The Marginal Cost of Public Funds in Developing Countries: An Application to 38 African Countries

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Abstract: In this paper we propose estimates of the marginal cost of public funds (MCF) in 38 African countries. We develop a simple general equilibrium model inspired by the '1-2-3' model of Devarajan, Go, Lewis, Robinson, & Sinko (1994) that can handle taxes on the five major tax classes, takes account of the informal sector, and can be calibrated with little more than national accounts data. Sensitivity analysis suggests that our base case estimates are reasonably robust for purposes of tax reform. Contrary to conventional wisdom, differences in MCF are not strongly related to the wealth of the country. We hence show that a reasonable estimate of the average MCF in Africa is 1.17. On the other hand, there is a strong relationship between the size of the informal sector and the value of MCF. This suggests that welfare could be improved by reducing red tape barriers to business entry into the formal sector.

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1 Introduction

Tax revenue as a proportion of GDP is typically 20% lower in African countries than in rich OECD countries. Yet one of the salient features of African countries is an apparent under-provision of public goods in areas such as health, basic education or infrastructure. If there are unrealized benefits from public spending, why isn't taxation increased to enable more public projects to be undertaken? There are various possible responses to this question, and the answer may be different in different countries. Some African governments may not have the objective of maximizing social well-being, and so may under-provide public goods. Perhaps the assumption of high returns to public spending is false in practice, possibly because of inefficient or corrupt implementation of projects. Or perhaps the opportunity cost of public funds is higher in African countries than in rich countries. The paper explores this last response. It examines the marginal cost of public funds (MCF) - the change in social welfare associated with raising an additional unit of tax revenue using a particular tax instrument - in 38 African countries.

Measures of MCF are central to cost-benefit analysis for public expenditures. At a theoretical optimum, the marginal benefit of public expenditure would be equal to the marginal cost of public funds, which would be equal across all tax instruments. Socially desirable projects yield social rates of return greater than the MCF. More importantly estimates of MCF are crucial to determine the next challenge in the reform of African tax structures: the same revenue can be obtained at lower welfare cost by increasing a tax with a low MCF and lowering a tax with a high MCF.

The central role of the MCF in public economy is well known. The literature on this topic dates back at least thirty years, but it is almost entirely focused on the tax systems of the high income countries. For methodological reasons existing estimates are not all truly comparable. Table 1 summarizes existing estimates. We know of only one African country, Cameroon, for which the MCF has been estimated. Perhaps the major reason for the relative paucity of MCF estimates in developing countries has been the amount of data required and the time expense of creating and calibrating the computable general equilibrium (CGE) models used for estimation. Substitution effects, which are at the heart of the dead-weight loss of taxation, are poorly captured by partial equilibrium analysis. CGEs are required because real tax systems are complex, and because it is necessary to take account of multiple interactions within tax systems. The general equilibrium interactions between tax instruments are illustrated in Figure 2 section 4.1.[1]

Thus, the challenge as we have conceived it is to develop a simple CGE model that can be calibrated with little more than national accounts data, and which can be used to provide consistent estimates comparable across African countries. To be useful for tax policy analysis, the model should take account of the major classes of taxes in African countries: taxes on domestic goods, exports, imports, corporate income tax, and personal income tax. On average, in the sub-Saharan African countries that we examine, these taxes represent respectively 25%, 3%, 35%, 13%, and 14% of tax revenue, with the remaining 10% coming from other sources.[2] Another key requirement for realism is that the model should take account of the existence of informal sectors, which occupy a larger part of the economy in Africa than elsewhere. For example, Schneider & Enste (2000) report the shadow economy occupied on average 15% of GDP in OECD countries in 1990, but 27% in Botswana and 76% in Nigeria.[3] We might suspect that in countries with larger informal sectors, it is easier for economic agents to shift from formal to informal activity. Greater substitutability would lead to higher marginal costs of taxation on formal activity. The presence of an informal sector has been shown to have a noticeable effect on MCF estimates in Canada.[4]

In addition to their economic effect, including informal sectors avoids a calibration problem. CGE models have sometimes applied 'effective' tax rates, calculated as tax revenues divided by sector size. This results in low modelled tax rates. Effective tax rates provide an average between taxpayers who pay tax at something like the legal rate, subject to some under-reporting, and informal producers or consumers who pay no tax. Effective tax rates underestimate the marginal tax rate incurred by those who actually pay tax. Models using effective tax rates are thus likely to underestimate MCFs.[5]

[1]To provide the intuition for the effects of key parameters, analytical models with minimal data requirements were used in initial development of the theory (e.g. Stuart (1984), Mayshar (1991), Snow & Warren (1996)). But there are limits to the complexity of models that can be solved analytically.

[2]Eighteen countries in our sample do not use export taxes. Among the twenty countries that do use them export taxes constitute 7% of tax revenue.

[3]These figures are estimated using physical input methods.

[4]Fortin & Lacroix (1994) suggest the informal sector accounts for around 0.02-0.05 of their MCF estimates of 1.39-1.53 for labour taxation in Canada. They note that although small, the impact of the informal sector increases rapidly with the level of the marginal tax rate. The importance of the informal sector when analyzing taxation in developing countries is also emphasized in other settings in recent papers by García Peñalosa & Turnovsky (2003) and Emran & Stiglitz (2005).

[5]It seems that the models used by Devarajan, Suthiwart-Narueput & Thierfelder (2001)

Our paper makes several contributions to the literature on MCFs. The first is the development of a simple general equilibrium model that can handle taxes on the five major tax classes, takes account of the existence of informal sectors, can be calibrated with little more than national accounts data, and can be easily reproduced by other researchers for other countries. After a brief review of the existing literature in section 2, we present the model in section 3. It is inspired by the minimal data requirements of the '1-2-3 model' of Devarajan et al. (1994). The basic 1-2-3 model has one country with two producing sectors and three goods: a domestic good, exports and imports. This model is extended to include production of an informal good, and four factors of production: formal capital, informal capital, formal labour and informal labour. Our definition of an informal good or factor is one on which no tax is paid. As a side product of the calibration exercise, the paper hence proposes several measures of the informal economy.

Section 4 contains the paper's second important contribution: application of the model to produce estimates of MCFs for taxes in 38 African countries, vastly increasing the number of developing countries for which MCF estimates exist. Sensitivity testing of the model reveals which elasticities are the most important in determining MCF magnitudes, and suggests that our base case estimates are reasonably robust for purposes of tax reform.

The paper's third innovation, presented in section 5, is an investigation of the marginal cost of raising funds in currently informal sectors, and the effect of administrative costs on MCF estimates. We find that quite high levels of administrative costs would be justified to extend taxation to currently exempt informal sectors. Finally, in section 6 we examine the relationship between dispersion of MCFs for different tax instruments and the scope for welfare improving tax reform. We conclude in section 7.

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apply 'effective' tax rates. For example their Cameroon model applies tax rates of 0.7% for food and forestry, 2.5% for intermediate goods, 3.4% for construction, 6% for services, 7.4% for food and consumption, and 19.1% for cash crops. We do not have access to the legal tax rates in force at the time the Cameroon model was created, but the current VAT rate (which has replaced the previous system of sales taxes) is 18%, suggesting that effective tax rates were indeed used in their model. The relatively high effective tax rate for cash crops may be explained by noting that cash crops are largely exported. Exports tend not to escape taxation, so the effective tax rate approximates a revenue-weighted average of legal tax rates.

Table 1: Selected MCF Estimates

Country	Tax type	Estimate	Source
Australia	Labour	1.19-1.24	Campbell & Bond (1997)
Australia	Labour	1.28-1.55	Findlay & Jones (1982)
Australia	Capital	1.21-1.48	Diewert & Lawrence (1998)
Australia	Capital	1.15-1.51	Benge (1999)
Bangladesh	Sales	0.95-1.07	Devarajan et al. (2001)
Bangladesh	Import	1.17-2.18	Devarajan et al. (2001)
Cameroon	Sales	0.48-0.96	Devarajan et al. (2001)
Cameroon	Import	1.05-1.37	Devarajan et al. (2001)
Canada	Commodity	1.25	Campbell (1975)
Canada	Labour	1.38	Dahlby (1994)
Canada	Labour	1.39-1.53	Fortin & Lacroix (1994)
China	Sales	2.31	Laffont & Senik-Leygonie (1997)
India	Excise	1.66-2.15	Ahmad & Stern (1987)
India	Sales	1.59-2.12	Ahmad & Stern (1987)
India	Import	1.54-2.17	Ahmad & Stern (1987)
Indonesia	Sales	0.97-1.11	Devarajan et al. (2001)
Indonesia	Import	0.99-1.18	Devarajan et al. (2001)
New Zealand	Labour	1.18	Diewert & Lawrence (1994)
Sweden	All taxes	1.69-2.29	Hansson & Stuart (1985)
United States	All taxes	1.17-1.33	Ballard, Shoven & Whalley (1985)
United States	Labour	1.21-1.24	Stuart (1984)
United States	Labour	1.32-1.47	Browning (1987)
United States	All taxes	1.47	Jorgenson & Yun (1990)
United States	Labour	1.08-1.14	Ahmed & Croushore (1994)

2 The Marginal Cost of Public Funds

The MCF measures the change in social welfare associated with raising an additional unit of tax revenue using a particular tax instrument:

$$MCF = -\frac{\Delta W}{\Delta R} \tag{1}$$

where ΔW is a monetary measure of the change in social welfare and ΔR is the change in tax revenue arising from a marginal change in a tax instrument. The change in social welfare is a measure such as the equivalent variation or change in consumer surplus. Useful reviews of the theoretical and empirical literature on MCFs can be found in Ballard & Fullerton (1992) and Devarajan et al. (2001). The literature is plagued by multiple definitions of the same concepts. Different measures of the MCF for the same tax instrument can be found according to:

- The nature of the tax experiment conducted. Ballard & Fullerton (1992) identify two broad classes of theoretical analysis: 'differential' and 'balanced budget.' In differential analysis, one tax is marginally increased and another is decreased sufficiently to maintain the budget balance. The usual experiment is to increase a distortionary tax, and to reduce a lump-sum tax (return the revenue to consumers as a lump-sum). The income effects of the two tax changes cancel, leaving only substitution effects. Estimates of the welfare change, ΔW, depend on compensated elasticities, while the change in revenue, ΔR, can be equated with the actual lump-sum transfer. In balanced-budget analysis, one tax is marginally increased and the revenue is spent on a public project. Income effects are included in the analysis, and MCF estimates are derived using uncompensated elasticities. These are not the only possible measures. Wildasin (1984) proposed a measure in which the compensated change in welfare is divided by the compensated change in tax revenue rather than the actual change in tax revenue.

- The choice of numeraire. Håkonsen (1998) has proposed an alternative measure derived from the dual of the government's optimal tax problem (maximize revenue subject to a given level of social welfare) that is invariant to the choice of numeraire.

- The attribution of some general equilibrium effects between benefit and cost. Consider a marginal tax increase that increases revenue by one dollar, before public spending occurs. Public spending could increase the tax base in a second round effect (for example, building highways increases petrol tax revenue). If this second round effect is attributed to the MCF, the increase in revenue is greater than one dollar, and the MCF is accordingly reduced. But the second round effect could equally well be attributed to a measure of the marginal benefit of public spending. Mayshar (1991) proposed that all revenue effects of public spending should be incorporated in the benefits measure (MBF), rather than the MCF.

