

Voter Turnout and Government's Legitimate Mandate*

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Abstract

The paper studies a group-mobilization model of costly voting in which citizens care about the legitimate mandate of the government formed by the winning group. This, as a function of the electorate's voting behavior, depends on both the margin of victory and the total turnout rate. Citizens prefer a high mandate when their own group forms the government but a low one if the government is formed by an opposing group. As such, the eventual losing group faces a trade-off: a higher participation from its members decreases the margin of victory but increases the total turnout. In equilibrium, a second fundamental trade-off arises, which overturns the supposed positive relationship between turnout and mandate: as the total turnout becomes more important for the government's mandate, the first decreases but the second strengthens. The key mechanism at play is a shift in the relative participation of the two groups, which favors the majority and raises its margin of victory, thus yielding a bandwagon effect. The implications for the evolution of turnout and the occurrence of election boycotts are discussed.

Keywords: voter turnout, legitimacy, mandate, costly voting, bandwagon.

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1 Introduction

Elections have policy consequences that depend not only on the identity of the winning party but also on the popular support shown by the election results. The notion of mandate, which refers to the power granted by a constituency to act on its behalf, is usually employed

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to explain how a large vote share helps the elected representatives to implement firmly the proposed policies (Conley 2001, Fowler and Smirnov 2007).

Accordingly, a few papers have recently modeled elections assuming that candidates and voters care about the margin of victory, as a measure of the electoral mandate (Castanheira 2003, Faravelli et al. 2015, McMurray 2017). However, when participation is voluntary, there is a second element that contributes to shaping the mandate and provides information about the electorate's approval for the competing candidates: the turnout rate. Indeed, in line with a view of elections as a mechanism for the generation of popular support for the government and its policies (Ginsberg and Weissberg 1978), scholars have often referred to the legitimizing function of electoral participation (Nadeau and Blais 1993). In light of this argument, the purpose of this paper is to study electoral participation in a unified framework, in which the government's mandate is affected both by the margin of victory and by the turnout rate, as two complementary indicators of popular support.

The relative importance of these two sources of mandate reasonably depends on the characteristics of the political regime in which the elections take place. As an extreme case, elections held in authoritarian regimes are generally considered a tool in the hands of the regime to foster its legitimacy via the popular participation. In such a context, the political science literature agrees that turnout is indeed a major dimension of political competition (Gonzalez-Ocantos et al. 2015). As a consequence, a typical form of protest by the opposition parties in non-democratic regimes is to call for an election boycott, with the aim of delegitimizing the outcome of a process that is deemed unfair (Buttorff and Dion 2017).

In established democracies, instead, the legitimacy effect given by a high turnout rate is arguably smaller and the extent to which it interacts with the institutional environment in determining parties' political strength has lacked a precise empirical investigation. The results of the 2017 UK general election, for example, have been unanimously interpreted in terms of a reduced mandate for May's future government, as a consequence of the decreased number of seats obtained by the conservative party. This interpretation prevailed despite a historically high turnout rate, which resulted in an increased popular vote - and even vote share - in favor of the conservative party relatively to the previous general election.

However, even in contexts in which parties' power is mostly determined by the electoral and institutional rules, voters might still partially perceive their act of voting as a way to give legitimacy or to show consent to the future government (Miles 2015), and thus decide their participation accordingly. Moreover, a concern for the legitimacy of representative democracy is becoming increasingly salient in western countries, given a steady pattern of declining turnout rates, which could also raise parties' efforts to ensure a high participation when they aim to form a legitimate government after the election.

This paper postulates that voters and parties care about the electoral mandate emerging from the election results and presents a theoretical model that relates different patterns of electoral participation to the extent to which the turnout rate contributes to the mandate. The model follows the group-voting literature in studying electoral participation as the outcome of a competition between voters' groups (Morton 1987, 1991, Shachar and Nalebuff 1999, Coate and Conlin 2004). As such, it features two groups, namely a majority and a minority, which compete in an election and trade off the electoral benefit of participation with the cost of mobilizing the members to vote. Both groups care not only about who wins but also about a measure of "legitimate mandate" assigned to the government formed by the winning group, as a convex combination between the total turnout rate and the margin of victory.¹

Concerning preferences, the model assumes that both groups would like to enjoy a large legitimate mandate if they form the government, but would prefer a small legitimate mandate for an opposing government. This assumption generates a specific trade-off faced by the eventual losing group: a higher participation by its members, on the one hand, reduces the margin of victory of the winning group, but, on the other hand, increases the total turnout rate; the total effect on the government's legitimate mandate is thus ambiguous. In equilibrium, both groups decide their turnout rates optimally with respect to this trade-off, taking into account the relative importance of the two components of the mandate and the mobilization cost.

The equilibrium - which exists under some conditions and is unique - features the majority group winning the election and thus forming the government. As such, the group facing the previous trade-off is (always) the minority. This follows from an assumption of no uncertainty, which makes the majority group always able and willing to ensure its victory. Importantly, this assumption also implies that local deviations in the participation decisions do not affect the groups' probability of winning but only the strength of the mandate enjoyed by the majority government.

Unable to win with positive probability, the minority solves its trade-off in favor of a complete withdrawal from voting, when the relative importance of the turnout for the government's mandate is high enough. Such behavior is consistent with the previous observation of frequent election boycotts in authoritarian regimes, as an opposition strategy to prevent leaders from claiming their legitimacy out of the popular participation.

For mid-range values of the parameters, the equilibrium is interior, with both groups participating at a rate between 0 and 1. Its comparative statics analysis reveals several insights. In particular, the main contribution of the paper is to highlight a non-trivial relationship between

¹Because the concept of mandate is specific to elections of citizens' representatives, the model assumes that the groups vote in order to elect a government; however, note that an alternative and similar interpretation can be developed to study voting behavior in referendums. Indeed, given their nature of instrument to elicit the popular will, the perceived success of referendums often depends both on a high turnout rate and a clear margin of victory.

turnout and mandate in equilibrium: in a large region of the parameter space, a second fundamental trade-off arises between the two variables, which move in opposite directions following variations in the relative weights of the mandate's components. For example, if the turnout becomes relatively more important than the margin of victory, the total turnout itself decreases but the government's mandate increases.

The mechanism at play is a shift in the relative participation of the two groups, which favors the majority and thus generates a bandwagon effect. As a consequence, the margin of victory increases to the point that it offsets the effect of a lower turnout and yields a positive net effect on the government's mandate. Hence, the supposed positive relationship between turnout and mandate can be overturned in equilibrium, given that both variables are determined endogenously. In this model, a more salient warning of the undermining effect of low turnout on the government's legitimacy produces stronger mandates, coming from higher margins of victory.

A conflicting relationship between turnout and mandate is a general property of the model, which holds not only when the equilibrium is interior but across the whole parameter space: in particular, the highest degree of legitimate mandate is attained for a set of parameters' values for which the majority participates at a full rate, while the minority turns out only partially.

The key determinant of the bandwagon effect is the assumption of turnout contributing to the mandate, which interacts with the different attitudes towards the mandate between the winning and losing group. This assumption crucially alters the relative participation rates of the groups. Indeed, when the mandate is mainly given by the margin of victory, the minority participates at a higher rate than the majority, offsetting the size disadvantage in line with an underdog effect often predicted by the theoretical literature on costly voting (Levine and Palfrey 2007, Herrera et al. 2016). If turnout matters as well, however, the relative participation of the majority increases, reducing the underdog effect until it shifts to a bandwagon effect, when the weight of the turnout is big enough. Bandwagon voting has been observed empirically and experimentally by the previous literature (e.g. Klor and Winter 2007, Morton et al. 2015); this paper identifies a new theoretical mechanism for this effect in the lower participation, driven by legitimacy concerns, of those citizens who do not support the likely government-forming party.

The aforementioned property of the total turnout decreasing when it matters relatively more for the government's mandate might be surprising. When a majority of the electorate prefers a large mandate and this is determined to a large extent by the turnout rate, one could expect participation to rise. Instead, an increase in the weight of turnout reduces participation in both groups (though disproportionately for the minority). The intuition for this result comes from the majority perceiving the participation rates of the competing groups as strategic complements: as such, when the minority lowers its turnout in order to minimize the mandate, the majority

also participates less to save on mobilization costs.

