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Nursing Homes' Competition and Distributional Implications when the Market is Two-Sided

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Abstract

We investigate the effect of competition in the nursing homes sector with a two-sided market approach. More precisely, we investigate the distributional implications across the three key actors involved (residents, nurses and nursing homes) that arise from the two-sidedness of the market. Within a Hotelling set up, nursing homes compete for residents and for nurses, who provide quality to residents, by setting residents price and nurses wage. Nurses are assumed altruistic and therefore motivated to provide quality. The market is two-sided because: i) a higher number of residents affects nurses workload, which affects their willingness to provide labour supply; and ii) a higher number of nurses affects residents' quality through a better matching process and by relaxing nurses time constraints. Our key findings are that i) the two-sidedness of the market leads to higher wages for nurses, which makes the nurses better off; ii) this is then passed to residents in the form of higher prices, which makes residents worse off; iii) nursing homes profits are instead unaffected. In contrast, when nurses wages are regulated, the two-sidedness of the market implies a transfer between residents and nursing homes. When residents price are regulated, it implies a transfer between nurses and nursing homes. These results are robust to institutional settings which employ pay-for-performance schemes (that reward either nursing homes or nurses): the two-sidedness of the market is strengthened and residents are still worse off.

Keywords: nursing homes; competition; two-sided markets; distribution.

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1 Introduction

Long term expenditure is expected to rise driven by an ageing population. Projections suggest that it might more than double by 2060 in several high-income countries (OECD, 2011). Nursing homes represent a significant proportion of long term care expenditure (around 0.5% and 1.5% of GDP in most countries; OECD, 2017) and this is likely to remain the case in the future despite governments policies that encourage informal care. Governments also increasingly encourage residents' choice: by developing quality ratings and spreading this information widely, they can encourage providers to compete. Quality is a key concern in the nursing homes sector and this is mainly driven by the care that nurses provide within the home. In the US for instance, nursing homes have been historically understaffed and the low levels of quality have motivated the introduction of minimum nurse staffing ratios in nursing homes.

This study investigates competition among nursing homes when providers compete both for residents and for nurses (who provide care to residents) under realistic assumptions which make the model *two-sided*. It then explores the distributional implications across the three key actors involved (residents, nurses and nursing homes) that arise from the two-sidedness of the market.

Nursing homes compete for residents on price and quality. Differently from the hospital sector, where prices are regulated in most OECD countries (except for the US outside of Medicare and Medicaid), nursing homes are free to set prices in most OECD countries within a competitive market. Nursing homes also compete on quality, which is an important aspect of residents experience.

One important way to influence the quality of care is by attracting a larger number of nurses. More nurses can improve quality of care through a better matching between residents and nurses (residents more likely to get along with the nurse) and a relaxed time constraint for nurses: for a given number of residents, more nurses implies that each nurse can spend more time with each individual resident which allows them to provide better care. The empirical evidence also supports that nurses staffing levels affect quality of care, as measured by deficiencies related to quality of care and quality of life (Lin, 2014), and incidence of pressure sores and urinary tract infections (Konetzka, Stearns and Park, 2008). The effect can be quantitatively large. Increasing nurses staffing by one standard deviation increases quality by more than 16% (Lin, 2014).

The importance of nurses staffing levels in affecting the demand for nursing homes is exemplified in the Medicare web portal *Nursing Home Compare* which provides case-mix adjusted staffing measures that prospective residents can use to choose the nursing home.[1] The empirical evidence also supports that demand responsiveness to quality can be enhanced by publicly reported quality information, which includes nurses staffing ratios in addition to clinical indicators (Werner *et al.*, 2012). In turn, this induces nursing homes to compete more aggressively on quality (Zhao, 2016). In summary, nursing homes have to compete not only on price but also for nurses in an attempt to increase their quality and attract more residents.

The nursing homes market is two-sided because: i) a higher number of nurses can affect demand for residents because it implies higher quality (relaxed time constraints for nurses and better matching between residents and nurses), and ii) a higher number of residents affects nurses labour supply by affecting nurses working conditions (nurses working under higher pressure with a larger volume of residents). Both effects create different types of network externalities between nurses and residents that are typical of two-sided markets.

The *distributional* consequences that arise in the presence of the two-sidedness of the market across residents, nurses and nursing homes, are as follows. The two-sidedness of the market leads to more intense competition for nurses since their number impacts positively the quality supplied by their nursing homes, which contributes to higher wages offered to nurses. As a result, nurses are better off. Such increases in wage are however passed to the residents in the form of higher prices, so that residents are worse off. Nursing-home profits are instead unaffected since the increase in nurses wages is offset by the increase in residents price. By offering a higher wage a nursing home increases nurses' utility directly but also indirectly by reducing the residents-nurse ratio which is valued by nurses (because of lower workload) and residents (because it implies a higher quality). These effects depend critically on the assumption that the marginal cost of providing quality for nurses depends on the resident-nurse ratio.

The two-sidedness of the market matters also if either residents prices or nurses wages are *regulated*, but has different distributional implications. When *nurses wages* are regulated, the two-sidedness of the market still increases residents' prices, which makes residents worse off,

[1] https://www.medicare.gov/nursinghomecompare/Data/About.html

if the regulated wages are not too high. In turn, this implies a transfer between residents and nursing homes (rather than between residents and nurses, as in the main model). More precisely, when the regulated wage is higher than its equilibrium value (without regulation), residents' price is lower, and both effects work at the expense of nursing homes' profits. Differently, when *residents price* are regulated, the two-sidedness of the market affects nurses wages and nursing homes profits, and it implies a transfer between nurses and nursing homes. A higher regulated price lowers residents utility, while it increases nurses wage with an ambiguous effect on nursing homes' profits. A fuller discussion dealing with the distributional implications of both regulations is given in the conclusion.

Pay for performance (P4P) schemes are increasingly used to incentivise quality of nursing homes (Miller and Singer Babiarz, 2014). We show that the key insights in terms of the effect of the two-sidedness of the market on residents prices and nurses wages also hold, and can be even strengthened, when P4P schemes are used to incentivise quality. We distinguish two types of P4P schemes. In the first scenario, a regulator uses a P4P scheme to reward nursing homes according to their performance on quality indicators. In the second scenario, nursing homes are allowed themselves to introduce P4P within their organization and reward nurses with higher quality. In both cases, we still find that the two-sidedness of the market makes the residents worse off. In the former case, quality remains unchanged (since nurses are not directly incentivised by the scheme) but P4P amplifies the competition effects arising from the two-sidedness of the market which in turn increases nurses wages even more, which are again passed to residents in the form of higher prices. In the latter, we show that the P4P scheme is such that the quality fee paid to nurses is equal to residents valuation of quality. Nurses are better off as a result since the fee more than compensates for their increase in effort. Residents are worse off since they are charged a higher price which does not compensate for the higher quality. Nursing homes pass the higher costs to the residents so that profits remain unchanged.[2]

Our model is related to two strands of literature: (i) the one that investigates quality

[2] In an extension we show that our results generally hold also in the presence of an uncovered market segment as long as this segment is small, which is in line with empirical evidence (Grabowski and Gruber, 2007; Mommaerts, 2018), and that the introduction of such uncovered market segment has ambiguous predictions on the relation between the two-sidedness parameter and equilibrium wage and price therefore providing limited additional insights to the model. This extension can be justified by some potential residents (with higher degree of independence in their daily activities) being taken care by informal carers if the price of the nursing home is too high or the quality too low.

and price competition within an horizontal differentiation framework (Hotelling/Salop models) within the health sector,[3] and (ii) the literature on two-sided markets that has dramatically grown during the last decade after the seminal articles of Rochet and Tirole (2003, 2006) but which has mainly been applied to sectors like banking or retail markets that are distinct from the nursing homes one. Nevertheless, there is a smaller literature which combines both elements. Bardey and Rochet (2010) analyse the competition between PPO and HMO which both compete for policyholders one side and to affiliate health care providers on the other side. We borrow from this study that consumers value having a larger pool of providers (doctors, nurses) but in order to work in an horizontal differentiation set-up we assume that all consumers value them in the same way.[4] Pezzino and Pignataro (2007) make a similar assumption that consumers value having access to more doctors in a context of hospital competition under regulated price. By contrast, we assume that the quality is not a decision variable from nursing homes but rather is endogenously decided by their nurses. Such a decision depends on their workload, which in turn depends on the resident/nurse ratio. As quality enters positively in nurses and customers utility function, this assumption yields some analogies to the common network externalities framework introduced by Bardey *et al.* (2012, 2014). We develop our two-sided analysis in a Hotelling framework, following Armstrong's (2006) two-sided model.

2 The model

We use a Hotelling set up with a market characterized by two nursing homes (providers) $i = \{1, 2\}$ which are located at the endpoints of the unit line $Y = [0, 1]$. Residents (consumers) are uniformly distributed on Y with a total mass normalised to 1. The utility of a resident located at $y \in Y$ and who chooses nursing home i is:

$$U_i(y) = \theta q_i - p_i - t_r y + 2\beta N_i \int_0^{\frac{1}{2N_i}} (v - z)\, dz, \qquad (1)$$

[3] See for example Ma and Burgess (1993), Gravelle (1999), and Brekke, Siciliani and Straume (2012).

[4] In other words, the adverse selection effect pointed out by these authors is assumed away here. Boilley (2012) extends this article and analyses the case where PPO and HMO compete for the same health care providers whereas Bardey and Rochet (2010) make a local monopoly assumption on this side.

where q_i is the quality of care provided to residents by nursing home i, p_i is the price charged by the nursing home to their residents, t_r is the marginal disutility of distance (for example related to distance to family and friends), θ is the marginal benefit of quality, and N_i is the number of nurses employed.

