Disinflation Shocks in the Euro Zone: A DSGE Perspective

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Abstract
This paper investigates the effects of disinflation policies on key macroeconomic variables. Using euro–area data and Structural VectorAutoregressions (SVARs), we first identify disinflation shocks as the only shocks that drive nominal variables to a lower level in the long–run. We find that in the immediate aftermath of a disinflation shock, the euro–area enters in a persistent recession. We then use the dynamic responses drawn from the SVAR model to estimate a medium–scale DSGE model with imperfect information about the disinflation shock. Using the estimated model, we perform counterfactual experiments. Our findings suggest that both nominal and real frictions and monetary policy gradualism have played a prominent role in the recessionary effect of disinflation shocks in the euro–area. Conversely, allowing for imperfect credibility does not yield a better fit, except when we shut key model’s frictions down.

Keywords: Disinflation shocks, SVAR, DSGE, Frictions, Imperfect information, Gradualism.

JEL Class: E31, E32, E52.

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1 Introduction

Many economists suspect disinflation policies to be a cause of recession in industrialized countries. For example, Ball (1994) argues that most of US downturns during the seventies and the beginning of the eighties are associated with periods of monetary tightening aimed at reducing inflation. In addition, it is commonly accepted that the Volker disinflation triggered one of the most severe contraction in post World War II U.S. history (Erceg and Levin, 2003). What more, empirical studies report sizeable output losses following a disinflation policy (see Ball, 1994, Cecchetti and Rich, 2001).

The euro–area as a whole has gone through such disinflation episodes. Indeed, the annual inflation rate dramatically fell from 12% in 1980 to 4% in the late nineties. In spite of differences, inflation dynamics in individual euro–area countries all display a significant downward trend during this period. This feature reflects either a more stable monetary policy (as in Germany) or an explicit change in monetary policy (competitive disinflation in France, see Blanchard and Muet, 1993). At the same time, unemployment sharply increased and real activity was depressed, especially so during the beginning of the eighties. Obviously, these adverse effects may result from other factors and/or disturbances (for example oil price increases in the late seventies). However, shocks to monetary policy driving inflation down seem to be legitimate candidates.

The empirical literature based on Structural Vector Autoregressions (SVARs), usually reports a delayed and hump–shaped response of real and nominal variables to monetary policy shocks. Typically, this literature focusses on transitory monetary policy changes, representing short–run deviations from normal policy (see Christiano et al., 1999, for a thorough survey). Thus, these SVAR models are not well suited to deal with permanent changes in inflation, as observed for many industrialized countries and more specifically, for our purpose, in the euro–area during the eighties. Yet, studying disinflation–induced recessions remains a highly attractive research topic. Indeed, many researchers view the economy’s dynamic responses to disinflation shocks as a natural assessment of monetary models’ empirical plausibility (e.g. Furher and Moore, 1995, Coenen and Wieland, 2005). This is even truer as standard New Keynesian models are usually thought to have trouble reproducing such dynamics (Mankiw, 2001).

The aim of this paper is twofold. First, we propose a SVAR model with a minimal set of restrictions in order to assess the dynamic effect of a permanent disinflation shock in the euro–area. Second, we use the SVAR–based responses to this shock in order to estimate a medium–scale structural model. This estimated DSGE model is then used to perform various counterfactual experiments designed to deepen our understanding of the effects of disinflation policy shocks.

In a first step, we follow the empirical strategy proposed by Bullard and Keating (1995) and adopted by Andres et al. (2002), Dolado et al. (2000), and Vlaar (2004). This approach adapts that proposed by Blanchard and Quah (1989) in the case of permanent supply shocks. The disinflation shock is identified as the only shock that can exert a permanent effect on the long–run level of
nominal variables but no long–run effect on real variables. In our SVAR model which includes both nominal and real variables, we obtain that the disinflation shock tracks remarkably well the common disinflation episodes experienced by France, Germany, and Italy. We also find that this shock has long–lasting and negative effects on real activity. In particular, output persistently decreases and its dynamic response is negative during almost five years. Likewise, hours worked and investment share a similar pattern, with a deeper negative response for investment. At the same time, the nominal interest rate displays such a short–run inertia that the real interest rate persistently increases. Moreover, nominal wages appear less sticky in the short–run than prices, so that the real wage decreases. Our results turn out to be robust to a number of perturbations to the benchmark specification. The main insight that can be drawn from this empirical analysis is that the behavior of the real interest rate seem to be associated with the disinflation–induced recession.

In a second step, we inspect the disinflation mechanisms through the lenses of a medium scale structural model à la Smets and Wouters (2003, 2005). Following Ireland (2007), Smets and Wouters (2005) and de Walque et al. (2006), the disinflation shock is modelled as a permanent negative shock to the central bank’s inflation target. We add to this benchmark framework by considering imperfect information of private agents about the disinflation policy. More precisely, in each and every period, they face a signal extraction problem because they cannot disentangle permanent disinflation shocks from standard transitory monetary policy shocks. We then use the estimated dynamic responses drawn from the SVAR model as moments to be matched by our DSGE model. Relying on a limited estimation information method, we estimate a set of structural and policy parameters to minimize the discrepancy between responses drawn from the SVAR model and their DSGE model counterparts. Our estimation suggests that the model fits reasonably well the dynamic behavior of aggregate data after a disinflation shock. Moreover, our results indicate that the imperfect information channel does not help the DSGE model to fit the data better. To some extent, this is quite a surprising result, given the alleged inability of New Keynesian models to reproduce such dynamics, according to Mankiw’s (2001) criticism of this class of models.

Finally, armed with the estimated model, we perform various counterfactual experiments, in the spirit of Boivin and Giannoni (2006), Smets and Wouters (2007), Christiano et al. (2008), and Sahuc and Smets (2008). In doing so, we resort to the estimated DSGE model as a positive assessment tool of disinflation policies. Indeed, as argued by Galí and Gertler (2007), the DSGE model can be used to run such experiments because it has explicit theoretical foundations. Our goal consists in characterizing quantitatively the propagation channels and recessionary effects of disinflation shocks. To this end, we pay particular attention to a single number called the sacrifice ratio that measures the output loss required to eliminate permanently one point of inflation. We thus make

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1This method has pros and cons. Since it concentrates exclusively on a particular observed phenomenon, it does not specify the whole model’ stochastic structure (other shocks and/or mechanisms). At the same time, the method does not pretend that the DSGE model represents the true Data Generating Process of actual data but only a useful approximation for the question under study (see Driddi, Guay and Renault, 2006).

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contact with a venerable literature that largely predates the current wave of DSGE models (Gordon, 1982, Gordon and King, 1982, Okun, 1978). We conduct two types of counterfactual experiments. First, we shut down one or several propagation channels by altering a parameter (or a combination thereof) and re-estimate the remaining parameters in order to minimize the distance between IRFs from our SVAR model and IRFs from the DSGE model. Second, we alter the same set of parameters while keeping all the other parameters at their estimated values in our benchmark case.\footnote{Christiano, Eichenbaum and Evans (2005) conduct the same type of quantitative evaluations, while Smets and Wouters (2007) concentrate their analysis on the first counterfactual experiment.}

We investigate three main issues that have been already discussed in the context of contractionary monetary policy. The first deals with nominal rigidities, the second with real rigidities and the third concerns the form of monetary policy during disinflation episodes. All these channels have been rounded up as the most likely suspects explaining disinflation--induced recessions, as shown by Ball (1994). The greatest benefit of resorting to a DSGE model is our implied ability to analyze the structural mechanisms at work after a disinflation shock. Our counterfactual DSGE experiments show that the main mechanisms accounting for the level of the sacrifice ratio are: (i) price rigidities in the form of the frequency of no adjustment and the degree of indexation (ii) real rigidities in the form of adjustment costs on investment and (ii) the low speed of disinflation policy.

The paper is organized as follows. In a second section, we present the identification of disinflation shocks and the empirical results. A third section is devoted to presenting the medium--scale structural model, the estimation results, and the counterfactual experiments obtained from the estimated DSGE model. A last section offers some concluding remarks.

## 2 Identification of Disinflation Shocks

We first describe the SVAR model and the long--run restriction used to identify the disinflation shocks. We then discuss our empirical findings with euro data. Finally, we perform a robustness analysis on the key identification restrictions used in the SVAR model.

### 2.1 The SVAR Model and its Theoretical Underpinnings

In order to identify the disinflation shock, we use the following procedure. Let the vector $Z_t$ include a set of aggregate variables. We assume that the stochastic process for $Z_t$ is of the form

$$Z_t = A_0 + \sum_{i=1}^{\ell} A_i Z_{t-i} + u_t$$  \hspace{1cm} (1)$$

where $\ell$ is the number of lags and $u_t$ is a zero mean disturbance with covariance matrix $\Sigma$. In the subsequent experiment, we select $\ell$ using standard information criteria.
Model (1) is estimated using euro–area quarterly data. The sample runs from 1970:1 to 2004:4. The choice of variables in $Z_t$ implies a trade-off. On the one hand, we would like to include as many variables as possible. However, this would imply estimating a very large number of parameters in a finite sample, thus yielding very imprecise estimates of the responses to a disinflation shock. On the other hand, a regression featuring too few variables in $Z_t$ could be corrupted by an omitted variable bias. We therefore choose to adopt an intermediate empirical strategy. In our benchmark experiment, $Z_t$ includes the following seven variables: the cyclical component of real per capita output ($\hat{y}_t$), the log of the consumption-output ratio ($c_t - y_t$), the log of the investment-output ratio ($x_t - y_t$), the cyclical component of total hours worked per capita ($\hat{h}_t$), the inflation rate ($\pi_t$), the short–run nominal interest rate ($R_t$), and wage inflation ($\pi^w_t$). Except for hours worked and population, all the data are extracted from the AWM database compiled by Fagan et al. (2005).

