d\bar{z}}{dw} = \frac{d(z^*/w)}{dw} = \frac{z^*}{w^2}[\varepsilon_{z^*,w} - 1],$$

it is straightforward to show that $d\bar{z}/dw > 0, < 0, = 0$ when $\alpha >, <, = \beta$ implies that $\varepsilon_{z^*,w} >, <, = 1$.

Assume now that $t^* = 0$, the FOC on z can be rewritten as follows

$$-\gamma[w - z^0]^{-\beta} + (z^0)^{-\alpha} = 0$$

Fully differentiating this expression with respect to w, and using the previous equation, yields the following equality:

$$\frac{\beta}{c_d}[1 - \frac{dz^0}{dw}] - \frac{\alpha}{z^0}\frac{dz^0}{dw} = 0$$

Rearranging terms, we obtain that

$$\varepsilon_{z^0,w} = \frac{dz^0}{dw}\frac{w}{z^0} = \frac{\beta w}{\beta z^0 + \alpha c_d} = \frac{c_d + z^0}{z^0 + \frac{\alpha}{\beta}c_d}.$$

which yields Lemma 1 for $\varepsilon_{z^0,w}$.

Proof of Proposition 6

The threshold productivity $\tilde{w}$ is defined in the proof of Proposition 5. Applying Cramer's rule on (12) and (13), one finds that $d\tau^*/dw < 0$ if $\beta < \alpha$.

Appendix B: Two-period model

Let us assume a two-period model where in the first period, the agent decides to save for his old days and to invest in a LTCI in case he becomes dependent in the second period. In the second period, if he is autonomous, he consumes his savings, while if he becomes dependent he receives in addition a LTC benefit. Under that alternative modeling, the agent's expected utility writes as follows:

$$EU(t, z) = u_a(x) + (1 - p)u_a(c_a) + p[u_d(c_d) - h(z)],$$

where $x = w - s - t$ is first-period consumption under autonomy (with certainty the agent is autonomous), $c_a = Rs$ is second-period consumption if the agent is autonomous while $c_d = Rs + \kappa\frac{t}{p} - z$ is second-period consumption if dependent.

First-order conditions with respect to savings, LTC expenditures, z and the premium paid, t are:[21]

$$\frac{\partial EU}{\partial s} = -u_a'(x) + R[pu_d'(c_d) + (1 - p)u_a'(c_a)] = 0 \quad (14)$$

$$\frac{\partial EU}{\partial z} = -u_d'(c_d) - h'(z) = 0, \quad (15)$$

$$\frac{\partial EU}{\partial t} = -u_a'(x) + \kappa u_d'(c_d) \leq 0. \quad (16)$$

[21] We assume that second-order conditions are satisfied.

For simplicity, assume that $R = 1$. Under Inada conditions ($h'(0) \to -\infty$ and $u'_a(0) \to \infty$), it is necessarily the case that $s^*, z^* > 0$.

Replacing for eq. (14), the FOC with respect to t, eq. (16), can be rewritten as follows:

$$\frac{\partial EU}{\partial t} = (\kappa - p)u'_d(c_d) - (1-p)u'_a(s) \leq 0.$$

A necessary condition for t to be positive is that the degree of actuarial fairness is greater than the probability to become dependent, i.e. $\kappa > p$. In the following, we make this assumption.

We denote by (s^*, z^*, t^*) the solution to this system of 3 equations, with the corresponding consumption levels c_a^* and c_d^* and x^*.

It will prove useful to denote by s^0 and z^0 the optimal levels of savings and LTC expenditures (satisfying equations 14 and 15) when $t = 0$. In that case, we denote the consumption bundle as (z^0, x^0, c_d^0, c_a^0).

Our proposition below provides results similar to those we obtained in the baseline model where we had assumed away savings.

Proposition 7 *(i) An agent chooses to (resp., not to) buy LTCI if* $\kappa u'_d(s^0 - z^0) > u'_a(w - s^0)$ *(resp.,*$\leq$*),*
or equivalently, if $(\kappa - p)u'_d(s^0 - z^0) > (1-p)u'_a(s^0)$ *(resp.,*$\leq$*).*

(ii) If the agent decides to insure herself, the level of LTCI coverage is incomplete, that is $c_d^* < c_a^*$ *and* $z^* > \kappa t^*/p$.

