The Price for Information about Probabilities and its Relation with Capacities♦

Giuseppe Attanasi[1] – *Toulouse School of Economics (LERNA), France*

Aldo Montesano – *Bocconi University, Milan, Italy*

September 2010

Abstract

In this paper ambiguity aversion is measured through the maximum price the decision maker is willing to pay in order to know the probability of an event. Two comparative problems are examined in which the decision maker faces an act: in one case buying information implies playing a lottery, while in the other case buying information gives also the option to avoid playing the lottery. In both decision settings, relying on Choquet expected utility model, we study how the decision maker's risk and ambiguity attitudes affect the reservation price for information. These effects are analyzed for different levels of ambiguity of the act.

JEL classification: D81; D83; C91.

Keywords: Ambiguity Aversion; Choquet Expected Utility; Information about probabilities.

♦ G. Attanasi gratefully acknowledges financial support by the Chair Georges Meyer in Mathematical Economics at Jean-Jacques Laffont Foundation (TSE). The research leading to these results has received funding from the European Research Council under the European Community's Seventh Framework Programme (FP7/2007-2013) Grant Agreement no. 230589.

[1] Corresponding author: 21 allée de Brienne, 31000 Toulouse. Email: attanasi@cict.fr. Phone: +33(0)561128713.

1. Introduction

The analysis of choice under ambiguity is one of the fundamental problems studied by decision theorists in the last decades. One of the first theorists addressing this issue was Knight (1921), who distinguished between "measurable" uncertainty or risk, with known probabilities, and "unmeasurable" uncertainty, with unknown probabilities. Even though Knight's distinctions did not play a role in the Savage expected utility model (Savage, 1954), the Ellsberg paradox (Ellsberg, 1961) reintroduced the importance of ambiguity in affecting decision making. This gave rise to experimental research on subjects' attitudes toward uncertainty, that is extensively reviewed in Camerer and Weber (1992).

Facing rich empirical evidence pointing to ambiguity aversion, various alternatives to Savage subjective expected utility theory have been proposed. Among these alternative theories, a clear formalization of ambiguity aversion is provided by Schmeidler's (1989) Choquet expected utility model with convex capacities, the multiple prior model of Gilboa and Schmeidler (1989), the smooth second-order prior model of Klibanoff et al. (2005) and the variational preferences model of Maccheroni et al. (2006). Each model allows to construct its own index of ambiguity aversion. However, these indexes have been rarely used in the experimental research, mainly because of the complexity in eliciting the fundamental variables on which they are based. [2]

In order to fill the gap between the theoretical characterization of ambiguity aversion and its experimental analysis, this paper links the subject's risk and ambiguity attitude to a variable that can be easily observed in the lab: the subject's reservation price for information about the probability of an unknown event. The idea that information reducing ambiguity has a positive value for ambiguity averse subjects has been already explored in the literature. Quiggin (2007), using Machina's (2004) concept of almost-objective acts, shows that ambiguity aversion may be defined in terms of the value of information. He states that, for expected utility preferences, the value of information with respect to almost-objective acts is asymptotically equal to zero. Snow (2010) studies the value of information in a non-expected utility model of ambiguity with second-order probabilities, an adaptation of the model of Klibanoff et al. (2005). He proves that the value of information that resolves ambiguity increases with greater ambiguity and with greater ambiguity aversion. However, he does not explore the effect of risk aversion on the value of information that resolves ambiguity. In this paper, instead, we focus on the

[2] Among recent experimental works aiming to estimate parametric models of ambiguity aversion, see Halevy (2007), Dominiak and Schnedler (2010), Ahn et al. (2010).

interplay between risk and ambiguity attitudes in determining the decision maker's reservation price for information that resolves ambiguity, for different levels of ambiguity. Furthermore, to provide a characterization of ambiguity aversion that can be easily adopted for an experimental test, we rely on Choquet expected utility model rather than on Klibanoff et al. (2005). In the former model, a rigorous experimental analysis of risk and ambiguity aversion would require to elicit the decision maker's utility function and her subjective beliefs on events, respectively. In the latter approach, given that ambiguity preferences are specifically captured by a nondecreasing transformation function defined on expected utility, we would need in addition to elicit this function. Finally, in the last part of the paper we examine the relation between the option value of information and its value in terms of ambiguity resolution.

We consider a simple decision problem under uncertainty, in which the subject should choose whether to buy or not buy information about the probability of an unknown event. As mentioned above, we measure her degree of ambiguity aversion through the maximum price she is willing to pay in order to know the objective probability of the event, for a generic utility function, that is for different risk aversion levels.

First, in a decision setting in which the subject should play an act, we derive an analytical relation between the reservation price for information about the objective probability of an event and the (subjective) capacity of this event. In this case, if the expected utility model holds, information should not have any economic value for the subject, given that even after buying information she should play the lottery. Since the choice to pay in order to know the objective probability cannot be rationalized in Savage's expected utility model, we move to Choquet expected utility and we find that an ambiguity averse subject is willing to pay a positive price for such information, since she prefers to know the probabilities, that is to participate in a lottery rather than to participate in an act.

Then, we slightly modify the decision setting, by allowing the subject to choose whether to play or not play the lottery in case she buys information about the probability of the event. In this setting, information does have an additional economic value, namely the option value in the subject's subsequent choice.

In both information settings, we analyze how the subject's willingness to pay for information changes when the level of ambiguity decreases. For a given level of ambiguity of the act the subject should play, once her reservation price for information is known, the corresponding capacity of the event can be easily calculated and ambiguity aversion can be measured accordingly. We argue that the two comparative decision problems proposed in this paper could be easily implemented in a laboratory experiment to measure subjects' ambiguity aversion through the price they are ready to pay to surely know the probability of an event.

2. The decision problem

Consider an urn with n_g *g*reen balls and n_b *b*lue balls. The Decision Maker (DM henceforth) knows the total number of balls $n = n_g + n_b$, but she does not know n_g and n_b. We are interested in the DM's valuation of the act $a_h = (\overline{x}, s_h; \underline{x}, s_{-h})$ with $h = g,b$ she has to participate in, where $\overline{x}$ represents her gain if the event s_h occurs (if the act is a_g, a *g*reen ball is drawn) and $\underline{x}$ is her gain if s_{-h} occurs (if the act is a_g, a *b*lue ball is drawn), with $\overline{x}, \underline{x} \in \mathbb{R}_+$ and $\overline{x} > \underline{x}$. The two events s_g, s_b are mutually incompatible $(s_g \cap s_b = \varnothing)$ and exhaustive ($s_g \cup s_b = S$, where S is the set of all possible events). Let us indicate with $\Pr(s_h)$ the probability that a *h*-color ball is drawn, with $\Pr(s_h) \geq 0$ for $h = g,b$, and $\Pr(s_g) + \Pr(s_b) = 1$.

Let V be the maximum price the DM is willing to pay in order to know the ratio between *h*-color balls and the total number of balls in the urn, i.e. the probability that a *h*-color ball is drawn, $\Pr(s_h) = \frac{n_h}{n}$ for $h = g,b$. Thus, *V* is the DM's reservation price for "full" information about the urn composition, i.e. for information that resolves ambiguity. We assume that the DM's choice is dichotomous: she can buy full information or not buy information at all. Therefore, throughout in the paper, the expression "buy information" always stands for "buy information that resolves ambiguity".