The consequence of this multiplicity of measures is that MCF estimates prepared using different methodologies are not comparable. Fortunately, Schöb (1994) has shown that standard MCF measures provide a valid basis for revenue-neutral tax reform, provided they are prepared using consistent methodologies. The levels of MCF estimates will depend on the estimation methodology, but these levels are not important in deciding directions for reform. What is important is which tax instruments have high MCFs and which have low MCFs. In other words the MCFs can loosely be thought of as ordinal measure. For our tax reform analysis, estimates of the MCF for different tax instruments in the same country will commonly be prepared using consistent methodology. That is, with the same numeraire, with changes in welfare measured using the equivalent variation, and with public spending consisting only of lump sum transfers. By virtue of Schöb (1994) result, our estimates indicate priorities for the reform of African tax structures.

3 The Model

Our model is formally set out in Appendix 1. Four goods are consumed in the economy: *leisure* (Z), *untaxed* (U), *domestic* (D), and *imports* (M). Investment occupies a large part of the economy in reality, so it needs to be incorporated somehow, although it serves no role in a static model. This is achieved in the model by creating an additional good, *investment* (I). The representative consumer derives utility by purchasing investment, which can be thought of as deriving utility from future consumption. We set preferences as Cobb-Douglas over investment and all other goods, meaning that consumers devote a constant proportion of their incomes to each of them. The representative consumer has endowments of leisure (which may be converted into labour), capital, and foreign exchange, which is used to purchase imports from the rest of the world.[6]

On the production side of the economy, three final goods are produced in the country: *untaxed* (U), *domestic* (D), and *exports* (E). Exports are not consumed directly. Instead they are used to purchase foreign exchange (at a constant exchange rate), which is used to purchase imports from the rest of the world. Firms receive funds from sales of their production and from investment. These funds are spent on factors of production. Thus the production sectors are modelled as using factor inputs, to produce jointly a final good (*untaxed*, *domestic* or *export*) together with *investment*. Sector output is divided between the good and investment according to a constant elasticity of transformation function. By setting the elasticity, denoted τ, equal to one (i.e., Cobb-Douglas), a constant proportion of each production sector's total income is derived from investment.

The production goods use four factors of production: *formal capital* (K^f), *informal capital* (K^i), *formal labour* (L^f) and *informal labour* (L^i).[7] One consequence of including informal factors is that capital taxation results in a deadweight loss, even though the total supply of capital is fixed. Labour taxation also induces a deadweight loss as a result of substitution between formal and informal labour, in addition to the deadweight loss arising from substitution between labour and leisure. It is assumed that production is competitive, so that all funds received by firms are paid out to factors. This assumption is consistent with empirical evidences on return to scale in African industries.[8]

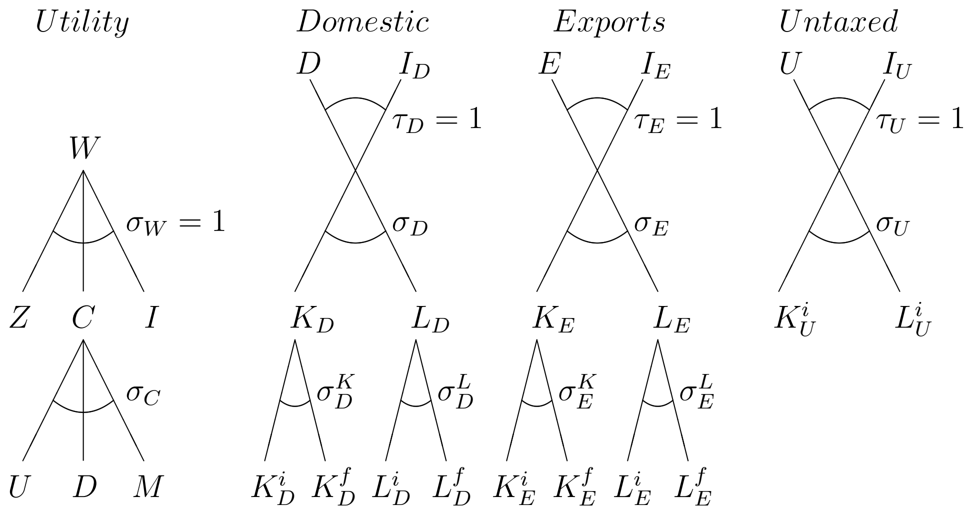
Constant elasticity of substitution (CES) functions are used in all production and utility functions. The structure of these functions is set out in Figure 1. The first panel is the utility function, where utility (W) is a Cobb-Douglas function of *leisure*, *investment*, and a consumption good C, and C is a composite good produced by a CES function of *untaxed*, *domestic*, and

[6] The endowment of foreign exchange represents the trade balance. In Africa, this is financed by borrowing and foreign aid. In a static model borrowing has no purpose, so the endowment can be thought of as foreign aid.

[7] The *informal* good uses only *informal capital* and *informal labour*.

[8] Tybout (2000) who surveyed the empirical literature on production in developing countries, reports returns to scale very close to unity in all the industries covered by the literature (including India, Indonesia and Africa). Regarding Africa he writes: "*Using firm-level African data collected by the Regional Program on Enterprise Development (RPED), Biggs, Shah, and Srivasava (1995) fit the same estimator to four manufacturing sectors in Ghana, Kenya, and Zimbabwe. Interestingly, even when the sample is limited to firms with three to twenty workers, they estimate returns to scale very close to unity.*" [...] "*when the entire stratified sample is used for each industry (covering the entire size spectrum), returns to scale are still close to unity in food and textiles/garments, while mild increasing returns are found in wood products and metal products.*"

Figure 1: Utility and Production Functions



imports. The remaining panels are the production functions for *domestic*, *export* and *untaxed*. The bottom parts of these diagrams represent the nested CES functions over the factors of production, with the top levels representing capital/labour substitution, and the bottom levels representing formal/informal substitution. The top parts of the diagrams represent the Cobb-Douglas transformation functions between the final good and investment. In the base case all the elasticities are set to 1 so that the functions are Cobb-Douglas. The relevant elasticities, used in robustness testing of the model, are labelled by sigma and appropriate sub- and super-scripts.

On the public side of the economy taxes are imposed on *domestic*, *exports*, *imports*, *formal capital*, and *formal labour*. There are no untaxed traded exports or imports. This is not meant to imply that no smuggling occurs in African countries. Rather, the official figures for trade are based on customs data, which typically reflects taxed goods. An implication is that the *untaxed* good is produced and consumed purely domestically.

Tax revenue received by the Government is transferred lump-sum to consumers. The experiment of increasing a distortionary tax rate and returning the revenue lump-sum can be interpreted in terms of both 'differential' and 'balanced-budget' analysis. But as emphasized in section 2, for purposes of MCF estimates the realism of the modelled public expenditure is not as important as the fact that a consistent experiment is conducted across tax instruments.

3.1 Data

The data are given in Appendix 2. Country-specific data comprise values of exports, imports and investment as a percentage of GDP, tax revenues for each of the five taxes, and legal tax rates for domestic goods, capital and labour. These data were obtained from IMF country report statistical Annexes.

Data for tax revenue from sales of *domestic* are derived from tax revenues from domestic VATs. In the absence of a VAT, general sales tax revenues are used. Corporate income tax revenues are interpreted as tax revenues from *formal capital*. Personal income tax revenues are equated with revenues from *formal labour*. Data for tax revenues from *exports* and *imports* are taken directly from the national accounts.[9] We ignore classes of tax revenue that do not fall into any of the five tax revenue classes of the model. For the countries in our sample, such other taxes represent on average 10% of total tax revenue. We assume that these other tax revenues are unaffected by shocks to the model's five tax rates, implicitly treating them as lump-sum taxes.

Our model has only a single tax instrument for each good or factor. This rate is treated as both the average and the marginal tax rate. The inclusion of an untaxed domestic good and informal factors allows us to assume that the average tax rates for *domestic*, *formal capital* and *formal labour* can be equated with the legal tax rates: either the legal tax bill is paid entirely or it is not paid at all.

For *domestic*, the legal tax rate used is the standard VAT or sales tax rate on domestic goods. In countries without a VAT, tax rates on intermediate goods cascade in the taxed price of final output. In such cases, the tax rate applied in the model is likely to be an underestimate of the marginal tax rate on many domestic goods.

Progressive personal income tax schedules provide multiple legal marginal tax rates for the tax on *formal labour*. Where the average public sector salary is known or can be calculated, this salary is treated as a proxy for the average wage paid to formal labour, and is used to determine the relevant marginal

[9]The countries of the Southern African Customs Union (Botswana, Lesotho, Namibia, South Africa and Swaziland) share imports tax revenue according to a formula that gives a reduced share of the revenue to South Africa. Our calibration method is inaccurate to the extent that the formula differs from the share of regional imports that is consumed by each country.

tax rate. If the average public sector salary is not known a marginal rate of income tax from the middle of the legally specified tax rates is chosen. To this rate is added the rate of any payroll tax. The standard rate of corporate income tax is used for the tax on *formal capital*.

For taxes on *exports* and *imports*, there are usually multiple legal tax rates, and there is no simple method of choosing between them to provide a single tax rate for the model. Instead we used average tax rates calculated as tax revenue divided by the tax base. To the extent that informal traded goods are represented in the official statistics for exports and imports, this procedure underestimates the average of multiple marginal tax rates.

Finally the data also include labour-output ratios. These labour-output ratios are, by assumption, common for all countries. They are the averages of ratios derived from social accounting matrices (SAMs) prepared by IFPRI for Malawi 1998, South Africa 1999, Tanzania 2001, Zambia 1995, and Zimbabwe 1991. The data are set out in Appendix 2, and the average ratios are: 37% for *untaxed*, 43% for *domestic* and 52% for *exports*.

3.2 Model Calibration

The economic relationships in the model can be represented by a rectangular SAM such as Table 2. The entries in the SAM are expressed as percentages of GDP at market value. All rows and columns sum to zero, reflecting a Walrasian equilibrium in which incomes equal expenditures. In the consumer's column positive entries are endowments or factor incomes, negative figures are expenditures on goods, including investment. In the production columns, positive entries are the receipt of sales revenue or investment, and negative entries are payments to factors or factor taxes. In the government's column, positive figures are tax revenues, the negative figure is the transfer to consumers. The 'Foreign' column represents the purchase of exports and the sale of imports by the rest of the world, using foreign exchange.

The calibration process is formally described in Appendix 3. It starts with a standard definition of GDP, the sum of consumption, investment and net exports: $GDP = C + I + (E - M)$. The figures for C and I incorporate government consumption and investment. This definition of GDP corresponds to the format usually given in IMF country reports. In our model the goods that are consumed (ignoring leisure), can be divided between *imports* (M) and non-imports (N), where non-imports comprise *domestic* and *untaxed*: $C = N + M$. Using the definition of GDP we calculate the amount of non-

Table 2: SAM for Guinea-Bissau 2001

	Consumer	Untaxed	Domestic	Exports	Imports	Foreign	Govt
Untaxed	-48.20	48.20					
Domestic	-5.67		4.93				**0.74**
Exports				24.60		**-27.21**	**2.61**
Imports	**-57.71**				53.41		**4.30**
Investment	**-18.92**	11.73	1.20	5.99			
Foreign Exchange	26.20				-53.40	27.20	
Formal Capital	2.36		-0.39	-1.97			
Informal Capital	48.36	-28.85	-2.93	-16.58			
Formal Labour	9.45		-1.58	-7.87			
Informal Labour	34.61	-31.08	-0.92	-2.61			
Capital Taxes			-0.15	-0.77			**0.92**
Labour Taxes			-0.16	-0.79			**0.95**
Transfers	9.51						-9.51

Figures represent a percentage of GDP at market value. Positive figures are revenues received by the column from the row. Negative figures are payments made by the column to the row. Figures in bold are taken directly from national accounts, all others are calculated using the calibration process.

imports consumed as $N = GDP - I - E$. Consumption of non-imports is equal to production of non-imports. Production of *domestic*, the taxed non-imported good in the model, is calculated by dividing VAT revenue by the tax rate, giving the tax base: $D = \frac{R_D}{T_D}$. Consumer expenditure on the *untaxed* good (equal to the value of production) is thus the residual of non-imports: $U = N - (1 + T_D) \times D$. This provides a first measure of the size of the informal economy in countries where such estimate does not exist.

