Optimal Carbon Capture and Storage Policies

Alain Ayong Le Kama
Equippe, Université of Lille 1.

Mouez Fodha*
LEO, University of Orleans and Paris School of Economics.

Gilles Lafforgue
LERNA (INRA) and Toulouse School of Economics.

May 26, 2011

Abstract

The IPCC recommends the use of carbon capture and sequestration (CCS) technologies in order to achieve the Kyoto environmental goals. This paper sheds light on this issue by assessing the optimal strategy regarding the long-term use of CCS technologies. The aim is to analyze the optimal CCS policy when the sequestration rate is endogenous, being hence one specific tool of the environmental policy. We develop a simple growth model and therefore exhibit the main driving forces that should determine the optimal CCS policy. We show that, under some conditions on the cost of extractions, CCS may be a long-term solution to curb carbon emissions. Besides, we show that the social planner choose to decrease over time the rate of capture and sequestration. Next, we derive the decentralized equilibrium outcome by considering the programs of the fossil resource-holder and of the representative consumer. Finally, we determine the environmental policy, i.e. the carbon tax scheme, as well as the the dynamics of the fossil fuel price that implement the optimum.

JEL classification: O44 - Q53 - Q58.

Key words: carbon capture and sequestration, optimal growth, environmental policy.

*Corresponding Author: mouez.fodha@univ-paris 1.fr.

1 Introduction

The IPCC's report [17] recommends the development and the use of carbon capture and sequestration (CCS) technologies in order to achieve the environmental goals imposed by the Kyoto Protocol. CCS is a geoengineering technique for the long-term storage[1] of carbon dioxide or other forms of carbon. Carbon dioxide is usually captured from the atmosphere through biological, chemical or physical processes. CO2 may be captured as a pure by-product in processes related to petroleum refining or from flue gases from power generation. The carbon dioxide might then be permanently stored away from the atmosphere. It has been argued that CCS applied to a conventional power plant could reduce CO2 emissions to the atmosphere by approximately 80-90% compared to a plant without CCS. The IPCC estimates that the economic potential of CCS could be between 10% and 55% of the total carbon mitigation effort until year 2100 (IPCC [17]). Despite the lack of certainty about the long-term economic efficiency of the CCS, many countries have already launched some experiences, which are still operating. For instance, three important industrial-scale storage projects are in operation: Sleipner is the oldest project and is located in the North Sea (Finland); the Weyburn-Midale CO2 Project is currently the world's largest carbon capture and sequestration project (Canada); the site of In Salah (Algeria) is a natural gas reservoir located in In Salah.

The aim of this paper is to study the optimal carbon capture and sequestration policy. We hence analyze what should be the CCS policy in a deterministic world.[2] The CCS technologies has motivated a number of empirical studies, via complex integrated assessment models (see McFarland et al. [21], Edmonds et al. [7], Kurosawa [18], Gitz et al. [10], Edenhofer et al. [6], Gerlagh [8], Gerlagh and van der Zwaan [9], Grimaud et al. [13]). These papers consider that the existing technology allows sequestrating a constant fraction of the carbon emissions. They generally conclude that the early introduction of sequestration can lead to a substantial decrease in the cost of environmental externality. A high level of complexity for such operational models, aimed at defining some specific climate policy, may be required so as to take into account the various interactions at hand. Conversely, in this paper, we consider a stylized model so as to exhibit the main driving forces that should determine the optimal CCS policy, in a very simple economy. While a generic abatement option can take

[1] It includes gaseous storage in various deep geological formations (including saline formations and exhausted gas fields), liquid storage in the ocean, and solid storage by reaction of CO2 with metal oxides to produce stable carbonates.

[2] One direct extension, among others, is to take into account the uncertainty linked to CSS efficiency. The CSS in action are still recent and we do not know exactly the full consequences of such abatement technologies, in terms of environmental consequences (on oceans for instance), or in terms of efficiency once we consider the leakage problems.

several forms, such as sequestration by forests or pollution reduction at the source, in this paper we are mainly concerned with the rate of carbon capture and sequestration, although we also introduce the limited size and an access cost of the reservoir. Thus, and by contrast to these previous models, we do not consider the optimal level of carbon emissions to capture and store to achieve a given goal, but we come out with an analytical value of the instantaneous rate of capture and sequestration, that is the optimal rate of storage.

Following Hotteling [16], Dasgupta and Heal [5] and Hartwick [14] who analyze the optimal use (i.e. exploitation and/or depletion) of environmental resources,[3] we consider an optimal growth path of an economy facing a dilemma of consumption vs. pollution. The framework introduced in this paper, the Ramsey model, is quite similar to the one used in the papers dealing with optimal pollution control (van der Ploeg and Withagen; [23], Gradus and Smulders [11], Ayong Le Kama [1], Ayong Le Kama and Schubert [3], [4]). Lafforgue et alii. ([19], [20]) and Ragot and Schubert [22] have already studied the theoretical consequences of the CCS for some specific cases. Lafforgue et alii.([19], [20]) consider the energy substitution issues when the economy faces a ceiling on the stock of pollution in the atmosphere. Ragot and Schubert ([22]) analyze the temporality of sequestration in agricultural soils by considering the asymmetric dynamic process. Finally, Grimaud and Rouge [12] study the implications of the CCS technology availability on the optimal use of polluting exhaustible resources and on optimal climate policies, within an endogenous growth framework. They conclude that CCS is detrimental to output growth. But these papers do not consider the CCS technology as a particular tool for the environmental policy. Moreover, in these papers, the rate of change of the stock of pollution or of the stock of the environmental resource, that is the natural rate of absorption/regeneration, is given. The framework introduced here is different since we determine endogenously the optimal rate of carbon sequestration, as if the rate of change of the stock of pollution becomes endogenous. This framework originates from Ayong Le Kama and Fodha [2] who study the optimal rate of nuclear waste storage, but in a partial equilibrium case.

