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Essays on Public Economics

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Declaration

I acknowledge financial support from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 669217). All remaining errors are mine.

Abstract

This thesis consists of three essays in public economics. They are concerned on the impact of government intervention in markets characterized by the presence of harmful goods.

The first chapter considers a setting where individuals can consume two types of sin goods differ in their consumption observability (taxability) by the government. As a benchmark, the first-best taxes for the observable and non-observable sin good are derived, considering homogeneous individuals. In the second-best setting, where observability on sin good consumption is limited, the rule for the taxable sin good is shown to depend on the degree of complementarity or substitutability with the unobservable sin good. Finally, redistributional considerations are incorporated by extending the analysis to a setting where individuals differ in their wealth and in their degree of misperception of the health damage caused by sin good consumption. Policy implications are illustrated considering physical inactivity and illicit drugs as non-taxable sin goods, while alcohol, tobacco, fat and sugar account for the taxable sin goods.

The second chapter studies the optimal policies related to the legalization of marijuana, in a setting where consumers differ in their utility from consumption of the psychoactive component of cannabis, THC, and suffer from misperception on the health damage it causes. We analyze this problem through a vertical differentiation model, where a public and a black market firm compete in prices and quality (THC content). A paternalistic government would like to correct for the misperceived health damage caused by marijuana consumption, as well as to reduce the size of the black market. We show that it is the undesirability of black market profits, rather than the health damage misperception, what makes the first-best allocation impossible to decentralize. We find two possible equilibria,

in which the public firm serves either the consumers with the highest or the lowest willingness to pay for quality. Allowing the public firm to move first, à la Stackelberg, does not provide it an advantage and social welfare remains second-best.

The third chapter analyses on how the scheme towards harmful drugs adopted by a symmetric neighboring jurisdiction, impacts in the domestic optimal drug policy in a imperfectly competitive market for harmful drugs, characterized by the presence of a black market firm and where consumers may engage in cross-border shopping. In our setting, a drug policy consists in adopting either a scheme of prohibition or one of legalization, and to decide how much to invest in enforcement activities to tackle black market supply. We consider a negative social valuation for consumption of harmful drugs, as well as for the profits generated in the black market. We find that for a low (high) concern for consumption of harmful drugs, both jurisdictions adopt in equilibrium a scheme of legalization (prohibition). More interestingly, for an intermediate social valuation for consumption of harmful drugs, different scenarios may arise, that can for instance explain why two symmetric jurisdictions may end up adopting different schemes towards harmful drugs. Furthermore, under some circumstances governments may face a prisoner's dilemma, where the resulting equilibrium is one where both jurisdictions legalize the harmful drug, despite that both sticking to a scheme of prohibition would yield a better outcome.

Résumé

Cette thèse est composée de trois essais en économie publique. Ils étudient l'intervention publique dans des marchés où il existe des biens nocifs pour la santé. Le premier chapitre considère une situation où les consommateurs peuvent acquérir deux types de biens, chacun nocifs pour la santé mais qui diffèrent par leur observabilité par le gouvernement (taxable ou non). Dans un premier temps, en considérant des individus homogènes, les impôts optimaux pour ces biens nocifs taxable et non taxable pour le gouvernement sont calculés. Ensuite, quand l'observabilité de la consommation est limitée, on montre que la règle de taxation optimale de second rang dépend du degré de complémentarité ou de substituabilité entre les deux biens, observables et non observables. Enfin, des questions redistributives sont analysées en considérant d'une part des inégalités de richesse et d'autre part des différences de perception des dommages causés par la consommation des biens nocifs pour la santé. Des recommandations politiques sont proposées en considérant l'inactivité physique et les drogues illégales comme des biens nocifs qui ne peuvent être taxés, et l'alcool, le tabac et les produits gras et sucrés comme des biens nocifs peuvent l'être.

Le deuxième chapitre est consacré aux politiques optimales de légalisation du cannabis. Les consommateurs diffèrent par l'utilité qu'ils tirent de la consommation de THC, l'élément psychoactif du cannabis, et ont une perception erronée des dommages de sa consommation sur leur santé. Le problème est analysé à l'aide d'un modèle différenciation verticale où deux firmes, l'une publique l'autre œuvrant dans le marché noir, se font concurrence en prix et en qualité (taux de THC). Un gouvernement paternaliste voudrait corriger, d'une part, l'excès de consommation lié à la mauvaise perception des consommateurs des dommages causés par le cannabis et, d'autre part, réduire la taille du marché noir. Nous montrons que c'est la volonté de réduire les profits du marché noir, plutôt que le manque de perception des consommateurs, qui explique que la consommation optimale

de premier rang n'est pas atteignable dans un marché décentralisé. Nous trouvons deux équilibres possibles, où la firme publique sert uniquement les consommateurs avec la plus haute ou bien la plus basse propension à payer pour la qualité (du cannabis). Autoriser la firme publique à agir la première ne lui procure aucun avantage et ne permet donc pas d'améliorer le bien-être social.

Le troisième chapitre analyse comment la politique d'une juridiction voisine, avec les mêmes caractéristiques que la juridiction domestique, affecte la politique domestique optimale de lutte contre les drogues. La concurrence sur le marché des drogues est supposée imparfaite, un marché noir est présent, et les consommateurs peuvent effectuer des achats transfrontaliers. Nous considérons qu'une politique de lutte contre les drogues consiste à choisir, d'une part, entre la légalisation et la prohibition de la vente de drogues et, d'autre part, de l'intensité des investissements pour lutter contre la production illégale. Nous faisons l'hypothèse que la consommation de drogues et les profits du marché noir ont tous deux une valeur sociale négative. À l'équilibre, si la préoccupation pour la consommation de drogues est faible (élevée), alors les deux juridictions adoptent une politique de légalisation (prohibition) des drogues. Plus intéressant, pour des niveaux intermédiaires de la valeur sociale de la consommation de drogues, les équilibres sont asymétriques, ce qui pourrait expliquer, par exemple, pourquoi deux juridictions symétriques adoptent des politiques opposées dans la lutte contre les drogues. Par ailleurs, dans certaines circonstances, les gouvernements font face à un dilemme du prisonnier. À l'équilibre, les deux juridictions légalisent la vente de drogues nocives alors même que maintenir deux politiques de prohibition serait socialement préférable.

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Contents

Chapter 1

Taxable and non-taxable sin goods *

Luis Rodrigo Arnabal

Abstract
Sin good consumption entails health damage, which is in general not fully perceived by individuals, what results in its overconsumption. One way to tackle this problem is to tax these unhealthy goods. However, not all the individual choices that affect health status can be easily observed by the government. This paper considers a setting where individuals can consume two types of sin goods that differ in their observability (taxability) by the government. As a benchmark, the first-best taxes for the observable and non-observable sin good are derived, considering homogeneous individuals. In the second-best setting, where observability on sin good consumption is limited, the rule for the taxable sin good is shown to depend on the degree of complementarity or substitutability with the unobservable sin good. Finally, redistributional considerations are incorporated by extending the analysis to a setting where individuals differ in their wealth and in their degree of misperception of the health damage caused by sin good consumption. Policy implications are illustrated considering physical inactivity and illicit drugs as non-taxable sin goods, while alcohol, tobacco, fat and sugar account for the taxable sin goods.

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1.1 Introduction

One of the biggest health concerns today is the increasing lifestyle-related disorders and sedentary habits. Sin good consumption and physical inactivity are the main reasons that account for the increasing risks of dying from a noncommunicable disease, which according to the World Health Organization (WHO) kill 41 million people every year.[1] Sin goods such as tobacco, alcohol, fat or sugar have been the target of different public policies aiming to discourage its consumption. These policies are justified by the fact that individuals fail to fully acknowledge the consequences caused by their sin good consumption, both to themselves and to the rest of the society through an increase in health care costs. One important measure to mitigate this overconsumption problem has been to increase their price through the so called *sin taxes*. However, not all the individual decisions that affect health status can be effectively taxed by the government. This is the case, for example, of physical inactivity, which represents today the fourth cause of death worldwide, accounting for 3.2 million death annually, being considered as one of the most important health problems of this century.[2] According to the WHO, engaging in a physical activity with moderate intensity by at least two hours and a half per week reduces between 20% and 30% the risk of all-cause mortality. Another prominent example of non-taxable sin goods are illicit drugs. Depending on the illicit drug in question, they may cause from minor health damages to death, due, for instance, to an overdose.[3] Regarding illicit drugs, one of the biggest concerns today is the public health impact of the increase in cannabis use, whose health effects are not yet well understood.[4]

Given this non-observability on some sin good consumption, introducing a tax on an observable sin good may create a distortion on the trade-off between consuming this particular sin good and another sin good that is unaffected by this policy. It may then induce agents to consume more of other types of unhealthy goods that are not so heavily taxed and that may be eventually more harmful than the targeted sin good, undermining the effectiveness of sin good taxation. The fact that consumption of some types of sin

[1] Data from http://www.who.int/news-room/fact-sheets/detail/noncommunicable-diseases, updated on june 2018.

[2] See Lim et al. (2013) for an analysis on the contribution of different risk factors to disease burden worldwide.

[3] One relevant aspect of the problem that is not analyzed in this work is the fact that sin good consumption may result in addiction.

[4] For a comprehensive survey on the health impacts of cannabis see National Academies of Sciences et al. (2017).

goods is more difficult to observe or to tax than others, together with the pattern of complementarity or substitutability that may exist between them is what motivates this work.

In this paper, we focus on the problem faced by a paternalistic government that wants to correct for the misperception that agents have of the health damage caused by sin good consumption, but faces a restriction on the set of sin goods it can tax. This misperception of health damage may be due, for instance, to ignorance or some cognitive inability that impairs the individual to perfectly understand the health consequences of sin good consumption. In this sense, the relationship between awareness of risk factors and education is well documented.[5] Moreover, based on a recent survey in the US, the analysis of Waters and Hawkins (2018) has found that while most individuals associate physical inactivity with cardiovascular and metabolic problems, they fail to acknowledge that the lack of physical activity may result in cancer and other diseases.

Our analysis begins by considering a setting where identical individuals derive utility from consuming a numeraire good and two types of sin goods that differ in their observability by the government. We abstract from the production side and focus our attention on the corrective role of the sin taxes on consumer behavior and how this role is affected by the non-observability on some sin good consumption. This simple setting will allow us to understand how this restriction affects the optimal tax on the observable sin good. Finally, in order to incorporate redistributional considerations, we extend the analysis to a setting where individuals differ in wealth and in their degree of misperception of the health damage caused by sin good consumption.

The present works aims to contribute to the growing literature on sin good taxation. In a general framework, O'Donoghue and Rabin (2003, 2006) show that when some agents suffer from misperception of the health damage caused by sin good consumption, introducing a sin tax improves welfare and it may even be Pareto improving under some circumstances. Allowing agents to mitigate the damage from sin good consumption by investing in health care, Cremer et al. (2012) derive optimal formulas for sin taxes and health care subsidies. They find that the optimal sin tax will depend on the degree of substitutability between health care investments and sin good consumption. Our analysis is

[5] For instance, Devaux et al. (2011) find a strong relationship between obesity and education, while Steptoe et al. (2002) identify the same pattern for tobacco consumption and education. Several studies suggest lower levels of education are associated to higher risks of alcohol disorder, (e.g. Crum et al. (1993); Gilman et al. (2008)).

formally similar, though we abstract from health investments or treatments and consider instead that health status is determined by the consumption of two different sin goods. We are concerned on how a tax on a particular sin good may affect the consumption of another sin good that the government cannot observe. Our study puts emphasis on the undesirable effects that may arise when introducing sin taxes. In this sense, it is similar in spirit to the studies of Yaniv et al. (2009) and Dragone et al. (2016). The former considers a situation where consumers can directly consume a fat good or buy a healthy good that entails an additional time cost to be consumed. Under some specific conditions they find that a fat tax may lead to a reduction of the time devoted to physical activity. The latter focuses on the interaction of smoking and eating behavior, suggesting that taxing junk food may jointly reduce tobacco smoking and obesity prevalence, as a decrease in body weight would make smoking less interesting as a dieting device. In our analysis, consumption of non-taxable sin goods will be considered as being a direct choice made by individuals, rather than being a consequence of their time allocation and our focus is on how to improve welfare, given the non-observability or the inability for the government to tax some sin goods.

We depart from the previous contributions by considering the case where an individual experiences utility from the joy of consumption of two types of sin goods, that differ in their observability by the government and whose damage on health status is mispercieved. We study how the optimal rule for the observable sin good ought to be adjusted in order to account for the fact that the government faces a restriction on the observability on the other sin good. The interaction between taxable and non-taxable sin goods in the utility of individuals is a crucial element in our analysis. A pattern of complementarity (substitutability) between a taxable and a non-taxable sin good will call for a higher (lower) sin tax. Ignoring this interaction for the design of the tax scheme will set the wrong incentives for consumption of the non-taxable sin good. When this interaction is of importance, the sin tax becomes a useful tool for the government to tackle the overconsumption of sin goods that cannot be directly taxed. The policy implications associated to this result are discussed in the last section of this paper, considering physical inactivity and cannabis as particular cases of non-taxable sin goods.

The remainder of this paper is organized in the following way. Section 2 presents the economic environment, the *laissez-faire*, the first-best allocation and its decentralization through taxes. In section 3 we derive the optimal sin tax when consumption of non-taxable sin goods is not observable. Section 4 extends the analysis to a setting with heterogeneous agents. Section 5 discusses the main results and policy implications of our analysis. Section 6 concludes.

1.2 The basic model

1.2.1 The economic environment

We begin by considering identical individuals with an initial endowment w. They derive utility from consumption of a numeraire good c and from two types of sin goods: a taxable sin good x and a non-taxable sin good f. Taxable sin goods are, for instance, alcohol, tobacco, fat or sugar, while examples of non-taxable sin goods are physical inactivity or illicit drugs. The utility from the joy of sin good consumption is given by $v(x, f)$, so that the joy that the agent gets from consuming the taxable sin good may depend also on the consumption of the non-taxable one. This specification captures the idea that consuming, for instance, a certain amount of alcohol may make more or less attractive to consume cannabis. In the case of physical inactivity as a non-taxable sin good, it can be interpreted as whether a higher level of consumption of tobacco, fat or sugar, makes being lazy more or less attractive. Sin good consumption entails health damage, that will be captured by a harm function $h(x, f)$, with the following partial derivatives: $h_x(x, f) \equiv h_x > 0$, $h_{xx}(x, f) \equiv h_{xx} > 0$ and $h_f(x, f) \equiv h_f > 0$, $h_{ff}(x, f) \equiv h_{ff} > 0$. The assumption that the harm function is convex in each of its arguments captures the fact that an additional unit of consumption of both types of sin goods will have a higher damage on health status than its previous ones. We further assume that the agent only perceives a fraction β of the true damage caused by sin good consumption, due, for instance, to ignorance, with $0 \leq \beta < 1$.[6] It is this misperception of health damage caused by sin good consumption what motivates government intervention.

[6]While our framework does not allow for time-inconsistent preferences, the rationale for government intervention is very related.

1.2.2 The *laissez-faire*

The problem of an individual is to decide how to allocate his initial wealth between consumption of the numeraire good and two types of sin goods, in order to maximize his perceived utility U:

$$U(c, x, f) = u(c) + v(x, f) - \beta h(x, f). \tag{1.1}$$

where u denotes the utility the agent gets from consumption of the numeraire and v from the joint consumption of both types of sin goods. We assume that u is strictly concave, with $u' > 0$, $u'' < 0$, while v is assumed to be strictly concave in each of its arguments. Using and equivalent notation than for the harm function, we have that $v_x > 0$, $v_f > 0, v_{xx} < 0$, $v_{ff} < 0$. The individual maximizes his utility in (1.1), subject to the following budget constraint:

$$w = c + x + f. \tag{1.2}$$

The first order conditions (FOCs) yield:

$$u'(c) \ = v_x - \beta h_x = v_f - \beta h_f. \tag{1.3}$$

The conditions above imply that the individual equates his marginal utility from consumption of the numeraire with his (mis)perceived utility from consumption of taxable and non-taxable sin goods. The agent undervalues the marginal damage that sin good consumption has on his health. As a result, he consumes too much of both types of sin goods. Notice that for simplicity we are considering the same degree of misperception of health damage caused by both types of sin goods. More in general, the overconsumption problem would be more relevant for one particular type of sin good if the misperception of the health damage it causes were to be higher than for the other type of sin good. In what follows, we consider a paternalistic government that will intervene in order to correct for this overconsumption problem.

1.2.3 The first-best problem

The first-best allocation corresponds to the one of a decentralized economy in the absence of misperception of the health damage caused by sin good consumption. The first-best problem amounts then to maximize the utility of the individual in (1.1) with $\beta = 1$, subject to the budget constraint in (1.2). The Lagrangian associated to the first-best problem is the following:

$$\mathcal{L} = u(c) + v(x, f) - h(x, f) + \lambda \cdot (w - c - x - f),$$

where $\lambda > 0$ is the Lagrange multiplier associated with the budget constraint.

FOCs yield:

$$u'(c) \ = v_x - h_x \ = v_f - h_f = \lambda. \tag{1.4}$$

The conditions above indicate that the marginal utility from consumption of the numeraire is now equal to the utility the agent gets from both sin goods when their true health damage is well accounted for. From these conditions we get the first-best values for consumption of the numeraire and for the taxable and non-taxable sin good that we denote respectively by c^{FB}, x^{FB}, f^{FB}.

1.2.4 The decentralization of the first-best

With full information, the government can observe all consumer choices. It will then be able to decentralize the first-best allocation by taxing both types of sin goods. In order to focus on the paternalistic role of the government, the proceedings from taxation will be returned to the individual through a lump sum transfer that we denote by a. The individual problem is to maximize his perceived utility in (1.1), subject now to the following budget constraint:

$$w = c + (1 + \theta_x)x + (1 + \theta_f)f - a, \tag{1.5}$$

where θ_x and θ_f are the tax rates for the observable and non-observable sin good respectively.

FOCs yield:

$$u'(c) = \frac{v_x - \beta h_x}{1 + \theta_x} = \frac{v_f - \beta h_f}{1 + \theta_f}. \tag{1.6}$$

Comparing the first order conditions of the first-best problem in (1.4) with the ones from the decentralized problem above leads to the following result:

Lemma 1.2.1 *The first-best allocation can be decentralized by imposing a tax on each type of sin good while returning the proceedings from taxation through a lump sum transfer. The optimal expressions for the taxes on the observable and non-observable sin goods are given by*

$$\theta_x^{FB} = \frac{1-\beta}{u'(c^{FB})} h_x(x^{FB}, f^{FB}) > 0, \tag{1.7}$$

$$\theta_f^{FB} = \frac{1-\beta}{u'(c^{FB})} h_f(x^{FB}, f^{FB}) > 0. \tag{1.8}$$

The expressions above are the Pigouvian taxes that restore the first-best allocation and that will be the benchmark for second-best analysis. They are proportional to the marginal health damage they cause and to its degree of misperception, while being inversely proportional to the marginal utility from consumption of the numeraire.

1.3 The second-best problem

In general, neither the level of physical (in)activity exerted by individuals nor the level of consumption of drugs that are forbidden by the law are easily observable. Likewise, it is not easy for the government to observe the health status of individuals. In this section we consider the situation where the government cannot make use of any instrument to target neither the health status of the individuals nor the consumption of non-taxable sin goods. The available tools for the government are a tax on the observable sin good and a lump sum transfer, while taxation on consumption of the numeraire is normalized to zero. In this setting, given the restriction faced by the government, the first-best allocation is no longer possible to decentralize. The optimal formula for the tax on the observable sin good will depend now on how the fiscal instruments that the government has at its disposal affect the level of consumption of the non-observable sin good chosen by the individual.

1.3.1 The individual problem

The individual will maximize his perceived utility in (1.1) subject to the budget constraint that results from the impossibility of the government to tax f:

$$w = c + (1+\theta_x)x + f - a.$$

FOCs yield:

$$u'(c) = \frac{v_x - \beta h_x}{1+\theta_x} = v_f - \beta h_f.$$

The first order conditions above together with the budget constraint will determine the Marshallian demand functions for consumption of the numeraire and for the taxable and non-taxable sin goods. They will depend on the choice variables of the government, so that: $c^* \equiv c(\theta_x, a)$, $x^* \equiv x(\theta_x, a)$, $f^* \equiv f(\theta_x, a)$. Making use of the Slutsky equation, we have that for the compensated demands $\tilde{x}$ and $\tilde{f}$, it holds that:[7]

$$\begin{aligned}
\frac{\partial \tilde{x}}{\partial \theta_x} &= \frac{u'(c^*)[v_{ff} - \beta h_{ff} + u''(c^*)]}{Q} < 0, \\
\frac{\partial \tilde{f}}{\partial \theta_x} &= \frac{-u'(c^*)[v_{xf} - \beta h_{xf} + (1+\theta_x)u''(c^*)]}{Q} \lesseqqgtr 0;
\end{aligned}$$

with $Q = (v_{xx} - \beta h_{xx})(v_{ff} - \beta h_{ff}) - (v_{xf} - \beta h_{xf})^2 +$
$u''(c^*)[v_{xx} - \beta h_{xx} + (1+\theta_x)^2(v_{ff} - \beta h_{ff}) - 2(1+\theta_x)(v_{xf} - \beta h_{xf})]$.

The derivation of the formulas for the comparative statics are provided in the Appendix 1.7.1. In order to have a clear interpretation of the effect of the sin tax on consumption of the sin goods, we make the assumption that the direct second derivatives associated to the utility from sin good consumption are of greater magnitude than the cross derivative terms, so that $Q > 0$.[8] While a higher sin tax will naturally reduce consumption of the taxable sin good, it will additionally modify the consumption of the non-taxable one. Its direction depends on how x and f interact in the utility of the individual. If both types of sin goods are economic substitutes, then consumption of the non-taxable sin

[7] Formally: $\frac{\partial \tilde{x}}{\partial \theta_x} = \frac{\partial x^*}{\partial \theta_x} + x^* \frac{\partial x^*}{\partial a}$; $\frac{\partial \tilde{f}}{\partial \theta_x} = \frac{\partial f^*}{\partial \theta_x} + x^* \frac{\partial f^*}{\partial a}$.

[8] More precisely, we assume that: $|v_{xx} - \beta h_{xx}| > v_{xf} - \beta h_{xf}$ and $|v_{ff} - \beta h_{ff}| > v_{xf} - \beta h_{xf}$.

good will increase. A higher sin tax will only decrease consumption of the non-taxable sin good when the degree of complementarity is sufficiently high.[9] The magnitude of these responses will depend on the concavity of the particular functions that describe the utility of the agent.

1.3.2 The government problem

The individual suffers from misperception of the harm caused by sin good consumption. Denoting the perceived indirect utility function of the individual as $V(\theta_x, a)$ and making use of Roy's identity we have that:

$$\frac{\partial V(\theta_x, a)}{\partial \theta_x} = -u'(c^*)x^*; \ \frac{\partial V(\theta_x, a)}{\partial a} = u'(c^*).$$

The government maximizes social welfare, that in our setting amounts to maximize the true indirect utility of the individual, subject to its resource constraint. In what follows, we assume that the objective function of the government is well behaved, so as to have interior solutions. The Lagrangian associated with the government problem is:

$$\mathcal{L}_G = V(\theta_x, a) - (1-\beta)h(x^*, f^*) + \mu \cdot (\theta_x x^* - a).$$

The first term on the objective of the government is the indirect utility function perceived by the individual, while the second term expresses the correction needed to restore the first-best allocation. The third term takes into account the resource constraint faced by the government, where μ is its associated Lagrange multiplier. Solving for the optimal value of the sin tax we get the following formula:

$$\theta_x^* = \frac{1-\beta}{\mu} \left[h_x(x^*, f^*) + h_f(x^*, f^*)\left(\frac{\partial \tilde{f}/\partial \theta_x}{\partial \tilde{x}/\partial \theta_x}\right)\right]. \tag{1.9}$$

The derivation of the optimal sin tax rule above is provided in the Appendix 1.7.2. The expression for θ_x^* now depends on how the taxable and non-taxable sin good interact in the utility function of the individual. The second-best sin tax can be interpreted as having two terms. The first term takes into account the *direct* effect that an additional

[9]In particular, from the comparative statics, this will only be the case when it holds that $v_{xf} - \beta h_{xf} > (1+\theta_x)|u''(c^*)|$.

unit of consumption of the taxable sin good has on health status, while the second term captures the *indirect* effect it has through the variation on the level of consumption of the non-taxable sin good. The *direct* effect has the same expression as in the first-best sin tax in (1.7), while now this new *indirect* effect accounts for the fact that if there is some interaction between the taxable and non-taxable sin good in the determination of the utility of the agent, the sin tax rule has to be modified accordingly. The relevance of this *indirect* effect is captured by the variation on the compensated demand for the taxable and non-taxable sin good due to a marginal increase in the sin tax. This is weighted by the amount of the marginal impact that an additional unit of consumption of the non-taxable sin good has on health status that is misperceived by the agent in terms of consumption of the numeraire. From the comparative statics in the Appendix 1.7.1, we have that the expression that captures the interaction between the taxable and non-taxable sin good in the optimal sin tax is as follows:

$$\frac{\partial \tilde{f}/\partial \theta_x}{\partial \tilde{x}/\partial \theta_x} = \frac{v_{xf} - \beta h_{xf} + (1+\theta_x)u''(c^*)}{-v_{ff} + \beta h_{ff} - u''(c^*)}. \tag{1.10}$$

The denominator above is always positive and it measures how fast it increases the marginal utility perceived by the agent from an additional unit of consumption of the non-taxable sin good, without taking into account the additional effect from the complementarity or substitutability with the taxable sin good. The direction in which the sin tax has to be modified is determined by the numerator. Its first two elements reflect the interaction between the taxable and the non-taxable sin good in the utility of the individual. Their sign will depend on whether the taxable and non-taxable sin good are economic complements or substitutes. The last term of the numerator is always negative and it depends on the price of the taxable sin good and on the concavity of the utility from consumption of the numeraire. The terms related to the concavity of the utility from consumption of the numeraire good capture the fact that introducing a variation on the price of the observable sin good will distort as well the trade-off between sin good consumption and consumption of the numeraire good. With a quasilinear utility function or if the non-taxable sin good would only have a utility cost, these terms related to the concavity of the utility from consumption of the numeraire good vanish.[10]

The *indirect* effect is the consequence that in this second-best setting, consumption of

[10]When considering a quasilinear utility or when the non-taxable sin good has no monetary cost, the expression in (1.10) depends only on how x and f interact in the utility function: $\frac{\partial \tilde{f}/\partial \theta_x}{\partial \tilde{x}/\partial \theta_x} = \frac{v_{xf} - \beta h_{xf}}{-v_{ff} + \beta h_{ff}}$.

the non-taxable sin good cannot be taxed anymore. The level of consumption of the non-taxable sin good can be however affected through the sin good that the government can effectively tax. The results related to the government problem can be summarized in the following proposition:

Proposition 1.3.1 *When the government cannot observe neither the health status nor the level of consumption of the non-taxable sin good, the optimal formula for the sin tax is given by the expression in* (1.9)*. The second-best sin tax will:*

1. *increase with the degree of misperception of the health damage caused by sin good consumption.*

2. *increase (decrease) with the degree of complementarity (substitutability) between the taxable and the non-taxable sin good.*

How the sin tax affects the level of consumption of the non-taxable sin good will depend on the degree of concavity of the utility from consumption of sin goods, as well as on the particular form of the harm function. When the agent decides the level of consumption of the non-taxable sin good, he takes into account the direct utility from consumption as well as his perceived damage on health status, that are both affected by the level of consumption of the taxable sin good. The fact that the health damage caused by consumption of both types of sin goods is mispercieved, together with the restriction on the observability of consumption of the non-taxable sin good, implies that the first-best allocation cannot be decentralized anymore. From the comparison between the expression of the sin tax that decentralizes the first-best allocation in (1.7) with the second-best sin tax expression in (1.9) we have the following result:

Proposition 1.3.2 *When the government cannot observe the level of consumption of the non-taxable sin good, the optimal sin tax must be adjusted to take into account the interaction between the taxable and non-taxable sin goods in the utility of the agent. In particular, when the taxable and non-taxable sin goods are substitutes (complements), the sin tax will be scaled down (up) with respect to the sin tax that decentralize the first-best allocation in* (1.7) *by the second term in* (1.9)*, as a response to the impossibility to target the non-taxable sin good directly.*

The results above illustrate the main contribution of this paper: once the government cannot tax the non-observable sin good anymore, the second-best sin tax has to be modified in order to account for the complementarity or substitutability between taxable and non-taxable sin goods. This distinction is caused by the interaction of taxable and non-taxable sin goods in the utility of the agent. This interaction happens at two levels: the joy that agents get from the joint consumption of both types of sin goods and the health damage they jointly cause. Whether an individual will purchase more or less of one sin good when there is an increase in the tax of the other sin good will depend on both the joy and the perceived health damage from their joint consumption.

The restriction on the observability of the non-taxable sin good departs from one of the assumptions behind the additivity property in Sandmo (1975). The "internality" generated by the non-observable sin good is now mitigated through the tax on the sin good the government can observe. A tax on this sin good will directly reduce its consumption, while indirectly modify the consumption of the non-taxable sin good. The optimal sin tax expression in this second-best setting will only coincide with the one of the first-best when the numerator of (1.10) is equal to zero. Its first two terms vanish when the utility the agent gets from consuming one type of sin good is independent of the level of consumption of the other type of sin good, while the last term arises as long as utility from consumption of the numeraire is non-linear and the non-taxable sin good has a monetary cost. In other words, the optimal sin tax will differ in our second-best setting from the first-best one as long as distorting the level of consumption of the observable sin good entails a variation on the consumption of the non-observable sin good.

1.4 The heterogeneous case

In the previous sections, we have studied a situation where identical individuals suffered from misperception of the health damage caused by two types of sin goods that differed in their observability by the government. This simple setting allowed us to focus on the corrective role of the sin tax and to highlight how a restriction on the observability or taxability of some sin goods affected the design of the optimal sin tax. In reality, individuals differ in several characteristics, such as their level of education and their wealth. Precisely, one of the relevant critics towards the use of sin taxes, besides the libertarian free will argument, is that they are deem to be regressive. As mentioned earlier, the perception of the harm caused by sin good consumption is also not the same across the population. This section incorporates redistributive considerations to the previous analysis by allowing agents to differ in their level of wealth and in their perception of the health damage caused by sin good consumption.

1.4.1 The *laissez-faire*

Consider N types of individuals indexed by i, in a proportion n_i, where the size of the total population is normalized to one. Agents differ now in their wealth w_i and in their degree of perception of the health damage caused by sin good consumption β_i, with $\beta_i \leq 1$, $\forall i$. Each type of individual has to decide how to allocate its initial wealth between consumption of the numeraire and both types of sin goods in order to maximize its utility U_i:

$$U_i(c_i, x_i, f_i) = u(c_i) + v(x_i, f_i) - \beta_i h(x_i, f_i). \tag{1.11}$$

subject to its budget constraint:

$$w_i = c_i + x_i + f_i. \tag{1.12}$$

FOCs yield:

$$u'(c_i) \; = v_x(x_i, f_i) - \beta_i h_x(x_i, f_i) = v_f(x_i, f_i) - \beta_i h_f(x_i, f_i). \tag{1.13}$$

Each type of agent i will equal his marginal utility from consumption of the numeraire to the utility perceived from consumption of both types of sin goods. Consumption of both x_i and f_i will increase with the degree of misperception of health damage caused by sin good consumption and with their wealth level.

1.4.2 The first-best problem

We consider a paternalistic utilitarian government, who consequently assigns an equal weight to the utility of each type of individual and aims to correct for the misperception that individuals suffer of the harm caused by sin good consumption. The first-best allocation corresponds now to the one where each type of individual takes into account the true damage caused by sin good consumption, *i.e.*, when $\beta_i = 1$, $\forall i$. The Lagrangian associated to the first-best problem for an individual of type i is as follows:

$$\mathcal{L}_i = u(c_i) + v(x_i, f_i) - h(x_i, f_i) + \lambda_i \cdot (w_i - c_i - x_i - f_i).$$

where $\lambda_i > 0$ denotes the Lagrange multiplier associated to the budget constraint of an individual of type i.

FOCs yield:

$$u'(c_i) \ = v_x(x_i, f_i) - h_x(x_i, f_i) \ = v_f(x_i, f_i) - h_f(x_i, f_i) \ = \lambda_i. \tag{1.14}$$

The conditions above determine the first-best allocation for consumption of the numeraire good and for the taxable and non-taxable sin goods when individuals differ in their perception of health damage caused by sin good consumption and in their wealth endowment. The first and second equality in (1.14) tell us that each individual will equate its marginal utility from consumption of the numeraire to the utility they get from both types of sin goods, taking into account the true damage sin good consumption has on their health. With full information, a paternalistic government will correct for the misperception that agents have of the health damage caused by sin good consumption through individualized taxes. The formulas for the taxes for each type of sin goods are the analogous to the expressions in (1.7) and (1.8) for the homogeneous case, but now targeted at the individual level. Once this misperception is corrected, a utilitarian government will put in place individualized transfers to redistribute wealth in a way such that each individual achieves the same bundle of consumption. While these instruments will decentralize the first-best allocation, since individualized taxes and transfers are not realistic, we will consider as a benchmark a situation where the government can observe all the choices made by the agents in the economy but it is restricted to linear taxes. The Lagrangian associated to the government problem when it has full information but is restricted to linear taxes is the following:

$$\mathcal{L}_H = \sum_{i=1}^{N} n_i V_i(\theta_x, \theta_f, a) - \sum_{i=1}^{N} n_i (1 - \beta_i) h(x_i^*, f_i^*) + \mu_H \cdot \Big[\sum_{i=1}^{N} n_i (\theta_x x_i^* + \theta_f f_i^*) - \sum_{i=1}^{N} n_i a \Big].$$

where V_i is the indirect utility function perceived by and individual of type i, while $\mu_H > 0$ is the Lagrange multiplier associated to the resource constraint of the government.

