Agency Costs, Firm Behavior and the Nature of Competition*

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1 Introduction

1.1 Motivation and punchline

This paper presents an agency model of firms where the need for outside finance interacts with product market behavior in a non-monotonic way: for low levels of outside finance, a rise in this need makes the firm's management "softer", since its incentive to work hard and raise profits are diminished. This effect has been familiar since the work of Jensen and Meckling (1976) on equity finance and Myers (1977) on debt finance. However, for higher levels of outside finance, the firm may be constrained by its capacity to attract outside investors. Then, in order to obtain additional funds - and thus to raise its outside finance capacity - the management of the firm may have to resort to (privately costly) commitment devices to make it "tougher" and thereby raise its future profits: the need to satisfy investors' repayment requirements introduces a role for outside finance as a discipline device.

The above story leads to a situation where, depending on its need for outside finance, a firm is in one of two regimes: when, ceteris paribus, this need is limited and the outside finance capacity not a concern, the firm is in a "*shirking regime*", where a rise in the level of outside finance reduces the incentive for managers to work; when the level of outside finance is high and the firm has to take actions to enhance its outside finance capacity, the firm is in a "*bonding regime*", where a rise in the level of outside finance starts requiring *more* work from management.

Interestingly, the behavior of the firm depends a lot on the regime it faces: for example, a worsening of demand conditions, or the arrival of new competitors, will typically make the firm *softer* in the shirking regime but *tougher* in the bonding regime, where it has to convince investors that it is not becoming insolvent.[1] Moving from one regime to the other can actually transform the nature of competition and strategic interaction, from strategic substitution to strategic complementarity, to follow the terminology of Bulow et al. (1985). As we know from Industrial Organization, this has also important consequences on the behavior of firms in terms of barriers to entry or exit inducement, particularly if competitors can observe the regime of the firm. A lesson from this paper is thus that the nature of competition is significantly influenced by firms' needs for outside finance.

The above themes are analyzed in this paper in a setup where the firm's key decision variable is a noncontractible effort, which determines the probability of success of its project. We allow the firm to influence its incentives to exert this effort by contracting on other variables. In reality, R&D expenditures, for example, can be more or less tangible: a firm can for example buy machines that make it easier or cheaper to switch to a new technological process; it can hire a number of research specialists, or even buy a small firm where this type of R&D is performed. Moreover, the firm can influence its future behavior by setting up monitoring schemes, like for example a board of directors, or by giving higher-powered incentives schemes, designing new franchise or patent contracts,

[1]In this bonding regime, the behavior of the entrepreneur can be described as "satisficing behavior:" when conditions become worse, the entrepreneur does "what it takes" to keep attracting investors.

etc. These various decisions can be explicitly detailed in a financial contract. Still there is always room for discretion for the firm in a number of dimensions of its internal efforts. In the end, one is thus faced with a moral hazard problem. Intuitively, when the firm's need for outside finance grows from zero, effort starts shrinking because the monetary return to the firm is reduced. This process will continue as long as the firm is not facing financing problems. When the need for outside finance becomes so high that investors become reluctant to lend more to the firm, it becomes necessary for the firm to *commit itself to raising effort.* This will be done through the contractible variables, even if this is expensive for the firm. Hence a non-monotonic (U-shaped) relationship between outside finance and effort.[2] In this paper, we derive this relationship in two sets of examples (one with closed-form solutions in sections 2 to 4 and one with simulations in section 5) which, together, point to its robustness. Further research should of course try and generalize the framework.[3]

1.2 Link with the literature

Our paper contributes to the analysis of the interaction between outside finance, managerial effort and product market competition. It is thereby connected to three strands of literature.

[2]Povel and Raith (2000) obtain a similar U-shaped relationship between output and net worth. In their framework, a small reduction in the net worth of the firm (and thus an increase in outside finance) first discourages the firm, which reduces its output. However, for very low (negative) levels of net worth (and thus for large needs for outside finance), a further reduction in the firm's net worth leads the firm to increase output, to generate earnings and repay the loan.

[3]Cestone and Fumagalli (2000) study the implications of the above insights for a firm's internal allocation of capital across business divisions.

There are first the agency models of corporate finance[4]. While this literature is mainly concerned with capital structure and the comparison between debt and equity financing, our model concentrates on a simple two-outcome case - "success" and "failure" - which does not distinguish between different *forms* of outside finance. We feel that this is a natural first step, which allows us to concentrate on the general effect of the *level* of outside finance. Still, our model generates predictions reminiscent from that literature: specifically, for low levels of external finance, more external finance discourages effort, as in Myers (1977) and Jensen-Meckling (1976); while, for high levels of external finance, more external finance encourages effort, as in Jensen (1986).

A second strand of related literature analyzes the impact of firms' capital structures in explicit IO models[5]. Our paper contributes to this literature by stressing that the effect of indebtedness on strategic behavior (e.g. on the slope of firms' reaction functions, on the firms' incentives to bar entry and induce exit, on the effects of market size on market concentration ...) will mainly depend on

[4]Jensen and Meckling (1976) show that equity leads to "shirking" and debt to "asset substitution", i.e. excessive risk-taking to take advantage of the fact that the payment to equity is convex in the value of the firm. Myers (1977) however stresses that the existence of debt will also reduce incentives for management to work/invest. On the other hand, Jensen (1986) stresses the disciplining role of debt in reducing the firm's "free cash flow". This has to be balanced against shareholders' ability to discipline management either directly, through proxy contests, or indirectly, through takeovers.

[5]For example, Brander and Lewis (1986) show that higher leverage induces equityholders to push for tougher firm behavior, in order to increase the riskiness of the firm's profit. Maksimovic (1988) also presents a model where leverage is positively correlated with toughness on product markets. By contrast, Bolton and Scharfstein (1990) show that leverage (in fact, outside finance) can make a firm the target of predatory behavior. And Chevalier (1995), in an empirical analysis of leverage buyouts in the supermarket industry, shows that these very significant increases in debt-equity ratios are "good news" for competitors and induce them to expand in the market.

the *extent* to which firms are constrained by their outside finance capacity.

Finally, our paper is closely connected to the literature that focuses on the impact of product market competition as a way to reduce agency problems. For example, Hart (1983) and Scharfstein (1988) show that a higher competitive pressure, in the form of a higher proportion of entrepreneurial firms (as opposed to managerial firms), has a positive impact on managerial behavior if managers are not very responsive to monetary incentives, but may have a negative impact otherwise[6]. Our analysis puts those insights into perspective: when the firm mainly relies on internal finance, its behavior is driven by its expected returns and resembles that predicted by Scharfstein; when instead the finance capacity becomes a concern, the firm behaves as predicted by Hart or as when the threat of bankruptcy dominates the other incentives. To sum up: whether more competition reduces or worsens agency problems will depend on the degree to which outside finance capacity constraints matter.

The structure of the paper is as follows. Section 2 presents a model for the behavior of a single firm, which captures in a simple way the nonmonotonic relation between effort and outside finance.

[6]More recently, Hermalin (1992) has stressed the sensitivity of the analysis to the presence of income effects in managers' preferences; Schmidt (1997) has emphasized another potentially positive impact of competition on managerial effort, through a greater threat of bankruptcy. Aghion et al. (1999) have developed a similar idea in the context of an economywide growth model in which product market competition and financial discipline can both push firms with "satisficing" or "empire-building" managers towards profit maximization. This work is in turn related to Nelson-Winter (1982); they consider a setup where firms keep their behavior unchanged as long as they earn nonnegative profits and investigate whether evolutionary forces could lead to convergence towards profit maximization.

Sections 3 and 4 extend this framework to multiple firms, in order to revisit a number of themes in the industrial organization literature: oligopolistic equilibria, barriers to entry and exit inducement (Section 3), and free-entry equilibria (Section 4). Section 5 argues that the coexistence of a shirking regime (for low levels of outside finance) and of a bonding regime (for large levels of outside finance) is fairly robust. Finally, Section 6 concludes.

2 A simple model of firm behavior

A firm has a project, which generates a (verifiable) profit Π in case of success and 0 otherwise. The firm is run by a risk-neutral entrepreneur who can influence the probability of success z through the effort he provides. By running the project, the entrepreneur gets private benefits B but incurs a private cost of effort C. The entrepreneur cannot contract on z but can contract on related variables. To fix ideas, throughout most of the paper we will suppose that there are two perfectly substitutable types of effort (Section 5 discusses how this setup can be generalized):

$$z \equiv e + a,$$

where a is contractible but costly whereas e is efficient but not contractible. To make things particularly simple, we assume that only the non-verifiable effort is cost-effective: the private cost of effort C is given by

$$C(e,a) \equiv \begin{cases} \gamma a & when e < \tilde{e}, \\ \frac{\beta}{2}(e-\tilde{e})^2 + \gamma a & when e \geq \tilde{e}, \end{cases}$$

with $0 < \tilde{e} < 1$ and $\gamma > \Pi$. With this cost structure, $a = 0$ is always efficient whereas the efficient level of e is above $\tilde{e}$, which can be interpreted as related to the private benefits of running the project: even in the absence of any monetary incentive, the entrepreneur will provide at least $e = \tilde{e}$.