The consumption–output ratio is measured as the ratio of real private consumption expenditures to real GDP. In the AWM database, these include nondurables, services and durables expenditures. While this variable is useful and significant in the estimated VAR model, it raises difficulties for our DSGE model. Indeed, consumption proves almost as volatile as output, due to expenditures on durables. In the remainder, we do not consider the dynamic responses of consumption as moments to be matched. The investment–output ratio is defined as the ratio of real gross investment to real GDP. We measure inflation using the growth rate of the GDP deflator. Wage inflation is simply the growth rate of nominal wage compensation. The population series is the working age population for the euro–area, extracted from the OECD Economic Outlook. Total hours worked are extracted from the Groningen database. The hours data are only available at annual frequency. Therefore, we resort to the Chow and Lin (1971) time disaggregation procedure to convert the series to quarterly frequency. More precisely, the growth rate of total hours is converted to quarterly frequency using the growth rate of employment as the informative covariate. The rationale for doing this is that the bulk of total hours variability is suspected to emanate essentially from changes in employment, as argued by Galí (2005).

In our benchmark specification, we adopt the following definitions. The cyclical component of output, $\hat{y}_t$, is the residual of a regression of the log of real per capita GDP on a constant and a linear trend. The cyclical component of total hours, $\hat{h}_t$, is obtained as the residuals of a regression on a quadratic trend. The latter is meant to capture low frequency movements in hours worked that are attributable to institutional features on the labor market (see Galí, 2005).

For the purpose of identifying disinflation shocks, it is important to assess, as a preliminary step, whether inflation and other nominal variables can be characterized as integrated processes over

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3The AWM mnemonics are as follows. Real output: YER, real consumption expenditures: PCR, real gross investment: ITR, nominal wage: WRN, short–run nominal interest rate: STN.

4These data are available at the website http://www.ggdc.net/.

5Notice that our results are robust to a level specification. However, such a specification yields much wider confidence intervals.
the sample period. To do so, we perform several Augmented Dickey Fuller (ADF) tests of unit root. More precisely, for each nominal variable, we regress the growth rate on a constant, the lagged level and lags of the first difference. For each variable, the number of lags is selected using standard information criteria. The ADF test statistic is equal to -1.52 for inflation, -2.01 for wage inflation, and -1.67 for the short-run nominal interest rate. The null hypothesis of a unit root thus cannot be rejected at the 10 percent level. Conversely, the null hypothesis of a unit root for nominal variables in first difference is rejected at conventional levels. We also investigate if there exist long-run relationships among nominal variables. To do this, we perform ADF tests on the ex-post real interest rate and real wage growth. While the difference between wage inflation and inflation unambiguously appears to be stationary (ADF test statistic of -3.91), we obtain mixed evidence when it comes to the ex-post real interest rate. An ADF test rejects its stationarity at conventional levels, but a Phillips-Perron test with a Bartlett correction is supportive of this hypothesis. Given our emphasis on a structural interpretation of disinflation shocks, we choose to impose the cointegration restriction among nominal variables. The vector \( Z_t \) is accordingly specified as follows:

\[
Z_t = (\Delta R_t, \hat{y}_t, c_t - y_t, x_t - y_t, R_t - \pi_t, \hat{h}_t, R_t - \pi^w_t)'
\]

Building on these empirical results, we follow Bullard and Keating (1995) and identify the disinflation shock as the only shock having a permanent effect on the nominal interest rate, inflation and wage inflation. This is similar in practice to the identification strategy originally proposed by Blanchard and Quah (1989) in the context of supply shocks. Formally, let us define \( B(L) = (I_m - A_1L - \cdots - A_L L^L)^{-1} \), where \( I_m \) is the identity matrix and \( m \) is the number of variables in \( Z_t \). Now, we assume that the canonical innovations are linear combinations of the structural shocks \( \eta_t \), i.e. \( u_t = S\eta_t \), for some non-singular matrix \( S \). As usual, we impose an orthogonality assumption on the structural shocks, which combined with a scale normalization implies \( E\{\eta_t\eta_t'\} = I_m \). Since we are only identifying a single shock, we need not impose a complete set of restrictions on the matrix \( S \). Let us define \( C(L) = B(L)S \). Given the ordering of \( Z_t \) in eq. (2), we simply require that \( C(1) \) be lower triangular, so that only disinflation shocks can affect the long-run level of inflation. This amounts to imposing that \( C(1) \) be the Cholesky factor of \( B(1)\Sigma B(1)' \). Given consistent estimates of \( B(1) \) and \( \Sigma \), we easily obtain an estimate for \( C(1) \). Retrieving \( S \) is then a simple task using the formula \( S = B(1)^{-1}C(1) \). The specification of \( Z_t \) uses the nominal interest rate for long-run identification. This implies that our identification restriction imposes that a disinflation shock is the only one that can permanently affect the nominal interest rate in the long-run. In our benchmark case, we use this variable rather than inflation, because

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6 We hasten to add that we view our results as a useful statistical approximation of inflation dynamics over this sample period. It is of course harder to make such an assumption for longer dataset, especially in the face of stable monetary policy, such as witnessed over the recent period.

7 See also Andres et al. (2002), the working paper version of Coenen and Vega (2001), Dolado et al. (2000), and Vlaar (2004) for a similar SVAR setup.
it delivers much less erratic short–run dynamic responses. In our robustness analysis, we present the dynamic responses when inflation (or wage inflation) is used to identify disinflation shocks. Moreover, because we impose one for one cointegration relationships among nominal variables, all the nominal variables share the exact same stochastic trend. As a consequence, the identified shock exerts no long–run effect on the real interest rate and should not be confused with other shocks that can impact permanently on this variables (e.g. permanent changes in the tax on capital income, permanent shifts in the subjective discount factor).

2.2 Results

The lag–length $\ell = 2$ in the SVAR is selected according to the Hannan-Quin criterion. We compute confidence intervals by standard bootstrap techniques. To simplify the interpretation, we report the dynamic responses of investment, inflation, short–run nominal interest rate and wage inflation. This requires that these variables be reconstructed from the dynamic responses of $Z_t$.

Before analyzing the impulse response functions, let us consider what our SVAR model implies in terms of the dynamics of the inflation target. Consistent with the DSGE model considered in the next section, we assume that the target $\pi_t^\star$ evolves according to a simple random walk

$$\pi_t^\star = \pi_{t-1}^\star + \sigma_\epsilon \epsilon_t^\star$$

where $\epsilon_t^\star$ is the structural disinflation shock and $\sigma_\epsilon$ denotes the corresponding entry in $S$. In the empirical literature, similar specifications have been widely adopted, see, e.g., Stock and Watson (2005) and Cogley and Sargent (2007). Figure 2 reports the induced sample path for $\pi_t^\star$. The figure also reports three shaded areas, corresponding to the common disinflation periods in the three big countries of the euro zone, namely France, Germany, and Italy, as reported by Caporale and Caporale (2008). As is clear, the inflation target shock declines during these periods, which is comforting. At the same time, it captures the low frequency movements in inflation, even though the figure makes clear that other forces than $\epsilon_t^\star$ have played a role in shaping inflation’s dynamics.

Figure 2 reports the dynamic responses of the nominal interest rate, real per capita output, investment, inflation, hours worked, and wage inflation to a permanent disinflation shock as identified.
above. These responses, which represent the key facts that we want to match later, are normalized so that inflation is permanently reduced by 1%. Let us first consider the responses of nominal variables. In the very short–run, inflation reaches a level in between 50% and 75% of its new long–run value and then slowly converges to its new, lower long–run value. At the same time, the nominal interest rate is almost unresponsive on impact, then gradually declines, and slightly overshoots its new steady state value. This implies a sizeable rise in the real interest rate in the immediate aftermath of the disinflation shock. In the short–run, wage inflation decreases at a quicker pace than inflation, implying a decrease in the real wage after a disinflation shock. In the medium–run, inflation and wage inflation share very similar patterns.

When it comes to the real variables, we obtain the following results. The disinflation shock has a long–lasting, negative, and significant effect on output. More precisely, a one percent asymptotic decrease in inflation translates into approximately a 2% decline in output after about seven quarters. After five years, this response is still negative. These dynamic responses imply large sacrifice ratios, defined as the cumulated responses of output divided by the annualized decrease in inflation (here, 4%). Indeed, after eight years, the sacrifice ratio is estimated to be 6.58% of cumulated forgone output, with a standard error of 2.85, thus significant at the 5% level. Investment experiences a drop more than twice as large as that of output. To gain some intuition, it is instructive to
Figure 2: Impulse Response Functions

**Notes:** The gray area corresponds to the 90% confidence interval obtained by standard bootstrap techniques. For ease of interpretation, the size of the disinflation shock is normalized so as to generate an asymptotic inflation decrease by one percentage point. The solid line represents the dynamic responses obtained from the SVAR model. The dashed line corresponds to the responses obtained from the DSGE model.

consider the response of the ex-ante real interest rate (not reported). The ex-ante rate is obtained as the difference between the nominal rate and the one-step ahead expected response of inflation, using the expected inflation rate implied by the SVAR model. The real rate displays a positive, persistent, and significant response to a disinflation shock. This suggests that disinflation policies have persistently increased the real cost of capital, thus leading to large declines in investment. Finally, hours worked do not react much on impact but display an inverted hump–shaped pattern in the subsequent periods. As for output, the disinflation shock has negative and significant long–lasting effect on hours worked. The next section presents a DSGE model that will help us to investigate the structural mechanisms at work following a disinflation shock.

