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General Equilibrium in a Labor-Managed Economy with Uncertainty and Incomplete Markets*

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This paper studies the free entry competitive equilibria of a labor managed economy when there are less than complete contingent claims markets. First, we prove the existence of a simple labor management equilibrium which is inefficient in the Diamond sense. Second, we introduce an insurance system internal to the firm and show that the resulting equilibrium exists and is Diamond efficient. This insurance system plays for a labor managed economy the same role as fixed obligation debt markets in an entrepreneurial stock market equilibrium.

Key Words: Labor-management; Uncertainty; Incomplete markets; General equilibrium.

JEL Classification Numbers: D5, D8, J5.

1. INTRODUCTION

The economic theory of labor management has clearly established the equivalence of long-run labor management equilibria and competitive equilibria in models characterized by competitive behavior of all agents, complete (non-labor) markets and free entry (see in particular Ward [1958], Domar [1966], Vanek [1970], Drèze [1976], Ichiishi [1977], and Greenberg [1979]]. As suggested by Drèze [1976], it is therefore appropriate to compare the two systems on other grounds.

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This paper studies labor management under uncertainty when there are less than complete contingent claims markets. Following the previous literature, we assume that workers receive a fixed share of the value added generated by the firm at which they are employed. In an incomplete market setting, this assumption can be interpreted as a constraint on the form of the contingent claims allocations that can be exchanged. The constraint is, in fact, closely related to that imposed by Diamond in his 1967 paper on the stock market. In that paper, there are markets for firm shares and for riskless debt. Thus the available contingent claims allocations are those which can be constructed as a sum of riskless payments and shares of the random outputs of firms. We will call contingent claims allocations which have this structure Diamond feasible.

The first section introduces the formal model of the economic environment. This model is essentially the same as that studied in Kihlstrom-Laffont [1982]. It differs from the earlier framework of that paper in only one respect: there are two productive inputs, capital and labor, rather than one. By employing this two-input model of the economic environment, we are able to compare the economic outcomes of the worker-management system with the outcomes generated by the capitalistic entrepreneurial stock market economy studied in Kihlstrom-Laffont [1982]. We begin the analysis in Section II by considering a labor-management system in which workers receive only a fixed share, proportional to their labor supply, of the value added of the single firm in which they work. The resulting equilibrium is called a Simple Labor Management Equilibrium(SLME). It is formally defined and shown to exist in that section. In an appendix, an example is discussed in which the equilibrium exhibits an inefficiency in the sense of Diamond that, in general, characterizes SLME. The problem arises because, in an SLME, although the sharing rule provides Diamond-feasible allocations to labor managers, it does not permit the exchange of risks required for Diamond efficiency when workers have different degrees of risk aversion. In a stock market economy, these exchanges are possible because of the existence of fixed obligation debt markets. In Section III, we show that the same possibilities for risk trading can be incorporated in a worker-management system by the introduction of an insurance system internal to the firm. The resulting equilibrium is called a Labor Management Equilibrium(LME) and is shown in an appendix to be equivalent to an entrepreneurial stock market equilibrium. Using this equivalence, a labor management equilibrium is shown to exist. The set of LME is also shown to coincide with the core in the set of Diamond feasible allocations. As a consequence, the LME is Diamond efficient.

It should be noted that the efficiency of the LME is attained even though workers do not "diversify;" i.e. , they satisfy the constraint which limits them to employment as worker-managers in only one firm. The imposition of this constraint creates a potential for inefficiency because it eliminates some Diamond-feasible allocations from the set of contingent claims allocations available to worker-managers. This potential for inefficiency is unrealized in the present analysis because of the strong assumption made about the statistical dependence of the output of different firms. It is assumed, in particular, that the same random factors influence the output of all firms. As a result of this hypothesis, there is no need for diversification. Thus workers, if given the option of diversifying by accepting employment as worker-manages in more than one firm, would not find it advantageous to exercise that option. If, however, the assumption of statistical dependence of firms' output were dropped, Diamond-efficient allocations would be characterized by diversification. Thus labor management equilibria would be efficient only if it were possible to be a labor-manager in several firms or if labor-managed firms were diversified conglomerates.

By reversing the roles of capital and labor, our analysis can be reinterpreted to yield a theory of capital management. The comparison between capital and labor management in terms of Diamond efficiency is then reduced to a comparison of the costs of operating a riskless debt market in a capitalist system with the costs of introducing internal insurance to a labor management system.

2. THE BASIC MODEL

First, we describe the economic environment within which the alternative economic systems to be considered will operate. Then we describe the main institutional features of the labor management systems studied below.

2.1. THE ECONOMIC ENVIRONMENT

We consider an economy with three commodities, a consumption good referred to as "income" and two production goods "labor" and "capital."

The set of agents is represented by the continuum $[0, 1]$,[1] Although there is a continuum of individuals, there is only a finite number, I, of types of individuals. It will often be convenient to let I represent the set of agent types $\{1, \cdots, I\}$ as well as the number of types. For $i = 1, \ldots, I$, we will let μ_i denote the Lebesgue measure of the set of type i individuals. We must then have $\sum_{i \in I} \mu_i = 1$.

A representative agent of type i is identified with his utility function $u_i(\cdot)$ and his vector of initial resource holdings $(\omega_{li}, \omega_{ki}, \omega_{ci})$ where

ω_{li} = a type i individual's initial labor allocation,

ω_{ki} = a type i individual's initial capital allocation,

and

[1] Throughout the paper, we will avoid all trivial theoretical statements.

ω_{ci} = a type i individual's initial income allocation.

We will assume for simplicity that the sole argument of the utility function is income .

The economy's total endowments of the resources are denoted by

$$\overline{\omega}_l = \sum_{i \in I} \mu_i \omega_{li},$$

$$\overline{\omega}_k = \sum_{i \in I} \mu_i \omega_{ki}$$

and

$$\overline{\omega}_c = \sum_{i \in I} \mu_i \omega_{ci}$$

Assumption 1.

a) $\omega_{ci} > 0;\ \omega_{li} > 0;\ \omega_{ki} > 0;\quad i = 1, \ldots, I.$

b) $u_i(\cdot)$ strictly increasing and strictly concave on $[0, +\infty),\quad i = 1, \ldots, I.$

A risky technology is available to any group of agents willing to pay a set-up cost s in labor units. It is defined by a production function: $g(k, l, \widetilde{x})$, where

k is the amount of capital used,

l is the amount of labor used (in addition to s), and

$\widetilde{x}$ is the random variable which is the same for any firm that uses the technology.

Assumption 2.

a) $\widetilde{x}$ takes its value in a finite set: $x \in \{x_1, \cdots, x_s\} = X$.

b) There exists $x \in X$ such that $g(k, l, x) = 0,\quad k \geq 0,\quad l \geq 0.$

c) $g(0, l, x) = 0,\quad l \geq 0,\ x \in X;$
$g(k, 0, x) = 0,\ k \geq 0,\ x \in X.$

d) For all $x \in X, g(\cdot, \cdot, x)$ is continuous, increasing, and strictly concave in (k, l) on the entire domain, $[0, +\infty)^2$.

e) For each x, the asymptotic cone of the production set defined by g (with the set-up cost included) is $\{k, l + s, y : k \geq 0, l \geq 0, y = 0\}$.

2.2. THE BASIC INSTITUTIONAL FRAMEWORK

Both types of labor-management systems considered below are assumed to be characterized by certain common institutional features.

First there are no markets for contingent claims to the consumption good. There is, however, a market in which capital is traded for riskless debt. The price of capital is denominated in income terms and is denoted by r. In order to guarantee that debt is, in fact, riskless, we want to avoid the possibility of bankruptcy. Thus we make

Assumption 3.

An agent never makes a decision which could lead to bankruptcy with a positive probability.

Worker-managed firms are, in essence, coalitions of workers who share the proceeds of production. Each worker's fractional share in the firm's value added is fixed before the firm's output is known, i.e. before the value taken by x is observed. A worker's share is, furthermore, proportional to the quantity of labor he supplies to the firm.

Firms created by coalitions are operated according to the wishes of the coalition's labor-managers. Any worker who disagrees with the actions taken by a coalition is free to leave and join another coalition. If there is no other existing coalition he would prefer to join, he is also free to attempt to create a new coalition. The only cost of creating a coalition is the set-up cost(s) of creating the firm(s) it will operate. Thus there is unlimited, but costly, entry.

3. THE SIMPLE LABOR MANAGEMENT EQUILIBRIUM

The absence of labor markets is a basic feature of the systems to which the term "labor management" is commonly applied. The present section proceeds with this strict interpretation of labor management. In order to simplify the current exposition, we also assume that each worker can supply his labor to only one firm. As we show in Appendix A, however, even if this assumption were dropped, it would be satisfied in equilibrium because of the statistical dependence of firm outputs.

