THÈSE

Envuedel'obtentiondu

DOCTORAT DE L'UNIVERSITE DE TOULOUSE

Délivré par l'Université Toulouse Capitole

Écoledoctorale : **Sciences Economiques-Toulouse School of Economics**

Présentée et soutenue par

HERESI José Ignacio

le4 mai 2020

A Multi-Product Model of Credit Cards with Naive Consumers

Discipline : **Sciences Economiques**

Unitéderecherche:**TSE-R(UMR CNRS 5314 – INRA 1415)**

Directeur de thèse: Monsieur Wilfried SAND-ZANTMAN, Professeur, Université Toulouse 1 Capitole

JURY

Rapporteurs Monsier Gary BIGLAISER, Professeur, University of North Carolina at Chapel Hill.
Monsieur Thibaud VERGE, Professeur, ENSAE-CREST

Suffragants Monsieur Wilfried SAND-ZANTMAN, Professeur, Université Toulouse 1 Capitole
Monsieur Andrew RHODES, Professeur, Université Toulouse 1 Capitole
Monsieur Regis RENAULT, Professeur, Université de Cergy-Pontoise

A multi-product model of credit cards with naive consumers*

José Ignacio Heresi†

January 2020

Abstract

We analyze the pricing problem of credit card issuers when setting per-transaction fees for payments and interest rates for the credit associated with these cards. By considering the issuer's incentives coming from the credit market, we provide a new explanation to the widely observed phenomenon of credit card rewards. Issuers induce higher demand for the credit function of their cards by lowering per-transaction fees, even to negative levels. In addition, by assuming that some consumers face a form of behavioral bias, consistent with recent empirical findings in financial markets, we develop a new explanation for the interest rate exhibiting some degree of stickiness in this market. Finally, we argue that interest rates are independent of interchange fees and that interchange fee regulation should take into account the costs and benefits of different forms of credit.

Keywords: credit card rewards, interchange fees, interest rates.

JEL Classifications: L11, E42.

1 Introduction

We study the relationship between the price charged by credit card issuers for the payment feature of their cards and the interest rate associated with the "revolving" credit feature of those cards.[1] Previous literature has often omitted the issuers'

*I'm grateful to Wilfried Sand-Zantman, Yassine Lefouili, Bruno Jullien, Martin Peitz, the participants of the EARIE 2016 conference in Lisbon, for helpful comments and suggestions. I would like to acknowledge financial support from CONICYT "Becas Chile" and the Jean-Jacques Laffont Foundation.

†Ph.D. Student, Toulouse School of Economics, University of Toulouse Capitole, Toulouse, France: jheresig@gmail.com.

[1]Revolving credit refers to a product that charges a given interest rate based on outstanding credit balance on credit accounts.

incentives arising from interest rate revenues, and we argue that such incentives can explain some of the observed outcomes in this market, such as consumer rewards and interest rate stickiness. In fact, about 40% of the revenues from credit card issuers in the U.S. come from interest rates.[2] In the U.S. market, where consumers possess more than 360 million credit card accounts, making, on average, 120 payments a year for a total value of USD 11,000 and holding more than 1,000 billion dollars in outstanding revolving credit, this issue is of significant importance.[3]

To explore this relationship, we develop a multi-product model of credit cards in which an issuer offers cards as a payment method and as a form of credit. Consumers decide first whether to own a credit card and then whether to use the credit associated with the card. They have outside options for both products, namely cash for the payment feature and an exogenous credit market for the loan. We include a behavioral assumption for some consumers, where they do not take into account the interest rate charged by issuers and make decisions based only on per-transaction fees, consistent with recent findings on consumer behavior in credit markets. For example, Zaki (2018) finds that consumers not only fail to take "hidden" prices into consideration when choosing credit cards but actually cannot translate interest rates to real financial obligations. Another recent example is Heidhues and Koszegi (2015), who investigate the welfare losses of firms exploiting behavioral biases from credit card consumers.

This model allows us to provide explanations for some interesting features of this market. First, per-transaction fees might be negative in equilibrium, meaning that issuers offer rewards to consumers for using cards as a payment instrument. Our model explains this broadly observed phenomenon by showing that issuers induce higher demand to the credit characteristic of their cards by lowering the per-transaction fees, even to negative levels. The more profitable the credit market is, the more incentives issuers have to offer credit card rewards to consumers. This result is consistent with the fact that issuers earn significant revenues from interest rates. Therefore, we provide a new explanation for the existence of credit card rewards in the credit card market.

Second, and as long as there exist at least a minimal fraction of naive consumers in the market, interest rates exhibit some stickiness: for a wide range of parameters, they remain independent of the marginal cost of credit, of the marginal cost of card payments and of the interchange fee. The existence of naive consumers makes it optimal for issuers to compete in per-transaction fees and not in interest rates.

[2] In 2016, interest income from credit cards was USD 63.4 billion from a total of USD 163 billion.

[3] Data from the year 2017. For more information see http://www.federalreserve.gov.

In equilibrium, the interest rate can only take one of two values, namely a "low" interest rate given by an indifference condition for sophisticated consumers with the exogenous credit market, or a "high" interest rate given by the maximum regulated rate that extracts the most profits from naive consumers. Given that the decision to own a card precedes the decision of asking for a loan, when setting interest rates, the only important information for issuers is the share of naive consumers and the relative profitability of both possible interest rates. Therefore, for a wide range of values, the equilibrium interest rate remains independent of the marginal cost of credit, and it's always independent of the interchange fee.

This result also allows us to evaluate interchange fee regulation. All around the world, antitrust authorities and regulators have capped interchange fees for debit and credit cards set by four-party systems. For example, after a long antitrust case in Europe against Mastercard, both Visa and Mastercard have agreed to cap their interchange fees to around 0.3% for credit cards, while the Reserve Bank of Australia cut to half the interchange fees in that country in 2003 (to around 0.5%).[4] In the United States, the Durbin Amendment of the Dodd-Frank Act in 2010 capped the debit card interchange fees.[5] However, the relationship between interchange fees and per-transaction fees with the credit market associated with credit cards has not been studied in detail. Our second result means that interest rates are independent of variations on the interchange fees, a result that contributes to the discussion of the regulation implemented in different countries.[6]

Finally, we extend the analysis of Rochet and Tirole (2011) and compare the socially optimal interchange fee with the interchange fee set by payment systems maximizing issuers' profits, under the assumption of full merchant internalization. Under such an assumption, retailers take into account the average consumer benefits of using cards when deciding whether to accept cards, and they show that this result holds in a variety of competitive settings. We extend their analysis and argue that, as higher interchange fees reduce the per-transaction fees charged to consumers, inducing a higher demand for both, the payment feature and the credit characteristic of credit cards, interchange fee regulation should take into account

[4]The interchange fees are usually paid from the acquiring bank to the issuing bank for each transaction made through the payment system.

[5]Other examples are Canada, Israel, Mexico, Spain, and others. For more details see Bradford and Hayashi (2008).

[6]See The Economic Impact of Interchange Fee Regulation in the UK (2013) available at http://www.europe-economics.com/publications/28062013-final-report–for-distribution.pdf, and The Effects of the Mandatory Decrease of Interchange Fees in Spain (2012), The Economic Impact of Interchange Fee Regulation in the UK (2013) available at http://www.europe-economics.com/publications/28062013-final-report–for-distribution.pdf, and The Effects of the Mandatory Decrease of Interchange Fees in Spain (2012), Reserve Bank of Australia Annual Report on Payment Systems available at http://www.rba.gov.au/publications/annual-reports/psb/2006/html/ for different arguments on this issuer

the social costs and benefits of the different forms of credit.

To simplify the exposition, we first build a monopoly issuer model and derive all of our results. Then, we extend to consider the cases of competition between differentiated issuers in a pure single-homing and a pure multi-homing settings, showing that our main results are consistent with different market structures. Following previous literature, consumers obtain a heterogeneous benefit of using cards as a payment device. To consider the credit market in our model, we assume that all consumers need a loan of an exogenous amount of D, and they always repay their debts. The acquiring side of the market is assumed to be competitive. Therefore, the interchange fee is fully passed-through to retailers. In the following section, we relate our work with the literature.

Related Literature

Several papers have analyzed the outcomes of the credit card market and the role of the interchange fee and its regulation. This paper borrows closely from Rochet and Tirole (2011) the way of modeling cardholder's and retailer's benefits of using cards over cash, while we extend to add the credit feature to the model. In their paper, the authors explain why cards are often understood as "must take" by merchants. They also provide a benchmark for the regulation of the interchange fee, the tourist test. They show that retailers may accept cards even when these cards raise their operational costs, due to partial internalization of buyer surplus. They also show that, in many cases, the privately optimal interchange fee will be too high, producing an excessive use of cards as a payment device. Wright (2012) goes further in the last argument and show conditions under which the privately set interchange fee will be unambiguously biased against retailers. Bedre-Defolie and Calvano (2013), in a model of usage and participation benefits heterogeneity on both consumers and merchants, also show that a network aiming to maximize profits oversubsidize the usage of cards by cardholders while overcharging merchants, mainly due to the fact that consumers choose both membership and usage while retailers only choose membership.

The closest papers in the literature are Chakravorti and To (2007) and Rochet and Wright (2010), who also take into account the credit functionality of cards. In the first paper, the authors show that merchants accept credit cards because they increase sales today in contrast with tomorrow. Thus merchants are willing to accept higher merchants discounts. In equilibrium, all merchants accept cards. Still, by doing so, they are worse off, as in a prisoner's dilemma situation. They focus more on retailers than on issuers' incentives and have no interest rate in their model. In the latter paper, the authors also take into account the credit functionality of credit cards and show that a monopoly network sets an interchange fee that is higher than the one maximizing consumer surplus. They also focus on the retailer side of the market by looking for equilibria where credit cards are accepted even when stores can provide credit for themselves. They analyze the impact of interchange fees on retail prices while simplifying the issuing side of the market. Therefore, there is no explicit interest rate either in their model. In contrast with those papers, this work focus in the issuing bank and on understanding the relationship between the per-transaction fees and the interest rates from banks perspectives, therefore providing a tool to explain credit card rewards and interest rate stickiness in the market, while also analyzing interchange fees in this context. Both works are thus complementary to this article.

Regarding the issue of credit card rewards, previous results in the literature show that rewards appear if the convenience benefit for merchants exceed the costs of the transactions (Rochet and Tirole, 2011, Beldre and Calvano, 2013) or due to oligopolistic competition by retailers, non-surcharging by merchants and network competition (Hayashi, 2008), among others. This work provides an alternative explanation for this widely observed phenomenon through an alternative approach.

Finally, the behavior of naive consumers borrows from the analysis of Gabaix and Laibson (2006) on shrouded attributes and add-on pricing. More recently, Heidhues and Koszegi (2015) evaluate the welfare losses of issuers exploiting naive consumers in the United States, and Heidhues and Koszegi (2017) analyze how firms can price discriminate according to consumers' level of "naivete". Their primary application is the credit market and focuses on price discrimination issues. In contrast, we focus more on multi-product dimensions and interchange fee analysis in a setting designed to explain outcomes in the credit card market. Armstrong and Vickers (2012) build a similar model relative to consumers' decision to have current accounts associated with overdraft fees, which some consumers fail to internalize. Finally, we extend the literature on credit card interest rates being sticky, as discussed by Ausubel (1991). However, the explanation of this phenomenon is different and related to the existence of naive consumers, as is empirically explored by Zaki (2018), who shows that naive consumers exist in the credit card market and provides some insights on the stickiness interest rates analogous to our theoretical findings.

The rest of the article is organized as follows. In Section 2, we set up the model, and we derive all of our results assuming there is a monopoly issuer, while the level of the interchange fee is exogenously given in the market. Then, in Section 3, we analyze the socially and privately optimal interchange fees. In Section 4, we show that our main results extend when we consider pure single-homing and pure multi-homing competition between two differentiated issuers. Finally, in Section 5, we provide some concluding remarks.

2 The model

Consider a continuum of mass 1 of consumers demanding two products, namely a payment device and some form of credit for an amount of money D, to use them to purchase a basket of goods. The issuing bank M offers a credit card, charging a per-transaction fee f_M for each payment made with the card and an interest rate r_M per unit of money borrowed by consumers. This per-transaction fee might be negative, meaning that the issuer gives rewards to consumers, in the form of money in their cards or as goods and services offered by external firms. Consumers use the credit to purchase a basket of goods providing utility v, and they experience a convenience benefit of b_b from the payment feature of credit cards. This benefit is assumed to be heterogeneous and distributed over a closed interval $[\underline{b_b}, \overline{b_b}]$ following a cumulative distribution function $H(\cdot)$, with increasing hazard rate. Both bounds can take positive or negative levels as long as $\overline{b_b} > \underline{b_b}$.

Consumers have outside options for both products. They can pay using cash, which cost is normalized to 0. Consumers also have access to an exogenous credit market that charges a fixed interest rate of r_0. If consumers decide to use the exogenous credit, they face a fixed and homogeneous cost of h_b, representing, for instance, within period impatience or transaction costs. Every consumer uses either form of credit and always repay their debts (no default allowed).

We assume there are two types of consumers exhibiting different behaviors. Following Gabaix and Laibson (2006), a fraction γ of naive consumers make decisions based only on per-transaction fees while still using some form of loan and paying interest rates. The remainder fraction $(1-\gamma)$ of sophisticated consumers choose their payment device and credit form taking into account both per-transaction fees and having rational expectations over future interest rates. We assume that $0 < \gamma < 1$ for simplicity in the exposition.[7] We also assume that there exists an exogenous maximum interest rate $\overline{r}$ that can be charged by banks due to regulation or other social constraints. Both naive and sophisticated consumers are assumed to have the same distribution of convenience benefits b_b.

Given the description above, consumers have three options: 1) to pay with the card and use the associated revolving credit, 2) to pay with the card and use the external loan, and 3) to use cash and use the external loan. The utility derived

[7]When $\gamma = 1$ the model is solved differently and the results are straightforward.

from each choice is respectively given by

$$U_1 = v + b_b - f_M - (r_M \cdot D) \tag{1}$$

$$U_2 = v + b_b - f_M - (r_0 \cdot D) - h_b \tag{2}$$

$$U_3 = v - (r_0 \cdot D) - h_b. \tag{3}$$

Retailers receive a homogeneous convenience benefit of b_r from payments with cards and they are not allowed to surcharge payments with those cards.[8] The acquiring market is assumed to be competitive, implying that the merchant discount rate m is given by $m = c_a + a$, where c_a is the per-transaction cost of the acquirer, and a is the interchange fee, defined as a payment from the acquiring bank to the issuing bank each time a card transaction is made.[9]

Assume in this section that the interchange fee a is exogenously given. The timing of the game is the following:

- t=1: The issuer sets the per-transaction fee f_M. Then, consumers choose whether to own a card or not.
- t=2: The issuer sets the interest rate for the revolving credit r_M. Then, consumers choose which form of credit to use and purchase the basket of goods. Consumers who do not own cards always use the external loan.

Consumers choosing to own a credit card in the first period will always use it as a payment device when they purchase, while they still choose which form of credit to use. We assume that the issuer cannot commit to a level of the interest rate in $t = 1$ and he cannot price discriminate between consumers. For an in-depth analysis of price discrimination issues with naive consumers, see Heidhues and Koszegi (2017). Our equilibrium concept is subgame perfect Nash equilibrium.

2.1 Equilibrium analysis

Define

$$\underline{r} \equiv r_0 + \frac{h_b}{D} \tag{4}$$

as the interest rate that makes sophisticated consumers holding a card indifferent between the revolving credit associated with the card and the exogenous loan choice in the second period, where we assume that $\overline{r} > \underline{r}$.[10] Then, in $t = 2$, it can

[8] Usually there is a non-surcharge rule by networks like Visa and Mastercard. In countries where this rule has been eliminated, there is little evidence of surcharging by merchants.

[9] The analysis below could be extended to acquirers having a positive margin. The critical assumption is that an increase in the interchange fee will increase the merchant discount rate charged by them.

[10] If they use the outside option they pay $D \cdot r_0 + h_b$ which is equal to $D \cdot \underline{r}$.

only be optimal for the monopolist to charge either $r_M = \underline{r}$ or $r_M = \overline{r}$. Given the card owning decision of consumers, in the second period, the monopolist chooses between serving every consumer in the credit market at a "low" interest rate or serving only naive consumers at a "high" interest rate. Any interest rate strictly greater than $\underline{r}$ would lead to only naive consumers using the revolving credit. Therefore, it would be in the monopolist interest to set the highest price possible, in this case, $\overline{r}$. If M sets any rate lower or equal than $\underline{r}$, every consumer uses the revolving credit, so it is more profitable to set $\underline{r}$.

We start by assuming that the monopolist sets $r_M = \underline{r}$ at $t = 2$, and sophisticated consumers correctly anticipate this price. Then, in the first period, the demand for card payments is derived from the condition $b_b \geq f_M$,[11] for both sophisticated and naive consumers. This implies a demand function equal to $1 - H(f_M)$. We assume that every consumer having a card will use the revolving credit at this interest rate, as they are indifferent between both types of loans. The profit function for the monopolist in $t = 1$ is given by

$$\Pi(f_M; \underline{r}) = (f_M + a - c_i)(1 - H(f_M)) + D(\underline{r} - c_c)(1 - H(f_M)), \tag{5}$$

where a is the interchange fee, c_i is the constant marginal cost of a payment transaction and c_c is the cost per unit of money lent. Taking the first-order conditions with respect to f_M and solving for the optimal per-transaction fee results in

$$f_M^* = c_i - a + \frac{1 - H(f_M^*)}{h(f_M^*)} - D(\underline{r} - c_c). \tag{6}$$

The increasing hazard rate assumption ensures that the first-order condition will provide a unique solution to the monopolist problem. Next, assume that the monopolist charges $\overline{r}$ in $t = 2$ and sophisticated consumers anticipate this price. In this case, sophisticated consumers expect to use the exogenous loan. The demands for cards, for both naive and sophisticated consumers, is again given by the condition $b_b \geq f_M$, but now only naive consumers will use the revolving credit. The monopolist's profit function is given by

$$\Pi(f_M; \overline{r}) = (f_M + a - c_i)(1 - H(f_M)) + \gamma D(\overline{r} - c_c)(1 - H(f_M)). \tag{7}$$

Solving for the optimal per-transaction fees we have

$$f_M^* = c_i - a + \frac{1 - H(f_M^*)}{h(f_M^*)} - \gamma D(\overline{r} - c_c). \tag{8}$$

The following result characterizes the monopolist optimal decision in the first stage:

[11] Details on the derivation of demand functions in appendix 1.

Lemma 1. *At $t = 1$, the monopolist sets per-transaction fees given by expression (6) or (8). In any case:*

- *The per-transaction fee charged by the monopolist is strictly decreasing on the interchange fee.*
- *If the expected revenues in the credit market are high enough, the per-transaction fee is negative.*

Proof. See the analysis above for the derivation of the optimal per-transaction fee for each possible interest rate. This fee is strictly decreasing in the interchange fee a directly from the implicit function theorem and the increasing hazard rate assumption for $H(\cdot)$. The revenues coming from the credit market are, for each possible interest rate, $D(\underline{r} - c_c)$ and $\gamma D(\overline{r} - c_c)$. Due to the increasing hazard rate assumption, if these terms are high enough, the equilibrium expression for the per-transaction fee is negative.

□

The per-transaction fee charged to consumers is decreasing with the profits made on the credit market, as the monopolist has incentives to induce demand to the credit feature of its credit card to increase its revenues through interest rate charges. As only consumers holding a credit card might use the revolving credit at $t = 2$, it is optimal for the monopolist to lower the per-transaction fee charged to consumers, even to negative levels, if the credit market is profitable enough. How profitable is the credit market depends on the amount of debt that consumers are expected to have, the conditions of the exogenous loan market and the number of naive consumers in the market. This result is consistent with the fact that issuers make a significant share of their income in the credit card market through the credit feature of their cards.

Lemma 1 provides a new explanation for the widely observed credit card rewards in the credit card market. So far, the literature had focused on the first terms of expressions (6) and (8), given by $c_i - a$. For example, Bedre and Calvano (2013), find that the optimal per-transaction fee set by the issuer is $c_i - a$. Therefore, rewards are less likely to exist in their model and only if the interchange fee is higher than the marginal cost of providing card payments on the issuer side.[12]

Next, we study whether the high or the low interest rate equilibrium is the unique subgame perfect Nash equilibrium of the game:

[12] Similarly, Rochet and Tirole (2011) find that the optimal per-transaction fee is $c_i - a + k$ where k is a fixed margin of the issuer, making it even more unlikely to obtain as a result the widely spread reward observed on the credit card markets. Rochet and Wright (2010) assume a similar expression for the per-transaction fee in their model.

Proposition 1. *Assume that the distribution $H(\cdot)$ has an increasing hazard rate. Then, for a given level of the interchange fee a, there are two possible equilibriums:*

- *A low interest rate equilibrium with $r_M^* = \underline{r}$ and f_M^* characterized by expression (6).*
- *A high interest rate equilibrium with $r_M^* = \overline{r}$ and f_M^* characterized by expression (8).*

The low interest rate equilibrium is the unique subgame perfect Nash equilibrium if and only if:

$$\gamma \leq \frac{(\underline{r} - c_c)}{(\overline{r} - c_c)} \leq 1 \tag{9}$$

Else, the high interest rate equilibrium is the unique subgame perfect Nash equilibrium.

Proof. See appendix 1. □

Whether the high or the low interest rate equilibrium hold, depends on the share of naive consumers and on the relative profitability on the credit market of charging a high or a low interest rate. If the share of naive consumers is high, the monopolist has more incentives to charge a high interest rate, as naive consumers will always pay this higher price. In contrast, the monopolist loses relatively little demand from not serving the sophisticated consumers, and vice versa.

This condition is similar to the one obtained by Armstrong and Vickers (2012) in their analysis of current accounts attached to overdraft fees. The main differences are that in their model, the goods are tied and have no outside option, while we allow outside options for both goods, and in their model, sophisticated consumers must incur in some effort to learn the less salient price. However, the intuition of the condition that compares the share of naive consumers with the relative profitability of different options for the firm is the same.

Finally, following the result derived in Proposition 1, we discuss the equilibrium interest rate and its determinants:

Lemma 2. *The equilibrium interest rate is independent of the marginal cost of the payment feature of the card and independent of the interchange fee. Moreover:*

- *If the marginal cost of providing credit increases, the high interest rate equilibrium becomes more likely.*
- *If the regulated interest rate $\overline{r}$ decreases, the high interest rate equilibrium becomes less likely.*

- *If the outside option credit market becomes more competitive, meaning r_0 decreases, the high interest rate equilibrium becomes more likely.*

Proof. Direct inspection of the condition leading to each possible equilibrium in Proposition 1.

□

The condition leading to a high or to a low interest rate equilibrium is independent of the interchange fee and of the marginal cost of providing a card payment c_i. Therefore, interchange fee regulation doesn't affect the equilibrium interest rate. The only effect of interchange fee regulation in our model is an increase in the per-transaction fee and lower usage of credit cards as a payment device and as a credit feature. We discuss the welfare implications of such a policy in the next section.