Our strategy for estimating the parameters of the subsequent DSGE model focuses on only that fraction of aggregate dynamics due to disinflation shocks. As a preliminary step, it is thus important to assess how large is this fraction. To do so, we compute the contribution of the disinflation shock to the variance of the seven variables included in model (1). Table 1 reports the variance decomposition at different horizons. Concerning the nominal variables, the disinflation shock is the dominant source of their fluctuations, even in the short–run. For example, this shock accounts for more than 30% of inflation, more than 30% of the nominal interest rate, and more than 40% for wage inflation after four quarters. At longer horizons, the shock mechanically explains all the fluctuations in the nominal variables, by construction. Though our identification strategy imposes
Table 1. Variance decomposition

<table>
<thead>
<tr>
<th></th>
<th>Forecast Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>11.06</td>
</tr>
<tr>
<td></td>
<td>[0.38, 56.41]</td>
</tr>
<tr>
<td>Output</td>
<td>1.64</td>
</tr>
<tr>
<td></td>
<td>[0.02, 25.85]</td>
</tr>
<tr>
<td>Investment</td>
<td>7.03</td>
</tr>
<tr>
<td></td>
<td>[0.06, 30.49]</td>
</tr>
<tr>
<td>Inflation</td>
<td>18.78</td>
</tr>
<tr>
<td></td>
<td>[0.46, 49.07]</td>
</tr>
<tr>
<td>Hours worked</td>
<td>43.21</td>
</tr>
<tr>
<td></td>
<td>[5.31, 65.57]</td>
</tr>
<tr>
<td>Wage inflation</td>
<td>29.88</td>
</tr>
<tr>
<td></td>
<td>[1.69, 50.04]</td>
</tr>
</tbody>
</table>

Notes: The figures in brackets are the 90% confidence interval, obtained by standard bootstrap techniques.

long-run neutrality of monetary policy shocks, the disinflation shock has a sizeable effect on real variables. It accounts for more than 22% of the variance of the cyclical component of output, more than 40% for hours, and more than 25% for investment after four quarters. At longer horizons, say after ten years, this contribution is in between 25% and 32% for the real variables. This reflects the gradual response of the production sector to monetary policy shocks. To sum up, even though the disinflation shocks might not represent the only source of aggregate fluctuations at business cycle frequencies, they still account for a non trivial portion of these and thus constitute a legitimate object of study.

2.3 Discussion and Robustness Analysis

The approach adopted here calls for comments, pertaining to our identification strategy and to the specification of \( Z_t \) in eq. (2).

First, when it comes to our identification of disinflation shocks, notice that our approach implies these shocks occur in each and every period. This is in contrast with the narrative approach which selects a small number of episodes associated to an active disinflation policy. As noticed by Christiano, Eichenbaum and Evans (1999), an advantage of the narrative approach (see Romer and Romer, 1989 and 1994) is that the econometrician does not have either to formally specify a monetary feedback rule or to impose a particular identification scheme to recover the responses of the economy. Such an approach is not an option for us. Indeed, analogues of the FOMC minutes are not available from either National Central Banks or the European Central Bank. Second, since
all the nominal variables included in $Z_t$ share the same stochastic trend in eq. (2), our long–run restriction also imposes that only disinflation shocks can exert a permanent effect on the nominal interest rate, inflation and wage inflation. In addition, by specifying the cyclical component of output and hours as deviations from deterministic trends, we implicitly impose that disinflation shocks cannot have any long–run effect whatsoever on these real variables. Moreover, by including the consumption–output and the investment–output ratios in $Z_t$, we also impose that these variables share the same trend as output and, thus, that the disinflation shocks cannot have long–run effects on either consumption or investment. In structural terms, the disinflation shocks can be interpreted as permanent monetary policy shocks that are neutral in the long–run. This restriction is consistent with the monetarist dictum that “inflation is always a monetary phenomenon in the long–run”, in the spirit of Friedman (1968).

The restrictions documented above obviously raise robustness problems. We examine these issues in various dimensions: omitted dynamics (SV AR with more lags), the definition of inflation (we replace the GDP deflator by the harmonized index of consumer prices), omitted variables in the SV AR model (1) susceptible to affect inflation dynamics (for example, the price of oil), the specification of output and hours (in first difference, allowing for permanent real effect of disinflation shocks), the monetarist dictum (allowing for an opportunistic approach to disinflation, as in Ireland, 2007) and the nominal variable used to identify the disinflation shock (we use inflation and wage inflation instead of the nominal interest rate). Figure 3 summarizes all our robustness experiments and compares them to our benchmark specification. To save space, this figure collects the point estimates of the dynamic responses without their confidence interval. However we place much emphasis on the precision of estimated responses in our discussion.\textsuperscript{10}

The number of lags $\ell = 2$ in the SV AR model has been selected using a statistical criterion. We investigate whether this relatively small lag–length in the VAR picks up sufficient interaction between variables and thus does not corrupt the identification of disinflation shock. We thus redo the exercise with more lags. The dynamic responses obtained from a VAR with four lags are reported in figure 3. We obtain broadly similar dynamic patterns. Yet, the IRFs profiles are much more erratic.

We now use the harmonized index of consumer prices (HICP) instead of the GDP deflator in our definition of inflation in the vector $Z_t$. As is well known, this is the price monitored by the European Central Bank (ECB) and also the price which served to define the requirements to participate to the euro zone. Indeed, as reported in ECB (2004), “The Governing Council of the ECB has defined price stability in terms of the HICP for the euro area.” In addition, this was also the price index monitored by the Deutsch Bundesbank prior to adopting the single currency. It is thus legitimate

\textsuperscript{10}We have also relaxed the restriction that nominal variables share a single stochastic trend associated to the disinflation shock. Without imposing a long–run restriction among nominal variables, the precision of the estimated dynamic responses for both nominal and real variables dramatically decreases. A technical appendix collecting and documenting this robustness analysis is available from the authors upon request.

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Figure 3: Robustness Analysis

(a) Nominal Variables

Nominal interest rate

Percentage deviation

Quarters after shock

Inflation

Percentage deviation

Quarters after shock

Wage Inflation

Percentage deviation

Quarters after shock

(b) Real Variables

Output

Percentage deviation

Quarters after shock

Investment

Percentage deviation

Quarters after shock

Hours

Percentage deviation

Quarters after shock

Legend:
- Benchmark
- Four Lags
- HICP
- Price of Oil
- Permanent Effect
- Opportunistic Approach
- LR with inflation
- LR with Wage inflation
to use this price for the purpose of identifying the effects of disinflation policy shocks. Again, the
dynamic responses of both nominal and real variables are not very sensitive to the choice of the
price index.

One may also argue that the decrease in the inflation rate during the eighties can be largely
attributable to the large decrease in oil price in the mid-eighties. To control for this decrease, we
include the growth rate of the price of oil as an additional variable in the SVAR model. This price
is computed as the product of the West Texas Crude Oil Price and the nominal Euro–$ exchange
rate. Figure 3 ("price of oil") shows that the dynamic responses are undistinguishable from those
of the benchmark specification. We have also investigated whether the estimated disinflation shock
in our benchmark specification of the SVAR model is correlated with oil inflation and Granger
cau sed by this variable. The contemporaneous correlation between $\epsilon_t^*$ and oil inflation is almost
zero. Moreover, when we perform a Granger causality test, we obtain a large P–value for the null
hypothesis on non–causality.

In our benchmark specification, the disinflation shock cannot have a permanent effect on real
variables. We relax this assumption and specify $Z_t$ as follows:

$$ Z_t = (\Delta R_t, \Delta y_t, c_t - y_t, x_t - y_t, R_t - \pi_t, \Delta h_t, R_t - \pi^w_t)' $$

The disinflation shock has a permanent detrimental effect on output and a highly persistent effect on
investment and hours worked (see “permanent effect” in figure 3). However the dynamic responses
of nominal variables are not affected too much.

We then relax the monetarist dictum, thus allowing for an opportunistic approach to disinflation.
To do so, we allow technology shocks, driving permanently labor productivity, to have a long–run
effect on nominal variables. Using the same identification strategy as before, we now specify the
vector $Z_t$ as follows

$$ Z_t = (\Delta(y_t - h_t), \Delta R_t, c_t - y_t, x_t - y_t, R_t - \pi_t, \hat{h}_t, R_t - \pi^w_t)' $$

In this specification of $Z_t$, we still maintain that the disinflation shock cannot have a long–run
effect on output and hours worked. In addition, we assume that only technology shocks can impact
on the long–run level of labor productivity. At the same time, technology shocks are allowed to
affect nominal variables permanently. With this new identification scheme, we obtain a persistent
recessionary effect of the disinflation shock, in line with the estimations in the benchmark case. In
this case, the disinflation is only moderately contaminated by permanent technology shocks, given
that the monetary dictum still accounts for 93% of the variance of the nominal variables in the
long–run.

Finally, we change the nominal variable used to identified the disinflation shock. This is referred to
as “LR with inflation” and “LR with wage inflation” in figure 3. The declines in real variables are
less pronounced, whereas the dynamic responses of inflation and wage inflation are more erratic
than in our benchmark case.
All in all, this robustness analysis gives us confidence in the dynamic effects of disinflation previously identified. It remains to be seen whether a standard DSGE model can match these moments.

3 Structural Analysis of Disinflation Shocks

We first briefly present the main ingredients of the DSGE model. We then discuss the estimation strategy and the empirical results. Finally, we investigate the key mechanisms at work during a disinflation shock using counterfactual analyses.

