Ordering the Extraction of Polluting Nonrenewable Resources

by

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Abstract

A well known theorem by Herfindahl states that if nonrenewable resources differ by their cost of extraction, then their use must follow the "least cost first" principle. The low cost resource must be exploited first. In this paper we consider resources that are differentiated not by cost but by their pollution content. For instance, both coal and natural gas are used to produce electricity, yet coal is more polluting. Environmental regulation is imposed in the form of a cap on the stock of pollution. We show that if the cap is binding, the "clean" resource must be used first, exactly as Herfindahl had predicted. However, when the cap is non-binding, the "dirty" resource coal may be used first. We may also get a complete preference reversal, i.e., coal is used for some time, followed by natural gas and again by coal. Such outcomes do not arise in models of cost heterogeneity. A perverse policy implication is that regulating the stock of pollution may accelerate use of the polluting resource.

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1. Introduction

Energy markets are often characterized by the use of multiple nonrenewable resources, such as coal and natural gas, with different pollution characteristics. The same resource may have different qualities, as when high and low sulfur coal is used to generate electricity. Multiple resources are used in providing transportation services, such as cars running on gasoline or hybrids fueled by electricity produced from coal.

When these nonrenewable resources contribute differentially to pollution, what is the sequence of extraction over time? Hotelling (1931) developed the classical theory of extraction of a nonrenewable resource. Herfindahl (1967) extended the Hotelling model by considering many resources with different unit costs of extraction and proposed the "least cost first" principle: extraction must be ordered by cost, with the cheapest resource used first. Others have examined whether Herfindahl's model remains valid in a variety of situations: in a general equilibrium setting (Kemp and Long, 1980; Lewis, 1982), in the presence of setup costs (Gaudet, Moreaux and Salant, 2001), under heterogenous demands (Chakravorty and Krulce, 1994) and when the extraction rate is constrained (Amigues et al, 1998).

In these studies following Hotelling and Herfindahl, resources were differentiated by cost alone. In this paper, we abstract from cost considerations and focus on how the sequence of extraction may be affected when resources are differentiated only by their pollution characteristics. We consider two resources, one more polluting than the other. Without loss of generality, consider coal to be the dirty resource and natural gas the clean one. Both may be used to generate electricity. Environmental regulation is imposed through a cap on the stock of pollution. This may be a stylized approximation of an international agreement such as the Kyoto Protocol which aims to stabilize the concentration of greenhouse gases in the atmosphere.

With heterogeneity in pollution, the results are non-intuitive and differ sharply from Herfindahl. When the economy is already at its allowable stock of pollution, the clean

natural gas is used first and use of the dirty coal is postponed to the future. This is what one would expect, armed with insights from the Herfindahl model. However, when the economy starts from below the ceiling and accumulates pollution, coal may be used first and use of the clean natural gas is postponed. The optimal strategy is to benefit from natural dilution by building the stock of pollution as quickly as possible. This is done by burning coal rather than natural gas. This phenomenon is the reverse of Herfindahl – the inferior resource is used first. Only when natural gas is abundant is it used before coal.

We also obtain a complete "preference reversal" over resources. That is, coal may be used for a period of time, then natural gas, and finally coal for another time period. Such phenomena with complete switching between resources does not occur in models with multiple nonrenewable resources. Several papers report the joint extraction of a low and a high cost resource under cost heterogeneity (e.g., Amigues (1998), Kemp and Long (1980) and Chakravorty and Krulce (1994) but not a complete reversal. We show that it may be efficient to extract the dirty resource first, then only the clean resource until exhaustion, and finally, the dirty resource.

The pattern of extraction is dependent upon the initial endowments of the two resources. If the stock of coal is relatively low, then the Hotelling rents of the two resources are exactly equal and regulation is never binding. However, if coal is abundant, it has a lower Hotelling rent than natural gas. With abundant resources, extraction paths have a turnpike feature in which both resources are jointly extracted at the maximum allowed level. We show that all paths must pass through this turnpike.

Stylized facts suggest that our planet has abundant supplies of coal but limited amounts of natural gas. Our results imply that if the economy is below the regulated stock of emissions, then it is optimal to use the more polluting resource first and get to the ceiling. Once the ceiling is achieved, we must use the cleaner fuel first. From a policy point of view, we thus obtain a perverse result: imposing a cap on the stock of pollution may trigger a race to the ceiling by burning coal. [2]

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[2] In a parallel effort, Smulders and van der Werf (2005) develop a model to examine the extraction of heterogenous resources when the flow of emissions (not the stock as in our case) is constrained. In their model, resources are imperfect substitutes. Their findings suggest that the economy may use more of the clean resource before the constraint is binding in order to use more of the dirty resource during the period

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Section 2 extends the textbook Hotelling model with two costless but polluting resources. Section 3 characterizes the sequence of extraction when the stock of pollution is already at the ceiling. Section 4 extends this analysis to stocks starting from below the ceiling. Section 5 concludes the paper.

2. The Model

Consider an economy in which energy consumption at any time is given by q and the corresponding gross surplus u is twice continuously differentiable, strictly increasing and strictly concave over the interval $[0, \overline{q})$ with $\overline{q} > 0$ and constant over the interval $[\overline{q}, \infty)$ with $u(q) = \overline{u}$ for $q \geq \overline{q}$. This implies that $\lim_{q \uparrow \overline{q}} u'(q) = 0$. Let $\lim_{q \downarrow 0} u'(q) = +\infty$.[3]

Denote the marginal suplus by $p(q) \equiv u'(q)$ and by $d(p)$ the corresponding inverse demand function. Welfare W is the sum of the gross surplus discounted at some constant rate $\rho > 0$ given by

$$W = \int_0^\infty u(q) e^{-\rho t} dt.$$

Since $u(q) \leq \overline{u}$, this integral is well defined. We consider two nonrenewable resources indexed by $i=1,2$ which are perfect substitutes in demand. Each resource is characterized by the vector $\{\theta_i, X_i^0\}$ where θ_i is the pollution generated by burning one unit of the resource and X_i^0 is its given initial stock. Let X_i be the residual stock at time t and x_i the extraction rate. Then

$$\dot{X}_i = -x_i, i = 1,2.\tag{1}$$

Let the cost of extraction of both resources be zero. Without loss of generality, let us assume that resource *1* (say, natural gas) is cleaner than resource *2* (coal), $0 < \theta_1 < \theta_2$.

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when the constraint is binding. It may be useful at a later stage to develop a more general framework in which the effect of different types of regulation (stock vs flow constraints) on the extraction path is better understood.

[3] To keep the notation simple, we avoid writing the time argument t whenever the context is clear.

Let $X^0 = X_1^0 + X_2^0$ be the total initial stock, $X \equiv X_1 + X_2$ the total residual stock at time t, and $x \equiv x_1 + x_2$ the aggregate extraction at t.

As in most Hotelling models, we assume an abundant renewable backstop resource (e.g., solar energy) that is non-polluting. Its unit cost is given by $c_r > 0$. Let y be the rate of extraction of the backstop resource. Once coal and natural gas are exhausted, the price of the renewable resource is equal to c_r and its consumption is determined by $\tilde{y}$, the solution to the equation $u'(y) = c_r$.

Burning of the fossil fuels increases the aggregate stock of pollution denoted by $Z(t)$. This may be the level of carbon in the atmosphere. We assume that there is a natural decay at the constant rate $\alpha > 0$ so that

$$\dot{Z} = \sum_i \theta_i x_i - \alpha Z, \quad Z(0) = Z^0 \leq \bar{Z} \text{ given,} \qquad (2)$$

where $Z(0) = Z^0$ and $\bar{Z}$ is the regulated limit on the stock of pollution.[4] This may be exogenously imposed by some regulatory authority or the outcome of a negotiated international agreement.[5] It may also approximate a specific form of a damage function that equals zero at stock levels below $\bar{Z}$ but imposes prohibitive damages beyond that threshold value. The inequality in (2) suggests that the initial level of pollution is below the ceiling.

Suppose only resource i is being used when the stock of pollution equals $\bar{Z}$ over a time period. Let $\bar{x}_i$ be this maximum extraction rate. By (2), $\bar{x}_i = \dfrac{\alpha \bar{Z}}{\theta_i}$ and $\bar{p}_i$, the marginal gross surplus is given by $\bar{p}_i = u'(\bar{x}_i)$. Since natural gas is less polluting than coal, more gas would be used at the ceiling, so $\bar{x}_1 > \bar{x}_2$ and $\bar{p}_1 < \bar{p}_2$.

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[4] This natural decay is an increasing function of the stock Z. The linear form αZ is chosen for analytical convenience.

[5] For example, a policy goal of 550 parts per million of carbon in the atmosphere may correspond to a specific value of $\bar{Z}$.

The social planner maximizes the net surplus by choosing extraction rates of natural gas and coal and the backstop resource at each time t as follows:

$$\underset{\{x_i, i=1,2; y\}}{Max} \int_0^\infty \{ u(\sum_i x_i + y) - c_r y \} e^{-\rho t} dt \tag{3}$$

subject to (1) and (2), $X_i^0, Z^0,$ and $\overline{Z}$ given, with $\overline{Z} - Z(t) \geq 0$. The corresponding current value Lagrangian can be written as

$$L = u(\sum_i x_i + y) - c_r y - \sum_i \lambda_i x_i + \mu[\sum_i \theta_i x_i - \alpha Z] + v[\overline{Z} - Z] + \sum_i \gamma_i x_i + \gamma_r y$$

where v is the multiplier attached to $\overline{Z} - Z \geq 0$, γ_i and γ_r are the multipliers for the non-negativity constraints $x_i \geq 0$ and $y \geq 0$. The first order conditions are

$$u'(\sum_i x_i + y) = \lambda_i - \theta_i \mu - \gamma_i, i = 1,2 \tag{4}$$

$$u'(\sum_i x_i + y) = c_r - \gamma_r. \tag{5}$$

The dynamics of the costate variables are given by

$$\dot{\lambda}_i(t) = \rho \lambda_i \text{ which gives } \lambda_i = \lambda_i^0 e^{\rho t}, i = 1,2 \tag{6}$$

$$\dot{\mu}(t) = (\rho + \alpha)\mu + v, \tag{7}$$

where we write λ_i^0 for $\lambda_i(0)$. The complimentary slackness conditions are

$$\gamma_i \geq 0, x_i \geq 0, \gamma_i x_i = 0, i = 1,2 \tag{8}$$

$$\gamma_r \geq 0, y \geq 0, \gamma_r y = 0, \tag{9}$$

$$v \geq 0, \overline{Z} - Z \geq 0, v(\overline{Z} - Z) = 0. \tag{10}$$

Finally, the transversality conditions are

$$lim_{t\uparrow\infty} e^{-\rho t}\lambda_i(t)X_i(t) = \lambda_i^0 \, lim_{t\uparrow\infty} X_i(t) = 0, i = 1,2 , \tag{11}$$

$$lim_{t\uparrow\infty} e^{-\rho t}\mu(t)Z(t) = 0 \tag{12}$$

Condition (6) implies that the scarcity rent of coal and gas must rise at the rate of discount. Define the price of resource i as its (full) marginal cost at time t, given by $p_i = \lambda_i - \theta_i\mu$. It has two components: the scarcity rent and the externality cost, the extraction cost being zero. Since μ is the shadow price of the pollution stock, it is negative or zero. If $Z < \bar{Z}$ over some time period, then by (10), $\nu = 0$ and $\mu(t) = \mu^0 e^{(\rho+\alpha)t}$ where $\mu(0)$ is written as μ^0. During this period, the shadow price of pollution grows at an exponential rate given by the sum of the discount rate and the dilution rate of the pollution stock. If regulation is never binding, then the initial shadow price μ^0 is zero hence $\mu \equiv 0$.