We assume that a higher number of nurses increases residents' utility and satisfaction as a result of higher chance of a good match between the resident and the nurse. This is captured by the last term in (1). Analytically, v gives the highest (gross) benefit to residents from staying in a nursing home which is reduced by an amount z if the resident is not matched with her ideal nurse. Therefore, z captures the cost of matching, which for analytical simplicity, we assume to be uniformly distributed between zero and one. This degree of satisfaction cannot be known *ex ante* as it depends on an *ex post* interaction between the nurse and the resident. When residents choose their nursing home, they will take into account that nursing homes with more nurses are associated in expected terms with a better match and therefore a higher utility. β is a preference parameter related to the marginal benefit of residents from a better nurse-resident match.[5] Solving for the integral in (1) residents' utility can be written as:

$$U_i(y) = \theta q_i - p_i - t_r y + \beta \left(v - \frac{1}{4N_i} \right). \tag{2}$$

Assuming that quality is high enough to ensure that the market is covered, the demand functions of nursing homes 1 and 2 are respectively:

$$D_1 = \frac{1}{2} + \frac{\theta}{2t_r}(q_1 - q_2) + \frac{1}{2t_r}(p_2 - p_1) - \frac{\beta}{8t_r}\left(\frac{1}{N_1} - \frac{1}{N_2} \right), \tag{3}$$

$$D_2 = 1 - D_1. \tag{4}$$

These demand functions suggest that nursing homes with higher quality, lower prices, and a higher number of nurses attract a larger number of residents.

We now turn to the market for nurses. Nurses are also uniformly distributed on Y with a total mass normalised to 1. The utility of a nurse located at $y \in Y$ who works for nursing home

[5] See Gal-Or (1997) for a similar assumption applied to healthcare markets.

i is:

$$V_i(y) = w_i + \alpha q_i - \frac{1}{2}\frac{c}{(k-\gamma)+\gamma\frac{N_i}{D_i}} q_i^2 - t_N y, \tag{5}$$

where w_i is the fixed wage paid to the nurse by nursing home i, and t_N is the transportation costs for nurses, which reflects the desire to work close from home. We assume that nurses are altruistic and care about the residents, as it is often argued that nursing is a vocational job. The assumption that nurses are motivated or altruistic has been recognized for long time within the health economics literature,[6] and more recently in the literature on motivated agents in the broader public sector.[7] Altruism is captured by the positive parameter α, and assume that residents value quality weakly more than nurses, $\theta \geq \alpha$. Providing quality is however costly to the nurse. The positive parameter c (k) is (inversely) related to the marginal disutility of providing quality.

We also critically allow for congestion effects through the positive parameter γ. We make the intuitive assumption that for nurses it is more costly to provide quality when the residents-nurses ratio is high, everything else equal. Nurses will have to work harder to offer the same attention to their residents if the number of residents per nurse increases. We enter γ in the cost function so that the marginal cost of quality is unchanged in equilibrium. Therefore, comparative statics with respect to γ is equivalent to the strength of the two-sidedness of the market in a *Common Network Externality* framework.[8]

Maximising nurse's utility with respect to quality, we obtain the first order condition for the optimal level of quality chosen by a nurse working for nursing home i:

$$q_i^* = \frac{\alpha}{c}\left(k - \gamma + \gamma\frac{N_i}{D_i}\right). \tag{6}$$

Quality increases in the nurses' altruism and decreases in the nurse-resident ratio of the nursing home since a higher ratio increases the marginal disutility of providing quality.

[6] See, *e.g.*, Ellis and McGuire (1986), Chalkley and Malcolmson (1998), Eggleston (2005), Heyes (2005), Jack (2005), Kaarbøe and Siciliani (2011).

[7] See, for example, Francois (2000), Murdock (2002), Glazer (2004), Besley and Ghatak (2005), and Francois and Vlassopoulos (2008) for a literature review.

[8] This type of two-sidedness introduced in Bardey *et al.* (2012, 2014) occurs when both sides value the quality provided and the quality depends positively on the number of providers (nurses) and negatively on the number of consumers (residents).

After substitution, the indirect utility function of the nurse working for nursing home i is:

$$V_i = w_i + \frac{\alpha^2}{2c}\left(k - \gamma + \gamma\frac{N_i}{D_i}\right) - t_N y, \tag{7}$$

which is decreasing in the residents-nurses ratio. *Ceteris paribus*, it is more pleasant to work in nursing home i if the nurse has fewer residents to take care of.

The supply functions of nurses in nursing homes 1 and 2 are given by:

$$N_1 = \frac{1}{2} + \frac{1}{2t_N}(w_1 - w_2) + \frac{\gamma\alpha^2}{4ct_N}\left(\frac{N_1}{D_1} - \frac{N_2}{D_2}\right), \tag{8}$$

$$N_2 = 1 - N_1. \tag{9}$$

Everything else equal, the nursing home that pays a higher wage will attract more nurses. Moreover, nurses are more willing to work for nursing homes that have low resident-nurse ratios, which could be interpreted broadly as better working conditions. After substituting for quality, we can also re-write the demand functions for residents as

$$D_1 = \frac{1}{2} + \frac{1}{2t_r}(p_2 - p_1) - \frac{\beta}{8t_r}\left(\frac{1}{N_1} - \frac{1}{N_2}\right) + \frac{\gamma\alpha\theta}{2t_r c}\left(\frac{N_1}{D_1} - \frac{N_2}{D_2}\right), \tag{10}$$

and $D_2 = 1 - D_1$. It is worth to emphasize the two-sided nature of the market by noticing that residents' demand and nurses' supply are inter-related. This arises because the number of nurses affects positively residents' demand through better matching (third term in (10)) and through higher quality, due to lower congestion (fourth term in (10). In turn, a higher number of residents affects nurses labour supply through higher workload and therefore worse working conditions (third term in (8)).

The comparative static of residents' demand and nurses' supply with respect to residents' price and nurses' wage is

$$\frac{dD_1}{dp_1} = -\frac{1}{2\Delta}\left(ct_N - \frac{\gamma\alpha^2}{4D_1(1-D_1)}\right), \tag{11}$$

$$\frac{dN_1}{dp_1} = \frac{\gamma\alpha^2}{8\Delta}\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right) > 0, \tag{12}$$

$$\frac{dD_1}{dw_1} = \frac{\beta c}{16\Delta}\left(\frac{1}{N_1^2} + \frac{1}{(1-N_1)^2}\right) + \frac{\gamma\alpha\theta}{4\Delta D_1(1-D_1)} > 0, \tag{13}$$

$$\frac{dN_1}{dw_1} = \frac{ct_r}{2\Delta} + \frac{\gamma\alpha\theta}{4\Delta}\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right) > 0, \tag{14}$$

where Δ is defined in Appendix 8.1 and is positive if the problem is well behaved. This highlights again the two-sidedness of nursing home competition. First, an increase in nurses' wage by nursing home 1 increases nurses' supply. This in turn increases residents' demand since residents value more nurses, through a better matching process. These two effects are respectively captured by dN_1/dw_1 and dD_1/dw_1 when setting $\gamma = 0$. There are however two additional effects at work: a higher number of nurses also reduces the resident-nurse ratio. This reduces the cost for nurses to provide quality, which further increases nurses' willingness to work for nursing home 1, and ultimately leads to an increase in quality, which is valued by residents.

An increase in prices charged to residents by nursing home 1 reduces residents' demand and therefore also reduces the resident-nurse ratio. The latter makes nurses more willing to work for nursing home 1 because of better working conditions, and also has a feedback effect on demand: although facing a higher price residents value the reduced resident-nurse ratio. We assume that the latter is a second order effect, so that an increase in price always decreases demand. This seems the most plausible and realistic scenario. At the symmetric equilibrium this implies $ct_N > \gamma\alpha^2$, which as shown below is required for the nursing home maximisation problem to be well behaved.

2.1 Optimal residents price and nurses wage

The profit function of nursing home 1 is

$$\Pi_1 = (p_1 - g)\, D_1(p_1, p_2, w_1, w_2) - w_1 N_1(p_1, p_2, w_1, w_2), \tag{15}$$

where g denotes the marginal cost of having a resident in a nursing home.

The first order conditions with respect to residents' price and nurses' wage are:

$$\frac{\partial \pi_1}{\partial p_1} = (p_1 - g)\frac{\partial D_1}{\partial p_1} + D_1 - w_1 \frac{\partial N_1}{\partial p_1} = 0, \tag{16}$$

$$\frac{\partial \pi_1}{\partial w_1} = (p_1 - g)\frac{\partial D_1}{\partial w_1} - N_1 - w_1 \frac{\partial N_1}{\partial w_1} = 0. \tag{17}$$

The first two terms of the optimality condition for residents' price p_1 are in line with the traditional monopolistic pricing rule. An increase in price raises revenues on all infra-marginal resident but also reduces demand. Moreover, this optimality condition (16) contains a new additional term, which is negative. When setting the price, nursing homes have to take into account that a higher price will reduce the resident-nurse ratio which will attract a larger number of nurses that will translate into higher nurses expenditure. This last effect therefore tends to reduce the price.

The first order condition for nurses' wage w_1 is such that it trades off the benefits from a larger residents' demand generated by an increase in quality (better matching and lower resident-nurse ratio) with the cost of higher wage for nurses. The two-sidedness of this optimality condition comes from the fact that residents' demand increases in the wage paid to nurses.[9]

Within a nursing home, prices charged to residents and wages paid to nurses tend to be *complement*:

$$\frac{\partial^2 \pi_1}{\partial p_1 \partial w_1} = \frac{\partial D_1}{\partial w_1} - \frac{\partial N_1}{\partial p_1}. \tag{18}$$

The two-sidedness of the nursing home market makes that a higher wage paid to nurses increases the number of nurses working for a nursing home, which in turn increases quality and decrease demand sensitivity, therefore allowing nursing homes to charge a higher price. There is however another effect going in the opposite direction. A higher wage makes a marginal increase in price more costly since there are more nurses willing to work for the nursing home when prices are higher due to the lower workload. The latter is generally smaller, so that residents' price and nurses' wage are generally complements. A sufficient condition to ensure that this is the

[9] The second-order conditions are given by: $\frac{\partial^2 \pi_1}{\partial p_1^2} = 2\frac{\partial D_1}{\partial p_1} < 0$, $\frac{\partial^2 \pi_1}{\partial w_1^2} = -2\frac{\partial N_1}{\partial w_1} < 0$, and $\frac{\partial^2 \pi_1}{\partial p_1^2}\frac{\partial^2 \pi_1}{\partial w_1^2} > \left(\frac{\partial^2 \pi_1}{\partial p_1 \partial w_1}\right)^2$ where $\frac{\partial^2 \pi_1}{\partial p_1 \partial w_1} = -\frac{\partial N_1}{\partial p_1} + \frac{\partial D_1}{\partial w_1}$.

case is that $\theta \geq \alpha$ and $D_1 \leq 0.75$ (see Appendix 8.2). We assume this is the case below.