3.1 The Medium-Scale Structural Model

In this section we briefly describe the structural model we will use in the subsequent quantitative analysis. It is a slightly modified version of the structural models presented in Smets and Wouters (2003, 2005, 2007) and Christiano et al. (2005). Households maximize a separable utility function in consumption and labor effort over an infinite life horizon. Consumption appears in the utility function relative to a time-varying internal habit that depends on past consumption. Each household provides differentiated labor inputs. Monopoly power in the labor market results in an explicit wage equation and allows for the introduction of sticky nominal wages as in the Calvo (1983) model (households are allowed to reset their wage each period with an exogenous probability). Households rent capital services to firms and decide how much capital to accumulate given certain costs of adjusting the capital stock. The introduction of variable capital utilization implies that as the rental price of capital changes, the capital stock can be used more or less intensively according to some cost schedule. Firms produce differentiated goods, decide on labor and capital inputs, and set prices according to the Calvo model. The Calvo model in both wage and price setting is augmented by the assumption that prices that are not re-optimized in a given period are partially indexed to past inflation rates. Prices are therefore set in function of current and expected marginal costs, but are also determined by the past inflation rate. The marginal cost of production depends on the wage and the rental rate of capital. Similarly, wages also depend on past and expected future wages and inflation. Finally, the model is closed with a Taylor-type rule with partial adjustment. Here, the desired nominal interest rate is set in function of the inflation target, the difference between expected inflation and the inflation target, and the output gap. In addition, we allow the nominal interest rate to react on impact to changes in the inflation target.

We further extend the DSGE model by allowing for imperfect information of private agents about monetary policy. More precisely, private agents cannot discriminate between two exogenous shocks to the monetary policy rule, namely a permanent inflation target shock and a transitory monetary policy shock. They have to solve a signal extraction problem using Kalman filtering techniques. Optimal expectations of the inflation target are then governed by a single parameter that depends on the relative size of the two monetary shocks. Finally, these expectations are plugged into the
decision rules of private agents when the central bank implements an unexpected change to its inflation target.

Model Summary

In what follows, we briefly describe the log-linearized version of the model.\(^{11}\)

The consumption equations:

\[
(1 - \beta b)(1 - b)\lambda_t = \beta b(E_t\{c_{t+1}\} - b c_t) - (c_t - b c_{t-1}),
\]

\[
\lambda_t = R_t + E_t\{\lambda_{t+1} - \pi_{t+1}\}.
\]

The marginal utility of wealth $\lambda_t$ is a weighted average of present, past, and expected future consumption ($c_t$). In turn, $\lambda_t$ is linked to the ex-ante real interest rate $R_t - E_t\{\pi_{t+1}\}$.\(^{12}\) The parameter $b$ captures the degree of internal habit formation in consumption and lies between zero and one. Finally $\beta$ is the subjective discount factor.

The capital accumulation equation:

\[
k_{t+1} = (1 - \delta) k_t + \delta i_t
\]

The capital stock $k_t$ depreciates with a constant rate $\delta$. Here, $i_t$ denotes investment.

The investment equation:

\[
\kappa (i_t - i_{t-1}) - \beta \kappa E_t\{i_{t+1} - i_t\} = p_{k,t}.
\]

Investment $i_t$ depends on past and expected future investment and the value of the existing capital stock $p_{k,t}$. The parameter $\kappa$ is related to the elasticity of the investment adjustment costs.

The $Q$ equation:

\[
E_t\{(1 - \beta (1 - \delta))r_{k,t+1}^k + \beta (1 - \delta) p_{k,t+1} + \lambda_{t+1}\} = \lambda_t + p_{k,t},
\]

The value of the capital stock depends negatively on the ex-ante real interest rate ($\lambda_t - E_t\{\lambda_{t+1}\}$) and positively on its expected future value and on the expected real rental rate $r_{k,t+1}^k$.

The price inflation equation:

\[
\pi_t - \gamma_p \pi_{t-1} = \frac{(1 - \beta \alpha_p)(1 - \alpha_p)}{\alpha_p(1 + \epsilon_p \theta_p)} s_t + \beta E_t\{(\pi_{t+1} - \gamma_p \pi_t)\} + (1 - \gamma_p)\pi_t^* - \beta (1 - \gamma_p) E_t\{\pi_{t+1}^*\},
\]

Inflation $\pi_t$ depends on its past and expected future values, on the current real marginal cost $s_t$, and on present and expected inflation targets $\pi_t^*$, to be defined later. The parameter $\alpha_p$ is the...

---

\(^{11}\)A technical appendix is available upon request.

\(^{12}\)Here and in the remainder of the paper, we refer to loglinearized nominal variables with a slight abuse in notations. What we do in fact consists in getting rid of the stochastic trend included in those variables (due to the dynamics of the inflation target). We then reconstruct the original variables levels from these loglinearized stationary equations. Here, we only report the reconstructed equations.
probability that prices cannot be reset in a given period while $\gamma_p$ is the degree of indexation of prices to past inflation. The parameter $\theta_p$ is the price elasticity of demand for goods and $\epsilon_\mu$ is the markup elasticity that emerges from a Kimball (1995)–type aggregate production function.

The factor prices, real marginal cost, and utilization equations

\[
\frac{1}{1-\mu_p s_x} s_t = w_t - \phi(u_t + k_t - h_t), \\
\frac{1}{1-\mu_p s_x} s_t = r^k_t - (\phi - 1)(u_t + k_t - h_t), \\
\mu_p s_x = \sigma_a u_t.
\]

The real wage $w_t$ and the rental rate of capital $r^k_t$ are linked to the real marginal cost according to the above equations. Here, $u_t$ is the utilization rate of capital and $h_t$ denotes the labor input. The parameter $\phi$ stands for the elasticity of output to the capital stock, $\mu_p$ is the steady-state price markup, and $s_x$ is the share of material goods in gross output. The last equation determines the optimal degree of utilization $u_t$ according to the real rental rate $r^k_t$ and the curvature of the utilization cost function, $\sigma_a$.

The wage inflation equation:

\[
\pi^w_t - \gamma_w \pi_{t-1} = \frac{(1-\beta \alpha_w)(1-\alpha_w)}{(1+\omega \theta_w)\alpha_w}(\omega h_t - \lambda_t - w_t) + \beta E_t \{\pi^w_{t+1} - \gamma_w \pi_t\} + (1-\gamma_w)\pi^*_t - \beta(1-\gamma_w)E_t\{\pi^*_t\},
\]

Nominal wage inflation $\pi^w_t$ is a function of its expected future value, past and present inflation, and depends also on the wage gap $\omega h_t - \lambda_t - w_t$. Here the parameter $\gamma_w$ accounts for the degree of indexation of nominal wages to lagged inflation. The parameter $\alpha_w$ is the probability that nominal wages cannot be reset in a given period; $\omega$ and $\theta_w$ denote the inverse elasticity of labor supply and the labor demand elasticity, respectively. Finally, inflation and wage inflation are linked together according to the identity $\pi^w_t = \pi_t + w_t - w_{t-1}$.

The goods market equilibrium conditions:

\[
y_t = s_c c_t + (1-s_c)\mu_t + \phi u_t, \\
\frac{1-\mu_p s_x}{\mu_p(1-s_x)} y_t = \phi(u_t + k_t + (1-\phi)h_t).
\]

The first equation is the resource constraint which links consumption, investment and utilization expenditures ($\phi u_t$) to aggregate output $y_t$. Here $s_c$ is the steady state consumption-output ratio.

The second equation is the production function. This equation derives from the assumption that the fixed cost parameter is pinned down so that the share of aggregate profits in output is zero in a deterministic steady state.

The monetary policy reaction function:

\[
R_t = \rho R_{t-1} + (1-\rho) [\pi^*_t + a_p E_t\{\pi_{t+1} - \pi^*_t\} + a_y \tilde{y}_t] + a_{\pi^*_t} \Delta \pi^*_t.
\]
This equation is a generalized Taylor rule featuring nominal interest rate smoothing at rate \( \rho \) and a variable inflation target \( \pi_t^* \). The parameters \( a_p \) and \( a_y \) govern the sensitivity of the desired nominal interest rate to expected inflation and the output gap \( \tilde{y}_t = y_t - y^n_t \), where \( y^n_t \) represent the natural rate of output. This variable corresponds to the level of output obtained in an economy without nominal frictions, given the inherited capital stock, as in Woodford (2003). In addition, we allow the nominal interest rate \( R_t \) to react contemporaneously to changes in the inflation target, with sensitivity parameter \( a_{\pi^*} \). This allows us to separate the consequences of monetary policy inertia from those of gradual disinflation. The coefficient \( a_{\pi^*} \) allows us to neutralize the effect of monetary policy inertia on the propagation of inflation target shocks. In the case when \( a_{\pi^*} = \rho \), the nominal interest rate reacts one for one to changes in the inflation target. To the contrary, suppose that \( a_{\pi^*} = 0 \) and that \( \rho \) is close to one. In this case, the nominal interest rate is disconnected from \( \pi_t^* \) on impact. This specification is sufficiently flexible to let the data sort out which of these competing configurations has the better fit.\(^{13}\)

Following Ireland (2007), Smets and Wouters (2005) and de Walque, Smets and Wouters (2006), the inflation target \( \pi_t^* \) is assumed to follow a random walk

\[ \pi_t^* = \pi_{t-1}^* + \sigma \epsilon_t^* \]

where \( \sigma > 0 \) and \( \epsilon_t^* \) is a zero mean and unit variance innovation to the inflation target. In what follows, we will investigate the quantitative effects of an unexpected decrease in \( \epsilon_t^* \).

