# Fixed Water Sharing Agreements Sustainable to Drought

Stefan Ambec* Ariel Dinar† and Daene McKinney‡§

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**Abstract**

By signing a fixed water sharing agreement (FWSA), countries voluntarily commit to release a fixed amount of river water in exchange for an agreed compensation. We examine the vulnerability of such commitments to reduced water flows. Among all FWSAs that are acceptable to riparian countries, we find out the one which is sustainable to the most severe drought scenarios. The so-called upstream incremental FWSA assigns to each country its marginal contribution to its followers in the river. Its mirror image, the downstream incremental FWSA, is not sustainable to reduced flow at the source. We apply our analysis to the Aral Sea basin agreement.

Keywords: international river treaty, water, stability, core, compliance, Aral sea.

JEL codes: D74, Q25, Q28, Q54.

*Toulouse School of Economics (INRA-LERNA), France, and Department of Economics, University of Gothenburg, Sweden, `stefan.ambec@toulouse.inra.fr`

†Water Science and Policy Center, Department of Environmental Sciences and University of California, Riverside, USA `adinar@ucr.edu`

‡Department of Civil, Architectural and Environmental Engineering, University of Texas at Austin, USA `daene@aol.com`

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# 1 Introduction

Water scarcity is becoming one of the major challenges worldwide. Because of population and economic growth, demand for water has tremendously increased. At the same time, water becomes less available in many parts of the world because of global warming. The higher world temperatures are expected to increase the hydrological cycle activity, leading to a general change in precipitation patterns and increase in evapotranspiration (IPCC 2007:7). Many semi-arid regions (e.g. Mediterranean, western United States, southern Africa and northeast Brazil) will suffer a decrease in water resources availability due to climate change (Bates et al. 2008). Moreover, other consequences of global warming such as the more frequent of extreme events of precipitation and dry periods and the early melting of glaciers would lead to an increase in the variance of water supply. In the next decades, global warming will not only reduce the mean flows of water supply but also its variance, especially in regions where water is scarce.

Since at least Hardin (1968) and Ostrom (1990), it has been established that the sustainable exploitation of common-pool natural resources, such as water, requires cooperation among users. In practice, users such as farmers, industries, cities or countries, coordinate water extraction through various arrangement from irrigation communities (Ostrom, 1990), to water markets (Libecap, 2011) or international river treaties (Dinar, 2008). Those arrangements are designed by users. They specify water releases and, sometime, payments through monetary transfers, taxes, prices and subsidies. Examples include international river sharing agreements in which countries commit to release water in exchange for compensations. For instance, by the Bishkek Treaty signed in 1998, compensation is paid for Kyrgyztan's compliance with release schedules that take into account winter energy needs and Uzbekistan's and Kazakhstan's summer irrigation water demands.

This paper addresses the vulnerability of existing water sharing arrangements to drought events. More precisely, it considers the problem of sharing water from a river with random water supplies where riparian countries coordinate water extraction through fixed water sharing agreements. Those agreements commit upstream countries to release fixed volumes of water in exchange for compensations by downstream users for any realized water supplies. We analyze the design of Fixed Water Sharing Agreements (FWSA) by sovereign countries. Countries agree on water releases and transfers based on their expected welfare before water supplies are realized. In case of low water supply, a country might be better off not releasing what it committed even if it has to renounce the compensation. We examine such defection strategies in case of droughts, where water supplies are below the long-term mean flow.

Examples of countries defection during droughts have been observed. Dinar et al. (2007) recorded complaints made during 1950-2005 regarding water sharing issues by states sharing international rivers. They found that a total of 112 complaints have been recorded regarding drought and floods between 1950-2005. One hundred and six of them regarding droughts and 6 regarding floods. In the Jordan River, while the Jordan-Israel water treaty of 1994 has mechanisms for dealing with shortages that cover a significant range of possible shortages, there is no stated mechanism for sharing shortages, mainly in prolonged droughts and extreme shortages, when they occur. This was the case in the 1998-2000 drought. Israel stated that it would not be possible to allow Jordan its water allocation according to the agreement, and it would have to reduce it.

Our framework is an extension of the river sharing problem introduced by Ambec and Sprumont (2002) to random water flows (see also Ambec and Ehlers 2008). We first study a cooperative game in which countries negotiate FWSAs based on expected water flows. In a negotiation among sovereign countries, the agreement should be accepted in a voluntary manner. In particular, countries are free to reject any water sharing agreement at the basin-wide level if they are better off signing agreements with a partial number of the basin riparians. To be accepted by all countries, the FWSA should make any group of countries better-off in terms of their expectations compared with any other partial agreement (including no agreement at all). In other words, the FWSA should be in the core of the cooperative game associated with the river sharing problem.

We first show that the cooperative game generated by the river sharing problem is convex. It implies that many river sharing agreements are in the core. One of them is the so-called downstream incremental FWSA introduced by Ambec and Sprumont (2002). It assigns to any country its marginal contribution to the set of predecessors in the river. By doing so, it maximizes lexicographically the welfare of the most downstream countries in the river in the set of core FWSAs. It thus favors downstream countries against upstream countries. We consider the FWSA opposite to the downstream incremental in the core: the upstream incremental FWSA. It assigns to each country its marginal contribution to its followers in the river.

We then examine the vulnerability of core FWSAs to defection in case of drought. A FWSA agreement specifies some amount of water to be released in exchange for monetary transfers. With water flows lower than the mean, a country is obliged to consume less than its water allocation under the FWSA in order to fulfill its commitment. Yet the payment it receives from the volume of water released is unchanged. With water being more valued by countries in case of drought, a country might be better off by not releasing the volume of water it committed, although at a cost of not getting the monetary transfer from downstream countries. For a given level of reduced flow, a FWSA is sustainable to some reduced water flows if no country chooses to defect by not releasing water. Among all core FWSAs, the upstream incremental FWSA is the most sustainable one in the sense that it maximizes the range of reduced flows for which no country defects. By maximizing payment for water released, it avoids defection in case of drought as much as possible. In contrast, since it assigns the lowest payment for water released, the downstream incremental FWSA is the less sustainable core FWSA. It is indeed not sustainable to drought for the first country in the river.

The economic literature includes several works that focus on various aspects of international water sharing issues and their stability in a basin setting. Several studies analyze river sharing agreements but with deterministic water flows (Ambec and Sprumont, 2002; Ambec and Ehlers, 2008, Wang, 2011, Ansink and Weikard, forthcoming). Yet, others introduce the water supply variability into their analysis. Kilgour and Dinar (2001) review several sharing rules that are common in international water treaties and demonstrate how they may not meet the treaty parameters under increased water variability. Alternative sharing rules are suggested and their sustainability is demonstrated, using the case of the annual flow of the Ganges River at Farakka, the flash point between India and Bangladesh. Focussing on interstate river compacts in the United States, Bennett et al. (2000) compare the efficiency of fixed versus proportional allocation of water with variable water flow in inter-state water compacts. They compute the optimal fixed water allocation taking into account flow variability, whereas, here we consider fixed water allocation based on mean flow. They do not address the issue of sustainability in case of drought, since the federal government has coercive power to enforce interstate compacts.

Ansink and Ruijs (2008) compare the performance of fixed and proportional agreements regarding their sustainability to reduce water flow. They rely on a two-country repeated game approach with self-enforcement constraints. Both types of agreements share the same division of welfare which translates into a payment from the downstream country to the upstream country. The authors show that fixed agreements are less sustainable than proportional agreements. Our paper departs from the last study in two features. First, we do not compare the performance of different types of agreements with similar exogenous surplus sharing rules (i.e. transfers) among countries. We rather focus on fixed agreements but endogenize the surplus sharing rule. We want to identify the surplus sharing rule (or equivalently the transfers among countries) that is more sustainable to drought. Our paper is thus more on the *design* of fixed water sharing agreements than on the comparison of different types of agreements. It aims to recommend transfers that are less vulnerable to defection in case of drought. Second, we do not restrict our analysis to bipartite agreements. We consider a river shared by $n \geq 2$ countries. By doing so, we allow for partial agreements in the river basin and coalition deviations during the negotiation. We also highlight the importance of the spatial structure in a river sharing problem. As suggested by Dinar (2008), geography is an important aspect that explains many of the outcomes of treaty stability as affected by water supply variability. We address the geography aspect in the design of the FWSA.