Assume that the DM's preferences are represented by the von Neumann–Morgenstern expected utility function in case of choice under risk (i.e., concerning lotteries, with known probabilities of the events) and by the Choquet expected utility function in case of choice under (risk and) ambiguity (i.e., referring to acts, with unknown probabilities of the events), with $u : \mathbb{R} \to \mathbb{R}$ being the correspondent utility function over the outcomes in both models. This function is assumed to be strictly increasing, that is $u(x') > u(x)$ if and only if $x' > x$.

2.1 Buying Information without using it

We assume that if the DM does not buy information, she has to participate in the act. Let us first consider the case in which, even if the DM buys information about $\Pr(s_g)$ and $\Pr(s_b)$, she has to play, i.e. she has to participate in the lottery. Hence, she does not have the option to give up playing the lottery if she does not "like" the revealed composition of the urn. The decision problem, with respect to act a_g, is described in Figure 1.

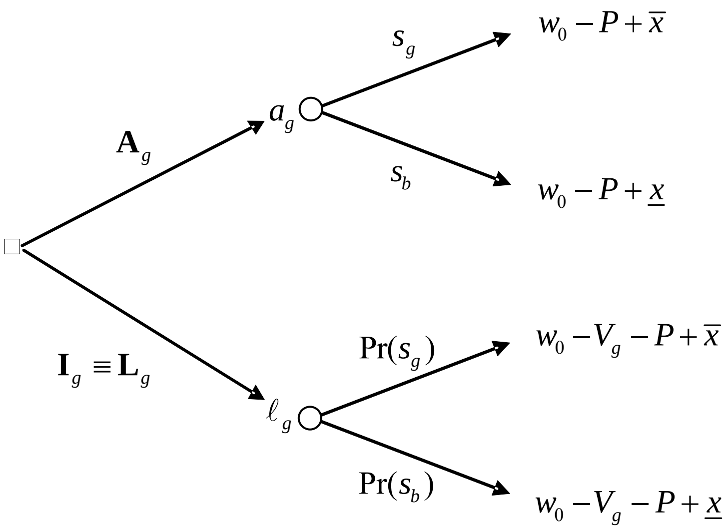

Figure 1. Choosing between an act and a lottery through buying information

In Figure 1, branch $\mathbf{A}_g$ indicates the choice of playing act a_g, so that the DM's final wealth is equal to the initial one, w_0, decreased by the price $P \geq 0$ paid to participate in the act, and increased by the random outcome of the act, $x(a_g)$. Branch $\mathbf{I}_g$ or, equivalently, $\mathbf{L}_g$, represents the choice of buying information about the probabilities of the events of act a_g, by paying the price V_g. In that case, the probabilities of the two events are fully revealed to the DM, so she plays a lottery determined by the act and the known probabilities of the events, $\ell_g = (\overline{x}, \Pr(s_g); \underline{x}, \Pr(s_b))$. Thus, her final wealth is equal to $w_0 - V_g - P + x(\ell_g)$, where $x(\ell_g)$ is the random outcome of the lottery. Notice that the participation price P does not influence the DM's choice in Figure 1. We introduce it here for a comparative analysis with the decision problem that will be discussed in section 2.2, in which P does play a role. For convenience, define also $w = w_0 - P$.

The condition for indifference between branch $\mathbf{A}_g$ and branch $\mathbf{L}_g$ determines the relation between the capacity of the favorable event, $v(s_g)$, and V_g, the maximum price the DM is willing to pay in order to know the probability of this event, $\Pr(s_g)$. The expected utility of branch $\mathbf{L}_g$ is $EU(w-V+\ell_g)=u(w+\underline{x}-V_g)\left(1-\Pr(s_g)\right)+u(w+\overline{x}-V_g)\Pr(s_g)$, that can be rewritten as $EU(w-V+\ell_g)=u(w+\underline{x}-V_g)+\left(u(w+\overline{x}-V_g)-u(w+\underline{x}-V_g)\right)\Pr(s_g)$.

We calculate the expected utility of branch $\mathbf{A}_g$ of the decision tree using the Choquet expected utility function $CEU(w+a_g)=u(w+\underline{x})+\left(u(w+\overline{x})-u(w+\underline{x})\right)v(s_g)$. [3]

Then, we impose the indifference condition between branch $\mathbf{A}_g$ and branch $\mathbf{L}_g$, $CEU(w+a_g)=EU(w-V_g+\ell_g)$, which leads to

$$v(s_g)=\frac{u(w+\underline{x}-V_g)-u(w+\underline{x})+\left(u(w+\overline{x}-V_g)-u(w+\underline{x}-V_g)\right)\Pr(s_g)}{u(w+\overline{x})-u(w+\underline{x})} \tag{1.a}$$

Let us now reframe the problem represented in Figure 1, by considering the mirrored act $a_b=(\overline{x},s_b;\underline{x},s_g)$, with the highest outcome $\overline{x}$ associated to the event s_b and the lowest outcome $\underline{x}$ to the event s_g. By imposing that the DM is indifferent between branch $\mathbf{A}_b$ and branch $\mathbf{L}_b$, we obtain a relation between the DM's reservation price for information and the capacity of the favorable event, that is similar to (1.a), with the substitution of the subscript *b* for *g*:

$$v(s_b)=\frac{u(w+\underline{x}-V_b)-u(w+\underline{x})+(u(w+\overline{x}-V_b)-u(w+\underline{x}-V_b))\Pr(s_b)}{u(w+\overline{x})-u(w+\underline{x})} \tag{1.b}$$

There is no reason for a Choquet expected utility DM to set $V_g \neq V_b$. Since symmetric information with respect to the occurrence of events should result in equal capacities (see Schmeidler, 1989), the equality between the two DM's reservation prices for the two mirrored acts is a minimal condition for rationality. Thus, we can assume, without loss of generality, that $V_g=V_b$, and indicate them with *V*. From relations (1.a) and (1.b), and taking into account that $\Pr(s_g)+\Pr(s_b)=1$, we obtain

$$v(s_g)+v(s_b)=\frac{u(w+\overline{x}-V)+u(w+\underline{x}-V)-2u(w+\underline{x})}{u(w+\overline{x})-u(w+\underline{x})} \tag{1.c}$$

and, consequently, the *index of ambiguity aversion*, based on Schmeidler's (1989) proposal of uncertainty premium, is

$$1-v(s_g)-v(s_b)=\frac{u(w+\overline{x})-u(w+\overline{x}-V)+u(w+\underline{x})-u(w+\underline{x}-V)}{u(w+\overline{x})-u(w+\underline{x})} \tag{2}$$

[3] Notice that, in the Choquet expected utility of act a_g, only capacity $v(s_g)$ appears. Then, in order to elicit capacity $v(s_b)$, we have to take into account act a_b in the analysis, as we do below.