In western democracies, a steady decline in turnout is sometimes attributed to a large process of dealignment or alienation. The previous result points out that if voting or mobilizing voters is costly, the total turnout can theoretically decrease mainly due to a complementarity effect: even when only a (small) minority abstains with a delegitimizing intention, the majority may follow the same trend despite a favorable attitude towards the government’s mandate.

The remainder of the paper is organized as follows. Section 2 summarizes the related literature. Section 3 presents the model and discusses the existence and properties of the equilibrium. Section 4 focuses on the comparative statics analysis of the key variables in the model. Section 5 extends the discussion of the trade-off between the turnout and the mandate across the whole parameter space. Section 6 concludes.

2 Related literature

This paper is related to different strands of literature. The idea that elections give newly elected governments a mandate from the voters to implement policy change is recurrent in political debate, although often vague and contentious. Peterson et al. (2003) point out that what matters is not whether true mandates exist but rather whether politicians - “who lack the scholarly luxury to wait for careful analyses”- act as if they do. Extensive discussion and evidence that mandates do matter for policy are provided by Conley (2001) and Fowler and Smirnov (2007). A few theoretical models of electoral behavior when voters care about mandates have been studied by Castanheira (2003), Faravelli et al. (2015), and McMurray (2017). These papers, however, only measure the electoral mandate in terms of the margin of victory.

By looking at how the turnout rate also affects the mandate, this paper connects the political economy literature on voting with the political science literature on the legitimizing function of electoral participation. The focus of this line of work in political science, however, has mainly been on describing a winner-loser gap in attitudes towards legitimacy (Anderson et al. 2005) and on whether participation enhances losers’ consent to the election outcome (Nadeau and Blais 1993).

This paper, instead, takes a different perspective and investigates how the electoral consequences in terms of the government’s mandate shape the participation’s incentives of the competing groups. A similar analysis in the previous literature has been restricted to the context of authoritarian regimes, where it highlighted the trade-off between participating and boycotting faced by the opposition parties (Beaulieu 2006, Gonzalez-Ocantos et al. 2015, Buttorff and Dion 2017). The extent to which a legitimacy effect given by the turnout matters for parties and voters in established democracies, instead, has not yet been studied empirically, to my knowledge. With respect to voters, however, Miles (2015) argues that, even in advanced

democracies, citizens might indeed vote to show consent to the political regime.

The paper presents a simplified model that studies electoral participation as an equilibrium outcome of the strategic interaction of competing groups of voters. Morton (1987, 1991) first investigated the role of groups in coordinating members to vote, in order to explain large turnout. The model is very general on how the groups internally provide members with the incentives to participate and is thus compatible with two distinct interpretations. The first is in line with the follow-the-leader model by Shachar and Nalebuff (1999), according to which groups' leaders engage in costly voter mobilization in order to reach a targeted turnout rate. The second refers to the group rule-utilitarian approach of Coate and Conlin (2004), in which individual voting is costly and the incentive takes the form of an ethical obligation to follow the turnout rule that maximizes the group's utility. As such, the model also resembles the group-voting models in Herrera et al. (2016).

A last strand of literature to which this paper is related is that on the effects of opinion polls on election outcomes. Part of this literature focuses on how the electoral participation of the competing groups depends on the groups' sizes, as could be revealed by an opinion poll. Following the literature's terminology, a model in which the turnout rate of the majority is greater than that of the minority is said to yield a bandwagon effect, i.e. an advantage to the candidate with the greatest support as a consequence of the poll's publication.² On the contrary, a model in which the turnout rate of the majority is smaller than that of the minority is said to yield an underdog effect.

The theoretical literature on costly voting has often converged to a prediction in favor of the underdog effect across different models (Herrera et al. 2016). In group-voting models, the reason for the underdog effect lies in the relatively higher cost of increasing the supporters' turnout borne by bigger groups. While there has been some experimental support for the underdog effect (Levine and Palfrey 2007), several empirical papers have also found evidence of bandwagon effects (Klor and Winter 2007, Grosser and Schram 2010, Morton et al. 2015). A few attempts to model theoretically the bandwagon effect have stressed the role of informational asymmetries (McKelvey and Ordeshook 1985), psychological factors (Callander 2007), other-regarding preferences (Morton and Ou 2015), or risk-aversion (Grillo 2017). This paper contributes to this literature by showing that, when the competing groups have opposite preferences over the government's mandate, a mandate-enhancing role of electoral participation can reduce the underdog effect or even yield a bandwagon effect, by affecting the groups' participation differently.

²A bandwagon effect could be due to changes in participation or in vote choices; this paper deals only with the first type of bandwagon.

3 Model

There is a continuum of citizens divided in two groups, A and B. The fraction of citizens in group A is μ while $1 - \mu$ citizens belong to group B. Without loss of generality, assume $\mu < \frac{1}{2}$ so that group A is the minority and group B is the majority. Citizens can vote or abstain in a majority-rule election contested by two parties representing the two groups, which is held in order to select the government.

By assumption, the turnout decision is made at the group-level, which implies that the model can be studied as a game between the two groups. I adopt here a group-mobilization interpretation à la Shachar and Nalebuff (1999), in a simplified model similar to Herrera et al. (2016): the two groups choose their desired level of turnout and they are supposed to achieve such level by exerting a costly effort to mobilize their members to vote. As mentioned before, the model is also compatible with a group rule-utilitarian interpretation à la Coate and Conlin (2004), according to which the groups' members ethically follow the turnout rule that maximizes their group's utility.³

The party that wins the election is entitled to form a government, which implements the group's preferred policy. Ties will not be equilibrium outcomes of the model for any tie-breaking rule, hence for the sake of exposition I assume that the government is formed by the majority party B in the case of a draw. A key element of the model is the definition of a function that assigns a measure of legitimate mandate to the government formed after the election, as a convex combination of two components: the total turnout rate and the margin of victory. Formally, given a turnout rate x chosen by group A and a turnout rate y by group B, the government is formed by group A if $\mu x > (1 - \mu)y$ and by group B if $\mu x \leq (1 - \mu)y$. Then the total turnout rate T and the margin of victory M are equal to

$$T = \mu x + (1 - \mu)y \quad , \quad M = \frac{|\mu x - (1 - \mu)y|}{\mu x + (1 - \mu)y}$$

while the legitimate mandate function is defined as

$$\mathcal{L} = \lambda T + (1 - \lambda)M$$

in which the relative weight $\lambda \in (0, 1)$ captures the extent to which the turnout rate affects the government's mandate.

The mobilization cost is assumed to be increasing and convex in the number of votes for both groups, according to a quadratic specification: given the targeted turnout rates (x, y) ,

³In a group rule-utilitarian model, the turnout rule specifies the probability of voting for each group's member, which determines the (expected) turnout rate of the group.

group A pays $c(\mu x)^2$ and group B pays $c((1 - \mu)y)^2$, where $c \in [0, \bar{c}]$ is a cost parameter.⁴

Both groups want to win the election in order to form the government and care about its realized legitimate mandate. In particular, each group prefers a large legitimate mandate for its own government but a small legitimate mandate if the government is formed by the opposing group. Hence, the groups' utility functions are as follows

$$U_A = \begin{cases} \mathcal{L} - c(\mu x)^2 & \text{if } \mu x > (1 - \mu)y \\ -\mathcal{L} - c(\mu x)^2 & \text{if } \mu x \leq (1 - \mu)y \end{cases}$$

$$U_B = \begin{cases} \mathcal{L} - c((1 - \mu)y)^2 & \text{if } \mu x \leq (1 - \mu)y \\ -\mathcal{L} - c((1 - \mu)y)^2 & \text{if } \mu x > (1 - \mu)y \end{cases}$$

The solution concept is Nash equilibrium in pure strategies. The turnout rates (x, y) chosen by the two groups determine the total turnout, the group entitled to form the government, and the legitimate mandate of the formed government.