The paper proceeds as follow. We introduce the model in Section 2. We analyze the design of river sharing agreements in Section 3. In particular, we define the constrained upstream incremental river sharing agreement. We show that it is in the core and it is fair. In Section 4, we study the vulnerability of river sharing agreements to defection in case of drought. We show that the constrained upstream incremental river sharing agreement is the more sustainable river sharing agreement among those that are fair and in the core. We then turn to a numerical application of the Aral Sea and conclude.

## 2 The river sharing problem

A set $N = \{1, ..., n\}$ of countries are located along a river and share its water. We identify countries by their locations along the river and number them from upstream to downstream: $i < j$ means that $i$ is upstream to $j$. A coalition of countries is a non-empty subset of $N$. Given two coalitions $S$ and $T$, we write $S < T$ if $i < j$ for all $i \in S$ and all $j \in T$. Given a coalition $S$, we denote by $\min S \equiv \min_i S$ and $\max S \equiv \max_i S$, respectively, the smallest and largest members of $S$, i.e. $S = \{minS, ..., maxS\}$. Let $Pi = \{1, \ldots, i\}$ denote the set of predecessors of country $i$ and $P^0 i = Pi \backslash \{i\}$ denote the set of strict predecessors of country $i$. Similarly, let $Fi = \{i, i+1, \ldots, n\}$ denote the set of followers of country $i$ and let $F^0 i = Fi \backslash \{i\}$ denote the set of strict followers of $i$. For any $n$-dimension vector $\boldsymbol{y} = (y_i)_{i \in N}$, we denote by $\boldsymbol{y}_S = (y_i)_{i \in S}$ the vector of its components in $S$ for any arbitrary $S \subset N$.

Each country $i \in N$ enjoys a benefit $b_i(x_i)$ from diverting $x_i$ units of water from the river. We assume that the benefit function $b_i$ is differentiable, increasing and strictly concave for all $x_i > 0$. Furthermore, $b_i'(x_i)$ goes to infinity as $x_i$ approaches 0. A country also values money linearly in the sense that the welfare realized by country $i$ with $x_i$ units of water and $t_i$ units of money (or welfare or any numerary good) is $b_i(x_i) + t_i$.[1]

Each country $i \in N$ controls a flow of water $e_i \geq 0$ with $e_1 > 0$ at the river source. It includes water supplied by tributaries or stored in a reservoir controlled by $i$. The controlled water flows are random. The controlled flow $e_i$ ranges in $[\underline{e}_i, \bar{e}_i]$ with $0 \leq \underline{e}_i \leq \bar{e}_i$ and $\underline{e}_1 > 0$. The vector of flows $\boldsymbol{e}$ is distributed according to a density $f$ and cumulative $F$ with $f(\boldsymbol{e}) > 0$ for every $\boldsymbol{e} \in \times_{i \in N} [\underline{e}_i, \bar{e}_i]$.

Countries might agree before the realization of $\boldsymbol{e}$ to release some fixed amounts of water in exchange of some payments. A Fixed Water Sharing Agreement (FWSA) $(\boldsymbol{w}, \boldsymbol{\tau})$ is a vector of water releases $\boldsymbol{w}$ and payments $\boldsymbol{\tau}$ where $w_i$ denotes the amount of water country $i$ commits to release downstream while $\tau_i$ is the payment received by country $i$ in exchange of $w_i$ for $i = 1, .., n$. Of course, $w_n = 0$ since the most upstream country has no reason to release water and, therefore, receives no payment from downstream, i.e. $\tau_n = 0$. Symmetrically, $\tau_0 = 0$ and $w_0 = 0$ because the most upstream country 1 does not receive water from other countries. For a realization of $e_i$, the FWSA $(\boldsymbol{w}, \boldsymbol{\tau})$ allows country $i$ to consume $x_i = e_i + w_{i-1} - w_i$ units of water and $t_i = -\tau_{i-1} + \tau_i$ units of money for every $i \in N$. Given $e_i$, the *ex post* utility or welfare of country $i$ with the FWSA $(\boldsymbol{w}, \boldsymbol{\tau})$ is:

$$b_i(x_i) + t_i = b_i(e_i + w_{i-1} - w_i) - \tau_{i-1} + \tau_i.$$

Countries are expected utility maximizers. The *ex ante* welfare of country $i$ with the FWSA $(w, t)$ is defined by its expected utility or welfare given the distribution of $e_i$:

$$E[b_i(x_i) + t_i] = E[b_i(x_i)] + t_i = E[b_i(e_i + w_{i-1} - w_i)] - \tau_{i-1} + \tau_i.$$

[1]In other words, the benefit of water consumption is expressed in money.

The concavity of $b_i$ makes country $i$ dislike the variability of water flow. A river problem with random water flows is defined by $(N, \boldsymbol{e}, \boldsymbol{b})$ where $\boldsymbol{e}$ is a random vector of water flows distributed according to $f$ on $\times_{i \in N}[\underline{e}_i, \bar{e}_i]$.

In this setting, non-cooperative water extraction is inefficient. Under laisser-faire, each country $i$ extracts water flowing down on its territory. Country 1 consumes $e_1$ leaving nothing to country 2 who itself extracts its controlled water flow and so on. Individual welfare is $b_i(e_i)$ ex post and $E[b_i(e_i)]$ ex ante for every $i \in N$. This outcome is inefficient: the welfare of two countries $i$ and $j$ with $i < j$ can be improved if country $i$ releases some water to supply country $j$ in exchange of some transfer.[2] In this transferable utility (TU) set-up, efficiency would require water transfers to maximize total welfare. Moreover, under random water flows, water transfers should be contingent on realized water flows. Here we don't allow for such contingencies. We restrict ourself to ex ante fixed water transfers as it improves welfare in many cases. Those agreements are widely observed in international rivers. They are also quite popular in irrigation communities. The fixed nature of water transfers make them vulnerable to drought.

We first characterize the efficient FWSA at the basin level. Since utility is transferable, the efficient water releases vector denoted $\boldsymbol{w}^*$ is uniquely defined as the one that maximizes total welfare ex ante subject to feasibility constraints. It defines a water consumption vector $\boldsymbol{x}^* = (\boldsymbol{x}^*)_{i \in N}$ where $x_i^* = e_i + w_{i-1}^* - w_i^*$ for any realization $e_i \in [\underline{e}_i, \bar{e}_i]$, for every $i \in N$. Formally, $\boldsymbol{w}^*$ solves the following maximization program:

$$\begin{aligned} \max_{\boldsymbol{w}} \quad & E\left[\textstyle\sum_{i \in N} b_i(e_i + w_{i-1} - w_i)\right], \\ & \text{subject to} \\ & w_i \geq 0 \text{ for every } i \in N, \\ & \underline{e}_i + w_{i-1} - w_i \geq 0 \text{ for every } i \in N. \end{aligned}$$

The first set of feasibility constraints $w_i \geq 0$ for every $i \in N$ are on water releases: since water can only be transferred from upstream to downstream water releases cannot be negative. The second set of feasibility constraints $\underline{e}_i + w_{i-1} - w_i \geq 0$ are on water consumption under the lowest water supply $\underline{e}_i$. These constraints make sure that consumption $x_i = e_i + w_{i-1} - w_i$ is non-negative for any realized water flows $e_i \in [\underline{e}, \bar{e}_i]$ so that country $i$ will always be able to

[2] Indeed there exists $\epsilon > 0$ such that $b_i(e_i - \epsilon) + b_j(e_j + \epsilon) > b_i(e_i) + b_j(e_j)$. The later condition of welfare improvement holds in many cases for instances if $b_i = b_j$ and $e_i > e_j$.

release what it committed to. This constraint should hold for every country $i \in N$. Denoting $\mu_i$ and $\lambda_i$ the Lagrangian multipliers of the first and the second set of feasibility constraints respectively, the first-order conditions yield:

$$E[b'_{i+1}(x^*_{i+1}) - b'_i(x^*_i)] = \lambda_i - \lambda_{i+1} - \mu_i,$$

for $i = 1, ..., n-1$. The above conditions implies for any $j > i$:

$$E[b'_j(x^*_j) - b'_i(x^*_i)] = \lambda_i - \lambda_j - \sum_{l=i}^{j} \mu_l, \tag{1}$$

The first-order conditions prescribe equalizing ex ante marginal benefits of water consumption whenever it is possible. If not, then one of the constraints is binding. It could be that the non-negative water release constraint is binding at say $i$, and, therefore, $\mu_i > 0$. Or the non-negative water consumption constraint is binding, in which case $\lambda_i > 0$.