When the shares of value added received by a firm's worker-managers are proportional to the labor they supply to the firm, a worker of type i will consume

$$\frac{\omega_{li}}{s+l}[g(k,l,\widetilde{x}) - rk] + r\omega_{ki} + \omega_{ci} \tag{1}$$

if he is a worker-manager of a firm that employs k units of capital and l units of labor.[2]

Consider now a worker-manager of type i. Since his consumption level is given in (l), since his actions must not lead to any possibility of bankruptcy, and since $g(k,l,x) = 0$ is always a possibility, an individual of type i will

[2]Note that, in (l), the worker's share of value added exceeds one if ω_{li} exceeds $s+l$. We want to permit such a situation. This will be possible if we introduce the possibility of firms which are, in effect, holding companies or coalitions of smaller firms. Each smaller firm is a branch of the coalition. We will permit these multiple branch coalitions to form, but we will assume that if a coalition does create more than one branch, it will choose to employ the same (k,l) in all of its branches. Because firm or branch outputs are statisfically dependent, it would be satisfied in an equilibrium if it were not imposed. We discuss this point in Appendix A.

prefer to join a coalition which employs a (k, l) vector that maximizes

$$Eu_i\left(\frac{\omega_{li}}{s+l}[g(k, l, \widetilde{x}) - rk] + r\omega_{ki} + \omega_{ci}\right) \tag{2}$$

subject to $k \geq 0, l \geq 0$, and the constraint

$$r\left[\omega_{ki} - \left(\frac{\omega_{li}}{s+l}\right)k\right] + \omega_{ci} \geq 0. \tag{3}$$

Let (k_i, l_i) represent such a vector. A variation on the argument in Appendix A implies uniqueness of (k_i, l_i). Coalitions which create firms that employ the vector (k_i, l_i) will be called type i coalitions. The firms they create are type i firms. Individuals of type i will join coalitions of type i; i.e., they will be worker-managers in these coalitions. Since there are μ_i agents of type i each of whom supplies ω_{li} labor units to a type i coalition, the total supply of management labor to type i firms is $\mu_i\omega_{li}$. If there are ν_i firms of type i, each of which employs $l_i + s$ units of labor, the total demand for management labor for type i firms will be $\nu_i(l_i + s)$. This demand will equal the supply if there are

$$\nu_i = \left(\frac{\omega_{li}}{l_i + s}\right)\mu_i \tag{4}$$

firms of type i. If, in addition,

$$\sum_{i\in I}\nu_i k_i = \sum_{i\in I}\left(\frac{\omega_{li}}{l_i + s}\right)k_i = \overline{\omega}_k, \tag{5}$$

there will also be equality of supply and demand in the capital market.

Having made these introductory remarks, we can now give the formal definition of a Simple Labor-Managed Economy (SLME).

DEFINITION 1 A Simple Labor-Managed Economy (SLME) is an interest rate, r^*, and a capital labor allocation, (k_i^*, l_i^*), for each type $i \in I$ such that

(i) for each i, (k_i^*, l_i^*) maximizes (2) subject to (3), and

(ii) supply equals demand in the capital market in the sense that (5) holds.

It should be noted that, because there is free entry in the sense that new coalitions can always be formed, there can never be any equilibrium different from those described in Definition l. Suppose, for example, that with r^* equal to the capital price, a significant number of firms, say ν, attempted to form and employ a (k, l) vector not included in the set $\{(k_i^*, l_i^*)\}_{i \in I}$. If these firms are to operate, some labor-managers must be willing to supply them with $\nu(s + l)$ labor units. But there will be no workers of any type willing to supply labor to these firms. Instead, individuals of type i are free to form coalitions which create firms that employ k_i^* capital units and l_i^* labor units. In fact, $\nu^* = \mu_i(\omega_{li}/s + l_i^*)$ will be created and will attract all of the labor supplied by type i workers; none will remain for the firm employing (k, l). When there are ν_i^* firms of type i, condition (5) guarantees that total capital demands equal supplies.

We now state formally the theorem which establishes the existence of SLME. The proof of this theorem is contained in Appendix B.

THEOREM 1. *Under A1, A2 and A3, there exists a SLME.*

We can now ask whether the Simple Labor-Management Equilibria are efficient in some appropriate sense. It should be immediately clear that the institutional structure implicit in the definition of these equilibria is not sufficiently rich to result in the attainment of first-best optima.

In spite of the fact that SLME fail to be efficient in a first-best sense, they may have other desirable efficiency properties. For the purpose of investigating this possibility, we follow the approach of Diamond. In his study of the market for firm shares, Diamond effectively asked whether it was possible for any system to use stock market institutions more efficiently than the stock market itself used them. As discussed in the introduction, Diamond imposes constraints on the contingent claims allocations under consideration. Diamond-efficient allocations are those which are optimal in this constrained set of contingent claims allocations. The constraints introduced by Diamond go beyond the conditions that supply equal demand imposed in complete contingent claims markets. They are intended to describe the institutional restrictions imposed on risk trading when exchanges are made in fixed obligation debt markets and firm stock markets.

The definition given below will embody two further restrictions. The first is that each individual invests in only one type of firm. The second is that individuals of each type are equally treated. This second restriction can be eliminated. It is used only to simplify the discussion. The first restriction can also be added without cost, because of the assumption that the same $\widetilde{x}$ enters all production functions. When this assumption is satisfied, allocations in only one type of firm Pareto dominate allocations in which individuals holds shares in several types of firms. The argument used to justify this remark is essentially the same as that outlined in Appendix A.

DEFINITION 2 A Diamond-feasible allocation is a contingent consumption vector $(C_i(x))_{i \in I, x \in X}$ and a vector $(k_i, l_i, a_i, b_i, \nu_i)_{i \in I} \in [0, +\infty)^3 \times$

$[-\omega_{ci}, +\infty) \times [0, +\infty)$ such that

$$\begin{aligned} C_i(x) &= a_i g(k_i, l_i, x) + b_i + \omega_{ci} \\ & b_i + \omega_{ci} \geq 0 \\ & \nu_i = a_i \mu_i \end{aligned} \tag{6}$$

$$\begin{aligned} \sum_{i \in I} \mu_i b_i &= 0 \\ \sum_{i \in I} \nu_i k_i &= \overline{\omega}_k \\ \sum_{i \in I} \nu_i (l_i + s) &= \overline{\omega}_l \end{aligned} \tag{7}$$

DEFINITION 3 A Diamond-efficient allocation is a Diamond-feasible allocation which is not Pareto dominated by any other Diamond-feasible allocation.

THEOREM 2. *There exist SLME which are not Diamond efficient.*

The example which establishes Theorem 2 is constructed in Appendix D.[3] Clearly the lack of Diamond efficiency is due to the fact that, regardless of his risk aversion or endowment, each agent must invest all of his labor endowment in a risky firm. Formally, a_i must equal $\omega_{li}/(s + l_i)$, because there exists no opportunity for the exchange of risk between workers with different attitudes toward risk. In the next section, we consider a labor-management system which makes such exchanges possible. Two interpretations are given to this system. In the first, coalitions include individuals of diverse types. The firm is interpreted as being composed of divisions within which all individuals are of the same type. These divisions, in effect, trade insurance in exchange for labor services. In the second interpretation, which is detailed in Appendix C, there are explicit labor markets internal to each coalition. Divisions trade labor for a riskless wage in these markets. Free entry guarantees that the wages of all of these internal labor markets are equated. Thus the equilibrium is the same as that which would result if the labor market were external.

[3] The example of Appendix D is also used to illustrate the equilibrium concepts defined in the section which follows. For this reason, the reader is advised to consult this appendix after reading Section 4.

4. LABOR MANAGEMENT EQUILIBRIUM

Suppose that we now broaden the range of possibilities open to coalitions. In particular, assume that these coalitions are composed of labor-managed divisions which insure one another by exchanging non-random output payments for labor. Each individual will now be a worker-manager of one division; he will supply all of his labor to that division and he will receive a predetermined share of the value added generated by that division.

It is now possible for divisions to obtain labor from the internal insurance market as well as from its labor managers. The labor obtained from its managers will be called management labor. If a division employ $s+l$ total labor units and receives δ of the units from other divisions, the management labor employed by the division is $s+l-\delta$. A worker of type i who supplies labor to such a division receives a share of output equal to

$$\frac{\omega_{li}}{s+l-\delta}$$

By varying the δ units of nonmanagement labor employed by this division, the type i worker's share can be varied; it is no longer fixed at

$$\frac{\omega_{li}}{s+l}$$

as it was in the SLME. Suppose that a particular division employs k capital units and l units of operating labor and that δ labor units are provided by other divisions in return for a nonrandom insurance payment η. The payment η is deducted from the value added in determining the output generated by the division. Thus the output divided by the division's worker managers is

$$g(k,l,x)-rk-\eta,$$

when $\widetilde{x}=x$. The expected utility of a type i worker is

$$Eu_i\left(\left[\frac{\omega_{li}}{s+l-\delta}\right]\left[g(k,l,\widetilde{x})-rk-\eta\right]+r\omega_{ki}+\omega_{ci}\right) \tag{8}$$

Assumption 3 requires that

$$\left[\frac{\omega_{li}}{s+l-\delta}\right]\left[-rk-\eta\right]+r\omega_{ki}+\omega_{ci}\geq 0. \tag{9}$$

Now let (k_i,l_i) be the input combination employed by divisions managed by type i workers. Let δ_i be the nonmanagement labor employed by these divisions and denote by η_i the insurance payment made by these divisions.