The first term of the Lagrangian is already maximized in the individual problem, so that making use of the envelope theorem we get the following FOCs:

$$\frac{\partial \mathcal{L}_H}{\partial \theta_x} = \sum_{i=1}^{N} n_i \Big\{ -x_i^* u'(c_i^*) - (1-\beta_i)\Big[h_x(x_i^*, f_i^*)\frac{\partial x_i^*}{\partial \theta_x} + h_f(x_i^*, f_i^*)\frac{\partial f_i^*}{\partial \theta_x}\Big] + \mu_H \Big[x_i^* + \theta_x \frac{\partial x_i^*}{\partial \theta_x} + \theta_f \frac{\partial f_i^*}{\partial \theta_x}\Big]\Big\} = 0, \quad (1.15)$$

$$\frac{\partial \mathcal{L}_H}{\partial \theta_f} = \sum_{i=1}^{N} n_i \Big\{ -f_i^* u'(c_i^*) - (1-\beta_i)\Big[h_x(x_i^*, f_i^*)\frac{\partial x_i^*}{\partial \theta_f} + h_f(x_i^*, f_i^*)\frac{\partial f_i^*}{\partial \theta_f}\Big] + \mu_H \Big[f_i^* + \theta_x \frac{\partial x_i^*}{\partial \theta_f} + \theta_f \frac{\partial f_i^*}{\partial \theta_f}\Big]\Big\} = 0, \quad (1.16)$$

$$\frac{\partial \mathcal{L}_H}{\partial a} = \sum_{i=1}^{N} n_i \Big\{ u'(c_i^*) - (1-\beta_i)\Big[h_x(x_i^*, f_i^*)\frac{\partial x_i^*}{\partial a} + h_f(x_i^*, f_i^*)\frac{\partial f_i^*}{\partial a}\Big] + \mu_H \Big[-1 + \theta_x \frac{\partial x_i^*}{\partial a} + \theta_f \frac{\partial f_i^*}{\partial a}\Big]\Big\} = 0. \quad (1.17)$$

Let's now define the following compensated derivatives:[11]

$$\frac{\partial \tilde{x}_i}{\partial \theta_x} = \frac{\partial x_i^*}{\partial \theta_x} + \bar{x}\frac{\partial x_i^*}{\partial a}; \quad \frac{\partial \tilde{x}_i}{\partial \theta_f} = \frac{\partial x_i^*}{\partial \theta_f} + \bar{f}\,\frac{\partial x_i^*}{\partial a}; \quad \frac{\partial \tilde{f}_i}{\partial \theta_x} = \frac{\partial f_i^*}{\partial \theta_x} + \bar{x}\frac{\partial f_i^*}{\partial a}; \quad \frac{\partial \tilde{f}_i}{\partial \theta_f} = \frac{\partial f_i^*}{\partial \theta_f} + \bar{f}\,\frac{\partial f_i^*}{\partial a};$$

where we have used that $\bar{x} = \sum_{i=1}^{N} n_i x_i^*$ and $\bar{f} = \sum_{i=1}^{N} n_i f_i^*$.

[11]Following Cremer et al. (2012), we make use of the concept of average compensation, so that the income the agent receives to compensate the tax increase depends on the average demand for the sin good in question, rather than on its own demand.

To make the expressions more tractable, let's additionally denote the impact of sin taxes on health status in the following way:

$$H_{\theta_{xi}} = h_x(x_i, f_i)\frac{\partial \tilde{x}_i}{\partial \theta_x} + h_f(x_i, f_i)\frac{\partial \tilde{f}_i}{\partial \theta_x};$$
$$H_{\theta_{fi}} = h_x(x_i, f_i)\frac{\partial \tilde{x}_i}{\partial \theta_f} + h_f(x_i, f_i)\frac{\partial \tilde{f}_i}{\partial \theta_f}.$$

Substituting (1.17) into (1.15) and (1.16) and making use of the Cramer's rule we get our benchmark sin taxes for the heterogeneous case:

$$\theta_x^{\mathrm{C}} = \frac{1}{\mu_H A}\Big\{\sum_{i=1}^{N} n_i \frac{\partial \tilde{f}_i}{\partial \theta_f}\Big(\mathrm{cov}[u'(c_i), x_i] + \sum_{i=1}^{N} n_i(1-\beta_i)H_{\theta_{xi}}\Big) - \sum_{i=1}^{N} n_i \frac{\partial \tilde{f}_i}{\partial \theta_x}\Big(\mathrm{cov}[u'(c_i), f_i] + \sum_{i=1}^{N} n_i(1-\beta_i)H_{\theta_{fi}}\Big)\Big\}, \quad (1.18)$$

$$\theta_f^{\mathrm{C}} = \frac{1}{\mu_H A}\Big\{-\sum_{i=1}^{N} n_i \frac{\partial \tilde{x}_i}{\partial \theta_f}\Big(\mathrm{cov}[u'(c_i), x_i] + \sum_{i=1}^{N} n_i(1-\beta_i)H_{\theta_{xi}}\Big) + \sum_{i=1}^{N} n_i \frac{\partial \tilde{x}_i}{\partial \theta_x}\Big(\mathrm{cov}[u'(c_i), f_i] + \sum_{i=1}^{N} n_i(1-\beta_i)H_{\theta_{fi}}\Big)\Big\}, \quad (1.19)$$

with $A = \Big(\sum_{i=1}^{N} n_i \frac{\partial \tilde{f}_i}{\partial \theta_f} \sum_{i=1}^{N} n_i \frac{\partial \tilde{x}_i}{\partial \theta_x} - \sum_{i=1}^{N} n_i \frac{\partial \tilde{f}_i}{\partial \theta_x} \sum_{i=1}^{N} n_i \frac{\partial \tilde{x}_i}{\partial \theta_f}\Big)$.[12]

The expressions θ_x^{C} and θ_f^{C} are the optimal sin taxes when the government has full information and it is constrained to use linear taxes and transfers. We then have the following result:

[12] Where we have used the following definitions:

$$\mathrm{cov}[u'(c_i), x_i] = \sum_{i=1}^{N} n_i x_i^* u'(c_i^*) - \sum_{i=1}^{N} n_i x_i^* \sum_{i=1}^{N} n_i u'(c_i^*),$$
$$\mathrm{cov}[u'(c_i), f_i] = \sum_{i=1}^{N} n_i f_i^* u'(c_i^*) - \sum_{i=1}^{N} n_i f_i^* \sum_{i=1}^{N} n_i u'(c_i^*).$$

Lemma 1.4.1 *When agents differ in their wealth and in their perception of the damage caused by sin good consumption, the first-best allocation can only be restored through individualized taxes and transfers. When restricted to linear taxation, the expressions* (1.18) *and* (1.19) *denote the optimal rules for the taxable and non-taxable sin goods that maximize welfare.*

The expressions (1.18) and (1.19) put in evidence that both sin taxes are necessary to achieve the welfare-maximizing allocation when restricted to linear taxes. They have two different components, one that corrects for the misperception of health damage and another that reflects redistributive considerations. The Pigouvian part calls for a positive sin tax to correct for the misperception of health damage. A pattern of substitutability between taxable and non-taxable sin goods would mitigate this effect, while a pattern of complementarity will reinforce it, what is reflected through the second terms in $H_{\theta_{xi}}$ and $H_{\theta_{fi}}$. The redistributive component depends on how the marginal utility from consumption covariates with consumption of the taxable and non-taxable sin goods in question. A positive relationship will call for a lower tax on both types of sin goods. This implies that if poorer individuals consume more sin goods *vis-à-vis* wealthier individuals, sin taxes should be lower. This is because redistribution takes place from those with higher consumption of sin goods to those with lower consumption.

1.4.3 The second-best problem

We now proceed to analyze the case where consumption of the non-taxable sin good is not observable anymore. The optimal formula for the second-best sin tax will still have a Pigouvian and a redistributive component, but now reflecting the fact that the government cannot directly influence the level of consumption of the non-taxable sin good. The objective of the government is as before, but restricted to the fact that now $\theta_f = 0$. The Lagrangian associated with this problem is the following:

$$\mathcal{L}_{HG} = \sum_{i=1}^{N} n_i V(\theta_x, a) - \sum_{i=1}^{N} n_i(1-\beta_i)h(x_i^*, f_i^*) + \mu_{HG} \cdot (\sum_{i=1}^{N} n_i(\theta_x x_i^*) - \sum_{i=1}^{N} n_i a).$$

where μ_{HG} is the Lagrange multiplier for the second-best problem with heterogeneity and where consumption of the non-taxable sin good is not observed by the government.

Solving in the same manner as the previous problem we get the following first order conditions:

$$\frac{\partial \mathcal{L}_{HG}}{\partial \theta_x} = \sum_{i=1}^{N} n_i \Big\{ - x_i^* u'(c_i^*) - (1-\beta_i)\Big[h_x(x_i^*, f_i^*)\frac{\partial x_i^*}{\partial \theta_x} + h_f(x_i^*, f_i^*)\frac{\partial f_i^*}{\partial \theta_x}\Big] + \mu_{HG}\Big[x_i^* + \theta_x \frac{\partial x_i^*}{\partial \theta_x}\Big]\Big\} = 0, \quad (1.20)$$

$$\frac{\partial \mathcal{L}_{HG}}{\partial a} = \sum_{i=1}^{N} n_i \Big\{ u'(c_i^*) - (1-\beta_i)\Big[h_x(x_i^*, f_i^*)\frac{\partial x_i^*}{\partial a} + h_f(x_i^*, f_i^*)\frac{\partial f_i^*}{\partial a}\Big] + \mu_{HG}\Big[- 1 + \theta_x \frac{\partial x_i^*}{\partial a}\Big]\Big\} = 0. \quad (1.21)$$

Substituting (1.21) into (1.20) and solving for θ_x we get the expression for the optimal second-best sin tax when the consumption of the non-taxable sin good is not observed by the government:

$$\theta_x^* = \frac{\sum_{i=1}^{N} n_i \frac{\partial \tilde{x}_i}{\partial \theta_x}(1-\beta_i)\Big[h_x(x_i^*, f_i^*) + h_f(x_i^*, f_i^*)\Big(\frac{\partial \tilde{f}_i/\partial \theta_x}{\partial \tilde{x}_i/\partial \theta_x}\Big)\Big] + \text{cov}[u'(c_i), x_i]}{\mu_{HG} \sum_{i=1}^{N} n_i \frac{\partial \tilde{x}_i}{\partial \theta_x}}. \quad (1.22)$$

The second-best sin tax with heterogeneous agents has also two components. The first term is the Pigouvian component that aims to correct for the overconsumption of sin goods due to the misperception of its health damage. The terms inside the square brackets capture the interaction between the taxable and non-taxable sin good in the determination of health status. While increasing the tax on the taxable sin good reduces its consumption, it may induce a lower or higher level of consumption of the non-taxable sin good, depending on the degree of complementarity or substitutability between them. These terms now take into account that individuals receive an average compensation for the increase in the sin tax. Consequently, agents that have a lower sin good consumption than the average receive an income improvement. This is due to the fact that proceedings from taxation are redistributed through a uniform lump sum transfer. The second term captures the redistributive part of the sin tax and depends only on the covariance between the marginal utility from consumption of the numeraire and consumption of the taxable sin good. Whether this term is positive or negative will depend on how the wealth and the degree of perception of health damage caused by sin good consumption correlates with consumption of the observable sin good. Wealthier individuals have more money to spend on all goods, what necessarily implies a negative relationship between the marginal utility from consumption of the numeraire and sin good consumption. This effect may be however offset if the awareness of health damage caused by sin good consumption is positively correlated with sin good consumption.[13] This two components are weighted by the marginal utility from public funds and the average reaction on the consumption of the taxable sin good for a marginal variation in the tax rate. To have a more clear interpretation regarding the second-best sin tax for the homogeneous case, we can rearrange terms in order to get:

$$\theta_x^* = \sum_{i=1}^{N} n_i \frac{(1-\beta_i)}{\mu_{HG}} \Big[h_x + h_f \Big(\frac{\partial \tilde{f}_i / \partial \theta_x}{\partial \tilde{x}_i / \partial \theta_x} \Big) \Big] + \frac{\operatorname{cov}[u'(c_i), x_i] + \operatorname{cov}\Big[\frac{\partial \tilde{x}_i}{\partial \theta_x}, \sum_{i=1}^{N} n_i (1-\beta_i) h_x + h_f \Big(\frac{\partial \tilde{f}_i / \partial \theta_x}{\partial \tilde{x}_i / \partial \theta_x} \Big) \Big]}{\mu_{HG} \sum_{i=1}^{N} n_i \frac{\partial \tilde{x}_i}{\partial \theta_x}}. \tag{1.23}$$

From the expression above we see that the first term corresponds to the expression for the Pigouvian component that corrects for the misperception of health damage that we had for the homogeneous case, expressed now on average terms. The second and third terms capture the redistributive concerns of the government. While the second term is the same as before, the third term now depends on the covariance of the sensitivity of demand of the observable sin good to a tax variation with the Pigouvian correction needed to restore the first-best allocation. For the same level of misperception of the harm caused by sin good consumption, a poorer individual will react more to an increase in the price of the observable sin good. If the correlation between wealth and perception of health damage is positive, this third term will likely be positive, as the correction needed to restore the first-best is higher for individuals with lower perception of the harm caused by sin good consumption. If this is true, then both covariance terms call for a lower sin tax to improve redistribution. The magnitude and eventual direction of the optimal sin tax

[13] See Lockwood and Taubinsky (2017) for a comprehensive analysis on how redistributive considerations may amplify or dampen the corrective role of sin taxes.

will then depend on how this trade-off between efficiency and equity concerns is resolved. The findings of this section can be summarized in the following proposition:

Proposition 1.4.2 *When individuals differ in their wealth and in their perception of the health damage caused by sin good consumption and when the government tools are restricted to a linear tax on the taxable sin good and a uniform lump sum transfer, it follows that the optimal tax on the observable sin good:*

(a) *is characterized by* (1.22)*.*

(b) *has a Pigouvian component that takes into account the average indirect effect that increasing the sin tax has on consumption of the non-taxable sin good. This impact will be more relevant:*

  (1) *the larger the misperception of the health damage caused by sin good consumption in the population.*

  (2) *the more convex is the damage from consumption of the non-taxable sin good.*

  (3) *the higher the sensitivity of consumption of the non-taxable sin good to an increase in the sin tax.*

(c) *has a redistributive component that does not depend directly on the level of consumption of the non-observable sin good.*

When the government had full information, the redistributive components of the taxes on both types of sin goods expressed by (1.18) and (1.19) took into account both the covariance of the taxable and non-taxable sin good with the marginal utility from consumption of the numeraire. With more limited tools, redistribution only takes place now regarding the consumption of the sin good that the government can tax. So that the restriction faced by the government regarding the observability on consumption of the non-taxable sin good diminishes the scope for redistribution. The second item of the previous proposition reminds us that when designing taxes on sin goods, we must take into account how this will impact on the consumption of other sin goods that the government may not be able to tax. The direction of this adjustment will depend on whether on average, these taxable and non-taxable sin good are economic complements or substitutes.

1.5 Discussion

In this section we discuss the policy implications of the main results of this paper as well as some remarks regarding the assumptions of the model.

1.5.1 Complementarity or substitutability between sin goods

The optimal sin tax formulas we have derived depend crucially on whether both types of sin goods are economic complements or substitutes. This pattern of consumption depends on the particular sin goods into consideration. In general, taxable and non-taxable sin goods are more likely to exhibit a pattern of complementarity in the harm function, *i.e.*, $h_{xf}(x_i, f_i) > 0$. This implies that the damage caused by an additional unit of consumption of one type of sin good is increasing in the level of consumption of the other type of sin good. The pattern regarding the joy from joint consumption of sin goods captured through $v_{xf}(x_i, f_i)$ is however not so clear. Whether an increase in the tax on the observable sin good will reduce or increase the consumption of the non-observable sin good, that is, whether they are economic complements or substitutes, will depend on how the consumer accounts for these two interactions. To illustrate the policy implications of our analysis we will restrict the attention to physical inactivity and illicit drugs as non-taxable sin goods, while alcohol, tobacco, fat and sugar will account for the taxable sin goods.

Physical inactivity

Several studies in the medicine literature suggest that physical activity has higher health gains for individuals that are more obese, smokers or alcohol consumers. In other words, the marginal benefits from doing physical activity are higher when we depart from a higher level of consumption of those sin goods. Engaging in moderate physical activity has, for instance, a greater effect in reducing the risk of type 2 diabetes among individuals with high body mass index (Manson et al. (1992); Gregg et al. (2003)). In the same line, the study of Ross et al. (2000) finds that performing physical activity can have health improvements due to a better fat distribution, even if there is no weight loss. A better fat distribution can possibly eliminate the excess risk of type 2 diabetes and cardiovascular diseases, (Appleton et al. (2013)). More in general, the study of Gordon-Larsen et al.

(2009) concludes that individuals with higher baseline weight benefit more from physical activity. Moreover, a recent study from Bell et al. (2015) point out that due to previous inaccurate measurement of the amount of physical activity actually performed, the actual health gains for the obese are probably higher than what has already been documented. In our model, this translates into a positive interaction between x and f on the harm function, that is, a pattern of complementarity on the health damage they cause.

Considering alcohol consumption, a recent study based on the UK population by Perreault et al. (2016) finds that performing two hours and a half of physical activity per week can offset all-cause mortality risks, as well as some of the cancers associated with the consumption of this sin good.[14] This study suggest then, that the marginal gains on physical activity are increasing with alcohol consumption, as they mitigate the health damage it causes. Regarding tobacco smokers, the analysis of Ferrucci et al. (1999) concludes that a moderate level of physical activity can reduce their disable life expectancy at old age, while only vigorous physical activity will increase significantly their life expectancy. In the same line, the study of Garcia-Aymerich et al. (2007) emphasizes the fact that physical activity can counteract the negative effects on health status of smoking. In particular, they find that moderate to vigorous physical activity reduces the lung function decline due to smoking, as well as the risk of chronic obstructive pulmonary disease.

According to these studies, smoking and physical inactivity may also be considered as complements in the determination of health damage. This implies that agents will trade off consumption between physical inactivity and tobacco, alcohol, fat or sugar, to attain a certain level of health status. On the other hand, regarding the utility from the joy of the joint consumption of physical inactivity and the taxable sin goods we are considering, the pattern is not so clear. It seems reasonable to think that a higher level of consumption of tobacco, fat or sugar makes more costly to engage in physical activity, making physical inactivity more attractive. However, alcohol consumption is sometimes associated with sport practices, which would imply the opposite relationship. The review of Lisha and Sussman (2010) shows that this is generally the case among high-school and college students. In sum, the complementarity pattern in the determination of health damage between physical inactivity and the taxable sin goods considered here calls for a lower sin

[14] These results hold true for women and men that consume less than 20 and 28 US drinks per week respectively.

tax, while the joy from the joint utility from its consumption may mitigate or reinforce this correction.

Illicit drugs: Cannabis

Regarding illicit drugs, it seems again reasonable to believe that the marginal damage from its consumption increases with the level of consumption of the taxable sin goods considered here, while the utility from its joint consumption may vary depending on the particular drug into consideration. We will restrict the attention to cannabis, as it is by far the most consumed illicit drug worldwide.[15]

The first study trying to identify the pattern of consumption between alcohol and marijuana was carried out by DiNardo and Lemieux (2001), who found that increasing the legal age for drinking has slightly increased the prevalence of marijuana consumption. The same pattern of substitutability was confirmed by Chaloupka and Laixuthai (1997) and more recently by Crost and Guerrero (2012). On the other hand, Williams et al. (2004), find a pattern of complementarity between alcohol and marijuana consumption among college students in the US. Using data from National Longitudinal Survey of Youth, Pacula (1998) arrives to the same conclusion. More recently, the study of Wen et al. (2015) suggests a pattern of complementarity between marijuana consumption and binge drinking on jurisdictions that have passed medical marijuana laws in the US. In general, whether cannabis and alcohol are economic complements or substitutes seems to depend on several characteristics of the population, such as age, education, ethnic groups, but also on the desired effect that the consumer seeks consuming alcohol and/or marijuana. Regarding the pattern of consumption between tobacco and cannabis, there is less empirical evidence, also with mixed results. While some studies document a pattern of complementarity, (e.g Farrelly et al. (2001); Cameron and Williams (2001)), a recent study by Choi et al. (2018) finds that enacting medical marijuana laws reduces tobacco consumption. It has also been documented an increase in appetite when consuming cannabis, in particular for sweet food, (Cota et al. (2003); Kirkham (2009)). This would imply a pattern of complementarity on the joy from consumption between marijuana and sugar and fast food. On the other hand, consumption of marijuana has been associated with lower prevalence of

[15] According to the recent study of the WHO, Organization (2016), at least 2.5% of the world population consume cannabis.

obesity in the US adult population and with smaller waist circumferences, (Le Strat and Le Foll (2011); Penner et al. (2013)).

The studies mentioned above try to estimate how demand for marijuana varies with the consumption of alcohol, tobacco, fat or sugar. In our formal analysis this pattern of consumption is decomposed in two parts: the joy from its joint consumption and their perceived damage on health status. If they were to be economic complements, it would mean that the joy from joint consumption must be quite strong, or eventually that the perception of the complementarity in the health damage they cause is too low. More empirical analysis is needed to understand in which direction the optimal taxes on alcohol or tobacco should be adjusted to improve welfare when considering marijuana consumption as the non-taxable sin good. In this sense, the recent experiences of marijuana legalization in some jurisdictions may shed more light on these patterns.

1.5.2 Final remarks

The non-taxable sin good has been introduced in the model as a good that the individual can obtain by paying a monetary cost and that it brings joy and health damage from its consumption. While this interpretation is straightforward for the case of illicit drugs, it is worth making some remarks for the case of physical inactivity. In particular, the joy from physical inactivity can be thought of as the utility an agent gets from not facing a utility cost associated to engage in physical activity. This utility cost will depend, for instance, on the level of consumption of tobacco, alcohol, fat or sugar. On the other hand, the monetary cost may account for some fee from joining a gym or for instance, for the cost of buying some sport gears. We are assuming that this monetary costs cannot be observed by the government. Removing this monetary cost from our formal analysis translates into the second-best sin tax formulas not having the term related to the concavity of the marginal utility from consumption of the numeraire good. Indeed, this term captures the fact that once the relative price of the observable sin good is affected by the introduction of a sin tax, the trade-offs between consumption of the numeraire good and sin good consumption are also distorted.

Our formal analysis has modeled the utility from consumption of the numeraire as separable from the joy from sin good consumption. If consumption of a numeraire good would also interact with the utility from the joy of sin good consumption, the government will find optimal to tax as well the numeraire good in order to distort the level of consumption of the non-observable sin good.

Throughout this work we have assumed that the individual understands perfectly how his health status is determined as a result of the choices made about consumption of taxable and non-taxable sin goods. Our model could be extended to a setting where the agents do not understand perfectly how health status is determined by, for instance, introducing different factors for the health damage perception of each type of sin good, as well for their interaction. If agents have on average a lower perception of the health damage caused by the non-taxable sin good, the *indirect* effect will be stronger and welfare gains of taking it into account in the design of sin taxes would be more significant.

When funding of the health care system is public or collective, sin good consumption may also cause an externality through an increase in health care costs. Introducing this feature into the model will amplify the effects described in our analysis, as long as health care costs depend on the health damage caused by both types of sin goods. In this sense, a tax on the observable sin good must take into account the *indirect* effect it causes on health damage through a variation on the consumption of the non-taxable sin good and its consequent impact on health care costs. Another concern regarding sin good consumption, is the effect that an individual may cause on future generations or on to their peers. The relevance and underlying mechanisms of intergenerational transmission of noncommunicable diseases has been formalized by Goulao and Pérez-Barahona (2014). Incorporating this dynamic feature into our analysis would be an interesting avenue for future research. Finally, another appealing extension of the analysis considered in this paper would be to endogenize labor supply, so that engaging in physical activity entails a time cost. More in general, both types of sin goods could be thought of as having a productivity cost.

1.6 Conclusion

We have begun by studying the case where identical individuals suffer from misperception of the health damage caused by consumption of two types sin goods. These sin goods differ in their observability by the government. Tobacco, alcohol, fat or sugar, are examples of sin goods that we consider the government can effectively tax, while physical inactivity or illicit drugs are on the other hand examples of non-taxable sin goods. In the full information case, the government can restore the first-best allocation by introducing Pigouvian taxes on each type of sin good. Once the government cannot target physical inactivity or illicit drugs directly, the second-best sin tax rule has to be modified accordingly. This simple setting allowed us to understand how the sin tax rule should be adjusted in order to incorporate this restriction faced by the government. The second-best sin tax expression has two parts, one that corrects for the overconsumption of the taxable sin good and another one that takes into account the *indirect* effect sin good consumption has on health damage through the level of consumption of the non-taxable sin good it induces. The second-best sin tax should then be scaled up or down, depending on whether the taxable and non-taxable sin goods are economic complements or substitutes. Our contribution makes emphasis on the fact that not taking this effect into account will set the wrong incentives for consumption of the non-taxable sin good and therefore lead to an undesired health outcome, with its consequent welfare loss. Policy makers should then bear in mind that taxing sin goods may have undesired effects on the consumption of non-observable sin goods and therefore on health status. Welfare implications of ignoring this indirect effect will be more relevant, the more sensitive is the variation in the consumption of the non-observable sin good to an increase in the tax on the observable sin good and the more significant is the marginal health damage it causes. In order to incorporate redistributive considerations, the analysis is extended to a situation where individuals differ in wealth and in their degree of perception of the health damage caused by sin good consumption. When consumption of the non-taxable sin good is not observed anymore, the government not only looses an instrument to correct for the overconsumption of the non-taxable sin good, but also a redistributive tool. Considering a utilitarian planner, the Pigouvian correction is now on average terms, so that the optimal sin tax considers the average indirect effect on consumption of the non-observable sin good induced by the sin tax. While in general taxable and non-taxable sin goods plausibly exhibit a pattern of complementarity in the health damage they cause, what calls for a lower sin tax, it may well be that the joy from its joint consumption has a complementarity pattern, which may mitigate this adjustment, or even make a higher sin tax desirable. Empirical evidence is however not conclusive on this pattern, when considering physical inactivity or cannabis as non-taxable sin goods and tobacco, alcohol, fat or sugar as examples of taxable sin goods. The direction in which the tax on the observable sin good ought to be adjusted will depend on the particular sin goods into consideration and more in general, on the characteristics of the population. Though our analysis has focused on the extreme situation where the government cannot observe consumption of some sin goods, it can also be applied to situations where taxation of some sin goods is restricted, for instance, due to some political reasons.

1.7 Appendix

1.7.1 Comparative statics

The Lagrangian associated with the problem faced by the individual when consumption of the non-taxable sin good is unobserved by the government is the following:

$$\mathcal{L} = u(c) + v(x, f) - \beta h(x, f) + \lambda \cdot (w + a - (1 + \theta_x)x - f).$$

FOCs yield:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c} &= u'(c) - \lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial x} &= v_x - \beta h_x - \lambda(1 + \theta_x) = 0, \\
\frac{\partial \mathcal{L}}{\partial f} &= v_f - \beta h_f - \lambda = 0.
\end{aligned}$$

The system of equations above together with the budget constraint determine the Marshallian demands for consumption of the numeraire and for the taxable and non-taxable sin good, c^*, x^* and f^*. The comparative statics with respect to the fiscal instruments available for the government in our second-best setting are the following:

$$\frac{\partial x^*}{\partial a} = \frac{u''(c^*)[(1+\theta_x)(v_{ff}-\beta h_{ff})-v_{xf}+\beta h_{xf}]}{Q} > 0,$$

$$\frac{\partial f^*}{\partial a} = \frac{u''(c^*)[v_{xx}-\beta h_{xx}+(1+\theta_x)(-v_{xf}+\beta h_{xf})]}{Q} \gtreqless 0,$$

$$\frac{\partial x^*}{\partial \theta_x} = \frac{(u'(c^*)-u''(c^*)(1+\theta_x)x)(v_{ff}-\beta h_{ff})+u''(c^*)(u'(c^*)+x(v_{xf}-\beta h_{xf}))}{Q} < 0,$$

$$\begin{aligned}\frac{\partial f^*}{\partial \theta_x} =& \frac{1}{Q} \times \Big\{u''(c^*)(u'(c^*)(1+\theta_x)+x(v_{xx}-\beta h_{xx})) \\ &+ (u''(c^*)x(1+\theta_x)-u'(c^*))(v_{xf}-\beta h_{xf})\Big\} \gtreqless 0,\end{aligned}$$

with $Q = (v_{xx}-\beta h_{xx})(v_{ff}-\beta h_{ff})-(v_{xf}-\beta h_{xf})^2 +$
$u''(c^*)[v_{xx}-\beta h_{xx}+(1+\theta_x)^2(v_{ff}-\beta h_{ff})-2(1+\theta_x)(v_{xf}-\beta h_{xf})]$.

Where we recall that in order to determine the signs of the expressions above we made the assumption that the direct second derivatives associated to the utility from sin good consumption are of greater magnitude than the cross derivative terms. From the expressions above we can see how the optimal choices of consumption of the taxable and the non-taxable sin good vary in response to a marginal increase in the fiscal tools used by the government. They can be decomposed into an income and a substitution effect. These effects will depend particularly on the degree of concavity of the functions defined to represent the utility of the agent. Making use of the Slutsky equation we can now see the effect that marginal change on the sin tax has on the compensated demands, $\tilde{x}$ and $\tilde{f}$:

$$\begin{aligned}\frac{\partial \tilde{x}}{\partial \theta_x} =& \frac{u'(c^*)[v_{ff}-\beta h_{ff}+u''(c^*)]}{Q} < 0, \\ \frac{\partial \tilde{f}}{\partial \theta_x} =& \frac{-u'(c^*)[v_{xf}-\beta h_{xf}+(1+\theta_x)u''(c^*)]}{Q} \lesseqgtr 0.\end{aligned}$$

As discussed in section **3.1**, a marginal increase in the sin tax reduces consumption of the taxable sin good, while it may reduce or increase consumption of the non-taxable sin good. The comparative statics above are used in order to simplify for the optimal sin tax formula in (1.10).

1.7.2 Second-best optimal tax formulas

Homogeneous agents

The Lagrangian associated to the government problem is the following:

$$\mathcal{L}_G = V(\theta_x, a) - (1-\beta)h(x^*, f^*) + \mu \cdot (\theta_x x^* - a),$$

where μ is the lagrangian multiplier associated to the resource constraint of the government.

The first term of the Lagrangian is already maximized in the individual problem, so that using the envelope theorem we get the following FOCs:

$$\frac{\partial \mathcal{L}_G}{\partial \theta_x} = -\, x^* u'(c^*) - (1-\beta)\Big[h_x(x^*, f^*)\frac{\partial x^*}{\partial \theta_x} + h_f(x^*, f^*)\frac{\partial f^*}{\partial \theta_x}\Big] + \mu\Big[x^* + \theta_x \frac{\partial x^*}{\partial \theta_x}\Big] = 0, \tag{1.24}$$

$$\frac{\partial \mathcal{L}_G}{\partial a} = u'(c^*) - (1-\beta)\Big[h_x(x^*, f^*)\frac{\partial x^*}{\partial a} + h_f(x^*, f^*)\frac{\partial f^*}{\partial a}\Big] + \mu\Big[-1 + \theta_x \frac{\partial x^*}{\partial a}\Big] = 0. \tag{1.25}$$

Substituting (1.25) in (1.24) we have that:

$$\frac{\partial \mathcal{L}_G}{\partial \theta_x} + x^*\frac{\partial \mathcal{L}_G}{\partial a} = -(1-\beta)\Big[h_x(x^*, f^*)(\frac{\partial x^*}{\partial \theta_x} + x^*\frac{\partial x^*}{\partial a}) + h_f(x^*, f^*)(\frac{\partial f^*}{\partial \theta_x} + x^*\frac{\partial f^*}{\partial a})\Big] + \mu\Big[\theta_x(\frac{\partial x^*}{\partial \theta_x} + x^*\frac{\partial x^*}{\partial a})\Big] = 0.$$

Solving for θ_x and making use of the Slutsky equation we get the optimal sin tax formula in (1.9):

$$\theta_x^* = \frac{1-\beta}{\mu}\,\Big[h_x(x^*, f^*) + h_f(x^*, f^*)\Big(\frac{\partial \tilde{f}/\partial \theta_x}{\partial \tilde{x}/\partial \theta_x}\Big)\Big].$$

Chapter 2

Legalizing harmful drugs: Government participation and optimal policies *

Luis Rodrigo Arnabal

Abstract

We are currently witnessing a shift in the approach to combat the traffic and consumption of illegal harmful drugs, being marijuana legalization a prominent example. In this paper, we study the optimal policies related to the legalization of marijuana, in a setting where consumers differ in their utility from consumption of the psychoactive component of cannabis, THC, and suffer from misperception on the health damage it causes. We analyze this problem through a vertical differentiation model, where a public and a black market firm compete in prices and quality (THC content). A paternalistic government would like to correct for the misperceived health damage caused by marijuana consumption, as well as to reduce the size of the black market. We show that it is the undesirability of black market profits, rather than the health damage misperception, what makes the first-best allocation impossible to decentralize. We find two possible equilibria, in which the public firm serves either the consumers with the highest or the lowest willingness to pay for quality. Allowing the public firm to move first, à la Stackelberg, does not provide it an advantage and social welfare remains second-best.