Obviously, as there was no single monetary policy (either in terms of adjustment speed, of inflation target or in terms of responsiveness to the economic environment) in the euro–area over the whole estimation sample adopted in our paper, the above specification warrants some words of caution. The latter is meant as a useful practical simplification of a much more complex decision process.

**Imperfect Information about Monetary Policy**

For expositional clarity, we have focused exclusively on the case when the disinflation policy is fully credible and fully understood by private agents. Recall that the empirical SVAR model does not rely on any such assumption. To allow for some credibility issues, we also consider the possibility that private agents face a signal extraction problem about monetary policy shocks.\(^{14}\) The monetary policy rule is now given by

\[ R_t = \rho R_{t-1} + (1 - \rho) \left[ \pi_t^* + a_p E_t \{ \pi_{t+1} - \pi_t^* \} + a_y \tilde{y}_t \right] + a_{\pi^*} \Delta \pi_t^* + \eta_t. \]

The variable \( \eta_t = \sigma_R \eta_{R,t+1} \) is the monetary policy shock, where \( \sigma_R > 0 \) and \( \eta_{R,t+1} \) is an iid random variable with zero mean and unit variance. In each and every period, private agents fail to observe

\(^{13}\)We thank an anonymous referee for suggesting us this flexible specification of the monetary policy rule.

\(^{14}\)We closely follow Erceg and Levin (2003), and Melecky, Palenzuela and Söderström (2008).
separately $\pi_t^*$ and $\eta_t$. However, they are perfectly informed about all other aspects of the economy. In particular, using the monetary policy rule, private agents observe the composite shock

$$v_t = \left[ (1 - \rho)(1 - a_p)\pi_t^* + a_{\pi^*}\pi_t^* \right] + \eta_t.$$

They then compute optimal forecasts of the inflation target $\pi_t^*$ and the monetary policy shock $\eta_t$ using Kalman filtering techniques. The Kalman filter is associated to the following state equation:

$$\begin{bmatrix} \pi_{t+1}^* \\ \eta_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_t^* \\ \eta_t \end{bmatrix} + \begin{bmatrix} \sigma_{\epsilon} \epsilon_{t+1}^* \\ \sigma_R \eta_{t+1} \end{bmatrix}$$

$$\equiv F \begin{bmatrix} \pi_t^* \\ \eta_t \end{bmatrix} + \begin{bmatrix} \sigma_{\epsilon} \epsilon_{t+1}^* \\ \sigma_R \eta_{t+1} \end{bmatrix}$$

and the observation equation

$$v_t = \left[ (1 - \rho)(1 - a_p) + a_{\pi^*} \right] \begin{bmatrix} \pi_t^* \\ \eta_t \end{bmatrix}$$

$$\equiv H' \begin{bmatrix} \pi_t^* \\ \eta_t \end{bmatrix}$$

From this representation, optimal forecasts of the inflation target and monetary policy shock are given by:

$$\begin{bmatrix} \hat{E}_t \pi_{t+1}^* \\ \hat{E}_t \eta_{t+1} \end{bmatrix} = \left( F - \kappa H' \right) \begin{bmatrix} \hat{E}_{t-1} \pi_{t}^* \\ \hat{E}_{t-1} \eta_t \end{bmatrix} + \kappa H' \begin{bmatrix} \pi_t^* \\ \eta_t \end{bmatrix}$$

where $\kappa$ is the Kalman gain. This gain is obtained by iterating on the system

$$\begin{align*}
\kappa_t &= FP_{t|t-1}H (H'P_{t|t-1}H)^{-1} \\
\kappa_t &= (F - \kappa_t H') P_{t|t-1} (F - \kappa_t H')' + \Sigma(7)
\end{align*}$$

where

$$\Sigma = \begin{bmatrix} \sigma^2_{\epsilon} & 0 \\ 0 & \sigma^2_R \end{bmatrix}$$

and

$$P_{t+1|t} = \text{E}[(\zeta_{t+1} - \hat{E}_t \zeta_{t+1})(\zeta_{t+1} - \hat{E}_t \zeta_{t+1})']$$

where $\zeta_{t+1} = [\pi_{t+1}^*, \eta_{t+1}]'$ and $\kappa_t$ is the Kalman gain at iteration $t$.

Given an arbitrary initial condition $P_{1|0}$, steady-state values for $\kappa$ and $P$ are obtained by iterating on (6) and (7) (see Hamilton, 1994). In our setup, given the sparsity of the matrix $F$, analytical expressions for $\kappa$ and $P$ can be derived. It is then straightforward to compute the matrices $\kappa H'$ and $F - \kappa H'$. Interesting properties associated to the elements of these matrices are summarized in the following proposition (the proof is reported in appendix):
Proposition. Under imperfect information, when the economy is perturbed by inflation target shocks only, optimal forecasts in (5) are given by:

\[ \hat{E}_t \pi^\star_{t+1} = \nu \hat{E}_{t-1} \pi^\star_t + (1 - \nu) \pi^\star_t \]

with

\[ \nu = \frac{2\bar{z}}{1 + \sqrt{1 + 4\bar{z} + 2\bar{z}}} \]

where

\[ z = (\sigma_R/\sigma_e) \quad c = ((1 - \rho)(1 - a_p) + a_{\pi^\star}) \quad \text{and} \quad \bar{z} = (z/c)^2 \]

Moreover, the parameter \( \nu \) satisfies

\[ \nu = 0 \quad \text{when} \quad z = 0 \quad , \quad \lim_{z \to \infty} \nu = 1 \quad \text{and} \quad \frac{\partial \nu}{\partial z} > 0 \quad \forall z \geq 0 \]

When the standard error of the transitory monetary policy shock \( \sigma_R \) is zero, the parameter \( \nu \) is equal to zero. In this case, there is no information problem because the monetary policy rule is only perturbed by the inflation target shock. As \( \sigma_R \) increases, the parameter \( \nu \) monotonically increases. For large \( z \), \( \nu \) is close to one. It follows that the optimal forecasts of the inflation target display large inertia and are disconnected from the inflation target itself in the short-run. In this case, imperfect information may contribute to enriching the propagation mechanisms of the DSGE model (see Erceg and Levin, 2003). In the limit, when \( z \to \infty \), the parameter \( \nu \) tends to 1. The parameter \( \sigma_R \) (relative to the parameters \( c \) and \( \sigma_e \)) and \( \nu \) thus contain the same information about the credibility of the disinflation policy.

In practice, we append the above learning equations to the set of structural model’s equations, which we complement with the identity \( \hat{E}_t \{ \pi^\star_{t+1} \} = \hat{E}_t \{ \pi^\star_t \} \). We then modify the structural equations so that each time \( \pi^\star_t \) shows up, the term is replaced with \( \hat{E}_t \{ \pi^\star_t \} \), except in the monetary policy rule. Thus we assume that the central bank responds to the true innovations while the private sector only reacts to optimal forecasts of these perturbations. We then solve the resulting system by resorting to standard methods.

3.2 Econometric Methodology and Estimation Results

We now turn to the estimation of the medium-scale structural model. Following Rotemberg and Woodford (1997) and Christiano et al. (2005), we adopt a Minimum–Distance Estimation technique of the structural parameters.\(^{15}\) The basic idea is to select the value of structural and monetary policy parameters so as to minimize the discrepancy between theoretical dynamic responses and their SVAR counterparts.

\(^{15}\)See also Altig et al. (2004), Amato and Laubach (2003), Boivin and Giannoni (2006) and Giannoni and Woodford (2005) among many others.
More formally, let $\psi$ denote the whole set of model parameters. We partition $\psi$ in two groups, $\psi = (\psi_1', \psi_2')$. Let $\psi_1' \equiv (\beta, \omega, s_x, \theta_p, \theta_w, \epsilon_\mu, \delta, \phi, \sigma_\epsilon)$ regroup the parameters calibrated prior to estimation. We set $\beta = 0.99$, implying a steady–state annualized real interest rate of 4 percent. We impose $\omega = 2$, a value similar to that obtained by Smets and Wouters (2003). The share of material goods $s_x$ is set to 0.5, which matches the euro area figure reported by Jellema et al. (2006). We choose to calibrate $\theta_p$, $\theta_w$ and $\epsilon_\mu$ because identification problems arise as long as the Calvo probabilities $\alpha_p$ and $\alpha_w$ are estimated (see Rotemberg and Woodford, 1997, and Amato and Laubach, 2003, for a discussion on this issue). We thus set $\theta_p = \theta_w = 11$, so that both markups charged by intermediate goods producers and households amount to 10%. We set $\epsilon_\mu = 1/11$ as suggested by Woodford (2003). The depreciation rate is $\delta = 0.025$ and the elasticity of output with respect to the capital stock is $\phi = 0.36$. Finally, the parameter $\sigma_\epsilon$ associated to the disinflation shock is set to 1% as in the SVAR model. Indeed, for the selected horizon of dynamic responses drawn from the SVAR model (20 quarters), all nominal variables have approximately converged to their new long–run values.