To undertake its project the firm needs to raise an amount I from the capital market. By assumption, (risk neutral) outside investors have to be repaid out of Π. To attract those investors, the entrepreneur[7] can offer them a share θ of the profit in case of success and also contract on a, in order to influence the probability z that they will indeed get $\theta\Pi$. The time line is as follows:

------- | ------------------------------ | ------------------------ | ------------ *time*

Contract (a, θ) *Entrepreneur chooses e* *Success/failure*

The optimal contract (a^*, θ^*) solves:[8]

$$
\begin{aligned}
e, a, \theta \max \quad & B + (1-\theta)(e+a)\Pi - C(e,a) \\
s.t. \quad & 0 \leq \theta \leq 1, e \geq 0, a \geq 0, e + a \leq 1 \\
& e \in \arg\max_{\widehat{e}} (1-\theta)(\widehat{e}+a)\Pi - C(\widehat{e}, a) \qquad (IC) \\
& \theta (e+a)\Pi \geq I \qquad (IR)
\end{aligned}
$$

where (IR) makes sure investors get their money back (all amounts are in initial-period money) for the effort level that satisfies (IC), the incentive constraint of the entrepreneur.

[7]Throughout the paper, we focus on the agency problem between the firm and its outside investors and neglect other potential agency problems. In particular, there is no assumed conflict between the owner of the firm (the "entrepreneur") and its manager, and we will thus refer indistinctly to the firm, the entrepreneur or the manager when discussing the behavior of the "firm".

[8]We assume for the moment that private benefits, say, induce the entrepreneur to undertake the project as long as he can convince outside investors to finance it. Obviously, this may not be the case if the entrepreneur must abandon an excessively high share of the profits and/or if private costs are too large as compared with private benefits. We explore this issue in Section 4.

When $I = 0$, the entrepreneur keeps all the profits ($\theta = 0$) and sets optimally $a^{FB} \equiv 0$, while the first-order condition with respect to e yields:

$$e^{FB} \equiv \min\left\{\tilde{e} + \frac{\Pi}{\beta}, 1\right\}.$$

When I rises, so does θ, the outsiders' share of profit. This in turn reduces the entrepreneur's incentive to exert effort e, up to the point where he may have to agree to a positive a, in order to keep attracting outside investors. To characterize the optimal solution $\{\theta^*, e^*, a^*\}$, the following lemma provides a useful step:

Assume:

$$H_1: \quad \beta\tilde{e} > \Pi and \gamma > \frac{\Pi}{1 - \Pi/\beta\tilde{e}}.$$

Then:

$$\forall I \geq 0, either e^* = \tilde{e} or a^* = 0.$$

See Appendix.

Intuitively, since a is inefficient, when I and thus θ are small and e close to its first-best level, the entrepreneur prefers to give a bigger share θ to the investor (and lower e), rather than increase a. Assumption H_1 ensures that this remains the case as long as $e > \tilde{e}$, because $\tilde{e}$ is sufficiently large and a is sufficiently more expensive than e. This lemma drastically simplifies the analysis and allows us to consider two distinct cases, according to the type of effort that is relied upon.

- Case $e^* > \tilde{e}, a^* = 0$

In this case, e^* is given by $((IR), (IC))$:

$$e^* \left[\Pi - \beta \left(e^* - \tilde{e}\right)\right] = I.$$

Under H_1, the left-hand side of the above equation increases from 0 to $\tilde{I} \equiv \tilde{e}\Pi$ when e decreases from e^m to $\tilde{e}$. Thus, as I increases from 0 to $\tilde{I}$, optimal effort e^* decreases from e^m to $\tilde{e}$ and the optimal share of the return going to the investor, θ^*, increases accordingly from 0 to 1. For $I > \tilde{I}$, it is not possible to sustain a level of effort $e > \tilde{e}$.

- Case $a^* > 0, e^* \leq \tilde{e}$

In that case, $\theta^* = 1$ and $e^* = \tilde{e}$, whereas a^* is given by:

$$\left(\tilde{e} + a^*\right) \Pi = I,$$

The entrepreneur's objective is in that case driven by the preservation of his private benefits, since he has given up all of the monetary returns to the outside investor. In what follows we will assume to simplify that those private benefits are large enough to ensure that the entrepreneur is always willing to run the project as long it can be financed, i.e. as long as $I \leq \Pi$.[9]

To sum up:

Characterization of the optimal contract

Under assumption H_1, for any $I \leq \Pi$ there exists a unique optimal contract (θ^*, e^*, a^*), characterized as follows:

- *Shirking regime*: For $I < \tilde{I} \equiv \tilde{e}\Pi$, $a^* = 0, \theta^* < 1$, and $z^* = e^*$ is the largest solution to

$$e^* \left[\Pi - \beta \left(e^* - \tilde{e}\right)\right] = I,$$

[9] That is, $B \geq \gamma \left(1 - \tilde{e}\right)$. If this condition is not satisfied, the range of the "bonding region" goes from $I = \tilde{I}$ to $I = \left(\tilde{e} + B/\gamma\right) \Pi$.

which decreases from e^{FB} to $\tilde{e}$ when I increases from 0 to $\tilde{I}$.

- *Bonding regime*: For $\tilde{I} \leq I \leq \Pi$, $\theta^* = 1, e^* = \tilde{e}$, and $z^* = \tilde{e} + a^*$ is given by

$$z^*\Pi = I,$$

which increases from $\tilde{e}$ to 1 when I increases from $\tilde{I}$ to Π.

Note that assumption H_1 is satisfied when $\gamma > \Pi$ and β is "large". Figure 1 illustrates the evolution of the optimal effort, as a function of the level of outside finance.

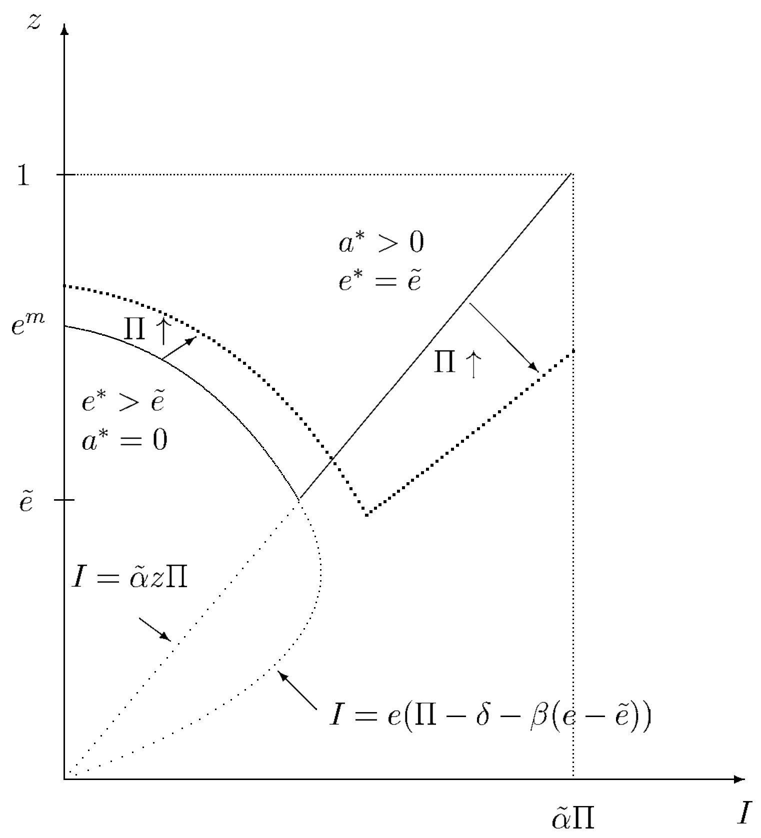

Figure 1

This analysis stresses several interesting features. First, the need for external finance creates an *agency situation*: when $I = 0$ the entrepreneur behaves as a standard profit-maximizer but, when $I > 0$, the entrepreneur must compensate the outside investor and is not fully residual claimant any more. As I increases the entrepreneur progressively departs from the profit-maximizing levels of effort, first by reducing the non-verifiable effort, then by turning to alternative, verifiable but more costly effort. The amount of outside finance I thus measures the magnitude of the agency problem between the entrepreneur and the external investor(s), indicating by "how much" the firm deviates from profit-maximization.

Second, whether an increase in the amount of outside finance will increase or decrease total effort z depends on the initial level of outside finance. When that level is small ($I < \tilde{I}$) the firm is in the *shirking* regime, where raising I *lowers* the amount of effort produced, as is standard in agency models, because the entrepreneur has to concede a higher share of profits to the investor; this regime corresponds to the classical "debt overhang" problem stressed by Myers (1977). Note that in this two-outcome case, the contract can also be interpreted as an outside equity contract, whose adverse incentive effect on effort has first been stressed by Jensen and Meckling (1976). Instead, for $I > \tilde{I}$, the firm is in a "bonding" regime where an increase in I *raises* the effort to innovate, because a higher level of outside finance forces the firm to raise a in order to credibly raise the expected repayment to the investor. This is somehow reminiscent of Jensen (1986)'s "no free cash" idea, where debt forces the firm to maximize monetary profits. In a previous paper (Aghion et al.,1999), we explored the role of debt in eliminating slack in innovation effort. There, as in the present bonding regime, a high debt can commit the firm to be more aggressive in effort. Section 5 shows that the existence of both the shirking regime (for low needs of outside finance) and the bonding regime (for large levels of outside finance) is fairly robust.

