# Assets relative risk for long-term investors

Christian Gollier
Toulouse School of Economics (LERNA)

July 14, 2007

**Abstract**

We show that, for all developed countries for which data are available, the relative riskiness of equity compared to bonds and bills goes down when the investment horizon increases. In particular, VAR setups show that this effect is very strong. This fact implies that investors with a longer investment horizon should invest relatively more in equity. These results are opposite to those presented by Lukassen and Pröpper (2007). They shed some critical light on the proposed Solvency II reform, which would force life insurers to inefficiently rebalance their portfolio towards safer assets.

# 1 Introduction

When they spend a fraction of their income to invest in life insurance or in pension funds, consumers obviously have long term objectives. It would therefore be inefficient to impose short term objectives to financial intermediaries and to provide solvency constraints that bias their time preferences towards short termism. By contemplating the possibility to calibrate all quantifiable risks on a one-year time horizon in Solvency II, CEIOPS' proposed approach to investment and equity investment does not take into account of the consumers' aspiration for a whole life span saving perspective. This failure to recognize the consumers' aspiration for a lifecycle perspective in the risk-return optimization may yield potentially lethal consequences for the life insurance industry, with a catastrophic impact for the current and future generations' welfare.

Since the seminal papers on dynamic portfolio management (Merton (1969), Samuelson (1969)), there has been a never-ending confusion about the effect of the time horizon on the optimal portfolio allocation. As documented by Lukassen and Pröpper (2007) for example, there is no doubt that the riskiness of a diversified stocks portfolio increases with the investment horizon $T$. But the expected total return of this portfolio is also increasing with $T$. This implies that we cannot in general determine the effect of the investment horizon on the optimal risk-return solution.

The problem implicit in dynamic portfolio choices is to determine how future investment opportunities affect instantaneous investment choices. Conventional wisdom states that long-horizon investors can tolerate more risk because they have more time to recoup transient losses. This dictum has not received the backing of scientific theory, however. As Samuelson (1963, 1989) in particular points out, this "time-diversification" argument relies on a fallacious interpretation of the Law of Large Numbers: repeating an investment pattern over many periods does not cause risk to wash out in the long run. This fallacy is illustrated by the following question raised by Samuelson (1963): "I offered some lunch colleagues to bet each \$200 to \$100 that the side of a coin they specified would not appear at the first toss. One distinguished scholar (...) gave the following answer: I won't bet because I would feel the \$100 loss more than the \$200 gain. But I'll take you on if you promise to let me make 100 such bets". This story suggests that independent risks are complementary. However, Samuelson went ahead and asked why it would be optimal to accept 100 separately undesirable bets. The scholar answered: "One toss is not enough to make it reasonably sure that the law of averages will turn out in my favor. But in a hundred tosses of a coin, the law of large numbers will make it a darn good bet." Obviously, this scholar misinterprets the Law of Large Numbers! It is not by accepting a second independent lottery that one reduces the risk associated with the first one. It is by subdividing — not adding — independent risks that they are washed away by diversification.

The picture would be quite different, and in fact more favorable for Paul Samuelson's colleague, if the various risk exposures would be correlated, as is the case when we apply this story to the case of intertemporal assets portfolios. We hereafter show that assets returns exhibit mean reversion (for equity) or the mean aversion (for bonds and bills), which implies that long term risk-averse investors should invest more in equity relative to myopic investors who evaluate the riskiness of their portfolio on a yearly basis.[1] This is why we strongly oppose any solvency rule that would measure portfolio risks on an annual basis for life insurers with long liabilities.

In Section 2, I review the benchmark case with serially uncorrelated equity returns. I document the existence of serial correlations in assets returns in Section 3. In Section 4, we summarize the findings of the more sophisticated vector autoregressive (VAR) setups.

# 2 The case of serially independent equity returns

The benchmark model in finance to examine this question is when there two assets, one being risk free and the other having serially independent returns. Let $x_t$ denote the logarithm of one plus the excess equity return in year $t$. Let $X_T$ denote the logarithm of one plus the cumulative excess equity return over $T$ years, i.e. $X_T = \Sigma_{t=1}^{T} x_t$. If $x_1, ..., x_T$ are i.i.d. with mean $\mu$ and variance $\sigma^2$, the first two moments of $X_T$ are

$$EX_T = T\mu \;\text{ and }\; Var(X_T) = T\sigma^2. \tag{1}$$

[1]Jagannathan and Kocherlakota (1996) and Gollier (2007) provide alternative arguments in favor of the positive relationship between the demand for stocks and the time horizon of the investor.