A few remarks are in order. First, note that a relative measure of margin of victory is employed, in which the total turnout T appears in the denominator of M . This means that the higher the total turnout, the less a change in participation by a group affects the margin of victory. This standard assumption does not play a relevant role in driving all main results, which would hold also with a linear definition of margin of victory as difference in votes.⁵

Second, in the model there is no uncertainty, which will imply that groups either win or lose for sure the election in equilibrium. Nonetheless, the assumption that the margin of victory matters provides both groups with an incentive to participate even if they cannot affect the probability of winning at the margin.⁶

Moreover, note that for both parties the utility function has a discontinuity point at $x = \frac{1-\mu}{\mu}y$, due to the switch in the group forming the government, which entails a change of sign for the legitimate mandate function in the groups' utility. This implies that the equilibrium should satisfy the best-reply condition both locally (fixing the identity of the winning group) and globally (checking for deviations that would change the identity of the winning group). As shown later, this is also the reason why the existence of an equilibrium will not be always guaranteed in the whole parameter space.

Before specifying the existence conditions for the equilibrium, the following result highlights an important necessary condition concerning the identity of the winning group.

Lemma 1. *In equilibrium $\mu x \leq (1 - \mu)y$, i.e. the majority forms the government.*

Proof. See the appendix. □

⁴All main results are robust to the specification of linear mobilization costs.

⁵I.e. all main results are robust to a definition of margin of victory as $M' = |\mu x - (1 - \mu)y|$.

⁶I will discuss later how the assumption of no uncertainty is important for the bandwagon result in section 4.

The intuition for the result is that if an equilibrium in which the minority wins existed, the majority would have an incentive to deviate by increasing its turnout rate in order to overcome the minority and win the election. In light of this result, we can first focus on the local best-replying behavior of the two groups: in order for an equilibrium to exist, the majority must be in best-reply to maximize the government's legitimate mandate and the minority must be in best-reply to minimize it. That is, the equilibrium must solve the following maximization problems:

$$\begin{aligned} \max_{x \in [0,1]} \quad & -\lambda(\mu x + (1 - \mu)y) - (1 - \lambda) \frac{(1 - \mu)y - \mu x}{\mu x + (1 - \mu)y} - c(\mu x)^2 \\ \max_{y \in [0,1]} \quad & \lambda(\mu x + (1 - \mu)y) + (1 - \lambda) \frac{(1 - \mu)y - \mu x}{\mu x + (1 - \mu)y} - c((1 - \mu)y)^2 \end{aligned} \tag{1}$$

The first equation in (1) highlights the trade-off faced by the minority: by increasing its turnout rate x , the minority lowers the margin of victory but increases the total turnout rate. This trade-off has to be weighted also by the cost of mobilizing the group's members. The majority instead only trades off the larger legitimate mandate obtained by increasing the turnout rate with the mobilization cost.

By the same reasoning as before, however, a pair of turnout rates (x, y) that solve system (1) is not an equilibrium of the model if the minority has a profitable deviation in increasing its turnout rate to the extent necessary to win the election. As shown in the proof of the next lemma, it turns out that whenever such a deviation is possible, it is also profitable. Hence, in order to have existence of the equilibrium, this global deviation by the minority has to be ruled out by the condition $\mu \leq (1 - \mu)y$. That is, the turnout rate of the majority has to be high enough so that even by turning out fully the minority would not change the identity of the winning group. Combining this "no-global-deviation" condition with the solution of system (1) yields the following result.

Lemma 2. *An equilibrium (x, y) exists if and only if $\mu \leq \max\{\frac{1}{2}\sqrt{\frac{(1-\lambda)}{c}} + \frac{\lambda}{4c}, \frac{\lambda}{2c}\}$ and is unique.*

Proof. See the appendix. □

The condition on μ in Lemma 2 translates into a condition on c , stating that an equilibrium does not exist, if the mobilization cost is too high. The intuition is that, with a high c , given a victory by the majority, the two groups would want to lower their turnout rates; but, if μ is high enough, the minority could then deviate in order to win the election. The proof also shows that the equilibrium is unique, when it exists, as the solution of system (1) is unique. Before turning to the characterization of the equilibrium, a last preliminary result concerns how the participation incentive of each group is affected by the turnout of the opposing group.

Lemma 3. *Consider the groups' best replies as functions of the opposing group's turnout $x(y)$ and $y(x)$, obtained from the first order conditions for system (1). We have that x is decreasing in y while y is increasing in x .*

Proof. See the appendix. □

Lemma 3 shows that the minority considers the groups' turnout rates as strategic substitutes, while the majority considers them as strategic complements. That is, the minority would like to decrease its turnout rate x if the majority participates more, while the majority would like to increase its turnout rate y if the minority participates more. This property is important for understanding the mechanisms behind the results in the next section and it is due to the fact that a change in the opposing group's participation affects the trade-off between the turnout, the margin of victory, and the mobilization cost differently depending on whether the government's mandate is valued positively or negatively.

We can now move on to the analysis of the equilibrium. A full graphical discussion of the possible types of equilibrium in terms of being interior or a corner solution is postponed to the appendix, as a function of the values of the three parameters λ , μ , and c . The core of the analysis, in the next section, presents the analytical solution for the interior equilibrium and shows its comparative statics properties. First, however, I discuss the interesting case of the corner solution equilibrium in which $x = 0$. Indeed, if the turnout weight λ is high enough, the minority withdraws completely from participating in equilibrium, which reproduces the election boycotts typically observed in authoritarian regimes.

Proposition 1. *There exists a threshold $\bar{\lambda}(c, \mu)$ above which the minority withdraws from participating in equilibrium, i.e. $x = 0$ if $\lambda > \bar{\lambda}(c, \mu)$. Moreover, the threshold $\bar{\lambda}(c, \mu)$ is (weakly) increasing in both c and μ .*

Proof. See the appendix. □

The minority boycotts the election because, given the high weight of the turnout λ in determining the legitimate mandate of the government, participating would increase the mandate, despite reducing the margin of victory. The majority responds by participating either partially or fully depending on the values of the parameters, as detailed in the proof. The most interesting exercise concerning the election boycott by the minority consists of studying how the whole region identified by $\bar{\lambda}(c, \mu)$ expands or shrinks as a function of the parameters, in order to analyze what makes an election boycott more likely. As argued in the proposition, election boycotts are more likely when the size of the minority μ is small and when the mobilization cost c is low. The result concerning μ provides theoretical support for the argument sometimes advanced, according to which parties' decision to boycott could be strategically motivated in order to avoid a large loss when the party is particularly weak (Beaulieu 2006). Instead, when the

mobilization cost increases, an election boycott becomes less likely because a higher c decreases the turnout of the majority, making participation more attractive for the minority despite the higher mobilization cost.

4 Interior equilibrium

Having stressed the importance of electoral participation as a source of mandate in order to observe an election boycott, this section focuses on the case in which the equilibrium is interior, i.e. $x, y \in (0, 1)$. Through the comparative statics analysis on the key variables of the model, it shows how a trade-off between turnout and mandate emerges in a large region of the parameter space and highlights the dynamic favoring a bandwagon effect.

4.1 Groups' turnout rates

The following proposition gives the conditions for the equilibrium of the model to be interior and specifies the analytical solution for the turnout rates of the two groups.

Proposition 2. *The equilibrium is interior and has turnout rates*

$$x = \frac{1}{\mu} \left(\frac{1}{2} \sqrt{\frac{(1-\lambda)}{c}} - \frac{\lambda}{4c} \right) \quad , \quad y = \frac{1}{(1-\mu)} \left(\frac{1}{2} \sqrt{\frac{(1-\lambda)}{c}} + \frac{\lambda}{4c} \right) \quad (2)$$

in the parameter region defined by $0 < \frac{1}{2} \sqrt{\frac{(1-\lambda)}{c}} - \frac{\lambda}{4c} < \mu \leq \frac{1}{2} \sqrt{\frac{(1-\lambda)}{c}} + \frac{\lambda}{4c} < 1 - \mu$.

Proof. See the appendix. □

Note that, for both groups, the turnout rate is decreasing in the size of the group, while the number of votes is independent of it: an increase in size (e.g. μ) determines a proportional decrease in the turnout rate (e.g. x), such that the number of votes (e.g. μx) remains the same.