We also analyze the impact of the marginal cost of credit c_c on the probability of having a high or a low interest rate equilibrium. If this cost goes up, the share of naive consumers needed to have a high interest rate equilibrium is smaller, making it more likely to hold. The intuition is that a higher marginal cost of giving credit makes the lower interest case relatively less profitable than the high interest rate case, increasing the incentives of the monopolist to charge a high interest rate. However, as long as this condition is not reversed, the interest rate doesn't change. Therefore, for a wide range of parameters and marginal costs, the interest rate is sticky with respect to changes in the marginal cost of credit. This result is consistent with the analysis of Ausubel (1991), but for different reasons. The explanation is consistent with the findings of Zaki (2018), where she argues that behavioral components, such as consumers being unable to transform interest rates into real financial obligations, might explain the stickiness of interest rates in the credit card market. In our model, the fact that the decision of owning a card is previous to the credit decision and that the interest rate is determined based either on an indifference condition with an outside option or either by the maximum regulated interest rate, results in interest rate stickiness.[13]

Using this result, we can also analyze the effects of potential regulation, such as reducing the maximum regulated interest rate $\overline{r}$. This regulation would make it more likely that the low interest equilibrium holds by reducing the profitability of the high interest rate equilibrium. This kind of regulation is often criticized based

[13]Note that we assume that the marginal cost of credit is constant. If this marginal cost would be decreasing in the amount of credit given to consumers, then capping interchange fees would increase per-transaction fees, reduce demand for cards and increase the marginal cost of credit for the monopolist. In this case, interchange fee regulation might generate a switch from low to high interest rates. Whether this effect is significant in reality is left as an open question for empirical work, and it is outside of the results of this paper.

on the argument that such a regulation would make issuers to improve other fees in response. Our multi-product model allows us to investigate this criticism and, in fact, show that such a regulation would increase per-transaction fees. Whether this is better for welfare and consumers is studied in the following section.

3 Socially and Privately Optimal Interchange Fees

3.1 Privately optimal interchange fees

In this section, we provide a simplified analysis to show how adding the credit feature of credit cards implies that additional variables should be taken into consideration when analyzing interchange fee regulation. For simplicity in the exposition, we assume that the low interest rate equilibrium holds, meaning that every consumer holding a card uses the card as a payment device and as a credit form.

Rochet and Tirole (2011) show that under different competitive environments, retailers are willing to accept a higher merchant discount rate m than their direct convenience benefits, due to what they call "merchant internalization", meaning that merchants at least partially internalize the cardholders benefits of their card acceptance policy. Following their work, define $v_b \equiv E[b_b - f | b_b \geq f]$ as the net cardholder benefit per card payment. These authors show that retailers accept cards if and only if

$$m \leq b_r + v_b(f), \tag{10}$$

where f is the per-transaction fee charged to consumers and b_r is the homogeneous benefit that retailers obtain from accepting a card payment. This means that merchants fully internalize consumers' benefits of using cards in their card acceptance decision. Also, both in their work and ours, issuers' profits are always increasing on the interchange fee, as it is an exogenous source of income. Therefore, a payment platform aiming to maximize issuers' profits, will set the highest interchange fee subject to merchants accepting cards, given by $m = b_r + v_b(f)$. Given the competitive acquiring market assumption, this implies an interchange fee equal to

$$a_p^* = b_r + v_b(f) - c_a. \tag{11}$$

This result is the same as in Rochet and Tirole (2011), as the privately optimal interchange is set according to the merchants' participation constraint and, as consumers are left indifferent between both credit options, there is no extra merchant internalization from the credit feature of credit cards.

3.2 Socially optimal interchange fees

To define the social welfare function, it's important to mention the sources of social benefit and cost in this model. Social benefits are given by convenience benefits b_b for consumers and b_r for retailers. The social costs are given by c_i, c_c, c_a and h_b. We also assume that the exogenous loan has a cost of c_l per unit of credit given. Then, the social welfare function as a function of the per-transaction fee f is given by

$$SW(f) = \int_f^\infty (b_b + b_r - (c_i + c_a + (D(c_c - c_l) - h_b)))dH(b_b). \tag{12}$$

Note that this welfare function is relative to a base case where consumers use cash and the outside credit option. Maximizing with respect to the equilibrium per-transaction fee we obtain

$$f^{W*} = \underbrace{c_i + c_a - b_r}_{\text{Same as Rochet and Tirole (2011)}} + \underbrace{D(c_c - c_l) - h_b}_{\text{Extra terms from credit feature}} . \tag{13}$$

The per-transaction fee that maximizes total welfare is divided into two terms. The first term aims at balancing the social benefits and costs of the payment feature of the card and is the same as the one obtained by Rochet and Tirole (2011). In addition, by considering the credit characteristic of cards, we get an additional term $D(c_c - c_l) - h_b$. The first part of this new term depends on the difference between the marginal cost of the revolving credit of credit cards and the marginal cost of the exogenous loan. If the marginal cost associated with credit cards is higher, the socially maximizing fee is also higher in order to reduce the demand for this form of credit that is more expensive to produce. The last term represents the transaction costs incurred by consumers when going to the exogenous loan market. If this is very costly for consumers, the per-transaction fee should go down to avoid such a cost for consumers.

This optimal per-transaction fee can be implemented by the regulator by setting the appropriate interchange fee depending on the distribution of match values b_b. For any distribution, we observe that the privately optimal interchange fee does not take into account the terms associated with the credit feature of cards, while the social maximizing per-transaction fee does. Therefore, when regulating interchange fees, the regulator must take into account the different costs of credit and transaction costs for consumers, as interchange fee regulation will affect per-transaction fees, which, in turn, will affect how demand is allocated between these different forms of credit.

4 Extension: issuer competition

In this section, we show how our main results extend to the cases of pure single-homing competition, that is, assuming that consumers choose and hold only one card, and to the case of pure multi-homing competition, meaning that consumers always own the cards of both competing issuers.

4.1 Pure single-homing competition

Consider two firms, A and B, that compete for consumers in a Hotelling line. The total amount of consumers is normalized to $M + 2K$. To introduce outside options to both products while taking into account product differentiation, instead of cash, we introduce two exogenous firms A' and B' offering only the payment product at an exogenously given per-transaction fee f_0.[14] As is standard in Hotelling models, all firms provide a homogeneous benefit b_b of the payment feature of cards minus the transport cost given by their location. These firms represent alternative payment cards, such as debit cards, that also provide the benefit b_b to consumers, but without giving the credit option. Firms A and B are located symmetrically in the middle of the line at a distance M from each other, while A' and B' are located in the extremes of each side of the line, at a distance K from the main firms, as we show in Figure 1.

The linear transport cost is given by t and consumers still have the choice of getting a loan at a rate r_0 at the exogenous credit market, incurring in a transaction cost of h_b. We impose in this section that consumers can only choose to have only one card between the offers of firm A and B and the outside options, representing a pure single-homing case. The timing of the game is the following:

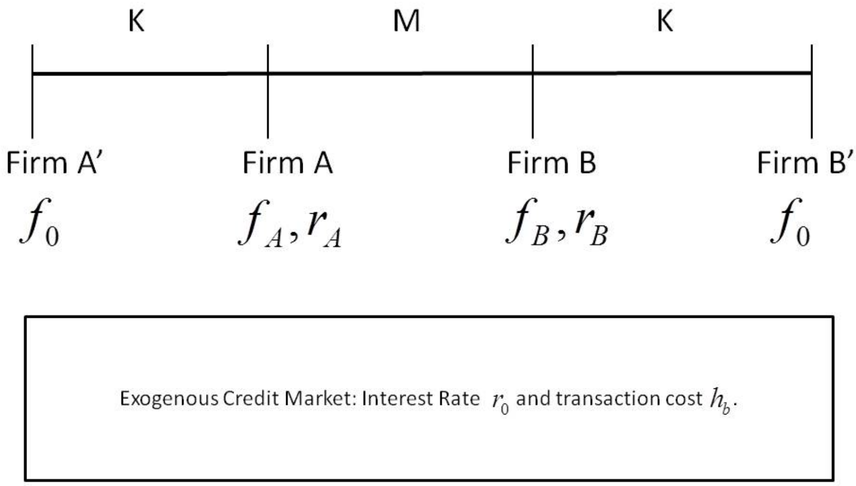
- t=1: Issuers A and B set per-transaction fees simultaneously. Then, consumers choose between the cards from firm A, firm B or one of the outside option firms.

- t=2: Issuers A and B set interest rate for the revolving credit simultaneously. Then, consumers choosing outside option firms use the exogenous loan, while consumers with cards from A or B choose between their firm's revolving credit and the exogenous loan.

Suppose that a given consumer chooses the card from firm A or B. In the second period, this consumer can only use the revolving credit associated with his card or the exogenous loan. At this stage, firms "stop" competing with each other

[14] This is a variation of the Hotelling model, similar to that in Rochet and Tirole (2003).

Figure 1: Hotelling Competition with Outside Options



and only compete with the outside option credit. Therefore, the only relevant options for them in $t = 2$ are to serve all of their consumers at the interest rate $\underline{r}$ or serve only their naive consumers in the credit market at $\overline{r}$.

Consumers' demand for cards at $t = 1$ now depends on their expectation on the interest rate charged by both firms in the second period. Focusing on a symmetric equilibrium,[15]and denoting by r_i^e and r_{-i}^e the expected interest rates of issuer i and its rival, the demand for cards in $t = 1$ for firm i is given by (details of this derivation are found in Appendix 2)

$$D_i(f_i; r_i^e) = \frac{M+K}{2} - \frac{f_i}{t} + \frac{f_{-i} + f_0}{2t} - \frac{D(1-\gamma)(2r_i^e - r_{-i}^e - \underline{r})}{2t}. \quad (14)$$

Suppose first that both firms charge the low interest rate at $t = 2$. Therefore, consumers expect to use the revolving credit associated with the card they pick. Then, the profit function of issuer i is

$$\Pi_i(f_i; \underline{r}) = D_i(f_i; r_i^e)(f_i + a - c_i) + D_i(f_i; r_i^e)D(\underline{r} - c_c). \quad (15)$$

Taking the first-order conditions for each firm with respect to f_i, and using the fact that consumers have rational expectations, implies a candidate symmetric

[15]In fact, an asymmetric equilibrium where one firm charges the high interest rate and the other the low interest rate does not exist in this game.

equilibrium per-transaction fee of

$$f_i^* = \frac{t(M+K) + f_0 - 2(a - c_i) - 2D(\underline{r} - c_c)}{3}. \tag{16}$$

Analogously, assuming that issuers will charge $\overline{r}$ in $t = 2$, sophisticated consumers expect to go to the exogenous loan. Therefore, the profit function of firm i is now

$$\Pi_i(f_i; \overline{r}) = D_i(f_i; \overline{r})(f_i + a - c_i) + \gamma D_i(f_i; \overline{r}) D(\overline{r} - c_c), \tag{17}$$

while the candidate per-transaction fees for a symmetric equilibrium is now

$$f_i^* = \frac{t(M+K) + f_0 - 2(a - c_i) - 2\gamma D(\overline{r} - c_c)}{3}. \tag{18}$$

The following Proposition characterizes the unique subgame perfect Nash equilibrium of the game:

Proposition 2. *In the described Hotelling model, when consumers single-home, and for a given level of the interchange fee a, there are two possible symmetric equilibriums:*

- *A low interest rate equilibrium with $r_i^* = \underline{r}$ and f_i^* given by (16), for $i \in \{A, B\}$.*
- *A high interest rate equilibrium with $r_i^* = \overline{r}$ and f_i^* given by (18), for $i \in \{A, B\}$.*

The low interest rate equilibrium is the unique subgame perfect Nash equilibrium if and only if:

$$\gamma \leq \frac{(\underline{r} - c_c)}{(\overline{r} - c_c)} \leq 1 \tag{19}$$

Else, the high interest rate equilibrium is the unique subgame perfect Nash equilibrium.

Proof. The argument follows the same steps as the ones for proposition 1 in appendix 1, using the demand functions for the competition case. □

As in the monopoly case, we have two possible equilibriums, one with a high interest rate and one with a low interest rate. The condition under which these equilibriums hold is the same as in the monopoly case because when consumers are assumed to single-home, they are captive to their own credit card supplier in $t = 2$. Therefore their credit choice reduces to using their card revolving credit or the credit market outside option. At this stage, each firm faces the same trade-off as the monopolist: whether to serve every consumer at a low interest rate or only to naive consumers at a high interest rate. This trade-off is again defined by the relative amount of naive consumers in the population relative to the possible margins in the credit market.

This result shows that, even in a competitive environment, if the credit market is profitable enough, per-transaction fees may be negative. Therefore, the explanation for credit card rewards extends to competition when single-homing is imposed. Moreover, as the condition leading to each equilibrium is the same, the result for interest rate stickiness also remains unchanged in this competitive environment.

4.2 Pure Multi-Homing Competition

Consider a variation of the Hotelling model of the last section, in which sophisticated consumers hold both cards from the beginning and must choose which one to use at each stage. In the case of naive consumers, this assumption is innocuous because they make decisions only based on per-transaction fees. The timing is now the following:

- t=1: Issuers A and B set per-transaction fees simultaneously. Both types of consumers hold cards from firms A and B.
- t=2: Issuers A and B set interest rate for the revolving credit simultaneously. Then, sophisticated consumers observe both per-transaction fees and interest rates and choose whether to pay using one of the firms' cards or the outside option and which form of credit to use. Naive consumers choose only based on per-transaction fees.

The only restriction for consumers in this section is that, in order to use the revolving credit associated with a card, they must use that card as a payment device. Any other combination of cards and outside options is allowed.

The main difference with the single-homing case is that sophisticated consumers will choose considering the best combination of fees and observed interest rates in $t = 2$. Therefore, different interest rates charged by issuers in $t = 2$ may influence sophisticated consumers' choice of their payment device.

Demands for card payments are the same as in the last section but considering observed interest rates instead of expected ones. The set of strategies for issuers is now broader, as now they can attract more consumers by charging an interest rate lower than $\underline{r}$. It's still never profitable for issuers to charge anything between $\underline{r}$ and $\overline{r}$, because sophisticated consumers will go to the outside option and naive consumers could be furthered exploited by charging $\overline{r}$. However, lowering the interest rate to levels below $\underline{r}$ will attract demand from sophisticated consumers, in contrast with the single-homing case. Therefore, firms choose interest rates either in the set $[0, \underline{r}]$ or equal to $\overline{r}$.

Suppose first that issuers set interest rates in the interval $[0, \underline{r}]$. Then, the demand for cards for firm i in $t = 2$ is given by

$$D_i(r_i; f_i) = \frac{M+K}{2} - \frac{f_i}{t} + \frac{f_{-i} + f_0}{2t} - \frac{D(1-\gamma)(2r_i - r_{-i} - \underline{r})}{2t}. \tag{20}$$

When interest rates belong to the mentioned interval, every consumer demanding the card of firm i will also use the revolving credit associated to that card. Therefore, the problem in $t = 2$ is to maximize the following profit function with respect to r_i for any value of per-transaction fees f_i, with the restriction that $r_i^* \in [0, \underline{r}]$. The profit function is

$$\Pi(r_i; f_i) = D_i(r_i; f_i)(f_i + a - c_i + D(r_i - c_c)). \tag{21}$$

Ignoring the restriction for the moment and solving for r_i^* as a function of per-transaction fees of both firms gives, as a result, a candidate interest rate of

$$r_i^* = \frac{5\Delta - (15 - 8\gamma)f_i + 2\gamma f_{-i}}{15D(1-\gamma)}, \tag{22}$$

with $\Delta \equiv (t(M+K) + f_0) + 5D(1-\gamma)(\underline{r} + 2c_c) - 10(1-\gamma)(a - c_i)$. This candidate symmetric equilibrium interest rate for issuer i is strictly decreasing on the per-transaction fee f_i.

Then, the maximization problem of the firms in $t = 1$ is to choose per-transaction fees to maximize

$$\Pi(f_i, r_i^*, r_{-i}^*) = D_i(f_i, r_r^*, r_{-i}^*)(f_i + a - c_i + D(r_i^* - c_c)). \tag{23}$$

This profit function is strictly decreasing on per-transaction fees. Therefore, issuers decrease per-transaction fees, which in turn increases interest rates charged in $t = 2$. When the per-transaction fee is low enough, the interest rate charged in $t = 2$ reaches $\underline{r}$. At this rate, issuers do not have incentives to increase the interest rate further, as they would lose all sophisticated consumers. However, by decreasing the per-transaction fee, they still attract demand to their cards. They continue to decrease per-transaction fees until they have no more incentives to do so (details on Appendix 3). In equilibrium, the per-transaction fee is given by

$$f_i^* = \frac{t(M+K) + f_0 - 2(a - c_i) - 2D(\underline{r} - c_c)}{3}. \tag{24}$$

Note that this per-transaction fee is exactly the same than in the single homing case. This means that the low interest equilibrium remains unchanged. The intuition for this result is as follows: firms, given how interest rates will be affected by per-transaction fees in $t = 2$, always have incentives to lower per-transaction fees in $t = 1$. Increasing them would lead to lower interest rates, lower demand and lower profits. However, when they reach the per-transaction fee consistent with the restriction of the maximization problem and with expression (22), firms still have incentives to lower per-transaction fees, even if they can't keep increasing the interest rate. This process goes on until both firms set per-transaction fees equal to expression (24) in equilibrium, which is the same expression for equilibrium per-transaction fees than in the single homing case.

The explanation of this result comes from the existence of an outside option in the credit market and the participation of naive consumers in the market. Even a very low fraction of these consumers give banks' incentives to always use per-transaction fees in order to attract demand, while setting the highest interest rate possible. The outside option in the credit market sets a cap on this interest rate at the level $\underline{r}$. These two factors make the multi-homing case equal to the single homing case in the low interest rate equilibrium.

Assume now that the firm charges a high interest rate at $t = 2$. This means that sophisticated consumers will use the outside option credit even when choosing the card of either firm A or B. This means that firms maximize in $t = 1$ the following profit function:

$$\Pi(f_i; \overline{r}) = D_i(f_i; \overline{r})(f_i + a - c_i) + D_i(f_i; \overline{r})\gamma D(\overline{r} - c_c)) \tag{25}$$

Solving for both firms:

$$f_i^* = \frac{t(M+K) + f_0 - 2(a - c_i) - 2D\gamma(\overline{r} - c_c)}{3} \tag{26}$$

And $r_i^* = \overline{r}$.

Proposition 3. *In the described Hotelling model, when consumers multi-home, and for a given level of the interchange fee a, there are two possible symmetric equilibriums:*

- *A low interest rate equilibrium with $r_i^* = \underline{r}$ and f_i^* given by (24), for $i \in \{A, B\}$.*
- *A high interest rate equilibrium with $r_i^* = \overline{r}$ and f_i^* given by (26), for $i \in \{A, B\}$.*

The low interest rate equilibrium is the unique subgame perfect Nash equilibrium if and only if:

$$\gamma \leq \frac{(\underline{r} - c_c)}{(\overline{r} - c_c)} \leq 1 \tag{27}$$

Else, the high interest rate equilibrium is the unique subgame perfect Nash equilibrium.

Proof. See appendix 3. □

We observe that, again, if the credit market is profitable enough, per-transaction fees may be negative. Therefore, the explanation for credit card rewards also extends to competition when multi-homing is imposed. Finally, as the condition leading to each equilibrium is the same, the result for interest rate stickiness also remains unchanged in this competitive environment.

5 Discussion and Final Remarks

We argue that studying the relationships between the payment market and its transaction fees and between the credit market and its interest rates is important mainly in two dimensions.

First, this relationship can explain some interesting phenomena observed in the credit card market. Issuers offer credit card rewards to attract demand to their cards and increase the usage of the associated form of credit, increasing their revenues. This explanation is consistent with a significant share of the revenues of a typical issuer in the U.S. coming from interest rates. Also, we provide an alternative explanation to credit card stickiness, which has been a long studied problem[16]. Our theoretical results are consistent with the empirical analysis of Zaki (2018).

Second, our model is useful when addressing credit card interchange fee regulation. Caps on interchange fees have been implemented due to some agreement on the fact that privately set interchange fees would be too high, generating an over usage of payment cards and excessive costs for merchants. However, this regulation may have a significant influence on the credit market associated with payment cards, and these implications should be taken into account when evaluating this kind of policy. Interchange fee caps reduce the incentives for issuers to induce more demand to their revolving credit associated with credit cards, and therefore it increments the number of consumers going to the exogenous loan market. If the transaction costs of going to the outside option market are too high or if the convenience benefit from retailers from the revolving credit is also high, reductions on interchange fees have the potential to reduce total welfare. Which effects dominate in practice depends on the consumers' and retailers' behavior of

[16]See Ausubel (1991) for a much earlier discussion of this issue.

a given market, and antitrust authorities should take these additional effects into account when regulating interchange fees.

Appendix 1: Monopoly Section

Demand For Cards

Naive consumers do not observe or value differences on interest rates, valuing only per-transaction fees. They will demand the card offered by the monopolist if and only if

$$v + b_b - f_M \geq v,$$

while sophisticated consumers only when

$$v + b_b - f_M - D\underline{r} \geq v - Dr_0 - h_b,$$

if they expect an interest rate equal to $\underline{r}$, meaning they will use the revolving credit, or

$$v + b_b - f_M - Dr_0 - h \geq v - f_0 - Dr_0 - h_b,$$

if they expect an interest rate strictly higher than $\underline{r}$, meaning they will use the outside option credit. In both cases, the condition defining the demand function is given by

$$b_b \geq f_M. \tag{28}$$

Proof of Proposition 1

The expressions for both per-transaction fees and interest rates are already derived above. To prove when each equilibrium holds, name f_M^{L*} to the per-transaction fee charged in the low interest rate and f_M^{H*} to the fee charged in the high interest equilibrium. Suppose first, that the low interest equilibrium is holding. Then, we need that

$$\Pi(f_M^{L*}, \underline{r}) \geq \Pi(f_M^{L*}, \overline{r}). \tag{29}$$

This means that the monopolist won't deviate to charge the high interest rate in the second period. This condition is equivalent to

$$(f_M^{L*} + a - c_i)(1 - H(f_M^{L*})) + \beta D(\underline{r} - c_c) \geq (f_M^{L*} + a - c_i)(1 - H(f_M^{L*})) + \beta\gamma D(\overline{r} - c_c), \tag{30}$$

which is equivalent to $\gamma \leq \frac{(\underline{r} - c_c)}{(\overline{r} - c_c)}$.

Appendix 2: Single-Homing Section

Demand For Cards

Call x_1 the consumer indifferent between A and B, x_2 the consumer indifferent between A and A' and x_3 the consumer indifferent between B and B'. Assume that a consumer to the left of A will never go to B and that a consumer to the right of B will never go to A. The conditions to calculate x_1, x_2 and x_3 are, for sophisticated consumers, respectively

$$b_b - f_A - Dr_A^e - x_1 t = b_b - f_B - Dr_B^e - (M - x_1)t$$

$$b_b - f_0 - Dr_0 - h - x_2 t = b_b - f_A - Dr_B^e - (K - x_2)t$$

$$b_b - f_B - Dr_B^e - x_3 t = b_b - f_0 - Dr_0 - h - (K - x_3)t.$$

Assuming that both r_A^e and r_B^e are weakly smaller than $\underline{r}$, meaning that sophisticated consumers using A or B expect to use the revolving credit. If the expected rate is higher than $\overline{r}$, consumers expect to go to the exogenous loan, meaning that these relationships change to

$$b_b - f_A - Dr_0 - h - x_1 t = b_b - f_B - Dr_0 - h - (M - x_1)t$$

$$b_b - f_0 - Dr_0 - h - x_2 t = b_b - f_A - Dr_0 - h - (K - x_2)t$$

$$b_b - f_B - Dr_0 - h - x_3 t = b_b - f_0 - Dr_0 - h - (K - x_3)t.$$

Finally, demands for sophisticated consumers are given by

$$D_A^s(f_A; r_A^e) = x_1 + K - x_2 \tag{31}$$

$$D_B^s(f_B; r_A^e) = M - x_1 + x_3. \tag{32}$$

These relationships along with the definition of $\underline{r}$ result in the demands used in section 3. Demands for naive consumers are derived analogously without taking into account interest rates or

$$D_A^n(f_A) = \frac{M + K}{2} - \frac{f_A}{t} + \frac{f_B + f_0}{2t}. \tag{33}$$

Finally, total demand for firms A and B is given by $D_i(f_i, r_i^e) = \gamma D_i^m(f_i) + (1 - \gamma)D_i^s(f_i; r_i^e)$.