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2.1 Introduction

There is an increasing consensus that the so called "War on Drugs" has failed. As a consequence, policymakers are exploring other alternatives to discourage consumption of illicit drugs.[1] In the last years, the idea that marijuana, the most consumed illicit substance in the world, should be legalized has gained strength and very recently some jurisdictions have started to put this idea into practice.[2] The aim of this paper is to address the question of how to optimally regulate the market for marijuana.

The arguments for marijuana legalization are many, standing out the reduction on enforcement costs and the increase in tax revenue. Perhaps more importantly, legalization is to reduce the contact of marijuana consumers with the black market. Removing revenues from the black market will reduce the strength of the mafias and their criminal activity, improving thus welfare for the whole population. This last point was very relevant for the case of Uruguay, where marijuana legalization was initially framed as a policy to combat crime, rather than a public health or consumers rights issue. The recent experiences in Canada, Uruguay and in some US states have shown however that marijuana legalization per se does not remove the black market and it may even fuel its demand.[3] Differences in prices, the amount of psychoactive component Δ9-tetrahydrocannabinol (THC) and the mistrust in the legal system are part of the explanation.

Legalization has some undesired consequences, as once the stigma of illegality is removed and marijuana becomes more accessible, its uptake is expected to increase, as documented for instance by Jacobi and Sovinsky (2016). Marijuana consumption raises public health issues that the government cannot ignore.[4] The damage caused by this harmful drug is in general not fully acknowledged by consumers. According to recent surveys carried out by Johnston et al. (2019), the perceived risk of using marijuana among US adolescents

[1] See Csete et al. (2016) for an analysis on the negative public health consequences of drug prohibition and a set of policy recommendations.

[2] According to the World Drug Report of 2019, around 188 million people have used marijuana at least once in 2017.

[3] According to the IRCCA, the public agency that regulates production and sales of marijuana in Uruguay, only 30% of those who consumed marijuana in 2018 did so through the legal channel.

[4] See Volkow et al. (2014) for a survey on the health damage caused by marijuana consumption.

has been declining since the mid 2000s. While for most consumers the amount of THC is perceived as the main measure of quality for recreational marijuana, this psychoactive component is also associated with potential brain damage.[5] Evidence from the medical literature suggest that health costs associated to marijuana consumption are convex in THC levels. For instance, a recent study conducted by Rigucci et al. (2016) shows that high-potency marijuana causes more severe damage than regular cannabis. It has also been documented that daily use of high potency marijuana accelerates the onset of psychosis, (*e.g.* Di Forti et al. (2014) and Di Forti et al. (2019)). More recently, the study of Arterberry et al. (2019) suggests that higher potency cannabis leads to an increase in the risk of progression from cannabis initiation to cannabis use disorder symptom onset.

This paper studies the optimal policies to be undertaken by an utilitarian paternalistic government upon the decision to legalize marijuana, in an economy characterized by the presence of a black market firm and where consumers differ in their taste for THC, while suffering from misperception on the health damage caused by its consumption.[6] To account for the fact that the presence of a black market is detrimental for society, a negative value is assigned for the profits generated in the black market. For a paternalistic government the motivation to intervene in the marijuana market is twofold: reduce black market activity and correct for the overconsumption of marijuana that arises due to health damage misperception suffered by consumers.

There has been several studies conjecturing what would be the consequences of marijuana legalization, for instance, Caulkins et al. (2012) and more in general, for illicit drugs, Galenianos et al. (2012). With the recent trend of legalization, the debate is now shifting towards how to regulate the market for cannabis, (*e.g.*, Pacula et al. (2014), Caulkins and Kilmer (2016)). This paper aims to contribute to this discussion by formally analyzing the question of what are the optimal policies for regulating the marijuana market. The study of Becker et al. (2006) formally describes how legalization of illicit drugs may achieve a better outcome than prohibition, through the combination of a tax and strong enforcement on underground production. Legalization brings the possibility for the government to target the quantity of illicit drugs through taxes, increasing revenues and reducing enforcement costs, thus improving welfare.

[5] Analyzing data from legal sales in Washington, the study of Smart et al. (2017) documents that the amount of THC is associated with higher prices per gram.

[6] We abstract from the possibility that marijuana consumption results in addiction or that it operates as a gateway drug.

In practice, the regulation for the market of marijuana has followed different paths. Marijuana is sold by regulated dispensaries in the US, by the government in Uruguay and by both the government and private firms in Canada.[7] The Uruguayan government has initially fixed the amount of THC per gram of marijuana at 2% and later increased it up to 9%, while assuring a minimum of 3% of Cannabidiol (CBD), the other principal component in cannabis, more common in medical marijuana. On the other hand, there are few restrictions regarding THC content in the US, that may go up to 32%.

Inspired by the Uruguayan case, our framework will consider that marijuana legalization translates into a direct participation of the government in the marijuana market through a public firm that aims to maximize welfare. Consumers differ in their marginal willingness to pay for quality (THC content) and suffer from misperception on the health damage caused by its consumption. In order to address the question of what levels of THC should be available for consumers upon legalization, competition will be described through a vertical differentiation model, where the government and the black market compete both in price and quality. We incorporate therefore a new dimension to the discussion related to the optimal qualities to be offered in a legalized market.

This paper relates also to the literature on mixed oligopolies under product differentiation. As it has been well documented, the presence of a public firm may be welfare improving in a market with an oligopolistic structure, (*e.g.*, De Fraja and Delbono (1989), Cremer et al. (1991), Grilo (1994)). We analyze the welfare impact of the participation of a public firm into a vertically differentiated market characterized by the presence of a black market firm, whose profits generate a welfare loss for society and where consumers misperceive the health damage caused by the product, which is increasing in its quality.

We assume that marginal production costs are increasing in quality and that they are the same for the public and for the black market firm. Regarding the timing, we will consider that firms compete first in qualities and then simultaneously in prices. We focus on pure strategies, where the concept of equilibrium is subgame perfect Nash equilibrium. We further assume that the market is fully covered and restrict our attention to situations where both firms are active in the market, and are restricted to offer only one quality.[8] In the last section, we remove the assumption of fixed demand, by allowing for a new set of

[7]Marijuana legalization sometimes includes legal home growing, as well as cannabis social clubs, which are not addressed in this work.

[8]Considering that each firm provides a unique quality allows us to focus on the interactions between the public and black market firm when deciding their optimal prices and qualities.

consumers to enter in the market, who will exclusively buy from the public firm, in order to capture the fact that legalization may increase the overall number of consumers.

We find that as long as black market profits are deemed socially undesirable, the first-best allocation cannot be decentralized. While the presence of a public firm allows to correct for the misperception on the health damage through Pigouvian prices, what would split consumers optimally between the two available qualities, the undesirability of black market profits drives the public firm to reduce its price, in an effort to attract more demand. For the same reason, the public firm distorts its quality to increase its market share, which in turn triggers a variation on the black market quality, in order to make more profits through a higher product differentiation. Assuming that both firms are active in the market, we find that there are two possible equilibrium configurations as a result of the competition between the public and the black market firm, depending on whether the public firm serves the consumers with higher or lower willingness to pay for quality. While they both yield the same level of welfare and profits for the black market firm, the one where the government supplies the higher quality product has the advantage that the public firm makes positive profits and the average health damage caused by marijuana consumption is lower. We also find that adding in the first stage of the game a first-mover advantage to the public firm does not modify the equilibria that result from simultaneous quality choice.

The remainder of the paper is organized as follows. Section 2 describes the economic environment, the normative benchmark and the *laissez-faire* situation. Section 3 studies the competition between a public and a black market firm in prices and qualities. Section 4 extends the analysis by expanding the demand faced by the public firm. Section 5 concludes.

2.2 The Model

Consider a continuum of consumers that purchase at most one unit of marijuana. Consumers differ in the marginal utility they get from the psychoactive component (THC), that we will consider to be the relevant aspect related to the quality of the product.[9]

[9] This assumption captures the idea that consumers prefer to get a “higher bang for the buck", though we acknowledge that there may be a horizontal differentiation pattern as well in the market for marijuana.

While THC content measures the quality of the product, it is also directly related to the health damage caused by consuming this drug. Consumers fail to fully internalize the damage caused by marijuana consumption, as well as the fact that profits generated in the black market have a detrimental value for society. We study the welfare impact of the direct participation of the government in the market for marijuana through a public firm, that competes in prices and quality with a black market firm.

2.2.1 Economic environment

There exists a unit mass of consumers that differ in their marginal valuation for quality (THC) denoted by θ, that is assumed to be positive and uniformly distributed between $[\underline{\theta}, \bar{\theta}]$, with $\bar{\theta} - \underline{\theta} = 1$. Individuals incur on a monetary cost $p_i > 0$, to purchase at most one unit of marijuana from firm i with a THC content $q_i > 0$. Marijuana consumption entails health damage that, in accordance to the medical literature, is considered to be convex in the amount of THC, what is captured by the following harm function: $h(q_i) = q_i^2/2$. Consumers are only able to perceive a fraction $\beta < 1$ of this damage. We consider a model of vertical differentiation, where to the utility from consumption described in Mussa and Rosen (1978), we add a health cost related to the perceived harm caused by marijuana consumption. So that the perceived utility from buying a unit of marijuana from firm i for a consumer of type θ and with a health damage perception of β is given by the following expression:

$$U^{\beta}(p_i, q_i) = \theta q_i - p_i - \beta \frac{q_i^2}{2}.$$

Marginal production costs are assumed to be quadratic on the quality level, $C(q_i) = cq_i^2/2$, with $c > 0$. This specification captures the fact that obtaining higher THC levels implies making use of more expensive inputs, as well as a more detailed care in the whole production process. Moreover, we make the assumption that each firm is restricted to offer a unique quality. A firm will set a level of THC q_i and a price p_i for its product, in order to maximize profits Π_i. For a consumer to participate in the market, it must be that his taste for quality is higher than the following participation threshold:

$$\theta_P = \frac{p_j}{q_j} + \beta \frac{q_j}{2}.$$

where here j corresponds to the firm offering the product with lowest quality.

We make the assumption that the market is fully covered, which implies that the consumer with the lowest marginal willingness to pay for THC $\underline{\theta}$, has a positive utility from purchasing the lowest quality available:

$$\underline{\theta} > \theta_P. \tag{2.1}$$

This assumption implies that the demand is fixed, so that government participation in the marijuana market would not attract additional consumers. In the last section we relax this assumption by allowing for new consumers to enter the market and exclusively demand from the public firm.

2.2.2 Normative Benchmark

Our analysis focuses on two sources of inefficiency that motivate government intervention: the misperception on the health damage that THC causes and the presence of a firm that operates in the black market. The tools available for the government are the price p_G and quality q_G of the product it will supply through the public firm. Additionally, the government may also make use of a lump sum transfer T to redistribute the profits or losses made by the public firm. The budget constraint faced by the government is therefore:

$$T \leq \Pi_G = \big(p_G - c\frac{q_G^2}{2}\big)D_G,$$

where D_G corresponds to the demand faced by the public firm.

First-best allocation and optimal splitting

As a benchmark, let's consider a situation where the government can offer two different qualities, q_L and q_H, with $q_H > q_L$.[10] While the government takes into consideration the true health damage caused by THC content, consumers only perceive a fraction β of this health damage. We consider here that the taste for quality is such that all consumers

[10] While a first-best allocation would imply to offer a different quality for each type of individual, we constraint the government to offer two qualities for comparative purposes.

would like to participate in the market, that is, that condition (2.1) holds. In order to derive the optimal allocation, we must first determine the demands for the low and high quality products. For the consumer that is indifferent between both varieties must verify that:

$$\theta q_L - \beta \frac{q_L^2}{2} - p_L = \theta q_H - \beta \frac{q_H^2}{2} - p_H,$$

so that the indifferent consumer $\hat{\theta}$ is such that:

$$\hat{\theta} = \frac{p_H - p_L}{q_H - q_L} + \beta \frac{q_H + q_L}{2}. \tag{2.2}$$

In order to have positive demands for both qualities in equilibrium, besides the participation constraint, we need to further assume that there is enough consumer heterogeneity such that $\bar{\theta} > \hat{\theta}$. This is equivalent to assume that the following condition holds:

$$\bar{\theta} > \frac{p_H - p_L}{q_H - q_L} + \beta \frac{q_H + q_L}{2}. \tag{2.3}$$

The expressions for demands, the indifferent consumer and the assumption above have been determined for prices p_L and p_H. We will refer to them throughout out the rest of the analysis.

A paternalistic government maximizes social welfare, considering the true health damage caused by marijuana consumption. The first-best allocation would then correspond to offer a low and a high quality product that maximize the true utility of consumers, *i.e*, with $\beta = 1$. The welfare function can be expressed in the following way:

$$W = \int_{\underline{\theta}}^{\hat{\theta}} \theta q_L - (1+c)\frac{q_L^2}{2}\, d\theta + \int_{\hat{\theta}}^{\bar{\theta}} \theta q_H - (1+c)\frac{q_H^2}{2}\, d\theta \tag{2.4}$$

The government will choose qualities q_L and q_H and how consumers should be split among these two qualities, what amounts to choose the indifferent consumer $\hat{\theta}$. To derive how consumers should be optimally split between qualities, let's take the first order condition of the welfare function in (2.4) with respect to the indifferent consumer:

$$\frac{\partial W}{\partial \hat{\theta}} = -(\hat{\theta} q_L - (1+c)\frac{q_L^2}{2}) + (\hat{\theta} q_H - (1+c)\frac{q_H^2}{2}) = 0$$

Solving for $\hat{\theta}$ yields the following optimal splitting condition:

$$\tilde{\theta} = (1+c)\frac{q_L + q_H}{2} \tag{2.5}$$

So that optimal splitting is achieved when the indifferent consumer is such that his marginal willingness to pay for quality equals the social cost of its production.

Let's now determine the first-best qualities. Taking the first order conditions of the welfare function with respect to qualities yields:

$$\begin{aligned}
\frac{\partial W}{\partial q_L} &= \left[\frac{\hat{\theta} + \underline{\theta}}{2} - (1+c)q_L\right](\hat{\theta} - \underline{\theta}) = 0 \\
\frac{\partial W}{\partial q_H} &= \left[\frac{\hat{\theta} + \bar{\theta}}{2} - (1+c)q_H\right](\bar{\theta} - \hat{\theta}) = 0
\end{aligned}$$

Solving for the system above and making use of the optimal splitting condition in (2.5) yields the following equilibrium qualities:

$$q_L^{FB} = \frac{4\underline{\theta} + 1}{4(1+c)}, \tag{2.6}$$

$$q_H^{FB} = \frac{4\underline{\theta} + 3}{4(1+c)}. \tag{2.7}$$

These are the first-best qualities that will be used as a benchmark to address the case where a public and a black market firm engage in price and quality competition. They correspond to the qualities that maximize welfare, once the true health damage is taken into account and they are derived under the assumptions expressed by conditions (2.1) and (2.3), that is, that the whole market is covered and that there is enough heterogeneity on the taste for THC such that both qualities face positive demand in equilibrium.

We observe from the denominator of the expressions above how the production costs and the health damage caused by marijuana consumption diminish the qualities offered in equilibrium. If the social planner were not to be paternalistic, the denominator of both expressions would instead be $4(\beta + c)$, what would imply higher equilibrium qualities.

Plugging in the expressions for the first-best qualities in (2.6) and (2.7) into the expression that characterizes the optimal splitting in (2.5), we have that:

$$\hat{\theta}^{FB} = \underline{\theta} + \frac{1}{2}$$

so that the optimal splitting between first-best qualities is such that the indifferent consumer lays at the middle of the distribution.

First-best decentralization

Decentralization of the optimal allocation will require individualized prices that induce consumers to make the "right" quality choices. This amounts to choose the prices p_L and p_H in such a way that the condition for optimal splitting in (2.5) is satisfied. Combining the expressions for the indifferent consumer in (2.2) and the optimal splitting condition in (2.5), we have that:

$$\hat{\theta} = \frac{p_H - p_L}{q_H - q_L} + \beta\frac{(q_H + q_L)}{2} = (1+c)\frac{q_L + q_H}{2}$$

so that the optimal prices are given by the following condition:

$$p_H^{FB} - p_L^{FB} = (1 - \beta + c)\frac{q_H^2 - q_L^2}{2}$$

From the condition above we see that what matters for the optimal splitting between consumers is the price difference between the two available products. The optimal allocation can be decentralized by making this price difference equal to the sum of the difference in the marginal production costs and the difference in the fraction of health damage that is misperceived. This can be achieved, for instance, by setting prices in the following way:

$$p_L = \frac{(1-\beta+c)q_L^2}{2},$$
$$p_H = \frac{(1-\beta+c)q_H^2}{2}.$$

For the prices above, budget balance could be achieved by setting, for instance, the following individualized lump sum transfers:

$$T_L = \frac{(1-\beta)(\hat{\theta}-\underline{\theta})q_L^2}{2},$$
$$T_H = \frac{(1-\beta)(\bar{\theta}-\hat{\theta})q_H^2}{2}.$$

If the government is restricted to uniform lump sum transfers, budget balance can be achieved by setting the following lump sum transfer:

$$T = \frac{(1-\beta)[(\hat{\theta}-\underline{\theta})q_L^2 + (\bar{\theta}-\hat{\theta})q_H^2]}{2}.$$

The latter would imply a monetary transfer from consumers with low taste for THC to those with high taste for THC.

2.2.3 Black Market Duopoly

In a *laissez-faire* situation, the market for marijuana would be in the hands of the black market. In order to compare with our normative benchmark, we will consider that there are two black market firms competing for the market of marijuana. Firms decide simultaneously in a first stage their respective quality and in a second stage their price. The resulting allocation will be socially inefficient, not only because profit-maximizing firms do not take into account how their quality choices affect consumer surplus, but also because they do not acknowledge the fact that consumers do not fully perceive the damage caused by marijuana consumption.

Each firm maximizes profits, where without loss of generality one firm offers the low quality product q_L and the other one the high quality product q_H. The profit function for a firm who chooses a quality i is given by:

$$\Pi_i(p_i, q_i) = [p_i - c\frac{q_i^2}{2}]D_i \quad \text{with } i = L, H.$$

where demands have the following expressions:

$$D_L = \frac{p_H - p_L}{q_H - q_L} + \beta\frac{(q_L + q_H)}{2} - \underline{\theta},$$
$$D_H = \bar{\theta} - \Big(\frac{p_H - p_L}{q_H - q_L} + \beta\frac{q_L + q_H}{2}\Big).$$

From FOC:

$$\frac{\partial \Pi_H}{p_H} = \frac{2p_L - 4p_H + 2(1+\underline{\theta})(q_H - q_L) - \beta(q_H^2 - q_L^2) + cq_H^2}{2(q_H - q_L)} = 0$$
$$\frac{\partial \Pi_L}{p_L} = \frac{2p_H - 4p_L - 2\underline{\theta}(q_H - q_L) + \beta(q_H^2 - q_L^2) + cq_L^2}{2(q_H - q_L)} = 0$$

Solving for prices, we get the following best reply correspondences:

$$\overline{p}_L = \frac{2p_H - 2\underline{\theta}(q_H - q_L) + \beta(q_H^2 - q_L^2)) + cq_L^2}{4},$$
$$\overline{p}_H = \frac{2p_L + 2(1+\underline{\theta})(q_H - q_L) - \beta(q_H^2 - q_L^2)) + cq_H^2}{4}.$$

Solving the system above, we get the equilibrium prices of the second stage of the game, that are given by:

$$p_L^{PD} = \frac{2(1-\underline{\theta})(q_H - q_L) + \beta(q_H^2 - q_L^2)) + c(q_H^2 + 2q_L^2)}{6},$$
$$p_H^{PD} = \frac{2(2+\underline{\theta})(q_H - q_L) - \beta(q_H^2 - q_L^2)) + c(2q_H^2 + q_L^2)}{6}.$$

Let's plug the equilibrium prices of the second stage game into the profit functions and take now the first order conditions with respect to qualities:

$$\frac{\partial \Pi_H(p_L^{PD}, p_H^{PD})}{q_H} = \frac{[4 + 2\underline{\theta} + (\beta + c)(q_L - 3q_H)][4 + 2\underline{\theta} - (\beta + c)(q_L + q_H)]}{36} = 0$$
$$\frac{\partial \Pi_L(p_L^{PD}, p_H^{PD})}{q_L} = \frac{[2 - 2\underline{\theta} + (\beta + c)(3q_L - q_H)][2 - 2\underline{\theta} + (\beta + c)(q_L + q_H)]}{36} = 0$$

Solving for q_L and q_H we get the following best reply functions:

$$\bar{q}_L = \frac{q_H}{3} + \frac{2(\underline{\theta} - 1)}{3(\beta + c)},$$
$$\bar{q}_H = \frac{q_L}{3} + \frac{2(\underline{\theta} + 2)}{3(\beta + c)}.$$

Solving for the system above leads to the following equilibrium qualities:

$$q_L^{PD} = \frac{4\underline{\theta} - 1}{4(\beta + c)}, \tag{2.8}$$
$$q_H^{PD} = \frac{4\underline{\theta} + 5}{4(\beta + c)}. \tag{2.9}$$

The associated equilibrium profits are:

$$\Pi_H^{PD} = \Pi_L^{PD} = \frac{3}{8(\beta + c)}.$$

While the indifferent consumer is at the middle of the distribution as in the first-best allocation, under a black market duopoly the quality differentiation is higher. The respective distances between qualities are given by the following expressions:

$$\Delta q^{FB} = q_H^{FB} - q_L^{FB} = \frac{4\underline{\theta} + 3}{4(1 + c)} - \frac{4\underline{\theta} + 1}{4(1 + c)} = \frac{1}{2(1 + c)},$$
$$\Delta q^{PD} = q_H^{PD} - q_L^{PD} = \frac{4\underline{\theta} + 5}{4(\beta + c)} - \frac{4\underline{\theta} - 1}{4(\beta + c)} = \frac{3}{2(\beta + c)}.$$

Consequently, compared with our benchmark scenario, with a black market duopoly there is too much quality differentiation:

$$\Delta q^{PD} - \Delta q^{FB} = \frac{3}{2(\beta + c)} - \frac{1}{2(1 + c)} > 0.$$

Introducing a profit-seeking firm rather than a welfare maximizing one leads to more quality differentiation in order to soften price competition. The fact that black market profits are socially undesirable makes the *laissez-faire* situation even more detrimental for welfare.

2.3 Mixed Duopoly

Consider now that as a result of legalization, the government participates in the market for marijuana through a public firm, competing in prices and qualities with a black market firm. To capture the fact black market profits are socially undesirable, the government will assign to them a negative value. A black market firm in our framework is then a profit-maximizing firm who does not account for the fact that consumers suffer from misperception on the health damage of the product they sell and whose profits have a negative value for society. Regarding the timing, we will begin by considering a situation where firms will first choose simultaneously which quality to provide and then compete in prices à la Bertrand, as in our *laissez-faire* situation. Then, in order to capture the fact that the government has a stronger commitment than the black market, we will address the case where the public firm has a first-mover advantage in the quality selection stage. We solve the games by backward induction, where the solution concept is sub-game perfect Nash equilibrium.

2.3.1 Simultaneous decisions

Let's begin by considering the case where the government and the black firm set first qualities and then prices simultaneously. Because the government and the black market firm have different objectives, whether one of the firms serves the lower or the upper-end of the demand will be of relevance for our analysis. We therefore have two possible scenarios: either the government offers the product with lower or higher quality.[11] We will present both cases separately and then compare the equilibrium outcomes.

Public firm supplies the low quality product

Let's begin by considering that the public firm offers a lower quality than the black market firm, which seems a priori the most plausible scenario, since in our setting the government cares about the true health damage caused by marijuana consumption, while the black

[11]Since we are considering a vertical differentiation model, the black market firm will always have an incentive to differentiate its quality in order to extract a higher surplus. A characterization of the best reply correspondence of the black market firm is presented in Appendix 2.6.1.

market firm is interested exclusively in making profits. This case resembles the strategy followed by the Uruguayan government in the legalization of cannabis.

Let's proceed to determine the demands for the low and high quality product when $q_B > q_G$. Assuming that the whole market is covered and restricting our attention to the situation where both firms are active, the government will serve consumers with a lower valuation for THC than the indifferent consumer, while the black market firm will serve the rest. Consequently, demands have the following expressions:

$$D_B = \bar{\theta} - \Big(\frac{p_B - p_G}{q_B - q_G} + \beta\frac{q_B + q_G}{2}\Big),$$
$$D_G = \frac{p_B - p_G}{q_B - q_G} + \beta\frac{(q_B + q_G)}{2} - \underline{\theta}.$$

The objective of the black market firm is to maximize profits:

$$\Pi_B = [p_B - c\frac{q_B^2}{2}]D_B. \tag{2.10}$$

The government, on the other hand, maximizes social welfare. For our analysis, we consider welfare as the sum of consumer surplus and profits of the public firm, while black market profits are instead weighted by a parameter λ, that captures how socially (un)desirable are black market profits for society. While for our results to hold we only require for the welfare weight on black market profits to be lower than one, to have a more clear interpretation, in what follows we consider λ to be negative. Thus we emphasize the negative externality that the presence of a black market generates to society through crime and violence.

The objective of the government is to maximize welfare, subject to its resource constraint:

$$\begin{aligned} \max_{p_G,q_G,T} W = & \int_{\underline{\theta}}^{\hat{\theta}} \theta q_G - (1+c)\frac{q_G^2}{2}\, d\theta + \int_{\hat{\theta}}^{\bar{\theta}} \theta q_B - (1+c)\frac{q_B^2}{2}\, d\theta \\ & - (1-\lambda)\Big[p_B - c\frac{q_B^2}{2}\Big]D_B + T \\ s.t. \quad & \Big[p_G - c\frac{q_G^2}{2}\Big]D_G - T \geq 0 \end{aligned}$$

We have expressed the welfare function as the sum of the social surplus that corresponds to the trade of the low and high quality product, subtracting the welfare loss associated to the profits generated in the black market. Prices are not anymore just transfers between agents the government cares for equally. As it will become clear, it is the fact that black market profits entail a welfare loss for society what explains that the first-best allocation cannot be restored. The resource constraint we are considering allows for the public firm to make a loss, as long as it can be covered through a lump sum tax to be imposed on all consumers. For the moment, we will consider that this is the case. Later, we will address the consequences of imposing a restriction on the economic result of the public firm.

In the second stage of the game, the black market firm and the government choose simultaneously their optimal prices to maximize profits and social welfare respectively, for any given qualities q_B and q_G.

The FOCs associated to the objective of the black market firm and the government are given respectively by:

$$\frac{\partial \Pi_B}{\partial p_B} = \frac{2(p_G - 2p_B) + 2\bar{\theta}(q_B - q_G) - \beta(q_B^2 - q_G^2) + cq_B^2}{2(q_B - q_G)} = 0$$
$$\frac{\partial W}{\partial p_G} = \frac{2(\lambda p_B - p_G) - (1-\beta)(q_B^2 - q_G^2) + c(q_G^2 - \lambda q_B^2)}{2(q_B - q_G)} = 0$$

Solving for p_B and p_G, we get the following best reply correspondences:

$$\overline{p}_B = \frac{cq_B^2 + 2p_G + 2\bar{\theta}(q_B - q_G) - \beta(q_B^2 - q_G^2)}{4}, \tag{2.11}$$
$$\overline{p}_G = \frac{cq_G^2 - (1-\beta)(q_B^2 - q_G^2) + \lambda(2p_B - cq_B^2)}{2}. \tag{2.12}$$

The first term in equation (2.11) refers to the marginal cost of production per unit of THC for the black market firm, while the second term expresses how it takes into account the decision of the public firm to set its own price. The third term shows how the quality differentiation allows for the firm to charge a higher price. The last term accounts for the fact that consumers are less willing to pay for a higher quality, as they acknowledge some of the health damage caused by higher levels of THC. Regarding the public firm, the first term in equation (2.12) corresponds to the marginal production cost for the public firm per unit of THC. The second term is a Pigouvian component, which is negative ($q_B > q_G$), so that as long as there is some misperception on the health damage caused by marijuana consumption in the population, the government would like to lower its price in order for consumers to internalize the misperceived difference in health damage associated to the difference in THC content of both products. The last component tell us that the government will reduce its price proportionally to the degree of undesirability of black market profits. So that the last two terms push the government to price the low quality product below marginal costs, which will require for a lump sum tax to have budget balance. This is a remarkable difference with respect to the price that would decentralize the first-best allocation in our normative setting, as in that case the government increases its price above marginal cost in order for consumers to internalize the health damage caused by marijuana consumption and split them optimally between the two available qualities. The price set by the government appears as a useful tool to attract demand towards the public firm and fight black market activity, as well as to correct for the misperception on the damage caused by marijuana consumption.

Combining the best reply correspondences for the black market and the government in the price sub-game given by (2.11) and (2.12), results in the following Nash equilibrium prices:

$$p_B^* = \frac{c(q_G^2 + (1-\lambda)q_B^2) + 2\bar{\theta}(q_B - q_G) - (q_B^2 - q_G^2)}{2(2-\lambda)}, \tag{2.13}$$

$$p_G^* = \frac{2cq_G^2 - 2(1-\beta)(q_B^2 - q_G^2) + \lambda[2\bar{\theta}(q_B - q_G) - \beta(q_B^2 - q_G^2) - cq_B^2]}{2(2-\lambda)}. \tag{2.14}$$

Before moving to the first stage of the game and derive the optimal qualities, let's analyze the implications of the equilibrium prices of the second stage of the game for the utility of consumers. For a consumer of type θ, we have that the utility difference between buying from the black market or public firm is the following:

$$U_B^\beta(p_B^*, q_B) - U_G^\beta(p_G^*, q_G) = \frac{1}{2-\lambda} \times \left[\theta(q_B - q_G) - (1+c)\frac{(q_B^2 - q_G^2)}{2} - [1-\lambda][\bar{\theta} - \theta](q_B - q_G)\right] \tag{2.15}$$

The first three terms in the numerator capture the utility difference from buying the high versus the low quality product in a first-best situation, that is, considering the differences in the true health damage and in marginal production costs. The last term accounts for the distortion that arises due to the undesirability of black market profits. The public firm distort its price downwards, in order to steal more consumers from the black market. This reduction is decreasing in the taste for quality and it becomes zero for the consumers with a marginal willingness to pay for quality $\bar{\theta}$. The choice between buying from the black market or the public firm is therefore not distorted for the consumer with highest taste for THC. We see from the denominator that for any given consumer, the utility difference from buying from the black market firm rather than from the public one is decreasing in the undesirability of black market profits, though it does not distort its choice between both qualities. Notice from the expression in (2.15) that the equilibrium prices are such that the misperception on health damage is corrected, as individuals take into account the true (rather than the perceived) difference in health damage caused by the two products. This is due to the fact that what matters for the consumers' choice is the price difference. If the black market firm were to generate no damage for society, that is if $\lambda = 1$, then the participation of the public firm in the market for marijuana achieves the optimal splitting between the two available qualities. It is this last inefficiency what makes the optimal splitting not possible. Indeed, plugging in the equilibrium prices in (2.13) and (2.14) into the expression for the indifferent consumer in (2.2) and rearranging terms we get:

$$\hat{\theta}^* = (1+c)\frac{q_B + q_G}{2} + \frac{1-\lambda}{2-\lambda}\Big(\bar{\theta} - (1+c)\frac{q_B + q_G}{2}\Big). \tag{2.16}$$

Taking the difference of the indifferent consumer above in (2.16), that results from the equilibrium prices (2.13) and (2.14), for any given qualities, with respect to the optimal splitting condition in (2.5), we have:

$$\hat{\theta}^* - \tilde{\theta} = \frac{1-\lambda}{2-\lambda}\Big(\bar{\theta} - (1+c)\frac{q_B + q_G}{2}\Big).$$

The expression above captures the deviation from the optimal splitting that arises due to the undesirability of black market profits. As long as λ is different from one, that is, as long as black market profits entail some welfare loss, the public firm will distort its price, making it impossible to achieve optimal splitting. The higher is the undesirability of black market profits, the more the government will distort its price to deviate more consumers to the public firm. We then have the following result:

Proposition 2.3.1 *As long as black market profits entail a welfare loss ($\lambda < 1$), the first-best allocation cannot be decentralized by the direct participation of the government through a public firm.*

For the result above, it is irrelevant whether the public firm serves the consumers with higher or lower willingness to pay for quality, as it is driven by the inefficiency caused by the undesirability of black market profits.