In the IID case, both the expectation and the variance of aggregate excess return increase proportionally with the investment horizon. Investing in equity for one year entails some risk, but investing in stocks for 40 years is 40 times riskier, assuming no serial correlation of stock returns! Whether this increased volatility of total return is compensated or not by the increased expected total return is an open question. Thus, it is not clear a priori that younger investors should bias their portfolio allocation towards stocks or bonds in such a model. If we consider an investor who follows a buy-and-hold strategy over $T$ years, we know that the optimal share $\alpha$ of wealth that should be invested in equity is approximately proportional to the ratio of the expectation over the variance of the aggregate excess return of equity:

$$\alpha_T = \frac{EX_T}{\gamma Var(X_T)}, \tag{2}$$

where $\gamma$ is an index of risk aversion (see for example Campbell and Viceira (2002 p. 29) or Gollier (2001 p. 57)). From equation (1), this is independent of the investment horizon. In short, a longer holding period of equity yields a more uncertain aggregate return. But at the same time, it yields a larger expected return. In the IID case, the two effects compensate each other perfectly, and the holding period has no effect on the optimal portfolio allocation. This is the main message of the early classical literature on this question (Mossin (1968), Merton (1969), Samuelson (1969)). If the assumption on which this theoretical result would be verified, I would have no reservation on the recommendation to use a one-year time horizon to quantify portfolio risks of financial intermediaries.

Many refinements of this basic result in the IID case can be provided, with no essential impact on the main message. For example, it is robust to allowing rebalancement of the assets portfolio in discrete or continuous time. In Gollier and Zeckhauser (2002), I show that a myopic portfolio strategy is optimal in the IID case for a wide set of the investor's utility function.

Suppose alternatively that assets returns are not IID over time. Suppose for example that an asset has a mean reversion behavior. Because mean reversion implies that the asset is relatively safer in the long run, the intuition suggests that a long horizon agent should have a positive "hedging demand" for that asset in the initial stage of the investment period. Mean reversion means that the variance of the aggregate return increases with the investment horizon $T$ at a decreasing rate. Using equation (2), this implies that the optimal investment in the asset is increasing with the investment horizon.

The absence of any predictability in asset returns has long been considered as a dogma in the theory of finance. A convergent flow of published empirical papers over the last twenty years that contradicts the assumption that equity returns are serially independent, in particular in the United States (Poterba and Summers (1988), Campbell (1996), Campbell, Lo and MacKinlay (1997), Campbell and Viceira (1999, 2002)). For example, Barberis (2000) estimates significant mean-reversion in U.S. stock returns: a high return of the risky portfolio in period $t$ implies a lower expected portfolio return period $t+1$. Campbell and Viceira (1999) and Barberis (2000) have shown that the hedging demand for stocks is surprisingly large. For an agent with a relative risk aversion equaling 10 and a ten-year time horizon, the optimal investment in stocks is about 40% of current wealth without predictability. It goes up to 100% when stock returns' mean-reversion is taken into account.

# 3 A simple statistical measure of serial correlation in assets returns

Following Campbell and Viceira (2002) and Lukassen and Pröpper (2007), a simple way to evaluate whether assets returns are serially correlated is to compute the annualized standard deviation of $T$-years return. Consider a portfolio whose log of the gross return in year $t$ is denoted $y_t$. The log of total return over $T$ years is $Y_T = \Sigma_{t=1}^{T} y_t$. In the IID case, the standard deviation of $Y_T$ should be proportional to $\sqrt{T}$. Following Campbell and Viceira (2002, p. 108), we annualize this standard deviation by dividing it by the square root of $T$:

$$\text{Annualized vol. of } T\text{-years returns} = \frac{\text{vol. of the log of total } T\text{-years returns}}{\sqrt{T}}.$$

Thus, in the IID case, this should be a constant. In the case of mean reversion, this should be decreasing with the time horizon. Equation (2) suggests that a longer time horizon should raise the demand for that asset.[2] In the case of mean aversion, the opposite would be true.