It is convenient to split the analysis into two parts. In section 3 we assume that the initial pollution stock is at the ceiling, $Z^0 = \bar{Z}$. In section 4, the initial stock is assumed to be strictly under the ceiling $Z^0 < \bar{Z}$.

3. The Initial Stock of Pollution is at the Ceiling

Let $Z^0 = \bar{Z}$. We plan to show that when the aggregate stock of the two resources is small, both coal and natural gas must have the same scarcity rent and extraction is "pure" Hotelling, i.e., regulation becomes a non-issue. When the stock of coal is small but natural gas is abundant, both resources have the same scarcity rent but the price paths are non-Hotelling. When coal is abundant, neither are resource rents equal nor the price paths Hotelling.

Extraction with Only One Resource

To understand how environmental regulation affects extraction, assume that there is only a single resource i, which may be coal or natural gas. Since extraction is costless, the resource price $p_i(t)$ is equal to the scarcity rent and is completely determined by its initial value λ_i^0. The Hotelling price path is given by $p_i(t) \equiv \lambda_i^0 e^{\rho t}$. Define T as the time at which

this Hotelling price equals the cost of the backstop c_r. Then $T = [ln\, c_r - ln\, \lambda_i^0\,]/\rho$.[6] The initial scarcity rent can be written as a decreasing function of the initial stock, $\lambda_i^0(X_i^0)$.

Since $Z^0 = \overline{Z}$, let the lowest value of the scarcity rent such that regulation is non-binding be given by $\lambda_i^0 = \overline{p}_i$. At resource prices lower than this level, initial resource extraction will lead to a pollution stock higher than the regulated level $\overline{Z}$ and the ceiling constraint will be binding over a non-zero time period. If regulation is non-binding, the resource price rises at the rate of discount starting from $\overline{p}_i$. Define the corresponding extraction period as $\Delta_i^H = \dfrac{[ln\, c_r - ln\, \overline{p}_i\,]}{\rho}$, where the superscript H stands for the Hotelling path.

Cumulative extraction is then given by $X_i^H = \int\limits_0^{\Delta_i^H} d(\,\overline{p}_i e^{\rho t}\,)dt$. Thus X_i^H is the highest stock of resource i that generates a pure Hotelling path unconstrained by environmental regulation. Since $\overline{p}_1 < \overline{p}_2$, the aggregate stock of coal that could be used up over this Hotelling path must be lower than the corresponding stock of natural gas, $X_1^H > X_2^H$.

If the initial resource stock is higher than this maximal Hotelling stock, $X_i^0 > X_i^H$, then at the beginning, extraction must be limited to $\overline{x}_i$, otherwise the pollution stock will exceed the ceiling (see Fig. 1). When the resource stock declines to X_i^H, the Hotelling path begins. The duration of the first phase is then given by $\Delta_i = \dfrac{X_i^0 - X_i^H}{\overline{x}_i}$. Until time Δ_i, the price is $\overline{p}_i$. Since regulation is binding, the shadow cost of pollution $\mu(\,t\,)$ is strictly negative and (4) yields $\overline{p}_i = \lambda_i^0 e^{\rho t} - \mu\theta_i$.[7] The gap between the resource price $\overline{p}_i$ and the shadow price λ_i in the figure is the externality cost per unit of the resource i, given by $-\mu\theta_i$. Beyond Δ_i, the ceiling is no longer binding hence $\mu \equiv 0$ and both resource price and scarcity rents are equal, as shown.

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[6] The optimal value of λ_i^0 is determined uniquely by the relationship $\int\limits_0^T d(\,\lambda_i^0 e^{\rho t}\,)dt = X^0$.

[7] Hence $\mu(\,t\,) = \dfrac{\lambda_i^0 e^{\rho t} - \overline{p}_i}{\theta_i}, t < \Delta_i$, with $\lambda_i^0 = \overline{p}_i e^{-\rho\Delta_i}$.

[Fig. 1 here]

Extraction with Both Resources

Consider that both resources are available, $X_i^0 > 0, i = 1,2$. If $X^0 \le X_2^H$, i.e., the aggregate stock is lower than the minimum of the two critical stocks, the solution must still be the Hotelling path with initial extraction lower than the maximum allowed at the ceiling, $\bar{x}_2$. The scarcity rents of the two resources are equal, and their extraction rates a matter of indifference.[8] Regulation is never active except possibly at the initial instant.

This is shown in $[X_1^0, X_2^0]$ space in Fig.2 where each point represents an initial endowment of the two resources. Points *A* and *B* denote the stock X_2^H on each axis. In zone I, which is the triangle bounded by the axes and the straight line *AB* with slope -1, the aggregate stock is always lower than X_2^H and endowments in this zone yield a Hotelling solution in which only the aggregate extraction is determinate but not its composition.

[Fig. 2 here]

Now let $X_2^H < X^0 < X_1^H$. The aggregate stock is higher than the maximum Hotelling-induced stock of coal, but lower than that of natural gas. As X^0 approaches X_1^H, given a stock of natural gas, we search for the maximum endowment of coal that generates a Hotelling price path, i.e., one unconstrained by regulation. In the limit, the pollution constraint can only bind at discrete points in time but not over an interval of non-zero duration. Consider the extraction sequence in which gas is consumed initially followed by coal. In the first interval, as price increases because of scarcity, extraction of gas falls below the critical $\bar{x}_1$ so that the pollution stock declines from the initial level of $\bar{Z}$. When

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[8] The extraction rates must satisfy $x(t) = d(\lambda^0 e^{\rho t})$ and $\int_0^{\Delta(X^0)} x_i dt = X_i^0, i = 1,2$, where λ^0 is the common value of the initial scarcity rent of the two resources and $\Delta(X^0)$ the duration of this extraction period, i.e., $\Delta(X^0) = \dfrac{ln\, c_r - ln\, \lambda^0}{\rho}$.

we switch from gas to coal, the stock is strictly lower than $\overline{Z}$. Therefore we could extract an aggregate stock of coal strictly larger than X_2^H and still not violate the ceiling. As shown in Fig.3, natural gas is consumed first to bring down the stock of pollution, then coal is extracted at rates higher than $\overline{x}_2$ to restore the stock of pollution to $\overline{Z}$. Exactly at that instant, the residual coal stock must equal X_2^H and the Hotelling path follows as before. Aggregate coal use in this case is strictly larger than X_2^H.

[Fig. 3 here]

There may be other such Hotelling paths along which both resources are consumed simultaneously while maintaining the stock of pollution at less than or equal to $\overline{Z}$ and again letting it rise to the ceiling $\overline{Z}$, followed by a phase during which only coal is consumed. A polar case of such a path is one in which both resources are used while the stock of pollution is maintained exactly at its maximal level $\overline{Z}$ in the first phase, followed by exclusive use of coal.[9] The locus of maximal resource stocks that lead to a Hotelling path is shown as curve AA' in Fig.2.

Point A represents $(0, X_2^H)$ and A' the endowment pair $(\overline{X}_1^0, \overline{X}_2^0)$ such that $\overline{X}_1^0 + \overline{X}_2^0 = X_1^H$. As we travel on AA' towards A' we increase the stock of natural gas and the corresponding maximal stock of coal that can be used to remain in the Hotelling path. The path AA' can not cross the 45°line through X_1^H because the aggregate stock would exceed the maximum stock of natural gas compatible with the Hotelling path. Because the stock increases along AA', the common initial scarcity rent declines from $\overline{p}_2$ at A to $\overline{p}_1$ at A'.[10]

The shaded region zone II in Fig.2 denotes the set of initial stocks that generate a Hotelling solution when the aggregate stock is higher than X_2^H. Each point on AA'

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[9] In Appendix A we show that this property is satisfied by the path that maximizes the consumption of coal for any given stock of natural gas such that $X^0 \in (X_2^H, X_1^H]$.

[10] Let Δ_{12} denote the interval when the two resources are consumed jointly. From A to A', Δ_{12} increases from zero to $\dfrac{ln\,\overline{p}_2 - ln\,\overline{p}_1}{\rho}$.

corresponds to a unique extraction path, as shown in Appendix A. Extraction proceeds along the locus $A'A$ towards A. Once A is reached, all natural gas is exhausted and the stock of coal equals X_2^H. In the next phase only coal is extracted.

If the set of initial stocks is in zone II but strictly below the curve AA', the amount of coal available is less than the maximal amount required and there may be some flexibility in sequencing the use of the two resources, since they are perfect substitutes on any Hotelling path not constrained by regulation. For instance, optimal extraction beginning at point C may involve use of both resources at a maximal rate while staying at the ceiling followed by the exclusive use of coal, and finally the residual stock of natural gas. This is shown by the path $CC'C''O$ where the segment CC' is the translation of AA' through point C. However, since we are in the strict interior of zone II, regulation does not bind, hence some natural gas extraction at the beginning may be substituted by a compensating amount of coal later in time.[11]

The union of zones I and II is the area $OAA'B'$, the set of all solutions that are pure Hotelling, i.e., unaffected by environmental regulation.

Only Natural Gas is Abundant

This case is depicted by zone III in Fig. 2, where $X^0 > X_1^H$ and $X_2^0 < \overline{X}_2^0$.[12] The pattern of resource use is a variation of the one described in Fig.1. Consider an initial vector of stocks at point D. Since natural gas is abundant, it is used first at the maximal level $\overline{x}_1$ until D' is reached. The resource price is constant at $\overline{p}_1$. The aggregate stock at D' equals X_1^H. From here the extraction sequence is similar to the pure Hotelling paths originating in zone II. The scarcity rents of the two resources are equal and their common initial value is given by $\lambda^0 = \overline{p}_1 e^{-\rho \Delta_I} = c_r e^{-\rho(\Delta_I + \Delta_I^H)}$ where Δ_I and Δ_I^H $(= \dfrac{[\ln c_r - \ln \overline{p}_1]}{\rho})$ are the durations of the first and second phases respectively. The shadow cost of pollution μ is non-zero until D' and zero beyond. As we see below, this is the only non-Hotelling path with equal scarcity rents for the two resources.