Let's move now to the second stage of the game. We first plug in the equilibrium prices in (2.13) and (2.14) into the objective function of the government and the black market firm and then solve for their respective qualities. The FOC for the black market firm is as follows:

$$\frac{\partial \Pi_B}{\partial q_B} = \frac{[2\bar{\theta} + (1+c)(q_G - 3q_B)][2\bar{\theta} - (1+c)(q_B + q_G)]}{4(2-\lambda)^2} = 0$$

Solving for q_B we get the best reply function of the black market firm for any given quality set by the government:

$$\overline{q}_B = \frac{q_G}{3} + \frac{2\bar{\theta}}{3(1+c)} \tag{2.17}$$

The FOC for the objective of the government is as follows:

$$\begin{aligned}\frac{\partial W}{\partial q_G} = & \frac{1}{8(2-\lambda)^2} \times \Big\{ -4\bar{\theta}^2 + (1+c)^2(q_B^2 - 2q_B q_G - 3q_G^2) \\ & - 4(2-\lambda)^2(1 + 2(\underline{\theta} - q_G(1+c))) - 8q_G(1+c)\bar{\theta}\Big\} = 0\end{aligned}$$

Solving for q_G we get the best reply function for any given quality set by the black market:

$$\begin{aligned}\overline{q}_G = & \frac{1}{3(1+c)} \times \Big\{4\bar{\theta} - q_B(1+c) - 4(2-\lambda)^2 + \\ & + 2\sqrt{[\bar{\theta} - q_B(1+c)]^2 + [2-\lambda]^2][4(2-\lambda)^2 - 2(\bar{\theta} - q_B(1+c)) + 3]}\Big\}.\end{aligned} \tag{2.18}$$

Solving the system of equations given by the best reply correspondences $\overline{q}_B$ and $\overline{q}_G$ we get the following equilibrium qualities:

$$q_B^* = \frac{8\bar{\theta} - 3(2-\lambda)^2 + (2-\lambda)\sqrt{1 + 9(\lambda^2 - 4\lambda + 3)}}{8(1+c)}, \tag{2.19}$$

$$q_G^* = \frac{8\bar{\theta} - 9(2-\lambda)^2 + 3(2-\lambda)\sqrt{1 + 9(\lambda^2 - 4\lambda + 3)}}{8(1+c)}. \tag{2.20}$$

The equilibrium when the public and the black market firm compete simultaneously, first in qualities and then in prices, is characterized by expressions (2.13), (2.14), (2.19) and (2.20). We recall that this equilibrium is derived under the assumptions of market coverage, expressed in condition (2.1) and that there is enough heterogeneity in the marginal willingness to pay for THC content such that both firms are active, expressed by condition (2.3), where in particular the public firm supplies the low quality product. For this equilibrium, no firm has incentive to deviate. Second order properties are presented in the Appendix 2.6.2.

Comparing the expressions for the qualities that result from simultaneous competition between a public and a black market firm above, with the expressions for first-best qualities in (2.8) and (2.9), we observe that they differ as long as λ is different from one. From the expression in (2.20) we see that the quality of the public firm is increasing in the degree of undesirability of black market profits. The public firm increases its market participation at the expense of deviating from the social optimum qualities. This increase in the quality of the public firm triggers a strategic increase in the quality of the black market firm, in order to extract higher rents through more differentiation. As a result, both qualities are higher than the first-best allocation. There is a limit of course, to the quality that can be offered by the public firm, as from a certain threshold on, the black market firm would be better off by switching to serve the consumers with lower marginal willingness to pay for THC.[12] Specifically, this threshold is given by the following government quality:

$$\tilde{q}_G = \frac{\bar{\theta} + \underline{\theta}}{2(1+c)}.$$

[12]The characterization of the black market firm best reply correspondence is presented in the Appendix 2.6.1.

The associated profits and demands to this equilibrium are the following:

$$
\begin{aligned}
D_B^* =& \frac{3(2-\lambda) - \sqrt{1+9(\lambda^2-4\lambda+3)}}{4} \\
D_G^* =& \frac{3\lambda - 2 + \sqrt{1+9(\lambda^2-4\lambda+3)}}{4} \\
\Pi_B^* =& \frac{(2-\lambda)[3(2-\lambda) - \sqrt{1+9(\lambda^2-4\lambda+3)}]^3}{64(1+c)} > 0 \\
\Pi_G^* =& \frac{D_G}{16(1+c)^2}\Big\{\Big[3(2-\lambda) - \sqrt{1+9(\lambda^2-4\lambda+3)}\Big]\Big[\lambda 4\bar{\theta}(1+c) - \\
& - \Big(4\bar{\theta} - 3(2-\lambda)^2 + (2-\lambda)\sqrt{1+9(\lambda^2-4\lambda+3)}\Big)\Big(2(1-\beta) + \lambda(c+\beta)\Big)\Big]\Big\} < 0
\end{aligned}
$$

While the public firm prices below marginal costs, thus making a loss, the black market firm, on the other hand, makes positive profits. We observe that both qualities are independent of β, the degree of health damage perception of the individuals. The government will only make use of its price instrument to correct for the health damage mispercieved by the individuals, as it would do in a first-best situation. While the inefficiency related to health damage misperception is only addressed with one instrument, the government makes use of both prices and qualities to fight black market activity. Once the optimal splitting is distorted due to the undesirability of black market profits, so are the qualities that maximize welfare. The quality offered by the public firm is higher than the low-quality product it would offer in a first-best situation. It is the undesirability of the black market what makes the government to deviate from its optimal quality and increase its market share. The equilibrium qualities in (2.19) and (2.20) that result from simultaneous competition, first in qualities and then in prices, are higher than our benchmark qualities in (2.6) and (2.7). In comparison with a black market duopoly, the presence of a public firm increases competition, what results in less quality differentiation.

The findings from the characterization of the equilibrium above when the public and black market firm engage in simultaneous competition, first in qualities and then in prices, can be summarized in the following proposition:

Proposition 2.3.2 *When the public firm supplies the low quality product we have the following results:*

- *The quality deviation with respect to the first-best allocation increases with the degree of undesirability of black market profits.*
- *By lowering its price and increasing its quality, the government is able to reduce black market participation.*
- *The resulting second-best allocation is one where both qualities are higher than the first-best allocation.*

We recall that these results are derived considering that the market is fully covered and that there is enough consumer heterogeneity such that both firms are active in equilibrium.

Legalization brings two sources of welfare variation: the first comes from the reduction in the participation and in the profits per unit sold in the black market, whose profits were a pure loss for welfare; while the second comes from a variation in the qualities offered in equilibrium, what results in less product differentiation.

While the public firm makes a loss, the surplus of consumers who buy from the public firm is enhanced, as the negative result of the public firm is financed through a lump sum tax imposed on all consumers. Moreover, participation and the profits per unit of the black market firm is reduced, so that less surplus from consumers is extracted by the black market firm. Despite profits made in the black market result in a welfare loss for society, the utility some consumers derive from the existence of a higher quality product justify its presence.

Public firm supplies the high quality product

We now turn to the case where the public firm offers a product with higher THC content than the black market firm, so that it serves the consumers with higher willingness to pay for quality. In order to determine the optimal prices and qualities we follow the same procedure as for the previous case, presenting here the main results. A more detailed analysis of the derivations can be found in the Appendix 2.6.3.

For the price sub-game, the best reply correspondences are now the following:

$$\begin{aligned}
\overline{p}_B =& \frac{2p_G + cq_B^2 - 2\underline{\theta}(q_G - q_B) + \beta(q_G^2 - q_B^2)}{4}, \\
\overline{p}_G =& \frac{cq_G^2 - (1-\beta)(q_B^2 - q_G^2) + \lambda(2p_B - cq_B^2)}{2}.
\end{aligned}$$

From the best reply correspondences above we see that while the government has the same best reply function than when it offers the low-quality product, the black market firm now adapts its price strategy to the fact that it serves consumers with lower willingness to pay for quality.

Solving for p_B and p_G we get the following Nash equilibrium prices:

$$p_B^{**} = \frac{c(q_G^2 + (1-\lambda)q_B^2) - 2\underline{\theta}(q_G - q_B) + q_G^2 - q_B^2}{2(2-\lambda)}, \tag{2.21}$$

$$p_G^{**} = \frac{2cq_G^2 + 2(1-\beta)(q_G^2 - q_B^2) + \lambda[-2\underline{\theta}(q_G - q_B) + \beta(q_G^2 - q_B^2) - cq_B^2]}{2(2-\lambda)}. \tag{2.22}$$

As before, through its price, the public firm corrects for the mispercieved difference on the damage caused by each type of product. However, unlike the case where it serves the consumers with lower willingness to pay for THC, the public firm makes now positive profits.

Plugging in the prices above into the objective functions of the black market firm and the government and solving for the optimal qualities, leads to the following equilibrium qualities:

$$q_B^{**} = \frac{8\underline{\theta} + 3(2-\lambda)^2 - (2-\lambda)\sqrt{1 + 9(\lambda^2 - 4\lambda + 3)}}{8(1+c)}, \tag{2.23}$$

$$q_G^{**} = \frac{8\underline{\theta} + 9(2-\lambda)^2 - 3(2-\lambda)\sqrt{1 + 9(\lambda^2 - 4\lambda + 3)}}{8(1+c)}. \tag{2.24}$$

When the government supplies the high quality product, the resulting equilibrium is characterized by prices (2.21), (2.22) and qualities (2.23) and (2.24). The demands and profits associated to this equilibrium are the following:

$$
\begin{aligned}
D_B^{**} =& \frac{3(2-\lambda) - \sqrt{1+9(\lambda^2-4\lambda+3)}}{4} \\
D_G^{**} =& \frac{3\lambda - 2 + \sqrt{1+9(\lambda^2-4\lambda+3)}}{4} \\
\Pi_B^{**} =& \frac{(2-\lambda)[3(2-\lambda) - \sqrt{1+9(\lambda^2-4\lambda+3)}]^3}{64(1+c)} > 0 \\
\Pi_G^{**} =& \frac{D_G}{16(1+c)^2}\Big\{\Big[3(2-\lambda) - \sqrt{1+9(\lambda^2-4\lambda+3)}\Big]\Big[-\lambda 4\underline{\theta}(1+c) + \\
& + \Big(4\underline{\theta} + 3(2-\lambda)^2 - (2-\lambda)\sqrt{1+9(\lambda^2-4\lambda+3)}\Big)\Big(2(1-\beta) + \lambda(c+\beta)\Big)\Big]\Big\} > 0
\end{aligned}
$$

As in the previous case, the fact that black market profits generate a welfare loss for society leads to a second-best outcome. The main features of this equilibrium can be summarized in the following proposition:

Proposition 2.3.3 *When the public firm supplies the high quality product we have the following results:*

- *The quality deviation with respect to the first-best allocation increases with the degree of undesirability of black market profits.*
- *By lowering its price and quality, the government is able to reduce black market participation.*
- *The resulting second-best allocation is one where both qualities are lower than the first-best allocation.*

Table 2.1 presents a comparison between the equilibrium outcomes, where for ease of presentation we have denoted $\Delta = 1 + 9(\lambda^2 - 4\lambda + 3)$. We observe that the two equilibria are symmetric, in the sense that they both yield the same market share, black market profits and welfare levels. While the quality difference between both equilibria is the same, when the government offers the low quality product, the resulting equilibrium qualities are higher. There is however a difference in the profits made by the public firm, who before made a loss and now makes positive profits.

	Public firm supplies low quality	**Public firm supplies high quality**
$\mathbf{q_B}$	$\frac{8\bar{\theta}-3(2-\lambda)^2+(2-\lambda)\sqrt{\Delta}}{8(1+c)}$	$\frac{8\underline{\theta}+3(2-\lambda)^2-(2-\lambda)\sqrt{\Delta}}{8(1+c)}$
$\mathbf{\Pi_B}$	$\frac{(2-\lambda)[3(2-\lambda)-\sqrt{\Delta}]^3}{64(1+c)}$	$\frac{(2-\lambda)[3(2-\lambda)-\sqrt{\Delta}]^3}{64(1+c)}$
$\mathbf{D_B}$	$\frac{3(2-\lambda)-\sqrt{\Delta}}{4}$	$\frac{3(2-\lambda)-\sqrt{\Delta}}{4}$
$\mathbf{q_G}$	$\frac{8\bar{\theta}-9(2-\lambda)^2+3(2-\lambda)\sqrt{\Delta}}{8(1+c)}$	$\frac{8\underline{\theta}+9(2-\lambda)^2-3(2-\lambda)\sqrt{\Delta}}{8(1+c)}$
$\mathbf{D_G}$	$\frac{3\lambda-2+\sqrt{\Delta}}{4}$	$\frac{3\lambda-2+\sqrt{\Delta}}{4}$
$\mathbf{W}$	$\frac{\underline{\theta}(1+\underline{\theta})+(2-\lambda)[36(8\lambda+4)+9\lambda^2(3\lambda-18)+\sqrt{\Delta}\Delta]}{64(1+c)}$	$\frac{\underline{\theta}(1+\underline{\theta})+(2-\lambda)[36(8\lambda+4)+9\lambda^2(3\lambda-18)+\sqrt{\Delta}\Delta]}{64(1+c)}$
$\mathbf{\lvert q_G - q_B \rvert}$	$\frac{3(2-\lambda)^2+\sqrt{\Delta}}{4(1+c)}$	$\frac{3(2-\lambda)^2+\sqrt{\Delta}}{4(1+c)}$
$\mathbf{\Pi_G}$	$\{[3(2-\lambda)-\sqrt{\Delta}][\lambda 4\bar{\theta}(1+c)-(4(\bar{\theta})-3(2-\lambda)^2+(2-\lambda)\sqrt{\Delta})(2(1-\beta)+\lambda(c+\beta))]\}\frac{D_G}{16(1+c)^2}$	$\{[3(2-\lambda)-\sqrt{\Delta}][-\lambda 4\underline{\theta}(1+c)+(4\underline{\theta}+3(2-\lambda)^2-(2-\lambda)\sqrt{\Delta})(2(1-\beta)+\lambda(c+\beta))]\}\frac{D_G}{16(1+c)^2}$

TABLE 2.1: Comparison of equilibrium outcomes.

Health damage comparison

In order to compare the health damage caused to the population in our two cases, we will focus on the average health damage H, defined in the following way:

$$H = D_L \times h(q_L) + D_H \times h(q_H).$$

When comparing the average health damage resulting from the two equilibria we have derived, we find that the harm caused to the population is lower when the public firm supplies the high quality product:

$$H^* - H^{**} = \frac{(1-\lambda)(1+2\underline{\theta})[9(2-\lambda)^2 - 4 - 3(2-\lambda)\sqrt{1+9(\lambda^2-4\lambda+3)}]}{8(1+c)^2} > 0 \quad (2.25)$$

where H^* and H^{**} correspond to the average health damage caused by marijuana consumption when the public firm offers the low and high quality product respectively.

Consequently, from the comparison of the equilibria where the government serves the consumers with lower or higher taste for THC, we have the following result:

Proposition 2.3.4 *While the resulting social welfare and market shares are the same, whether the government supplies the low or the high quality product, the average health damage is lower when it offers a higher THC content product than the black market.*

Comparing both cases, we observe that equilibrium qualities are higher when the government supplies the low quality product, *i.e.*, $q_G^* > q_B^{**}$ and $q_B^* > q_G^{**}$. The intuition is that when the government supplies the low (high) quality product, it has to increase (decrease) its quality in order to attract more demand. However, it is always the public firm who serves more consumers, so that a priori it is not clear which equilibrium is more beneficial in terms of the average health damage caused to the population. It turns out that the average health damage is lower when the government supplies the high quality product. Notice from the health damage comparison in (2.25) that when $\lambda = 1$ the total health damage is the same, whether the government supplies the lower or the higher quality product. This is because when black market firm profits do not imply a welfare loss, we are back to the scenario where the government is able to decentralize the first-best allocation.

2.3.2 Public firm has a first-mover advantage on quality selection

In this section we study the impact of allowing the public firm to have a first-mover advantage in the first stage of the game, when firms choose their qualities. As before, in the second stage of the game both firms compete simultaneously in prices. We will present here the case where the government supplies the low quality product, while the other case is described in Appendix 2.6.4.

As nothing changes in the second-stage of the game, the optimal prices conditional on qualities are given by as before by expressions (2.13) and (2.14). In the second stage of the game, the black market firm maximizes profits, so that its best reply function remains also unchanged and is given by (2.17). On the other hand, the government now observes the best reply function of the black market and maximizes its objective function, taking into account this new information. So that the government maximizes:

$$W(p_G^*, p_B^*, \overline{q}_B, q_G) = \int_{\underline{\theta}}^{\hat{\theta}} \theta q_G - (1+c)\frac{q_G^2}{2}\, d\theta + \int_{\hat{\theta}}^{\bar{\theta}} \theta \overline{q}_B - (1+c)\frac{\overline{q}_B^2}{2}\, d\theta - (1-\lambda)\Big[p_B^* - c\frac{\overline{q}_B^2}{2}\Big] D_B$$

From FOC:

$$\frac{\partial W}{\partial q_G} = \frac{9(2-\lambda)^2[1+2\underline{\theta} - 2q_G(1+c)] - 8[q_G(1+c) - \bar{\theta}]^2}{18(2-\lambda)^2} = 0$$

Solving for q_G yields the same equilibrium quality for the public firm than when qualities were chosen simultaneously:

$$q_G^* = \frac{8\bar{\theta} - 9(2-\lambda)^2 + 3(2-\lambda)\sqrt{1 + 9(\lambda^2 - 4\lambda + 3)}}{8(1+c)}$$

Since the best reply function of the black market firm and the equilibrium prices are also the same, we arrive to the same equilibrium as when we had simultaneous competition in qualities. The same holds true for the case where the government supplies the higher quality. The formal analysis is relegated to the Appendix 2.6.4.

From the analysis above we have the following result:

Proposition 2.3.5 *Adding a first-mover advantage to the public firm at the quality selection stage does not improve welfare with respect to a situation where qualities are chosen simultaneously.*

The intuition for this result lays in the fact that the public firm chooses its quality in such a way that the resulting low and high quality products maximize welfare. The public firm internalizes how its quality choice ends up determining both equilibrium qualities, selecting optimally its own quality in such a way that welfare is maximized.

In the context of a mixed duopoly under vertical differentiation, the analysis of Grilo (1994) shows that direct public intervention in the market suffices to decentralize the social optimum allocation. In our framework, the fact that black market profits generate a welfare loss for society makes the decentralization of the first-best allocation unfeasible. However, direct public intervention is sufficient to achieve the second-best allocation, that is, the welfare-maximizing allocation constrained to the fact that black market profits generate a welfare loss. Adding a first-mover advantage to the public firm does not help to improve welfare, because the constrained optimum has already been attained. Even if profits generated in the black market imply a welfare loss, they generate value to society, as some consumers obtain a higher surplus by buying a different quality. As shown in the Appendix 2.6.5, the presence of a black market firm brings a welfare improvement compared to a situation where a public monopoly that offers only one quality, due to the enhance in consumer welfare that comes with having a new quality available. Nonetheless, a first-mover advantage for the public firm will guarantee that out of the two possible equilibria, the most desired one is attained. As we saw previously, both equilibria yield the same levels of welfare, however the one where the government offers the high quality product has the advantage that the public firm makes profits and the average health damage caused by marijuana consumption is lower.

2.3.3 Restriction on the profitability of the public firm

In the previous section we allowed for the public firm to make a loss. We saw that when the public firm supplies the low quality product, it finds optimal to set its price below marginal costs. This may not be feasible, for instance, for political reasons. In this section we consider that the public firm is constrained to make non-negative profits and we study the impact of this restriction on the equilibrium qualities. This constraint is of course only binding when the public firm serves the consumers with lower willingness to pay for quality. We will describe how this price restriction impacts on the optimal qualities, first when quality choice is made simultaneously and then when the public firm has a first-mover advantage.

If the public firm is constrained not to make losses and sets its quality below the black market quality, the price restriction is binding. On the other hand, the black market firm will set its price according to (2.11) as before. We then have the following Nash equilibrium prices:

$$\begin{aligned} p_B^r &= \frac{c(q_B^2 + q_G^2) + 2\bar{\theta}(q_B - q_G) - \beta(q_B^2 - q_G^2)}{4}, \\ p_G^r &= \frac{cq_G^2}{2}. \end{aligned}$$

Due to the fact that the government is constrained to set prices at marginal costs, we see that the equilibrium prices of the second stage of the game do not depend anymore on the undesirability of black market profits λ. This price restriction faced by the public firm also removes the possibility for the government to correct for the misperception on health damage. The profitability constraint forces the public firm to set a higher price than what it would find optimal, what in turn lets the black market firm to charge a higher price for its product. So that the price restriction results in higher prices for both products for any given qualities.

The formal analysis of the first stage of the game is presented in the Appendix 2.6.6, while we will describe next the qualitative impact of the price restriction on the equilibrium qualities.

Simultaneous competition

Since the government cannot correct for the misperception on health damage, its qualities will be adjusted in such a way to tackle both inefficiencies. Both a higher degree of misperception of the health damage and a higher undesirability of black market profits, increase the quality offered by the public firm. Compared with a situation where the public firm is unconstrained, the fact that it is forced to increase its price above what it would find optimum also increases the equilibrium qualities, in an effort to correct for the health damage misperception and the negative welfare impact of black market profits.

Stackelberg Competition

With respect to the previous situation, where there is simultaneous competition in qualities and the public firm faces a restriction on the price it can set, the quality offered when having a first-mover advantage will be lower, what translates into a lower quality for the black market firm. This is true as long as consumers suffer from misperception on the health damage caused by marijuana consumption. If consumers fully perceive the damage caused by THC, that is if $\beta = 1$, then adding a first-mover advantage does not improve welfare. By comparing the qualities offered by the public firm when it engages in simultaneous competition with the black market firm, versus a situation where it has a first-mover advantage, we have the following proposition:

Proposition 2.3.6 *If the public firm supplies the low quality product and is restricted to make non-negative profits, then adding a first-mover advantage in the quality selection stage improves welfare, provided that consumers suffer from misperception on the health damage caused by THC.*

The intuition is that as long as the government is not able to use its price to correct for the inefficiency related to the misperception on health damage, adding a first-mover advantage to the public firm helps to improve welfare by reducing both equilibrium qualities. By having a first-mover advantage, the public firm can commit to a lower quality than what would result from simultaneous competition, what in turn triggers a strategic reduction on the quality supplied by the black market firm.

2.4 Expanding Demand

This section aims to capture the fact that making marijuana more accessible will attract new consumers, which is the main argument against its legalization. Consider now that the black market firm and the government compete for a mass $0 < \gamma < 1$ of consumers uniformly distributed between $[\underline{\theta}, \bar{\theta}] \in \mathbf{R}_{+}$, with $\bar{\theta} - \underline{\theta} = 1$. Moreover, there is an additional mass of consumers $1 - \gamma$ of the same characteristics that will exclusively buy from the public firm, as a consequence of marijuana legalization. This parameter γ can be therefore interpreted as a measure of how easy is to acquire marijuana. A situation where marijuana would be easy to come by, would depict a situation where γ is close to one, while if this parameter is close to zero, legalization makes marijuana available for a large number of consumers that before found it very difficult to get access to. We then consider a situation where demand for marijuana is divided in two groups: a group of size $1 - \gamma$, that exclusively buy from the public firm or otherwise do not buy the product at all and a market of size γ, for which the public and black market firm compete. Except for this new badge of consumers that demand exclusively from the public firm after legalization, the assumptions considered in the previous section remain the same.

2.4.1 Public firm supplies the low quality product

The objective functions of the black market firm and the government are now given respectively by:

$$\begin{aligned}\hat{\Pi}_B =& \Big[p_B - c\frac{q_B^2}{2}\Big]\hat{D}_B \\ \hat{W} =& \gamma\Big[\int_{\underline{\theta}}^{\hat{\theta}} \theta q_G - (1+c)\frac{q_G^2}{2}\, d\theta + \int_{\hat{\theta}}^{\bar{\theta}} \theta q_B - (1+c)\frac{q_B^2}{2}\, d\theta\Big] - (1-\lambda)\Big[p_B - c\frac{q_B^2}{2}\Big]\hat{D}_B \\ &+ (1-\gamma)\Big[\int_{\underline{\theta}}^{\bar{\theta}} \theta q_G - (1+c)\frac{q_G^2}{2}\, d\theta\Big]\end{aligned}$$

where $\hat{D}_B = \gamma[\hat{\theta} - \underline{\theta}]$

In the second stage, the black firm and the government choose their prices to maximize profits and welfare respectively. The equilibrium of the price sub-game is the same as when we did not consider an additional demand and is given by (2.13) and (2.14). The fact that legalization may attract a new badge of consumers to the public firm does not affect the optimal pricing of both firms. This is because these new consumers will exclusively buy from the government and since for them there is only one quality available, the price chosen by the government will not affect total welfare, provided that all consumers participate. However, for the consumers that the public firm disputes with the black market, the government would like to correct for the difference in the health damage that is misperceived. For given prices, we solve for the optimal qualities as in the previous section, what yields:

$$\hat{q}_B^* = \frac{8\gamma\bar{\theta} - 3(2-\lambda)^2 + (2-\lambda)\sqrt{9(2-\lambda)^2 - 8\gamma)}}{8\gamma(1+c)}, \tag{2.26}$$

$$\hat{q}_G^* = \frac{8\gamma\bar{\theta} - 9(2-\lambda)^2 + 3(2-\lambda)\sqrt{9(2-\lambda)^2 - 8\gamma)}}{8\gamma(1+c)}. \tag{2.27}$$

The derivation of the equilibrium qualities is presented in the Appendix 2.6.7. The fact that legalization brings along a new set of consumers to the public firm, impacts in the quality it offers, with the consequent strategic reaction of the black market firm. The public firm will now offer a higher quality than when there is no additional demand accruing from legalization. This is due to the fact that the new set of consumers have only access to the product offered by the government, what makes the public firm to adjust its quality upwards, to better serve the taste of these consumers. In response, the black market firm will strategically increase its quality.

2.4.2 Public firm supplies the high quality product

In the case where the public firm supplies the high quality product, at the price selection stage we are again in the same situation as when there was no additional demand, as for the new consumers the price set by the public firm will only affect their surplus, but not their quality choice. The equilibrium of the price sub-game is then given by expressions (2.21) and (2.22). Following the same procedure as in previous sections yields the following equilibrium qualities:

$$\tilde{q}_B^{**} = \frac{8\gamma\underline{\theta} + 3(2-\lambda)^2 - (2-\lambda)\sqrt{9(2-\lambda)^2 - 8\gamma)}}{8\gamma(1+c)}, \tag{2.28}$$

$$\tilde{q}_G^{**} = \frac{8\gamma\underline{\theta} + 9(2-\lambda)^2 - 3(2-\lambda)\sqrt{9(2-\lambda)^2 - 8\gamma)}}{8\gamma(1+c)}. \tag{2.29}$$

The derivation of the equilibrium qualities is presented in the Appendix 2.6.7.

We observe now that the public firm reduces its quality with respect to the situation where there was no additional demand after legalization. The reason to do this is analogous to the previous case, that is, to better serve these new consumers, who on average at the price fixed by the government are better off with a lower quality. In response, the black market firm also lowers the quality offered.

The main result of this section can be summarized in the following proposition:

Proposition 2.4.1 *When legalization attracts a new badge of consumers with the same characteristics as those who were already in the market and who exclusively demand from the public firm, the quality offered by the public firm is adjusted towards the one that would maximize the utility of the average consumer.*

All in all, the impact of the new exclusive customers is that the quality set by the public firm moves towards the monopoly quality, given by the following expression: $q_M = \frac{\bar{\theta}+\underline{\theta}}{2(1+c)}$. The effect of this new badge of consumers who exclusively buy form the public firm is then to adjust its quality towards the one that maximizes surplus when there is only one quality available in the market.

2.5 Conclusion

Motivated by the recent legalization of marijuana in several jurisdictions, we have analyzed the optimal policies for the government to undertake when directly participating in a vertically differentiated market, characterized by the presence of a black market firm and with the peculiarity that a higher quality of the product increases both the joy from consumption and the health damage it causes, where the latter is misperceived by consumers. A black market firm is considered to cause harm to society through the increase in crime and violence, what is captured in the model by considering that black market profits generate a welfare loss. With respect to the first-best allocation, a black market duopoly would generate too much quality differentiation and moreover, the profits generated would entail an additional welfare loss. Consequently, for a paternalistic government the reason to intervene is twofold: to correct for the misperception caused by marijuana consumption and to diminish black market profits.

We find two possible equilibrium configurations that depend on whether the government supplies a product with lower or higher quality than the one supplied by the black market firm. These equilibria are symmetric, in the sense that they yield the same market shares, black market profits and welfare levels. However, they differ in the profits made by the public firm and in the resulting average health damage. Paradoxically, the average health damage will be lower if the public firm supplies the high quality product. In practice, this results suggest that in terms of the average health damage, a government would be better off by offering a product with higher THC rather than one with low THC content. This policy recommendation is in sharp contrast with the strategy followed by the Uruguayan government.

We also find that introducing a first-mover advantage for the public firm in the quality selection stage does not improve welfare, as simultaneous competition already achieves the second-best allocation. With these instruments, the first-best allocation cannot be restored, as long as black market profits entail a welfare loss. Regarding the possibility that legalizing marijuana leads to an increase in the number of users, provided that this new consumers have the same characteristics than the old ones and that they only buy from the legal market, we find that the government should keep the same price policy but it ought to adjust its quality towards the one that would offer, if it were to be the only supplier.

Our analysis allowed us to understand the role of the price and quality selection by the public firm in order to correct for the misperception on health damage and to fight black market activity. In particular, the price is sufficient to correct for the misperceived difference on the health damage caused by the two quality levels offered in equilibrium and it is also useful as an instrument to reduce black market participation. As long as the public firm faces no constraints on the price it can set, the government can effectively correct for the health damage misperception through a Pigouvian component in the public firm price that accounts for the difference in mispercieved health damage caused by the two available qualities. It is the strategic interaction between the public and black market firm what makes it possible for the government to correct for the health damage misperception, by only setting its own price. Because black market profits are undesirable, the government will distort its price in order to attract more demand, so that due to this inefficiency, consumers do not split optimally between qualities anymore. Moreover, the undesirability of black market profits causes the government to deviate from first-best qualities in an effort to steal more demand from the black market.

We have focused on a situation where the government competes directly with the black market. To complement the analysis, it could also be interesting to consider the case of a regulated supplier and focus on the optimal tax policy that the government should set to maximize welfare. In that scenario, an additional tool for the regulator would be to set, for instance, a cap on the THC content for the regulated firm, what should be sufficient to achieve the second-best allocation.

There are several policy implications that result from our analysis. First, the price of the public firm should be set in accordance to how undesirable is the presence of black market activity and it should also be adjusted to take into account that consumers misperceive the health damage caused by the amount of THC they consume, what may make the product in the black market more attractive to them. This seems to be in line with the approach followed by the Uruguayan government, who sells its product at a very competitive price, but in contrast with introducing high taxes on legal marijuana purchases, as it is the case in the US. Second, regarding quality, the THC content of the product of the public firm should also be set in accordance to how important is for the government to diminish the presence of the black market. To increase its market share, the public firm should adjust its quality upwards if it opts to sell a low quality product and downwards if it opts for a high one. Finally, we find that if the public firm provides the high THC content product, a lower average health damage is achieved, together with positive profits for the public firm.

Our results were derived under the assumptions that the market is fully covered and restricting our attention to situations where the taste for quality is sufficiently heterogeneous that both firms are active in equilibrium. We have focused on THC as a proxy for quality and it has been considered as the main driver for health damage, though the quantity consumed of this harmful drug is also relevant from a public health perspective. Relaxing the assumption that the consumer only buys one unit of the harmful drug would enrich the analysis, as it would allow us to understand how variations in quality may affect the total quantity demanded, though this is beyond the scope of this paper. We have also abstracted from redistributional considerations, that may arise, for instance, due to differences in income or in health damage perception.

2.6 Appendix

2.6.1 Characterization of the best reply correspondences in a mixed duopoly

Consider first that the public firm supplies the low quality product, that is, $q_G < q_B$. To decide how to optimally split consumers between both qualities, the government solves the following problem:

$$\max_{\hat{\theta}} W = \int_{\underline{\theta}}^{\hat{\theta}} \theta q_G - (1+c)\frac{q_G^2}{2}\, d\theta + \int_{\hat{\theta}}^{\bar{\theta}} \theta q_B - (1+c)\frac{q_B^2}{2}\, d\theta - (1-\lambda)\Big[p_B - c\frac{q_B^2}{2}\Big][\bar{\theta} - \hat{\theta}]$$

FOC yields:

$$\frac{\partial W}{\partial \hat{\theta}} = \hat{\theta} q_G - (1+c)\frac{q_G^2}{2} - \hat{\theta} q_B + (1+c)\frac{q_B^2}{2} + (1-\lambda)\Big[p_B - c\frac{q_B^2}{2}\Big] = 0$$

So that for any given price of the black market firm, the indifferent consumer must be such that:

$$\hat{\theta} = (1+c)\Big(\frac{q_B + q_G}{2}\Big) + \Big(\frac{1-\lambda}{q_B - q_G}\Big)\Big(p_B - c\frac{q_B^2}{2}\Big) \tag{2.30}$$

For the case where the public firm supplies the high quality product, that is, when $q_G > q_B$ the expression for the indifferent consumer that splits consumers optimally between both qualities is the same one. The only difference is that the second term is positive when the government supplies the low quality product and negative in the other case.

