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Stephen F. Hamilton, Philippe Bontems and Jason Lepore

INSTITUT D'ECONOMIE INDUSTRIELLE

Toulouse School of Economics

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Stephen F. Hamilton*
California Polytechnic State University San Luis Obispo

Philippe Bontems
Toulouse School of Economics (GREMAQ, INRA, IDEI)

Jason Lepore
California Polytechnic State University San Luis Obispo

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Abstract

Policy design in oligopolistic settings depends critically on the mode of competition between firms. We develop a model of oligopoly intermediation that reveals the mode of competition to be an equilibrium outcome that depends on the relative degree of rivalry between firms in the upstream and downstream markets. We examine two forms of sequential pricing games: Purchasing to stock (PTS), in which firms select input prices prior to setting consumer prices; and purchasing to order (PTO), in which firms sell forward contracts to consumers prior to selecting input prices. The equilibrium outcomes of the model range between Bertrand and Cournot depending on the relative degree of rivarly between firms in the upstream and downstream markets. Prices are strategic complements and the equilibrium prices coincide with the Bertrand outcome when the markets are equally rivalrous, while prices are strategic substitutes when the degree of rivalry is sufficiently high in one market relative to the other. Cournot outcomes emerge under circumstances in which prices are strategically independent in either the upstream or downstream market. We derive testable implications for the mode of competition that depend only on primitive conditions of supply and demand functions.

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*Correspondence to: S. Hamilton, Department of Economics, Orfalea College of Business, California Polytechnic State University, San Luis Obispo CA 93407. Voice: 805-756-2555, FAX: 805-756-1471, email: shamilto@calpoly.edu. Part of this paper was written while Stephen F. Hamilton was visiting the Toulouse School of Economics and the author thanks members of that department for their hospitality. We are indebted to Robert Innes and Richard Sexton for invaluable advise on an earlier draft of the paper, and thank seminar participants at UC Berkeley, Toulouse School of Economics, the University of Nebraska and the University of Kiel for helpful comments.

1 Introduction

An obstacle to deriving policy implications from the oligopoly model is the sensitivity of strategic pre-commitment devices to the mode of competition. As observed by Fudenberg and Tirole (1984) and Bulow, Geanakoplos, and Klemperer (1985), the strategic underpinnings of the oligopoly model depend fundamentally on the manner in which firms' choice variables alter the marginal profit expressions of rivals, a feature that has essential implications for the design of effective policy. For strategic trade policy, export subsidies are optimal when firms' choice variables are strategic substitutes (Brander and Spencer 1985); however, export taxes are optimal when firms' choice variables are strategic complements (Eaton and Grossman 1986). For strategic delegation between the owner and manager of a firm, the optimal contract overcompensates sales when firms compete in strategic substitutes, but overemphasizes cost when agents compete in strategic complements (Fershtman and Judd 1987, Sklivas 1987). For contracts between vertically-aligned suppliers, the optimal contract involves lump sum transfers from manufacturers to retailers when retailers compete in strategic complements (Shaffer 1991), but involves negative lump sum payments when retailers compete in strategic substitutes (Vickers 1985).[1]

In this paper, we develop a model of oligopoly intermediation that generates testable hypotheses on the mode of competition. We frame our model around duopoly intermediaries who purchase an input from a common upstream market and sell finished goods in a common downstream market. Both the input and output prices selected by oligopoly firms are strategically interdependent, so that a pricing strategy pursued by a firm in one market, for instance a price promotion on the finished good designed to increase market share, has implications for both the supply and demand conditions facing the rival. The intermediated oligopoly model provides a convenient way to represent strategic interaction between firms: It reduces to the

[1] The mode of competition of the oligopoly model also has important implications for first-mover advantage in sequential games between firms. It has been recognized since von Stackelberg (1934) that advantage goes to the first-mover when choice variables are strategic substitutes; however, being the first-mover is disadvantageous when firms compete in strategic complements (Gal-Or 1985).

usual oligopoly (oligopsony) model when the supply (demand) function is infinitely elastic, yet encompasses general forms of strategic interaction when prices are interdependent in both supply and demand functions.

We characterize the strength of the oligopoly pricing interaction between firms in the upstream and downstream markets in terms of the degree of market rivalry. The products procured and produced in each market are differentiated, as would be the case when manufacturers rely on specialized inputs to produce branded consumer goods, and a greater extent of product differentiation softens the degree of market rivalry. Our setting thus extends Stahl's (1988) analysis of the intermediated oligopoly model to differentiated product markets. We follow Stahl (1988) in examining two forms of sequential price competition: (*i*) "purchasing to stock" (PTS), in which the firms select input prices prior to setting output prices; and (*ii*) "purchasing to order" (PTO), in which the firms sell forward contracts to consumers prior to selecting input prices.

Our observations on the mode of competition depend critically on the "relative degree of market rivalry" in the upstream and downstream markets. We define the relative rivalry of the upstream market to be the difference (in absolute terms) between the ratio of the cross-price to own-price elasticity of supply and the ratio of cross-price to own-price elasticity of demand. It is a measure of the relative strength of the oligopoly interaction at each point of market contact between firms. If the cross-price elasticity is zero in a market (the monopoly case), the market is non-rival, whereas if the ratio of cross-price elasticity to own-price elasticity is approximately one in a market (the case of commoditized products), the market is highly rivalrous. Markets are equally rivalrous when there is no difference in the intensity of the oligopoly interaction in the upstream and downstream markets, as would be the case in the homogeneous product setting considered by Stahl (1988).

Our main results can be summarized as follows. First, we find that an outcome of Bertrand merchants emerges only under circumstances in which the upstream and downstream markets are equally rivalrous. Such an outcome occurs irrespective of the degree of product differentiation in each market. Thus, our analysis reveals the underpinning of the Stahl (1988) outcome of Bertrand merchants to be determined by the *relative* degree of market rivalry rather than the *absolute* degree of rivalry in the upstream and downstream markets.

Second, we show firm profits to be greater under PTS than under PTO when the downstream market is relatively rivalrous, whereas the PTO outcome Pareto dominates the PTS outcome in terms of firm and industry profits when the upstream market is relatively rivalrous. In either case, setting prices in the less rivalrous market serves as a pre-commitment device for prices in the more rivalrous market, providing firms with the ability to soften price competition in the market where the degree of oligopoly interaction is most intense.

Third, we demonstrate that when one market is more rivalrous than the other, prices can be either strategic complements or strategic substitutes depending on the relative degree of market rivalry. As the relative degree of market rivalry increases, the equilibrium outcome under PTS and PTO converges to Cournot as prices become strategically independent in the less rivalrous market. For the PTS game, this result relates to the finding of Kreps and Scheinkman (1983) that the Cournot outcome emerges in a two-stage game where firms first choose capacity and then select prices. In general, the PTS game differs from the setting considered by Kreps and Scheinkman (1983) in the sense that firms' supply function in the upstream input (capacity) market have rival's input prices as arguments; however, the models are isomorphic when each firm has monopsony control of an independent upstream market.

Fourth, we provide conditions under which our observations on oligopoly intermediation are robust to inventory-holding behavior in symmetric markets with linear supply and demand functions. Our analysis thus formally extends the finding of Kreps and Scheinkman (1983) to markets with differentiated products and interdependent upstream markets.

Finally, we expand our analysis of sequential pricing outcomes to an extended game in which firms first choose the timing of pricing decisions prior to selecting prices. For the case of symmetric markets with linear supply and demand functions, we verify that the Pareto dominant outcomes in the pricing sub-game represent equilibrium strategies in the extended game. Namely, under circumstances in which industry profits are largest under PTS (PTO), we show that such a pricing strategy is also the equilibrium outcome of the extended game.

Our observations are related to previous research by Maggi (1996) that endogenizes the mode of competition in the oligopoly model.[2] In Maggi's (1996) model, firms make capacity commitments before trading occurs in a downstream international market, but can subsequently relax their prior capacity commitments subject to an *ex post* adjustment cost parameter. This sequential capacity adjustment process produces a continuum of outcomes that spans the modes of competition between Bertrand and Cournot according to the cost of capacity adjustment. Our model departs from this framework by specifying oligopsony interaction in the input (capacity) market in place of *ex post* adjustment cost. An advantage of our approach is that it results in testable hypotheses on the mode of competition that are readily estimable from market data.

We illustrate the policy implications of the model for the case of a contract between a principle (a domestic trade authority, a firm, or a controlling shareholder) and an agent (a domestic firm, a supplier/consumer, or a manager) that imposes a unit tax or subsidy on the input procured from the upstream market. We show that the optimal contract to maximize oligopoly profits involves taxing the input when the upstream and downstream markets are relatively equal in terms of rivalry, but subsidizing the input when the degree of relative rivalry is sufficiently large. We numerically characterize these policy outcomes for perturbations in the relative degree of market rivalry under linear supply and demand conditions and show that the optimal value of the policy variable follows an inverted u-shaped pattern: As the

[2] We follow the convention of Maggi (1996) in characterizing the mode of competition according to whether prices are strategic substitutes or strategic complements in the oligopoly model.

upstream market becomes increasingly rivalrous, the optimal policy switches from a subsidy to a tax under PTS before reverting back to a subsidy under PTO.

The remainder of the paper is structured as follows. In the next Section we present the model and characterize equilibrium prices under PTS and PTO. In Section 3, we compare the outcomes for firm and industry profits and classify the Pareto dominant Nash equilibrium according to the relative degree of rivalry in the upstream and downstream markets. In Section 4, we extend these outcomes to consider inventory-holding behavior and games with endogenous timing under symmetric market conditions with linear supply and demand functions. In Section 5, we derive the mode of competition in the linear model and characterize conditions for prices to be strategic substitutes that depend only on observed market prices and estimable supply and demand elasticities in a given industry. In Section 6, we numerically illustrate the implication of our findings for the case of vertical contracts between firms and their upstream suppliers, and in Section 7, we conclude. The proofs of all Propositions appear in the Appendix.

2 The Model

We consider duopoly intermediaries who compete against each other in prices. The firms purchase differentiated stocks ("inputs") from suppliers in an upstream market, and sell finished products ("outputs") derived from the inputs to consumers in a downstream market. Both input suppliers and consumers are price-taking agents in their respective markets.

To clarify the implications of the model for oligopoly pricing outcomes, we consider fixed proportions technology. Specifically, letting x_i denote the quantity of the input purchased in the upstream market by firm $i \in \{1, 2\}$, we scale units such that $y_i = x_i$ denotes the quantity of the output sold by the firm in the downstream market under circumstances where inventory is not held. Products in each market are differentiated, and the degree of rivalry between firms potentially differs at their points of contact in the upstream and downstream markets according to the intensity of cross-price effects between firms in each market. One interpretation of the nature of product differentiation in the model is that demand and supply functions depend on the spatial location of suppliers, consumers, and firms. Another interpretation is that the firms require specialized inputs to produce differentiated consumer goods.

Let $\mathbf{p} = (p_1, p_2)$ denote the vector of output prices in $\mathbb{R}^2$. Consumer demand for product i is given by

$$D^i = D^i(\mathbf{p}), \tag{1}$$

where D^i is in the class of smooth functions on $\mathbb{R}^2$. We assume $D^i_i \equiv \partial D^i / \partial p_i < 0$ and $D^i_j \equiv \partial D^i / \partial p_j \geq 0$, where the latter condition confines attention to the case of substitute goods. We restrict output prices to $\mathbb{P} = [0, \overline{p}] \times [0, \overline{p}]$, a convex and compact subset $\mathbb{R}^2$, where $\overline{p} > 0$ is a symmetric price such that $D^i(p, p) = 0$ for all $p \geq \overline{p}$ and $D^i(p, p) > 0$ for all $p \in [0, \overline{p})$.

Let $\mathbf{w} = (w_1, w_2)$ denote the vector of input prices. The supply function facing firm i in the upstream market is

$$S^i = S^i(\mathbf{w}), \tag{2}$$

where S^i is in the class of smooth functions on $\mathbb{R}^2$ and $S^i_i \equiv \partial S^i / \partial w_i > 0$ and $S^i_j \equiv \partial S^i / \partial w_j \leq 0$ (i.e., products in the upstream market are substitutes). As no firm can earn positive rents with an input price greater than its output price, we accordingly restrict input prices to be in $\mathbb{P}$.

Throughout the paper, we assume that the direct effect of a price change outweighs the indirect effect in each market; that is, $\Delta \equiv D^i_i D^j_j - D^i_j D^j_i > 0$ and $\Sigma \equiv S^i_i S^j_j - S^i_j S^j_i > 0$. These conditions ensure that the system of demand and supply equations is invertible.

