Risk Aversion and Herd Behavior in Financial Markets*

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Abstract

We show that differences in market participants risk aversion can generate herd behavior in stock markets where assets are traded sequentially. This in turn prevents learning of market's fundamentals. These results are obtained without introducing multidimensional uncertainty or transaction cost. (JEL: G1, G14, C11, D82)

Keywords: Herd Behavior, Risk Aversion.

1 Introduction

The literature on rational herding pioneering by Bikhchandani, Hirshleifer and Welch (1992) and Banerjee (1992) among others, proves that sequential interaction of rational investors can generate imitative behavior (herding) that prevents learning of the economy's fundamentals. However, in the herding models transaction prices are exogenous and constant, therefore their predictions cannot be directly extended to stock markets. To what extent the endogeneity of trading prices in financial markets can prevent herding phenomena and guarantee full information aggregation?

Avery and Zemsky (1998) (AZ henceforth) and Lee (1998) study the occurrence of herding in stock markets when trading is sequential and prices are endogenous. AZ show that the presence of multidimensional uncertainty, in the short run can generate herd behavior as well as large differences between an asset's trading price and its fundamental value. Nevertheless, in the long run all these phenomena vanish and all private

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information is eventually incorporated into prices. Lee (1998) shows that the presence of an exogenous transaction cost leads to information aggregation failure.

Within a simple sequential trade model, we show that herding, contrarian behavior[1] and information aggregation failure can occur even in the absence of both multidimensional uncertainty and transaction cost. Differently from AZ, in our model traders and market makers interpret past histories in the same way. Nevertheless, when market makers and traders differ in their risk aversion, the same information affects market makers' quotes and traders' valuations differently. This is sufficient to generate herding, contrarian behaviors, long run informational inefficiency and mispricing.

2 The economy

We consider a sequential trade model similar to Glosten and Milgrom (1985): a risky asset is exchanged for money among market makers and traders. At each trading period, a randomly selected trader has a unique opportunity to buy or sell one unit of the asset at the most attractive ask (A_t) or bid price (B_t) respectively. Prices are competitively posted by market makers. We denote with $\mathbf{v} = \mathbf{V} + \boldsymbol{\varepsilon}$ the liquidation value of the asset, where $\boldsymbol{\varepsilon}$ has a normal distribution $N(0, \sigma)$ with $\sigma > 0$, $\mathbf{V} \in \{\underline{V}, \overline{V}\}$ with $\underline{V} < \overline{V}$ and $\Pr(V = \overline{V}) = \pi_0$. $\mathbf{V}$ and $\boldsymbol{\varepsilon}$ are independently distributed. Each trader receives a private signal $\mathbf{s} \in \{l, h\}$ with $\Pr(\mathbf{s} = l | \mathbf{V} = \underline{V}) = \Pr(\mathbf{s} = h | \mathbf{V} = \overline{V}) = p \in (\frac{1}{2}, 1)$. Signals are conditionally i.i.d. across traders and independent from $\boldsymbol{\varepsilon}$. Note that we have $\underline{V} < E[\mathbf{v}|s = l] < E[\mathbf{v}] < E[\mathbf{v}|s = h] < \overline{V}$.

Let H_t be the history of trade (past quantities and prices) up to date $t-1$. All agents observe H_t and update their beliefs according to Bayes' rule. We denote $\pi_t = \Pr\left[\mathbf{V} = \overline{V} | H_t\right]$. We denote $\pi_t^s = \Pr\left[\mathbf{V} = \overline{V} | H_t, s\right]$, $s \in \{h, l\}$, an informed traders' belief at time t. A trader's action $\mathcal{A} \in \{buy, sell, no\ trade\}$ is said to be *not informative* at date t if it does not affect the public belief: $\Pr\left[\mathbf{V} = \overline{V} | H_t, \mathcal{A}\right] = \pi_t$. Note that the learning process in the economy regards only the realization of $\mathbf{V}$ and not $\boldsymbol{\varepsilon}$, still $E\left[v|H_t\right] = E\left[\mathbf{V}|H_t\right]$.

