INSTITUT D'ECONOMIE INDUSTRIELLE

IDEI Report # 2

Rail Transport

May 2003

Entry in the Passenger Rail Industry
A Theoretical Investigation

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Acknowledgements

The authors acknowledge with gratitude the advice and research support of Deutsche Bahn, but note that the views expressed remain those of the authors alone and in no way represent the official or unofficial views of Deutsche Bahn or its representatives.

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1 Main results

The following results emerge from our analysis.

- Price competition with homogenous services triggers a vigorous price war and erodes the profits of operators. Therefore, taking entry as given, there exists a strong incentive for both the incumbent and the entrant to differentiate their product in order to recover some profits. Differentiation is used strategically by competitors in order to avoid a fierce price competition.

- There exists different notions of differentiation and therefore different ways for the operators to differentiate their products. Each notion is related to particular assumptions on the preferences of the passengers and crucially depends on the heterogeneity in the passenger population.

- Heterogeneity in the population of passengers is different from market size. Heterogeneity refers to the variance in the willingness-to-pay for train services across different types of passengers.[1] For instance, business passengers have a larger valuation for travel time than leisure passengers. The parameter of differentiation can be used to target specific segments of the population of (potential) passengers.

- Under monopoly, the incumbent has an incentive to restrict demand in order to impose a higher price: Excluding the passengers with lower valuations for train services from the market enables the incumbent to target its offer to the most profitable passengers (i.e., those with the highest willingness-to-pays). Since passengers' willingness-to-pay decreases with travel time, the incumbent offers a low travel time in order to raise the price of train services. If the cost of lowering the travel time and/or if passengers are sufficiently sensitive to travel time reduction, then the monopolist incumbent implements the lowest travel time consistent with the network constraints.

- Depending on the efficiency gap between the entrant and the incumbent and on the differentiation opportunities, different entry scenarios are possible.
  - If the incumbent anticipates a large differentiation between its product and the entrant's offer, then the best strategy for the incumbent depends on the relative cost efficiency. If the incumbent is not too inefficient compared to entrant, then it should continue to offer a service targeted toward the most profitable segments (i.e., passengers with high willingness-to-pays). On the contrary, if the incumbent is much less efficient than the entrant, then it should target its offer towards a niche market (less profitable segments, lower willingness-to-pays passengers) and let the most profitable segments to the entrant. This strategy enables the incumbent to save on its cost.
  - By contrast, if the incumbent anticipates that the entrant will not differentiate much its product, then it should continue to target the most

[1] For instance, we can have a small market size with a large heterogeneity in the population.

profitable segment of higher willingness-to-pay passengers, even if it is less efficient than the entrant.

  - If the entrant is much more efficient than the incumbent, then the incumbent should exit the market.

- Entry shortens the incumbent's market power and increases the variety of services; this enhances consumers' surplus. However, if entry cost are non-negligible, then social welfare might not always be improved with entry. Moreover, if the incumbent benefits from large returns to scale and/or returns to density, then competition may deteriorate welfare since the erosion of the incumbent's market share translates into a lower efficiency for this operator. We deliberately abstract from this effect and refer to the first IDEI report on Passenger Rail Transport for a thorough analysis of these effects.

- Assume that heterogeneity in the population is small, i.e., that the differences in willingness-to-pays for train services across passengers are small. 1) There is only one operator present on the market. Two configurations are possible: Either the entrant replaces the incumbent, or the entrant does not enter the market. 2) Which configuration prevails depends on the ability to credibly commit to target the most profitable segments of passenger population (i.e., to offer a sufficiently high frequency level) in order to threaten the competitor if it attempts to enter/stay active.

- Assume that the demand mainly emanates from potential passengers with low willingness-to-pays for train transport services and that this demand has a high price-elasticity (i.e., a small increase of price generates a large fall in demand). 1) The incumbent with a monopoly position serves the entire market. 2) It is not possible to have both competitors simultaneously active on the market because both competitors seek to offer products which are not sufficiently differentiated (to be close to the demand) and enter in a tough price competition.

- The withdrawal decision by the incumbent should take into account not only the direct gains (such as the saving of some part of fixed costs) or direct costs (such as the loss of the local market where entry occurs) but also the opportunity cost and gain associated to such a decision. There is an opportunity loss of staying on the local market and competing head-to-head with the entrant since this might force the entrant's price to be very low, thereby limiting the pricing decisions of the incumbent on other markets when products are imperfectly differentiated.

- In a network with a hub-spoke structure the incumbent has a strategic advantage over the potential entrants and may not withdraw from the local market where entry occurs. Entry is therefore likely to be deterred when potential entrants do not enjoy a substantial cost advantage over the incumbent and when the irretrievable entry costs are sufficiently high.

Indeed, entry on a local market has a positive spillover effect on the connecting markets served by the incumbent when the network exhibits a pattern of complementarities between the incumbent's and the entrant's services, which arises when the network is a hub-spoke. By staying on the market where entry occurred, the incumbent engages in a vigorous price competition (which also erodes the profit of operators on this market). Since connecting services are complements due to the hub-spoke feature of the network, the price competition increases the demand on the complementary markets. Therefore, following entry, two cases happen: (i) If the incumbent exits, then the entrant enjoys a monopoly position on its market and sets a high price on this market and on the connecting markets jointly serviced with the incumbent; (ii) If the incumbent stays on the market where entry has occurred and shares the connecting markets with the entrant, then it will engage in a price war on the market where entry occurs. The incumbent prefers the latter strategy to the former since the smaller price on the market where entry occurs increases the demand in the connecting and complementary markets in which the incumbent still enjoys a monopoly position.

Figure 1 summarizes the main results and offers a road map for the analysis undertaken in this report.

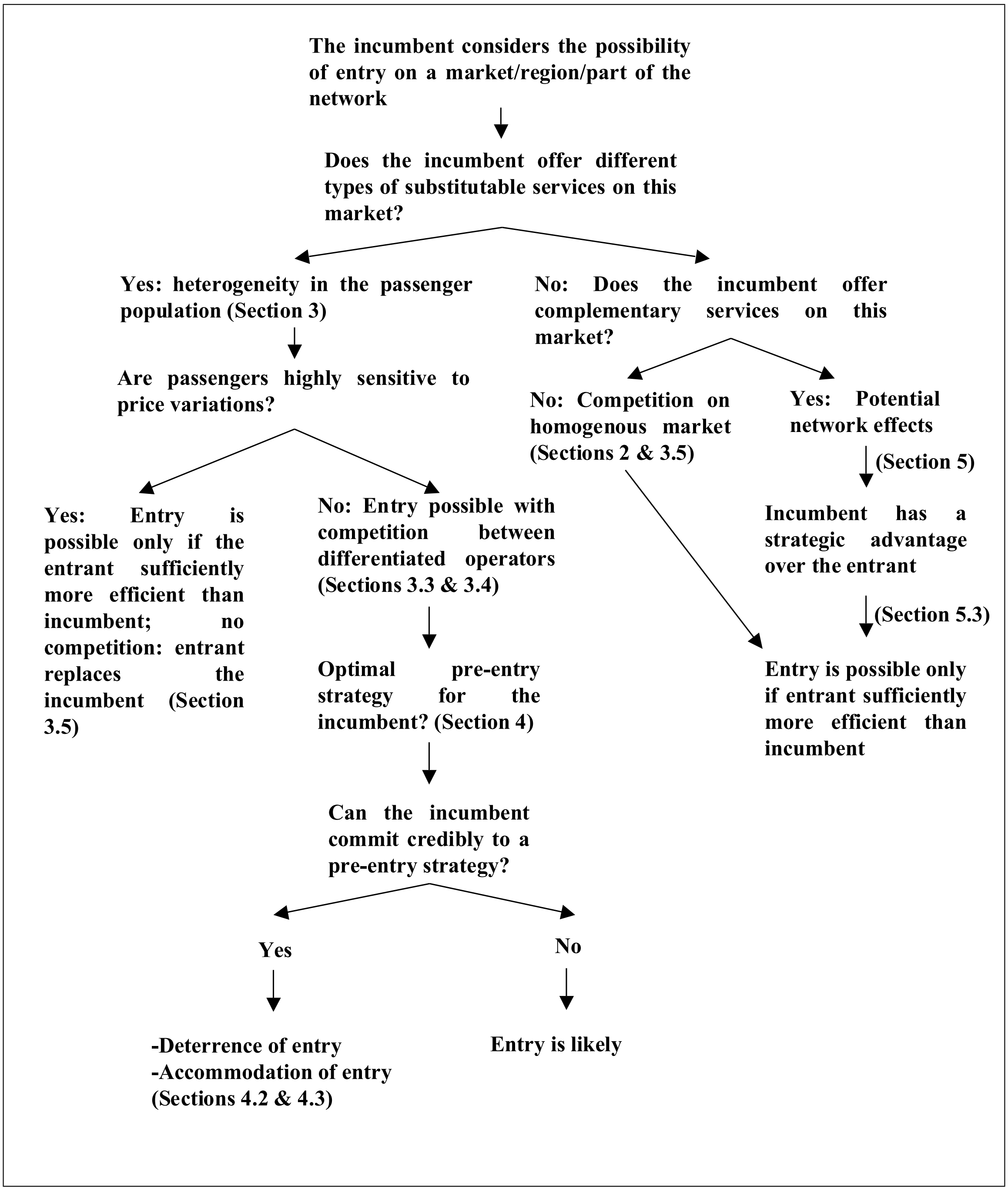

Figure 1: Road map of the analysis and main results.

2 Introduction

This report focusses on two aspects that play a crucial role on the impact and nature of entry in the railway industry. First, it sheds light on the entry process by taking into account the opportunity of services differentiation between operators. Second it deals with the role of externalities provided by the particular structure of railways network on the possibility of entry.

The first part (Section 3) studies the conditions under which competitors effectively have an incentive to differentiate their services. We argue that differentiation by a competitor has both a *strategic impact* on the rival's price setting behavior and a *direct effect* on the demand that this competitor faces. Strategic differentiation aims at softening the price war between competitors and depends on some conditions on the pattern of demands that prevails on the market in which competition takes place. The analysis of these issues requires a thorough knowledge of variables such as travel demand, heterogeneity among the different types of passengers, cost functions. Indeed, any meaningful analysis of entry and differentiation between competitors in the passenger transport services is dependent on the pattern of demands that prevails on the network.

Section 4 discusses the case of an incumbent in an asymmetric position vis-à-vis a potential competitor when competition is still at an infant stage as in the railway industry today. In order to prevent entry, the incumbent must commit in a credible way to an intense price competition in the event of entry. The strategic interaction between the incumbent's and the entrant's decision variables is studied, as well as the link between the incumbent's possible strategies, i.e., adaptation to or prevention of entry.

Section 5 focusses on the effects created by the structure of a railway network. We develop a simple model that accounts for such network externalities and study the impact of entry in that framework. Once again, we show that the analysis of entry depends on the pattern of demands. We also consider the role of credible commitments by the incumbent. For instance, in absence of high exit costs, this credibility depends on the interaction between the different services offered by the incumbent. If a multiproduct incumbent offers substitutable services to passengers, we show that it has more incentive to withdraw from the market when entry occurred than the entrant; by contrast, when the incumbent and the entrant offer complementary services, which typically occurs when the incumbent operates a network with a hub-spoke structure, then we argue that the incumbent has an incentive to stay on the market where entry occurred. Therefore, entry is more likely to be successful in the former case than in the latter one.

As there is no single and simple answers to the complex question of entry in the railway industry, the theoretical discussion must be completed with an empirical approach. Indeed, it is required to assess the sets of parameters that trigger the different answers.

This work sheds light on some issues of entry in the railway industry. It differs drastically from Preston, Whelan and Wardman (1998). They analyze the impact of entry in a very simple network consisting of eight different towns in line. Their methodology can be exposed as follows. First, they estimate a demand system with a nested logit model; second, they estimate a cost model; third, they offer different scenarios related to entry. In this approach, the strategies of the different operators are not endogenous. They are postulated by the analysts although it should be noted that they look at a large number of different scenarios. Then what might be optimal for a certain demand system might no longer be optimal when the focus is on another part of the network with probably different demands and products. Therefore, a more theoretical analysis is also warranted as it should enable to understand the different effects at work.

We finally emphasize that we do not take explicitly into account the universal service obligations that must be satisfied by the incumbent operator. An ad hoc way to account for these constraints is to consider that they translate into higher costs for the incumbent with respect to the entrant.

Now, we briefly overview the report.

2.1 Competition and differentiation

We first highlight the role of product differentiation in the competition process. In a broad sense, differentiation arises when passengers have different valuations/willingness-to-pays for the service offered by an operator. However, there exists different ways of differentiation, and therefore different ways for the operators to differentiate their products. Each way involves particular assumptions on the preferences of passengers. For the sake of illustration, we will consider passengers which have different valuations or willingness-to-pays for transport services stemming from differences in their value of time. However, the analysis can be applied to any possible ways of differentiation like comfort, in-board services, flexibility in changing tickets or electronic ticketing. Summing up, differentiation means that the entrant and the incumbent target different segments of the passenger population with their different products. The important point to notice is that the decision related to the differentiation of services affects the pricing decisions.

Were the products of the different competitors not differentiated, price competition would trigger a vigorous price war and would erode the profits of the operators. Therefore, there exists a potentially strong incentive for both incumbents and entrants to differentiate their products in order to recover profits.[2] Differentiation is used strategically by the competitors to avoid an intense price competition: This is the *strategic effect* of differentiation in a competitive environment. However, a competitor must also consider the impact on its demand when it decides the differentiation level of its product: This is the *direct effect*.

The possibility and the incentive for competitors to differentiate their services depend on the heterogeneity in the passenger population. Heterogeneity traduces the degree of differences in the willingness-to-pay for train services across different types of passengers. For instance, business passengers have larger valuations for travel time (or for comfort, flexibility or frequency) than leisure passengers. The

[2]Bonanno (1986) et Gal-Or (1983) study vertical differentiation under Cournot (i.e. quantity) competition (which is typically 'less intense' than price competition), and show that the competitors' incentives to differentiate their services are still strong.

scope of this difference measures the heterogeneity in the passenger population.[3] Note that heterogeneity differs from the notion of market size.

First, we study the situation in which the incumbent has a monopoly situation on a given market. The monopoly has an incentive to ration demand in order to be able to impose a higher price on the residual demand. Excluding from the market the passengers with lower valuations for train services enables the incumbent to target its offer on the more profitable passengers (i.e., those with higher willingness-to-pays). Since passengers' willingness-to-pay decreases with travel time, the incumbent offers a low travel time/high speed in order to raise the price of train transportation services. The level of differentiation chosen by the monopolist incumbent simply obeys a 'marginal cost equals marginal benefit' rule.

Second, we consider the competition between the incumbent and the entrant. Note that entry is taken for granted. The entrant and the incumbent are treated symmetrically.

To begin with, note that it is typically the case that introducing competition enhances the demand for transport services: passengers with lower willingness-to-pay for transport services and who were not travelling by train under monopoly are no longer rationed and decide to travel by train.

The choice of differentiation depends now on the strategic interaction, i.e., on the way the incumbent's differentiation parameter (travel time) affects the entrant's pricing decision, and vice-versa since both competitors are treated symmetrically. The strategic interaction depends on characteristics of the demand function, in particular, the distribution of heterogeneity in the population. In our model, prices will be strategic complements, implying that each competitor has an incentive to differentiate its service in order to trigger a price increase from its rival, i.e., in order to soften the price competition.

Competition leads the industry to different equilibrium configurations: Either the incumbent still targets the segment of passengers with higher willingness-to-pay and the entrant focuses on the segment of passengers with lower valuations, or the incumbent switches to the low valuation segments and the entrant 'skims the cream'. Which configuration prevails typically depends on the discrepancy between the incumbent and the entrant in terms of efficiency.

If the entrant is more efficient than the incumbent, then the former can credibly threaten the latter to undertake a price war, which would be unprofitable for the incumbent if it remains on the high valuation segments of the passenger population. In that scenario, the entrant picks the cherries and the incumbent finds it more profitable to service the low valuation segments than to enter into a fierce price war.[4] When the efficiency gap is sufficiently large, the incumbent might exit the market. This is not because the demand on the market under consideration is too small; rather the efficient entrant attracts almost all the different segments of the passenger population because its price is relatively low, even if differentiation between operators is important.

[3] This statement will be made clearer later on.

[4] More precisely, if the incumbent stays on the high valuation segments, then the efficient entrant would put a strong competitive pressure on the incumbent; therefore, the latter prefers to target the low valuation segments.

When the discrepancy in efficiency is not too large, two scenarios become simultaneously possible: One operator services the most profitable segments of the population while the other operator targets the less profitable ones. Indeed, since (i) competitors have an incentive to differentiate their products in order to soften the price competition and (ii) they are both attracted by the more profitable segments of the passenger population, multiple industry configurations become possible. This suggests a 'race-to-be-the-first' between competitors in order to target the most profitable segments of the population. This also suggests that the industry configuration is likely to be changing repeatedly over time.

We show that if the heterogeneity in the population is small, i.e., the differences in willingness-to-pay for train services across passengers are small, then

- There will be only one operator present on the market. Two configurations are then possible: Either the entrant replaces the incumbent, or the entrant does not enter the market.

- Which configuration prevails depends on the capacity to credibly commit to target the most profitable segments of the passenger population (i.e., to offer a sufficiently high frequency level) in order to threaten the competitor. If, for instance, the incumbent has committed credibly to target the most profitable segments of the population, then the incentive to differentiate its services leads the entrant to target the less profitable segments.

Finally, we emphasize that heterogeneity is different from market size. If the size of the market is sufficiently small so that on-the-track competition is not viable per se, obviously the entrant is not willing to enter the market. Weak or small heterogeneity means that there is not enough variance or scope in the passenger willingness-to-pay for the competitors to differentiate sufficiently and to escape from an intense price competition.

When demand emanates mainly from potential passengers with a low willingness-to-pay for train transport services and has a high price-elasticity (i.e., a small increase of price generates a large fall in demand), the incumbent with a monopoly position serves the entire market. In this case, there is not enough space in terms of product differentiation to accommodate two competitors. Either the incumbent exits the market or the entrant replaces the incumbent.