If there are, in total, ν_i type i divisions, the demand for type i management labor will be $(s + l_i - \delta_i)$. This demand will equal the supply, $\mu_i \omega_{li}$, if

$$\nu_i = \frac{\mu_i \omega_{li}}{s + l_i - \delta_i}. \tag{10}$$

Supply and demand will be equated in the labor, capital, and insurance markets respectively when

$$\sum_{i \in I} \nu_i (s + l_i) = \overline{\omega}_l, \tag{11}$$

$$\sum_{i \in I} \nu_i k_i = \overline{\omega}_k, \tag{12}$$

and

$$\sum_{i \in I} \nu_i \eta_i = 0, \tag{13}$$

Using (10) to substitute for ν_i in (11), (12), and (13), these equations reduce to

$$\sum_{i \in I} \mu_i \left(\frac{\omega_{li} \delta_i}{s + l_i - \delta_i} \right) = 0, \tag{14}$$

$$\sum_{i \in I} \mu_i \left(\frac{\omega_{li} k_i}{s + l_i - \delta_i} \right) = \overline{\omega}_k, \tag{15}$$

and

$$\sum_{i \in I} \mu_i \left(\frac{\omega_{li} \eta_i}{s + l_i - \delta_i} \right) = 0. \tag{16}$$

Each division's choices can be reinterpreted to yield a relatively convenient definition of equilibrium. Specifically, we can define new variables β_i and α_i by

$$\frac{\omega_{li} + \beta_i}{s + l_i} = \frac{\omega_{li}}{s + l_i - \delta_i} \tag{17}$$

and

$$\alpha_i = - \left(\frac{\omega_{li} \eta_i}{s + l_i - \delta_i} \right). \tag{18}$$

Equation (17) can be solved for β_i to yield

$$\beta_i = \frac{\omega_{li} \delta_i}{s + l_i - \delta_i}. \tag{19}$$

When β_i is negative, δ_i will be negative also. In this case, type i divisions supply δ_i labor units to divisions of other types. The amount supplied per type i worker is β_i. The type i division receives the fixed payment $-\eta_i$ from the other divisions. The amount received per type i worker is α_i. The payment α_i can be interpreted as an insurance payment to type i workers for which they pay by supplying β_i labor units. This insurance is supplied by those divisions for which α_i is negative and β_i is positive. Since β_i and δ_i are positive for divisions which supply insurance, these firms employ labor in amounts, $s + l_i$, which exceed their supply of management labor $s + l_i - \delta_i$.

With this reformulation, the equilibrium conditions (11), (12), and (13) are replaced by

$$\sum_{i \in I} \mu_i \beta_i = 0, \tag{20}$$

$$\sum_{i \in I} \mu_i \left(\frac{\omega_{li} + \beta_i}{s + l_i} \right) k_i = \overline{\omega}_k, \tag{21}$$

and

$$\sum_{i \in I} \mu_i \alpha_i = 0 \tag{22}$$

respectively. The expressions (10) and (8) for ν_i and expected utility become

$$\nu_i = \mu_i \left(\frac{\omega_{li} + \beta_i}{s + l_i} \right). \tag{23}$$

and

$$Eu_i \left(\left[\frac{\omega_{li} + \beta_i}{s + l_i} \right] [g(k_i, l_i, \widetilde{x}) - rk_i] + \alpha_i + r\omega_{ki} + \omega_{ci} \right). \tag{24}$$

The non-negativity restriction (9) becomes

$$r \left(\omega_{ki} - \left[\frac{\omega_{li} + \beta_i}{s + l_i} \right] k_i \right) + \alpha_i + \omega_{ci} \geq 0. \tag{25}$$

The description of the expanded concept of labor management equilibrium will be complete when we add the optimality conditions satisfied by the equilibrium choice for $(k_i, l_i, \beta_i, \alpha_i)_{i \in I}$ to the supply equal demand conditions (20), (21), and (22). The coalitions which exist in equilibrium must make these optimal choices because of the possibility of free entry by new coalitions. As in a SLME, if a coalition did not make the optimal choice, new coalitions would form and offer the optimal choice. As a consequence, the coalitions making non-optimal choices could not attract management labor. The formal definition of the equilibrium follows.

Definition 4 A Labor Management Equilibrium (LME) is an interest rate r^* and a vector $(k_i^*, l_i^*, \alpha_i^*, \beta_i^*, \nu_i^*)_{i\in I}$ satisfying (20), (21), (22), and (23) such that there exists no set (of strictly positive measure) of agents $T = \bigcup_{j\in J} T_j$ formed as a union of subsets T_j of agents of types $j \in J \subseteq I$ and no alternative allocation $(k_j, l_j, \alpha_j, \beta_j, \nu_j)_{j\in J}$ for which (25) holds and

$$\begin{aligned} & Eu_j\left(\left[\frac{\beta_j+\omega_{lj}}{s+l_j}\right][g(k_j,l_j,\widetilde{x})-r^*k_j]+\alpha_j+r^*\omega_{kj}+\omega_{cj}\right) \\ \geq\ & Eu_j\left(\left[\frac{\beta_j+\omega_{lj}}{s+l_j}\right][g(k_j^*,l_j^*,\widetilde{x})-r^*k_j^*]+\alpha_j^*+r^*\omega_{kj}+\omega_{cj}\right) \end{aligned} \tag{26}$$

for all $j \in J$ with at least one strict inequality and for which

$$\omega_j = \mu(T_j)\left(\frac{\omega_{lj}+\beta_j}{s+l_j}\right) \tag{27}$$

$$\sum_{j\in J}\alpha_j\mu(T_j) = 0 \tag{28}$$

and

$$\sum_{j\in J}\beta_j\mu(T_j) = 0. \tag{29}$$

We first prove that an LME exists. The proof is given in Appendix D, where it is shown that an LME is equivalent to the Walrasian equilibrium of an auxiliary exchange economy which is then shown to exist.

Theorem 3. *Under (A1), (A2) and (A3), there exists an LME.*

Next we establish the Diamond efficiency of the LME. In fact, we will obtain a somewhat stronger result by proving that the set of LME coincides with what will be referred to as the Diamond core. This is a second-best concept of the core. It is defined analogously to the usual core concept. That is, a definition of blocking by coalitions of individuals is introduced and the core allocations are those which are unblocked. In defining the Diamond core, the contingent claims allocations under consideration are restricted to be Diamond feasible. Thus a coalition is permitted to block with only reallocations of its own resources that are Diamond feasible for the coalition. As in the discussion of Diamond efficiency, these restrictions are imposed as a means of introducing the institutional restrictions inherent in a stock market economy or a labor-management system. Therefore, it is impossible, for any coalition of individuals to improve on a Diamond core allocation using only stock market or labor management institutions.

We will now give a formal definition of the blocking concept and of the core. In this formal discussion, a Diamond-feasible allocation $((C_i(x))_{x\in X}, k_i, l_i, a_i, b_i)_{i\in I}$ will be identified with the imputation vector; i.e. the vector of utilities

$$(\overline{u}_i = Eu_i(a_i g(k_i, l_i, \widetilde{x}) + b_i + \omega_{ci}))_{i\in I}\,.$$

A coalition is a subset $T = \bigcup_{j\in J} T_j$, where T_j is a subset of type j individuals and $J \subseteq I$. A vector $((C_i(x))_{x\in X}, k_j, l_j, b_j, \nu_j)_{j\in J}$ is Diamond feasible for coalition T if

$$C_j(x) = a_j g(k_j, l_j, x) + b_j + \omega_{cj}, \tag{30}$$

$$b_j + \omega_{cj} \geq 0 \tag{31}$$

and

$$\nu_j = a_j \mu(T_j) \tag{32}$$

for all $j \in J$ and if

$$\sum_{j\in J} b_j \mu(T_j) = 0 \tag{33}$$

$$\sum_{j\in J} \nu_j k_j = \sum_{j\in J} \mu(T_j)\omega_{kj} \tag{34}$$

and

$$\sum_{j\in J} \nu_j (l_j + s) = \sum_{j\in J} \mu(T_j)\omega_{lj}. \tag{35}$$

Definition 5 A Diamond-feasible imputation vector $(\overline{u}_j)_{i\in I}$ is blocked if there exists a coalition T and a vector $((C_j(x))_{x\in X}, k_j, l_j, a_j, b_j, \nu_j)_{j\in J}$ that is feasible for T such that

$$Eu_j\left(a_j g(k_j, l_j, \widetilde{x}) + b_j + \omega_{cj}\right) \geq \overline{u}_j \tag{36}$$

for all $j \in J$ and such that the strict inequality holds in (36) for some $j \in J$.

Definition 6 A Diamond feasible allocation (or imputation) is in the Diamond core if it is not blocked.

In Appendix C, we will prove the following result:

Theorem 4. *The set of LME coincides with the Diamond core.*

Note that, since one possible blocking coalition is the coalition of the whole, a Diamond core allocation is Diamond efficient. Thus as a corollary of Theorem 4 we obtain the Diamond efficiency of an LME.

In the description of the LME, we have interpreted the exchange of sure payments for labor between divisions as a form of insurance. Divisions operated by risk averse workers obtain insurance by supplying labor for use in other divisions rather than their own. If this labor were used in their own division, these workers would bear the risks associated with its employment. Because, however, their division receives a nonrandom output transfer which they share, these risk averse workers are insured against the risks associated with the use of the labor in the divisions to which it is transferred.