Recall from condition (2.2) that for $q_H > q_L$, the indifferent consumer $\hat{\theta}$ is such that:

$$\hat{\theta} = \frac{p_H - p_L}{q_H - q_L} + \beta\frac{q_H + q_L}{2}.$$

If $q_G < q_B$, the black market firm solves the following problem:

$$\max_{p_B} \Pi_B = \Big[p_B - c\frac{q_B^2}{2}\Big][\bar{\theta} - \hat{\theta}]$$

FOC yields:

$$\frac{\partial \Pi_B}{\partial p_B} = \bar{\theta} - \hat{\theta} - \frac{p_B - c\frac{q_B^2}{2}}{q_B - q_G} = 0 \tag{2.31}$$

Using condition (2.31), we have that:

$$\Pi_B = \Big[p_B - c\frac{q_B^2}{2}\Big][\bar{\theta} - \hat{\theta}] = (q_B - q_G)[\bar{\theta} - \hat{\theta}]^2$$

From conditions (2.30) and (2.31) we get the following equilibrium splitting condition when $q_G < q_B$:

$$\hat{\theta}^* = \frac{1}{2-\lambda}\Big[(1+c)\frac{q_B + q_G}{2} + (1-\lambda)\bar{\theta}\Big] \tag{2.32}$$

When $q_G > q_B$, the black market firm solves the following problem:

$$\max_{p_B} \Pi_B = [p_B - cq_B^2/2][\hat{\theta} - \underline{\theta}]$$

FOC yields:

$$\frac{\partial \Pi_B}{\partial p_B} = \hat{\theta} - \underline{\theta} + \frac{p_B - c\frac{q_B^2}{2}}{q_B - q_G} = 0 \tag{2.33}$$

Using condition (2.33), we have that:

$$\Pi_B = \left[p_B - c\frac{q_B^2}{2}\right][\hat{\theta} - \underline{\theta}] = (q_G - q_B)[\hat{\theta} - \underline{\theta}]^2$$

From conditions (2.30) and (2.33) we get the following equilibrium splitting condition when $q_G > q_B$:

$$\hat{\theta}^{**} = \frac{1}{2-\lambda}\left[(1+c)\frac{q_B + q_G}{2} + (1-\lambda)\underline{\theta}\right] \tag{2.34}$$

The objective of the black market firm is then given by:

$$\Pi_B = \begin{cases} (q_B - q_G)(\bar{\theta} - \hat{\theta})^2 & \text{if } q_G \leq q_B; \\ (q_G - q_B)(\hat{\theta} - \underline{\theta})^2 & \text{if } q_G > q_B. \end{cases}$$

where for the first case the expression for $\hat{\theta}$ is given by (2.32) and for the second by (2.34).

To find the quality supplied by the public firm that makes the black market indifferent between offering the low or the high quality product, we must first determine the best reply function of the black market firm. So that maximizing black market profits with respect to its own quality, for any given quality offered by the public firm, we have that:

$$\bar{q}_B = \begin{cases} \dfrac{q_G}{3} + \dfrac{2\bar{\theta}}{3(1+c)} & \text{if } q_G \leq q_B; \\ \dfrac{q_G}{3} + \dfrac{2\underline{\theta}}{3(1+c)} & \text{if } q_G > q_B. \end{cases}$$

We can now substitute the best reply correspondences into the expressions for the black market profits and find the quality of the public firm that makes the black market firm indifferent between offering the low or the high quality product. Following this procedure we find that this quality is given by:

$$\tilde{q}_G = \frac{\bar{\theta} + \underline{\theta}}{2(1+c)}$$

so that for any given quality of the public firm, the best reply function of the black market firm is given by:

$$\overline{q}_B = \begin{cases} \dfrac{q_G}{3} + \dfrac{2\bar{\theta}}{3(1+c)} & \text{if } q_G \leq \dfrac{\bar{\theta}+\underline{\theta}}{2(1+c)}; \\ \dfrac{q_G}{3} + \dfrac{2\underline{\theta}}{3(1+c)} & \text{if } q_G > \dfrac{\bar{\theta}+\underline{\theta}}{2(1+c)}. \end{cases}$$

From the expression above, it becomes clear that both qualities will never be equal in equilibrium, as the black market profits depend crucially on quality differentiation.

The same procedure can be followed to characterize the best reply correspondence of the public firm.

2.6.2 Second order properties

We want to verify that in the first stage of the game, the objective functions for the government and the black market firm are concave in their respective qualities, for the given price equilibrium in the second stage of the game.

Public firm supplies the low quality product

The second order properties for the black market firm are as follows:

$$\begin{aligned} \frac{\partial^2 \Pi(q_B, q_G)}{\partial q_B^2} &= \frac{1+c}{2(\lambda-2)^2}\big[-4\bar{\theta} + (1+c)(3q_B + q_G)\big], \\ \frac{\partial^2 \Pi(q_B, q_G)}{\partial q_B \partial q_G} &= \frac{(1+c)^2}{8}(q_B - q_G) > 0. \end{aligned}$$

For the black market profits to be concave on its own quality it must verify that:

$$q_B < \frac{4\bar{\theta}}{3(1+c)} - \frac{q_G}{3}.$$

The second order properties for government are as follows:

$$\frac{\partial^2 W(q_B, q_G)}{\partial q_G^2} = \frac{(1+c)}{4(2-\lambda)^2}[4(\bar{\theta} - (2-\lambda)^2) - (1+c)(q_B + 3q_G)],$$
$$\frac{\partial^2 W(q_B, q_G)}{\partial q_G \partial q_B} = \frac{(1+c)^2}{4(2-\lambda)^2}(q_B - q_G) > 0.$$

So that the welfare function is concave in the quality chosen by the government as long as it verifies that:

$$q_G > \frac{4(\bar{\theta} - (2-\lambda)^2)}{3(1+c)} - \frac{q_B}{3}.$$

There exists then a pattern of complementarity between qualities offered by the public and black market firm. For the equilibrium qualities, both objective functions satisfy the conditions for concavity.

Public firm supplies the high quality product

The second order properties associated to the profit function of the black market firm are as follows:

$$\frac{\partial^2 \Pi(q_B, q_G)}{\partial q_B^2} = \frac{1+c}{2(\lambda-2)^2}\left[4\underline{\theta} - (1+c)(3q_B + q_G)\right],$$
$$\frac{\partial^2 \Pi(q_B, q_G)}{\partial q_B \partial q_G} = \frac{(1+c)^2}{8}(q_G - q_B) > 0.$$

For the black market profits to be concave on its own quality it must hold that:

$$q_B > \frac{4\underline{\theta}}{3(1+c)} - \frac{q_G}{3}.$$

The second order properties associated to the welfare function are as follows:

$$\frac{\partial^2 W(q_B, q_G)}{\partial q_G^2} = \frac{(1+c)}{4(2-\lambda)^2}[-4(\underline{\theta} + (2-\lambda)^2) + (1+c)(q_B + 3q_G)],$$
$$\frac{\partial^2 W(q_B, q_G)}{\partial q_G \partial q_B} = \frac{(1+c)^2}{4(2-\lambda)^2}(q_G - q_B) > 0.$$

So that the welfare function is concave in the quality chosen by the government as long as it holds that:

$$q_G < \frac{4(\underline{\theta} + (2-\lambda)^2)}{3(1+c)} - \frac{q_B}{3}$$

There exists then a complementarity pattern between the qualities offered by the public and black market firm. For the equilibrium qualities, both objective functions satisfy the conditions for concavity.

2.6.3 Characterization of the equilibrium in a mixed duopoly with simultaneous competition when the public firm supplies the high quality product.

When the public firm supplies the high quality product, the demands faced by the black market and public firm are given respectively by:

$$D_B = \hat{\theta} - \underline{\theta},$$
$$D_G = \bar{\theta} - \hat{\theta}.$$

The public firm now serves the consumers with higher taste for quality, so that the objective functions of the black market firm and the government are now given respectively by:

$$\Pi_B = \left[p_B - c\frac{q_B^2}{2}\right][\hat{\theta} - \underline{\theta}]$$
$$W = \int_{\underline{\theta}}^{\hat{\theta}} \theta q_B - (1+c)\frac{q_B^2}{2}\, d\theta + \int_{\hat{\theta}}^{\bar{\theta}} \theta q_G - (1+c)\frac{q_G^2}{2}\, d\theta - (1-\lambda)\left[p_B - c\frac{q_B^2}{2}\right][\hat{\theta} - \underline{\theta}]$$

In the second stage of the game, the black market and public firm set their prices, for given qualities. The FOCs for their respective objective functions with respect to prices are given respectively by:

$$\frac{\partial \Pi_B}{\partial p_B} = \frac{4p_B - 2p_G + 2\underline{\theta}(q_G - q_B) + \beta(q_B^2 - q_G^2) - cq_B^2}{2(q_B - q_G)} = 0$$
$$\frac{\partial W}{\partial p_G} = \frac{2(p_G - \lambda p_B) + (1-\beta)(q_B^2 - q_G^2) + c(\lambda q_B^2 - q_G^2)}{2(q_B - q_G)} = 0$$

Solving for p_B and p_G we get the best reply correspondences:

$$\overline{p}_B = \frac{2p_G + cq_B^2 - 2\underline{\theta}(q_G - q_B) + \beta(q_G^2 - q_B^2)}{4}$$
$$\overline{p}_G = \frac{cq_G^2 - (1-\beta)(q_B^2 - q_G^2) + \lambda(2p_B - cq_B^2)}{2}$$

Solving the system of equations above we get the Nash equilibrium prices that correspond to the expressions (2.21) and (2.22) in the main text:

$$p_B^{**} = \frac{c(q_G^2 + (1-\lambda)q_B^2) - 2\underline{\theta}(q_G - q_B) + q_G^2 - q_B^2}{2(2-\lambda)},$$
$$p_G^{**} = \frac{2cq_G^2 + 2(1-\beta)(q_G^2 - q_B^2) + \lambda[-2\underline{\theta}(q_G - q_B) + \beta(q_G^2 - q_B^2) - cq_B^2]}{2(2-\lambda)}.$$

Following the same procedure as before, we plug in the prices above into the objective function of the government and the black market firm and then solve for their respective qualities. The FOC for the black market firm is as follows:

$$\frac{\partial \Pi_B}{\partial q_B} = \frac{[2\underline{\theta} + (1+c)(q_G - 3q_B)][2\underline{\theta} - (1+c)(q_B + q_G)]}{4(2-\lambda)^2} = 0$$

Solving for q_B we get the best reply function of the black market firm for any given quality set by the public firm:

$$\overline{q}_B = \frac{q_G}{3} + \frac{2\underline{\theta}}{3(1+c)}$$

The FOC for the government is as follows:

$$
\begin{aligned}
\frac{\partial W}{\partial q_G} =& \frac{1}{8(2-\lambda)^2} \times \Big\{ 4\underline{\theta}^2 + (1+c)^2(-q_B^2 + 2q_B q_G + 3q_G^2) \\
&+ 4(2-\lambda)^2(1+2\underline{\theta} - 2q_G(1+c)) - 8\underline{\theta} q_G(1+c) \Big\} = 0
\end{aligned}
$$

Solving for q_G we get the best reply function for any given quality set by the black market:

$$
\begin{aligned}
&\overline{q}_G = \frac{1}{3(1+c)} \times \Big\{ 4\underline{\theta} - q_B(1+c) + 4(2-\lambda)^2 + \\
&+ 2\sqrt{[\underline{\theta} - q_B(1+c)]^2 + (2-\lambda)^2[4(2-\lambda)^2 - 2(\underline{\theta} - q_B(1+c)) - 3]} \Big\}
\end{aligned}
$$

Combining the best reply functions above, we get the equilibrium qualities when the public firm supplies the high quality product:

$$
\begin{aligned}
q_B^{**} =& \frac{8\underline{\theta} + 3(2-\lambda)^2 - (2-\lambda)\sqrt{1+9(\lambda^2 - 4\lambda + 3)}}{8(1+c)}, \\
q_G^{**} =& \frac{8\underline{\theta} + 9(2-\lambda)^2 - 3(2-\lambda)\sqrt{1+9(\lambda^2 - 4\lambda + 3)}}{8(1+c)}.
\end{aligned}
$$

For the qualities above, that corresponds to the expressions (2.23) and (2.24) in the main text, no firm has incentives to deviate.

2.6.4 Optimal prices and qualities when the public firm has a first-mover advantage

Here we analyze in more detail the first stage of the game, when the government has a first-mover advantage. The second stage of the game is identical to the one presented before, when firms compete simultaneously in prices.

Public firm supplies the low quality product

From the second stage of the game we had the following equilibrium prices that correspond to the expressions given by (2.13) and (2.14) in the main text:

$$p_B^* = \frac{c(q_G^2 + (1-\lambda)q_B^2) + 2\bar{\theta}(q_B - q_G) - (q_B^2 - q_G^2)}{2(2-\lambda)},$$
$$p_G^* = \frac{2cq_G^2 - 2(1-\beta)(q_B^2 - q_G^2) + \lambda[2\bar{\theta}(q_B - q_G) - \beta(q_B^2 - q_G^2) - cq_B^2]}{2(2-\lambda)}.$$

Moving now to the first stage of the game, let's plug in the prices above into the objective functions of the black market firm and the government. Since nothing has changed for the black market firm, the best reply function is the same as with simultaneous competition and given by:

$$\overline{q}_B = \frac{q_G}{3} + \frac{2\bar{\theta}}{3(1+c)}$$

The government now chooses its quality taking into account the best reply of the black market firm. We then plug in the best reply above into the objective function of the government:

$$W(p_G^*, p_B^*, \overline{q}_B, q_G) = \int_{\underline{\theta}}^{\hat{\theta}} \theta q_G - (1+c)\frac{q_G^2}{2}\, d\theta + \int_{\hat{\theta}}^{\bar{\theta}} \theta \overline{q}_B - (1+c)\frac{\overline{q}_B^2}{2}\, d\theta - (1-\lambda)[p_B^* - c\frac{\overline{q}_B^2}{2}]D_B$$

FOC yields:

$$\frac{\partial W}{\partial q_G} = \frac{9(2-\lambda)^2[1 + 2\underline{\theta} - 2q_G(1+c)] - 8[q_G(1+c) - \bar{\theta}]^2}{18(2-\lambda)^2} = 0$$

Solving for q_G we get the following equilibrium quality for the public firm:

$$q_G^* = \frac{8\bar{\theta} - 9(2-\lambda)^2 + 3(2-\lambda)\sqrt{1 + 9(\lambda^2 - 4\lambda + 3)}}{8(1+c)}$$

which is the same quality offered when the public and the black market firm engage in simultaneous competition.

Since the best reply function of the black market is the same than with simultaneous competition and the equilibrium prices from the first stage of the game are also the same, we arrive to the conclusion that adding a first-mover advantage in qualities leads to the same equilibrium than when we had simultaneous competition.

Public firm supplies the high quality product

We proceed in the same way as in the previous section. From the second stage of the game the equilibrium prices are given by:

$$\begin{aligned} p_B^{**} =& \frac{c(q_G^2 + (1-\lambda)q_B^2) - 2\underline{\theta}(q_G - q_B) + q_G^2 - q_B^2}{2(2-\lambda)} \\ p_G^{**} =& \frac{2cq_G^2 + 2(1-\beta)(q_G^2 - q_B^2) + \lambda[-cq_B^2 - 2\underline{\theta}(q_G - q_B) + \beta(q_G^2 - q_B^2)]}{2(2-\lambda)} \end{aligned}$$

As before, nothing has changed for the black market firm with respect to the situation where firms engage in the second stage of the game in simultaneous competition, so that the best reply function for the black market firm remains the same and is equal to:

$$\overline{q}_B = \frac{q_G}{3} + \frac{2\underline{\theta}}{3(1+c)}$$

The government now chooses its quality taking into account the best reply of the black market firm. As before, we plug in the best reply above into the objective function of the government:

$$W(p_G^{**}, p_B^{**}, \overline{q}_B, q_G) = \int_{\underline{\theta}}^{\hat{\theta}} \theta\overline{q}_B - (1+c)\frac{\overline{q}_B^2}{2}\, d\theta + \int_{\hat{\theta}}^{\bar{\theta}} \theta q_G - (1+c)\frac{q_G}{2}\, d\theta - (1-\lambda)[p_B^{**} - c\frac{\overline{q}_B^2}{2}]D_B$$

FOC yields:

$$\frac{\partial W}{\partial q_G} = \frac{9(2-\lambda)^2[1 + 2\underline{\theta} - 2q_G(1+c)] + 8[q_G(1+c) - \underline{\theta}]^2}{18(2-\lambda)^2} = 0$$

Solving for q_G we get:

$$q_G^{**} = \frac{8\underline{\theta} + 9(2-\lambda)^2 - 3(2-\lambda)\sqrt{1 + 9(\lambda^2 - 4\lambda + 3)}}{8(1+c)}$$

which is the same quality offered when the public and black market firm engage in simultaneous competition, leading to the same equilibrium as before.

2.6.5 Welfare comparison between a public monopoly and a mixed duopoly

Let's consider a situation where a public firm is a monopolist and offers only one quality, versus the situation where it competes with the black market firm.

If the taste for THC content of the lowest consumer is such that he wants to participate in the market, then the government problem amounts to set the quality that maximizes social surplus. The government solves then the following problem:

$$\max_{q_G} W_M = \int_{\underline{\theta}}^{\bar{\theta}} \theta q_G - (1+c)\frac{q_G^2}{2} \, d\theta$$

Solving for the objective function of the government, we get the following FOC:

$$\frac{\partial W_M}{\partial q_G} = \frac{\bar{\theta}^2 - \underline{\theta}^2}{2} - q_G(\bar{\theta} - \underline{\theta} + c) = 0$$

The quality level that solves the problem above is then:

$$q_M = \frac{\bar{\theta} + \underline{\theta}}{2(1+c)} \tag{2.35}$$

This is the level of THC that maximizes the average utility of consumers. This allocation can be decentralized, for instance, by setting the price at marginal costs. The correspondent welfare level is given by:

$$W_M^* = \frac{(\bar{\theta} + \underline{\theta})^2}{8(1+c)}$$

The welfare that results from the mixed duopoly, presented in table 2.1, has the following expression:

$$W^*_{MD} = \frac{\underline{\theta}(1+\underline{\theta}) + (2-\lambda)[36(8\lambda+4) + 9\lambda^2(3\lambda-18) + (1+9(\lambda^2-4\lambda+3))^{3/2}}{64(1+c)}$$

Comparing welfare levels, we see that a mixed duopoly leads to a better outcome than a public monopoly who is restricted to offer only one quality:

$$\begin{aligned} W^*_{MD} - W^*_{M} =& \frac{1}{64(1+c)} \times 8(\bar{\theta}+\underline{\theta})^2 - 32\underline{\theta}(1+\underline{\theta}) + (\lambda-2)\Big(9\left(3\lambda^3 - 18\lambda^2 + 32\lambda - 16\right) + \\ & (9(\lambda-4)\lambda+28)\sqrt{9(\lambda-4)\lambda+28}\Big) > 0. \end{aligned}$$

From the welfare comparison above we see that the social surplus of introducing a new quality outweighs the costs associated to the existence of a black market.

2.6.6 Optimal qualities with price restriction

Simultaneous Competition

When supplying the low quality product, the government would like to set prices below marginal cost according to (2.12). If the public firm is restricted to make non-negative profits, then this restriction becomes binding, so that its best reply function has the following expression:

$$\overline{p}^r_G = \frac{cq_G^2}{2} \tag{2.36}$$

The objective of the black market firm is to maximize profits. Its best reply function is unaffected by this restriction and given by (2.11) as before. Combining the expressions (2.11) and (2.36), we have the following Nash equilibrium prices:

$$p^{r*}_B = \frac{c(q_B^2+q_G^2) + 2\bar{\theta}(q_B-q_G) - \beta(q_B^2-q_G^2)}{4}, \tag{2.37}$$

$$p^{r*}_G = \frac{cq_G^2}{2}. \tag{2.38}$$

In the first stage of the game the government solves the following problem:

$$\max_{q_G} W = \int_{\underline{\theta}}^{\hat{\theta}} \theta q_G - (1+c)\frac{q_G^2}{2}\, d\theta + \int_{\hat{\theta}}^{\bar{\theta}} \theta q_B - (1+c)\frac{q_B^2}{2}\, d\theta - (1-\lambda)[p_B - c\frac{q_B^2}{2}]D_B$$

while the black market firm maximizes profits as before.

Let's plug in the expressions for p_B^{r*} and p_G^{r*} into the profit function of the black firm given by (2.10) and in the expression for social welfare above and take first order conditions with respect to their respective qualities:

$$\begin{aligned}
\frac{\partial \Pi_B}{\partial q_B} =& \frac{[2\bar{\theta} + (\beta+c)(q_G - 3q_B)][2\bar{\theta} - (\beta+c)(q_B + q_G)]}{16} = 0 \\
\frac{\partial W}{\partial q_G} =& \frac{1}{32} \times \Big\{16(1+2\underline{\theta}) - 4\bar{\theta}^2(2\lambda+1) + \\
& (\beta+c)(q_B^2 - 2q_Bq_G - 3q_G^2)(\beta(2\lambda-3) + c(1+2\lambda) + 4) \\
& + 8\underline{\theta} q_G(\beta(2\lambda-1) + (2\lambda+1)c + 2) - 2q_G(-8\lambda(\beta+c-1) + 4\beta + 12c)\Big\} = 0
\end{aligned}$$

Solving the system above yields the optimal qualities when the government faces a price restriction and supplies the low quality product. The optimal qualities are given by the following expressions:

$$\begin{aligned}
q_B^{r*} =& \frac{1}{4(\beta+c)(4-3\beta+c+2\lambda(\beta+c))} \times \Big\{\bar{\theta}(7-9\beta+4c+8\lambda(\beta+c)) - 2c \\
& - \sqrt{17 + 9\underline{\theta}^2(\beta-1)^2 + \beta^2(-15+16\lambda) + \beta(26-32\lambda-4c+2\underline{\theta}(\beta-1)(5+\beta(-15+16\lambda) + 2(-5+8\lambda)c) + 4c(15+7c-4\lambda(2+c))}\Big\}
\end{aligned} \qquad (2.39)$$

$$\begin{aligned}
q_G^{r*} =& \frac{1}{4(\beta+c)(4-3\beta+c+2\lambda(\beta+c))} \times \Big\{\bar{\theta}(7-3\beta+4c+8\lambda(\beta+c)) - 14c - 18 \\
& + 3\sqrt{17 + 9\underline{\theta}^2(\beta-1)^2 + \beta^2(-15+16\lambda) + \beta(26-32\lambda-4c+2\underline{\theta}(\beta-1)(5+\beta(-15+16\lambda) + 2(-5+8\lambda)c) + 4c(15+7c-4\lambda(2+c))}\Big\}
\end{aligned} \qquad (2.40)$$

So that the equilibrium when the public firm supplies the low quality product and is constrained not to make non-negative profits is characterized by equations (2.37), (2.38), (2.39) and (2.40).

Stackelberg Competition

The second stage of the game remains the same, so that p_B^{r*} and p_G^{r*} are the optimal prices conditional on quality choices.

For given p_B^{r*} and p_G^{r*}, the black market firm solves the following problem:

$$\max_{q_B} \Pi_B(p_B^{r*}, p_G^{r*}, q_B, q_G) = [p_B^{r*} - c\frac{q_B^2}{2}][\bar{\theta} - \hat{\theta}]$$

For the black market firm nothing has changed, having the following best reply function:

$$\overline{q}_B^r = \frac{q_G}{3} + \frac{2\bar{\theta}}{3(\beta + c)}$$

To set the optimal quality q_G, the government takes into account the best reply function of the black market firm. In the first stage of the game the government solves the following problem:

$$\max_{q_G} W(p_B^{r*}, p_G^{r*}, q_B, q_G) = \int_{\underline{\theta}}^{\hat{\theta}} \theta q_G - (1+c)\frac{q_G^2}{2}\, d\theta + \int_{\hat{\theta}}^{\bar{\theta}} \theta q_B^r - (1+c)\frac{{q_B^r}^2}{2}\, d\theta - (1-\lambda)[p_B^{r*} - c\frac{{q_B^r}^2}{2}]D_B$$

FOC yields:

$$\begin{aligned}\frac{\partial W}{\partial q_G} =& \frac{1}{18} \times -2\bar{\theta}^2(2\lambda + 1) + 2\underline{\theta}(2q_G(\beta(2\lambda - 1) + c(1 + 2\lambda) + 2) + 9) \\ &- 2q_G^2(\beta + c)(\beta(2\lambda - 3) + c(1 + 2\lambda) + 4) + 2q_G(\beta(4\lambda - 2) + c(4\lambda - 7) - 5) + 9 = 0\end{aligned}$$

Solving condition above for q_G yields the optimal quality for the public firm. The equilibrium qualities are given by:

$$\begin{aligned}
q_B^{S*} &= \frac{q_G}{3} + \frac{2\bar{\theta}}{3(\beta+c)} \\
q_G^{S*} &= \frac{2\bar{\theta}(2+c(1+2\lambda)+\beta(-1+2\lambda)-9-7c}{4(\beta+c)(4-3\beta+c+2\lambda(\beta+c))} \\
&\quad \frac{\sqrt{16\underline{\theta}^2(\beta-1)^2+4\underline{\theta}(\beta-1)(\beta(18\lambda-19)+18\lambda c-9c+10)}}{4(\beta+c)(4-3\beta+c+2\lambda(\beta+c))} \\
&\quad + \frac{\sqrt{\beta^2(36\lambda-38)+\beta(76-72\lambda)-9(4\lambda-7)c(c+2)+25}}{4(\beta+c)(4-3\beta+c+2\lambda(\beta+c))}
\end{aligned}$$

Comparing the equilibrium quality when the government engages in simultaneous competition q_G^{r*}, with the quality that results from having a first-mover advantage q_G^{S*}, we see that they will only be equal when $\beta = 1$, what is behind the result in proposition 2.3.6.

2.6.7 Optimal qualities when legalization attracts additional consumers

Consider now that legalization brings along a new badge of consumers that exclusively buy from the public firm. The public and black market firm set first their respective qualities and then they set a price for their products. As before, we solve the game by backward induction.

Public firm supplies the low quality product

Demands faced by the black market and public firm are given respectively by:

$$\begin{aligned}
\hat{D}_B &= \gamma(\bar{\theta} - \hat{\theta}) \\
\hat{D}_G &= \gamma(\hat{\theta} - \underline{\theta}) + (1-\gamma)(\bar{\theta} - \underline{\theta})
\end{aligned}$$

where $\hat{\theta}$ is the indifferent consumer in (2.2).

So that the black market firm and the government compete for a market of size γ, while the government exclusively serves a market of size $1-\gamma$.

The objective functions of the black market firm and the government are now given respectively by:

$$
\begin{aligned}
\hat{\Pi}_B =& \Big[p_B - c\frac{q_B^2}{2}\Big]\gamma[\hat{\theta} - \underline{\theta}] \\
\hat{W} =& \gamma\Big[\int_{\underline{\theta}}^{\hat{\theta}} \theta q_G - (1+c)\frac{q_G^2}{2}\, d\theta + \int_{\hat{\theta}}^{\bar{\theta}} \theta q_B - (1+c)\frac{q_B^2}{2}\, d\theta\Big] - (1-\lambda)[p_B - c\frac{q_B^2}{2}]\hat{D}_B \\
&+ (1-\gamma)\Big[\int_{\underline{\theta}}^{\bar{\theta}} \theta q_G - (1+c)\frac{q_G^2}{2}\, d\theta\Big]
\end{aligned}
$$

In the second stage of the game, the black market and public firm set their prices, for given qualities. The FOCs for their respective objective functions with respect to prices are given respectively by:

$$
\begin{aligned}
\frac{\partial \hat{\Pi}_B}{\partial p_B} =& \gamma\frac{2(p_G - 2p_B) + 2\bar{\theta}(q_B - q_G) - \beta(q_B^2 - q_G^2) + cq_B^2}{2(q_B - q_G)} = 0 \\
\frac{\partial \hat{W}}{\partial p_G} =& \gamma\frac{2(\lambda p_B - p_G) - (1-\beta)(q_B^2 - q_G^2) + c(q_G^2 - \lambda q_B^2)}{2(q_B - q_G)} = 0
\end{aligned}
$$

Solving for p_B and p_G we get the best reply correspondences, that coincide with expressions (2.11) and (2.12) in the main text. Consequently, we get the Nash equilibrium prices that correspond to the expressions (2.21) and (2.22) in the main text. Let's plug in these prices into the objective function of the government and the black market firm and then solve for their respective qualities. The FOC for the black market firm is as follows:

$$
\frac{\partial \hat{\Pi}_B}{\partial q_B} = \gamma\frac{[2\bar{\theta} + (1+c)(q_G - 3q_B)][2\bar{\theta} - (1+c)(q_B + q_G)]}{4(2-\lambda)^2} = 0
$$

Solving for q_B we get the best reply function of the black market firm for any given quality set by the government:

$$
\bar{\hat{q}}_B = \frac{q_G}{3} + \frac{2\bar{\theta}}{3(1+c)}. \tag{2.41}
$$

The FOC for the objective of the government is as follows:

$$\begin{aligned}\frac{\partial \hat{W}}{\partial q_G} =& \frac{1}{8(2-\lambda)^2} \times \Big\{\gamma\left(-4\bar{\theta}^2 + 8\bar{\theta}q_G(1+c) + (1+c)^2\left(q_B^2 - 2q_Bq_G - 3q_G^2\right)\right) \\ & + 4(2-\lambda)^2(1+2\underline{\theta} - 2q_G(1+c))\Big\} = 0\end{aligned}$$

Solving for q_G we get the best reply function for any given quality set by the black market:

$$\begin{aligned}\bar{\hat{q}}_G =& \frac{1}{3\gamma(1+c)} \times \Big\{\gamma(4\bar{\theta} - q_B(1+c)) - 4(2-\lambda)^2 + \\ & + 2\sqrt{\gamma^2[\bar{\theta} - q_B(1+c)]^2 + [2-\lambda]^2[4(2-\lambda)^2 - \gamma(2(\bar{\theta} - q_B(1+c)) + 3)]}\Big\}. \end{aligned} \tag{2.42}$$

Solving the system of equations given by the best reply correspondences $\overline{q}_B$ and $\overline{q}_G$ we get the following equilibrium qualities:

$$\begin{aligned}\hat{q}_B^* =& \frac{8\gamma\bar{\theta} - 3(2-\lambda)^2 + (2-\lambda)\sqrt{9(2-\lambda)^2 - 8\gamma)}}{8\gamma(1+c)}, \\ \hat{q}_G^* =& \frac{8\gamma\bar{\theta} - 9(2-\lambda)^2 + 3(2-\lambda)\sqrt{9(2-\lambda)^2 - 8\gamma)}}{8\gamma(1+c)}.\end{aligned}$$

Qualities above correspond respectively to expressions (2.26) and (2.27) in the main text.

Public firm supplies the high quality product

The public firm now serves the consumers with more willingness to pay for quality so that demands are given by:

$$\begin{aligned}\tilde{D}_B &= \gamma(\hat{\theta} - \underline{\theta}) \\ \tilde{D}_G &= \gamma(\bar{\theta} - \hat{\theta}) + (1-\gamma)(\bar{\theta} - \underline{\theta})\end{aligned}$$

where $\hat{\theta}$ is the indifferent consumer in (2.2).

The objective functions of the black market firm and the government are now given respectively by:

$$\begin{aligned}
\tilde{\Pi}_B =& \Big[p_B - c\frac{q_B^2}{2}\Big]\gamma[\hat{\theta} - \underline{\theta}] \\
\tilde{W} =& \gamma\Big[\int_{\hat{\theta}}^{\bar{\theta}} \theta q_G - (1+c)\frac{q_G^2}{2}\, d\theta + \int_{\underline{\theta}}^{\hat{\theta}} \theta q_B - (1+c)\frac{q_B^2}{2}\, d\theta\Big] - (1-\lambda)[p_B - c\frac{q_B^2}{2}]\tilde{D}_B \\
&+ (1-\gamma)\Big[\int_{\underline{\theta}}^{\bar{\theta}} \theta q_G - (1+c)\frac{q_G^2}{2}\, d\theta\Big]
\end{aligned}$$

In the second stage of the game, the black market and public firm set their prices, for given qualities. The FOCs for their respective objective functions with respect to prices are given respectively by:

$$\begin{aligned}
\frac{\partial \tilde{\Pi}_B}{\partial p_B} =& \gamma\frac{2(p_G - 2p_B) + 2\bar{\theta}(q_B - q_G) - \beta(q_B^2 - q_G^2) + cq_B^2}{2(q_B - q_G)} = 0 \\
\frac{\partial \tilde{W}}{\partial p_G} =& \gamma\frac{2(\lambda p_G - p_B) - (1-\beta)(q_B^2 - q_G^2) + c(q_G^2 - \lambda q_B^2)}{2(q_B - q_G)} = 0
\end{aligned}$$

Solving for p_B and p_G we get the best reply correspondences:

$$\begin{aligned}
\bar{\tilde{p}}_B =& \gamma\frac{2p_G + cq_B^2 - 2\underline{\theta}(q_G - q_B) + \beta(q_G^2 - q_B^2)}{4} \\
\bar{\tilde{p}}_G =& \ \gamma\frac{cq_G^2 - (1-\beta)(q_B^2 - q_G^2) + \lambda(2p_B - cq_B^2)}{2}
\end{aligned}$$

Consequently, we get the Nash equilibrium prices that correspond to the expressions (2.21) and (2.22) in the main text. Let's plug in these prices into the objective function of the government and the black market firm and then solve for their respective qualities. The FOC for the black market firm is as follows:

$$\frac{\partial \tilde{\Pi}_B}{\partial q_B} = \gamma\frac{[2\underline{\theta} + (1+c)(q_G - 3q_B)][2\underline{\theta} - (1+c)(q_B + q_G)]}{4(2-\lambda)^2} = 0$$

Solving for q_B we get the best reply function of the black market firm for any given quality set by the government:

$$\bar{\tilde{q}}_B = \frac{q_G}{3} + \frac{2\underline{\theta}}{3(1+c)}. \tag{2.43}$$

The FOC for the objective of the government is as follows:

$$\frac{\partial \tilde{W}}{\partial q_G} = \frac{1}{8(2-\lambda)^2} \times \Big\{ \gamma \left(4\underline{\theta}^2 - 8\underline{\theta} q_G (1+c) - (1+c)^2 \left(q_B^2 - 2q_B q_G - 3q_G^2\right)\right) + 4(2-\lambda)^2 (1 + 2\underline{\theta} - 2q_G(1+c)) \Big\} = 0$$

Solving for q_G we get the best reply function for any given quality set by the black market:

$$\bar{\tilde{q}}_G = \frac{1}{3\gamma(1+c)} \times \Big\{ \gamma(4\underline{\theta} - q_B(1+c)) + 4(2-\lambda)^2 + 2\sqrt{\gamma^2[\underline{\theta} - q_B(1+c)]^2 + [2-\lambda]^2[4(2-\lambda)^2 - \gamma(2(\underline{\theta} - q_B(1+c)) + 3)]} \Big\}. \tag{2.44}$$

Solving the system of equations given by the best reply correspondences $\bar{\tilde{q}}_B$ and $\bar{\tilde{q}}_G$, we get the following equilibrium qualities:

$$\tilde{q}_B^{**} = \frac{8\gamma\underline{\theta} - 3(2-\lambda)^2 + (2-\lambda)\sqrt{9(2-\lambda)^2 - 8\gamma)}}{8\gamma(1+c)},$$
$$\tilde{q}_G^{**} = \frac{8\gamma\underline{\theta} - 9(2-\lambda)^2 + 3(2-\lambda)\sqrt{9(2-\lambda)^2 - 8\gamma)}}{8\gamma(1+c)}.$$

Qualities above correspond respectively to expressions (2.28) and (2.29) in the main text.