Expression (2) relates subject's ambiguity attitude with her reservation price for information about the probability of the unknown event. The index of subject's ambiguity attitude is an increasing function of V and is positive (negative) if the DM is ambiguity averse (lover). The maximum price she is willing to pay to be informed is zero if and only if the DM is ambiguity neutral (whatever her attitude towards risk). In this case, (2) gives $v(s_g)+v(s_b)=1$, that is capacity is additive, i.e. it is a probability.

If the DM is risk neutral but not ambiguity neutral ($V \neq 0$), relation (2) becomes

$$1-v(s_g)-v(s_b)=\frac{2V}{\overline{x}-\underline{x}} \tag{3}$$

which is the *index of attitude towards ambiguity* for a *risk neutral* DM. In this case, ambiguity aversion is directly measured through the DM's reservation price in relative terms. If the DM is not risk neutral, then, using the Taylor expansion of the DM's utility function, (2) can be rewritten as [4]

$$1-v(s_g)-v(s_b)\simeq\frac{2V}{\overline{x}-\underline{x}}\cdot\frac{1+0.5\dfrac{u''(w)}{u'(w)}(\overline{x}+\underline{x}-V)}{1+0.5\dfrac{u''(w)}{u'(w)}(\overline{x}+\underline{x})} \tag{4}$$

so that if $\dfrac{u''(w)}{u'(w)}<(>)0$, then $1-v(s_g)-v(s_b)>(<)\dfrac{2V}{\overline{x}-\underline{x}}$ in (4). Notice that $-\dfrac{u''(w)}{u'(w)}$ is the de Finetti-Arrow-Pratt measure of risk aversion. Therefore, for an ambiguity averse DM, given $V>0$, the reservation price of a risk averse (lover) DM in (4) is lower (higher) than the one in (3) and it decreases (increases) in the de Finetti-Arrow-Pratt index. This means that the maximum price an ambiguity averse DM is willing to pay for ambiguity resolution is, *ceteris paribus* (i.e. for a given degree of ambiguity aversion) reduced by risk aversion, as formally stated below.[5]

Result 1. *For an ambiguity averse Choquet expected utility DM, the reservation price for information about the probability of the unknown event depends positively on her degree of ambiguity aversion and negatively on her degree of risk aversion. If the DM is*

[4] Relation (4) holds under the condition that the Taylor expansion provides a sufficiently good approximation of the DM's utility function. Therefore, it has been obtained by assuming $u(w+y)\simeq u(w)+u'(w)y+0.5u''(w)y^2$ and applying it for $y=\underline{x}-V,\underline{x},\overline{x}-V,\overline{x}$.

[5] In case of an ambiguity lover DM, given that $(-V)$ is her reservation price for information reception, relation (4) can be rewritten as $v(s_g)+v(s_b)-1\simeq\dfrac{2(-V)}{\overline{x}-\underline{x}}\cdot\dfrac{1+0.5\frac{u''(w)}{u'(w)}(\overline{x}+\underline{x}+(-V))}{1+0.5\frac{u''(w)}{u'(w)}(\overline{x}+\underline{x})}$.

ambiguity lover, given that her reservation price for information is negative, the minimum price she would be willing to accept in order to receive information is increasing both in her love for ambiguity and in her aversion to risk.

The intuition behind Result 1 can be summarized as follows. An ambiguity averse DM would pay for ambiguity resolution. However, according to (4), this payment represents a sure loss in order to switch from an act to a lottery which, although unambiguous, is as risky as the act. Thus, her willingness to pay is lower the higher her aversion to risk. If instead she is ambiguity lover, she prefers not to know the probabilities of the events, so she must be paid to be willing to accept ambiguity resolution: the more risk averse she is, the higher the sure amount she asks for.

Let us examine how the DM's reservation price for ambiguity resolution changes with the level of ambiguity of the decision setting. Consider two comparative information settings concerning the urn with n_g green balls and n_b blue balls introduced at the beginning of section 2. In the first setting, (α), the DM knows only that in the urn there are n green and blue balls, but she does not know how many of them are green and how many are blue. In the second setting, (β), the DM knows that in the urn there are at least $\underline{n}_g$ green balls and at least $\underline{n}_b$ blue balls, with $\underline{n}_g, \underline{n}_b \in \{0,1,...,n-1\}$ and $\underline{n}_g + \underline{n}_b < n$. Therefore, in setting (α) the DM faces a more ambiguous act than the one faced in setting (β). Correspondingly, if she buys information, the reduction in ambiguity is greater in (α) than in (β).

Let us indicate with V^j and $v^j(s_h)$ respectively the DM's reservation price and capacity for the event $h = g,b$ in the (j) information setting, with $j = \alpha, \beta.$ The relation determining the DM's reservation price for ambiguity resolution about the urn composition is (2), with $v^\alpha(s_h) \in [0,1]$ and $v^\beta(s_h) \in \left[\frac{\underline{n}_h}{n}, \frac{n-\underline{n}_{-h}}{n}\right]$, $h = g,b$. If the DM is ambiguity averse, then $v^\alpha(s_g) + v^\alpha(s_b) \in [0,1)$ and $v^\beta(s_g) + v^\beta(s_b) \in \left[\frac{\underline{n}_g + \underline{n}_b}{n}, 1\right)$. If instead she is ambiguity lover, then $v^\alpha(s_g) + v^\alpha(s_b) \in (1,2]$ and $v^\beta(s_g) + v^\beta(s_b) \in \left(1, \frac{2n - \underline{n}_g - \underline{n}_b}{n}\right]$.

Therefore, if the DM is "highly" ambiguity averse when facing the greater ambiguity level of the (α) setting, i.e. $v^{\alpha}(s_g)+v^{\alpha}(s_b)\in\left[0,\frac{\underline{n}_g+\underline{n}_b}{n}\right]$, we can easily state that $V^{\alpha}\geq V^{\beta}$. Similarly, if she is "highly" ambiguity lover when facing the greater ambiguity level of the (α) setting, i.e. $v^{\alpha}(s_g)+v^{\alpha}(s_b)\in\left[\frac{2n-\underline{n}_g-\underline{n}_b}{n},2\right]$, then $-V^{\alpha}\geq -V^{\beta}$. For all other values of the DM's capacities, there is no rationality condition that can be imposed on the capacities emerging in the (β) setting. Thus, in order to provide a deeper understanding of how the DM's risk and ambiguity attitudes affect V for different levels of ambiguity of the decision setting, we determine the threshold values of V, namely for the two extreme cases of *maximum aversion to ambiguity* and *maximum love for ambiguity*. Relying on the (α) setting as a benchmark, we examine how the smaller ambiguity characterizing the (β) setting affects these values.

Let us indicate with $\overline{V}^{j}$ $(\underline{V}^{j})$ the reservation price for ambiguity resolution of the most ambiguity averse (lover) DM in the (j) information setting, with $j=\alpha,\beta$.