[2]Gollier (2004) shows that this is formally true only if relatuve risk aversion is larger than unity, when the investor can dynamically rebalance his portfolio.

As Lukassen and Pröpper (2007), we base our analysis on Dimson-Marsh-Staunton nominal annual return indexes for the period 1900-2006 (Source: Morningstar). We consider three different assets: stocks, bonds and bills. We consider time horizons $T$ between 1 and 5 years, and 16 countries. For each country and each value of $T$, we computed the series of total log returns $(y_0 + ... + y_{T-1}, y_1 + ... + y_T, ...)$, where $y_t$ equals the log gross return of year 1900+$t$, and we computed the standard deviation of this series. We finally divided this volatility by $\sqrt{T}$ to get the annualized standard deviation. We also report the volatility of global indexes for bills, bonds and stocks that aggregate these 16 financial markets, as done by Dimson-Marsh-Staunton. Tables 1 and 2 show the annualized standard deviation of $T$-years returns for these countries computed for period 1900-2006.[3]

Let us first observe that for all countries and for all holding horizons, stocks are riskier than bonds, and bonds are riskier than bills.

We now examine the volatility of stocks returns. Observe first that we have not been able to reproduce the numbers that have been presented by Lukassen and Pröpper (2007). Using the same data set (except for year 2006 which is added in this study), they exhibited a strong positive relationship between the annualized historical volatility and the holding period in most countries. For example, for Italy, they obtained an annualized volatility of 34% for a holding period of 1 year, and an annualized volatility of 85% for a holding period of 5 years. We rather obtained a flat volatility around 26%.[4] The overall picture resulting from our computations is that the annualized standard deviation of stocks returns is slightly decreasing with the holding periods for some countries like the U.K., the U.S. and Germany, whereas it is almost independent of the holding period in the other countries. This is confirmed with the global stocks index, with an annualized volatility of 15.8% and 15.7% respectively for $T = 1$ and $T = 5$. We conclude, that contrary to what is claimed by Lukassen and Pröpper (2007), there is no mean aversion in the behavior of stocks return. Quite the contrary, there is some tendency

[3]For Germany, we only considered period 1930-2006, because of the extremely high volatility of the German inflation during the twenties.

[4]Lukassen and Propper (2007) do not explain what formula did they used to obtain their numbers, so it is hard the sources of the huge discrepancies between the results of these two studies. A clear difference is that they decided to compute the volatility of net returns, whereas we use the more logical and classical approach based on the log of gross returns. But, we have not been able to reproduce their Table 1 using net returns.

for stocks returns to exhibit mean reversion, among else in the United States. This confirms the theoretical findings for the United States by the authors mentioned above.

We now turn to the analysis of the riskiness of bonds relative to the holding period. Except for Germany, Japan and Switzerland, bonds are relatively riskier in the long run than in the short run. For example, for France, the annualized volatility is equal to 8% and 11.5% respectively for holding periods $T = 1$ years and $T = 5$ years. Thus, bonds returns exhibit some degree of mean aversion. This is an argument in favor of providing incentives for financial intermediaries with long liabilities (as life insurers) to invest more in stocks.

This point is reinforced when we look at the relative riskiness of bills in the long term. It is universally true that, during the 20th century, bills returns exhibited a strong mean aversion behavior, which implied that the relative riskiness of bills increases with the holding horizon. Using the global index, the annualized volatility of bills for a holding period of 5 years is double the annualized volatility based on a one-year holding period. The mean aversion of bill returns is due to the persistent nature of shocks to the real interest rate, which amplifies the volatility of returns when bills are reinvested over long horizons.

Our conclusion at this stage is that a simple statistical treatment of the observed assets returns over the 20th century demonstrate that the relative riskiness of stocks with respect to either bills or bonds goes down when the holding period is increased. In Figure 1, we describe this relationship. For a holding horizon of one year, stocks are 6 times riskier than bills, and twice as riskier than bonds. For a holding horizon of 5 years, these relative degrees of riskiness go down to 300% and 150% respectively. Thus, the use of a one-year horizon to measure the riskiness of assets inefficiently force long term investors to invest in relatively safer assets. This is socially inefficient.