The calibration process can be illustrated with data from Guinea-Bissau. All quantities in the SAM of Table 2 can be derived from the data described in section 3.1. The value of *investment*, as a percentage of GDP at market prices, available directly from the national accounts, is: $I = 18.9$. The producer value of exports is derived by subtracting export tax revenue from the market value of exports: $E = 27.21 - 2.61 = 24.6$. The producer value of *domestic* is determined by dividing tax revenue for domestic goods (0.74% of GDP) by the standard sales tax rate (15%): $D = \frac{0.74}{0.15} = 4.93$. The value of *untaxed* is then the residual of GDP less investment and the taxed value of *domestic* and *exports*, all expressed as a percentage of GDP: $U = 100 - (D + 0.74) - (E + 2.61) - I = 48.20$. For estimates of the size of the informal economy in all the countries see Table 11 in the Appendix.

Total funds received by the firms in a sector is the sum of sales and investment. For example, firms producing exports receive $E + I_E = 24.60 + 5.99 = 30.59$. Since production is assumed to be competitive, all funds received by firms are paid out to factors. Capital and labour requirements for production are determined using the average labour-output ratios of Appendix 2. Thus, for example, with a labour-output ratio of 0.37 for exports, total capital used to produce exports is $K_E = (1 - 0.37) \times (E + I_E) = 18.55$. Since *untaxed* uses only *informal capital* and *informal labour*, this procedure is all that is required to determine the allocation of capital and labour to *untaxed*.

For *domestic* and *exports*, factor usage must be divided between formal and informal factors. The amounts of formal capital and labour are determined by dividing tax receipts by the legal tax rate. With a 39% corporations tax, total *formal capital* is $K^f = \frac{R_K}{T_K} = \frac{0.92}{0.39} = 2.36$. Applying a personal income tax rate of 10% implies total *formal labour* of $L^f = \frac{0.945}{0.1} = 9.45$. These amounts are divided between *domestic* and *exports* in proportion to output. For example, *formal capital* in *exports* is $K^f_E = K^f \times \frac{E}{E+D} = 2.36 \times \frac{24.60}{24.60+4.93} = 1.97$. Informal capital and labour are then the residuals of total capital and labour for each industry less formal capital and labour and the taxes paid on these factors. For example *informal capital* used to produce *exports* is $K^i_E = K_E - K^f_E \times (1 + t_K) = 18.55 - 1.97 \times (1 + 0.39) = 16.58$.

The SAM of Table 2 does not show *leisure*. The quantity of leisure is determined by the representative consumer's endowment of time less the sum of formal and informal labour. Since the utility function is Cobb-Douglas, the elasticity of labour supply is $\eta = \frac{T-L}{L}$ where T is the time endowment and L is total labour supply. The value of L is fixed by the data – in the case of Guinea-Bissau $L = L^i + L^f = 9.45 + 34.61 = 44.06$. The calibration approach is to choose a value for the labour supply elasticity, η, which then fixes the value of T. There appear to be very few published estimates of the elasticity of labour supply in developing countries. For rich countries, estimates of the elasticity of labour supply are close to zero, when estimated for those who are employed. Nevertheless at the extensive margin, the decision whether or not to take a job, the elasticity of labour supply is positive. For our developing countries, we felt that some small responsiveness of labour supply to changes in taxation was appropriate, and thus chose an elasticity of $\eta = 0.05$, implying that $T = 1.05 \times L = 46.263$.

National accounts figures represent values: price $\times$ quantity. Following the Harberger convention, units of aggregate goods are chosen such that quantities equal values. This implies that for untaxed goods or factors, benchmark prices are equal to one. For taxed goods and factors, we choose quantity units such that either the net of tax price or the gross of tax price is equal to one at the benchmark equilibrium. Specification of the tax rates allows calculation of the missing prices.

With all benchmark prices and quantities determined, calibration finishes with the selection of elasticities for the CES utility and production functions. Our base case uses unitary elasticities: constant returns to scale Cobb-Douglas functions. This appears to be reasonable for the elasticity of demand for imports (see Senhadji (1998)). But we do not have evidence on the empirical magnitude of other necessary elasticities. Faced with a data vacuum we preferred a simple functional specification, and performed sensitivity checks by changing elasticities. Using the quantities in the SAM, and the assumed elasticities, the production and utility functions are calibrated. The same methodology is applied for all 38 African countries.

3.3 Discussion

While this account of the calibration process is straight-forward, it glosses over several problems with the calibration of informal sectors. It is likely that these problems affect all countries to some degree, but the issue is particularly acute in eight countries where, using the procedure described above, the calibrated values of sections of the informal economy are negative. The *untaxed* good is negative in Equatorial Guinea (-74% of GDP), Gabon (-2 %), Mauritania (-3%), Namibia (-22%), and Swaziland (-3%). In addition, the calibration process gives negative values for *informal labour* used to produce *domestic* and *exports* in Cape Verde (-5%), South Africa (-13%) and Zimbabwe (-13%) (see Table 11 in the Appendix).

The first problem is a general question about the reliability of GDP data in small developing economies. Statisticians dealing with developing economies make efforts to incorporate the informal economy, but to the extent that some informal activities escape the measure of GDP, our calibrated informal sectors are too small. Data reliability may affect results for all countries, but it does not give rise to negative calibrated values for *untaxed* goods.

The second problem arises in countries with considerable re-exportation of imports. The problem can be seen when we consider the definition $GDP = C + I + (E - M)$. Suppose that initially there are no re-exports, and then an additional \$100 of goods is imported and immediately re-exported without any value-added. The GDP figure, which is the sum of value-added, is unchanged. But the increase in measured exports reduces our measure of non-imports by \$100, since $N = GDP - I - E$. With the size of *domestic* calibrated independently, any re-export of goods results in a corresponding diminution of the *untaxed* sector: $U = N - D$. With sufficient re-exportation, the *untaxed* sector becomes negative. An extreme case is Equatorial Guinea, where exports constitute 106% of GDP. For most countries it seems likely that re-exports constitute a sufficiently small part of total exports that the calibration method is not seriously compromised. Unfortunately we did not have the necessary data to eliminate the problem by separating total exports between those of domestic origin and those of foreign origin.

The final problem in calibrating *untaxed* arises from the method of calculating *domestic*, by dividing sales tax revenue by the tax rate. Where the sales tax is a VAT, with full rebating of tax paid on inputs and a single rate, the calibration method probably gives a fairly accurate measure of the *domestic* tax base. But if sales taxes cascade, without rebates for taxes paid on inputs, the effective marginal tax rate can be much higher than the standard sales tax rate. In such cases, the tax rate used for calibration purposes is too low, so the estimated *domestic* tax base is too large. Then, noting that $U = N - D$, when the estimated size of D is too large, the estimated size of the *untaxed* sector is too small. The fact that Namibia and Swaziland do not have VATs may help to understand the small calibrated values of their *untaxed* sectors. The further a country's sales tax system departs from a 'pure' VAT (a single tax rate, no exemptions, and full refunds for taxed inputs), the less accurate is the calibration methodology for identifying the *domestic* tax base.

Similar issues arise in calibrating informal factor supplies. If the assumed labour-output ratios are not sufficiently large, the total labour supply is less than the calibrated value of *formal labour*. If the personal income tax rate used for calibration is too small, the calibrated value of *formal labour* is too large. In principle the same problems could arise in calibrating *informal capital*, but in practice the problem did not arise.

Our solution is pragmatic. The basic calibration methodology suggests 'small' values for the *untaxed* good and *informal labour* used to produce taxed goods. At the same time, strictly positive values are required for the experiments conducted in section 5.1. For the five countries with negative *untaxed*, we set the value of *untaxed* at 1% of GDP.[10] For the three countries

[10]For these countries, the calibrated market value of the three produced goods thus exceeds 100% of GDP.

with negative *informal labour* requirements in *exports* and *domestic*, we adjusted the labour-output ratio to ensure that *informal labour* constituted 1% of *formal labour*. It should be emphasized that a process of adjustment of the values in the SAM is typical in the development of CGEs: real-world data rarely match the internal consistency requirements of a SAM. Nevertheless, our results should be interpreted with particular caution for the 'problem' countries, and for all countries lacking VATs (these are reported in the data appendix).

4 Results

We calculated the MCFs associated with six different shocks to tax rates. In the first five experiments we increased each tax rate individually by 1% of the existing tax rate.[11] In the sixth experiment we increased all five tax rates simultaneously by 1%. In each case the additional tax revenue, ΔR, was redistributed to consumers as a lump sum transfer. The new equilibrium was established using a computable general equilibrium model written using GAMS MPSGE. The welfare change induced by the combined tax and spend experiment was measured in terms of the numeraire using the equivalent variation, denoted EV.[12] The MCF of the experiment was calculated as

$$MCF = \frac{EV}{\Delta R}. \tag{2}$$

4.1 Base Case Estimates

The resulting MCF estimates are presented in Table 3. The estimates provide a basic blueprint for tax reform in each country. For any pair of tax instruments, the same total revenue could be achieved for lower deadweight loss by lowering tax rates associated with a high MCF and increasing low-MCF tax rates.

[11] Where the tax rate was 0% we increased it to 1%.

[12] In theoretical papers on the MCF changes in utility are converted to a monetary measure by dividing by the marginal utility of income. When the utility function is linearly homogeneous (as is the case in our model) the equivalent variation gives the same measure of welfare change. GAMS measures the change in welfare, ΔW_G, after the tax revenues has been redistributed lump sum to the representative consumer. Since taxation is distorting in general ΔW_G is negative. In order to get the value express in (2) we compute $EV = \Delta R - \Delta W_G$.