Chapter 3

Optimal drug policy under cross-border shopping *

LUIS RODRIGO ARNABAL

Abstract
The recent wave of marijuana legalization brought along unintended consequences for neighboring jurisdictions where this harmful drug remained illegal. This paper studies how the scheme towards harmful drugs adopted by a symmetric neighboring jurisdiction, impacts in the domestic optimal drug policy in a imperfectly competitive market for harmful drugs, characterized by the presence of a black market firm and where consumers may engage in cross-border shopping. In our setting, a drug policy consists in adopting either a scheme of prohibition or one of legalization, and to decide how much to invest in enforcement activities to tackle black market supply. We consider a negative social valuation for consumption of harmful drugs, as well as for the profits generated in the black market. We find that for a low (high) concern for consumption of harmful drugs, both jurisdictions adopt in equilibrium a scheme of legalization (prohibition). More interestingly, for an intermediate social valuation for consumption of harmful drugs, different scenarios may arise, that can for instance explain why two symmetric jurisdictions may end up adopting different schemes towards harmful drugs. Furthermore, under some circumstances governments may face a prisoner's dilemma, where the resulting equilibrium is one where both jurisdictions legalize the harmful drug, despite that both sticking to a scheme of prohibition would yield a better outcome.

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3.1 Introduction

We are currently witnessing a wave of marijuana legalization across the world, where the main arguments that justify this drug policy shift are the increase in tax revenue and the reduction of black market activity.[1] On the other hand, marijuana legalization is expected to increase its consumption with its consequent impact on health costs. While legalization of a harmful drug such as marijuana has its advantages and disadvantages for the local authority, it causes some undesired consequences for neighboring jurisdictions. Our analysis will focus on how the decision of a neighboring jurisdiction of whether to legalize or forbid a harmful drug affects the domestic optimal drug policy.

When one jurisdiction makes a harmful drug legal, while its neighbor sticks to a scheme of prohibition, consumers from the latter jurisdiction may find attractive to engage in cross-border shopping. By doing so, legalization of a harmful drug in a neighboring jurisdiction undermines the efforts made by the local government to discourage its consumption. Indeed, strong spillover effects of marijuana legalization to neighboring states have been recently documented in the US. For instance, Hansen et al. (2017) find a significant decline in legal marijuana sales in Washington, upon the legalization of recreational marijuana in the neighboring state of Oregon. In this line, Hao and Cowan (2017) find an increase in self-reported cannabis use in neighboring US states where recreational marijuana has been legalized. Potential spillovers may also arise in Europe, with the current prospect of legalization of production and consumption of recreational marijuana in Luxembourg, which is for instance in sharp contrast with the French policy, where consumption of illegal substances is currently fined with 200 euros.

In this paper we study the interplay between the optimal policy towards harmful drugs adopted by two symmetric neighboring jurisdictions, in an imperfectly competitive market characterized by the presence of a black market monopolist in each jurisdiction. A drug policy consist in first to decide whether to adopt a scheme of prohibition or one of legalization towards harmful drugs, and second to decide how much to invest in enforcement activities, that target exclusively black market supply. The interaction between the optimal policies chosen by the neighboring economies will be processed through the channel of cross-border shopping. We characterize the different equilibrium configurations regarding

[1]By 2020 recreational cannabis was already legal in Canada, Georgia, South Africa, and Uruguay, as well as in several US states, the District of Columbia, and on the Australian Capital Territory.

the optimal drug policy, that will depend on the social valuation for harmful drugs and on transportation costs. We find that for an intermediate social valuation for consumption of harmful drugs, the equilibria that arise are not always socially optimal. Moreover, even when governments have the same social valuation for consumption of harmful drugs, under some conditions they opt for different schemes towards harmful drugs, which may be a rationale for the existence of different regimes across neighboring jurisdictions. Though our analysis is static, this asymmetric equilibrium configuration, where a government profits from unilaterally deviating to a scheme of legalization, could be a potential explanation for the recent race towards the development of a legal marijuana industry in North America.

In our formal analysis we consider that under a scheme of prohibition there will be one black market monopolist in each jurisdiction.[2] In practice, marijuana legalization has restricted the number of regulated suppliers, giving retailers significant local market power.[3] To make the analysis tractable, we consider that legalization translates into the participation of one regulated supplier into the market for harmful drugs. These assumptions will allow us to understand how the decision to legalize a harmful drug by a local authority affects consumption in each jurisdiction. Moreover, we make the simplifying assumption that both legal and illegal firms face the same marginal production costs, being their objective to maximize profits, competing in quantities à la Cournot. The social valuation for consumption of harmful drugs is considered to be negative. This is for instance the case when a government deems that health costs associated to its consumption outweigh the perceived benefits. Government intervention is also motivated by the fact that black market profits generate a negative externality to society through an increase in crime and violence. This is in line with the study of Gavrilova et al. (2017), who document a reduction in violence on Mexican border states as a consequence of medical marijuana legalization. This violence reduction is explained by the fact that an increase in supply of harmful drugs reduces its profitability, making it less worthy to compete for monopolizing the market.[4]

[2]The drug dealing industry is usually dominated by cartels who restrict entry or limit competition by exerting violence, see Miron and Zwiebel (1995) for a brief discussion on the link between cartelization and violence.

[3]For an estimation of the market power in the legal marijuana industry in the US, see Hollenbeck and Uetake (2018).

[4]See for instance Castillo et al. (2014), for an analysis on how the reduction of cocaine supply increases violence in Mexico, particularly when there are at least two competing cartels.

Regarding the timing, each government will first choose whether it will adopt a scheme of prohibition or one of legalization towards harmful drugs, and then will set their optimal enforcement investments accordingly. For a given scheme towards harmful drugs and enforcement investments adopted in each jurisdiction, firms will decide their supply. For a given total supply, consumers will decide how much of the harmful drug to consume. We focus on pure strategies, where the concept of equilibrium is subgame perfect Nash equilibrium.

This paper relates to the literature on optimal taxation of harmful goods in the presence of cross-border shopping. In this line, the study of Aronsson and Sjögren (2010) shows how the possibility of importing alcohol or allocating time to produce it illegally limits the scope of a corrective tax on alcohol. A similar point is raised by Goolsbee et al. (2010) who argue that US consumers may avoid domestic taxes on cigarettes through tax-free purchases on the internet. The study of Kotakorpi (2009) illustrates how cross-border shopping reduces the feasibility of implementing paternalistic taxation. In our work, the possibility to engage in cross-border shopping will not only undermine the efficacy of the investment in enforcement activities that target local black market supply, but also affect the decision for the local authority to legalize or forbid the harmful drug. This policy brings to the picture a new trade-off: introducing a regulated supplier will increase competition, and therefore reduce black market profits while generating tax revenue, but it will do so at the expense of increasing total supply, what in turn reduces prices and consequently boosts total consumption of the harmful drug. While the choice of enforcement investments resembles the tax choice in a tax competition setting, the possibility to make a harmful drug legal or illegal adds new dimension into the analysis, bringing new insights regarding the optimal drug policy.

More generally, this paper aims to contribute to the hot debate regarding the regulation of harmful drugs. Indeed, legalization of harmful drugs does not imply however lack of regulation, which is from a public health perspective desirable, Pacula et al. (2014). Even if consumption of harmful drugs is not socially desirable, the analysis of Becker et al. (2004) puts forward that legalizing drug use may lead to a better outcome than prohibition through savings in enforcement costs and an increase in tax revenue. The study of Caulkins and Kilmer (2016), points out that while legal marijuana may initially attract demand from neighboring jurisdictions, thus generating additional tax revenue, it may result in lower tax revenues via tax competition if neighboring jurisdictions also decide to legalize. We analyze formally the optimal drug policy in an imperfectly competitive market for harmful drugs characterized by the presence of black market firms and where consumers may engage in cross-border shopping in a neighboring jurisdiction.

The remainder of this paper is organized as follows. The following section presents the basic model and derives the optimal drug policy under autarky. In section 3 we derive the optimal drug policy when consumers may engage in cross-border shopping under different scenarios. Finally, section 4 compares the welfare outcomes of adopting different regimes towards harmful drugs. Section 5 concludes.

3.2 The basic model

3.2.1 Economic Environment

Consider two symmetric neighboring jurisdictions, where in each of them there exists a representative consumer who needs to decide how to allocate its initial wealth ω, between a harmful good q and a numeraire good z. Consumer from jurisdiction i will only engage in cross-border shopping in foreign neighboring jurisdiction j if the price for the harmful good is lower, *i.e.*, when $p_i > p_j$. In order to acquire the harmful good in the neighboring jurisdiction, the consumer must incur in additional transportation costs. We assume that this costs have the following quadratic form: $T(q_{ij}) = tq_{ij}^2/2$, with $t > 0$, and where the first subindex refers to the jurisdiction of origin of the consumer who purchases the good, while the second subindex denotes the jurisdiction where it has been produced, a notation that will be used through out the rest of the analysis.

Consumer from jurisdiction i maximizes his utility function U_i, that we consider to have the following expression:

$$U_i(q_i^D, z_i) = q_i^D - \frac{{q_i^D}^2}{2} + z_i, \tag{3.1}$$

subject to its budget constraint given by:

$$\omega_i = p_i q_{ii} + p_j q_{ij} + t\frac{q_{ij}^2}{2} + z_i + a_i, \tag{3.2}$$

where q_i^D denotes the total amount of the harmful drug demanded in jurisdiction i, while a_i denotes a lump sum tax imposed on consumer of jurisdiction i.

Solving the problem faced by consumers in jurisdictions i and j, we get following demands for harmful drugs:

$$q_i^D = q_{ii} + q_{ij} = 1 - p_i, \tag{3.3}$$

$$q_j^D = q_{jj} + q_{ji} = 1 - p_j. \tag{3.4}$$

The expressions for the direct demands above will be used throughout the rest of our analysis, where for ease of presentation its derivation is presented in the Appendix 3.6.1.

Regarding supply, there will be in each jurisdiction one black market monopolist, while the decision to legalize the harmful drug implies the introduction of one regulated firm into the market. The illegal and legal supply will be denoted respectively by x and y. We assume for simplicity that both types of firms will face the same marginal production costs c. The government will decide how much to invest in costly enforcement activities e, that target exclusively black market supply. The illegal and legal firms maximize profits Π, having respectively the following objective functions:

$$\Pi^x(q_i^S, q_j^S) = \Big[p(q_i^S, q_j^S) - (c+e)\Big]x, \tag{3.5}$$

$$\Pi^y(q_i^S, q_j^S) = \Big[p(q_i^S, q_j^S) - c\Big]y. \tag{3.6}$$

where the supraindex on the profit function denotes whether it refers to a legal or an illegal firm, while q_i^S and q_j^S refer respectively to the quantities supplied in the local and foreign economy.

Solving the problems of the firms together with the problems of the consumers, we get the equilibrium demands and supplies for given enforcement investments, that we denote respectively by $q^{D*}(e)$ and $q^{S*}(e)$.

For the government, the costs associated to the investment in enforcement activities are assumed to have the following quadratic expression: $C(e) = e^2/2$. The objective of the government in each jurisdiction is given by:

$$W(e) = -\alpha q^{D*}(e) - \Pi^x(e) - a, \tag{3.7}$$

where $\alpha > 0$ is a parameter that captures how socially undesirable is the consumption of harmful drugs.

The associated resource constraint to the government problem is the following:

$$a \geq \frac{e^2}{2} - \Pi^y(e). \tag{3.8}$$

The first term in the objective of the government in (3.7), refers to the negative social valuation that the government has for consumption of harmful drugs. While we consider the parameter α to be positive, the reader must bear in mind that the government has a negative valuation for consumption of harmful drugs, so that a higher α translates into a higher undesirability for consumption of harmful drugs. The second term captures the fact that black market profits generates a welfare loss for society. Notice that the government assigns a unitary weight on the undesirability of black market profits for society. The last term refers to the cost imposed on the consumer through the lump sum tax used to fund enforcement activities, which is also assigned a unitary weight. How undesirable is the consumption of harmful drugs, should be then interpreted in relative terms to the social valuation for illegal profits and to the utility loss caused to the consumer by reducing its wealth through a lump sum tax. Notice also, from the resource constraint, that profits from the regulated firm, that are taxed away, are also devoted to fund enforcement investments, reducing the burden imposed on the consumer.

Since we consider that the social valuation for consumption of illicit drugs is negative, in a first-best situation consumption of harmful drugs would be zero. For welfare considerations, rather than taking into account the indirect utility of the consumer in their objective function, the government will care about the total consumption of harmful drugs as well as on the utility loss due to the lump sum tax imposes to fund enforcement activities in an effort to reduce supply of illegal drugs. It is this negative valuation for consumption of illicit drugs, and the externality generated by black market profits what motivates government intervention.

While our focus is in the interaction between the policies followed by the symmetric governments, we will first study the autarky case in order to understand the underlying trade-off between adopting a scheme of legalization or one of prohibition towards harmful drugs. Throughout our study the focus will be from the point of view of the domestic economy.

3.2.2 Autarky

We now proceed to compare the welfare outcomes of adopting a scheme of prohibition versus a scheme of legalization towards harmful drugs when consumers cannot engage in cross-border shopping.[5]

Prohibition

Under a scheme of prohibition there is only a black market firm supplying the harmful drug. The consumer in jurisdiction i will maximize his utility function in (3.1) subject to its budget constraint in (3.2). Solving the consumer problem we get the following demand for the harmful good:

$$q_i^D = 1 - p_i(x_i).$$

We then have the following inverse demand function:

$$p_i(x_i) = 1 - q_i^D = 1 - x_i.$$

To solve the black market firm problem we plug the inverse demand function above into its profit function in (3.5), and take the first order condition (FOC), what yields:

$$\frac{\partial \Pi_i^x}{\partial x_i} = 1 - 2x_i - c - e_i = 0.$$

Since there is only one supplier, the equilibrium quantity under a scheme of prohibition is given by:

$$q_i^{S*}(e_i) = q_i^{D*}(e_i) = \frac{1 - c - e_i}{2}. \tag{3.9}$$

Notice from the expression above that for the black market firm to remain active in equilibrium, enforcement investments must be such that $e_i < 1 - c$.

[5] The case of autarky is equivalent to a situation where consumers do not engage in cross-border shopping because transportation costs are too high.

Let's now analyze the government problem. The government maximizes its objective function in (3.7), subject to its resource constraint in (3.8). Taking FOC yields:

$$\frac{\partial W_i}{\partial e_i} = \frac{\alpha + (1 - c - e_i) - 2e_i}{2} = 0.$$

Consequently, the expression for the optimal enforcement under a scheme of prohibition is given by:

$$e_I^* = \frac{\alpha + (1 - c)}{3}. \tag{3.10}$$

Combining the equilibrium condition in (3.9) with the expression for the optimal enforcement above in (3.10) we see that if $\alpha \geq 2(1-c)$, the enforcement level that maximizes welfare will be such that the black market firm does not participate in the market, and in particular it will be given by:

$$\bar{e}_I = 1 - c. \tag{3.11}$$

To focus on the interesting case where supply of harmful goods is positive, we will assume through out the rest of our analysis that the social valuation for consumption of harmful drugs is such that under a scheme of prohibition the black market firm always remains active in equilibrium, *i.e.*, that α is lower than the following threshold:

$$\alpha_I = 2(1 - c). \tag{3.12}$$

Otherwise, if the social valuation for consumption of harmful drugs is bigger than the threshold above, the government would set the enforcement level such that the black market firm makes no profits, thus eliminating all supply of the harmful drug. Under this circumstance, legalization would always result in a welfare loss.

For $e_i = e_i^*$, the equilibrium quantities are given by:

$$q_i^*(e_I^*) = \frac{2(1-c) - \alpha}{6}. \tag{3.13}$$

Plugging the optimal enforcement level in (3.10) into the objective function in (3.7), we have that following expression for welfare under a scheme of prohibition:

$$W_I^*(e_I^*) = \frac{\alpha^2 - 4\alpha(1-c) - 2(1-c)^2}{12} < 0. \tag{3.14}$$

We now proceed to determine the welfare outcome of adopting a scheme of legalization, in order to be able to establish under which conditions the government will be better off by legalizing the harmful drug.

Legalization

Legalization materializes in the introduction of a regulated firm into the market for harmful drugs that is not subject to the enforcement activities carried out by the government. We assume that the consumer gets the same utility from buying from the legal or the illegal supplier, in other words, the their products are perceived as perfect substitutes:

$$q_i^D = x_i + y_i.$$

Solving the consumer problem in the same fashion as in the prohibition case yields the following inverse demand function under a scheme of legalization:

$$p_i(x_i, y_i) = 1 - x_i - y_i.$$

The black market and the regulated firm maximize their respective profits functions in (3.5) and (3.6). The FOCs are given respectively by:

$$\begin{aligned}\frac{\partial \Pi_i^x}{\partial x_i} =& 1 - 2x_i - y_i - c + e_i = 0, \\ \frac{\partial \Pi_i^y}{\partial y_i} =& 1 - 2y_i - x_i - c = 0.\end{aligned}$$

Solving the system of equations given by the FOCs of the black market and the legal firm above, yields the following equilibrium quantities as a function of the enforcement level chosen by the government:

$$x_L^*(e_i) = \frac{1-c-2e_i}{3}, \tag{3.15}$$

$$y_L^*(e_i) = \frac{1-c+e_i}{3}. \tag{3.16}$$

From the expression in (3.15) we now observe that under a scheme of legalization the black market firm will remain active as long as the enforcement is below the following threshold:

$$\bar{e}_L = \frac{1-c}{2}. \tag{3.17}$$

Legalization, by increasing supply and consequently lowering the price of the harmful drug, requires for a lower marginal cost per unit of enforcement to wipe out the black market firm. Indeed, if $e_i \geq \bar{e}_L$, the black market firm is inactive, and the regulated firm will set the monopoly quantity: $y_i^m = (1-c)/2$. Under a scheme of legalization total quantities will then be given by:

$$q_L^*(e_i) = \begin{cases} \dfrac{2(1-c)-e_i}{3} & \text{if } e_i < \bar{e}_L, \\ \dfrac{1-c}{2} & \text{if } e_i \geq \bar{e}_L. \end{cases} \tag{3.18}$$

Solving the government problem as for the prohibition case, we have that the expression for the optimal enforcement under a scheme of legalization is given by:

$$e_L^* = \begin{cases} \dfrac{\alpha+2(1-c)}{5} & \text{if } \alpha < \dfrac{1-c}{2}, \\ \dfrac{1-c}{2} & \text{if } \alpha \geq \dfrac{1-c}{2}. \end{cases} \tag{3.19}$$

Notice that the participation threshold for the black market firm under a scheme of legalization is now given by:

$$\alpha_L = \frac{1-c}{2}. \tag{3.20}$$

We then have the following equilibrium quantities:

$$x_L^*(e_L^*) = \begin{cases} \dfrac{-2\alpha + (1-c)}{15} & \text{if } \alpha < \alpha_L, \\ 0 & \text{if } \alpha \geq \alpha_L. \end{cases}$$

$$y_L^*(e_L^*) = \begin{cases} \dfrac{\alpha + 7(1-c)}{15} & \text{if } \alpha < \alpha_L, \\ \dfrac{1-c}{2} & \text{if } \alpha \geq \alpha_L. \end{cases}$$

We observe that legalization reduces black market supply dramatically, and that now a much lower social valuation for consumption of harmful drugs is required to eliminate black market supply.

Substituting the optimal enforcement given by (3.19) into the objective function of the government in (3.7) yields the following welfare outcome under a scheme of legalization:

$$W_L^*(e_L^*) = \begin{cases} \dfrac{\alpha^2 - 16\alpha(1-c) + 4(1-c)^2}{30} & \text{if } \alpha < \alpha_L, \\ \dfrac{-4\alpha(1-c) + (1-c)^2}{8} & \text{if } \alpha \geq \alpha_L. \end{cases} \tag{3.21}$$

Comparing the expression for welfare under a scheme of prohibition in (3.14) with the one in (3.21) above we have that:

$$\Delta W_A = W_L^*(e_L^*) - W_I^*(e_I^*) = \begin{cases} \dfrac{-\alpha^2 - 4\alpha(1-c) + 6(1-c)^2}{20} > 0 & \text{if } \alpha < \alpha_L, \\ \dfrac{-2\alpha^2 - 4\alpha(1-c) + 7(1-c)^2}{24} \gtreqless 0 & \text{if } \alpha \geq \alpha_L. \end{cases}$$

Whether legalization will end up being detrimental or beneficial for welfare will depend on how strong is the social valuation for consumption of harmful drugs. In particular, from the expression above it can be seen that to adopt a scheme of legalization will be detrimental for welfare if the social valuation for consumption of harmful drugs is above the following threshold:

$$\alpha^* = \left[\frac{3}{\sqrt{2}} - 1\right] \times (1-c). \tag{3.22}$$

The main result of this section is summarized in the following proposition:

Proposition 3.2.1 *If the social valuation for consumption of harmful drugs is above the threshold given by the expression in* (3.22)*, adopting a scheme of prohibition will be welfare improving versus a strategy of legalization of harmful drugs.*

We recall that the result above is derived under a setting where to tackle consumption of harmful drugs, the government's tools are limited to decide whether to legalize or forbid the harmful drug, and to choose how much to invest in enforcement activities that target black market supply exclusively.

3.3 Model with cross-border shopping

In this section we proceed to study how the decision to adopt a scheme of prohibition or one of legalization towards harmful drugs may be affected by the possibility to engage in cross-border shopping. The inverse demand functions faced by suppliers in jurisdiction i and j have now the following (symmetric) expressions:

$$p_i(q_i^S, q_j^S) = \frac{1 - q_j^S + (1+t)(1 - q_i^S)}{2+t}, \tag{3.23}$$

$$p_j(q_i^S, q_j^S) = \frac{1 - q_i^S + (1+t)(1 - q_j^S)}{2+t}. \tag{3.24}$$

The expressions above represent the inverse demand functions faced by the domestic and foreign firms respectively. They result from the consumers' problem when there is the possibility to engage in cross-border shopping, and where for ease of presentation its derivation is presented in the Appendix 3.6.1. We see from these expressions, that they depend on the local and foreign supply, where the weights assigned to each market depend on transportation costs.

In order to determine the equilibrium configurations, we first derive the optimal investment in enforcement activities chosen by each jurisdiction, conditional on the scheme towards harmful drugs adopted. We split the analysis into the three possible scenarios that arise depending on the scheme towards harmful drugs adopted by each government, and derive for each case the resulting welfare levels. In the following section we proceed to compare the resulting welfare outcomes depending on the schemes towards harmful drugs adopted.

3.3.1 Symmetric prohibition

Consider first the situation where both governments adopt a scheme of prohibition, meaning that the market for harmful drugs is dominated in each jurisdiction by a black market monopolist.

In jurisdiction i the black market firm solves the following problem:

$$\max_{x_i} \ \Pi_i^x = \big[p_i(x_i, x_j) - (c + e_i)\big] x_i$$

FOC yields:

$$\frac{\partial \Pi_i^x}{\partial x_i} = \frac{-(1+t)x_i + 1 - x_j + (1+t)(1-x_i)}{2+t} - c - e_i = 0.$$

From the condition above we have that for a given supply of the black market firm in the neighboring jurisdiction, the best reply function of the black market firm in jurisdiction i is given by:

$$\bar{x}_i(x_j) = \frac{1 - c - e_i}{2} + \frac{1 - x_j - c - e_i}{2(1+t)}.$$

We have a symmetric situation in neighboring jurisdiction j.

The first term in the best reply function corresponds to the quantities the black market firm would supply under autarky. The second term captures how the possibility to engage in cross-border shopping impacts on domestic supply, where the foreign market appears as an additional source of demand. An increase in foreign supply reduces local output due to the substitutability pattern between both products, where the magnitude of this impact will depend on how significant are transportation costs. Combining the symmetric best reply functions of the black market firms in jurisdictions i and j we have that the equilibrium quantities are given respectively by:

$$q_i^{S*} = x_i^* = \frac{1 - c - e_i}{2} \times \left[1 + \frac{1}{3+2t}\right] - \frac{(2+t)(e_i - e_j)}{3 + 4t(2+t)}, \tag{3.25}$$

$$q_j^{S*} = x_j^* = \frac{1 - c - e_j}{2} \times \left[1 + \frac{1}{3+2t}\right] - \frac{(2+t)(e_j - e_i)}{3 + 4t(2+t)}. \tag{3.26}$$

For given symmetric enforcement levels across jurisdictions, the magnitude of increase in domestic supply due to the possibility to engage in cross-border shopping is captured by the second terms inside the square brackets of expressions (3.25) and (3.26), that depend crucially on transportation costs. The increase in competition, reduces prices, which in turn translates into an increase in consumption of the harmful drug in both jurisdictions. The possibility to engage in cross-border shopping per se increases local supply with respect to autarky. The last term of both expressions captures how enforcement investments of both jurisdictions interact in the supply decision of each black market firm. Differences in enforcement investments across jurisdictions would create an arbitrage opportunity, that would trigger an opposite response in each jurisdiction. The black market firm in the jurisdiction who is subject to less enforcement by its government, would then increase supply in the same amount that it would be reduced for the foreign black market firm. To better understand how enforcement investments affect domestic and foreign consumption and supply, let's take a look at the comparative statics:

$$\frac{\partial x_i}{\partial e_i} = -\frac{1}{2} - \frac{5+4t}{2[3+4t(2+t)]} < 0, \tag{3.27}$$

$$\frac{\partial q_i^D}{\partial e_i} = -\frac{1}{2} + \frac{1}{2[3+4t(2+t)]} < 0, \tag{3.28}$$

$$\frac{\partial x_i}{\partial e_j} = \frac{2+t}{3+4t(2+t)} > 0, \tag{3.29}$$

$$\frac{\partial q_i^D}{\partial e_j} = -\frac{1+t}{3+4t(2+t)} < 0. \tag{3.30}$$

With respect to autarky, enforcement activities become more efficient to reduce domestic black market supply, what is being captured by the second term in (3.27). On the other hand, they are now less efficient to reduce domestic consumption of harmful drugs, due to the possibility to engage in cross-border shopping, what can be seen from the second term in the expression (3.28). This second term captures how much the efficacy of enforcement activities, as a tool to reduce consumption of harmful drugs, is undermined by the possibility to engage in cross-border shopping. Regarding how enforcement investments interact across jurisdictions, we see from the comparative statics above that an increase in foreign enforcement boosts local supply, while at the same time reducing total consumption of domestic citizens. The increase in the foreign price makes consumers substitute foreign for domestic consumption, resulting in a lower level of total consumption. Enforcement investments are complements across jurisdictions in reducing total consumption of harmful drugs. Therefore, while the increase of enforcement activities in one jurisdiction increases the incentive to engage in cross-border shopping in the margin, it is still effective as a tool to reduce total consumption of its residents. This effectiveness of enforcement activities increases with transport costs. Taking the difference in supply across jurisdictions we have that:

$$\Delta q_I^S = q_i^{S*} - q_j^{S*} = -\frac{2+t}{1+2t} \times (e_i - e_j).$$

From the expression above we see that black market supply will only be different across jurisdictions, if enforcement activities taken by the respective governments differ. With no transportation costs and no difference in enforcement levels across jurisdictions, we are in the traditional Cournot setting, while as transportation cost increase, we converge to the autarky situation described in the previous section.

For a given supply, each government will choose how much to invest in enforcement activities in order to maximize welfare. In a setting where both governments forbid the harmful drug, their objective is given by:

$$W_{II}(e) = -\alpha q^{D*}(e) - \Pi^x(e) - \frac{e^2}{2}, \tag{3.31}$$

where from (3.3) and (3.4) demands in jurisdictions i and j are given respectively by $q_i^{D*} = 1 - p_i$ and $q_j^{D*} = 1 - p_j$.

The FOC associated to the problem of government i is then given by:

$$\frac{\partial W_{II}(e_i)}{\partial e_i} = \frac{1}{3+4t(2+t)} \times \Big\{\alpha(1+2t(2+t)) + \\ \frac{4(1+t)^2(2+t)[(1+2t)(1-c-e_i)+(e_j-e_i)]}{3+4t(2+t)}\Big\} - e_i = 0. \tag{3.32}$$

The first term inside the curly brackets indicates the impact of a marginal variation in the local enforcement on total consumption of the harmful good of domestic citizens, weighted by the social valuation for consumption of harmful drugs. The following terms refer to the impact enforcement has on profits of the local illegal firm. This profits depend on the interaction of the illegal suppliers as well as on the difference on enforcement activities taken by governments. The last term outside the curly brackets refers to the marginal cost of enforcement activities.

Solving the condition in (3.32) for the enforcement investments of government i yields the following best reply function:

$$\begin{aligned}\bar{e}_i(e_j) =& \frac{1}{24t^4 + 104t^3 + 160t^2 + 104t + 25} \times \Big\{(2t+1)[\alpha\left(4t^3 + 14t^2 + 14t + 3\right) \\ &+ (1-c)\left(4t^3 + 16t^2 + 20t + 8\right)] + 4e_j(t+1)^2(t+2)\Big\}\end{aligned}$$

We have a symmetric situation for jurisdiction j.

Solving the system of equations given by the best reply functions of governments i and j, we get the following symmetric optimal enforcement investments under a symmetric scheme of prohibition:

$$e^*_{II} = \frac{\alpha}{3}\left[1 - \frac{2t^2 + 16t + 8}{12t^3 + 44t^2 + 50t + 17}\right] + \frac{1-c}{3}\left[1 + \frac{4t^2 + 10t + 7}{12t^3 + 44t^2 + 50t + 17}\right]. \tag{3.33}$$

Comparing the expression above with the optimal enforcement investments under a scheme of prohibition in a situation of autarky given by the expression in (3.10), we see that the difference is explained by the second terms inside the square brackets. This difference captures how the possibility to engage in cross-border shopping affects the two arguments that motivate government intervention. The optimal enforcement is now driven more by the objective to reduce black market profits, than for the concern on total consumption of harmful drugs with respect to the situation we had previously in autarky under a scheme of prohibition. With a low (high) concern for consumption of harmful drugs, the enforcement level is higher (lower) with cross-border shopping. This is explained by the fact that with cross-border shopping, the impact of a marginal increase on enforcement activities is now less efficient to reduce total consumption, but on the other hand it is more efficient to reduce black market profits, due to the increase in competition.

Combining the expression for black market supply in (3.25) with the one for the enforcement level in (3.33), we see that for the black market firm to remain active we now require for the social valuation for consumption of harmful drugs to be below the following threshold:

$$\alpha_{II} = 2(1-c) \times \left[1 + \frac{1}{2 + 4t(2+t)}\right]. \tag{3.34}$$

The second term inside the square brackets represents how much the participation threshold for the black market firm is modified, by the possibility to engage in cross-border shopping with respect to the one under autarky, given by the expression in (3.12). Comparing both expressions we see that for a given social valuation for consumption of harmful drugs, the possibility to engage in cross-border shopping makes less interesting for the government to eliminate domestic illegal supply than what it would under autarky.

Plugging the equilibrium values for the enforcement investments under a symmetric scheme of prohibition in (3.33) into the welfare function in (3.31) yields the welfare levels $W^*_{II}(e^*_{II}, e^*_{II})$, that is achieved in each jurisdiction if both governments adopt a scheme of prohibition. Since the algebraic expressions for these welfare levels are quite cumbersome, they are presented in the Appendix 3.6.2. These welfare levels will be compared with those that arise from switching to a scheme of legalization to be derived next. This comparison will allow us to understand which is the configuration of the scheme towards harmful drugs and optimal enforcement investments adopted in equilibrium. With respect to autarky, the possibility to engage in cross-border shopping is detrimental for welfare.

3.3.2 Symmetric legalization

Consider now that the domestic and foreign jurisdictions adopt a scheme of legalization, meaning that there is in each of them a regulated and a black market firm who engage in Cournot competition.

The black market firm in jurisdiction i solves the following problem:

$$\max_{x_i} \; \Pi_i^x = \left[p_i(q_i^S, q_j^S) - (c + e_i)\right] x_i$$

where he inverse demand functions are given as before by the expressions in (3.23) and (3.24).