# 4 A VAR setup

The analysis presented in the previous section has at least three deficiencies. First, the holding horizon is limited to a maximum of 5 years, in spite of the fact that the duration of the life insurers' liabilities often exceed twenty years. Second, simple statistic analysis can be seen only as a first step into the study of the complex dynamic relationships that link various assets returns together with other economic variables. Third, the above analysis overlooked the fact that if assets returns are predictable, it is optimal for investors to time their allocation strategy. For example, if stocks returns mean revert, it is optimal to reallocate the portfolio towards safer assets after a long period of historically large stocks returns. This is overlooked in the previous analysis because we only considered buy-and-hold strategies.

As observed by Campbell and Viceira (2002), the method based on vector autoregressive (VAR) dynamics is very useful to explore such a problem. Their analysis can be summarized as follows. A VAR(1) system is considered:

$$\mathsf{z}_t = \Phi_0 + \Phi_1 \mathsf{z}_{t-1} + \mathsf{v}_t, \tag{3}$$

where $\mathsf{z}_t$ is a $6 \times 1$ vector containing two sets of variables. The first set of variables contains the real return of bills, the excess return of stocks and the excess return of bonds. The second set of variables in $\mathsf{z}_t$ contains variables which have been identified as returns predictors in existing empirical analysis, such as the nominal short rate, the dividend-price ratio and the yield spread between long-term and short-term bonds. In equation (3), $\Phi_0$ is the $6 \times 1$ vector of intercepts and $\Phi_1$ is the $6 \times 6$ matrix of slope coefficients. Finally, $\mathsf{v}_t$ is the $6 \times 1$ vector of innovations in asset returns and return forecasting variables, which is assumed to be IID normally distributed.

Campbell and Viceira (2002) estimate this model with quarterly data of US market performances over period 1952-1999. The fact that many elements in matrix $\Phi_1$ are different from zero means that asset returns are predictable. For example, these authors show that the dividend-price ratio is a good predictor of future stock returns. From this work, they plotted the annualized standard deviations of real returns of stocks, bills and bonds for the US for investment horizons up to 100 quarters. This is reproduced in Figure 2. Contrary to what was done in the previous section, these standard deviations are conditional, in the sense that they take out changes in returns that are predictable in advance (and which therefore do not represent risk). We see mean-reversion in real stock returns, which cuts the annualized standard deviation of returns from 16% to 8% as one moves from one-quarter horizon to a 25-year horizon. On the contrary, bonds held to maturity exhibit mean-averting returns. With an investment horizon of 25 years, the annualized standard deviation of returns of respectively stocks and bonds held to maturity are respectively 8% and 6%. This implies that there is no reason to use very different risk weights for stocks and bonds in capital requirement constraints of financial intermediaries with a long duration of their liabilities.

We suggested in the previous section that the United States is a special case, with a phenomenon of mean reversion for stocks returns that is stronger than in other countries. However, Bec and Gollier (2007) have shown that a similar phenomenon exists on French financial markets. They used quarterly data of French financial markets over period 1970-2006, together with the same three predictors than in Campbell and Viciera (2002). Their finding is reproduced in Figure 3. The degrees of riskiness of stocks and bonds held to maturity are almost the same when considering investment horizons above 10 years.

# 5 Conclusion

Using different methods, we have shown that the relative riskiness of equity compared to bonds and bills goes down when the investment horizon increases. This is true for all countries for which data are available. In particular, VAR setups show that this effect is very strong, at least for the United States and France, the only two countries for which such setup has been developed. This fact implies that investors with a longer investment horizon should invest relatively more in equity. The current proposition in Solvency II is to limit the measurement of risk to a one-year horizon. This will force life insurers to rebalance their portfolio towards safer assets, which clearly goes opposite to the interests of their customers. We urge the European institutions to reconsider this specific aspect of their proposed new regulation of the insurance sector.

## References


Barberis, N., (2000), Investing for the long run when returns are predictable, *Journal of Finance*, 55, 225-64.

Bec, F., and C. Gollier, (2007), Assets returns volatility and investment horizon: The french case, mimeo, University of Toulouse 1.

Campbell, J.Y., A.W. Lo and A. C. MacKinlay, (1997), *The Econometrics of Financial Markets*, Princeton University Press, Princeton.