The next expressions summarise whether the strategic complementarity or substitutability between nursing homes' decision variables, residents' prices and nurses' wages.

$$\frac{dp_1}{dp_2} = -\frac{1}{\Lambda}\left(\underbrace{\frac{\partial D_1}{\partial p_2}}_{+}\underbrace{\frac{\partial^2 \pi_1}{\partial w_1^2}}_{-} + \underbrace{\frac{\partial^2 \pi_1}{\partial p_1 w_1}}_{+}\underbrace{\frac{\partial N_1}{\partial p_2}}_{-}\right), \tag{19}$$

$$\frac{dw_1}{dw_2} = \frac{1}{\Lambda}\left(\underbrace{\frac{\partial N_1}{\partial w_2}}_{-}\underbrace{\frac{\partial^2 \pi_1}{\partial p_1^2}}_{-} + \underbrace{\frac{\partial^2 \pi_1}{\partial p_1 w_1}}_{+}\underbrace{\frac{\partial D_1}{\partial w_2}}_{-}\right), \tag{20}$$

where $\Lambda > 0$ by the second order conditions (see Appendix 8.2), $\partial D_1/\partial p_2 = -\partial D_1/\partial p_1 > 0$, $\partial N_1/\partial p_2 = -\partial N_1/\partial p_1 < 0$, $\partial N_1/\partial w_2 = -\partial N_1/\partial w_1 < 0$ and $\partial D_1/\partial w_2 = -\partial D_1/\partial w_1 < 0$. It suggests that prices are strategic complements. Nurses wages are also strategic complements if the complementarity between residents prices and nurses wages (captured by $\partial^2 \pi_1/\partial p_1 w_1$) or the residents demand responsiveness to competitor's nurses wage is not too high.

2.2 Symmetric equilibrium

The symmetric equilibrium is summarized in the following proposition:

Proposition 1 *Suppose that $t_N - \gamma\alpha(\alpha + 2\theta)/c < \beta \leq t_N + t_r$ to ensure equilibrium existence. At a symmetric equilibrium, residents' prices and nurses' wages are such that:*

$$\begin{aligned} p^* &= g + t_r + \frac{\gamma\alpha(\alpha + 2\theta)}{c}, \\ w^* &= \beta - t_N + \frac{\gamma\alpha(\alpha + 2\theta)}{c}, \end{aligned}$$

and nursing homes' profits are:

$$\Pi^* = \frac{1}{2}(t_r + t_N - \beta).$$

See Appendix 8.3 for proof. The price mark-up charged by nursing homes on residents' side depends on the transportation cost (t_r), which is in line with the standard Hotelling model. Lower transportation costs imply more competition and a more responsive demand function to price, which in turn reduce the price. Similarly, the wage paid to nurses is negatively related to nurses transportation cost (t_N). Lower transportation costs for nurses imply that more nurses are more willing to switch provider for a given increase in wage, which implies that nursing homes will compete more aggressively to attract nurses by offering a higher wage. The wage paid to nurses is positively related to the marginal residents evaluation of having more nurses through better matching process.

Both residents prices and nurses wages have an additional term, which is due to the two-sided nature of the market. The parameter γ plays a critical role in the analysis, which recall measures the degree to which nurses' workload (through the resident-nurse ratio) affects the marginal cost of quality. If γ is zero, the marginal cost of providing quality is independent of resident-nurse ratio, and the solution reduces to $p^* = g + t_r$, $w^* = \beta - t_N$. It is therefore this parameter that introduces the two-sidedness of the market. When γ is positive and the marginal cost of quality depends on the resident-nurse ratio, nursing homes will compete more aggressively on wages: by offering a higher wage a nursing home can attempt to attract more nurses, which also has the potential additional effect of reducing the resident-nurse ratio which is valued directly by nurses (because of lower workload) and residents (because it implies a higher quality). Residents however are charged a higher price as a result, and this higher price is entirely transferred from residents to nurses, such that nursing homes' profit is ultimately independent of any quality considerations. This arises because both sides, *i.e.* residents like nurses, value quality. This quality component enters into the class of common network externalities studied in Bardey *et al.* (2014) and as it depends on the nurse/resident ratio.[10] Only residents' valuation for additional nurses affects nursing homes' profit: a higher evaluation for nurses (β) triggers more competition for nurses which translates into higher wages and lower profits.

Perhaps counter-intuitively, higher altruism leads to higher nurses' wage. From a contract theory perspective we could have expected that nursing homes would take advantage of nurses' vocations to pay them a lower wage, as pointed out in Heyes (2005). Higher altruism amplifies the mechanisms introduced by the two-sidedness of the market (captured by γ) leading to an even more aggressive competition for nurses.

[10] More generally, if common network externalities (CNE hereafter) are represented by an homogeneous function, Bardey *et al.* (2014) show that the rents obtained by the providers, here the nursing homes, at the symmetric (and covered market) equilibrium depend on the homogeneity degree of such CNE. As the ratio is a 0-degree homogeneous function, profits are independent of this quality component.

Finally, a note on equilibrium existence. We require profits to be weakly positive and nurses' wage to be strictly positive. The first requires residents' evaluation of quality to be not too high ($\beta < t_r + t_N$). Otherwise, competition leads to ruinous competition where nursing homes make negative profits in equilibrium. The second requires that residents' valuation is not too low that it leads to negative wages ($\beta > t_N - \alpha\gamma\left(\alpha + 2\theta\right)/c$).

2.3 Residents' and nurses' utility and welfare

In the symmetric equilibrium, quality provided is equal to $q^* = \alpha k/c$, and does not depend on the two-sidedness of the market parameter γ. Residents' utility at the symmetric equilibrium is:

$$U_i^*(y) = \theta q^* - p^* + \beta\left(v - \frac{1}{2}\right) - t_r y = \frac{\alpha\left(\theta k - \gamma\left(2\theta + \alpha\right)\right)}{c} - g - t_r + \beta\left(v - \frac{1}{2}\right) - t_r y \quad (21)$$

which unambiguously decreases in γ. If the market is two-sided, quality does not vary, neither it changes the number of nurses in the market, but it increases residents price and therefore reduces residents' utility.

The nurses' utility is given by:

$$V_i^*(y) = \beta - t_N + \frac{\alpha}{2c}\left(\alpha k + 2\gamma\left(\alpha + 2\theta\right)\right) - t_N y. \quad (22)$$

Nurses utility instead increases in γ. If the market is two-sided, their workload is unaffected in equilibrium but they receive a higher wage.

Finally, the total welfare is given by the sum of residents and nurses utility and nursing homes profits:

$$U^* + V^* + \Pi^* = \left(\alpha + 2\theta\right)\frac{\alpha k}{2c} - \frac{3}{2}t_N - \frac{3}{2}t_r - g + v\beta. \quad (23)$$

The two-sidedness of the market only implies a transfers from residents to nurses, and does not affect profits, so that the social welfare remains unchanged.

3 Price or wage regulation

In this section, we compare the results of the main model with two other institutional settings. First, we investigate the scenario when nurses wages are regulated and residents prices are endogenously determined. We show that the two-sidedness of the market still affects residents' prices despite nurses wages being regulated. Second, we investigate the scenario when prices are fixed or regulated and nurses wage are endogenous. Again, the two-sidedness of the market affects nurses wages despite prices being regulated.

3.1 Symmetric equilibrium under regulated nurses wages

Suppose that nurses wages are regulated and takes a value $\overline{w}$. Using the first-order condition for price (16) we obtain:

Proposition 2 *At a symmetric equilibrium with a regulated wage $\overline{w}$, the price charged to residents is:*

$$\bar{p} = g + t_r + \frac{\gamma\alpha}{ct_N - \gamma\alpha^2}\left(\alpha\left(\beta - \bar{w}\right) + 2\theta t_N\right). \tag{24}$$

Note that $ct_N - \gamma\alpha^2 > 0$ for the second order condition of the price to hold.[11] If the market is not two-sided, we obtain the traditional mark-up pricing: $\bar{p} = g + t_r$. Instead, if the market is two-sided, compared to the familiar benchmark, the price charged to residents at equilibrium depends on the regulated wage. An increase in regulated nurses' wage reduces residents' price. This arises because a higher price tends to attract nurses at the margin through a more favorable resident-nurse ratio and therefore tends to exacerbate wage expenditure for nurses. This latter effect, which is analogous to the one described in Section 2, is larger (in absolute terms) the higher is the regulated wage. This leads to the (possibly) counter-intuitive result, that the price charged to residents decreases with the regulated nurses' wage (when intuitively we may expect the higher nurses wage to be passed on to the resident through higher prices). As a result, residents are always better off when nurses' wage increase.

Whether the price under regulated wages is higher or lower compared to the scenario where nursing homes compete on nurses' wage depends on the level of the regulated wage. Re-writing

[11] The Second Order Condition is $\frac{\partial^2 \pi_1}{\partial p_1^2} = 2\frac{\partial D_1}{\partial p_1} < 0$, and $\frac{dD_1}{dp_1} = -\frac{1}{2\Delta}\left(ct_N - \frac{\gamma\alpha^2}{4D_1(1-D_1)}\right)$, which evaluated at the symmetric equilibrium gives: $\frac{dD_1}{dp_1} = -\frac{1}{2\Delta}\left(ct_N - \gamma\alpha^2\right) < 0$.

(24) as:

$$\bar{p} = p^* + \frac{\gamma\alpha^2}{ct_N - \gamma\alpha^2}\left(w^* - \overline{w}\right) \tag{25}$$

suggests that the price under regulated wages will be lower whenever the regulated wage is higher than the wage when nursing homes compete for nurses.