Table 3: MCF Estimates

Country	Domestic	Exports	Imports	Capital	Labour	All
Benin	1.11	1.28	1.15	1.68	1.60	1.23
Botswana	1.03	1.02	1.03	1.07	1.05	1.05
Burkina Faso	1.15	-10.82	1.18	1.55	1.55	1.25
Burundi	1.10	0.76	1.10	1.66	1.88	1.22
Cameroon	1.10	1.08	1.07	1.53	1.27	1.14
Cape Verde	1.11	1.96	1.21	1.72	1.79	1.37
Central African Rep.	1.14	1.31	1.14	1.62	1.71	1.23
Chad	1.19	1.26	1.09	1.87	2.01	1.33
Dem. Rep. of Congo	1.01	1.02	1.01	1.43	1.38	1.10
Côte d'Ivoire	1.05	1.06	1.06	1.43	1.36	1.12
Eq. Guinea	1.05	1.00	1.00	1.24	1.10	1.13
Eritrea	1.02	0.63	1.03	1.20	1.14	1.09
Ethiopia	1.13	3.14	1.23	1.75	1.60	1.31
Gabon	1.03	1.02	1.05	1.39	1.35	1.10
Gambia	0.98	1.12	1.08	1.45	1.21	1.12
Ghana	1.03	1.17	1.10	1.50	1.26	1.17
Guinea	1.11	1.01	1.00	1.49	1.43	1.12
Guinea-Bissau	1.09	1.35	1.13	2.03	1.49	1.26
Kenya	1.02	1.20	1.06	1.30	1.11	1.08
Madagascar	1.13	1.16	1.11	1.58	1.32	1.17
Malawi	1.20	1.04	1.01	1.39	1.43	1.23
Mali	1.11	1.27	1.14	1.66	1.64	1.21
Mauritania	1.08	0.97	0.98	1.24	1.22	1.10
Mozambique	1.04	1.17	1.07	1.60	1.22	1.11
Namibia	0.97	1.10	1.06	1.39	1.09	1.10
Niger	1.17	1.29	1.15	1.90	1.80	1.24
Nigeria	0.99	1.02	1.02	1.30	1.19	1.08
Rwanda	1.14	-92.74	1.14	1.87	1.80	1.28
São Tomé	1.27	1.21	1.07	1.54	1.31	1.15
Senegal	1.07	1.27	1.13	1.55	1.80	1.19
South Africa	1.09	1.02	1.00	1.29	1.11	1.12
Sudan	1.09	1.92	1.23	1.87	1.57	1.26
Swaziland	1.11	1.01	1.01	1.34	1.30	1.09
Tanzania	1.17	1.59	1.20	1.76	1.73	1.27
Togo	1.06	1.11	1.07	1.40	1.50	1.12
Uganda	1.10	0.82	0.97	1.40	1.30	1.11
Zambia	1.06	1.09	1.03	1.44	1.14	1.09
Zimbabwe	1.08	1.10	1.03	1.28	1.10	1.11
Average	1.09	-1.58	1.08	1.52	1.42	1.17
Max	1.27	3.14	1.23	2.03	2.01	1.37
Min	0.97	-92.74	0.97	1.07	1.05	1.05
Std. Dev.	0.06	15.32	0.07	0.22	0.26	0.08

On average, the MCF associated with a marginal increase in all five tax rates is 1.17, indicating a required rate of return of 17% for African public projects. Among individual tax instruments, on average the lowest MCFs are associated with taxes on the two taxed consumption goods (*domestic* and *imports*), and the highest MCFs are associated with the two taxed factors (*formal capital* and *formal labour*). The *exports* MCFs exhibit the greatest variation.

The high variance of *exports* MCFs requires comment. Much of the variation comes from two countries: Burkina Faso and Rwanda. The *exports* MCFs in these two countries are negative because the tax shock reduces tax revenue. The high absolute values occur because the exports tax rate shock induces a marginal change in total tax revenue that is close to zero.[13] Excluding these two countries, the remaining countries can be divided into two groups. There are 16 countries with zero exports taxes (see the data in Appendix 7), that as a group have a low average *exports* MCF of 1.07. The remaining 20 countries have positive exports taxes, and have an average *exports* MCF of 1.32. The relatively high MCF associated with positive exports taxes can be understood by noting that *exports* is effectively an input into *imports*, so that distortions introduced in the market for *exports* amplify distortions in the market for *imports*.

The general equilibrium interactions between tax instruments are illustrated in Figure 2 for the case of Guinea-Bissau. MCFs for different tax instruments are plotted as a function of the import tariff T_M. It shows that relying on partial equilibrium measures (i.e., focusing only on the import market) to compute a single MCF would be misleading. The diagram provides various insights into the interactions within the system.

- The *imports* MCF increases with the import tariff T_M.
- The MCFs for *exports*, *capital* and *labour* are greater than the *imports* and *domestic* MCFs, and they increase at a faster rate than the *imports* MCF. *Exports*, *capital* and *labour* are inputs into the production of the taxed consumption goods, *imports* and *domestic*, that enter directly into the consumer's welfare function. Increases in taxation of these inputs causes increased distortion of the taxed prices of the outputs, in addition to substitution of informal factors for formal factors.

[13] Although not apparent in the reported results, the exports tax shock also reduces total tax revenue in Burundi and Eritrea. In these countries, which have zero exports tax rates, the marginal reduction in tax revenue is associated with a positive *exports* MCF, because the marginal welfare change is positive.

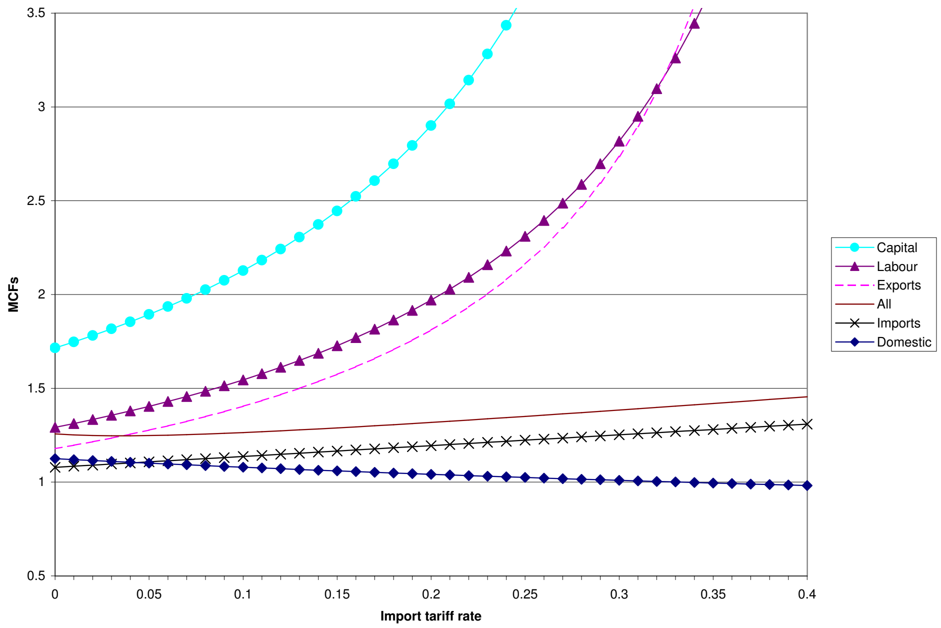


Figure 2: MCFs in Guinea-Bissau as a Function of the Import Tariff

- The *domestic* MCF decreases as T_M increases. When $T_M \geq 34\%$ the *domestic* MCF is less than one, indicating that marginally increasing the *domestic* tax helps to correct the distortions induced by high import tariffs, thereby increasing welfare. *Domestic* and *imports* are substitutes in consumption, so as *import* tariffs increase, consumption of *domestic* increases, helping to offset the effect of existing *domestic* taxation.

- Reflecting the conflicting effects of the individual taxes, the MCF obtained by simultaneously varying *all* tax rates decreases for values of $T_M < 4\%$ and increases for $T_M > 4\%$. One implication is that this measure of the MCF does not necessarily increase with total tax revenue. The illustrated values of T_M are on the 'good' side of the Laffer curve: total revenue is increasing in T_M.

The diagram also illustrates that over wide ranges of the import tariff rate the ordering of MCFs for individual taxes is unchanged. The next section explores this robustness result further.

4.2 Robustness of Results

Our results are dependent on the specified elasticity parameters. We experimented with alternative elasticities to see the effect on our estimates. The notation for the relevant elasticities is set out in Figure 1. In testing different elasticities we maintained the following values equal to one: σ_W, τ_D, τ_E, and τ_U. This ensures that the representative consumer invests a constant proportion of income, and that each production process receives a constant proportion of its total funds from investment.

We tested several combinations of elasticities, and the results are set out in Table 4. For each experiment, any unmentioned elasticity is equal to 1. The first five experiments increase elasticities. The first experiment increases the consumer's elasticity of substitution between consumption goods. The second increases the elasticity of substitution between capital and labour in all goods. The third increases the elasticity of substitution between consumption goods and between capital and labour. The fourth increases substitution between formal and informal factors. The fifth increases substitution between formal and informal labour, but leaves unchanged the degree of substitution between formal and informal capital. The final five experiments perform corresponding reductions in the elasticities of substitution.

Changing elasticities had the expected effect on the magnitude of our MCF estimates: higher substitutability results in higher MCFs. The MCF associated with a simultaneous shock of all five taxes, $MCF(All)$, varied between 1.11 and 1.32. $MCF(All)$ is more sensitive to variation in the consumer's elasticity of substitution between consumption goods than the producers' elasticity of substitution between capital and labour. On the production side, MCF estimates are more sensitive to changes in the elasticity of substitution between between formal and informal factors than the elasticity of substitution between capital and labour. It is clearly important to have a good estimate of the relevant elasticities if MCF estimates are to be used for cost-benefit analysis.

But for partial revenue-neutral tax reform, it is the ordering of MCF estimates that is important, not their magnitudes. For such reform, the aim is to compare two tax instruments, increase the tax rate on the low MCF tax, and decrease the tax rate on the high MCF tax. Table 4 reports the number of countries in which the ordering of MCFs is robust to changes in elasticities. Three measures of robustness are used. The first measure supposes that reform is concentrated on the most extreme MCFs. The two tax instruments with the highest and lowest MCFs are identified. If these

Table 4: Sensitivity Testing

Experiment	D	E	M	K	L	All	(1)	(2)	(3)
$\sigma_C = 2$	1.19	1.88	1.17	1.93	1.89	1.31	29	25	35
$\sigma_D = \sigma_E = \sigma_U = 2$	1.09	-0.26	1.08	1.50	1.45	1.17	33	33	38
$\sigma_C = \sigma_D = \sigma_E = \sigma_U = 2$	1.19	2.02	1.18	1.93	1.98	1.32	29	25	36
$\sigma_D^K = \sigma_D^L = \sigma_E^K = \sigma_E^L = 2$	1.09	-1.58	1.08	2.53	1.89	1.24	35	34	37
$\sigma_D^L = \sigma_E^L = 2$	1.09	-1.58	1.08	1.52	1.89	1.19	22	20	37
$\sigma_C = 0.5$	1.05	1.15	1.05	1.40	1.29	1.11	35	32	35
$\sigma_D = \sigma_E = \sigma_U = 0.5$	1.09	1.13	1.08	1.52	1.39	1.17	36	34	37
$\sigma_C = \sigma_D = \sigma_E = \sigma_U = 0.5$	1.05	1.14	1.04	1.41	1.28	1.11	34	32	35
$\sigma_D^K = \sigma_D^L = \sigma_E^K = \sigma_E^L = 0.5$	1.09	-1.58	1.08	1.29	1.28	1.14	31	28	37
$\sigma_D^L = \sigma_E^L = 0.5$	1.09	-1.58	1.08	1.52	1.28	1.16	31	27	38

Columns D, E, M, K, L, and All report average MCFs for the 38 countries. Columns (1), (2) and (3) indicate the number of countries that are 'close' to the base case estimates in the sense of giving the same recommendations for tax reform. In column (1) the tax instruments found to have the highest and lowest MCFs are the same as in the base case. In column (2) all five individual MCFs are ranked in the same order as in the base case. In column (3) at most one pairwise comparison of MCFs gives a different recommendation from the base case estimates.

two instruments are the same as in the base case, the ordering is considered robust. Among the five tax instruments there are ten pairwise comparisons that are possible, identifying for each pair the high and low MCF instruments. The second measure of robustness requires that all ten such comparisons report the same pairwise ordering. This implies that all five instruments follow the same order as in the base case. The third measure of robustness permits just one pairwise comparison to give a different ordering from the base case. On the basis of the first measure the ordering of MCF estimates was robust to variations in elasticities in a minimum of 22 out of 38 countries. On the basis of the third measure it is 35 out of 38 countries. That is, we can be reasonably confident of the recommendations for tax reform that are implied by our base case estimates.