Risk averse agents of our economy have utility function $u(\mathbf{v}x + m) = -\gamma e^{-\gamma(\mathbf{v}x+m)}$, where x and m are respectively the amount of risky asset (*inventory* henceforth) and money in his portfolio. We denote with β (resp. α) the agent's buy (resp. sell) *reservation price* that corresponds to the asset's price such that this agent is indifferent between buying (resp. selling) one asset or not trading at all. The reservation prices for a risk averse agents whose initial inventory is x and that attaches probability π to the event $\{\mathbf{V} = \overline{V}\}$ are

$$\beta(\pi, x) = \frac{1}{\gamma}\left(-\frac{\gamma^2\sigma^2(2x+1)}{2} + \ln\left(\frac{\pi e^{-\gamma\overline{V}x} + (1-\pi)e^{-\gamma\underline{V}x}}{\pi e^{-\gamma\overline{V}(x+1)} + (1-\pi)e^{-\gamma\underline{V}(x+1)}}\right)\right), \quad (1)$$

$$\alpha(\pi, x) = \frac{1}{\gamma}\left(-\frac{\gamma^2\sigma^2(2x-1)}{2} + \ln\left(\frac{\pi e^{-\gamma\overline{V}(x-1)} + (1-\pi)e^{-\gamma\underline{V}(x-1)}}{\pi e^{-\gamma\overline{V}x} + (1-\pi)e^{-\gamma\underline{V}x}}\right)\right). \quad (2)$$

[1]See next section for a precise definition of these behaviors.

We adopt exactly the same definition of herding, contrarian behavior and informational cascade as in Avery and Zemsky (1998):

A trader with private signal s engages in buy (sell) *herding behavior* if: (i) initially he strictly prefers not to buy (resp. not to sell); (ii) after observing a positive history of trades H_t, i.e. $\pi_t > \pi_0$ (resp. negative history, i.e. $\pi_t < \pi_0$), he strictly prefers to buy (resp. sell).

A trader engages in buy (sell) *contrarian behavior* if: (i) initially he strictly prefers not to buy (resp. not to sell); (ii) after observing a negative (resp. positive) history of trades H_t, he strictly prefers to buy (resp. sell).

An *informational cascade* occurs when the actions of all informed traders are not informative.[2]

Suppose first that **market makers are risk neutral and traders are risk averse**. Then, in any given period t, the competition among market makers leads to bid and ask quotes (B_t, A_t) that are equal to the expectation of $\mathbf{v}$ conditional to the information provided by the past and current trades. If time-t-trader has signal s and inventory x, then he will buy if $\beta(\pi_t^s, x) \geq A_t$, he will sell if $\alpha(\pi_t^s, x) \leq B_t$ and he will not trade elsewhere.

Proposition 1 *A trader whose inventory is bounded away from* $-1/2$ *and* $1/2$ *will engage in herd or contrarian behavior with positive probability. Moreover, if there is a zero probability that a trader's inventory is close to either* $1/2$ *or* $-1/2$*, then an informational cascade occurs almost surely.*

Suppose now that **traders are risk neutral and market makers are risk averse**. Consider market maker i at time t, and let B_t^i, A_t^i and x_t^i be his bid and ask reservation prices and his inventory respectively. Taking into account the information provided by the trade at time t, we have:

$$\begin{aligned} A_t^i &= \alpha(\Pr(\mathbf{V} = \overline{V} | H_t, buy), x_t^i), \\ B_t^i &= \beta(\Pr(\mathbf{V} = \overline{V} | H_t, sell), x_t^i). \end{aligned}$$

We assume for simplicity that there are just two market makers, that they are symmetrically informed and behave myopically. Then, following Ho and Stoll (1983), in equilibrium, it results $A_t = \max\{A_t^1, A_t^2\}$ and $B_t = \min\{B_t^1, B_t^2\}$. If time-$t$-trader has signal s, then he will buy if $E[\mathbf{V}|H_t, s] \geq A_t$, sell if $E[\mathbf{V}|H_t, s] < B_t$ and he will not trade elsewhere.

Proposition 2 *If market makers' inventories are bounded away from* $\frac{1}{2}$ *and* $-\frac{1}{2}$ *then, eventually all traders take the same action. Consequently an informational cascade occurs, and traders engage in herding or contrarian behavior with positive probability.*

[2] Note that if an informational cascade never occurs, then π_t eventually converges to 1 (resp. to 0) for $\mathbf{V} = \overline{V}$ (resp. $\mathbf{V} = \underline{V}$).