The effect of the two-sidedness parameter on residents' price is, differently from Section 2, ambiguous:

$$\frac{\partial\bar{p}}{\partial\gamma} = \frac{\alpha ct_N\left(\alpha\left(\beta - \overline{w}\right) + 2\theta t_N\right)}{\left(ct_N - \gamma\alpha^2\right)^2}. \tag{26}$$

On one hand, it increases the price charged to customers because a higher price tends to attract nurses at the margin through a more favorable resident-nurse ratio. On the other hand, it exacerbates the wage expenditure for nurses. The first effect dominates when the regulated wage is not too high ($\overline{w} < \overline{w}_\gamma \equiv \beta + 2\theta t_N/\alpha$), and the two-sidedness works in the same direction as in Section 2. The difference is that this price increase is not transferred to nurses wage anymore.

Equilibrium existence requires weakly positive profits:

$$\bar{\Pi} = \frac{1}{2}\left(\bar{p} - g - \bar{w}\right) = \frac{1}{2}\left(t_r + \gamma\alpha\frac{\alpha\left(\beta - \bar{w}\right) + 2\theta t_N}{ct_N - \gamma\alpha^2} - \bar{w}\right) \geq 0, \tag{27}$$

which implies that nurses' (regulated) wage cannot be too high: $\bar{w} \leq \bar{w}_\pi \equiv t_r + 2\theta/c + \gamma\alpha^2\left(\beta - t_r\right)/ct_N$. Differently from Section 2, the two-sidedness of the market affects profits, through its effect on prices. Similarly, residents' utility is only affected through changes in prices. Since nurses' wages are regulated, nurses' utility in equilibrium is not affected by the two-sidedness parameter. Total welfare is also not affected. The two-sidedness of the market implies a transfer between residents and the nursing home, rather than between the residents and the nurses when wages are endogenously determined (as in Section 2).

We conclude by analysing how the degree of competition in the residents or nurses market, and nurses' altruism, affects prices.

$$\frac{\partial\bar{p}}{\partial t_r} > 0,\ \frac{\partial\bar{p}}{\partial t_N} = -\gamma\alpha^2\frac{2\theta\alpha\gamma + c\left(\beta - \bar{w}\right)}{\left(ct_N - \gamma\alpha^2\right)^2},\ \frac{\partial\bar{p}}{\partial\alpha} = 2\gamma ct_N\frac{\theta t_N\left(1 + \alpha^2\right) + \alpha\left(\beta - \bar{w}\right)}{\left(ct_N - \gamma\alpha^2\right)^2}. \tag{28}$$

As expected, lower residents transportation costs imply a more responsive demand function to price, which reduce residents' price. Similarly to Section 2, higher altruism increases residents' prices when the markets is two-sided if the regulated wage is not too high. Differently from Section 2, nurses' transportation costs now affect residents price when the market is two-sided, and a more responsive nurses' supply function implies an increase in residents' price if the wage is not too high.

3.2 Symmetric equilibrium under regulated prices

Suppose now that prices are regulated with $p_1 = p_2 = \widehat{p}$ but nursing homes compete for nurses. Using the first-order condition for nurses wage (17) we obtain:

Proposition 3 *At a symmetric equilibrium with a regulated price $\widehat{p}$, nurses' wage $\widehat{w}$ is:*

$$\widehat{w} = \beta \frac{(\widehat{p} - g)\, c - \gamma \alpha^2}{c t_r + 2\alpha\theta\gamma} - t_N + \gamma\alpha \frac{\alpha t_r + 2\theta\,(\widehat{p} - g)}{c t_r + 2\alpha\theta\gamma}. \tag{29}$$

We assume that the regulated price is always sufficiently high to rule out corner solutions, *i.e.* $\widehat{p} - g \geq A / (c\beta + 2\alpha\theta\gamma)$, where $A := c t_r t_N + \alpha\gamma\,(\alpha\,(\beta - t_r) + 2\theta t_N)$. The optimal nurses' wage increases with the price mark up. The higher the mark up, the stronger is the incentive for nursing homes to attract nurses to induce higher quality, higher demand and revenues. The incentive to increase nurses wage is reinforced when this translates into a larger marginal increase in residents' demand (through better matching process, and higher quality due to more favorable resident-nurse ratio).

Whether the wage under regulated prices is higher or lower compared to the scenario where nursing homes compete on residents' price depends on the level of the regulated price:

$$\widehat{w} = w^* + \frac{c\,(\beta + 2\gamma\alpha\theta)}{c t_r + 2\gamma\alpha\theta}\,(\widehat{p} - p^*)\,. \tag{30}$$

If the regulated price is below the price when nursing homes compete in an unregulated market, then nurses wages will also be lower.

The effect of the two-sidedness parameter on nurses wages price is, differently from Section 2, ambiguous:

$$\frac{\partial \widehat{w}}{\partial \gamma} = \frac{\alpha c\left(t_r - \beta\right)\left(2\theta\left(\widehat{p} - g\right) + \alpha t_r\right)}{\left(ct_r + 2\gamma\theta\alpha\right)^2}. \tag{31}$$

This effect is positive only if residents transportation costs are high compared to their valuation of the benefits from a better match between nurses and residents. Since residents prices are regulated, residents' utility in equilibrium is not affected by the two-sidness parameter. The two-sidedness of the market implies a transfer between the nurses and the nursing home, rather than between the nurses and the residents when both prices and wages are endogenously determined (as in Section 2).

We conclude by analysing how the degree of competition in the residents or nurses market, and nurses' altruism, affects wages.

$$\frac{\partial \widehat{w}}{\partial t_N} = -1; \frac{\partial \widehat{w}}{\partial t_r} = \frac{\alpha^2\gamma\left(c\beta + 2\alpha\theta\gamma\right) - c(\widehat{p} - g)}{\left(ct_r + 2\alpha\theta\gamma\right)^2}; \frac{\partial \widehat{w}}{\partial \alpha} = \frac{2\gamma\left(t_r - \beta\right)\left[\theta c(\widehat{p} - g) + \alpha\left(2ct_r + \alpha\theta\gamma\right)\right]}{\left(ct_r + 2\alpha\theta\gamma\right)^2}. \tag{32}$$

Lower nurses transportation costs imply a more responsive supply function to wages, which reduce nurse's wage. Differently from Section 2, residents transportation costs affect nurses wages: lower residents transportation costs (more competition for residents) have an indeterminate effect on nurses wages. Similarly to Section 2, altruism affects nurses wages but the effect is indeterminate. It is positive only if and only if residents' transportation costs are high compared to the benefits valuation from matching residents with nurses.

Equilibrium profit is given by

$$\widehat{\Pi} = \frac{1}{2}\left(\widehat{p} - g - \widehat{w}\right) = \frac{1}{2}\left(t_N + \left(t_r - \beta\right)\frac{\left(\widehat{p} - g\right)c - \gamma\alpha^2}{ct_r + 2\alpha\theta\gamma}\right), \tag{33}$$

which is increasing in the regulated price only if the direct effect on profits is not offset by an increase in nurses wage. To ensure at least zero profit the following condition has to hold $(\widehat{p} - g) \geq A/c\left(\beta - t_r\right)$.[12]

[12] This condition is always satisfied if $\beta < t_r$. But if $\beta > t_r$, then this condition is more stringent than the one required for a weakly positive nurses salary. Therefore, to ensure that both profits and salaries are weakly positive the following has to hold: $\widehat{p} - g \geq \max\left\{A/c\left(\beta - t_r\right), A/(c\beta + 2\alpha\theta\gamma)\right\}$.

4 Pay for performance

P4P schemes which reward quality have become increasingly popular within the health and long-term care settings. However, the empirical evidence in relation to its effectiveness is mixed (Miller and Singer Babiarz, 2014). For instance, Werner *et al.* (2013) did not find that nursing homes significantly improved quality following the introduction of P4P. The authors argue that current P4P programs may fail to achieve quality improvements because the incentives were paid to the nursing homes, rather than to their individual staff members.

In this section, we extend the main model in Section 2 by introducing P4P in two plausible scenarios. First, we consider a P4P scheme financed by a public funder (*e.g.* a local or central goverment) under which nursing homes receive financial incentives that depend on the level of quality provided. Second, we assume that nursing homes can remunerate nurses not only through the fixed wage but also through a pay-for-performance scheme. Therefore, nursing homes compete for nurses not only through the wage but through a fee, set by the nursing home, for each unit of quality provided.

4.1 Pay for performance at the nursing home level

In a partial equilibrium environment, we assume that nursing homes receive a financial incentive τ, which is paid by a public funder, for each unit quality provided to residents. In such a case, nursing home 1 profit function is:

$$\Pi_1 = (p_1 - g)\, D_1\,(p_1, p_2, w_1, w_2) - w_1 N_1\,(p_1, p_2, w_1, w_2) + \tau q_1. \tag{34}$$

The first order conditions for residents price and nurses wage are:

$$\frac{\partial \pi_1}{\partial p_1} = (p_1 - g)\frac{\partial D_1}{\partial p_1} + \frac{1}{2} - w_1\frac{\partial N_1}{\partial p_1} + \tau\frac{\gamma\alpha}{cD_1^2}\left(\frac{\partial N_1}{\partial p_1}D_1 - \frac{\partial D_1}{\partial p_1}N_1\right) = 0, \tag{35}$$

$$\frac{\partial \pi_1}{\partial w_1} = (p_1 - g)\frac{\partial D_1}{\partial w_1} - \frac{1}{2} - w_1\frac{\partial N_1}{\partial w_1} + \tau\frac{\gamma\alpha}{cD_1^2}\left(\frac{\partial N_1}{\partial w_1}D_1 - \frac{\partial D_1}{\partial w_1}N_1\right) = 0. \tag{36}$$

Compared to the benchmark case, we have an additional term that captures the price and wage effects on quality according to demand and supply responses.