The sketch of the model is the following. We consider an economy with only one good which is fossil energy fuel. This good comes from the extraction of a non-renewable and given resource stock. Its consumption generates environmental damages due to the release of carbon flows into the atmosphere. For simplification, we assume that the flows of carbon are proportional to the level of consumption. Consumption and pollution enter in a separable way into the utility function. Besides, we assume that the social planner can capture and store a part of the carbon flow in some appropriately deep sinks. Hence, the

[3] See for example Heal [15] for a survey on these topics.

social planner goal is to choose the optimal CCS rate policy. We show, under some conditions on the cost of extractions, that CCS may be a long-term solution to curb carbon emissions. Next, we derive the decentralized equilibrium outcome by considering the programs of the fossil resource-holder and of the representative consumer. Finally, we determine the environmental policy, i.e. the carbon tax scheme, as well as the the dynamics of the fossil fuel price that implement the optimum.

The paper is organized as follows. Section 2 describes the model. Section 3 studies the optimal dynamics of fossil resource extraction and sequestration. It also provides an illustration of optimal trajectories by using some specified functional forms. Section 4 derives the decentralized equilibrium outcome and characterizes the carbon tax trajectory that implements the optimum. The last section concludes.

2 The model

Let us consider an economy in which, at each date t, the unique consumption good is a flow of fossil energy fuel x_t. This good exhibits two main properties. First, it comes from the extraction of a non-renewable and initially given resource stock X_0. The current fossil resource stock X_t thus evolves over time as follows:

$$\dot{X}_t = -x_t \tag{1}$$

We denote by $c(X_t)$ the full marginal delivery cost of the fossil fuel, which includes the extraction cost of the resource, the cost of industrial processing (refining) and the transportation cost, so that the resource is ready for use by the consumer. This cost is assumed to be decreasing and convex in X_t, thus growing as the resource is depleted in order to reflect the fact that the more accessible deposits are exploited first.

Second, consumption of fossil energy provides utility but it also generates some environmental damages due to the release of carbon flows into the atmosphere associated to the combustion of the fossil fuel. For the sake of simplicity, we assume additive separability between utility and damage (i.e. marginal utility is not impacted by pollution). We denote by $u(x_t)$ the instantaneous flow of utility provided by the consumption of x_t units of fossil energy and by $v(P_t)$ the instantaneous flow of damage associated with the atmospheric carbon stock P_t. We assume that $u(.)$ has the standard properties (increasing, concave, Inada) and that function $v(.)$ is increasing and convex. We also assume that the utility and damage flows are expressed in monetary terms and can thus be viewed as the consumer surplus. Finally, net utility flows are discounted at rate ρ, where ρ is the pure rate of time preferences.

The unitary carbon content of fossil fuel is β so that, without any abatement at the pollution source, the instantaneous carbon emissions would be βx_t. We assume that a CCS device is available from the initial date and we note γ_t the rate of sequestration, i.e. the proportion of carbon emissions that is captured and stored into geological reservoirs. The instantaneous flow of carbon sequestration is then equal to $s_t = \gamma_t \beta x_t$ and the dynamics of storage is given by:

$$\dot{S}_t = s_t = \gamma_t \beta x_t \tag{2}$$

where S_t is the cumulated quantity of carbon stored into carbon sinks, S_0 been given. We also assume that the maximum amount of CO_2 that can be captured and stored is limited by the physical capacity $\bar{S}$:

$$S_t \leq \bar{S} \quad \forall t \tag{3}$$

To motivate this assumption, we can argue that carbon emissions are mainly stockpiled into empty geological deposits, such as oil sinks or gas fields, and those potential reservoirs are available themselves in finite quantities. CCS is costly and we assume that the sequestration cost $D(\gamma_t, x_t)$ depends both on the level of emissions and the rate of sequestration.[4] For simplicity, we impose $D(\gamma_t, x_t) = \beta x_t d(\gamma_t)$, where function $d(.)$ is increasing and convex in γ_t. Then, sequestration costs are linear in carbon emissions, but not in the rate of sequestration in order to reflect decreasing returns in the associated CCS technology.

Finally the atmospheric carbon accumulation process is captured by the following dynamic constraint:

$$\dot{P}_t = (1 - \gamma_t)\beta x_t - \alpha P_t, \quad P_0 \textit{ given} \tag{4}$$

where $(1-\gamma_t)\beta x_t$ measures the carbon emissions net of abatement and α is the natural rate of decay of the atmosphere.

3 The optimal extraction and CCS paths

3.1 Optimal program and first-order conditions

The program of the social planner consists in choosing a fossil fuel consumption profile $\{x_t\}_{t\geq 0}$ and a sequestration rate trajectory $\{\gamma_t\}_{t\geq 0}$ that maximize the sum of the discounted net current surplus:

$$\max_{\{x_t, \gamma_t\}} \int_0^{\infty} \left[u(x_t) - v(P_t) - c(X_t)x_t - \beta x_t d(\gamma_t)\right] e^{-\rho t} dt \tag{5}$$

[4] For the sake of computational conveniences, we do not assume here that the sequestration cost depends on the cumulated past storage S_t.

subject to constraints (1)-(4) and to:

$$x_t \geq 0 \tag{6}$$
$$0 \leq \gamma_t \leq 1 \tag{7}$$

Here we make several points before solving this optimal program. As usually assumed, we will not consider the non-negativity constraints on the state variables. From the Inada conditions, the non-negativity constraint on x_t will never be binding, except asymptotically, so we do not consider it further here. Finally, we examine the case where the economy is not constrained by (7) and we will check this condition ex-post.

The corresponding Hamiltonian in current value writes:

$$\begin{aligned} H &= u(x_t) - v(P_t) - c(X_t)x_t - \beta x_t d(\gamma_t) \\ &\quad -\lambda_t x_t + \mu_t \gamma_t \beta x_t + \eta_t \left[(1-\gamma_t)\beta x_t - \alpha P_t\right] + \xi_t \left(\bar{S} - S_t\right) \end{aligned}$$

where λ_t, μ_t, η_t are the co-state variables associated with state equations (1), (2) and (4). Those variables read respectively as the scarcity rent of the fossil resource ($\partial W_t / \partial X_t$), the implicit (social) marginal value of carbon capture and storage ($\partial W_t / \partial S_t$), the implicit (social) marginal cost of releasing CO_2 into the atmosphere ($\partial W_t / \partial P_t$). Intuitively, along any optimal path, we may obtain non-negative values for λ_t and non-positive values for μ_t and η_t. Moreover, ξ_t denotes the social cost of sequestration coming from a tightening in the limited capacity constraint of reservoir, formally the Lagrange multiplier associated with constraint (3).