Appendix 3: Multi-Homing Section

Demand For Cards

Demands for cards are calculated just as in appendix 2 but with observed interest rates instead of expected interest rates. Therefore, the conditions to calculate x_1, x_2 and x_3 are

$$b_b - f_A - Dr_A - x_1 t = b_b - f_B - Dr_B - (M - x_1)t$$

$$b_b - f_0 - Dr_0 - h - x_2 t = b_b - f_A - Dr_B - (K - x_2)t$$

$$b_b - f_B - Dr_B - x_3 t = b_b - f_0 - Dr_0 - h - (K - x_3)t.$$

If observed interest rates are lower than $\underline{r}$ and

$$b_b - f_A - Dr_0 - h - x_1 t = b_b - f_B - Dr_0 - h - (M - x_1)t$$

$$b_b - f_0 - Dr_0 - h - x_2 t = b_b - f_A - Dr_0 - h - (K - x_2)t$$

$$b_b - f_B - Dr_0 - h - x_3 t = b_b - f_0 - Dr_0 - h - (K - x_3)t.$$

If observed interest rates are higher than $\underline{r}$. The rest of the calculations follows the same steps as in the appendix 2.

Proof of proposition 3

Solving the problem of the firms in $t = 2$, means maximizing:

$$\Pi(r_i; f_i) = D_i(f_i, r_i)(f_i + a - c_i + D(r_i - c_c)), \tag{34}$$

with respect to r_i, which first order condition is given by

$$tD_i(f_i, r_i) = (1 - \gamma)(f_i + a - c_i + D(r_i - c_c), \tag{35}$$

for each firm. Solving this system yields the following interest rates as a function of per-transaction fees

$$r_i^* = \frac{5(t(M + K) + f_0) + 5D(1 - \gamma)(\underline{r} + 2c_c) - 10(1 - \gamma)(a - c_i) - (15 - 8\gamma)f_i + 2\gamma f_{-i}}{15D(1 - \gamma)}. \tag{36}$$

Replacing these expressions in demand functions and solving the maximization in $t = 1$ of the following profit function

$$\Pi(f_i, r_A^*, r_B^*) = D_i(f_i, r_A^*, r_B^*)(f_i + a - c_i + D(r_i^* - c_c)). \tag{37}$$

This profit function is strictly decreasing on per-transaction fees, therefore a candidate equilibrium is the largest interest rate and the corresponding per-transaction fee, that is

$$\overline{f_i^*} = \frac{C + D(1-\gamma)(\underline{r} + 2c_c) - 2(1-\gamma)(a - c_i) - 3D(1-\gamma)\underline{r}}{(3 - 2\gamma)}, \tag{38}$$

and $r_A^* = r_B^* = \underline{r}$. However, even if firms can't keep increasing interest rates (because they would loose all sophisticated consumers), they still have incentives to lower per-transaction fees. This will happen until the following equilibrium condition hold, for any positive Δ

$$\Pi(f_i*, \underline{r}) \geq \Pi(f_i* - \Delta, \underline{r}), \tag{39}$$

which means that per-transaction fees are given by

$$f_i^* = \frac{t(M+K) + f_0 - 2(a - c_i) - 2D(\underline{r} - c_c)}{3} \tag{40}$$

At this value for per-transaction fees, firms have no incentives to deviate either setting slightly higher or lower per-transaction fees.

Now, in $t = 2$ assume you are in the low interest rate equilibrium. Firms are playing best responses to per-transaction fees, but they can still deviate to a high interest rate. They won't do so if and only if

$$\gamma \leq \frac{\underline{r} - c_c}{\overline{r} - c_c}. \tag{41}$$

By the same argument as in the single-homing case. Note that for any given per-transaction fees charged by firms in the first period, this condition makes setting a low interest rate a dominant strategy.

Now, assume the high interest rate equilibrium holds. Firms won't deviate to $\underline{r}$ if and only if

$$\gamma \geq \frac{\underline{r} - c_c}{\overline{r} - c_c}. \tag{42}$$

However, now firms can deviate to even lower interest rates because, in contrast with the single homing case, they can obtain more demand by lowering interest rates. Suppose the firm deviates and Δ less than $\underline{r}$ in $t = 2$. The profits of sticking to the Nash equilibrium strategies and deviating are given by

$$(f_i^* + a - c_i)D_i(f_i^*, \overline{r}) + \gamma D(\overline{r} - c_c)D_i(f_i^*, \overline{r}) \tag{43}$$

$$(f_i^* + a - c_i)D_i(f_i^*, \underline{r} - \Delta) + \gamma D(\overline{r} - c_c)D_i(f_i^*, \underline{r} - \Delta). \tag{44}$$

The deviating profits are higher than the Nash equilibrium profits if and only if

$$0 \geq (3 - 2\gamma)D(\underline{r} - \overline{r}), \tag{45}$$

which can never hold, so this deviation is not profitable, if $\overline{r} > \underline{r}$.

Finally, note that deviations in $t = 1$ and in $t = 2$ by some firm can't occur in any of both equilibriums because, given any per-transaction fees charged in the first period, the equilibrium condition for γ defines the interest rate charged in the second period and charging this interest rate will be a dominating strategy for each firm. Therefore, firms won't have incentives to deviate in the first period either.

References

Agarwal, S., Chakravorti, S., and Lunn, A. (2010). Why do banks reward their customers to use their credit cards?

Agarwal, S., Chomsisengphet, S., Mahoney, N., and Stroebel, J. (2013). Regulating consumer financial products: Evidence from credit cards. Technical report, National Bureau of Economic Research.

Armstrong, M. and Vickers, J. (2012). Consumer protection and contingent charges. *Journal of Economic Literature*, 50(2):477–93.

Ausubel, L. M. (1991). The failure of competition in the credit card market. *The American Economic Review*, pages 50–81.

Bedre-Defolie, Ö. and Calvano, E. (2013). Pricing payment cards. *American Economic Journal: Microeconomics*, 5(3):206–231.

Bourguignon, H., Gomes, R. D., and Tirole, J. (2014). Shrouded transaction costs.

Bradford, T. and Hayashi, F. (2008). Developments in interchange fees in the united states and abroad. *Payments System Research Briefing*.

Chakravorti, S. and To, T. (2007). A theory of credit cards. *International Journal of Industrial Organization*, 25(3):583–595.

Chang, H., Evans, D. S., and Garcia Swartz, D. D. (2005). The effect of regulatory intervention in two-sided markets: An assessment of interchange-fee capping in australia. *Review of Network Economics*, 4(4).

Gabaix, X. and Laibson, D. (2006). Shrouded attributes, consumer myopia, and information suppression in competitive markets. *The Quarterly Journal of Economics*, pages 505–540.

Hayashi, F. (2008). The economics of payment card fee structure: what drives payment card rewards? *Federal Reserve Bank of Kansas City Working Paper*, (08-07).

Hayashi, F. (2009). Do us consumers really benefit from payment card rewards? *Economic Review, First Quarter, Federal Reserve Bank of Kansas City.*

Heidhues, P. and Kőszegi, B. (2015). On the welfare costs of naiveté in the us credit-card market. *Review of Industrial Organization*, 47(3):341–354.

Heidhues, P. and Kőszegi, B. (2017). Naivete-based discrimination. *The Quarterly Journal of Economics*, 132(2):1019–1054.

Iranzo, J., Fernández, P., Matías Clavero, G., and Delgado, M. (2012). The effects of the mandatory decrease of interchange fees in spain.

Rochet, J.-C. and Tirole, J. (2002). Cooperation among competitors: Some economics of payment card associations. *Rand Journal of Economics*, pages 549–570.

Rochet, J.-C. and Tirole, J. (2011). Must-take cards: Merchant discounts and avoided costs. *Journal of the European Economic Association*, 9(3):462–495.

Rochet, J.-C. and Wright, J. (2010). Credit card interchange fees. *Journal of Banking & Finance*, 34(8):1788–1797.

Schmalensee, R. (2002). Payment systems and interchange fees. *The Journal of Industrial Economics*, 50(2):103–122.

Semeraro, S. (2012). Assessing the costs & benefits of credit card rewards: A response to who gains and who loses from credit card payments-theory and calibrations. *Loy. Consumer L. Rev.*, 25:30.

Wright, J. (2003a). Optimal card payment systems. *European Economic Review*, 47(4):587–612.

Wright, J. (2003b). Pricing in debit and credit card schemes. *Economics Letters*, 80(3):305–309.

Wright, J. (2004). The determinants of optimal interchange fees in payment systems. *The Journal of Industrial Economics*, 52(1):1–26.

Wright, J. (2012). Why payment card fees are biased against retailers. *The RAND Journal of Economics*, 43(4):761–780.

Zaki, M. (2018). Interest rates: Prices hidden in plain sight. *Available at SSRN 3168043*.

Platform Price Parity Clauses and Consumer Obfuscation*

José Ignacio Heresi†

January 2020

Abstract

We analyze the impact of platform price parity clauses when sellers set final prices and platforms set fixed fees. In this environment, platforms often rely on different tools to affect the degree of competition between sellers and influence final prices charged to consumers. We develop a model in which platforms can decide the unitary search cost faced by consumers, and show when it is profitable for platforms to obfuscate consumers through high search costs. Then, we show that price parity clauses, when exogenously given, can reduce or increase prices and consumer surplus. Even though platforms' demands become independent of prices, price parity clauses make the equilibrium price set by sellers less responsive to the search cost. Finally, when price parity clauses are endogenous, we show that in the unique equilibrium, platforms set price parity clauses if and only if they lead to higher obfuscation, prices, and lower consumer welfare.

Keywords: price parity rules, consumer obfuscation, platforms.

JEL Classifications: D18, D83, L81.

1. Introduction

Platform price parity clauses (PPCs) restrict sellers joining a platform not to charge a lower price on their alternative distribution channels. In this article, we

*I'm grateful to Wilfried Sand-Zantman, Andrew Rhodes, Bruno Jullien, Gary Biglaiser, Yassine Lefouili, Jidong Zhou, Chris Wilson, Xavier Lambin, Mounu Prem, Mario Giarda, the participants of the EARIE 2018 conference in Athens, the participants of the CRESSE 2018 conference in Crete, the participants of seminars at University of North Carolina at Chapel Hill and at the Toulouse School of Economics, for helpful comments and suggestions. I would like to acknowledge financial support from CONICYT "Becas Chile" and the Jean-Jacques Laffont Foundation.

†Ph.D. Student, Toulouse School of Economics, University of Toulouse Capitole, Toulouse, France: jheresig@gmail.com.

propose a mechanism under which PPCs can increase or decrease final prices and consumer welfare. As platforms are adopting the agency model, in which the final prices of the goods are set by the sellers, platforms often rely on complementary tools to influence the degree of competition between sellers to affect the final price charged to consumers. Then, they extract sellers' profits through different kinds of fees. We show that PPCs reduce the ability of a platform to influence the final prices set by sellers, as these clauses make sellers internalize that a price increase must also occur on their other distribution channels. This "reduced pass-through" effect reduces the incentives of platforms to increase prices through these complementary tools. In contrast, these clauses have been argued to reduce platform competition, leading to platforms setting higher fees, and therefore, sellers charging higher prices to consumers. Which effect dominates critically depends on the shape of the demand function faced by both sellers and platforms.

These clauses are widely used by several platforms, such as Amazon, Booking.com, Expedia, or Apple on its e-books market and have been contested by Antitrust authorities around the world. Some of the main antitrust investigations include Expedia and Booking.com[1], Amazon[2] and Apple's e-books market.[3] These clauses have been argued to reduce competition between platforms, while the defendants argue that these restrictions prevent showrooming, where consumers search in a platform and then buy directly from the seller (or another platform) at a lower price. We contribute to this discussion by showing that, even in the absence of showrooming arguments, PPCs might lead to lower prices and higher consumer surplus.

Our analysis is based on the fact that many online platforms rely on alternative tools to shape competition between sellers, and have stopped using traditional instruments such as linear fees. Linear fees used to provide a simple tool to affect prices set by sellers, but have lost popularity among platforms for different reasons.[4] Some examples of alternative tools used by platforms to influence competition between sellers and extract profits include: the auctioning of prominent positions in the platform's search engine,[5] advertising products on their websites,

[1]In 2015, several European authorities lead to Expedia and Booking removing their wide PPCs. For more details see Wright and Wang (2018).

[2]Amazon removed these clauses in Europe after Antitrust investigations in the U.K. and Germany.

[3]The U.S. Department of Justice contested Apple's switch to the agency model in conjunction with PPCs after Apple entered the e-books market in 2010. For more details see Foros, Kind and Shaffer (2017) for more details.

[4]See Johnson (2017) and Wang and Wright (2017) for some explanations on why linear fees are no longer used by online platforms.

[5]See Athey and Ellison (2011) and Chen and He (2011) for a detailed discussion on this issue.

or selling their own products while competing with independent sellers,[6] among others. We focus on platforms' incentives to obfuscate consumers by increasing their search cost when sampling sellers in the platform.

While obfuscation practices by firms have been extensively studied in the literature, the reasons why platforms acting as marketplaces would obfuscate are less clear. We argue that platforms may use obfuscation in order to reduce competition between sellers, and then extracting sellers' profits through complementary fees, such as fixed or ad-valorem fees. One of the main examples of obfuscating practices observed online is the use of partitioned and drip pricing,[7] where the total price is divided into several subcategories, including shipping fees, handling fees, or credit card surcharges, among others. Huck and Wallace (2015) show, through experimental evidence, that the way prices are showed or "framed" may have detrimental effects on consumers' search process. In this experiment, just by adding two clicks to find out the total price of the product, consumers end up paying higher prices while their consumer surplus is reduced by 22%.[8] Another relevant example of drip pricing, that has brought the attention of the regulators in the United States is the use of "resort fees" by hotels, especially when posting their rooms in online travel agents. These are per-room and per-night mandatory fees charged by some hotels and are usually disclosed separately from the room rate. An analysis from the Federal Trade Commission finds that these separate resort fees are "*likely to harm consumers by increasing the search costs and cognitive costs of finding and choosing hotel accommodations*".[9]

To investigate these issues, we develop a stylized model that matches the features of many online markets. We focus on an equilibrium where a large number of sellers join two horizontally differentiated platforms, while consumers join and search only from their preferred platform. This kind of "bottleneck" equilibrium is representative of many of these markets. We assume that consumers incur in costly search to learn both the price and their valuation for a product, and make optimal

[6] For example, Amazon is known to sell its own products in competition with independent sellers.

[7] Drip pricing is a particular case of partitioned prices where the different categories composing the price are showed after in the search process.

[8] There is an extensive body of empirical research reaching similar results. For example, Hossain and Morgan (2006) use data from eBay and show that increasing the shipping fee relative to the base price, increases the number of bidders and the revenue for firms selling through this website. Also, Brown et al. (2010) make a field experiment using the Yahoo Taiwan and the eBay Ireland platforms, and find that increasing shipping fees can boost revenues if these fees are shrouded. An interesting example is the case of taxes that are sometimes shown separately and later on in the purchasing process. Chetty et al. (2009) show, while studying the salience of taxes in a supermarket, that taxes included in posted prices have more significant effects on consumer demand than the ones added at the register in the end. Greenleaf et al. (2016) provide a survey of this literature.

[9] See "Economic analysis of hotel resort fees" by Mary Sullivan from the Federal Trade Commission, January 2017. Quote from Executive Summary p.V.

decisions relative to which platform to join. Sellers are horizontally differentiated and compete in prices. Platforms design a search environment for consumers and sellers to interact, defined by the unitary search cost faced by consumers when sampling sellers. For simplicity, we assume that platforms choose a single level of search cost in the model. Platforms also charge a fixed fee to sellers.

Our first contribution is to show that, in the absence of PPCS, when competition between platforms is not too intense, there will be obfuscation in equilibrium, interpreted as search costs in equilibrium being higher than a minimum exogenous level. The intuition is that when platforms increase their search cost, they reduce price competition between sellers, because consumers, on average, sample fewer firms. Therefore, the equilibrium price set by sellers increase. This reduces platforms' demands due to: the higher equilibrium price and consumers finding, on average, worse products, due to the higher search costs. If platform differentiation is not too small, the equilibrium involves obfuscation.

Our second contribution is to analyze the impact of PPCs. We start by showing that, when these clauses are exogenously imposed, the level of obfuscation and prices may decrease. The intuition is that PPCs affect platforms' incentives to obfuscate in two opposite ways. First, when PPCs are imposed, platforms' demands stop depending on prices, as consumers expect the same price in both platforms, increasing platforms' incentives to obfuscate to increase the price set by sellers. Second, PPCs reduce the pass-through rate from search costs to final prices, as sellers facing an increase in the search cost by a given platform, now internalize that increasing their price also means increasing it in the rival platform. Therefore, the price increase or a marginal increment in the search cost of a given platform is lower, decreasing its incentives to obfuscate. We show that which effect dominates depends on the curvature of the distribution of match values, understood as the curvature of the Mills ratio of the match values distribution around the symmetric equilibrium.

Then, we extend the model, and we allow platforms to choose whether to impose PPCs or not as an endogenous variable. We show that these clauses are imposed if it is individually profitable for each platform to do so. Therefore, in the unique equilibrium of the game, if price parity clauses are set by both platforms, equilibrium prices are higher, consumer surplus is lower, and platforms profits increase, relative to the case with no PPCs. On the contrary, if PPCs are not set in equilibrium, it is because they lead to lower equilibrium prices and profits for platforms.

Literature review

This work is directly related to the platform price parity clauses literature. Wang and Wright (2018) focus on an environment where consumers can search for products sold through platforms or directly trough the sellers' direct channel. Price parity clauses in this context would prevent consumers from searching in a platform and then switching to buy directly in a seller's direct channel at a lower price. They analyze two types of PPCs. First, narrow PPCs, where sellers are cannot charge lower prices in their direct channel. Second, wide PPCs, where sellers cannot charge lower prices in any other channel. In this setting, when a monopoly platform exists, PPCs lead to higher prices and lower consumer surplus. When there are competing platforms, a narrow PPC might be good for consumers as long as platform competition is sufficiently intense.

Johansen and Vergé (2017) also model a case with platform competition and a direct channel. In their model, wide PPCs can be better for all participants, depending on the level of differentiation between goods and between platforms. Boik and Corts (2016), on their part, find that prices are always higher under PPCs, but it might be that this effect is so strong, that even platforms are worse in equilibrium. Edelman and Wright also develop a model in which sellers can sell in their direct channel, and PPCs prevent showrooming by increasing the price if this channel. Platforms over-invest in ancillary services when PPCs are in place, and consumer surplus is lower.

Calzada, Manna, and Mantovani (2019) focus on the online travel agents market and analyze segmentation issues, where hotels can choose to delist from a platform and sell directly through their own channels. In their model, online travel agents set PPCs when showrooming is intense or when substitutability between them is high. When PPCs are set, hotels choose to single-home and sell their products only in one online travel agent. The main difference between these papers and ours is that we focus on a setting with no direct channel but with ad-valorem fees instead of linear fees and that in our model, platforms indirectly affect competition through an additional competitive variable.

Johnson (2017) and Foros, Jarle Kind, and Shaffer (2017) study the impact of PPCs in the decision of upstream and downstream firms to adopt the agency model, where suppliers instead of retailers set final prices, and the revenues are split according to an ad-valorem fee. While their focus is mainly related to the effects of the agency model on final prices and welfare relative to the traditional wholesale model and to when this model is adopted, we take as given that platforms use the agency model and set the ad-valorem fees and focus on the effect of PPCs when platforms use alternative tools to control the final price charged by sellers indirectly. Therefore, we consider all this literature complementary to our work.

This article is also related to the literature of obfuscation, in particular, related to obfuscation practices by intermediaries. Hagiu and Jullien (2011, 2014) study the incentives o an information intermediary, that collects per-click fees, to divert search. The intermediary has superior information about that match between a consumer and a firm, and they derive conditions under which the intermediary guides the consumer to search their less preferred firm first. Eliaz and Spiegler (2011) explain how a search engine chooses a pool of sellers to show when a consumer submits a query and is also paid on a per-click basis. They also find that it may be optimal for the engine to contaminate the search pool with non-relevant firms for consumers. Teh and Wright (2018) also consider a case where an intermediary has higher information on the match between consumers and firms, but its revenue comes from sellers paying a commission conditional on consumers making a purchase. The intermediary provides a ranking of firms for consumers. In equilibrium, even if the recommendation is not distorted, competition between sellers to offer higher commissions to the intermediary increase final prices, and consumer surplus is lower. Other papers focusing on obfuscation by intermediaries that give recommendations to consumers are Inderst and Ottaviani (2012a, 2012b), Murooka (2015), and de Cornière and Taylor (2016). Our work extends and complements this literature by showing how obfuscation, through increased search costs, might be optimal in an environment where platforms set ad-valorem fees and sellers set final prices.

The rest of the article is organized as follows. In Section 2, we describe the model and its main assumptions. In Section 3, we analyze the platforms' obfuscation game, when PPCs are not imposed. Then, in section 4, we analyze the platforms' game when PPCs are exogenously imposed, and when PPCs are a decision variable for each platform. Finally, in Section 5, we conclude.

2. The model

A continuum of consumers has unit demand for a single good produced at zero marginal cost by a continuum of sellers (firms). We normalize the measures of both consumers and sellers to one. Consumers can only interact with sellers through one of two competing platforms acting as marketplaces, called A and B. The value of a seller's product is idiosyncratic to consumers, and they must engage in costly search to learn both the match value and the price of a given seller. The surplus of buying from seller j, net of search costs, is $\epsilon_{ij} - p_{jk}$, where ϵ_{ij}

is the idiosyncratic match utility of consumer i derived from firm j (the same in both platforms) and p_{jk} is the price of firm j on platform k. We assume that the distribution of ϵ_{ij} is independent across consumers and firms and has a smooth, positive everywhere and log-concave density function g on the interval $[0, \overline{v}]$, with cumulative distribution function G.

Consumers also have a heterogeneous fixed cost of joining each platform denoted by t_A and t_B. We assume that half of consumers have $t_A = 0$ and t_B distributed uniformly on $[0, t]$, while the other half has $t_B = 0$ and t_A following the same distribution. This assumption represents a situation where half of the consumers are already familiar with one of the platforms and must face a fixed cost if they decide to join the other platform and vice versa. Therefore, the parameter t is a measure of platform differentiation or market power, because a higher value of t makes consumers, on average, more reluctant to visit one of the platforms.[10] Consumers' outside option to both platforms is 0.

Platforms mediate transactions at zero cost. Platform $k \in \{A, B\}$ charges a publicly displayed fixed fee τ_k to all sellers. The platforms also design a search environment characterized by the unitary search cost s_k faced by consumers when sampling sellers in their marketplace. There is a minimum exogenous search cost $\underline{s}$ that can be set by the platforms, meaning that even if they create the simplest search environment possible, consumers must still incur a small cost to sample each firm.[11] We define obfuscation as the equilibrium search cost set by both platforms being higher than $\underline{s}$. Finally, platforms set no fees to consumers.

Sellers compete in prices and decide whether to participate in one or both platforms after observing the fixed fee τ_k, and the unitary search costs s_k set by A and B. The search process for consumers is assumed to have perfect recall and no replacement. Consumers observe the search cost of each platform before making their participation decision and have rational expectations on equilibrium prices and sellers' participation decisions.[12] We assume the search process is random, meaning there is no particular order in which consumers sample sellers. Consumers hold passive beliefs about equilibrium prices when observing a price deviation from

[10] This specification allows us to derive Hotelling-like demands for platforms and focus on platform competition in an equilibrium where all consumers participate in the market, without the technical complications of adding an extensive margin decision when platforms choose their strategic variables. See Moraga-González, Sándor and Wildenbeest (2017) for a search model with heterogeneous search costs where both the intensive and extensive margin play a role in market outcomes.

[11] The model could be solved ignoring this assumption. The main insights would not be affected. This assumption helps to make exposition simpler by ignoring the case where search costs are negligible, and the model becomes the one studied by Perloff and Salop (1985).