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[11] See Appendix B for further characterization of paths starting from zone II.
[12] Detailed characterization of the paths originating from zone III and from zones IV and V defined below, are given in Appendix C.

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When Coal is Abundant

If coal, the more polluting resource is abundant, extraction may be constrained because of environmental regulation. The scarcity rents of the two resources may no longer be equal. It is easier to first discuss the case when both resources are abundant (see zone IV in Fig. 2), before considering abundant coal with limited stocks of natural gas, the clean resource (zone V).

Endowments in zone IV are defined by $X_1^0 > \bar{X}_1^0$ and $X_2^0 > \bar{X}_2^0$, such as point E in Fig.2. As in zone III, natural gas is used at the maximal rate $\bar{x}_1$ until its stock is reduced to $\bar{X}_1^0$ at point F'. The duration of this phase equals $\Delta_1 = \dfrac{X_1^0 - \bar{X}_1^0}{\bar{x}_1}$. The corresponding price paths are shown in Fig. 4. During this first phase the price of energy $\bar{p}_1$ is lower than that of coal given by $p_2 = \lambda_2^0 e^{\rho t} - \mu \theta_2$. It is constant until F' (Fig. 2) when both resource prices are equal and a period of joint use begins. Extraction proceeds along a path $F'F$ that is a vertical translation of $A'A$ until the price of energy reaches $\bar{p}_2$ at F. Denote this period of joint use by Δ_{12}.

[Fig. 4 here]

Natural gas is exhausted at location F. The cleaner resource has a higher scarcity rent, which is to be expected given that they have the same (zero) extraction cost. [13] Coal is

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[13] When both resources are consumed, $x_i > 0$ and $\gamma_i = 0, i = 1,2$. From (4), $\lambda_1 - \mu \theta_1 = \lambda_2 - \mu \theta_2$

which implies that $\lambda_1 - \lambda_2 = -\mu(\theta_2 - \theta_1)$ hence $\mu = \dfrac{-(\lambda_1^0 - \lambda_2^0)e^{\rho t}}{\theta_2 - \theta_1}$. Thus $\lambda_1^0 > \lambda_2^0$. Aggregate

supply must equal demand, $x_1 + x_2 = d(\lambda^0 e^{\rho t})$ and the pollution stock stays at the ceiling. From (2),

$\theta_1 x_1 + \theta_2 x_2 = \theta_1 \bar{x}_1 = \theta_2 \bar{x}_2$ so that

$$x_1 = \frac{\theta_2[d(\lambda^0 e^{\rho t}) - \bar{x}_2]}{\theta_2 - \theta_1}, \quad x_2 = \frac{\theta_1[\bar{x}_1 - d(\lambda^0 e^{\rho t})]}{\theta_2 - \theta_1}. \tag{13}$$

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used at the maximum level (see Fig.4). Its price $\bar{p}_2$ is constant until time $\Delta_1 + \Delta_{12} + \Delta_2$. This is followed by the terminal phase when regulation is non-binding and the price of coal follows a Hotelling path $p(t) = \lambda_2 = \lambda_2^0 e^{\rho t}$ until it is exhausted and the backstop is used. The stock of coal at the starting location E (in Fig.2) determines the precise path $F'F$ each of which is a vertical translation of $A'A$.

Now consider zone V with smaller endowments of natural gas, specifically $X_1^0 \le \bar{X}_1^0$. If it is a strict equality, the first period of exclusive natural gas use in zone IV disappears completely. If the inequality is strict, as in point G in Fig.2, extraction begins with both resources and a common price in the interval $[\bar{p}_1, \bar{p}_2]$ that depends upon the location of G on the curve FF'. The higher the endowment of natural gas, the closer is G to F' and the closer is the initial price to $\bar{p}_1$ (Fig.4). Extraction proceeds along the curve GF until all gas is exhausted at F. The remaining phases are as in zone IV.

The demarcation of the different zones depends on $\bar{Z}$. If $\bar{Z}$ is higher, point A moves up in Fig.2 and B' moves to the right, enlarging the set of endowments that result in a Hotelling path (zones I and II). On the other hand, if $\bar{Z}$ approaches zero, the Hotelling set shrinks towards the origin until in the limit, none of the resources may be used. The pollution content of the resources also determines the set of Hotelling paths. If coal was more polluting and gas cleaner, point A will move down and B' will shift to the right, leading to a flat and elongated Hotelling set.

The main insight when regulation is binding at the beginning is that excess natural gas must be used first. When both resources are used, natural gas extraction declines and that of coal increases. This "turnpike" feature with joint use allows for a smooth transition from the clean to the polluting resource until only coal is used. Loosely speaking, the order of extraction is according to pollution content. The more polluting resource is used latter. This trend is driven by time preference. Using the cleaner resource allows for higher current extraction rates and higher profits earlier in time. However, as we see

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Differentiating gives $\dot{x}_1(t) = \dfrac{\theta_2}{\theta_2 - \theta_1} \dot{d}(\lambda^0 e^{\rho t}) < 0$ and $\dot{x}_2(t) = -\dfrac{\theta_1}{\theta_2 - \theta_1} \dot{d}(\lambda^0 e^{\rho t}) > 0$. The

extraction rates move in opposite directions.

below, this tendency does not always hold when the initial pollution stock is below the regulated level.

4. The Initial Stock of Pollution is Below the Ceiling

For the stock to build up to the ceiling, there must be a period when emissions are strictly greater than the maximum allowed at the ceiling. We adopt the same approach as before, differentiating between resource endowments that lead to an unconstrained Hotelling path and those that do not. However, the analysis in this section has an added dimension, the initial stock of pollution Z^0. This pollution stock together with the endowment of resources determines the approach to the ceiling. Now all variables are also a function of Z^0.

Hotelling Paths with a Single Resource

Let λ_i^0 be the initial scarcity rent. Since it increases over time, emissions decline. If emissions at scarcity rent λ_i^0 are lower than the natural dilution rate, than future emissions will also be lower, hence the stock of pollution will permanently decline. However, if initial emissions are higher than the dilution rate, then the stock of pollution $Z(t)$ will increase, reach a peak value than decline steadily. The lower the value of λ_i^0, the higher the peak value of $Z(t)$. Thus there exists some value of the initial scarcity rent $\lambda_i^0(Z^0)$ such that the peak $Z(t)$ equals the ceiling $\bar{Z}$, i.e., when $\lambda_i^0 e^{\rho t} = \bar{p}_i$.[14] In a Hotelling path, the pollution stock must be off the ceiling both before and after this instant. Thus $\lambda_i^0(Z^0)$ is the lowest possible scarcity rent that depends upon the initial stock Z^0. A lower initial rent will mean higher emissions, so the path will be constrained by regulation and will no longer be Hotelling. A higher rent will imply that the pollution stock will be strictly below the ceiling. This scarcity rent $\lambda_i^0(Z^0)$ must increase with the initial stock of pollution Z^0. The cumulative extraction until the ceiling is reached is given by $X_i^H(Z^0) = \int_0^{\Delta_i^0} d(\lambda_i^0(Z^0)e^{\rho t})dt$ where Δ_i^0 is the time it takes to get to the ceiling

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[14] At this instant, $\bar{x}_i$ units of resource are extracted.

given by $\Delta_i^0 = \frac{\ln c_r - \ln \lambda_i^0}{\rho}$.[15] Then for a given Z^0, we have $X_1^H > X_2^H$ and $\lambda_1^0 < \lambda_2^0$ since

more of the clean fuel can be used on the way to the ceiling.[16] Let Δ_i^H be the time duration

during which resource i is extracted along a Hotelling path. We have $\Delta_1^H > \Delta_2^H$. For an

initial stock of resource i lower than the critical level X_i^H, the stock does not reach the

ceiling at all.

When the stock exceeds X_i^H, the scarcity rent is lower and regulation binds over a non-

zero time period. The pollution stock rises from Z^0 to $\overline{Z}$. The resource is then extracted at

the maximum $\overline{x}_i$ until the Hotelling stock X_i^H remains and extraction declines gradually to

zero. The price at the ceiling is a constant $\overline{p}_i$. Suppose it takes time Δ_i^0 to go to the ceiling

and the phase at the ceiling is of duration Δ_i. In the first phase the price of the resource

rises from a lower level to $\overline{p}_i$ and the shadow price of pollution μ is non-zero. Beyond

time $\Delta_i^0 + \Delta_i$, regulation does not matter so the value of μ is zero and extraction is pure

Hotelling.

Extraction with Both Resources

As in section 3, if Hotelling paths were to hold, for initial aggregate stocks lower than

X_2^H, regulation will not bind, so which resource is extracted is a matter of indifference.

This solution is shown by stocks in zone I in Fig.5, which is a generalization of Fig.2 for

a given Z^0. This new zone I is strictly larger than the one in section 3, since it must

include additional resource stocks that take the initial stock of pollution to the ceiling.

For aggregate stocks $X^0 \in [X_2^H, X_1^H]$ we may obtain pure Hotelling paths as in section

3, provided the stock of coal is not too large. The Hotelling set corresponding to zone II

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[15] All variables are a function of the initial stock of pollution Z^0 so we sometimes avoid writing it explicitly. Variables such as X_i^H in this section are defined for a initial stock Z^0 and therefore are different from those in section 3, where the initial stock of pollution is the constant $\overline{Z}$ (see Fig.5).

[16] When Δ_i^0 equals zero, the initial stock is at the ceiling as in section 3.

in section 3 is given by figure $KLL'MN$.[17] These vectors are all functions of the initial

stock of pollution Z^0 . As Z^0 approaches $\bar{Z}$ the curve LL' approaches AA' and

$L'M$ collapses to the vertex A' . The new zone II reduces to the one from section 3.