Let $\psi_2' \equiv (\gamma_p, \gamma_w, \alpha_p, \alpha_w, b, \kappa, \sigma_a, \rho, a_p, a_y, a_\pi, \nu)'$ regroup all the remaining free parameters. As mentioned above, since $\sigma_R$ is a free parameter, we can estimate $\nu \in [0, 1]$ directly. Given that our MDE approach focuses exclusively on shocks to the inflation target, this procedure is straightforward. The parameters are estimated by minimizing a measure of the distance between the empirical responses of $Z_t$ and their model counterparts. We define $\theta_j$ the vector of responses of the variables in $Z_t$ to a disinflation shock at horizon $j \geq 0$, as implied by the above SVAR. The object which we seek to match is $\theta = \text{vec}([\theta_0, \theta_1, \ldots, \theta_k])'$ where $k$ is the selected horizon. Then let $h(\cdot)$ denote the mapping from the structural parameters $\psi_2$ to the DSGE counterpart of $\theta$. Our estimate of $\psi_2$ is obtained by minimizing

$$(h(\psi_2) - \hat{\theta}_T)V_T(h(\psi_2) - \hat{\theta}_T)'$$

where $\hat{\theta}_T$ is an estimate of $\theta$, $T$ is the sample size, and $V_T$ is a weighting matrix which we assume is the inverse of a matrix containing the asymptotic variances of each element of $\theta$ along its diagonal and zeros elsewhere. As suggested by Christiano et al. (2005), this choice of weighting matrix ensures that the model-based dynamic responses lie as much as possible inside the confidence interval of their SVAR-based counterparts.

The estimated structural parameters are reported in table 2 and the estimated dynamic responses of the structural model in figure 2. Two parameters hit their constraint during the course of estimation: the sensitivity of the desired nominal interest rate to the output gap ($a_y = 0$) and the recessionary parameter ($\nu = 0$). First, setting $a_y = 0$ is necessary for the purpose of matching the recessionary effects of a disinflation shock. We started with different positive initial condition for $a_y$ and we always obtain that this parameter hits its lower bound. A possible explanation for this is that positive values for $a_y$ always result in lower output, investment and hours losses after a disinflation policy. Our results thus suggest that during disinflation periods, monetary policy in the euro–area as a whole was conducted without any explicit concern for real activity. The second
result is even more interesting since it suggests that within our setup, imperfect credibility is not necessary to match the long-lasting recessionary effect of a disinflation shock. We have checked the robustness of this results using different set of initial conditions (for example, lower initial values for the $\gamma$’s and a value close to unity for $\nu$) and the minimization of the loss function always selects the lower value of zero for the credibility parameter. This means that when indexation and imperfect credibility are two competing mechanisms, our estimation results favor clearly the first channel. Notice, however, that when important propagation mechanisms are shut down, we obtain positive values for $\nu$. See the next section (counterfactual analyses) for further details.

The probability of no price reoptimization ($\alpha_p$) is larger than previous estimates found in the literature (see, e.g., Smets and Wouters, 2003, 2005). This estimated value $\alpha_p = 0.849$ is fairly large especially when we acknowledge that the model includes different features that allow for a lower estimated value of this parameter (material goods, Kimball-type technology). This can reflect that high levels of price rigidity are necessary to properly match the very persistent effects of disinflation shocks. The probability of no wage reoptimization ($\alpha_w$) is smaller ($\alpha_w = 0.637$), a value which is consistent with preliminary results reported by the ECB’s Wage Dynamics Network, as summarized by Druant et al. (2008). These estimates suggest more price stickiness than wage stickiness. This result shows that the behavior of the real wage cannot be considered as the major suspect for the protracted recession triggered by disinflation shocks, since real wages will actually decrease in the immediate aftermath of this shock. In addition, nominal wage displays less intrinsic parameter estimates

### Table 2. Structural parameter estimates

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_a$</td>
<td>2.200</td>
<td>Curvature of the utilization cost function</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>12.027</td>
<td>Adjustment cost parameter</td>
</tr>
<tr>
<td>$b$</td>
<td>0.891 (0.184)</td>
<td>Habit parameter</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>0.849 (0.615)</td>
<td>Proba. of no price reoptimization</td>
</tr>
<tr>
<td>$\alpha_w$</td>
<td>0.637 (2.098)</td>
<td>Proba. of no wage reoptimization</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>0.844 (0.843)</td>
<td>Price indexation parameter</td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>0.731 (6.810)</td>
<td>Wage indexation parameter</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.906 (0.483)</td>
<td>Interest rate smoothing</td>
</tr>
<tr>
<td>$a_p$</td>
<td>1.291 (7.370)</td>
<td>Interest rate elasticity wrt $\hat{\pi}_t$</td>
</tr>
<tr>
<td>$a_y$</td>
<td>0.000 (+)</td>
<td>Interest rate elasticity wrt $\hat{y}_t$</td>
</tr>
<tr>
<td>$a_{\pi^*}$</td>
<td>0.573 (2.288)</td>
<td>Implementation of disinflation shock</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.000 (+)</td>
<td>Credibility of the Disinflation Policy</td>
</tr>
</tbody>
</table>

Notes: The values in parentheses are standard errors. A star refers to a parameter which hits a constraint during the course of the first stage estimation.
The estimated value for the habit parameter is large $b = 0.891$ compared to previous findings with euro data. The estimated investment adjustment cost parameter is also large ($\zeta = 11.736$), but it is not precisely estimated. The capital utilization cost parameter ($\sigma_a = 2.200$) is very comparable to previous estimated values. The estimated smoothing parameter in the interest rate equation is $\rho = 0.906$, suggesting a substantial degree of policy inertia. The estimated value of the parameter related to the implementation of the disinflation policy $a_{\pi^*} = 0.573$ is lower than $\rho$, suggesting that this policy has been conducted in a gradual way in the euro zone. It is important to keep these last two results in mind for our counterfactual analysis. The sensitivity of the desired nominal interest rate to the expected inflation ($a_p = 1.291$) is broadly consistent with the estimates obtained by Clardia et al. (1998) for the three main euro–area countries (Germany, France, Italy).

For these estimated values, the dynamic responses of the structural model are reported in figure 2 (dashed line) together with those from the SVAR model and with the associated confidence interval. This figure shows that the model mimics well the estimated dynamic responses from the SVAR model. More precisely, the DSGE model successfully reproduces the recessionary effects of the disinflation shock, together with the short–run inertia of the nominal interest rate and the dynamic responses of real variables. For example, the estimated structural model implies a large sacrifice ratio for output (about 7.11) in accordance with actual data.

### 3.3 Counterfactual Analysis

Having shown that the medium–scale structural model replicates reasonably well the effects of a permanent disinflation shock on a set of real and nominal variables, we now investigate the key mechanisms at work during a disinflation policy. To do so, we conduct two types of counterfactual experiments. First, we shut down one or several propagation channels by altering a parameter (or a combination thereof) and re–estimate the remaining parameters in order to minimize the above loss function. Second, we perturb the same set of parameters while keeping all the other parameters at the estimated values obtained in our benchmark case. If the fit of the model is strongly deteriorated in both types of counterfactual experiments, the isolated mechanism plays an essential role in the transmission of disinflation shocks. This simply means that the other parameters that can potentially alter these effects are not adjusted, so that perturbing the parameter under control properly identifies the essential mechanism at work. Alternatively, if other estimated parameters adjust to fit the data, there exist other forces that propagate disinflation shocks. Consequently, we can obtain very different responses in the two types of counterfactual experiments.

We concentrate our analysis on various issues that have been previously discussed in the context of contractionary monetary policy: (i) nominal rigidities, (ii) real rigidities and (iii) monetary policy. All our experiments are reported in figures 4, 5 and 6. To save space, these figures include only the IRFs of the nominal interest rate, output, and inflation for the benchmark case and the
Table 3. Sacrifice ratio

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>7.108</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counterfactual #1</td>
<td>Counterfactual #2</td>
</tr>
<tr>
<td>Flexible Price</td>
<td>6.383</td>
</tr>
<tr>
<td>Flexible Wage</td>
<td>7.219</td>
</tr>
<tr>
<td>No Nominal Rigidities</td>
<td>0.002</td>
</tr>
<tr>
<td>No Variable Capital Utilization</td>
<td>6.781</td>
</tr>
<tr>
<td>No Habit Formation</td>
<td>6.960</td>
</tr>
<tr>
<td>Small Adjustment Costs</td>
<td>6.886</td>
</tr>
<tr>
<td>No Real Rigidities</td>
<td>6.691</td>
</tr>
<tr>
<td>Faster Implementation</td>
<td>7.000</td>
</tr>
<tr>
<td>No Policy Inertia</td>
<td>7.497</td>
</tr>
<tr>
<td>More Aggressive Policy</td>
<td>7.202</td>
</tr>
</tbody>
</table>

Note: In counterfactual # 1, we set a parameter (or a combination of parameters) and we re-estimate the remaining parameters in order to minimize the loss function. In counterfactual # 2, we set this parameter while keeping the other parameters at the estimated values obtained in our benchmark case.

two counterfactual experiments. In addition, for each counter-factual experiment, we compute the sacrifice ratio as a simple way to compare the cost of disinflation (see table 3). Recall that this ratio is equal to 7.108 in the benchmark case. As before, we view the sacrifice ratio as a useful summary statistic of the recession deepness. This number, however, is given no normative content. Finally, table 4 reports the estimation of the DSGE model in the different cases we choose to investigate.

Nominal Rigidities

The first issue concerns the role played by nominal rigidities, either on the labor or on the goods market. Indeed, as argued by Gordon (1982), nominal wage rigidities (low frequency of adjustment and/or high degree of indexation) might be held responsible for the large recessionary effects of disinflation. Moreover, Mankiw (1990) argues that a higher degree of price rigidities contributes to increasing the cost of disinflation. We investigate these candidate explanations by perturbing the values of parameters which governing the degree of nominal rigidities.