In Section 4.2, we derive sufficient conditions for no strategic inventory-holding behavior. For now, we streamline the exposition of equilibrium outcomes under PTS and PTO by suppressing inventory-holding behavior and the destruction or removal of goods. Without the possibility of holding inventory, the demand and supply functions facing each firm are linked by the material balance equations,

$$\mathbf{D}(\mathbf{p}) = \mathbf{S}(\mathbf{w}). \tag{3}$$

There are two important subgames we define below: the PTS game and the PTO game. In the PTS game, the two firms simultaneously and independently choose their input prices. Given that the firms are not able to hold inventory, the unique set of output prices associated with the input price pair $\mathbf{w} = (w_1, w_2)$ are determined by condition (3), which we denote by $\mathbf{p}(\mathbf{w})$. Firm i's profit in the PTS game is

$$\pi^{i,s}(\mathbf{w}) = \big(p^i(\mathbf{w}) - w_i\big)\, S^i(\mathbf{w}) - F_i, \qquad i \in \{1, 2\},$$

where F_i denotes fixed costs, a portion of which may be sunk.[3]

To guarantee existence and uniqueness of the PTS equilibrium, we impose the following regularity conditions on profits.

Axiom 1 $\pi^{i,s}_{ii}(\mathbf{w}) < 0$, *and* $\pi^{i,s}_{ii}(\mathbf{w}) + \pi^{i,s}_{ij}(\mathbf{w}) < 0$ *for all* $\mathbf{w} \in \mathbb{P}$ *and* $i \in \{1, 2\}$.

In the PTO game, the two firms simultaneously and independently choose their output prices. We assume that firms must fill all orders and cannot hold forward contracts as inventory. As a consequence, the unique set of input prices are determined by condition (3) under PTO as $\mathbf{w}(\mathbf{p})$. Firm i's profit in the PTO game is

$$\pi^{i,o}(\mathbf{w}) = \big(p_i - w^i(\mathbf{p})\big)\, D^i(\mathbf{p}) - F_i, \qquad i \in \{1, 2\},$$

We impose the following regularity condition on profits under PTO.

Axiom 2 $\pi^{i,o}_{ii}(\mathbf{p}) < 0$, *and* $\pi^{i,o}_{ii}(\mathbf{p}) + \pi^{i,o}_{ij}(\mathbf{p}) < 0$ *for all* $\mathbf{p} \in \mathbb{P}$ *and* $i \in \{1, 2\}$.

At times, we wish to compare equilibrium prices under PTS and PTO to those that emerge under the "two-sided Cournot game" in which quantity choices determine both upstream and downstream market prices at once. To do so, we define

[3] It is straightforward to show that continuity and differentiability of the the supply and demand functions imply that $\pi^{i,s}$ is also in the class of smooth functions on $\mathbb{R}^2$.

inverse demand and inverse supply from equations (1) and (2) as $P^i(\mathbf{y})$ and $W^i(\mathbf{y})$, respectively.

The profit of firm i under Cournot competition is given by

$$\pi^{i,C}(\mathbf{y}) = \left(P^i(\mathbf{y}) - W^i(\mathbf{y})\right) y_i - F_i, \qquad i \in \{1,2\}, \tag{4}$$

where $\mathbf{y} = (y_1, y_2)$ denotes the vector of retail (and wholesale) quantities under quantity competition. To facilitate comparison of the PTS and PTO equilibrium with the Cournot outcome, we make the following assumptions about the Cournot profit functions.

Axiom 3 *$\pi_{ii}^{i,C}(\mathbf{y}) < 0$, and $\pi_{ii}^{i,C}(\mathbf{y}) + \pi_{ij}^{i,C}(\mathbf{y}) < 0$ for all $\mathbf{y} \in [0, D(0,\overline{p})] \times [0, D(0,\overline{p})]$ and $i \in \{1,2\}$.*

This assumption guarantees existence and uniqueness of equilibrium in the Cournot game. In addition, we assume that the collective profit is strictly concave in symmetric quantities.

Axiom 4 *$\pi_{ii}^{i,C}(\mathbf{y}) + 2\pi_{ij}^{i,C}(\mathbf{y}) + \pi_{jj}^{i,C}(\mathbf{y}) < 0$ for all symmetric $\mathbf{y}$ $(y_1 = y_2)$.*

As a convenient benchmark for this analysis, we first examine the case in which firms simultaneously select prices in the upstream and downstream markets. Throughout the paper, we refer to this object as the Bertrand outcome.

2.1 Bertrand Outcomes

Consider the case in which firms select prices simultaneously in the upstream and downstream markets. Firm i seeks to maximize profits

$$\pi^{i,B}(\mathbf{p}, \mathbf{w}) = p_i D^i(\mathbf{p}) - w_i S^i(\mathbf{w}) - F_i, \qquad i \in \{1,2\} \tag{5}$$

subject to the inventory constraint (3). Evaluating the first-order conditions with respect to w_i and p_i, we describe the Bertrand outcome in the sense of Stahl (1988) as the simultaneous solution to

$$p_i - w_i = \frac{S^i(\mathbf{w})}{S_i^i(\mathbf{w})} - \frac{D^i(\mathbf{p})}{D_i^i(\mathbf{p})}, \qquad i \in \{1,2\}, \tag{6}$$

where subscripts refer to partial derivatives.

Simultaneously solving equations (6) subject to the inventory constraint (3) yields the Bertrand prices, which we define in the symmetric case as (w^B, p^B). For future reference, it is helpful to express the symmetric price-cost margin of Bertrand merchants as

$$p^B - w^B = \frac{p^B}{\varepsilon_o^d} + \frac{w^B}{\varepsilon_o^s}. \tag{7}$$

2.2 Relative Rivalry

The concept of relative rivalry is important for the analysis to follow. We define the relative rivalry of markets as a measure of the strength of the oligopoly interaction in the upstream market relative to the downstream market, where a more rivalrous market is one in which a change in the input (output) price selected by a firm leads to a greater change in supply (demand) for the rival. Focusing on the symmetric market equilibrium, we describe the relative rivalry of markets in terms of supply and demand elasticities. Specifically, let $\epsilon_s = \frac{\varepsilon_c^s}{\varepsilon_o^s} < 1$ denote the absolute value of the ratio of cross-price elasticity of supply ($\varepsilon_c^s = -S_j^i \frac{w_j}{S^i} > 0$) to own-price elasticity of supply ($\varepsilon_o^s = S_i^i \frac{w_i}{S^i} > 0$), and let $\epsilon_d = \frac{\varepsilon_c^d}{\varepsilon_o^d} < 1$ denote the corresponding ratio of demand elasticities in the downstream market, where $\varepsilon_c^d = D_j^i \frac{p_j}{D^i} > 0$ and $\varepsilon_o^d = -D_i^i \frac{p_i}{D^i} > 0$.

Definition 1 *The relative rivalry of the upstream market is given by* $\Theta = \epsilon_s - \epsilon_d$.

We measure the relative rivalry of the upstream market according to equilibrium values for the prices in the Bertrand outcome described above. We refer to the upstream market as being more rivalrous than the downstream market when $\Theta > 0$, the downstream market as being more rivalrous than the upstream market when $\Theta < 0$, and the markets as being equally rivalrous when $\Theta = 0$.

We are now ready to examine the equilibrium outcomes of the intermediation model under two forms of sequential pricing behavior: (i) Purchasing to Stock (PTS); and (ii) Purchasing to Order (PTO).

2.3 Purchasing to Stock (PTS)

In the purchasing to stock (PTS) game, firms first select input prices and acquire stocks in the upstream market, and then subsequently select output prices for finished goods in the downstream market. Let $\mathbf{w} = (w_1, w_2)$ denote the vector of input prices selected by the firms and let $p^i(\mathbf{w})$ and $p^j(\mathbf{w})$ denote the associated output prices implicitly defined by equations (3).

Under PTS, firm i selects w_i to maximize profits,

$$\pi^{i,s}(w_i, w_j) = \left(p^i(\mathbf{w}) - w_i\right) S^i(\mathbf{w}) - F_i, \qquad i \in \{1, 2\}, \tag{8}$$

where F_i denotes fixed costs, a portion of which may be sunk. The first-order necessary condition for a profit maximum is

$$\pi_i^{i,s} \equiv (p^i(\mathbf{w}) - w_i)S_i^i(\mathbf{w}) + (p_i^i(\mathbf{w}) - 1)S^i(\mathbf{w}) = 0, \qquad i \in \{1, 2\}. \tag{9}$$

The effect of an input price change by firm i on output prices can be derived by totally differentiating equations (3). Holding $dw_j = 0$, this yields

$$\begin{bmatrix} D_i^i & D_j^i \\ D_i^j & D_j^j \end{bmatrix} \begin{bmatrix} dp_i \\ dp_j \end{bmatrix} = \begin{bmatrix} S_i^i \\ S_i^j \end{bmatrix} dw_i.$$

It follows that

$$\begin{aligned} p_i^i(\mathbf{w}) &\equiv \frac{\partial p^i}{\partial w^i} = \frac{D_j^j S_i^i - D_j^i S_i^j}{\Delta} < 0 \\ p_i^j(\mathbf{w}) &\equiv \frac{\partial p^j}{\partial w^i} = \frac{D_i^i S_i^j - D_i^j S_i^i}{\Delta}. \end{aligned} \tag{10}$$

Equation (2.3) represents the own-price effect of an increase in the input price on the output price of firm i. This term is negative. When firm i raises his input price, the firm procures a greater quantity of the input, and this drives down the firm's output price and narrows his price-cost margin.

Equation (10) is the cross-price effect of a input price increase by firm i on the output price of the rival firm j. The sign of this term is important for the results to follow. Firm j responds to a higher input price by firm i ($dw_i > 0$) by increasing his output price ($dp_j > 0$) whenever $D_i^i S_i^j > D_i^j S_i^i$, and otherwise holds constant or decreases his output price.

Under PTS, a change in a firm's input price has two offsetting effects on the output price of the rival. The first effect is the "supply effect", $D_i^i S_i^j dw_i \geq 0$. Selecting a higher input price bids stocks away from the rival in the upstream input market, which leads to an inward shift of the rival's supply function. The supply effect reduces the rival's procurement of the input, thereby reducing the quantity sold in the downstream market by firm j and raising the rival's output price. The second effect is the "demand effect", $D_i^j S_i^i dw_i \geq 0$. A rise in the input price of firm i raises input procurement for firm i in the upstream market, leading to a commensurate increase in production and sales for firm i in the downstream market and a decrease in p_i. The demand effect results in an inward shift of the rival's demand function, placing downward pressure on p_j. The relative magnitude of the supply and demand effects depends on the relative rivalry of the upstream market. When products in the downstream market are commoditized, for example, the demand effect dominates the supply effect, as total output would rise following a unilateral increase in the input price of one firm, flooding the downstream market with finished goods.

In the symmetric market equilibrium ($D^i = D^j, S^i = S^j, p_1 = p_2 = p, w_1 = w_2 = w$), the cross-price effect in equation (10) can be expressed in terms of relative rivalry as

$$p_i^j(\mathbf{w}) \overset{s}{=} \Theta,$$

where " $\overset{s}{=}$ " denotes"equal in sign". Under circumstances in which the upstream market is relatively rivalrous ($\Theta > 0$), an increase in the input price by firm i increases the output price of firm j. When the downstream market is relatively rivalrous ($\Theta < 0$), an increase in the input price by firm i decreases the output price of firm j, and for an equal degree of rivalry, $\Theta = 0$, the output price of firm j is independent of the input price selection of firm i.

Relative rivalry has essential strategic implications for the oligopoly model. To see this, note that

$$S^i(w_i, w_j) = D^i(p_i(\mathbf{w}), p_j(\mathbf{w}))$$

under the inventory constraint (3). Differentiating this expression with respect to w_i and dropping arguments for notational convenience, we have

$$S_i^i = D_i^i p_i^i + D_j^i p_i^j.$$

When firm i increases his input price in the upstream market, this leads to a direct increase in the output sold by firm i, $D_i^i p_i^i > 0$. But the input price increase also facilitates a strategic response by the rival firm, $D_j^i p_i^j \lesseqgtr 0$, the sign of which depends on p_i^j (and thus on Θ). Specifically, firm i perceives a smaller supply response to a rise in w_i when $\Theta < 0$, which dampens the value of an input price increase. Thus, engaging in PTS behavior serves as a commitment device to refrain from increasing input prices when $\Theta < 0$. It is of course in the interest of both firms to maintain lower input prices in the upstream market, as this supports correspondingly higher output prices in the downstream market and higher price-cost margins, making sequential pricing behavior under PTS a facilitating practice whenever $\Theta < 0$.

The equilibrium under PTS is determined by the simultaneous solution of equations (9). Let $\mathbf{w}^s = (w_1^s, w_2^s)$ denote the equilibrium input price vector that solves these equations and let $\mathbf{p}^s = (p_1^s, p_2^s)$ denote the associated vector of equilibrium output prices implied by equations (3).

The equilibrium price-cost margin for symmetric firms can be written

$$p^s - w^s = k^s \frac{p^s}{\varepsilon_o^d} + \frac{w^s}{\varepsilon_o^s}, \tag{11}$$

where $k^s = 1 + \frac{D_i^j D_j^j \Theta}{\Delta}$. Notice that the second term on the right-hand side of equation (11) is identical to the second term on the right hand-side of equation (7), but that the first term differs from the Bertrand margin. Playing PTS introduces a weight on the "demand-side" portion of the equilibrium price-cost margin that jointly accounts for the relative rivalry of the upstream market. By inspection, $k^s > 1$ if and only if $\Theta < 0$. That is, equilibrium price-cost margins are higher for firms in the PTS game than in the Bertrand outcome when the upstream market is less rivalrous than the downstream market ($\Theta < 0$). The intuition for this finding is quite clear: When $\Theta < 0$, a decrease in the input price by a firm facilitates an increase in the rival's output price, thereby softening price competition in the downstream market.