Campbell, J., and L. Viceira, (1999), Consumption and portfolio decisions when expected returns are time varying, *Quarterly Journal of Economics*, 114, 433-95.

Campbell, J.Y., and L.M. Viceira, (2002), *Strategic asset allocation*, Oxford University Press, Oxford.

Gollier, C. and R.J. Zeckhauser, (2002), Horizon Length and Portfolio Risk, *Journal of Risk and Uncertainty*, 24 (3), 195-212.

Gollier, C., (2001), *The economics of risk and time*, MIT Press, Cambridge MA.

Gollier, C., (2004), Optimal Portfolio Risk with First-Order and Second -Order Predictability, *Contributions to Theoretical Economics*, Vol 4 No 1, www.bepress.com/bejte/contributions/vol4/iss1/art4.

Gollier, C., (2007), Optimal portfolio management for individual pension plans, in *Pension Strategies in Europe and the United States*, ed. by G. de Menil and P. Pestieau, MIT Press, forthcoming.

Jagannathan, R., and N.R. Kocherlakota, (1996), Why should older people invest less in risky assets than younger people?, *Federal Reserve Bank of Minneapolis Quarterly Review*, 20, 11-23. http://minneapolisfed.org/research/qr/qr2032.pdf

Lukassen, R., and M. Pröpper, (2007), Equity risk at the horizon, *Life and Pensions*, April, pp. 43-47.

Merton, R.C., (1969), Lifetime portfolio selection under uncertainty: The continuous- time case, *Review of Economics and Statistics*, 51, pp. 247-257.

Mossin, J., (1968), Optimal multiperiod portfolio policies, *Journal of Business*, 215-229.

Poterba, J., and L.H. Summers, (1988), Mean reversion in stock returns: Evidence and implications, *Journal of Financial Economics*, 22, 27-60.

Samuelson, P.A., (1963), Risk and uncertainty: the fallacy of the Law of Large Numbers, *Scientia*, 98, 108-113.

Samuelson, P.A., (1969), Lifetime portfolio selection by dynamic stochastic programming, *Review of Economics and Statistics*, 51, 239-246.

| Holding periods (years) | | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Australia | Bills | 3,7% | 5,2% | 6,3% | 7,2% | 8,0% |
| | Bonds | 10,5% | 11,4% | 11,5% | 11,4% | 11,5% |
| | Stocks | 16,1% | 15,5% | 14,4% | 13,8% | 13,4% |
| Belgium | Bills | 2,8% | 3,8% | 4,7% | 5,3% | 5,9% |
| | Bonds | 9,3% | 9,6% | 9,6% | 9,6% | 9,8% |
| | Stocks | 20,1% | 22,1% | 21,8% | 21,1% | 20,3% |
| Canada | Bills | 3,4% | 4,6% | 5,6% | 6,4% | 7,0% |
| | Bonds | 8,3% | 8,5% | 9,1% | 9,6% | 10,1% |
| | Stocks | 15,6% | 16,9% | 16,6% | 16,1% | 15,0% |
| Denmark | Bills | 3,1% | 4,2% | 5,1% | 5,9% | 6,5% |
| | Bonds | 9,0% | 8,8% | 9,3% | 9,9% | 10,6% |
| | stocks | 17,3% | 16,7% | 16,3% | 15,8% | 15,0% |
| France | Bills | 2.2% | 3.0% | 3.6% | 4.1% | 4.5% |
| | Bonds | 8.0% | 9.1% | 10.1% | 10.9% | 11.5% |
| | stocks | 21.2% | 22.5% | 22.6% | 22.5% | 22.0% |
| Germany | Bills | 2,0% | 2,7% | 3,2% | 3,6% | 3,9% |
| | Bonds | 28,4% | 27,9% | 27,8% | 27,8% | 27,7% |
| | stocks | 35,8% | 33,7% | 34,2% | 33,4% | 32,4% |
| Ireland | Bills | 3,8% | 5,2% | 6,4% | 7,3% | 8,1% |
| | Bonds | 11,4% | 11,7% | 12,4% | 12,5% | 12,9% |
| | stocks | 19,1% | 18,2% | 17,7% | 18,0% | 17,6% |
| Italy | Bills | 3,0% | 4,2% | 5,2% | 5,9% | 6,5% |
| | Bonds | 8,1% | 7,9% | 8,2% | 8,6% | 9,1% |
| | stocks | 26,1% | 26,6% | 26,7% | 26,4% | 25,9% |
| Japan | Bills | 2.2% | 3.0% | 3.5% | 3.9% | 4.3% |
| | Bonds | 13.8% | 11.1% | 10.8% | 9.7% | 10.7% |
| | stocks | 24.1% | 25% | 24.3% | 24.6% | 25.6% |
| Netherlands | Bills | 2,2% | 3,0% | 3,6% | 4,0% | 4,4% |
| | Bonds | 7,4% | 7,3% | 7,7% | 7,8% | 8,0% |
| | stocks | 19,3% | 20,4% | 20,4% | 20,8% | 21,1% |
| Spain | Bills | 3,6% | 5,1% | 6,1% | 6,9% | 7,7% |
| | Bonds | 9,4% | 9,1% | 9,5% | 9,8% | 10,0% |
| | stocks | 19,5% | 21,7% | 22,7% | 22,8% | 22,1% |
| South Africa | Bills | 5,2% | 7,3% | 8,8% | 10,1% | 11,2% |
| | Bonds | 8,4% | 9,2% | 9,9% | 10,7% | 11,6% |
| | stocks | 19,4% | 20,3% | 20,1% | 19,6% | 18,9% |