Considering that entry is costly and that it takes time for the entrant to build a reputation, entry is more likely to occur only in markets characterized by (i) a sufficiently large degree of heterogeneity in the passenger population allowing both the entrant and the incumbent to offer differentiated services and (ii) a sufficiently low price elasticity of the demand in order that each competitor benefits from a non negligible market power.

2.2 Entry barriers and the incumbent's behavior faced to the threat of entry

So far, the focus was on the competition between an incumbent operator and a competitor, which successfully enters the market. This analysis aimed at understanding the different aspects of competition through price and non-price variables, and to identify the conditions under which competition is not viable. One assumption was to treat the different operators symmetrically: The incumbent affects as much the competitor's decisions as does the competitor's decisions impact on the incumbent's different choices.

However, since competition in the railway industry has not yet developed or remains at an early stage, it becomes natural to treat the incumbent and the potential entrant asymmetrically: The incumbent can undertake actions today that will affect competition tomorrow.

With the previous framework in mind, assume that now the potential competitor has not yet entered the market and must decide whether or not to compete with the incumbent on that market. What is the optimal strategy for the incumbent operator facing the threat of entry by a potential competitor? Differently stated, what kind of actions can the incumbent undertake today in order to accommodate or to prevent entry tomorrow?

In order to deter entry, the incumbent needs to credibly commit to enter in an intense price competition if entry occurs. For instance, crowding out the spectrum of services, i.e., producing more than one service when passengers are sufficiently heterogenous, reduces the opportunity for the entrant to earn positive profits if it enters the market. Indeed, in that event, the potential competitor anticipates that it will face more substitutable services to compete with. If the incumbent offers a sufficiently large number of different services on the same market, then entry will be deterred. However, such a strategy might be too costly to undertake, in which case the incumbent prefers to adapt to entry. Its optimal strategy is then aimed at softening the price competition that will take place on the market once entry has occurred. The effectiveness of this strategy depends on the strategic interaction.

Summing up, by producing different substitutable services, the incumbent commits to behave aggressively in the event of entry. However, it is crucial that this commitment be credible as we explain in the last section.

2.3 Entry, fixed cost network externalities

We relax now the assumption of credible commitments. More precisely, we assume that, after entry has taken place, the incumbent can decide to interrupt one of its services if it finds it profitable to do so. The novelty here is to account for the network structure of railways and the externalities created by the network architecture. This allows us to consider another asymmetry between the incumbent operator and the potential competitors: The incumbent operates the whole network whereas potential entrants are likely to enter only on some segments of this network.

To illustrate, consider first a case in which (i) there is one market with heterogenous passengers and (ii) the incumbent decides, prior to the entry decision by a potential competitor, to produce two services, one targeted toward the low-profitability segments, the other targeted toward the high-profitability segments. The incumbent is therefore a multi-product operator which offers imperfectly differentiated and substitutable services. If entry occurs on the (sub-)market of one of the services offered by the incumbent, then the entrant and the incumbent engage in an intense price competition which results in zero profits for both operators (on the segment targeted both by the entrant and the incumbent).

Should the potential competitor decide to enter the market? The answer is positive, since it anticipates that the incumbent will withdraw from the segment where entry occurred, transforming the industry into a differentiated duopoly. The argument goes as follows. Assuming that exit costs are not too large, the incumbent's commitment to stay on both segments in order to deter entry is not credible. Stated differently, the multi-product incumbent has more incentive to withdraw from the segment where entry occurred than the potential competitor. Indeed, if the incumbent stays on both sub-markets, then the entrant is indifferent between staying and exiting since profits are driven down to zero by the price competition on that sub-market. If the incumbent stays on the sub-market, it has no direct benefit since it looses the profits on the sub-market where entry occurred. In addition, it entails an opportunity cost due to the price war with the entrant that depreciates the demand prevailing on the sub-market serviced by the incumbent since products are demand substitutes. Therefore, the incumbent has more incentive to exit than the entrant since this would increase the price on the segment where entry occurred, thereby increasing the demand that the incumbent faces in the other substitutable sub-market. Anticipating this, the potential entrant decides to enter, thereby triggering the exit of the incumbent on that segment. More generally, if the incumbent cannot commit credibly not to exit, crowding out the product spectrum might not always succeed in deterring entry.

Therefore, when considering the entry decision by a potential competitor, the incumbent must account not only for the direct costs associated to entry (i.e., the loss of profit in the market where entry has occurred) but also the opportunity costs/benefits associated to entry (i.e., the fall in demand for the substitutable services). These costs/benefits determine the entrant's incentive to enter or not on a given market.

Consider now the following setting. An incumbent operates a three-node network, the nodes being denoted by A, B and C. There is a demand for transport services from A to C, A to B and B to C. The incumbent offers a service between each city pair. In order to be viable, it must be the case that passengers from A to C prefer a direct (non stop) trip than an indirect travel from A to B and then from B to C. The incumbent must therefore satisfies an arbitrage condition: The price of the AC trip must not be higher than the sum of the price to travel from A to B and then from B to C.[5] We study the possibility of entry in the local AB-market by a competitor.

In this context, entry involves again a direct cost and an opportunity cost supported by the incumbent. The direct cost is the loss of profit on the AB-market: Price competition erodes the profit of both operators. The opportunity cost stems from the fact that the decrease in the AB-price tends to harden the arbitrage constraint for the incumbent. This constraint creates an interaction between the pricing

[5] One could add an additional cost of waiting time between trains for the AC-passengers without affecting our results. The mere fact that the incumbent operates a whole network puts constraints on its pricing decisions which create links between the different markets of services operated by the incumbent.

decisions of the incumbent and generates an additional opportunity cost associated to entry. In that case, the incumbent has an incentive to withdraw from the market where entry occurred in order to soften the distortions on the related markets.

Is this opportunity cost always negative? The answer is negative, as we now illustrate. In order to capture the idea of a hub-spoke network, we consider a variation of the three-nodes network studied previously. To go from one point to another, the incumbent operator goes through a hub city. In this framework, we consider the impact of entry on the local market AB by a competitor and we identify some conditions under which such an entry is unlikely.

One obvious explanation for the barrier to entry is that interlining is costly. Passengers may incur time costs for switching between train in the hub-station; companies may experience coordination costs in arranging schedules and joint fares for connecting passengers. The explanation we offer enhances other effects of entry. We show that, even if interlining is costless, network externalities make it difficult for an entrant to survive.

To understand the effect of entry on the profitability of the operators, two elements are important

- Transport services on the same link/market are (perfectly) substitutable services. The entry of an operator on one link clearly decreases the profitability of this market for the incumbent.

- Transport services on different (but connected) links are complementary goods. Therefore, an increased competition on one link increases the demand on the connected markets, even if these services are not offered by other companies than the incumbent.

If a competitor enters on one link and the incumbent does not concede the market, competition between the two operators will lower the price on that market. The incumbent suffers losses in this market that it can partially offset by adjusting its prices on the complementary services (i.e., on the connecting markets). When the size of the network is large enough and more precisely, when the induced traffic is sufficiently high, the incumbent's optimal response to entry on one link is not to withdraw its service on that link. As a result, if we assume that competition on only one link cannot be profitable and that the competitor is as efficient as the incumbent, then the competitor is forced to exit and entry is deterred. If the entrant is sufficiently more efficient than the incumbent, then entry on a local market may still occur. However, the market share of the entrant is likely to remain small since the incumbent will be willing to engage in an intense price competition to benefit from the positive spillovers over the rest of the network. The logic of this argument is similar to the one developed in the case of an incumbent producing multiple substitutable services. The difference is that with complementary services, the opportunity cost of entry is now an opportunity gain for the incumbent since competition in a local market spreads out in the connecting market. Moreover, we argue that different industry configurations following entry are possible: One in which the incumbent shares the connecting markets with the entrant, the other in which the incumbent does not share the market. Another implication is that entry is more likely to occur 'on-the-rim', that is, on a new line so that it does not directly compete with the incumbent.

3 Competition and differentiation

Before presenting a theoretical analysis of oligopolistic competition under differentiation, we discuss first the meaning and the role of product differentiation.

3.1 Why is differentiation relevant?

Consider the simplest setting in which an incumbent and an entrant compete in price on a given point-to-point transport service. Assume that passengers have no other possibilities than to demand the point-to-point service to this incumbent or this entrant (i.e., there are no substitutes for train transportation).

To understand the role of differentiation, consider first that competitors offer a perfectly homogenous good: All attributes of the services offered by the incumbent and the entrant are identical (travel time, frequency, comfort/on-board services). From the passengers' point of view the product of the entrant is perfectly identical to the product offered by the incumbent.

Let p_i and p_e be respectively the price offered by the incumbent and the entrant for this travel. Let $D(p_i, p_e)$ be the total demand associated to these prices. To further simplify the analysis, we assume that both competitors have no fixed costs and identical constant marginal costs of production.

What is the outcome? In such a situation, price competition between competitors offering a perfectly homogenous good/service leads to zero profit at equilibrium. Indeed, consider that the incumbent proposes a price p_i greater than the price p_e offered by the entrant; in this case the entrant has the entire demand[6]: $D_i(p_i, p_e) = 0$ and $D_e(p_i, p_e) = D(p_i, p_e)$. The incumbent earns zero profit while the entrant has a monopoly position over the entire demand. Then, the incumbent would be better off lowering its price slightly below the entrant's price. As the entrant makes simultaneously a similar reasoning and would then have an incentive to undercut the incumbent's price, the process leads through progressive adjustments to a price war which erodes the profits of both competitors. At equilibrium, prices are such that both competitors price at marginal cost and share equally the demand. Thus, neither the incumbent nor the entrant make strictly positive profits.[7]

The reason is that, with an homogenous product, each competitor has a strong incentive to undercut the price of its rival, since it is anticipated that such a price reduction will enable to attract all the demand for transport services.

The analysis remains qualitatively unchanged when operators have positive and identical costs, whatever the cost structures. At equilibrium, prices are then equal to average costs and operators make no profits. If costs are different, then the lowest cost operator offers a price equal to the average cost of the highest cost operator and

[6] A subscript k indicates that we refer to operator k.

[7] In this simple framework, such a situation, in which operators price at their marginal cost, would also be socially optimal. This is no longer the case when marginal costs are constant but different or when Bertrand competition leads to average cost pricing at equilibrium (see below).

earns some profits; the highest cost operator earns no profit since it cannot offer a price smaller than its rival as its marginal cost is higher.

The picture would be different if the entrant's product is differentiated from the incumbent's product. In this case, some passengers would intrinsically value more the entrant's product while other passengers would have a larger valuation for the incumbent's product[8]. As opposed to the perfectly homogenous products case, if the entrant decreases its price slightly below the incumbent's price for instance, then this would not lead anymore to a drastic shift of demand: Only a fraction (which, of course, depends on the price gap) of total demand would be captured by the entrant. To a certain extent, differentiation will enable the operators to escape from an intense price competition and to earn positive profits at equilibrium.[9]

> *Summary 1: Price competition with homogenous services triggers a vigorous price war and erodes the profits of operators. There exists a strong incentive for both the incumbent and the entrant to differentiate their product in order to recover some profits. Differentiation is used strategically by competitors in order to avoid a fierce price competition.*

While the previous reasoning is immediate, many issues remain to be understood. In particular, the following questions are critical: What factors do influence the way operators differentiate their products/services? What is the outcome of the competitive game with endogenous product differentiation?

3.2 Ways of differentiation

First, we recall the economic definitions of differentiation. There are two types of differentiation that we illustrate now with the following situation: An incumbent operator I acts as a monopoly and produces a transport service which is then sold to a population of passengers.

3.2.1 Vertical differentiation

At an abstract level, vertical differentiation arises when passengers agree on the ranking of preferences.[10] For instance, all passengers prefer fast than slow trains. However, passengers may have different valuations[11] for the product offered by the

[8] This statement is vague. Even though some passengers may value more one service than the other, this depends also on the difference between the prices of the incumbent's product and the entrant's service as we explain later on.

[9] Price competition is the most extreme form of competition. For instance, quantity (i.e., Cournot) or capacity competition is usually less intense.

[10] See Gabszewicz and Thisse (1979, 1980), Shaked and Sutton (1982).

[11] The terms 'valuation' and 'willingness-to-pay' define the maximal price that a given passenger is willing to pay for the service offered by an operator; this price depends on the characteristics of that service.

incumbent or different willingness-to-pay for the incumbent's product. These differences in valuations may come from differences in the value of time: Business passengers usually have higher valuations for the time spent on travelling than leisure passengers. These differences may also come from differences in valuations of alternative options: Some passengers may be more willing to substitute train transportation to another transport mode than other passengers.

Another instance of vertical differentiation concerns flexibility: Some passengers are more willing to benefit from flexibility (i.e., to be allowed to change their ticket for another train), while other passengers may prefer a lower price associated restricted possibilities to change their ticket once bought. Finally comfort or on-board services might also be valued differently by different segments of passengers.

The simplest way to model such a situation goes as follows. Consider passengers who decide to travel from point A to point B either by train or by car (or any alternative transport mode). If they travel by car, they earn an exogenous utility level that we normalize to 0. If they travel by train, then the utility level they get depends on the attributes of the service offered by the train operator. Assume for the sake of simplicity that the only relevant attribute is the time length T of the trip from A to B. The utility of passengers is given by

$$U = \begin{cases} \theta\nu(T) - p & \text{if they travel by train,} \\ 0 & \text{otherwise.} \end{cases}$$

Parameter θ describes the heterogeneity in the population of passengers: Some of them have a higher utility derived from the train service with travel time T than others. The function $\nu(T)$ expresses how passengers derive utility from a train service with attribute/travel-time T.[12]

In that framework, the operator chooses both the travel time T and the associated price p. Given the constraints inherited from the physical network, it is reasonable to assume that only a restricted set of travel times can be implemented by the train operator, i.e., $T \in [\underline{T}, \overline{T}]$. In other words, for the trips considered, it is not possible to offer a travel time smaller than $\underline{T}$ and larger than $\overline{T}$ because, for instance, it would affect the other services offered on the whole network. Note that these constraints on the set of travel time values that can be implemented limit the set of services that can be offered by the incumbent.

When we consider oligopolistic competition later on, it is reasonable to assume that both operators face those constraints. By contrast, the choice of a travel time may have distinct impacts on the competitors depending on their 'production technology' (i.e., their cost function).

[12]For instance, the smaller the travel time is, the higher is the utility derived by passengers. However, for a given θ, the marginal utility with respect to time might be decreasing, because passengers might value more a reduction of the travel time when the travel time is initially high than when it is low.

3.2.2 Horizontal differentiation

Horizontal differentiation arises when the choice made by passengers for identical prices of the services offered by the operator depends on the passengers.[13] Suppose that the incumbent faces an heterogenous population of passengers. Now, each passenger would like to buy an 'ideal product'.[14] As an example, consider the case of two distinct populations of passengers: business and leisure passengers. Assume that business passengers prefer travelling early in the morning and going back in the evening, whereas leisure passengers are more indifferent on the departure and return time.

Passengers are distributed on the segment $[0, 1]$. A passenger located in $x \in [0, 1]$ has the following utility

$$U = \begin{cases} s - \alpha(x-a)^2 - p & \text{if it travels by train} \\ 0 & \text{otherwise.} \end{cases}$$

Parameter s is the gross utility of of a trip from A to B by train. Parameter a describes the location of service (i.e., its attributes) offered by the train operator in the product space. To fix ideas, it might be a particular time slot for the AB-trip. Then, the value $\alpha(x-a)^2$ gives the disutility for the passenger located in x to use the train operator's particular service that differs from the 'ideal' service that this passenger would like to buy. For instance, some passengers would like to arrive at point B at 11:00 AM whereas others would prefer to arrive at 12:00 AM. If the incumbent offers a service with an arrival time at 11:30 AM it cannot perfectly coincide with the 'ideal product' of these two types of passengers.

As in the vertical differentiation setting, the constraints inherited from the physical network reduce the spectrum of products that can be offered by the different operators. This is captured through the assumption that each operator offers only one product, i.e., each operator is located on a particular point a of the $[0, 1]$ segment.

3.2.3 Differences and similarities

Under oligopolistic competition, these two ways of product differentiation generally lead to similar qualitative results. There are however some exceptions that we describe later on. Note that these two ways involve different assumptions on the preferences of passengers and therefore on the demand functions.

Summary 2: There exists different notions of differentiation and therefore different ways for the operators to differentiate their products. Each notion is related to particular assumptions on the preferences of the passengers.

[13]See Hoteling (1929), D'Aspremont, Gabszewicz and Thisse (1979).

[14]In the vertical differentiation case, if two services are offered at the same price, then all passengers choose the service with the highest quality/lowest travel time.

3.3 A model of entry and vertical differentiation

Assume that passengers' gain or utility when they travel from A to B by train with travel time T is given by $U = \theta\nu(T) - p$ and 0 otherwise. The frequency T belongs to the interval $[\underline{T}, \overline{T}]$.

Parameter θ is the valuation for travel time and takes values in the range $[\underline{\theta}, \overline{\theta}]$. This reflects the heterogeneity among passengers: Lower valuations for time are associated to lower willingness-to-pay than for passengers with higher valuations. The smaller the travel time is, the larger will be the willingness-to-pay for the service. Roughly speaking, the parameter of differentiation is a variable that enables the operator(s) to target specific classes or segments of the population of passengers.

We assume that θ is distributed according to a density $g(.)$ and a cumulative distribution $G(.)$ on the interval $[\underline{\theta}, \overline{\theta}]$. The distribution of the heterogeneity provides the weights of different categories of passengers in the population. For instance, a uniform distribution means that there are the same proportions of the different types of passengers. If the distribution is decreasing over $[\underline{\theta}, \overline{\theta}]$, there are more passengers with relatively low valuations for train services than passengers with relatively high willingness-to-pay. We also assume that the total demand (provided that all passengers decide to travel by train) is normalized to 1 (without loss of generality). This is represented in Figure 2.