In Section 3, we provide a more complete comparison of PTS and the Bertrand outcomes. Before turning to this analysis, we derive the market equilibrium for the remaining case of PTO.

2.4 Purchasing to Order (PTO)

Suppose the firms sell forward contracts for delivery of finished goods to consumers prior to procuring inputs in the upstream market. Forward contracts are widely used in practice, including imported goods sold to retail distributors and a significant portion of wholesale trade (Stahl 1988).

Let $\mathbf{p} = (p_1, p_2)$ denote the vector of output prices selected by the firms and let $w^i(\mathbf{p})$ and $w^j(\mathbf{p})$ denote the associated input prices defined by equations (3). In the PTO game, firm i selects his output price to maximize profits of

$$\pi^{i,o}(p_i, p_j) = \left(p_i - w^i(\mathbf{p})\right) D^i(\mathbf{p}) - F_i, \qquad i \in \{1, 2\}.$$

The first-order necessary condition for a profit maximum is

$$\pi_i^{i,o} \equiv (p_i - w^i(\mathbf{p}))D_i^i(\mathbf{p}) + (1 - w_i^i(\mathbf{p}))D^i(\mathbf{p}) = 0, \qquad i \in \{1, 2\}. \tag{12}$$

We evaluate the PTO equilibrium by proceeding as above. Making use of the implicit function theorem on equations (3) gives the input price responses

$$w_i^i(\mathbf{p}) \equiv \frac{\partial w^i}{\partial p^i} = \frac{D_i^i S_j^j - D_i^j S_j^i}{\Sigma} < 0 \tag{13}$$

$$w_i^j(\mathbf{p}) \equiv \frac{\partial w^j}{\partial p^i} = \frac{D_i^j S_i^i - D_i^i S_i^j}{\Sigma}. \tag{14}$$

Equation (13) measures the own-price effect of an increase in the output price, and equation (14) measures the cross-price effect of a output price change by firm i on the input price selected by firm j. As in the case of PTS, the sign of the cross-price effect depends on the relative magnitude of the supply effect, $D_i^i S_i^j dp_i \geq 0$, and the demand effect, $D_i^j S_i^i dp_i \geq 0$.

Notice that the cross-price effect in equation (14) always takes the opposite sign of the cross-price effect in equation (10); that is, $p_i^j(\mathbf{w}) \stackrel{s}{=} -w_i^j(\mathbf{p})$. Under conditions in which an increase in a firm's input price increases the output price of his rival in the PTS game, an increase in the output price decreases the input price of his rival in the PTO game. In the symmetric market equilibrium ($D^i = D^j, S^i = S^j, p_1 = p_2 = p, w_1 = w_2 = w$),

$$w_i^j(\mathbf{p}) \stackrel{s}{=} -\Theta.$$

The equilibrium under PTO is determined by the simultaneous solution of equations (12). Define the equilibrium output price vector that solves these equations as $\mathbf{p}^o = (p_1^o, p_2^o)$ and the associated vector of equilibrium input prices as $\mathbf{w}^o = (w_1^o, w_2^o)$.

The equilibrium price-cost margin for symmetric firms can be written

$$p^o - w^o = \frac{p^o}{\varepsilon_o^d} + k^o \frac{w^o}{\varepsilon_o^s}.$$

where $k^o = 1 - \frac{S_i^j S_j^j}{\Sigma}\Theta$. Inspection of this term reveals that $k^o > 1$ if and only if $\Theta > 0$. Equilibrium price-cost margins are higher in the PTO game than in the Bertrand outcome when the upstream market is relatively rivalrous ($\Theta > 0$). The reason is that a rise in the output price by a firm decreases the input price set by his rival when $\Theta > 0$, which facilitates higher price-cost margins.

3 Equilibrium Outcomes

In this Section we compare firm profits in the PTS and PTO games to the Bertrand outcome and identify Pareto dominant strategies in the symmetric market equilibrium. To do so, we consider the symmetric Bertrand prices (w^B, p^B) and examine multilateral defections from the Bertrand outcome that increase the profits of firms.[4]

[4] With slight abuse of notation, we write demand, supply, and profit as functions of the scalar values of input and output prices in the symmetric equilibrium.

3.1 PTS Versus PTO

Consider first the PTS game. Evaluating the input price condition (9) at the symmetric Bertrand values (w^B, p^B) gives

$$\pi_i^{i,s}(w^B, p^B) \equiv \left(\frac{D_j^j S_i^i - D_j^i S_i^j}{\Delta} - 1 \right) S^i - D^i \left(\frac{S_i^i}{D_i^i} \right).$$

Making use of the market-clearing condition (3) and factoring terms yields

$$\pi_i^{i,s}(w^B, p^B) \equiv \frac{S^i D_j^i S_i^i}{\Delta} \Theta \stackrel{s}{=} \Theta,$$

where all terms are evaluated at (w^B, p^B). In the PTS game, firms select higher input prices than the symmetric Bertrand level when $\Theta > 0$ and select lower input prices when $\Theta < 0$.

Proceeding similarly in the case of PTO, evaluating the output price condition (12) at the symmetric Bertrand position (w^B, p^B) gives

$$\pi_i^{i,o}(w^B, p^B) \equiv \frac{D^i D_i^i S_j^i}{\Sigma} \Theta \stackrel{s}{=} \Theta.$$

In the PTO game, firms select higher output prices than the symmetric Bertrand level when $\Theta > 0$ and set lower output prices when $\Theta < 0$.

Proposition 1 *For the intermediated oligopoly model:*

(i) *If $\Theta = 0$ the equilibrium market outcome under either PTO or PTS is Bertrand*

(ii) *If $\Theta > 0$ (< 0) the Pareto dominant equilibrium is PTO (PTS).*

In Stahl's (1988) model, intermediaries compete in homogeneous product markets ($\Theta = 0$), and an outcome with Bertrand merchants emerges under both PTO and PTS. Proposition 1 extends this outcome to encompass any market that satisfies $\Theta = 0$. Thus, the Bertrand outcome represents an envelope of oligopoly equilibria characterized by equal degrees of rivalry in the upstream market and downstream market. The essential underpinning of the Bertrand outcome is that firms' input (output) price choices are independent of the resulting output (input) prices selected by the rival. This outcome depends on the relative degree (rather than the absolute degree) of market rivalry.

When $\Theta \neq 0$, firm profits are larger in the symmetric equilibrium in cases where the firms enjoy wider price-cost margins. Under circumstances where the downstream market is relatively rivalrous ($\Theta < 0$), the PTS game facilitates this outcome, because the rival responds to a lower input price by selecting a higher output price in equation (10). Equilibrium price-cost margins and profits are accordingly higher under PTS than under Bertrand. Under circumstances where the upstream market is relatively rivalrous ($\Theta > 0$), profits are higher in the PTO game, as the rival firm responds to a higher output price in this case by selecting a lower input price in equation (14). In either case, the Pareto dominant equilibrium involves selecting prices in the relatively less rivalrous market as a facilitating practice to soften price competition in the remaining market.

The anatomy of the intermediated oligopoly model can be illustrated by describing circumstances in which playing the PTS (PTO) game produces Cournot outcomes. In the following section we characterize the Cournot equilibrium and derive formal conditions under which the PTS and PTO games produce Cournot outcomes.

3.2 Cournot Outcomes

The Cournot outcome is characterized by maximizing expression (4) with respect to y_i. The first-order necessary condition for a maximum is

$$\pi_i^{i,C}(\mathbf{y}) = P^i - W^i + \left(\frac{\partial P^i}{\partial y_i} - \frac{\partial W^i}{\partial y_i}\right) y_i = 0, \qquad i \in \{1, 2\}. \tag{15}$$

Simultaneously solving equations (15) in the symmetric equilibrium gives the equilibrium quantity, $y^C = y_1^C = y_2^C$, which can be used to recover the symmetric Cournot equilibrium prices (w^C, p^C).

Proposition 2 *The Cournot outcome emerges in the PTS game when $\epsilon_s = 0$ and in the PTO game when $\epsilon_d = 0$.*

Proposition 2 clarifies the essential finding of Kreps and Scheinkman (1983) as an outcome that depends on the strategic independence of prices in the upstream input (capacity) market in the PTS game. The Cournot outcome also emerges in the PTO game when the output prices of firms are strategically independent. In the following section, we illustrate the robustness of these outcomes by considering circumstances in which firms can hold inventories.

To see the intuition for Proposition 2, consider the price-cost margins under Cournot in the symmetric market equilibrium. Making use of equations (2.3) and (13), the symmetric equilibrium price-cost margin can be written as

$$p^C - w^C = \frac{p^C}{\varepsilon_o^d \left(1 - \epsilon_d^2\right)} + \frac{w^C}{\varepsilon_o^s \left(1 - \epsilon_s^2\right)}. \tag{16}$$

The essential difference between this outcome and the outcome under PTS and PTO is that quantity-setting firms jointly consider the strategic effect of a quantity increase on raising their procurement cost in the upstream market and on reducing their sales revenue in the downstream market. In contrast, firms in the PTS equilibrium, who set prices sequentially in the upstream market prior to selecting prices in the downstream market, can consider only the implication of their first-stage input price choice on their subsequent level of sales. When prices in the upstream market are strategically independent, $\epsilon_s = 0$, however; the only interaction that remains with the rival in this case occurs through the interdependence of prices in the downstream oligopoly market, so that the inability of the firm to account for the strategic interdependence of input prices in the upstream market no longer has any consequence. When $\epsilon_s = 0$, the second term on the right-hand side of equation (16) reduces to $\frac{w^C}{\varepsilon_o^s}$ and the weight on the demand-side portion of the equilibrium price-cost margin in the PTS game reduces to $k^s = \frac{1}{1-\epsilon_d^2}$, resulting in the Cournot outcome. For a similar reason, the PTO game results in the Cournot oligopsony outcome when prices are strategically independent in the downstream market, $\epsilon_d = 0$.

4 Model Extensions

In this Section we consider two extensions of the model. We first extend the game structure to a setting in which firms endogenously select the timing of their pricing decisions from the choice set $\{PTS, PTO\}$ prior to choosing prices. We then extend the model to consider inventory-holding behavior by allowing the supply procured by each firm to exceed demand in equations (3).

For each extension, we examine the symmetric market equilibrium under conditions of linear supply and demand. Specifically, we consider the linear specialization,

$$D^i(\mathbf{p}) = \max\{a - bp_i + cp_j, 0\}, \tag{17}$$

$$S^i(\mathbf{w}) = \max\{\beta w_i - \gamma w_j, 0\}, \tag{18}$$

where $i = 1, 2, i \neq j$, and where $a, b, \beta > 0$ and $c, \gamma \geq 0$ are positive constants. We restrict the each firm's prices to the interval $[0, P]$, where $P \geq 2a/(b - c)$. Demand (supply) conditions reduce to local monopoly (monopsony) markets when $c = 0$ ($\gamma = 0$) and products in the downstream (upstream) market become increasingly commoditized as $c \longrightarrow b$ ($\gamma \longrightarrow \beta$).

4.1 Endogenous Timing

Our observations above on the profit motive of firms to select prices sequentially in intermediated oligopoly markets raises the question of whether such behavior is also an equilibrium strategy in settings where coordination on timing is not possible prior to selecting prices. Here, we extend the game structure to a setting in which firms first choose whether to engage in PTS or PTO prior to selecting prices in each market.

Consider the following three stage game. In stage 1, firms select the timing of their pricing decisions from the choice set $\{PTS, PTO\}$. In stage 2, firms select prices in the relevant subgame according to the timing of pricing decisions determined in stage 1. The firms face symmetric market conditions in the second stage according to the demand and supply functions in equations (17) and (18).

	PTS_2	PTO_2
PTS_1	$\frac{S^j}{S^j_j} - \frac{D^j}{D^j_j}k^s, \frac{S^j}{S^j_j} - \frac{D^j}{D^j_j}k^s$	$\frac{S^j}{S^j_j}k^o - \frac{D^j}{D^j_j}, \frac{S^j}{S^j_j} - \frac{D^j}{D^j_j}k^s$
PTO_1	$\frac{S^j}{S^j_j} - \frac{D^j}{D^j_j}k^s, \frac{S^j}{S^j_j}k^o - \frac{D^j}{D^j_j}$	$\frac{S^j}{S^j_j}k^o - \frac{D^j}{D^j_j}, \frac{S^j}{S^j_j}k^o - \frac{D^j}{D^j_j}$

Table 1: Price-Cost Margins for Firms in Response to Timing Choices

Our comparison of stage 2 outcomes involves examining equilibrium prices under several possible alternatives: (*i*) firm i plays PTS while firm j plays PTO; (*ii*) both firms play PTS; and (*iii*) both firms play PTO. We have completely characterized the equilibrium outcomes of the latter two subgames and we turn here to cases in which firms select asychronous prices (PTS, PTO). Our analysis of the extended game is more transparent when making use of the following, general result.