FOC yields:

$$\frac{\partial \Pi_i^x}{\partial x_i} = \frac{-(1+t)x_i + (1+t)(1 - x_i - y_i) + (1 - x_j - y_j)}{2+t} - c - e_i = 0$$

The best reply function for the black market firm is then given by:

$$\bar{x}_i(x_j, y_j) = \frac{1 - y_i - c - e_i}{2} + \frac{1 - x_j - y_j - c - e_i}{2(1+t)}.$$

The legal firm in jurisdiction i solves the following problem:

$$\max_{y_i} \ \Pi_i^y = \left[p_i(q_i^S, q_j^S) - c\right] y_i$$

The FOC yields:

$$\frac{\partial \Pi_i^y}{\partial y_i} = \frac{-(1+t)y_i + (1 - x_j - y_j) + (1+t)(1 - x_i - y_i)}{t+2} - c = 0$$

The best reply function for the legal firm is then given by:

$$\bar{y}_i(x_j, y_j) = \frac{1 - x_i - c}{2} + \frac{1 - x_j - y_j - c}{2(1+t)}.$$

Combining the best reply functions above we have that the equilibrium quantities are now given by:

$$x_i^* = \left(\frac{2+t}{5+3t}\right)\left(1 - c - e_i\left[2 + \frac{1}{1+t}\right] - \frac{e_i - e_j}{1+3t}\right), \tag{3.35}$$

$$y_i^* = \left(\frac{2+t}{5+3t}\right)\left(1 - c + e_i\left[1 + \frac{1}{1+t}\right] - \frac{e_i - e_j}{1+3t}\right). \tag{3.36}$$

Total supply in jurisdiction i is given by:

$$q_i^{S*} = x_i^* + y_i^* = \left(\frac{2+t}{5+3t}\right)\left(2(1-c) - e_i - \frac{2(e_i - e_j)}{1+3t}\right).$$

We have a symmetric situation in neighboring jurisdiction j.

We have the following comparative statics for the enforcement investments when both governments adopt a scheme of legalization:

$$\frac{\partial x_i}{\partial e_i} = \left(\frac{2+t}{1+t}\right)\left[-\frac{2}{3} - \frac{2}{3\,(5+9t(2+t))}\right] < 0, \tag{3.37}$$

$$\frac{\partial y_i}{\partial e_i} = \left(\frac{2+t}{1+t}\right)\left[\frac{1}{3} - \frac{2}{3\,(5+9t(2+t))}\right] > 0, \tag{3.38}$$

$$\frac{\partial q_i^D}{\partial e_i} = -\frac{1}{3} + \frac{2}{3[5+9t(2+t)]} < 0, \tag{3.39}$$

$$\frac{\partial x_i}{\partial e_j} = \frac{\partial y_i}{\partial e_j} = \frac{2+t}{5+9t(2+t)} > 0, \tag{3.40}$$

$$\frac{\partial q_i^D}{\partial e_j} = -\frac{1+t}{5+9t(2+t)} < 0. \tag{3.41}$$

As in the previous case, with respect to autarky, enforcement now becomes a more efficient tool to reduce black market supply, but on the other hand it becomes less efficient as a tool to reduce total consumption. This can be seen from the expression in (3.39), where the first term is the pure effect from legalization, that adds a regulated firm in the domestic market, while the second term captures the effect of the possibility to engage in cross-border shopping. With respect to the previous scenario where both jurisdictions adopt a scheme of prohibition, enforcement activities are now less efficient to reduce total consumption. This result is mainly driven by the direct substitution from the illegal supplier towards the regulated firm, as the effect of cross-border shopping plays now a smaller role in this aspect. From (3.40) we see that an increase in foreign enforcement affects equally the legal and the illegal firm, what is explained by the fact that consumers do not distinguish between both products.

Taking the difference in supply across jurisdictions we now have that:

$$\Delta q_L = q_i^{S*} - q_j^{S*} = -\frac{2+t}{1+3t} \times (e_i - e_j).$$

From the expression above we see that as in the symmetric prohibition case, it is the difference in enforcement activities what determines the difference in supply across jurisdictions. Notice that as before, if both governments have the same enforcement investments, there is no cross-border shopping and each consumer only buys from its own jurisdiction.

Let's now analyze the problem faced by the governments. Each government will maximize the following welfare function:

$$W_{LL} = -\alpha q^D - \Pi^x + \Pi^y - \frac{e^2}{2}. \tag{3.42}$$

Solving for the enforcement in each jurisdiction as in the symmetric prohibition case we get the following symmetric expression for the optimal enforcement:

$$e^*_{LL} = \frac{\alpha}{5}\left(1 - \frac{4(2+t)}{15t^2+34t+13}\right) + \frac{2(1-c)}{5}\left(1 - \frac{3-t}{15t^2+34t+13}\right). \tag{3.43}$$

From the expression for the local black market firm in (3.35) and for the optimal enforcement level when both governments adopt a scheme of legalization in (3.43), we have that for the black market firm to remain active, the social valuation for consumption of the harmful drug must be lower than the following threshold:

$$\alpha_{LL} = \frac{1-c}{2}\left[1 - \frac{15t^2+26t+1}{6t^3+21t^2+20t+3}\right]. \tag{3.44}$$

The second term in the expression above captures how the participation threshold for the black market firm is reduced when consumers may engage in cross-border shopping under a scheme of legalization. This threshold is also lower than the one that results when both jurisdictions adopted a scheme of prohibition in (3.34).

We have two different situations depending on the social value for consumption of harmful drugs, one where with legalization the black market firm remains active, and other where it doesn't.

Let's first analyze the case where $\alpha < \alpha_{LL}$, so that both black market firms remain active. In this case, plugging the expression for the optimal enforcement in (3.34) into the welfare function in (3.42) yields the welfare levels when both black market firms are active, $W^*_{LL}(e^*_{LL}, e^*_{LL})$, whose expression is presented in the Appendix 3.6.2.

Let's now analyze the case where $\alpha \geq \alpha_{LL}$, what translates into both black market firms being inactive. From the expressions in (3.35) and given the symmetry across jurisdictions we have that the enforcement will be given by:

$$\hat{e}^*_{LL} = \frac{(1-c)(1+t)}{3+2t}.$$

Where the equilibrium quantities are now given by:

$$\begin{cases} \hat{x}^*_{LL} & = 0, \\ \hat{y}^*_{LL} & = \hat{q}^*_{LL} = \dfrac{(1-c)(2+t)}{3+2t}. \end{cases} \tag{3.45}$$

Without black market firms, the expression for welfare is given by:

$$\hat{W}^*_{LL}(\hat{e}^*_{LL}, \hat{e}^*_{LL}) = \frac{-\alpha(1-c)(4t^2+14t+12)+(1-c)^2(t^2+4t+3)}{2(3+2t)^2}. \tag{3.46}$$

The expressions for welfare derived in this section will be compared later with the ones resulting from the different possible combinations of schemes towards harmful drugs adopted by the neighboring jurisdictions.

3.3.3 Asymmetric case

Consider now that the government in jurisdiction i adopts a scheme of prohibition, while the neighboring jurisdiction j moves towards a scheme of legalization. Domestic and foreign supply are now given respectively by:

$$\begin{cases} q_i^S & = x_i, \\ q_j^S & = x_j + y_j. \end{cases}$$

We have an asymmetric situation regarding the number of regulated suppliers in each jurisdiction. It is worth noting that given this asymmetry, in equilibrium prices will now be different across jurisdictions, resulting in a positive amount of cross-border shopping. This implies that quantities consumed, as well as the utility of consumers will now differ across jurisdictions.

The inverse demand functions are given as before by the expressions in (3.23) and (3.24). The illegal firm in jurisdiction i, where the harmful good remains illegal, solves the following problem:

$$\max_{x_i} \ \Pi_i^x = \big[p_i(q_i^S, q_j^S) - (c + e_i)\big]x_i$$

FOC yields:

$$\frac{\partial \Pi_i^x}{\partial x_i} = \frac{-(1+t)x_i + (1+t)(1-x_i) + (1-x_j-y_j)}{2+t} - c - e_i = 0$$

The best reply of the domestic black market firm is given by:

$$\bar{x}_i(x_j, y_j) = \frac{1-c-e_i}{2} + \frac{1-x_j-y_j-c-e_i}{2(1+t)}. \tag{3.47}$$

Solving the problems of the illegal and legal firm in jurisdiction j in the same fashion yields the following best reply functions:

$$\bar{x}_j(x_i, y_j) = \frac{1-y_j-c-e_j}{2} + \frac{1-x_i-c-e_j}{2(1+t)}, \tag{3.48}$$

$$\bar{y}_j(x_i, x_j) = \frac{1-x_j-c}{2} + \frac{1-x_i-c}{2(1+t)}. \tag{3.49}$$

Legalization in the foreign economy j intensifies competition in its own market, but also in the neighboring jurisdiction i via cross-border shopping. The asymmetry in the number and characteristics of firms across jurisdictions explains the difference in the best reply functions. Solving the system of best reply functions given by (3.47), (3.48) and (3.49) we have that for given enforcement levels, the firms supply the following quantities:

$$x_i^* = \left(\frac{(2+t)(1+3t)}{4+6t(2+t)}\right) \times \left(1 - c - e_i\left[1 + \frac{1}{1+3t}\right] - \frac{e_i - e_j}{1+3t}\right), \tag{3.50}$$

$$x_j^* = \left(\frac{(2+t)(1+2t)}{4+6t(2+t)}\right) \times \left(1 - c - e_j\left[2 + \frac{t}{(1+t)(1+2t)}\right] + \frac{e_i - e_j}{1+2t}\right), \tag{3.51}$$

$$y_j^* = \left(\frac{(2+t)(1+2t)}{4+6t(2+t)}\right) \times \left(1 - c + e_j\left[1 + \frac{t}{(1+t)(1+2t)}\right] + \frac{e_i}{1+2t}\right). \tag{3.52}$$

We then have the following supply in each jurisdiction:

$$
\begin{aligned}
q_i^{S*} =& x_i^* = \left(\frac{(2+t)(1+3t)}{4+6t(2+t)}\right) \times \left(1 - c - e_i\left(1+\frac{1}{1+3t}\right) - \frac{e_i - e_j}{1+3t}\right), \\
q_j^{S*} =& x_j^* + y_j^* = \left(\frac{(2+t)(1+2t)}{4+6t(2+t)}\right) \times 2\left(1 - c - \frac{(1+t)e_j - e_i}{1+2t}\right).
\end{aligned}
$$

We have the following comparative statics for the enforcement investments when jurisdiction i adopts a scheme of prohibition while neighboring jurisdiction j one of legalization:

$$
\begin{aligned}
\frac{\partial x_i}{\partial e_i} &= -\frac{1}{2} - \frac{3t+4}{4+6t(2+t)} < 0, && (3.53) \\
\frac{\partial x_i}{\partial e_j} &= \frac{\partial x_j}{\partial e_i} = \frac{\partial y_j}{\partial e_i} = \frac{2+t}{4+6t(2+t)} > 0, && (3.54) \\
\frac{\partial x_j}{\partial e_j} &= \left(\frac{2+t}{1+t}\right) \times \left(-\frac{2}{3} - \frac{1}{3(4+6t(2+t))}\right) < 0, && (3.55) \\
\frac{\partial y_j}{\partial e_j} &= \left(\frac{2+t}{1+t}\right) \times \left(\frac{1}{3} - \frac{1}{3(4+6t(2+t))}\right) > 0, && (3.56) \\
\frac{\partial q_i^D}{\partial e_i} &= -\frac{1}{2} + \frac{1}{4+6t(2+t)} < 0, && (3.57) \\
\frac{\partial q_j^D}{\partial e_j} &= -\frac{1}{3} + \frac{1}{3(4+6t(2+t))} < 0, && (3.58) \\
\frac{\partial q_i^D}{\partial e_j} &= \frac{\partial q_j^D}{\partial e_i} = -\frac{1+t}{4+6t(2+t)} < 0. && (3.59)
\end{aligned}
$$

From the comparative statics in (3.54) we see that a variation in the enforcement investments of the neighboring economy affects equally supply of domestic black market firms, no matter which scheme towards harmful drugs has been adopted. Firms react equally to a variation in foreign enforcement, as consumers do not distinguish between products. From the second term in the expressions (3.57) and (3.58) we see that the possibility to engage in cross-border shopping reduces more the efficiency of enforcement activities as a tool to reduce total consumption in the jurisdiction who adopted a scheme of prohibition. Nonetheless, the main factor that explains how efficient is enforcement to reduce total consumption is given by the number of suppliers in each jurisdiction, captured by the first term of these expressions. Therefore, what matters more in terms of the efficiency of enforcement investments as a tool to reduce total consumption of harmful drugs, it is the domestic decision of whether to legalize or forbid the harmful drug, rather than the possibility to engage in cross-border shopping. Comparing both expressions we see that when two neighboring jurisdictions adopt different schemes towards harmful drugs, enforcement is less efficient to reduce domestic consumption of the harmful drug in the jurisdiction who legalized versus the one who forbid the harmful drug. We also see from expression (3.59) that despite having different number of suppliers across jurisdictions, the effect that an increase on enforcement in one jurisdiction has in reducing total consumption of the harmful drug in the neighboring jurisdiction is of the same magnitude.

Taking the difference in supply across jurisdictions we have that:

$$\Delta q_{IL} = q_i^{S*} - q_j^{S*} = -\frac{(2+t)[(1-c)(1+t)+(2+t)e_i+(3+2t)(e_i-e_j)]}{4+6t(2+t)}.$$

Notice that now, if both jurisdictions were to set the same enforcement investments, supply in the jurisdiction who adopted a scheme of prohibition would be lower than in the one who legalized the harmful drug. For the same enforcement across jurisdictions, legalization results in a higher supply, being this is the main reason why a local authority would not desire to legalize the harmful drug. If the price in the jurisdiction who legalized the harmful drug is lower, the total amount of harmful drugs purchased from consumers of the neighboring jurisdiction will be given by:

$$q_{ij}(e_i, e_j) = \frac{(1-c)(1+t)+(2+t)e_i+(3+2t)(e_i-e_j)}{4+6t(2+t)},$$

we have symmetric expression for q_{ji}, in the case that the price were to be lower in jurisdiction i, who adopted a scheme of prohibition.[6]

Let's analyze the participation of black market firms in each jurisdiction. From the expressions in (3.50) and (3.51) we have that for the black market firms to remain active in their respective jurisdictions, we require for the enforcement to be lower than the

[6]The derivation of the demand for harmful drugs in the neighboring jurisdiction is presented in the Appendix 3.6.1.

following thresholds:

$$\hat{e}_i(e_j) = \frac{e_j + (1-c)(1+3t)}{3(1+t)}, \tag{3.60}$$

$$\hat{e}_j(e_i) = \frac{(1+t)[e_i + (1-c)(1+2t)]}{3+4t(2+t)}. \tag{3.61}$$

Notice that since the level of enforcement required to eliminate local black market supply differ across jurisdictions, we may have in equilibrium only one black market firm active, both, or none.

Let's now address the government problem. The welfare function in jurisdictions i and j are given respectively by:

$$W_{IL} = -\alpha q_i^D - \Pi_i^x - \frac{e_i^2}{2}, \tag{3.62}$$

$$W_{LI} = -\alpha q_j^D - \Pi_j^x + \Pi_j^y - \frac{e_j^2}{2}. \tag{3.63}$$

Solving for the enforcement level as in the previous sections, we have that the best reply functions for each government are given respectively by:

$$\bar{e}_i(e_j) = \frac{1}{27t^4 + 117t^3 + 177t^2 + 111t + 26} \times \Big\{ e_j \left(3t^3 + 12t^2 + 15t + 6\right) + \alpha \left(9t^4 + 36t^3 + 45t^2 + 18t + 2\right) + (1-c)\left(9t^4 + 39t^3 + 57t^2 + 33t + 6\right) \Big\}, \tag{3.64}$$

$$\bar{e}_j(e_i) = \frac{e_i(2t+4) + \alpha\left(2t^2 + 4t + 1\right) + (1-c)\left(4t^2 + 10t + 4\right)}{10t^2 + 24t + 12}. \tag{3.65}$$

The best reply functions are not symmetric anymore. From the expressions above we see that there is a pattern of complementarity between the enforcement activities carried out by each government, since a higher level of enforcement on the neighboring jurisdiction makes more difficult for the local government to wipe out the domestic black market firm. Under this asymmetry in the scheme adopted towards harmful drugs, we may have different equilibrium configurations depending on the number of active firms.

Solving for the optimal enforcement level in each jurisdiction as in the previous case yields:

$$
\begin{aligned}
e^*_{IL} = &\frac{\alpha}{3}\left(1-\frac{72t^5+522t^4+1407t^3+1742t^2+975t+198}{270t^6+1818t^5+4896t^4+6726t^3+4970t^2+1884t+288}\right)\\
&+\frac{1-c}{3}\left(1+\frac{36t^5+180t^4+348t^3+304t^2+96t}{270t^6+1818t^5+4896t^4+6726t^3+4970t^2+1884t+288}\right), \qquad (3.66)
\end{aligned}
$$

$$
\begin{aligned}
e^*_{LI} = &\frac{\alpha}{5}\left(1-\frac{18t^5+111t^4+321t^3+525t^2+429t+118}{270t^6+1818t^5+4896t^4+6726t^3+4970t^2+1884t+288}\right)+\\
&+\frac{2(1-c)}{5}\left(1+\frac{36t^5+177t^4+327t^3+285t^2+118t+16}{135t^6+909t^5+2448t^4+3363t^3+2485t^2+942t+144}\right). \qquad (3.67)
\end{aligned}
$$

Combining the expressions for the supply of black market firms in (3.50) and (3.51), together with the optimal enforcement levels in (3.66) and (3.67) above, we see that for black market firms to remain active in jurisdiction i and j, we require for the social valuation for consumption of harmful drugs to be lower than the following respective thresholds:

$$
\hat{\alpha}_{IL} = 2(1-c)\left(1-\frac{18t^4+63t^3+66t^2+17t-2}{45t^5+237t^4+456t^3+386t^2+135t+14}\right); \qquad (3.68)
$$

$$
\alpha_{LI} = \frac{1-c}{2}\left(1-\frac{81t^5+387t^4+678t^3+532t^2+181t+18}{36t^6+225t^5+559t^4+702t^3+464t^2+149t+18}\right). \qquad (3.69)
$$

The participation threshold for the black market firm in the jurisdiction who adopted a scheme of legalization α_{LI}, is below than the one in the neighboring jurisdiction who adopted a scheme of prohibition $\hat{\alpha}_{IL}$, so that the former is the one that matters in order to have both black market firms active in equilibrium. From the expressions above we see that when marginal transportation costs are zero, it suffices for the social valuation for consumption of harmful drugs to be positive for the government who legalized to choose an enforcement level that wipes out the black market firm. A more detail analysis of the participation thresholds for black market firms is presented in the following section.

When $\alpha < \alpha_{LI}$, both black market firms are active and the optimal enforcement levels are those given by the expressions in (3.66) and (3.67). Substituting these values into the welfare functions in (3.62) and (3.63), we get the welfare outcomes when jurisdiction i has adopted a scheme of prohibition and j one of legalization, where both black market firms are active in equilibrium. The correspondent expressions for welfare $W^*_{IL}(e^*_{IL}, e^*_{LI})$ and $W^*_{LI}(e^*_{IL}, e^*_{LI})$ are presented in the Appendix 3.6.2.

Let's now analyze the scenario where legalization eliminates the black market firm in the jurisdiction j who opted for a scheme of legalization, that is, when $\alpha_{LI} \leq \alpha$. Then the enforcement in jurisdiction j is given by the expression in (3.61), while the government in jurisdiction i sticks to its best reply function in (3.64). Solving the system given by these best reply functions yields the following optimal enforcement levels:

$$\tilde{e}^*_{IL} = \frac{\alpha}{3}\left(1 - \frac{12t^3 + 55t^2 + 74t + 27}{36t^4 + 156t^3 + 238t^2 + 152t + 36}\right) + \tag{3.70}$$

$$\frac{1-c}{3}\left(1 + \frac{3t^3 + 7t^2 + 5t}{18t^4 + 78t^3 + 119t^2 + 76t + 18}\right), \tag{3.71}$$

$$\tilde{e}^*_{LI} = \frac{\alpha}{2}\left(\frac{3t^3 + 9t^2 + 7t + 1}{18t^4 + 78t^3 + 119t^2 + 76t + 18}\right) + \frac{1-c}{2}\left(1 - \frac{6t^3 + 15t^2 + 10t + 2}{18t^4 + 78t^3 + 119t^2 + 76t + 18}\right). \tag{3.72}$$

The participation threshold for the local black market firm in jurisdiction i is now:

$$\alpha_{IL} = 2(1-c)\left(1 - \frac{3t^2 + 4t}{6t^3 + 18t^2 + 14t + 2}\right). \tag{3.73}$$

Plugging in the optimal enforcement levels above into the respective welfare function yields the welfare levels $\tilde{W}^*_{IL}(\tilde{e}^*_{IL}, \tilde{e}^*_{LI})$ and $\tilde{W}^*_{LI}(\tilde{e}^*_{IL}, \tilde{e}^*_{LI})$ when $\alpha_{LI} \leq \alpha < \alpha_{IL}$, and whose expressions are presented in the Appendix 3.6.2.

Finally, when the social valuation is such that black market firms are eliminated in the two neighboring economies, that is, when $\alpha \geq \alpha_{IL}$, the best reply for the governments in jurisdiction i and j are given respectively by expressions (3.60) and (3.61). Solving the system of these best reply functions yields the following equilibrium enforcement levels:

$$\hat{e}^*_{IL} = \frac{(1-c)(1+2t)}{2(1+t)},$$
$$\hat{e}^*_{LI} = \frac{1-c}{2}.$$

For the equilibrium enforcement levels above we have the following welfare outcomes when $\alpha > \alpha_{LI}$:

$$\hat{W}_{IL}(\hat{e}^*_{IL}, \hat{e}^*_{LI}) = \frac{-(1-c)^2\,(4t^2+4t+1) - 4\alpha(1-c)(1+t)}{8(1+t)^2}, \tag{3.74}$$

$$\hat{W}_{LI}(\hat{e}^*_{IL}, \hat{e}^*_{LI}) = \frac{(1-c)^2(3+t) - 4\alpha(1-c)(1+t)}{8(1+t)}. \tag{3.75}$$

We are now in conditions to compare the welfare outcomes of adopting a scheme of legalization versus one of prohibition.

3.4 Comparison of regimes

In the previous section we have derived the welfare outcomes for all the possible combination of schemes towards harmful drugs adopted by two neighboring jurisdictions. By comparing them, we can now determine which combination will be adopted in equilibrium. Since the welfare expressions derived are quite cumbersome, in particular due to how transportation costs are modeled, we present the main results of our analysis graphically.

As it has been shown in the previous section, whether a government will find optimal or not to remove the domestic black market firm from the market, will depend on the social valuation for consumption of harmful drugs, as well as on how costly it is for consumers to engage in cross-border shopping. Figure 3.1 below depicts the different thresholds above which a government would find optimal to exert a level of enforcement such that the illegal firm is removed from the market.[7] We recall that the first and second subindex denote the scheme towards harmful drugs adopted by the domestic and foreign jurisdiction respectively.

[7] The participation thresholds are depicted neglecting marginal production costs, to incorporate them will only affect the scale of the vertical axis. Additional graphs regarding this aspect are presented in the Appendix 3.6.3.

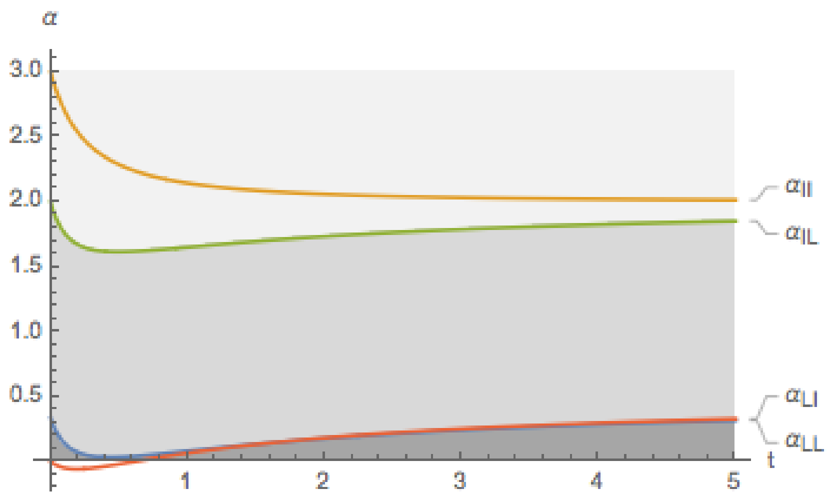

FIGURE 3.1: Participation thresholds for black market firms.

For instance, for a social valuation for consumption of harmful drugs above the participation threshold for black market firms under a symmetric scheme of prohibition α_{II}, both governments exert a level of enforcement that eliminates supply of harmful drugs altogether. In a given jurisdiction, we observe that there is a sharp decline in the participation threshold for the black market firm when the government decides to legalize the harmful drug. This is explained by the fact that legalization implies an increase in supply, what reduces profitability of the black market firm, making less costly for the government to eliminate black market supply. Moreover, with legalization, enforcement investments become a more efficient tool to reduce black market supply, due to the strategic interaction between firms. We have seen in the previous section that when it is less costly for consumers to engage in cross-border shopping, enforcement becomes a less efficient tool to tackle consumption of harmful drugs. In the graph above, this corresponds to the higher levels for the social valuation for harmful drugs required to eliminate local black market supply for small marginal transportation costs.

In the following, we split the analysis into three according to how strong is the social valuation for consumption of harmful drugs, and we proceed to compare the different welfare levels depending on the scheme towards harmful drugs adopted. The welfare levels to be compared in what follows were derived in the previous section and their algebraic expressions are presented in the Appendix 3.6.2.

3.4.1 Low social valuation for consumption of harmful drugs

Let's first consider the case where governments do not value consumption of harmful drugs as something very detrimental for welfare. In particular, let's consider that the social valuation for consumption of harmful drugs is such that under a situation where at least one government has decided to adopt a scheme of legalization, both black market firms remain active in equilibrium. This corresponds to the darkest gray area in figure 3.1, where the social valuation for consumption is such that: $\alpha < \max\{\alpha_{LL}, \alpha_{LI}\}$. In this scenario, when both governments adopt a scheme of prohibition, the resulting welfare for both jurisdictions is given by the expression in (3.76), while when both opt for a scheme of legalization it is given by the expression in (3.77). For the asymmetric situation where one jurisdiction forbids and the other legalizes the harmful drug, the resulting welfare levels are given respectively by the expressions in (3.78) and (3.79). Figure 3.2 illustrates these welfare outcomes as a function of the marginal transportation costs.[8]

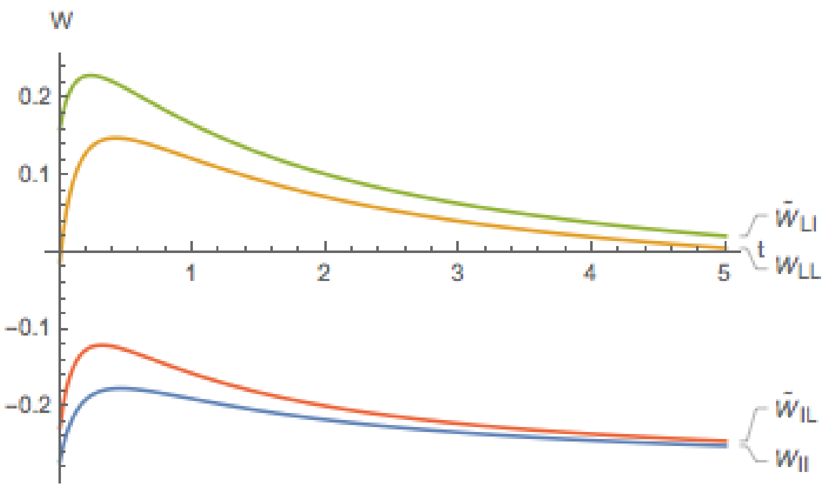

FIGURE 3.2: Welfare comparison for $\alpha \sim \alpha_{LL}$.

From figure 3.2 it becomes clear that, for a low social valuation for consumption of harmful drugs, both governments are better off by adopting a scheme of legalization rather than one of prohibition. It is in particular a dominant strategy to do so, as no matter which scheme is adopted in the neighboring jurisdiction, legalization yields a better outcome

[8] The welfare outcomes represented in figure 3.2 have been drawn neglecting marginal production costs, and for a social valuation for consumption of harmful drugs close to the upper threshold, though the same ordering among welfare levels holds for any $\alpha < \max\{\alpha_{LL}, \alpha_{LI}\}$, and for any $c \in [0, 1)$.

than prohibition. This result is not surprising, since legalization brings along tax revenue together with the reduction of black market profits, and the only reason in our setting to adopt a scheme of prohibition is given by the fact that the government has a negative valuation for consumption of harmful drugs, which in this case is too low to offset the benefits aforementioned.

3.4.2 Moderate social valuation for consumption of harmful drugs

Let's consider now that the social valuation for consumption of harmful drugs belongs to the following interval: $\max\{\alpha_{LL}, \alpha_{LI}\} \leq \alpha < \alpha_{IL}$, that corresponds to the gray area situated in the middle of figure 3.1. For this interval, the welfare expressions when governments adopt a symmetric strategy with respect to the scheme towards harmful drugs are given by (3.46) in the case of legalization, and by (3.76) in the case of prohibition. For the asymmetric case, where one jurisdiction forbids and the other legalizes the harmful drug, the resulting welfare outcomes are given respectively by the expressions in (3.80) and (3.81). For an intermediate social valuation for consumption of harmful drugs, whether adopting a scheme of prohibition or one of legalization yields a better outcome will depend on the parameters of the model. For relative low social values for consumption of harmful drugs, the welfare ordering is the one described by figure 3.2, where the Nash equilibrium is one where both jurisdictions adopt a scheme of legalization.

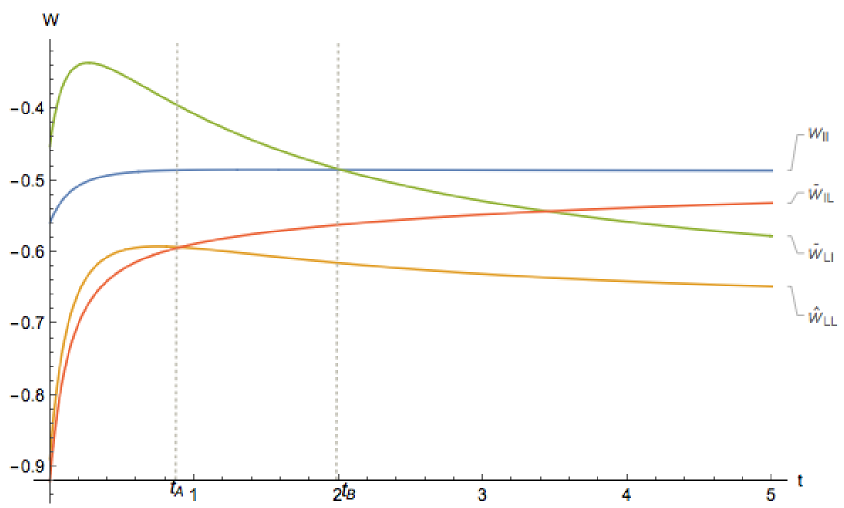

FIGURE 3.3: Welfare comparison for $(\alpha_{LL} + \alpha_{IL})/2$.

As the social valuation for consumption of harmful drugs increases, we approach a second scenario depicted by figure 3.3 above.[9] What is interesting about this second scenario is that if transportation costs are not too high ($t < t_A$), governments face a prisoner's dilemma, in the sense that no matter the scheme adopted in the foreign jurisdiction, the domestic government is better off by legalizing rather than prohibiting the harmful drug. However, if both governments adopt a scheme of legalization, we end up in a situation where welfare is lower than when both governments adopt a scheme of prohibition. This finding can be summarized in the following proposition:

Proposition 3.4.1 *While for some intermediate social valuation for consumption of harmful drugs, adopting a scheme of prohibition results in the highest welfare outcome possible for both neighboring jurisdictions, the possibility to engage in cross-border shopping creates incentives to deviate from this welfare-maximizing equilibrium, what results in both governments adopting a scheme of legalization.*

The result above makes emphasis on the fact that under some circumstances, both governments would be better off by adopting a scheme of prohibition, but since there exists a profitable deviation, this equilibrium is not sustainable, leading as a result to a situation where the harmful drug is legalized in both jurisdictions. The incentive to deviate is explained by the fact that by doing so, the government who legalizes first benefits from additional tax revenue from neighboring consumers who engage in cross-border shopping. Moreover, as each government only takes into account for welfare, consumption of its own citizens, cross-border shoppers only bring revenue to the economy. On the other hand, the economy who sees its neighbor switching to a scheme of legalization, suffers from a positive externality related to the reduction of black market profits, due to an increase in competition. However, it does not compensate for the negative welfare impact caused by the increase in total consumption of its citizens, who now have access to the harmful drug at a lower price. Enforcement has now become a less efficient tool, due to the possibility to engage in cross-border shopping. At this point, legalization also appears as an attractive option, as while this strategy would come with an increase in total consumption, it will bring back the tax revenue home, and it will furthermore reduce local black market profits through an increase in competition. Once the neighboring economy also decides to

[9] The welfare levels are drawn considering zero marginal production costs. Additional graphs for different values of marginal production costs and social valuation for consumption of harmful drugs are presented in the Appendix 3.6.3.

legalize, cross-border shopping disappears altogether, with the original incentive to deviate. As a result, both governments end up with a higher total consumption than under a scheme of prohibition, which is now unjustified for the lower benefits from the regulated firm and for the insufficient decrease of black market profits.