5 Tax Reform in Africa

Since on average the lowest MCFs are associated with the two taxed consumption goods (*domestic* and *imports*), and the highest MCFs are associated with the two taxed factors (*formal capital* and *formal labour*), the dead-weight loss of taxation could be lowered by increasing goods taxation and decreasing factor taxation. However this result is partial because it overlooks the possibility to conduct structural reforms to enlarge the taxation base. Analysis of African tax systems quickly confronts the significance of informal sectors. Excluding countries in which our basic calibration method gave a negative *untaxed* good, the value of *untaxed* production represents on average 30% of GDP, with a maximum of 61% in Niger. Adding in the value of untaxed factors used to produce *domestic* and *exports*, the average informal economy in these countries represents 69% of GDP, with a maximum of 90% in the Democratic Republic of Congo (see Table 11 in the Appendix).

In this section we examine the impact of the informal economy on the MCF in the formal economy, and then measure MCFs of potential taxes in different parts of the informal economy, ignoring any administrative costs. We then examine how the presence of administrative costs affects our estimates for taxes in the formal economy, and we consider the level of administrative costs that could be justified to impose taxes in the informal economy.

5.1 The Informal Economy

At the heart of the dead-weight loss of taxation is the substitution effect. It is presumably higher when it is easier to escape taxation. This suggests that economies with large informal sectors are likely to have high MCFs. Figure 3 plots our base case estimates of $MCF(All)$ against the calibrated value of *untaxed*, suggesting the strength of this intuition. A simple regression of our 38 estimates of $MCF(All)$ on *untaxed* and a constant has an adjusted R^2 of 0.75. Some of the unexplained variation in MCF values in the diagram is due to variation in tax structures. To capture this in a simple way, we created a variable, *structure*, which measured the proportion of exports and factor tax revenue in total tax revenue (using the same figures used for calibration of the model). Adding this variable to the regression we obtained:

$$\begin{array}{llllll} MCF(All) = & \underset{(0.024)}{1.024} & + & \underset{(0.036)}{0.402} & Untaxed + & \underset{(0.049)}{0.116} \; Structure \end{array}$$

The adjusted R^2 of this regression is 0.78. Standard errors are reported in brackets, with the coefficients on the constant and *untaxed* significant at 1% and the coefficient on *structure* significant at 2.5%. In contrast a regression of $MCF(All)$ on just *structure* and a constant gives no statistically significant results. These results suggest that, at least for the African countries we have studied, the magnitude of an economy's average MCF is dominated by the size of the informal economy, with the tax structure playing only a relatively

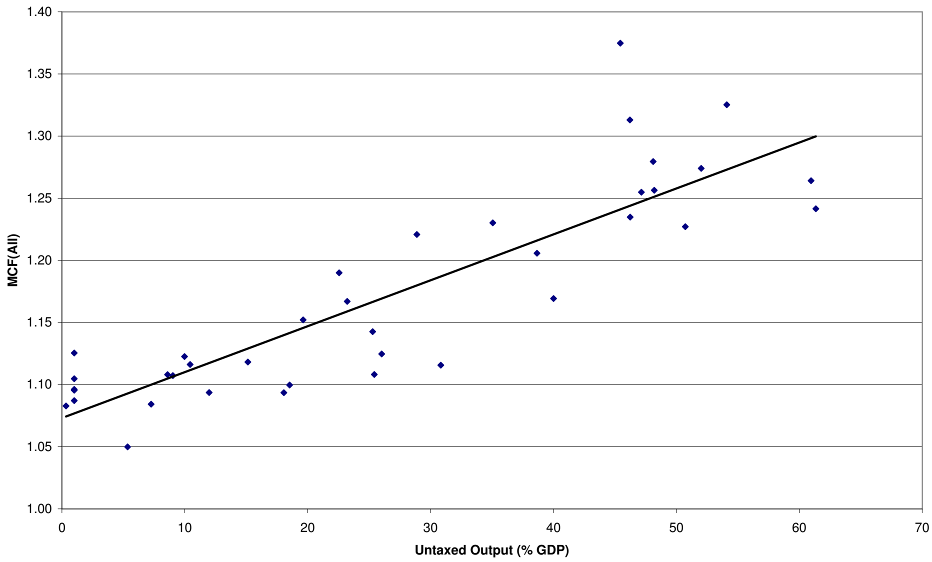


Figure 3: MCF(All) as a Function of the Value of the Untaxed Good

small role.

Intuitively, goods that are more lightly taxed than goods in the rest of the economy will tend to have low MCFs, since increasing their taxes will shift relative prices towards the undistorted ratios. Thus, taxes imposed on parts of the informal economy should have very low MCFs. We confirmed this intuition by imposing hypothetical taxes on informal parts of the economy in our model. We supposed it was possible to impose a tax in these sectors, and marginally increased the tax from an initial rate of zero. We calculated MCFs when 1% taxes were imposed on: production of the *untaxed* good (U); *informal capital* used anywhere in the economy (K^i); *informal capital* used to produce *domestic* or *export* goods (K^i_{DE}); *informal labour* used anywhere in the economy (L^i); and *informal labour* used to produce *domestic* or *export* goods (L^i_{DE}). We distinguished between taxing a factor wherever it is used and taxing a factor when it is used to produce *domestic* or *export* goods because the latter seems more plausible. It seems more likely that the administration will be able to tax a firm's accounting profits (returns to capital) and labour inputs in cases where the firm's output is already taxed. The results of these experiments are reported in Table 5.

Table 5: MCFs in the Informal Sector

Country	U	K^i	K^i_{DE}	L^i	L^i_{DE}
Benin	0.87	0.95	1.07	0.94	1.12
Botswana	0.86	0.90	0.91	0.92	0.95
Burkina Faso	0.89	0.94	1.21	0.92	1.19
Burundi	0.83	0.95	1.05	0.87	0.99
Cameroon	0.89	0.97	1.02	0.97	1.06
Cape Verde	0.81	0.79	1.22	0.80	1.26
Central African Republic	0.92	0.98	1.15	0.97	1.16
Chad	0.93	0.97	1.12	0.97	1.15
Democratic Republic of Congo	0.97	0.99	1.00	0.99	1.00
Côte d'Ivoire	0.83	0.95	0.98	0.95	1.00
Equatorial Guinea	0.98	0.98	0.98	1.00	1.00
Eritrea	0.87	0.88	0.92	0.93	0.99
Ethiopia	0.87	0.89	1.17	0.92	1.21
Gabon	0.85	0.97	0.97	0.99	0.99
Gambia	0.84	0.95	0.99	0.93	1.01
Ghana	0.87	0.95	1.03	0.92	1.04
Guinea	0.93	1.00	1.05	0.98	1.06
Guinea-Bissau	0.89	0.97	1.22	0.92	1.26
Kenya	0.81	0.92	0.94	0.87	0.93
Madagascar	0.91	0.97	1.07	0.96	1.11
Malawi	0.96	0.98	1.03	0.98	1.05
Mali	0.89	0.96	1.10	0.96	1.15
Mauritania	0.89	0.96	0.97	0.94	0.95
Mozambique	0.86	0.98	1.02	0.95	1.03
Namibia	0.79	0.92	0.92	0.85	0.86
Niger	0.94	0.98	1.16	0.98	1.19
Nigeria	0.92	0.98	0.98	0.97	0.97
Rwanda	0.90	0.94	1.14	0.96	1.18
São Tomé	0.84	0.95	1.08	0.80	1.14
Senegal	0.83	0.95	1.03	0.93	1.05
South Africa	0.78	0.89	0.92	0.71	0.77
Sudan	0.95	0.97	1.18	1.00	1.21
Swaziland	0.78	0.96	0.96	0.89	0.89
Tanzania	0.91	0.98	1.21	0.93	1.21
Togo	0.89	0.96	1.02	0.98	1.05
Uganda	0.92	0.98	1.03	0.98	1.05
Zambia	0.82	0.97	1.00	0.81	0.92
Zimbabwe	0.76	0.88	0.91	0.69	0.76
Average	0.87	0.95	1.05	0.92	1.05
Max	0.98	1.00	1.22	1.00	1.26
Min	0.76	0.79	0.91	0.69	0.76
Std. Dev.	0.06	0.04	0.09	0.07	0.12

In all countries the MCF of a tax on the *untaxed* good is less than 1. The negative welfare shock suffered by households (before the revenue is returned lump-sum) is smaller than the increase in government revenue. Once lump-sum redistribution of the revenue occurs, households are better off than before the tax shock. Increasing taxes in the informal sector helps to counteract existing taxes in other sectors, moving relative prices towards their undistorted levels.

When taxation of informal factors is restricted to inputs to *domestic* and *export* goods, the MCF is higher than when informal factors are taxed wherever they are used: $MCF(K^i_{DE}) > MCF(K^i)$ and $MCF(L^i_{DE}) > MCF(L^i)$. Two effects operate. Taxing an untaxed factor helps to restore prices to their undistorted levels, tending to lower the MCF. When the untaxed factors are inputs to taxed goods, the relative costs of these goods rise, the distortion associated with goods taxation increases and the MCF tends to be higher. The second of these effects is relatively stronger when the tax shock is concentrated on inputs to *domestic* and *export* goods.

Ignoring distributional issues, on average among the fictional tax instruments for taxing the informal sectors, the best way of raising money is by increasing taxes on *untaxed* goods. The average $MCF(U)$ is 0.87, lower than the average MCF elsewhere in the informal economy. But governments may hesitate to increase taxes of the informal sector if they care about distributional issues and poor households are concentrated in production of *untaxed* goods or they consume a lot of *untaxed* goods. When distributional issues are considered, a promising part of the informal economy is *informal capital* used to produce taxed goods: average $MCF(K^i_{DE})$ is 1.05. That is, imposing taxes on companies that produce taxed goods but do not pay company tax generally offers a lower cost of public funds than increasing existing taxes elsewhere in the formal economy. In many cases such companies have legal tax exemptions, which can be removed with low administrative expense.[14] Removing such exemptions has the potential for a low marginal cost of taxation, without obvious major effects on the poorest households. An alternative

[14]In a survey of 197 businesses in Cameroon, Gauthier & Gersovitz (1997) report that 4 were legally exempt from sales tax, while 30 were legally exempt from the business profits tax. In Gauthier & Reinikka (2001) a similar survey of 158 businesses in Uganda reports 17 exemptions from sales tax and 41 exemptions from the corporate income tax. Both studies found that exemptions tended to be granted to large firms, while smaller firms were more likely to evade tax illegally. Legal tax exemptions may be the result of corruption. Fjeldstad (2002) reports that in the mid 1990s senior Tanzanian officials accepted bribes in return for tax exemptions: "within the Ministry of Finance, the *Revenue Department* went under the nickname of the 'Tax Exemption Department.' "

means of increasing capital taxation may be to encourage firms to move from the informal to the formal sector by lowering license fees and time costs for establishing formal enterprises (see Auriol & Warlters (2005)).