We show that, under infinite marginal benefit at zero water consumption, the non-negative water consumption constraints are not binding. If it was binding for, say, country $j$, then water consumption in case of extreme drought $\underline{e}_j$ would be set to zero for $i$ which implies an infinite marginal benefit in this case, formally $b'_j(\underline{x}^*_j) = +\infty$ where $\underline{x}^*_j = \underline{e}_j + w^*_{j-1} - w^*_j$. Since $\underline{e}_j$ occurs with strictly positive probability, it implies that $j$'s marginal benefit is also infinite in expectation: $E[b'_j(x^*_j)] = +\infty$. On the other hand, since $\underline{e}_1 > 0$, at least some country $i$ consumes water in all states of nature. For this country $i$, expected marginal benefit is finite: $E[b'_i(x^*_i)] < +\infty$. The last two conditions on expected marginal benefits for $i$ and $j$ contradicts the first-order condition (1). We therefore conclude that $\lambda_i = 0$ for every $i \in N$ so that the first-order condition (1) becomes:

$$E[b'_i(x^*_i) - b'_j(x^*_j)] = \sum_{l=i}^{j} \mu_l, \tag{2}$$

Ex ante marginal benefits are equalized whenever the non-negative water release constraint is not binding between $i$ and $j$. If it is at some location $l$ with $i < l < j$ then $\mu_l > 0$ which implies $E[b'_i(x^*_i)] > E[b'_j(x^*_j)]$: country $i$ enjoys a higher ex ante marginal benefit from water consumption than country $j$. Moreover, in this case, $w_l = 0$ and, therefore, no water transferred from $i$ to $j$. Indeed, binding the constraint at $l$ would imply not releasing water from country $l$. It is optimal to do that if water is relatively more abundant downstream $l$ in expectation.

Since marginal benefits reflect water scarcity in the sense that more water leads to a lower marginal benefit, ex ante marginal benefit is lower downstream.

Similarly to Ambec and Sprumont (2002) for deterministic water flows $e$ (see also Kilgour and Dinar, 2002), the best FWSA partitions the set of agents $N$ into consecutive subsets $\{N_k\}_{k=1}^{K}$ with $N_k < N_{k+1}$ for $k = 1, ..., K-1$. It defines the ex ante shadow value of water $\{\beta_k\}_{k=1}^{K}$ at each segment $N_k$ of the river with $\beta_k > \beta_{k+1}$. Ex ante marginal benefits from water consumption are equalized among countries within $N_k$. They are equal to the ex ante shadow value of water $\beta_k$ at $N_k$. Countries in $N_k$ share the water flows they control $\sum_{i \in N_k} e_k$ and, therefore, do not transfer water downstream of $N_k$. Ex ante marginal benefit decreases moving from $N_k$ to $N_{k+1}$ as well as the shadow value of water. Formally, the following conditions hold:

$$N_k < N_{k'} \text{ and } \beta_k > \beta_{k'} \text{ whenever } k < k'$$

$$E[b_i'(x_i^*)] = \beta_k = \sum_{j \leq i} \mu_j \text{ for every } i \in N_k \text{ and } k = 1, ...K,$$

$$w_{maxN_k}^* = 0 \text{ for } k = 1, ...K.$$

Having fully described the efficient vector of water releases $w^*$, we now come back to the design of the FWSA. We consider the following timing for the design and compliance to a FWSA.

1. Countries agree on a FWSA $(\boldsymbol{w}, \boldsymbol{\tau})$.

2. Water flows $\boldsymbol{e}$ are realized.

3. Each country decides to comply or not with the FWSA.

We first model the negotiation among countries on a FWSA as a cooperative game. We do not impose any structure on the bargaining game. Instead we assign minimal restrictions on the outcome of the negotiation given by the bargaining power of countries represented by their *worth*. The worth of a coalition of countries is the welfare that the coalition can secure by itself (i.e. by leaving the negotiation). A FWSA at the basin level should at least assign its worth to any coalition of countries. Otherwise, a coalition would block the FWSA, objecting that it can achieve a higher welfare on its own. In the next section, we describe the set of core FWSAs defined as FWSAs that are not blocked by any coalition. We then discuss the emergence of some of the core FWSAs in specific non-cooperative bargaining games or water markets.

# 3 The design of FWSAs

When leaving the negotiation for a basin-wide FWSA, a coalition $S \subset N$ can still rely on its controlled water flows $e_S = (e_i)_{i \in S}$. The welfare that coalition $S$ can secure is the highest welfare achieved by designing a partial FWSA $(\boldsymbol{w}_S, \boldsymbol{\tau}_S)$ among the members of $S$. Denoted $v(S)$, the worth of a coalition $S$ can easily be defined for a connected coalition. A coalition $S$ is connected if for all $i, j \in S$ and all $k \in N$, $i < k < j$ implies $k \in S$. For a connected coalition $S$,

$$v(S) = \max_{\boldsymbol{w}_S} \quad \begin{array}{l} E\left[\sum_{i \in S} b_i(e_i + w_{i-1} - w_i)\right], \\ \text{subject to} \\ w_i \geq 0 \text{ for every } i \in S, \\ \underline{e}_i + w_{i-1} - w_i \geq 0 \text{ for every } i \in S, \end{array} \tag{3}$$

where $w_{minS-1} = 0$. In particular, the stand-alone welfare of country $i$ is simply $v(\{i\}) = E[b_i(e_i)]$. Let us denote by $\boldsymbol{w}_S^S$ the solution to (3). It is the efficient vector of water releases of the reduced game $(S, \boldsymbol{e}_S, \boldsymbol{b}_S)$.

For a disconnected coalition, we first need to decompose the coalition into its connected components. Let $\mathcal{P}(S) = \{S_l\}_{l=1}^L$ be the unique partition of $S$ into its connected components. Since water cannot be transferred between two components $S_l$ and $S_{l+1}$ of $\mathcal{P}(S)$, the worth of coalition $S$ is obtained by summing up the worth of its connected components:

$$v(S) = \sum_{S_l \in \mathcal{P}(S)} v(S_l), \tag{4}$$

where $v(S_l)$ is given by (10). A FWSA $(\boldsymbol{w}, \boldsymbol{\tau})$ is not blocked by coalition $S \subset N$ if

$$\sum_{i \in S} \left(E\left[b_i(e_i + w_{i-1} - w_i)\right] - \tau_{i-1} + \tau_i\right) \geq v(S). \tag{5}$$

We say that a FWSA is in the core of the cooperative game generated by the problem $(N, \boldsymbol{b}, \boldsymbol{e})$ if the no-blocking condition (5) holds for every $S \subset N$. We call $v(S)$ the core lower bound for coalition $S$ for every $S \subset N$.