Proposition 4 *In the symmetric equilibrium when a P4P scheme is financed by a public funder, we have:*

$$
\begin{aligned}
q' &= \frac{\alpha k}{c}, \\
p' &= g + t_r + \frac{\gamma\left(\alpha + 2\left(\theta + \tau\right)\right)\alpha}{c}, \\
w' &= \beta - t_N + \frac{\gamma\left(\alpha + 2\left(\theta + \tau\right)\right)\alpha}{c}, \\
\pi' &= \frac{1}{2}\left(t_r + t_N - \beta + \tau\frac{\alpha k}{c}\right).
\end{aligned}
$$

The proof is provided in the Appendix (section 8.4). When P4P is paid to nursing homes by a public funder, we find that P4P does not affect quality. This result is in line with the study by Werner *et al.* (2013) that finds that nursing homes did not significantly improve quality following the introduction of P4P. Nevertheless, P4P has some distributional implications. Nursing homes profit increases by the amount of the financial incentives. Moreover, the P4P scheme exacerbates the positive (respectively negative) externality that nurses (resp. residents) generate on quality. Thus, nurses also benefit from a wage increase while residents face a higher price. Anatically, the effect of an increase in the P4P fee τ on prices and wages is equivalent to an increase in the marginal valuation of quality θ, but has a different effect on profits, which increase with the fee but are not affected by residents valuation of quality.

4.2 Pay for performance at the staff level

Consider now that the P4P scheme are paid to nurses by nursing homes. This can be thought as the nursing home using P4P as an internal management tool to increase quality. Each nursing home can set a fee f_i for each unit of quality provided. The utility of a nurse located at y who works for nursing home i becomes:

$$V_i(y) = w_i + \left(\alpha + f_i\right) q_i - \frac{c}{k - \gamma + \gamma\frac{N_i}{D_i}}\frac{q_i^2}{2} - t_N y, \qquad (37)$$

and the level of quality provided is: $q_i^* = (\alpha + f_i)\left(k - \gamma + \gamma\frac{N_i}{D_i}\right)/c$ implying a nurse indirect utility:

$$V_i(y) = w_i + \frac{1}{2c}(\alpha + f_i)^2\left(k - \gamma + \gamma\frac{N_i}{D_i}\right) - t_N y. \tag{38}$$

An increase in the fee increases quality and increases nurses' indirect utility. The resident demand function and nurse supply function are now given by:

$$D_1 = \frac{1}{2} + \frac{1}{2t_r}(p_2 - p_1) - \frac{\beta}{8t_r}\left(\frac{1}{N_1} - \frac{1}{N_2}\right) + \frac{\theta}{2t_r}(q_1 - q_2), \tag{39}$$

$$\begin{aligned} N_1 = {} & \frac{1}{2} + \frac{1}{2t_N}(w_1 - w_2) + \frac{\gamma\alpha^2}{4ct_N}\left(\frac{N_1}{D_1} - \frac{N_2}{D_2}\right) \\ & + \frac{1}{4ct_N}\left((f_1)^2\left(k - \gamma + \gamma\frac{N_1}{D_1}\right) - (f_2)^2\left(k - \gamma + \gamma\frac{N_2}{D_2}\right)\right) \end{aligned} \tag{40}$$

with $D_2 = 1 - D_1$, $N_2 = 1 - N_1$.

Proposition 5 *In the symmetric equilibrium when nursing homes also compete in P4P we have:*

$$\begin{aligned} f^{**} &= \theta, \\ q^{**} &= \frac{(\alpha + f^{**})\,k}{c}, \\ p^{**} &= g + t_r + \frac{[\gamma(\alpha + 2\theta) + kf^{**}](\alpha + f^{**})}{c}, \\ w^{**} &= \beta - t_N + \frac{\gamma(\alpha + f^{**})(\alpha + 2\theta)}{c}, \\ \pi^{**} &= \frac{1}{2}(t_r + t_N - \beta). \end{aligned}$$

The proof is provided in the Appendix (section 8.4). The key result is that nursing homes set a quality fee which is equal to the residents' quality valuation. The quality fee does not depend on the nurses' altruism so that the level of quality reflects the sum of residents and nurses valuation, *i.e.* $(\alpha + \theta)$. Nursing homes profit is not affected by the introduction of the quality fee. Nurses overall payment, given by $w^{**} + f^{**}q^{**}$, is increased in the presence of the P4P; the quality fee is paid on top of the fixed wage, and moreover the introduction of the P4P scheme stimulates competition for nurses, which increases their fixed wage as well. However, the increase in nurses' payment is passed to the residents through an increase in price so that nurses' profits remain unchanged.

It is straightforward to show that the solution in the absence of the P4P scheme is obtained when the quality fee is set to zero, so that $p^{**} > p^*$, $w^{**} > w^*$ and $\pi^{**} = \pi^*$, with $p^{**} - p^* = f^{**}q^{**} + w^{**} - w^*$. Nurses always gain from the introduction of the P4P scheme:

$$V^{**} - V^* = \frac{\theta}{c}\left[\gamma\left(2\theta + \alpha\right) + \frac{k}{2}\left(2\alpha + \theta\right)\right] > 0. \tag{41}$$

Whether residents gain or lose is in principle indeterminate: they gain from higher quality but pay a higher price:

$$U^{**} - U^* = \theta\left(q^{**} - q^*\right) - \left(p^{**} - p^*\right) = -\frac{\theta}{c}\left[k\alpha + \gamma\left(\alpha + 2\theta\right)\right] < 0. \tag{42}$$

The price effect however dominates, and residents have lower utility in the presence of the P4P scheme. The increase in nurses utility is however higher than the reduction in residents utility, so that welfare is increased:

$$W^{**} - W^* = \Delta V + \Delta U = \frac{\theta^2 k}{2c} > 0. \tag{43}$$

We summarise with the following proposition.

Proposition 6 *When the pay for performance scheme is paid by nursing homes to nurses, the quality fee is equal to residents valuation of quality. The scheme induces an increase in quality and nurses wage which is passed on to residents prices so that profits are unchanged. Residents are worse off as a result of higher prices and despite the higher quality. Nurses are better off and total welfare is increased.*

See Appendix 8.4 for proof. The proposition highlights that quality and welfare enhancements may come at the cost of higher prices and lower residents utility.

5 Comparison

Table 1 compares the results obtained under the unregulated market with the two regulated markets covered in section 3 and the two P4P schemes explored in section 4. We use the equilibrium under the unregulated market as the benchmark. Under nurses *wage* regulation, if the regulated wage is higher (lower) than under the unregulated market, then residents and nurses are both better (worse) off but nursing homes are worse (better) off. Therefore, wage regulation can improve both residents and nurses utility at the expense of nursing homes profits.

Interestingly, *price* regulation on residents side works in a very different way. In such a case, a reduction (increase) in the price charged to residents *reduces* (increases) nurses wage, and therefore regulation has opposite effects on nurses and residents utility, while the effect on nursing homes profits is indeterminate and is determined by whether the price effects dominates on the wage effect.

The introduction of pay-for-perfomance schemes increases quality only when it is paid directly to nurses, otherwise quality is unchanged and P4P only has distributional implications. Perhaps surprisingly, when P4P are paid to nurses then residents are worse off under P4P because the increase in quality is more than offset by an increase in price. This arises because nurses succeed to extract residents surplus through two channels. On the one hand, they receive an additional fee for this quality increase which is just equal to residents valuation of quality. On the other hand, it also stimulates competition between nursing homes to attract nurses that contributes to increase their wage. Both effects are contribute to increase residents price.

Table 1. Comparison of different scenarios

	ΔU	ΔV	$\Delta \pi$	ΔW
$\bar{w} > w^*$, wage regulation	> 0	> 0	< 0	$= 0$
$\bar{w} < w^*$, wage regulation	< 0	< 0	> 0	$= 0$
$\bar{p} < p^*$, price regulation	> 0	< 0	$\gtrless 0$	$= 0$
$\bar{p} > p^*$, price regulation	< 0	> 0	$\gtrless 0$	$= 0$
$\tau > 0$, introduction of P4P at nursing homes level	< 0	> 0	> 0	$= 0$
$f^* > 0$, introduction of P4P at nurses level	< 0	> 0	$= 0$	> 0

6 Uncovered residents market

In this section, we investigate the scenario when the reservation utility of some residents is not high enough to guarantee a covered market. We assume that there are two types of residents $m \in$

$\{H, L\}$ who differ in their gross valuation of treatment in proportion λ and $1-\lambda$, respectively. Their utility function is

$$U_i(y) = \begin{cases} S + \theta q_i - p_i - t_r y + \beta\left(v - \frac{1}{4N_i}\right) & \text{if } m = H, \\ s + \theta q_i - p_i - t_r y + \beta\left(v - \frac{1}{4N_i}\right) & \text{if } m = L, \end{cases} \tag{44}$$

where $S > s$. This parameter could be related to the degree of autonomy (e.g. degree of independence in their daily activities) so that some potential residents are not willing to go to a nursing home if the price is too high or the quality too low relative to living by themselves or with an informal carer. The nurses supply function remains unchanged. We characterize the symmetric equilibrium in such an environment. We focus on equilibria where the H-segment is fully covered, and the L-segment is only partially covered.

For the H-segment the market is covered and the demand functions for the two nursing homes are:

$$\begin{aligned} D_1^H &= \frac{1}{2} + \frac{1}{2t_r}(p_2 - p_1) - \frac{\beta}{8t_r}\left(\frac{1}{N_1} - \frac{1}{N_2}\right) + \frac{\theta}{2t_r}\frac{\alpha\gamma}{c}\left(\frac{N_1}{D_1} - \frac{N_2}{D_2}\right), \\ D_2^H &= 1 - D_1^H. \end{aligned} \tag{45}$$

For the L-segment, the market is uncovered and the demand functions are:

$$D_i^L = \frac{1}{t_r}\left[\frac{\alpha\theta}{c}\left(k - \gamma + \gamma\frac{N_i}{D_i}\right) - p_i + \beta\left(v - \frac{1}{4N_i}\right)\right], \qquad \forall i = \{1, 2\}. \tag{46}$$

To keep the computations simpler we set $\beta = 0$, given that the role of this parameter has been explored in Section 2. Total demand for nursing home 1 is given by

$$D_1 = \lambda\left[\frac{1}{2} + \frac{1}{2t_r}(p_2 - p_1) + \frac{\theta}{2t_r}\frac{\alpha\gamma}{c}\left(\frac{N_1}{D_1} - \frac{N_2}{D_2}\right)\right] + \frac{1-\lambda}{t_r}\left[\frac{\alpha\theta}{c}\left(k - \gamma + \gamma\frac{N_1}{D_1}\right) - p_1\right], \tag{47}$$

and an analogous demand can be obtained for nursing home 2, D_2.