This insurance scheme can also be interpreted as a labor market internal to the firm in which η_i is the cost to the division i of employing the δ_i labor units supplied by other divisions. The implicit wage paid by division i is η_i/δ_i. Because of the free entry assumption which permits new coalitions to form and because workers can freely leave one firm to join a new coalition, this wage will be the same for all firms and for all divisions within each firm. Thus, in an LME as in an economy with an external labor market, all labor will be exchanged formally in Appendix D, where we define an equilibrium in which a fixed wage labor market and labor management co-exist. This hybrid system is introduced for two reasons. On the one hand, it is useful as a step in the proof of existence and in demonstrating the core equivalence theorem. On the other hand, in this equilibrium the labor market plays the same role as the market for fixed obligation debt in the Diamond stock market economy.[4] Since, as we have just noted, the external labor market could be viewed as a replacement for the internal insurance system in a pure worker-management system, we can interpret the insurance system as playing the same role in the labor management economy as the bond market in a stock market economy.

APPENDIX A

We first outline the proof that an individual will never choose to supply labor to two different firms in a SLME.

[4]In fact, by reversing the roles played by labor and capital, the labor management system with a labor market can be interpreted as a capital management system with a fixed obligation debt market in which the debt market is the analog of the fixed wage labor market in the hybrid labor-management equilibrium.

Consider a type i individual who supplies m labor units to a firm employing (k, l) and $\hat{m} = \omega_{li} - m$ labor units to a firm that employs $(\hat{k}, \hat{l}) \neq (k, l)$. When $\widetilde{x} = x$, this individual will receive

$$\left(\frac{m}{s+l}\right)[g(k,l,x) - rk] + \left(\frac{\hat{m}}{s+\hat{l}}\right)[g(\hat{k},\hat{l},x) - r\hat{k}] \tag{A.1}$$

consumption good units from these firms. When $x \neq \underline{x}$, g is strictly concave and the amount in (A.l) is exceeded by

$$\left(\frac{m}{s+l} + \frac{\hat{m}}{s+\hat{l}}\right)[g(\overline{k},\overline{l},x) - r\overline{k}] = \left(\frac{\omega_{li}}{s+\hat{l}}\right)[g(\overline{k},\overline{l},x) - r\overline{k}], \tag{A.2}$$

where

$$(\overline{k},\overline{l}) = \frac{\left(\frac{m}{s+l}\right)(k,l) + \left(\frac{\hat{m}}{s+\hat{l}}\right)(\hat{k},\hat{l})}{\left(\frac{m}{s+l}\right) + \left(\frac{\hat{m}}{s+\hat{l}}\right)}$$

If $\widetilde{x} = \underline{x}$, the amounts in (A.1) and (A.2) are equal. Thus a labor-manager of type i who supplies labor to a single firm employing $(\overline{k},\overline{l})$ always does at least as well as he would if he were to divide his time between two firms employing (k, l) and $(\hat{k},\hat{l})$.

For similar reasons, individuals will always prefer to be part of a coalition whose firms all employ the same input combinations. Suppose to the contrary that a coalition created ν firms which employed (k, l) and $\hat{\nu}$ firms which employed $(\hat{k},\hat{l})$. As a member of this coalition, a type i individual would receive

$$\frac{\omega_{li}}{\nu(s+l) + \widetilde{\nu}(s+\hat{l})}\left\{\nu[g(k,l,x) - rk] + \hat{\nu}[g(\hat{k},\hat{l},x) - r\hat{k}]\right\} \tag{A.3}$$

if $\widetilde{x} = x$. If instead this coalition used the same amount of labor to create $(\nu + \hat{\nu})$ firms each employing

$$(\overline{k},\overline{l}) = \frac{\nu(k,l) + \hat{\nu}(\hat{k},\hat{l})}{\nu + \hat{\nu}},$$

the type i labor-manager would receive

$$\frac{\omega_{li}}{\nu(s+l) + \hat{\nu}(s+\hat{l})}\left\{(\nu+\hat{\nu})[g(\overline{k},\overline{l},x) - r\overline{k}]\right\} = \frac{\omega_{li}}{s+\hat{l}}[g(\overline{k},\overline{l},x) - r\overline{k}] \tag{A.4}$$

if $\widetilde{x} = x$. When $\widetilde{x} = \underline{x}$, the amounts in (A.3) and (A.4) are equal. In all other cases, (A.4) exceeds (A.3). Thus the type i workers prefer to join a coalition whose $(\nu + \hat{\nu})$ firms all employ $(\overline{k},\overline{l})$ in preference to one which has ν firms employing (k, l) and $\hat{\nu}$ firms employing $(\hat{k},\hat{l})$.

APPENDIX B

Proof of the existence of a SLME (Theorem l):

We reduce the SLME existence problem to a more familiar problem; viz., one of finding a Walrasian equilibrium in an exchange economy. To accomplish this translation of the problem, we first define

$$\xi_i = \frac{\omega_{li}}{s + l_i} k_i$$

so that

$$\frac{\xi_i}{k_i} = \frac{\omega_{li}}{s + l_i}$$

and

$$l_i = \frac{k_i}{\xi_i}\omega_{li} - s.$$

We then note that the problem of finding an SLME is equivalent to one of finding a vector $\langle r, (\xi_i, k_i)_{i \in I} \rangle$ such that, for each $i \in I$, (ξ_i, k_i) maximizes

$$Eu_i \left(\xi_i \frac{g(k_i, (k_i/\xi_i)\omega_{li} - s, \widetilde{x})}{k_i} + r[\omega_{ki} - \xi_i] + \omega_{ci} \right) \tag{B.1}$$

subject to non-negativity constraints on ξ_i and k_i and subject to

$$r[\omega_{ki} - \xi_i] + \omega_{ci} \geq 0, \tag{B.2}$$

and such that

$$\sum_{i \in I} \mu_i \xi_i = \overline{\omega}_k. \tag{B.3}$$

This problem is translated into the problem of finding the Walrasian equilibrium of a two-good exchange economy by defining

$$\zeta_i = r[\omega_{ki} - \xi_i] + \omega_{ci} \tag{B.4}$$

and

$$\nu_i(\xi_i, \zeta_i) = \max_{k_i \geq 0} Eu_i \left(\xi_i \frac{g(k_i, (k_i/\xi_i)\omega_{li} - s, \widetilde{x})}{k_i} + \zeta_i \right). \tag{B.5}$$

From (Al), (A2), and (A3), we can derive the existence and the continuity of the function V_i on $[0, +\infty)^2$.

The exchange economy considered is one in which there are μ_i type i individuals and in which a type i individual begins with the initial endowment $(\xi_i, \zeta_i) = (\omega_{ki}, \omega_{ci})$ and maximizes the utility function V_i defined in

(B.5). When we interpret ζ_i as the numeraire and let r be the price of ξ_i, equation (B.4) becomes the budget constraint subject to which V_i is maximized in the Walrasian equilibrium. Solving the problem of choosing $(\xi_i, \zeta_i) \in [0, +\infty)^2$ to maximize V_i subject to (B.4) is in fact equivalent to finding a $(\xi_i, k_i) \in [0, +\infty)^2$, which maximizes (B.1) subject to (B.2). The Walrasian requirement that supply and demand for ξ be equated is simply (B.3). Taken together, the observations just made imply that a Walrasian equilibrium $\langle r, (\xi_i, \zeta_i) \rangle_{i \in I}$ for the exchange economy yields an LME. The proof of existence of the Walrasian equilibrium is now accomplished using standard techniques as described, for example, in Aumann [1966]. The singlevaluedness of the demand functions is obtained from the uniqueness of the maximizing input vectors (k_i, l_i) derived by arguments like those in Appendix A. Continuity of demand functions follows then from (A1)-(A2).

APPENDIX C

We begin by defining three auxilliary concepts which are useful in establishing the existence and core equivalence theorems for LME. The first of these concepts is the labor-management equilibrium with an external labor market, in which workers are paid a nonrandom wage (LMEFW). These are the equilibria referred to in the concluding section. The other concepts are the Walrasian equilibria (WE) and the Core (EE Core) of an appropriately defined exchange economy. The structure of the argument used to establish the existence and core equivalence is described in the diagram below.

$$\begin{array}{ccc}
\text{LMEFW} & \subset^{(1)} & \text{LME} \\
(2)\ ||| & & \\
\text{WE} & & \cap^{(5)} \\
(3)\ ||| & & \\
\text{EE Core} & \equiv^{(4)} & \text{Diamond Core}
\end{array}$$

In this diagram, $\equiv$ means is identical to.

Step (1) consists of showing that an LMEFW is an LME.

Step (2) identifies the set of LMEFW and the set of WE and demonstrates that these sets are non-empty.

Step (3) is Aumann's core equivalence theorem.

Step (4) establishes the equivalence of the exchange economy core and the Diamond core.

Step (5) shows that an LME belongs to the Diamond core implying, in combination with the other steps, that LME Diamond core.

STEP 1

We begin by defining the LMEFW. In a labor-managed economy with an external labor market workers may spend only a part, m_i, of their time working as labor managers. The remainder of their time $\omega_{li} - m_i$ can then be supplied for a fixed wage which we denote by w. When a type i worker supplies m_i units of management-labor to a firm employing k_i capital units and l_i labor units, all of which is supplied by labor managers, his share of value added is $\frac{m_i}{s+l_i}$ and his wealth is

$$\frac{m_i}{s+l_i}[g(k_i, l_i, x) - rk_i] + r\omega_{ki} + w(\omega_{li} - m_i) + \omega_{ci} \tag{C.1}$$

when $\widetilde{x} = x$.