To explain how x and y change when the parameters λ or c vary, instead, it is useful to recall the result of Lemma 3, according to which the turnout rates of the two groups are strategic complements for the majority but strategic substitutes for the minority. As we can see from (2), an increase in the mobilization cost c decreases the turnout rate of the majority y but has an ambiguous effect on the turnout rate of the minority x . Indeed, for the minority, besides the negative direct effect of a higher c , there is an indirect positive effect due to the substitutability of the groups' turnout rates: as a higher c decreases the majority turnout y , the minority finds participating more attractive. Which effect prevails depends on the parameters' values.⁷

⁷In particular, $\frac{\partial x}{\partial c} > 0$ iff $c < \frac{\lambda^2}{1-\lambda}$, i.e. an increase in c increases the minority's turnout if c is low relatively to λ .

Similarly, an increase in the weight of the total turnout λ decreases the turnout rate of the minority x , consistently with the objective of minimizing the mandate, but has, in theory, an ambiguous effect on the turnout rate of the majority y . On one side, the majority has a direct positive effect from increasing the turnout rate in terms of a larger mandate; on the other, as the minority participates less, there is a negative indirect effect due to the complementarity of the turnout rates for the majority. In practice, however, the first effect prevails only if both λ and c are low, in which case the equilibrium is not interior.⁸ Hence, for the interior equilibrium, also the turnout rate of the majority y decreases if the weight of the total turnout rate λ increases.

The comparative statics analysis is particularly interesting with respect to the other variables of the model that depend on the groups' turnout rates, as shown in the next subsections.

4.2 Total turnout, margin of victory, legitimate mandate

Consider first the total turnout rate, which is equal to

$$T = \mu x + (1 - \mu)y = \sqrt{\frac{1 - \lambda}{c}} \quad (3)$$

Note that T is decreasing in λ and c , and independent of μ . The most interesting feature is that the more the total turnout matters for the mandate, the more its value decreases in equilibrium, as it happens although a majority of citizens prefer a large mandate. This is however a natural consequence of the fact that both groups reduce their participation when λ increases: as argued before, it is the complementarity in the groups' turnout rates that makes the majority willing to save on mobilization costs, when the minority participates less following an increase in λ . The result stresses how a complementarity effect can, when voting is costly, decrease the total turnout even if a small minority abstains to delegitimize the government while the majority of the electorate values the positive effect of participation on the mandate.

The second component of the mandate, i.e. the margin of victory, is instead equal to

$$M = \frac{(1 - \mu)y - \mu x}{\mu x + (1 - \mu)y} = \frac{\lambda}{2\sqrt{c(1 - \lambda)}} \quad (4)$$

The margin of victory is also decreasing in c and independent of μ . However, it is crucially increasing in λ . This is a consequence of the fact that, when λ increases, the minority reacts more drastically in cutting its turnout, given the different preferences for the government's mandate. Note that since the weights of the two components of the mandate sum to one, the result implies that also the margin of victory decreases in equilibrium when its relative importance for the mandate increases. As such, the model has the somehow counterintuitive

⁸Specifically $\frac{\partial y}{\partial \lambda} > 0$ iff $\lambda < 1 - c$, which is never satisfied for the interior equilibrium, given the conditions in Proposition 2.

property that the more a component of the mandate matters, the more its value decreases in equilibrium.

Finally, the equilibrium value of the legitimate mandate is equal to

$$\mathcal{L} = \lambda T + (1 - \lambda)M = \frac{3\lambda}{2} \sqrt{\frac{1 - \lambda}{c}} \quad (5)$$

As both T and M , the legitimate mandate \mathcal{L} is decreasing in c and independent of μ . Moreover, it is non-monotone with respect to λ , because of the combination of the opposing effects on the total turnout and the margin of victory. In particular, however, if λ is not too high (i.e. $\lambda < \frac{2}{3}$), the positive effect of a marginal increase in λ on the margin of victory M is greater than the negative effect on the total turnout rate T , and even if the total turnout becomes more important for the government's legitimate mandate, this increases in equilibrium.⁹

Overall, thus, the previous analysis shows an interesting trade-off between turnout and mandate arising in equilibrium, highlighted in the following proposition.

Proposition 3. *If the weight of the total turnout in the legitimate mandate is not too high, a marginal change in the weights of the two components of the mandate move the interior equilibrium values of the total turnout rate and the degree of legitimate mandate in opposite directions.*

Hence, while starting from the simple assumption of a positive contribution of the turnout to the mandate, the model delivers a more conflicting relationship between the two variables in equilibrium, as they are both determined endogenously. As such, despite the undermining effect of a low voter turnout on the government's legitimacy, the model warns that stronger mandates coming from higher margins of victory could be observed, if the lower turnout is due to different attitudes toward the government's mandate that make minority groups abstain disproportionately more.

Section 5 will investigate the relationship between the total turnout rate and the government's mandate in the whole parameter space, showing that the previous trade-off does not emerge only when the equilibrium is interior but it is a more general property of the model.

4.3 Groups' relative participation: a bandwagon effect

The key mechanism behind the dynamic analyzed above is that the more turnout is important for the government's mandate, the more the relative participation of the two groups shifts in favor of the majority. This relates to the literature on electoral behavior studying how the turnout rates of competing groups depend on the groups' sizes, and thus on the information revealing them, e.g. the opinion polls.

⁹Indeed, $\frac{\partial \mathcal{L}}{\partial \lambda} = \frac{3}{2\sqrt{c(1-\lambda)}}(1 - \frac{3}{2}\lambda)$, which is positive if $\lambda < \frac{2}{3}$.

According to the literature’s terminology, a model in which the participation rate of the majority is greater than that of the minority yields a bandwagon effect, while a model in which the opposite is true yields an underdog effect. As discussed in Section 2, while the theoretical literature generally predicts the prevalence of the underdog effect, the empirical evidence is mixed and bandwagon effects have also been observed.

To study the implications of the model with respect to such effects, let us compare the turnout rates of the two groups, x and y , in equation (2). First, note that in the absence of a turnout component of the mandate, i.e. if $\lambda = 0$, the equilibrium is such that $\mu x = (1 - \mu)y$, thus displaying a full underdog effect. The result is due to the fact that a marginal increase in the turnout rate is more costly for the bigger group and thus the optimization by the two groups yields a toss-up election in equilibrium. This is in line with a general prediction of underdog effects in models of costly voting.

However, a key contribution of this paper consists of showing that the underdog effect is not robust to the introduction of a turnout component of the government’s mandate, if its weight is big enough. Indeed, define the difference in turnout rates $D = y - x$, whose equilibrium value is equal to

$$D = y - x = \frac{1}{2\mu(1 - \mu)} \left(\frac{\lambda}{2c} - (1 - 2\mu) \sqrt{\frac{1 - \lambda}{c}} \right) \quad (6)$$

Such difference D is increasing in λ , which implies, as detailed in the next proposition, that the more important turnout is for the mandate, the more the underdog effect is reduced until the prediction switches to a bandwagon effect if λ is big enough.

Proposition 4. *There exists a threshold λ^* in the turnout’s weight below which the difference in turnout rates $D = y - x$ is negative and above which D is positive: $D < 0$ if $\lambda < \lambda^*$, $D > 0$ if $\lambda > \lambda^*$. Moreover, the threshold λ^* is increasing in c and decreasing in μ .*

Proof. See the appendix. □

The proposition also shows that a higher value for λ is necessary to observe a bandwagon effect when the mobilization cost is higher and when the size of the minority is smaller. Overall, the result highlights that a concern for the mandate-enhancing role of the turnout contributes to reducing the underdog effect or even to generating a bandwagon effect, when the minority group lowers its participation with the aim of reducing the majority government’s mandate.

I conclude the analysis of the interior equilibrium with an important caveat on the bandwagon effect displayed in the model. In the previous literature, the underdog effect often occurs when a minority can increase its (little) chance of winning by putting more effort on participation. In this model, this channel through the probability of winning is shut down by the assumption of no uncertainty. Verifying whether the bandwagon effect is robust to the in-

roduction of uncertainty involving the possibility of a minority win would require additional assumptions to model uncertainty and is left as an interesting avenue for future research.¹⁰

5 Legitimate mandate in the full parameter space

While the previous section focused on explaining how a change in the turnout's weight λ moves the turnout and the mandate in opposite directions when the equilibrium is interior, it also showed that both groups' turnout rates increase when the mobilization cost c decreases. One could then wonder whether it is possible, by decreasing c , to move to an equilibrium of full turnout (i.e. $x = y = 1$) at which the government's legitimate mandate is also maximized.