Panel (a) in figure 4 reports the dynamic responses when prices are assumed to be flexible ($\alpha_p = 0.01$ and $\gamma_p = 0$). When all the remaining parameters are re-estimated, we obtain that the output loss, although still sizeable, is substantially reduced. This is highlighted by the lower sacrifice ratio (see the column Counterfactual # 1 in table 3). The fact that the real loss is not totally eliminated comes from the increase in the estimated value of $\nu$ ($\nu = 0.745$), implying that credibility issues matter a lot when we shut down price rigidities (see the second row of table 4). This finding illustrates the role played by slow price adjustments in the propagation of a disinflation policy. Moreover, the Calvo parameter for nominal wage rigidity $\alpha_w$ is not altered much, compared to
the benchmark case. At the same time, the wage indexation parameter $\gamma_w$ is now zero. Thus, wage stickiness does not appear to be an essential channel of the propagation of disinflation shocks.

In the second counterfactual experiment, all parameters are set to their benchmark values. For this experiment, $\nu$ is maintained to zero and thus the output loss is significantly reduced and the sacrifice ratio is much smaller (see the column Counterfactual # 2 in table 3). In this case, both the nominal interest rate and the inflation rate reach their common long–run value at a faster pace.

Panel (b) of figure 4 reports the IRFs when nominal wages are flexible, i.e. $\alpha_w = 0.01$ and $\gamma_w = 0$. In this case, re-estimating the model leads to higher values for the Calvo probability $\alpha_p = 0.893$ and the price indexation $\gamma_p = 0.901$. Consequently, the decrease in output is similar to what obtains in the benchmark case and the sacrifice ratio is almost left unchanged. Notice, that the credibility issue no longer matters in this case since the estimated value of $\nu$ hits the lower bound constraint 0 during the estimation. In the second counterfactual experiment, the output loss is reduced because the price rigidities are kept to a smaller level. Consequently, the sacrifice ratio decreases. Comparing panels (a) and (b), price rigidity plays more important a role than wage rigidity after a disinflation shock.

Finally, in panel (c) of figure 4, we shut the two sources of nominal rigidities down. In the counterfactual experiments, the fit of the model is deeply altered since output remains almost constant and the nominal interest rate drops immediately to its long–run value. This results from the new estimated value of $\alpha_{\pi^*} (0.981)$ which is equal to the smoothing parameter $\rho (0.981)$ in the monetary policy rule. Notice that, due to numerical failures, we must set most of the parameters to their benchmark values at the estimation stage. This means that when there is no nominal rigidities, there seems to exist no other combination of driving forces that allows to replicate the dynamic responses drawn form the SVAR model. In this situation, the sacrifice ratio is almost zero in both counterfactual experiments.

**Real Rigidities**

Our second set of experiments deals with the role played by real rigidities in the propagation of disinflation shocks. Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007) have already shown that such rigidities are central in creating long–lasting effects of transitory shocks to monetary policy. Therefore, we just want to assess what form of real frictions matter for the propagation of a disinflation shock.

Panel (a) of figure 5 reports the dynamic responses when there is no variable capital utilization ($\sigma_a = 100$). This figure suggests that this real rigidity plays only a minor role: the three responses are very close and the parameter estimates are almost unchanged (see the fifth row of table 4). This is also confirmed by the computed sacrifice ratio (see table 3).
Table 4. Structural parameter estimates under the first counterfactual analysis

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_a$</th>
<th>$\kappa$</th>
<th>$b$</th>
<th>$\alpha_p$</th>
<th>$\alpha_w$</th>
<th>$\gamma_p$</th>
<th>$\gamma_w$</th>
<th>$\rho$</th>
<th>$a_p$</th>
<th>$a_{\pi*}$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>2.200</td>
<td>12.027</td>
<td>0.891</td>
<td>0.849</td>
<td>0.637</td>
<td>0.844</td>
<td>0.731</td>
<td>0.906</td>
<td>1.291</td>
<td>0.573</td>
<td>0.000</td>
</tr>
<tr>
<td>Flexible Price</td>
<td>0.632</td>
<td>4.713</td>
<td>0.873</td>
<td>0.010</td>
<td>0.645</td>
<td>0.000</td>
<td>0.000</td>
<td>0.879</td>
<td>3.433</td>
<td>0.306</td>
<td>0.745</td>
</tr>
<tr>
<td>Flexible Wage</td>
<td>1.429</td>
<td>12.027</td>
<td>0.908</td>
<td>0.893</td>
<td>0.010</td>
<td>0.901</td>
<td>0.000</td>
<td>0.944</td>
<td>1.288</td>
<td>0.885</td>
<td>0.000</td>
</tr>
<tr>
<td>No Nominal Rigidities</td>
<td>2.199</td>
<td>12.027</td>
<td>0.891</td>
<td>0.010</td>
<td>0.010</td>
<td>0.000</td>
<td>0.000</td>
<td>0.981</td>
<td>1.291</td>
<td>0.981</td>
<td>0.861</td>
</tr>
<tr>
<td>No Variable Capital Utilization</td>
<td>100</td>
<td>13.293</td>
<td>0.862</td>
<td>0.639</td>
<td>0.870</td>
<td>0.694</td>
<td>0.914</td>
<td>1.293</td>
<td>0.607</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>No Habit Formation</td>
<td>2.220</td>
<td>2.345</td>
<td>0.000</td>
<td>0.949</td>
<td>0.727</td>
<td>0.607</td>
<td>0.571</td>
<td>0.962</td>
<td>1.291</td>
<td>0.961</td>
<td>0.000</td>
</tr>
<tr>
<td>Small Adjustment Costs</td>
<td>0.838</td>
<td>0.100</td>
<td>0.189</td>
<td>0.010</td>
<td>0.000</td>
<td>0.998</td>
<td>0.000</td>
<td>0.930</td>
<td>1.291</td>
<td>0.927</td>
<td>0.760</td>
</tr>
<tr>
<td>No Real Rigidities</td>
<td>100</td>
<td>0.100</td>
<td>0.000</td>
<td>0.262</td>
<td>0.713</td>
<td>0.000</td>
<td>0.000</td>
<td>0.895</td>
<td>1.291</td>
<td>0.911</td>
<td>0.761</td>
</tr>
<tr>
<td>Faster Implementation</td>
<td>3.107</td>
<td>4.698</td>
<td>0.766</td>
<td>0.894</td>
<td>0.687</td>
<td>0.783</td>
<td>0.630</td>
<td>0.949</td>
<td>1.291</td>
<td>0.949</td>
<td>0.000</td>
</tr>
<tr>
<td>No Policy Inertia</td>
<td>2.878</td>
<td>2.181</td>
<td>0.748</td>
<td>0.864</td>
<td>0.348</td>
<td>0.878</td>
<td>0.679</td>
<td>0.000</td>
<td>1.291</td>
<td>0.617</td>
<td>0.892</td>
</tr>
<tr>
<td>More Aggressive Policy</td>
<td>2.383</td>
<td>11.603</td>
<td>0.891</td>
<td>0.849</td>
<td>0.652</td>
<td>0.825</td>
<td>0.656</td>
<td>0.954</td>
<td>3.000</td>
<td>0.744</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: The parameter $a_p$ is constrained to its benchmark value in the following model’s versions: no nominal rigidities, no habit formation, small adjustment costs, no real frictions, fast implementation and no policy inertia. The parameter $\kappa$ is constrained to its benchmark value in the flexible and no nominal rigidities versions. The parameters $\sigma_a$ and $b$ are constrained to their benchmark values in the no nominal rigidities version.
Panel (b) is devoted to the no habit formation case ($b = 0$). In this experiment, in order to match the decrease in output (and other related real variables), some parameters are adjusted so as to generate more persistence: the Calvo probabilities on prices and nominal wages increase and monetary policy inertia is more pronounced. Setting $b = 0$ and leaving the other parameters unaffected, we obtain a larger short–run decrease in output together with a quicker recovery. This finding highlights the role played by this type of real friction in shaping the gradual effect of disinflation shocks. The computed sacrifice ratio also summarizes well this property of habit formation, since the output loss increases by 34 % in the second counterfactual experiment.

Panel (c) reports the dynamic responses when adjustment costs on investment are small ($\kappa = 0.1$). In such a case, the re–estimated model faces severe difficulties in matching the data. The mechanism that allows the DSGE model to replicate the persistent decrease in output is now related to imperfect credibility, since $\nu = 0.760$. Notice that the model does not match the gradual adjustment of the nominal interest rate since the estimated value of $a_{\pi^*} (0.927)$ is very close to $\rho (0.930)$. When the remaining parameters are set to their benchmark values, we obtain a huge short–run decrease in output but a very quick recovery. This huge drop in the short–run also implies a large sacrifice ratio (27.281).

Finally, panel (d) considers the case of no real rigidities. Results are very similar to the previous experiment, meaning that the key propagation mechanism is related to the adjustment cost parameter.

**Monetary Policy**

The third important issue relates to the form of monetary policy, either in terms of adjustment speed or responsiveness. The literature has largely dwelt on the central and controversial question of the optimal speed of disinflation, *i.e.* the choice between “gradualism” and “cold turkey”. On the one hand, a gradual disinflation is less costly because wages and prices display inertia and cannot adjust quickly to a permanent and sudden change in monetary policy (see Taylor, 1983). On the other hand, Sargent (1983) argues that a quick disinflation is less costly because a sharp regime change in monetary policy makes the new policy credible and thus changes downward inflation expectations. Notice also that empirical studies suggests that the speed of disinflation allows to reduce the sacrifice ratio (see Ball, 1994 and Boschen and Weise, 2001). To investigate this issue, we perturb the smoothness parameters $a_{\pi^*}$ and $\rho$ in the Taylor–type rule. In addition, the responsiveness of monetary policy to the expected inflation is often put forth as a key ingredient of a successful disinflation policy. To investigate this, we alter the sensitivity $a_p$ of the desired nominal interest rate to inflation.