Proposition 3 *The selection of a given timing strategy* $\{PTS, PTO\}$ *by firm* i *provides the rival firm* j *with an equilibrium price-cost margin that corresponds with the strategy selected by firm* i.

Table 1 describes the different possibilities for the equilibrium price cost margin for firm j depending on the firm i's strategy. The entries in Table 1 have important strategic implications in the linear case. Recall that $k^o = 1 - \frac{S^i_j S^j_j}{\Sigma}\Theta$ and $k^s = 1 + \frac{D^j_i D^j_j}{\Delta}\Theta$. These terms are constant under linear demand and supply functions in equations (17) and (18). If firm i defects from the PTO game to play PTS, the defection has no implications for firm i's price-cost margin, which remains at the PTO level; however, rival firm j responds to the initial input price selected by firm i by adjusting his price-cost margin. In equilibrium, firm j selects the wider price-cost margin of the PTS game when $\Theta < 0$. The wider equilibrium price-cost margin that emerges for the PTO player in the (PTS, PTO) subgame subsequently increases the volume of sales for the PTS player at the PTO equilibrium price-cost margin, making the choice of PTS a facilitating practice. The opposite is true –defecting from PTS to the PTO subgame is a facilitating practice– when $\Theta > 0$.

Now consider the outcome for profits under the system of equations (17) and (18). Evaluating the symmetric pay-off matrix in the first stage of the game yields:

Proposition 4 *Under linear supply and demand conditions:*

(i) *(Bertrand,Bertrand) is an equilibrium if and only if* $\Theta = 0$*;*

(ii) *(PTS,PTS) is the unique equilibrium of the game when* $\Theta < 0$*; and*

(iii) *(PTO,PTO) is the unique equilibrium of the game when* $\Theta > 0$.

The intuition for this result is that the choice of timing allows firms to employ prices in the relatively less rivalrous market to serve as a facilitating practice to soften price competition in the more rivalrous market. When $\Theta < 0$, a firm that chooses to play PTS causes his rival to adjust his price-cost margin upward in equilibrium, because $k^s > 1$. Moreover, as demonstrated by the entries in Table 1, this adjustment occurs irrespective of the timing of price-setting behavior chosen by the rival firm. The larger price-cost margin selected by the rival firm benefits the firm playing PTS, and as a result, both firms select PTS in the first stage of the game. A comparable outcome occurs when $\Theta > 0$, allowing both firms to coordinate on PTO.

Before examining outcomes with inventory-holding behavior, it is worthwhile to consider the case in which input prices are strategically independent in the upstream market, $\epsilon_s = 0$. When $\epsilon_s = 0$, (PTS, PTS) is the unique equilibrium outcome in the linear case and the margin adjustment by each firm is $k^s = 1 + (D_i^j)^2/\Delta$. The unique equilibrium outcome of the game is Cournot. If the firms instead engaged in PTO, k^o reduces in this case to $k^o = 1$, and the PTO equilibrium would produce the Bertrand outcome. As in Kreps and Scheinkman (1983), the firms would wish to defect from selecting output prices that result in Bertrand profits by first committing to the purchase of stocks ("capacity"), which results in the Cournot outcome.

4.2 Inventory-Holding

Now consider a setting in which inventory-holding behavior is possible. For expositional clarity, we limit our attention to market conditions that satisfy $\Theta < 0$, which provides strategic incentives for PTS to emerge as the unique equilibrium outcome in the case where holding inventory is not possible. Analogous conditions apply for PTO in settings where forward contracts do not represent binding commitments to deliver finished goods to consumers in the downstream market.[5]

Suppose stocks procured from the upstream input market in the PTS game can be freely disposed. The possibility of free disposal of inputs relaxes the inventory constraint in equation (3), which becomes $D^i(\mathbf{p}) \leq S^i(\mathbf{w})$ for $i = 1, 2$.

To study the pricing subgame with fixed supplies and potentially slack inventory constraints, we follow Kreps and Scheinkman (1983) and assume that residual demand is efficiently rationed. We then characterize the profitability of defection strategies from the no-inventory equilibrium. This analysis results in the following:

Proposition 5 *The inventory constraint in equation (3) is always binding when* [6]

$$P_j^i(\mathbf{y})S_i^j(\mathbf{w}) \leq 1 + \varepsilon_o^s. \tag{19}$$

The implication of Proposition 5 is that the PTS equilibrium described above under the assumption of no inventory-holding coincides with the equilibrium outcome in a general setting with inventory-holding behavior provided that condition (19) holds. The interpretation of this condition is as follows. After input procurement has taken place, the procurement cost of the firm is sunk, leaving the firm with the potential to sell less than the procured quantity with no additional cost under the assumption of free disposal. Proposition 5 describes market conditions under which a firm selecting quantities in a setting with no production cost would wish to select an output level that is (at least weakly) greater than the output level implied by PTS, thereby resulting in a binding inventory constraint.

[5] The possibility of holding negative inventory under PTO implicitly assumes that forward contracts with consumers can be renegotiated. In the event that forward contracts are non-renegotiable, free disposal of forward contracts would not be possible, and an additional deterrent would exist to holding inventory under PTO.

[6] In the linear case, all terms in condition (19) are constant and the inequality can be written, $\gamma c(\beta - \gamma) \leq \Delta(2\beta - \gamma)$, which holds for sufficiently "small" c, γ.

5 Mode of Competition

Characterizing and measuring the mode of competition in the oligopoly model is essential for deriving policy prescriptions in settings with strategic pre-commitment. It is also important for deriving inferences on the type of market conditions that warrant antitrust scrutiny, for instance market features that favor the use of slotting allowances as a practice to soften price competition.[7] Our goal in this section is to develop testable hypotheses on the mode of competition in industrial settings.

To characterize the mode of competition in intermediated oligopoly settings, consider the second partials of profit under PTS. Dropping arguments for notational convenience, the effect of an input price change by firm j on the marginal profit of firm i under PTS is

$$\pi_{ij}^{i,s} \equiv (p^i - w_i)S_{ij}^i + (p_i^i - 1)S_j^i + p_j^i S_i^i + p_{ij}^i S^i, \qquad i \in \{1,2\}. \tag{20}$$

Prices may be strategic complements ($\pi_{ij}^{i,s} > 0$) or strategic substitutes ($\pi_{ij}^{i,s} < 0$) in the sense of Bulow, Geanakoplos, and Klemperer (1985).

To provide an intuitive characterization of these outcomes, consider the first-order effects in expression (20). Classifying the mode of competition in the linear model is important for deriving testable hypotheses for empirical work that relies on linear estimation techniques. On substitution of equations (2.3) and (10), the mode of competition in the PTS game can be expressed as

$$\pi_{ij}^{i,s} = \left(\frac{D_j^j S_i^i - D_j^i S_i^j}{\Delta} - 1 \right) S_j^i + \left(\frac{D_j^j S_j^i - D_j^i S_j^j}{\Delta} \right) S_i^i.$$

In the symmetric market equilibrium, this condition becomes

$$\pi_{ij}^s \stackrel{s}{=} \frac{(p-w)}{p} \varepsilon_o^d \epsilon_s \left(1 - \epsilon_d^2\right) + \Theta, \tag{21}$$

where the first term on the right hand side of equation (21) is positive and $\Theta < 0$ is negative in the PTS game by Proposition 1. It can be seen immediately upon inspection of terms in equation (21) that prices are strategic complements when markets

[7] The Federal Trade Commission (FTC), which regulates the grocery industry, refused to provide guidelines for slotting allowances, citing the need for further investigation on the efficiency effects of the practice (FTC 2001).

are equally rivalrous, $\Theta = 0$ (Bertrand), whereas prices are strategic substitutes in the case where prices are strategically independent in the upstream market, $\epsilon_s = 0$ (Cournot). In general, $\Theta < 0$ is a necessary but not a sufficient condition for prices to be strategic substitutes in the PTS game.

In the PTO game, the effect of an output price change by firm j on the marginal profit of firm i is

$$\pi_{ij}^{i,o} = (p_i - w^i - t_i)D_{ij}^i + (1 - w_i^i)D_j^i - w_j^i D_i^i - w_{ij}^i D^i, \qquad i \in \{1,2\}. \tag{22}$$

Confining attention to first-order effects in expression (22) and making substitutions from (13) and (14) in the symmetric market equilibrium yields

$$\pi_{ij}^o \overset{s}{=} \frac{(p-w)}{w} \varepsilon_o^s \epsilon_d \left(1 - \epsilon_s^2\right) - \Theta. \tag{23}$$

As in the PTS game, output prices may be strategic complements ($\pi_{ij}^{i,o} > 0$) or strategic substitutes ($\pi_{ij}^{i,o} < 0$) depending on the magnitude of $\Theta > 0$. When $\Theta > 0$, the first term on the right-hand side of the expression is positive and the second term is negative. The relative rivalry of the upstream market influences the mode of competition under PTO in the opposite manner as under PTS: The model reduces to Bertrand when $\Theta = 0$ and to Cournot oligopsony when $\epsilon_d = 0$. A relatively rivalrous upstream market ($\Theta > 0$) is necessary but not sufficient for output prices to be strategic substitutes in the PTO game.

Proposition 6 *Under conditions of linear supply and demand, prices are strategic substitutes in the intermediated oligopoly model when:*

(i) $\epsilon_s \leq \frac{\epsilon_d}{\left[1 + \frac{(p-w)}{p}\varepsilon_o^d\left(1 - \epsilon_d^2\right)\right]}$; *or*

(ii) $\epsilon_d \leq \frac{\epsilon_s}{\left[1 + \frac{(p-w)}{w}\varepsilon_o^s(1 - \epsilon_s^2)\right]}$.

Under circumstances in which $\Theta < 0$ ($\epsilon_s < \epsilon_d$), the upstream market is relatively less rivalrous than the downstream market. Firms are able to soften downstream price competition by selecting input prices prior to output prices in the PTS game. Part

(i) of Proposition 6 is the relevant criteria for the mode of competition, and prices are strategic substitutes when ϵ_s is sufficiently small relative to ϵ_d. The opposite is true when $\Theta > 0$ ($\epsilon_d < \epsilon_s$). Part (ii) of Proposition 6 is the relevant criteria for the mode of competition, and prices are strategic substitutes when ϵ_d is sufficiently small relative to ϵ_s. In either case, the mode of competition is determined by the relative rivalry of the upstream market.

6 Policy Implications

The mode of competition in the oligopoly model has important implications in a number of policy settings. In this section, we highlight policy implications of the model for the case of contract design between a principal (a domestic trade authority, a firm, or a controlling shareholder) and an agent (a domestic firm, a supplier/consumer, or a manager). For expositional clarity, we consider strategic policies by the principal that tax or subsidize input procurement by the agent in the upstream market under conditions of linear supply and demand. This allows us to numerically compute the optimal tax level for variation in the relative rivalry of the upstream market by perturbing c and γ in the system of equations (17) and (18).

We consider the following three-stage game. In stage 1, the principal of firm i imposes a unit tax t_i on inputs procured in the upstream market by agent i. In stage 2, agents take principals' policy decisions parametrically and select the timing of pricing decisions, and in stage 3, agents select prices in the relevant subgame.

Letting $\pi^i(\mathbf{p}, \mathbf{w})$ denote the profit of principal i, we denote agent i's profit

$$\Pi^i(\mathbf{p}, \mathbf{w}) = \pi^i(\mathbf{p}, \mathbf{w}) - t_i S^i(\mathbf{w}) + \Omega_i, \qquad i \in \{1, 2\}, \tag{24}$$

where Ω_i is a lump-sum transfer that subsumes fixed costs. Maximizing the agent's profit gives the first-order condition

$$\Pi_i^{i,s} \equiv \left(p^i(\mathbf{w}) - w_i - t_i\right) S_i^i(\mathbf{w}) + (p_i^i(\mathbf{w}) - 1) S^i(\mathbf{w}) = 0, \qquad i \in \{1, 2\}, \tag{25}$$

for the PTS game and

$$\Pi_i^{i,o} \equiv (p_i - w^i(\mathbf{p}) - t_i) D_i^i(\mathbf{p}) + (1 - w_i^i(\mathbf{p})) D^i(\mathbf{p}) = 0, \qquad i \in \{1, 2\}, \tag{26}$$

for the PTO game.

Proposition 7 *The choice of PTO or PTS as an equilibrium strategy of agent i is invariant to the principal's choice of policy variable t_i. Thus, the market equilibrium is Bertrand if $\Theta = 0$, PTO if $\Theta > 0$, and PTS if $\Theta < 0$.*

The magnitudes of the policy variables, t_i, $i = 1, 2$ influence neither the sign nor the value of Θ. An implication of Proposition 7 is that our observations on strategic pre-commitment devices are robust to different policy structures that alter the marginal returns facing oligopoly agents.