[12] If consumers would anticipate instead of observing the search costs, then platforms would deviate by increasing the search costs once consumers have joined their marketplace, leading to a Diamond paradox like equilibrium with maximum search costs.

the equilibrium path.[13]

The timing of the game is as follows: at stage 1, platforms simultaneously design their search environment by choosing s_k. At stage 2, they set their fixed fee τ_k after observing the search design of their rival. At stage 3, firms decide which platform to join and simultaneously set prices, possibly different, in each of them. Finally, at stage 4, consumers decide which platform to join and search randomly for their preferred product.

We focus on a symmetric Perfect Bayesian Equilibrium in which platforms set symmetric fixed fees and obfuscation levels, sellers join both platforms and consumers search and buy in only one, usually referred to as a bottleneck equilibrium.

2.1. Discussion of modeling assumptions

Our model tries to capture the fact that platforms create search environments, where it can be easier or harder for consumers to search. While part of the literature focuses on the composition of the pool of firms shown to consumers after a query, or the ordering of such firms,[14] we focus on the unitary search cost, interpreted as how difficult is to learn the relevant information of a particular product. For example, which features of the product are immediately shown and which features are discovered after a few clicks. Such features include price, product characteristics, shipping fees, etc. In practice, platforms give different options to sellers, and they decide how to show their information within the framework that the platform allows. For simplicity, we assume that the platform chooses a single frame that must be followed by all firms.

Regarding platforms' fee structure, we focus on a fixed fee aimed at extracting sellers' profits. In practice, many platforms have switched to a business model where they set ad-valorem fees, defined as a percentage of the price charged by sellers. Our results would be unaffected under the assumption of an exogenous ad-valorem fee instead of a fixed fee. For example, Apple uses a fixed ad-valorem fee of 30% for all firms in every one of the markets they serve under the agency model.[15] Given that our main intuition comes from the fact that platforms are

[13]This means that observing a price deviation from a seller does not change the expectation prices charged by unsampled sellers. We also assume that the search process has perfect recall, meaning that consumers can come back at any time to buy from a previously sampled firm.

[14]See, for example, Athey and Ellison (2011), Eliaz and Spiegler (2011), Chen and He (2011), Chen and Zhang (2017), among others.

[15]Even when considering an endogenous ad-valorem fee, it is generally the case that platforms increase such fees to extract most profits from sellers, maintaining our main intuitions. From a technical perspective, when price parity clauses are imposed, such fees generate the unrealistic deviation from equilibrium in which a platform reduces the equilibrium price in the other platform, therefore sellers do not join, and that platform becomes inactive.

not using linear fees,[16] and, therefore, must rely on other variables to affect price competition and equilibrium prices, assuming a fixed fee simplifies the analysis without losing important features of these markets.

With respect to the equilibrium concept, we focus on a symmetric equilibrium where both platforms are active while sellers multi-home and consumers single-home. If consumers single-home, firms have incentives to join both platforms to access as many consumers as possible, as long as they cover their fixed costs. This kind of competitive bottleneck equilibrium, where the side that faces differentiation (consumers) single-homes and the side that considers platforms as homogeneous (sellers) multi-homes, naturally arises in this kind of setting, as discussed by Armstrong and Wright (2007). Also, there are less interesting equilibria where only one or none platform is active because consumers expect no firms to join, and firms do not join because they expect no consumer to participate.

Finally, we assume that consumers search randomly among the firms in a given platform. Even if in reality firms are usually ordered, and this order influences consumers' search behavior, the central intuition where a higher search cost reduces competition between sellers and therefore increases equilibrium prices, stays unchanged.[17] Hence, assuming that firms are ordered would not affect the qualitative results of our model.

2.2. Preliminaries

In this section, we briefly recapitulate some known results from the search literature that are instrumental in understanding the analysis that follows.[18]

Consumers' behavior: consumers' behavior is characterized by whether to participate in the market or not, and which platform to use, along with their search and purchasing behavior once they are in a platform.

Once on a platform, as is known from Kohn and Shavell (1974) and Weitzman (1979), consumer optimal search is characterized by a stationary stopping rule, based on a constant reservation value that we denote a, that depends on the

[16]There are several explanations on why platforms are migrating to the agency model with ad-valorem fees. For example, Wang and Wright (2017, 2018) show that these fees are used to achieve efficient price discrimination when platforms sell different goods with different costs and valuations by consumers.

[17]textFor example, Armstrong, Vickers and Zhou (2009) show that in a model where a firm is prominent, and therefore sampled first by every consumer, equilibrium prices of every firm are still increasing in the search cost of consumers.

[18]The derivation of all of the following results are found in Wolinsky (1986) and Anderson and Renault (1999).

search cost. This reservation value a is given by the unique solution to

$$\int_{a}^{\overline{v}} (x-a)dG(x) = s. \tag{1}$$

When consumers are in a platform, they start sampling firms randomly and suppose all the firms join each platform. Every time they sample a firm, say firm j, it is optimal for them to stop and purchase that good if $\epsilon_{ij} - p_j > a - p^*$, where p^* is the expected equilibrium price of the other firms. If they purchase the product, they leave the market. If not, they go on and sample the next firm and follow the same decision rule. Therefore, equation (1) derives the reservation value such that the incremental benefit of one additional search is equal to the search cost. Given that there is a continuum of firms in each platform, consumers never return and buy from a previously sampled firm and never switch platforms.

Before joining either platform, consumers must decide whether to participate in the market and which platform to join. The expected consumer surplus of joining platform k, that sets reservation a value a_k (or equivalently a search cost s_k) is given by

$$\phi_k = \underbrace{\frac{\int_{a_k}^{\overline{v}} x dG(x)}{1-G(a_k)}}_{\text{Expected match value}} - \underbrace{\frac{s_k}{1-G(a_k)}}_{\text{Expected search cost}} - \underbrace{p_k^*(s_k) - t_k}_{\text{Expected price and fixed cost}} . \tag{2}$$

This expression is derived as follows. Given that there is a continuum of firms in the platform, consumers eventually find a suitable product. Therefore, their expected match value is given by $\mathbb{E}[\epsilon | \epsilon \geq a]$. With respect to the expected search costs, in a symmetric price equilibrium, a consumer stops and buys in a given firm with probability $1 - G(a_k)$, and keeps searching with probability $G(a_k)$. Therefore, the expected search cost is given by $\sum_{l=0}^{\infty} G^l(a_k)s = \frac{s}{1-G(a_k)}$.

After some manipulations, we can rewrite (2) as:[19]

$$\phi_k = a_k(s_k) - p_k^*(s_k) - t_k, \tag{3}$$

where we observe that the expected match value and the expected search cost simplify to a_k. Consumers join the platform yielding the highest expected consumer surplus based on the observed search cost, expected prices, and idiosyncratic fixed cost, as long as this value is non-negative.

Firms' pricing: for a given level of a_A and a_B, denote D_A and D_B as the demands of platforms A and B, respectively. As all sellers join both platforms, the expected number of consumers searching a given seller in platform k is D_k,

[19]Integrating by parts the first term and using (1) on both terms to derive an expression only as a function of a_k.

given that the number of buyers and sellers are normalized to 1. In a symmetric price equilibrium between sellers, the probability of a consumer buying in any given seller when it is sampled is $1 - G(a)$. Therefore, the expected number of consumers sampling any seller in their second round is $G(a)D_k$, in their third round is $G^2(a)D_k$, and so son. Now, suppose seller j deviates from the equilibrium. A consumer buys from firm j if and only if $e_{ij} - p_j \geq a - p_k^*$, where p_k^* is the equilibrium price in platform k. Therefore, seller j's demand in platform k is given by

$$\sum_{l}^{\infty}[G^l(a)][1 - G(p_{jk} + a - p_k^*)]D_k = \frac{1 - G(p_{jk} + a_k - p_k^*)}{1 - G(a)}D_k. \tag{4}$$

Note that in a symmetric equilibrium, each seller's demand is equal to D_k. As consumers join platforms based on expected prices, the value of D_k is fixed for sellers, and they only compete for consumers that have decided to join that platform. The profit function of firm j is given by

$$\Pi_j = \Pi_{jA} + \Pi_{jB}, \tag{5}$$

where, for $k \in \{A, B\}$,

$$\Pi_{jk} = p_{jk}\frac{[1 - G(p_{jk} + a_k - p_k^*)]}{1 - G(a_k)}D_k - \tau_k. \tag{6}$$

The prices every firm sets in each platform are independent of each other, as the price set in one platform does not affect a given seller's demand on the other platform. Therefore, taking the first-order conditions for a symmetric price equilibrium in each platform leads to

$$p_k^* = \frac{1 - G(a_k)}{g(a_k)}. \tag{7}$$

This is the unique symmetric price equilibrium of the sellers' pricing stage for a given a_k, under the assumption that g is log-concave.

We define $\lambda(a) \equiv \frac{1-G(a)}{g(a)}$ as the Mills ratio of the distribution g. Therefore, the equilibrium price is equal to the Mills ratio of the distribution. Given that g is log-concave, the equilibrium price is decreasing in a_k or, equivalently, increasing in s_k. Also, $\lambda'(a)$ represents the pass-trough rate of an infinitesimal change on the reservation value a_k to the equilibrium price.

Given that sellers cannot affect the share of consumers participating in a given platform and given that the equilibrium price in a platform is independent of the strategic variables set by the rival platform, sellers will participate in each platform as long as they make non-negative in each platform.

3. Consumer obfuscation

In this section, we characterize the symmetric Perfect Bayesian Equilibrium of the game, focusing on a bottleneck equilibrium in which sellers join both platforms and consumers search and buy in only one platform. We explain why and when platforms obfuscate in equilibrium, meaning that they design a search environment with a unitary search cost higher than the minimum exogenous level $\underline{s}$.

To begin the analysis, we highlight the relationship between the search cost s_k set by a platform k, and the corresponding reservation value a_k associated with that value of the search cost. A higher search cost implies a lower reservation value and leads to a higher equilibrium price. This leads to a consumers' optimal search rule associated with consumers searching less, and therefore, finding, on average, lower match values. Thus, their expected consumer surplus is lower. For technical simplicity, we take advantage of this one-to-one inverse relationship between search costs and reservation values, and from now on, we characterize the stage 1 of the game as the platforms' choice of reservation values.

Now, we describe the feasible set for reservation values for both platforms. Define $\underline{a}$ as the unique value of a such that $\underline{a} = \lambda(\underline{a})$.[20] Given the expression for expected consumer surplus, $\phi(a_k) = a_k - \lambda_k(a_k) - t_k$, and given that $a_k - \lambda_k(a_k)$ is strictly increasing in a_k, any a_k lower than $\underline{a}$ would leave consumers with negative expected consumer surplus, even if their fixed cost is 0. Therefore, platforms will never set a reservation value $a_k < \underline{a}$. Also, the minimum search cost $\underline{s}$ generates a maximum possible reservation value $\overline{a}$, given by equation (1) evaluated at $\underline{s}$. Hence, platforms' feasible set for reservation values is $[\underline{a}, \overline{a}]$.[21]

To ensure the existence and uniqueness of the symmetric bottleneck equilibrium, we make the following assumption:

Assumption 1 The Mills ratio $\lambda(a)$ is log-concave, meaning that $\lambda(a)\lambda''(a) - \lambda'^2(a) \leq 0, \quad \forall a \in [\underline{a}, \overline{a}]$.

This assumption means that the Mills ratio is not "too convex" in the relevant support for the platforms choice. This assumption is satisfied by any distribution with constant curvature, such as the Generalized Pareto Distribution (GPD). It also holds the Power Function distribution and some parametrizations of the Beta distribution. Moreover, we provide a result in the Appendix (see Lemma 3), showing that right-hand truncations of log-concave distribution functions generate log-concave Mills ratios, as long as the truncation point is not too high, therefore satisfying assumption 1.

[20] It is unique due to the log-concavity of the distribution of match values.
[21] This set is non-empty as long as the value of $\underline{s}$ is not too high.

To derive platforms' demands, note that consumers expect the continuum of firms to be active in each platform, so they rationally anticipate that joining a platform will lead to a purchase with probability 1 in that platform. Therefore, they will join platform A, rather than platform B, if and only if $\phi_A(a_A) \geq \phi_B(a_B)$ or, equivalently, if $(a_A - p_A^* - t_A \geq a_B - p_B^* - t_B)$. Consider the case where $a_A \geq a_B$. Then, all consumers that have $t_A = 0$ go to platform A. Consumers with $t_B = 0$ also buy from A as long as $(t_A \leq a_A - a_B + p_B^* - p_A^*)$. Therefore

$$D_A(a_A) = \underbrace{\frac{1}{2}}_{\text{Consumers with } t_A = 0} + \underbrace{\frac{1}{2}\Pr(t_A \leq a_A - a_B + p_B^* - p_A^*)),}_{\text{Consumers with } t_B = 0} \tag{8}$$

which is equal to

$$D_A(a_A) = \frac{1}{2} + \frac{a_A - a_B}{2t} + \frac{p_B^*(a_B) - p_A^*(a_A)}{2t}. \tag{9}$$

We obtain the same expression for D_A when $a_B > a_A$. Therefore, D_A is given by expression (9). The demand for platform B is derived following the same steps.

The reservation value a_A has two effects on demand. First, a traditional price effect, where a lower reservation value increase the equilibrium price, and therefore, reducing consumers expected consumer surplus. Second, an effect on consumers' optimal search rule, where lower reservation value increases the expected match value net of search costs they obtain in the search process.

Platform k extracts all profits from sellers joining that platform, by setting a fixed fee $\tau_k = p_k^* D_k$ at t=2. Therefore, $t = 1$, the maximization problem of a given platform, say A, is given by

$$\max_{a_A} \Pi_A(a_A, \tau_A) = p_A^*(a_A) D_A(a_A). \tag{10}$$

The maximization problem of platform B is analogous. The first-order condition for platform A with respect to the reservation value a_A is given by

$$\lambda'(a_A) D_A(a_A) + \frac{(1 - \lambda'(a_A))}{2t} \lambda(a_A) = 0, \tag{11}$$

and similarly for platform B. In a symmetric equilibrium of the obfuscation game between platforms, we have

$$\lambda(a^*) = -\frac{t\lambda'(a^*)}{(1 - \lambda'(a^*))}. \tag{12}$$

The following proposition characterizes the unique symmetric bottleneck equilibrium of the game:

Proposition 1. *Suppose assumption 1 holds. Then, for any $t > 0$, there exists a unique symmetric bottleneck equilibrium. Sellers multi-home and make zero* *profits in each platform, and consumers always search and buy from their preferred platform. Moreover, there exist values $\underline{t}$ and $\overline{t}$ such that the equilibrium reservation value (search cost) is as follows*

- *If $t \leq \underline{t}$, then $a^* = \overline{a}$.*
- *If $\underline{t} < t < \overline{t}$, then $a^* \in [\underline{a}, \overline{a}]$, given by the solution of (12).*
- *If $\overline{t} < t$, then $a^* = \underline{a}$.*

The equilibrium reservation value is non-increasing in t, and the equilibrium search cost is non-decreasing in t.

Proof. See Appendix. □

This result shows how obfuscation can be a useful tool to increase prices and profits for platforms. The main economic trade-off highlighted by proposition 1 is as follows. If a platform decreases its reservation value, or equivalently, increases its search cost, competition between sellers is reduced, and the equilibrium price increases. The cost of doing so is a loss of demand due to this higher price and also due to a lower expected consumer surplus because of lower match values. When differentiation between platforms is high enough, the equilibrium reservation value is lower than $\overline{a}$, and the equilibrium search cost is higher than $\underline{s}$, result that we interpret as obfuscation.

In the symmetric bottleneck equilibrium described in Proposition 1, all consumers join the platform where they face zero fixed cost. Therefore, consumer welfare is proportional to the equilibrium reservation value (inversely proportional to the equilibrium search cost) as a higher reservation value lowers the equilibrium price and increases the match value that consumers obtain on average. This means that increased competition between platforms increases consumer surplus. Platforms profits, on the contrary, decrease if t is lower, while sellers are always fully extracted.

In our model, the platform sets a unique search cost that consumers face when searching for products. In reality, platforms implement this by allowing their sellers to use different forms of obfuscation, such as using drip pricing or concealing relevant information to consumers. This forces consumers to click several times on a product to find out all the relevant information before purchasing, increasing their search cost. In doing so, consumers search, on average, fewer firms, softening competition between sellers, and increasing the prices they charge. Finally, platforms extract these profits through fixed fees.

A relevant example of such obfuscation behavior is the use of "resort fees" when hotels post their offers in online travel agents (OTAs). These mandatory per-night fees charged by some hotels are often concealed from consumers when searching for hotel rooms and are showed at late stages of the purchasing process. This behavior brought the attention of the Federal Trade Commission in the United States, that concluded that this conduct "*is likely to harm consumers by increasing the search costs and cognitive costs of finding and choosing hotel accommodations*".[22] Even if hotels are setting the resort fees, OTAs design the search environment where it is possible for the hotels to obfuscate through the use of such fees. We interpret this as OTAs obfuscating consumers.

This result also highlights that increased platform competition in our model always leads to lower obfuscation and equilibrium prices. Whether competition is enough to solve obfuscation practices has been a relevant question in the literature. While Hagiu and Jullien (2014) also find that platform competition reduces obfuscation practices, other articles find opposite results. For example, de Cornière and Taylor (2016) find that competition between biased intermediaries might lead to the same outcome as in a case with a monopolist because all consumers use only one of the intermediaries in equilibrium.[23]

Finally, we explain the effect of the curvature of the match value distribution in our result. For this purpose, we provide an example by assuming that the distribution of match values follows a Generalized Pareto Distribution with a density function given by $g(x,\xi) = (1+\xi x)^{-(1+\xi)/\xi}$ and a cumulative distribution function given by $G(x,\xi) = 1-(1+\xi x)^{-1/\xi}$. This distribution has the characteristic of having a linear Mills ratio given by $\lambda(x,\xi) = 1+\xi x$. We assume that $\xi < 0$, implying the Mills ratio is strictly decreasing in x.

Example 1. *If the distribution of match values follows a GPD with paramter $\xi < 0$, we can analytically solve for the equilibrium reservation value, given by:*[24]

$$a^* = \frac{\xi(1-t)-1}{\xi(1-\xi)}. \tag{13}$$

We observe that the equilibrium reservation value is decreasing in t and increasing in ξ in the interval $[\underline{a}, \overline{a}]$. The intuition is that if the curvature of the Mills ratio, given by the value of ξ, becomes smaller in absolute value, the price equilibrium in the sellers' stage is less responsive to changes in reservation values by platforms. This lower pass-through from search costs to the equilibrium prices set by

[22]See "Economic analysis of hotel resort fees" by Mary Sullivan, Bureau of Economics, Federal Trade Commission 2017.

[23]This result is often found in the literature on obfuscation by individual firms. For example, Gabaix and Laibson (2006) find that competition will not eliminate shrouding of attributes by firms, and in Ellison and Wolitsky (2012) the levels of obfuscation might increase in the number of firms.

[24]The conditions of existence and uniqueness of equilibrium hold as for the GPD $\lambda'' = 0$ and therefore assumption 1 holds for any $\xi < 0$.

sellers decreases the incentives of platforms to induce higher prices through lower reservation values (through higher obfuscation).

For distributions without a constant Mills ratio, how the curvature of demand affects the equilibrium depends on the shape of the distribution around the symmetric equilibrium, which in turn depends on the value of t. In the next section, we study this issue when comparing the result of Proposition 1 with a case where price parity clauses are set by the platforms.

4. Price parity clauses

In this section, we analyze the effects of price parity clauses on the level of search costs and equilibrium prices. We focus on "wide" price parity clauses, where sellers joining a platform that imposes such a clause cannot charge a lower price in the other platform. We start by characterizing the case where price parity clauses are exogenously imposed in both platforms, to understand their effects on consumer behavior, sellers' pricing, and platforms' obfuscation strategies. Then, we extend the model to allow platforms to decide whether to impose such clauses or not, allowing us to understand platforms' incentives to impose those restrictions.

Exogenous price parity clauses: suppose first that PPCs are exogenously imposed by both platforms. Therefore, as we continue to focus on a symmetric bottleneck equilibrium, sellers set a uniform price. We show that PPCs influence platforms' search design decisions through two effects. First, a demand effect, given by the fact that consumers expect the same price in both platforms. Therefore, platforms' demands become less elastic to changes in their reservation values (search costs), as platforms' demands become independent of equilibrium prices. This effect decreases the level of competition between platforms and increases platforms' incentives to implement higher equilibrium prices through higher obfuscation. Second, a pass-through effect, where the sensibility of the equilibrium price with respect to the reservation values is lower. This reduces platforms' incentives to increase prices through obfuscation. We show how the shape of the distribution of match values is critical to determine which effect dominates. In equilibrium, PPCs may lead to higher or lower obfuscation and prices.

To understand the demand effect, note that consumers' participation decision and search behavior once they join a platform is the same as in the previous section. The only difference is that they expect the same price on both platforms. Thus, platform A's demand is given by

$$D_A(a_A) = \frac{1}{2} + \frac{a_A - a_B}{2t}. \tag{14}$$

The demand for platform B is derived analogously. Platforms' demands become less elastic to changes in reservation values, as they are now independent of the uniform equilibrium price. This reduces platform competition when designing their search environment, increasing their incentives to obfuscate.

On the seller side, a given seller j's profit function is now given by

$$\Pi_j = p_j \left[D_A \frac{1 - G(a_A + p_j - p_p^*)}{1 - G(a_A)} + D_B \frac{1 - G(a_B + p_j - p_p^*)}{1 - G(a_B)} \right] - \tau_A - \tau_B, \quad (15)$$

where p_p^* is the symmetric price equilibrium in both platforms and p_j is the uniform price set by seller j. The profit function is the same as the one derived in section 2, but now sellers must charge the same price on both platforms. The following result characterizes the unique equilibrium of the sellers sub-game at $t = 3$, for given values of a_A, a_B, when price parity clauses are exogenously imposed on both platforms:

Lemma 1. *Suppose sellers join both platforms, and price parity clauses are exogenously imposed. Then, for given a_A and a_B, if $\underline{s}$ is not too small, the unique symmetric price equilibrium in the sellers' stage is given by sellers charging*

$$p_p^* = 2\frac{\lambda(a_A)\lambda(a_B)}{\lambda(a_A) + \lambda(a_B)} \quad (16)$$

in both platforms. The equilibrium price is increasing in a_A and a_B. Moreover, if $a_A > a_B$, we have

$$p_A^* < p_p^* < p_B^*, \quad (17)$$

where p_A^* and p_B^* are the equilibrium prices in platforms A and B when PPCs are not imposed. Finally, if $a_A = a_B$, we have

$$p_A^* = p_B^* = p_p^*. \quad (18)$$

Proof. See Appendix. □

The equilibrium price is a composition of the prices that would be set in each platform in the absence of price parity clauses. This price now depends on the reservation values set by both platforms. The reservation values' effect on the uniform equilibrium price is twofold. First, there is a composition effect, where a platform setting a higher reservation value than its rival attracts more consumers, and therefore, becomes more important for sellers. Second, we have a competition effect, where a higher reservation value, equivalent to a lower search cost, increases competition between sellers in that platform. Both effects go in the same direction and imply that an increase in any of the reservation values decreases the equilibrium price. Note that Lemma 1 assumes that the exogenous minimum search cost is not too small. This assumption is needed to ensure that the sellers' profit functions are quasiconcave with respect to their own price.[25]

The equilibrium price in Lemma 1 allows us to understand how the pass-through from reservation values to the equilibrium price is affected when PPCs are in place. Around a symmetric equilibrium, we have

$$\frac{\partial p_p^*}{a_k}(a_A = a_B) = \frac{\lambda'}{2}, \tag{19}$$

while in the case with no PPCs, the pass-through from reservation values to the equilibrium price in a platform is λ'. This means that the pass-through rate from reservation values to the equilibrium price is lower, meaning that a marginal decrease in the reservation value of a platform, or, equivalently, a marginal increase in the search cost of a platform, increases the price charged by sellers by a smaller value than in the last section. Therefore, the incentives of platforms to increase prices through higher obfuscation are lower.