5.2 Costs of Tax Administration

The preceding section shows that large informal sector are associated with inefficiency in taxation. If governments were able to enlarge their tax base, they could simultaneously increase tax collection and decrease the dead weight loss of taxation. One may wonder why governments in developing countries do not enlarge their taxation base. One plausible explanation is because of high administration costs. There is indeed evidence of a relatively high cost of tax administration in Africa. Table 6 provides an international comparison of administrative costs, measured by dividing the expenses of tax collection agencies by the revenue collected. The average for rich countries is 1.36%, 1.88% for Latin American countries, and 2.35% for African countries.[15]

A taxpayer paying a dollar of taxes suffers the same loss of utility regardless of whether the administration has paid 2 cents or 50 cents to enforce the collection. Further, the administration costs are not lost to society. They are paid to civil servants and other providers of goods and services. Thus, tax administration costs do not alter ΔW in our MCF formula (1).[16] Administration costs do, however, alter ΔR, by reducing the net revenue available for government spending. If we suppose that administration costs constitute $\mu\%$ of tax revenue collected, a tax shock that changes gross revenue by ΔR changes net revenue by $\Delta R(1-\mu)\%$. Incorporating administrative costs in our MCF estimates is thus a simple matter of multiplying our existing estimates by $\frac{1}{1-\mu}$.

Table 6 indicates that on average $\mu = 2.35\%$ in Africa. On this evidence, our base case results should be multiplied by 1.024. Incorporating administration costs increases the average $MCF(All)$ for African countries from 1.17 to 1.20. Although Africa has more costly tax administrations than other regions, this alone is unlikely to result in substantially higher marginal costs of public funds.

[15]Not all of the tax agencies represented in these figures collect all types of taxes. Since the costs of collection vary by tax type, this may give a distorted impression of the efficiency of some agencies.

[16]We treat as negligibly small the marginal change in consumer surplus forgone on goods that could have been produced using the factors of production involved in tax administration.

Table 6: Tax Administration Costs

Country	Year(s)	Cost/Collections
Australia	2001-2002	1.2%
Canada	2001-2002	2.3%
New Zealand	2001-2002	1.2%
UK	2001-2002	1.6%
US	2002	0.5%
Guatemala	1999-2001	1.9%
Mexico	1995, 1997-98	1.7%
Peru	1996-1998	1.9%
Venezuela	1995-1998	2.0%
Ghana	1993	2.8%
Kenya	1995-2000	1.2%
Namibia	2001-2002	1.3%
South Africa	1998-2001	1.1%
Tanzania	1996-1997	3.0%
Uganda	1991-2000	3.6%

The cost/collections ratio reports the annual cost of tax collection agencies divided by the amount of money collected. For data sources, see Appendix 7.

One explanation for the failure to tax the informal sector may be large administrative costs associated with taxing the first marginal unit of the tax base. We do not have data on the magnitude of administrative costs for the informal sector (μ^i). However, noting that in an optimized tax system the MCF of all tax instruments would be equal, we can calculate the μ^i that would equate the MCF of a tax in the informal sector, MCF^i, with the base case $MCF(All)$ when administrative costs are considered, according to the following formula:

$$\mu^i = 1 - \frac{MCF^i(1-\mu)}{MCF(All)} \tag{3}$$

Using μ=0.0235, we calculated the threshold administrative costs, μ^i for the parts of the informal economy discussed in section 5.1. Table 7 reports the averages across our sample of countries.

On average, increased efforts to enforce taxes on the *untaxed* good would be justified up to the point where administrative costs consume 27% of the revenue collected. This is more than 10 times the actual average cost of

Table 7: Administrative Cost Thresholds

U	K^i	K^i_{DE}	L^i	L^i_{DE}
0.27	0.21	0.13	0.23	0.12

collection. Taxing currently untaxed profits of businesses that produce taxed goods would be justified with administrative costs of 13%. These figures are far from negligible. They suggest that large benefits can be gained from a reform of the tax system. However in some cases they might not be large enough, at least in the short run, to cover the fixed cost of the reform. For example, there are high sunk costs associated with identifying taxpayers. In practice not only may administrative costs be higher for taxes on the informal sector than for taxes in other sectors, but the administrative cost function may exhibit non-convexity.

6 MCF Dispersion and Tax System Inefficiency

When only distorting taxes are available, the MCFs of optimal taxes are all equal. This suggests that MCF dispersion could be used as a measure of tax system inefficiency. We explore the relevance of this intuition in what follows. We leave aside informal sector tax reform. We focus on actual taxes. To assess the potential benefits of tax reform, we derived optimal taxes to achieve each country's existing revenue. We iteratively reduced high MCF taxes and increased low MCF taxes until MCFs were equalized or remaining high MCF taxes had zero rates. The resulting tax structure involves zero taxes on *exports*, *capital* and *labour*. The only taxes are on the taxed goods that are consumed: *domestic* and *imports*. This is an illustration of the Diamond & Mirrlees (1971) production efficiency result. We calculated the welfare gain from moving to optimal taxes (subject to the limited set of distorting tax instruments), and calculated the resulting MCFs of the taxes on *domestic* and *imports*. For comparison, we also measured the welfare gain from entirely removing all distorting taxes, or equivalently, only using lump-sum taxation.

The results are reported in Table 8. Countries are listed in order of the potential welfare gains from moving to optimal taxes to achieve the same tax revenue using only distorting taxes. The column 'ΔW (optimal)' reports these welfare gains in percentage terms, where the change in welfare is measured using the equivalent variation. The average potential welfare gain is

Table 8: Measures of Tax System Inefficiency (% of GDP)

Country	ΔW (optimal)	ΔW (zero taxes)	Std. Dev MCFs	MCF (optimal)	Revenue
Botswana	0.02	0.38	0.02	1.05	12.50
Eq. Guinea	0.11	0.11	0.10	1.00	3.42
Dem. Rep. Congo	0.12	0.13	0.21	1.01	2.57
Guinea	0.19	0.39	0.23	1.06	6.96
Madagascar	0.25	0.73	0.20	1.11	9.40
Uganda	0.26	0.41	0.24	1.04	8.88
Nigeria	0.28	0.33	0.13	1.01	6.45
Sudan	0.28	0.59	0.37	1.14	5.23
Malawi	0.29	0.33	0.19	1.03	3.78
Mozambique	0.29	0.50	0.22	1.05	11.18
Eritrea	0.29	0.38	0.22	1.02	13.96
Niger	0.30	0.69	0.36	1.14	6.69
Cameroon	0.33	0.75	0.20	1.08	10.43
Togo	0.34	0.66	0.21	1.06	11.30
Gabon	0.36	0.75	0.19	1.06	10.20
Mauritania	0.36	0.42	0.13	1.01	10.52
C.A.R.	0.37	0.72	0.27	1.11	7.75
Zambia	0.38	0.72	0.17	1.05	17.30
Tanzania	0.41	1.05	0.28	1.16	9.40
São Tomé	0.43	0.70	0.17	1.05	15.18
Kenya	0.43	0.76	0.11	1.04	19.48
Mali	0.43	0.96	0.27	1.12	11.19
Gambia	0.44	0.88	0.18	1.06	16.57
Zimbabwe	0.45	1.05	0.10	1.06	23.34
Burkina Faso	0.47	0.93	5.45	1.12	10.00
Ghana	0.48	0.79	0.18	1.06	11.86
South Africa	0.50	1.09	0.11	1.06	21.17
Côte d'Ivoire	0.52	0.98	0.19	1.06	15.74
Guinea-Bissau	0.56	0.78	0.38	1.07	9.51
Chad	0.58	0.78	0.42	1.08	6.22
Benin	0.59	1.27	0.26	1.12	12.90
Swaziland	0.60	0.86	0.15	1.03	22.69
Rwanda	0.63	1.03	42.14	1.11	9.66
Namibia	0.72	0.96	0.16	1.03	25.19
Ethiopia	0.75	1.47	0.81	1.14	13.28
Senegal	0.76	1.49	0.31	1.10	17.23
Cape Verde	1.12	1.79	0.38	1.12	18.32
Burundi	1.13	1.74	0.46	1.08	18.04
Average	0.44	0.80	1.48	1.07	12.25

worth 0.44% of GDP in money terms. This is not negligible in light of the low level of tax collection in African countries. For instance Chad suffers a dead-weight loss of 0.6 % of GDP for a collection level of 6.2% of GDP.

The column 'ΔW (zero taxes)' reports the total welfare cost of the tax system. On average, replacing all taxes by a lump sum tax (or equivalently in the model, eliminating all taxes), welfare could be increased by 0.8% of GDP. This suggests that more than half of the deadweight loss of African tax systems is due to the tax structure, rather than the revenue target.

The column 'Std. Dev. MCFs' reports the standard deviation of the base case MCF estimates for the five tax instruments. To examine the correlation between this measure of inefficiency and the potential welfare gains of optimal distortionary taxes we performed a simple regression of the form $\Delta W_i = aStDev_i + b$, where ΔW_i is the deadweight loss of current taxes relative to optimal taxation, $StDev_i$ is the standard deviation of base case MCFs, and a and b are coefficients to be estimated. The coefficient a was positive but the result was statistically insignificant. We re-ran the regression without the two outliers; Rwanda and Burkina Faso. The result was highly significant (figures in parentheses are t-statistics) with $R^2 = 0.27$:

$$\begin{array}{lcccc} \Delta W(optimal) = & 0.22 & + & 0.91 & StDev. \\ & (3.18) & & (3.61) & \end{array}$$

As expected high variance of MCFs indicates scope for welfare-enhancing revenue-neutral tax reform. We conclude that the standard deviation of MCFs is a good predictor of the potential benefits of tax reform.

The column 'MCF' reports the MCF associated with optimal taxation. The average figure is 1.07. This is well below the average figure for *MCF(All)* reported in the base case estimates (1.17). Tax reform can potentially lower the cost of public funds by a significant amount. If the MCF were actually used as a criterion for project approval, moving to optimal taxes would permit many more public projects to go ahead.

The 'Revenue' column reports tax revenue as a percentage of GDP. We explored the relationship between tax revenue and tax system inefficiency using simple linear regressions with tax revenue as the single explanatory variable. We found tax revenue to be also a significant determinant of the two deadweight loss measures.[17]

[17] Significant results are: $\Delta W(optimal) = 0.02(4.05)Revenue + 0.16(2.05)$, $R^2 = 0.31$;

Since they were both significant, we finally ran a regression with tax revenue and with standard deviation of the base case MCF estimates as explanatory variables.

$$\begin{array}{lccccccc} \Delta W(optimal) = & -0.15 & + & 0.027 & Revenue + & 1.06 & StDev \\ & (0.039) & & (0.000) & & (0.000) & \end{array}$$

The adjusted R^2 of this regression is 0.68. Standard errors are reported in brackets. The result shows that the taxation dead-weight loss increases with the taxation level and with the dispersion of MCFs. The result is highly significant. Revenue only explains part of the differences in the efficiency cost of taxation, even when the effects of tax structure are removed (by finding optimal imports and domestic taxes to achieve the given revenue). MCFs dispersion explain another part. Contrary to what is often assumed in the theoretical literature on regulation or in cost-benefit analysis, low MCFs do not necessarily signal an efficient tax system. In fact the opposite may be the case, as illustrated by our MCFs on the informal sector. The level of MCFs is not an indication of efficiency. On the other hand, the dispersion of MCFs is a valid signal of tax system efficiency.[18]

Collection levels in Africa are among the lowest in the world (see the 'Revenue' column in table 8). Because of the size of the African informal economy, we suspect that dispersion of their MCFs is among the widest in the world, though we lack data to compare with other regions. Taxation reforms should simultaneously aim at decreasing MCFs dispersion and increasing tax revenue. Based on the preceding analysis this should be performed preferably by targeting the informal economy.