Clearly, the core lower bound for the "grand coalition" $N$ forces the FWSA to be efficient. Indeed, since $v(N) = \sum_{i \in N} E[b_i(x_i^*)] = \sum_{i \in N} E[b_i(e_i + w_{i-1}^* - w_i^*)]$, the core lower bounds determine fully water releases to $\boldsymbol{w} = \boldsymbol{w}^*$. Monetary transfers $\boldsymbol{\tau}$ still need to be defined. To do so, it is convenient to work with welfare distributions instead of payments. Let us define

$\boldsymbol{u} = (u_i)_{i\in N}$ a distribution of the total ex ante welfare $v(N)$ with $\sum_{i\in N} u_i = v(N)$. There is a mapping between welfare distributions and transfers. To a given transfer scheme $\boldsymbol{\tau}$ corresponds a unique distribution of the welfare $\boldsymbol{u}$ where $u_1 = E[b_1(x_1^*)] + \tau_1$, $u_i = E[b_i(x_i^*)] - \tau_{i-1} + \tau_i$ for $i = 2, ..., n-1$ and $u_n = E[b_n(x_n^*)] - \tau_{n-1}$. Hence, from the welfare distribution $\boldsymbol{u}$ with $\sum_{i\in N} u_i = v(N)$, one can compute the monetary transfers defined as $\tau_i = \sum_{j\leq i}(u_i - E[b_j(x_j^*)])$ for $i = 1, ..., n-1$.

We will say that a welfare distribution $\boldsymbol{u}$ satisfies the core lower bounds if for every $S \subset N$:

$$\sum_{i\in S} u_i \geq v(S).$$

A welfare distribution that satisfies the core lower bounds is called a core welfare distribution. We now establish a useful property of the characteristic function $v$, namely its convexity. The proof is in Appendix A.

**Proposition 1** *The cooperative game $v$ is convex in the sense that $v(S) - v(S\backslash i) \geq v(T) - v(T\backslash i)$ for every $i \in T \subset S \subset N$.*

The above proposition allows us to describe the full set of core welfare distributions. Shapley (1971) has shown that the core of a convex game is the convex hull of the so-called *marginal contribution vectors*. A marginal contribution vector assigns to each agent its marginal contribution to the coalition composed by its strict predecessors in a specific ordering of all agents. Let us define such an ordering by $\gamma$ which is a bijection from $N$ to $N$. The vector of marginal contributions of the ordering $\gamma$ assigns $u_i = v(P\gamma(i)) - v(P^0\gamma(i))$ to agent $i$ for $i = 1, ..., n$. All these marginal contribution vectors are in the core. Moreover, the core contains all linear combinations of marginal contribution vectors. One example is the *Shapley value* which assigns to every agent $i$ its marginal contribution to all possible orderings weighted by the probability of such an ordering. It is indeed the barycenter of the core. An other interesting element of the core is the so-called *downstream incremental* distribution proposed by Ambec and Sprumont (2002). Denoted $\boldsymbol{u}^d$, it considers the natural ordering along the river $\gamma(i) = i$. It assigns to any country $i$ its marginal contribution to the coalition composed by its predecessors along the river: $u_i^d = v(Pi) - v(P^0 i)$ for $i = 1, ..., n$. It is the unique welfare distribution in the core that maximizes lexicographically the welfare of the most downstream users $n, n-1, ..., 1$. Given the above definition of $u_i^d$ for every $i \in N$, the upstream incremental distribution determines the payments for water releases $\tau_i^d$ for every $i \in N$ as:

$$\tau_i^d = v(Pi) - E\left[\sum_{j \in P_i} b_j(e_j + w_{j-1}^* - w_j^*)\right]. \tag{6}$$

Payments are based on losses for upstream countries. The compensation paid by country $i+1$ to country $i$ is the expected loss from releasing $w_i^*$ units of water at $i$ for all upstream countries.

The welfare distribution opposite to the downstream incremental in the core is the *upstream incremental* distribution. It considers the reverse ordering of agents $\gamma(i) = n - i$. Defined as $u_i^u = v(Fi) - v(F^0 i)$ for $i = 1, ..., n$, it assigns to country $i$ its marginal contribution to its successors along the river. The upstream incremental distribution is the core welfare distribution that maximizes lexicographically the welfare of the most upstream agents $1, 2, ...n$. The upstream incremental distribution determines the payments for water releases $\tau_i^u$ for every $i \in N$:

$$\tau_i^u = E\left[\sum_{j \in F_i^0} b_j(e_j + w_{j-1}^* - w_j^*)\right] - v(F^0 i). \tag{7}$$

Payments are based on gains for downstream countries. The compensation paid by country $i+1$ to country $i$ is the expected gain from releasing $w_i^*$ units of water at $i$ for all downstream countries.

Although we did not put any structure in the bargaining game when deriving the above welfare distribution solutions, it is easy to show that they might emerge as an outcome of particular non-cooperative game. Consider for instance water trading among countries in which most upstream countries have bargaining power. More precisely, supposer that country 1 makes a take-it-or-leave-it water trade offer $(w_1, \tau_1)$ to country 2 who then does the same to country 3 and so on up to country $n$. Then backward induction shows that subgame perfect equilibrium share welfare according to the upstream incremental distribution. The most downstream country $n$ would accepts any water release $w_{n-1}$ and payment $\tau_{n-1}$ that yields its at least its stand alone ex ante welfare $v(n)$.[3] To maximize its ex ante welfare, country $n-1$ would leave country $n$ on its participation constraint and, therefore assigns exactly $v(n)$ to $n$. Now move backward to the bargaining between country $n-2$ and $n-1$. Country $n-1$ would accept any water release $w_{n-2}$ and payment $\tau_{n-2}$ that assigns at least what it would get by

[3] $v(n)$ is a slight abuse of the notation $v(\{n\})$ which is the worth of coalition $\{n\}$.

itself while bargaining with country $n$, that is $v(n-1,n) - v(n)$. Country $n-2$ leaves country $n-1$ on its participation constraint by assigning $v(n-1,n) - v(n)$ to $n-1$. Doing so country $n-2$ achieves at least $v(n-2,n-1,n) - v(n-1,n)$ which is its outside option if it refuses an offer to country $n-3$. Moving backward again and gain leads to the upstream incremental distribution as an outcome of the subgame perfect payoffs of the bargaining game.[4]

A similar argument holds if bargaining power is assigned to downstream countries during bilateral trades. Suppose that country $n$ makes a take-it-or-leave-it offer to country $n-2$ who them makes a take-it-or-leave-it offer to country $n-3$ and so on up to country 1. Then, by backward induction, one can easily show that subgame perfect equilibrium payoffs share the total welfare $v(N)$ according to the downstream incremental distribution. At the last bargaining stage, country 1 accepts any offer $w_1$ and $\tau_1$ from country 2 assigning itself at least $v(1)$. Given that, country 2 would leave exactly $v(1)$ to country 1. It would accept any $w_2$ and $t_2$ from country 3 that assigns at least $v(1,2) - v(1)$ and so on. The arguments follows up to country $n$.

Which bargaining structure seems most natural here is debatable.[5] In particular, being upstream does not necessarily provide more bargaining power. Downstream users might have access to other tributaries or storage facilities located within their territories. They might also be able to value water more efficiently through irrigation and hydropower infrastructures. All these elements are embedded in the $e_i$ parameters and $b_i$ functions which determine the outside option of countries. Although downstream countries demand water, upstream countries need the payment from downstream countries to be able to extract more from water management. During the negotiation among two countries, there is no clear justification for assuming more bargaining power to the one who provides water (upstream) rather than the one who provides money (downstream).

A last solution is the welfare distribution that emerges if countries are assigned property rights on their control flows $e$ and exchange water in a competitive market. Let us call it the Walrasian welfare distribution. It relies on the assumption of price-taker countries which, therefore, have limited bargaining power when they decide how much to buy upstream and how

[4] Notice that it implies that not only transfers $\boldsymbol{t}^u$ are implemented but also efficient water releases $\boldsymbol{w}^*$.