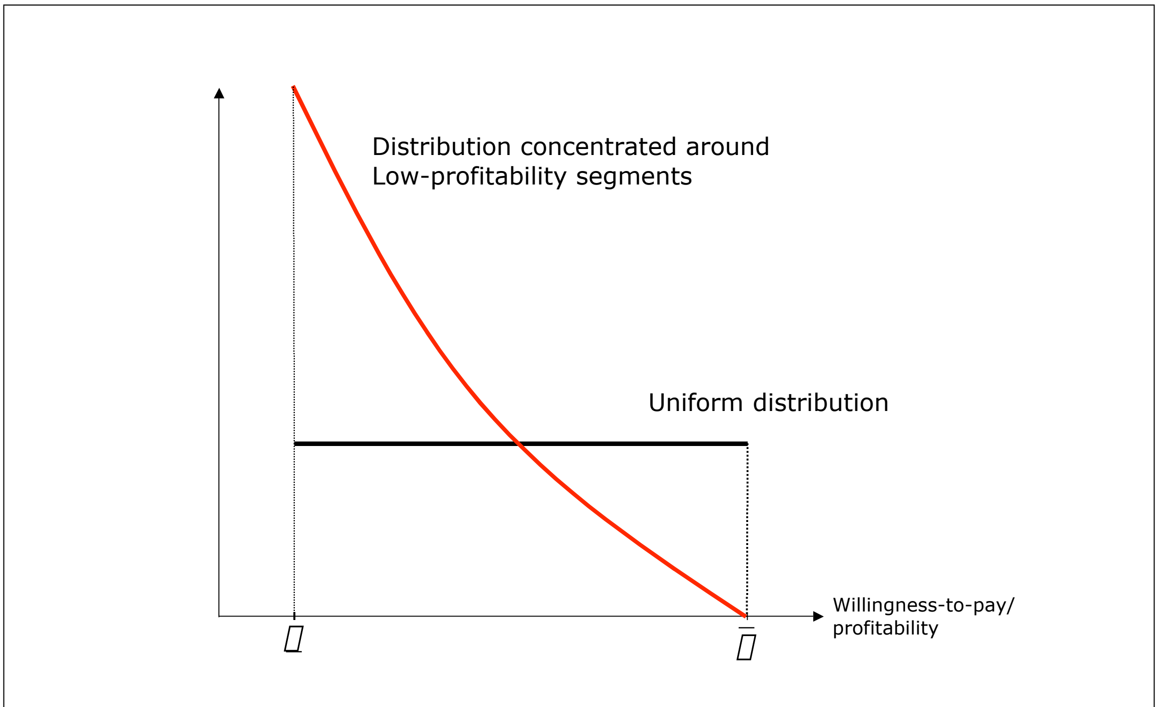

Figure 2: Different distributions of the heterogeneity in the passenger population.

Note that market size (here equal to 1) is different from heterogeneity, here measured by $\Delta\theta = \overline{\theta} - \underline{\theta}$ and the distribution $g(.)$. The market may be large or small independently of the heterogeneity being low ($\Delta\theta \approx 0$) or high ($\Delta\theta \gg 0$).

Summary 3: Heterogeneity in the population of passengers is different from market size. Heterogeneity refers to the variance in the willingness-to-pay for train services across different types of passengers. For instance, business passengers have a larger valuation for travel time than leisure passengers. The parameter of differentiation can be used to target specific segments of the population of (potential) passengers.

3.3.1 The monopoly situation

In the sequel, the monopoly case serves as a benchmark. The incumbent's problem is to find a pair travel time-price which maximizes its profit. For a given pair $\{T_i, p_i\}$, there exists a passenger with valuation $\tilde{\theta}$ who is exactly indifferent between travelling with the incumbent or not, i.e., $U(\tilde{\theta}) = \tilde{\theta}\nu(T_i) - p_i = 0$ or $\tilde{\theta} = p_i/\nu(T_i)$. It is the 'marginal passenger'. The demand for transport services faced by the monopolist is $D_i(p_i, T_i) = 1 - G(\tilde{\theta})$ where $G(\tilde{\theta})$ is the share of potential passengers that do not travel by train and prefer another transport mode when the monopoly offers the pair $\{T_i, p_i\}$ and is represented in Figure 3.

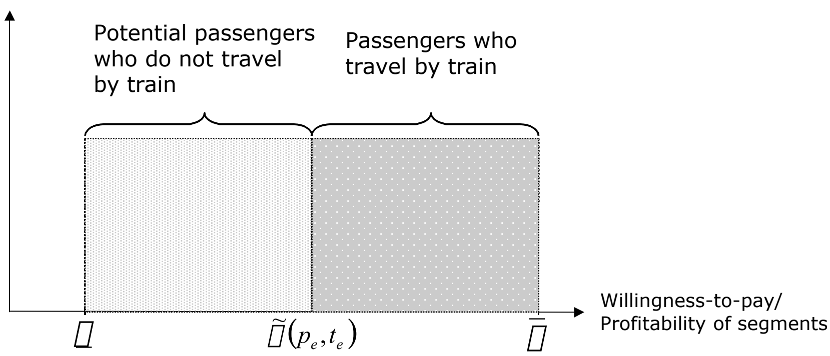

Figure 3: The demand that addresses to the monopoly with vertical differentiation.

Consider that the incumbent's cost is given by $C_i(Q_i, T_i)$ where Q_i is the demand for transport services, i.e., $Q_i = D_i(p_i, T_i)$.

The incumbent's problem is written as

$$\max_{\{T_i, p_i\}} \quad \pi_i \equiv p_i D_i(p_i, T_i) - C_i(D_i(p_i, T_i), T_i).$$

The price chosen by the monopolist is such that the marginal revenue equals the marginal cost, i.e.,

$$\frac{\partial \pi_i}{\partial p_i} = 0 \Leftrightarrow \underbrace{p_i \frac{\partial D_i}{\partial p_i} + D_i}_{\text{Marginal Revenue}} = \underbrace{\frac{\partial C_i}{\partial Q_i}\frac{\partial D_i}{\partial p_i}}_{\text{Marginal Cost}}, \tag{1}$$

which can be rewritten as follows

$$\underbrace{\frac{p_i - \frac{\partial C_i}{\partial Q_i}}{p_i}}_{\text{Relative Mark-up}} = \underbrace{\frac{1}{\epsilon_i}}_{\text{Inverse of Demand Elasticity}}, \tag{2}$$

where $\epsilon_i \equiv -\frac{\partial D_i/\partial p_i}{D_i/p_i}$ is the demand elasticity.

Equation (1) tells us that an increase in price has two effects. First, it reduces the demand since the number of passengers willing to travel by train decreases, i.e., the marginal passenger is changed: $\frac{\partial \tilde{\theta}}{\partial p_i} > 0$ implies that $\frac{\partial D_i}{\partial p_i} = -g(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial p_i} < 0$. Second, it increases the gain earned on the types that are still willing to travel with the incumbent operator, i.e., the so-called inframarginal passengers. At equilibrium, the price chosen by the monopoly is such that the sum of these effects is equal to the marginal cost of an additional passenger.

Equation (2) tells that the incumbent's market power, given by the relative markup imposed on the final price of service, is inversely proportional to the price elasticity of the demand for transport services. The higher the price elasticity of the demand is, the smaller is the distortion on the final price created by the market power of the incumbent. It is an intuitive result since the elasticity provides the sensitivity of passengers with respect to price, for a given travel time.

As it is illustrated in Figure 4, the pricing decision by the monopoly is socially inefficient.

Consider now the choice of travel time by the incumbent operator. Simple manipulations show that the travel time chosen by the incumbent obeys the following formula

$$\frac{\partial \pi_i}{\partial T_i} = 0 \Leftrightarrow \left(p_i - \frac{\partial C_i}{\partial D_i}\right)\frac{\partial D_i}{\partial T_i} = \frac{\partial C_i}{\partial T_i}. \tag{3}$$

The incumbent chooses the travel time such that the marginal gain of reducing the travel time by a small amount in terms of increased demand and revenue is equal to the marginal cost borne by the incumbent operator to reduce that travel time by this amount.

Note that from the optimal pricing decision (Equation (1)), the markup of the final price over the marginal cost is positive, i.e., $p_i - \frac{\partial C_i}{\partial D_i} > 0$. Consequently, if passengers are sufficiently sensitive to the change in the travel time (i.e., $\frac{\partial D_i}{\partial T_i}$ is sufficiently negative), or if the marginal cost of travel time is sufficiently low (i.e., $\frac{\partial C_i}{\partial T_i}$ is sufficiently small), then the monopolist incumbent offers the smallest travel time consistent with the technical constraint

$$\frac{\partial \pi_i}{\partial T_i} = \left(p_i - \frac{\partial C_i}{\partial D_i}\right)\frac{\partial D_i}{\partial T_i} - \frac{\partial C_i}{\partial T_i} < 0 \Leftrightarrow T_i = \underline{T}. \tag{4}$$

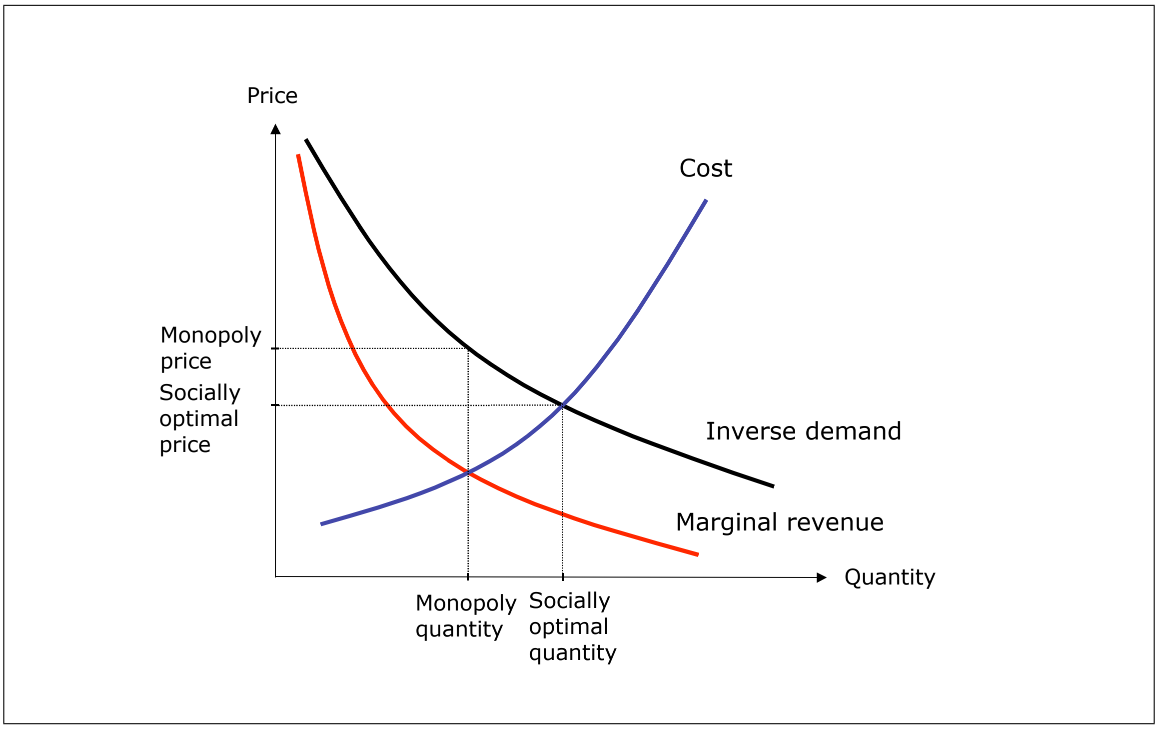

Figure 4: The pricing and production decisions of the monopoly: Marginal benefit equals marginal cost.

In this abstract framework, the incumbent operator with a monopoly position has an incentive to offer a service with high quality (low travel time) in order to raise passengers' willingness-to-pay for that service. If the direct cost of doing so is not too high, then the incumbent will set the smallest possible travel time.

Finally, demand at equilibrium is rationed (i.e., $\tilde{\theta} > \underline{\theta}$). Indeed, by focusing on passengers with larger valuations, i.e., by rationing of the demand, the incumbent can impose a higher price on the more profitable segments of the population.[15]

Summary 4: Under monopoly, the incumbent has an incentive to ration demand in order to impose a higher price on the residual demand: Excluding the passengers with lower valuations for train services from the market enables the incumbent to target its offer on the more profitable passengers (i.e., those with higher willingness-to-pay). Since passengers' willingness-to-pay decreases with travel time, the incumbent offers a low travel time in order to raise the price of train services. If the cost of lowering the travel time and/or if passengers are sufficiently sensitive to travel time reduction, then the monopolist incumbent implements the lowest travel time consistent with the network constraints.

[15]This depends on characteristics of demand function. We consider later a case in which all passengers decide to travel by train when the incumbent has a monopoly position.

3.3.2 The monopoly situation: An illustration

As an illustration of these different points, consider an example based on the following assumptions

- Heterogeneity among the population of passengers is uniformly distributed. Then $g(\theta) = 1/\Delta\theta$ and $G(\theta) = (\theta - \underline{\theta})/\Delta\theta$. Roughly speaking, a uniform heterogeneity means that there is the same proportion of different types of passengers. We return later on this assumption when we deal with the impossibility of effective competition.

- The cost function of the incumbent operator is given by: $C_i(Q_i, T_i) \equiv c_i Q_i + \tilde{c}_i(T_i)$. The cost function is assumed to be separable in the volume of passenger-kilometers and the travel time (the differentiation variable). Indeed, we view the cost of transporting passenger and adopting a technology that enables to operate faster on the AB-trip as mainly unrelated. In particular, the decision to operate a faster train on a segment of the network does not affect the marginal cot of transporting an additional passenger on that train (if the capacity constraint of the train is not binding of course).

Equations (1) and (3) become

$$\frac{\partial \pi_i}{\partial p_i} = 0 \Leftrightarrow p_i = \frac{1}{2}\left[c_i + \overline{\theta}\nu(T_i)\right], \tag{1'}$$

$$\frac{\partial \pi_i}{\partial T_i} = 0 \Leftrightarrow (p_i - c_i)\,\frac{p_i}{\nu(T_i)^2}\nu'(T_i) - \Delta\theta\tilde{c}_i'(T_i) = 0. \tag{3'}$$

We can make the following comments on these equations.

First, the monopolist incumbent's price will be above its marginal cost c_i if the passengers' willingness-to-pay is sufficiently large, i.e., $\overline{\theta}\nu(T_i) > c_i$, an assumption that we keep from now on.

Second, this decision on travel time trades off two effects. The first effect is the increase in the willingness-to-pay of passengers. The second effect is related to the increase in cost associated to a marginal reduction of time travel. If the marginal cost of decreasing travel time is large (i.e., $\tilde{c}_i'(.)$ strongly negative), then the monopoly may prefer setting the largest travel time consistent with the network constraints, i.e., $T_i = \overline{T}$. A similar result occurs when passengers are not very sensitive to a reduction of travel time, i.e., $\nu'(T_i)$ is very small, in which case the incumbent sets a large travel time. By contrast, when the cost of reducing travel time is low or when passengers are highly sensitive to a reduction of the travel time, then the incumbent operator sets the smallest possible travel time consistent with the network constraints: $T_i = \underline{T}$. For intermediate situations, the travel time implemented by the monopolist lies in between these two extreme values.

Third, demand is rationed, i.e., the market is not entirely covered, when $\tilde{\theta} > \underline{\theta}$ or $c_i + (\overline{\theta} - 2\underline{\theta})\nu(T_i) > 0$. This condition is intuitive: When the marginal cost is high, reducing the demand saves on cost. More interesting is the role of the 'spread' of passengers' willingness-to-pay: When heterogeneity in the population of passengers is large, i.e., when $\overline{\theta} - 2\underline{\theta} > 0$, then the monopoly has an incentive to ration demand in order to be able to impose a higher price on the residual demand (remember that the set of passengers that travel by train is $[\tilde{\theta}, \overline{\theta}]$); by contrast, when the population of passengers is homogenous, then there is almost no gain to exclude passengers with lower valuations for train services in order to impose a higher price on passengers with higher valuations, since the latters' willingness-to-pay is close to the formers' willingness-to-pay.

Finally, we emphasize the following point. When the incumbent benefits from a monopoly position, the 'quality' offered, which is represented here by the travel time, tends to be high, except when that incumbent operator is relatively cost inefficient. Importantly, the price and travel time are set by simple 'price equals marginal cost'-like rules.

3.3.3 The impact of entry

Now consider the impact of entry by a competitor E. Let $\{p_i, T_i\}$ and $\{p_e, T_e\}$ be the price and travel time offered by the incumbent operator and the entrant, respectively.

To study this oligopolistic competition under quality differentiation, we first analyze price competition assuming a given travel time per operator. Then, we determine the choice of travel time by each operator. This timing traduces the fact that changing prices might be done in a very short period of time, whereas changing travel time values might involve a longer period of time (planning time). This two-stage game allows us to understand how competition does affect both the pricing decisions and the choices of travel time values by the different operators.

Assume for the moment that $T_i \leq T_e$: The entrant offers a larger travel time on the AB-trip. Given the different interpretations of the differentiation parameter, the following interpretation also holds. The entrant targets the less profitable segments of the population (passengers with lower valuations for train services) and the incumbent targets the most profitable segments of the population. This implies that prices must be such that $p_i \geq p_e$.

The demand functions. What is the demand faced by each operator? First consider the passenger with valuation $\tilde{\theta}$ who is indifferent between travelling with the incumbent and the entrant: $\tilde{\theta}\nu(T_i) - p_i = \tilde{\theta}\nu(T_e) - p_e$ or $\tilde{\theta} = (p_i - p_e)/(\nu(T_i) - \nu(T_e))$. This is represented in Figure 5.

Note that we could also consider that there exists a (different) threshold $\hat{\theta}$ such that passengers with type $\theta \leq \hat{\theta}$ decide not to travel by train. This would put an additional constraint on the pricing/travel time decisions of the operator which targets passengers with lower willingness-to-pay. Our analysis can be extended to account for such a case. We can show that, under fairly weak assumptions, the low frequency operator has no incentive to ration the demand. Indeed, price competition already drastically reduces the demand faced by each operator and there would be no or a small gain for the low frequency operator to further reduce its demand through rationing. Therefore competition will usually lead to a larger number of passengers that decide to travel by trains: Competition tends to increase the total demand for transport services.