The following proposition is obtained.

Proposition 7 *At the symmetric equilibrium ($D_1^* = D_2^* = D^*$ and $N_1^* = N_2^* = 1/2$), residents* *price and nurses wage are:*

$$p^* - g = 2t_r \frac{D^*(p^*,\gamma)}{(2-\lambda)} + \frac{\alpha\gamma\left(\alpha + 4\theta D^*(p^*,\gamma)\right)}{8\left(D^*(p^*,\gamma)\right)^2},$$
$$w^* = \frac{\alpha\gamma}{c}\left(\frac{\alpha}{2D^*(p^*,\gamma)} + \frac{2\theta}{2-\lambda}\right) - t_N,$$

where equilibrium demand is implicitly defined by the function

$$F(D^*,p^*,\gamma) \equiv D^*(p^*,\gamma) - \frac{1-\lambda}{t_r}\left[\frac{\alpha\theta}{c}\left(k - \gamma + \gamma\frac{1}{2D^*(p^*,\gamma)}\right) - p^*\right] = 0,$$

and is decreasing in price and increasing in the two-sidedness parameter:

$$\frac{dD^*}{dp^*} = -\frac{1-\lambda}{t_r}\frac{1}{\partial F/\partial D^*} < 0, \quad \frac{dD^*}{d\gamma} = \frac{1-\lambda}{t_r}\frac{\alpha\theta}{c}\left(\frac{1}{2D^*} - 1\right)\frac{1}{\partial F/\partial D^*} > 0,$$

where $\partial F/\partial D^* = 1 + \left((1-\lambda)\gamma\alpha\theta\right)/\left(2ct_r\left(D^*\right)^2\right) > 0.$

See Appendix (Section 8.5) for proof. The effect of the two-sidedness parameter on residents price is given by:

$$\frac{dp^*}{d\gamma} = \frac{1}{1 - \frac{dG}{dD^*}.\frac{dD^*}{dp^*}}\left[\frac{\alpha\left(\alpha + 2\theta D^*(p^*,\gamma)\right)}{8c\left(D^*(p^*,\gamma)\right)^2} - \frac{dG}{dD^*}.\frac{dD^*}{d\gamma}\right], \tag{48}$$

where

$$G(D^*,p^*,\gamma) \equiv p^* - g - \frac{2t_r D^*(p^*,\gamma)}{2-\lambda} - \frac{\alpha\gamma\left(\alpha + 4\theta D^*(p^*,\gamma)\right)}{8\left(D^*(p^*,\gamma)\right)^2} = 0, \tag{49}$$

with $dG/dD^* = -2t_r/\left(2-\lambda\right) + \alpha\gamma\left(\alpha + 4\theta\right)/4(D^*)^3 \lesseqgtr 0$ (by the implicit function theorem). Compared to Section 2, there are now some additional effects which relate to changes in equilibrium demand.

When the size of uncovered market is small (so that λ is close to 1), then dD^*/dp^* and $dD^*/d\gamma$ are small and the two-sidedness parameter increases price, $dp^*/d\gamma > 0$, as in the main model. Therefore, the results obtained in Section 2 are robust to the inclusion of an uncovered market as long as this market is relatively small. Moreover, given that $dG/dD^* \lesseqgtr 0$, the introduction of an uncovered market segment could in principle reinforce or weaken the relation between the two-sidedness parameter and equilibrium price.

The effect of the two-sidedness parameter on nurses wage is given by:

$$\frac{dw^*}{d\gamma} = \alpha\left(\frac{\alpha}{2D^*} + \frac{2\theta}{(2-\lambda)}\right) - \frac{\alpha^2\gamma}{2\,(D^*)^2}\left(\frac{dD^*}{dp^*}\frac{dp^*}{d\gamma} + \frac{dD^*}{d\gamma}\right). \tag{50}$$

Again, when the size of uncovered market is small (so that λ is close to 1), then dD^*/dp^* and $dD^*/d\gamma$ are small and the two-sidedness parameter increases nurses wage, $dw^*/d\gamma > 0$, as in the main model. Given that $dp^*/d\gamma \lesseqgtr 0$, the introduction of an uncovered market segment could reinforce or weaken the relation between the two-sidedness parameter and equilibrium wage.

The empirical evidence does indeed confirm that demand for nursing homes is generally inelastic (Grabowski and Gruber, 2007; Mommaerts, 2018) and that it is only residents with lower dependence who are like to respond to changes in market conditions.

7 Conclusion

This study has investigated the market for nursing homes using a *two-sided market* approach. Our key assumptions, which are in line with the empirical evidence (see Introduction) are that i) a higher number of nurses can affect demand for residents because it potentially implies higher quality (through better matching and relaxed time constraints), and ii) a higher number residents affects nurses labour supply by affecting nurses working conditions (nurses working under higher pressure with a larger volume of residents). It is the combination of these two assumptions which makes the market two-sided.

Our main result is that the two-sidedness in the market has *distributional* implications as it leads to more intense competition for nurses and to higher wages, so that nurses are better off. Such increases in wage are then passed to the residents in the form of higher price, which makes the residents worse off. Nursing homes profits are instead unaffected since the increase in nurses wages is exactly offset by the increase in price. By offering a higher wage a nursing home increases nurses utility directly but also indirectly by reducing the residents-nurse ratio which is valued by nurses (because of lower workload) and residents (because it translates into a higher quality). These incentive effects depend critically on how the resident-nurse ratio affects nurses utility and therefore the quality they provide. When the resident-nurse ratio does not affect directly nurses utility, both nurses wages and residents prices tend to be lower.

The two-sidedness of the market matters and has different distributional implications if either residents price or nurses wage is *regulated*. When *nurses wages* are regulated, the two-sidedness of the market implies a transfer between residents and nursing homes. When *residents price* is regulated, it instead implies a transfer between nurses and nursing homes.

Our results have therefore implications for the regulation of nursing homes sector. Suppose that an unregulated market, where prices and wages are endogenous, leads to resident prices which are considered excessive by the regulator (*e.g.* an anti-trust authority, health ministry, local and federal authorities). Two regulatory interventions are possible. We show, counter-intuitively, that an increase in the regulated nurses wage implies a *reduction* in residents price. Then, regulating the nurses wages at a level which is higher than the wage in the unregulated market, will also reduce residents price. Both nurses and residents are better off as a result, while nursing homes profits will correspondingly reduce. An alternative way to reduce residents price is for the regulator to introduce price regulation. The introduction of a regulated price, which is below the price in an unregulated market, will make residents better off, but in this scenario will instead compress nurses wage, and might also reduce nurses profits. This form of price regulation therefore generates a transfer from nurses (and potentially nursing homes) to residents.

Finally, we show that policy interventions which facilitate the introduction of pay for performance schemes (*e.g.* by developing reliable quality metrics) at the nursing homes level, rather than the staff level, do not affect the quality provided, which seems in line with mixed empirical evidence in relation to its effectiveness (Miller and Singer Babiarz, 2014). In contrast, P4P which are paid by nursing homes to nurses have the intended effect of improving quality. They also increase the scope for competition for nurses, which tranlates into larger wages for nurses and higher nurses utility. Our analysis however highligths an adverse effect for residents. Despite benefiting from higher quality, residents are worse off since the higher price that they are charged does not compensate for the higher quality, while nursing homes profits are unchanged. Again, the two-sidedness of the market does not favour residents. Overall, our analysis highlights that policy evaluations that empirically test the effect of the introduction of P4P schemes in nursing homes markets should not only focus on quality outcomes but also on its effect on nursing homes profits and residents prices, and more broadly its distributional consequences.

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8 Appendix

8.1 Demand function analysis

The demand functions system is given by:

$$\begin{aligned} D_1 &= \frac{1}{2} + \frac{\theta}{2t_r}(q_1 - q_2) + \frac{1}{2t_r}(p_2 - p_1) - \frac{\beta}{8t_r}\left(\frac{1}{N_1} - \frac{1}{N_2}\right), \\ D_2 &= 1 - D_1, \\ N_1 &= \frac{1}{2} + \frac{1}{2t_N}(w_1 - w_2) + \frac{\alpha^2\gamma}{4ct_N}\left(\frac{N_1}{D_1} - \frac{N_2}{D_2}\right), \\ N_2 &= 1 - N_1. \end{aligned} \tag{51}$$

Consider the following functions:

$$\begin{aligned} \Gamma_D &= D_1 - \frac{1}{2} - \frac{\alpha\theta\gamma}{2ct_r}\left(\frac{N_1}{D_1} - \frac{1-N_1}{1-D_1}\right) - \frac{1}{2t_r}(p_2 - p_1) + \frac{\beta}{8t_r}\left(\frac{1}{N_1} - \frac{1}{1-N_1}\right), \\ \Gamma_N &= N_1 - \frac{1}{2} - \frac{1}{2t_N}(w_1 - w_2) - \frac{\alpha^2\gamma}{4ct_N}\left(\frac{N_1}{D_1} - \frac{1-N_1}{1-D_1}\right). \end{aligned} \tag{52}$$

Totally differentiating, we obtain:

$$\begin{bmatrix} 1 + \frac{2\alpha\theta\gamma}{ct_r}\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right) & -\frac{\alpha\theta\gamma}{2ct_r D_1(1-D_1)} - \frac{\beta}{8t_r}\left(\frac{1}{N_1^2} + \frac{1}{(1-N_1)^2}\right) \\ \frac{\alpha^2\gamma}{4ct_N}\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right) & 1 - \frac{\alpha^2\gamma}{4ct_N D_1(1-D_1)} \end{bmatrix} \begin{bmatrix} dD_1 \\ dN_1 \end{bmatrix} = - \begin{bmatrix} \frac{1}{2t_r} \\ 0 \end{bmatrix} dp_1. \tag{53}$$