This section gives a negative answer to such a question and shows that a trade-off between turnout and mandate is a general property of the model. Indeed, for any mobilization cost c , the value of the parameter λ that maximizes the government's legitimate mandate is such that the total turnout T is smaller than one in the corresponding equilibrium. In particular, the minority participates only partially in the equilibrium that maximizes the government's mandate with respect to the parameters (λ, c) .

Proposition 5. *Fix $\mu \in (0, \frac{1}{2})$. (i) For any c , the value of λ that maximizes the government's legitimate mandate \mathcal{L} is such that in the corresponding equilibrium $T < 1$. (ii) The highest possible equilibrium value of \mathcal{L} is attained for a couple of parameters' values (λ, c) , for which the corresponding turnout rates satisfy $x \in (0, 1)$ and $y = 1$.*

Proof. See the appendix. □

Note that μ is fixed because, uninterestingly, for c low enough, both the turnout and the legitimate mandate would tend to one if μ tends to zero, i.e. if there is only one group. The proof of the proposition consists of showing that starting from anywhere in the parameter space, the degree of government's legitimate mandate increases if the parameters λ and c move in the direction of the region in which the equilibrium satisfies $x \in (0, 1)$ and $y = 1$. Thus, even starting from an equilibrium of full turnout ($x = y = 1$), for any μ and any c , an increase in λ that makes the minority participate less, reducing the total turnout, increases the majority government's legitimate mandate. This confirms the trade-off between the turnout and the mandate as a main result of the model.

¹⁰Note however that such an extension would add the probability of winning as a fourth variable affected by groups' participation decisions, besides the total turnout rate, the margin of victory, and the mobilization costs. Hence, the analysis of the groups' trade-offs between these four variables is likely to be complicated.

6 Conclusion

The paper has presented a model of electoral participation in which two groups compete to win an election and care about the mandate of the government formed by the winning group. The main novelty of the model with respect to the previous literature is that the government's mandate is not measured only in terms of the margin of victory but it is also positively affected by the total turnout rate. The model highlights two different trade-offs.

The first concerns the optimal participation decision of the minority, given the objective of minimizing the majority's mandate: on the one hand, increasing its turnout rate decreases the margin of victory and thus the mandate; on the other hand, it increases the total turnout rate and thus the mandate. If the weight of the turnout for the mandate is high enough, this trade-off is even resolved in favor of a complete withdrawal from participating. Such behavior replicates the election boycotts that are often observed in authoritarian regimes, where turnout is indeed considered the main dimension of political competition.

The second trade-off is the one that emerges in equilibrium between the total turnout rate and the government's mandate. While, *ceteris paribus*, an increase in the total turnout affects positively the mandate, both variables are determined endogenously in the model by the participation choices of the two groups. The trade-off consists then of the fact that, in a large region of the parameter space, a marginal variation in the relative importance of the two mandate's components move the equilibrium values of the total turnout and the government's mandate in opposite directions. In particular, if the relative weight of the total turnout is not too high, an increase in such weight decreases the total turnout but increases the government's mandate. Moreover, the value of the turnout weight λ that maximizes the government's mandate in equilibrium is such that the corresponding total turnout rate is smaller than one.

In the interior equilibrium, the total turnout always decreases when it becomes more important for the government's mandate, although the majority group aims at maximizing the mandate. This happens because, for the majority, the direct incentive to increase its turnout rate when this becomes more important for the mandate is outweighed by the indirect incentive to save on mobilization costs, given the concurrent lower turnout by the minority. This result favors a particular interpretation of the much discussed decrease in turnout rates in established democracies worldwide. While such decrease is often attributed to a large process of alienation of the electorate, my model points out that even when only a small minority of voters reduces its participation rate with the aim of delegitimizing the government, the total turnout can decrease significantly due to the indirect effect of complementarity.

Because the reduction in participation affects disproportionately the minority, the margin of victory of the majority increases when the total turnout becomes more important. This effect on the margin of victory offsets the one on the total turnout if the turnout weight is not

too high, yielding a positive net effect on the government's legitimate mandate and thus the uncovered equilibrium trade-off. Such dynamics identify the mandate-enhancing role played by the turnout as a new theoretical driver of a bandwagon effect. This contrasts with a standard prediction of underdog effect in models of costly voting, which prevails only when turnout does not matter much for the mandate. A limitation for the general validity of this result is, however, that the model assumes the absence of uncertainty and thus a sure victory for the majority in equilibrium. Assessing the robustness of the bandwagon effect in a framework in which a win by the minority is possible, even if unlikely, represents an interesting direction for extending the model, which is left for future research.

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A Equilibrium types, conditions, and graphical representation

System (1) in section 3 has a unique solution (x, y) , which can be interior or a corner solution, depending on the values of the three parameters λ , μ , and c . Together with the no-global-deviation

condition $\mu \leq (1 - \mu)y$, this yields six possible types of equilibrium in terms of being interior or corner solutions, which are detailed in Table 1 with their conditions on the parameters.

	Eq. Types	Eq. Conditions		Eq. Types	Eq. Conditions
I	$x \in (0, 1)$ $y \in (0, 1)$	$0 < \frac{1}{2}\sqrt{\frac{(1-\lambda)}{c}} - \frac{\lambda}{4c} < \mu$ $\mu \leq \frac{1}{2}\sqrt{\frac{(1-\lambda)}{c}} + \frac{\lambda}{4c} < 1 - \mu$	IV	$x \in (0, 1)$ $y = 1$	$2(1 - \lambda)(1 - \mu) - 2\mu c < \lambda < \frac{2(1-\lambda)}{1-\mu}$ $\frac{1}{2}\sqrt{\frac{(1-\lambda)}{c}} + \frac{\lambda}{4c} \geq (1 - \mu)$
II	$x = 1$ $y = 1$	$2(1 - \mu)c \leq \lambda + 2(1 - \lambda)\mu$ $2\mu c \leq -\lambda + 2(1 - \lambda)(1 - \mu)$	V	$x = 0$ $y \in (0, 1)$	$2c\mu \leq \lambda < 2c(1 - \mu)$ $\frac{1}{2}\sqrt{\frac{(1-\lambda)}{c}} - \frac{\lambda}{4c} \leq 0$
III	$x = 1$ $y \in (0, 1)$	$2(1 - \mu)c > \lambda + 2(1 - \lambda)\mu$ $\frac{1}{2}\sqrt{\frac{(1-\lambda)}{c}} - \frac{\lambda}{4c} \geq \mu$	VI	$x = 0$ $y = 1$	$2c(1 - \mu) \geq \lambda$ $(1 - \mu)\lambda \geq 2(1 - \lambda)$

Table 1: Possible equilibrium types

Figure 1 then plots the regions of the parameter space where the different types of equilibrium exist in the (λ, c) plane, for two values of μ , namely $\mu = 0.4$ and $\mu = 0.25$.