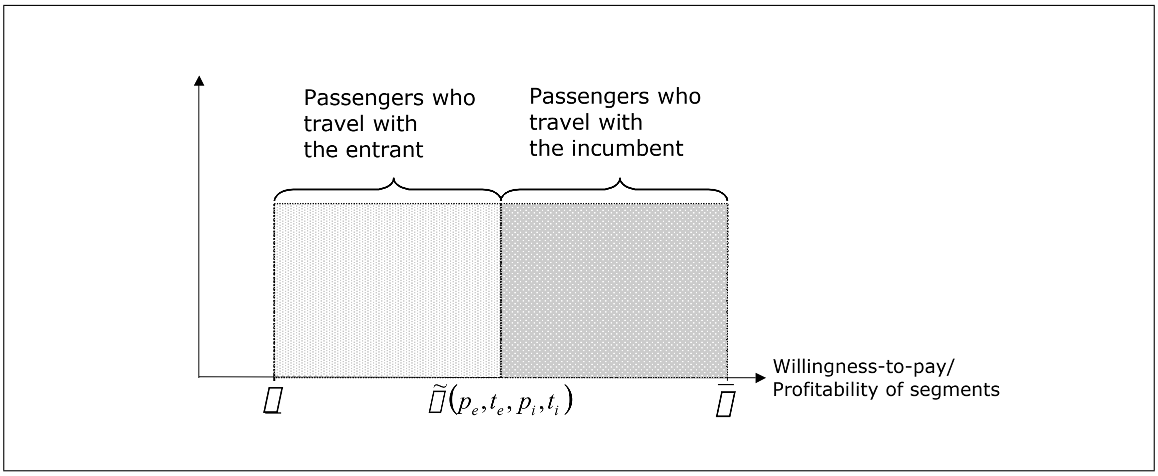

Figure 5: The demand that addresses to the different competitors with vertical differentiation.

The demand faced by the entrant is now $D_e(p_i, p_e) = G(\tilde{\theta})$ and the demand faced by the incumbent is $D_i(p_i, p_e) = 1 - G(\tilde{\theta})$. The profits of the two competitors are given by

$$\begin{cases} \pi_i = p_i D_i(p_i, p_e) - C_i(D_i(p_i, p_e), T_i), \\ \pi_e = p_e D_e(p_i, p_e) - C_e(D_e(p_i, p_e), T_e). \end{cases}$$

Define $\Delta\nu(T) \equiv \nu(T_i) - \nu(T_e) \geq 0$.

The price competition stage. Consider the incumbent's choice of price. The first-order condition associated with p_i is

$$p_i \frac{\partial D_i}{\partial p_i} + D_i = \frac{\partial C_i}{\partial Q_i} \frac{\partial D_i}{\partial p_i},$$

or

$$\frac{p_i - \frac{\partial C_i}{\partial Q_i}}{\Delta\nu(T)} = \frac{1 - G(\tilde{\theta})}{g(\tilde{\theta})}. \tag{5}$$

The pricing decision is similar to the one under monopoly. The modification is that the incumbent's demand is reduced and depends on the pair price-frequency offered by the entrant (through $\tilde{\theta}$).

Similarly, the price chosen by the entrant is given by

$$\frac{p_e - \frac{\partial C_e}{\partial Q_e}}{\Delta\nu(T)} = \frac{G(\tilde{\theta})}{g(\tilde{\theta})}. \tag{6}$$

Solving simultaneously (5) and (6) enables to obtain the prices as function of the frequencies: $p_e(T_i, T_e)$ and $p_e(T_i, T_e)$.

When setting its price, a competitor simply maximizes its profit which depends on the residual demand it faces; this residual demand in turn is a function of the price set by the competitor and taken as given. Once we account for the 'modified' demand function, the pricing policy of the incumbent operator is the same as in the monopoly situation. The important assumption here is that prices are chosen simultaneously by the entrant and the competitor.

The travel time choice stage. Consider now the choice of travel time by the operators at the first stage of the game. Consider the incumbent. The choice of T_i has a direct effect on its profit π_i via the effect on the demand faced by the incumbent and on its costs. This choice also affects the setting of price by the entrant at the second stage of the game through a strategic effect. To understand these effects, consider the total impact of T_i on π_i

$$\frac{d}{dT_i}\pi_i(p_i(T_i,T_e),p_e(T_i,T_e),T_i,T_e) = \underbrace{\frac{\partial \pi_i}{\partial T_i}}_{\text{Direct effect}} + \underbrace{\frac{\partial \pi_i}{\partial p_e}\frac{dp_e}{dT_i}}_{\text{Strategic effect}}. \qquad (7)$$

The direct effect corresponds to the 'marginal benefit minus marginal cost of travel time' effect that was guiding the choice of travel time under monopoly. A decrease in the travel time T_i has a direct cost, but also comes with a benefit since it expands the demand faced by the incumbent.

Competition forces the incumbent to account for an additional effect when deciding its differentiation level. This strategic effect corresponds to the impact of its choice of differentiation level on the competitor's behavior in the price competition stage. When the incumbent looses its monopoly position, the setting of the attributes of the services offered to passengers must account for a new strategic effect.[16]

Notice that $\frac{\partial \pi_i}{\partial p_e} > 0$. Indeed, if the entrant's quality/travel time is fixed, then a decrease of the entrant's price tends to erode the demand, and therefore the profit, of the incumbent operator. Conversely, if the entrant's final price increases, then the price competition becomes softer and the incumbent's profit increases.

Now, direct computations show that[17]

$$\frac{dp_e}{dT_i} = \frac{\partial p_e}{\partial T_i} + \frac{\partial p_e}{\partial p_i}\frac{\partial p_i}{\partial T_i} \propto \underbrace{-H(\tilde{\theta})\nu'(T_i)}_{\geq 0} + \underbrace{\Delta\nu(T)}_{\geq 0}\underbrace{H'(\tilde{\theta})}_{?}\underbrace{\frac{\partial \tilde{\theta}}{\partial T_i}}_{\leq 0},$$

where $H = \frac{1-G}{g}$. Notice that $H' = \frac{(1-G)(1-G)''-[(1-G)']^2}{[(1-G)']^2}$. The sign of H' depends on the log-concavity/convexity of $1-G(\theta)$. Hence the nature of the strategic effect strongly depends on the distribution of the heterogeneity among passengers. We come back on this point later on.

Note that this point is more general than it may appear at a first sight. In oligopolistic models of competition, the strategic interaction between the competitors' decision variables depends on the characteristics of the demand functions. In our case, it depends on the curvature (i.e., the second derivative) of the measure of the heterogeneity given by $H(.)$. This strategic interaction is fundamental to de-

[16] Since the incumbent and the entrant are treated symmetrically, the same considerations hold for the entrant.

[17] We assume that $\frac{\partial^2 C_e}{\partial Q_e^2} > 0$, which is sufficient for the entrant's maximization problem to have an interior solution. This does not drastically affect the main points of this analysis.

termine how competitors will differentiate their services in order to soften the price competition at the subsequent stage of the game.

Indeed, it might be the case that $\frac{dp_e}{dT_i} \geq 0$, in which case the incumbent has an incentive to increase the travel time of its service for strategic purpose, i.e., to ensure that the price competition with the entrant will not be too intense. By contrast, when $\frac{dp_e}{dT_i} \leq 0$ then the incumbent has an incentive to reduce the travel time of its service in order to soften the entrant's behavior.

The important point to note is that the precise way to soften the entrant's behavior at the price competition stage is dictated by the distribution of heterogeneity, i.e., by the characteristics of the demand function.

3.4 The impact of entry: An illustration of the possible scenarios

To illustrate the impact of entry, we return to the example that we introduced previously.

We modify specification of the cost function by assuming that the cost of the travel time consists only of a fixed cost: $\tilde{c}_k(T_k) = F_{T_k}$. Differently stated, implementing a shorter AB-trip involves only a fixed cost. The shorter the time travel is, the larger is the fixed cost. Since travel time belongs to the interval $[\underline{T}, \overline{T}]$, the fixed cost belongs to $[F_{\overline{T}} \equiv \underline{F}, F_{\underline{T}} \equiv \overline{F}]$.

We emphasize that similar qualitative results would be achieved with different specifications of the cost functions. The basic message that is conveyed by this analysis is that the entry scenarios might not be unique and that differentiation (in a broad sense) enables to soften the price competition between the incumbent and the entrant under certain conditions. In our case, it is typically the case that differentiation is maximal. With different cost specifications, a significant amount of differentiation between operators would occur in order to soften the price competition game.

3.4.1 First scenario: Incumbent (entrant) on the more (less) profitable segments of the passenger population

Solving the price competition stage leads to the following expressions

$$\begin{cases} p_i(T_i, T_e) = \frac{1}{3}\left[2c_i + c_e + (2\overline{\theta} - \underline{\theta})\Delta\nu(T)\right], \\ p_e(T_i, T_e) = \frac{1}{3}\left[2c_e + c_i + (\overline{\theta} - 2\underline{\theta})\Delta\nu(T)\right]. \end{cases}$$

It is then immediate to compute the incumbent's and the entrant's profit from the viewpoint of the first stage of the game. Then, simple manipulations show that the operators will differentiate as much as possible their services by setting $T_i = \underline{T}$ and $T_e = \overline{T}$ since $\frac{dp_i}{dT_e} > 0$ and $\frac{dp_e}{dT_i} < 0$. Profits are then given by

$$\begin{cases} \pi_i = \left[p_i(\underline{T}, \overline{T}) - c_i\right] D_i(p_i(\underline{T}, \overline{T}), p_e(\underline{T}, \overline{T}), \underline{T}, \overline{T}) - \overline{F}, \\ \pi_e = \left[p_e(\underline{T}, \overline{T}) - c_e\right] D_e(p_i(\underline{T}, \overline{T}), p_e(\underline{T}, \overline{T}), \underline{T}, \overline{T}) - \underline{F}. \end{cases}$$

	Sufficiently high travel time differential	Sufficiently low travel time differential
Variation of incumbent's profit	$\pi_i^{\text{competition}} \geq \pi_i^{\text{monopoly}}$	$\pi_i^{\text{competition}} \leq \pi_i^{\text{monopoly}}$

Table 1: Incumbent's profit under monopoly and competition when the entrant targets the less profitable segments.

To further simplify the analysis, we assume from now on that $c_i = c_e = 0$.

Impact of competition on the incumbent's profit. With respect to the monopoly situation, since the incumbent sticks to the lowest travel time, it has the same total cost under monopoly and under competition. Therefore, the variation of the incumbent's profit only depends on the travel time differential, as shown in Table 3.

The intuition is the following. Under monopoly, the incumbent can only offer a limited number of services, that we set equal to 1: The monopolist incumbent rations the demand in order to increase revenue earned from the residual demand (i.e., the passengers who effectively travel with the incumbent operator). In this competitive scenario, competitors seek to differentiate as much as possible their services in order to soften the price competition. The entrant then targets the low profitability segments of passengers and might benefit from a low market share if it is relatively inefficient. The entrant's offer (price and travel time) now defines in part the demand that addresses to the incumbent which targets the higher profitability segments. Therefore, the demand that addresses to the incumbent might be larger than under monopoly in particular when services are sufficiently differentiated.

To fix ideas, consider that the marginal production costs of both competitors are zero: $c_i = c_e = 0$. Then, the demand that the monopolist incumbent faces is equal to $\frac{\overline{\theta}}{2}$; in this competitive scenario, its demand becomes equal to $\frac{\underline{\theta}+\overline{\theta}}{3}$, which is larger than the demand under monopoly when the heterogeneity is sufficiently large.[18] With entry, when the travel time differential is very large, i.e., when the differentiation between operators is very large, the entrant does not put a strong competitive pressure on the incumbent and only deals with a limited fraction of the market with low profitability levels; the incumbent has no longer to ration the demand to increase passengers' willingness-to-pay and fully exercises its market power on the large part of the market that addresses to it.

However, the entry of a new competitor also triggers a price competition, whose

[18]Rigorously, we need to check that the entrant has no incentive to ration the demand, a requirement which puts an additional constraint on the travel time values. Simple computations show that in our example it is always possible to find a pair of travel time values $\{\underline{T}, \overline{T}\}$ such that the entrant does not ration the demand and the incumbent's demand increases.

intensiveness depends on the differentiation between products. If the differentiation between the products offered by the entrant and the incumbent is too low, then a vigorous price competition destroys the benefit created by the increase in total demand, leading to a smaller profit for the incumbent in that entry scenario than under monopoly.

3.4.2 Second scenario: Incumbent (entrant) on the less (more) profitable segments of the passenger population

In this case, the roles of the entrant and the incumbent are reversed. Computations are immediately adapted from the analysis of the preceding scenario.

The interesting situation arises in the case of a high fixed cost of travel time and a large travel time differential. A high fixed cost means that the incumbent's profit under monopoly is relatively low. Moreover, remember that under monopoly the incumbent offers the lowest travel time.

With that competitive scenario, the entrant 'replaces' the incumbent as it offers low travel time trains while the incumbent now offers high travel time trains. When the travel time differential is large enough, this means that price competition is not too intense and that the incumbent still benefits from a sufficiently large demand; simultaneously, the incumbent is now able to drastically reduce its cost since it offers the highest travel time. In that case, the introduction of competition does not reduce the incumbent's profit. When the differential in travel time is rather small, however, the incumbent's profit decreases after the entry of a competitor because the most profitable segments of the population of passengers adopt the entrant and/or because price competition is intense.

This result has a broader interpretation. When the incumbent adapts to entry, then it is sometimes profitable to let the entrant target the most profitable segments of the population of passengers. Indeed, those segments require to set a high level of 'quality' (frequency, comfort, speed...) and therefore generate a high cost. If the differentiation between competitors is sufficiently large so that price competition is not too intense, then the incumbent prefers to focus on the low profitability segments to reduce its costs.

3.4.3 Discussion

Which scenario is likely to occur? We can only partially reply to this question.

First, based on our most simple example (no cost associated to the volume of passenger-kms, separable costs, identical cost functions, uniform heterogeneity) assume that there are no fixed costs associated with different travel time values. These are strong assumptions. However, an interesting result arises in that case: the two entry scenarios are simultaneously possible. Differently stated, the equilibrium of the game is not unique since both operators have the incentive to target the most profitable segments of the population of passengers and there is no cost associated to a decrease in the travel time.

Therefore, when the difference between the incumbent's and the entrant's efficiency is rather small, a 'race-to-be-the-first' is likely to emerge in order to target the most profitable segments of the population. This case yields an unstable industry configuration over the time: The competitor dealing with the less profitable segments is likely to try repeatedly to attack the operator dealing with the more profitable segments in the hope of switching from one industry equilibrium to another.[19]

Second, what does happen when one competitor is much more efficient than the other? In this case, the efficient competitor can credibly commit to target the most profitable segments of passengers. Indeed, if the efficient operator was serving segments of low profitability, then it would put too strong a pressure on the other competitor; a limit case arise when the most efficient operator's price is so low (despite product differentiation) that the least efficient operator is forced to exit the market. However, when the efficiency differential is not too large, then it might be possible for the least efficient operator to target the most profitable segments of the population of passengers since both competitors have a strong incentive to differentiate their services.

Third, when the differentiation between services is low, then both competitors anticipate an intense price war. The operator which targets the higher-valuations segments is likely to earn a larger profit than the other operator.

[19]This implicitly assumes that the profit of serving the higher valuations segments is larger than the profit associated to the service of the lower valuations segments, which is the case when the difference between the costs associated to a low travel time and a high travel time is not too large. Otherwise, it suffices to change the labelling of the different segments.

Summary 5: The incumbent has a privileged position in the industry in the sense that when multiple industry configurations following entry are possible, the incumbent can influence the realized configuration.
If the incumbent anticipates a large differentiation between its product and the entrant's offer, then the best strategy for the incumbent depends on the relative cost efficiency. If the incumbent is not too inefficient compared to entrant, then it should continue to offer a service targeted toward the most profitable segments (i.e., passengers with high willingness-to-pay). On the contrary, if the incumbent is much less efficient than the entrant, then it should target its offer towards a niche market (less profitable segments, lower willingness-to-pay passengers) and let the most profitable segments to the entrant. This strategy enables the incumbent to save on its cost.
By contrast, if the incumbent anticipates that the entrant will not differentiate much its product, then it should continue to target the most profitable segment of higher willingness-to-pay passengers, even if it is less efficient than the entrant.
If the entrant is much more efficient than the incumbent, then the incumbent should exit the market.

3.5 Impact of competition on profits and welfare

We consider that the distribution of heterogeneity is uniformly distributed on its support. We also assume that the entrant's cost of travel time is equal to zero. To further simplify the analysis we assume (without loss of generality) that $\underline{\theta} = 1$ and $\overline{\theta} = 5$. In order to obtain simple analytical expressions, we assume that $\nu(T) = 1/T$ and $c_i(T) = 1/TF_i$ for all $T > 0$, where $c_i(T)$ is the incumbent's cost of implementing a trip with travel time equal to T.

3.5.1 First case: Entrant (incumbent) on the more (less) profitable segments

The monopoly benchmark. Let $\tilde{\theta}^m = p_i^m T_i^m$ be the marginal passenger. The demand faced by the incumbent is given by $D_i(p_i^m, T_i^m) = \frac{1}{\Delta\theta}(\overline{\theta} - \tilde{\theta}^m)$. The profit of the incumbent under monopoly is given by $\pi_i^m(p_i^m, T_i^m) = D_i^m p_i^m - F_{T_i^m}$.

Optimization with respect to price yields

$$p_i^m(T_i^m) = \frac{1}{T_i^m}\frac{5}{2}.$$

This implies that $\tilde{\theta}^m = \frac{5}{2}$: The incumbent rations more than half of the market ($\frac{5}{2} > \frac{5-1}{2} = 2$). The incumbent's profit can be rewritten as follows

$$\pi_i^m(T_i^m) = \frac{1}{T_i^m}\left(\frac{25}{16} - F_i\right).$$

Assuming that $\frac{25}{16} - F_i > 0$, the incumbent will choose $T_i^m = \underline{T}$.