In panel (a) of figure 6, we consider the case of a faster implementation, *i.e.* when we set $a_{\pi^*} = \rho$. Notice that the estimated model now produces more inertia in monetary policy ($\rho = 0.949$) compared to the benchmark case. Indeed, a higher degree of inertia is needed to produce a persistent
Figure 4: Nominal Rigidities

(a) Flexible Price ($\alpha_p = 0.01; \gamma_p = 0$)

(b) Flexible Wage ($\alpha_w = 0.01; \gamma_w = 0$)

(c) No Nominal Rigidity ($\alpha_p = \alpha_w = 0.01; \gamma_p = \gamma_w = 0$)

Notes: The solid line corresponds to the benchmark case, the dashed line corresponds to the counterfactual # 1 and the dotted line corresponds to the counterfactual # 2. In counterfactual # 1, we set a parameter (or a combination of parameters) and we re-estimate the remaining parameters in order to minimize the loss function. In counterfactual # 2, we set this parameter while keeping the other parameters at the estimated values obtained in our benchmark case.
Figure 5: Real Rigidities

(a) No variable capital utilization ($\sigma_a = 100$)

(b) No habit formation ($b = 0$)

(c) Small investment adjustment costs ($\kappa = 0.1$)

(d) No real friction ($\frac{1}{\sigma_a} = b = 0; \kappa = 0.1$)

Notes: The solid line corresponds to the benchmark case, the dashed line corresponds to the counterfactual # 1 and the dotted line corresponds to the counterfactual # 2. In counterfactual # 1, we set a parameter (or a combination of parameters) and we re-estimate the remaining parameters in order to minimize the loss function. In counterfactual # 2, we set this parameter while keeping the other parameters at the estimated values obtained in our benchmark case.
decrease in real variables and not too quick an adjustment of nominal variables. When the remaining parameters are set to their benchmark values, we obtain a sizeable reduction in the output loss, suggesting that a faster disinflation policy can reduce the associated output costs. This is also confirmed by the computed sacrifice ratio in table 3.

Panel (b) of figure 6 reports the dynamic responses when there is no monetary policy inertia ($\rho = 0$). When parameters are re-estimated with this constraint, we observe that the imperfect credibility of the policy is essential to replicate the IRFs drawn from the SVAR model. The parameter $\nu$ now takes on a large value (0.892), while parameters related to nominal rigidities are similar to what obtained in the benchmark case. When the remaining parameters are unchanged (notably $\nu = 0$), we obtain a very small, negative effect of the disinflation policy. This confirms that a gradual implementation of monetary policy is an important suspect. In this case, the sacrifice ratio is equal to 0.963.

Finally, panel (c) of figure 6 reports the dynamic responses when monetary policy is more aggressive with respect to expected inflation. In the two counterfactual experiments, aggressive monetary policy has very little impact and thus it cannot be considered as a key propagation mechanism of disinflation shocks. To sum up, the gradual implementation of monetary policy seems to be a key element of the long–lasting and negative effects of disinflation shocks.

4 Conclusion

This paper characterizes the dynamic responses of aggregate variables to a permanent disinflation shock in the euro–area. Using a SVAR model, we obtain that this type of monetary policy has a sizeable recessionary effect on aggregate variables. Our results show that a standard medium-scale DSGE model, as currently much used in central banks, manages to match fairly well the impulse responses drawn from the SVAR. While the SVAR embeds the sufficient amount of economic theory needed to identify the disinflation shock, it is not well suited to give more than intuitions as to the economic mechanisms responsible for the overall aggregate effects of these shocks. To the contrary, the DSGE model offers a natural framework that lends itself to highlighting these mechanisms, provided that it fits the data reasonably well. Indeed, our medium–scale DSGE model turns out to capture reasonably well the aggregate dynamics triggered by a disinflation shock. Our counterfactual exercises then show that the main mechanisms at work are the following: (i) price rigidities in the form of frequency of no adjustment and the degree of indexation, (ii) real rigidities in the form of adjustment costs on investment and (iii) the speed of disinflation policy. Interestingly, our findings echo empirical results emphasized by Ball (1994). As explained above, however, going to the hassle of specifying and estimating a DSGE model has a high payoff in terms of structural interpretation of disinflation policies.
Figure 6: Monetary Policy

(a) Fast implementation ($\rho = a_{\pi^*}$)

(b) No interest rate inertia ($\rho = 0$)

(c) More aggressive policy ($a_p = 3$)

Notes: The solid line corresponds to the benchmark case, the dashed line corresponds to the counterfactual #1 and the dotted line corresponds to the counterfactual #2. In counterfactual #1, we set a parameter (or a combination of parameters) and we re-estimate the remaining parameters in order to minimize the loss function. In counterfactual #2, we set this parameter while keeping the other parameters at the estimated values obtained in our benchmark case.
References


Appendix

Proof of the Proposition  Let the steady–state Kalman gain $\kappa$ and steady–state matrix of the mean–squared forecast error be partitioned according to:

$$
\kappa = \begin{bmatrix} \kappa_1 \\ \kappa_2 \end{bmatrix} \quad P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}
$$

where $\kappa_1$, $\kappa_2$, $p_{11}$, $p_{12}$ = $p_{21}$ and $p_{22}$ are scalars that will be determined latter.

The matrices $\kappa$ and $P$ are solution to the steady–state equations associated to (6) and (7):

$$
\kappa = FP\left(H'PH\right)^{-1} \quad \text{(8)}
$$

$$
P = (F - \kappa H') P (F - \kappa H')' + \Sigma \quad \text{(9)}
$$

From (8), $\kappa$ satisfies

$$
\kappa = \begin{bmatrix} \frac{cp_{11} + p_{12}}{c^2p_{11} + 2cp_{12} + p_{22}} \\
0 \end{bmatrix}
$$

where $c = ((1 - \rho)(1 - a_p) + a_{\pi^*})$. It follows immediately that $\kappa_2 = 0$. This result is a direct consequence of the sparsity of the matrix $F$. From (9), the $P$ matrix obeys:

$$
P = \begin{bmatrix} p_{11}(1 - c\kappa_1)^2 - 2p_{12}(1 - \kappa_1c) + p_{22}\kappa_1^2 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2_R \end{bmatrix}
$$

Term by term identification of matrix $P$ implies

$$
p_{12} = 0 \quad p_{22} = \sigma^2_R \quad \text{and} \quad p_{11} = p_{11}(1 - c\kappa_1)^2 + \sigma^2_R\kappa_1^2 + \sigma^2_e
$$

Now, using $p_{12} = 0$ and $p_{22} = \sigma^2_R$, the Kalman gain $\kappa$ is given by:

$$
\kappa = \begin{bmatrix} \frac{cp_{11}}{c^2p_{11} + \sigma^2_R} \\
0 \end{bmatrix}
$$

After replacement of $\kappa_1$ into $p_{11}$ and some straightforward algebraic manipulations, it follows that $p_{11}$ is solution to the second–order polynomial equation

$$
p_{11}^2 - \sigma^2_e p_{11} - (\sigma_e\sigma_R/c)^2 = 0.
$$

This yields the two roots

$$
p_{11} = \frac{\sigma^2_e \left(1 \pm \sqrt{1 + 4(z/c)^2}\right)}{2}
$$

where $z = (\sigma_R/\sigma_e)$. One root is excluded from the solution, since it implies a negative value for $p_{11}$. The selected root is then given by:

$$
p_{11} = \frac{\sigma^2_e \left(1 + \sqrt{1 + 4(z/c)^2}\right)}{2}
$$

Now, we replace the expression of $p_{11}$ into $\kappa_1$:

$$
\kappa_1 = \frac{1 + \sqrt{1 + 4(z/c)^2}}{c[1 + \sqrt{1 + 4(z/c)^2} + 2(z/c)^2]}
$$

Using this and eq. (5) yields

$$
\kappa H' = \begin{bmatrix} \kappa_1 c & \kappa_1 \\ 0 & 0 \end{bmatrix}
$$

and

$$
F - \kappa H' = \begin{bmatrix} 1 - \kappa_1 c & -\kappa_1 \\ 0 & 0 \end{bmatrix}
$$
Denoting $\nu = 1 - \kappa_1 c$ and $\tilde{z} = (z/c)^2$, we obtain:

$$\nu = \frac{2\tilde{z}}{1 + \sqrt{1 + 4\tilde{z}} + 2\tilde{z}}$$

We directly deduce the dynamics of optimal forecasts from (5)

$$\begin{bmatrix} \hat{E}_t\pi_{t+1}^r \\ \hat{E}_t\eta_{t+1} \end{bmatrix} = \begin{bmatrix} \nu & \frac{\nu - 1}{c} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{E}_{t-1}\pi_t^r \\ \hat{E}_{t-1}\eta_t \end{bmatrix} + \begin{bmatrix} 1 - \nu & \frac{1 - \nu}{c} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_t^r \\ \eta_t \end{bmatrix}$$

For $z = 0$ (i.e. $\tilde{z} = 0$), it must be the case $\nu = 0$. Now, dividing both the numerator and the numerator of $\nu$ by $\tilde{z}$, we obtain

$$\nu = \frac{2}{2 + (1/\tilde{z}) + (\sqrt{1 + 4\tilde{z}}/\tilde{z})}$$

When $z \to \infty$, we have $(1/\tilde{z}) \to 0$ and $(\sqrt{1 + 4\tilde{z}}/\tilde{z}) \to 0$. It follows that when $z \to \infty$, we have $\nu \to 1$. The derivative of $\partial \nu / \partial z$ is of the sign of

$$2\sqrt{1 + 4\tilde{z}} + 4\tilde{z} + 2$$

which is strictly positive $\forall z \geq 0$. 

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