In the (α) setting, maximum aversion to ambiguity is represented by $v(s_g)=v(s_b)=0$. Then, taking into account relation (2), $\overline{V}^{\alpha}$ is implicitly determined by the relation

$$\sum_{x=\underline{x},\overline{x}} u(w+x-\overline{V}^{\alpha})=2u(w+\underline{x}) \tag{5}$$

If the DM is risk neutral, then $\overline{V}^{\alpha}=\frac{\overline{x}-\underline{x}}{2}$. Maximum love for ambiguity is represented by $v(s_g)=v(s_b)=1$. Then, $\underline{V}^{\alpha}$ is implicitly determined by the relation

$$\sum_{x=\underline{x},\overline{x}} u(w+x-\underline{V}^{\alpha})=2u(w+\overline{x}) \tag{6}$$

In order for (6) to hold, $\underline{V}^{\alpha}$ must be negative. Thus, $-\underline{V}^{\alpha}$ is the price the most ambiguity lover DM would pay in order to avoid receiving information. Notice that for all $V\notin\left[\underline{V}^{\alpha},\overline{V}^{\alpha}\right]$ we can state that the DM is not rational according to Choquet expected utility theory. In fact, $V<\underline{V}^{\alpha}$ implies $v(s_g)+v(s_b)>2$. Similarly, $V>\overline{V}^{\alpha}$ implies $v(s_g)+v(s_b)<0$. If the DM is risk neutral, then $\underline{V}^{\alpha}=-\frac{\overline{x}-\underline{x}}{2}$, so that $\overline{V}^{\alpha}+\underline{V}^{\alpha}=0$. If the DM is not risk neutral, this equality does not hold. In particular, the following result can be proved.

Result 2. *If the Choquet expected utility DM is strictly risk averse, that is, if her utility function is strictly concave, then* $\overline{V}^{\alpha} + \underline{V}^{\alpha} < 0$*, i.e. the reservation price for information of the most ambiguity averse DM is lower in absolute value than that of the most ambiguity lover DM.*

Proof. Let us prove the statement by contradiction. Assume that $\overline{V}^{\alpha} + \underline{V}^{\alpha} = z \geq 0$. Then relations (5) and (6) would require

$$2(u(w+\overline{x}) - u(w+\underline{x})) = \sum_{x=\underline{x},\overline{x}} \left(u(w+x+\overline{V}^{\alpha} - z) - u(w+x-\overline{V}^{\alpha}) \right) \tag{7}$$

Strict concavity of u and $z \geq 0$ would require

$$u(w+\overline{x}+\overline{V}^{\alpha} - z) - u(w+\overline{x}-\overline{V}^{\alpha}) > 2(u(w+\overline{x}+\overline{V}^{\alpha} - z) - u(w+\overline{x})) \tag{8.a}$$

$$u(w+\underline{x}+\overline{V}^{\alpha} - z) - u(w+\underline{x}-\overline{V}^{\alpha}) > 2(u(w+\underline{x}+\overline{V}^{\alpha} - z) - u(w+\underline{x})) \tag{8.b}$$

Considering (8.a) and (8.b) together, we would obtain

$$\sum_{x=\underline{x},\overline{x}} \left(u(w+x+\overline{V}^{\alpha} - z) - u(w+x-\overline{V}^{\alpha}) \right) > 2 \sum_{x=\underline{x},\overline{x}} \left(u(w+x+\overline{V}^{\alpha} - z) - u(w+x) \right) \tag{9}$$

The left-hand side of (9) is equal to $2(u(w+\overline{x}) - u(w+\underline{x}))$, because of (7). Also the right-hand side, with $\underline{V}^{\alpha}$ in place of $-\overline{V}^{\alpha} + z$, is equal to $2(u(w+\overline{x}) - u(w+\underline{x}))$, because of (6). Therefore, contradiction implies that $z < 0$, that is $\overline{V}^{\alpha} + \underline{V}^{\alpha} < 0$. ■

Result 2 strengthens Result 1 in suggesting that risk aversion produces different effects on the reservation price of an ambiguity averse and of an ambiguity lover subject, respectively. Consider two equally risk averse DMs, having the same (strictly concave) utility function, but with opposite and mirrored ambiguity attitude. That is, the former is ambiguity averse, with $1 - v(s_g) - v(s_b) = c \in [0,1]$ and the latter is ambiguity lover, with $v(s_g) + v(s_b) - 1 = c$. Then, the reservation price for ambiguity resolution of the ambiguity lover DM will be greater, in absolute value, than the one set by the ambiguity averse DM. This is not true when the two DMs with opposite and mirrored ambiguity attitude are both risk neutral.

Let us now find the set of possible reservation prices in the less ambiguous setting, (β).

In the (β) setting, maximum aversion to ambiguity is represented by $v(s_g) = \frac{n_g}{n}$, $v(s_b) = \frac{n_b}{n}$. Then, $\overline{V}^{\beta}$ is implicitly determined by the relation

$$\sum_{x=\underline{x},\overline{x}} u(w+x-\overline{V}^{\beta}) = 2u(w+\underline{x}) + (u(w+\overline{x}) - u(w+\underline{x}))\frac{\underline{n}_g + \underline{n}_b}{n} \tag{10}$$

If the DM is risk neutral, then $\overline{V}^{\beta} = \frac{n - \underline{n}_g - \underline{n}_b}{2n}(\overline{x} - \underline{x})$. Maximum love for ambiguity is represented by $v(s_g) = \frac{n - \underline{n}_b}{n}$, $v(s_b) = \frac{n - \underline{n}_g}{n}$. Then, $\underline{V}_{\beta}$ is implicitly determined by the relation

$$\sum_{x=\underline{x},\overline{x}} u(w+x-\underline{V}^{\beta}) = 2u(w+\overline{x}) - (u(w+\overline{x}) - u(w+\underline{x}))\frac{\underline{n}_g + \underline{n}_b}{n} \tag{11}$$

If the DM is risk neutral, then $\underline{V}^{\beta} = -\frac{n - \underline{n}_g - \underline{n}_b}{2n}(\overline{x} - \underline{x})$, so that again $\overline{V}^{\beta} + \underline{V}^{\beta} = 0$. Result 2 holds also in the (β) setting, i.e. $\overline{V}^{\beta} + \underline{V}^{\beta} < 0$ if the DM is strictly risk averse.

The four threshold values for the DM's reservation price in settings (α) and (β) are related through the following expressions:

$$\begin{aligned}\left(u(w+\overline{x}) - u(w+\underline{x})\right)\frac{\underline{n}_g + \underline{n}_b}{n} &= \sum_{x=\underline{x},\overline{x}} \left(u(w+x-\overline{V}^{\beta}) - u(w+x-\overline{V}^{\alpha})\right) = \\ &= \sum_{x=\underline{x},\overline{x}} \left(u(w+x-\underline{V}^{\alpha}) - u(w+x-\underline{V}^{\beta})\right)\end{aligned} \tag{12}$$

By comparing relation (5) to relation (10) we conclude that it is $\overline{V}^{\beta} < \overline{V}^{\alpha}$, and, analogously, $\underline{V}^{\beta} > \underline{V}^{\alpha}$ from the comparison between (6) and (11). It follows $\left[\underline{V}^{\beta}, \overline{V}^{\beta}\right] \subset \left[\underline{V}^{\alpha}, \overline{V}^{\alpha}\right]$. Therefore, the interval of possible values that *V* can take is smaller the smaller the ambiguity of the setting, as formally stated below. Notice that this result holds whatever the DM's risk attitude. If the DM is risk averse, we can also conclude that $\left|\underline{V}^{\alpha} - \underline{V}^{\beta}\right| > \left|\overline{V}^{\alpha} - \overline{V}^{\beta}\right|$, i.e. when ambiguity is smaller the size of the decrease (in absolute value) of the DM's reservation price if she is ambiguity lover is greater than the size of the decrease if she is ambiguity averse. This second statement follows from (12) and from the strict concavity of the utility function. Both these results are formally summarized below.