[Fig. 5 here]

Consider an initial endowment $F = (F_1, F_2)$ located on the AA' curve. From section 3, we

already know the resource use profile beginning from F where the pollution stock is at

the ceiling. With some abuse of notation, let the current resource price at F be given

by $p(F)$ which lies in the interval $(\bar{p}_2, \bar{p}_1)$ as in Fig.4. Suppose only coal is used to go to

the ceiling. As before, let Δ^0 denote the time taken from an initial endowment F' to reach

the ceiling at F. Then the price path beginning from F' is shown in Fig. 6.[18]

[Fig. 6 here]

The curve LL' is the locus of all such points F' that map to points F lying on the curve

AA' . However LL' is not a vertical translation of AA' . Since $p(L') < \bar{p}_1 < \bar{p}_2 < p(L)$,

resource prices are lower starting from L' relative to L. This will lead to higher coal use

and higher emissions starting from L' given the same initial pollution stock Z^0 . The

distance $F'F$ is smaller, the closer F is to A' . From the ceiling at F, extraction follows the

sequence described in section 3.[19]

Points on the segment $L'M$ (in Fig.5) can be explained in the same fashion. At

vertex M with aggregate endowment X_1^H only natural gas is used during the transition to

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[17] where $K = (X_2^H, 0), L = (0, X_2^H), L'$ is located vertically above A', $M = (X_1^H - \bar{X}_2^0, \bar{X}_2^0)$ and $N = (X_1^H, 0)$.

[18] This path solves $Z(\Delta^0) = \bar{Z}$ where $Z(t) = Z^0 e^{-\alpha t} + \theta_2 \int_0^t d(p(F) e^{-\rho(\Delta^0 - \tau)}) e^{-\alpha(t - \tau)} d\tau$

and $F_2' = F_2 + \int_0^{\Delta^0} d(p(F) e^{-\rho(\Delta^0 - t)} dt)$ where $F' = (F_1', F_2')$.

[19] This is a Hotelling path with $\lambda^0 = \lambda_1^0 = \lambda_2^0 = p(F) e^{-\rho \Delta^0}$ and $\mu = 0, \upsilon = 0$ for $t \in [0, \Delta^0]$. To show that the extraction path $F'FAO$ is optimal, it is easy to check that all the necessary conditions are satisfied. For $t \geq \Delta^0$, the analysis from section 3 applies.

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the ceiling at point A'. Since this is again a Hotelling path, both resource rents are equal. Consider an intermediate point J on the path $A'M$, where by construction, the stock of pollution is below the ceiling. Starting from M only gas is extracted to arrive at J. But starting from J' only coal is extracted. Since both of these paths must start from pollution stock Z^0, follow a Hotelling path and arrive at the same stock of resources and pollution, more natural gas needs to be used than coal. Thus traveling to J will take longer from M than from J'. This explains why point J' is located to the left of the extension of the line NM which is the locus of points $X_1^0 + X_2^0 = X_1^H$.

In the area of zone II above the line $AA'M$, coal is used first. For example, from F' only coal is used until the ceiling is reached at F followed by simultaneous extraction. For points located on the line $J'J$, coal is used first to point J, followed by natural gas until the ceiling is attained at A'.

Why will coal, the more polluting resource be used *before* natural gas on the way to the ceiling? Consider two instants of time t_1 and t_2 with $t_1 < t_2$. Since the two resources are perfect substitutes and their scarcity rents are equal in zone II, suppose one unit of coal is used at t_1 and an unit of gas later at t_2. Then at some time $t > t_2$ the increment to the pollution stock will be $[\theta_2 e^{-\alpha(t_2-t_1)} + \theta_1]e^{-\alpha(t-t_2)}$. Now consider the alternative – using an unit of gas at t_1 and coal later at t_2. The addition to pollution is $[\theta_1 e^{-\alpha(t_2-t_1)} + \theta_2]e^{-\alpha(t-t_2)}$. Subtracting the latter from the former gives $(\theta_2 - \theta_1)(e^{-\alpha(t-t_2)} - 1)e^{-\alpha(t-t_2)} < 0$. Using coal rather than natural gas will imply a lower pollution stock in the future because a higher Z implies increased dilution which is costless.

Only Gas is Abundant

Here the initial coal stock is less than $\overline{X}_2^0$ while the aggregate stock is larger than X_1^H, shown as zone III in Fig. 5. Consider a starting point G. Natural gas is used until the pollution stock reaches the ceiling. The ceiling is achieved before arriving at the frontier $A'B'$, at point G'.[20] Next gas is extracted at the maximum level $\bar{x}_1$ until the stocks

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[20] If the stock of gas is sufficiently large, the point G' may lie to the right of the line segment MN.

reach the boundary $A'B'$ of zone II from section 3. The remaining path is Hotelling. Shadow prices of the two resources are still equal.

When Coal is Abundant

There are three alternative paths to the ceiling with abundant coal, given by a stock located higher than the line $LL'MQ$ shown in Fig.7. These are demarcated by zones IV, V and VI. It is easier to discuss them in reverse order starting with zone VI. Coal is abundant with only a small stock of natural gas lower than $\overline{X}_1^0$. We know from section 3 that once the ceiling is attained both resources are used until natural gas is exhausted. Consider a point R where the stock of pollution is exactly at the ceiling. Then R must be on some path HH' that is parallel to AA' and the sequence of extraction from R is well-defined.

[Fig.7 here]

Let $p(R)$ be the price of energy at R and Δ^0 be the time at which the ceiling is achieved exactly at R. Since both resources are used beginning from this location, (4) implies that $p(R) = p(\Delta^0) = \lambda_i(\Delta^0) - \mu(\Delta^0)\theta_i, i = 1,2$ with $p(R) \in (\overline{p}_1, \overline{p}_2)$. Since $\theta_2 > \theta_1$, we have $\lambda_1(\Delta^0) > \lambda_2(\Delta^0)$. Before Δ^0 the ceiling is non-binding, so $\mu(t) = \mu(\Delta^0)e^{-(\rho+\alpha)(\Delta^0-t)}$. Then for $t < \Delta^0$, $(\lambda_1(t) - \mu(t)\theta_1) - (\lambda_2(t) - \mu(t)\theta_2) = (\lambda_1(\Delta^0) - \lambda_2(\Delta^0))e^{-\rho(\Delta^0-t)}$ $-\mu(\Delta^0)(\theta_1 - \theta_2)e^{-(\rho+\alpha)(\Delta^0-t)} > 0$. That is, before location R, the marginal cost of coal is lower than that of gas. Thus only coal is used until regulation binds.

There exists a vector of stocks R' (see Fig.7) with an initial stock of pollution Z^0 such that coal is used until R. Once at R, both resources are used jointly along the curve $H'H$ as in section 3. Conversely any starting vector R' in zone VI will imply an exclusive use of coal until the ceiling is reached. Because natural gas is not abundant, it is relatively expensive, hence coal is used on the "fast track" to the ceiling. As there is a small stock of natural gas, it is efficient to use it to relax the consumption constraint at the ceiling than for slowing down the growth of the pollution stock before the ceiling is achieved.

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Now consider a higher stock of natural gas so that $X_1^0 \geq \overline{X}_1^0$, as shown by endowments in zone V (see Fig.7). Then as in zone VI, we use coal at the beginning, but because there is a higher endowment of natural gas, some of it is used to get to the ceiling. From an initial vector S, coal is used until location S', then natural gas is used until S'' when the ceiling is reached. Beyond this location, gas continues to be used at its maximum level $\overline{x}_1$ and extraction proceeds as in zone IV in section 3.

Why not only coal to the ceiling as in zone VI? With increased gas resources, there is competition between coal and gas. The benefit of coal use on the access to the ceiling is that it allows for increased pollution and therefore, increased dilution which is costless. However, with higher reserves of gas, its scarcity rent is lower and it is less polluting, so it makes sense to use some gas exclusively at the beginning of the ceiling period and additional gas resources to ensure continuity of the price path as the pollution stock approaches the ceiling. These prices are shown in Fig.8. Coal is cheaper at first followed by natural gas, until the ceiling is reached at time Δ^0.[21]

[Fig.8 here]

Finally consider zone IV where both resources are abundant (Fig.7). As the endowment of natural gas increases, its scarcity rent decreases. Even though the scarcity rent of gas is higher than coal, large stocks of gas will imply that its marginal cost may be lower than for coal, $\lambda_1^0 - \mu^0\theta_1 < \lambda_2^0 - \mu^0\theta_2$. During the initial phase, gas is used exclusively all the

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[21] To prove that this is the only possible sequence, we can show that the price of coal is strictly higher than that of natural gas and the differential increases as we move recursively from $\Delta^0 + \Delta_1$ to Δ^0 (see Fig.8). Then coal can not be used in the preceeding interval and gas must be used. Consider the price differential $p_2(t) - p_1(t)$ at time $\tau \in [0, \Delta_1]$ so that $\Delta^0 + \Delta_1 - \tau \in [\Delta^0, \Delta^0 + \Delta_1]$, i.e., τ is measured from $\Delta^0 + \Delta_1$. In this interval by (4), $\overline{p}_1 = \lambda_1 - \mu\theta_1$ which gives $\mu(t) = \dfrac{\lambda_1 - \overline{p}_1}{\theta_1}$ We have

$$p_2(\Delta^0 + \Delta_1 - \tau) - \overline{p}_1 = \lambda_2(\Delta^0 + \Delta_1 - \tau) - \frac{\lambda_1((\Delta^0 + \Delta_1 - \tau) - \overline{p}_1)\theta_2}{\theta_1} - \overline{p}_1$$

$$= \lambda_2^0 e^{\rho(\Delta^0 + \Delta_1 - \tau)} + \frac{\theta_2[\overline{p}_1 - \lambda_1^0 e^{\rho(\Delta^0 + \Delta_1 - \tau)}]}{\theta_1} - \overline{p}_1 \text{ which after some manipulation yields}$$

$$p_2 - \overline{p}_1 = \frac{(\theta_2 - \theta_1)\overline{p}_1}{\theta_1}[1 - e^{-\rho\tau}] \text{ so that } \frac{d}{d\tau}(p_2 - \overline{p}_1) = \frac{(\theta_2 - \theta_1)\overline{p}_1}{\theta_1}[\rho e^{-\rho\tau}] > 0.$$

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way to the ceiling.[22] This is followed by maximal use of gas until the boundary of zone IV. The extraction path is shown starting from an initial endowment V in Fig. 7.

5. Concluding Remarks

We extend Hotelling theory to resources when they are differentiated by their pollution characteristics. Herfindahl showed that when there is cost heterogeneity among resources, extraction follows the "least cost first" principle. We show that when resources are differentiated by pollution there is no such ordering of extraction. With a "clean" and a "dirty" resource, the order of extraction suggested by Herfindahl breaks down. When the economy starts with the regulated level of pollution, the clean resource is used first, analogous to the Herfindahl principle. However, when the economy starts from a lower level of pollution, it may use the dirty resource first and build the pollution stock as quickly as possible. In this manner, it benefits from natural dilution of the pollution stock, which is "free." However, if the stock of the clean resource is large, it may be used from the beginning.