[5] On this issue, Van der Brink and al. (2007) criticize the downstream incremental distribution for not giving "any incentive to a player $i$ to cooperate with its successors."

much to release downstream. The Walrasian welfare distribution and corresponding Walrasian FWSA $(\boldsymbol{w}^*, \boldsymbol{\tau}^w)$ assigns $u_i = E[b_i(e_i^* + w_{i-1}^* - w_i^*)] + \beta_k(w_{i-1}^* - w_i^*)$ to every country $i \in T_k$ for $k = 1, ..., K$. It decomposes the river into $K$ local markets for water where $\beta_k$ is the price of water at $T_k$ for $k = 1, ..., K$. Country $i \in T_k$ buys $w_{i-1}^*$ units of water to country $i-1$ and sells $w_i^*$ units of water to country $i$ at price $\beta_k$. It thus pays $\tau_{i-1}^w = \beta_k w_{i-1}^*$ to country $i-1$ and receives $\tau_i^w = \beta_k w_i^*$ from country $i$. Notice that if $i - 1 \in T_{k-1}$ then $w_{i-1}^* = 0$ and country $i$ does not buy water to country $i-1$. Similarly, if $i \in T_{k+1}$ then $w_i^* = 0$ and country $i$ does not sell water to country $i+1$. Notice that, under deterministic water flows, Wang (2011) shows that bilateral trading among price-takers countries leads to the Walrasian welfare distribution.

## 4 Sustainable FWSAs

We examine compliance with FWSAs. By signing an FWSA $(\boldsymbol{w}, \boldsymbol{\tau})$, countries commit to release water against money independently of the realized water flows. For some realized water flow, some countries might be tempted not to fulfill their obligations. Indeed, even if signing a FWSA is welfare increasing ex ante, ex post some countries might be better off not not complying with the FWSA. First, a country might be better off not releasing what it committed to the next downstream country. Second, a country might be better off not buying the water it committed to the next upstream country. The former defection arises with lower water flows than expected whereas the later is tempting when water is more abundant than expected. We focus on defection in case of drought as defined below.

**Definition 1** *An FWSA $(\boldsymbol{w}, \boldsymbol{\tau})$ is sustainable to reduced flow $e_i \leq E[e_i]$ if:*

$$b_i(e_i + w_{i-1} - w_i) + \tau_i - \tau_{i-1} \geq b_i(e_i + w_{i-1}) - \tau_{i-1}.$$

The above no-defection constraints insures that country $i$ is better-off by releasing $w_i$ rather than consuming all water. The no-defection constraint for country $i$ and realized flow $e_i$ simplifies to,

$$\tau_i \geq b_i(e_i + w_{i-1}) - b_i(e_i + w_{i-1} - w_i). \tag{8}$$

The transfer paid for $w_i$ should not exceed the relative value of $w_i$ for country $i$ for any realized water flow. Since the left-hand side is decreasing with $e_i$, one need to consider only the lowest water flow $\underline{e}_i$ to asses the sustainability of a FWSA.

**Definition 2** *An FWSA* $(\boldsymbol{w}, \boldsymbol{\tau})$ *is sustainable if*

$$\tau_i \geq b_i(\underline{e}_i + w_{i-1}) - b_i(\underline{e}_i + w_{i-1} - w_i) \text{ for every } i \in N.$$

We are now able to establish the main result of the paper. It characterizes the upstream incremental FWSA as the (unique) core FWSA that is the most sustainable to drought (the proof is in appendix).

**Proposition 2** *The upstream incremental FWSA* $(\boldsymbol{w}^*, \boldsymbol{\tau}^u)$ *is the most sustainable FWSA in the sense that it is sustainable to more severe droughts than any other core FWSA.*

Proposition 2 allows us to determine the minimal flow of water such that a FWSA might be sustainable. It implies that, if a realized water flow is not sustainable under the FWSA, no FWSA is sustainable. Combining the definition of $\boldsymbol{\tau}_i^u$ in (7) with the no-defection constraint (8) defines the minimal flow $\tilde{e}_i$ such that $(\boldsymbol{w}^*, \boldsymbol{\tau}^u)$ is sustainable:

$$b_i(\tilde{e}_i) - b_i(\tilde{e}_i + w_{i-1}^* - w_i^*) = \sum_{j \in F^0 i} E\left[b_j(x_j^*)\right] - v(F^0 i) \tag{9}$$

We thus obtain the following Corollary.

**Corollary 1** *A FWSA can be made sustainable if and only if* $\underline{e}_i \geq \tilde{e}_i$ *for every* $i \in N$.

In the particular case where $\underline{e}_i \geq \tilde{e}_i$ for every $i \in N$ then, among all the core FWSA, only the upstream incremental FWSA is sustainable. The minimal flow $\tilde{e}_i$ for $i = 1, ..., n$ is a measure of sustainability for FWSAs. It indicates weather compliance in case of drought is a serious issue or not. If it is, the upstream incremental FWSA should be selected. If not, other FWSAs might be sustainable. Therefore, other considerations such as fairness concerns might be invoked to selected a FWSA among those who satisfy the core lower bounds. For instance, Ambec and Sprumont (2002) proposes a fairness criteria called the aspiration welfare upper bounds that selects the downstream incremental FWSA under deterministic flows. Under random water flows, the next proposition shows that the downstream incremental FWSA is not a good candidate among all core FWSAs to insure sustainability (the proof is in appendix).

**Proposition 3** *The downstream incremental FWSA* $(\boldsymbol{w}^*, \boldsymbol{\tau}^d)$ *is the less sustainable FWSA in the sense that all other core FWSAs are sustainable to more severe droughts. It is not sustainable to reduced flow at the source.*

Proposition 3 provides another characterization of the downstream incremental FWSA: among all core FWSA, it is the less sustainable one. Since the downstream incremental FWSA is the only FWSA that satisfies the core lower bounds the fairness (aspiration welfare) upper bounds, Proposition 3 implies that, with random water flows, no core FWSA is fair and sustainable to drought.

# 5 Application to the Aral Sea Basin

We illustrate our approach with a simple example of 3 players, calibrated to the Aral Sea basin. More precisely, we focus on the Bishkek international agreement signed in 1998 by Kyrgyzstan (KG), Uzbekistan (UZ) and Kazakhstan (KZ) on the Syr Darya river. The Syr Darya is one of the main streams that create the Aral Sea Basin in Central Asia. A description of the various features of the Syr Darya River, within the Aral Sea Basin are provided in Dinar et al, (2007). Dukhovny and de Schutter (2011) estimate the total annual river runoff between 1951-1975 to be 37.2 km3. Of that volume, the runoff formed within KG, UZ and KZ is 74.2, 16.6, and 6.5 percent, respectively. Tajikistan contributes a miniscule amount of 2.7 percent, and for practical purposes it is not considered a riparian to this river. KG is the upstream riparian, using the water for electricity generation. UZ and KZ are both downstream riparians that use the water for irrigation of field crops (mainly wheat and cotton). The heart of the conflict between the three riparians stems from the reciprocal need-period of water for production of electricity (winter) and irrigation (summer). These conflicts are exacerbated by two factors related to climate change, namely variation in water availability across years, and extreme temperature low values in winter experienced by the upstream riparian KG. After several conflict incidents that followed the 1991 Soviet Union collapse the riparian states reached several agreements, including the 1998 Bishkek Barter Agreement. Without entering the agreement features and usefulness, the barter details (Dinar et al., 2007) suggest that KG receives from KZ the equivalent of 1.1 billion kWh of electric power in the form of coal (valued at 22 million dollars) and 400 million kWh + 500 million m3 of natural gas (valued at 48.5 million dollars) from UZ. In return, KG releases 3.25 billion m3 of water from the Toktogul Reservoir in monthly flows during the irrigation season and 2.2 billion kWh of summer electricity (from its hydropower facility on the Toktogul reservoir and downstream cascade) to KZ and UZ. Water release in summer was renegotiated to 1.3 billion m3 in 2000 and 2.5 billion m3 in 2001.[6] The 2000 agreement specifies that the summer water release should be allocated equally between KZ and UZ.