Suppose then that (k_i^*, l_i^*, m_i^*) maximizes

$$Eu_i\left(\frac{m_i}{s+l_i}[g(k_i, l_i, \widetilde{x}) - rk_i] + r\omega_{ki} + w(\omega_{li} - m_i) + \omega_{ci}\right), \tag{C.2}$$

subject to

$$r\left[\omega_{ki} - \left(\frac{m_i}{s+l_i}\right)k_i\right] + w(\omega_{li} - m_i) + \omega_{ci} \geq 0. \tag{C.3}$$

Type i individuals will supply m_i^* units of management labor to firms employing (k_i^*, l_i^*), i.e. to type i firms. The supply of and demand for management labor for type i firms will be equated when there are

$$\nu_i^* = \frac{\mu_i m_i^*}{s + l_i^*} \tag{C.4}$$

of these firms. The equilibrium wage level and capital price will be such that the corresponding $(\nu_i^*, k_i^*, l_i^*)_{i\in I}$ results in equality of supply and demand for capital and labor.

Definition C.1. A Labor Management Equilibrium with a Fixed Wage Labor Market (LMEFW) is an interest rate, wage vector (r^*, w^*), and a vector $(k_i^*, l_i^*, m_i^*, \nu_i^*)_{i\in I}$ such that

i) for each $i \in I$, (k_i^*, l_i^*, m_i^*) maximizes (C.2) subject to (C.3) when $w = w^*$ and $r = r^*$,

ii) ν_i^*, m_i^*, and l_i^* satisfy (B.4), and

iii) the supply equal demand conditions (11) and (12) hold.

Using an argument analogous to the standard proof that a Walrasian equilibrium is in the core, it can easily be shown that a LMEFW is a LME. We state this formally in the following proposition.

Proposition C.1. *If $\langle r^*, w^*, (k_i^*, l_i^*, m_i^*, \nu_i^*)_{i\in I}\rangle$ is an LMEFW, then $\langle r^*, (k_i^*, l_i^*, \alpha_i^*, \beta_i^*, \nu_i^*)_{i\in I}\rangle$ is an LME when α_i^* and β_i^* are defined by*

$$\alpha_i^* = w^*[\omega_{li} - m_i^*] \tag{C.5}$$

and

$$\beta_i^* = m_i^* - \omega_{li}. \tag{C.6}$$

Proof. Suppose that $\langle r^*, w^*, (k_i^*, l_i^*, m_i^*, \nu_i^*)_{i\in I}\rangle$ is an LMEFW. Define α_i^*, β_i^* by (C.5) and (C.6). Now suppose that there exists a coalition $T = \bigcup_{j\in J} T_j$ and a vector $(k_j, l_j, \alpha_j, \beta_j, \nu_j)_{j\in J}$ such that (26) holds for all $j \in J$, with a strict inequality for some $j \in J$. Using (C.5), (C.6) and the fact that (k_i^*, l_i^*, m_i^*) maximizes (C.2) subject to (A3.3), we obtain

$$\begin{aligned} & Eu_j\left[\left(\frac{\beta_j^* + \omega_{lj}}{s + l_j^*}\right)[g(k_j^*, l_j^*, \widetilde{x}) - rk_j^*] + \alpha_j^* + r^*\omega_{kj} + \omega_{cj}\right] \qquad \text{(C.7)} \\ = \;& Eu_j\left[\left(\frac{m_j^*}{s + l_j^*}\right)[g(k_j^*, l_j^*, \widetilde{x} - rk_j^*] + w^*[\omega_{lj} - m_j^*] + r^*\omega_{kj} + \omega_{cj}\right] \\ \geq \;& Eu_j\left[\left(\frac{m_j}{s + l_j}\right)[g(k_j, l_j, \widetilde{x}) - rk_j] + w^*[\omega_{lj} - m_j] + r^*\omega_{kj} + \omega_{cj}\right] \end{aligned}$$

where

$$m_j = \beta_j + \omega_{lj}. \tag{C.8}$$

When we use (C.8) to substitute for m_j in the right side of (C.7) and then combine the resulting inequality with (26), the result is

$$\begin{aligned} & Eu_j\left[\left(\frac{\beta_j + \omega_{lj}}{s + l_j}\right)[g(k_j, l_j, \widetilde{x}) - rk_j] + \alpha_j + r^*\omega_{kj} + \omega_{cj}\right] \qquad \text{(C.9)} \\ \geq \;& Eu_j\left[\left(\frac{\beta_j + \omega_{lj}}{s + l_j}\right)[g(k_j, l_j, \widetilde{x}) - rk_j] - w^*\beta_j + r^*\omega_{ki} + \omega_{cj}\right]. \end{aligned}$$

This inequality holds for all $j \in J$, and for at least one $j \in J$ it holds with a strict inequality. Thus, for all $j \in J$,

$$\alpha_j \geq -w^*\beta_j$$

and the inequality is strict for some $j \in J$. As a consequence,

$$-\sum_{j\in J} \alpha_j \mu(T_j) < w^* \sum_{j\in J} \beta_j \mu(T_j). \tag{C.10}$$

Inequality (C.10) implies that the conditions (28) and (29) cannot hold simultaneously. Thus $(k_j, l_j, \alpha_j, \beta_j, \nu_j)$ cannot satisfy (28) and (29) when

(C.10) holds for all $j \in J$ with a strict inequality for some $j \in J$. The vector r^*, $(k_i^*, l_i^*, \alpha_i^*, \beta_i^*, \nu_i^*)_{i \in I}$ must therefore be an LME. ▮

The interpretation of an LMEFW as defined is somewhat strained when m_i exceeds ω_{li}. In this case, type i workers in effect hire workers of other types to work, on behalf of the type i individuals, as labor-managers in a type i firm. It seems unlikely that such arrangements will be observed in an explicit form. There is, however, a version of the Modigliani-Miller theorem which can be established for LMEFW and which can be used to yield conveniently interpretable equilibria equivalent to LMEFW.

For the purpose of discussing this issue, suppose that firms hire labor directly at the wage w. Let ϕ_i be the amount of labor hired by type i firms. The Modigliani-Miller theorem referred to above asserts that the choice of ϕ_i is irrelevant. Specifically a change in ϕ_i can be compensated for by changing m_i so as to leave income unchanged in all states $x \in X$. Suppose, for example, that we start with a situation in which $\phi_i = 0$ and $(k_i, l_i, m_i) = (k_i^*, l_i^*, m_i^*)$. If we now let $\phi_i = \overline{\phi}_i$, where $\overline{\phi}_i$ is an arbitrarily chosen non-zero amount, we can compensate for this change in ϕ_i from zero to $\overline{\phi}_i$ by changing m_i to

$$\overline{m}_i = \left[\frac{m_i^*}{s + l_i^*}\right][s + l_i^* - \phi_i].$$

It is easily verified that, for each $x \in X$,

$$\begin{aligned}&\left[\frac{\overline{m}_i}{s + l_i^* - \overline{\phi}_i}\right][g(k_i^*, l_i^*, m_i^*) - rk_i^* - w\overline{\phi}_i] + r\omega_{ki} + w[\omega_{li} - \overline{m}_i] + \omega_{ci}\\ = &\left[\frac{m_i^*}{s + l_i^*}\right][g(k_i^*, l_i^*, m_i^*) - rk_i^*] + r\omega_{ki} + w[\omega_{li} - m_i^*] + \omega_{ci}.\end{aligned}$$

Thus, for each state $x \in X$, a type i individual receives the same amount of income when $(\phi_i, m_i) = (\overline{\phi}_i, \overline{m}_i)$ as he does when $(\phi_i, m_i) = (\phi_i^*, m_i^*)$.

As a consequence of the result just proved, the opportunity to choose ϕ_i is of no value and can be ignored as we have done in the preceeding discussion. The irrelevance of ϕ_i can, however, be exploited to provide a more convenient interpretation of the LMFW. In particular, we can assume that m_i always equals ω_{li} and that firms hire ϕ_i^* labor units where ϕ_i^* is such that

$$\frac{m_i^*}{s + l_i^*} = \frac{\omega_{li}}{s + l_i^* - \phi_i^*}. \tag{C.11}$$

In (C.11), the starred values for m_i^* and ϕ_i^* are the LMEFW values, i.e. the equilibrium values when $\phi_i = 0$ when we use this approach, it is never necessary for type i individuals to employ other laborers to work as managers on their behalf. All hiring in the labor market is done explicitly by the firms.

At this stage, it should be noted that, in defining the economic environment, the only asymmetry in the roles played by capital and labor arises because fixed costs are borne in the form of labor. When there are markets for both inputs in which payments to suppliers are nonrandom obligations, this asymmetry in the economic environment does not give a managerial advantage to the suppliers of either input. Thus, in the above definition, the roles of capital and labor can be reversed to obtain a definition of capital management with a fixed obligation debt market. The set of these capital management equilibria can be shown to coincide with the (LMEFW). We will not formally perform this exercise, but we will outline the argument.