We show that in a model with pollution heterogeneity, a resource may be used over two disjoint intervals. Coal may be used exclusively at the beginning, followed by exclusive use of gas, then again later in time, the exclusive use of coal, as in zone V. This sort of complete "preference reversal" over resources does not emerge in models with cost differentiation among resources.

Unlike in the literature following Hotelling, the sequence of extraction in this pollution model depends strongly upon the initial endowment of the resources. A common feature of extraction when both resources are abundant is a turnpike property, in which the two resources are used jointly at their maximal levels until the clean resource is completely depleted. Which resource will be used to get to the ceiling depends on their relative abundance.

These results have implications for the order of extraction when an economy has to meet environmental goals. For instance, a stated aim of the Kyoto Treaty is to stabilize the

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[22] The case of natural gas use followed by coal to the ceiling can be eliminated by arguments made earlier for endowments in zone V.

atmospheric concentration of carbon at approximately 550 parts per million. Currently it is about 370 parts per million. Since reserves of cleaner fuels (such as crude oil and natural gas) are limited while polluting fuels such as coal are abundant, our results suggest that coal should be used exclusively to get to the ceiling. This seems counter-intuitive, but by getting to the ceiling as quickly as possible, we also get costless dilution of a larger part of the pollution stock. Once the target ceiling is achieved, we may use both resources jointly for a period until all gas reserves are exhausted. Finally only coal will be used until exhaustion and transition to the backstop.

An important assumption in the model is that both resources are homogenous with respect to cost. In future work, it may be useful to generalize Herfindahl's framework to examine how cost heterogeneity interacts with pollution heterogeneity. To keep the analysis tractable, we also use a very specific form of environmental regulation which is perfectly inelastic at some exogenously fixed level of pollution. However, more realistic damage functions may be used in later work to explore how the sequence of extraction may be sensitive to the specification of damages. For instance if damages were strictly positive at all levels of the pollution stock, we may not get these sharp transitions between resources. The present paper may thus be thought of as a first step towards increased understanding of how environmental regulation affects the use of nonrenewable resources.

Appendix A

Maximizing Cumulative Extraction of Coal along a Hotelling Path implies that the Stock of Pollution is Always at the Ceiling

For $X^0 \in (X_2^H, X_1^H)$, we have a common scarcity rent $\lambda(X^0) \in (\bar{p}_1, \bar{p}_2)$. Consider the Hotelling path $d(\lambda^0(X^0)e^{\rho t})$, written in reduced form as $d(t)$, over the time interval $[0, \Delta_{12})$ during which $p(t) = \lambda^0(X^0)e^{\rho t}$ increases from $\lambda^0(X^0)$ to $\bar{p}_2$. Then

$$\Delta_{12} = \frac{\ln \bar{p}_2 - \ln \lambda^0(X^0)}{\rho}.$$ Let the extraction sequence be given by $\{x_1, x_2\}$ such

that $x_1 + x_2 = d(t)$. We show that among paths starting from $Z^0 = \bar{Z}$ and satisfying the constraint $Z(t) \leq \bar{Z}$, $Z(\Delta_{12}) = \bar{Z}$, maximizing the extraction of coal given a stock of gas implies that the ceiling constraint is always binding. That is, we must have $Z(t) = \bar{Z}$ over the entire interval $[0, \Delta_{12})$. Define $\hat{x}_1(t)$ and $\hat{x}_2(t)$ to be the extraction rates of gas and coal when the pollution stock is at the ceiling as defined in equation (13). The maximization problem can be written as:

$$\text{Maximize}_{x_2} \int_0^{\Delta_{12}} x_2(t) dt$$

subject to

$$\dot{Z}(t) = \theta_1(d(t) - x_2(t)) + \theta_2 x_2(t) - \alpha Z(t),$$

$$Z^0 = \bar{Z}, \bar{Z} - Z(t) \geq 0, t \in [0, \Delta_{12}), Z(\Delta_{12}) = \bar{Z},$$

$$d(t) - x_2(t) \geq 0, \quad x_2(t) \geq 0.$$

The Lagrangian can be written as

$$L = x_2 + \pi[\theta_1(d - x_2) + \theta_2 x_2 - \alpha Z] + \nu[\bar{Z} - Z] + \bar{\gamma}_2[d - x_2] + \underline{\gamma}_2 x_2.$$

The first order conditions are

$$\frac{\partial L}{\partial x_2} = 0 \Leftrightarrow 1 + \pi(\theta_2 - \theta_1) = \bar{\gamma}_2 - \underline{\gamma}_2, \tag{A1}$$

$$\dot{\pi} = -\frac{\partial L}{\partial Z} \Leftrightarrow \dot{\pi} = \alpha\pi + v, \tag{A2}$$

$$v \geq 0, \ v[\bar{Z} - Z] = 0, \tag{A3}$$

$$\bar{\gamma}_2 \geq 0, \ \bar{\gamma}_2(d - x_2) = 0, \text{and} \tag{A4}$$

$$\underline{\gamma}_2 \geq 0, \ \underline{\gamma}_2 x_2 = 0. \tag{A5}$$

The shadow price of pollution π must be non-positive. First we show that $\hat{x}_1(t)$ and $\hat{x}_2(t), t \in (0, \Delta_{12})$ satisfy the above first order conditions (A1-A5). Since $\hat{x}_2(t) \in (0, d(t))$, both $\bar{\gamma}_2(t), \underline{\gamma}_2(t)$ equal zero. Then (A1) implies $\pi = \frac{-1}{\theta_2 - \theta_1} < 0$.

Substituting in (A2) and using the fact that π is a constant yields

$$v = \frac{\alpha}{\theta_2 - \theta_1} > 0, t \in (0, \Delta_{12}).$$

We show below that if $Z < \bar{Z}$ over any interval $(t_1, t_2) \subseteq (0, \Delta_{12})$ it leads to a contradiction. On this interval, let $Z(t_1) = \bar{Z} = Z(t_2)$. From (A2) and (A3), we have

$$\pi(t) = \pi(t_1)e^{\alpha(t - t_1)}, \ t \in (t_1, t_2) \tag{A6}$$

There are five possible cases:

(i). $\exists t_0$ and $t_3 : 0 \leq t_0 < t_1$ and $t_2 < t_3 \leq \Delta_{12}$ such that $Z(t) < \bar{Z}$ for $t \in [t_0, t_1] \cup [t_2, t_3]$.

Then by definition, $x_2(t) = \hat{x}_2(t)$ hence $\pi(t) = -\frac{1}{\theta_2 - \theta_1}$ for $t \in [t_0, t_1] \cup [t_2, t_3]$. By

(A6), we have $lim_{t \uparrow t_2} \pi(t) = -\frac{1}{\theta_2 - \theta_1}e^{\alpha(t_2 - t_1)} < \frac{-1}{\theta_2 - \theta_1}$, which implies a discontinuity in

the path of π at $t = t_2 < \Delta_{12}$, a contradiction.

(ii). $t_1 = 0$ and $\exists t_3 : t_2 < t_3 \leq \Delta_{12}$ with $Z(t) = \overline{Z}, t \in [t_2, t_3]$. We then have

$$\pi(t_2) = -\frac{1}{\theta_2 - \theta_1} = \pi^0 e^{\alpha t_2} \text{ where } \pi(0) \text{ is written as } \pi^0. \text{ Thus for } t \in (0, t_2),$$

$1 + \pi(t)(\theta_2 - \theta_1) = 1 + \pi(t_2)e^{-\alpha(t_2-t)}(\theta_2 - \theta_1) = 1 - e^{-\alpha(t_2-t)} > 0$. From (A1), (A4) and

(A5), $\overline{\gamma}_2(t) > 0$ hence $x_2(t) = d(t) > \overline{x}_2$ since $Z^0 = \overline{Z}$. This implies $Z(t) > \overline{Z}, t \in (0, t_2)$ a

contradiction.

(iii). $\exists t_0 : 0 \leq t_0 < t_1$ with $Z(t) = \overline{Z}, t \in [t_0, t_1], t_2 = \Delta_{12}$. By (A6), $\pi(t) = \dfrac{-e^{\alpha(t-t_1)}}{\theta_2 - \theta_1}$ so

$1 + \pi(t)(\theta_2 - \theta_1) < 0$ hence $\gamma_2(t) > 0$ so that by (A5), $\underline{\gamma}_2(t) > 0$. Since $Z(t_1) = \overline{Z}$ and

$x_1(t) = d(t) < \overline{x}_1$ because $Z < \overline{Z}$ on (t_1, t_2), we have $lim_{t \uparrow \Delta_{12}} Z(t) < \overline{Z}$ which violates the

terminal condition $Z(\Delta_{12}) = \overline{Z}$.

(iv). $t_1 = 0, t_2 = \Delta_{12}$. This yields $\pi(t) = \pi^0 e^{\alpha t}, t \in (0, \Delta_{12})$. Suppose $\pi^0 > \dfrac{-1}{\theta_2 - \theta_1}$. Then

$1 + \pi^0(\theta_2 - \theta_1) < 0$. Since π is non-positive, by (A6), there exists $\varepsilon > 0$ such that

$1 + \pi(\varepsilon)(\theta_2 - \theta_1) > 0$. By (A1), $\overline{\gamma}_2(t) > 0$ and we get the same contradiction as in part

(ii). Now consider $\pi^0 \leq \dfrac{-1}{\theta_2 - \theta_1}$. Then $1 + \pi(t)(\theta_2 - \theta_1) < 0$ so that $\underline{\gamma}_2(t) > 0$ and

$x_2(t) = 0, t \in (0, \Delta_{12})$. But $x_2(t)$ must be positive over some subinterval of $(0, \Delta_{12})$ since

we maximize its integral over $[0, \Delta_{12}]$.

(v). Suppose $Z(t) < \overline{Z}$ except at a finite number of discrete points $0 \leq t_1 \leq t_2 \leq .. \leq t_n < \Delta_{12}$

such that $Z(t_i) = \overline{Z}, i = 1, 2, .., n$. At time t_n the problem becomes the same initial problem

but restricted to the interval $[t_n, \Delta_{12}]$ The argument from case (iv) applies.

Appendix B

Characterization of Optimal Hotelling Paths Starting from the Ceiling in zone II

Consider point *C* in Fig. B1. It is chosen so that the stock of coal at *C* is higher than at *A*. Let $A_C A'_C$ be the translation of the *AA′* curve through *C*. One possible path from *C* is to extract both resources while keeping the stock of pollution at the ceiling, $Z(t) = \bar{Z}$. This is a Hotelling path in which the common scarcity rent λ^0 corresponds to the initial aggregate stock at *C*. This rent must equal the one starting from point D on the *AA′* curve, since the global aggregate stock is equal for both and resources are perfect substitutes and the paths are Hotelling. From *C*, extraction proceeds along the $A'_C A_C$ curve. At any point on this curve, extraction rates are exactly equal to the corresponding points on the *AA′* curve obtained by drawing a 45° line as shown for *C*. This program ends at A_C which has the same aggregate stock X_2^H as in point *A*. The price of energy at A_C is $\bar{p}_2$, although the residual coal stock is lower than X_2^H. In general, any path from A_C to the origin may now be followed provided the ceiling constraint is not violated and $x_1 + x_2 = d(\lambda^0 e^{\rho t})$.