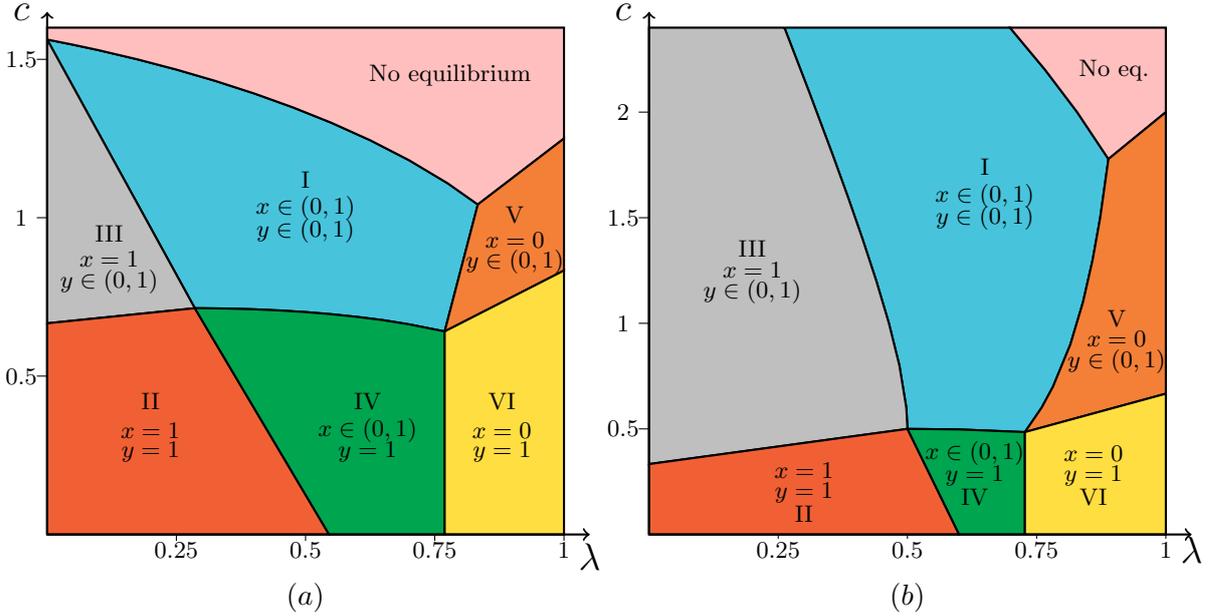


Figure 1: Equilibrium types in (λ, c) plane for (a) $\mu = 0.4$ and (b) $\mu = 0.25$

The contour lines of the regions in Figure 1 represent the equilibrium conditions in Table 1. In particular, the contour lines of the region where the equilibrium does not exist represent the condition $\mu \leq \max\{\frac{1}{2}\sqrt{\frac{1-\lambda}{c}} + \frac{\lambda}{4c}, \frac{\lambda}{2c}\}$ in Lemma 2, which implies that the equilibrium does not exist if the mobilization cost c is too high.

As we can see, the minority turns out at a full rate (i.e. $x = 1$) if λ is low and hence the

margin of victory is more important for the mandate, while the majority does it (i.e. $y = 1$) for any λ if the mobilization cost c is low. The parameter region for the interior equilibrium - the focus of section 4 - is at the center of the graph, where λ and c take intermediate values. Instead, as discussed in the main text, if λ is high enough the minority withdraws completely from participating, boycotting the election (i.e. $x = 0$).

The comparison between the two figures 1a and 1b shows how the different regions evolve when μ varies. Note that, if the size of the minority decreases, an equilibrium exists for a bigger range of the cost parameter c . Indeed, even if c is higher and thus the majority would lower its turnout in equilibrium, the minority could not deviate in order to win the election, because of its limited group size.

B Proofs

PROOF OF LEMMA 1. Assume that there exists an equilibrium in which $\mu x > (1 - \mu)y$. Then, to be local best replies, x, y have to satisfy

$$\begin{aligned} \max_{x \in [0,1]} \quad & \lambda(\mu x + (1 - \mu)y) + (1 - \lambda) \frac{\mu x - (1 - \mu)y}{\mu x + (1 - \mu)y} - c(\mu x)^2 \\ \max_{y \in [0,1]} \quad & -\lambda(\mu x + (1 - \mu)y) - (1 - \lambda) \frac{\mu x - (1 - \mu)y}{\mu x + (1 - \mu)y} - c((1 - \mu)y)^2 \end{aligned} \quad (7)$$

There can be four cases: (1) $x, y \in (0, 1)$; (2) $x = 1, y \in (0, 1)$; (3) $x \in (0, 1), y = 0$; (4) $x = 1, y = 0$. I show that in all four cases it is profitable for group B to deviate to $y = \frac{\mu x}{1 - \mu}$ and win the election.

Case (1): $x, y \in (0, 1)$. The first order conditions for an interior solution are

$$\begin{cases} \lambda + (1 - \lambda) \frac{2(1 - \mu)y}{(\mu x + (1 - \mu)y)^2} - 2c\mu x = 0 \\ -\lambda + (1 - \lambda) \frac{2\mu x}{(\mu x + (1 - \mu)y)^2} - 2c(1 - \mu)y = 0 \end{cases}$$

which imply

$$\mu x = \frac{1}{2} \sqrt{\frac{1 - \lambda}{c}} + \frac{\lambda}{4c}, \quad (1 - \mu)y = \frac{1}{2} \sqrt{\frac{1 - \lambda}{c}} - \frac{\lambda}{4c}$$

Group B's utility in equilibrium is thus $U_B^{eq} = -\frac{5\lambda}{4} \sqrt{\frac{1 - \lambda}{c}} - \frac{1 - \lambda}{4} - \frac{\lambda^2}{16c}$, while deviating to $y = \frac{\mu x}{1 - \mu}$ yields $U_B^{dev} = \frac{3\lambda}{4} \sqrt{\frac{1 - \lambda}{c}} - \frac{1 - \lambda}{4} + \frac{7\lambda^2}{16c}$. The deviation is profitable since

$$U_B^{dev} - U_B^{eq} = 2\lambda \sqrt{\frac{1 - \lambda}{c}} + \frac{\lambda^2}{2c} > 0$$

Case (2): $x = 1, y \in (0, 1)$. The first order condition for y satisfies

$$-\lambda + (1 - \lambda) \frac{2\mu}{(\mu + (1 - \mu)y)^2} - 2c(1 - \mu)y = 0 \quad (8)$$

while in order to have $x = 1$ the derivative of group A's utility with respect to x has to be positive at $x = 1$, i.e.

$$\lambda + (1 - \lambda) \frac{2(1 - \mu)y}{(\mu + (1 - \mu)y)^2} - 2c\mu \geq 0 \quad (9)$$

Call $z = (1-\mu)y$, $z < \mu$ by assumption. Then group B's utility in equilibrium is $U_B^{eq} = -\lambda(\mu+z) - (1-\lambda)\frac{\mu-z}{\mu+z} - cz^2$, while deviating to $z = \mu$ yields $U_B^{dev} = \lambda 2\mu - c\mu^2$. The difference in utility is

$$U_B^{dev} - U_B^{eq} = \lambda 2\mu + \lambda(\mu+z) + (1-\lambda)\frac{\mu-z}{\mu+z} - c(\mu^2 - z^2) \quad (10)$$

which has to be positive for the deviation to be profitable. Now, rewriting (8) as

$$c = -\frac{\lambda}{2z} + (1-\lambda)\frac{\mu}{z(\mu+z)^2} \quad (11)$$

and replacing (11) in (9) yields

$$\begin{aligned} -\frac{\lambda}{2z} + (1-\lambda)\frac{\mu}{z(\mu+z)^2} &\leq \frac{\lambda}{2\mu} + (1-\lambda)\frac{z}{\mu(\mu+z)^2} \\ \Rightarrow (1-\lambda)(z-\mu) + \frac{\lambda}{2}(\mu+z)^2 &\geq 0 \end{aligned} \quad (12)$$

Replacing (11) in (10), imposing (10) to be positive and dividing by $(\mu^2 - z^2)$ yields

$$\frac{\lambda 2\mu}{\mu^2 - z^2} + \frac{\lambda}{\mu - z} + \frac{(1-\lambda)}{(\mu+z)^2} + \frac{\lambda}{2z} - \frac{\mu(1-\lambda)}{z(\mu+z)^2} > 0$$

and, after some algebra,

$$(1-\lambda)(z-\mu) + \frac{\lambda}{2}(\mu+z)^2 > -\frac{2\lambda\mu z(\mu+z)}{\mu-z} - \frac{\lambda z(\mu+z)^2}{\mu-z} \quad (13)$$

which is always verified, since the left-hand side is positive by (12) and the right-hand side is negative.

Case (3): $x \in (0, 1)$, $y = 0$. The first order condition for x is

$$\lambda - 2c\mu x = 0 \quad \Rightarrow \mu x = \frac{\lambda}{2c}$$

Group B's utility in equilibrium is thus $U_B^{eq} = -\frac{\lambda^2}{2c} - (1-\lambda)$, while deviating to $y = \frac{\mu x}{1-\mu}$ yields $U_B^{dev} = \frac{\lambda^2}{c} - \frac{\lambda^2}{4c} = \frac{3\lambda^2}{4c}$. The deviation is profitable since $U_B^{dev} - U_B^{eq} > 0$.