Using an integrated hydrologic-agronomic-economic model of the Syr Darya basin (Cai, McKinney and Lasdon, 2003), we estimated a quadratic water benefit function for each of the three countries. Releasing $D_1$ billion cubic meters from the Toktogul Reservoir allows KG to produce hydropower with an estimated benefit of $B_1(D_1) = 10.9D_1 - 0.032D_1^2$ in millions of dollars. From the $D_1$ billion m3 released by KG, let us denote UZ and KZ's water consumption in billion m3 by $D_2$ and $D_3$ respectively with $D_2 + D_3 = D_1$. The agricultural benefit from KG's water releases is $B_2(D_2) = 12.749 + 538D_2 - 22D_2^2$ for UZ and $B_3(D_3) = 3.148 + 540D_3 - 23D_3^2$ for KZ. The intercepts 12.749 and 3.148 represent the value of crop produced with the water inflows controlled by UZ and KZ, respectively. Under the above benefit functions, we estimate the upstream and downstream incremental transfers paid to KG for the 1998, 2000 and 2001 agreements. Consistent with theory, under the downstream incremental transfer $t^d$, the most upstream country is compensated exactly for its loss of welfare. That means that KG is paid for the loss of hydropower in winter due to water release in summer. The transfer $t^d$ is thus defined as the expected loss of welfare for KG due to summer water releases. If KG has to release 3.25 billion m3 in summer in compliance with the 1998 agreement, then the downstream incremental transfer is the difference between the expected value of hydropower production with and without 3.25 billions m3.[7] Symmetrically, the upstream incremental transfer is the increased welfare due to summer water releases in UZ and KZ. Since the intercept evaluates the benefit without (summer) water releases, it is simply the difference between the benefit with $3.25/2 = 1.625$ billion m3 and the intercept for each country.[8] We sum up the two differences to obtain the transfer received by KG under the upstream incremental distribution. The estimated transfers are presented in Table 1 below.

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[6] Sources: www.ce.utexas.edu/prof/mckinney/papers/aral/central_asia_regional_water.htm and www.cawater-info.net/bk/water_law/part3_e.htm

[7] More precisely, we compute the expected benefits $E[B_1(D_1)]$ with water releases $D_1$ corresponding to the water inflows described in Table 2 (with the probabilities computed in the first column) and the expected benefit with the same water releases minus 3.25 billion m3.

[8] Consistently with the 2000 agreement, water released by KG $D_1$ is shared equally between UZ and KZ: $D_2 = D_3 = D_1/2$. It is also approximatively the optimal split of $D_1$ given UZ's and KZ's benefit functions.

| Date | Delivery in billion m3 | $t^d$ in million $ | $t^u$ in million $ |
|---|---|---|---|
| 1998 | 3.25 | 33.3 | 1633 |
| 2000 | 1.3 | 13.2 | 682 |
| 2001 | 2.5 | 25.5 | 1277 |

Table 1: Water and monetary transfers

Our Aral Sea basin example illustrates the magnitude of the difference between the two solutions. It also suggests that the range of acceptable transfers defined as $[t^u, t^d]$ is quite significant. The transfer negotiated in the 1998 agreement which is $22 + 48.5 = 70.5$ million dollars, turns out to be included in this range. In Table 2 below, we compute the loss of welfare for all water inflows under the agreements signed in 1998, 2000 and 2001. That is the difference between $B_1(q)$ and $B_1(q - R)$ for any realized water inflows $q$ under the committed release $R$ with $R = 3250$ for 1998, $R = 1300$ for 2000 and $R = 2500$ for 2001.

| Probability of higher inflow $P(Q > q)$ | Water Inflow $q$ in million m3 | Loss 1998 agreement in million \$ | Loss 2000 agreement in million \$ | Loss 2001 agreement in million \$ |
|---|---|---|---|---|
| 0.990 | 6525 | 34.4 | 13.7 | 26.4 |
| 0.980 | 7478 | 34.2 | 13.6 | 26.3 |
| 0.970 | 7750 | 34.2 | 13.6 | 26.2 |
| 0.945 | 8290 | 34.0 | 13.5 | 26.1 |
| 0.895 | 8810 | 33.9 | 13.5 | 26.0 |
| 0.830 | 9232 | 33.8 | 13.5 | 26.0 |
| 0.780 | 9714 | 33.7 | 13.4 | 25.9 |
| 0.720 | 10267 | 33.6 | 13.4 | 25.8 |
| 0.605 | 10763 | 33.5 | 13.3 | 25.7 |
| 0.495 | 11286 | 33.4 | 13.3 | 25.6 |
| 0.430 | 11746 | 33.3 | 13.2 | 25.6 |
| 0.390 | 12130 | 33.2 | 13.2 | 25.5 |
| 0.330 | 12755 | 33.1 | 13.2 | 25.4 |
| 0.260 | 13207 | 33.0 | 13.1 | 25.3 |
| 0.210 | 13686 | 32.9 | 13.1 | 25.3 |
| 0.165 | 14329 | 32.8 | 13.0 | 25.2 |
| 0.110 | 14702 | 32.7 | 13.0 | 25.1 |
| 0.065 | 15152 | 32.6 | 13.0 | 25.0 |
| 0.050 | 15763 | 32.5 | 12.9 | 24.9 |
| 0.041 | 16250 | 32.4 | 12.9 | 24.9 |
| 0.035 | 16590 | 32.3 | 12.8 | 24.8 |
| 0.030 | 17250 | 32.2 | 12.8 | 24.7 |
| 0.027 | 17750 | 32.1 | 12.7 | 24.6 |
| 0.023 | 18250 | 32.0 | 12.7 | 24.5 |
| 0.020 | 18754 | 31.9 | 12.7 | 24.4 |
| 0.017 | 19250 | 31.8 | 12.6 | 24.4 |
| 0.015 | 19750 | 31.7 | 12.6 | 24.3 |
| 0.010 | 20725 | 31.5 | 12.5 | 24.1 |

Table 2: Loss of welfare due to water release depending on water inflow under the three agreements

As expected, the loss of benefit is increasing with a decline in water inflow. For a given inflow $q$, KR is better-off defecting if the loss of benefit from releasing water is higher than the transfer it receives. Consider the two transfers $t^d$ and $t^u$ computed in Table 1. None of the agreement would be sustainable with the transfer $t^d$ when inflow is lower than 11 billions m3 (approximately) which occurs 40% of the time. However, all agreements are sustainable with $t^u$ for any potential inflow according to our estimations. Furthermore, the 70.5 million dollars compensation for KG stipulated from the 1998 agreement seems also to prevent KG from defecting for any realized inflow.

# 6 Conclusion

By signing international river sharing treaties voluntarily, countries agree to release some fixed amount of water in exchange for some compensation. They have a self-interest in complying with the releases when water inflow is high enough. Even if an agreement specifies water supply to downstream countries, a country is better off by releasing what it had committed to, since the payment it receives from downstream countries offset its welfare loss from releasing water. This is not always the case under water drought conditions within its territory. To release the same amount of water, the country is obliged to consume less water. It might be tempted to defect if the payment ir receives does not compensate its welfare loss from releasing the water.

In this paper, we analyze the design of fixed water sharing agreements under variable water flow and their robustness to the above defection strategy by countries. We first fully characterize the set of agreements that are acceptable by all groups of riparian countries. They all prescribe the same water releases: those which maximize the expected welfare of water extraction along the river. In contrast, many monetary transfers can be part of an acceptable water agreement including the ones defined by the Shapley value, the Walrasian allocation and the downstream incremental welfare distribution. They might emerge from a negotiation process among countries.

Among the set of acceptable monetary transfers, we identify the one which is the most robust to defection in case of drought. It is the upstream incremental transfer scheme which requires that each country receives the marginal contribution of its water releases to all the countries located downstream. It maximizes lexicographically the welfare of the most upstream countries in the set of acceptable transfers. Opposite in this set is the downstream incremental transfer scheme which maximizes lexicographically the welfare of the most downstream countries. The downstream incremental transfer scheme turns out to be less robust to defection than any other acceptable transfer scheme. Our computation from a simple representation of the Aral Sea basin provides evidence that the two types of solutions can differ substantially. It thus suggests that picking the right agreement can greatly reduce the vulnerability of fixed water sharing agreements to global warming.

# A Proof of Proposition 1

The proof generalizes Ambec and Sprumont (2000) to random water flows. For any coalition $S$, let $\boldsymbol{w}_S^S$ denote the water releases solution to the program (3) defined by $v(S)$. We use the following notation: for any two coalitions $R$ and $S$, $R < T$ (resp. $R > T$) means $R$ is strictly upstream (resp. strictly downstream) $S$ in the sense that $j < i$ (resp. $j > i$) for any $j \in R$ and $i \in S$. We first proof the following Lemma.