Third, those two regimes also differ with respect to the impact of Π: in the shirking regime, e decreases as Π decreases, both because the profitability of effort is reduced and also because the share of verifiable profits going to the investor has to increase in order for the constraint to remain satisfied, which in turn further reduces incentives. Instead, in the bonding regime, when Π decreases, a has to increase to keep the expected repayment to the investor from falling. This comparative statics underlies the strategic analysis detailed in the subsequent sections.

Remark *Robustness of the analysis to monetary costs:* Assume that a also involves monetary costs, equal to fix ideas to λa. The entrepreneur's program is then the same as before, except that the outsiders' participation constraint now becomes:

$$\theta\left(e+a\right)\Pi = I + \lambda a.$$

IR(1)

The previous analysis then remains valid, provided that monetary costs are not too high (namely, $\lambda < \Pi$, so that an increase in a still generates a net increase in expected monetary profits): the behavior is represented by the same shirking regime as before for $I < \tilde{e}\Pi$, whereas for $I > \tilde{e}\Pi$, it is characterized by a bonding regime where the levels of efforts are $e^* = \tilde{e}$ and a^* is such that

$$\tilde{e}\Pi + a^* (\Pi - \lambda) = I,$$

so that $z^* = \tilde{e} + a^*$ increases from $\tilde{e}$ to 1 when I increases from $\tilde{I}$ to $\Pi - (1 - \tilde{e}) \lambda$.

3 Oligopolistic competition

We now analyze a symmetric n-firm oligopoly. Each firm is as described above, with the same need for external finance I. We will assume that a firm receives Π only when no other project is successful (as for R&D projects for a new product, with downstream Bertrand competition if several firms innovate); entrepreneur i's objective thus becomes:

$$(1 - \theta) \left(\prod j = 1 j \neq i^n (1 - e_j - a_j) \right) (e_i + a_i) \Pi - C(e_i, a_i).$$

We analyze the strategic interactions in this oligopolistic setting. In a first step we consider a game where firms' characteristics are common knowledge, but firms sign their contracts, of the same form (θ_i, a_i) as before, and choose their levels of effort, e_i, simultaneously. This framework rules out "commitment effects" that could be attached to the contracts: firms' decisions are based on their expectations regarding the other firms' contracts and efforts, not on the actual contracts and efforts chosen by the other firms; hence a firm will not try to manipulate its rivals' strategies when determining the terms of its own contract. We thus focus in this first step on the impact of agency problems (measured by the firms' need for outside finance) on the strategic interaction between competitors, leaving aside the possible strategic use of the contracts themselves. We briefly explore such strategic use at the end of the section.

3.1 Strategic complements versus strategic substitutes

We will focus on symmetric Nash equilibria and first consider one firm's best response to its rivals' contracts, taking those rivals' contracts, and thus the rivals' levels of effort, as given. If the firm expects each competitor to exert the same effort $\widetilde{z} = \widetilde{e} + \widetilde{a}$, its best response solves:

$$
\begin{aligned}
e, a, \theta \max \qquad & B + (1-\theta)(1-\widetilde{z})^{n-1}(e+a)\Pi - C(e,a) \\
s.t. \quad & 0 \leq \theta \leq 1, e \geq 0, a \geq 0, e + a \leq 1 \\
& e \in \arg\max_{\widehat{e}} (1-\theta)(1-\widetilde{z})^{n-1}(\widehat{e}+a)\Pi - C(\widehat{e},a) \qquad (IC) \\
& \theta(1-\widetilde{z})^{n-1}(e+a)\Pi \geq I \qquad (IR)
\end{aligned}
$$

This program, which characterizes the firms' best response function, $e^r(\widetilde{z})$, $a^r(\widetilde{z})$ and thus $z^r(\widetilde{z})$, is similar to the one analyzed for the monopoly case, replacing Π with $\Pi' = (1-\widetilde{z})^{n-1}\Pi$. Furthermore, an increase in the other firms' effort $\widetilde{z}$ (or, for that matter, of any of the other firms' effort $\widetilde{z}_j$) will have the same impact on the firm as a *decrease* in the profit Π, since $\widetilde{z}$ and Π are relevant for the firm only through $\Pi' = (1-\widetilde{z})^{n-1}\Pi$. Hence for I small, the firm will be in a shirking regime and will respond to an increase in the other firms' effort $\widetilde{z}$ by a decrease in its own effort, in the same way as it would respond to a decrease in Π: firms' efforts are in that case *strategic substitutes*, as they would be if the firms were behaving here as profit-maximizers. But for larger values of I the firm may be in a bonding regime and respond to the same increase in $\widetilde{z}$ by increasing its own effort, because it needs to do so in order to keep attracting outside investors: firms' efforts then become *strategic complements.*

Building on the previous analysis, and noting that H_1 remains satisfied when replacing Π with $(1-\tilde{z})^{n-1}\Pi$, the firm's best response is given by:

- For $I \leq \tilde{e}(1-\tilde{z})^{n-1}\Pi$, $a^r(\tilde{z}) = 0$, $\theta^r(\tilde{z}) < 1$ and $z^r(\tilde{z}) = e^r(\tilde{z})$ is the largest solution to:[10]

$$z^r\left[(1-\tilde{z})^{n-1}\Pi - \beta(z^r - \tilde{e})\right] = I \tag{2}$$

  and $dz^r/d\tilde{z} < 0$, since the left-hand side of (2) decreases with $\tilde{z}$ and is concave in z^r. In the context of R&D for example, this corresponds to the Schumpeterian view that competition (in the form of more aggressive rivals) discourages innovation.

- For $\tilde{e}(1-\tilde{z})^{n-1}\Pi \leq I \leq (1-\tilde{z})^{n-1}\Pi$, $e^r(\tilde{z}) = \tilde{e}$, $\theta^r(\tilde{z}) = 1$ and $z^r(\tilde{z}) = \tilde{e} + a^r(\tilde{z})$ is given by:

$$z^r(1-\tilde{z})^{n-1}\Pi = I \tag{3}$$

  and $dz^r/d\tilde{z} > 0$ since the left-hand side of (3) again decreases with z but increases in z^r. In the context of R&D, this corresponds to the "Darwinian" view that competition (in the form of more aggressive rivals) fosters innovation, as a means of survival.

3.2 Oligopolistic equilibria

We now characterize the symmetric Nash equilibria in contracts. When $I = 0$, the equilibrium is such that all firms choose $z = e_n^c$, characterized by:

$$(1-e_n^c)^{n-1}\Pi = \beta(e_n^c - \tilde{e}).$$

[10] Assuming $(1-z)^{n-1}\Pi < \beta(1-\tilde{e})$; otherwise, $z^r = 1$ as long as I is not too large.

This level of effort is larger than what would maximize the joint expected profit,

$$e_n^m \equiv \arg\max_e \pi(z) - \frac{\beta}{2}(e-\tilde{e})^2 \mid_{e>\tilde{e}},$$

where:

$$\pi(z) \equiv z(1-z)^{n-1}\Pi$$

represents the expected firm profit when all firms choose the same z.

Consider now the case $I > 0$, and suppose that all firms choose the level $z = e \geq \tilde{e}$. Using the first-order condition

$$(1-\theta)(1-z)^{n-1}\Pi = \beta(e-\tilde{e})$$

to determine θ, the repayment available for the investor is then equal to:

$$R(e) \equiv \theta\pi(e) = \pi(e) - \beta(e-\tilde{e})e.$$

Under H_1, $R(e)$ is a decreasing function of e. Also, $\pi(z)$ increases from 0 to $\bar{I}_n \equiv \max_z \pi(z)$ when z increases from 0 to $1/n$, and then decreases when z further increases. The bonding equilibrium, where all firms choose $z = \tilde{e} + a > \tilde{e}$, can thus only appear if $\tilde{e} < 1/n$:

Characterization of the symmetric equilibria

Assume H_1 and $\tilde{e} < 1/n$. Then for $I \leq \bar{I}_n$ there exists a unique symmetric stable equilibrium:[11]

[11] Stability refers here to the standard notion of stable oligopolistic equilibria. That is, starting from the equilibrium configuration, consider a slight exogenous increase in one firm's R&D effort, and compute the change it induces in the equilibrium best response of the other firms (that is, in the partial equilibrium of the other firms, taking the R&D of the first firm as given); then, compute the first firm's own best response to this new R&D level for the other firms. The equilibrium is stable if this best response is closer than the exogenous initial

- For $I < \tilde{I}_n \equiv \pi(\tilde{e})$, $z^* = e^*$ is the largest solution to:

$$R(z^*) = I, \tag{4}$$

which decreases with I from e_n^c to $\tilde{e}$.

- For $I > \tilde{I}_n$, $z^* = \tilde{e} + a^*$ is the smallest solution to:

$$\pi(z^*) = I, \tag{5}$$

which increases with I from $\tilde{e}$ to $1/n$.

See Appendix.

There are thus two distinct "regimes", as before. When the agency problem is small, firms are in a *shirking* regime: they choose the "efficient type" of effort (e rather than a) but, being less than full residual claimants and responding to their own incentives, they choose a lower level of effort than would be fully efficient. When the agency problem is more important, firms are in a *bonding* regime: they rely on the inefficient but contractible type in order to guarantee a sufficient return to the outside investor. We now study the impact of various changes in the environment on the characteristics of the equilibrium.