Case (4): $x = 1$, $y = 0$. The first order condition for x is

$$\lambda - 2c\mu \geq 0 \quad (14)$$

Group B's utility in equilibrium is $U_B^{eq} = -\lambda\mu - (1-\lambda)$, while deviating to $y = \frac{\mu}{1-\mu}$ yields $U_B^{dev} = \lambda 2\mu - c\mu^2$. The difference in utility is

$$U_B^{dev} - U_B^{eq} = 3\lambda\mu - c\mu^2 + (1-\lambda) = (5\lambda + \lambda - 2c\mu)\frac{\mu}{2} + (1-\lambda)$$

which is positive, given (14). Hence the deviation is profitable.

PROOF OF LEMMA 2. I first show that there exists a unique local optimum that solves system (1) in the parameter region in which $\mu x < (1-\mu)y$. This local optimum can be of six different types, depending on whether the solution is interior or a corner solution. I then impose the impossibility of a global deviation for the minority for the different types, which ensures that the local optimum is the unique equilibrium of the model (if part of the lemma). Finally I show that when the global deviation is possible, it is also profitable, which means that an equilibrium does not exist (only if part of the lemma).

Existence and uniqueness of the local optimum: To be locally optimal, the equilibrium must solve system (1), i.e.

$$\begin{aligned} \max_{x \in [0,1]} \quad & -\lambda(\mu x + (1 - \mu)y) - (1 - \lambda) \frac{(1 - \mu)y - \mu x}{\mu x + (1 - \mu)y} - c(\mu x)^2 \\ \max_{y \in [0,1]} \quad & \lambda(\mu x + (1 - \mu)y) + (1 - \lambda) \frac{(1 - \mu)y - \mu x}{\mu x + (1 - \mu)y} - c((1 - \mu)y)^2 \end{aligned}$$

Note that the first objective function is concave with respect to x , while the second is concave with respect to y . Hence, the first order conditions are sufficient to have a solution of system (1). The derivative of the first objective function with respect to x is

$$-\lambda\mu + (1 - \lambda) \frac{2\mu(1 - \mu)y}{(\mu x + (1 - \mu)y)^2} - 2c\mu^2 x \quad (15)$$

which is decreasing in x and changes sign in \mathbb{R} . Hence, there exists a unique $x^*(y)$ such that (15)=0. Such function $x^*(y)$ is decreasing in y when $\mu x < (1 - \mu)y$ (and increasing when $\mu x > (1 - \mu)y$). It follows that the best reply $x(y)$ is

$$x(y) = \begin{cases} 0 & \text{if } x^*(y) < 0 \\ x^*(y) & \text{if } x^*(y) \in (0, 1) \\ 1 & \text{if } x^*(y) > 1 \end{cases} \quad (16)$$

which is a decreasing function of y in the region $\mu x < (1 - \mu)y$.

The derivative of the second objective function in system (1) with respect to y is

$$\lambda(1 - \mu) + (1 - \lambda) \frac{2\mu(1 - \mu)x}{(\mu x + (1 - \mu)y)^2} - 2c(1 - \mu)^2 y \quad (17)$$

which is decreasing in y and changes sign in \mathbb{R} . Hence, there exists a unique $y^*(x)$ such that (17)=0. Such function $y^*(x)$ is increasing in x when $\mu x < (1 - \mu)y$ (and decreasing when $\mu x > (1 - \mu)y$). Note that $y^* > 0$ when $x = 0$. It follows that the best reply $y(x)$ is

$$y(x) = \begin{cases} y^*(x) & \text{if } y^*(x) \in (0, 1) \\ 1 & \text{if } y^*(x) > 1 \end{cases} \quad (18)$$

which is an increasing function of x in the region $\mu x < (1 - \mu)y$. The two best replies cross at most once in the region $\mu x < (1 - \mu)y$, since one is increasing and the other is decreasing. To make sure that they always cross each other in that region, we have to rule out the case in which $y^*(x)$ crosses the $\mu x = (1 - \mu)y$ line for a smaller value of x than that at which $x^*(y)$ crosses the same line. Both $y^*(x)$ and $x^*(y)$ cross the line only once. Calling x_1 the value for which $y^*(x_1) = \frac{\mu}{1-\mu}x_1$ and x_2 the value for which $x_2 = \frac{1-\mu}{\mu}y^*(x_2)$, this means checking that $x_1 > x_2$. Now, x_1 solves $\lambda + \frac{1-\lambda}{2\mu x_1} = 2c\mu x_1$, while x_2 solves $-\lambda + \frac{1-\lambda}{2\mu x_2} = 2c\mu x_2$. Subtracting the second from the first, we obtain

$$2\lambda + \frac{(1 - \lambda)(x_2 - x_1)}{2\mu x_1 x_2} = 2c\mu(x_1 - x_2)$$

which implies $x_1 > x_2$, otherwise the left-hand side would be positive and the right-hand side negative. Hence the two best replies cross once and only once in the parameter region in which $\mu x < (1 - \mu)y$, which means that there exists a unique local optimum in such region.

Equilibrium types and no-global-deviation condition: The local optimum can be of six different types: (1) $x \in (0, 1)$, $y \in (0, 1)$; (2) $x = 1$, $y = 1$; (3) $x = 1$, $y \in (0, 1)$; (4) $x \in (0, 1)$, $y = 1$; (5) $x = 0$, $y \in (0, 1)$; (6) $x = 0$, $y = 1$. The conditions on the parameters for the different types are analyzed in Appendix A. For the local optimum to be an equilibrium of the model, the no-global-deviation condition $\mu \leq (1 - \mu)y$ must hold, ensuring that group A cannot deviate to $x = 1$ and win the election (since, as shown below, such deviation would always

be profitable). The condition holds by assumption whenever x or y are equal to 1 and thus needs to be imposed only for cases (1) and (5):

Case (1): $x \in (0, 1)$, $y \in (0, 1)$. The first order conditions are

$$\begin{cases} -\lambda + (1-\lambda) \frac{2(1-\mu)y}{(\mu x + (1-\mu)y)^2} - 2c\mu x = 0 \\ \lambda + (1-\lambda) \frac{2\mu x}{(\mu x + (1-\mu)y)^2} - 2c(1-\mu)y = 0 \end{cases}$$

which imply

$$\mu x = \frac{1}{2} \sqrt{\frac{1-\lambda}{c}} - \frac{\lambda}{4c}, \quad (1-\mu)y = \frac{1}{2} \sqrt{\frac{1-\lambda}{c}} + \frac{\lambda}{4c} \quad (19)$$

The no-global-deviation condition $\mu \leq (1-\mu)y$ holds if

$$\mu \leq \frac{1}{2} \sqrt{\frac{1-\lambda}{c}} + \frac{\lambda}{4c} \quad (20)$$

Case (5): $x = 0$, $y \in (0, 1)$. The first order conditions are

$$\begin{cases} -\lambda + (1-\lambda) \frac{2}{(1-\mu)y} \leq 0 \\ \lambda - 2c(1-\mu)y = 0 \end{cases} \quad (21)$$

From the second equation of (21) we obtain $(1-\mu)y = \frac{\lambda}{2c}$ and thus the no-global-deviation condition $\mu \leq (1-\mu)y$ holds if

$$\mu \leq \frac{\lambda}{2c} \quad (22)$$

We can combine (20) and (22) in the condition $\mu \leq \max\{\frac{1}{2}\sqrt{\frac{1-\lambda}{c}} + \frac{\lambda}{4c}, \frac{\lambda}{2c}\}$ expressed in Lemma 2, which ensures the existence of a unique equilibrium (see also Figure 1 in Appendix A).

It remains to show that when the condition in Lemma 2 does not hold, the minority has a profitable (global) deviation and hence the equilibrium does not exist.