Suppose then that we begin with a LMEFW $\langle r^*, w^*, (k_i^*, l_i^*, m_i^*, \nu_i^*)_{i \in I} \rangle$. To define an equivalent capital management equilibrium, we let n_i^* represent the amount of management capital supplied by a type i individual. To insure that these individuals receive the same output share as they would in the LMEFW, it must be the case that

$$n_i^* = k_i^* \frac{m_i^*}{s + l_i^*}. \tag{C.12}$$

when (C.12) is satisfied, i's income is easily seen to equal

$$\frac{n_i^*}{k_i^*}[g(k_i^*, l_i^*, x) - w(l_i^* + s)] + r[\omega_{ki} - n_i^*] + w\omega_{li} + \omega_{ci} \tag{C.13}$$

when $\widetilde{x} = x$. Furthermore, (k_i^*, l_i^*, m_i^*) maximizes the expected utility (C.2) subject to (C.3) if and only if (k_i^*, l_i^*, m_i^*) maximizes the expected type i utility of (C.13) subject to the appropriate analog of inequality (C.3). When n_i^* satisfies (C.12), ν_i^* is still given by (C.4), and the supply equal demand conditions are therefore still satisfied.

The argument just outlined establishes the equivalence of labor and capital management. Capital management equilibrium can also be identified with the stock market equilibria of Kihlstrom-Laffont [1982]. This identification is achieved by interpreting

$$\gamma_i = \frac{n_i}{k_i} \tag{C.14}$$

as the share of a type i firm held by a type i individual. The choice of (k_i, l_i, n_i) can then be shown to be equivalent to the choice of (k_i, l_i, γ_i) . when γ_i and n_i are related by (C.14), the stock market equilibrium and capital management equilibrium result in the same number of firms of each type and the equality of input supplies and demands in a stock market equilibrium is equivalent to equality of these supplies and demands in the capital market. In making the translation to the stock market equilibrium, it is assumed that s is the price in labor terms of a full share in a firm which employs k capital units and l labor units and finances these input purchases entire1y with debt.Thus the labor value of the firm is the labor cost of creating it. It is argued in Kihlstrom-Laffont[1982] that this assumption about the share price of a firm can be justified as a form of the rational expectations hypothesis. There may be other stock market equilibria in which the price of shares in some firms not observed in equilibrium is below s, the cost of creating these firms. These "non-rational expectations" equilibria can never be capital management or labor management equilibria, however. Thus the capital and labor management equilibria discussed in this section will yield only those stock market equilibria in which expectations about firm share prices are rational

STEP 2

We can now identify the LMEFW with the Walrasian equilibria of an appropriately defined pure trade economy.

This pure trade economy also has I types of individuals who will now be interpreted as consumers of three commodities. Again there are μ_i consumers of type i. The amounts of the commodities consumed by type i consumers are denoted by m_i, ξ_i and ζ_i. At this point, the utility function of a type i consumer is simply denoted by V_i and is assumed to have domain $[0, +\infty)^3$. V_i will be related to u_i below. The initial endowment of a type i consumer is the vector $(m_i, \xi_i, \zeta_i) = (\omega_{li}, \omega_{ki}, \omega_{ci})$, where ω_{li}, ω_{ki} and ω_{ci} are as defined in the specification of the economic environment in Section 2. Summarizing, the pure trade economy is formally represented by the vector $\langle V_i, (\omega_{li}, \omega_{ki}, \omega_{ci}), \mu_i\rangle)_{i\in I}$.

In defining the Walrasian equilibrium of this pure trade economy, ζ is interpreted as the numeraire. The wage w and interest rate r of an LMEFW are now interpreted as the Walrasian prices of commodities m and ξ respectively.

Definition C.2. A Walrasian equilibrium of the pure trade economy $\langle V_i, (\omega_{li}, \omega_{ki}, \omega_{ci}), \mu_i\rangle_{i\in I}$ is a vector $\langle (m_i^*, \xi_i^*, \zeta_i^*)_{i\in I}, w^*, r^*\rangle$ such that

i) $(m_i^*, \xi_i^*, \zeta_i^*)$ maximizes V_i on $[0, +\infty)^3$ subject to the budget constraint

$$\zeta_i + wm_i + r\xi_i = \omega_{ci} + w\omega_{li} + r\omega_{ki}, \tag{C.15}$$

and

ii) supplies equal demands in all markets; i.e.

$$\sum_{i \in I} \mu_i \zeta_i = \overline{\omega}_c, \tag{C.16}$$

$$\sum_{i \in I} \mu_i m_i = \overline{\omega}_l, \tag{C.17}$$

and

$$\sum_{i \in I} \mu_i \xi_i = \overline{\omega}_k. \tag{C.18}$$

An LMEFW can now be translated into a Walrasian equilibrium. We first identify the m_i in a LMEFW with the same variable in the pure trade economy. The ξ_i and ζ_i are then related to the variables in the LMEFW by the equations

$$\xi_i = m_i \left(\frac{k_i}{s + l_i} \right) \tag{C.19}$$

and

$$\zeta_i = r(\omega_{ki} - \xi_i) + w(\omega_{li} - m_i) + \omega_{ci}. \tag{C.20}$$

Note that (C.20) is simply a rearrangement of the budget constraint (C.15). Also note that (C.19) can be inverted to obtain an expression for l_i. Specifically,

$$l_i = \frac{m_i k_i}{\xi_i} - s. \tag{C.21}$$

We now define V_i by letting

$$V_i(m_i, \xi_i, \zeta_i) = \max_{k_i \geq 0} E u_i \left(\zeta_i + \xi_i \left(\frac{g\left(k_i, \frac{m_i}{\xi_i} k_i - s, \widetilde{x}\right)}{k_i} \right) \right) \tag{C.22}$$

The fact that V_i is well-defined and continuous on $[0, +\infty)^3$ follows from (A1),(A2), and (A3).

Using the expressions for l_i, ξ_i and ζ_i given in (C. 21), (C.19), and (C. 20), it is easily demonstrated that the problem of choosing (k_i, l_i, m_i) to maximize (C.2) subject to (C.3) is equivalent to choosing $(m_i, \xi_i, \alpha_i) \in [0, +\infty)^3$ to maximize V_i subject to the budget constraint (C.15). We complete the translation from a LMEFW by noting that the conditions (11) and (12) requiring no excess demands in the input markets are equivalent to the conditions (C.l7) and (C.18) equating m and ξ supplies and demands in the Walrasian equilibrium. Walras' law guarantees the equality of demand

and supply in the ζ market. In making the translation to (C.17) and (C.18),it must be recalled that the ν_i's in (11) and (12) are defined by (C.4).

The existence of an LMEFW and, in view of proposition 1,of an LME will be established if we can demonstrate the existence of a Walrasian equilibrium of the pure trade economy. The proof is easily derived from Aumann [1966l.

STEP 3

From Aumann [1964], we know that the set of Walrasian equilibria of the exchange economy coincides with the core of the exchange economy.

STEP 4

We now show that the core of the exchange economy coincides with the Diamond core.

Let $T = \bigcup_{j \in J} T_j$ be a coalition. A consumption vector $(m_j, \xi_j, \zeta_j)_{j \in J} \in [0, +\infty)^3$ is feasible in the exchange economy for this coalition if it satisfies

$$\sum_{j \in J} (\zeta_j - \omega_{lj}) \mu(T_j) = 0, \tag{C.23}$$

$$\sum_{j \in J} (m_j - \omega_{lj}) \mu(T_j) = 0 \tag{C.24}$$

and

$$\sum_{j \in J} (\xi_j - \omega_{kl}) \mu(T_j) = 0 \tag{C.25}$$

As in the discussion of the Diamond core, we identify the consumption vector $(m_j, \xi_j, \zeta_j)_{j \in J}$ with the imputation vector

$$(\overline{u}_j = V_j(m_j, \xi_j, \zeta_j))_{j \in J}.$$

Thus we will sometimes refer to an imputation vector $(\overline{u}_j)_{j \in J}$ as being feasible for T. When T is the coalition of the whole, (C.23), (C.24) and (C.25) become (C.16), (C.17) and (C.18). We denote by EF the set of consumption vectors or imputation vectors which are feasible for the coalition of the whole.

An allocation, equivalently an imputation vector $(\overline{u}_i)_{i \in I}$, in EF is blocked by a coalition $T = \bigcup_{j \in J} T_j$ if there exists a consumption vector $(m_j, \xi_j, \zeta_j)_{j \in J}$ feasible for T such that

$$V_j(m_j, \xi_j, \zeta_j) \geq \overline{u}_j$$

for all $j \in J$ with a strict inequality for at least one $j \in J$.

An allocation or imputation vector in EF is in the exchange economy core if it is not blocked by any coalition of positive measure.