Confining attention to first-order effects, the second partial of Π^i with respect to w_i and w_j is

$$\Pi^{i,s}_{ij} = p^i_j(\mathbf{w})S^i_i(\mathbf{w}) + (p^i_i(\mathbf{w}) - 1)S^i_j(\mathbf{w}) = \pi^{i,s}_{ij}, \qquad i \in \{1, 2\},$$

for the PTS game and the second partial of Π^i with respect to p_i and p_j is

$$\Pi^{i,o}_{ij} = (1 - w^i_i(\mathbf{p}))D^i_j(\mathbf{p}) - w^i_j(\mathbf{p})D^i_i(\mathbf{p}) = \pi^{i,o}_{ij}, \qquad i \in \{1, 2\}.$$

In the symmetric market equilibrium, it is straightforward to show that the optimal values of the policy variables satisfy

$$t^*_i \stackrel{s}{=} \pi^{i,s}_{ij} \stackrel{s}{=} \pi^{i,o}_{ij}, \qquad i \in \{1, 2\}.^8 \tag{27}$$

The optimal value of the policy variable for each agent depends on the mode of competition between oligopoly firms. Taxes ($t^*_i > 0$) are optimal when prices are strategic complements, whereas subsidies ($t^*_i < 0$) are optimal when prices are strategic substitutes.

The intermediated oligopoly model allows the optimal value of the strategic precommitment policy to be explicitly computed according to supply and demand conditions facing firms. We illustrate this outcome for the case of linear supply and demand functions by calculating the (unique) Nash equilibrium in policy variables

[8] Full derivation of the optimal policy varibles is provided in the Web Appendix.

for variation in the degree of pricing interdependence in each market, c and γ. For our numerical analysis, we select parameters $a = b = \beta = 1$ and identify the optimal tax policy in the symmetric pre-commitment equilibrium, $t^* = t_i^* = t_j^*$ for variations in $c \in (0, 1]$ and $\gamma \in (0, 1]$.

Figure 1 depicts the contour lines of the symmetric trade policy equilibrium in the space (c, γ). Note that PTS is the unique equilibrium of the game when $\gamma > c$, while PTO is the unique equilibrium of the game when $\gamma < c$. The optimal trade policy is a tax ($t^* > 0$) in the shaded region of the figure and a subsidy ($t^* < 0$) in the non-shaded area of the figure. The contour line where the laissez-faire outcome is optimal ($t^* = 0$) is represented by the contour line that separates these two areas. For a fixed γ (respectively c) the optimal policy reveals an inverted u-shaped pattern as c (respectively γ) increases from 0 to 1.

7 Concluding Remarks

The oligopoly intermediation model results in testable hypotheses on the mode of competition that can be used to derive policy implications in industrial settings. Our analysis reveals the prevailing mode of competition in a particular industry to be an empirical question that depends on estimable parameters of supply and demand functions in the upstream and downstream markets where firms interact.

We have demonstrated that oligopoly firms have an incentive to select input prices prior to choosing output prices in the PTS game when the downstream market is relatively rivalrous, but to select output prices prior to choosing input prices in the PTO game when the upstream market is relatively rivalrous. Under reasonably mild conditions, we have shown these outcomes to be robust to the possibility of strategic inventory-holding behavior of firms. For both the PTS and PTO games, prices are strategic complements when the upstream market and the downstream market are relatively equal in terms of rivalry, whereas prices are strategic substitutes under circumstances where the upstream (downstream) market is sufficiently rivalrous relative

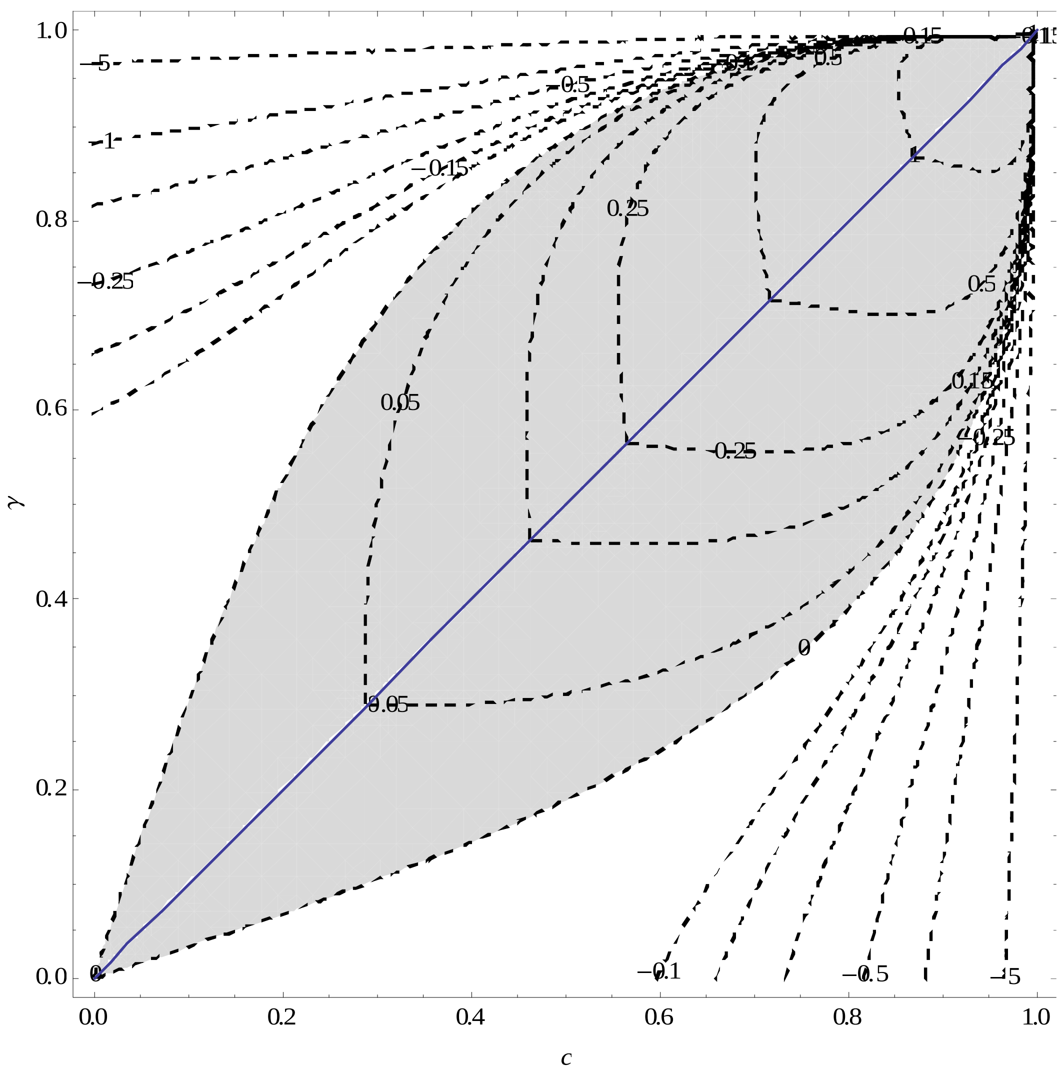

Figure 1:

to the remaining market. In the case where the markets are equally rivalrous, the equilibrium outcome of the oligopoly model is Bertrand and in cases where prices are strategically independent in either the upstream market or the downstream market, the equilibrium outcome of the oligopoly model is Cournot.

Our model provides empirical direction for examining the mode of competition in industrial settings that can be used to tailor pre-commitment policies to the particular conditions facing firms. This feature of the model can provide a useful tool for policy formulation as well as for the enforcement of anti-trust regulations in a given industry.

An interesting avenue for future research is to consider an active role for firms in contributing to market rivalry through product differentiation. To the extent that oligopoly firms engage in activities to differentiate themselves from rivals, whether by creating differentiated products from common inputs or by discovering new techniques for producing existing products from specialized inputs, differentiation can provide a tool to soften price competition with rivals. But, perhaps surprisingly, our analysis clarifies that relaxing market rivalry is not necessarily a globally desirable endeavor. The reason is that it is the relative degree of market rivalry – not the absolute degree– that is the source of strategic advantage. Indeed, a firm that seeks to specialize input requirements would have a greater ability to soften price competition with rivals when downstream products are commoditized than when downstream products are highly differentiated. Our model suggests the potential for "strategic standardization" to occur under circumstances in which inputs (outputs) in the upstream (downstream) market are highly specialized.

References

[1] Bertrand, Joseph. 1883. "Theorie Mathematique de la Richesse Sociale," *Journal des Savants*: 499-508.

[2] Brander, James A. and Barbara J. Spencer. 1985. "Export Subsidies and International Market Share Rivalry," *Journal of International Economics* 18(1-2): 83-100.

[3] Bresnahan, Timothy F. 1981. "Duopoly Models with Consistent Conjectures," *American Economic Review* 71(5): 934-45.

[4] Bulow, Jeremy I., John D. Geanakoplos, and Paul D. Klemperer. 1985. "Multimarket Oligopoly: Strategic Substitutes and Complements," *Journal of Political Economy* 93(3): 488-511.

[5] Cournot, Augustine. 1838. *Recherches sur les Principes Mathematiques de la Theorie des Richesses*. Paris.

[6] Eaton, Jonathan and Gene M. Grossman. 1986. "Optimal Trade and Industrial Policy under Oligopoly," *Quarterly Journal of Economics* 101(2): 383-406.

[7] Fershtman, Chaim and Kenneth L. Judd. 1987. "Equilibrium Incentives in Oligopoly," *American Economic Review* 77(5): 927-40.

[8] Fudenberg, Drew, and Jean Tirole. 1984."The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean-and-Hungry Look," *American Economic Review* 74(2): 361-68.

[9] Gal-Or, Esther. 1985. "First Mover and Second Mover Advantages," *International Economic Review* 26(3): 649-53.

[10] Kreps, David M. and Jose A. Scheinkman. 1983. "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," *Bell Journal of Economics* 14(2): 326-37.

[11] Maggi, Giovanni. 1996. "Strategic Trade Policies with Endogenous Mode of Competition," *American Economic Review* 86(2): 237-58.

[12] Martin, Stephen. 1999. "Kreps And Scheinkman With Product Differentiation: An Expository Note," University of Copenhagen, Centre for Industrial Economics Discussion Paper no. 1999-11.

[13] Shaffer, Greg. 1991. "Slotting Allowances and Resale Price Maintenance: A Comparison of Facilitating Practices," *RAND Journal of Economics* 22(1): 120-36.

[14] Singh, Nirvikar and Xavier Vives. 1984. "Price and Quantity Competition in a Differentiated Oligopoly," *RAND Journal of Economics* 15(4): 546-54.

[15] Sklivas, Steven D. 1987. "The Strategic Choice of Managerial Incentives," *RAND Journal of Economics* 18(3): 452–458.

[16] von Stackelberg, Heinrich. 1934. *Market Structure and Equilibrium (Marktform und Gleichgewicht).* 1st Edition Translation into English, Springer: Bazin, Urch & Hill, 2011.

[17] Stahl, Dale O. 1988. "Bertrand Competition for Inputs and Walrasian Outcomes," *American Economic Review* 78(1): 189-201.

[18] Vickers, John. 1985. "Delegation and the Theory of the Firm," *Economic Journal* 95 (Supplement): 138-47.

[19] Vives, X. 1999. *Oligopoly Pricing: Old Ideas and New Tools.* (Cambridge: MIT Press).

Appendix

A Proof of proposition 1

The proof is constructed by comparing equilibrium prices under PTO and PTS to Bertrand prices and the prices that emerge from symmetric joint profit maximization. The joint profit maximizing solution involves selecting a symmetric quantity pair (y, y) to maximize

$$\pi^M(y) = \sum_{i \in \{1,2\}} \pi^{i,C}(\mathbf{y}),$$

where $\pi^{i,C}(\mathbf{y})$ is given by expression (4). Collective profit, $\pi^M(y)$, is concave in the symmetric quantities under Assumption 4. This is immediate since,

$$\frac{\partial^2 \pi^M(y)}{\partial y^2} = \sum_{i \in \{1,2\}} \Big(\pi_{11}^{i,C}(\mathbf{y}) + 2\pi_{12}^{i,C}(\mathbf{y}) + \pi_{22}^{i,C}(\mathbf{y}) \Big) < 0$$

To complete the proof, we show that the prices that maximize collective industry profits in the symmetric market equilibrium (p^M, w^M) satisfy $w^M < w^s \leq w^B$ and $p^M > p^o \geq p^B$. When this condition holds, firm profits are rising for defections from w^B (p^B) that involve $w < w^B$ ($p > p^B$) under PTS (PTO).

The first-order necessary condition for a joint profit maximum is

$$\frac{\partial \pi^M(y)}{\partial y} \equiv P^i - W^i + \left(\frac{\partial P^i}{\partial y_i} - \frac{\partial W^i}{\partial y_i} \right) y + \left(\frac{\partial P^j}{\partial y_i} - \frac{\partial W^j}{\partial y_i} \right) y = 0, \qquad i \in 1, 2, i \neq j \tag{28}$$

Simultaneously solving (28) for symmetric quantities yields $y^M (= y_1^M = y_2^M)$, which can be used to recover the joint profit maximizing prices.