7 Conclusion

Our results suggest that a reasonable estimate of the average MCF in Africa is 1.17 (the average of *MCF(All)* across the 38 countries). Among the various sensitivity tests that we conducted, this figure varied between 1.11 and 1.32. The estimate was most sensitive to changes in the consumer's elasticity of substitution between *imports*, *domestic* and *untaxed*. Senhadji's (1998)

and $\Delta W(zero\ taxes) = 0.04(4.26) Revenue + 0.31(2.42)$, $R^2 = 0.34$. Figures in parentheses are t-statistics.

[18] At the theoretical optimum there is one MCF equal across all tax instruments. This MCF increases with tax revenue.

estimates of the elasticity of demand for imports supports our assumption of unitary elasticity of substitution between *imports* and other goods. So the major uncertainty concerning our base case estimates is the elasticity of substitution between *domestic* and *untaxed*.

On average, taxes on factors have high MCFs and taxes on imports and domestic goods have low MCFs. This outcome follows from the Diamond & Mirrlees (1971) result for optimal taxation, that production decisions should not be distorted. A major focus of tax reform in recent years has been the introduction of VATs. Our results suggest that welfare could be improved by increased reliance on these VATs, and reduced reliance on exports and factor taxes.

An important finding is the strong relationship between the size of the untaxed sector and the value of $MCF(All)$ in the taxed sector. Moreover the existence of MCFs lower than one in the informal sector suggests scope for increasing welfare and tax revenue simultaneously. Measures to bring currently informal activities within the tax base would be justified even if a large proportion (up to 27%) of the additional revenue were consumed in enforcement and administration. Auriol & Warlters (2005) suggest that governments could reduce substantially the size of their informal sectors by reducing red tape barriers to business entry into the formal sector. Such a policy would not only help to enhance revenue by enlarging the tax base, but would also reduce the marginal costs of public funds.

An obvious question is whether MCFs in Africa are higher or lower than in rich countries. It is beyond the scope of this paper to estimate the MCF in rich countries,[19] but we suggest that differences in the MCF are probably not strongly related to the wealth of the country. Our results suggest that $MCF(All)$ tends to be greater with higher tax revenue, greater use of corporate and personal income taxes, higher administrative costs, larger informal sectors and greater use of export taxes. The first two of these factors tend to occur in rich countries, while the final three factors tend to occur in poor countries.

The potential extensions of this paper are numerous. Extensions to take account of the dynamic effects of taxation on savings, investment and growth;

[19]There are several difficulties in adapting our model to rich countries. For example, the European customs union must be modelled to examine import tariffs, and in the United States, state level taxes vastly complicate the analysis. Social security taxes also need to be taken into account. These difficulties are not insurmountable, but we leave them for future research.

or distributional considerations; or non-tax distortions such as labour-market rigidities or regulated prices[20] would all be useful. We hope, however, that in the first instance the model we have presented can be applied to further countries and across time periods, to provide a panel of MCF estimates. These estimates would not only be useful for public policy analysis, but also for the testing of numerous economic theories in which the MCF plays a role.

[20]Devarajan et al. (2001) have empirically shown that tax and non-tax distortions are cumulative. Taxes in sectors with less total distortions tend to have lower MCFs because they push resources towards highly-distorted sectors. The relative importance of tax and non-tax distortions is a subject for further research.

APPENDIX 1: Model Specification

The single representative consumer maximizes a CES utility function with five goods: *leisure* (Z), *untaxed* (U), *domestic* (D), *imports* (M), and *investment* (I) subject to the income constraint.

$$Max\ W = W(Z, U, D, M, I)\ subject\ to\ P_L Z + \tilde{p}_u U + \tilde{P}_D D + \tilde{P}_M M + \tilde{P}_I I \leq Y$$

A tilde over a price indicates that it is tax-inclusive:

$$\tilde{P}_j = (1 + T_j)P_j, \forall j \in \{D, E, M, K_D^f, K_E^f, L_D^f, L_E^f\}$$

Consumer income is the value of the endowments of foreign exchange ($\bar{a}$), time ($\bar{T}$), and capital ($\bar{K}$) plus the transfer received from the government (R).

$$Y = \bar{a} + P_L \bar{T} + P_K \bar{K} + R$$

Leisure plus labour supply equals the time endowment.

$$Z + L = \bar{T}$$

The consumer's first order conditions are:

$$\frac{\partial W/\partial Z}{P_L} = \frac{\partial W/\partial U}{\tilde{P}_U} = \frac{\partial W/\partial D}{\tilde{P}_D} = \frac{\partial W/\partial M}{\tilde{P}_M} = \frac{\partial W/\partial I}{P_I}$$

Factors are combined by CES production functions to produce intermediate goods for *untaxed* (ψ_U), *domestic* (ψ_D), and *exports* (ψ_E). The factors used are capital and labour, each of which may be formal (taxed) or informal (untaxed). The notation for factors is s_r^q : the amount of factor $s \in \{K, L\}$ used to produce good $r \in \{U, D, E\}$, where $q \in \{i, f\}$ indicates whether the factor is informal or formal.

$$\psi_U = \gamma_U(K_U^i, L_U^i)$$

$$\psi_D = \gamma_D(K_D^i, K_D^f, L_D^i, L_D^f)$$

$$\psi_E = \gamma_E(K_E^i, K_E^f, L_E^i, L_E^f)$$

The 10 first order conditions determining factor usage in production are given by:

$$\frac{\partial \psi_r}{\partial s_r^q} = \frac{\tilde{P}_{s_r^q}}{P_r}$$

The intermediate goods are divided between final goods and investment using CET production functions.

$$\psi_U = \delta_U(X_U, I_U)$$

$$\psi_D = \delta_D(X_D, I_D)$$
$$\psi_E = \delta_E(X_E, I_E)$$

The value of imports is equal to the value of exports plus the endowment of foreign exchange.

$$\tilde{P}_M X_M = \tilde{P}_E X_E + \bar{A}$$

Factor demand equals factor supply:

$$K_U^i + K_D^i + K_E^i + K_D^f + K_E^f = \bar{K}$$
$$L_U^i + L_D^i + L_E^i + L_D^f + L_E^f = L$$

Factors receive the same after-tax return wherever employed:

$$P_{s_r^q} = P_s, \forall s \in \{K, L\}, \forall q \in \{i, f\}, \forall r \in \{U, D, E\}$$

Taxes are zero for the informal good and factors:

$$T_j = 0 \ \forall j \in \{U, \{s_r^i\}\}, \forall s \in \{K, L\}, \forall r \in \{U, D, E\}.$$

Formal factors face the same tax rates whether producing *exports* or *formal* goods. This permits simpler notation:

$$T_K \equiv T_{K_r^f}, \ T_L \equiv T_{L_r^f}, \forall r \in \{D, E\}$$

The numeraire is foreign exchange:

$$P_M^w = 1$$

Goods supply equals demand.

$$X_U = U$$
$$X_D = D$$
$$X_M = M$$
$$I_U + I_D + I_E = I$$

The transfer to the consumer is equal to tax revenue.

$$R = T_E P_E X_E + T_M P_M X_M + T_D P_D X_D + T_L P_L (L_D^f + L_E^f) + T_K P_K (K_D^f + K_E^f)$$

Parameters in the model are: production and utility function parameters; endowments of time, capital and foreign exchange; and tax rates. Parameter values are determined by the calibration process.

APPENDIX 2: DATA

The data required for the model are:

E	exports (% of GDP)
M	imports (% of GDP)
R_D	tax revenue from VATs and sales taxes (%GDP)
R_E	tax revenue from export taxes (% GDP)
R_M	tax revenue from import taxes (% GDP)
R_K	tax revenue from capital taxes (% GDP)
R_L	tax revenue from labour taxes (% GDP)
T_D	tax rate on domestic goods and services
T_K	tax rate on capital (corporate tax rate)
T_L	tax rate on labour
α_U	labour-output ratio in production of *untaxed*
α_D	labour-output ratio in production of *domestic*
α_E	labour-output ratio in production of *exports*

Country-specific data are set out in Table 9. All country-specific data were obtained from IMF Statistical Annexes to country reports, available on the internet at www.imf.org. Table 9 also reports the year for which the data apply, and whether the country had a VAT in that year.

In the absence of data on labour-output ratios for all countries, the model uses the average ratios for the five countries of Table 10, supposing that the ratios are constant across all countries. These data are derived from social accounting matrices prepared by the International Food Policy Research Institute. Among other data, the SAMs provide for each commodity: the value-added by each factor; exports of each commodity; domestic demand for the commodity; and goods taxes paid. Let L_i denote the labour share of value added for each commodity i. Let E_i denote exports of each commodity. The first row in the table, the labour-output ratio for exports is then $\frac{\sum_i (L_i \times E_i)}{\sum_i E_i}$. Commodities are classified as *untaxed* if the taxes paid on the commodity constitute less than 5% of the value of domestic output of the commodity, and *domestic* if more than 5% is taxed. Let U_j be the value-added of untaxed goods. The labour-output ratio for *untaxed* is then calculated as $\frac{\sum_j (L_j \times U_j)}{\sum_j U_j}$. The same procedure is used to calculate the labour-output ratio for *domestic*.

The administrative costs of tax collection presented in Table 6 are derived from various sources. For the United States: IRS Data Book, FY2002, available at: www.irs.gov/pub/irs-soi/02db30cs.xls; For other OECD countries