The intuition for this reduced pass-through is that, when a platform decreases its reservation value (or increases its search cost), sellers now internalize that if they increase their price, they must also increase their price on the other platform. Therefore, this price increase is lower, and the corresponding pass-through rate is also lower.

Next, we characterize the platforms' optimal strategies regarding their fixed fees and search design. At $t = 2$, for a given a_A and a_B, platforms set their fixed fees to attract sellers to participate while extracting all of their profits. Therefore, the fixed fee is such that the participation constraint for sellers is binding in each platform, given by $\tau_k = p_p^* D_k$. Therefore, at $t = 1$, platform A maximizes

$$\max_{a_A} \Pi_A(a_A, a_B) = p_p^*(a_A) D_A(a_A, a_B), \tag{20}$$

with respect to a_A. Taking the first-order condition with respect to a_A leads to

$$\frac{\partial p_p^*}{\partial a_A} D_A + \frac{p_p^*}{2t} = 0. \tag{21}$$

Following the same steps for platform B, around a symmetric equilibrium we have

$$\lambda(a_p^*) = -\frac{t\lambda'(a_p^*)}{2}, \tag{22}$$

where a_p^* is the symmetric equilibrium reservation value. The following result compares the unique symmetric bottleneck equilibrium of the game when PPCs are exogenously imposed in both platforms, with the case where PPCs are not

[25] For very asymmetric values of a_A and a_B, one platform becomes irrelevant for sellers as it attracts only a few consumers, and profit functions for sellers may fail to be quasiconcave. To avoid such a case, we assume that $\underline{s}$ is not too small, so that $\overline{a}$ is not too high and therefore the profit functions are well behaved even for the extreme values of a_A and a_B given by $\underline{a}$ and $\overline{a}$.

enforced. It shows that, when the pass-through effect dominates, equilibrium obfuscation and prices are lower when PPCs are imposed, and when the demand effect dominates, obfuscation and prices are higher:

Proposition 2. *Suppose assumption 1 holds and price parity clauses are exogenously imposed by both platforms. Then, for any $t > 0$, there is a unique symmetric bottleneck equilibrium. The equilibrium reservation value is non-increasing in t and the equilibrium search cost is non-decreasing in t. Moreover, when comparing interior solutions:*

- *If $\lambda'(a_p^*) < -1$, the equilibrium reservation value and consumer surplus is lower, while the equilibrium price and platform profits are higher under price parity clauses.*
- *If $\lambda'(a_p^*) > -1$, the equilibrium reservation value and consumer surplus is higher, while the equilibrium price and platform profits are lower under price parity clauses.*
- *If $\lambda'(a_p^*) = -1$, the equilibrium reservation value, consumer surplus, equilibrium price and platform profits are the same as when no PPCs are imposed.*

Proof. See Appendix. □

Note that Proposition 2 is stated for interior solutions when comparing the cases with and without PPCs. As the platforms always set reservation values inside of the interval $[\underline{a}, \overline{a}]$, and in both cases, reservation values decrease with t, there exist values for t for which both cases lead to corner solutions and price parity clauses do not make a difference in the market outcomes. From now on, we focus on the comparison between interior solutions.

The intuition for the result is as follows. Consider the case where $\lambda'(a_p^*) > -1$. When decreasing the reservation value (increasing the search cost), the pass-through rate of a given platform around a symmetric equilibrium goes from λ' to $\frac{\lambda'}{2}$, while the marginal loss in demand goes from $\frac{1-\lambda'}{2t}$ to $\frac{1}{2t}$. The smaller $\lambda'(a_p^*)$ is in absolute value, meaning that λ' is also very close to zero, implies that the demand effect when no PPCs are in place is mainly explained by reduced match values that consumers face and is barely explained by the price difference between the platforms. Therefore, the demand effect when imposing PPCs is very small in comparison to the case without PPCs. Hence, a value of $\lambda'(a_p^*)$ smaller than -1 in absolute value means that the pass-through effect is relatively more important when comparing the case with no PPCs to the case with exogenous PPCs. This implies lower obfuscation and prices when these clauses are in place. The opposite argument holds when $\lambda'(a_p^*) < -1$. When $\lambda'(a_p^*) = -1$, these two effects exactly cancel, for any level of t, and PPCs don't make a difference in prices, consumer welfare, and platform profits.

Whether PPCs lead to higher or lower obfuscation and prices, depends on the value of the pass-through from reservation values to equilibrium prices around the symmetric equilibrium given by $\lambda'(a_p^*)$. A simple case is given by the Generalized Pareto Distribution that has a constant Mills ratio, as in example 1. We use the GPD to illustrate the result of Proposition 2:

Example 2. *Suppose the distribution of match values follows a GPD with parameter $\xi < 0$. Thus, we can analytically solve for the equilibrium reservation value when PPCs are in place and compare it with the case without PPCs. Using (22), we obtain*

$$a_p^* = \frac{-t\xi - 2}{2\xi}, \tag{23}$$

while in the case with no PPCs, in Example 1, we obtained $a^* = \frac{\xi(1-t)-1}{\xi(1-\xi)}$. We have that $a^* > a_p^*$ if and only if

$$t\xi(1+\xi) > 0 \tag{24}$$

which is equivalent to $\xi < -1$. As for the GPD $\lambda'(a_p^*) = \xi$, the equilibrium reservation value is lower, or equivalently, the equilibrium search cost and price are higher under PPCs when $\xi < -1$, as stated in Proposition 2.

For other distributions, the value of λ' depends on the shape of λ and on the level of product differentiation t, as this parameter defines the equilibrium level of the reservation value. When t increases, both the equilibrium reservation values with and without PPCs decrease, while the equilibrium search costs and prices increase. Therefore, how the parameter t influences the result of Proposition 2, depends on the concavity or convexity of λ, as explained in the following Lemma:

Lemma 2. *Suppose assumption 1 holds, and that $\lambda'(a_1) > -1$ and $\lambda'(a_2) < -1$ for some values $a_1, a_2 \in (\underline{a}, \overline{a})$. Then:*

- *When λ is strictly concave, if t is high enough, PPCs leads to lower obfuscation and prices.*
- *When λ is strictly convex, if t is high enough, PPCs leads to higher obfuscation and prices*

Proof. See Appendix. □

If $\lambda'(a_p^*)$ is always greater or smaller than -1, the result is always the one explained in Proposition 2, as showed in Example 2. Therefore, this result focuses on cases where both values are possible for different reservation values. In this case, whether λ is concave or convex is critical to understand the effect of t in the result. Suppose that λ is concave. Hence, λ decreases with a at an increasing rate. As a higher t leads to a lower equilibrium reservation value a_p^*, this implies a greater value of $\lambda(a_p^*)$, where $\lambda'(a_p^*)$ is also higher (closer to 0) due to the concavity of λ. Therefore, if t is high enough and λ is concave, we have $\lambda'(a_p^*) > -1$ and equilibrium obfuscation and prices are lower under PPCs.

In comparison with some of the literature, this result shows how price parity clauses that restrict competition between platforms might lead to lower prices and higher consumer surplus. Johansen and Vergé (2017) also find that clauses restricting competition between platforms might benefit consumers when interbrand competition is strong. In contrast, Boik and Corts (2016) find that PPCs always lead to higher prices, while Wang and Wright (2018) find that wide PPCs allow competing platforms to reach the same outcome as a monopolist, always hurting consumers.

Endogenous price parity clauses: now, we endogenize the decision of setting a PPC as a choice variable for both platforms in the first stage of the game. We continue to focus on a symmetric bottleneck equilibrium. The timing of the new game is as follows. At stage 1, platforms simultaneously design their search environment by choosing s_k and decide whether to impose a PPC. At stage 2, they set their fixed fee τ_k after observing the search design and PPC choice of their rival. At stage 3, firms decide which platform to join and set prices restricted by the PPC (if any) set by platforms. Finally, at stage 4, consumers decide which platform to join and search randomly for their preferred product.

Note that any asymmetric case, given by a platform setting PPCs and the rival not setting such a clause, can be analyzed as the case with no PPCs from Proposition 1 or the case with both platforms setting PPCs from Proposition 2. Consider, for example, that $a_A > a_B$. In the absence of PPCs, the equilibrium price in platform B would be higher than in platform A. Therefore, if B sets a PPC and A does not, the PPC from platform B would be binding, and the resulting game is the one described by Proposition 2. Thus, for the purpose of characterizing the equilibrium, when $a_A > a_B$, it is equivalent that only the platform with a higher price sets a PPC with both platforms setting PPCs.

We denote such a case as one with a binding PPC. If B does not set a PPC and A does, this PPC is not binding, and sellers would set different prices in each platform, as in Proposition 1. With this in consideration, we can derive the equilibrium of the endogenous game:

Proposition 3. *Suppose assumption 1 holds. Then, for any $t > 0$:*

- *If $\lambda'(a_p^*) > -1$, the unique symmetric bottleneck equilibrium is one with no platform setting price parity clauses, and is described by Proposition 1.*
- *If $\lambda'(a_p^*) < -1$, the unique symmetric bottleneck equilibrium is one with a binding price parity clause, and is described by Proposition 2.*
- *If $\lambda'(a_p^*) = -1$, both cases are an equilibrium.*

Proof. See Appendix. □

The intuition of this result is as follows. Consider the first case where the unique equilibrium is one with no platform setting PPCs. In the first stage, say platform A, wants to deviate and set a PPC. If platform A increases its reservation value, its price parity clause is not binding, and the game remains as described in Proposition 1. But then, the initial reservation value that platform A was choosing is the best response for that platform. Therefore, this deviation is never profitable. On the contrary, if platform A decreases its reservation value with the objective of increasing the price its sellers are charging, its PPC becomes binding, and the game becomes as described by Proposition 2. This deviation can only be profitable if price parity clauses lead to higher prices, or when $\lambda'(a_p^*) < -1$. Therefore, when$\lambda'(a_p^*) > -1$, both platforms not setting PPCs is the unique equilibrium.

Now, consider the equilibrium when price parity clauses are binding. For simplicity, assume both platforms are initially setting a PPC. If a platform deviates and stops imposing the clause, and it decreases the price charged by its sellers by increasing the reservation value it sets, the PPC of the rival platform still binds, and this cannot be a deviation. If it decreases the reservation value to increase the price, the game becomes one with no price parity, and this deviation can only occur when prices and profits are higher when no price parity clauses are set. Therefore, we have that the unique equilibrium is individually profitable for each platform, and its always related to higher prices and lower consumer surplus, despite if it involves PPCs or not.

To conclude this section, we show that when PPCs are endogenously given, they are only imposed when they lead to higher prices and lower consumer surplus. Therefore, our analysis is consistent with the suspicions that PPCs might hurt consumers. However, we also show that, when PPCs are exogenously given, prices and obfuscation might go down, benefiting consumers. This case is still useful for policymakers. For example, suppose that PPCs are mainly set to avoid showrooming from consumers. Then, our analysis would be relevant, meaning that eliminating PPCs could cause prices to increase.

5. Conclusion

We developed a model in which two differentiated platforms attract buyers and sellers to interact in their marketplace. As sellers choose final prices, and in the absence of linear fees, platforms must rely on alternative strategic variables to shape competition between sellers. We showed how obfuscation, defined as increasing unitary search costs of consumers, can be a useful tool for platforms to increase prices charged by sellers and then extract these profits through a fixed fee. We find that increased platform differentiation leads to higher obfuscation and higher prices charged to consumers.

Then, we use this model to study the effect of price parity clauses on the level of obfuscation and equilibrium prices. We show that, when price parity clauses are exogenously imposed in both platforms, price parity clauses can increase or decrease obfuscation and prices. Therefore, in some cases, these clauses may be beneficial for consumers. However, when price parity clauses are endogenously chosen by platforms, we find that the unique equilibrium involves price parity clauses being set if and only if this leads to higher obfuscation and prices. This confirms the traditional suspicions on these clauses.

Nevertheless, we raise two important factors that should be considered by regulators when analyzing price parity clauses. First, as platforms are switching to the agency model, where sellers set final prices, platforms are relying on additional tools to extract profits, such as obfuscation, auctions for prominent positions, advertising, among many others. We showed in the particular case of search design, that PPCs influence the profitability of these practices for both platforms and sellers, through a pass-through effect. Second, platforms normally sell many different products, which are likely to have different shapes of demand. As our results suggest, this shape is critical to determine the effects of PPCs, which is likely to be very heterogeneous. Therefore, even when PPCs are set by a platform, forbidding such clauses may cause an unexpected increase in prices in many products. The same logic applies if a platform is setting PPCs mainly to avoid showrooming.

There are several alleys of research to understand both obfuscation by platforms and price parity clauses. Regarding obfuscation, issues such as asymmetric platforms, the existence of superstar firms, and prominence issues in consumer's search behavior are likely to influence obfuscation decisions by these platforms. In relation to price parity clauses, we show how alternative strategic variables may be used by platforms to affect prices chosen by sellers. These clauses are likely to affect both platforms' and sellers' behavior, as the effectiveness of these variables is likely to be affected by these clauses. Further research on how these clauses affect different strategies, such as auctioning positions in the platform, advertising products, among others, is important to understand the effect of these restrictions on widely used platforms such as Amazon, eBay, and Airbnb.

Appendix

Proof of proposition 1

We show that the platforms' profit functions at $t = 1$ are quasiconcave in their own reservation values and that, for a given t, the symmetric equilibrium value a^* characterizes the unique bottleneck equilibrium of the game.

i) Quasiconcavity: the first-order condition for a given platform, say platform A, is given by

$$\lambda'(a_A)D_A(a_A, a_B) + \frac{(1 - \lambda'(a_A))}{2t}\lambda(a_A) = 0. \tag{25}$$

Due to the log-concavity assumption on the distribution of match values, we have that $\lambda' < 0$. We take the left-hand side of the first-order condition and divide it by $\lambda'(a_A)$, obtaining the following function

$$\psi(a_A, a_B) = D_A(a_A, a_B) + \lambda(a_A)\frac{(1 - \lambda'(a_A))}{2t\lambda'(a_A)}. \tag{26}$$

If this function intersects 0 at most at a unique value of a_A, the original first-order condition also intersects 0 at most once. Differentiating $\psi(a_A, a_B)$ with respect to a_A we get

$$\frac{\partial\psi(a_A, a_B)}{\partial a_A} = \frac{1 - \lambda'(a_A)}{2t} + \frac{1}{2t\lambda'(a_A)^2}[\lambda'^2(a_A) - \lambda'^3(a_A) - \lambda\lambda''(a_A)], \tag{27}$$

which is strictly greater than 0 due to Assumption 1. Therefore, $\psi(a_A, a_B)$ is strictly increasing in a_A and must intersect 0 at most once. Therefore, the first-order condition for profit maximization for platform $A1$ intersects 0 at most once. This also means that if the first-order condition intersects 0, it must be from positive to negative. This implies that the profit function is quasiconcave on its own reservation value.

ii) To show that the equilibrium is unique, we take the first-order condition at a symmetric equilibrium

$$\frac{\lambda'(a^*)}{2} + \frac{(1 - \lambda'(a^*))}{2t}\lambda(a^*) = 0 \tag{28}$$

Following the same steps as in part i), we divide by $\lambda'(a^*)$ and show that the new function

$$\phi(a_A, a_B) = \frac{1}{2} + \frac{(1-\lambda'(a^*))}{2t\lambda'(a^*)}\lambda(a^*) \tag{29}$$

is strictly increasing in a^*. We have

$$\frac{\partial \phi(a^*)}{\partial a^*} = \frac{1}{2t\lambda'^2}[\lambda'^2 - \lambda'^3 - \lambda\lambda''], \tag{30}$$

which is strictly greater than 0 due to Assumption 1. Therefore, the first-order condition in a symmetric equilibrium intersect 0 at most once. In case the first order conditions in a symmetric equilibrium intersect 0, it must be from positive to negative, implying the equilibrium is unique.

Finally, we describe when the unique equilibrium is defined by the symmetric first order-conditions for both platforms or by corner solutions. Define $\underline{t}$ such that the first-order conditions at a symmetric equilibrium evaluated at $a^* = \overline{a}$ are equal to 0 and $\overline{t}$ such that the first order conditions at a symmetric equilibrium evaluated at $a^* = \underline{a}$ are equal to 0. We have that $\underline{t} < \overline{t}$ from the fact that a^* is decreasing in t. Now, we have three cases:

Case i) $t < \underline{t}$: in this case, each platform setting $a_k = \overline{a}$ (or equivalently $s_k = \underline{s}$) is the unique equilibrium at the platforms stage. If a given platform would deviate by setting a lower reservation value, its first-order condition would become positive, which cannot be a profitable deviation. Also, by technological restrictions, platforms cannot set a higher a_k. Therefore, the unique equilibrium is characterized by platforms setting $a_A = a_B = \overline{a}$.

Case ii) $\underline{t} < t < \overline{t}$: in this case, the equilibrium value of a^* is given by the unique solution of the symmetric first order-conditions.

Case iii) $\overline{t} < t$: in this case, each platform setting $a_k = \underline{a}$ (or equivalently $s_k = \overline{s}$) is the unique equilibrium at the platforms stage. If a given platform would deviate by setting a higher reservation value, its first-order condition would become negative, which cannot be a profitable deviation. If a given platform would deviate by setting a lower reservation value, no consumer would join the platform, which cannot be a profitable deviation either. Therefore, the unique equilibrium is characterized by platforms setting $a_A = a_B = \underline{a}$.

Proof of lemma 1

The sellers maximize:

$$\Pi_j = p_j[\kappa_A(1 - G(a_A + p_j - p_p^*)) + \kappa_B(1 - G(a_B + p_j - p_p^*))] - \tau_A - \tau_B, \tag{31}$$

where $\kappa_k = \frac{D_k}{1-G(a_k)}$. Define $D_j = [\kappa_A(1-G(a_A+p_j-p_p^*))+\kappa_B(1-G(a_B+p_j-p_p^*))]$ as the total demand of firm j when joining both platforms. Following Caplin and Nalebuff (1991), if we show that $1/D_j$ is convex, then the profit function of each seller is quasiconcave in its own price. This condition is equivalent to $2D_j'^2 - D_j D_j'' > 0$. We have that:

$$D_j' = -\kappa_A g(a_A + p_j - p_p^*) - \kappa_B g(a_B + p_j - p_p^*). \tag{32}$$

$$D_j'' = -\kappa_A g'(a_A + p_j - p_p^*) - \kappa_B g'(a_B + p_j - p_p^*). \tag{33}$$

Therefore, the condition can be written as

$$\begin{aligned}
&2[\kappa_A^2 g^2(a_A + p_j - p_p^*) + \kappa_B^2 g^2(a_B + p_j - p_p^*) + 2\kappa_A\kappa_B g(a_A + p_j - p_p^*)g(a_B + p_j - p_p^*)] \\
&+ \kappa_A^2 g'(a_A + p_j - p_p^*)(1 - G(a_A + p_j - p_p^*)) + \kappa_B^2 g'(a_B + p_j - p_p^*)(1 - G(a_B + p_j - p_p^*)) \\
&+ \kappa_A\kappa_B[g'(a_A + p_j - p_p^*)(1 - G(a_B + p_j - p_p^*)) + g'(a_B + p_j - p_p^*)(1 - G(a_A + p_j - p_p^*))] > 0
\end{aligned}$$

Because $g(x)$ is logconcave, then $1 - G(x)$ is logconcave, meaning that $g^2(x) + g'(x)(1 - G(x)) \geq 0$. Therefore, taking all the terms proportional to κ_A^2 and to κ_B^2 we have

$$\kappa_A^2[2g^2(a_A + p_j - p_p^*) + \kappa_A^2 g'(a_A + p_j - p_p^*)(1 - G(a_A + p_j - p_p^*))] \geq 0,$$

and

$$\kappa_B^2[2g^2(a_B + p_j - p_p^*) + \kappa_B^2 g'(a_B + p_j - p_p^*)(1 - G(a_B + p_j - p_p^*))] \geq 0.$$

Then, it is sufficient to show that the remainder terms (all proportional to $\kappa_A\kappa_B$) are strictly greater than 0. This reduces to (the terms $\kappa_A\kappa_B$ cancel out as they are always positive)

$$\begin{aligned}
4g(a_A + p_j - p_p^*)g(a_B + p_j - p_p^*) + g'(a_A + p_j - p_p^*)(1 - G(a_B + p_j - p_p^*)) \\
+g'(a_B + p_j - p_p^*)(1 - G(a_A + p_j - p_p^*)) > 0.
\end{aligned}$$

This condition holds directly if $g'(x)$ is positive. We derive a condition on $\underline{s}$ such that this condition holds even for the minimum value for $g'(x)$. The logconcavity of $1 - G(x)$ implies that $g'(x) \geq -\frac{g(x)}{(1-G(x))}$. We replace this lower bound for $g'(x)$ and rearranging terms we get

$$4 - \frac{\lambda(a_B + p_j - p_p^*)}{\lambda(a_A + p_j - p_p^*)} - \frac{\lambda(a_A + p_j - p_p^*)}{\lambda(a_B + p_j - p_p^*)} > 0.$$

Note that this expression always hold in a symmetric equilibrium in the obfuscation game between platforms where $a_A = a_B$. As the difference between reservation values increase, the condition might not hold. Given that the maximum difference between $\lambda(a_A + p_j - p_p^*)$ and $\lambda(a_B + p_j - p_p^*)$ depends on the maximum difference between a_A and a_B for a fixed p_j, if $\underline{s}$ is not too small, then $\overline{a}$ is not too big and the condition is satisfied as even when a platform, say A, sets $a_A = \underline{a}$, then the biggest value that platform B can choose is $a_B = \overline{a}$ and the condition is still satisfied.

Given that all sellers's profit function are quasiconcave, the unique equilibrium price equilibrium at the sellers' stage is given by taking the first order conditions of the sellers' maximization problem, assuming symmetry and we obtain expression (16).

Proof of proposition 2

First, we show that there exists a unique symmetric bottleneck equilibrium when PPCs are exogenously given following the same steps than in Proposition 1. Then, we compare it with the case with no price parity clauses.

i) Quasiconcavity: The first-order condition for a given platform, say A, is given by

$$\frac{\partial p_p^*}{\partial a_A} D_A + \frac{p_p^*}{2t} = 0. \tag{34}$$

Due to the log-concavity assumption on the distribution of match values, we have that $\lambda' < 0$. We take the left hand side of the first order-condition and divide it by $\frac{\partial p_p^*}{\partial a_A}$, obtaining the following function:

$$\psi(a_A, a_B) = D_A + \frac{p_p^*}{2t\frac{\partial p_p^*}{\partial a_A}}. \tag{35}$$

If this function intersects 0 at most at a unique value of a_A, the original first-order condition also intersects 0 at most once. Differentiating $\psi(a_A, a_B)$ with respect to a_A we get (we omit a positive dividing term)

$$\frac{\partial \psi(a_A, a_B)}{\partial a_A} = \lambda'^2(a_A) - \lambda(a_A)\lambda''(a_A) + \frac{\lambda'^2(a_A)\lambda(a_A)\lambda(a_B)}{\lambda(a_B)(\lambda(a_A) + \lambda(a_B))}, \tag{36}$$

which is strictly greater than 0 if $g(x)$ is log-concave, due to assumption 1. Therefore, $\psi(a_A, a_B)$ is strictly increasing in a_A and must intersect 0 at most once, and therefore the original first order-condition also intersects 0 at most once. In case that the first-order conditions intersect 0, it must be from positive to negative, implying that the profit function is quasiconcave on its own reservation value.

ii) To show that the equilibrium is unique, we take the first-order conditions at a symmetric equilibrium

$$\frac{\lambda'}{4} + \frac{\lambda}{2t} = 0. \tag{37}$$

Following the same steps as in i), we divide by $\lambda'(a^*)$ and show that the new function $\phi(a_p^*)$ is strictly increasing in a^*. Differentiating with respect to a_A we have

$$\frac{\partial \phi(a_p^*)}{\partial a_p^*} = \frac{1}{2t\lambda'^2}[\lambda'^2 - \lambda\lambda''] \tag{38}$$

which is strictly greater than 0 due to assumption 1. Therefore, the first-order conditions in a symmetric equilibrium intersect 0 at most once. In case the first order conditions in a symmetric equilibrium intersect 0, it must be from positive to negative, implying the equilibrium is unique. Following the same steps as in Proposition 1, it can be shown that there exist $\underline{t}_p < \overline{t}_p$ such that if t is greater or lower than these boundaries, the unique equilibrium is given by corner solutions with $a_p^* = \underline{a}$ or $a_p^* = \overline{a}$

To compare the equilibrium with and without price parity clauses, we focus on interior solutions for both cases, as at corner solutions the levels of obfuscation are the same. Replacing the value a_p^* that solves the first-order conditions for a symmetric equilibrium when PPCs are set in the first-order conditions for a symmetric equilibrium without PPCs we have

$$\frac{\lambda'(a_p^*)}{4} + \frac{\lambda'^2(a_p^*)}{4}. \tag{39}$$

This term is positive if $\lambda'(a_p^*) < -1$, meaning that when replacing a_p^* in the first-order condition when PPCs are not imposed leads to a positive first-order condition. Therefore, to satisfy the condition, the value of a^* when PPCs are not set must be higher than a_p^*, in order to reduce the first-order condition back to 0. This implies that $a^* > a_p^*$ and $s^* < s_p^*$ and equilibrium prices are higher when PPCs are imposed. The other case is solved analogously. When $\lambda'(a_p^*) = -1$, both first-order conditions are satisfied and PPCs make no difference in equilibrium.