Result 3. *With reference to a Choquet expected utility DM, the set of rational reservation prices for ambiguity resolution shrinks with* $(\underline{n}_g + \underline{n}_b)$*, i.e. its size depends negatively on the level of ambiguity. If the DM is risk averse, the decrease in her reservation price is greater if she is ambiguity lover than if she is ambiguity averse.*

From Result 3 it follows that, since $v^{\beta}(s_g)+v^{\beta}(s_b)\in\left[\frac{\underline{n}_g+\underline{n}_b}{n},\frac{2n-\underline{n}_g-\underline{n}_b}{n}\right]$, the DM, in stating V^{β}, only takes into account the size of the ambiguity reduction. She does consider whether $\underline{n}_g$ is greater or smaller than $\underline{n}_b$. This means that, in switching from a more ambiguous to a less ambiguous act (e.g., from setting (α) to (β)), any variation of the likelihood of the favorable event in the act a_g due to the reduction in the ambiguity level with respect to (α) does not influence the DM's reservation price for information in (β), if it is compensated by the opposite variation of the likelihood of the favorable event in the act a_b ($\underline{n}_g+\underline{n}_b$ is constant). Suppose that the act is $a_g=(\overline{x},s_g;\underline{x},s_b)$. Given that in the (β) setting the DM has no information about the minimum number of balls of *h*-color in the urn, she rationally perceives that the two events of the act are equally likely. Now, consider two different specifications of the (β) setting. In the first one, before buying information, the DM knows that there are at least $\underline{n}_g^{(1)}$ and $\underline{n}_b^{(1)}$ balls in the urn, with $\underline{n}_g^{(1)}>\underline{n}_b^{(1)}$; thus, in the act the favorable event is more likely. In the second one, the DM knows that there are at least $\underline{n}_g^{(2)}$ and $\underline{n}_b^{(2)}$ balls in the urn, with $\underline{n}_g^{(2)}<\underline{n}_b^{(2)}$; thus, in the act the unfavorable event is more likely. Then, if it is $\underline{n}_g^{(1)}=\underline{n}_b^{(2)}$ and $\underline{n}_b^{(1)}=\underline{n}_g^{(2)}$, for given DM's risk and ambiguity attitudes we would have the same V^{β} in the two cases.

2.2 Buying Information and using it in the subsequent choice

Consider an extension of the decision problem described in section 2.1. In this new version of the problem, if the DM buys the information about $\Pr(s_g)$ and $\Pr(s_b)$, she has the option to give up playing the lottery in case she does not "like" the revealed composition of the urn. The new decision problem, with respect to act a_g, is represented in Figure 2. Branch $\mathbf{A}_{\boldsymbol{g}}$ represents the choice of playing the act $a_g=(\overline{x},s_g;\underline{x},s_b)$, so that the DM's final wealth is equal to the initial one, w_0, decreased by the price $P\geq 0$ paid to participate in the act, and increased by the random outcome of the act, $x(a_g)$. Branch $\mathbf{I}_{\boldsymbol{g}}$ represents the choice of buying information about the probabilities of the events of the act a_g, by paying the price V_g.

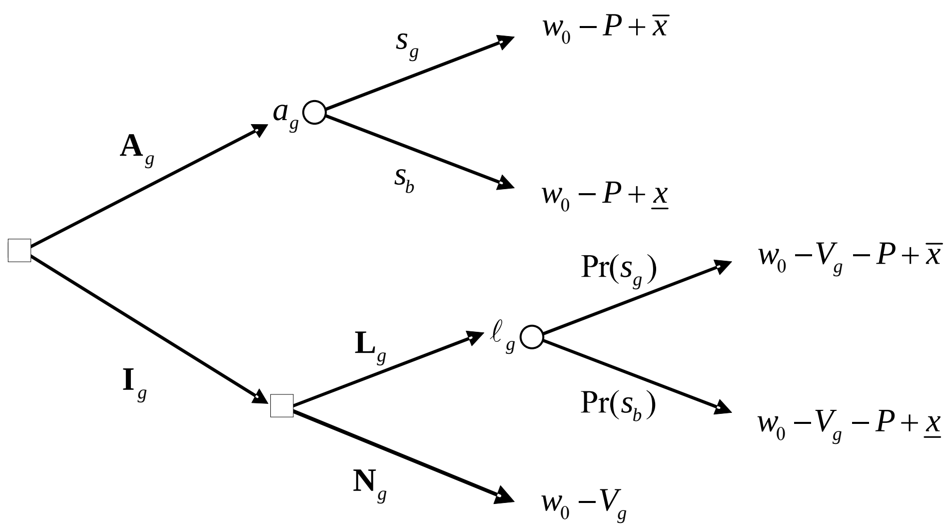

Figure 2. Choosing between an act, a lottery, or none of the two

In case the DM decides to buy information, the probabilities of the events are fully revealed to her, so she faces a lottery determined by the act and the known probabilities of the events, namely $\ell_g = (\overline{x}, \Pr(s_g); \underline{x}, \Pr(s_b))$. In contrast to the problem analyzed in section 2.1, after having known the values of $\Pr(s_g)$ and $\Pr(s_b)$, the DM can choose between playing the resulting lottery ℓ_g (branch $\mathbf{L}_g$) and not playing it (branch $\mathbf{N}_g$). If she chooses $\mathbf{L}_g$, her wealth is $w_0 - V_g - P + x(\ell_g)$, where $x(\ell_g)$ indicates the random outcome of the lottery; otherwise, she gives up with the lottery and saves *P*, so that her wealth is $w_0 - V_g$. Notice that the participation price *P* does not necessarily belong to the interval $[\underline{x}, \overline{x}]$.[6] However, in the analysis below we focus on the relevant case when $P \in [\underline{x}, \overline{x}]$. Notice also that the option of refusing to play is introduced only in branch $\mathbf{I}_g$, not in branch $\mathbf{A}_g$. Therefore, as in the previous decision problem (Figure 1), the DM cannot avoid playing the act if she does not buy information.[7]

Again, let us determine the relation between the capacity of the favorable event, $v(s_g)$, and V_g, the DM's reservation price for ambiguity resolution, by imposing that she is indifferent between branch $\mathbf{A}_g$ and branch $\mathbf{I}_g$.

[6] For example, if participating in the lottery were free, it would be $P=0$ even with $\underline{x} > 0$.

[7] A different setting (different from the setting represented in Figure 2 and analyzed in section 2.2) could be proposed, where the DM has the option not to play even when not buying information. In this setting, the DM is left with w_0 if she decides not to play the act.