Table 9: Country-Specific Data

Country	year	E	M	I	R_D	R_E	R_M	R_K	R_L	T_D	T_K	T_L	VAT
Benin	2001	27.69	35.40	19.18	2.76	0.14	6.54	1.90	1.57	0.18	0.38	0.28	✓
Botswana	2001	56.20	33.20	15.70	2.07	0.00	2.95	5.00	2.48	0.10	0.15	0.10	✓
Burkina Faso	2001	9.34	23.38	19.50	3.66	0.42	3.32	1.28	1.32	0.18	0.20	0.15	✓
Burundi	2000	7.79	21.23	8.39	9.16	0.00	3.40	1.57	3.91	0.20	0.40	0.50	-
Cameroon	2001	31.80	29.20	17.80	3.96	0.08	3.91	1.46	1.03	0.19	0.39	0.17	✓
Cape Verde	2001	30.82	65.35	17.36	0.58	0.78	9.78	3.59	3.59	0.10	0.20	0.20	-
CAR	1999	16.78	23.83	14.41	2.76	1.12	2.36	0.52	1.00	0.18	0.30	0.31	-
Chad	2000	16.58	32.03	17.02	1.88	0.22	1.85	1.13	1.15	0.18	0.45	0.48	✓
Congo	1999	21.90	25.59	24.10	1.03	0.05	0.74	0.27	0.49	0.03	0.40	0.35	-
Côte d'Ivoire	1999	44.33	37.46	16.30	4.82	2.56	4.60	2.08	1.68	0.20	0.35	0.27	✓
Eq. Guinea	2001	106.02	125.60	56.79	0.63	0.45	0.28	1.67	0.39	0.06	0.25	0.10	-
Eritrea	2000	5.26	67.37	18.50	2.77	0.00	4.37	4.84	1.98	0.05	0.25	0.12	-
Ethiopia	2001	15.36	31.18	18.04	2.66	0.18	6.22	2.90	1.32	0.15	0.30	0.15	-
Gabon	2000	66.96	32.65	21.84	1.99	0.79	5.16	1.54	0.72	0.18	0.35	0.30	✓
Gambia	1998	50.53	61.32	18.30	1.46	0.00	11.18	2.09	1.85	0.10	0.35	0.15	-
Ghana	1998	34.27	46.66	24.67	1.62	2.32	4.30	1.85	1.77	0.10	0.35	0.15	✓
Guinea	1998	21.60	23.40	17.10	4.65	0.02	1.45	0.20	0.65	0.18	0.35	0.29	✓
Guinea-Bissau	2001	27.21	57.71	18.92	0.74	2.61	4.30	0.92	0.95	0.15	0.39	0.10	-
Kenya	2000	26.47	36.39	15.55	6.62	0.00	5.96	3.45	3.45	0.15	0.30	0.15	✓
Madagascar	2001	28.59	31.75	15.50	2.65	0.00	4.79	0.99	0.97	0.20	0.35	0.15	-
Malawi	1999	27.04	49.32	17.08	1.61	0.00	0.61	0.71	0.86	0.20	0.30	0.32	-
Mali	2000	25.50	37.50	20.60	2.32	0.22	6.22	1.31	1.12	0.18	0.35	0.29	✓
Mauritania	2002	38.54	66.93	33.04	3.87	0.00	2.34	2.01	2.30	0.14	0.25	0.26	✓
Mozambique	2001	21.90	38.50	41.64	3.99	1.98	3.22	0.73	1.27	0.17	0.45	0.15	✓
Namibia	1997	52.62	58.17	19.75	4.54	0.00	9.62	5.03	6.00	0.10	0.40	0.20	-
Niger	2000	17.80	25.33	10.78	1.61	0.18	3.61	0.56	0.73	0.19	0.43	0.34	✓
Nigeria	2000	53.27	37.54	17.70	1.37	0.00	2.77	1.22	1.09	0.05	0.30	0.20	✓
Rwanda	2001	9.30	25.80	18.40	3.16	0.00	3.41	1.91	1.19	0.15	0.40	0.30	-
São Tomé	2000	35.32	82.44	43.49	0.41	0.05	10.20	1.81	2.71	0.35	0.30	0.13	-
Senegal	2000	30.50	39.60	19.10	4.64	0.00	8.48	1.91	2.20	0.20	0.35	0.44	✓
South Africa	1999	25.70	24.40	16.50	5.87	0.00	1.00	3.60	10.70	0.14	0.30	0.30	✓
Sudan	1999	8.11	15.13	16.70	1.29	0.07	2.85	0.90	0.12	0.10	0.40	0.20	-
Swaziland	2001	70.20	83.30	18.10	3.55	0.00	13.20	2.15	3.78	0.32	0.30	0.33	-
Tanzania	2002	15.90	24.50	17.00	2.51	0.00	4.67	0.66	1.55	0.20	0.30	0.25	✓
Togo	1998	33.72	40.39	14.20	3.20	0.00	5.60	1.60	0.90	0.14	0.30	0.33	✓
Uganda	1998	11.78	28.48	17.65	6.56	0.01	1.00	0.66	0.66	0.17	0.30	0.20	✓
Zambia	1998	29.30	38.40	14.40	6.60	0.00	4.60	1.40	4.70	0.18	0.35	0.20	✓
Zimbabwe	2000	29.81	31.00	15.55	6.00	0.00	2.59	3.89	10.86	0.15	0.30	0.30	-

Table 10: Labour-Output Ratios (%)

	Malawi	South Africa	Tanzania	Zambia	Zimbabwe	Average
export	38.08	56.14	33.58	19.39	37.09	36.86
domestic	48.34	48.28	33.59	39.37	46.82	43.28
untaxed	58.66	53.99	43.79	53.15	49.70	51.86

– authors' calculations based on the following tax agency annual reports: Australia – ATO Annual Report 2002, available at www.ato.gov.au; United Kingdom – Inland Revenue, Annual Report for the year ending 31st March 2002, available at www.inlandrevenue.gov.uk/pdfs/report2002.pdf; New Zealand – Inland Revenue Annual Report 2001-2002, available at: www.ird.govt.nz/aboutir/reports/annual-02.pdf; and Canada: Canada Customs and Revenue Agency, 2001-2002 Annual Report to Parliament, Financial Statements, available at www.cra-arc.gc.ca/agency/ annual/2001-2002/. For Guatemala: simple average of adjusted figures from Mann (2002). Ghana: revenue-weighted average of figures cited by Terkper (1995). Namibia: statistics provided by Klaus Schade of the Namibian Economic Policy Research Unit. Tanzania: statistics provided by Odd-Helge Fjeldstad of the Chr. Michelsen Institute. Remaining countries: Taliercio (2004). We are very grateful to Klaus Schade, Odd-Helge Fjeldstad and Robert Taliercio for their assistance in obtaining these data.

APPENDIX 3: Model Calibration

Benchmark quantities of goods and factors (entries in the SAM) are calculated using the following equations:

$X_D = \frac{R_D}{T_D}$	production of *domestic*
$X_U = 100 - E - I - X_D - R_D$	production of *untaxed*
$X_E = E - R_E$	production of *exports*
$X_M = M - R_M$	quantity of *imports*
$\bar{A} = X_M - E$	endowment of foreign exchange
$I_U = I \frac{X_U}{X_U + X_D + X_E}$	investment in *untaxed*
$I_D = I \frac{X_D}{X_U + X_D + X_E}$	investment in *domestic*
$I_E = I \frac{X_E}{X_U + X_D + X_E}$	investment in *exports*
$K_D^f = \frac{X_D + I_D}{X_D + I_D + X_E + I_E} \frac{R_K}{T_K}$	*formal capital* used to produce *domestic*
$K_E^f = \frac{X_E + I_E}{X_D + I_D + X_E + I_E} \frac{R_K}{T_K}$	*formal capital* used to produce *exports*
$L_D^f = \frac{X_D + I_D}{X_D + I_D + X_E + I_E} \frac{R_L}{T_L}$	*formal labour* used to produce *domestic*
$L_E^f = \frac{X_E + I_E}{X_D + I_D + X_E + I_E} \frac{R_L}{T_L}$	*formal labour* used to produce *exports*

$L^i_D = \alpha_D(X_D + I_D) - (1 + T_L)L^f_D$	*informal labour* used to produce *domestic*
$L^i_E = \alpha_E(X_E + I_E) - (1 + T_L)L^f_E$	*informal labour* used to produce *exports*
$L^i_U = \alpha_U(X_U + I_U)$	*informal labour* used to produce *untaxed*
$K^i_D = (1-\alpha_D)(X_D+I_D)-(1+T_K)K^f_D$	*informal capital* used to produce *domestic*
$K^i_E = (1-\alpha_E)(X_E+I_E)-(1+T_K)K^f_E$	*informal capital* used to produce *exports*
$K^i_U = (1 - \alpha_U)(X_U + I_U)$	*informal capital* used to produce *untaxed*
$\bar{K} = K^f_E + K^f_D + K^i_E + K^i_D + K^i_U$	total capital endowment
$L = L^f_E + L^f_D + L^i_E + L^i_D + L^i_U$	total labour supply
$\bar{T} = 1.05L$	endowment of time

Tax rates on *exports* and *imports* are calibrated, rather than being drawn directly from the legal tax rates:

$T_E = \frac{R_E}{X_E}$	tax rate on *exports*
$T_M = \frac{R_M}{X_M}$	tax rate on *imports*

We do not observe price or quantities of goods, but we do observe the total amount of money spent on each good (values as a percentage of GDP). Following the Harberger convention we choose units of the aggregate goods such that quantities equal values. This implies that initial prices equal one. Where goods are taxed, goods units can be chosen such that either the gross of tax or net of tax price equals one. We chose units such that the agent supplying the good or factor received a price of one, with remaining prices implied by tax rates:

$P^w_M = 1$	world price of imports (and foreign exchange)
$P_I = 1$	price of *investment*
$P_U = 1$	price of *untaxed*
$P_L = 1$	wage received by labour (formal or informal); also the wage paid by producers for *informal labour*
$P_K = 1$	wage received by capital (formal or informal); also the wage paid by producers for informal capital

$P_D = 1$	producer price of *domestic*
$P_E = 1$	producer price of *exports*

An n-factor CES production function, $F = A(\sum_{j=1}^{n} \theta_j X_j^{\rho})^{\frac{1}{\rho}}$, with factors X_j, share parameters θ_j, scale parameter A, and elasticity of substitution $\sigma = \frac{1}{1-\rho}$, can be rewritten in calibrated form as

$$F = \bar{F}\left[\sum_{i=1}^{n}\left(\frac{\bar{p}_i\bar{X}_i}{\sum_{j=1}^{n}\bar{p}_j\bar{X}_j}\right)\left(\frac{X_i}{\bar{X}_i}\right)^{\rho}\right]^{\frac{1}{\rho}}$$

where a bar over a variable indicates the observed benchmark level. The benchmark factor demands, factor prices and product outputs, combined with the elasticities of substitution fully specify the three production functions. Indeed theory implies that the Cobb-Douglas coefficient in the base case are $\theta_j = \frac{\bar{p}_i\bar{X}_i}{\sum_{j=1}^{n}\bar{p}_j\bar{X}_j}$ $j = 1, ..., n$. The same methodology can be used for the CES utility function, where the X_is represent goods consumed, the p_is are goods prices, and the benchmark utility level is normalized to unity.

The calibration process is completed with the selection of substitution elasticities for production and utility functions. In the absence of reliable evidence on the magnitude of these elasticities for Africa we chose a base case of $\sigma = 1$ for all four functions, and then ran sensitivity checks with different values for σ.

Table 11: Size of the Informal Sector (% GDP)

Country	U Untaxed good sales	$K^i_{DE}+ L^i_{DE}$ informal factor in formal good	Total
Benin	35.06	39.36	74.42
Botswana	5.34	25.92	31.26
Burkina Faso	47.16	18.93	66.09
Burundi	28.88	41.81	70.69
Cameroon	25.29	52.46	77.75
Cape Verde	45.44	-4.86	40.58
Central African Republic	50.72	29.99	80.71
Chad	54.11	25.22	79.33
Democratic Republic of Congo	18.53	71.60	90.13
Côte d'Ivoire	10.43	64.08	74.51
Equatorial Guinea	-73.92	159.74	85.82
Eritrea	18.05	32.28	50.33
Ethiopia	46.22	17.72	63.94
Gabon	-1.83	89.70	87.87
Gambia	15.13	57.74	72.87
Ghana	23.21	44.12	67.33
Guinea	30.83	54.11	84.94
Guinea-Bissau	48.20	23.05	71.25
Kenya	7.26	43.30	50.56
Madagascar	40.01	38.53	78.54
Malawi	46.23	35.85	82.09
Mali	38.66	38.35	77.02
Mauritania	-3.05	77.49	74.44
Mozambique	9.03	65.76	74.80
Namibia	-22.35	64.03	41.68
Niger	61.34	24.56	85.90
Nigeria	0.34	86.37	86.72
Rwanda	48.11	25.60	73.71
São Tomé	19.62	33.31	52.94
Senegal	22.56	52.57	75.13
South Africa	9.97	6.75	16.71
Sudan	60.97	21.37	82.34
Swaziland	-3.07	74.72	71.65
Tanzania	52.02	23.72	75.74
Togo	26.02	55.74	81.77
Uganda	25.42	55.31	80.74
Zambia	11.99	45.63	57.62
Zimbabwe	8.61	6.16	14.77
Average	23.33	45.21	68.54

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