Proof of lemma 2

From Proposition 2, we know that

$$\lambda(a_p^*) = -\frac{t\lambda'(a_p^*)}{2}. \tag{40}$$

Therefore, $\lambda(a_p^*)$ increases with t (a_p^* decreases with t). Suppose λ is strictly concave. Then, a higher t increases λ and λ' becomes smaller in absolute value. Thus, if t is high enough, λ' becomes greater than -1 and PPCs lead to lower obfuscation and prices. The analysis when λ is convex is analogous.

Proof of proposition 3

We prove each case separately. Define a^* as the equilibrium reservation value given in proposition 1 when no price parity clauses are in place and a_p^* as the equilibrium reservation value when price parity are exogenously given as in proposition 2.

Price parity equilibrium: suppose both platform set price parity clauses, set search costs according to (22) and leave sellers with 0 profits through their fixed fee. This is the unique bottleneck equilibrium if both platforms set price parity clauses, according to proposition 2. The only possible deviation would be a platform not setting price parity clauses.

Suppose platform A deviates and does not set a price parity clause:

i) If A increases a_A, there is a downward pressure on p_A^*, but because of the price parity of platform B, the prices must remain the same in both platforms, as long as sellers keep multi-homing. Therefore, the initial value of $a_A = a_p^*$ was already a best response for A, so this cannot be a deviation.

However, this deviation may be profitable if sellers leave platform B and therefore A gets all the market. However, in this case, at $t = 2$ platform B will respond by lowering their fixed fee such that sellers don't leave their platform. Therefore, this deviation is not profitable.

ii) If A decreases a_A, there is an upward pressure on p_A^*, leading to $p_A^* > p_B^*$, therefore the price parity clause set by platform B is not binding and equilibrium prices in the sellers subgame is given by the base case with no price parity. However, if $a_p^* < a^*$, meaning price parity leads to higher obfuscation and prices, the first order condition of the no price parity case for platform A, starting from $a_A = a_B = a_p^*$ is positive, meaning that lowering a_A cannot be a deviation. If it was the case that $a_p^* > a^*$, the first order condition would be negative and this indeed would be a profitable deviation. There is no possible delisting possibility from either platform when price parity clauses are not binding.

Therefore, if $a_p^* < a^*$, both platforms setting price parity clauses is the unique bottleneck equilibrium of the game.

No price parity equilibrium: suppose both platform do not set price parity clauses, set search costs according to (12) and leave sellers with 0 profits through their revenue sharing fee. This is the unique bottleneck equilibrium if both platforms do not set price parity clauses, according to proposition 2. The only possible deviation would be a platform setting price parity clauses.

Suppose platform A deviates and sets a price parity clause:

i) If A increases a_A, there is a a downward pressure on p_A^*, but then the price parity clause set by A is not binding. Therefore, the initial value of $a_A = a^*$ was already a best response for A, so this cannot be a deviation. There is no possible delisting possibility from either platform when price parity clauses are not binding.

ii) If A decreases a_A, there is an upward pressure on p_A^*, leading to $p_A^* > p_B^*$, and then the price parity clause set by A becomes binding, as long as sellers continue to multi-home. However, if $a_p^* > a^*$, meaning price parity leads to lower obfuscation and prices, the first order condition of the price parity case for platform A, starting from $a_A = a_B = a^*$ is positive, meaning that lowering a_A cannot be a deviation.

However, this deviation may be profitable if sellers leave platform B and therefore A gets all the market. However, in this case, at $t = 2$ platform B will respond by lowering their fixed fee such that sellers don't leave their platform. Therefore, this deviation is not profitable.

If it was the case that $a_p^* < a^*$, the first order condition would be negative and this indeed would be a profitable deviation.

Therefore, if $a_p^* > a^*$, both platforms setting price parity clauses is the unique bottleneck equilibrium of the game.

Lemma 3

The following Lemma shows that right-hand truncations of log-concave distributions functions satisfy Assumption 1, if the truncation point is small enough (but strictly greater than 0):

Lemma 3. *Assume $g(x)$ is a strictly positive everywhere and log-concave density function in $[0, \overline{v}]$. Let $g_t(x, \overline{h})$ be the right hand truncation of $g(x)$ at $\overline{h} < \overline{v}$ and define $\lambda_t(x, \overline{h})$ as the Mills ratio of the truncated distribution. Then:*

- *$\lambda_t(x, \overline{h})$ is strictly increasing in $\overline{h}$.*
- *There exists a $\hat{h} > 0$ such that $\forall\, \overline{h} < \hat{h}$, $\lambda_t(x, \overline{h})$ is strictly log-concave $\forall x \in (0, \overline{h}]$.*

Proof. When a distribution is truncated on the right at $\overline{h}$, the new Mills ratio is given by

$$\lambda_t(x, \overline{h}) = \frac{G(\overline{h}) - G(x)}{g(x)} \tag{41}$$

where $G(\overline{h}) < 1$ if $\overline{h} < \overline{v}$. This expression is clearly strictly increasing in $\overline{h}$ as $G(x)$ is a cumulative distribution function and the density function is strictly positive everywhere. Differentiating with respect to x we have

$$\lambda_t'(x, \overline{h}) = -1 - \frac{g'(x)}{g(x)} \lambda_t(x, \overline{h}), \tag{42}$$

and

$$\lambda_t''(x, \overline{h}) = -\left[\left(\frac{g'(x)}{g(x)} \right)' \lambda_t(x, \overline{h}) + \lambda_t'(x, \overline{h}) \frac{g'(x)}{g(x)} \right]. \tag{43}$$

The condition for $\lambda_t(x, \overline{h})$ to be log-concave is given by $\lambda_t''(x, \overline{h})\lambda_t(x, \overline{h}) - \lambda_t'^2(x, \overline{h}) < 0$. Some algebra leads to

$$1 + \lambda_t(x, \overline{h}) \frac{g'(x)}{g(x)} + \lambda_t^2(x, \overline{h}) \left(\frac{g'(x)}{g(x)} \right)' > 0.$$

As $\lambda_t(x, \overline{h})$ is strictly increasing in $\overline{h}$ and we know that, because $\overline{h} < \infty$, then $|g'(x)| < \infty$ and $|g''(x)| < \infty$ (bounded derivatives in a compact set), if $\overline{h}$ is small enough, the second and third terms become small and the condition is satisfied. □

References

Anderson, S. P. and Renault, R. (1999). Pricing, product diversity, and search costs: A bertrand-chamberlin-diamond model. *The RAND Journal of Economics*, pages 719–735.

Armstrong, M. (2006). Competition in two-sided markets. *The RAND Journal of Economics*, 37(3):668–691.

Armstrong, M., Vickers, J., and Zhou, J. (2009). Prominence and consumer search. *The RAND Journal of Economics*, 40(2):209–233.

Armstrong, M. and Wright, J. (2007). Two-sided markets, competitive bottlenecks and exclusive contracts. *Economic Theory*, 32(2):353–380.

Athey, S. and Ellison, G. (2011). Position auctions with consumer search. *The Quarterly Journal of Economics*, 126(3):1213–1270.

Brown, J., Hossain, T., and Morgan, J. (2010). Shrouded attributes and information suppression: Evidence from the field. *The Quarterly Journal of Economics*, 125(2):859–876.

Calzada, J., Manna, E., and Mantovani, A. (2019). Platform price parity clauses and segmentation.

Chen, Y. and He, C. (2011). Paid placement: Advertising and search on the internet. *The Economic Journal*, 121(556):F309–F328.

Chen, Y. and Zhang, T. (2018). Intermediaries and consumer search. *International Journal of Industrial Organization*, 57:255–277.

Chetty, R., Looney, A., and Kroft, K. (2009). Salience and taxation: Theory and evidence. *American economic review*, 99(4):1145–77.

De Corniere, A. and Taylor, G. (2019). A model of biased intermediation. *The RAND Journal of Economics*, 50(4):854–882.

Edelman, B. and Wright, J. (2015). Price coherence and excessive intermediation. *The Quarterly Journal of Economics*, 130(3):1283–1328.

Ellison, G. and Wolitzky, A. (2012). A search cost model of obfuscation. *The RAND Journal of Economics*, 43(3):417–441.

Greenleaf, E. A., Johnson, E. J., Morwitz, V. G., and Shalev, E. (2016). The price does not include additional taxes, fees, and surcharges: a review of research on partitioned pricing. *Journal of Consumer Psychology*, 26(1):105–124.

Hagiu, A. and Jullien, B. (2011). Why do intermediaries divert search? *The RAND Journal of Economics*, 42(2):337–362.

Hagiu, A. and Jullien, B. (2014). Search diversion and platform competition. *International Journal of Industrial Organization*, 33:48–60.

Hagiu, A. and Wright, J. (2015). Multi-sided platforms. *International Journal of Industrial Organization*, 43:162–174.

Hossain, T. and Morgan, J. (2006). ... plus shipping and handling: Revenue (non) equivalence in field experiments on ebay. *Advances in Economic Analysis & Policy*, 5(2).

Huck, S. and Wallace, B. (2015). The impact of price frames on consumer decision making: Experimental evidence. Technical report.

Johnson, J. P. (2017). The agency model and mfn clauses. *The Review of Economic Studies*, 84(3):1151–1185.

Kohn, M. G. and Shavell, S. (1974). The theory of search. *Journal of Economic Theory*, 9(2):93–123.

Loertscher, S. and Niedermayer, A. F. (2012). Fee-setting mechanisms: On optimal pricing by intermediaries and indirect taxation.

Moraga-González, J. L., Sándor, Z., and Wildenbeest, M. R. (2017). Prices and heterogeneous search costs. *The RAND Journal of Economics*, 48(1):125–146.

Muthers, J. and Wismer, S. (2013). Why do platforms charge proportional fees? commitment and seller participation.

Petrikaitė, V. (2018). Consumer obfuscation by a multiproduct firm. *The RAND Journal of Economics*, 49(1):206–223.

Rochet, J.-C. and Tirole, J. (2003). Platform competition in two-sided markets. *Journal of the european economic association*, 1(4):990–1029.

Rochet, J.-C. and Tirole, J. (2006). Two-sided markets: a progress report. *The RAND journal of economics*, 37(3):645–667.

Shy, O. and Wang, Z. (2011). Why do payment card networks charge proportional fees? *American Economic Review*, 101(4):1575–90.

Wang, Z. and Wright, J. (2017). Ad valorem platform fees, indirect taxes, and efficient price discrimination. *The RAND Journal of Economics*, 48(2):467–484.

Wang, Z. and Wright, J. (2018). Should platforms be allowed to charge ad valorem fees? *The Journal of Industrial Economics*, 66(3):739–760.

Weitzman, M. L. (1979). Optimal search for the best alternative. *Econometrica: Journal of the Econometric Society*, pages 641–654.

Wilson, C. M. (2010). Ordered search and equilibrium obfuscation. *International Journal of Industrial Organization*, 28(5):496–506.

Wolinsky, A. (1986). True monopolistic competition as a result of imperfect information. *The Quarterly Journal of Economics*, 101(3):493–511.

Coalition Loyalty Programs and Consumer Search*

José Ignacio Heresi†
Xavier Lambin‡

January 2020

Abstract

We provide a new interpretation for coalition loyalty programs in an environment where consumers search for their preferred products. A dominant firm in one market can generate a prominent position for another firm in an unrelated market, by offering a reward to consumers conditionally on buying in both firms. We show when it is optimal for consumers to sample first the firms in the coalition. We derive conditions under which this coalition is profitable in comparison with a case with no coalition and in comparison with a case where the dominant firm creates a loyalty program only in its own market. We find that the coalition can be profitable if and only if it increases consumer surplus. Then, we explore the case where a continuum of coalitions compete, and show that in this case, consumer surplus is lower, industry profits are higher, and total welfare is ambiguous, relative with a case with no coalitions.

Keywords: coalition loyalty programs, consumer search, prominence.

JEL Classifications: D83, L10, M21.

1 Introduction

Coalition loyalty programs (CLPs), also known as joint or partnership loyalty programs, allow consumers to collect points from any merchant affiliated to the

*We are gratefull to James Dana, Thomas Ross, Andrew Rhodes, Wilfried Sand-Zantman, Jidong Zhou, for useful comments and suggestions.

†Ph.D. Student, Toulouse School of Economics, University of Toulouse Capitole, Toulouse, France: jheresig@gmail.com.

‡Grenoble Ecole de Management Univ Grenoble Alpes ComUE, 12 rue Pierre Sémard, 38000 Grenoble, France. xavier.lambin@grenoble-em.com

network and exchange them for rewards and discounts in other merchants from the same coalition. While the different explanations for standalone loyalty programs (SLPs) are extensively discussed in the literature, CLPs remain surprisingly unexplored, despite their growing popularity around the world.[1] We provide a new explanation for why merchants engage in CLPs and offer a new framework to understand their effects on consumers' behavior and welfare. In our interpretation, as consumers must engage in costly search in order to find their preferred products, a CLP generates prominent positions to their merchants in consumers' search order.

Consider, for example, that two firms from different markets form a CLP. After a consumer buys in either firm, when he searches in the other market, it is natural that he will begin searching the firm from the same CLP. Therefore, this firm becomes "prominent" for these consumers. The effects of prominence on search markets are studied by Armstrong, Vickers and Zhou (2009), where they show that prominence has adverse effects on consumer surplus and total welfare when the prominent position is exogenously given. We contribute to this analysis by considering that a CLP can generate endogenous positions of prominence. However, coalitions must offer (costly) rewards and benefits to consumers to obtain such positions. Therefore, the final effects on equilibrium prices, profits and welfare are ambiguous.

There are many examples of successful coalition loyalty programs. An eminent case is observed in the airline industry, where there exist three main coalitions, namely OneWorld, SkyTeam, and Star Alliance. Each of these coalitions is composed of several airlines, and reward points can be earned and redeemed in different airlines from the same coalition. Card coalitions, such as Nordstrom, Payback in Germany, and Nectar in the UK, gather several merchants from different markets into the same loyalty program with success.

There are still several open questions regarding CLPs in the literature. In Basso et al. (2015), the authors explore these questions and summarize the open alleys for research in this area. For example, CLPs can be formed by a dominant firm organizing the program or by several equal-level partners organized by an independent managing firm. The relative profitability of both types of programs and the benefits for their participants are still open questions: the number of firms that should belong to the coalition, from which markets, the rewards structure and the cost-sharing of these rewards, the behavior of consumers under such programs, and the effects on consumer surplus and total welfare, among many others.

We take a step in answering these questions by assuming that consumers search

[1] From the 3.8 billion LP memberships in the U.S., around 16% of the memberships are with CLPs. In Canada, CLPs grew 78% between 2014 and 2016, reaching 27 million memberships. See Colloquy report (2017).

for their preferred goods and that CLPs might affect their search behavior. We consider a dominant firm in a given market forming a coalition with a firm from another market. This firm is dominant in our model as we assume that all consumers start searching in this firm's market. There are many reasons why consumer searches might be ordered in such a way. For example, some goods can be chronologically bought before others, as is the case with plane tickets and hotel rooms or car rentals. Some products are in high demand, and therefore most consumers buy those products before buying products in less tight markets, as is the case with prominent airline routes with respect to routes with low demand.

In our model, there are two markets, and we take a coalition of one firm from each market as given. In each market, there is a continuum of competitors outside of the CLP. The coalition offers a reward T if consumers buy the product of both firms in the coalition, and offering this reward costs C to the member firms. How to offer a higher reward of T at a lower cost of C is the source of considerable attention in the marketing literature. To focus on our main contribution, which is the impact of a CLP on search behavior, we assume that these parameters are exogenously given. Also, firms share the cost of the reward, where the firm in the second market pays a share of α of this reward. We characterize the equilibrium of the game by taking α also as an exogenous parameter.

Under these assumptions, we derive an equilibrium where the reward T acts as a demand shifter in the second market, as consumers that already bought from the dominant firm only get the reward if they buy from the second firm in the coalition. In addition, the cost of the reward is passed through to the final prices. Therefore, the price of the second firm is higher than without a CLP. Under a simple condition, even if the price is higher, the reward compensates for this fact, and consumers are, on average, better off. However, as the price is higher, only consumers that bought from the dominant firm will ever sample the coalition's firm in the second period. In the first period, when consumers sample the dominant firm, they take into account the "expected reward" of buying from this firm, which also acts as a demand shifter. Therefore, the price of the dominant firm is also higher. In equilibrium, consumer surplus increases on average if and only if the coalition is profitable for the firms, with respect to a case with no loyalty program. However, consumers that bought from the dominant firm and not from the partner are worse off. The coalition is profitable if and only if, for a given level of reward T, the cost C is not too high. In fact, if $T = C$, representing a case where the reward is cash or a discount, the CLP is never profitable. Therefore, if the CLP is not creating value, it cannot be profitable.

We then compare the outcome of the dominant firm CLP with a case where this firm decides to offer the reward T directly upon purchase, and not conditional on buying in another firm, in what we call a standalone loyalty program (SLP). We compare the profit of the dominant firm in both cases and analyze when it is more profitable to create each type of loyalty program. We derive a simple condition under which each program is the most profitable.

Then, we analyze the case where a continuum of CLPs compete with each other. Every firm in market A forms a CLP with a firm in market B, and offers the same reward T at the same cost C. When the level of reward is relatively high, in equilibrium, every consumer buys from firms in the same CLP. After choosing a firm in market A, they become "locked" to the partner firm in market B. Therefore, on average, they buy worse products and pay higher prices, even if they get the reward. Consumer surplus goes down, while industry profits increase. Total welfare increases if and only if the cost of the reward C is not too high. Thus, competition between CLPs hurt consumers relative to the case with only one CLP.

Finally, as an extension, we provide some preliminary results for the case when half of the consumers start in each market. We call this case a symmetric CLP. We show that the equilibrium price from firms in the CLP lies between the prices of the case of a dominant firm CLP. We also show that, for some parameters, this CLP increases consumer welfare, and it is indeed profitable for consumers to sample first the firms in the CLP. A full characterization of this case is left for further research.

Related Literature

This article is the first to explore coalition loyalty programs under a consumer search environment. Gardete and Lattin (2018), in a similar setting but without search, also study CLPs in a setting where there are two markets and two firms in each of them. Each of the four firms has a segment of loyal consumers, and they leverage their market power to another market through the CLP. The CLP is profitable if rewards are chosen after prices, due to firms leveraging their market power to the other market. Their model considers a symmetric case while ours focuses on a dominant firm loyalty program. We consider this work as highly complementary to ours. A relevant difference between this work and ours is that they find that the CLP can be profitable even when the reward is given as a cash discount to consumers, while in our model, this is never profitable.

There is a wide literature analyzing individual or standalone loyalty programs. The main explanations for loyalty programs include price discrimination, switching costs, barriers to entry, database marketing and collusion. Basso, Clements and

Ross (2009) summarize the main literature and provide an alternative explanation based on a moral hazard problem between employers that pay for the tickets who book travels and have incentives to increase their loyalty rewards. Another part of the literature is focused on whether loyalty programs effectively increase demand. For example, Lederman (2007, 2008) shows that enhancements to airlines' coalition loyalty programs increase demand on routes departing from airports at which the airline is dominant. In the present paper, we focus on CLPs to highlight a specific benefit of CLPs relative to SLPs: the LP may generate prominence for partner firms who otherwise would not be prominent.

Gans and King (2006) and Armstrong (2013) study the possibility of firms of different markets offering a discount when consumers buy the products together. The main difference with our paper is that consumers know valuations from all firms when buying the bundle, while in our model, consumers must sequentially and costly search markets and products. The first paper studies a setting two markets and two firms in each market, and a pair of firms offering a discounted price for both products can create interdependence between products otherwise independent. One pair of firms offering a bundle earns greater profits, but if both pairs of firms bundle, profits are the same as in a no bundling case while consumer surplus and total welfare are reduced due to distorted decisions by consumers. The second paper shows that firms can unilaterally have incentives to offer a discount for their product conditional on a consumer buying the other product when the demand for the bundle is more elastic than the demand of the firm's product. Another important difference with our work is that we assume that the cost of the reward can be lower than the reward itself, while in their models, the cost of the discount for buying the bundle is equal to the level of reward.

Our article is greatly related and borrows from the analysis of prominence on search markets of Armstrong, Vickers and Zhou (2009). They show, using the search environment developed by Wolinsky (1986) and Anderson and Renault (1999), how a position of prominence exogenously given in the search order of consumers, leads to a lower price and higher profits from the prominent firm, while the other firms increase their prices and consumer surplus and total welfare is reduced, as long as there is a finite number of firms on the market. When there is an infinite number of firms in the market, prominence only increases profits of the prominent firm, while prices, consumer surplus and total welfare are unchanged. Our model describes a situation where a firm can endogenously generate a prominent position for another firm in another market through a CLP, and when this strategy is profitable. As we assume that there is an infinite number of firms in our model, the channel through which the non-prominent firms increase prices is shut down.

The remainder of this article is organized as follows. In Section 2, we set up the model of a dominant firm CLP, and derive our main results. In Section 3, we study the case of competing CLPs. Then, in Section 4, we extend the base model to consider a symmetric setting where half of the consumers start searching in market A and half start in market B. Finally, in Section 5, we conclude.

2 The model

Consider two product markets, A and B, each one served by a unit mass of horizontally differentiated sellers producing at 0 marginal cost. A unit mass of consumers has unit demand for each good. The value of each firm's product is idiosyncratic to consumers, and it is only learned after costly search. The surplus of buying from firm j, net of search costs, is $\epsilon_{ij} - p_j$, where ϵ_{ij} is the idiosyncratic match utility of consumer i derived from firm j and p_j is the price of firm j. We assume that ϵ_{ij} is distributed uniformly in the interval $[0, 1]$. Consumers face a search cost s each time the sample a firm. Consumers have rational expectations and passive beliefs over the sequence of equilibrium prices. This means that consumers correctly anticipate equilibrium prices and that this expectation is not affected when a price deviation is observed in the search process. The search process is without replacement, and there is costless recall, so that consumers can always come back and buy from a previously sampled firm.