Before writing down this indifference condition, we need to calculate the expected utility when choosing branch $\mathbf{I}_g$. This value is obtained through the comparison between branches $\mathbf{L}_g$ and $\mathbf{N}_g$. In particular, the DM chooses $\mathbf{L}_g$ if the probability of the favorable event is sufficiently high, otherwise she chooses $\mathbf{N}_g$. The indifference condition between these two choices allows to determine the probability threshold $\Pr^*(s_g)$ (and the threshold number of green balls n_g^*), such that $EU(w-V_g+\ell_g(\Pr^*(s_g)))=u(w-V_g+P)$, i.e.

$$\frac{n_g^*}{n}=\Pr^*(s_g)=\frac{u(w+P-V_g)-u(w+\underline{x}-V_g)}{u(w+\overline{x}-V_g)-u(w+\underline{x}-V_g)} \tag{13}$$

which exists if $P\in[\underline{x},\overline{x}]$.[8] Then, if the information received is $\Pr(s_g)>\Pr^*(s_g)$, i.e. $n_g>n_g^*$, the DM chooses $\mathbf{L}_g$; if instead it indicates $\Pr(s_g)<\Pr^*(s_g)$, i.e. $n_g<n_g^*$, the DM chooses $\mathbf{N}_g$; and if it indicates $\Pr(s_g)=\Pr^*(s_g)$, i.e. $n_g=n_g^*$, the DM is indifferent between $\mathbf{L}_g$ and $\mathbf{N}_g$.

If the DM is risk neutral, then $\frac{n_g^*}{n}=\Pr^*(s_g)=\frac{P-\underline{x}}{\overline{x}-\underline{x}}$. If she is not risk neutral, then, using the Taylor expansion of the DM's utility function (as already done to find relation (4)), (13) can be rewritten as

$$\frac{n_g^*}{n}=\Pr^*(s_g)=\frac{P-\underline{x}}{\overline{x}-\underline{x}}\cdot\frac{1+0.5\frac{u''(w)}{u'(w)}(P+\underline{x}-2V_g)}{1+0.5\frac{u''(w)}{u'(w)}(\overline{x}+\underline{x}-2V_g)}$$

so that if $\frac{u''(w)}{u'(w)}<(>)0$, then $\frac{n_g^*}{n}>(<)\frac{P-\underline{x}}{\overline{x}-\underline{x}}$ and $\frac{n_g^*}{n}$ is an increasing function of the de Finetti-Arrow-Pratt measure of risk aversion, $-\frac{u''(w)}{u'(w)}$. Therefore, Result 4 can be stated.

Result 4. *Suppose that the ambiguity averse Choquet expected utility DM decides to buy information that resolves ambiguity. Then, the minimum probability of the favorable outcome in order to accept playing the lottery is an increasing function of her degree of risk aversion.*

If the DM is not risk neutral, her probability threshold $\Pr^*(s_g)$ depends also on her ambiguity attitude, through V_g. This is due to the fact that the DM's reservation price for

[8] In fact, if $P<\underline{x}$, it is $\Pr^*(s_g)<0$, hence the DM chooses $\mathbf{L}_g$ for any information received. If it is $P>\overline{x}$, then $\Pr^*(s_g)>1$ and the DM chooses $\mathbf{N}_g$ for any information received.

ambiguity resolution is derived from the indifference condition between branch $\mathbf{A}_{\boldsymbol{g}}$ and branch $\mathbf{I}_{\boldsymbol{g}}$, as a function of $v(s_g)$.

Recalling that, by definition, $w = w_0 - P$, the expected utility of branch $\mathbf{I}_{\boldsymbol{g}}$ is

$$EU(I_g) = \Pr(n_g \le n_g^*) \cdot u(w + P - V_g) +$$
$$+ \Pr(n_g > n_g^*) \cdot \left\{ u(w + \underline{x} - V_g) + \left[u(w + \overline{x} - V_g) - u(w + \underline{x} - V_g) \right] \Pr(s_g)_{n_g > n_g^*} \right\}$$

where $\Pr(s_g)_{n_g > n_g^*}$ is the probability that a green ball is drawn provided that the number of green balls in the urn is larger than n_g^*.

Let us now write down the indifference condition between branch $\mathbf{A}_{\boldsymbol{g}}$ and branch $\mathbf{I}_{\boldsymbol{g}}$, $CEU(w+a_g) = EU(I_g)$. Being $CEU(w+a_g) = u(w+\underline{x}) + (u(w+\overline{x}) - u(w+\underline{x}))v(s_g)$, we can rewrite the indifference condition as

$$v(s_g) = \frac{u(w+P-V_g) - u(w+\underline{x}) - \left[u(w+P-V_g) - u(w+\underline{x}-V_g) \right] \Pr(n_g > n_g^*)}{u(w+\overline{x}) - u(w+\underline{x})} + \\ + \frac{\left[u(w+\overline{x}-V_g) - u(w+\underline{x}-V_g) \right] \Pr(n_g > n_g^*) \Pr(s_g)_{n_g > n_g^*}}{u(w+\overline{x}) - u(w+\underline{x})} \qquad (14.a)$$

If $P \le \underline{x}$, that is $n_g^* = 0$, so that $\Pr(n_g \ge n_g^*) = 1$ and $\Pr(s_g)_{n_g > n_g^*} = \Pr(s_g)$, we find that $v(s_g)$ is determined by (1.a), the relation obtained for the decision problem in section 2.1. Thus, we have the same expression for $v(s_g)$ for the two decision problems, because for $P \le \underline{x}$ it is always profitable for the DM to participate in the lottery.

If $P \in [\underline{x}, \overline{x}]$, the maximum price V_g the DM is willing to pay in order to know the probability $\Pr(s_g)$ depends not only on her degree of ambiguity aversion, but also on the possibility of not participating to the lottery if $\Pr(s_g)$ is too low.

Let us now consider the analogous problem with the act $a_b = (\overline{x}, s_b; \underline{x}, s_g)$ in place of $a_g = (\overline{x}, s_g; \underline{x}, s_b)$. The probability threshold for indifference between $\mathbf{L}_{\boldsymbol{g}}$ and $\mathbf{N}_{\boldsymbol{g}}$ is

$$\frac{n_b^*}{n} = \Pr^*(s_b) = \frac{u(w + P - V_b) - u(w + \underline{x} - V_b)}{u(w + \overline{x} - V_b) - u(w + \underline{x} - V_b)}$$

and the indifference condition between the two branches leads to

$$v(s_b)=\frac{u(w+P-V_b)-u(w+\underline{x})-\left[u(w+P-V_b)-u(w+\underline{x}-V_b)\right]\Pr(n_b>n_b^*)}{u(w+\overline{x})-u(w+\underline{x})}+$$
$$+\frac{\left[u(w+\overline{x}-V_b)-u(w+\underline{x}-V_b)\right]\Pr(n_b>n_b^*)\Pr(s_b)_{n_b>n_b^*}}{u(w+\overline{x})-u(w+\underline{x})} \qquad (14.b)$$