Consider first the impact of an increase in the magnitude in the *agency problem*, measured by the need for external funds. When I is initially small, an increase in I leads each firm to reduce its effort: e^* decreases with I since $R(.)$ is decreasing over the relevant

change to the equilibrium value.

The symmetric equilibrium is actually unique and stable in the shirking region, whereas in the bonding region there exists a second, unstable symmetric equilibrium in which the firms choose the largest solution to $\pi(z) = I$.

range. Since the competitive level exceeds the cartel level when firms are pure profit-maximizers (i.e., $e_n^c > e_n^m$), when I increases from 0 aggregate profits first increase as e^* gets closer to the cartel level e_n^m,[12] and then decrease (if $e_n^m > \tilde{e}$). If instead the need for external funds is already important, then a further increase in I forces each firm to increase its contractible effort: a^* increases and profits thus become smaller, since a^* is already excessive.

Consider now a change in the *number of firms*. If agency problems are small $\left(I \leq \tilde{I}_n\right)$, the addition of a new competitor induces each firm to *decrease* its effort. It may even be the case that the overall probability of innovation decreases, that is, the depressing effect of competition on each firm's effort may dominate the fact that more firms are exerting effort.[13] In all cases, individual profits

[12]However, they increase less than I. If the need for outside finance derives from the size of the fixed cost of the project, F, the firm's net profit thus decreases when the fixed cost F increases:

$$\frac{d}{dF}\left[\pi\left(e^*\right) - F\right] = \frac{d}{dI}\left[\pi\left(e^*\right)\right] - 1 = \frac{\pi'\left(e^*\right)}{R'\left(e^*\right)} - 1 < 0.$$

[13]For example, for $I = 0$:

$$\frac{\partial e_n^c}{\partial n} = \frac{\log\left(1-e\right)\left(1-e\right)^{n-1}\Pi}{\beta + \left(n-1\right)\left(1-e\right)^{n-2}\Pi}.$$

The probability of innovation is given by:

$$P = 1 - \left(1 - e_n^c\right)^n$$

and (writing e for e_n^c)

$$\begin{aligned} \frac{dP}{dn} &= \frac{\partial P}{\partial n} + \frac{\partial P}{\partial e}\frac{\partial e_n^c}{\partial n} \\ &= -\log\left(1-e\right)\left(1-e\right)^n + n\left(1-e\right)^{n-1}\frac{\log\left(1-e\right)\left(1-e\right)^{n-1}\Pi}{\beta + \left(n-1\right)\left(1-e\right)^{n-2}\Pi} \end{aligned}$$

is negative whenever (using the first-order condition $\left(1-e\right)^{n-1}\Pi = \beta\left(e - \tilde{e}\right)$) $e_n^c > \left(1 + \tilde{e}\right)/2$.

decrease with the number of competitors: even when the decrease in the equilibrium level of effort is profitable (i.e., when $e^* > e_n^m$), the direct negative effect of the increase in the number of competitors always dominates.[14]

If instead agency problems are large $\left(I > \tilde{I}_n\right)$, then an increase in the number of competitors results in an *increase* in each firm's commitment to verifiable efforts (π (.) increases in a in the relevant range). In that case, the industry-wide probability of innovation thus increases with the number of competitors, both because more firms provide effort and because each firm provides more effort. This changing impact of competition on effort can be related to the importance of managers' responsiveness to monetary incentives and to bankruptcy threats, as emphasized by Hart (1983), Scharfstein (1988) and Schmidt (1997). When firms have little need for outside finance, they are almost fully residual claimants and respond well to monetary incentives; in that case, an increase in competition, which adversely affects the profitability of effort, reduces effort levels. In contrast, when firms rely a lot on outside finance, they have to give

[14] ¿From the above Proposition, we have:

$$\frac{\partial R}{\partial e} de^* + \frac{\partial R}{\partial n} dn = 0.$$

Each firm's net surplus is given by:

$$V = \pi\left(e; n\right) - \frac{\beta}{2}\left(e - \tilde{e}\right)^2 + B - F.$$

Hence (using $\partial R / \partial n = \partial \pi / \partial n$):

$$\begin{aligned} \frac{dV}{dn} &= \frac{\partial \pi}{\partial n} + \frac{\partial V}{\partial e}\frac{de^*}{dn} = \left(\frac{\partial \pi}{\partial n} / \partial R \partial e\right)\left(\partial R \partial e - \partial V \partial e\right) \\ &= -\beta e \left(\frac{\partial \pi}{\partial n} / \partial R \partial e\right) < 0. \end{aligned}$$

away most of the monetary profits to repay outside investors; in that case, the entrepreneurs' incentives are driven by the preservation of their activity (not being able to attract outside investors can be seen as a form of "ex ante bankruptcy") and an increase in competition leads firms to provide more effort, in order to maintain their outside finance capacity.

Finally, consider the impact of a *government subsidy* aimed at encouraging innovative investments by a small number of firms, say by *one* firm. When agency problems (i.e. I) are small, the impact of such a subsidy policy will be limited due to a crowding out effect, since the other firms react to the subsidy by reducing their own R&D investments. In contrast, when agency problems are important, the same subsidy policy will lead to a multiplier effect, as the other firms will react by increasing their own R&D efforts.

4 Free entry

We now analyze the positive and normative properties of free-entry symmetric equilibria. To fix ideas, we will assume that the need for outside finance derives from the necessity of funding the fixed cost of the project:

$$I = F - A,$$

where F denotes the fixed cost of the project and A represents the firm's initial capital endowment. The number of firms is then given by a standard free-entry condition:

$$(1-\theta)\pi\left(e+a;n\right) - C\left(e,a\right) + B - (F - I) = 0.$$

When in equilibrium firms are in the shirking regime ("shirking equilibrium", for short), the equilibrium effort e^* is given by (4) while the free-entry condition becomes (using $\theta\pi(z) = I$):

$$\pi(e;n) - \frac{\beta}{2}(e-\tilde{e})^2 + B - F = 0. \tag{6}$$

When firms are instead in the bonding regime ("bonding equilibrium", for short), the equilibrium effort $z^* = \tilde{e} + a^*$ is determined by the investor's participation constraint (5) while the free-entry equation can be written as:[15]

$$\pi(\tilde{e}+a;n) - \gamma a + B - F = 0. \tag{7}$$

We shall now perform a number of comparative statics exercises, while the welfare analysis of the free-entry equilibrium can be found in appendix D.

4.1 Increasing the agency problem

Consider first an increase in the agency problems between firms and creditors, measured by the need for outside finance I.

In a bonding equilibrium, (5) and (7) directly yield the free-entry equilibrium level of effort a^B:

$$I - F + B = \gamma a^B, \tag{8}$$

which clearly increases with I. Since the left-hand side of the free-entry equation (7) decreases with n and with a under assumption H_1, we have:

[15] The existence of free-entry equilibria can easily be guaranteed. Note first that, for $I = F - B$, $z^0 = z^B = \tilde{e}$ (that is, $e^0 = \tilde{e}$, $a^B = 0$) and $n^O = n^B = \tilde{n}$, solution to $\pi(\tilde{e}, n) = \tilde{e}(1-\tilde{e})^{n-1}\Pi = F - B$. For $\tilde{e} < 1/2$ and F, B such that:

$$\tilde{e}\Pi < F - B < \tilde{e}(1-\tilde{e})^{\frac{1-\tilde{e}}{\tilde{e}}}\Pi,$$

such $\tilde{n}$ exists and satisfies $\tilde{e} < 1/\tilde{n} < 1$. Then, for each level z slightly above $\tilde{e}$, there exist two values for I, $I^O < F - B$ and $I^B > F - B$, such that a symmetric free-entry equilibrium exists and yields z in both cases, but is of the shirking type for $I = I^O$ (with $n^O < \tilde{n}$) and of the bonding type for $I = I^B$ (with $n^B > \tilde{n}$).