The first-order conditions are:

$$\frac{\partial H}{\partial x_t} = 0 \Rightarrow u'(x_t) = c(X_t) + \lambda_t + \beta[d(\gamma_t) - \gamma_t \mu_t] - (1-\gamma_t)\beta\eta_t \tag{8}$$
$$\frac{\partial H}{\partial \gamma_t} = 0 \Rightarrow d'(\gamma_t) - \mu_t = -\eta_t \tag{9}$$
$$\frac{\partial H}{\partial X_t} = \rho\lambda_t - \dot{\lambda}_t \Rightarrow \dot{\lambda}_t = \rho\lambda_t + x_t c'(X_t) \tag{10}$$
$$\frac{\partial H}{\partial S_t} = \rho\mu_t - \dot{\mu}_t \Rightarrow \dot{\mu}_t = \rho\mu_t + \xi_t \tag{11}$$
$$\frac{\partial H}{\partial P_t} = \rho\eta_t - \dot{\eta}_t \Rightarrow \dot{\eta}_t = (\rho + \alpha)\eta_t + v'(P_t) \tag{12}$$

The complementary slackness condition and the transversality conditions are:

$$\xi_t(\bar{S} - S_t) = 0, \quad \xi_t \geq 0 \tag{13}$$
$$\lim_{t\to\infty} \lambda_t X_t e^{-\rho t} = 0 \tag{14}$$
$$\lim_{t\to\infty} \mu_t S_t e^{-\rho t} = 0 \tag{15}$$
$$\lim_{t\to\infty} \eta_t P_t e^{-\rho t} = 0 \tag{16}$$

Equation (8) equates the marginal utility of consuming one unit of fossil energy with its full marginal cost. This last term can be decomposed in: i) the marginal extraction cost $c(X_t)$, ii) the resource scarcity rent λ_t, iii) the full cost of sequestration $\beta[d(\gamma_t) - \gamma_t \mu_t]$ by unit of fossil fuel use, and iv) the marginal social cost of augmenting the atmospheric carbon stock by the flow of residual emissions, i.e. $(1 - \gamma_t)\beta\eta_t$. Equation (9) says that the full marginal cost of carbon burying (left-hand-side) must be equal to its social marginal gain in terms of atmospheric carbon concentration reduction (right-hand side). Equation (10) is no other than the Hotelling rule in the case of stock-dependent extraction costs. Equation (11) implies that the implicit marginal value of CCS must grow at the pure rate of time preferences ρ, augmented by ξ_t which reflects the limited capacity of carbon sinks. Note that, from (13), this last term is nil as long as the reservoir is not fulfilled and non-negative otherwise, which means that μ_t obeys to the Hotelling rule only during the phase along which CCS is active. Finally, equation (12) says that the social marginal cost of atmospheric CO_2 accumulation must grow at a rate equal to the sum of the pure rate of time preference augmented by the natural rate of decay,[5] and the marginal damage.

Finally, remark that replacing $(\mu - \eta)$ by $d'(\gamma)$ from (9), the first-order condition (8) can be rewritten as:

$$u'(x_t) = c(X_t) + \lambda_t + \beta[d(\gamma_t) - \gamma_t d'(\gamma_t) - \eta_t] \tag{17}$$

3.2 Optimal dynamics

First, we solve the non-homogeneous differential equations (11) and (12) by using the associated transversality conditions (15) and (16), respectively, in order to identify initial values μ_0 and η_0. For any t, solutions are given by (stars in exponent refer here to optimality):

$$\mu_t^* = -\int_t^\infty \xi_s e^{-\rho(s-t)} ds \tag{18}$$

$$\eta_t^* = -\int_t^\infty v'(P_s) e^{-(\rho+\alpha)(s-t)} ds \tag{19}$$

Since $\xi_t \geq 0$ from (13) and $v'(.) > 0$ by assumption, we can then check that μ_t^* and η_t^* are non-positive for any t. The social marginal cost of sequestration (by unit of CO_2 emitted) is equal to the discounted sum over time of the instantaneous costs of the reservoir capacity constraint, from t up to ∞. The social

[5] This first term can be seen as a "modified" discount rate in order to take into account that emitting an additional unit of carbon today yields a marginal return ρ tomorrow, but it also increases the future marginal regeneration of the atmosphere by α.

marginal cost of atmospheric carbon concentration is equal to the discounted sum (at rate $\rho+\alpha$) over time of instantaneous marginal damages, from t up to ∞.

Next, to solve the optimal program in this deterministic case, we need to find the optimal expression of ξ_t. Let us assume that the carbon reservoir is fulfilled at a finite date $\bar{t} << \infty$. We will discuss about an eventual asymptotic fulfilling up of the reservoir later. Obviously, $\bar{t}$ is determined from $S_{\bar{t}} = \bar{S}$ and thus depends on the size $\bar{S}$ of the reservoir. For any date $t > \bar{t}$, we have $\dot{S}_t = 0$, which implies $\gamma_t \beta x_t = 0$. But due to the Inada conditions that we have imposed, the fossil resource stock can be exhausted only asymptotically, which finally implies $\gamma_t = 0$ for any $t \geq \bar{t}$. Since, in that case, (9) writes $d'(0) = \mu_t - \eta_t$, we must have $\dot{\mu}_t - \dot{\eta}_t = 0$, $\forall t > \bar{t}$. From (11) and (12), we thus obtain $\xi_t = \alpha\eta_t + v'(P_t) - \rho d'(0)$, $\forall t \geq \bar{t}$, which, by using (19), implies:

$$\xi_t^* = \begin{cases} 0 & , t < \bar{t} \\ v'(P_t) - \rho d'(0) - \alpha \int_t^\infty v'(P_s) e^{-(\rho+\alpha)(s-t)} ds & , t \geq \bar{t} \end{cases} \tag{20}$$

Recall that ξ_t^* reads as the optimal social value of the limited capacity constraint of carbon reservoirs or, in other words, as the marginal increase of social welfare coming from a marginal increase of $\bar{S}$. It is equal to zero as long as the reservoir is not filled, and it takes some positive value thereafter. Moreover, from the non-negativity condition (13), we must impose the following constraint:

$$\frac{1}{\rho}\left[v'(P_t) - \alpha \int_t^\infty v'(P_s) e^{-(\rho+\alpha)(s-t)} ds\right] > d'(0), \quad \forall t > \bar{t} \tag{21}$$

which states that it is optimal to fulfill up the carbon sink in finite time if and only if the net marginal damage divided by the social discount rate at some future date after fulfilling is larger than the initial marginal sequestration cost by unit of CO_2 emission. Here, the net marginal damage at date t is defined by the instantaneous marginal damage $v'(P_t)$ at this date, diminished by the discounted sum (at rate $\rho+\alpha$) from t up to infinity of all the marginal damages that will be avoided owing to natural cleaning-up of the atmosphere.