Applying the Cramer's rule we obtain:

$$\frac{dD_1}{dp_1} = -\frac{1}{2\Delta}\left(ct_N - \frac{\alpha^2\gamma}{4D_1(1-D_1)}\right), \tag{54}$$

where

$$\begin{aligned} \Delta &: = ct_r t_N \det = ct_r t_N + \alpha\gamma\left(\frac{\alpha\beta}{32t_r}\left(\frac{1}{N_1^2} + \frac{1}{(1-N_1)^2}\right)\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right) - \alpha t_r\left(\frac{1}{D_1} + \frac{1}{1-D_1}\right)\right. \\ &\left. +2\theta t_N\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right)\right), \end{aligned} \tag{55}$$

which is positive under minimal regularity conditions. Similarly, we have:

$$\frac{dN_1}{dp_1} = \frac{\alpha^2\gamma}{8\Delta}\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right). \tag{56}$$

Differentiating with respect to w_1 and applying Cramer's rule gives:

$$\frac{dD_1}{dw_1} = \frac{\alpha\theta\gamma}{4\Delta D_1(1-D_1)} + \frac{\beta c}{16\Delta}\left(\frac{1}{N_1^2} + \frac{1}{(1-N_1)^2}\right), \tag{57}$$

$$\frac{dN_1}{dw_1} = \frac{ct_r}{2\Delta} + \frac{\alpha\theta\gamma}{4\Delta}\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right). \tag{58}$$

8.2 Profit complementarity and substitutability in price and wage

The effect of an increase in wage on the marginal profitability of price is:

$$\frac{\partial^2\pi_1}{\partial p_1\partial w_1} = \frac{\gamma\alpha}{4\Delta}\frac{2\theta D_1(1-D_1) - \alpha\left[(1-N_1)D_1^2 + N_1(1-D_1)^2\right]}{2D_1^2(1-D_1)^2} + \frac{\beta c}{16\Delta}\left(\frac{1}{N_1^2} + \frac{1}{(1-N_1)^2}\right). \tag{59}$$

Suppose that we set altruism at the highest possible value, $\alpha = \theta$, then:

$$\frac{\partial^2\pi_1}{\partial p_1\partial w_1} = \frac{\gamma\alpha\theta}{4\Delta}\frac{(2D_1-1)N_1 + D_1(2-3D_1)}{2D_1^2(1-D_1)^2} + \frac{\beta c}{16\Delta}\left(\frac{1}{N_1^2} + \frac{1}{(1-N_1)^2}\right), \tag{60}$$

which is always positive when $D_1 \leq 0.75$.

Differentiating the first-order conditions with respect to p_2 gives:

$$\begin{bmatrix} \frac{\partial^2\pi_1}{\partial p_1^2} & \frac{\partial^2\pi_1}{\partial p_1 w_1} \\ \frac{\partial^2\pi_1}{\partial w_1 p_1} & \frac{\partial^2\pi_1}{\partial w_1^2} \end{bmatrix} \begin{bmatrix} dp_1 \\ dw_1 \end{bmatrix} = -\begin{bmatrix} \frac{\partial D_1}{\partial p_2} \\ -\frac{\partial N_1}{\partial p_2} \end{bmatrix} dp_2. \tag{61}$$

Applying Cramer's rule yields:

$$\frac{dp_1}{dp_2} = \frac{1}{\Lambda}\begin{vmatrix} -\frac{\partial D_1}{\partial p_2} & \frac{\partial^2\pi_1}{\partial p_1 w_1} \\ \frac{\partial N_1}{\partial p_2} & \frac{\partial^2\pi_1}{\partial w_1^2} \end{vmatrix} = -\frac{1}{\Lambda}\left(\frac{\partial D_1}{\partial p_2}\frac{\partial^2\pi_1}{\partial w_1^2} + \frac{\partial^2\pi_1}{\partial p_1 w_1}\frac{\partial N_1}{\partial p_2}\right) > 0, \tag{62}$$

where

$$\Lambda := \frac{\partial^2\pi_1}{\partial p_1^2}\frac{\partial^2\pi_1}{\partial w_1^2} - \left(\frac{\partial^2\pi_1}{\partial p_1 w_1}\right)^2 > 0. \tag{63}$$

Differentiating the first-order conditions with respect to w_2 gives:

$$\begin{bmatrix} \frac{\partial^2\pi_1}{\partial p_1^2} & \frac{\partial^2\pi_1}{\partial p_1 w_1} \\ \frac{\partial^2\pi_1}{\partial w_1 p_1} & \frac{\partial^2\pi_1}{\partial w_1^2} \end{bmatrix} \begin{bmatrix} dp_1 \\ dw_1 \end{bmatrix} = - \begin{bmatrix} \frac{\partial D_1}{\partial w_2} \\ -\frac{\partial N_1}{\partial w_2} \end{bmatrix} dw_2. \tag{64}$$

Applying Cramer's rule yields:

$$\frac{dw_1}{dw_2} = \frac{1}{\Lambda} \begin{vmatrix} \frac{\partial^2\pi_1}{\partial p_1^2} & -\frac{\partial D_1}{\partial w_2} \\ \frac{\partial^2\pi_1}{\partial w_1 p_1} & \frac{\partial N_1}{\partial w_2} \end{vmatrix} = \frac{1}{\Lambda}\left(\frac{\partial N_1}{\partial w_2}\frac{\partial^2\pi_1}{\partial p_1^2} + \frac{\partial^2\pi_1}{\partial p_1 w_1}\frac{\partial D_1}{\partial w_2}\right). \tag{65}$$

8.3 Symmetric equilibrium

The first-order conditions evaluated at the symmetric equilibrium are:

$$\frac{\partial\pi_1}{\partial p_1} = (p_1 - g)\frac{\partial D_1}{\partial p_1} + \frac{1}{2} - w_1\frac{\partial N_1}{\partial p_1} = 0, \tag{66}$$

$$\frac{\partial\pi_1}{\partial w_1} = (p_1 - g)\frac{\partial D_1}{\partial w_1} - \frac{1}{2} - w_1\frac{\partial N_1}{\partial w_1} = 0. \tag{67}$$

where

$$\frac{dD_1}{dp_1} = -\frac{ct_N - \gamma\alpha^2}{2A}, \; \frac{dN_1}{dp_1} = \frac{\gamma\alpha^2}{2A}, \tag{68}$$

$$\frac{dN_1}{dw_1} = \frac{ct_r + 2\alpha\theta\gamma}{2A}, \frac{dD_1}{dw_1} = \frac{\beta c + 2\alpha\theta\gamma}{2A}, \tag{69}$$

with

$$A =: ct_r t_N + \alpha\gamma\left(\alpha\left(\beta - t_r\right) + 2\theta t_N\right). \tag{70}$$

We obtain:

$$p_1^* - g = \frac{w_1^*\frac{\partial N_1}{\partial p_1} - \frac{1}{2}}{\frac{\partial D_1}{\partial p_1}}, \; w_1^* = \frac{\left(p_1^* - g\right)\frac{\partial D_1}{\partial w_1} - \frac{1}{2}}{\frac{\partial N_1}{\partial w_1}}. \tag{71}$$

Substituting for the price into the wage equation, we obtain

$$w_1^* = \frac{\frac{\partial D_1}{\partial p_1} + \frac{\partial D_1}{\partial w_1}}{2\left(\frac{\partial N_1}{\partial p_1}\frac{\partial D_1}{\partial w_1} - \frac{\partial N_1}{\partial w_1}\frac{\partial D_1}{\partial p_1}\right)} = \beta - t_N + \frac{\gamma\alpha\left(\alpha + 2\theta\right)}{c}. \tag{72}$$

Substituting the equilibrium wage into the price equation, we obtain

$$p_1^* - g = \frac{w_1 \frac{\partial N_1}{\partial p_1} - \frac{1}{2}}{\frac{\partial D_1}{\partial p_1}} = t_r + \frac{\alpha\gamma\left(2\theta + \alpha\right)}{c}. \tag{73}$$

At the symmetric equilibrium, nursing homes' profit are:

$$\Pi_1 = \frac{1}{2}\left(t_r + \frac{\alpha\gamma\left(2\theta+\alpha\right)}{c} + t_N - \frac{\alpha\gamma\left(2\theta+\alpha\right)+\beta c}{c}\right) = \frac{1}{2}\left(t_r + t_N - \beta\right). \tag{74}$$

8.4 Pay for performance

Pay for performance at the nursing home level. The first order condition for price is:

$$(p_1 - g) = \frac{\frac{1}{2} - w_1 \frac{\partial N_1}{\partial p_1} + \tau \frac{\gamma\alpha}{cD_1^2}\left(\frac{\partial N_1}{\partial p_1} D_1 - \frac{\partial D_1}{\partial p_1} N_1\right)}{\frac{\partial D_1}{\partial p_1}}. \tag{75}$$

Substituing in the first order condition for wage, we obtain:

$$\frac{1}{2} - w_1 \frac{\partial N_1}{\partial p_1} + \tau \frac{\gamma\alpha}{c}\left[\frac{\frac{\partial N_1}{\partial p_1} D_1 - \frac{\partial D_1}{\partial p_1} N_1}{D_1^2}\right] \frac{\frac{\partial D_1}{\partial w_1}}{\frac{\partial D_1}{\partial p_1}} - \frac{1}{2} - w_1 \frac{\partial N_1}{\partial w_1} + \tau \frac{\gamma\alpha}{c}\left[\frac{\frac{\partial N_1}{\partial w_1} D_1 - \frac{\partial D_1}{\partial w_1} N_1}{D_1^2}\right] = 0.$$

Thus, we have:

$$\begin{aligned} w' &= \frac{\frac{\partial D_1}{\partial p_1} + \frac{\partial D_1}{\partial w_1}}{2\left(\frac{\partial N_1}{\partial p_1}\frac{\partial D_1}{\partial w_1} - \frac{\partial N_1}{\partial w_1}\frac{\partial D_1}{\partial p_1}\right)} + \frac{2\gamma\tau\alpha}{c}, \\ p' &= \frac{\frac{\partial N_1}{\partial p_1} + \frac{\partial N_1}{\partial w_1}}{2\left(\frac{\partial N_1}{\partial p_1}\frac{\partial D_1}{\partial w_1} - \frac{\partial N_1}{\partial w_1}\frac{\partial D_1}{\partial p_1}\right)} + \frac{2\gamma\tau\alpha}{c}. \end{aligned} \tag{76}$$