PROPOSITION C.2. *The Diamond core and the exchange economy core coincide.*

This proposition is an immediate corollary of the following lemma which applies to any coalition $T = \bigcup_{j\in J} T_j$

LEMMA C.1. *If $(\overline{u}_j)_{j\in J}$ is an imputation vector that is Diamond-feasible for T, then there exists an imputation $(\overline{u}'_j)_{j\in J}$ that is feasible for T in the exchange economy and that Pareto dominates or is Pareto indifferent to $(\overline{u}_j)_{j\in J}$. Similarly, if $(\overline{u}_j)_{j\in J}$ is feasible for T in the exchange economy, it is also Diamond-feasible for T.*

Proof. If $(\overline{u}_j)_{j\in J}$ is Diamond-feasible for $T = \bigcup_{j\in J} T_j$, then

$$\overline{u}_j = Eu_j(a_j g(k_j, l_j, \widetilde{x}) + b_j + \omega_{cj}) \tag{C.26}$$

where $((C_j(x))_{x\in X}(k_j, l_j, a_j, b_j, \nu_j))_{j\in J}$ is Diamond-feasible for T. We can then define (m_j, ξ_j, ζ_j) by

$$\begin{aligned} \zeta_j &= b_j + \omega_{cj}, &\text{(C.27)}\\ \xi_j &= a_j k_j, &\text{(C.28)} \end{aligned}$$

and

$$m_j = (l_j + s)(\xi_j / k_j). \tag{C.29}$$

Substituting these expressions in (C.26), we can write

$$\begin{aligned} \overline{u}_j &= Eu_j\left(\zeta_j + \xi_j\left(\frac{g(k_j, (m_j/\xi_j)k_j - s, \widetilde{x})}{k_j}\right)\right) \\ &\leq \max_{k_j\geq 0} Eu_j\left(\zeta_j + \xi_j\left(\frac{g(k_j, (m_j/\xi_j)k_j - s, \widetilde{x})}{k_j}\right)\right) \\ &= V_j(m_j, \xi_j, \zeta_j). \end{aligned}$$

The vector (m_j, ξ_j, ζ_j) defined by (C.29), (C.28) and (C.27) is feasible for T in the exchange economy since (C.23), (C.24), and (C.25) are implied by (33), (34) and (35) when ν_j satisfies (32). Thus $(V_j(m_j, \xi_j, \zeta_j))_{j\in J}$ is an imputation vector which is Pareto superior or indifferent to $(\overline{u}_j)_{j\in J}$ and which is feasible for T in the exchange economy.

When $(\overline{u}_j)_{j\in J}$ is feasible for T in the exchange economy, we let k_j be such that

$$\begin{aligned} & Eu_j\left(\zeta_j + \xi_j\left(\frac{g_j(k_j,(m_j/\xi_j)k_j - s, x)}{k_j}\right)\right) \\ = & \max_{k_j\geq 0} Eu_j\left(\zeta_j + \xi_j\left(\frac{g_j(k_j,(m_j/\xi_j)k_j - s, x)}{k_j}\right)\right) \\ = & \ \overline{u}_j \end{aligned}$$

and then define l_j, a_j and b_j by

$$\begin{aligned} l_j &= (m_j/\xi_j)k_j - s, \\ a_j &= \xi_j/k_j, \end{aligned}$$

and

$$b_j = \zeta_j - \omega_{cj}.$$

The vector $(k_j, l_j, a_j, b_j, \nu_j)$ defined in this way yields an allocation which is Diamond-feasible for T since (C.23), (C.24) and (C.25) imply (33), (34) and (35) when ν_j satisfies (32). Thus $(\overline{u}_j)_{j\in J}$ is also Diamond-feasible for T. ▮

STEP 5

PROPOSITION C.3. *An LME is in the Diamond core.*

Proof. Suppose that $\langle r^*, (k_i^*, l_i^*, \alpha_i^*, \beta_i^*, \nu_i^*)_{i\in I}\rangle$ is an LME. Define a_i^*, b_i^* by

$$a_i^* = \frac{\beta_i^* + \omega_{li}}{s + l_i^*} \tag{C.30}$$

and

$$b_i^* = \alpha_i^* + r^*\left[\omega_{ki} - \left(\frac{\beta_i^* + \omega_{li}}{s + l_i^*}\right)k_i^*\right]. \tag{C.31}$$

Now assume that the Diamond-feasible allocation associated with $(k_i^*, l_i^*, a_i^*, b_i^*, \nu_i^*)$ is not in the Diamond core; i.e., assume that there is a coalition $T = \bigcup_{j\in J} T_j$ and a $((C_j(x))_{x\in X}, k_j, l_j, a_j, b_j, \nu_j)_{j\in J}$ feasible for T, such that

$$\begin{aligned} & Eu_j(a_j g(k_j, l_j, \widetilde{x}) + b_j + \omega_{cj}) \\ \geq & \ Eu_j(a_j^* g(k_j, l_j, \widetilde{x}) + b_j^* + \omega_{cj}) \\ = & \ Eu_j\left(\left[\frac{\beta_j^* + \omega_{lj}}{s + l_j^*}\right][g(k_j^*, l_j^*, \widetilde{x}) - rk_j^*] + \alpha_j^* + r\omega_{kj} + \omega_{cj}\right) \end{aligned} \tag{C.32}$$

holds for all $j \in J$ and such that the inequality (C.32) is strict for some $j \in J$.

We now define β_j and α_j by

$$\beta_j = a_j[s + l_j] - \omega_{lj} \tag{C.33}$$

and

$$\alpha_j = b_j - r^*[\omega_{kj} - a_j k_j]. \tag{C.34}$$

The fact that (C.32) holds for all $j \in J$ implies that (26) holds for all $j \in J$. For the j at which the inequality (C.32) is strict, the inequality (26) is also strict. The fact that $(k_j, l_j, a_j, b_j)_{j \in J}$ is feasible for T means that the inequalities (33), (34) and (35) hold when ν_j satisfies (32). These inequalities imply (27), (28) and (29) when β_j and α_j are related to (k_j, l_j, a_j, b_j) by (C.33) and (C.34). As a consequence, $r^*, (k_j^*, l_j^*, \alpha_j^*, \beta_j^*, \nu_j^*)_{j \in J}$ cannot be an LME as originally assumed. ▌

APPENDIX D

Example of a Diamond-inefficient SLME:

We provide an example which illustrates the equilibrium notions defined above and which shows why an SLME may fail to be Diamond-efficient.

Technology: $g(k, l, \widetilde{x}) = h(k, l)\widetilde{x}$, where

$$h(k, l) = k^\delta l^\sigma, \text{ with } \delta + \sigma < 1 \text{ and } \delta > 0, \sigma > 0. \tag{D.1}$$

Utility functions:

$$u_i(c) = -e^{-\rho_i c} \quad \rho_i > 0. \tag{D.2}$$

LME and LMEFW:

We consider the LMEFW. The first-order conditions satisfied by the (k_i, l_i, m_i) vector which maximizes

$$Eu_i\left(m_i \frac{h(k_i, l_i)\widetilde{x} - rk_i}{s + l_i} + w(\omega_{li} - m_i) + r\omega_{ki} + \omega_{ci}\right)$$

are

$$Eu_i'(\cdot)[h_k\widetilde{x} - r] = 0 \tag{D.3}$$

$$Eu_i'(\cdot)[h_l\widetilde{x} - w] = 0 \tag{D.4}$$

and

$$Eu_i'(\cdot)\widetilde{x}[h - h_k k_i - h_l(l_i + s)] = 0. \tag{D.5}$$

Combining (D.3) and (D.4) yields

$$\frac{\delta l_i}{\sigma k_i} = \frac{h_k}{h_l} = \frac{r}{w}. \tag{D.6}$$

(D.5) reduces to

$$h - h_k k_i - h_l(l_i + s) = 0, \tag{D.7}$$

which can be further simplified when (D.1) holds. The result is

$$h\left[1 - \delta - \sigma\left(1 + \frac{s}{l_i}\right)\right] = 0, \tag{D.8}$$

which implies that

$$l_i = \frac{\sigma s}{[1 - (\delta + \sigma)]}. \tag{D.9}$$

Equation (D.9) implies that l_i is independent of i. If we now let ν be the total number of firms, the requirement that supply equal demand in the labor market combines with (D.9) to imply that

$$\nu\left\{s + \frac{\sigma s}{[1 - (\delta + \sigma)]}\right\} = \overline{\omega}_l. \tag{D.10}$$

When solved for ν, (D.10) becomes

$$\nu = \frac{\overline{\omega}_l[1 - (\delta + \sigma)]}{(1 - \delta)s}. \tag{D.11}$$

Since l_i is independent of i , (D.6) implies that k_i is also. The fact that all firms choose to employ the same amounts of capital and labor is a consequence of the fact that there are stochastic constant returns to scale because $g(k, l, x) = h(k, l)x$. This is an example of the well-known "unanimity" theorem of Ekern-Wilson [1974], Leland [1974] and Radner [1974]. The requirement that supply equal demand in the capital market implies that $k_i = k$, where k satisfies

$$\nu k = \overline{\omega}_k = \sum_{i \in I} \omega_{ki}\mu_i. \tag{D.12}$$

when (D.11), the expression for ν, is substituted in (D.12), the result is the following expression for k:

$$k = \left[\frac{\overline{\omega}_k}{\overline{\omega}_l}\right]\left[\frac{(1 - \delta)s}{[1 - (\delta + \sigma)]}\right] \tag{D.13}$$

To obtain an expression for r/w, we first use (D.9) and (D.13) to obtain the following expression for l/k:

$$l/k = \overline{\omega}_l/\overline{\omega}_k \left(\frac{\delta}{1-\delta}\right). \tag{D.14}$$

When (D.14) is substituted in (D.6), the result is the desired expression for r/w. Specifically,

$$r/w = \frac{\overline{\omega}_l}{\overline{\omega}_k}\left(\frac{\delta}{1-\delta}\right). \tag{D.15}$$

Up to this point, we have determined the equilibrium values for l, v, k and r/w in equations (D.9), (D.11), (D.l3), and (D.15) respectively. We can now compute m_i as well as the equilibrium price of capital, r. First note that, because (D.3) holds for all i,

$$h_k \frac{Eu_i'(\cdot)\widetilde{x}}{Eu_i'(\cdot)} = r = h_k \frac{Eu_1'(\cdot)\widetilde{x}}{Eu_1'(\cdot)} \tag{D.16}$$

must hold for all i. When (D.2) is used in (D.16), the result is

$$h_k \frac{E\widetilde{x}e^{-\rho_i m_i(\frac{k^\delta l^\sigma}{s+l})\widetilde{x}}}{Ee^{-\rho_i m_i(\frac{k^\delta l^\sigma}{s+l})\widetilde{x}}} = r = h_k \frac{E\widetilde{x}e^{-\rho_1 m_1(\frac{k^\delta l^\sigma}{s+l})\widetilde{x}}}{Ee^{-\rho_1 m_1(\frac{k^\delta l^\sigma}{s+l})\widetilde{x}}} \tag{D.17}$$

where l and k are given by (D.9) and (D.13) respectively.