We assume that a firm in each market, denoted as $A1$ and $B1$, form a coalition loyalty program (CLP), offering a reward of value T if a consumer buys from both firms. The cost of this reward is C. A share $\alpha \in [0, 1]$ of this cost is paid by firm $B1$ and the remainder $1 - \alpha$ is paid by $A1$. We assume that T, C and α are exogenous. We assume all consumers start their search process in market A, and after purchasing, they search and buy in market B. This assumption represents real-life situations such as consumers buying a plane ticket in a prominent route, and at a later date, they buy a ticket in another route. Another example is consumers buying plane tickets and afterward being offered the possibility to rent a hotel room or a car at a discounted price. We discuss, as an extension, the case where half of the consumers start searching in each market.

As all consumers start searching in market A, we assume that firm $A1$ is the dominant firm of the CLP. This firm charges a fixed fee F to firm $B1$. The timing of the game is the following:

- t=1: taking the coalition as given, sellers simultaneously set prices in both markets.

- t=2: consumers search and purchase in market A and then in market B.

Our equilibrium concept is subgame perfect Nash equilibrium.

□ **Preliminaries.** We start by characterizing the search behavior of consumers in the absence of the CLP, as it is already studied in the literature. Suppose all firms are charging a symmetric equilibrium price p^* in a given market. As all prices charge the same price, the consumers' search order is random. As shown in Weitzman (1979), once in the market, when a consumer samples a firm, say firm j, he will stop and buy from that firm if the net surplus $\epsilon_{ij} - p_j$ is greater than a reservation utility $a - p^*$, where the reservation value a satisfies

$$\int_a^1 (x-a)dx = s, \tag{1}$$

implying that $a = 1 - \sqrt{2s}$ due to our assumption on the distribution. Given that there exists an infinite number of firms, consumers eventually find a suitable product. Therefore it is never optimal to come back and buy from a previously sampled firm. It also means that once consumers join the market, consumers always purchase a product, and the market is covered.

In a symmetric price equilibrium between sellers, the expected number of consumers searching a given seller is 1 (due to the normalization of the number and firms and consumers) in the first round of consumers' search. The probability of a consumer buying in any given seller when it is sampled is $1-a$. Therefore, the expected number of consumers sampling any seller in the second round is a, in the third round is a^2, and so son. Now, suppose seller j deviates from the equilibrium. A consumer buys from firm j if and only if $e_{ij} - p_j \geq a - p_k^*$, where p_k^* is the equilibrium price in platform k. Therefore, seller j's demand is given by

$$\sum_l^\infty [a^l][1 - p_j + a - p^*)] = \frac{1 - a - p_j + p^*}{1-a}. \tag{2}$$

Therefore, firm j's profit function is given by

$$\Pi_j = p_j \frac{[1 - a - p_j + p^*]}{1-a}. \tag{3}$$

Maximizing with respect to p_j, leading to a symmetric equilibrium with $p^* = 1-a$. The expected consumer surplus of the full process is given by $a - p^* = 2a - 1$. Therefore, we need that $a \geq \frac{1}{2}$ (equivalently $s \geq \frac{1}{8}$) so consumers participate in the market. Industry profit is p^*, while individual firms make 0 profits.

Armstrong, Vickers, and Zhou (2009) extend this analysis and show that, when a firm is exogenously prominent, meaning that it is sampled first by all consumers, the equilibrium price is unchanged, provided there is an infinite number of firms in the market. The only difference, in this case, is that the prominent firm makes positive profits, while consumer surplus and industry profits remain the same. When there is a finite number of firms in the market, however, the prominent firm charges a lower price while non-prominent firms charge a higher price, leading to lower output, consumer surplus, and total welfare, while the prominent firm still increases its profits. We abstract from the strategic price response from non-prominent firms by assuming markets with infinite numbers of firms.

□ **The impact of a coalition loyalty program.** We characterize an equilibrium where the firms that belong to the CLP charge higher prices than the rest of the firms. At the same time, it is still optimal for consumers to search these firms due to the offered reward. Non-prominent firms charge symmetric prices in each market. We rule out potential equilibria where the firms from the CLP are never sampled because consumers expect very high prices at these firms, and firms charge those prices as they expect 0 demand in any case. To avoid corner solutions and ensure that consumers participate in the market even in the case without CLP, we make the following assumptions on the parameters of the model:

Assumption 1: $T, C \leq 1$ *and* $a \geq \frac{1}{2}$.

To start the analysis, we describe firm $B1$'s behavior at $t = 2$. In equilibrium, consumers expect firm $B1$ to charge a higher price than the rest of the firms in market B, as this firm has higher demand from consumers who bought from $A1$ and because it passes through some of the cost of this reward to final prices. Therefore, only consumers that bought from firm $A1$ will sample firm $B1$. Assume that consumers sample firm $B1$ first (we show later that this is the optimal behavior in equilibrium). Then, firm $B1$ maximizes

$$\Pi_{B1}(p_{B1}) = (p_{B1} - \alpha C)D_{B1}(p_{B1}) \tag{4}$$

with respect to p_{B1}. The number of consumers that bought from firm $A1$ is taken as given at this stage. To derive D_{B1}, note that consumers sampling $B1$ buy from this firm if and only if $\epsilon_{B1} - p_{B1} \geq a - p^*_{2B} - T$, where p^*_{2B} is the equilibrium price of the rest of the firms in market B. If consumers obtain a low enough match value at firm $B1$, they randomly search the rest of the firms. At this point, the incentives of non-prominent firms are the same as in a market without a CLP. Therefore, the symmetric equilibrium price of non-prominent firms is given by $p^*_{2B} = 1 - a$, as shown in the preliminaries. Therefore, firm $B1$'s demand is given by the condition $\Pr(\epsilon_{B1} \geq 2a - 1 + p_{B1} - T)$, times the number of consumers that buy from $A1$. This is equal to

$$D_{B1} = D_{A1}(2(1 - a) + T - p_{B1}). \tag{5}$$

Maximizing Π_{B1} with respect to p_{B1} leads to

$$p^*_{B1} = 1 - a + \frac{T + \alpha C}{2}. \tag{6}$$

This price is higher than the one in the absence of a CLP for two reasons. First, the reward T is analogous to a shift in demand for product $B1$, and second, part of the cost of the reward paid by firm $B1$ is passed through to the final price. The reason why $B1$ cannot fully pass-through the shift in demand and cost to the final price is that if it charges a too high price, consumers keep on searching the rest of the firms, even if they do not provide a reward.

Even if this price is higher, consumers obtaining the reward pay an effective price at $t = 2$ of $p^*_{B1} - T = 1 - a - \frac{(T-\alpha C)}{2}$. Therefore, if

$$T - \alpha C \geq 0, \tag{7}$$

it is optimal for consumers that previously bought from firm $A1$ to sample firm $B1$ first in the second period. Thus, firm $B1$ becomes prominent for consumers, conditional on having bought from firm $A1$. Therefore, $T - \alpha C \geq 0$ is a necessary condition for the CLP to be profitable.

Given that consumers get a reward if they buy in firm $B1$, they are willing to accept a lower match value from this firm, compared to a case without a CLP. Firm $B1$'s demand in equilibrium is

$$D^*_{B1} = D_{A1}\left[(1 - a) + \frac{T - \alpha C}{2}\right], \tag{8}$$

while $B1$'s profit is given by

$$\Pi^*_{B1} = D_{A1}\left[(1 - a) + \frac{T - \alpha C}{2}\right]^2. \tag{9}$$

This result highlights how the CLP can make firm $B1$ prominent in market B for consumers that buy from firm $A1$, increasing its profits relative to the case with no CLP, where $B1$ made negligible profits. Then, firm $A1$ can extract these profits using the fixed fee F.

At $t = 1$, consumers buying from firm $A1$ anticipate that they have the possibility of earning a reward in the following period. This expected reward (ER) represents the difference in the continuation payoffs for consumers of buying from firm $A1$ relative to buying in any other firm. If a consumer buys from $A1$, he will sample firm $B1$ first at $t = 2$. Then, he has the possibility of earning a reward if the match value is not too low. If the match value is low, then he keeps searching for the rest of the firms. The expected utility of this process relative to the expected utility of only sampling no CLP firms and never earning a reward represents the expected reward for consumers from buying in $A1$. This expected reward increases the demand of firm $A1$, as long as the ER is positive. The expression for the ER is derived in the following Lemma (proofs for all results are provided in the Appendix):

Lemma 1. *Define* $\phi_B = \frac{T-\alpha C}{2}$. *Suppose that* $T > \alpha C$. *Then, the expected reward for consumers buying from firm* $A1$ *is given by*

$$ER = \phi_B(1-a) + \frac{\phi_B^2}{2} \tag{10}$$

The expected reward is increasing in T, *decreasing in* C, *decreasing in the reservation value* a *and increasing in the search cost* s. □

This expected reward is increasing in T and decreasing in C, as this cost is passed through to the price of firm $B1$, decreasing both the value of buying from that firm and the probability of doing so. A higher search cost increases the expected reward as consumers know it is more likely that they will buy in firm $B1$ in the future, while the price of firm $B1$ increases in the same amount than the price of the other firms in market B. Note that the ER is higher than 0 if and only if $T - \alpha C > 0$, which is the same condition for firm $B1$ to be prominent once consumers reach that market. From now on, we assume this condition holds, so the CLP is active in equilibrium.

With this expression for the ER, we can derive firm $A1$'s demand. A consumer sampling firm $A1$ buys from that firm if and only if $\epsilon_{A1} - p_{A1} \geq a - p_{2A}^* - ER$. Following the same logic as in market B, the price of the rest of the sellers is $p_{2A}^* = 1 - a$. Therefore

$$D_{A1} = 2(1-a) - p_{A1} + ER. \tag{11}$$

The profit function is given by

$$\Pi_{A1} = p_{A1}D_{A1} - (1-\alpha)CD_{B1}^*. \tag{12}$$

Maximizing with respect to p_{A1} leads to

$$p_{A1}^* = (1-a) + \frac{ER}{2} + \frac{(1-\alpha)C(1-a+\phi_B)}{2}, \tag{13}$$

In a similar fashion than for firm $B1$, firm $A1$ charges a higher price due to the higher demand generated by the ER, and because it passes through some of the cost of this reward to final consumers. Note that for firm $A1$, the price increase relative to a case with no CLP is proportional to $1-a$ through the expression for the ER, because the value of the expected reward is proportional to the value of being prominent for firm $B1$ in the following period, which is increasing in the search cost (decreasing in the reservation value a). Additionally, even if $\alpha = 1$, meaning that firm $B1$ pays fully for the reward C, this price depends on C. The intuition is that, even if firm $A1$ does not pay for the reward in this particular case, the probability of buying from firm $B1$, and therefore the value of the expected reward, is affected by C through the price charged by firm $B1$.

The equilibrium demand for firm $A1$ is given by

$$D^*_{A1} = (1-a) + (1-a)\left(\frac{T - C(2-\alpha))}{4}\right) + \frac{\phi_B}{2}\left(\frac{T - C(4-3\alpha)}{4}\right), \tag{14}$$

while its profit equal to $\Pi^*_{A1} = (D^*_{A1})^2$. As firm $A1$ extracts all profits generated by firm $B1$, the total profit of $A1$ through the CLP is given by $\Pi_{AB} = \Pi^*_{A1} + \Pi^*_{B1}$, which is equal to

$$\Pi_{AB} = D^*_{A1}\left[D^*_{A1} + \left((1-a) + \frac{T - \alpha C}{2}\right)^2\right], \tag{15}$$

Define

$$\phi_A \equiv (1-a)\left(\frac{T - C(2-\alpha))}{4}\right) + \frac{\phi_B}{2}\left(\frac{T - C(4-3\alpha)}{4}\right), \tag{16}$$

meaning that $D^*_{A1} = 1 - a + \phi_A$. With the equilibrium prices and demands, we can compute the expected consumer surplus:

Proposition 1. *If consumers sample the CLP, their expected consumer surplus is given by*

$$CS = 4a - 2 + \frac{\phi_A^2}{2} + \phi_A(1-a). \tag{17}$$

Consumer surplus increases with T, decreases with C, increases with the reservation value a and decreases with the search cost s. $\square$

As in Armstrong, Vickers and Zhou (2009), the existence of a prominent firm in each market decreases the match value that consumers are willing to accept in those firms, as the price charged to consumers, in their analysis, is lower than the price in the rest of the firms. In our model, the price charged by $A1$ and $B1$ is greater than the base price charge by firms outside of the CLP (or by firms in absence of a CLP), but the effective price, considering the reward, is lower. Therefore, on average, consumers find worse products when the CLP is active. Also, as the prominent firms charge a lower effective price, the average number of searches made by consumers is lower, meaning they face lower total search costs. On average, consumer surplus is given by expression (17). A relevant difference between their model and ours is that consumers that buy at $A1$ but then get a low match value in $B1$, and therefore purchase from another firm in the B market, pay a higher price in the first period without enjoying the reward in the second stage. Therefore, some groups of consumers end up worse than others.

With Proposition 1, we can analyze the profitability of the CLP for firm $A1$. Note that the expected consumer surplus in the absence of a CLP is given by $4a-2$. Therefore, the expression for the expected consumer surplus acts as a restriction for the CLP, as consumers only participate in the CLP and sample firms $A1$ and $B1$ if their expected consumer surplus increases, or if $\frac{\phi_A^2}{2} + \phi_A(1-a) > 0$. Otherwise, consumers never sample any firm in the coalition.

Now, we analyze the profitability of the CLP for firm $A1$. As a benchmark, we use the case where firm $A1$ is exogenously prominent in market A, and analyze whether it is profitable for this firm to create the CLP.[2] We derive the following result:

Proposition 2. *The CLP is profitable if and only if it (weakly) increases consumer surplus. Thus, for every $\alpha \in [0,1]$, if C is not too high, there exists a threshold $\tilde{T}(\alpha, a, C)$, such that if $T > \tilde{T}(\alpha, a, C)$, the CLP is profitable. The threshold $\tilde{T}(\alpha, a, C)$ is decreasing in α and increasing in a and in C. In particular, if $\alpha = 1$, it is enough that $T > C$ for the CLP to be profitable.* □

This result highlights the two main ingredients for the CLP to be profitable. First, it must increase consumer surplus to encourage consumers to participate. Second, this happens only if the level of reward is high enough relative to its cost. Else, the pass-through rate from the cost of the reward to prices is too high, and consumers will not participate. If firm $A1$ would be integrated with firm $B1$, even if the level of reward is not too high, the integrated firm could charge prices in both markets such that consumers would participate. As firm $B1$ is an independent firm, this is not possible, and therefore, when T is relatively small, the CLP fails to be profitable.

A particular case of interest is given by $T = C$, meaning that the cost of the reward is equal to the reward itself. This represents a case where the reward is a cash discount on the price of firm $B1$ conditional on buying from both firms in the coalition. We obtain the following result:

Corollary 1. *If $T = C$, meaning that the reward is given as a cash discount, the CLP is never profitable.* □

Proof. If $T = C$, for any α, $\phi_A < 0$. Therefore, consumer surplus goes down, consumers do not participate in the CLP, and therefore it is not profitable. □

[2]If we consider that firm $A1$ is non-prominent in market A as a benchmark case, it would mean that it makes 0 profits and the CLP would always be profitable. We are interested in whether extending prominence to another market is profitable.

When $T = C$, for any level of α, the pass-through from the cost of the reward to final prices is both markets is so high that consumers do not obtain any benefit from the CLP. As consumer surplus goes down, consumers never sample any firm in the CLP. This means that a CLP cannot be profitable if it does not create some value, given by a level of reward greater than its cost. This result contrasts with the one obtained by Gardete and Lattin (2018), wherein a model with no search and with two markets and two firms in each market, CLPs might be profitable even with $T = C$. The main difference with our model is that in our case, the CLP is constrained by optimal search behavior, and therefore, it must leave some surplus to consumers to make them participate.

Finally, we analyze the effect of the CLP in total industry profits and total welfare:

Proposition 3. *If the CLP is profitable for the members for the coalition, total industry profits and total welfare increase.*

This result is straightforward and doesn't need a formal proof. If the CLP is profitable, it means that firm $A1$ it is increasing its profits while leaving firm $B1$ indifferent with respect to a case without a CLP. The rest of the firms in both markets charge the same price as before. Therefore, total industry profits increase in both markets. If total profits increase, and so does consumer surplus, naturally total welfare increases.

Next, we compare the profits for firm $A1$ with a case where that firm offers a reward T only conditional on consumers buying from that firm, and not contingent on also buying from a firm in another market.

Coalition vs standalone loyalty program: now, we compare the CLP with a standalone loyalty program (SLP). Suppose firm $A1$ can decide to make a loyalty program that offers a reward T at cost C conditional on consumers buying its product (and not conditional on also buying from another firm). This could also be interpreted as firm $A1$ increasing the quality of its product by T at a cost C. The timing of the game, in this case, is the following:

- t=1: Firm $A1$ and the rest of the firms in market A simultaneously set prices.
- t=2: Consumers search and purchase in market A.

We start by assuming that firm $A1$ is prominent. Then we check that, in fact, it is optimal for consumers to sample $A1$ first. Therefore, firm $A1$'s demand is given by

$$D_{A1}(p_{A1}) = 1 - a + T - p_{A1} + p^*_{2A}, \tag{18}$$

where, by the same logic than in the previous section, $p^*_{2A} = 1-a$ is the symmetric expected equilibrium price of the non-prominent firms. Firm $A1$'s profit function is given by

$$\Pi_{A1}(p_1) = [p_{A1} - C]D_{A1}(p_{A1}). \tag{19}$$

Taking the first-order condition leads to an equilibrium price equal to

$$p^*_{A1} = (1-a) + \frac{(T+C)}{2}, \tag{20}$$

The effective price payed by consumers is $p^*_{A1} - T$, or

$$p^*_{A1} - T = (1-a) - \frac{(T-C)}{2}. \tag{21}$$

Therefore, if $T > C$, it is indeed optimal for consumers to sample firm $A1$ first. The equilibrium profit of firm $A1$ is

$$\Pi^{S*}_{A1} = \left[(1-a) + \frac{(T-C)}{2}\right]^2, \tag{22}$$

The following result compares the profitability of a SLP with a CLP:

Proposition 4. *If C and the reservation value a are not too high, there exists a $\overline{T}(\alpha, a, C)$ such that if $T > \overline{T}(\alpha, a, C)$, the CLP is more profitable than the SLP. The threshold $\overline{T}(a, C)$ is increasing in a and C. Moreover, we have that $\overline{T} > \tilde{T}$.*
□

The intuition for this result is as follows. When forming a CLP, firm $A1$ reduces the reward offered to consumers by making it conditional on also buying from another firm in another market. Therefore, firm $A1$ charges a lower price and gets a lower profit from market A. However, by creating a CLP, firm $A1$ generates a prominent position for firm $B1$ in market B, and then extracts these extra profits through the fixed fee F. Therefore, forming a CLP is more profitable than the SLP if the value of the prominent position in market B is higher than the direct profit obtained by $A1$ by directly offering a reward to its consumers. As firm $B1$ is only prominent for consumers that buy from $A1$, its prominent position is valuable only if the demand for firm $A1$ is high enough. This happens when the reward T is sufficiently big, for fixed values of C and a.

The threshold for the level of the reward depends on the cost C, and the reservation value a. If the cost is high, the pass-through to the equilibrium price of $A1$ is high, and its demand is lower. Thus, a higher cost needs a higher level of T to make the CLP profitable relative to the SLP. When the reservation value is higher, or, equivalently, the search cost is smaller, the prominent position of firm $B1$ is less valuable. This reduces the expected reward for consumers from buying at firm $A1$. Therefore, a higher reservation value needs a higher T to make the CLP profitable. Finally, if the combination of C and a are too high, the CLP is never more profitable than the SLP.

3 Competing CLPs

In this section, we extend the analysis to the case of competition between CLPs. Suppose that every firm Aj in market A forms a CLP with a firm Bj in market B. Every CLP offers the same reward T at the same cost C, and the share α of the cost borne by each firm Bj in market B is the same for every coalition. We look for a symmetric equilibrium where the price in each market is the same for every firm.

There are two possible equilibria, depending on the value of the reward T relative to the reservation value a. If $T \geq a$, all consumers that buy from a firm j in market A, purchases from the firm in the same CLP in market B. When $T < a$, some consumers having bought in firm Aj will keep on searching after firm Bj in market B. For simplicity, we characterize the first case, where consumers always buy from firms in the same coalition.

At stage 2, any firm Bj, given the demand of its partner firm D_{Aj}, maximizes

$$\Pi_{Bj} = (p_{Bj} - \alpha C) D_{Aj} (1 - a + p_B^* - p_{Bj} + T), \tag{23}$$

where p_B^* is the expected equilibrium price in market B. Taking the first-order condition with respect to p_{Bj} and assuming symmetry we obtain

$$p_B^* = 1 - a + \alpha C + T. \tag{24}$$

In equilibrium, firms in market B fully pass-through to the equilibrium price, not only their share of the cost of the CLP, but also the reward T. In contrast, when only one CLP is active in the market, only half of the reward is passed through to the final price of the firm in the CLP. The intuition is that, when only one CLP is active, consumers expect lower prices from firms outside the CLP in market B, constraining the price that the CLP can charge in market B. When there is a continuum of CLPs, consumers expect every firm to charge a higher price.

The equilibrium demand of every firm B is given by $D_{Aj}(1-a+T)$. Therefore, if $T \geq a$, every consumer that buys in firm Aj buys in the firm of the same coalition Bj. At stage 1, we also look for a symmetric price equilibrium. Firm Aj maximizes

$$\Pi_{Aj} = (p_{Aj} - (1 - \alpha) C) \frac{(1 - a + p_A^* - p_{Aj} + ER_j - ER^*)}{(1 - a)}, \tag{25}$$

where p_A^* is the equilibrium price in market A, ER_j is the expected reward derived from buying from Aj, and ER^* is the equilibrium expected reward from firms in market A. As every firm forms a CLP, there is no prominent firm in market A. Therefore, consumers search randomly for firms in market A. Taking the first-order condition with respect to p_A^* leads to

$$p_A^* = 1 - a + (1 - \alpha)C. \tag{26}$$

Just as in market B, the price in market A increases with the share of the cost paid by firms in market A. However, in contrast with the case with one CLP, the expected reward for consumers is not translated to higher prices. The reason is that the expected reward is defined as the difference in the continuation payoffs between buying in one firm compared to the other firms. As every firm is offering the same potential benefit at period 2, in a symmetric equilibrium, this expected reward is 0 for every firm. Therefore, there is no shift in demand, and the equilibrium price doesn't increase due to the potential benefit of obtaining a reward in market B.

The following proposition analyzes consumer surplus, industry profits, and total welfare when comparing competing CLPs with a case with no CLP in any of both markets:

Proposition 5. *When there is a continuum of competing CLPs offering the same reward T at the same cost C, consumer surplus goes down, and industry profits go up. Total welfare goes up if and only if C is not too high.*

Proof. See Appendix. □

A continuum of competing CLPs, when T is relatively high, generates that each firm in market B becomes prominent for some consumers, due to the possibility of obtaining the reward. However, as all firms charge a high price, consumers are effectively locked-in into these firms. This means that they are willing to accept lower products to obtain the reward. This generates that the value of the reward is fully passed through to final prices, along with the cost of providing such rewards, and the final price paid by consumers is higher than in the absence of CLPs, even net of the benefit T. This added to the fact that they accept worse products on average, leads to consumer surplus going down. Given that consumer surplus goes down, it may be that consumers do not find it profitable to join the market at all. The market is active if and only if consumer surplus is non-negative (derivation of CS is in the Appendix), or

$$CS = 4a - 2 - \frac{a^2}{2} - C \geq 0. \tag{27}$$

With respect to industry profits, it is straightforward to see that industry profits go up (as long as the market is active), as the monopolistic competition in each market in the absence of CLPs is replaced with an equilibrium with high prices and locked in consumers in market B. Finally, as consumers are getting worse products on average, total welfare can only go up if the value created by the rewards, represented by the difference between T and C is high enough, or, in other words, if the cost of such reward is not too high.