[Fig. B1 here]

Since the vector of endowments at *C* is under the *AA′* curve, there exist alternative extraction sequences that will not violate the ceiling. For example, we may use only natural gas at first. Since the aggregate endowment at *C* is strictly lower than X_1^H (*C* lies left of the 45° line through B'), scarcity rent will be higher at *C* and the extraction rate of natural gas $x_1(t) = d(\lambda^0(X^0))$ lower than $\bar{x}_1$. This path may cross the *AA′* curve and reach a point such as *E*. As the price of the resource increases, extraction of gas decreases, and the stock of pollution also decreases. At *E* the stock of pollution is lower than the ceiling and thus smaller than on the path $A'_C A_C$. Since the aggregate resource stock is higher at *E* relative to *A*, the common shadow price is lower. Coal can be used at rates higher than $\bar{x}_2$ beginning from *E* to go back to the *AA′*curve at point *F*. The stock of pollution will rise from *E* towards point *F*. The lengths of these two periods can be so

chosen that the stock at F is exactly $\overline{Z}$. The remaining path may follow the curve AA' until A followed by coal use until exhaustion.

Yet another alternative may be to use gas until point G on the AA' curve where $Z(t) < \overline{Z}$, then use coal until some location H where $Z(t) = \overline{Z}$. From there extraction can follow the translation of the AA' curve through H. Alternative sequences are possible including single or joint use of the two resources such that $\overline{Z}$ is not exceeded. Once the vector of stocks achieves the boundary AB of zone I, the proportion of each resource that can be used in response to the common scarcity rent is no longer restrained. For instance, from location A_C, only coal can be used until exhaustion, and the ceiling will not be violated. This is not possible for initial coal stocks larger than X_2^H such as from point C.

An important feature of extraction from any location C in zone II is that the residual vector of stocks must stay either on or above the $A_C' A_C$ curve for some initial period. Paths such as CJK are not allowed since they imply extraction of the polluting resource at rates higher than $\overline{x}_2$ and violation of the ceiling constraint.

Appendix C

Determining Optimal Paths for Initial Endowments in Zones III, IV and V

C1. *Initial Endowments in zone III*

On the line $A'B'$ (see Fig.C1) aggregate stocks of the two resources must sum to X_1^H. The common value of the initial scarcity rent λ^0 equals $\overline{p}_1$. Consider point D in zone III, with stocks $D = (D_1, D_2)$ and point $D' = (D_1', D_2)$ on line $A'B'$ with $D_1 > D_1'$. Then starting from D gas is consumed first at the maximal rate $\overline{x}_1$ over a time interval $\Delta_1 = \dfrac{D_1 - D_1'}{\overline{x}_1}$ until D' is reached. The price of energy at D' is $\overline{p}_1$. The initial value of the common scarcity rent is $\lambda^0 = \overline{p}_1 e^{-\rho \Delta_1}$. In this first period, $\lambda(t) = \overline{p}_1 e^{-\rho(\Delta_1 - t)}$. Since $p(t) = \overline{p}_1 = \lambda(t) - \mu(t)\theta_1$, we have

$$\mu(t) = \begin{cases} \dfrac{-\overline{p}_1(1 - e^{-\rho(\Delta_1 - t)})}{\theta_1}, & t \in [0, \Delta_1) \\ 0 & t \in [\Delta_1, +\infty) \end{cases}.$$

All points on any line parallel to $A'B'$ must have the same scarcity rent as well as the same length of the first period when only gas is extracted. The further right the location of this line, the lower the scarcity rent, the longer is the period of gas extraction and higher in absolute terms the starting value of $\mu(t)$.

[Fig. C1 here]

C2. *Initial Endowments in zone V*

Consider an initial endowment F (Fig.2) detailed in Fig.C2, with endowments $(0, F_2)$ such that $F_2 > X_2^H$. Coal is used at the maximum rate $\overline{x}_2$ until the stock decreases to X_2^H.

The energy price is constant at $\overline{p}_2$. The length of this phase is given by $\Delta_2 = \dfrac{F_2 - X_2^H}{\overline{x}_2}$.

The initial scarcity rent of coal is $\lambda_2^0 = \overline{p}_2 e^{-\rho \Delta_2}$. The larger the value of F_2, the longer is the duration of this phase and smaller the scarcity rent of coal. The next phase is pure

Hotelling of duration $\Delta_2^H = \dfrac{\ln c_r - \ln \overline{p}_2}{\rho}$. For a phase with joint use to occur at the

beginning, initial resource endowments must be higher than at F.

[Fig. C2 here]

Suppose Δ_{12} is the duration of this first phase starting from G. Then the additional stocks

required for the segment G to F are given by $\int_0^{\Delta_{12}} x_i(t)dt, i = 1,2$ where $x_i(t)$ are given by

(13). The maximum length of this phase is $\dfrac{\ln \overline{p}_2 - \ln \overline{p}_1}{\rho}$ because the initial price of

energy is $\overline{p}_1$ and the final price $\overline{p}_2$. Consider point H with a higher stock of coal. Then starting from H, the duration of the intermediate phase Δ_2 will be longer. Moreover, consider point K with the same stock of gas as in G. The duration of the phase from K to

H is exactly the same as from G to F. The consumption of the two resources is exactly equal and the stock of pollution is at the ceiling. However, the resource prices and scarcity rents are not equal. The maximum length of this phase is $\frac{\ln \bar{p}_2 - \ln \bar{p}_1}{\rho}$ which corresponds to starting stocks at F' and G'. Thus the curve HH' is a vertical translation of FF' and of AA' which of course has no intermediate phase Δ_2.

During the period when resources are jointly extracted, their marginal cost must be equal, i.e., $\lambda_1(t) - \mu(t)\theta_1 = \lambda_2(t) - \mu(t)\theta_2$. Define the terminal time for this phase as Δ_{12}. Then

$p(t) = \bar{p}_2 e^{-\rho(\Delta_{12}-t)}$. As in (13), we can write $x_1 = \frac{\theta_2[d(\bar{p}_2 e^{-\rho(\Delta_{12}-t)}) - \bar{x}_2]}{\theta_2 - \theta_1}$ and

$x_2 = \frac{\theta_1[\bar{x}_1 - d(\bar{p}_2 e^{-\rho(\Delta_{12}-t)})]}{\theta_2 - \theta_1}$. Thus $\frac{dx_1}{dt} < 0, lim_{t\uparrow\Delta_{12}} x_1(t) = 0$ and

$\frac{dx_2}{dt} > 0, lim_{t\uparrow\Delta_{12}} x_2(t) = \bar{x}_2$. Note that extraction depends upon the time variable

$\Delta_{12} - t$ and not on calendar time. Equating the marginal costs of gas and coal at time Δ_{12} gives $\lambda_1(\Delta_{12}) - \mu(\Delta_{12})\theta_1 = \lambda_2(\Delta_{12}) - \mu(\Delta_{12})\theta_2$. Since the shadow prices and $-\mu$ all grow at the rate of discount, we have $\lambda_1^0 - \lambda_2^0 = -\mu^0(\theta_2 - \theta_1)$. Substituting the initial value of the scarcity rent of coal given by $\lambda_2^0 = \bar{p}_2 e^{-\rho(\Delta_{12}+\Delta_2)}$, we get

$\mu^0 = \mu(\Delta_{12})e^{-\rho\Delta_{12}} = \frac{-\bar{p}_2 e^{-\rho\Delta_{12}}(1 - e^{-\rho\Delta_2})}{\theta_2}$. For points located on the AA' curve where

$\Delta_2 = 0$, we have $\mu^0 = 0$ so that $\lambda_1^0 = \lambda_2^0$. Both resources are perfect substitutes and regulation is non-binding.

Finally we show that coal is used exclusively beyond Δ_{12}, i.e., the marginal cost of gas is higher than $\bar{p}_2$ in the interval $(\Delta_{12}, \Delta_{12} + \Delta_2)$. In this period, we have

$\lambda_1(t) - \mu(t)\theta_1 - \bar{p}_2 = \lambda_1(\Delta_{12})e^{\rho(t-\Delta_{12})} - \mu(t)\theta_1 - \bar{p}_2 =$

$[\lambda_2(\Delta_{12}) - \mu(\Delta_{12})(\theta_2 - \theta_1)]e^{\rho(t-\Delta_{12})} - \mu(\Delta_{12})e^{\rho(t-\Delta_{12})}\theta_1 - \bar{p}_2 =$

$[\lambda_2(\Delta_{12}) - \mu(\Delta_{12})\theta_2]e^{\rho(t-\Delta_{12})} - \bar{p}_2 = \bar{p}_2 e^{\rho(t-\Delta_{12})} - \bar{p}_2 = \bar{p}_2(e^{\rho(t-\Delta_{12})} - 1) > 0$. In the final

interval $(\Delta_{12} + \Delta_2, \Delta_{12} + \Delta_2 + \Delta_2^H)$ regulation is no longer active hence $\mu(t) = 0$. The

marginal cost of the resource is its scarcity rent. Since $\lambda_1(t) > \lambda_2(t)$, coal is cheaper than natural gas.

C3. *Initial Endowments in zone IV*

Consider the vertical line through $\overline{X}_1^0$ (points such as A', F', H' in Fig.C2) where the resource price is $\overline{p}_1$ and the phase of joint use at the ceiling is of maximum duration. And points with a higher stock of gas, such as $E = (E_1, F_2')$. The path must be dynamically consistent and we already know the extraction sequence from location F'. We show that only gas is consumed at its maximum rate $\overline{x}_1$ from E to F'. The duration of this phase is given by $\Delta_1 = \dfrac{E_1 - F_1'}{\overline{x}_1}$. We only need to show that the marginal cost of coal is higher than that of gas which equals $\overline{p}_1$ in this period. The proof mimics the one above but on the interval $[0, \Delta_1)$. We have $\lambda_2(t) - \mu(t)\theta_2 - \overline{p}_1 = \lambda_2(\Delta_1)e^{\rho(t-\Delta_1)} - \mu(t)\theta_2 - \overline{p}_1 =$

$[\lambda_1(\Delta_1) + \mu(\Delta_1)(\theta_2 - \theta_1)]e^{\rho(t-\Delta_1)} - \mu(\Delta_1)e^{\rho(t-\Delta_1)}\theta_2 - \overline{p}_1 =$

$[\lambda_1(\Delta_1) - \mu(\Delta_1)\theta_1]e^{\rho(t-\Delta_1)} - \overline{p}_1 = \overline{p}_1 e^{\rho(t-\Delta_1)} - \overline{p}_1 = \overline{p}_1(e^{\rho(t-\Delta_{12})} - 1) > 0$.