In a bonding equilibrium, increasing the agency problem forces firms to provide more effort in order to meet the outside investor's participation constraint:

$$\frac{da^B}{dI} > 0.$$

This, in turn, has a deterrent effect on entry:

$$\frac{dn^B}{dI} < 0.$$

In a shirking equilibrium, intensifying the agency problem instead discourages the firms' efforts. Combining equations (4) and (6), we obtain:

$$\frac{\beta}{2}\left(e^2 - \tilde{e}^2\right) + I - F + B = 0, \tag{9}$$

which defines the free-entry equilibrium level of non-contractible effort, e^O, as a decreasing function of I. Since the left-hand side of the free-entry equation (6) is: (a) decreasing in n; and (b) increasing in e for $e < e_n^m$ and decreasing in e for $e > e_n^m$, we have:

In a shirking equilibrium, a marginal increase in the agency problem, measured by I, has a deterrent effect on the firms' individual levels of efforts:

$$\frac{de^O}{dI} < 0.$$

If e is already small ($e < e_n^m$), a symmetric reduction in effort lowers expected profits more than it reduces costs and:

$$\frac{dn^O}{dI} < 0.$$

If instead e is sufficiently large ($e > e_n^m$):

$$\frac{dn^O}{dI} > 0.$$

4.2 Increasing market size

Consider now the effect of an increase in the size of the monopoly rents Π that can be captured by the successful innovator. Note first that Π does not affect the individual effort levels, since (8) and (9) do not depend on its value. An increase in Π however attracts additional firms, as can be inferred from (6) for n^O or (7) for n^B, and thus does generate more effort in this way. This effect is the same in shirking and bonding equilibria. Consider for example two values I^O and I^B for which the individual levels of efforts are the same but the equilibrium is respectively of the shirking and bonding types ($z^O(I^O) = z^B(I^B) = z$). Then from equations (6) and (7) the number of competitors is larger in the first case ($n^O(I^O) > n^B(I^B)$, since contractible effort costs more than non-contractible effort), but the effect of an increase in Π is the same:

$$\frac{dn^O}{d\Pi} = \frac{dn^B}{d\Pi} \left(= \frac{1}{-\log(1-z)} \right) > 0. \tag{10}$$

Note however that the result that market size does not affect the level of effort provided by each firm is somewhat surprising and actually specific to the functional forms adopted here. Intuitively, one would expect that the effect of Π on z detailed in a previous section carries on to the free-entry situation. This is indeed the case when slight variants of the model are considered. Assume for example that the private benefits, B, are obtained only when the firm is the (sole) innovator; the entrepreneur's objective then becomes:

$$(e+a) \prod j = 1 j \neq i^n (1 - z_j) \left[(1-\theta) \Pi + B \right] - C(e,a) - (F - I).$$

In that case, $z^*(n)$ is still determined by conditions (4) and (5), but the free-entry conditions become respectively:

$$\left(1+\frac{B}{\Pi}\right)\pi\left(e\right)-\frac{\beta}{2}\left(e-\tilde{e}\right)^{2} = F, \tag{11}$$

$$\left(1+\frac{B}{\Pi}\right)\pi\left(z\right)-\gamma a = F, \tag{12}$$

so that (8) and (9) become:

$$\left(1+\frac{B}{\Pi}\right)I+\frac{B}{\Pi}\left(e-\tilde{e}\right)e+\frac{\beta}{2}\left(e^{2}-\tilde{e}^{2}\right) = F, \tag{13}$$

$$\left(1+\frac{B}{\Pi}\right)I-\gamma a = F. \tag{14}$$

In both cases, the left-hand side decreases with Π; and as before, it increases with e in the first case but decreases with a in the second case. Hence:

$$\frac{de^{O}}{d\Pi}>0>\frac{da^{B}}{d\Pi}.$$

This in turn implies that an increase in Π induces fewer firms to enter the market in a shirking equilibrium than in a bonding equilibrium. Considering as before the cases $I=I^{O}$ and $I=I^{B}$, for which the individual levels of effort are the same but the equilibrium is respectively of the shirking and bonding types; it is still the case that the number of competitors is larger in the first case $\left(n^{O}(I^{O})>n^{B}(I^{B})\right)$, but the effect of an increase in Π is now larger in the bonding case:

$$\frac{dn^{B}}{d\Pi}>\frac{dn^{O}}{d\Pi}>0,$$

due to the fact that, in the bonding equilibrium, the increase in Π not only makes the market more profitable, but it moreover induces each incumbent to exert less effort, thereby attracting additional firms.

5 Robustness of the existence of the shirking and bonding regimes

We now consider a more general description of firms' efforts and argue that the existence of a shirking regime, for low levels of outside finance, and of a bonding regime, for larger levels of outside finance, are both fairly robust. We first provide a partial characterization (for very low and very large levels of outside finance) and then present simulations showing that the impact of outside finance on firm effort has a U shape (first decreasing, then increasing) for a large class of cost functions.

5.1 A more general framework

We maintain the assumption that the entrepreneur cannot contract on z but now assume that the contractible variable a summarizes various dimensions which influence the entrepreneur's *incentives* to exert effort. For example, in the case of a high-tech firm, where z can be thought of as the effective overall R&D investment, many dimensions are difficult to contract upon: research agendas, quality of researchers, efforts provided both by the researchers and the managerial team, etc.). The parameter a instead reflects actions like hiring additional R&D specialists, buying a research lab, giving high-powered incentive schemes to employees, setting up monitoring systems like a board of directors, and so forth. Such actions, taken *before* z is to be decided, often increase total cost (for example, buying an R&D lab involves a potentially large fixed cost, and high-powered incentives schemes leave large rents to the employees), but typically help the firm to commit itself to higher levels of

R&D.[16]

To capture the idea that the firm can use a to commit itself to a higher level of effort z, we will assume that an increase in a lowers the marginal cost of effort of the entrepreneur; more precisely, the private cost of effort is now of the form $C(z,a)$ and satisfies:[17]

$$C_{za}(z,a) < 0.$$

Assuming that the cost function is convex in z,[18] the entrepreneur will offer investors a contract (θ^*, a^*) and thereby commit himself to an effort z^* that solves:

$$\max_{0 \leq \theta, z \leq 1, a} \quad (1-\theta)\, z\Pi - C(z,a),$$

$$s.t. \ \theta z \Pi \geq I \qquad (IR)$$

$$(1-\theta)\Pi = C_z(z,a) \qquad (IC)$$

[16]For simplicity we maintain the assumption that I is independent of a, which rules out monetary costs. As already noted at the end of Section 2, the analysis is similar when I depends positively on a, which is likely if a refers to tangible investments, as long as increasing a raises not only the net monetary profits of the firm (through a higher z), but also the private cost of the entrepreneur, thereby introducing a difference of objectives between the entrepreneur and investors.

[17]Imposing restrictions, e.g., on research topics, jobs assignments or on how time must be spent also constitute actions that indirectly affect the amount of effort provided. While the assumption $C_{za} < 0$ can account for some of these effects, others would better be rendered by a constraint of the form $z \geq \underline{z}(a)$, with $\underline{z}'(a) > 0$. What is relevant for the analysis is that, ceteris paribus, an increase in a induces a higher z.

[In the model of the earlier section, the cost function can be expressed in terms of (z,a) as:

$$C(z,a) \equiv \gamma a + \frac{\beta}{2}(z - \tilde{e} - a)^2 \mid_{z \geq \tilde{e}+a}$$

and thus satisfies $C_{za} = -\beta < 0$ for $z > \tilde{e} + a$ (for $z < \tilde{e} + a$, $C_{za} = 0$).]

[18]The convexity assumption with respect to z may appear counterfactual in contexts such as R&D, given the often-mentioned existence of scale economies. What needs to be assumed here is simply that, for any given α, there exists a well-defined optimal level of efforts (z,a). A renormalization ($p(z)$ concave in z) could account for scale economies.

where the entrepreneur's incentive constraint, (IC), simply equates the marginal benefit of z (which decreases in θ) to its marginal cost (which decreases in a).[19]

The first-best levels for z and a, z^{FB} and a^{FB}, are obtained when the entrepreneur relies solely on internal finance ($\theta = I = 0$) and solves:

$$0 \leq z \leq 1, a \max z \Pi - C(z, a).$$

When $I > 0$, the entrepreneur must offer a positive share $\theta > 0$; even if this is partially compensated by an increase in a, the overall effect is first a reduction in the incentives to provide effort:

Assume $0 < z^{FB} < 1$. Then, starting from $I = 0$, a small increase in I does not affect a but decreases z:

$$\left.\frac{\partial \theta^*}{\partial I}\right|_{I=0} = 0, \left.\frac{\partial a^*}{\partial I}\right|_{I=0} = 0 and \left.\frac{\partial z^*}{\partial I}\right|_{I=0} < 0.$$

See Appendix.

[19] If one interprets the variable a as the intensity of monitoring schemes like the number of seats for the investor on a Board of Directors, one could replace $C(z, a)$ with

$$K(z) + M(a) + S(\widetilde{z}, z, a)$$

where z now represents a "contractually agreed upon" level of effort, $K(z)$ is the cost of effort and $M(a)$ the cost of monitoring for the entrepreneur (loss of autonomy); in this interpretation, $S(\widetilde{z}, z, a)$ represents the cost of "shirking" (providing an effort $\widetilde{z}$ lower than the agreed upon level z): this is the cost to "hide" one's underprovision of effort, which increases with the monitoring intensity a. If for example

$$S(\widetilde{z}, z, a) = \min\{0; a(z - \widetilde{z})\},$$

the minimal amount of monitoring needed to ensure that the agreed effort z is indeed provided is given by

$$(1 - \alpha)\pi = K'(z) - a,$$

which yields the positive relation between z and a, given α.

As I grows, the entrepreneur can satisfy (IR) by either raising θ (which however adversely affects z) and/or by raising z, through an increase in a. The lemma establishes that, when I rises from 0, the entrepreneur first puts the emphasis on θ more than on a, so that θ^* rises but a^* remains essentially constant: the reason is that for θ close to 0, a rise in a (which reduces z) has almost no impact on the entrepreneur's repayment capacity $\theta z \Pi$.

By continuity, dz^*/dI remains negative for I small. However, this cannot remain the case for I large: the outside investors' participation constraint (IR) implies

$$z^* \geq \frac{I}{\theta \Pi} \geq \frac{I}{\Pi},$$

so that for I large enough (e.g., $I > z^{FB}\Pi$), z^* must *exceed* the first-best level in order to convince investors to lend all this money to the firm.