This allocation yields a profit for the incumbent, surplus for consumers and social welfare equal to

$$\begin{aligned}
\pi_i^m &= \frac{1}{\underline{T}}\left(\frac{25}{16} - F_i\right), \\
SC^m &= \frac{25}{32}\frac{1}{\underline{T}}, \\
SW^m &= \int_{\tilde{\theta}^m}^{\overline{\theta}} \left[\frac{1}{\underline{T}}\theta - p_i^m\right] \frac{1}{\Delta\theta} d\theta = \frac{1}{\underline{T}}\left(\frac{75}{32} - F_i\right).
\end{aligned}$$

Competition. We assume for simplicity that $F_e = 0$. With a uniform distribution, the entrant is likely to offer a lower travel time (and therefore a larger price) than the incumbent since the former has a lower cost than the latter and that (roughly speaking) there are the same proportions of travellers with a high willingness to pay than of travellers with a low willingness to pay. The demand faced by the entrant is then $D_e^c = \frac{1}{\Delta\theta}(\overline{\theta} - \tilde{\theta}_c)$; the demand for the incumbent becomes $D_i^c = \frac{1}{\Delta\theta}(\tilde{\theta}_c - \underline{\theta})$. Superscript 'c' stands for 'competition'.

Solving the price competition stage yields

$$\begin{aligned}
p_i^{c,1} &= \left(\frac{1}{T_e^{c,1}} - \frac{1}{T_i^{c,1}}\right), \\
p_e^{c,1} &= 3\left(\frac{1}{T_e^{c,1}} - \frac{1}{T_i^{c,1}}\right).
\end{aligned}$$

Plugging the values of the prices in the profit functions and optimizing with respect to the frequencies leads to

- For the entrant: $\frac{d\pi_e^{c,1}}{dT_e^{c,1}}(T_i^{c,1}, T_e^{c,1}) < 0$. Therefore, the entrant will choose the smallest travel time, i.e., $T_e^c = \underline{T}$.
- For the incumbent: $\frac{d\pi_i^{c,1}}{dT_i^{c,1}}(T_i^{c,1}, T_e^{c,1}) > 0$ and the incumbent will choose the highest travel time, i.e., $T_i^{c,1} = \overline{T}$.

At the equilibrium of this first entry scenario, profits of the different operators,

	Sufficiently high travel time differential	Sufficiently low travel time differential
Sufficiently high fixed cost	$\pi_i^{c,1} \geq \pi_i^m$	$\pi_i^{c,1} \leq \pi_i^m$
Sufficiently low fixed cost	$\pi_i^{c,1} \leq \pi_i^m$	$\pi_i^{c,1} \leq \pi_i^m$

Table 2: Incumbent's profit under monopoly and entry when the entrant targets the most profitable segments.

consumers' surplus and social welfare are equal to

$$\begin{aligned}
\pi_i^{c,1} &= \frac{1}{\overline{\overline{T}}}\left(\frac{\overline{T}-\underline{T}}{4\underline{T}} - F_i\right), \\
\pi_e^{c,1} &= \frac{9}{4\underline{T}\overline{\overline{T}}}\left(\underline{T}-\overline{T}\right), \\
SC^{c,1} &= \frac{23\underline{T}+\overline{T}}{8\overline{T}\underline{T}}, \\
SW^{c,1} &= \frac{21\overline{T}+3\underline{T}}{8\overline{T}\underline{T}} - \frac{1}{\overline{\overline{T}}}F_i.
\end{aligned}$$

There are some constraints to ensure that these expressions are positive. In particular, we must have $F_i \leq \min\{\frac{25}{16}; \frac{1}{4}\frac{\overline{T}-\overline{T}}{\underline{T}}\}$.

Depending on the level of fixed cost of the incumbent F_i and the travel time differential $\overline{T}-\underline{T}$, we obtain the Table 2

The most interesting situation concerns the case of a high fixed cost and a high frequency differential. A high fixed cost means that the incumbent under monopoly does not earn a lot of profit. Moreover, remember that under monopoly the incumbent offers the smallest travel time $\underline{T}$.

With competition, and in that scenario, the entrant replaces the incumbent in the following sense: It will offer low travel time trains while the incumbent will now offer high travel time trains. When the travel time differential is large enough, this means that price competition is not too intense and that the incumbent still benefits from a sufficiently large demand; simultaneously, the incumbent is now able to drastically reduce its cost since it offers the largest travel time. In that case, the introduction of competition does not reduce the incumbent's profit. In the other case, however, the incumbent's profit will decrease after the entry of a competitor.

We emphasize that this result has a broader interpretation. When the incumbent adapts to entry, then it is sometimes profitable to let the entrant target the most profitable segments of the travellers population. Indeed, those segments require to

	Sufficiently high travel time differential	Sufficiently low travel time differential
Variation of incumbent's profit	$\pi_i^{c,2} \geq \pi_i^m$	$\pi_i^{c,2} \leq \pi_i^m$

Table 3: Incumbent's profit under monopoly and entry when the entrant targets the less profitable segments.

set a high level of 'quality' (frequency, comfort, speed) and therefore generate a high cost. If the differentiation between competitors is sufficiently large so that price competition is not too intense, then the incumbent prefers to focus on the low profitability segments to reduce its costs.

Finally, as concerns welfare, we show that

$$SW^{c,1} \geq SW^m.$$

Indeed, competition enables to attenuate the incumbent's market power and to reduce the incumbent's cost: Both effects tend to increase unambiguously welfare.

3.5.2 Second case: Entrant (incumbent) on the less (most) profitable segments

Consider now the other possible equilibrium. Since the benchmark case is unchanged, we directly go to the analysis of competition.

Competition. The computations can be immediately adapted from the previous case and will not be repeated here. We simply note that the incumbent's profit and social welfare are equal to

$$\pi_i^{c,2} = \frac{9}{4\overline{T}} \frac{\overline{T} - \underline{T}}{\underline{T}} - \frac{1}{\underline{T}} F_i,$$
$$SW^{c,2} = \frac{21\overline{T} + 3\underline{T}}{8\overline{T}\underline{T}} - \frac{1}{\underline{T}} F_i.$$

As previously, we note that $SW^{c,2} \geq SW^m$: Competition always improves welfare with respect to the monopoly situation.

Consider the variation of the incumbent's profit in that case. Since the incumbent sticks to the smallest travel time level, it has the same total cost under monopoly and under competition. Therefore, the variation of the incumbent's profit only depends on the travel time differential, as shown in Table 3. The intuition is the following. The introduction of competition leads to an increase of the total demand for train transport services: More travellers are now willing to travel by train. This increase in total demand translates into an increase in the demand that addresses to each operator.

However, the entry of a new competitor also triggers a price competition, whose intensiveness depends on the differentiation between products. If the entrant's product is sufficiently differentiated from the incumbent's one (and reciprocally) then the price competition is not too intense. In that case, the incumbent's profit increases. If, on the contrary, the differentiation between the products offered by the entrant and the incumbent is too low, then a vigourous price competition destroys the benefit created by the increase in total demand.

3.5.3 Discussion

It should be emphasized that the illustration presented previously is strongly biased in favor of competition. We should emphasize that the gain generated by the reduction of the incumbent's market power can be more than offset if the entry costs are sufficiently high: In that case, the monopoly situation is preferred from the viewpoint of social welfare. In the same spirit, if the incumbent benefits from sufficiently large returns to scale and/or density then competition may deteriorate welfare since the erosion of the incumbent's market share translates into a lower efficiency for this operator.

3.5.4 Intermediate summary

Summing up, we have derived the following insights.

- When choosing a differentiation level (e.g., through the travel time for instance) each operator must take into account two effects: The direct effect of the chosen level of 'quality' on its cost and the strategic effect, that is to say, the opportunity gain/cost associated with this choice, i.e., the amount which reflects how this choice of the product's attributes affect the behavior of the entrant at the price competition stage.

- The strategic effect depends on characteristics of the demand function, on the heterogeneity among the passenger population, on the technical constraints that limit the set of product attributes that can be offered by competitors.

- Since the entrant and the incumbent are treated symmetrically, provided that the efficiency differential is not too large, then different equilibrium configurations are possible. In all configurations, differentiation is maximal since competitors have a strong incentive to soften the price competition. Which competitor targets the most profitable segments may depend on the possibility for one competitor to commit credibly to target those segments.

- When one competitor is much more efficient than the other, then the least efficient operator is likely to be forced to exit the market even with maximal service differentiation.

- The possibility of multiple industry configurations suggests a race to be the first in order to target the most profitable segments.

3.6 Extension: Impossibility of effective competition

3.6.1 The role of heterogeneity

The previous analysis requires some assumptions on the parameter values. For instance, the demands of operators under competition and prices must be positive. Instead of going through the whole list of different cases, we focus on one case, which enables us to highlight a critical feature of the vertical differentiation model.

Consider that the willingness-to-pay of passengers is distributed uniformly on $[\underline{\theta}, \overline{\theta}]$. Assume for convenience that the operators' marginal costs are identical: $c_i = c_e \equiv c$. In this case, solving the price competition stage yields (still assuming that the entrant offers a higher travel time level than the incumbent)

$$p_i(T_i, T_e) = \frac{1}{3}\left[3c + (2\overline{\theta} - \underline{\theta})\Delta\nu(T)\right],$$
$$p_e(T_i, T_e) = \frac{1}{3}\left[3c + (\overline{\theta} - 2\underline{\theta})\Delta\nu(T).\right]$$

Note that the demand that the incumbent (or more generally, the operator that offers the highest frequency level) faces is always positive since $\overline{\theta} > \underline{\theta}$. However, for the demand addressed to the entrant (or more generally, the operator that offers the lowest frequency level) to be positive, the range of valuations in the population must be large enough: $\overline{\theta} > 2\underline{\theta}$.

If this condition is not satisfied, we have the following interesting situation. At the price equilibrium, the entrant faces no demand, and the incumbent charges a strictly positive price. Therefore, the entrant is forced to exit the market. The reverse scenario is also possible if the entrant succeeds to credibly commit to be present on the high-valuations segments of the passenger population.

Hence, even when entry is costless, there is only one operator in the market at equilibrium. When heterogeneity among passengers is small, intense price competition drives the high travel time operator out of the market. The intuition is that, when the lower quality is very low then the demand of the low quality operator is too low; if it is high, then it triggers an intense price competition among operators. When heterogeneity is small, there is not enough 'room' for competitors to differentiate their products without triggering an intense price competition.

Accordingly we should not expect the market shares of the different operators to equalize as competition develops. This depends on the heterogeneity of the market, i.e., on the possibility for competitors to soften the price competition.

Moreover, we should expect competition to develop only in the markets characterized by a sufficiently large degree of heterogeneity in the passenger population. Indeed, in such markets, competitors find enough demand, even with services differentiated from the rivals' services which soften price competition. It is an important difference with the horizontal differentiation setting, where, when there are no entry cost, there is always an incentive to enter the market since any entrant anticipates a small but positive market power.

Summary 6: Assume that the heterogeneity in the population is small, i.e., that the differences in willingness-to-pay for train services across passengers are small. 1) There is only one operator present on the market. Two configurations are possible: Either the entrant replaces the incumbent, or the entrant does not enter the market. 2) Which configuration prevails depends on the possibility of committing credibly to target the most profitable segments of the passenger population (i.e., to offer a sufficiently high 'quality' level) in order to threaten the competitor if it attempts to enter/stay active. Finally, we emphasize that heterogeneity is different from market size. If the size of the market is sufficiently small so that on-the-track competition is not viable per se, then the entrant is obviously not willing to enter the market. Small heterogeneity means that there is not enough diversity in the passengers' willingness-to-pay for the competitors to differentiate sufficiently and to escape from an intense price competition.

Finally, note that a more general point has been made by Shaked and Sutton (1983). Oligopolistic competition with vertical differentiation leads to a finite number of operators that enter the market in the long run. Indeed, price competition between operators offering high quality products erodes the demand for low quality products. Therefore, when the number of operators present on the market is sufficiently large, there is no room for operators offering lower quality products and entry does no longer occurs even when it is costless.

3.6.2 The role of 'concentration' and demand elasticity

In the previous analysis, we considered that passengers' valuations were uniformly distributed. As we explained, this had some consequences on the choice of differentiation level by the incumbent and the entrant.

In this part, we study the case where demand is concentrated around the lowest valuation type. This means that there are much more potential passengers with small willingness-to-pay for transport services than passengers with a large willingness-to-pay.

Consider that the density is distributed according to $g(\theta) = \frac{\overline{\theta}\underline{\theta}}{\Delta\theta}\frac{1}{\theta^2}$ on $[\underline{\theta}, \overline{\theta}]$. Therefore, the willingness-to-pay are mostly concentrated around $\underline{\theta}$ and the proportion of passengers with high willingness-to-pay decreases quickly.

Monopoly. Consider first the case of an incumbent with a monopoly position. Given a threshold $\tilde{\theta} = \frac{p_i}{\nu(T_i)}$, the demand for the incumbent is given by

$$D(p_i, T_i) = \frac{\underline{\theta}}{\Delta\theta}\frac{\overline{\theta} - \tilde{\theta}}{\tilde{\theta}},$$

which decreases as $\tilde{\theta}$ increases. At equilibrium, we have $\tilde{\theta} = \underline{\theta}$ or $p_i = \underline{\theta}\nu(T_i)$. The intuition goes as follows. If the monopoly increases its price and excludes from the market passengers with the lowest willingness-to-pay in order to charge a higher price on the remaining passengers, then it looses a lot of demand. In a concentrated market, it is not a profitable strategy. So the incumbent prefers to serve the whole market.

As a complement way, the elasticity of demand for the monopoly is

$$\epsilon = \frac{\overline{\theta}}{\overline{\theta} - \tilde{\theta}}. \tag{8}$$

For all passengers with types $\theta \in [\tilde{\theta}, \overline{\theta}]$, the incumbent acts as a monopoly and offers a price which depends on the demand elasticity. An increase in the price reduces the demand but enables to charge a higher price on the other passengers. When the demand elasticity is sufficiently large, the incumbent prefers offering a low price to attract more passengers. Then, the incumbent chooses the highest travel time, $T_i = \overline{T}$.

Competition. Assume now that a competitor has entered the market. Simple computations show that it is never possible to have a situation in which the incumbent and the entrant simultaneously compete on the market. The intuition is the following. Since demand is concentrated around $\underline{\theta}$, there is a strong incentive for both competitors to offer a low price in order to attract the large mass of passengers with low willingness-to-pay for transport services. Assume that the entrant replaces the incumbent (i.e., it targets the high-valuations segments while the incumbent switches to the low-valuations segments). Since both operators are willing to serve the same passengers, this ends up with the incumbent being forced to offer a negative price to compensate for its lower quality, i.e., to exit the market. The reciprocal scenario occurs when we assume that the incumbent offers the high quality: in that case, the entrant is forced to exit the market.

The demand elasticity when the distribution is uniform is $\epsilon = (\overline{\theta} - \tilde{\theta})^{-1}$, which is (much) smaller than the one defined in Equation (8) when $\overline{\theta}$ is large. We have determined the equilibrium when the distribution is decreasing over its support in a linear way (the market is therefore still concentrated around $\underline{\theta}$). In this situation, we show that entry with both competitors present on the market is possible (this will not be detailed here). This comparison highlights the role of the demand elasticity to determine the entry scenario.

Summary 7: Assume that the demand mainly emanates from potential passengers with low willingness-to-pays for train transport services and that this demand has a high price-elasticity (i.e., a small increase of price generates a large fall in demand). 1) The incumbent with a monopoly position serves the entire market. 2) It is not possible to have both competitors simultaneously active on the market because both competitors seek to offer products which are not sufficiently differentiated (to be close to the demand) and enter in a tough price competition.

4 Entry barriers

4.1 Introduction

In the previous section, we considered situations in which the incumbent and the entrant were symmetric in the following sense: They take their decisions about the price and the differentiation level simultaneously. This might be a good approximation of a mid-term situation, when some competitors have successfully entered some particular markets.

However, the incumbent has a privileged position in the industry, in particular at the outset of the process of opening markets to competition when competition is at an infant stage.

What is the optimal strategy of the incumbent faced to the threat of entry and what is the impact of the incumbent's strategy on the potential entrant?

First, some elements of market structure can prevent a free entry in an industry. Bain (1956) defined entry barriers as a characteristic that enables operators to realize supra-normal profits despite the entry threat. For instance, scale economies and absolute cost advantages provide operators already present on the market with an advantage with respect to potential entrants. Another entry barrier might be the need for capital. For instance, banks might be less eager to lend to new entrants because they are less known than the incumbent; or entrants might be prevented from growing as the incumbent inflicts losses on the product market in order to reduce their ability to finance new investments.

More interesting, the incumbent's behavior might also be an entry barrier for potential entrants. Three types of strategy are available for the incumbent threaten by entry.

- Entry is blockaded: The incumbent does not modify its behavior (i.e., it acts as if there were no entry threat) but entrants do not find it attractive to enter the market. For instance, when the demand that the incumbent faces has a very high price elasticity, the entrant might not find it profitable since the market might not allow two operators to operate without loss.

- Entry is prevented: In that case, entry cannot be blockaded but the incumbent modifies its behavior in order to prevent entry by new competitor.

- Adaptation to entry: The incumbent finds it preferable to let the potential competitors entering the market rather than building costly entry barriers to prevent entry. However, its behavior is modified to account for entry.

4.2 An illustration of entry barriers: Crowding out the product spectrum

In order to introduce different notions, we present the following example. Consider the vertical differentiation setting of the previous section. Passengers demand transport services from point A to point B; they are vertically differentiated and the heterogeneity parameter θ is uniformly distributed on $[\underline{\theta}, \overline{\theta}]$. The AB-trip is initially serviced by the incumbent operator only. A potential competitor may enter the market. If it enters the market, then it must pay a strictly positive entry fixed cost γ_{entry}. Letting aside the question of effects of competition on product differentiation, we assume that the entrant has chosen a given travel time $\overline{T}$.