3 Discussion

As a general rule, herd or contrarian behavior and informational cascade occur when market makers' quotes and traders valuations for the asset react differently to an history of trade. In our model this happens because of the difference in risk aversion between dealers and traders. Take for example, a positive history that increases π_t. As π_t approaches 1 a risk neutral agent's valuation for the asset converges to $\overline{V}$. By contrast, a risk averse agent's buy and sell reservation prices will converge toward levels that are in general different from $\overline{V}$. If market makers are risk neutral and traders are risk averse, then expressions (1) and (2) imply that when π_t is close to 1 (or to 0) all traders whose inventory is $x < -\frac{1}{2}$, $x \in (-\frac{1}{2}, \frac{1}{2})$, or $x > \frac{1}{2}$ will sell, not trade or buy the asset respectively no matter their signal. Thus, a trader who initially would have sold the asset, and whose $x > \frac{1}{2}$, will engage in buy herding behavior after a sufficiently long positive history. If market makers are risk averse and traders are risk neutral, then expressions (1) and (2), imply that when π_t is close to 1 all traders will buy (resp. sell) the asset if $A_t < \overline{V}$, i.e., $\min\{x_t^1, x_t^2\} > 1/2$ (resp. $B_t > \overline{V}$, i.e., $\max\{x_t^1, x_t^2\} < -\frac{1}{2}$) no matter the signal they received. Thus, a trader who initially would have sold the asset will engage in buy herding with positive probability. Note also that when the inventory of all risk averse agents in the economy (dealers or traders) is bounded away form $-1/2$ or $1/2$ then eventually all traders order will not be informative and an informational cascade occurs. In the presence of an informational cascade, when traders are risk averse the bid-ask spread is equal to zero and prices are steady, by contrast spread continues to evolve when market makers are risk averse as quotes move for inventory purposes even when trades do not conceal any new information. For this reason an informational cascade never ends when market makers are risk neutral, whereas it might end if market makers are risk averse. In a related paper (Decamps and Lovo (2003)) we show that information aggregation failure does not relay on the restriction to CARA utility functions nor on the assumption that agents can just choose the sign of their trade but not its size.

Appendix

Proposition 1 and 2 are direct consequences of the following Lemma

Lemma 1 *Take π_t is sufficiently close to 1 or to 0, and let $x > 1/2$ and $x' < -1/2$. Then for any triple of signals s, s' and s'' it results*

$$\begin{aligned}\alpha(\pi_t^s, x) &< E[V|H_t, s'] < \alpha(\pi_t^{s''}, x'),\\ \beta(\pi_t^s, x) &< E[V|H_t, s'] < \beta(\pi_t^{s''}, x').\end{aligned}$$

Proof: First, as $\pi_t^l = \frac{\pi_t(1-p)}{\pi_t(1-p)+(1-\pi_t)p}$ and $\pi_t^h = \frac{\pi_t p}{\pi_t p+(1-\pi_t)(1-p)}$, π_t^s is continuous in π_t. Second, from expressions (1) and (2), α and β are continuous in π. Third, $\alpha(1, \frac{1}{2}) = \beta(1, -\frac{1}{2}) = \overline{V}$, $\alpha(0, \frac{1}{2}) = \beta(0, -\frac{1}{2}) = \underline{V}$ and α and β are decreasing in x. The result follows then from an easy continuity argument .

Proofs of Propositions 1 and 2: Without loss of generality we reason with sell orders. Take an informed trader who strictly prefer not to sell at date 0. Assume first that the trader is risk averse with inventory $x' < -\frac{1}{2}$ whereas dealers are risk neutral. From lemma 1, as π_t is sufficiently close to 0, we have $\alpha(\pi_t^s, x') > B_t$ for $s \in \{l, h\}$. Thus, our trader will engage in sell herding and his trade will not be informative. Second, suppose the trader is risk neutral and dealers are risk averse. If at date t, the public belief π_t is close to 0 and dealers inventories satisfy $\max(x_t^1, x_t^2) < -\frac{1}{2}$ then, from lemma 1, $B_t = \min\{B_t^1, B_t^2\} > E[V|H_t, s]$ for $s \in \{l, h\}$ and the trader will engage in sell herding. Once again trades are not informative. Similarly, π_t close to 1 leads to sell contrarian behavior. Using an analogous argument it can be easily checked that if all risk averse agents' inventories are bounded away from $\frac{1}{2}$ and $-\frac{1}{2}$ then, as soon as π_t is close to 1 or to 0, trades are not informative and therefore an informational cascade occurs almost surely.

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