This observation is the key behind the nonmonotonicity of the entrepreneur's effort with respect to the level of outside finance: outside finance reduces the entrepreneur's claim to monetary returns (which tends to discourage non-contractible effort), but also has a "disciplinary" role, by requiring higher (contractible) efforts in order to allow investors to get their money back. The first effect dominates for small levels of outside finance, while the latter effect prevails for larger financial needs. Hence, a shirking regime obtains for small levels of I, while a bonding regime obtains for large levels of I.

Note also that the first-best level of effort z^{FB} increases with the value of the innovation (Π); hence, z^* can also be expected to increase in I when I is small and z^* close to z^{FB}. In contrast, when I is large an increase in Π relaxes the (IR) constraint and may well allow the entrepreneur to reduce his effort z^*. The following subsection presents simulations that confirm this intuition.

Finally, while we have focused on a standard moral hazard problem (namely, inducing the entrepreneur to provide effort), in many situations, and particularly in the case of start-ups and venture capital, a key issue is rather to ensure that the entrepreneur provides the *right type of effort* –enhancing the prospects for monetary payoffs, as opposed to undertaking fancy projects with little commercial value. However, we can easily adapt the framework to account for such issues. For example, suppose that the entrepreneur must choose a value $e \in [0, 1]$; a value close to 1 increases the entrepreneur's private benefits B whereas a value close to 0 increases the probability of commercial success: $z = 1 - e$ and $B = B(e, a)$, with $B_e > 0$. Then, the above analysis goes through, assuming that the contractible parameter a reduces the private benefits derived from e ($B_{ea} < 0$) (alternatively, it could impose an upper limit on e), so that the entrepreneur's objective becomes:

$$B(e, a) + (1 - \theta)(1 - e)\Pi = B(1 - z, a) + (1 - \theta) z\Pi,$$

and is thus as before of the form $(1 - \theta) z\Pi - C(z, a)$, with $C(z, a) = -B(1 - z, a)$ satisfying $C_{za} < 0$.

5.2 Simulations

The previous subsection shows that, under general conditions, firms are in a shirking (respectively, bonding) regime for low (resp. sufficiently high) levels of outside finance. The stylized example introduced in Section 2 further suggests a U-shape relationship between entrepreneurial effort and the need for outside finance. We now present simulations that suggest that this U-shape extends to a broader class of situations. We stick to the general framework outlined above, and consider the following class of cost functions:

$$C(z,a) = \frac{z^\gamma}{\gamma}/a^\lambda \lambda + \beta a^\rho \rho, \tag{15}$$

where $\lambda, \gamma, \beta, \rho$ are positive numbers. The parameter γ measures its degree of diminishing returns to non-contractible effort, while λ is related to the effectiveness of the commitment technology, and the parameters β and ρ affect the slope and convexity of the cost of contractible effort.

5.2.1 Individual firm behavior

Figure 2 depicts an individual firm's optimal R&D effort z^* as a function of I , in the benchmark case where $\beta = 1/2$, $\lambda = 1$, $\gamma = \rho = 2$, and $\Pi = .8$. The function is clearly U-shaped, with the shirking region corresponding to the downward-sloping part of the curve and the bonding region corresponding to its upward-sloping part.

Starting from this benchmark case, we have performed several simulations to investigate how changes in the parameters γ, λ and ρ affect the $z^*(I)$ curve. These simulations[20] all generate a U-shaped relationship between R&D effort and outside finance. In addition, the bonding region increases when the commitment technology becomes more efficient (i.e., when λ increases or ρ decreases). Similarly, the shirking region expands when γ (the degree of diminishing returns to effort) increases. Last, an increase in either λ or ρ increases the firm's R&D effort, whereas an increase in the para-

[20]The following values have been used: $(1.8, 2, 2.2)$ for γ and ρ, and $(0.8, 1, 1.2)$ for λ and β. Due to space concerns, the results of these simulations are not displayed here but are available upon request.

meter γ increases effort in the shirking regime but decreases it in the bonding regime.

5.2.2 Oligopolistic competition

We now consider the same class of cost functions in an oligopolistic context where n firms engage in simultaneous R&D investments. A firm that expects each of its $(n-1)$ competitors to exert the same R&D effort $\tilde{z}$, will determine its best response by solving:

$$0 \leq \alpha, z \leq 1, a\max \qquad (1-\alpha)\, z\varepsilon\Pi - C\,(z,a)\,,$$

$$s.t.\ \alpha z\varepsilon\Pi \geq I \qquad (IR)$$

$$(1-\alpha)\varepsilon\Pi = C_z(z,a) \qquad (IC)$$

where $\varepsilon \equiv \phi + (1-\phi)\,(1-\tilde{z})^{n-1}$: a positive ϕ reflects the possibility that the value of a firm's innovation be unaffected by the success or failure of its competitors;[21] Section 3 concentrated on the benchmark case $\phi = 0$, where R&D projects are perfect substitutes.

Figures 3a and 3b depict the reaction function $z\,(\tilde{z})$ .for different values of I, respectively when $\phi = 0$ and $\phi = 0.2$. Figures 4a and 4b depict the same reaction functions for different values of Π. The reaction function is always U-shaped: it is first decreasing for low values of rivals' efforts (which implies a high "adjusted profitability" $\Pi' = \varepsilon\Pi$) and then increasing for high values of the rivals' efforts (or low adjusted profitability). Furthermore, an increase in the need for outside finance (I) increases the slope of the reaction function and may even transforms locally a decreasing reaction function (strategic substitutability) into an increasing one

[21] For example, an innovation by a given firm could lead to a new product that is not only better than this firm's previous product but also not perfectly substitutable for the new products generated by other firms.

(strategic complementarity). Also note that reaction functions corresponding to different values of I or Π cross when $\tilde{z}$ increases; in other words, whilst an increase in I or a decrease in Π increases the R&D best response $z(\tilde{z})$ for low $\tilde{z}$ (when there is high strategic substitutablity), it reduces it for high $\tilde{z}$ (when there is high strategic complementarity).[22]

The intersection of those reaction functions with the 45^0 line, produces symmetric equilibria that may lie both, in the strategic substitutability and the strategic complementarity part of the reaction functions; we refer to these equilibria respectively as "shirking" and "bonding" equilibria. These figures show that an increase in I, or a decrease in Π, makes it more likely that the symmetric equilibrium when, it exists, lies in the bonding region. Furthermore, whilst a "shirking" equilibrium is always stable, there may exist unstable bonding equilibria. This occurs when the reaction function intersects the 45^0 line twice: in that case, the first equilibrium (which may be of the shirking or bonding type) is stable while the second is unstable and of the bonding type. The comparison between figures 3 and 4 suggests that these unstable equilibria disappear when ϕ increases, that is, when R&D efforts are less substitutable.

6 Applying the framework: barring entry and inducing exit

The analysis in the above sections stresses the fact that, depending on the level of outside finance firms require, competition can

[22] In addition, our simulations show that the impact of the parameters λ, γ, and ρ on the size of the shirking and bonding regions are similar to those obtained in the single-firm case.

switch from a situation of strategic substitutes to one of strategic complements. As is well-known in the IO literature (see for example Fudenberg-Tirole, 1984), if one allows for an earlier stage of the game where firms can take *observable* actions that credibly alter their reaction functions, then firms can influence the continuation game and thereby enjoy strategic gains. The typical analysis takes a two-stage perspective and considers two firms, one of which can move in the first stage to influence its second-stage reaction function. The other firm can be seen as a potential entrant, which has to pay a fixed cost to enter in stage 2; equivalently, it can already be in the market, but leaves in stage 2 if its profit is insufficient. This can thus be interpreted as a problem of barriers to entry or, equivalently, of exit inducement.

What are the new insights obtained by performing this otherwise standard exercise in our model ? The key lesson from the IO literature is that firms want to commit to being *tougher* (expand their own output) in order to bar entry or induce exit, or when they are simply trying to influence their existing competitors' output in the case of *strategic substitutes* (downward-sloping reaction functions), but they want to commit to being *softer* (restrict their own output) when they want to influence their competitors' output in the case of *strategic complements* (upward-sloping reaction functions). In the literature, the nature of competition is moreover given by the strategic form of the game firms play: strategic substitutability with traditional quantity competition (Cournot) and strategic complementarity with traditional price competition (Bertrand).

We thus end up with a taxonomy that contains three dimensions: (i) the nature of competition; (ii) the need for the firm to share the market versus its ability to bar entry/induce exit; and (iii) whether the relevant strategic instrument makes the firm tougher or softer.

In our model, it is (i) that is fundamentally affected: first, *without* changing the strategic form of the game, the reaction function of the firm is either upward- or downward-sloping depending on its level of outside finance; second, the sign of the slope of the reaction function is *firm-specific*: it will be negative for firms with low external finance (shirking regime) and positive for firms with high external finance (bonding regime).

Once this is understood, we can proceed quickly with the usual exercise. For example, we can consider two instruments to bar entry or induce exit. First, as in the IO literature, a standard capital investment: specifically, assume that by paying $h(K)$, the firm raises its success probability from z to $K + z$. Second, a policy to distribute or retain prior earnings, which thus raises or reduces the necessary amount of outside finance I.