**Lemma 1** *For any two connected coalitions $T, S$ with $T \subset S$,*

*(a) If $S \backslash T < T$, $w_j^S \geq w_j^T$ and $x_j^S \geq x_j^T$ for every $e_j \in [\underline{e}_j, \bar{e_j}]$, for every $j \in T$.*

*(b) If $S \backslash T > T$, $w_j^S \leq w_j^T$ and $x_j^S \leq x_j^T$ for every $e_j \in [\underline{e}_j, \bar{e_j}]$, for every for every $j \in T$.*

We first prove (a). Let $t = minT \equiv \min_{i \in T} T$. First, remark that $\boldsymbol{w}_T^S$ is solution to the program

$$\begin{aligned} \max_{\boldsymbol{w}_T} \quad & E[b_t(e_t + w_{t-1}^S - w_t)] + E\left[\textstyle\sum_{i \in T \backslash t} b_i(e_i + w_{i-1} - w_i)\right], \\ & \text{subject to} \\ & w_i \geq 0 \text{ for every } i \in T, \\ & \underline{e}_t + w_{t-1}^S - w_t \geq 0, \\ & \underline{e}_i + w_{i-1} - w_i \geq 0 \text{ for every } i \in T \backslash t. \end{aligned} \tag{10}$$

The solution of the above program $\boldsymbol{w}_T^S$ is the best vector of water releases in the reduced game $(T, \boldsymbol{e}'_T, \boldsymbol{b}_T)$ where $e'_t = e_t + w_{t-1}^S$ and $e'_j = e_j$ for every $j \in T \backslash t$. The first-order conditions of the program (10) imply for every $j \in T$, $j > t$:

$$E[b'_t(e_t + w_{t-1}^S - w_t^S)] \geq E[b'_j(e_j + w_{j-1}^S - w_j^S)], \tag{11}$$

Similarly, $\boldsymbol{w}_T^T$ is the best vector of water releases of the reduced game $(T, \boldsymbol{e}_T, \boldsymbol{b}_T)$. Therefore, the first-order conditions of the program defined by $v(T)$ imply that $\exists l \in T$ such that for every $j : t < j < l$:

$$E[b'_t(e_t - w_t^T)] = E[b_j(e_j + w_{j-1}^T - w_j^T)] = E[b_l(e_l + w_{l-1}^T)]. \tag{12}$$

Suppose that $\boldsymbol{w}_t^S < \boldsymbol{w}_t^T$. We show that $w_j^S < w_j^T$ for every $j : t < j \leq l$. Since $e_t + w_{t-1}^S - w_t^S > e_t - w_t^T$ for every $e_t \in [\underline{e}_t, \bar{e}_t]$, $E[b'_t(e_t + w_{t-1}^S - w_t^S)] < E[b'_t(e_t - w_t^T)]$ by concavity of $b_t$. By (12) and (11) for $j = t + 1$, the last inequality implies $E[b'_{t+1}(e_{t+1} + w_t^S - w_{t+1}^S)] <$

$E[b'_{t+1}(e_{t+1}+w_t^T-w_{t+1}^T)]$. Now because $e_{t+1}+w_t^S < e_{t+1}+w_t^T$ for every $e_{t+1} \in [\underline{e}_{t+1}, \bar{e}_{t+1}]$, for the previous inequality to hold, we must have $w_{t+1}^S > w_{t+1}^T$. Proceeding the same argument for $t+1, t+2, ..$ up to $l-1$ shows that if $w_t^S < w_t^T$ then $w_j^S < w_j^T$ for $j = t+1, t+2, ..., l-1$. Now we have $w_{l-1}^S < w_{l-1}^S$ implies $e_l + w_{l-1}^S < e_l + w_l^T$ for every $e_l \in [\underline{e}_l, \bar{e}_l]$ which, in turn, implies $e_l + w_{l-1}^S - w_l^S < e_l + w_l^T$ for every $e_l \in [\underline{e}_l, \bar{e}_l]$. Therefore $E[b'_l(e_l + w_{l-1}^S - w_l^S)] > E[b'_l(e_l + w_{l-1}^T)]$. Combining the last inequality with (12) and (11) contradicts our starting assumption that $E[b'_t(e_t + w_{t-1}^S - w_t^S)] < E[b'_t(e_t - w_t^T)]$. Therefore $w_t^S \geq w_t^T$. The same arguments show that $w_i^S \geq w_i^T$ for $i = t+1, t+2, ..., maxT$.

Now we prove $x_j^S \geq x_j^T$ for every $e_j \in [\underline{e}_j, \bar{e_j}]$, for every $j \in T$. First, note that since $w_{maxT}^S \geq w_{maxT}^T$, $x_{maxT}^S = e_{maxT} + w_{maxT}^S \geq e_{maxT} + w_{maxT}^T$ for every $e_{maxT} \in [\underline{e}_{maxT}, \bar{e}_{maxT}]$. Therefore all we need to show is: $x_k^S \geq x_k^T$ implies $x_{k-1}^S \geq x_{k-1}^T$ for an arbitrary $k \in T \backslash minT$. Assume $x_k^S \geq x_k^T$. By concavity of $b_k$, $E[b'_k(x_k^S)] \leq E[b'_k(x_k^T)]$. If $w_{k-1}^S > 0$ then the first-order conditions of the maximization program defined by $v(S)$ and $v(T)$ imply $E[b'_{k-1}(x_{k-1}^S)] = E[b'_k(x_k^S)]$ and $E[b'_{k-1}(x_{k-1}^T)] \geq E[b'_k(x_k^T)]$ respectively. The last three inequalities imply $E[b'_{k-1}(x_{k-1}^S)] \leq E[b'_{k-1}(x_{k-1}^T)]$ which, in turns, imply $x_{k-1}^S \geq x_{k-1}^T$ for every $e_{k-1} \in [\underline{e}_{k-1}, \bar{e}_{k-1}]$. Now if $w_{k-1}^S = 0$, then $w_{k-1}^T = 0$ because $w_{k-1}^S \geq w_{k-1}^T$. In this case, $x_{k-1}^S = e_{k-1} + w_{k-2}^S \geq e_{k-1} + w_{k-2}^T$ for every $e_{k-1} \in [\underline{e}_{k-1}, \bar{e}_{k-1}]$ because $w_{k-2}^S \geq w_{k-2}^T$.

The Proof of (b) proceeds similarly, starting with $maxT$ instead of $minT$.

We are now ready to proof Proposition 1. Fix $i \in T \subset S \subset N$. Let $R$ be the (unique) connected sub-coalition of $T$ containing $i$ and let $Q$ be the (unique) coalition in $S$ containing $i$. Note that $R \subset Q$. Given (4), all we need to show is:

$$v(R) - v(R \backslash i) \leq v(Q) - v(Q \backslash i). \tag{13}$$

Let $R_p \equiv R \cap P^0 i$, $R_F \equiv R \cap F^0 i$, and define $Q_P$ and $Q_F$ similarly. Note that $v(R \backslash i) = v(R_P) + v(R_F)$ and $v(Q \backslash i) = v(Q_P) + v(Q_F)$. Moreover, $R_P \subset Q_P$, $R_F \subset Q_F$, and $R$, $R_P$, $R_F$ as well as $Q$, $Q_P$, $Q_F$ are connected.