Figure 1: Annualized standard deviation of $T$-years returns, $T$ =1, 2, 3, 4, and 5, using Dimson-Marsh-Staunton data over period 1900-2006 (except Germany, where period 1930-2006 is considered).

| Holding periods (years) | | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Sweden | Bills | 2,9% | 4,0% | 4,8% | 5,5% | 6,0% |
| | Bonds | 8,7% | 8,1% | 8,4% | 8,6% | 9,3% |
| | stocks | 20,8% | 22,5% | 22,8% | 22,2% | 21,3% |
| Switzerland | Bills | 1,7% | 2,3% | 2,7% | 3,0% | 3,2% |
| | Bonds | 4,5% | 4,6% | 4,4% | 4,2% | 4,0% |
| | stocks | 17,5% | 18,6% | 18,1% | 17,4% | 16,9% |
| United Kingdom | Bills | 3.5% | 4.9% | 5.9% | 6.8% | 7.5% |
| | Bonds | 10.7% | 11.1% | 11.8% | 11.9% | 12.2% |
| | stocks | 18.2% | 17.9% | 17.3% | 16.6% | 16.5% |
| Unites States | Bills | 2.7% | 3.7% | 4.4% | 5.0% | 5.5% |
| | Bonds | 7.4% | 7.1% | 7.6% | 8.0% | 8.5% |
| | stocks | 19.0% | 19.3% | 18.1% | 17.6% | 17.2% |
| Global | Bills | 2,7% | 3,7% | 4,4% | 5,0% | 5,5% |
| | Bonds | 8,0% | 8,9% | 9,4% | 9,9% | 10,3% |
| | stocks | 15,8% | 17,1% | 16,9% | 16,4% | 15,7% |

Figure 2: Annualized standard deviation of $T$-years returns, $T$ =1, 2, 3, 4, and 5, using Dimson-Marsh-Staunton data over period 1900-2006.