In the previous section we made the assumption that the incumbent, as well as the entrant, could produce only one service (a service being identified with its travel time and its price). This is a reasonable assumption if the cost of implementing an additional service is prohibitively high for the incumbent.[20] However, there are instances in which the incumbent operator may be willing to set up a new service in anticipation of entry of the segment AB. In this section, we allow the incumbent to introduce a new service before entry may eventually take place, although at a fixed and irretrievable fixed cost $\gamma_{\text{new service}}$.

Summing up, the timing of decisions is as follows. In a first time, the incumbent decides whether to set up a new service with given attributes on the AB-trip or not. Then, in a second time, the entrant decides whether to invade the AB segment or not. Finally, in the third stage, price competition between the entrant and the incumbent takes place if entry occurred; otherwise, the incumbent behaves as a monopoly.

The incumbent sets up only one service. It has decided not to implement a new service at the first stage of the game. The second stage of the game has an immediate solution. If the potential competitor has not entered the market, the incumbent has monopoly position on the AB market. If the competitor has entered the market, then, given that the attribute of the entrant's product is fixed, i.e., $T_e = \overline{T}$, the outcome is the same as in the previous section on oligopolistic competition with endogenous differentiation. Finally, provided that the heterogeneity is sufficiently large (in order to ensure that the entrant targeting the lower-valuations segments will earn a positive profit gross of the entry cost), or more generally that the market is sufficiently attractive, the entrant is ready to pay the cost to enter the market.

The incumbent sets up two services. It provides an additional service. Assume that the travel time associated with this new service is the largest possible travel

[20] This might be the case if the planning schedule is tight and cannot afford another service on the trip considered.

time.[21]

There is a potentially non negligible cost ($\gamma_{\text{new service}}$) to implement a new service by the incumbent. Moreover, since this service is identical to the product that would be offered by the entrant if the latter decides to enter the market, the price competition between the entrant and the incumbent on this market will be very tough: Price competition between these two homogenous goods leads to average cost pricing and zero profit.[22] For the sake of simplicity, assume that the entrant and the incumbent have identical cost functions. Therefore, the incumbent makes no profit or loss on the new service.

On the basis of the direct cost of implementing the new service, the incumbent operator does not find it profitable to run this service. However, there is also an opportunity gain to be considered. Indeed, the decision of producing a second service affects the potential competitor's entry decision at the second stage of the game. Indeed, if the entrant decides to enter the market and the incumbent has formerly decided to set up a new product, then the entrant expects an intense price competition which undermines the chances to recover the fixed cost of entry. In this case, the entrant may decide not to enter the market since the entry cost is irretrievable.

By crowding out the products spectrum, the incumbent has managed to prevent entry. Through the introduction of a new service, the incumbent has committed to an aggressive behavior ex post. When entry is sufficiently costly, the commitment to an intense price competition ex post deters the potential competitor from entering the market.

4.3 Analysis of strategies

It should be kept in mind that the 'entry deterrence' strategy has a cost for the incumbent: The commitment to an intense price competition ex post requires that the incumbent sets up a new product, which involves the fixed cost $\gamma_{\text{new service}}$. If that cost is too high, then the incumbent might prefer the 'adapt to entry' strategy, in which case it does not prevent entry by the competitor. However, the incumbent might still be willing to undertake some decisions to soften the ex post price competition.

In general, which strategy is the best for the incumbent depends on the values of different parameters. We do not describe the different cases here. More importantly, while the 'entry deterrence' strategy only depends on the impact of the incumbent's before-entry decision on the entrant's profit, the 'adaptation of entry' strategy depends on the strategic interaction between the operators' decision variables at the competition stage. We illustrate these points now.

[21] Given our timing, this choice should of course be endogenous. However, this is not important to illustrate our point.

[22] We implicitly assume the the cost function of the incumbent operator is separable in the different prices.

4.3.1 A simple model

Consider the following highly stylized model.[23] There are two operators, an incumbent and a potential entrant. There are two periods in order to account for the capacity of the incumbent to commit to undertake certain actions today to affect competition with the entrant that may take place tomorrow.

In the first period, the incumbent chooses an investment variable K_i. This investment should be understood in a broad sense: It might be the decision to offer a new service, or the decision to adopt a particular differentiation level for instance. Then, the entrant observes the decision K_i undertaken by the incumbent and decides whether to enter the market or not.[24] If it does not enter, then the entrant makes no profit; the incumbent enjoys a monopoly position and makes profit

$$\pi_i^m(K_i, x_i^m(K_i)),$$

where $x_i^m(K_i)$ is the monopoly choice at the second period as function of the first period investment K_i. For instance, x_i can be the price or the output associated to the incumbent's service. If the potential operator has entered the market, then they simultaneously decide the second period variables x_i and x_e and earn respectively

$$\begin{cases} \pi_i(x_i(K_i), x_e(K_i)) & \text{for the incumbent,} \\ \pi_e(x_i(K_i), x_e(K_i)) & \text{for the entrant.} \end{cases}$$

If the entrant has to bear a fixed cost of entry, then we assume that this cost is part of π_e. Let $x_i^*(K_i)$ and $x_e^*(K_i)$ be the second period equilibrium decision variables respectively. Finally, $R_i(x_e)$ (respectively, $R_e(x_i)$) is the optimal choice of second period decision by the incumbent (respectively, the entrant) for a given decision x_e of the entrant (respectively, for a given decision x_i by the incumbent). That is to say: $R_i(x_e) \equiv \arg\max_{x_i} \pi_i(K_i, x_i, x_e)$ (respectively, $R_e(x_i) \equiv \arg\max_{x_e} \pi_e(K_i, x_i, x_e)$). The second period equilibrium decisions x_i^* and x_e^* are such that each competitor does not want to deviate from this decision given the decision of its rival: $x_i^* = R_i(x_e^*)$ and $x_e^* = R_e(x_i^*)$.

4.3.2 First strategy: Deterrence of entry

Entry is deterred when the incumbent chooses K_i at the first period such that the profit of the entrant is negative, i.e., $\pi_e(K_i, x_i^*(K_i), x_e^*(K_i)) \leq 0$. Thus, in order to prevent entry, the incumbent must choose a level of investment K_i^d such that

$$\pi_e(K_i^d, x_i^*(K_i^d), x_e^*(K_i^d)) = 0.$$

[23] This is a simplified version of Chapter 8 of Tirole (1988).

[24] Once again, we consider a single market. More complicated situations stemming from the network structure for instance are addressed later on.

In order to determine which strategy should the incumbent adopt in order to prevent entry, look at the impact of K_i on the entrant's profit

$$\frac{d\pi_e}{dK_i} = \underbrace{\frac{\partial \pi_e}{\partial K_i}}_{\text{Direct Effect}} + \underbrace{\frac{\partial \pi_e}{\partial x_e}\frac{\partial x_e^*}{\partial K_i}}_{=0} + \underbrace{\frac{\partial \pi_e}{\partial x_i}\frac{\partial x_i^*}{\partial K_i}}_{\text{Strategic Effect}}.$$

Note that, at the second period of the game, the entrant optimally chooses its decision variable x_e, implying that $\frac{\partial \pi_e}{\partial x_e} = 0$.

The incumbent's investment K_i may have a direct impact on the entrant's profit. For instance, if K_i is the value of the clientele accumulated by the incumbent before the entry of the other operator, a larger clientele reduces the size of the market and thus lowers the entrant's profit independently of the strategic effect. If the investment K_i modifies the incumbent's technology then there is no direct effect: $\frac{\partial \pi_e}{\partial K_i} = 0$.

The strategic effect comes from the fact that the choice of K_i at the first period of the game changes the incumbent's behavior ex post by $\frac{\partial x_i^*}{\partial K_i}$, thus affecting the entrant's profit in proportion to $\frac{\partial \pi_e}{\partial x_i}$.

The total effect is given by the addition of the direct and strategic effects. In the sequel, the incumbent operator is said to be *tough* when $\frac{d\pi_e}{dK_i} < 0$ and *soft* when $\frac{d\pi_e}{dK_i} > 0$. In order to deter entry, the incumbent wants to look tough.

4.3.3 Second strategy: Accommodation of entry

Suppose now that the incumbent finds it too costly to deter entry. Whereas the incumbent's first period behavior was dictated by the entrant's profit which had to be driven down to zero, it is now driven by the incumbent's profit. The impact of the first period investment on the incumbent's profit can be understood via the following expression

$$\frac{d\pi_i}{dK_i} = \underbrace{\frac{\partial \pi_i}{\partial K_i}}_{\text{Direct Effect}} + \underbrace{\frac{\partial \pi_i}{\partial x_i}\frac{\partial x_i^*}{\partial K_i}}_{=0} + \underbrace{\frac{\partial \pi_i}{\partial x_e}\frac{\partial x_e^*}{\partial K_i}}_{\text{Strategic Effect}}.$$

The logic underlying the different terms of this expression is the same as in the entry deterrence case.

4.3.4 Links between the two strategies and discussion

The sign of the strategic effect in the case of accommodation to entry can be related to the nature of the strategic interaction and to the investment making the incumbent tough or soft.[25] Indeed, we have

$$\frac{dx_e^*}{dK_i} = \frac{dx_e^*}{dx_i} \times \frac{dx_i^*}{dK_i} = R_e'(x_i^*) \times \frac{dx_i^*}{dK_i}.$$

[25]We also assume that $\frac{\partial \pi_i}{\partial x_e}$ and $\frac{\partial \pi_e}{\partial x_i}$ have the same sign. This arises in most models of oligopolistic competition and is not a restrictive assumption.

Then we obtain

$$\text{Sign}\left(\frac{d\pi_i}{dK_i}\right) = \text{Sign}\left(\frac{d\pi_e}{dK_i}\right) \times R_e'(x_i^*).$$

Whereas the strategy in the entry deterrence case is simply contingent on the investment affecting the strength of the incumbent, the strategy in the entry accommodation case is now more 'complex': It depends not only on the investment effect on the power of the incumbent but also on the strategic interaction (i.e., the sign of slopes of reaction functions).

To illustrate this point, consider the following game: Choice of differentiation level by the incumbent at the first stage, followed by price competition at the second stage. Assume that prices are strategic complements, a characteristic stemming from the demand and cost functions. The investment is taken here to be the choice of differentiation by the incumbent: This choice makes the incumbent soft. Therefore the incumbent has an incentive to overinvest: Indeed, over-investment at the first period involves a higher price for the incumbent at the second period; since prices are strategic complements, this entails a higher price for the entrant. An opposite conclusion holds when prices are strategic substitutes, since an increase in the incumbent's price triggers a decrease in the entrant's price.

Recall that the important assumption here is the credibility of the first period decision by the incumbent. Differently stated, it must bind the incumbent to act in a certain way. This implies that the entrant must anticipate that the incumbent will stick to this strategy. This will typically be the case if the investment decision involves substantial sunk cost, i.e., costs which are not retrievable.

When the credibility of the investment is not ascertained, which occurs for instance when the investment is interpreted as the decision to offer a new service and when the incumbent can decide to withdraw one of its services following the entry decision by the competitor, then the investment decision looses its commitment value. The purpose of the next section is to focus on such cases and to show that entry might nevertheless be deterred because of network externalities.

5 Entry, fixed costs and the role of network externalities

5.1 Introduction

The previous sections apply to any industry. We consider a situation which is more relevant for the actual railway industry. It involves an incumbent operating a network. The structure of the network generates a pattern of substitutabilities and complementarities between the different services offered by the incumbent. Depending on the interaction between these services, the incentives to enter the market by a potential entrant are different. An entrant is more (respectively, less) likely to enter a part of the network in which the incumbent offered multiple services which are substitutes (respectively, complements) from the viewpoint of passengers. With substitutes, the incumbent has incentives to withdraw from one market in order to soften the competition with the entrant. On the contrary, with complements, the incumbent benefits from competition in one segment on the remaining parts of the network through spillovers effects. It is therefore more likely to remain active on that segment than the entrant.

5.2 Entry and exit in the case of substitutes

5.2.1 The intuition behind the main result

An entrant is likely to invade a part of the network where the incumbent produces substitutable services. Indeed the incumbent has an incentive to withdraw from the market where entry occurred in order to reduce the impact of entry on the substitutable services that the incumbent continues to offer. This impact may come either from constraints that affect the pricing decisions of the incumbent (the interdependency or arbitrage constraint) or from simple effects of demand substitution between different types of services offered by the incumbent.

5.2.2 The model

We study the conditions under which an entrant may successfully enter a given segment of the railway network and the impact of such entry on the profitability of incumbent operator. The emphasis is on the role of the structure of the network operated by the incumbent.

The network structure and the pattern of demands. The railway network is highly simplified: There are only three cities denoted by A, B and C. All nodes are connected. There are three services: AB, BC and AC.

There are two types of demand for round trip services. First, some passengers travel between A and B or between B and C. The demand on Origin-Destination od is denoted by $D_{od}(p_{od})$ if the price on this market is p_{od}, $od \in \{AB, BC\}$. There are passengers who travel from A to C; the demand on this market is denoted by $D_{AC}(p_{AC})$ where p_{AC} is the price on this market. There are no substitutabilities among $O - D$ markets. Differently stated, passengers on a specific OD do not substitute this trip for another one.

The incumbent offers direct services between all city pairs, i.e., ODs.

Cost structure. Assume that, on each OD-market, a proportion γ of the cost for establishing a service between two cities is sunk. That is, γF is the sunk entry cost and $(1 - \gamma)F$ can be recovered upon exit, i.e., if the operator decides to withdraw its service for the pair of cities considered. Differently stated, γF is a fixed entry cost while $(1 - \gamma)F$ is a fixed production cost. We assume that exit costs are equal to zero.[26]

The incumbent (respectively, the entrant) operator has a marginal cost denoted c_i (respectively, c_e). The entrant is a priori more efficient than the incumbent, i.e., $c_e \leq c_i$.

[26]Our model could be easily modified to account for exit costs.

Market structure. Consider the following possible market structures. The benchmark is the monopoly situation where an incumbent operates the whole network. As already mentioned, competition is modelled in a simplified manner, namely à la Bertrand (i.e., price competition). As an alternative, we allow for entry by a competitor on a local market, AB for instance. Notice that on a given market/segment there is no differentiation between the services offered by the different operators.

This is represented in Figure 6.

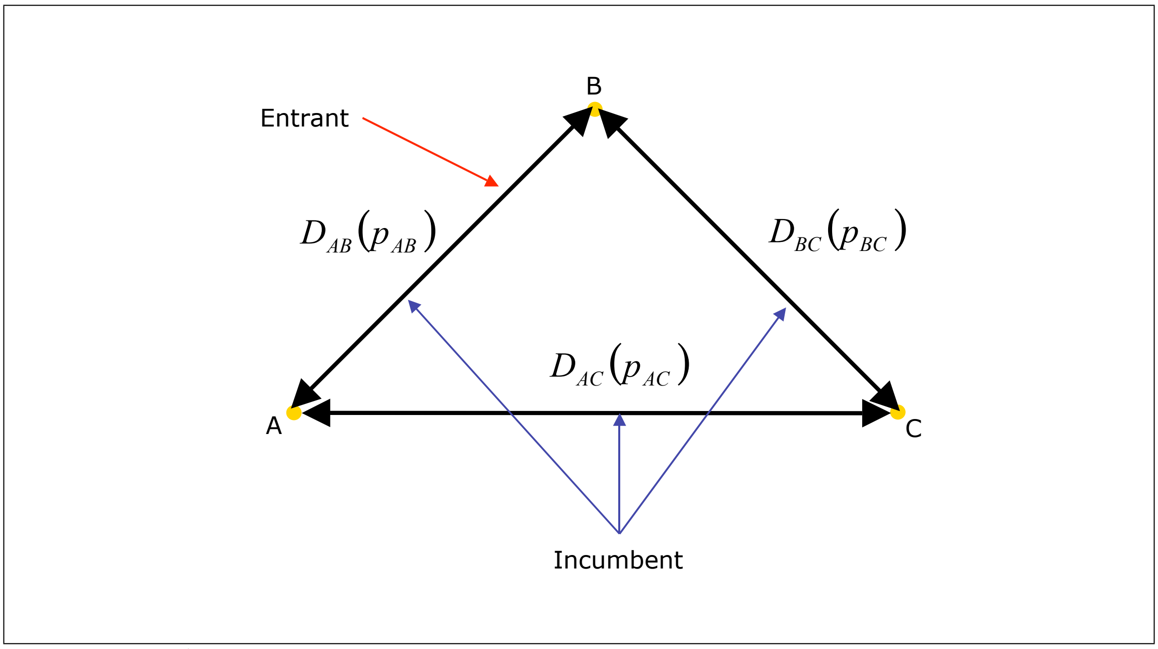

Figure 6: A three-city network: The incumbent operates the whole network, the entrant may enter only in a local market.

5.2.3 No entry

Consider the case in which the incumbent operator has a monopoly position over all the different markets. It sets the different prices p_{AB}, p_{BC} and p_{AC} in order to maximize its profit given by[27]

$$\pi_i^m = 2\,(p_{AB} - c_i)\,D_{AB}(p_{AB}) + 2\,(p_{BC} - c_i)\,D_{BC}(p_{BC}) + 2\,(p_{AC} - 2c_i)\,D_{AC}(p_{AC}) - 3F,$$

where superscript m stands for 'monopoly'. The first two terms represent the incumbent's profit on the local markets, the third one represents the profit on the AC-market. Since it is present on the three markets, the incumbent bears a total fixed cost equal to $3F$.

The monopolist incumbent sets the prices such that

$$\frac{p_{od}^m - c_i}{p_{od}^m} = \frac{1}{\epsilon_{od}(p_{od}^m)} \qquad \text{for } od \in \{AB, BC\},$$

[27] In this model, we consider that the marginal cost of AC is twice the marginal cost of AB or BC.

and

$$\frac{p^m_{AC} - 2c_i}{p^m_{AC}} = \frac{1}{\epsilon_{AC}(p^m_{AC})}.$$

These rules are standard: The monopoly imposes a price such that the relative markup of the price over the marginal cost is inversely proportional to the demand elasticity.