Replacing ξ_t^* by its expression (20) into (18), expanding computations and after simplifications, we obtain:

$$\mu_t^* = \begin{cases} -\int_{\bar{t}}^\infty v'(P_s) e^{-\rho(s-t)} e^{-\alpha(s-\bar{t})} ds + d'(0) e^{-\rho(\bar{t}-t)} & , t < \bar{t} \\ -\int_t^\infty v'(P_s) e^{-(\rho+\alpha)(s-t)} ds + d'(0) & , t \geq \bar{t} \end{cases} \tag{22}$$

We determine now the optimal dynamics of the two control variables x_t and γ_t. We start with γ_t by differentiating (9) with respect to time and by replacing

$\dot{\mu}_t$ and $\dot{\eta}_t$ by their expressions coming from (11) and (12), respectively:

$$d''(\gamma_t)\dot{\gamma}_t = \rho\mu_t + \xi_t - (\rho+\alpha)\eta_t - v'(P_t) \tag{23}$$

Using (9) again, it comes:

$$d''(\gamma_t)\dot{\gamma}_t = \rho d'(\gamma_t) + \xi_t - \alpha\eta_t - v'(P_t) \tag{24}$$

Proceeding in the same way with x_t (i.e. differentiating (17) with respect to time and replacing the time derivatives of the co-state variables by their corresponding expressions), we obtain after simplifications:

$$u''(x_t)\dot{x}_t = \rho(\lambda_t - \beta\eta_t) - \beta\gamma_t d''(\gamma_t)\dot{\gamma}_t - \beta\left[\alpha\eta_t + v'(P_t)\right] \tag{25}$$

which, once have been used (24), becomes:

$$u''(x_t)\dot{x}_t = \rho[\lambda_t - \beta\eta_t - \beta\gamma_t d'(\gamma_t)] - \beta\left\{\gamma_t\xi_t + (1-\gamma_t)[\alpha\eta_t + v'(P_t)]\right\} \tag{26}$$

Using (17) again and replacing ξ_t by its expression coming from (24), we finally get:

$$u''(x_t)\dot{x}_t = \rho[u'(x_t) - c(X_t) - \beta d(\gamma_t) + \beta\gamma_t d'(\gamma_t)] - \beta\left[\gamma_t d''(\gamma_t)\dot{\gamma}_t + \alpha\eta_t + v'(P_t)\right] \tag{27}$$

where η_t is defined by (19).

In the next subsection, we provide an analytical example of optimal sequestration and consumption trajectories.

3.3 Analytical example

We first postulate that the marginal damage is constant over time: $v'(P_t) = v$, $\forall t \geq 0$. With this analytical simplification, expressions (19), (20) and (22) become:

$$\eta_t^* = \frac{-v}{\rho+\alpha} \tag{28}$$

$$\xi_t^* = \begin{cases} 0 & , t < \bar{t} \\ \rho\left[\frac{v}{\rho+\alpha} - d'(0)\right] & , t \geq \bar{t} \end{cases} \tag{29}$$

$$\mu_t^* = \begin{cases} -\left[\frac{v}{\rho+\alpha} - d'(0)\right] e^{-\rho(\bar{t}-t)} & , t < \bar{t} \\ -\left[\frac{v}{\rho+\alpha} - d'(0)\right] & , t \geq \bar{t} \end{cases} \tag{30}$$

Assuming a constant marginal damage leads to constant implicit costs of reservoir limited capacity constraint ξ^* and atmospheric CO_2 concentrations η^*. This last result implies that the net present value of future damages, discounted at rate $(\rho+\alpha)$ is constant over time. Moreover, the implicit cost of CCS μ_t^* becomes constant at the date at which the carbon reservoir is filled. Beforehand, it is increasing over time and continuity condition at $t = \bar{t}$ is satisfied. Remark that the existence condition (21) writes now:

$$\frac{v}{\rho + \alpha} > d'(0) \tag{31}$$

which implies that the net present value of future damages must be larger than the initial marginal sequestration cost in order to provide incentives enough for CCS.

We have next recourse to the same quadratic CCS cost function than in Gerlagh and van der Zwaan [9], and which is defined as follows:

$$d(\gamma_t) = \gamma_t \left(1 + \frac{\kappa}{2}\gamma_t\right) \tag{32}$$

where κ, $\kappa > 0$ is the index of convexity of this function, i.e. $d''(.) = \kappa$. Remark that the initial marginal sequestration cost by unit of emission is now unitary, i.e. $d'(0) = 1$. Consequently, the existence condition (21) is reduced to $v > \rho + \alpha$, i.e. the instantaneous marginal damage must be larger than the "modified" social discount rate. Introducing these specifications into (24) yields to:

$$\dot{\gamma}_t = \rho\gamma_t + \frac{1}{\kappa}\left[\xi^* - \rho\left(\frac{v}{\rho + \alpha} - 1\right)\right] \tag{33}$$

Given (29) and the fact that $\gamma_t = 0$ for any $t \geq \bar{t}$ by definition of $\bar{t}$, the solution of the non-homogeneous differential equation (33) is:

$$\gamma_t^* = \begin{cases} \frac{1}{\kappa}\left(\frac{v}{\rho+\alpha} - 1\right)\left[1 - e^{-\rho(\bar{t}-t)}\right] & , t < \bar{t} \\ 0 & , t \geq \bar{t} \end{cases} \tag{34}$$

where the initial value of γ_t^* is given by:

$$\gamma_0^* = \frac{1}{\kappa}\left(\frac{v}{\rho + \alpha} - 1\right)\left(1 - e^{-\rho\bar{t}}\right) \tag{35}$$