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Smulders, Sjak and Edwin Van der Werf (2005), "Climate Policy and the Optimal Extraction of High and Low Carbon Fuels," Unpublished manuscript, Tilburg University and CentER.
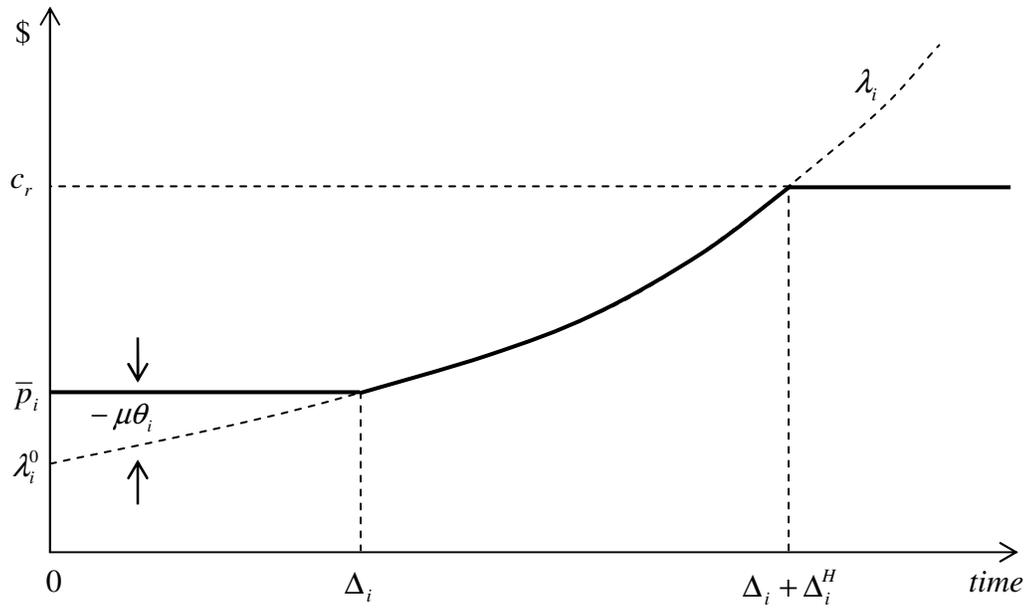
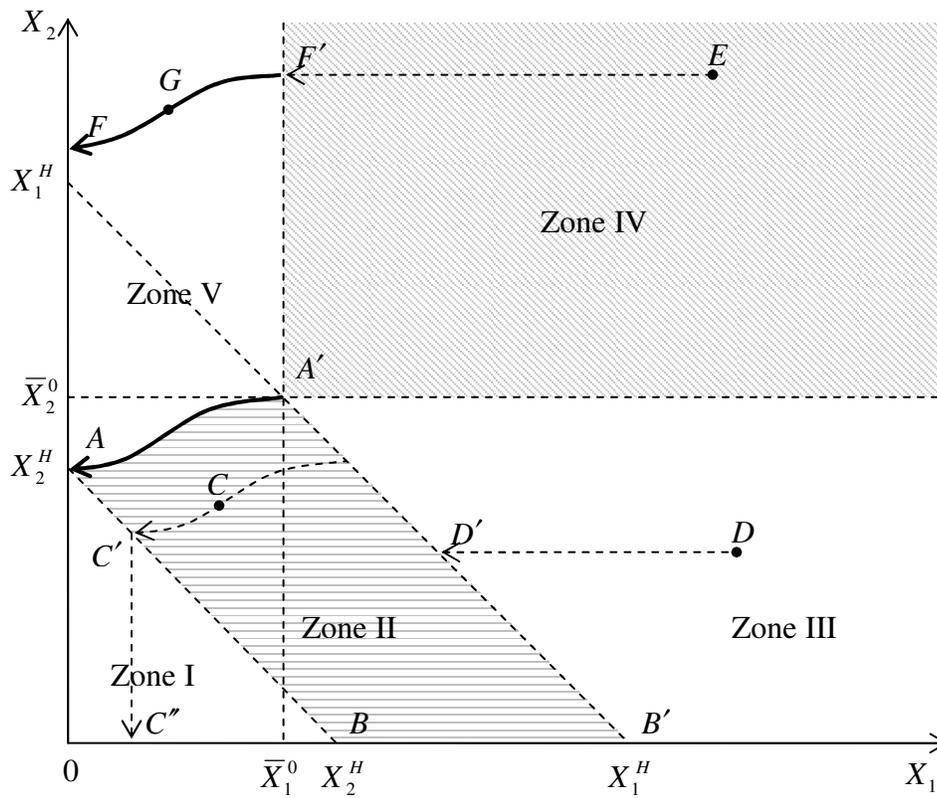Fig.1. Prices with one abundant resource

Fig.2. The sequence of extraction depends on initial endowments $(A': \overline{X}_1^0 + \overline{X}_2^0 = X_1^H)$
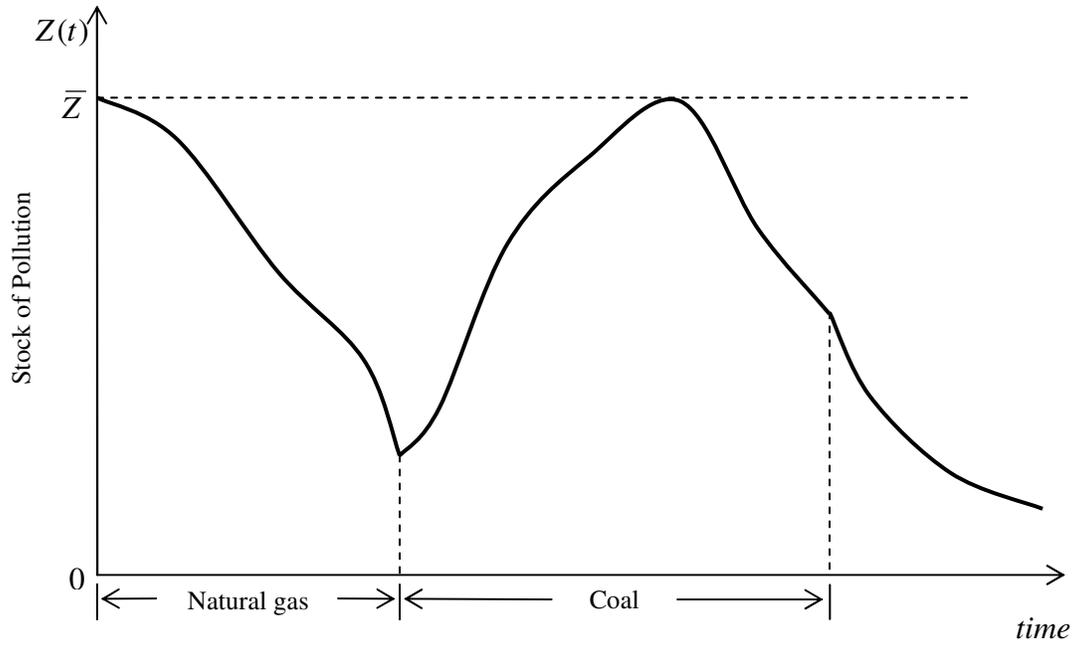
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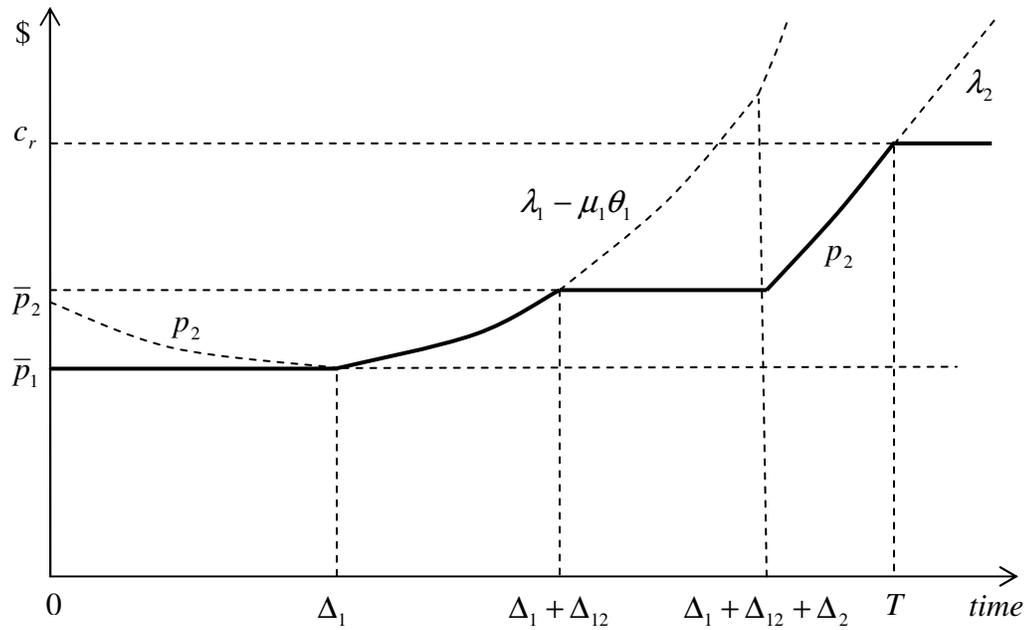Fig.3. Possible extraction sequence for $X^0 \in [X_2^H, X_1^H]$
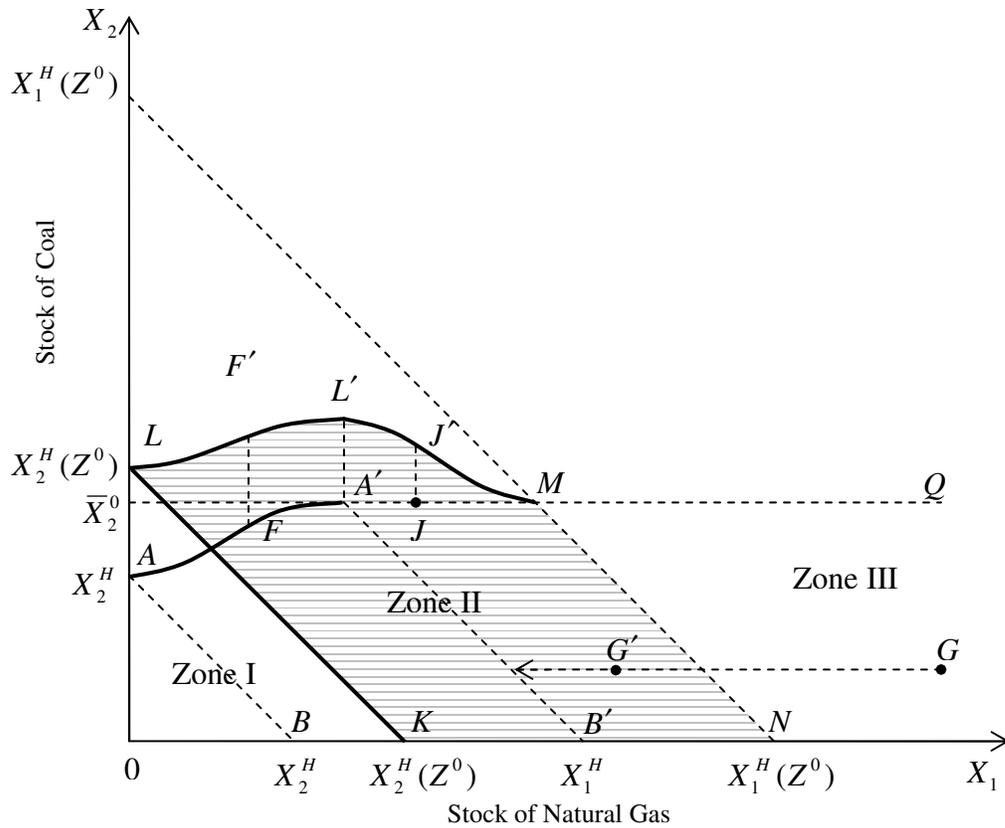