When demands for each OD markets are identical, $D_{AB} = D_{BC} = D_{AC} \equiv D$, assuming that the elasticity $\epsilon(p)$ of demand $D(.)$ is (weakly) increasing in the price p, it can be shown that

$$p_{AB} + p_{BC} \geq p_{AC}$$

holds for the monopoly prices p^m_{od}, $od \in \{AB, BC, AC\}$.[28] Herein, we call this inequality on the prices the interdependency constraint (IC). It says that the AC-passengers are not willing to substitute the direct AC trip for an indirect "AB plus BC" trip. Note that this result would be reinforced had we assumed that passengers from A to C with a change in B must bear a positive waiting cost.

This interdependency constraint might become binding when demands on the different markets are not equal. If demands are different between the AC-market and the local markets AB and BC, then we could obtain that the monopoly price on the AC market is much larger than the sum of the monopoly prices on the AB and BC markets. When this is the case, the interdependency constraint is not satisfied by the previous prices; the optimal monopoly prices constrained by the interdependency constraint are defined as follows

$$\frac{\tilde{p}^m_{od} - c_i}{\tilde{p}^m_{od}} = \frac{1}{\epsilon_{od}(\tilde{p}^m_{od})} + \frac{\lambda}{\epsilon_{od}(\tilde{p}^m_{od}) D_{od}(\tilde{p}^m_{od})} \qquad \text{for the local market } od \in \{AB, BC\},$$

$$\frac{\tilde{p}^m_{AC} - 2c_i}{\tilde{p}^m_{AC}} = \frac{1}{\epsilon_{AC}(\tilde{p}^m_{AC})} - \frac{\lambda}{\epsilon_{AC}(\tilde{p}^m_{AC}) D_{AC}(\tilde{p}^m_{AC})} \qquad \text{for the long distance market,}$$

where $\lambda \geq 0$ is the Lagrange multiplier associated to the interdependency constraint. This variable gives the shadow cost of the constraint, i.e., how much the incumbent looses profit by distorting the prices away from the unconstrained monopoly prices in order to satisfy the interdependency constraint. The larger λ is, the more costly it is for the incumbent to ensure that the interdependency constraint is satisfied.

The impact of the interdependency constraints on the final prices is clear: the prices on the local markets increase, $\tilde{p}^m_{od} \geq p^m_{od}$ for $od \in \{AB, BC\}$, and the price on the long distance market decreases, $\tilde{p}^m_{AC} \leq p^m_{AC}$. The extent of the increase on a given local market depends both on the elasticity of the demand associated with this market and on the size of this market. If the demand elasticity in the AB market is very large, i.e., passengers in this market are highly sensitive to small price variations, the incumbent does not distort too much this price and ensures that the interdependency constraint is satisfied through distortions on the other markets.

[28] Consider that demands are identical. From the first-order conditions, $2p_{od}(1 - 1/\epsilon(p_{od})) = p_{AC}(1 - 1/\epsilon(p_{AC}))$. As the price elasticity is nondecreasing, then $2p_{od} \leq p_{AC}$.

5.2.4 Entry on the local market

Now that a potential competitor enters the local market AB. This entrant only enters a local market.

The incumbent may either adapt to entry or withdraw from the local market. In the former case, head-to-head price competition on the local market AB takes place. In the latter case, the incumbent is present on the AC and BC markets while the entrant has a monopoly position on AB.

The incumbent withdraws from the local market where entry occurred. Specifically, the incumbent decides to withdraw from the AB market following the entry. The entrant faces no constraints and will offer a price p^e_{AB} given by

$$\frac{p^e_{AB} - c_e}{p^e_{AB}} = \frac{1}{\epsilon_{AB}(p^e_{AB})}.$$

Obviously, we have $p^e_{AB} \geq c_e$.

Concerning the incumbent, different cases may occur.

- *Small efficiency differential:* When the entrant is slightly more efficient than the entrant, then we have p^e_{AB} close to p^m_{AB}. Then, if the interdependency constraint were not binding before entry, it is likely that this constraint is not binding after entry. In that case, the incumbent enjoys the unconstrained monopoly profit on the BC and AC markets and saves on the fixed cost of production since it has withdrawn from the AB market. If the interdependency constraint were initially binding, then it is likely to be still binding after entry. Since the incumbent has withdrawn its service on the AB market, it has less instrument at his disposal to ensure that this constraint is satisfied: this implies that the distortion on the BC and AC markets is amplified.

- *Large efficiency differential:* Consider now that the entrant is much more efficient than the incumbent. In that case, even the monopoly price of the entrant on the AB market might be relatively small as compared to the monopolist incumbent's price. Therefore, the interdependency constraint is likely to be binding after entry, even if it was not the case before entry took place.

From the incumbent's point of view, the cost associated with the entry of a competitor not only involves the loss of the market (when the incumbent withdraws) but the effects on the different markets which are related by the interdependency constraint.

For further reference, note that provided that the entrant's monopoly price on AB is not too different from the incumbent's monopoly price, the incumbent's pricing decisions on the other markets might not be affected: The incumbent looses the AB market but saves on the fraction of fixed cost that can be recovered without distorting too much the prices on the other markets.

The incumbent stays on the local market where entry occurred. Specifically the incumbent decides to stay on the AB market following the entry by the competitor. Price competition on the homogenous good AB market occurs and is fierce. The competitor with the lowest price wins the entire market.

First, suppose that the interdependency constraint is not binding whatever the price on the AB market. Since markets are independent, the outcome is easy to draw: On the AB market, price competition between the incumbent and the entrant leads the entrant to win the entire market at a price $p^e_{AB} = c_i$. If the entrant is more efficient than the incumbent, it can always undercut the incumbent. In this case, the incumbent makes no profit on the AB segment, still enjoys a (unconstrained) monopoly position on the other markets, but has to pay the totality of the fixed cost (entry and production fixed costs). In other words, assuming that the interdependency constraint is not binding, the incumbent has always an incentive to withdraw from the AB market since it is not able to save on the fixed cost.

Second, assume that the interdependency constraint is binding. Note that the incumbent cannot propose a price p^i_{AB} on the AB-market that is smaller than its marginal cost c_i. Therefore, when the entrant is more efficient than the incumbent, it is always able to undercut the incumbent by proposing a lower price on the AB market and still makes a positive profit on that market. Now suppose that the incumbent offers a price on the AB-market such that $p^i_{AB} = \tilde{p}^m_{od}$; then the entrant undercuts the incumbent's price and offers p^e_{AB} slightly below $\tilde{p}^m_{ij}$. However, the incumbent would have an incentive to undercut the entrant's price since it would win the AB-market at a price strictly below $\tilde{p}^m_{od}$ which, since the AB-price would be reduced, would entail larger distortions on the other markets. Therefore, there exists a threshold $\hat{p}^i_{AB}$, with $\tilde{p}^m_{AB} > \hat{p}^i_{AB} > c_i$, above which the incumbent is not willing to win the entire AB-market. A low price on AB implies larger distortions both on AC and BC in order that the interdependency constraint be satisfied.

Therefore, when the incumbent remains on the AB market, it looses that market. The entrant then offers a price which enable to win the market, i.e., $p^e_{AB} = \hat{p}^i_{AB}$.

The entry and exit decisions. From the above analysis, provided that (i) the entrant is more efficient than the incumbent or (ii) the incumbent is constrained by the interdependency constraint, the entrant enters the local market AB and the incumbent will withdraw its services from this market.

The intuition goes as follows. If the incumbent stays on the local AB-market, then Bertrand competition (with perfectly substitutable services) on the AB-market drives prices to marginal cost. Therefore, even if it wins the market, the incumbent makes no profit on that local market since the price is equal to the marginal cost; simultaneously, such a low price on the local AB market strongly affects the pricing decisions on the other markets when the interdependency constraint is binding. Consequently, a withdrawal from the AB-market would enable the incumbent not only to save on the fraction of fixed costs which are retrievable, but would also triggers a higher price on the AB market since the entrant would enjoy a monopoly position, thereby giving more freedom to the incumbent in its pricing decisions on the other markets. This holds even if both competitors have the same efficiency.

5.2.5 Discussion and extension

Summary 8: The withdrawal decision by the incumbent should take into account not only the direct gains (such as the saving on part of the fixed costs) or direct costs (such as the loss of the AB market) but also the opportunity cost and gain associated to such a decision. In our model, there is an opportunity loss of staying on the local market AB and competing head-to-head with the entrant since this might force the entrant's price to be very low, thereby limiting the pricing decisions of the incumbent on other markets.

Entry when the incumbent offers multiple substitutable services without the capacity to credibly commit. The same insight appears in a model in which the incumbent is a multiproduct firm that produces imperfectly substitutable services. Indeed, consider the following modification to the previous model.

The incumbent faces an heterogenous population of passengers. Some passengers have relatively large valuations of time and are willing to pay a high price in order to benefit from a fast transport service between two cities; other passengers have relatively low valuations of time and are willing to travel with a slower train for a lower price. Another possibility is to consider that the incumbent offers different time slots for the same trip.

In those cases, the incumbent produces multiple products which are imperfect substitutes from the passengers' viewpoint. The fast/high quality service offered by the incumbent might attract some passengers when the price differential becomes not too large; conversely, the slow train might attract some passengers with high valuations of time if the price differential starts being too large. Similarly, some passengers might substitute one time slot to the other if the price differential starts to be too important. To simplify, let us denote these two substitutable services A and B.

Under monopoly, the incumbent sets its price for the different substitutable products by accounting for the externalities across services.

Now consider the possibility of entry by a competitor on one of the services, say product A, offered by the incumbent operator.

If the incumbent stays on the market of product A, then price competition on that market between the entrant and the incumbent leads to a low price on that market. If the incumbent decides to stay on the market, then it becomes likely that both operators will suffer losses on that market. [For this analysis, it is only required that price competition on one market leads to sufficiently low profits on that market for the operators.]

The important point is that the incumbent has more incentive to withdraw from the A-market than the entrant. Indeed, consider the entrant first. If it exits, it can retrieve part of the total fixed cost. Consider now the incumbent. If it withdraws, it can recoup part of the fixed cost of production, as does the entrant. However, since the situation is now a differentiated oligopoly (the entrant produces the A-service, the incumbent the B-service, and services are substitutes), the entrant's price on the A-market will be equal to the corresponding monopoly price and will therefore be larger than the price that would prevail on that market under Bertrand competition. Since services are substitutes, the increase in the price of the A-market leads to an increase of the demand on the B-market, where the incumbent enjoys a monopoly position. Therefore, such a withdrawal by the incumbent benefits the incumbent on the B-market. Anticipating this, the entrant effectively stays on the A market leading the incumbent to exit from that market.

The lesson from the analysis can be summarized as follows. The incumbent cannot preempt entry by entering all markets first if exit costs are sufficiently small, if services are sufficiently substitutes from the passengers' viewpoint, and if post-entry competition is sufficiently intense. Similarly, preempting entry by crowding out the products spectrum is not possible when the commitment to remain present on the different substitutable services is not credible.[29]

A by-product of this analysis is the following. When an incumbent cannot commit to stay on the different markets of substitutes that it offers, an entrant may find that preferable to enter on a service already offered by the incumbent than on a new service which would be only imperfectly substitutable with the ones offered by the incumbent. The reason is that such a strategy reinforces the incumbent's incentive to withdraw from the market where entry occurred. Indeed, if entry occurred on a differentiated service, then the impact of the substitutable markets is less important than if entry occurred on a service already offered by the incumbent.

5.3 Entry and exit in the case of complements

Wel now address another issue, which also concerns the role of the network structure on the incentive to enter.

It has been acknowledged that the railways network has the feature of a so-called hub-spoke network. We shall build on a modified version of the model developed in the previous subsection to highlight some of the implications of this characteristics.[30]

Basically, we keep the structure of the network (with three cities denoted by A, B, and C). We simplify the demand pattern by assuming that the round trip demands for each city-pair markets are identical (i.e., $D_{AB} = D_{AC} = D_{BC} \equiv D$). The cost structure is the same. The important change is that there is no direct service from A to C: In order to travel from A to C, passengers must stop at B. Therefore, a train service from A to C may also be used to transport passengers from A to B and from B to C.

As previously, we consider the impact of entry on a local segment AB by a competitor on the incumbent incentive to withdraw or stay on the local market.

The structure of the model is shown in Figure 7.

[29]See Judd (1985) for a more detailed model.

[30]A more detailed analysis can be found in Hendricks, Piccione and Tan (1997).

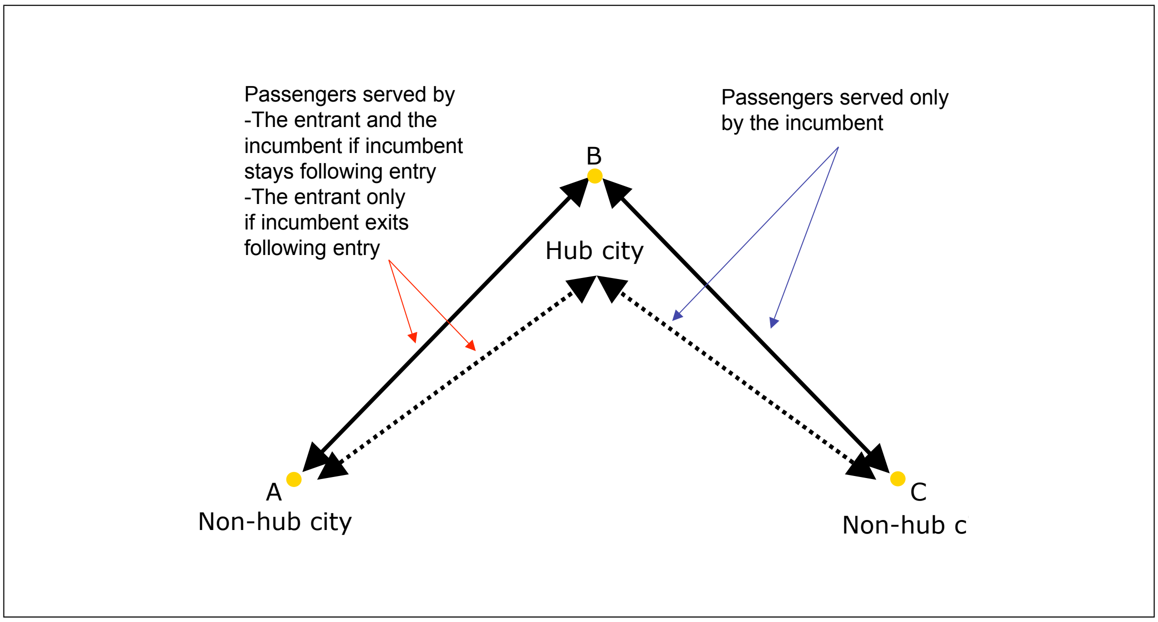

Figure 7: A three-city network with a hub-city and no direct trip between A and C.

5.3.1 Main result and intuition

The main result can be stated as follows. In a network with a hub-spoke structure, the incumbent has a strategic advantage over the potential entrant and may not withdraw from the local market where entry occurred. As a result, entry is likely to be deterred when the potential entrant does not enjoy a substantial cost advantage over the incumbent.

Intuitively, entry on the local market AB has a spillover effect on the connecting market AC. With the new specification of the services, the network exhibits a pattern of complementarities between the incumbent's and the entrant's services. Therefore, the logic of the argument developed in Section 4.2 is reversed. By staying on the market where entry occurred, the incumbent engages in a vigorous price competition (which also erodes the profit of the winning operator on that market). Since connecting services are complements due to the hub-spoke feature of the network, the price competition increases the demand on the complementary markets. Indeed, when the entrant wins the AB-market[31], two cases appear: (i) if the incumbent exits, then the entrant enjoys a monopoly position on the AB-market and sets a high price on that market and on the connecting markets jointly serviced with the incumbent; (ii) if the incumbent stays on the AB-market and shares the connecting markets with the entrant, then it forces the entrant to reduce its prices. The incumbent prefers the latter strategy to the former since the smaller price on the AB-market increases the demand in the connecting and complementary markets in which the incumbent still enjoys a monopoly position.

[31]Recall that, on the AB-market, the operators are offering perfectly homogenous services. Therefore, Bertrand competition on that market leads to only one operator servicing the whole demand. However, we would obtain a similar conclusion if, for instance, the competition between the incumbent and the entrant is less 'extreme'.

Summary 9: In a network with a hub-spoke structure the incumbent has a strategic advantage over the potential entrants and may not withdraw from the local market where entry occurs. Entry is therefore likely to be deterred when the potential entrants do not enjoy a substantial cost advantage over the incumbent and when the irretrievable entry cost are sufficiently high.
Intuitively, entry on a local market has a positive spillover effect on the connecting markets served by the incumbent when the network exhibits a pattern of complementarities between the incumbent's and the entrant's services, which arises when the network is a hub-spoke. By staying on the market where entry occurred, the incumbent engages in a vigorous price competition (which also erodes the profit of operators on this market). Since connecting services are complements due to the hub-spoke feature of the network, the price competition increases the demand on the complementary markets. When the entrant enters, two cases appear: (i) If the incumbent exits, then the entrant enjoys a monopoly position on its market and sets a high price on this market and on the connecting markets jointly serviced with the incumbent; (ii) If the incumbent stays on the market where entry has occurred and shares the connecting markets with the entrant, then it forces the entrant to reduce its prices. The incumbent prefers the latter strategy to the former since the smaller price on the market where entry occurs increases demand in the connecting and complementary markets in which the incumbent still enjoys a monopoly position.

5.3.2 The model

Prices are based upon cities of origin and destination. We denote by p^e_{od} the price that the entrant charges to od-passengers. If those passengers travel with the incumbent exclusively then they pay p^i_{od}; if they use both operators then they pay $p^e_{od} + s^i_{od}$ where s^i_{od} is the price charged by the incumbent for its segment of the trip.

To summarize, AB-passengers can either travel with the incumbent and pay p^i_{AB} or they have the option to travel with the entrant and pay p^e_{AB} (if entry occurred of course). BC-passengers can only travel with the incumbent in which case they are charged p^i_{BC}. Finally, AC-passengers have the possibility to travel with the incumbent only and pay p^i_{AC} or to use both operators and pay $p^e_{AC} + s^i_{AC}$ for the total travel.

5.3.3 Assumptions

Let $p_m(c_k)$ and $\pi_m(c_k)$ be the price and the monopoly profit respectively associated to a marginal cost c_k.