(iii) $z^0 \leq z^*$ *and* $s^0 \leq s^* + t^*\kappa/p$ *with strict inequalities iff* $t^* > 0$. *We also have that* $c_a^0 \geq c_d^0 \geq c_d^* \geq c_d^0$ *with strict inequalities iff* $t^* > 0$.

Proof. (i) The agent decides to buy insurance if and only if her marginal gain from buying insurance is positive when $t = 0$, namely

$$\kappa u'_d(s^0 - z^0) > u'_a(w - s^0). \tag{17}$$

(ii) We now assume that $\kappa u'_d(s^0 - z^0) > u'_a(w - s^0)$ so that the agent buys LTCI at equilibrium. In that case, the FOC with respect to t holds with equality and t^* is defined by

$$u'_a(w - s^* - t^*) = \kappa u'_d(c_d^*) \tag{18}$$

where s^* is defined by eq. (14), z^* is defined (15). Since $\kappa \leq 1$ and $u_a'(.) > u_d'(.)$, we obtain from equation (18), that

$$x^* = w - s^* - z^* > c_d^* = s^* + \kappa \frac{t^*}{p} - z^*.$$

We further use equation (14), and since $u_a'(x^*) \leq u_d'(c_d^*)$, we can deduce that $u_d'(c_d^*) \geq u_a'(x^*) \geq u_a'(c_a^*)$, and thus that $c_d^* < x^* \leq c_a^*$ (where the equality holds if and only if $\kappa = 1$). This in turn implies that $s^* > s^* + \frac{\kappa t}{p} - z^*$ and thus that $z^* > \frac{\kappa t^*}{p}$.

(iii) If $t^* = 0$, we have by definition that $z^0 = z^*$ and $s^0 = s^* + t^*\kappa/p$. Assume then that $t^* > 0$.

Assume by contradiction that $z^0 > z^*$. We then have that $-h'(z^0) < -h'(z^*)$. Comparing the equation (15) with z^* and z^0, we obtain that

$$\begin{aligned} &\Rightarrow \quad u_d'(s^0 - z^0) < u_d'(s^* - z^* + t^*\kappa/p) && (19) \\ &\Leftrightarrow \quad s^0 - z^0 > s^* - z^* + t^*\kappa/p \\ &\Rightarrow \quad s^0 > s^* + t^*\kappa/p. && (20) \end{aligned}$$

under $z^0 > z^*$.

Making use of the FOCs on s, evaluated for $t = 0$ and for t^*:

$$\frac{\partial EU}{\partial s}|_{t=0} = -u_a'(w - s^0) + pu_d'(s^0 - z^0) + (1-p)u_a'(s^0) = 0, \quad (21)$$
$$\frac{\partial EU}{\partial s}|_{t=t^*} = -u_a'(w - s^* - t^*) + pu_d'(s^* - z^* + t^*\kappa/p) + (1-p)u_a'(s^*) = 0, \quad (22)$$

together with (19), we obtain

$$u_a'(w - s^0) - (1-p)u_a'(s^0) < u_a'(w - s^* - t^*) - (1-p)u_a'(s^*).$$

Note that (20) implies that $s^0 > s^*$, which in turn leads to

$$u_a'(w - s^0) < u_a'(w - s^* - t^*),$$

or equivalently that $s^0 < s^* + t^*$, a contradiction, with (20) since $\kappa > p$ (a necessary condition to have $t^* > 0$).

Proceeding as above (see eq. (19)) while assuming that $z^0 < z^*$, we obtain that $c_d^0 = s^0 - z^0 < c_d^* = s^* - z^* + t^*\kappa/p$ and that $s^0 < s^* + t^*\kappa/p$.

We now prove that $s^0 > s^*$(so that $c_a^0 > c_a^*$). Assume rather that $s^0 < s^*$, so that $u_a'(c_a^0) > u_a'(c_a^*)$.

Recall that we have just proved that $c_d^0 < c_d^*$, so that $u_d'(c_d^0) > u_d'(c_d^*)$. Using these two inequalities together with the FOCs on s, eqs. (21) and (22), we obtain that $u_a'(x^0) > u_a'(x^*)$, implying that $s^0 > s^* + t^*$, a contradiction with the assumption that $s^0 < s^*$.
Given that $s^0 > s^*$, we are unable to compare $x^0$and x^* (so that we may have $s^0 \lesseqgtr s^* + t^*$).
Putting all this together, we have proved that

$$c_a^0 \geq c_a^* \geq c_d^* \geq c_d^0$$

with strict inequalities iff $t^* > 0$. ■