Step 1 We show that:

$$v(R_P \cup i) - v(R_P) \leq v(Q_P \cup i) - v(Q_P), \tag{14}$$

$$v(R_F \cup i) - v(R_F) \leq v(Q_F \cup i) - v(Q_F). \tag{15}$$

Let $d_j = w_j^{R_P} - w_j^{R_P \cup i}$ for each $j \in R_P \backslash i-1$ where $w_{R_P}^{R_P}$ and $w_{R_P \cup i}^{R_P \cup i}$ are the best fixed water releases vectors for coalitions $R_P$ and $R_P \cup i$ respectively. This quantity is nonnegative because of Lemma 1. Since $\sum_{j \in R_P} d_j = \sum_{j \in R_P} w_j^{R_P \cup i} = w_{i-1}^{R_P \cup i}$, by definition,

$$v(R_P \cup i) - v(R_P) = \sum_{j \in R_P} \Big( E[b_j(x_j^{R_P} - d_{j-1} + d_j)] - E[b_j(x_j^{R_P})] \Big) + b_i(e_i + \sum_{j \in R_P} d_j). \quad (16)$$

where $x_j^{R_P} \equiv e_j + w_{j-1}^{R_P} - w_j^{R_P}$ for every $j \in R_P$ is the vector of water consumption with $w_{R_P}^{R_P}$. Next, define $w'_{Q_P}$, a vector of water releases in $Q_P$, as follow:

$$w'_j = \begin{cases} w_j^{Q_P} & if \quad j \in Q_P \backslash R_P \\ w_j^{Q_P} - d_j & if \quad j \in R_P \backslash i-1 \\ w_j^{Q_P} + \sum_{j \in R_P} d_j & if \quad j = i-1 \end{cases}$$

Notice that, by Lemma 1, $w'_{Q_P}$ is feasible in $Q_P$. Therefore

$$v(Q_P \cup i) - v(Q_P) \geq \sum_{j \in R_P} \Big( E[b_j(x_j^{Q_P} - d_{j-1} + d_j)] - E[b_j(x_j^{Q_P})] \Big) + b_i(e_i + \sum_{j \in R_P} d_j). \quad (17)$$

Moreover, for any $j \in R_P$, since $-d_{j-1} + d_j = x_j^{R_P \cup i} - x_j^{R_P} \leq 0$ and $x_j^{Q_P} \geq x_j^{R_P}$ by Lemma 1, by concavity of $b_j$, $b_j(x_j^{R_P}) - b_j(x_j^{R_P} - d_{j-1} + d_j) \geq b_j(x_j^{Q_P}) - b_j(x_j^{Q_P} - d_{j-1} + d_j)$ for every $e_j \in [\underline{e}_j, \bar{e}_j]$. Taking the expectation of the last inequality with respect $e_j$ and re-arranging terms leads $E[b_j(x_j^{Q_P} - d_{j-1} + d_j)] - E[b_j(x_j^{Q_P})] \geq E[b_j(x_j^{R_P} - d_{j-1} + d_j)] - E[b_j(x_j^{R_P})]$ for every $j \in R_P$. Combining these inequalities with (16) and (17) leads to (14). The same argument establishes (15).

<u>Step 2</u>. By repeated application of (14) and (15), we obtain $v(R_P \cup Q_F \cup i) - v(R_P \cup R_F \cup i) \geq v(Q_F \cup i) - v(R_F \cup i)$ and $v(Q_P \cup Q_F \cup i) - v(R_P \cup Q_F \cup i) \geq v(Q_P \cup i) - v(R_P \cup i)$. Therefore:

$$\begin{aligned} & v(Q) - v(R) \\ & = (v(Q_P \cup Q_F \cup i) - v(R_P \cup Q_F \cup i)) + (v(R_P \cup Q_F \cup i) - v(R_P \cup R_F \cup i)) \\ & \geq (v(Q_P \cup i) - v(R_P \cup i)) + (v(Q_F \cup i) - v(R_F \cup i) \\ & \geq v(Q_P) - v(R_P) + v(Q_F) - v(R_F) = v(Q \backslash i) - v(R \backslash i), \end{aligned}$$

where the second inequality holds again because of (14) and (15). We are done.

## B Proof of Proposition 2

Suppose that a core FWSA $(\boldsymbol{w}^*, \boldsymbol{\tau}')$ with $\boldsymbol{\tau}' \neq \boldsymbol{\tau}^u$ is sustainable to $e_i < E[e_i]$ while $(\boldsymbol{w}^*, \boldsymbol{\tau}^u)$ is not. Then, by (8), we have:

$$\tau_i' \geq b_i(e_i + w_{i-1}^*) - b_i(e_i + w_{i-1}^* - w_i^*) > \tau_i^u.$$

By the definition of $\tau_i^u$ in (7), the above inequality implies:

$$\tau_i' > \sum_{j \in F^0 i} E\left[b_j(e_j + w_{j-1}^* - w_j^*)\right] - v(F^0 i),$$

or, equivalently,

$$v(F^0 i) > \sum_{j \in F^0 i} E\left[b_j(e_j + w_{j-1}^* - w_j^*)\right] - \tau_i'. \tag{18}$$

Now the ex ante welfare of country $j$ with $(\boldsymbol{w}^*, \boldsymbol{\tau}')$ is defined by:

$$u_j' = E\left[b_j(e_j + w_{j-1}^* - w_j^*)\right] - \tau_{j-1}' + \tau_j'.$$

The total ex ante welfare of coalition $F^0 i = \{i+1, ..., n\}$ is then:

$$\sum_{j \in F^0 i} u_j = \sum_{j \in F^0 i} E[b_j((e_j + w_{j-1}^* - w_j^*)] - \tau_i'.$$

Combined with (18), it leads to $v(F^0 i) > \sum_{j \in F^0 i} u_j$ which contradicts that $(\boldsymbol{w}^*, \boldsymbol{\tau}')$ is a core FWSA.

## C Proof of Proposition 3

The proof of the first part of Proposition 3 is similar than the proof of Proposition 2. Suppose that a core FWSA $(\boldsymbol{w}^*, \boldsymbol{\tau}')$ with $\boldsymbol{\tau}' \neq \boldsymbol{\tau}^d$ is not sustainable to $e_i < E[e_i]$ while $(\boldsymbol{w}^*, \boldsymbol{\tau}^d)$ is. Then, by (8), we have:

$$\tau_i^d \geq b_i(e_i + w_{i-1}^*) - b_i(e_i + w_{i-1}^* - w_i^*) > \tau_i'.$$

By the definition of $\tau_i^d$ in (7), the above inequality implies:

$$v(Pi) - \sum_{j \in Pi} E\left[b_j(e_j + w_{j-1}^* - w_j^*)\right] > \tau_i',$$

or, equivalently,

$$v(Pi) > \sum_{j \in Pi} E\left[b_j(e_j + w_{j-1}^* - w_j^*)\right] + \tau_i'. \tag{19}$$

The ex ante welfare of country $j$ with $(\boldsymbol{w}^*, \boldsymbol{\tau'})$ is defined by:

$$u_j' = E\left[b_j(e_j + w_{j-1}^* - w_j^*)\right] - \tau_{j-1}' + \tau_j'.$$

The total ex ante welfare of coalition $Pi = \{1, ..., i\}$ is then:

$$\sum_{j \in Pi} u_j = \sum_{j \in Pi} E[b_j((e_j + w_{j-1}^* - w_j^*)] + \tau_i'.$$

Combined with (19), it yields $v(Pi) > \sum_{j \in Pi} u_j$ which contradicts that $(\boldsymbol{w}^*, \boldsymbol{\tau'})$ is a core FWSA.

For the second part of Proposition 3, first remark that, since $b_1(e_1) - b_1(e_1 - w_1^*)$ is decreasing with $e_1 \in [\underline{e}_1, \bar{e}_1]$ for every $w_1^* \in (0, e_i)$, $b_1(\underline{e}_1) - b_1(\underline{e}_1 - w_1^*) \geq b_1(e_1) - b_1(e_1 - w_1^*)$ for every $e_1 \in [\underline{e}_1, \bar{e}_1]$ with a strict inequality for $e_1 > \underline{e}_1$. The last inequalities imply $b_1(\underline{e}_1) - b_1(\underline{e}_1 - w_1^*) > E[b_1(e_1) - b_1(e_1 - w_1^*)]$. Since, by (6), $\tau_1^d = v(1) - E[b_1(\underline{e}_1 - w_1^*)]$ and $v(1) = E[b_1(e_1)]$, it leads to $b_1(\underline{e}_1) - b_1(\underline{e}_1 - w_1^*) > \tau_1^d$ which shows that the downstream incremental FWSA is not sustainable to reduced flow at the source for any minimal flow $\underline{e}_1 < \bar{e}_1$, that is as long as $e_1$ is random.

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