To facilitate the comparison of the joint profit maximizing outcome with the Bertrand, PTS, and PTO equilibria, note that $\frac{\partial P^i}{\partial y_i} = \frac{D_j^j}{\Delta} < 0$, $\frac{\partial P^j}{\partial y_i} = \frac{-D_i^j}{\Delta} < 0$, $\frac{\partial W^i}{\partial y_i} = \frac{S_j^j}{\Sigma} > 0$ and $\frac{\partial W^j}{\partial y_i} = \frac{-S_i^j}{\Sigma} > 0$ for all prices in $\mathbb{P}$. Incorporating these terms in (28) and evaluating the slope of the joint profit function at the symmetric Bertrand output level, y^B, as defined by $(p^B - w^B) = y^B\left(\frac{1}{S_i^i} - \frac{1}{D_i^i}\right)$, gives

$$\begin{aligned}\pi_i^M(y^B) &= y^B\left[\frac{1}{S_i^i} - \frac{1}{D_i^i} + \frac{D_j^j - D_i^j}{\Delta} - \frac{\left(S_j^j - S_i^j\right)}{\Sigma}\right] \\ &= y^B\left[\frac{S_i^j(S_i^i - S_j^i)}{S_i^i\Sigma} + \frac{D_i^j(D_j^i - D_i^i)}{D_i^i\Delta}\right] < 0.\end{aligned}$$

Hence the output level that maximizes joint profits in the symmetric case satisfies $y^M < y^B$. By concavity of $\pi_i^M(y)$, it follows that $w^M < w^B$ and $p^M > p^B$.

It remains to compare the joint profit maximization solution to the equilibrium outcomes under PTS and PTO. Our conjecture is that profitable defections from the Bertrand prices involve $w < w^B$ and $p > p^B$. This conjecture holds provided that $w^M < w^s < w^B$ and $p^M > p^o > p^B$. We have already demonstrated that firms select $w^s < w^B$ in the PTS game when $\Theta < 0$ and that firms select $p^o > p^B$ in the PTO game when $\Theta > 0$. It remains to be shown that $w^M < w^s$ and $p^M > p^o$.

To verify this conjecture, consider the slope of the joint profit function at the symmetric PTS equilibrium position, y^s, as implicitly defined by equation (9): $(p^s - w^s) = \frac{y^s}{S_i^i}\left(1 - \frac{(D_j^j S_i^i - D_j^i S_i^j)}{\Delta}\right)$. Collecting terms in (28) yields

$$\pi_i^M(y^s) = \frac{y^s}{S_i^i}\left[1 - \frac{(D_j^j S_i^i - D_j^i S_i^j)}{\Delta}\right] + y^s\left(\frac{D_j^j - D_i^j}{\Delta} - \frac{\left(S_j^j - S_i^j\right)}{\Sigma}\right).$$

Factoring this expression yields

$$\pi_i^M(y^s) = \frac{y^s\left(S_i^i - S_i^j\right)}{S_i^i}\left[\frac{S_i^j}{\Sigma} - \frac{D_i^j}{\Delta}\right] < 0.$$

The output level that maximizes joint profits in the symmetric case satisfies $y^M < y^s$. By concavity of $\pi_i^M(y)$, it follows that $w^M < w^s$ and $p^M > p^s$. Thus, $w^M < w^s < w^B$ for $\Theta < 0$.

Proceeding similarly in the case of PTO competition, evaluating the slope of collective profit at the symmetric PTO equilibrium position gives

$$\pi_i^M(p^o, w^o) = \frac{y\left(D_i^i - D_j^i\right)}{D_i^i}\left[\frac{S_i^j}{\Sigma} - \frac{D_i^j}{\Delta}\right] < 0.$$

By inspection, $y^M < y^o$, so that $w^M < w^o$ and $p^M > p^o$ in the PTO game. It follows that $p^M > p^o > p^B$ for $\Theta > 0$.

$\square$

B Proof of Proposition 2

First consider PTS. We wish to show that the PTS game produces Cournot outcomes when $\epsilon_s = 0$. Note that $\frac{\partial P^i}{\partial y_i} = \frac{D^j_j}{\Delta} < 0$ and $\frac{\partial W^i}{\partial y_i} = \frac{S^j_j}{\Sigma} > 0$ for all prices in $\mathbb{P}$. Incorporating these terms in optimality condition (15), the Cournot equilibrium is characterized in the symmetric case by

$$\pi^{i,C}_i(\mathbf{y}) = P^i - W^i + \left(\frac{D^j_j}{\Delta} - \frac{S^j_j}{\Sigma}\right) y^i = 0. \tag{29}$$

The proof is constructed by showing that the equilibrium condition under PTS coincides with (29) when $\epsilon_s = 0$. Noting that $\epsilon_s = 0$ holds if and only if $S^i_j = S^j_i = 0$, condition (9) in the PTS game reduces to

$$\pi^{i,s}_i \equiv (p^i - w_i)S^i_i + \left(\frac{D^j_j S^i_i}{\Delta} - 1\right) S^i = 0 \Leftrightarrow p^i - w_i + \left(\frac{D^j_j}{\Delta} - \frac{1}{S^i_i}\right) S^i = 0.$$

To verify that the symmetric equilibrium in the PTS game produces Cournot outcomes when $\epsilon_s = 0$, note that the Cournot equilibrium satisfies $p^C - w^C = \left(\frac{1}{S^i_i} - \frac{D^j_j}{\Delta}\right) y^C$ when $S^i_j = S^j_i = 0$. Evaluating the first-order condition for the PTS game at the Cournot equilibrium output level, $S^i = y^C$ completes the proof.

Proceeding similarly for the case of PTO, note that $\epsilon_d = 0$ holds if and only if $D^i_j = D^j_i = 0$, which reduces equilibrium condition (12) to

$$\pi^{i,o}_i \equiv (p_i - w^i)D^i_i + \left(1 - \frac{D^i_i S^j_j}{\Sigma}\right) D^i = 0 \Leftrightarrow p_i - w^i + \left(\frac{1}{D^i_i} - \frac{S^j_j}{\Sigma}\right) D^i = 0.$$

When $D^i_j = D^j_i = 0$, the Cournot equilibrium satisfies $p^C - w^C = \left(\frac{S^j_j}{\Sigma} - \frac{1}{D^i_i}\right) y^C$. Evaluating the first-order condition from the PTO game at the Cournot equilibrium output level, $D^i = y^C$ completes the proof.

$\square$

C Proof of Proposition 3

The proof is constructed by sequentially evaluating the asynchronous pricing outcome in the stage 2 subgame in which one firm plays PTS and the rival plays PTO.

Lemma 1. *When firm i plays PTS and firm j plays PTO the equilibrium price-cost margins are $p^i - w_i = \frac{S^i}{S^i_i}k^o - \frac{D^i}{D^i_i}$ and $p_j - w^j = \frac{S^j}{S^j_j} - \frac{D^j}{D^j_j}k^s$, where $k^o = 1 - \frac{S^i_j S^j_j}{\Sigma}\Theta$ and $k^s = 1 + \frac{D^j_i D^j_j}{\Delta}\Theta$.*

Proof. Totally differentiating the binding inventory constraints yields

$$\begin{bmatrix} D^i_i & -S^i_j \\ D^j_i & -S^j_j \end{bmatrix} \begin{bmatrix} dp_i \\ dw_j \end{bmatrix} = \begin{bmatrix} S^i_i & -D^i_j \\ S^j_i & -D^j_j \end{bmatrix} \begin{bmatrix} dw_i \\ dp_j \end{bmatrix},$$

and the corresponding responses

$$\frac{\partial p^i}{\partial w_i} = \frac{-S^j_j S^i_i + S^i_j S^j_i}{-S^j_j D^i_i + S^i_j D^j_i} = -\frac{\Sigma}{\Psi} < 0$$

$$\frac{\partial w^j}{\partial w_i} = \frac{S^j_i D^i_i - S^i_i D^j_i}{-S^j_j D^i_i + S^i_j D^j_i} = -\frac{D^j_j S^j_j \Theta}{\Psi} \overset{s}{=} \Theta$$

$$\frac{\partial p^i}{\partial p_j} = \frac{S^j_j D^i_j - S^i_j D^j_j}{-S^j_j D^i_i + S^i_j D^j_i} = \frac{D^i_i S^i_i \Theta}{\Psi} \overset{s}{=} -\Theta$$

$$\frac{\partial w^j}{\partial p_j} = \frac{-D^j_j D^i_i + D^i_j D^j_i}{-S^j_j D^i_i + S^i_j D^j_i} = -\frac{\Delta}{\Psi} < 0$$

where $\Psi = -S^j_j D^i_i + S^i_j D^j_i > 0$, $\Sigma = S^j_j S^i_i - S^i_j S^j_i > 0$, $\Delta = D^j_j D^i_i - D^i_j D^j_i > 0$ and using symmetry, $\Theta = \frac{S^i_i D^j_i - S^j_i D^i_i}{D^j_j S^j_j}$.

Next, consider the problems of firms i and j in stage 1. The profit function for firm i is

$$\max_{w_i} \pi^{i,s} = \left(p^i(w_i, p_j) - w_i\right) S^i(w_i, w^j(w_i, p_j)),$$

with the corresponding first-order necessary condition

$$\left(p^i(w_i, p_j) - w_i\right)\left(S^i_i + S^i_j \frac{\partial w^j}{\partial w_i}\right) + \left(\frac{\partial p^i}{\partial w_i} - 1\right) S^i = 0.$$

Factoring this expression and making use of the inventory constraint gives

$$p^i - w_i = \frac{S^i}{S_i^i}\left(\frac{S_i^i}{\tilde{S}_i^i}\right) - \frac{D^i}{D_i^i}\left(-\frac{D_i^i\Sigma}{\tilde{S}_i^i\Psi}\right)$$

with $\tilde{S}_i^i = S_i^i - S_j^i \frac{D_j^j S_j^j \Theta}{\Psi}$. Expanding terms, we have

$$-\frac{D_i^i\Sigma}{\tilde{S}_i^i\Psi} = \frac{-D_i^i\left(S_j^j S_i^i - S_j^i S_i^j\right)}{S_i^i\left(-S_j^j D_i^i\right) + S_j^i\left(S_i^j D_i^i\right)} = 1$$

and

$$\frac{S_i^i}{\tilde{S}_i^i} = \frac{S_i^i\Psi}{D_i^i\left(S_j^i S_i^j - S_i^i S_j^j\right)} = -\frac{S_i^i\Psi}{D_i^i\Sigma} = 1 - \frac{S_j^i S_j^j}{\Sigma}\Theta.$$

Substitution of terms yields the equilibrium price-cost margin of firm i:

$$p^i - w_i = \frac{S^i}{S_i^i}k^o - \frac{D^i}{D_i^i}.$$

The profit function of firm j is

$$\max_{p_j} \pi^{j,o} = \left(p_j - w^j(w_i, p_j)\right) D^j\left(p^i(w_i, p_j), p_j\right)$$

The corresponding first-order necessary condition is

$$\left(p_j - w^j(w_i, p_j)\right)\left(D_j^j + D_i^j\frac{\partial p^i}{\partial p_j}\right) + \left(1 - \frac{\partial w^j}{\partial p_j}\right)D^j = 0,$$

which can be written on substitution of terms as

$$\begin{aligned}
p_j - w^j &= \frac{-D^j\left(\frac{\Delta}{\Psi}+1\right)}{D_j^j + D_i^j\frac{D_i^i S_i^i\Theta}{\Psi}} \\
&= \frac{S^j}{S_j^j} - \frac{D^j}{D_j^j}\left(-\frac{D_j^j\Psi}{S_i^i\Delta}\right) \\
&= \frac{S^j}{S_j^j} - \frac{D^j}{D_j^j}\left(1 + \frac{D_i^j D_j^j\Theta}{\Delta}\right) \\
&= \frac{S^j}{S_j^j} - \frac{D^j}{D_j^j}k^s
\end{aligned}$$

■

D Proof of Proposition 4

For the generic case, the equilibrium profit level of firm i is described by four equations

$$\begin{aligned} \frac{p_i - w_i}{S^i} &= \Gamma_i, \qquad i = 1, 2. \\ D^i &= S^i, \qquad i = 1, 2. \end{aligned}$$

The equilibrium profit level of firm i satisfies $\pi^i = (p_i - w_i)S^i = \Gamma_i (S^i)^2$, where

$$\Gamma_i \in \left\{ \Gamma^o = \frac{k^o}{S_i^i} - \frac{1}{D_i^i}, \Gamma^s = \frac{1}{S_i^i} - \frac{k^s}{D_i^i} \right\}.$$

For linear supply and demand, these terms all reduce to constants. Specifically,

$$\Gamma_i \in \left\{ \Gamma^o = \frac{k^o}{\beta} + \frac{1}{b}, \Gamma^s = \frac{1}{\beta} + \frac{k^s}{b} \right\},$$

where $k^o = \frac{\beta(\beta b - \gamma c)}{b(\beta^2 - \gamma^2)}$ and $k^s = \frac{b(\beta b - \gamma c)}{\beta(b^2 - c^2)}$.