Consumer surplus goes down, as the benefit from buying from a CLP, given by the reward T, is fully passed-through to prices in the second period, given that consumers are "locked" with the firm in the same CLP, due to the reward being high. In contrast with the case where only one CLP exists, consumers do not find optimal to search other firms in market B if the price is too high or the match values are low, given that every firm is charging a high price in equilibrium. Therefore, consumers end up worse off when CLPs compete with each other.

4 Extension: non-dominant firm CLP

The previous analysis assumes that consumers always start searching product A first. While this assumption is relevant in some cases, in other situations, such as purely independent goods, there is no natural search order for consumers. In this section, we assume that the product search order is random. More precisely, half of the consumers start searching in market A, and the other half start searching for products in market B.

We look for a symmetric equilibrium where the equilibrium price charged by both firms in the CLP, which we denote p_1^*, is the same in both markets. All the other firms are searched randomly. The price of the rest of the firms is $p_2^* = 1 - a$, just as in Section 2. Given the symmetry of this setting, we assume that $\alpha = \frac{1}{2}$.

The timing of the game is the following:

- t=1: taking the coalition as given, sellers simultaneously set prices in both markets.
- t=2: half of consumers start searching in each market, and then proceed to search in the other market.

We derive the results for firm $A1$. The maximization problem for firm $B1$ is derived analogously. The profit function of firm $A1$ is given by

$$\begin{aligned}\Pi_{A1} =& \frac{1}{2}\left(p_{A1} - \frac{C}{2}(2(1-a) - p_{B1} + T)\right)[2(1-a) - p_{A1} + ER^s)] \\ &+ \frac{1}{2}\left(p_{A1} - \frac{C}{2}C\right)[2(1-a) - p_{B1} + ER^s)]\,[2(1-a) - p_{A1} + T],\end{aligned}$$

where $ER^s(p_{B1})$ corresponds to the consumer's expected reward of buying from the CLP when the setting is symmetric. This profit function is explained as follows. The first term corresponds to consumers that start their search in market A. The term $[2(1-a) - p_{A1} + ER^s]$ is the demand from consumers buying from $A1$ in the first period. The cost paid by $A1$ for those consumers is adjusted by the probability that these consumers buy from firm $B1$ in the second period. The second term corresponds to consumers that buy from $B1$ in the first period. The number of such consumers is $[2(1-a) - p_{B1} + ER^s]$, while $[2(1-a) - p_{A1} + T]$ is the demand of firm $A1$ for those consumers in the second period. As all of these consumers get the reward, firm $A1$ pays $\frac{C}{2}$ for all of them.

Define $D_{A1}^T = 2(1-a) - p_{A1} + T$, $D_{A1}^E = 2(1-a) - p_{A1} + ER^s$, $D_{B1}^T = 2(1-a) - p_{B1} + T$, and $D_{B1}^E = 2(1-a) - p_{B1} + ER^s$. The profit function of firm $A1$ may be re-written as

$$\begin{aligned}\Pi_{A1}(p_{A1}, p_{B1}) =& \frac{1}{2}\left(p_{A1} - \frac{C}{2} D_{B1}^T(p_{B1})\right) D_{A1}^E(p_{A1}) \\ &+ \frac{1}{2}\left(p_{A1} - \frac{C}{2}\right) D_{A1}^T(p_{A1}) D_{B1}^E(p_{B1}).\end{aligned} \tag{28}$$

Taking the first-order condition with respect to p_{A1}, and using the symmetry of the model ($p_{A1} = p_{B1} = p_1^*$), leads to

$$D_{A1}^E(p_1^*) + D_{A1}^T(p_1^*) D_{B1}^E(p_1^*) + \frac{C}{2}(D_{B1}^T(p_1^*) + D_{B1}^E(p_1^*)) - p_1^*(1 + D_{B1}^E(p_1^*)) = 0. \tag{29}$$

The first-order condition provides an implicit relationship between the equilibrium price p_1^* and the equilibrium expected reward. As we cannot get an explicit solution for p_1^*, we compute the expected reward as a function of p_1^* to have another relationship between these variables. As in Section 2, the expected reward is the difference between the continuation payoffs for consumers buying from $A1$ compared to buying from any other firm in market A. This expected reward is given in the following Lemma:

Lemma 2. *In a symmetric equilibrium, the expected reward at $t = 1$ of buying from a CLP firm, as a function of p_1^*, is given by*

$$ER^s(p_1^*) = \frac{1}{2}\left(3(1-a)^2 + (p_1^* - T)^2 - 4(1-a)(p_1^* - T)\right) \tag{30}$$

The expected reward is decreasing in the equilibrium price p_1^*. □

As we do not have an explicit solution for p_1^*, we obtain the expected reward as a function of this symmetric equilibrium price. The system of equation (29) and (30) defines the equilibrium price of prominent firms p_1^* and the equilibrium expected reward. As this system does not admit an explicit solution, we provide some simulation results. Figure 1 displays the relation between the equilibrium price as a function of the reward level T.

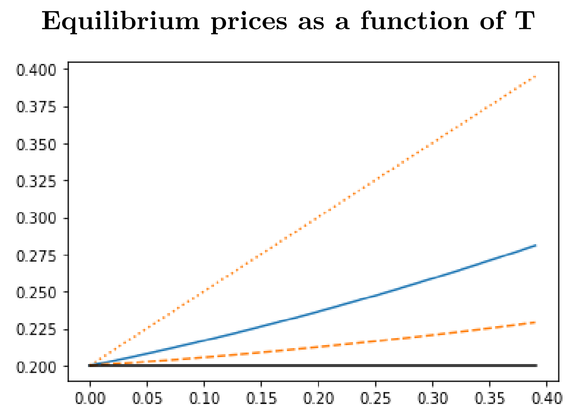


Figure 1: Equilibrium prices p_1^* (blue), p_{A1}^* (orange, dots), p_{B1}^* (orange, dashed), p_2^* (black), as a function of reward T. Model parameters: $a = 0.8$, $C = 0$, $\alpha = 0.5$.

As expected, the equilibrium price of the firms in the CLP increases with T. The price p_1^* of dominant firms, when the setting is symmetric, lies between that of the dominant firm in an asymmetric setting p_{A1}^*, and that of its partner p_{B1}^*. This is intuitive, as both firms now play the role of the dominant firm for half of the market, and the role of the partner firm for the other half.

Now, we derive the expected consumer surplus for consumers and show that it may be profitable, for some values of the parameters, for consumers to sample the CLP. The expected consumer surplus is expected to decrease in market A and increase in market B, due to the fact the the symmetric equilibrium price of firms in the CLP is lower in market A and higher in market B, relative to the case of a dominant firm CLP. To understand which effect dominates, Figure 2 displays the relation between the expected consumer surplus and the reward level T.

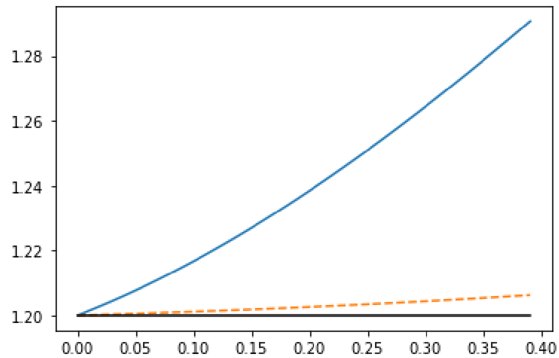


Figure 2: Expected consumer surplus in symmetric setting (blue), asymmetric setting (orange, dashed) and without a CLP (black), as a function of reward T. Model parameters: $a = 0.8$, $C = 0$, $\alpha = 0.5$.

The expected consumer surplus is greater when there is no preferred order search (symmetric setting). Indeed, simulations show that $2p^* < p_{A1} + p_{B1}$, which means that the actual price faced by consumers is, on average lower than when the CLP is asymmetric. As the consumer surplus without a CLP is given by $4a - 2 = 0.6$ for the assumed parameters, we have that the CLP might indeed be used by consumers in equilibrium. In turn, consumer surplus might increase with symmetric CLPs relative to dominant firm CLP.

We derived some preliminary results for a symmetric CLP, and show that the main intuitions are maintained, relative to the dominant firm CLP. A full characterization of the profitability and welfare analysis for all the possible parameters is left for further research.

5 Conclusion

In this paper, we take the first step to interpret coalition loyalty programs based on costly consumer search. As consumers often search for their preferred products, a prominent firm in one market can extend its prominence to another market, by offering a reward to consumers conditional on buying from both firms in the coalition. Then, the prominent firm can extract the extra profits generated in the other market through a fixed fee. In equilibrium, even if the coalition firms charge higher prices, both consumers and the coalition are better off. In contrast, the rest of the firms in both markets are worse off. The main intuition on why consumers are better is that the coalition is profitable only if consumer surplus is higher when consumers sample their firms first. Otherwise, consumers would never sample the coalition. They would only search for the other firms that charge lower prices and offer no reward.

Then, we extend our model to study competing CLPs. We show that, when the value of the reward offered to consumers is relatively high, consumers always buy from firms in the same coalition. As in this case, every firm charges a high price, and consumers are locked into the firms in the CLP to get the reward, consumers end up worse than in a case without CLPs. Therefore, consumer surplus goes down when CLPs compete.

As an extension, we study the case where half of the consumers start in each market. We show that the main intuitions remain the same as in the case with a dominant firm CLP. A full characterization of this case and the relative profitability of this CLP relative to the dominant firm CLP is left for further research.

For simplicity, we assumed that each market has an infinite number of competitors outside the CLP. This simplifies the analysis and allows us to understand the basic incentives of consumers and the firms in the CLP while shutting down the strategic reaction from the other firms in each market. In a more general analysis, a finite number of competitors in each market would generate a price reaction, where the non-prominent firms would likely increase their prices in reaction to the CLP, as shown by Armstrong, Vickers and Zhou (2009). In such a context, while our results on the profitability of the CLP should be qualitatively similar, our results concerning consumer surplus and total welfare might change. Also, we assumed that both the search cost and the distribution of match values are the same in both markets. Further research should take into account these potential differences, as they should be important to understand the effects of CLPs.

Appendix

Proof of Lemma 1

The expected reward of a consumer buying from firm $A1$ is the expected consumer surplus in $t = 2$ relative to the case where he doesn't buy from firm $A1$. It is composed by the expected match value he will obtain in his search process, the expected price he will pay and the expected search costs he will face. As a reference, we compute first the expected match value of not buying in firm $A1$. In this case, a consumer will follow the standard search rule and buy when he finds a match value greater than a. Therefore, his expected match value is $\frac{1+a}{2}$ which is the conditional expectation on the match value being greater than a. The expected price he will pay is $p^*_{2A} = 1 - a$ and the expected search cost is $\frac{s}{1-a}$, which is the weighted probability of stopping at the first firm sampled, the second, etc. Using the fact that $s = \frac{(1-a)^2}{2}$, summing the three terms lead to an expected consumer surplus of $2a-1$, which is the standard result in Wolinsky (1986) and Armstrong, Vickers and Zhou (2009) with infinite firms and uniform distribution.

With the CLP, if a consumer bought in $A1$, he will buy in $B1$ if and only if $\epsilon_{iB1} - p^*_{B1} \geq a - p^*_{2B} - T$ or if and only if

$$\epsilon_{B1} \geq a - \phi_B, \tag{31}$$

where $\phi_B = \frac{T-\alpha C}{2}$. Therefore, its expected match value is $M_{B1} = \frac{1+a-\phi_B}{2}$ if he buys from firm $B1$ and $\frac{1+a}{2}$ if he doesn't. The probability of him buying from firm $B1$ is $1-a+\phi_B$. The expected price he pays is $p^*_{B1} - T$ if he buys from $B1$ and $1-a$ if he doesn't. He faces one time the search cost s if he buys from firm $B1$ and $s(1+\frac{1}{1-a})$ if he doesn't. Summing all terms weighted by their probability leads to an expected consumer surplus of $2a-1+\phi_B(1-a)\frac{\phi_B^2}{2}$. The expected reward is therefore the difference between this and the case where a consumer doesn't buy from $B1$ given by $2a-1$.

Proof of Lemma 2

We follow the same steps as in Lemma 1. Condition on buying from the CLP, in the second period consumers buy if and only if $\epsilon \geq a - p^*_2 + p^*_1 - T$. Therefore, their expected match value is given by $\frac{2a+p^*_1-T}{2}$ if the buy from the CLP. This happens with probability $P_1 = 2(1-a) - p^*_1 + T$. If they buy from any of the other firms, consumers get an expected match value of $\frac{1+a}{2}$. This happens with probability $P_2 = 2a - 1 + p^*_1 - T$. The expected price is given by $p^*_1 - T$ if the buy from the CLP and $1-a$ otherwise. The expected search costs are given by s if the buy in the CLP and $s + \frac{s}{1-a}$ if they don't. We weighted sum of these terms is given by:

$$(T - p^*_1)(2(1-a) + \frac{T-p^*_1}{2}) + \frac{3}{2}a^2 - a + \frac{1}{2} \tag{32}$$

The expected reward is therefore the difference between this and the case where a consumer doesn't buy from $B1$ given by $2a-1$, as shown in Lemma 2.

Proof of Proposition 1

The expected consumer surplus before joining any market is computed in a similar fashion than in Lemma 1 but also considering the different possibilities in $t=1$. A consumer will buy from $A1$ if and only if $\epsilon_{iA1} - p_{A1} \geq a - p_{2A} - ER$ or

$$\epsilon_{iA1} \geq a - \phi_a, \tag{33}$$

where

$$\phi_A = (1-a)\left(\frac{T - C(2-\alpha))}{4}\right) + \frac{\phi_B}{2}\left(\frac{T - C(4-3\alpha)}{4}\right). \tag{34}$$

Therefore, the probability of buying from $A1$ is $1 - a + \phi_A$. Following Lemma 1, the probability of buying from $2B$ conditional on having bought from $A1$ is $1 - a + \phi_B$. Therefore, there are three options for consumers:

- Buy from $A1$ and $B1$, with probability $P_{AB} = (1 - a + \phi_A)(1 - a + \phi_B)$.
- Buy from $A1$ and in a non-prominent firm in market B with probability $P_{AN} = (1 - a + \phi_A)(a - \phi_B)$.
- Buy from non-prominent firms in both markets with probability $P_{NN} = a - \phi_A$.

Note that consumers never buy from a non-prominent firm in market A and then in firm $B1$, as this firm is never sampled by those consumers as it is expected to charge a higher price and consumers can not obtain a reward. The expected consumer surplus is composed by three terms: i) expected match value, ii) expected price and iii) expected search costs.

Expected match value: If consumers buy from those firms, they obtain a match value given by the expectation of the distribution of match values conditional on $\epsilon_{iA1} \geq a - \phi_a$ which is given by $M_{A1} = \frac{1+a-\phi_A}{2}$. Similarly, if the buy from firm $B1$ it is given by $M_{B1} = \frac{1+a-\phi_B}{2}$, and from buying from a non-prominent firm equal to $M_N = \frac{1+a}{2}$. Therefore, the expected match value is given by

$$EMV = P_{AB}(M_{A1} + M_{B1}) + P_{AN}(M_{A1} + M_N) + P_{NN}(2M_N) \tag{35}$$

which is equal to

$$EMV = 1 + a - \frac{\phi_A}{2}(1 - a + \phi_A) - \frac{\phi_B}{2}(1 - a + \phi_A)(1 - a + \phi_B) \tag{36}$$

Expected price: The expected price is given by $p^*_{A1} + p^*_{B1} - T$ if they buy from both firms, $p^*_{A1} + 1 - a$ if the buy only from firm $A1$ and $2(1-a)$ if they buy only from prominent firms. Therefore, the expected price is given by

$$EP = P_{AB}(p^*_{A1} + p^*_{B1} - T) + P_{AN}(p^*_{A1} + 1 - a) + P_{NN}(2(1-a)) \tag{37}$$

which is equal to

$$EP = 2(1-a) - (1 - a + \phi_A)\left((1-a)\frac{T - (2-\alpha)C}{4} + \phi_B\frac{3T - C(4-\alpha)}{8}\right) \tag{38}$$

Expected search costs: In the first case, consumers face a search cost of $2s$, in the second case $2s+\frac{s}{1-a}$ and in the third case $\frac{2s}{1-a}$. Therefore, the expected search costs is given by

$$ESC = P_{AB}(2s) + P_{AN}(2s + \frac{s}{1-a}) + P_{NN}(\frac{2s}{1-a}) \tag{39}$$

which is equal to

$$ESC = 1 - a - \frac{\phi_A}{2}(1-a) - \frac{\phi_B(1-a)(1-a+\phi_A)}{2} \tag{40}$$

Using the fact that $s = \frac{(1-a)^2}{2}$ and summing the three terms leads to $4a - 2 + \phi_A(1-a) + \frac{\phi_A^2}{2}$. Note that consumers are buying in two markets, the benchmark case without CLP is given by $4a - 2$.

Proof of Proposition 2

First, note that $D^*_{A1} = 1 - a + \phi_A$ and $CS = 4a - 2 + \phi_A(1-a) + \frac{\phi_A^2}{2}$. Consumer surplus is greater than in a case with no CLP if $\phi_A \geq 0$ or if ϕ_A is negative enough. But if ϕ_A is that negative, then $D^*_{A1} \leq 0$. Therefore CS increases if and only if $\phi_A \geq 0$. This means that D^*_{A1} increases and therefore Π^*_{A1} increases and so the profits of firm $B1$. This condition is equivalent to

$$(4T - 8C)(1-a) + T(T - 4C) + \alpha C(4(1-a) + 2T + 4C) - 3\alpha^2 C^2 \geq 0. \tag{41}$$

If this condition holds, the both consumer surplus and the CLP profits increase. Therefore, for a fixed level of α and C, if T is high enough, the left hand side is greater than 0, provided C is not too high.

For the comparative statics, it is enough to differentiate with respect to α, T and C and see that the LHS increases with α and decreases with a and C.

Proof of Proposition 3

Total industry profit is given by the sum of the profits from the non-prominent firms in each market plus the profits of the firms in the coalition. The profits from the non-prominent firms in market A are given by $(a - \phi_a)(1-a)$, from the non-prominent firms in market B by $(1 - (1 - a + \phi_A)(1 - a + \phi_B))$, from firm $A1$ by $(1 - a + \phi_A)^2$ and from firm $B1$ by $(1 - a + \phi_A)(1 - a + \phi_B)^2$. Summing all terms lead to a total industry profit of

$$\Pi = 2(1-a) + \phi_A(1 - a + \phi_A) + (1 - a + \phi_A)(1 - a + \phi_B)\phi_B \tag{42}$$

which is greater than in the case without CLP if the program is profitable. As industry profits and consumer surplus increase in this case, total welfare also increases.

Proof of Proposition 4

For simplicity in the exposition, we provide the proof assuming that $\alpha = 1$ and then explain why it extends to any α. When $\alpha = 1$, we have that the profits of firm $B1$ in the CLP case can be written as a function of the profits of $A1$ in the SLP case, or $\Pi^*_{B1} = D^*_{A1}\Pi^{S*}_{A1}$. As the joint profits of the CLP are given by $\Pi^*_{A1} + \Pi^*_{B1}$, we have that if D^*_{A1} is big enough, then the profits of the CLP will exceed the profits of the SLP.

We need to verify that this can indeed happen under the parametric restrictions of our model. Suppose $a = \frac{1}{2}$, which is the case generating the least profits for the CLP. Taking a value of C very small, approaching zero, and a big value of T, approaching one, we have that $D^*_{A1} = \frac{11}{16}$ and $D^*_{B1} = 1$. The joint profits of the CLP are given by $D^*_{A1}\left[D^*_{A1} + \left((1-a) + \frac{T-\alpha C}{2}\right)^2\right]$ or $\frac{11}{16}\frac{27}{16} \approx 1.16$ while the profits of the SLP are equal to 1. Therefore, if C is not too high, if T is high enough then the CLP is more profitable.

This result extends for any α because, if $\alpha = 1$ is not maximizing the profits of the CLP, than the optimal α increases its profits. and the argument still holds.

Proof of Proposition 5

First we compute the consumer surplus. We have $p^*_A = 1 - a + (1-\alpha)C$ and $p^*_B = 1 + \alpha C + T - a$.

Expected match value: At $t = 2$, consumers always buy the product from the firm in the coalition they bought in market A. Therefore, their expected match value is always $\frac{1}{2}$. In the first period, as all firms charge the same price and offer the same reward, their expected match value is the same is the case with no CLPs, or $\frac{1+a}{2}$.

Expected price: For every consumer the expected price is $p^*_A + p^*_B - T = 2(1-a) + C$.

Expected search cost: in the first period, the expected search cost is the same as in the case with no CLP, given by $\frac{s}{1-a}$. In the second period, the only search once, therefore their expected search cost is s. Remembering that $s = \frac{(1-a)^2}{2}$ we have that the expected search cost is $\frac{(1-a)^2}{2} + \frac{1-a}{2}$.

Summing all terms leads to

$$CS = 4a - 2 - C - \frac{a^2}{2}, \tag{43}$$

while the CS in the case with no CLP is $4a - 2$. Therefore, CS always goes down with competing CLPs.

Industry profits are equal to $(1 - a)$ in market A and equal to 1 in market B. Total industry profits are $2 - a$ which is greater than $2 - 2a$ in the case with no CLPs.

Total welfare is equal to

$$W = 3a - \frac{a^2}{2} - C, \tag{44}$$

while in the case with no CLPs is $2a$. Therefore, total welfare increases if and only if

$$a - \frac{a^2}{2} - C \geq 0. \tag{45}$$

For the parametric restrictions on a, it is easy to check that if C is small enough (close to 0), this condition is always satisfied, while if $C > \frac{1}{2}$, this condition is never satisfied. Therefore, total welfare can increase or decrease depending on the value of C.

References

Anderson, S. P. and Renault, R. (1999). Pricing, product diversity, and search costs: A bertrand-chamberlin-diamond model. *The RAND Journal of Economics*, pages 719–735.

Armstrong, M. (2013). A more general theory of commodity bundling. *Journal of Economic Theory*, 148(2):448–472.

Armstrong, M., Vickers, J., and Zhou, J. (2009). Prominence and consumer search. *The RAND Journal of Economics*, 40(2):209–233.

Basso, L. J., Clements, M. T., and Ross, T. W. (2009). Moral hazard and customer loyalty programs. *American Economic Journal: Microeconomics*, 1(1):101–23.

Breugelmans, E., Bijmolt, T. H., Zhang, J., Basso, L. J., Dorotic, M., Kopalle, P., Minnema, A., Mijnlieff, W. J., and Wünderlich, N. V. (2015). Advancing research on loyalty programs: a future research agenda. *Marketing Letters*, 26(2):127–139.

Caillaud, B. and De Nijs, R. (2014). Strategic loyalty reward in dynamic price discrimination. *Marketing Science*, 33(5):725–742.

Dorotic, M., Bijmolt, T. H., and Verhoef, P. C. (2012). Loyalty programmes: Current knowledge and research directions. *International Journal of Management Reviews*, 14(3):217–237.

Gans, J. S. and King, S. P. (2006). Paying for loyalty: Product bundling in oligopoly. *The Journal of Industrial Economics*, 54(1):43–62.

Gardete, P. and Lattin, J. (2018). Coalition loyalty program not working? perhaps you're doing it wrong.

Lederman, M. (2007). Do enhancements to loyalty programs affect demand? the impact of international frequent flyer partnerships on domestic airline demand. *The RAND Journal of Economics*, 38(4):1134–1158.

Lederman, M. (2008). Are frequent-flyer programs a cause of the "hub premium"? *Journal of Economics & Management Strategy*, 17(1):35–66.

Wolinsky, A. (1986). True monopolistic competition as a result of imperfect information. *The Quarterly Journal of Economics*, 101(3):493–511.