Condition (D.17) will hold if

$$m_i = \frac{\rho_1}{\rho_i} m_1 \tag{D.18}$$

for all i. Furthermore, in equilibrium the m_i's must satisfy

$$\overline{\omega}_l = \nu(s+l) = \sum_{i \in I} \mu_i m_i. \tag{D.19}$$

When (D.18), the expression for m_i, is substituted in (D.19) the result is an expression for m_1 which, because of (D.18), also yields an expression for m_i. Specifically, (D.18) and (D.l9) imply that, for all i,

$$m_i = (1/\rho_i)\overline{\omega}_l \overline{\rho} \tag{D.20}$$

where

$$\overline{\rho} = \left[\sum_{t \in I} \mu_t (1/\rho_t)\right]^{-1} \tag{D.21}$$

is a measure of average risk aversion. Substituting (D.2O) in (D.17) yields the following expression for r

$$r = \delta k^{\delta-1} l^{\sigma} \frac{E\widetilde{x}e^{-\overline{\omega}_l \overline{\rho}(\frac{k^{\delta}l^{\sigma}}{s+l})\widetilde{x}}}{Ee^{-\overline{\omega}_l \overline{\rho}(\frac{k^{\delta}l^{\sigma}}{s+l})\widetilde{x}}} \tag{D.22}$$

This completes the description of the LME in this case.

SLME:

In this case, (k_i, l_i) are chosen to maximize

$$Eu_i\left(\omega_{li}\left[\frac{h(k_i,l_i)\widetilde{x} - rk_i}{s+l_i}\right] + r\omega_{ki} + \omega_{ci}\right).$$

The first-order conditions satisfied by the optimal (k_i, l_i) choice are (D.3) and (D.5). Recall that these are also first-order conditions for the LMEFW. But note that the third LMEFW equilibrium condition, (D.4), does not hold in an SLME. This condition arises in an LMEFW when m_i is optimally chosen. In the SLME, m_i is not chosen optimally because it is constrained to equal ω_{li}.

Conditions (D.3) and (D.5) can now be used to give an explicit description of the SLME when (D.1) and (D.2) describe the technology and preferences respectively. We first show that, as was the case with an LME, (D.5) determines l_i and ν. As noted above, (D.5) implies (D.7) which reduces to (D.8) when (D.1) holds. Equation (D.8) can then be solved to obtain the expression (D.9) for l_i which is independent of i. Thus in the SLME, as in the LMEFW, each firm employs the same amount of labor and that amount is, as in the LMEFW, given by equation (D.9). The number of firms, ν, is determined by the equality of labor supply and demand. When (D.9) holds, as it does in both the LMEFW and SLME, the supply-demand equality is expressed by (D.10) and the number of firms is the ν value given by (D.11). This argument establishes that there are an equal number of firms in the SLME and the LME in this example. It is possible to make this argument because each firm in an SLME employs the same amount of labor as each firm in an LME. In these two respects then, the SLME and LMEFW are the same.

We will now show that the allocation of capital is different in the two equilibria. This occurs because m_i is, in effect, restricted to equal ω_{li} in the SLME. Recall that m_i determines a type i individual's share of the risk associated with a firm of type i. Thus the share which any individual may bear of the risk associated with a firm is restricted. It is possible for individuals to compensate for these restrictions by varying k_i to increase or decrease the risk being shared. In an LMEFW, the m_i's varied across types as a reflection of the differences in risk aversion across types. Since the m_i's are restricted to equal ω_{li} in an SLME, the differences in risk aversion across types results in differences in k_i across types. In this way, variations in the level of capital employed serve as substitutes in an SLME for the variations in m_i which are permitted in an LMEFW. In a SLME, k_i choices are thus forced to serve two purposes: they determine output and they allocate risks. In an LMEFW, the k_i choice simply determines output; the m_i choice allocates risk. Because the k_i choice is only asked to perform a single function in an LMEFW, it can do so perfectly. As a consequence, each firm chooses the technologically efficient k value determined in (D.13). Because the k_i choice is used to determine the allocation of risk in an SLME, the resulting k_i choices are different for each i. They therefore differ from the technologically efficient level described in (D.13).

Because the k_i's play a role in the SLME which is analogous to the role played by m_i in the LMEFW, the equilibrium SLME values of k_i can now be obtained from the analysis previously used to obtain the equilibrium LMEFW values for m_i. We first recall that (D.3) implies that (D.16) holds for all i. In the study of the LMEFW, (D.16) was used to obtain (D.l7). In an SLME, (D.17) is replaced by

$$h_k \frac{Exe^{-\rho_i \omega_{il}(\frac{k_i^\delta l^\sigma}{s+l})\widetilde{x}}}{Ee^{-\rho_i \omega_{il}(\frac{k_i^\delta l^\sigma}{s+l})\widetilde{x}}} = r = h_k \frac{Exe^{-\rho_1 \omega_{1l}(\frac{k_1^\delta l^\sigma}{s+l})\widetilde{x}}}{Ee^{-\rho_1 \omega_{1l}(\frac{k_1^\delta l^\sigma}{s+l})\widetilde{x}}} \tag{D.23}$$

Equation (D.23) will hold for all i if

$$\rho_i \omega_{il} k_i^\delta = \rho_1 \omega_{1l} k_1^\delta \tag{D.24}$$

holds for all i. Equation (D.24) is the SLME analog of (D.18) in the LME. This equation can be solved for k_i in terms of k_1. Specifically, (D.24) implies that

$$k_i = \left[\frac{\rho_1 \omega_{1l}}{\rho_i \omega_{il}}\right]^{1/\delta} k_1. \tag{D.25}$$

The relationship displayed in (D.25) between the SLME equilibrium values k_i and k_1 parallels the relationship, expressed in (D.l9), between the LMEFW values m_i and m_1. Note that k_i equals k_j if and only if $\rho_i \omega_{il}$ equals $\rho_j \omega_{jl}$. Thus k_i is in general different from k_j. As a consequence, not all SLME firms employ the technologically efficient capital level determined in (D.13).

The exact amount of capital has not yet been determined, since (D.25) only specifies how k_i and k_1 must be related. In order to determine k_1, we use the fact that, in equilibrium, capital supply and demand must be equal. We must also determine the number of firms of type i. This is not necessary in an LME because all firms are the same; they all employ the same amounts of capital and labor. In an SLME, in which k_i is different for each type, the labor supplied by type i workers to type i firms will be $\omega_{li}\mu_i$. If there are ν_i firms of type i, they will demand $\nu_i(l+s)$ units of labor where l is given by (D.9). The demand for labor by type i firms will equal the supply of labor to these firms when

$$\nu_i\left(s+\frac{\sigma s}{1-(\delta+\sigma)}\right)=\omega_{li}\mu_i. \tag{D.26}$$

Equation (D.26) implies that the equilibrium number of type i firms is

$$\nu_i=\frac{\omega_{li}\mu_i[1-(\delta+\sigma)]}{(1-\delta)s}. \tag{D.27}$$

Using (D.27) and (D.25), the total demand for capital is

$$\sum_{i\in I}\nu_i k_i=\frac{[1-(\delta+\sigma)]}{(1-\delta)s}(\rho_1\omega_{1l})^{1/\delta}k_1\sum_{i\in I}\left(\frac{1}{\rho_i\omega_{il}}\right)^{1/\delta}\mu_i\omega_{li} \tag{D.28}$$

This demand can now be equated to $\overline{\omega}_k$ the supply of capital, and the resulting equation can then be solved for k_1. When the resulting k_1 is substituted in (D.25), the equilibrium k_i value is found to equal

$$k_i=\frac{\overline{\omega}_k s(1-\delta)}{[1-(\delta+\sigma)]}\left(\frac{1}{\rho_i\omega_{il}}\right)^{1/\delta}\left[\sum_{t\in I}\left(\frac{1}{\rho_t\omega_{tl}}\right)^{1/\delta}\mu_t\omega_{lt}\right]^{-1}. \tag{D.29}$$

This k_i value equals the efficient k level give in (D.13) if and only if $\rho_i\omega_{il}$ equals $\rho_j\omega_{jl}$ for all i and j.

The description of the SLME is completed when the equilibrium r is computed by substituting the expression for k_i in (D.23).

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