Let us finally turn to the computation of the optimal energy consumption path. Using the analytical specifications introduced above, (27) reduces to:

$$u''(x_t)\dot{x}_t = \rho\left[u'(x_t) - c(X_t) - \beta\gamma_t\left(1 + \frac{\kappa}{2}\gamma_t\right) - \beta(1 - \gamma_t)\frac{v}{\rho + \alpha}\right] - \beta\gamma_t\xi^* \tag{36}$$

which, by using (29), can be expanded as follows:

$$\dot{x}_t = \begin{cases} \frac{\rho}{u''(x_t)}\left[u'(x_t) - c(X_t) + \beta\gamma_t^*|_{t<\bar{t}}\left(\frac{v}{\rho+\alpha} - 1 - \frac{\kappa}{2}\gamma_t^*|_{t<\bar{t}}\right) - \frac{\beta v}{\rho+\alpha}\right] & , t < \bar{t} \\ \frac{\rho}{u''(x_t)}\left[u'(x_t) - c(X_t) - \frac{\beta v}{\rho+\alpha}\right] & , t \geq \bar{t} \end{cases} \tag{37}$$

where $\gamma_t^*|_{t<\bar{t}}$ is determined from (34). Once γ_t have been replaced replaced by its optimal expression (34) into (36), we get an autonomous system of non-homogeneous differential equations in $(x_t, X_t)_{t\geq 0}$, together with (1), that can be solved. From the determination of optimal controls γ_t^* and x_t^*, we will next be able to characterize S_t^* and P_t^*. Finally, from the continuity condition on stock S_t, we will characterize the optimal date $\bar{t}$ of carbon reservoir filling up, depending upon the limited capacity $\bar{S}$ and the other parameters of the model. However, at this step, we need functional forms for $u(.)$ and $c(.)$ to solve the problem at the end. We then assume that utility is CES and that the marginal cost of extraction is constant:

$$u(x_t) = \frac{x_t^{1-\epsilon}}{1-\epsilon} \quad \text{and} \quad c(X_t) = c \tag{38}$$

where ϵ and c are strictly positive parameters.

Eliminating the stock effect on fossil resource extraction significantly simplifies the problem since it make differential equation (10) homogeneous. When the marginal extraction cost is constant, we recover the standard Hotelling rule $\dot{\lambda}_t = \rho\lambda_t$, whose solution is $\lambda_t^* = \lambda_0 e^{\rho t}$, where λ_0 is such that $\int_0^\infty x_t^* = X_0$. First-order (17) condition can be thus rewritten as:

$$x_t^{-\epsilon} = c + \lambda_0 e^{\rho t} - \beta\left(\frac{\kappa\gamma_t^{*2}}{2} + \eta_t^*\right) = c + \frac{\beta v}{\rho+\alpha} + \lambda_0 e^{\rho t} - \frac{\beta\kappa\gamma_t^{*2}}{2} \tag{39}$$

Given (34), this expression is equivalent to:

$$x_t^* = \begin{cases} \left[c + \frac{\beta v}{\rho+\alpha} + \lambda_0 e^{\rho t} - \frac{\beta}{2\kappa}\left(\frac{v}{\rho+\alpha} - 1\right)^2 \left(1 - e^{-\rho(\bar{t}-t)}\right)^2\right]^{-\frac{1}{\epsilon}} & , t < \bar{t} \\ \left[c + \frac{\beta v}{\rho+\alpha} + \lambda_0 e^{\rho t}\right]^{-\frac{1}{\epsilon}} & , t \geq \bar{t} \end{cases} \tag{40}$$

and then, the initial energy consumption level is:

$$x_0^* = \left[c + \frac{\beta v}{\rho+\alpha} + \lambda_0 - \frac{\beta}{2\kappa}\left(\frac{v}{\rho+\alpha} - 1\right)^2 \left(1 - e^{-\rho\bar{t}}\right)^2\right]^{-\frac{1}{\epsilon}} \tag{41}$$

All these findings are summarized into the following proposition.

Proposition 1 *The optimal sequestration rate and the optimal energy consumption $\{\gamma_t^*, x_t^*\}$ are characterized by:*

$$\gamma_t^* = \begin{cases} \frac{1}{\kappa}\left(\frac{v}{\rho+\alpha} - 1\right)\left[1 - e^{-\rho(\bar{t}-t)}\right] & , t < \bar{t} \\ 0 & , t \geq \bar{t} \end{cases} \tag{42}$$

$$x_t^* = \left(c + \frac{\beta v}{\rho+\alpha} + \lambda_0^* e^{\rho t} - \frac{\beta\kappa\gamma_t^{*2}}{2}\right)^{-\frac{1}{\epsilon}}, \ t \geq 0 \tag{43}$$

where the couple of variables $\{\bar{t}, \lambda_0^*\}$ is determined by the following system of equations:

$$\int_0^\infty x_t^* = X_0 \tag{44}$$

$$\beta \int_0^{\bar{t}} \gamma_t^* x_t^* dt = \bar{S} \tag{45}$$

Those solutions[6] are illustrated in Figure (1). Since $\dot{\gamma} = -\frac{\rho}{\kappa}\left(\frac{v}{\rho+\alpha} - 1\right) e^{-\rho(\bar{t}-t)} < 0$, the optimal sequestration rate starts from its initial value γ_0^* as defined by (35), and declines over time up to 0 which is reached at date $\bar{t}$, as showed in the northeast quadrant of Figure (1). The northwest quadrant draws the optimal fossil energy as a function of the optimal sequestration rate. From (42), for any $t < \bar{t}$, (43) can be rewritten as:

$$x^*(\gamma_t^*) = \left[c + \frac{\beta v}{\rho+\alpha} + \lambda_0^* e^{\rho\bar{t}} - \lambda_0^* \kappa \left(\frac{\rho+\alpha}{v-\rho-\alpha}\right)\gamma_t^* - \frac{\beta\kappa}{2}\gamma_t^{*2}\right]^{-\frac{1}{\epsilon}} \tag{46}$$

that is as an optimal energy consumption policy function depending on the sequestration rate. It is easy to verify that such a function is increasing and convex in γ^*.