The (symmetric) pay-off matrix is

	PTS	PTO
PTS	$\Gamma^s(S^{ss})^2, \Gamma^s(S^{ss})^2$	$\Gamma^o(S_1^{so})^2, \Gamma^s(S_2^{so})^2$
PTO	$\Gamma^s(S_1^{os})^2, \Gamma^o(S_2^{os})^2$	$\Gamma^o(S^{oo})^2, \Gamma^o(S^{oo})^2$

where we denote S_i^{hl} the equilibrium output of firm i when firm i plays strategy $h \in \{o, s\}$ and firm j plays strategy $l \in \{o, s\}$, where o stands for PTO and s stands for PTS. By symmetry, we denote also $S_i^{hh} = S_j^{hh} = S^{hh}$ for any h.

Note that $\Theta < 0 \Leftrightarrow \Gamma^s > \Gamma^o$ and $\Theta > 0 \Leftrightarrow \Gamma^s < \Gamma^o$. The system of equilibrium conditions can be rewritten as

$$\begin{aligned} p_i &= (1 + \beta\Gamma_i)w_i - \gamma\Gamma_i w_j, \qquad i \in 1, 2, \\ D^i &= S^i, \qquad i \in 1, 2. \end{aligned}$$

Replacing p_i by its value in the last two equations, then solving in w_i and replacing in S^i, we get

$$S^i = \frac{A + B\Gamma_j}{C + D\Gamma_i\Gamma_j + E(\Gamma_i + \Gamma_j)}$$

with $A = a(\beta - \gamma)(b + c + \beta + \gamma) > 0$, $B = a(b + c)\Sigma > 0$, $C = \Sigma + \Delta + 2\Psi > 0$, and $D = \Delta\Sigma > 0$, $E = \beta\Delta + b\Sigma > 0$. It follows that

$$\Gamma_j \geq \Gamma_i \Leftrightarrow S^i \geq S^j.$$

We get

$$\begin{aligned}
S^{ss} &= \frac{A+B\Gamma^s}{C+D(\Gamma^s)^2+2E\Gamma^s} \\
S^{oo} &= \frac{A+B\Gamma^o}{C+D(\Gamma^o)^2+2E\Gamma^o} \\
S_1^{os} &= S_2^{so} = \frac{A+B\Gamma^o}{C+D\Gamma^s\Gamma^o+E(\Gamma^s+\Gamma^o)} \\
S_2^{os} &= S_1^{so} = \frac{A+B\Gamma^s}{C+D\Gamma^s\Gamma^o+E(\Gamma^s+\Gamma^o)}
\end{aligned}$$

For part (i), to verify that (Bertrand,Bertrand) is the equilibrium outcome of all subgames (PTS,PTS), (PTS, PTO), (PTO,PTO) when $\Theta = 0$, notice $\Gamma^s(S^{ss}) = \Gamma^s(S^{so}) = \Gamma^s(S^{os}) = \Gamma^o(S^{oo}) = \Gamma^o(S^{os}) = \Gamma^o(S^{so}) = \Gamma^b$, where $\Gamma^b = \frac{1}{\beta} + \frac{1}{b}$ denotes the Bertrand margin. It follows immediately that $S^i = S^j = y^B$, which completes the proof of part (i).

Next consider part (ii). For (PTS,PTS) to be an equilibrium of the game with symmetric firms, it must be true that

$$\Gamma^s(S^{ss})^2 \geq \Gamma^s(S_1^{os})^2$$

or equivalently

$$S^{ss} \geq S_1^{os}$$

We have:

$$\begin{aligned}
S^{ss} - S_1^{os} &= \frac{A+B\Gamma^s}{C+D(\Gamma^s)^2+2E\Gamma^s} - \frac{A+B\Gamma^o}{C+D\Gamma^s\Gamma^o+E(\Gamma^s+\Gamma^o)} \\
&\stackrel{s}{=} (A+B\Gamma^s)\left(C+D\Gamma^s\Gamma^o+E(\Gamma^s+\Gamma^o)\right) - (A+B\Gamma^o)\left(C+D(\Gamma^s)^2+2E\Gamma^s\right) \\
&\stackrel{s}{=} AC + AD\Gamma^s\Gamma^o + AE(\Gamma^s+\Gamma^o) + BC\Gamma^s + BD(\Gamma^s)^2\Gamma^o + BE(\Gamma^s+\Gamma^o)\Gamma^s \\
&\quad -AC - AD(\Gamma^s)^2 - 2AE\Gamma^s - BC\Gamma^o - BD\Gamma^o(\Gamma^s)^2 - 2BE\Gamma^o\Gamma^s \\
&\stackrel{s}{=} (\Gamma^o - \Gamma^s)\left((AD-BE)\Gamma^s + AE - BC\right) \\
&\stackrel{s}{=} (\Gamma^s - \Gamma^o)
\end{aligned}$$

and the result follows that (PTS,PTS) is the unique equilibrium of the extended game when $\Theta < 0$.

The proof of part (*iii*) follows identical logic as in part (*ii*). For (PTO,PTO) to be an equilibrium of the game with symmetric firms, it must be true that

$$\Gamma^o (S^{oo})^2 \geq \Gamma^o (S_1^{so})^2$$

or equivalently

$$S^{oo} \geq S_1^{so}$$

We have:

$$\begin{aligned} S^{oo} - S_1^{so} &= \frac{A + B\Gamma^o}{C + D(\Gamma^o)^2 + 2E\Gamma^o} - \frac{A + B\Gamma^s}{C + D\Gamma^s\Gamma^o + E(\Gamma^s + \Gamma^o)} \\ &\overset{s}{=} (\Gamma^o - \Gamma^s) \,. \end{aligned}$$

It follows that (PTO,PTO) is the unique equilibrium of the extended game when $\Theta > 0$.

□

E Proof of Proposition 5

Consider the PTS game that allows for inventory holding. We begin by describing the equilibrium of the output pricing game when supply is fixed by wage choice in the initial stage of the game. Denote these levels by $x_i = S^i(\mathbf{w})$. Notice that the second stage analysis with fixed supplies x_i is exactly the same as the Bertrand-Edgeworth pricing game with fixed capacities.

Before turning to the Bertrand-Edgeworth pricing game, we need to define two other games that will be useful for our analysis: the zero-cost Bertrand game and the zero-cost Cournot game.

In the zero-cost Bertrand game, each firm i has the profit function

$$D^i(\mathbf{p})p_i = \max\{a - bp_i + cp_j, 0\}p_i,$$

The best response function of firm i is

$$R^i(p_j) = \max\left\{\frac{a + cp_j}{2b}, 0\right\}.$$

In the symmetric market equilibrium, this game has a unique, symmetric Nash equilibrium in pure strategies. We label the symmetric equilibrium price

$$p^b = \frac{a}{2b - c}.$$

In the zero-cost Cournot game, each firm i has the profit function

$$P^i(\mathbf{x})x_i = \max\left\{\frac{1}{b^2 - c^2}\left(a(b+c) - bx_i - cx_j\right), 0\right\} x_i$$

where $\mathbf{x} = (x_1, x_2)$ are the firms' quantities. Each firm has a best response function to the other firm's quantity given by

$$r^i(x_j) = \max\left\{\frac{1}{2b}(a(b+c) - cx_j), 0\right\}.$$

In the symmetric market equilibrium, the unique Nash equilibrium in this game is symmetric in pure strategies, with each firm's quantity given by

$$q^c = \frac{a(b+c)}{2b+c}.$$

The symmetric, Cournot equilibrium price is

$$p^c = \frac{ab}{b(2b-c) - c^2} > p^b$$

E.1 Bertrand-Edgeworth Pricing Game

Given that residual demand is efficiently rationed, the residual demand of firm i is

$$d^i(p_i, x_j) = \max\left\{\frac{1}{b}\left((b+c)\left(a - (b-c)p_i\right) - cx_j\right), 0\right\}$$

The price that maximizes the residual profit $p_i d^i(p_i, x_j)$ is

$$p^R(x_j) = \frac{1}{2(b-c)}\left(a - \frac{c}{b+c}x_j\right),$$

and the maximum residual profit is

$$\pi^R(x_j) = \frac{b+c}{4b(b-c)}\left(a - \frac{c}{b+c}x_j\right)^2.$$

Now we turn to characterizing the equilibrium of the pricing game with fixed capacities.

E.1.1 Cournot Pricing

Without loss of generality we assume that $x_1 \geq x_2$. We separate the analysis of the pricing subgame into two regions of capacities. The first is such that $x_1 \leq r(x_2)$. Note that $x_1 \geq x_2$ and $x_1 \leq r(x_2)$ imply that $x_2 \leq r(x_1)$. In this region of capacities, the pricing subgame has a unique pure strategy Nash equilibrium with market clearing prices.

Lemma 4. The unique Nash equilibrium of the of the pricing subgame is $p_1^* = P^1(\mathbf{x})$ and $p_2^* = P^2(\mathbf{x})$ if $x_1 \leq r(x_2)$.

Proof. **Step 1.** $p^* = \big(P^1(\mathbf{x}), P^2(\mathbf{x})\big)$ **is a Nash equilibrium**

Take $x_1 \leq r(x_2)$ and $p_j = P^j(\mathbf{x})$. If $p_i \leq P^i(\mathbf{x})$, then the profit of firm i is $p_i x_i$. Clearly, the profit of firm i is maximized at $p_i = P^i(\mathbf{x})$ if we constrain $p_i \leq P^i(\mathbf{x})$. If $p_i \geq P^i(\mathbf{x})$, then the profit of firm i is $p_i \min\{d^i(p_i, x_j), x_i\}$. By the definition of efficient rationing $\max_{p_i} \big\{p_i \min\{d^i(p_i, x_j), x_i\}\big\} = P^i(\mathbf{x})x_i$. This must be true since $x_i \leq r(x_j)$, which implies that $p^R(x_j) = P^i(r(x_j), x_j) \leq P^i(\mathbf{x})$. Thus, the maximum price must be at the boundary where $d^i(p_i, x_j) = x_i$, which is the prices $P^i(\mathbf{x})$. Consequently, if $p_j^* = P^j(\mathbf{x})$, then it is a best response for firm i to pick $p_i^* = P^i(\mathbf{x})$.

Step 2. Uniqueness of pure strategy equilibrium

Suppose to the contrary that there is another equilibrium $\mathbf{p}^o$. We will show a contradiction.

Case i. There is a firm i such that $D^i(\mathbf{p}^o) > x_i$. Firm i gains profit by picking p_i greater such that $D^i(p_i, p_j^o) \geq x_i$.

Case ii. There is an i such that $D^i(\mathbf{p}^o) < x_i$ for at least one firm $i \in \{1, 2\}$.

Suppose that the other firm j is also pricing such that $D^j(\mathbf{p}^o) < x_j$. If this is true the profits of each firm are the same as in the zero cost Bertrand game at $\mathbf{p}^o$. Since there is no zero cost Bertrand equilibrium price such that $D^j(\mathbf{p}^b) < x_j$ for both firms $j \in \{1, 2\}$ when $x_i \leq r(x_j)$, we know that $\beta(p_j^o) < p_j^o$ for $j \in \{1, 2\}$. Thus, either $\beta(p_j^o)$ or $P^j(\mathbf{x})$ leads to higher profit than p_i^o.

Suppose that the other firm j is pricing such that $D^j(\mathbf{p}^o) = x_j$. Then firm i is operating on the residual demand, which is uniquely maximized at $P^i(\mathbf{x})$.

Case iii. $D^i(\mathbf{p}^o) = x_i$ for both firms i. Then at least one firm i must be playing $p_i^o < P^i(\mathbf{x})$. The other firm j's best response is to maximize the residual demand by which is uniquely done by picking $P^j(\mathbf{x})$. So $p_j^o = P^j(\mathbf{x})$. Consequently, firm i's unique best response to $P^i(\mathbf{x})$ is to maximize the residual profit with $P^i(\mathbf{x})$.

Step 3. Uniqueness of equilibrium

The argument is based on showing that if $x_1 \leq r(x_2)$, then the game has *strategic complementaries* and then appealing to Theorem 12 in Milgrom and Shannon (1994). Adapting Milgrom and Shannon definition of a game with *strategic complementarities* to our notation, we have: for every player i

1. Player i's strategy space is a compact lattice;

2. Player i's payoff function is upper semi-continuous in p_i for p_j fixed, and continuous in p_j for fixed p_i;

3. Player i's payoff function is quasisupermodular in p_i and satisfies the single crossing property in $(p_i; p_j)$.

The first two requirements are satisfied for the pricing game, since the interval $[0, P] \subset \mathbb{R}$ is a compact lattice and the profits are continuous functions in both arguments. We are left to show that property three is satisfied.

The connection between monotonic nondecreasing best responses and the third requirement for a game with strategic complementarities is based on the following special case of Theorem 4 in Milgrom and Shannon (1994) modified to our notation.

[Milgrom and Shannon] Let $\pi : [0, P] \times [0, P] \mapsto \mathbb{R}$. Then $\phi(\rho)$ is monotonic nondecreasing in ρ if and only if π is quasisupermodular in p and satisfies the single crossing property in $(p; \rho)$.