Fig.4. Prices when both resources are abundant
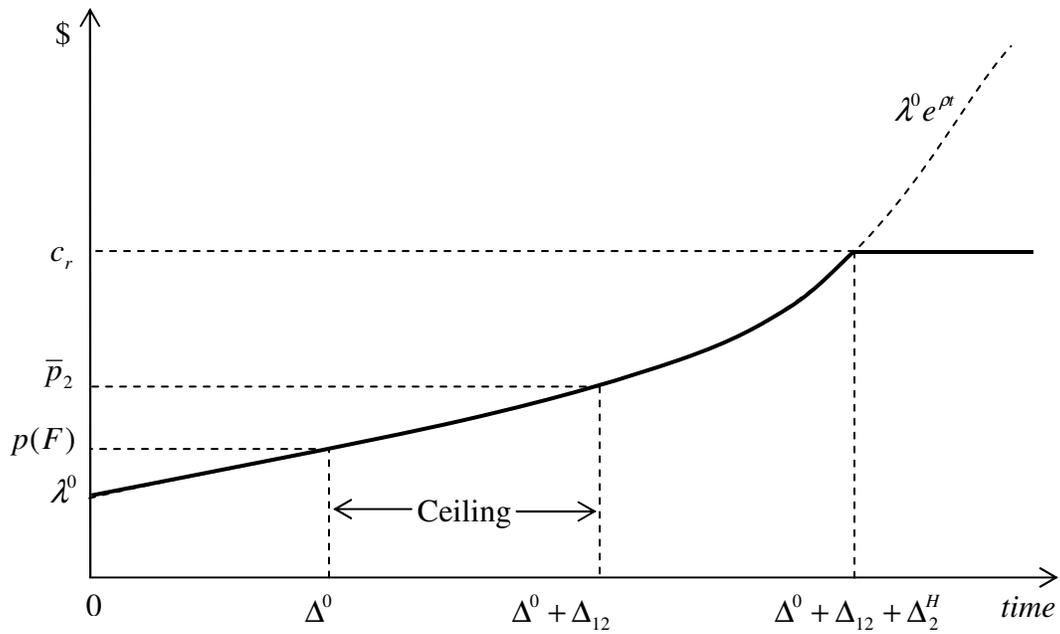
Fig.5. Hotelling paths starting from below the ceiling

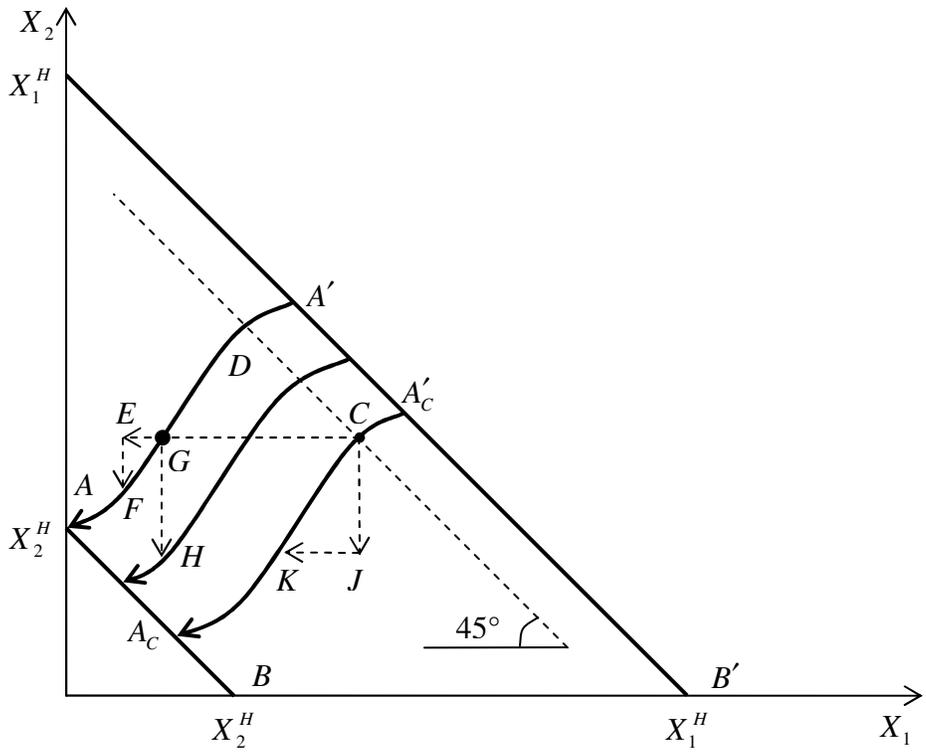
Fig.6. Resource Prices when Natural Gas is Abundant

Fig.7. Approach to the ceiling with abundant resources

Fig.8. Prices with abundant coal and limited gas reserves

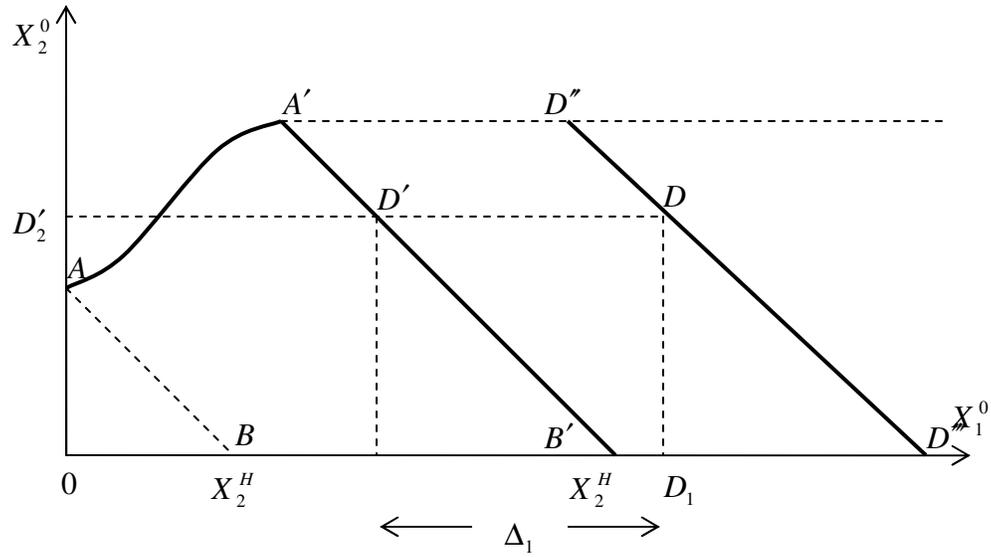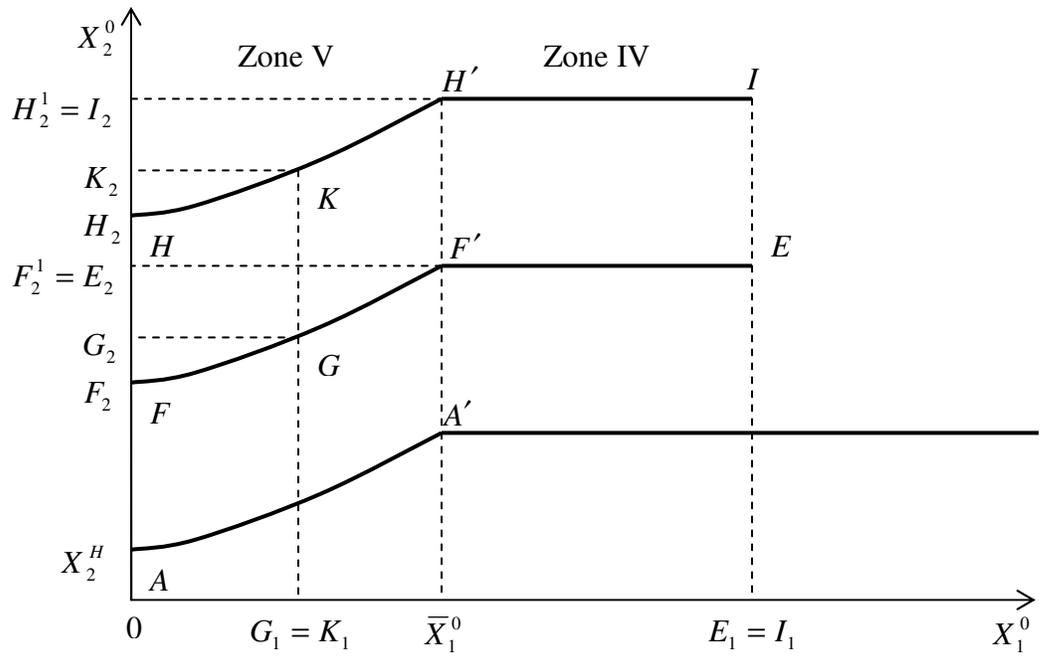Fig.B1. Hotelling paths starting from Zone II

Fig.C1. Iso-scarcity rent loci in Zone III

Fig.C2. Endowments in zones IV and V