Assume that the monopoly profit from the direct traffic between two cities is not large enough to cover the fixed cost of supplying that service. To ensure profitability, the operators have to service other city-pair markets as well.

Finally, and not surprisingly, the strategic interaction plays an important role in this analysis. This strategic interaction between the operators' decision variables arises when the operators jointly service the AC-market. In that case, the profit (gross of fixed cost) of the incumbent on that market is $(s^i_{AC} - c_i)D(s^i_{AC} + p^e_{AC})$ and the entrant's profit (gross of fixed cost) is given by $(p^e_{AC} - c_i)D(s^i_{AC} + p^e_{AC})$. The first-order condition associated to, say, the entrant's profit maximization problem yields

$$(p^e_{AC} - c_i)D'(s^i_{AC} + p^e_{AC}) + D(s^i_{AC} + p^e_{AC}) = 0. \tag{9}$$

The second-order condition associated to this maximization problem is

$$(p^e_{AC} - c_i)D''(s^i_{AC} + p^e_{AC}) + 2D'(s^i_{AC} + p^e_{AC}) \leq 0. \tag{10}$$

Totally differentiating (9) and using (10), we obtain

$$\frac{dp^e_{AC}}{ds^e_{AC}} \propto (D')^2 - DD''. \tag{11}$$

Therefore, prices are strategic complements when the demand is linear or, more generally, not too convex. If demand is sufficiently convex, then prices are strategic substitutes. To be consistent with the previous part of our analysis, assume that prices are strategic complements. The analysis remains unchanged qualitatively in the case of strategic substitutes.[32]

In addition, assume that the efficiency differential between the incumbent and the entrant is not too large, implying that the monopoly price of the entrant is larger than the marginal cost of the incumbent, i.e., $p_m(c_e) > c_i$.

5.3.4 No entry: The incumbent has a monopoly position over the whole network

In that case, the monopolist incumbent's profit maximization problem has a simple solution. It consists in charging the monopoly price associated to marginal cost c_i in each direct market, i.e. $p^i_{AB} = p^i_{BC} = p_m(c_i)$, and the monopoly price corresponding to twice the marginal cost in the connecting market AC, i.e., $p^i_{AC} = p_m(2c_i)$.

From the analysis in Subsection 5.2, we know that some constraints must be satisfied, in particular those that ensure that AC-passengers are not willing to buy two tickets, one for the AB-travel the other for the BC-travel, instead of the AC-ticket. This imposes the following constraint to be satisfied

$$p^i_{AB} + p^i_{BC} \geq p^i_{AC}. \tag{12}$$

It is immediate to show that this constraint is indeed satisfied for the monopoly prices defined previously when the demand elasticity is increasing in the price. Therefore,

[32] See Hendricks, Piccione and Tan (1997).

the monopoly profit under no entry is

$$\pi_i^{\text{no entry}} = 4\pi_m(c_i) + 2\pi_m(2c_i) - 3F.$$

5.3.5 Entry on a local segment: The incumbent withdraws its services from the market where entry occurred

We consider now that a potential entrant has decided to enter the AB-market and that the incumbent reacts by withdrawing its services from that segment. Each operator is then a monopoly on the local segment that it services: AB for the entrant and BC for the incumbent. As concerns the AC-segment, the operators jointly service this travel.

The profit of the entrant can be written as follows

$$\pi_e = 2\,(p^e_{AB} - c_e)\,D(p^e_{AB}) + 2\,(p^e_{AC} - c_e)\,D(p^e_{AC} + s^i_{AC}) - F.$$

Note that we implicitly assumed that the entrant is able to discriminate between the AB-passengers and the AC-passengers that use the entrant's service on the AB-segment of their trip. Moreover, we must ensure that

$$p^e_{AB} \geq p^e_{AC} \tag{13}$$

since AC-passengers always have the possibility to claim that they are AB-passengers only.

Now consider the incumbent. Its profit can be rewritten as follows

$$\pi_i = 2\left(p^i_{BC} - c_i\right)D(p^i_{BC}) + 2\left(s^i_{AC} - c_i\right)D(p^e_{AC} + s^i_{AC}) - F - \gamma F.$$

The first term represents the incumbent's profit in the local market where it is a monopolist; the second term represents its profit on the AC-market jointly serviced with the entrant. Finally, since the incumbent has withdrawn from the AB segment, it saves a fraction $1-\gamma$ of the fixed cost F that it would have paid if it were staying on that segment.

As for the entrant, the incumbent must ensure that the following constraint is satisfied

$$p^i_{BC} \geq s^i_{AC}. \tag{14}$$

The important feature of the price equilibrium in that configuration is that *the entrant's price in every market exceeds the marginal cost of the incumbent*: $p^e_{AB} > c_i$ and $p^e_{AC} > c_i$. Indeed, remember that prices are strategic complements. This implies that the best-response of the entrant is strictly increasing and starts from the monopoly price $p_m(c_e)$. Therefore, in equilibrium, we have $p^e_{AC} \geq p_m(c_e)$ and $p^e_{AB} \geq p_m(c_e)$. Using the assumption $p_m(c_e) > c_i$, we obtain the result.

Since best-responses are increasing and equilibrium prices in the shared AC-market are above the monopoly prices, the constraints defined in (14) and (13) are binding at equilibrium. Therefore, *competition in the shared market AC spills over into the direct markets AB and BC* since the incumbent cannot discriminate between BC- and AC-passengers and must charge them the same price.

5.3.6 Entry on a local segment: The incumbent stays on the market where entry occurred

It remains to study the case in which entry occurs on AB and the incumbent does not withdraw its service from that market. The first point to notice is that on the AB market the entrant and the incumbent offers perfectly substitutable products. Therefore, the operator that offers the lowest price wins the entire market (Bertrand competition). The situation in the AC-market is slightly more complex. Indeed, the incumbent can decide to share the market with the entrant or to service the market alone, depending on how it sets its price p^i_{AC} higher or lower than $p^e_{AC} + s^i_{AC}$.

The entrant's profit can be written as follows

$$\begin{aligned}\pi_e = 2 \times \mathbb{1}_{\{p^e_{AB} \leq p^i_{AB}\}} \left(p^e_{AB} - c_e\right) D\left(\min\{p^e_{AB}, p^i_{AB}\}\right) \\ + 2 \times \mathbb{1}_{\{p^e_{AC}+s^i_{AC} \leq p^i_{AC}\}} \left(p^e_{AC} - c_e\right) D\left(\min\{p^e_{AC} + s^i_{AC}, p^i_{AC}\}\right) - F,\end{aligned}$$

where $\mathbb{1}$ is the indicator function[33]. The entrant chooses the price p^e_{AB} and possibly the price p^e_{AC} when the incumbent shares the AC-market with the entrant, subject to the constraint (13).

Consider now the incumbent profit maximization's problem. Its profit can be written as follows

$$\begin{aligned}\pi_i =& 2 \times \left(1 - \mathbb{1}_{\{p^e_{AB} \leq p^i_{AB}\}}\right) \left(p^i_{AB} - c_i\right) D\left(\min\{p^e_{AB}, p^i_{AB}\}\right) \\ &+ 2 \times \mathbb{1}_{\{p^e_{AC}+s^i_{AC} \leq p^i_{AC}\}} \left(s^i_{AC} - c_i\right) D\left(\min\{p^e_{AC} + s^i_{AC}, p^i_{AC}\}\right) \\ &+ 2 \times \left(1 - \mathbb{1}_{\{p^e_{AC}+s^i_{AC} \leq p^i_{AC}\}}\right) \left(p^i_{AC} - 2c_i\right) D\left(\min\{p^e_{AC} + s^i_{AC}, p^i_{AC}\}\right) \\ &+ 2 \times \left(p^i_{BC} - c_i\right) D(p^i_{BC}) - 3F.\end{aligned}$$

The first term represents the profit on the market where entry occurred if the incumbent's price is smaller than the entrant's price. The second term is the incumbent's profit on the AC-market in a shared configuration, i.e., when the incumbent and the entrant jointly service that market. By contrast, the third term is the incumbent's profit on the long distance AC-market when it does not share that market with the entrant. Finally, the fourth and last term is the profit earned on the market where the incumbent benefits from a monopoly position. The incumbent chooses its prices in order to maximize its profit subject to the constraints (12) and (14).

The important result that we obtain can be stated as follows: The presence of a perfect substitutes in the AB-market implies that the entrant's prices in markets AC and AB cannot exceed the marginal cost of the incumbent. The proof goes along the following steps.

First, we show that $p^e_{AC} \leq c_i$. Indeed, if $p^e_{AC} > c_i$, then $p^i_{AC} < p^e_{AC} + s^i_{AC}$ since the incumbent can always obtain the entire AC-market and increase its by lowering p^i_{AC}. The constraints (14) and (13) implies that $p^i_{BC} + p^e_{AB} \geq s^i_{AC} + p^e_{AC}$. Therefore, $p^i_{BC} + p^e_{AB} > p^i_{AC}$.

Consequently, the incumbent can always undercut the entrant's price in the AB-market without violating the constraint (12). The entrant can also undercut

[33] $\mathbb{1}_{\{a \leq b\}}$ is equal to 1 if $a \leq b$ and 0 otherwise.

the incumbent's price in the AB-market and possibly make positive profits in the AC market setting $p^e_{AC} = p^e_{AB}$ to satisfy (13). This implies that $p^e_{AB} \leq c_i$ and a contradiction is obtained by (13). Thus, $p^e_{AC} \leq c_i$.

Second, we show that $p^e_{AB} \leq c_i$. Suppose that $p^e_{AB} > c_i$. Then, we have to consider two cases. In the first, $p^i_{AB} \geq p^e_{AB}$ and the incumbent makes zero profits in the AB-market. If $p^i_{AC} \leq p^e_{AC} + s^i_{AC}$, then it follows from (14) and the above result that $p^i_{AC} \leq c_i + p^i_{BC}$. Thus the incumbent can make positive profits by undercutting the entrant's price in the AB-market without violating the constraint (12). If $p^i_{AC} > p^e_{AC} + s^i_{AC}$, then set $p^i_{AC} = p^e_{AC} + s^i_{AC}$ and repeat the above argument. In the second case, $c_i \leq p^i_{AB} < p^e_{AB}$. The entrant can increase profits by setting $p^e_{AB} = p^i_{AB}$ without violating (13) since $p^e_{AC} \leq c_i$. In both cases, a contradiction is obtained.

Notice that we are not explicit on the nature of the price equilibrium that emerges in this game. In general, either an accommodating equilibrium emerges, in which the incumbent shares the AC-market with the entrant, or an isolating equilibrium emerges, in which the incumbent does not share the AC-market. If the fixed cost F is sufficiently large, then it is natural to assume that the entrant makes losses in an isolating equilibrium: if the entrant only services the local market AB then it will not be able to make positive profits. Therefore, in the following, we shall concentrate on the accommodating equilibrium.

5.3.7 The strategic advantage of the incumbent created by the complementarities of services on the network

We consider now the exit decision by the incumbent and the entry decision by the potential competitor.

From the previous analysis, it appears that if the incumbent withdraws its service from the AB-market, then this has the following consequences

- First, it enables the incumbent to save on the part of the fixed cost which is retrievable, $(1 - \gamma)F$.
- Second, since it withdraws from that market, it looses any potential profits that it could have earned if it were staying and competing with the entrant on that market. However, since we assumed that competition is à la Bertrand, there is no differentiation between the incumbent's and the entrant's services on the AB-market, and the entrant is more efficient than the incumbent, even if the incumbent stays on the AB-market it will not make any positive profits on that market.
- Third, this leads the entrant to increase its price in the connecting AC-market which is shared with the incumbent. This has a negative impact on the incumbent's profit since these services are demand complements.

Therefore, depending on parameters values, the incumbent might decide to withdraw or not. If the fixed production cost $(1 - \gamma)F$ is large, then it becomes more interesting for the incumbent to withdraw from the market where entry occurred. More importantly, if the size of the connecting market AC is sufficiently large, then the incumbent has no incentive to exit the local AB market since this would triggers an increase in price of the entrant on the AC market.

Note that the logic is the opposite to the one developed in Section 5.2. There we argued that a multiproducts incumbent, offering substitutable services from the viewpoint of passengers, has an incentive to withdraw from the market where entry occurred since this leads to an increase in the demand of the market where the incumbent remains present. In the current analysis, we already noticed that under entry, the incumbent's and the entrant's services are complementary from the viewpoint of passengers. Therefore, the incumbent has an incentive to stay active on the market where entry occurred since the intense competition on the local market will force the incumbent to reduce its price on the connecting market (which is jointly serviced by both operators); this strategy has a positive spillover effect for the incumbent on the connecting markets.

Now consider the entry decision by the potential competitor. Since the incumbent will decide to stay on the AB-market, if the competitor decides in turn to enter, then its price p^e_{AC} on the connecting AB-market will be capped by the marginal cost of the incumbent. Given our assumption that the monopoly profit on the local AB-market $\pi_m(c_e)$ does not enable the entrant to cover the total fixed cost F, we assume the price equilibrium that will realize ex post is a shared market equilibrium in which the incumbent and the entrant service jointly the AC-market. Then, we obtain the following conclusions:

- If the efficiency differential between the entrant and the incumbent is sufficiently small, then the potential competitor decides not to enter the local market. Indeed, in that case, the prices charged by the entrant cannot exceed the marginal cost of the incumbent; therefore, since the entrant's marginal cost is above but close to the incumbent's marginal cost, the entrant's profit will not enable to cover the fixed costs of production and entry.

- If the efficiency differential is sufficiently large, then the potential competitor may decide to enter. Entry is possible only in this case.

5.3.8 Discussions

Consider a more complex network. What is needed for this analysis is the hub-spoke feature of the network operated by the incumbent. If there are more than three cities but still one single hub city, then the result presented above are reinforced, because the complementarities created by the network effects are reinforced. Indeed, we argued that by staying on the local market where entry occurred, the incumbent triggers an intense price competition on that local market, which spreads out on every connecting markets that the entrant may serve. Therefore, the larger is the number of connecting markets from the hub city, the stronger will be the network externalities and the stronger become incentives of the incumbent to stay on the market where entry has occurred. This is represented in Figure 8.

This suggests that entry is likely to occur on the 'rim', which is possible and profitable only if the direct traffic between the two non-hub cities is sufficiently large.

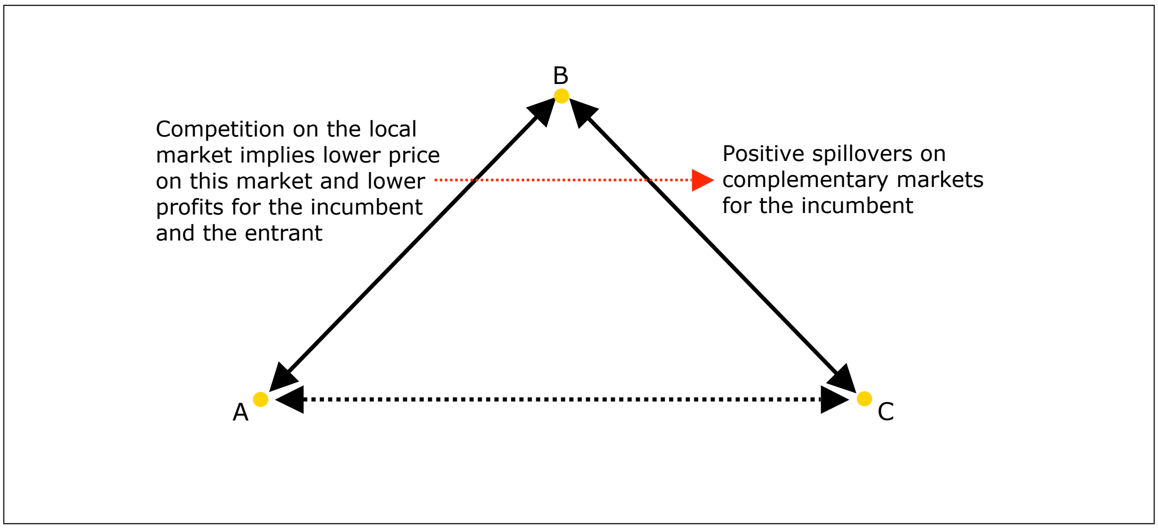

Figure 8: Entry with competition in a local market creates spillover on complementary markets served by the incumbent.

6 References

Bain, J., 1956, *Barriers to new competition*, Cambridge, Mass.: Harvard University Press.

Bonanno, G., 1986, "Vertical Differentiation with Cournot Competition", *Economic Notes*, 15: 68-91.

d'Aspremont, C., J. Gabszewicz and J.-F. Thisse, 1979, "On Hotelling's Stability in Competition", *Econometrica*, 17: 1145-1151.

Gal-Or, E., 1983, "Quality and Quantity Competition", *Bell Journal of Economics*, 14: 590-600.

Gabszewicz, J. and J.-F. Thisse, 1979, "Price Competition, Quality and Income Disparities", *Journal of Economic Theory*, 20: 340-359.

Gabszewicz, J. and J.-F. Thisse, 1980, "Entry (and Exit) in a Differentiated Industry", *Journal of Economic Theory*, 22: 327-338.

Hendricks, K., M. Piccione and G. Tan, 1997, "Entry and Exit in Hub-spoke Networks", 1997, *Rand Journal of Economics*, 28(2): 291-303.

Hotelling, H., 1929, "Stability in Competition", *Economic Journal*, 39: 41-57.

IDEI, 2003, "The Economics of Passenger Rail Transport: A Survey".

Judd, K, 1985, "Credible Spatial Preemption", *Rand Journal of Economics*, 16(2): 153-166.

Preston, J., G. Whelan and M. Wardman, 1998, "An Analysis of the Potential for On-track Competition in the British Passenger Rail Industry", *Journal of Transport Economics and Policy*, vol. 33(1): 77-94.

Shaked, A. and J. Sutton, 1982, "Relaxing Price Competition through Product Differentiation", *Review of Economic Studies*, 49: 3-13.

Shaked, A. and J. Sutton, 1983, "Natural Oligopolies", *Econometrica*, 71: 1469-1484.

Tirole, J., 1988, *Théorie de l'Organisation Industrielle*, 2 tomes, Eds Economica.