Notice also from figure 3.3 that for some relative higher transportation costs ($t_A < t < t_B$), we may also end up in a situation where once a neighboring jurisdiction has legalized, the other jurisdiction is better off by sticking to a scheme of prohibition rather than to switch to a scheme legalization of harmful drugs. This situation leads to the following result:

Proposition 3.4.2 *Depending on how strong are marginal transportation and production costs, for some intermediate social valuation for consumption of harmful drugs, there exists an equilibrium where two symmetric neighboring jurisdictions adopt different schemes towards harmful drugs.*

Proposition above is explained by the fact that foreign legalization increases competition in both economies, that though it results in higher levels of consumption of harmful drugs, it also reduces domestic black market profits. The jurisdiction who legalizes the harmful drug benefits from cross-border shopping through an increase tax revenue. However, for the neighboring jurisdiction where the harmful drugs remains illegal, while legalization would bring back consumers who are engaging in cross-border shopping, it will never result in additional tax revenue from foreign consumers. It will in turn increase even further consumption of the harmful drugs of its citizens. At this point, the cost of switching regime does not compensate the benefits. As transportation costs increase, the economies converge to a situation of autarky, where the additional benefits from stealing tax revenue due to cross-border shopping vanish.

In our analysis, governments choose simultaneously the scheme towards harmful drugs to be adopted, rising some coordination issues for the asymmetric equilibrium. While our analysis is static and the resulting equilibrium will depend on the coordination among governments, this last case suggests that under some circumstances, taking the decision to legalize first seems a good strategy, as it increases domestic welfare at the expense of the neighboring economy.

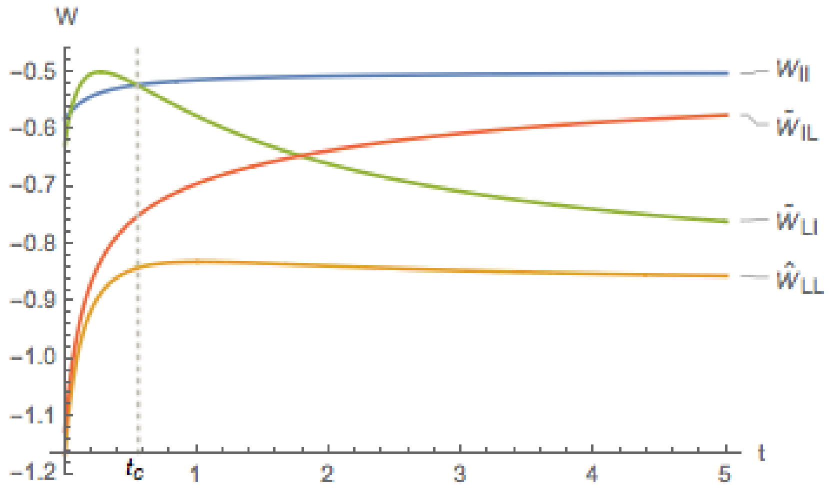

FIGURE 3.4: Welfare comparison for $\alpha \sim \alpha_{IL}$.

Finally, as the social valuation for consumption of harmful drugs approaches the upper-threshold α_{IL}, we arrive to the situation described in figure 3.4 above, where the prisoner's dilemma arises only for low marginal transportation costs ($t < t_c$). For higher marginal transportation costs the resulting equilibrium is one where both jurisdictions adopt a scheme of prohibition, which yields the best possible welfare outcome.

3.4.3 High social valuation for consumption of harmful drugs

Let's now consider the case where the social valuation for consumption of harmful drugs is high, that is, when $\alpha > \alpha_{IL}$. As the social valuation for consumption increases, we move from the situation in figure 3.4, to the one described by the graph below:

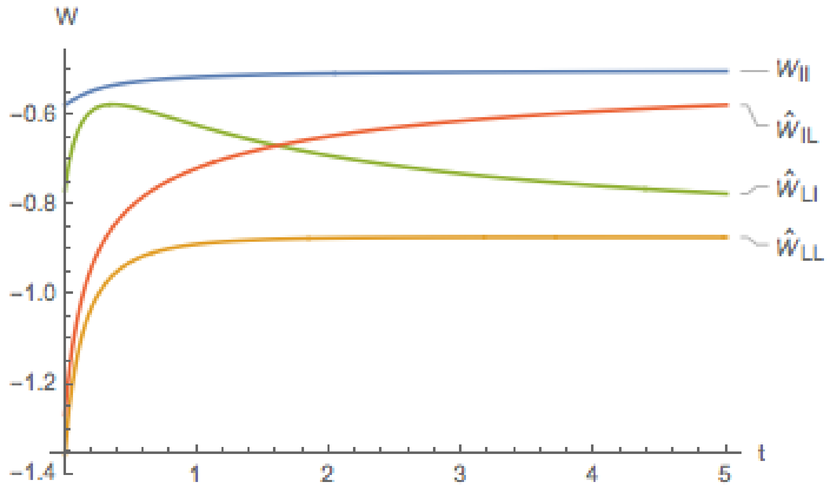

FIGURE 3.5: Welfare comparison when $\alpha \sim \hat{\alpha}_{IL}$.

For a sufficiently high valuation for consumption of harmful drugs, the best outcome for both governments is to adopt a scheme of prohibition.[10] Any deviation from this equilibrium will result in a welfare loss for both economies. For the case where the social valuation for consumption of harmful drugs is higher than α_{II}, then adopting a scheme of prohibition is a dominant strategy, as welfare levels will be higher no matter the scheme adopted by the neighboring jurisdiction. The difference with the previous case is that now enforcement investments are set such that the black market firms are inactive in equilibrium.

The different scenarios discussed in this section hold true for any $c \in [0, 1)$, and they can be summarized as follows:

In the presence of two neighboring jurisdictions where consumers may engage in cross-border shopping to acquire a harmful drug, we have that:

- for a high (low) social valuation of consumption of harmful drugs, the respective governments are better off by adopting a scheme of prohibition (legalization).
- for an intermediate social valuation of consumption of harmful drugs:
  - whether legalization or prohibition yields a better outcome depends on transportation and marginal production costs.

[10] While figure 3.5 considers zero marginal production costs, the welfare ordering remains the same for any $c \in [0, 1)$ and for any $\alpha > \hat{\alpha}_{IL}$.

– the resulting equilibrium may be different from the socially optimal one.

The following table summarizes the cases analyzed according to the different levels of the social valuation for consumption of harmful drugs:

Cases	**Social Valuation**	**Symmetric Prohibition**	**Asymmetric case**	**Symmetric Legalization**
$\alpha < \alpha_{LL}$	$\alpha_{LI} < \alpha_{LL}$	(W_{II}, W_{II})	$(\tilde{W}_{IL}, \tilde{W}_{LI})$	$(\mathbf{W_{LL}}, \mathbf{W_{LL}})$
	$\alpha_{LI} \geq \alpha_{LL}$	(W_{II}, W_{II})	(W_{IL}, W_{LI})	$(\mathbf{\hat{W}_{LL}}, \mathbf{\hat{W}_{LL}})$
$\alpha_{LL} < \alpha < \alpha_{IL}$	$\alpha_{LI} < \alpha_{LL}$	(W_{II}, W_{II})	$(\tilde{W}_{IL}, \tilde{W}_{LI})$	$(\hat{W}_{LL}, \hat{W}_{LL})$
	$\alpha_{LI} \geq \alpha_{LL}$	(W_{II}, W_{II})	(W_{IL}, W_{LI})	$(\hat{W}_{LL}, \hat{W}_{LL})$
$\alpha \geq \alpha_{IL}$		$(\mathbf{W_{II}}, \mathbf{W_{II}})$	$(\hat{W}_{IL}, \hat{W}_{LI})$	$(\hat{W}_{LL}, \hat{W}_{LL})$

For a sufficiently low (high) social valuation for consumption of harmful drugs, the resulting equilibrium is one where both governments adopt a scheme of legalization (prohibition) for any marginal production costs.[11] On the other hand, for an intermediate social valuation for consumption of harmful drugs, the resulting scheme adopted in equilibrium will depend as well on how relevant are transportation costs to acquire the harmful drug in the neighboring jurisdiction. It is worth mentioning that despite that we have focused our attention on the scheme towards harmful drugs to be adopted by each government, the equilibrium strategy consist as well on the choice of the optimal enforcement investments, that were already derived in the previous section, conditional on the schemes adopted in equilibrium.

3.5 Conclusion

We have analyzed the optimal policy towards harmful drugs in an imperfectly competitive market characterized by the presence of a black market firm, and where consumers may engage in cross-border shopping in a symmetric neighboring jurisdiction. Governments have a negative valuation for consumption of harmful drugs, while black market profits also generate a welfare loss to society. To aim to reduce consumption of harmful drugs, governments have to decide whether to adopt a scheme of prohibition or one of legalization.

[11] The minimum level of social valuation for consumption of harmful drugs that guarantees that in equilibrium both jurisdictions will adopt a scheme of prohibition is between α_{IL} and $\hat{\alpha}_{IL}$.

Besides the choice of the scheme towards harmful drugs to adopt, the main tool available for governments to reduce black market supply is to invest in enforcement activities.

Our analysis shows that when a government deems that consumption of harmful drugs is very bad for society, that is, when the (negative) social valuation for consumption of harmful drug is high, then adopting a scheme of prohibition is a dominant strategy and in equilibrium both jurisdictions adopt a scheme of prohibition. Depending on how strong is this valuation, governments may invest in enforcement activities up to the point that black market firms are eliminated. We have the opposite situation when the social valuation for consumption of harmful drugs is too low, where the resulting equilibrium is one where both governments legalize the harmful drug. When the social valuation for consumption of harmful drugs is moderate, whether a scheme of prohibition or one of legalization leads to a better outcome will depend on how strong is this valuation, as well as on the marginal transportation and production costs. More interestingly, the neighboring governments may end up trapped in a prisoner's dilemma, where while the welfare-maximizing outcome is one where both jurisdictions adopt a scheme of prohibition, the incentives to deviate in order to attract consumers from the neighboring jurisdiction to the regulated firm and thus generate tax revenue, lead to a lower welfare outcome where both governments legalize the harmful drug. This point stresses the importance of coordination among neighboring jurisdictions towards the fight on harmful drugs. It may also be a potential explanation for the decision to abandon a scheme of prohibition towards marijuana, followed by several US states who saw their neighbors adopting a scheme of legalization. Another possible scenario is one where despite two neighboring jurisdictions having the same social valuation for consumption of harmful drugs, they end up adopting different schemes towards harmful drugs. This is explained by the fact that, departing from a situation of prohibition, the jurisdiction who switches first to a scheme of legalization benefits from additional tax revenue due to cross-border shopping. However, the neighboring economy by legalizing second will never be able to steal any consumer from an already legalized market, finding itself in a situation where legalization would only bring more harm. This scenario suggest that under some circumstances, legalizing a harmful drug before my neighbor does may be a good idea. This could also be part of the explanation for the recent race towards marijuana legalization between some US states and Canada.

To make the analysis tractable, we have considered that the market for harmful drugs under a scheme of prohibition is dominated in each jurisdiction by a black market monopoly, and where legalization translates into the introduction of one regulated firm. This assumption has helped us to understand how enforcement investments and the scheme towards harmful drugs adopted affect welfare. Adding more firms into the analysis will increase total consumption and reduce profits, what would make enforcement less effective, and legalization less desirable.

The conclusions presented in this work have been derived considering that both governments have the same valuation for consumption of harmful drugs, and assumption that would be interesting to relax for future research. Moreover, adding more tools to governments such as the possibility to tax supply of the regulated firm, or to affect transportation costs would enrich the analysis. Our study focuses on cross-border shopping as the main channel through which the drug policies of neighboring jurisdictions interact. Another possible avenue for future research is to allow for black market firms to migrate from one jurisdiction to another in response to the policies towards harmful drugs followed by different jurisdictions.

3.6 Appendix

3.6.1 Demands

In order to derive the demands for harmful drugs, we first proceed to solve the consumer problem. The utility of consumer in jurisdiction i is given by:

$$U_i = \begin{cases} q_{ii} - \dfrac{q_{ii}^2}{2} + z_i & \text{if } p_i \leq p_j, \\ q_{ii} + q_{ij} - \dfrac{(q_{ii} + q_{ij})^2}{2} + z_i & \text{if } p_i > p_j; \end{cases}$$

where q_{ii} and q_{ij} denote the amount of the harmful good demanded from the domestic and foreign jurisdiction respectively.

Consumer in jurisdiction i will engage in cross-border shopping in neighboring economy j only if the neighboring price is lower, that is, only if $p_i > p_j$. The budget constraint of consumer in jurisdiction i is given by:

$$\omega_i - a_i \geq p_i(q_i^S, q_j^S)q_{ii} + p_j(q_i^S, q_j^S)q_{ij} + t\frac{q_{ij}^2}{2} + z_i, \quad \text{with } q_{ij} = 0 \text{ if } p_i \leq p_j.$$

Consumers will maximize their utility subject to their budget constraints. For $p_i > p_j$, the FOCs associated to the consumer problem in jurisdiction i are given by:

$$\begin{aligned}\frac{\partial U_i}{\partial q_{ii}} =& 1 - q_{ii} - q_{ij} = p_i;\\ \frac{\partial U_i}{\partial q_{ij}} =& 1 - q_{ii} - q_{ij} = p_j + tq_{ij}.\end{aligned}$$

We have a symmetric situation for the consumer from the foreign neighboring jurisdiction j.

Conditions above together with the budget constraint determine the demands for harmful drugs. If $p_i > p_j$, consumers engage in cross-border shopping and have a positive demand for the foreign harmful good. Combining conditions above yields the following arbitrage condition:

$$p_i = p_j + tq_{ij},$$

When $p_i > p_j$, the amount of the harmful drug demanded from the neighboring economy is consequently given by:

$$q_{ij} = \frac{p_i - p_j}{t}.$$

When $p_i \leq p_j$, the expressions for demands are given by:

$$\begin{cases} q_{ii} = 1 - p_i \\ q_{ij} = 0 \\ q_{ji} = \dfrac{p_j - p_i}{t} \\ q_{jj} = 1 - p_j - q_{ji} = 1 - \Big[\dfrac{(1+t)p_j - p_i}{t}\Big] \end{cases}$$

while if $p_i > p_j$, they are given by:

$$\begin{cases} q_{ii} = 1 - \left[\dfrac{(1+t)p_i - p_j}{t}\right] \\ q_{ij} = \dfrac{p_i - p_j}{t} \\ q_{ji} = 0 \\ q_{jj} = 1 - p_j \end{cases}$$

In equilibrium, firms will set their supply according to the demand they face form both the local and the foreign consumer:

$$q_i^S = q_{ii} + q_{ji} = 1 - p_i + \frac{p_j - p_i}{t} = 1 + \frac{p_j - (1+t)p_i}{t},$$
$$q_j^S = q_{jj} + q_{ij} = 1 - p_j + \frac{p_i - p_j}{t} = 1 + \frac{p_i - (1+t)p_j}{t}.$$

Solving the systems of equation above for the prices in jurisdiction i and j, yields the inverse demand functions faced by suppliers in jurisdiction i and j, that have the following expressions:

$$p_i(q_i^S, q_j^S) == \frac{t(q_j^S - 1) + t(1+t)(q_i^S - 1)}{1 - (1+t)^2} = \frac{1 - q_j^S + (1+t)(1 - q_i^S)}{2+t}; \tag{3.23}$$
$$p_j(q_i^S, q_j^S) == \frac{t(q_i^S - 1) + t(1+t)(q_j^S - 1)}{1 - (1+t)^2} = \frac{1 - q_i^S + (1+t)(1 - q_j^S)}{2+t}. \tag{3.24}$$

Regarding consumption of harmful drugs in each jurisdiction, as long as the exogenous income is big enough to cover for transportation costs and for the lump sum tax imposed on consumers, what we assume holds true, the expressions for the total amount of harmful drugs demanded by consumer in jurisdiction i and j are given respectively by:

$$q_i^D = q_{ii} + q_{ij} = 1 - p_i,$$
$$q_j^D = q_{jj} + q_{ji} = 1 - p_j.$$

The expressions above will matter are useful when considering the welfare outcome.

3.6.2 Welfare expressions

In this section we provide the expressions for the welfare levels depending on the scheme towards harmful drugs adopted in each jurisdiction.

Symmetric prohibition

For both jurisdictions for $\alpha < \alpha_{II}$:

$$\begin{aligned} W_{II}(e^*_{II}, e^*_{II}) = \frac{1}{A_I} \times \Big\{ & \alpha^2 \left(24t^6 + 200t^5 + 660t^4 + 1092t^3 + 938t^2 + 384t + 55\right) \\ & - \alpha(1-c)\left(96t^6 + 752t^5 + 2368t^4 + 3808t^3 + 3264t^2 + 1394t + 228\right) \\ & - 2(1-c)^2 \left(24t^6 + 176t^5 + 520t^4 + 792t^3 + 657t^2 + 283t + 50\right) \Big\}. \end{aligned} \tag{3.76}$$

with $A_I = 2(17 + 50t + 44t^2 + 12t^3)^2$.

Symmetric legalization

For both jurisdictions when $\alpha < \alpha_{LL}$:

$$\begin{aligned} W_{LL}(e^*_{LL}, e^*_{LL}) = \frac{1}{A_L} \times \Big\{ & \alpha^2 \left[45t^5 + 303t^4 + 750t^3 + 818t^2 + 361t + 43\right] \\ & - 4\alpha(1-c)\left[180t^5 + 1167t^4 + 2829t^3 + 3129t^2 + 1537t + 278\right] \\ & + 4(1-c)^2 \left[45t^5 + 303t^4 + 759t^3 + 857t^2 + 416t + 68\right] \Big\}. \end{aligned} \tag{3.77}$$

with $A_L = 2(3t + 5)(15t^2 + 34t + 13)^2$.

For both jurisdictions when $\alpha \geq \alpha_{LL}$:

$$\hat{W}^*_{LL}(\hat{e}^*_{LL}, \hat{e}^*_{LL}) = \frac{-\alpha(1-c)(4t^2 + 14t + 12) + (1-c)^2(t^2 + 4t + 3)}{2(3 + 2t)^2}. \tag{3.46}$$

Asymmetric legalization

Case where $\alpha < \alpha_{LI}$.

For the jurisdiction who adopts a scheme of prohibition:

$$W_{IL}(e^*_{IL},e^*_{LI}) = \frac{1}{B} \times \{\alpha^2(12150t^{12} + 166050t^{11} + 1012878t^{10} + 3640626t^9 + 8575140t^8 \\ +13926360t^7+15974743t^6+13031884t^5+7498932t^4+2965342t^3+763419t^2+114516t+7532)- \\ - \alpha(1-c)(48600t^{12} + 693360t^{11} + 4434480t^{10} + 16804908t^9 + 42017124t^8 + 73025508t^7+ \\ + 90502480t^6 + 80654860t^5 + 51360940t^4 + 22825868t^3 + 6731520t^2 + 1184944t + 94336)- \\ - (1-c)^2(24300t^{12} + 317520t^{11} + 1858572t^{10} + 6438528t^9 + 14691336t^8 + 23248752t^7+ \\ + 26154644t^6 + 21073640t^5 + 12070100t^4 + 4792976t^3 + 1252432t^2 + 193280t + 13312)\}. \tag{3.78}$$

For the jurisdiction who adopts a scheme of legalization:

$$W_{LI}(e^*_{IL},e^*_{LI}) = \frac{1}{B}\times\Big\{\alpha^2(4860t^{12}+74844t^{11}+511704t^{10}+2052972t^9+5377533t^8+9672756t^7 \\ + 12224437t^6 + 10906592t^5 + 6793915t^4 + 2868624t^3 + 774703t^2 + 119148t + 7788) \\ - \alpha(1-c)(77760t^{12} + 1051704t^{11} + 6385824t^{10} + 22996224t^9 + 54664440t^8 + 90337848t^7 \\ + 106450384t^6 + 90200912t^5 + 54637480t^4 + 23127008t^3 + 6512960t^2 + 1099584t + 84480) \\ + (1-c)^2(19440t^{12} + 279936t^{11} + 1812240t^{10} + 6970752t^9 + 17738736t^8 + 31462944t^7 \\ + 39899968t^6 + 36478400t^5 + 23884224t^4 + 10932224t^3 + 3322624t^2 + 602112t + 49152)\Big\}, \tag{3.79}$$

with $B = 145800t^{12}+1963440t^{11}+11897928t^{10}+42867792t^9+102220704t^8+169898544t^7+ 201822120t^6+172703472t^5+105729128t^4+45202272t^3+12824352t^2+2170368t+165888.$

Case when $\alpha_{LI} \leq \alpha < \alpha_{IL}$.

For the jurisdiction who adopts a scheme of prohibition:

$$\tilde{W}_{IL}(\tilde{e}^*_i,\tilde{e}^*_j) = \frac{1}{C} \times \{\alpha^2(216t^8 + 1800t^7 + 6240t^6 + 11664t^5 + 12763t^4 + 8324t^3 + 3146t^2 \\ + 620t + 47) - \alpha(1-c)(864t^8 + 8064t^7 + 31944t^6 + 70140t^5 + 93528t^4 + 77880t^3 \\ + 39800t^2 + 11492t + 1448) - (1-c)^2(432t^8 + 3600t^7 + 12588t^6 + 24120t^5 \\ + 27752t^4 + 19712t^3 + 8484t^2 + 2032t + 208)\}. \tag{3.80}$$

For the jurisdiction who adopts a scheme of legalization:

$$\tilde{W}_{LI}(\tilde{e}_i^*, \tilde{e}_j^*) = \frac{1}{C} \times \{\alpha^2(216t^7 + 1593t^6 + 4812t^5 + 7665t^4 + 6884t^3 + 3427t^2 + 848t + 75) \\ - \alpha(1-c)(1296t^8 + 11556t^7 + 43452t^6 + 89988t^5 + 112528t^4 + 87500t^3 + 41700t^2 \\ + 11268t + 1344) + (1-c)^2(324t^8 + 3240t^7 + 13464t^6 + 30672t^5 \\ + 42280t^4 + 36440t^3 + 19340t^2 + 5824t + 768)\}, \quad (3.81)$$

with $C = 2592t^8 + 22464t^7 + 82944t^6 + 170400t^5 + 213320t^4 + 167168t^3 + 80480t^2 + 21888t + 2592$.

Case when $\alpha \geq \alpha_{IL}$.

For the jurisdiction who adopts a scheme of prohibition:

$$\hat{W}_{IL}(\hat{e}_{IL}^*, \hat{e}_{LI}^*) = -\frac{(1-c)^2(4t^2+4t+1) + 4\alpha(1-c)(1+t)}{8(t+1)^2}; \qquad (3.74)$$

while for the one who adopts a scheme of legalization:

$$\hat{W}_{LI}(\hat{e}_{IL}^*, \hat{e}_{LI}^*) = \frac{(1-c)^2(t+3) - 4\alpha(1-c)(t+1)}{8(t+1)}. \qquad (3.75)$$

3.6.3 Additional graphs

Here we provide some graphs that complement those presented in section 3.4, that consider different values of the marginal production costs and of the social valuation for consumption of harmful drugs.

Participation of black market firms

The graphs below illustrate how marginal production costs affect the level of the social valuation for harmful drugs required to eliminate black market firms.

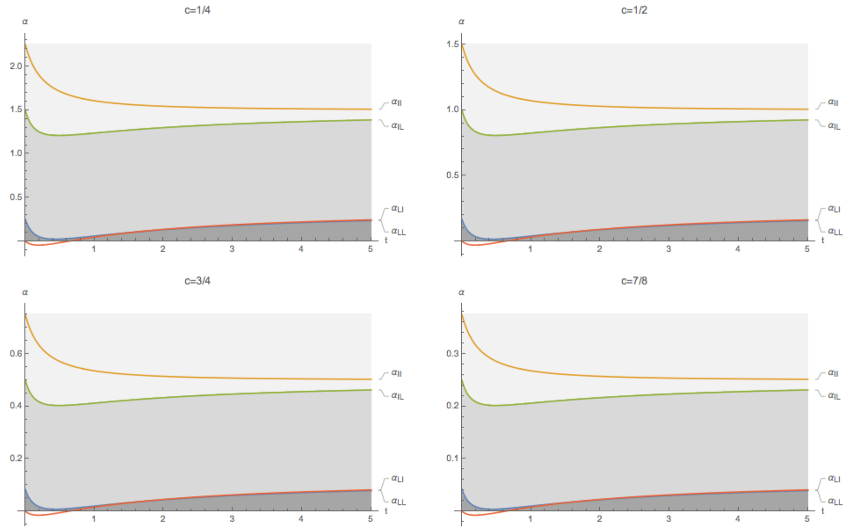

FIGURE 3.6: Participation thresholds for black market firms for different marginal production costs.

From the graphs above we see that how the participation thresholds for black market firms are ranked is not affected by marginal production costs, that only affect their levels.

Welfare comparison

Additional graphs are presented below in order to illustrate that the different equilibrium configurations resulting from comparing the welfare levels do not vary with the parameters of the model. Marginal production costs only play a role in the magnitude of the welfare outcomes, but not in its ordering with respect to the scheme towards harmful drugs adopted. For the graphs to be presented in what follows, the social valuation for consumption of harmful drugs decreases from top to bottom, while those to the left consider a smaller marginal production costs than those to the right.

Low social valuation for consumption of harmful drugs

We consider for the social valuation for consumption of harmful drugs to be low, if it belongs to the following interval: $0 < \alpha < \max\{\alpha_{LL}, \alpha_{LI}\}$.

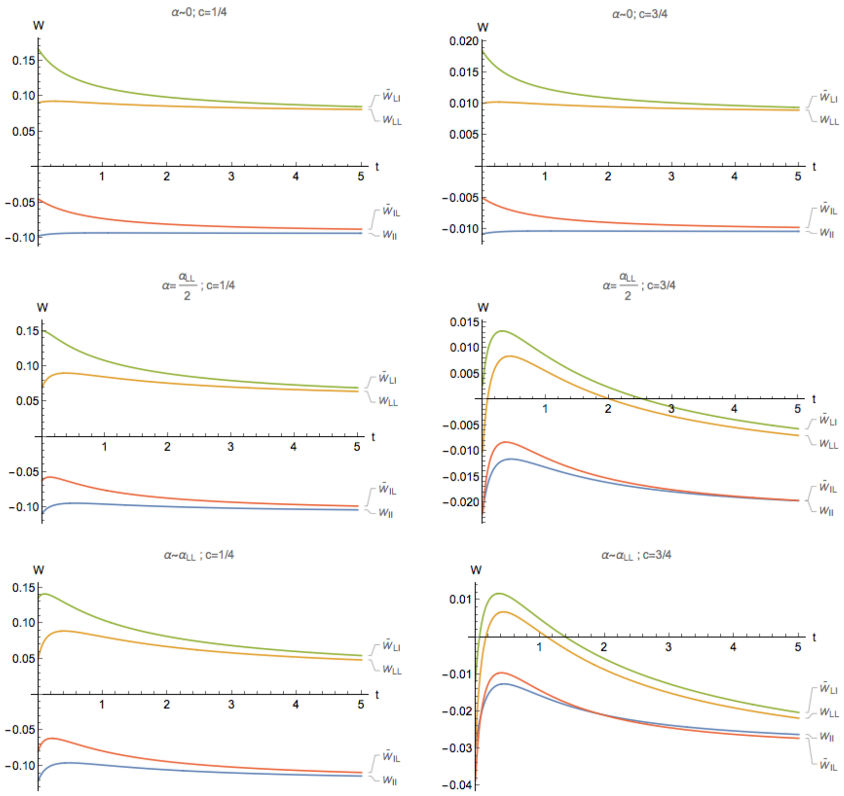

FIGURE 3.7: Welfare comparison for a low social valuation for consumption of harmful drugs.

The graphs above illustrate the same situation described in section 3.4: if consuming harmful drugs is not a big concern for the government, adopting a scheme of legalization is a dominant strategy and the resulting equilibrium is one where both jurisdiction legalize the harmful drug.

Intermediate social valuation for consumption of harmful drugs

Here we consider that the social valuation for consumption of harmful drugs is such that it belongs to the following interval: $\max\{\alpha_{LL}, \alpha_{LI}\} < \alpha < \alpha_{IL}$.

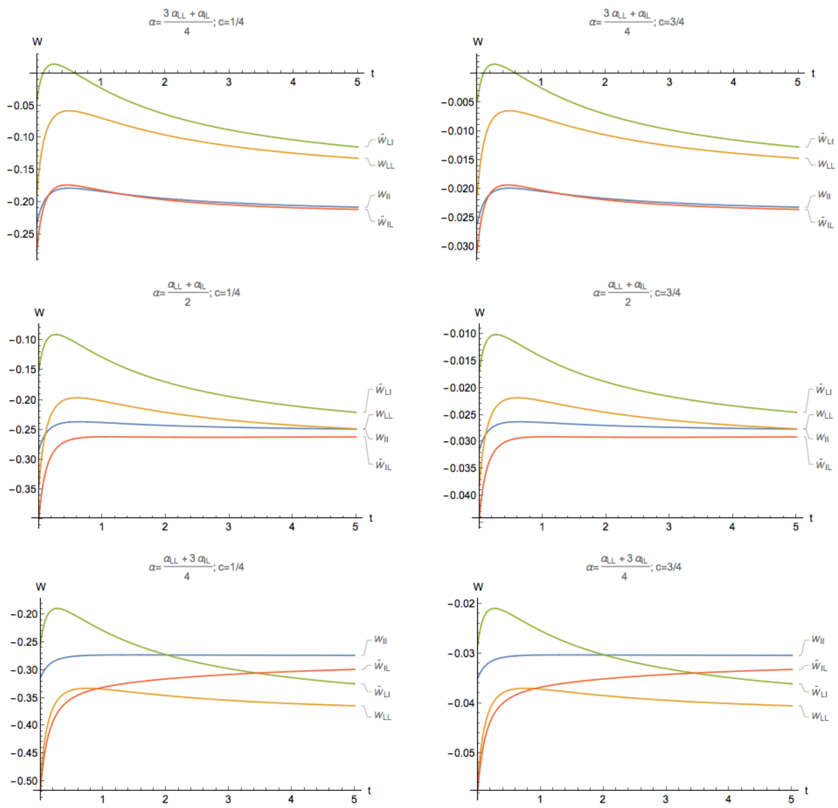

FIGURE 3.8: Welfare comparison for an intermediate social valuation for consumption of harmful drugs.

The graphs above present the different equilibria that may arise depending on the social valuation for consumption of harmful drugs. Those in the top resemble the situation for a low social valuation of harmful drugs, where the equilibrium is such that both governments adopt a scheme of legalization. We have the same situation for the graphs in the middle, that consider a higher social valuation for consumption of harmful drugs. The graphs in the bottom capture first, for low marginal transportation costs, the prisoners' dilemma, where the equilibrium is one where both jurisdictions legalize the harmful drug, despite both adopting a scheme of prohibition yields a better outcome. As transportation costs increase, we move to an asymmetric equilibrium, in terms of the schemes adopted towards harmful drugs. Then, for higher marginal transportation costs, adopting a scheme of prohibition becomes a dominant strategy and the equilibrium is such that both governments adopt a scheme of prohibition.

High social valuation for consumption of harmful drugs

We consider a high social valuation for consumption of harmful drugs when it is such that $\alpha \geq \alpha_{IL}$.

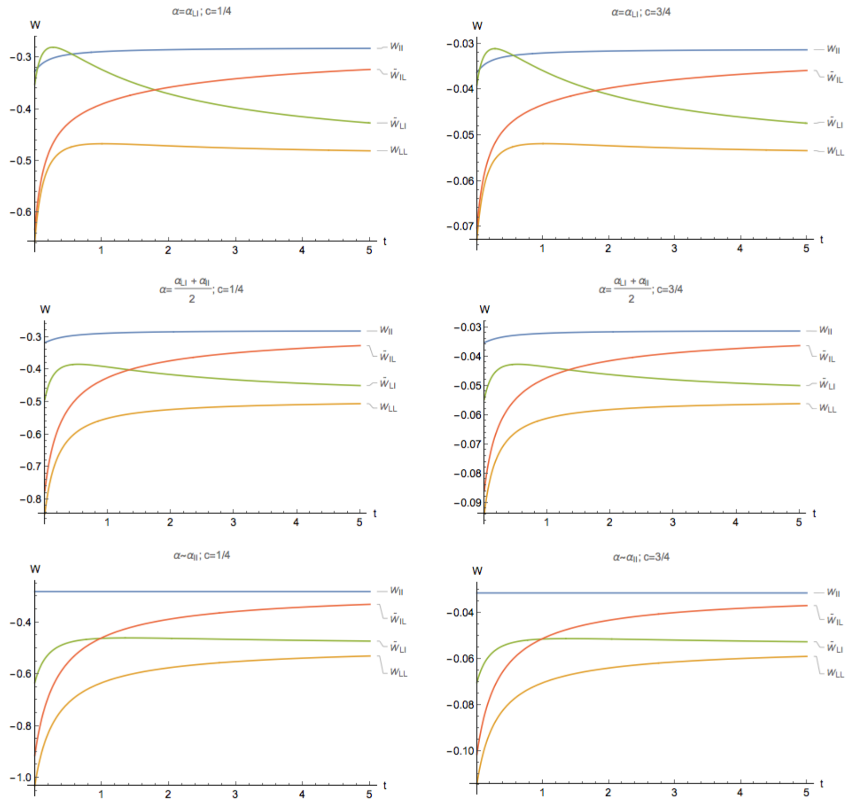

FIGURE 3.9: Welfare comparison for a high social valuation for consumption of harmful drugs.

From the two graphs in the top we see that when the social valuation for consumption of harmful drugs is not too high, marginal transportation cost cannot be too low, for the scheme of prohibition to be a dominant strategy. Otherwise, there will be a profitable deviation towards adopting a scheme of legalization. As the social valuation for consumption of harmful drugs increases, we arrive to a situation where the equilibrium is such that both governments adopt a scheme of prohibition. These situations are not particularly sensitive to a variation in the marginal production costs.

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