Let us assume, as in section 2.1, that $V_g = V_b$, and indicate them with *V*. Thus, taking into account that $\Pr(s_g)+\Pr(s_b)=1$ and $n_b^* = n_g^* = n^*$, we obtain from relations (14.a) and (14.b) the *index of ambiguity aversion*

$$1-v(s_g)-v(s_b)=\frac{u(w+\overline{x})-u(w+\overline{x}-V)+u(w+\underline{x})-u(w+\underline{x}-V)}{u(w+\overline{x})-u(w+\underline{x})}+$$
$$-\frac{\left[u(w+P-V)-u(w+\underline{x}-V)\right]\left(2-\sum_{h=g,b}\Pr(n_h>n^*)\right)}{u(w+\overline{x})-u(w+\underline{x})}+ \qquad (15)$$
$$+\frac{\left[u(w+\overline{x}-V)-u(w+\underline{x}-V)\right]\left(1-\sum_{h=g,b}\Pr(n_h>n^*)\Pr(s_h)_{n_h>n^*}\right)}{u(w+\overline{x})-u(w+\underline{x})}$$

Relation (15) implies that *V* is an increasing function of the index of ambiguity aversion. This conclusion can be easily proved by deriving the right-hand side of relation (15) with respect to *V* and noting that the derivative is positive.

The analytical expression (15) of the index of ambiguity aversion in the decision problem in Figure 2 differs from relation (2) because of the presence of additional terms which depend on the option value of information. With respect to the decision problem in Figure 1, the option value raises the DM's maximum willingness to pay for the information, given her capacities, since the possibility of avoiding an unfavorable lottery is advantageous. When knowing n_h / n (the probability of the favorable outcome) she can avoid playing the lottery when n_h is too low with respect to the threshold number of *h*-color balls leading her to accept playing the lottery. The option value effect vanishes if $P \le \underline{x}$, because for such a low lottery price the DM never uses this option.

If the DM is risk neutral (but not ambiguity neutral), we obtain

$$1-v(s_g)-v(s_b)=\frac{2V}{\overline{x}-\underline{x}}-\frac{P-\underline{x}}{\overline{x}-\underline{x}}\left(2-\sum_{h=g,b}\Pr(n_h>n^*)\right)+\left(1-\sum_{h=g,b}\Pr(n_h>n^*)\Pr(s_h)_{n_h>n^*}\right) \qquad (16)$$

which differs from (3) because of the option value (in relative terms) of information generated by the two choice problems respectively referred to act $a_g=(\overline{x},s_g;\underline{x},s_b)$ and act $a_b = (\overline{x}, s_b; \underline{x}, s_g)$. The option value is equal to the expected loss generated by the DM's participation to the lotteries ℓ_h when $n_h \le n_h^*$, with $h = g, b$. [9]

In order to set V according to relation (15) or (16) we should know the probabilities $\Pr(n_h > n^*)$ and $\Pr(s_h)_{n_h > n^*}$ for $h = g, b$. Given that they are determined by the particular information setting of the decision problem, we must take into account what the DM knows about the act before she decides whether to buy information. Let us assume that the urn to which act a_g and a_b refer is randomly drawn from a "big urn", composed of N_i^g urns with *i* *g*reen balls inside for $i = 0, 1, ..., n$, so that the total number of urns in the big urn is equal to $N = \sum_{i=0}^{n} N_i^g$. Consequently, with respect to the *b*lue balls, it is $N_i^b = N_{n-i}^g$, for $i = 0, 1, ..., n$, since each urn contains only green and blue balls. Recall that $n^* = n \frac{u(w + P - V) - u(w + \underline{x} - V)}{u(w + \overline{x} - V) - u(w + \underline{x} - V)}$ is the threshold number of balls that makes the DM indifferent between playing and not playing the lottery after having bought information. Let n^{**} be the greatest integer smaller than or equal to n^*, that is $n^{**} = \max_{i=0,1,...,n} \left\{ i : i \le n^* \right\}$.

Let us now resume the two comparative information settings introduced at the end of section 2.1, namely (α) and (β). We reinterpret the DM's knowledge about the likelihood of the events of the act in terms of her knowledge about the composition of the big urn.

In the (α) setting, it is $N_i^g = N_i^b = k \in \mathbb{N}$: the big urn is composed of an equal number of the different possible urns. Then, $N = (n+1)k$, $\Pr(n_h > n^*) = \frac{1}{N} \sum_{i=n^{**}+1}^{n} N_i^h = \frac{n - n^{**}}{n+1}$ for $h = g, b$, and $\Pr(n_h > n^*) \Pr(s_h)_{n_h > n^*} = \frac{1}{N} \sum_{i=n^{**}+1}^{n} \frac{i}{n} N_i^h = \frac{(n - n^{**})(n + n^{**} + 1)}{2n(n+1)}$ for $h = g, b$.

Taking into account these values, relation (15) becomes

[9] This expected loss is

$$\sum_{h=g,b} \left((P - \underline{x}) - (\overline{x} - \underline{x}) \Pr(s_h)_{n_h \le n^*} \right) \Pr(n_h \le n^*) =$$

$$\sum_{h=g,b} \left((P - \underline{x}) - (\overline{x} - \underline{x}) \frac{\Pr(s_h) - \Pr(s_h)_{n_h > n^*} \Pr(n_h > n^*)}{1 - \Pr(n_h > n^*)} \right) \left(1 - \Pr(n_h > n^*) \right) =$$

$$= (P - \underline{x}) \left(2 - \sum_{h=g,b} \Pr(n_h > n^*) \right) - (\overline{x} - \underline{x}) \left(1 - \sum_{h=g,b} \Pr(s_h)_{n_h > n^*} \Pr(n_h > n^*) \right).$$

$$1-v(s_g)-v(s_b)=\frac{u(w+\overline{x})+u(w+\underline{x})-u(w+\overline{x}-V^{\alpha})-u(w+\underline{x}-V^{\alpha})}{u(w+\overline{x})-u(w+\underline{x})}+$$

$$-\frac{n^{**}+1}{n+1}\cdot\frac{2\left[u(w+P-V^{\alpha})-u(w+\underline{x}-V^{\alpha})\right]-\frac{n^{**}}{n}\left[u(w+\overline{x}-V^{\alpha})-u(w+\underline{x}-V^{\alpha})\right]}{u(w+\overline{x})-u(w+\underline{x})}.$$

Taking into account also that for a risk neutral DM it is $\frac{P-\underline{x}}{\overline{x}-\underline{x}}=\frac{n^*}{n}$ (by definition of $\Pr^*(s_g)$), relation (16) becomes $1-v(s_g)-v(s_b)=\frac{2V^{\alpha}}{\overline{x}-\underline{x}}-\frac{n^{**}+1}{n+1}\left(2\frac{n^*}{n}-\frac{n^{**}}{n}\right)$. Since $n^{**}\le n^*$, with $n^*=n^{**}+\varepsilon$ and $\varepsilon<1$, then $\frac{n^{**}+1}{n+1}\left(2\frac{n^*}{n}-\frac{n^{**}}{n}\right)\in[0,1]$, where 0 corresponds to $n^*=0$ and 1 to $n^*=n$. Hence, for a risk neutral DM and for a given level of ambiguity aversion, the option value effect results in V increasing with n^*, i.e. with the participation price *P*. Notice that this result, formally stated below, has been obtained in the (α) setting, where the level of ambiguity is the greatest possible, given that every possible urn composition is equally likely.