Remember that, to have strategic value, the firms' choices of investment and/or earning policy have to be observable by the rival, an assumption which may be more or less realistic depending on the specific market context.

		The competitor		
		low I	high I	barely profitable
The firm	low I	Overinvest Underdistribute	Underinvest Overdistribute	Overinvest Underdistribute
	high I	Overinvest Overdistribute	Underinvest Underdistribute	Overinvest Overdistribute

Table 1: Strategic incentives and external finance

Table 1 displays the optimal strategy of the firm in the various cases. It focuses on strategic incentives, so that "Overinvest"/"Underinvest" concerns incentives relative to the optimal investment (K) in the absence of any effect on the firm's competitor, and the same is true in terms of distribution of prior earnings. Concerning the competitor, the last column assumes that barring entry or inducing exit is the optimal strategy, while sharing the market is the optimal strategy in the first two columns.

Table 1 is easily understood given the comparative statics of z^* with respect to I and Π: *first*, when the incumbent can hope to avoid having to share the market with the competitor, or when instead the competitor enjoys a low level of outside finance, the incumbent's incentive is to be tough; instead, when the incumbent firm faces a competitor saddled with a high amount of outside finance, the incumbent's incentive is to be soft. Second, while investment always makes one tougher, retaining earnings (lowering I) makes one tougher under low outside finance, but softer under high outside finance.

An interesting point that did not come out in earlier sections is the non-monotonicity of strategic incentives with respect to the level of outside finance of the competitor: namely, the incumbent wants to be tough if the competitor has a low I, soft when the competitor has a higher I, and tough again when its I becomes so high that barring entry or inducing exit becomes possible.

7 Conclusion

We have analyzed a simple moral hazard model where firms can use two instruments to influence their repayments to investors: profit-sharing, and verifiable investments (in technology or monitoring schemes) which influence the marginal cost of effort. As the need for outside finance rises, the impact on firms' effort is *non-monotonic*: initially, firms mainly raise the share of profits going to investors, which *lowers* effort; but, as the need for external finance rises, at some point the firm's effort must increase as well, which can be achieved through a commitment to a higher level of verifiable investment. This nonmonotonicity has significant implications for firm behavior, changing the slope of reaction functions, and thus the nature of strategic interactions: in our model, the *same* game form leads to strategic substitutability or strategic complementarity depending on the level of outside finance. This stands in contrast with the existing IO literature, where the nature of strategic interactions hinges on a given game form (e.g. quantity competition versus price competition).

Our paper delivers a number of predictions relating for example the level of outside finance to:

- managerial effort: as just mentioned, it first decreases and then increases with the need for outside finance;
- the structure of investments (verifiable vs intangible): firms relying more on outside finance invest relatively more on verifiable investments;
- the nature of strategic interaction in oligopolies: there is a trend towards more strategic complementarity (upward-sloping investment reaction functions) when firms rely more on external finance;

- the impact of product market competition on firms' investments: competition which may otherwise discourage investment instead boosts it when firms rely on external finance;
- the reaction to demand fluctuations: firms relying mostly on internal finance invest more in response to a positive shock on demand;
- the relation between market size and market concentration: as market size increases, we would expect concentration to decrease more when firms in the industry rely more on external finance.

Testing the various predictions of our model goes obviously beyond the scope of this paper. However, we can already point to some encouraging indirect evidence: on the relationship between product market competition and incentives, the main prediction from Section 3 is that whilst competition may discourage effort and therefore have a detrimental effect on performance for low levels of outside finance, competition enhances investment incentives and performance for high levels of outside finance. This prediction is in fact consistent with recent empirical work by Nickell et al. (1997) who show that the positive effect of competition on productivity growth tends to be reduced and even inverted for firms with a dominant shareholder (and therefore with a *small* agency problem), but that this effect remains substantial in firms with dispersed shareholding (and therefore more subject to agency problems).

The setup analyzed in this paper can be extended in several directions. First, we have limited ourselves to a success/failure framework. This has allowed us to stress the fundamental trade-offs of external finance on entrepreneurial effort: on the one hand, the incentive constraint of the entrepreneur implies that more external finance should reduce effort as it reduces the extent of his residual claim; on the other hand, the participation constraint of the investor implies that more external finance may force the entrepreneur to increase his commitment to effort provision[23]. This success/failure framework should however be generalized in order to investigate in more details the *form* of outside finance and revisit the important question of the relationship(s) between capital structure and the various types of strategic interactions emphasized by the IO literature.

Another extension would be to investigate the implications of our analysis in a macroeconomic context. In this respect, in a previous paper (Aghion et al. (1999)), we have contrasted the two polar cases of - respectively - profit-maximizing firms (with no outside finance) and "conservative" firms (with implicitly high levels of outside finance) in the context of an infinite horizon, general equilibrium model of endogenous technical change. While focusing on a static/partial equilibrium analysis, the present paper provides microfoundations to that and other forthcoming analyses of the relationship between agency costs, market competition and the dynamic evolution of aggregate output.

[23]Note that this fundamental trade-off also drives the incentive effects of debt and equity in a multi-outcome set-up, in particular in the agency models of corporate finance mentionned in the introduction.

A Proof of Lemma 2

The proof is by contradiction. Note first that for $e > \tilde{e}$, the incentive-compatibility condition can be replaced with its first-order condition:

$$(1-\theta)\,\Pi = \beta\,(e-\tilde{e})\,.$$

Hence, if $e^* > \tilde{e}$ and $a^* > 0$, the solution $\{e^*, a^*\}$ solves (using (IC) to eliminate θ):

$e, a\max \quad (e+a)\,\Pi - \beta 2\,(e-\tilde{e})^2 - \gamma a + B - F$

$$s.t.\ (IR)$$
$$(e+a)\,[\Pi - \beta\,(e-\tilde{e})] \geq I$$

Denoting by $\lambda > 0$ the Lagrange multiplier associated with (IR), the first-order conditions with respect to e and a are:

$$\begin{aligned}
\Pi - \beta\,(e-\tilde{e}) + \lambda\,[\Pi - \beta\,(e-\tilde{e}) - \beta\,(e+a)] &= 0, \\
\Pi - \gamma + \lambda\,[\Pi - \beta\,(e-\tilde{e})] &= 0,
\end{aligned}$$

which imply $\Pi > \beta\,(e-\tilde{e})$ (since $\lambda > 0$) and (eliminating λ):

$$[\Pi - \beta\,(e-\tilde{e})]^2 + (\gamma - \Pi)\,[\Pi - \beta\,(2e-\tilde{e})] = \beta\,(\gamma-\Pi)\,a,$$

and thus, since by assumption $e > \tilde{e}$:

$$\beta\,(\gamma-\Pi)\,a < \Pi^2 + (\gamma-\Pi)\,[\Pi - \beta\tilde{e}]\,.$$

But under H_1 the right-hand side is negative, a contradiction.

B Proof of Proposition 3.2

We first establish two useful preliminary results:

The profit function $\pi(z)$ is quasi-concave with respect to z on the relevant range $z \leq 1$: it increases with z for $z < 1/n$ and decreases for $z > 1/n$. Under H_1, the repayment function $R(e)$ is decreasing on the relevant range $\tilde{e} < e < 1$.

It suffices to analyze the derivatives of these two functions:

$$\pi'(z) = (1 - nz)(1 - z)^{n-2}\,\Pi.$$

is positive for $z < 1/n$ and negative for $z > 1/n$.

$$R'(e) = (1 - ne)(1 - e)^{n-2}\,\Pi - \beta(2e - \tilde{e})$$

is negative for $e > \tilde{e}$, since then $\beta(2e - \tilde{e}) > \beta\tilde{e} > \Pi > (1 - e)^{n-1}\,\Pi$.

We now characterize symmetric (pure strategy) equilibria. By construction, because of the properties of the best response functions, symmetric equilibria are of two possible types: $(e^* > \tilde{e}, a^* = 0)$ or $(e^* = \tilde{e}, a^* > 0)$.

In equilibria of the first type, $R(e^*) = I$; since $e^* > \tilde{e}$ and $R(e)$ decreases with e on this range, e^* must moreover be the largest solution to $R(e) = I$, and $I \leq R(\tilde{e}) = \pi(\tilde{e}) = \tilde{I}_n$; conversely, for any $I \leq \tilde{I}_n$, there exists a unique $e^* > \tilde{e}$ satisfying $R(e) = I$ and each firm is on its best-response when they all choose $z = e^*$ (i.e., $e^* = z^r(e^*)$).[24] Hence, for any $I \leq \tilde{I}_n$, there exists a unique

[24]It suffices to note that

$$I = R(e) = e(1 - e)^{n-1}\,\Pi - \beta(e - \tilde{e})\,e$$

implies

$$I \leq \tilde{I}_n^r(e) = \tilde{e}(1 - e)^{n-1}\,\Pi,$$

since under H_1 the function $\hat{R}(e, z) \equiv \left[(1 - z)^{n-1}\,\Pi - \beta(e - \tilde{e})\right] e$ decreases with e.

symmetric equilibrium of the type $(e^* > \tilde{e}, a^* = 0)$.