Pay for performance at the staff level. When P4P are paid to nurses, the demand and supply function are modified. At the symmetric equilibrium we obtain:

$$\begin{aligned}
\frac{dD_1}{dp_1} &= \frac{-1}{2ct_r t_N \det}\left[ct_N - (\alpha+f)^2\gamma\right], \\
\frac{dN_1}{dp_1} &= \frac{1}{2ct_r t_N \det}\left((\alpha+f)^2\gamma\right), \\
\frac{dD_1}{dw_1} &= \frac{1}{2ct_r t_N \det}\left[2(\alpha+f)\theta\gamma + \beta c\right], \\
\frac{dN_1}{dw_1} &= \frac{1}{2ct_r t_N \det}\left[ct_r + 2(\alpha+f)\theta\gamma\right], \\
\frac{dD_1}{df_1} &= \frac{k}{2ct_r t_N \det}\left[\theta t_N + (\alpha+f)\left(\theta\frac{\gamma}{c}(\alpha+f)+\beta\right)\right], \\
\frac{dN_1}{df_1} &= \frac{(\alpha+f)k}{2ct_r t_N \det}\left[t_r + \theta(\alpha+f)\frac{\gamma}{c}\right].
\end{aligned} \tag{77}$$

Since the quality provided by nurses is received by each resident, each nursing home i has to pay $f_i q_i D_i$. Then, nursing home i profit function is:

$$\pi_1 = (p_1 - g - f_1 q_1) D_1(p_1, p_2, w_1, w_2, f_1, f_2) - w_1 N_1(p_1, p_2, w_1, w_2, f_1, f_2). \tag{78}$$

Substituting quality in nursing home profit yields:

$$\begin{aligned}
\pi_1 &= \left(p_1 - g - \frac{f_1}{c}(\alpha+f_1)\left(k-\gamma+\gamma\frac{N_1}{D_1}\right)\right)D_1 - w_1 N_1, \\
&= \left(p_1 - g - \frac{f_1(\alpha+f_1)}{c}(k-\gamma)\right)D_1 - \left(w_1 + \frac{f_1(\alpha+f_1)\gamma}{c}\right)N_1.
\end{aligned} \tag{79}$$

The first-order conditions with respect to p_1, w_1 and f_1 are respectively:

$$\begin{aligned}
\frac{\partial\pi_1}{\partial p_1} &= \left(p_1 - g - \frac{f_1(\alpha+f_1)}{c}(k-\gamma)\right)\frac{\partial D_1}{\partial p_1} + D_1 - \left(w_1 + \frac{(\alpha+f_1)f_1\gamma}{c}\right)\left(\frac{\partial N_1}{\partial p_1}\right), \\
\frac{\partial\pi_1}{\partial w_1} &= \left(p_1 - g - \frac{f_1(\alpha+f_1)}{c}(k-\gamma)\right)\frac{\partial D_1}{\partial w_1} - N_1 - \left(w_1 + \frac{(\alpha+f_1)f_1\gamma}{c}\right)\frac{\partial N_1}{\partial w_1}, \\
\frac{\partial\pi_1}{\partial f_1} &= \left(p_1 - g - \frac{f_1(\alpha+f_1)(k-\gamma)}{c}\right)\frac{\partial D_1}{\partial f_1} - \left(w_1 + \frac{(\alpha+f_1)f_1\gamma}{c}\right)\frac{\partial N_1}{\partial f_1} \\
&\quad -\frac{(2f_1+\alpha)}{c}\left(\gamma N_1 + (k-\gamma)D_1\right).
\end{aligned} \tag{80}$$

Differently from Section 2, nursing home 1 here also chooses the fee f_1 to maximize its profit. It is such that the marginal marginal revenue, *i.e.* the mark-up $p_1 - g - (k-\gamma)f_1(\alpha+f_1)/c$

multiplied by the marginal increase in demand $(\partial D_1/\partial f_1)$, is equal to the marginal cost due to the increase in the fee, given by $(2f_1+\alpha)(\gamma N_1+(k-\gamma)D_1)/c$, and the marginal cost of increasing the number of nurses, given by $(w_1+(\alpha+f_1)f_1\gamma/c)(\partial N_1/\partial f_1)$.

8.5 Uncovered residents market

The residents demand and nurses supply functions are implicitly given by

$$\begin{aligned}\Gamma_D &= D_1-\frac{\lambda}{2}-\frac{(1-\lambda)\,\alpha\theta\,(k-\gamma)}{t_r c}+\left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right)p_1-\frac{\lambda}{2t_r}p_2 \qquad (81)\\ &\quad -\frac{\theta\alpha\gamma}{c}\left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right)\frac{N_1}{D_1}+\lambda\frac{\theta}{2t_r}\frac{\alpha\gamma}{c}\frac{1-N_1}{D_2}=0,\\ \Gamma_N &= N_1-\frac{1}{2}-\frac{1}{2t_N}(w_1-w_2)-\frac{\alpha^2\gamma}{4ct_N}\left(\frac{N_1}{D_1}-\frac{1-N_1}{D_2}\right)=0. \qquad (82)\end{aligned}$$

Differentiation of this system with respect to p_1 and w_1 yields respectively:

$$\begin{pmatrix} 1+\frac{\theta\alpha\gamma}{c}\left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right)\frac{N_1}{(D_1)^2} & -\frac{\theta\alpha\gamma}{c}\left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right)\frac{1}{D_1}-\lambda\frac{\theta}{2t_r}\frac{\alpha\gamma}{c}\frac{1}{D_2} \\ \frac{\alpha^2\gamma}{4ct_N}\frac{N_1}{(D_1)^2} & 1-\frac{\alpha^2\gamma}{4ct_N}\left(\frac{1}{D_1}+\frac{1}{D_2}\right)\end{pmatrix}\begin{bmatrix} dD_1\\ dN_1\end{bmatrix}=-\begin{bmatrix}\left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right)\\ 0\end{bmatrix}dp_1,$$

$$\begin{pmatrix} 1+\frac{\theta\alpha\gamma}{c}\left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right)\frac{N_1}{(D_1)^2} & -\frac{\theta\alpha\gamma}{c}\left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right)\frac{1}{D_1}-\lambda\frac{\theta}{2t_r}\frac{\alpha\gamma}{c}\frac{1}{D_2} \\ \frac{\alpha^2\gamma}{4ct_N}\frac{N_1}{(D_1)^2} & 1-\frac{\alpha^2\gamma}{4ct_N}\left(\frac{1}{D_1}+\frac{1}{D_2}\right)\end{pmatrix}\begin{bmatrix} dD_1\\ dN_1\end{bmatrix}=-\begin{bmatrix}0\\ -\frac{1}{2t_N}\end{bmatrix}dw_1.$$

Applying the Cramer's rule, we obtain:

$$\begin{aligned}\frac{dD_1}{dp_1} &= -\frac{1}{\widetilde{\det}}\frac{1}{8t_r c t_N}\left[(2-\lambda)\left(4ct_N-\alpha^2\gamma\left(\frac{1}{D_1}+\frac{1}{D_2}\right)\right)\right], \qquad (83)\\ \frac{dN_1}{dp_1} &= \frac{1}{\widetilde{\det}}\frac{(2-\lambda)\,\alpha^2\gamma}{8t_r c t_N}\frac{N_1}{D_1^2}, \qquad (84)\\ \frac{dD_1}{dw_1} &= \frac{1}{\widetilde{\det}}\frac{\theta\alpha\gamma}{4ct_r t_N}\left(\frac{2-\lambda}{D_1}+\frac{\lambda}{D_2}\right), \qquad (85)\\ \frac{dN_1}{dw_1} &= \frac{1}{\widetilde{\det}}\frac{1}{4ct_r t_N}\left[2ct_r+\theta\alpha\gamma\,(2-\lambda)\frac{N_1}{D_1^2}\right]. \qquad (86)\end{aligned}$$

The determinant is given by:

$$\widetilde{\det} = \left[1+\frac{\theta\alpha\gamma}{c}\left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right)\frac{N_1}{D_1^2}\right]*\left[1-\frac{\alpha^2\gamma}{4ct_N}\left(\frac{1}{D_1}+\frac{1}{D_2}\right)\right] \tag{87}$$
$$+\frac{\alpha^2\gamma}{4ct_N}\frac{N_1}{D_1^2}*\left[\frac{\theta\alpha\gamma}{c}\left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right)\frac{1}{D_1}+\lambda\frac{\theta}{2t_r}\frac{\alpha\gamma}{c}\frac{1}{D_2}\right]$$
$$= \frac{1}{8c^2t_rt_N}\left[8c^2t_rt_N-2ct_r\alpha^2\gamma\left(\frac{1}{D_1}+\frac{1}{D_2}\right)\right.$$
$$\left.+\theta\alpha\gamma\left(2-\lambda\right)\frac{N_1}{\left(D_1\right)^2}4ct_N-2\theta\alpha\gamma\left(1-\lambda\right)\frac{N_1}{D_1^2}\alpha^2\gamma\frac{1}{D_2}\right]. \tag{88}$$

Symmetric equilibrium

We define

$$\widetilde{\Delta} := \frac{\partial N_1}{\partial p_1}\frac{\partial D_1}{\partial w_1}-\frac{\partial N_1}{\partial w_1}\frac{\partial D_1}{\partial p_1}=\frac{1}{\widetilde{\det}}\frac{2-\lambda}{4t_rt_N}. \tag{89}$$

By substitution, the equilibrium price and wage are:

$$p_1-g = \frac{1}{\widetilde{\Delta}}\left(\frac{1}{2}\frac{\partial N_1}{\partial p_1}+D_1\frac{\partial N_1}{\partial w_1}\right)=\frac{2t_rD_1}{(2-\lambda)}+\frac{\alpha\gamma\left(\alpha+4D_1\theta\right)}{8cD_1^2}, \tag{90}$$
$$w_1 = \frac{1}{\widetilde{\Delta}}\left(\frac{1}{2}\frac{\partial D_1}{\partial p_1}+D_1\frac{\partial D_1}{\partial w_1}\right)=\frac{\alpha\gamma}{c}\left(\frac{\alpha}{2D_1}+\frac{2\theta}{(2-\lambda)}\right)-t_N. \tag{91}$$