Profitability of a global deviation when feasible: Case (1): $x, y \in (0, 1)$. Group A's utility in equilibrium is $U_A^{eq} = -\frac{5\lambda}{4} \sqrt{\frac{1-\lambda}{c}} - \frac{1-\lambda}{4} - \frac{\lambda^2}{16c}$, while deviating to $x = \frac{(1-\mu)y}{\mu} + \epsilon$ yields $U_A^{dev} = \frac{3\lambda}{4} \sqrt{\frac{1-\lambda}{c}} - \frac{1-\lambda}{4} + \frac{7\lambda^2}{16c} + \epsilon'$. The deviation is profitable since

$$U_A^{dev} - U_A^{eq} = 2\lambda \sqrt{\frac{1-\lambda}{c}} + \frac{\lambda^2}{2c} + \epsilon'' > 0$$

Case (5): $x = 0$, $y \in (0, 1)$. Group A's utility in equilibrium is $U_A^{eq} = -\frac{\lambda^2}{2c} - (1-\lambda)$, while deviating to $x = \frac{(1-\mu)y}{\mu} + \epsilon$ yields $U_A^{dev} = \frac{\lambda^2}{c} - \frac{\lambda^2}{4c} + \epsilon' = \frac{3\lambda^2}{4c} + \epsilon'$. The deviation is profitable since $U_A^{dev} - U_A^{eq} > 0$.

PROOF OF LEMMA 3. The best reply function of the minority $x(y)$ is the implicit function that solves

$$-\lambda + \frac{(1-\lambda)2(1-\mu)y}{(\mu x + (1-\mu)y)^2} - 2c\mu x = 0$$

By implicitly differentiating with respect to y , we obtain $\frac{\partial x}{\partial y} = \frac{2(1-\lambda)(1-\mu)(\mu x - (1-\mu)y)}{2c\mu(\mu x + (1-\mu)y)^3 + 4(1-\lambda)\mu(1-\mu)y} < 0$, since $\mu x - (1-\mu)y < 0$ in light of Lemma 1.

The best reply function of the majority $y(x)$ is the implicit function that solves

$$\lambda + \frac{(1-\lambda)2\mu x}{(\mu x + (1-\mu)y)^2} - 2c(1-\mu)y = 0$$

By implicitly differentiating with respect to x , we obtain $\frac{\partial y}{\partial x} = \frac{2(1-\lambda)\mu(1-\mu)y - \mu x}{2c(1-\mu)(\mu x + (1-\mu)y)^3 + 4(1-\lambda)(1-\mu)\mu x} > 0$, since

$(1 - \mu)y - \mu x > 0$ in light of Lemma 1.

PROOF OF PROPOSITION 1. As discussed in the proof of Lemma 2, the two cases in which $x = 0$ in equilibrium are case (5) $x = 0, y \in (0, 1)$ and case (6) $x = 0, y = 1$. The condition on the parameters for case (5) can be obtained from (21) and (22) and are

$$\begin{aligned}\lambda^2 &\geq 4c(1 - \lambda) \\ 2c\mu &\leq \lambda < 2c(1 - \mu)\end{aligned}\tag{23}$$

while the conditions on the parameters for case (6) can be derived from the first order conditions as

$$\begin{aligned}\lambda &\geq \frac{2}{3 - \mu} \\ \lambda &\leq 2c(1 - \mu)\end{aligned}\tag{24}$$

Combining (23) and (24) we can derive that $x = 0$ in equilibrium if $\lambda > \bar{\lambda}(c, \mu) := \max\{\frac{2}{3 - \mu}, 2c\mu, 4(c + \sqrt{c^2 + c})\}$, from which we can see that $\bar{\lambda}(c, \mu)$ is weakly increasing in both c and μ . See also Appendix A for a graphical representation of $\bar{\lambda}(c, \mu)$ in Figure 1.

PROOF OF PROPOSITION 2. It follows from case (1) in the proof of Lemma 2.

PROOF OF PROPOSITION 4. We have

$$D = y - x = \frac{1}{1 - \mu} \left(\frac{1}{2} \sqrt{\frac{1 - \lambda}{c}} + \frac{\lambda}{4c} \right) - \frac{1}{\mu} \left(\frac{1}{2} \sqrt{\frac{1 - \lambda}{c}} - \frac{\lambda}{4c} \right) = \frac{1}{2\mu(1 - \mu)} \left(\frac{\lambda}{2c} - (1 - 2\mu) \sqrt{\frac{1 - \lambda}{c}} \right)$$

which is strictly increasing in λ . Then λ^* solves $D = 0$, i.e.

$$\frac{\lambda^*}{\sqrt{1 - \lambda^*}} = 2(1 - 2\mu)\sqrt{c}$$

from which we obtain $\frac{\partial \lambda^*}{\partial c} > 0$ and $\frac{\partial \lambda^*}{\partial \mu} < 0$.

PROOF OF PROPOSITION 5. Part (i): In the region where $T=1$, we have $\mathcal{L} = \lambda + (1 - \lambda)(1 - 2\mu)$ which is increasing in λ . Hence \mathcal{L} increases moving from the region in which $T = 1$ to the one in which $x \in (0, 1), y = 1$. It remains to check that \mathcal{L} is still increasing in λ on the border line between the two regions, that is moving to the interior of the region in which $x \in (0, 1), y = 1$. In such region

$$\mathcal{L} = \lambda(z + 1 - \mu) + (1 - \lambda) \frac{1 - \mu - z}{z + 1 - \mu}\tag{25}$$

where $z = \mu x$ solves

$$-\lambda + (1 - \lambda) \frac{2(1 - \mu)}{(z + 1 - \mu)^2} - 2cz = 0\tag{26}$$

from which we can obtain $\frac{\partial z}{\partial \lambda} < 0$. On the border line between the two regions we have

$$z = \mu \quad , \quad 2\mu c = -\lambda + 2(1 - \lambda)(1 - \mu)\tag{27}$$

Now, taking the derivative of \mathcal{L} with respect to λ in (25) we get

$$\frac{\partial \mathcal{L}}{\partial \lambda} = (z + 1 - \mu) + \lambda \frac{\partial z}{\partial \lambda} - \frac{1 - \mu - z}{z + 1 - \mu} - (1 - \lambda) \frac{2(1 - \mu) \frac{\partial z}{\partial \lambda}}{(z + 1 - \mu)^2}\tag{28}$$

which, given (27), simplifies to

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 2\mu - \frac{\partial z}{\partial \lambda}(2\mu c) > 0 \quad (29)$$

To prove Part (ii), it suffices to show that \mathcal{L} increases moving from any region to the one in which $x \in (0, 1)$ and $y = 1$. Consider the six different regions corresponding to the six cases of Table 1 in Appendix A.

Case (1): $x, y \in (0, 1)$. We have $\mathcal{L} = \frac{3\lambda}{2} \sqrt{\frac{1-\lambda}{c}}$, which increases when c increases, moving to the region in which $x \in (0, 1)$ and $y = 1$.

Case (2): $x = y = 1$. By part (i), \mathcal{L} increases when λ increases, moving to the region in which $x \in (0, 1)$ and $y = 1$.

Case (3): $x = 1, y \in (0, 1)$. We have $\mathcal{L} = \lambda(\mu + z) + (1 - \lambda) \frac{z - \mu}{\mu + z}$, where $z = (1 - \mu)y$ solves

$$\lambda + \frac{(1 - \lambda)2\mu}{(\mu + z)^2} = 2cz \quad (30)$$

From (30) we can obtain $\frac{\partial z}{\partial c} < 0$ and thus $\frac{\partial \mathcal{L}}{\partial c} < 0$, which means that \mathcal{L} increases when c decreases, moving to the region in which $x = y = 1$.

Case (5): $x = 0, y \in (0, 1)$. We have $(1 - \mu)y = \frac{\lambda}{2c}$ and $\mathcal{L} = \frac{\lambda^2}{2c} + (1 - \lambda)$, which decreases in c , moving to the region in which $x = 0, y = 1$.

Case (6): $x = 0, y = 1$. We have $\mathcal{L} = \lambda(1 - \mu) + (1 - \lambda)$ which decreasing in λ , moving to the region in which $x \in (0, 1), y = 1$.

It remains to check that \mathcal{L} is still decreasing in λ on the border line between the region in which $x = 0, y = 1$ and the region in which $x \in (0, 1), y = 1$, that is moving to the interior of the region in which $x \in (0, 1), y = 1$. In such region (25) and (26) hold, and on the border we have

$$z = \mu x = 0 \quad , \quad \lambda(1 - \mu) = 2(1 - \lambda) \quad (31)$$

Thus (28) becomes

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -\mu + \frac{\partial z}{\partial \lambda}(\lambda\mu) < 0 \quad (32)$$

since $\frac{\partial z}{\partial \lambda} < 0$ given (26).