In equilibria of the second type, profit-sharing rules are then $\theta^* = 1$, and $z^* = \tilde{e} + a^*$ must thus satisfy: $\pi(z^*) = I$. Since $\pi(.)$ is quasi-concave and achieves its maximum for $z = 1/n$, as long as I remains smaller than $\overline{I}_n \equiv \max_z \pi(z)$, there exist two candidate values for z^*, one (z') below $1/n$ and another one (z'') above $1/n$.

We now show that z'' cannot constitute a stable equilibrium. To see this, consider a small increase in firm 1's effort z_1, starting from a candidate equilibrium configuration. The other firms's collective best response will be to choose $\hat{z}$ such that:

$$\hat{z}(1-\hat{z})^{n-2}(1-z_1)\Pi = I,$$

yielding, when evaluated at the equilibrium point $z_1 = \hat{z} = z^*$:

$$\frac{d\hat{z}}{dz_1} = \left.\frac{(1-\hat{z})\hat{z}}{(1-z_1)(1-(n-1)\hat{z})}\right|_{z_1=\hat{z}=z^*} = \frac{z^*}{1-(n-1)z^*}.$$

This change in the other firms' efforts would in turn induce firm 1 to choose $\tilde{z}_1$ such that:

$$\tilde{z}_1(1-\hat{z})^{n-1}\Pi = I,$$

yielding, at the equilibrium point:

$$\frac{d\tilde{z}_1}{d\hat{z}} = \left.\frac{(n-1)\tilde{z}}{1-\hat{z}}\right|_{\tilde{z}_1=\hat{z}=z^*} = \frac{(n-1)z^*}{1-z^*}.$$

The equilibrium is stable if and only if $\frac{d\hat{z}}{dz_1}.\frac{d\tilde{z}_1}{d\hat{z}} < 1$, that is, if and only if:

$$z^*(n-1)z^* < (1-(n-1)z^*)(1-z^*),$$

or:

$$z^* < 1/n.$$

Therefore, if $\tilde{e} < 1/n$, for any $I \in]\tilde{I}_n, \overline{I}_n]$ there exists one candidate symmetric stable equilibrium of the second type, and there is no such candidate if I lies outside this range (note in particular that $I < \tilde{I}_n = R(\tilde{e}) = \pi(\tilde{e})$ would imply either $z^* < \tilde{e}$ or $z^* > 1/n$). Conversely, if for $I \in]\tilde{I}_n, \overline{I}_n]$ all firms choose $z^* = \tilde{e} + a^* \in]\tilde{e}, 1/n]$ such that $\pi(z^*) = z^*(1-z^*)^{n-1}\Pi = I$, they are all on their best response functions since then $I > \tilde{I}_n^r(z^*) = \tilde{e}(1-z^*)^{n-1}\Pi$. Hence, the candidate equilibrium is indeed an equilibrium.

Putting those arguments together completes the proof of the Proposition.

C Welfare analysis

Let $S \geq \Pi$ denote the social value of innovation.[25] The social welfare generated by n firms that invest the same amount of R&D effort (e or a), can then be expressed as:

$$W(n,e,a) = [1-(1-(e+a))^n]\,S - n\left[F - B + \frac{\beta}{2}(e-\tilde{e})^2 + \gamma a\right].$$

Since $\gamma > 1$, the unconstrained social optimum entails $a = 0$ and $(n,e) = \arg\max_{n,e} W(n,e,0)$. However, if, as seems plausible, neither e nor a can be monitored by a social planner, the (constrained) socially optimal number of firms may instead be defined, following von Weizsäcker (1980) or Mankiw-Whinston (1986), by:

$$n^S = \arg\max_n W(n, e^*(n), a^*(n)),$$

[25]The difference $S - \Pi$ can for example reflect knowledge externalities which private firms do not necessarily take into account when deciding on R&D investments.

where $e^*(n)$ and $a^*(n)$ are the equilibrium efforts chosen by n firms in a symmetric equilibrium.

What can we say about the comparison between the *equilibrium number* and the *socially optimal number* of firms? As usual, this comparison depends on the appropriability of social surplus (S relative to Π): the closer profits are to social surplus, the higher the risk of excessive entry due to a "business stealing" effect (see e.g. Tirole (1988), chapters 7 and 8); in particular, there is always excessive entry when $S = \Pi$. But in addition, entry exerts a negative externality on firms' effort levels in a shirking equilibrium, and a positive externality in a bonding equilibrium. Consequently, for some values of S and Π entry will be excessive in the first regime and insufficient in the second.

To see this more formally, we address the following question: if the social planner could (slightly) modify the number of firms above or below the free-entry equilibrium, should he increase or decrease that number?

In a shirking equilibrium, $a^* = 0$, $e^*(n)$ is defined by (4) and:

$$\frac{dW}{dn} = \frac{\partial W}{\partial n} + \frac{\partial W}{\partial e}\frac{de^*}{dn},$$

where:

i) $\partial W \partial n = -\log(1-e)(1-e)^n S - \left[F - B + \beta 2 (e - \tilde{e})^2\right]$

or, using the free-entry condition (6):

$$\frac{\partial W}{\partial n} = (1-e)^{n-1}\left[-\log(1-e)(1-e)S - e\Pi\right].$$

The expression in brackets is negative for $S = \Pi$;[26] therefore, by

[26] The function $g(.)$, defined by:

$$g(x) = -x - (1-x)\log(1-x)$$

is negative for all $x \in [0,1)$.

continuity, there exists $S_n > \Pi$ such that $\partial W / \partial n < 0$ for $S < S_n$. However, for S large enough relative to Π, the expression is positive.

ii) $\partial W \partial e = n \left[(1-e)^{n-1} S - \beta (e - \tilde{e}) \right]$ or, using the incentive condition:

$$\frac{\partial W}{\partial e} = n (1-e)^{n-1} \left[S - (1-\theta) \Pi \right] > 0.$$

Since entry discourages effort in the shirking regime ($de^* / dn < 0$), the indirect effect, through a decrease in the equilibrium level of effort e^*, is always negative.

Consider now the polar case where under *laissez-faire* the equilibrium is in the bonding regime. Then $e^*(n) = 0$ and $a^*(n)$ is determined by (5). Proceeding as above, we have:

$$\frac{dW}{dn} = \frac{\partial W}{\partial n} + \frac{\partial W}{\partial a} \frac{da^*}{dn},$$

where:

i)

$$\partial W \partial n = (1-z)^{n-1} \left[-\log (1-z) (1-z) S - z\Pi \right]$$

is again negative if and only if $S < S_n$ (and we have the same S_n as above if the levels of R&D are the same, i.e., $z^B = \tilde{e} + a^B = e^O$).

ii)

$$\frac{\partial W}{\partial a} = n \left[(1-z)^{n-1} S - \gamma \right]$$

is negative under H_1 for S close to Π but positive for S sufficiently large. Since $da^* / dn > 0$, we obtain:

Starting from the free-entry equilibrium, a small increase in the number of firms:

i) has a direct effect which is negative for S close to Π and positive for S large enough, whether the equilibrium is of the shirking or bonding type;

ii) has also an indirect effect, which is negative in a shirking equilibrium, through a decrease in e^*, but can be positive for S large enough in a bonding equilibrium, through an increase in a^*.

That is, equilibrium entry is always excessive when a successful innovator can appropriate the entire social surplus ($S = \Pi$), but can be insufficient when there are externalities that the innovator cannot appropriate ($S > \Pi$). Furthermore, whereas an increase in the number of competitors has a negative indirect impact in a shirking equilibrium, where it adversely affects the firms' efforts, it may have a positive indirect effect in a bonding equilibrium, by inducing each firm to commit itself to larger levels of (contractible) efforts; since contractible effort is too costly to be privately efficient, this indirect effect can only be positive when the social value of the innovation is sufficiently important, that is, when S is sufficiently larger than Π.

D Proof of Lemma 5.1

Eliminating θ by combining (IC) and (IR) –which is clearly binding at the optimum– allows to rewrite the entrepreneur's problem as:

$$a, z\max \qquad z\Pi - C(z,a) \qquad .$$

$$s.t.\ z\left(\Pi - C_z(z,a)\right) = I \quad (*)$$

The first-order conditions with respect to z and a are respectively (denoting λ the Lagrange multiplier associated with the constraint $(*)$):

$$\begin{aligned}
\Pi - C_z(z,a) &= \frac{\lambda}{1+\lambda} z C_{zz}(z,a), \\
C_a(z,a) &= -\lambda z C_{za}(z,a),
\end{aligned}$$

implying:

$$(C_a - zC_{za})(\Pi - C_z) = zC_{zz}C_a$$

Differentiating the constraint $(*)$ and this last condition with respect to (z,a,I) yields, when evaluated at $I=0$ (where $C_z = \Pi$ and $C_a = 0$):

$$\begin{aligned}
z(C_{zz}dz + C_{za}da) &= -dI \\
zC_{zz}(C_{az}dz + C_{aa}da) &= -C_{za}dI,
\end{aligned}$$

which imply (since $C_{zz} > 0$ and $C_{aa}C_{zz} > C_{az}C_{za}$ from the convexity of $C(.,.)$):

$$\left.\frac{\partial z^*}{\partial I}\right|_{I=0} = -\frac{1}{zC_{zz}} < 0, \left.\frac{\partial a^*}{\partial I}\right|_{I=0} = 0,$$

and thus

$$\left.\frac{\partial \theta^*}{\partial I}\right|_{I=0} = 0.$$

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