The southwest quadrant is a purely technical device to show how the energy consumption is derived from the sequestration rate at the same time, which is finally illustrated in the southeast quadrant of Figure (1). We thus obtain an optimal consumption trajectory that is declining over time and continuous at $t = \bar{t}$, but that exhibits a kink at $t = \bar{t}$ (i.e. its slope is discontinuous at this point of time). Note that, for any $t \geq \bar{t}$, the rest of the optimal trajectory is characterized by (43) when $\gamma_t^* = 0$. As expected, the fossil resource exhaustion occurs then asymptotically.

Let us now check the existence conditions of the optimal solutions mentioned in Proposition 1. First, a direct implication of $\xi_t^* \geq 0$ is that $\gamma_t^* \geq 0$ for any t. Second, a necessary and sufficient condition for having γ_t^* smaller than 1 is $\gamma_0^* \leq 1$. Such a condition leads to:

$$\bar{t} \leq -\frac{1}{\rho} \ln\left[1 - \frac{\kappa(\rho+\alpha)}{v-\rho-\alpha}\right] \tag{47}$$

Condition (47) also provides some insights about the value of $\bar{t}$. If $v > (\rho+\alpha)(\kappa+1)$, (47) guarantees that carbon reservoir is fulfilled in finite time. Else, $\bar{t}$ can be finite as well as infinite. Finally, we have $x_t^* \geq 0$ for any $t \geq \bar{t}$ and $x_{\bar{t}}^* > 0$,

[6] We have here derived the analytical solution using a CES utility function. However, we can also write the optimal extraction for any class of utility function $u(x)$. In the general case, expression (43) can be rewritten as: $x_t^* = u'^{-1}\left(c + \frac{\beta v}{\rho+\alpha} + \lambda_0^* e^{\rho t} - \frac{\beta\kappa\gamma_t^{*2}}{2}\right)$.

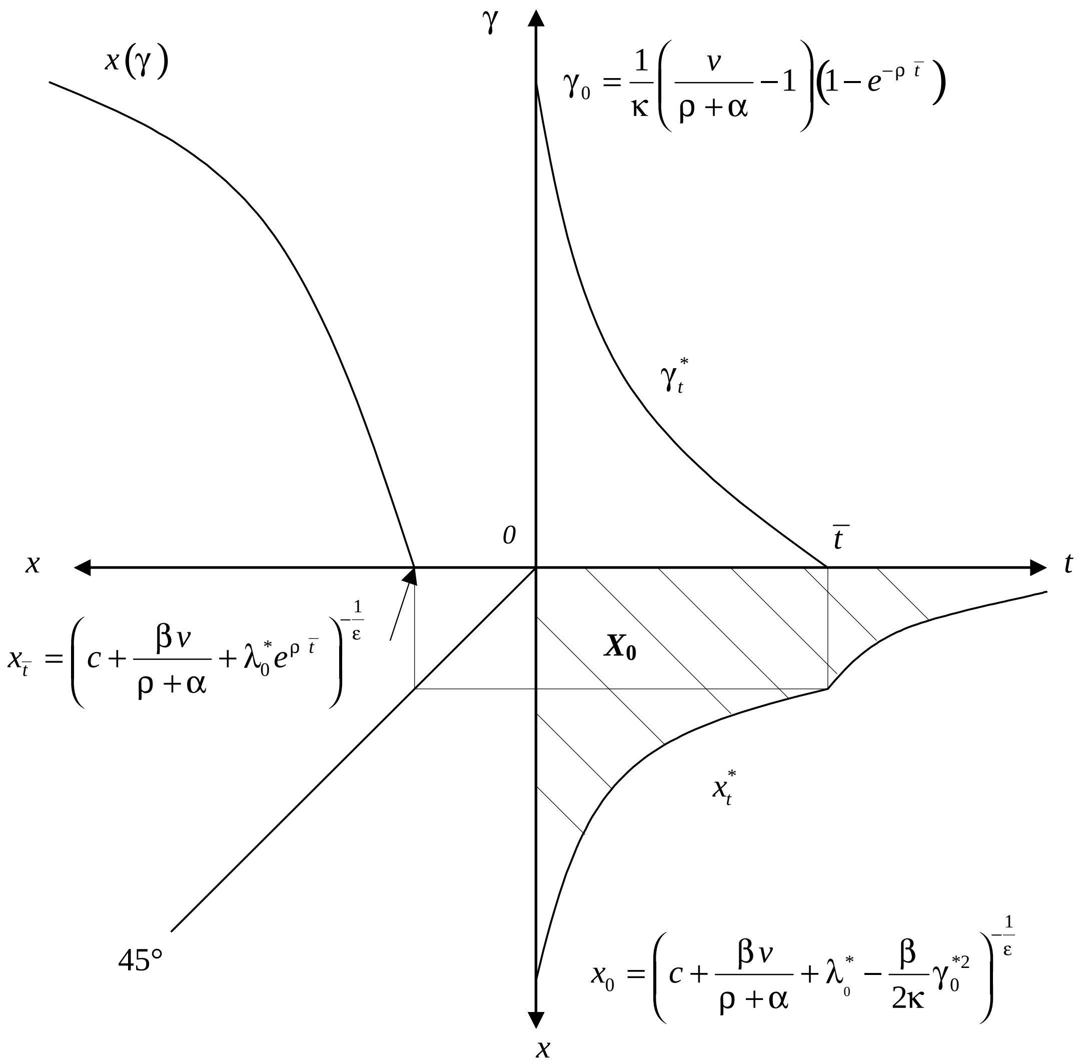

Figure 1: Optimal sequestration rate and fossil energy consumption

Condition for:	$v \leq \rho + \alpha$	$\rho + \alpha < v \leq (\kappa + 1)(\rho + \alpha)$	$(\kappa + 1)(\rho + \alpha) < v$
$\gamma_t^* > 0$	No	Yes	Yes
$\gamma_t^* \leq 1$	Yes	Yes	Yes
$\bar{t}$ finite	Not necessary	Not necessary	Yes

Table 1: Conditions of existence

which insures that $x_t^* > 0$ for any $t < \bar{t}$ since x_t^* is monotonically decreasing over time. Table (1) summarizes these findings.