Result 5. *For a risk neutral (but not ambiguity neutral) Choquet expected utility DM facing an act with the greatest level of ambiguity, the option value effect results in her reservation price for ambiguity resolution increasing with the participation price.*

In the (β) setting, recalling that $\underline{n}_h$ is the minimum number of *h*-color balls in each urn, we have $N_i^g=0$ for $i\notin\left[\underline{n}_g,n-\underline{n}_b\right]$ and $N_i^g=k$ for $i\in\left[\underline{n}_g,n-\underline{n}_b\right]$. Then, it is $N_i^b=0$ for $i\notin\left[\underline{n}_b,n-\underline{n}_g\right]$ and $N_i^b=k$ for $i\in\left[\underline{n}_b,n-\underline{n}_g\right]$. Moreover, $N=(n+1-\underline{n}_g-\underline{n}_b)k$,

$$\Pr(n_h>n^*)=\frac{1}{N}\sum_{i=n^{**}+1}^{n}N_i^h=\frac{1}{N}\sum_{i=\max\left(n^{**}+1,\underline{n}_h\right)}^{n-\underline{n}_{-h}}k=\frac{n+1-\underline{n}_{-h}-\max\left(n^{**}+1,\underline{n}_h\right)}{n+1-\underline{n}_g-\underline{n}_b}\quad\text{for}\quad h=g,b,$$

and $\Pr(n_h>n^*)\Pr(s_h)_{n_h>n^*}=\frac{1}{N}\sum_{i=n^{**}+1}^{n}\frac{i}{n}N_i^h=\frac{1}{N}\sum_{i=\max\left\{n^{**}+1,\underline{n}_h\right\}}^{n-\underline{n}_{-h}}\frac{i}{n}k$, that is equal to

$$\frac{\left(n+1-\underline{n}_h-\max\left\{n^{**}+1,\underline{n}_h\right\}\right)\left(n-\underline{n}_h+\max\left\{n^{**}+1,\underline{n}_h\right\}\right)}{2n(n+1-\underline{n}_g-\underline{n}_b)}$$

for $h=g,b$. By substituting these values into relation (15), it is possible to characterize the DM's ambiguity aversion index in the (β) setting, for a given attitude towards risk. If the DM is risk neutral, from relation (16) we obtain

$$1-v(s_g)-v(s_b)=\frac{2V^{\beta}}{\overline{x}-\underline{x}}-\frac{n^*}{n}\frac{\sum_{h=g,b}\max\left(n^{**}+1,\underline{n}_h\right)-\underline{n}_g-\underline{n}_b}{n+1-\underline{n}_g-\underline{n}_b}+$$

$$+\frac{\sum_{h=g,b}\left[\left(\max\left(n^{**}+1,\underline{n}_h\right)-1\right)\max\left(n^{**}+1,\underline{n}_h\right)-\underline{n}_h(\underline{n}_h-1)\right]}{2n(n+1-\underline{n}_g-\underline{n}_b)}=$$

$$=\frac{2V^{\beta}}{\overline{x}-\underline{x}}-\frac{\sum_{h=g,b}\left[\left(2n^*-\underline{n}_h+1-\max\left(n^{**}+1,\underline{n}_h\right)\right)\left(\max\left(n^{**}+1,\underline{n}_h\right)-\underline{n}_h\right)\right]}{2n(n+1-\underline{n}_g-\underline{n}_b)}$$

Suppose that $\underline{n}_g=\underline{n}_b$. Then, if $\underline{n}_g=\underline{n}_b\geq 1+n^{**}$, we find again relation (3) as in section 2.1, since the no-play option is never exerted for any information obtained. If $\underline{n}_g=\underline{n}_b<1+n^{**}$, we have $V^{\beta}=\frac{2}{\overline{x}-\underline{x}}\left[\left(1-v(s_g)-v(s_b)\right)+\frac{\left(2n^*-\underline{n}_h-n^{**}\right)\left(n^{**}+1-\underline{n}_h\right)}{n(n+1-\underline{n}_g-\underline{n}_b)}\right]$, which characterizes the risk neutral DM's reservation price in terms of her degree of ambiguity aversion and of the option value effect. The term embedding the option value effect, namely $\frac{\left(2n^*-\underline{n}_h-n^{**}\right)\left(n^{**}+1-\underline{n}_h\right)}{n(n+1-\underline{n}_g-\underline{n}_b)}$, is nonnegative (it is zero for $\underline{n}_g=\underline{n}_b<1+n^{**}$, only if $\underline{n}_h=n^{**}=n^*$) and decreasing in $\underline{n}_h$. Therefore, the option value effect is maximum for $\underline{n}_h=0$ ($h=g,b$), i.e. in the (α) information setting, where the ambiguity level of the act is the greatest possible. This last result is formally stated below.

Result 6. *For a risk neutral (but not ambiguity neutral) Choquet expected utility DM, the reservation price (in relative terms) for ambiguity resolution is the sum of her degree of ambiguity aversion and the option value effect. The latter increases with the level of ambiguity of the decision setting.*

3. Conclusion

Using the Choquet expected utility function, we have proposed a way to measure a subject's ambiguity aversion by letting her reveal the maximum price she is willing to pay in order to receive information about the probability of an unknown event. Once this maximum willingness to pay is determined, it is possible to measure the subject's ambiguity aversion through the implied capacity of the unknown event.

We have presented two possible decision problems in which ambiguity aversion can be easily captured in a laboratory experiment. In the first problem, an ambiguity averse decision maker prefers paying in order just to know probabilities, given that she cannot use this information for a subsequent choice. In this simple setting, the additional effect of risk aversion on the decision maker's reservation price for ambiguity resolution has been studied. In particular, we have shown how ambiguity aversion and risk aversion have opposite effects on the decision maker's reservation price for ambiguity resolution. Furthermore, risk aversion produces different effects on the reservation price of an ambiguity averse and of an ambiguity lover subject.

In the second problem, due to the possibility to use the information received in order to choose whether to play or not play the resulting lottery, the traditional option value of information emerges. In this slightly complex setting, we have examined how the option value affects the decision maker's willingness to pay for ambiguity resolution. We have found that, in the case where the decision maker buys information, the minimum probability of the favorable outcome in order to accept playing the lottery is an increasing function of her degree of risk aversion. With respect to the reservation price, we have found its relevant relations with the decision maker's risk and ambiguity attitudes and with the option value. However, we have not found qualitative results for the case when the decision maker is risk averse. If instead risk neutrality prevails, the role of ambiguity aversion and that of the option value can be disentangled. Due to the option value effect, the risk neutral decision maker's reservation price for ambiguity resolution increases with the participation price (in the act or in the lottery). This effect increases with the level of ambiguity in the decision setting.

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