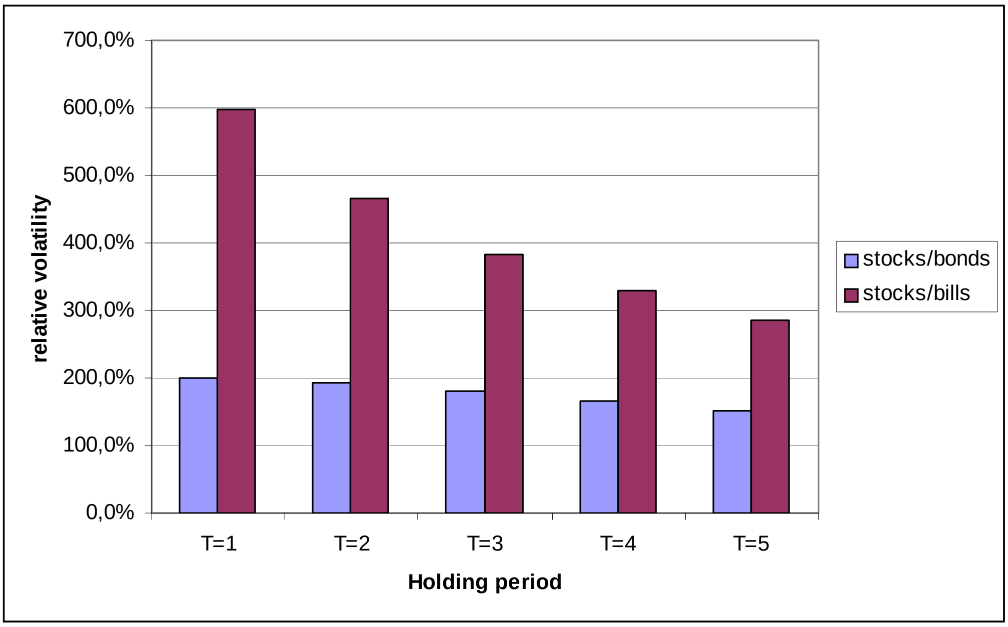


Figure 3: Annualized volatility of stocks returns expressed as a percentage of the annualized volatility of bond returns and bill returns respectively, Global index.

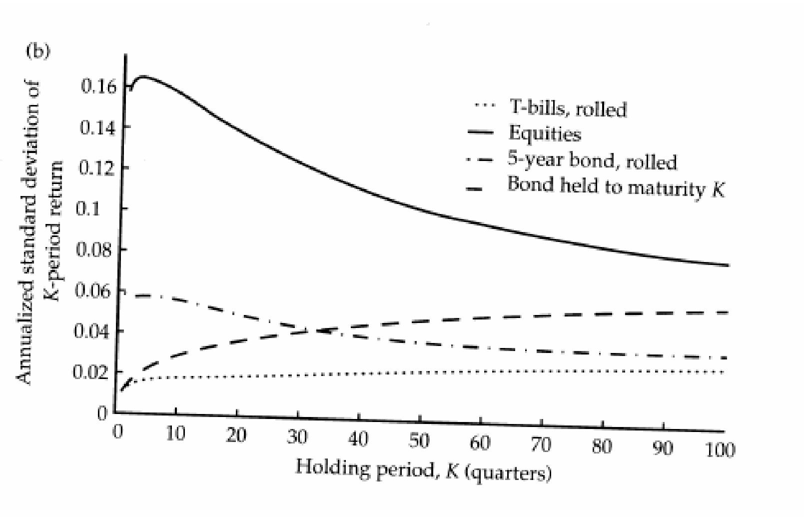


Figure 4: Annualized standard deviations of real returns on US markets, VAR setup, period 1952-1999. Source: Campbell and Viceira (2002).

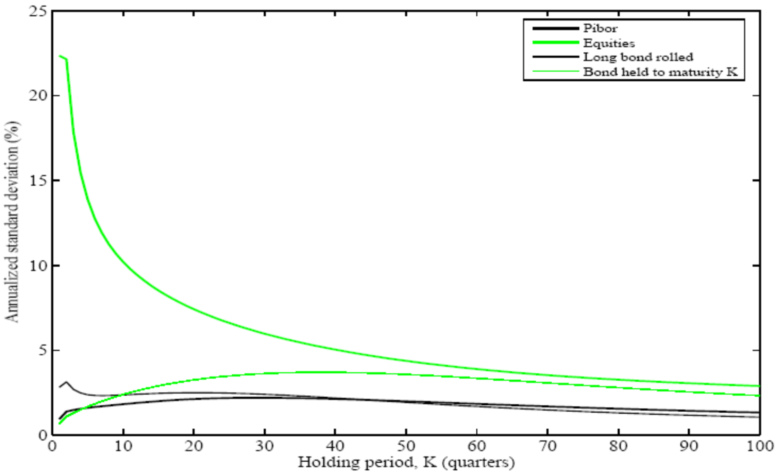


Figure 5: Annualized percent standard deviations of real returns on French markets, VAR setup, period 1970-2006. Source: Bec and Gollier (2007).