4 Decentralization of the economy and implementation of the optimum

In this section, we decentralize the economy which have been studied above by considering the individual programs of the fossil resource-holder and of the consumer. We assume perfect competitive markets and we denote by p_t and r_t the fossil fuel price and the real interest rate on financial markets, respectively. In order to correct the environmental externality, we introduce a carbon tax profile $\{\tau_t\}_{t=0}^{\infty}$. Note that, due to the CCS device, the tax applies on the sole part of carbon emissions which are effectively released into the atmosphere after sequestration. In that sense, carbon taxation is disconnected from the fossil resource use.

The resource-holder chooses the extraction path $\{x_t\}_{t=0}^{\infty}$ that maximizes the discounted sum over time of its current profits $\int_0^{\infty}[p_t - c(X_t)]x_t \exp\left(-\int_0^t r_s ds\right) dt$ subject to the constraint (1). First-order conditions imply:

$$\dot{p}_t = r_t[p_t - c(X_t)] \tag{48}$$

which is no other than the standard Hotelling rule with extraction costs, and which states that the resource rent must grow at the real interest rate.

The program of the resource-user consists in choosing the consumption and sequestration rate trajectories $\{x_t\}_{t=0}^{\infty}$ and $\{\gamma_t\}_{t=0}^{\infty}$ that maximize $\int_0^{\infty}[u(x_t) - \beta x_t d(\gamma_t) - p_t x_t - \tau_t(1-\gamma_t)\beta x_t] \exp(-\rho t) dt$, subject to constraints (2), (3) and (7). As in the previous section, we examine the case where the decision-maker is not constrained by (7), which leads to the following first-order conditions:

$$u'(x_t) = \beta d(\gamma_t) + p_t + \tau_t \beta(1-\gamma_t) - \beta \gamma_t \mu_t^e \tag{49}$$

$$d'(\gamma_t) = \tau_t + \mu_t^e \tag{50}$$

$$\dot{\mu}_t^e = \rho \mu_t^e + \xi_t^e \tag{51}$$

where, by analogy with the optimal program, μ_t^e and ξ_t^e are, respectively, the multipliers associated with constraints (2) and (3), but now expressed at the equilibrium. Differentiating (49) with respect to time and using (50), it comes:

$$u''(x_t)\dot{x}_t = \dot{p}_t + \beta\dot{\tau}_t - \beta\gamma_t d''(\gamma_t)\dot{\gamma}_t \tag{52}$$

Along any equilibrium trajectory, p_t is governed by the dynamic condition (48) so that, after simplifications and using (49) again, (52) can be rewritten as:

$$u''(x_t)\dot{x}_t = r_t\left\{u'(x_t) - c(X_t) - \beta[d(\gamma_t) - \gamma_t d'(\gamma_t) + \tau_t]\right\} + \beta\dot{\tau}_t - \beta\gamma_t d''(\gamma_t)\dot{\gamma}_t \tag{53}$$

There exists a particular equilibrium $\{x_t^e(\tau_t), \gamma_t^e(\tau_t)\}$ associated with any carbon tax trajectory. This set of equilibria is characterized by the condition (53) above. By analogy between this condition and the corresponding condition (27), we can determine the carbon tax trajectory that implements the optimum. Noting that the optimal interest rate must be equal to the social time preference index, i.e. $r_t^o = \rho$.[7] Then, $\forall t \geq 0$, the optimal tax scheme τ_t^o is such that:

$$\dot{\tau}_t^o = \rho\tau_t^o - \alpha\eta_t^* - v'(P_t^*) \tag{54}$$

where η_t^* is defined by (19) and where P_t^* denotes the trajectory of atmospheric carbon accumulation that is followed at the optimum. Results about the implementation of the optimum are summarized in the following proposition.[8]

Proposition 2 *The optimal environmental policy and the associated interest rate and fossil fuel price are given by:*

$$\tau_t^o = \tau_0^o e^{\rho t} - \int_0^t [\alpha\eta_s^* + v'(P_s^*)]e^{-\rho(s-t)}ds \tag{55}$$

$$r_t^o = \rho \tag{56}$$

$$p_t^o = p_0^o e^{\rho t} - \rho\int_0^t c(X_s^*)e^{-\rho(s-t)}ds \tag{57}$$

[7] We would obtain the same implementation requirement by considering the budget constraint of the consumer who can save money and hold a stock of bonds B_t. In its simplest form, this constraint would be $\dot{B}_t = rB_t - p_t x_t - \beta x_t d(\gamma_t) - \tau_t(1-\gamma_t)\beta x_t$.

[8] Another decentralization option is to consider that the representative energy user directly consumes final energy services and that the CCS deployment is undertaken by the fossil resource-holder (now the energy sector). This last sector also bears the carbon tax burden so that its program becomes:

$$\max \int_0^\infty \{[p_t - c(X_t)]x_t - \beta x_t d(\gamma_t) - \tau_t(1-\gamma_t)\beta x_t\}\, e^{-\int_0^t r_s ds}dt$$

subject to constraints (1), (2) and (3). Static and dynamic first order conditions of this program can be reduced in:

$$\dot{p}_t = r_t\left\{p_t - c(X_t) - \beta[\tau_t + d(\gamma_t) - \gamma_t d'(\gamma_t)]\right\}$$

The consumer intertemporal simple calculus gives $u'(x_t) = p_t$ and it is easy to verify that Proposition 2 remains valid. In the case where the budget constraint of the consumer, $\dot{B}_t = rB_t - p_t x_t$, is taken into account, we would obtain:

$$u''(x_t)\dot{x}_t = u'(x_t)\left[\rho - r_t + \frac{\dot{p}_t}{p_t}\right]$$

and conditions (54) and (56) would be sufficient to insure the coincidence between the equilibrium outcome and the optimum.

where the initial values of τ_t^o and p_t^o are determined by:

$$\tau_0^o = \frac{1}{\beta}\left[u'(x_0^*) - \beta d(\gamma_0^*) + \beta\gamma_0^* d'(\gamma_0^*) - p_0^o\right] \tag{58}$$

$$p_0^o = c(X_0) + \lambda_0^*, \quad \text{with } \lambda_0^* \text{ s.t. } \int_0^\infty x_t^* dt = X_0 \tag{59}$$

Finally, we illustrate those findings by using the same analytical forms than the ones that have been introduced in the previous section. Under specifications, we obtain $p_t^o = c + \lambda_0^* \exp(\rho t)$ from (57) and (59). From (55), the specified optimal tax trajectory writes $\tau_t^o = \tau_0^o \exp(\rho t) + (1 - \exp(\rho t))v/(\rho + \alpha)$. However, from (58), we find an initial level of tax equal to $v/(\rho + \alpha)$. This implies that, in the specified version of the model, the optimal carbon tax is constant over time and equal to $\tau_t^o = v/(\rho + \alpha)$, $\forall t \geq 0$. The optimal carbon price at any time t should be equal to the sum from t up to ∞ of the future marginal damage involved by the emission at time t of one unit of carbon and discounted at rate $(\rho + \alpha)$ in order to take into account that carbon is naturally absorbed into the atmosphere at rate α by unit of time. Obviously, in the special case where instantaneous marginal damages are considered as constant, the optimal carbon tax should also be constant over time.