Now we appeal to a special case of Milgrom and Shannon (1994), Theorem 12 modified to our notation:

[Milgrom and Shannon] In a game with strategic complementarities and a unique pure strategy Nash equilibrium, the Nash equilibrium strategies are the unique serial undominated strategies.

We will show that each firm's best response is a nondecreasing function on $[0, P]$ and then uniqueness follows from Fact 2. To do this, we show that the firms' best response correspondences are contained within monotonic nondecreasing correspondence that have the unique pure strategy equilibrium $\left(P^1(\mathbf{x}), P^2(\mathbf{x})\right)$. Denote by $\boldsymbol{\beta}^{\mathbb{C}}(\mathbf{p})$ the best response correspondence for the pricing subgame.

Define another correspondence

$$\widetilde{\beta}_j^{\mathbb{C}}(p) = \begin{cases} [\beta(p), p^R(x_j)] & p < p^R(x_j) \\ [p^R(x_j), \beta(p)] & p \geq p^R(x_j) \end{cases} .$$

$\widetilde{\beta}_i^{\mathbb{C}}$ is a monotonic nondecreasing correspondence on $[0, P]$ and is such that $\beta_i^{\mathbb{C}}(p) \in \widetilde{\beta}_i^{\mathbb{C}}(p)$ for all $p \in [0, P]$ for either firm i. Further, the unique pure strategy equilibrium of the game with best responses $\widetilde{\boldsymbol{\beta}}^{\mathbb{C}}$ is $\left(P^1(\mathbf{x}), P^2(\mathbf{x})\right)$. ■

E.1.2 Mixed Pricing

In this section, we only prove what is necessary for the proof of our main proposition about the PTS game with inventory-holding. We only need to consider $x_2 \leq q^c$ and without loss of generality $x_1 \geq x_2$. We will use the following nomenclature throughout the proofs of this subsection: As denoted previously, the maximal *residual profit* is $\pi^R(x_j)$, we will refer to this as simply the residual profit. We denote the *front-side profit* of firm i as the largest profit such that the firm j is best off taking the residual demand.

Lemma 5. If $x_1 > r(x_2)$, then $\pi_1^* \geq \pi^R(x_2)$

Proof. By picking $p_1 = p_1^R(x_2)$ can always get at least the residual demand $d^1(p_1, x_2)$ and hence at least the profit $\pi^R(x_2)$. ■

Lemma 6. Firm 1's front-side profit is less than its residual profit.

Proof. Take $x_1 \geq x_2$ and $r(x_2) = x_1$ (note that it must be that $x_2 \leq q^c$). This is the Cournot pricing region characterized in Lemma E.1.1. In the Cournot pricing region the front-side profit equals the residual profit, which equals the Cournot profit. Now we will show an increase to x_1' such that $x_1' > r(x_2)$ decreases firm 1's front-side profit while not affecting firm its residual profit. First notice that the residual profit $\pi^R(x_2)$ is not affected by increasing x_1. Now we show the front-side profit is decreasing in x_1. The front-side profit for firm 1 is bounded above by $\rho_1^f x_1$ where $\rho_1^f = \max\{0, p_1^f\}$ and $D^2(\widetilde{p}_2, p_1^f) = x_2$, where $\widetilde{p}_2 = \pi^R(x_1)/x_2$. We do not need to consider the case that $p_1^f \leq 0$, since in that case the front-side profit is zero, which is clearly less than a positive residual profit. Let us show that $p_1^f x_1$ is decreasing in x_1. Using our demand specification and solving for p_1^f we have

$$p_1^f = -\frac{(a - x_2)}{c} + \frac{b}{c}\widetilde{p}_2.$$

We plug in with the functional forms and differentiate the front-side profit of firm 1 with regards to x_1 to find

$$-\frac{(a-x_2)}{c} + \frac{b}{cx_2}\pi^R(x_1) + \frac{b}{cx_2}\frac{\partial \pi^R(x_1)}{\partial x_1}x_1 < 0 \tag{30}$$

The first term is negative, since $x_2 \leq q^c < a$. Thus, if we can show that the sum of the second two terms is negative than the whole expression must be negative as well. Plugging in with the functional forms of $\pi^R(x_1)$ and $\partial\pi^R(x_1)/\partial x_1$, (30) reduces to the inequality

$$\begin{aligned}
\frac{b}{cx_2}\left(\frac{b+c}{4b(b-c)}\left(a - \frac{c}{b+c}x_1\right)^2\right) &< \frac{b}{cx_2}\left(\frac{b+c}{2b(b-c)}\left(a - \frac{c}{b+c}x_1\right)\frac{c}{b+c}x_1\right) \\
a - \frac{c}{b+c}x_1 &< \frac{2c}{b+c}x_1 \\
\frac{a(b+c)}{3c} &< x_1.
\end{aligned}$$

The final inequality must be true because to be in $x_1 < r(x_2)$, $x_1 > q^c$ and $q^c > a(b+c)/3c$. ■

Lemma 7. For $x_1 > r(x_2)$, $\pi_2^* > \pi^R(x_1)$

Proof. Suppose to the contrary that firm 2 does not have an equilibrium profit higher than $\pi^R(x_1)$. Suppose that firm 2 plays $\widetilde{p}_2$, then only if firm 1 plays $p_1 \leq p_1^f$ it gets the front-side profit. We know that $p_1^f x_1 < \pi^R(x_2)$ based on Lemma E.1.2. Take $\epsilon > 0$ and suppose firm 2 prices at $\widetilde{p}_2 + \epsilon$, then firm 1 can price at most $p_1^f + b\epsilon/c$ and get the front-side profit. For small enough $\epsilon > 0$, $p_1^f x_1 + b\epsilon x_1/c < \pi^R(x_2)$. This implies that there exists $p_2' > \widetilde{p}_2$ such that firm 1 will always prefer to maximize the residual demand over undercutting and getting the front-side profit. Consequently, firm 2 can guarantee itself a profit greater than maximizing the residual by picking such a p_2', since $p_2' x_2 > \widetilde{p}_2 x_2 = \pi^R(x_1)$. ■

Lemma 8. For $x_1 > r(x_2)$, $\pi_1^* \leq \pi^R(x_2)$.

Proof. Define $\overline{p}_i$ as the least upper bound of the equilibrium best responses of any firm i. The key aspect of this price $\overline{p}_i$ is: Does firm i get residual for sure at $\overline{p}_i$? Note that at least one firm i must get residual for sure at their $\overline{p}_i$. Denote by $\mathrm{Pr}_i(p)$ the equilibrium probability that firm i does not get the residual demand at the price p.

The following cases can overlap.

Case i. At $\overline{p}_1$ firm 1 gets residual for sure. Consider the case that firm 1 has an atom at $\overline{p}_1$ or firm 1 has best responses approaching $\overline{p}_1$. If firm 1 has an atom at $\overline{p}_1$, then $\pi_1^* = \pi^R(x_2) \geq \overline{p}_1 d^1(\overline{p}_1, x_2)$. If firm 1 has best responses approaching $\overline{p}_1$, then and take any sequence of best responses $p_1^k \uparrow \overline{p_1}$ and $\lim_{k\to\infty}\{\mathrm{Pr}_1(p_1^k)p_1^k x_1 + (1 - \mathrm{Pr}_1(p_1^k))p_1^k d^1(p_1^k, x_2)\} = \overline{p}_1 d^1(\overline{p}_1, x_2) \leq \pi^R(x_2)$.

Case ii. At $\overline{p}_2$ firm 2 gets residual for sure. Consider the case that firm 2 has an atom at $\overline{p}_1$ or firm 2 has best responses approaching $\overline{p}_2$. We show that this case cannot be. If firm 2 has an atom at $\overline{p}_2$, then $\pi_2^* = \pi^R(x_2) \geq \overline{p}_2 d^2(\overline{p}_2, x_2)$, which contradicts Lemma E.1.2. Take any sequence of best responses $p_2^k \uparrow \overline{p}_2$, then $\lim_{k\to\infty}\{\mathrm{Pr}_2(p_2^k)p_2^k x_2 + (1 - \mathrm{Pr}_2(p_2^k))p_2^k d^2(p_2^k, x_1)\} = \overline{p}_2 d^2(\overline{p}_2, x_1) \leq \pi^c(x_1)$. But, again, based on Lemma E.1.2 we know that $\pi_2^* > \pi^R(x_1)$, a contradiction. ■

We now turn to our primary results on inventory-holding behavior in the PTS game.

E.2 PTS equilibrium with inventory holding

The PTS equilibrium wage $\widehat{w}$ is such that

$$S^i(\widehat{\mathbf{w}}) \leq q^c \tag{31}$$

Alternatively, we can define w^c such that $S^i(\mathbf{w}^c) = q^c$ and rewrite Condition 31 as

$$P^i_j(\mathbf{q}^c)S^j_j(\mathbf{w}^c) \leq 1 + \varepsilon^S_o(\mathbf{w}^c) \tag{32}$$

We wish to show that if Condition E.2 holds, then the Nash equilibrium outcome in the PTS game with a binding inventory constraint is a Nash equilibrium outcome of the PTS game with inventory holding.

Proof. If $S^i(\widehat{\mathbf{w}}) \leq q^c$, then only defections such that $S^i(w'_i, \widehat{w}_j) > r(S^j(\widehat{w}_j, w'_i))$ are potentially more profitable in the PTS game with inventory holding than the PTS game. Any other defection results in exactly the same profit as the PTS game. We first show that the profit of firm i is strictly concave over all $w'_i \geq \widehat{w}_i$ such that $S^i(w'_i, \widehat{w}_j) > r(S^j(\widehat{w}_j, w'_i))$ by showing using the fact that the profit is twice continuously differentiable.

$$\frac{\partial^2\left(\pi^R(S^2) - w_1 S^1\right)}{\partial w_1^2} = \frac{-\gamma c^2}{2b(b^2 - c^2)} - \beta < 0. \tag{33}$$

Thus, the profit of firm 1 for all $S^i(w'_i, \widehat{w}_j) > r(S^j(\widehat{w}_j, w'_i))$ is concave.

At the boundary of $S^i(w'_i, \widehat{w}_j) = r(S^j(\widehat{w}_j, w'_i)$ is the only point where the profit of firm i is not differentiable. Consequently, for such a defection to be profitable the one at the boundary must be marginally better.

We know at $w'_i \geq \widehat{w}_i$ such that $S^i(w'_i, \widehat{w}_j) = r(S^j(\widehat{w}_j, w'_i)$ that (based on $\widehat{\boldsymbol{w}}$ as an equilibrium of the PTS game)

$$P^i_j(S^i, S^j)S^j_i S^i - S^i - w'_i S^i_i < 0 \tag{34}$$

at such a w'_i, it must be that $S^i = r(S^j)$. Thus, the inequality 34 implies that

$$P^i_j(r(S^j), S^j)r(S^j)S^j_i - S^i - w_i S^i_i < 0.$$

Based the strict concavity of the profit for all $w_i' \geq \widehat{w}_i$ such that $S^i(w_i', \widehat{w}_j) > r(S^j(\widehat{w}_j, w_i'))$

$$P_j^i(r(S^j), S^j)r(S^j)S_i^j - S^i - w_i S_i^i$$

is non decreasing in w_i'. Therefore, the expected profit is decreasing at all w_i' such that $S^i(w_i', \widehat{w}_j) > r(S^j(\widehat{w}_j, w_i'))$. This is a sufficient condition for no profitable defection w_i' from $\widehat{\mathbf{w}}$. ■

F Proof of Proposition 6

Noting that $\Theta = \epsilon_s - \epsilon_d$, the proof holds by inspection upon substitution by rearranging terms in equations (21) and (23). □

G Proof of Proposition 7

The proof is constructed by considering price movements from the Bertrand outcome under PTS and PTO. In the case of an interior solution, the Bertrand outcome for an agent facing a contract of the form in (24) is defined by the first-order condition

$$p_i - w_i - t_i = \frac{S^i(\mathbf{w})}{S_i^i(\mathbf{w})} - \frac{D^i(\mathbf{p})}{D_i^i(\mathbf{p})}, \qquad i \in 1, 2.$$

In the PTS game, evaluating the agent's input price condition (25) at the symmetric Bertrand equilibrium position (w^B, p^B) gives

$$\Pi_i^{i,s}(w^B, p^B) \equiv \frac{S^i D_j^i S_i^i}{\Delta}\Theta \stackrel{s}{=} \Theta. \tag{35}$$

In the PTO game, evaluating the output price condition (26) at the symmetric Bertrand equilibrium position (w^B, p^B) gives

$$\Pi_i^{i,o}(w^B, p^B) \equiv \frac{D^i D_i^i S_j^i}{\Sigma}\Theta \stackrel{s}{=} \Theta. \tag{36}$$

By Proposition 1, oligopoly profits rise with defections from the Bertrand equilibrium (p', w') that involve $p' > p^B$ and $w' < w^B$. Letting the adjusted margin per unit of output of firm i be denoted $\widehat{\Gamma}_i = \frac{p_i - w_i - t_i}{S^i}$, the equilibrium timing of the game is determined by the adjusted margins according to Proposition 4. □