5 Conclusion

Following the IPCC's report [17], which recommended the development and the use of carbon capture and sequestration (CCS) technologies in order to achieve the environmental goals, defined by the Kyoto Protocol, the issue we have addressed in this paper concerns the optimal strategy regarding the long-term use of carbon capture and sequestration technologies.

The aim of this paper was to study the optimal carbon capture and sequestration policy. We then tried to analyze what should be the CCS optimal policy in a deterministic world. We have shown within this simple model that, under some conditions on the cost of extractions, CCS may be a long-term solution for the carbon emissions problem. Besides, it is also shown that the social planner will optimally choose to decrease the rate of capture and sequestration. We have also introduced a decentralized economy by considering the individual program of the fossil resource-holder and the one of the representative consumer. This helped us to compute analytically the optimal environmental policy, that is the optimal tax scheme, and also the optimal fossil fuel price profile.

However, all this results are obtained in a deterministic world. One direct and natural extension of the model, among others, might be to take into account the uncertainty linked to CCS efficiency. The CCS technologies in action are still recent and we do not know exactly the full consequences of such abatement technologies, in terms of environmental consequences (on oceans for instance), or in terms of efficiency once we consider the leakage problems.

References

[1] Ayong Le Kama, A., 2001. Preservation and exogenous uncertain future preferences. Economic Theory 18, 745-752.

[2] Ayong Le Kama, A., Fodha, M., 2010. Optimal Nuclear Waste Burial Policy under Uncertainty. Optimal Control Applications and Methods 31, 67-76.

[3] Ayong Le Kama, A., Schubert K., 2004. Growth, Environment and Uncertain Future Preferences. Environmental and Resource Economics 28, 31-53.

[4] Ayong Le Kama, A., Schubert, K., 2006. Ressources renouvelables et incertitude sur les préférences des générations futures. Revue d'Economie Politique 2, 229-250.

[5] Dasgupta, P., Heal, G., 1974. The Optimal Depletion of Exhaustible Resources, Review of economic studies 41, 1-28.

[6] Edenhofer, O., Bauer, N., Kriegler, E., 2005. The impact of technological change on climate protection and welfare: insights from the model MIND. Ecol. Econ. 54, 277-292.

[7] Edmonds, J., Clarke, J., Dooley, J., Kim, S.H., Smith, S.J., 2004. Stabilization of CO2 in a B2 world: insights on the roles of carbon capture and disposal, hydrogen, and transportation technologies. Energy Economics. 26, 517-537.

[8] Gerlagh, R., 2006. ITC in a global growth-climate model with CCS. The value of induced technical change for climate stabilization. Energy Journal, 1, 55-72 Special issue.

[9] Gerlagh, R., van der Zwaan, B.C., 2006. Options and instruments for a deep Cut in CO2 emissions: carbon capture or renewable, taxes or subsidies? Energy Journal 27, 25-48.

[10] Gitz, V., Ambrosi, P., Magné, B., Ciais, P., 2009. Is there an optimal timing for sequestration to stabilize future climate? American Geophysical Union, Geophysical Monograph, Washington, DC.

[11] Gradus, R., Smulders, S., 1996. Pollution Abatement and Long-term Growth. European Journal of Political Economy 12, 505-532.

[12] Grimaud, A., Rouge, L., 2009. Séquestration du carbone et politique climatique optimale. Economie et Prévision 190-191, 53-69.

[13] Grimaud A., Lafforgue G., Magné, B., 2010. Climate change mitigation options and directed technical change: A decentralized equilibrium analysis. Resource and Energy Economics. In press.

[14] Hartwick, R., 1977. Intergenerational Equity and the Investing of Rents from Exhaustible Resources. American Economic Review 67, 972-974.

[15] Heal G. M., 1993. The Optimal Use Of Exhaustible Resources, in Handbook of Natural Resource and Energy Economics, vol. III, A.V. Kneese and J.L. Sweeney (Eds), Elsevier Science Publishers.

[16] Hotteling, H., 1931. The Economics of Exhaustible Resources. The Journal of Political Economy 39, 137-175.

[17] IPCC, 2005. Special Report on Carbon Dioxide Capture and Storage, Contribution of Working Group III, Report of the Intergovernmental Panel on Climate Change. Cambridge Univ. Press, Cambridge.

[18] Kurosawa, A., 2004. Carbon concentration target and technological choice. Energy Economics 26, 675-684.

[19] Lafforgue, G., Magne B., Moreaux M., 2008. Energy substitutions, climate change and carbon sinks. Ecological Economics 67, 589-597.

[20] Lafforgue, G., Magne B., Moreaux M., 2009. The optimal sequestration policy with a ceiling on the stock of carbon in the atmosphere. In The Design of Climate Policy (Chapter 14, pp. 273-304), R. Guesnerie and H. Tulkens eds, Boston: The MIT Press.

[21] McFarland, J.R., Herzog, H.J., Reilly, J.M., 2003. Economic modelling of the global adoption of carbon capture and sequestration technologies. In: Gale, J., Kaya, Y. (Eds.), Proceedings of the Sixth International Conference on Greenhouse Gas Control Technologies. Elsevier Science, Oxford.

[22] Ragot, L., Schubert K., 2008. The optimal carbon sequestration in agricultural soils: do the dynamics of the physical process matter? Journal of Economic Dynamics and Control 32, 3847-3865.

[23] van der Ploeg and Withagen C., 1991. Pollution Control and the Ramsey Problem. Environmental and Resource Economics 1, 215-236.